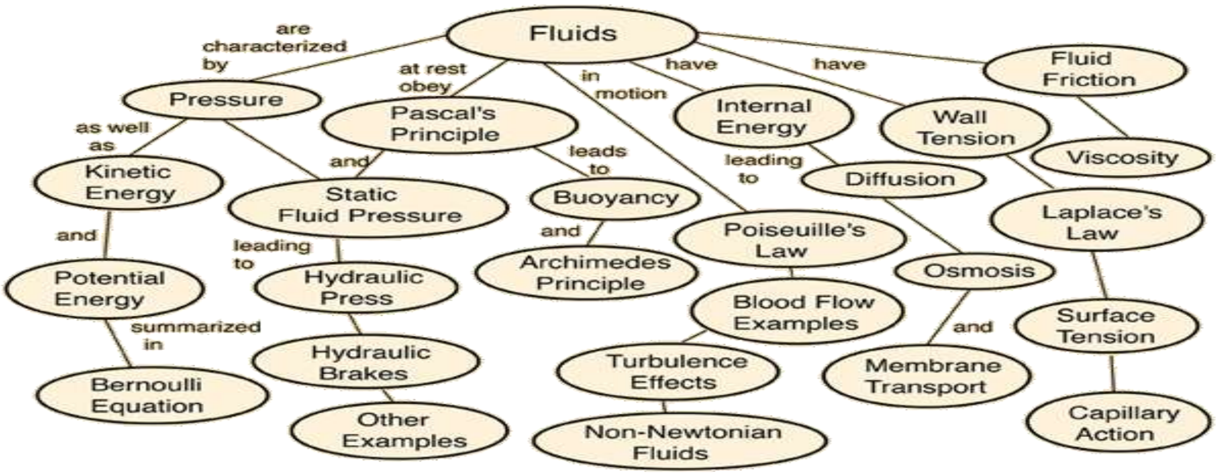
Ministry of Higher Education & Scientific Research

University of Baghdad – College of Engineering

Mechanical Engineering Department

Incompressible Fluid Mechanics



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Preface

The present book is a handout lectures for the B.Sc. Course ***ME202 : Fluid Mechanics / I***. The course is designed for B.Sc. Sophomore Students in the Mechanical Engineering Discipline. The time schedule needed to cover the course material is 32 weeks, 3 hrs. per week. The course had been taught by the author (course tutor) for more than 25 years. A short c.v. for the author is given below;

- ***Prof. Dr. Ihsan Y. Hussain*** **/ Baghdad - 1964**
- **B.Sc. (1986), M.Sc. (1989), & Ph.D. (1997) in Mechanical Engineering from the Mech. Engr. Dept. – University of Baghdad**
- **Professor of Mechanical Engineering – Thermo-Fluids**
- **Teaching Undergraduate Courses and Laboratories in Various Iraqi Universities (Baghdad, Al-Kufa, Babylon, Al - Nahrain ….) in Various Subjects of Mechanical Engineering**
- **Teaching Advanced Graduate Courses (M.Sc. and Ph.D.) in various Iraqi Universities (Baghdad, Technology, Babylon, Al-Kufa, Al-Mustansyrya, Al-Nahrain…) in the Areas of (Fluid Mechanics, Heat Transfer, CFD, Porous Media, Gas Dynamics, Viscous Flow, FEM, BEM**
- **Lines of Research Covers the Following Fields ;**
  - **Aerodynamics**
  - **Convection Heat Transfer (Forced, Free, and Mixed)**
  - **Porous Media (Flow and Heat Transfer)**
  - **Electronic Equipment Cooling**
  - **Heat Transfer in Manufacturing Processes (Welding, Rolling, … etc.)**
  - **Inverse Conduction**
  - **Turbomachinery (Pumps, Turbines, and Compressors)**
  - **Heat Exchangers**
  - **Jet Engines**
  - **Phase-Change Heat Transfer**
  - **Boundary Layers (Hydrodynamic and Thermal)**
- **Head of the Mech. Engr. Dept. / College of Engineering - University of Baghdad (December / 2007 – October / 2011)**
- **Member of Iraqi Engineering Union (Official No. 45836).**
- **ASHRAE Member (8161964)**
- **Head of (Quality Improvement Council of Engineering Education in Iraq QICEEI)**
- **Supervised (41) M.Sc. Thesis and (20) Ph.D. Dissertations**
- **Publication of more than (70) Papers in the Various Fields Mentioned above**
- **Member in the Evaluation and Examining Committees of more than (300) M.Sc. and Ph.D. Students in their Theses and Dissertations**
- **Member of the Editing Committee of a number of Scientific Journals**
- **Evaluation of more than (700) Papers for Various Journals and Conferences**
- **Working within the Mechanical Groups in the Consulting Engineering Bureau, College of Engineering, University of Baghdad, and in the Dept. of Mech. Engr. , in the Mechanical Design of Various Projects**
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"Table of Contents"

I. Preface

I.1 General Approach

The present “Lecture Notes" are intended for use in a first course in “Fluid Mechanics" for under - graduate engineering student in the sophomore years. It represents the revision of lecture notes for about 25- years experience in teaching the subject of (**Fluid Mechanics / I**) in the second year at the Mechanical Engineering Department / College of Engineering - University of Baghdad and other Iraqi universities. The text covers the basic principles of fluid mechanics with a broad range of engineering applications.

The students are assumed to have completed the subject of (Engineering Mechanics / Statics & Dynamics) and (Mathematics / I) courses. The relevant concepts from theses two subjects are introduced and reviewed as needed. The emphasis throughout the text is kept on the "physics" and the “physical arguments" in order to develop an “intuitive understanding" of the subject matter.

Throughout the text, we tried to match between engineering education and engineering practice. The text covers all the standard topics in fluid mechanics with an emphasis on physical mechanisms and practical applications, while de – emphasizing heavy mathematical aspects, which are being left to computers. We try to encourage “Creative Thinking" and development of a “Deeper Understanding" of the subject matter by the students.

I.2 Learning Tools

The emphasis in the text is on "developing a sense of underlying physical mechanism" and a “mastery of solving practical problems" an engineer is likely to face in the real world. Thus, the text covers more material on the fundamentals and applications of fluid mechanics. This makes fluid mechanics a more pleasant and worthwhile experience for the student.

The principles of fluid mechanics are based on our “everyday experiences" and " experimental observations". A more physical intuitive approach is used through the text.

Frequently, “Parallels are drawn" between the subject matter and students' every day experiences so that they can relate the subject matter to what they already know.

The material in the text is introduced at a level that an average student can follow comfortably. It speaks "to" the students, not "over" the students. All- derivations in the text are based on physical arguments with simple mathematics, and thus they are easy to follow and understand.

Figures are important learning tools that help the students “get the picture". The text makes effective use of graphics.

A number of worked –out examples are included in the chapters of the text. These examples clarify the material and illustrate the use of basic principles. An intuitive and systematic approach is used in the solution of the example problems.

A number of sheets of solved problems that covers the material of the subject are also included in the appendices. These problems are focusing on the application of concepts and principles covered in the text in solving engineering problems related to important application of the subject.

At the end of each chapter, a number of selected unsolved problems from related chapters in the “textbook" used during the course (reference (1)) in the list of references) are listed. The problems are grouped under specific topics (articles) in the order they are covered to make problem selection easier for the student.

A collection of examination paper is also included in the appendices. The collection covers quizzes, mid-term exams, comprehensive exam and final exams. It serves as a tool to improve students’ way of thinking and to make them familiar with the nature of examinations questions.

A number of quizzes are usually made during the teaching period of the subject (about 32 weeks). These many quizzes serve the following;

1. Make these students familiar with the environment of the examinations.
2. Continuous study and follow-up from students to the material of the subject.
3. A “bank" of miscellaneous questions covering the various applications of the subject material will be available for both students and instructor.

4. The final average mark will be distributed on large number of quizzes and tests, so that, the shortage in one or more of these quizzes will not affect the evaluation greatly.

A number of laboratory experiments (7-10) are made to cover the principle and concepts of the subject, experimentally. These experiments help to give the student skills of dealing with the devices set up, acquiring and recording data, writing reports, analysis and discussion of the results, conclusions and recommendations.

I.3 Contents

The lecture note comprises seven chapters and a number of appendices. The first five chapters cover the basic fundamentals of the material, while the last two ones involve the important applications of the subject.

In chapter -1- we introduce the basic fundamental concepts and definitions of fluid mechanics as a science and its applications in the real world. Chapter -2- presents the basic concepts of fluid statics and its application in engineering and industry. It includes the derivation of hydrostatic pressure variation, pressure measurements, forces on submerged surfaces, buoyancy and stability of immersed and floating bodies and relative equilibrium. In chapter-3-, the fundamentals of fluid motion are studied. The governing continuity, energy and momentum equations are derived by using the Reynolds-transport theorem. The various applications of these equations are introduced. Chapter-4- covers the dimensional analysis and similitude principles and their applications. In chapter-5-, the real (viscous) fluid flow is considered. Various applications of this type of fluid flow are investigated, which include: laminar flow between parallel plates and through circular tubes and annuli, boundary-layer flow, drag and lift, Moody diagram and simple pipe problems. Chapter-6- concerns the measurements of fluid and flow properties. These include density, pressure, velocity, flow rate and viscosity measurements. In chapter -7-, the analysis of pipes and pumps connection is introduced. Series, parallel and branching pipes networks are analyzed. Besides, series and parallel pumps connection in pumping stations is considered. The appendices include solved sheets of problems, collection of examination papers sheets of problems, collections, collection of examination papers and test questions.

I-4 Time Scheduling Table

The subject teaching period is (32 weeks), (4 hrs.) per week (3hrs. theory and 1 hr. tutorial). The following is the time scheduling table for the subject weekly outlines, see **Table (I.1)**. This time schedule is for the theoretical part of the subject only.

Table (I.1): Time Scheduling Table

Week	Covered Articles		Tests	Week	Covered Articles		Week
1	Chapter -1-	1.1+1.2+ 1.3+1.4		17	Chapter - 4-	4.1+4.2+4.2.1	
2		1.5	Q1	18		4.2.2+4.3+4.3.1	Q11
3	Chapter -2-	2.1+2.2	Q2	19	Chapter -5-	4.3.2+4.4	Q12
4		2.3		20		5.1+5.2+5.3	Q13
5		2.4.1+2.4.2	Q3	21		5.4+5.5	Q14
6		2.4.3		22		5.6+5.6.1+5.6.2	Q15
7		2.5	Q4	23		5.6.3+5.7	Q16
8		2.6.1+2.6.2	Q5	24		5.8+5.8.1	Q17
9		2.6.3		25		5.8.2	Q18
10		2.7.1	Q6	26		5.8.3+5.8.4	Q19
11		2.7.2		27	Chapter -6-	6.1+6.2+6.3	Q20
12	Chapter -3-	3.1+3.2+3.3	Q7	28		6.4	Q21
13		3.4		29		6.5	Q22
14		3.5+3.5.1.1 to 3.5.1.3	Q8	30	Chapter -7-	7.1+7.2+7.3	Q23
15		3.5.1.4 to 3.5.1.6+ 3.5.2+3.5.3	Q9	31		7.4	Q24
16		3.6+3.7	Q10 + TEST I	32		7.5+7.6	Q25 + Test II

__Note:__ The experimental part of the subject (1 hr. per week) in addition to the 4 hrs. (3 theory + 1 tutorial), see the following article**.**

I.5 Laboratory Experiments

We should mention here that the experimental part of the subject (1 hr per week) is included in a separate general “Mechanical Engineering Laboratories/II" course. It involves a number of experiments covering the principles and concepts of the subject material. As was mentioned earlier, the experimental part aims to acquire the student a skill of dealing with the devices, recording test data, writing a report, analysis and discussion of the results, presentation of the experimental observations and results, and conclusions and recommendations drawn from the results. The following is a list of some of these experiments;

1. Dynamic Similarity.
2. Meta centric Height.
3. Impact of Jet.
4. Friction in Pipes.
5. Minor Losses.
6. Flow Measurements (Orifice and Venturi Meters).
7. Stability of Floating Bodies.
8. Bernoulli's Equation Demonstration.

I.6 Textbook and References

The text adopted in teaching the subject is;

"Fluid Mechanics", by victor L.Streeter and E. Benjamin Wylie; First SI metric Edition, Mc.Graw Hill, 1988

Other references which may be used are listed below;

1. "Elementary Fluid Mechanics", by John k.Vennard and Robert L. Street, 5th edition, John Wylie and Sons,1976
2. "Engineering Fluid Mechanics", by John A. Roberson and Clayton T. Crow, 2nd Edition, Houghton Mifflin Co.,1998

I.7 Assessments

The final mark (100%) is distributed along the teaching period as follows;

Activity	Mark
Quizzes (15-20 Nos.)	15%
Comprehensive Tests (2Nos.)	10%
Extracurricular Activities	5%
Final Test	70%
Total Sum	100%

The Passed Average Required is (50%)

Chapter -1-

Introductory Concepts

1.1 Definition of Fluids and Fluid Mechanics

According to the variation of volume and shape with pressure, the state of matter is usually classified in to three states, solid, liquid and gas, see **Fig. (1)**. Fluids in clued the liquids and gases.

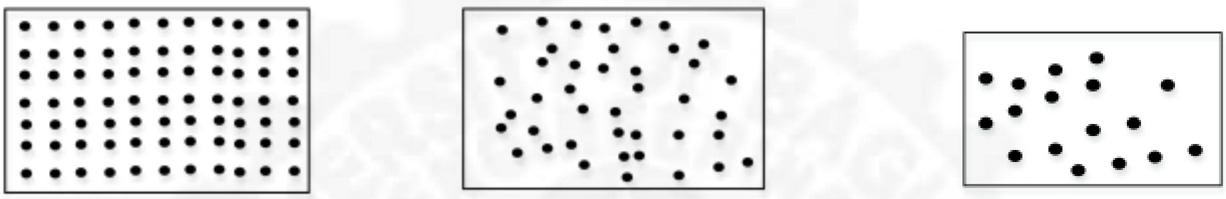


Fig. (1)

Fluid

It is a substance that deforms continuously when subjected to shear stress, no matter how small that shear stress may be. Fluids may be either <u>liquids</u> or <u>gases</u>. Solids, as compared to fluids, cannot be deformed permanently (plastic deformation) unless a certain value of shear stress (called the yield stress) is exerted on it.

According to the variation of density of the fluids with pressure, fluids are classified in to "<u>incompressible</u>" and "<u>compressible</u>" fluids.

Incompressible Fluids

They are the fluids with constant density, or the change of density with pressure is so small that can be neglected and considers the density as constant. The incompressible fluids are basically the **"LIQUIDS"**. Gases at low velocities are usually considered as incompressible fluids also.

There are no exact incompressible fluids in practice. For example, the density of water at atmospheric pressure (0.1 MPa) is (1000 kg/m^3). When the pressure is increased to (20 MPa), the density becomes (1010 kg/m^3). Thus, increasing the pressure by a factor of (200) increases the density by only (1%)!! For this reason, it is reasonable to consider the liquids as incompressible fluids with constant density.

Compressible Fluids:

They are the fluids with variable density, or the change of density with pressure is large and cannot be neglected. These include basically the **"GASES"**. In some liquids problems, such as "water hammer", the compressibility of liquids must be considered.

Fluid Mechanics

It is the science that deals with the action of forces on fluids, which may be either liquids or gases. Fluid mechanics, as a science, is a branch of the "Engineering Mechanics" as a general science deals with the action of forces on bodies or matter (solids, fluids (liquids of gases)), see **Fig. (2).**

Engineering Mechanics

- Mechanics of Rigid Bodies
  - Kinematics
  - Kinetics
- Mechanics of Deformable Bodies (Strength of Material)
- Mechanics of Fluids (Liquids & Gases)

Solids — Fluids

Fig. (2)

1.2 Scope and Applications of Fluid Mechanics

Fluids and Fluid Mechanics play a vital role in our daily life. Large and diverse applications which are based on fluid mechanics exist. Typical examples are listed below;

1- Irrigation.

2- Navigation.

3- Power Generation (Hydraulic, Gas and Steam Power Plant).

4- Ships, Boats and Submarines.

5- Airplanes and Hovercrafts:

   a. Wing Surfaces to Produce Lift.
   b. Jet Engines to Produce Thrust.
   c. Fuselage Design for Minimum Drag.
   d. Various Systems in the Air Craft (A/c, Fuel, Oil, Pneumatic).
   e. Control of the Airplane (Tail, Flaps, Ailerons, …).

6- Cars and Motorcycles.

   a. Pneumatic tires.
   b. Hydraulic Shock Absorbers.
   c. Fuel System (Gasoline + Air).
   d. Air Resistance Grates Drag on Car.
   e. Lubrication System.
   f. Cooling System.
   g. Aerodynamic Design of Car Profile for Minimum Drag.

7- Design of Pipe Networks.

8- Transport of Fluids.

9- Air – Conditioning and Refrigeration Systems.

10- Lubrication Systems.

11- Design of Fluid Machinery (Fans, Blowers, Pumps, Compressors, Turbines, Windmills, ….).

12- Bioengineering (Flow of Blood through Veins and Arteries).

13- Fluid Control Systems.

14- All Living Creatures Need Water (Fluid) for Life (We Made from Water Every Living Thing).

1.3 Dimensions and Units

"Dimensions" are physical variables that specify the behavior and the nature of a certain system, whereas the "Units" are used to specify the amount of these dimensions. Generally, two systems of units exist. These are the International System of units (SI) and the United States of units (U.S.). The base quantities in the (SI) system are the Mass (M), length (L) and time (T), therefore it is called the (MLT- system), whereas in the (U.S.) system, the base quantities are the force (F), Length (L) and time (T), and is called the FLT- system. The dimensions are expressed by capital letters. **Table (1)** lists the dimensions and units of the most important variables in Fluid Mechanics, in both (SI) and (U.S.) systems, followed by conversion factors between the two systems.

Table (1): Dimensions and Units

Physical Quantity	Dimension	Units	
		SI	U.S.
Mass	M	kg	Slug
Length	L	m	ft
Time	T	s	sec.
Force	F	N	lb_f
Temperature	θ	^oC K	^{o} F (Ordinary Temp.) ^{o} R (Absolute Temp.)
Area	L^2	m^2	ft^2
Velocity	LT^{-1}	m/s	ft/sec. (fps)
Acceleration	LT^{-2}	m/s^2	ft/sec^2
Work	FL (ML^2T^{-2})	N.m (J)	lb_f . ft
Power	FLT^{-1} ML^2T^{-3}	N.m /s ($\frac{J}{s}$), W	lb_f.ft / sec.
Pressure	FL^{-2} $ML^{-1}T^{-2}$	N/m^2 (Pa)	lb_f / ft^2 (psf)
Density	ML^{-3}	kg / m^3	$Slug / ft^3$

Conversion Factors

1- Slug = 14.59 kg

Slug= 32.2 lbm

1 lb_m = 0.4536 kg

2- 1 ft = 0.3048 m

1 ft = 12 in

1 in = 2.54 cm

1 mile = 5280 ft

1 mile = 1609 m.

3- 1 lb_f = 4.448 N

1 N = (1 kg) (1 m/s^2)

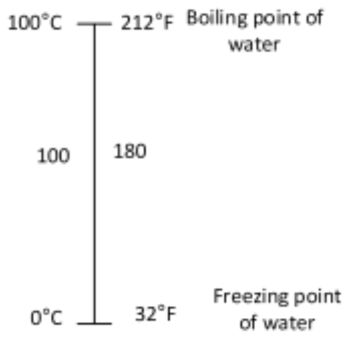
1 lb_f = (1 Slug) (1 ft/sec^2)

4- K= ^{o}C + 273

^{o}R= ^{o}F + 460

^{o}F = 1.8 ^{o}C + 32

^{o}R=1.8 K



Prefixes

G = 10^9	gega	m = 10^{-3}	milli
M = 10^6	mega	μ = 10^{-6}	micro
k = 10^3	kilo	n = 10^{-9}	nano
c = 10^{-2}	centi	p = 10^{-12}	pico

1.4 Newton's Law of Viscosity for Fluids

Consider a fluid placed between two closely spaced parallel plates so large that conditions at their edges may be neglected. The lower plate is fixed and a force (F) is applied at the upper plate which exerts a shear stress (F/A) on any fluid between the plates, see **Fig. (3)**. The force (F) causes the upper plate to move with a steady velocity (U), no matter how small the magnitude of (F)

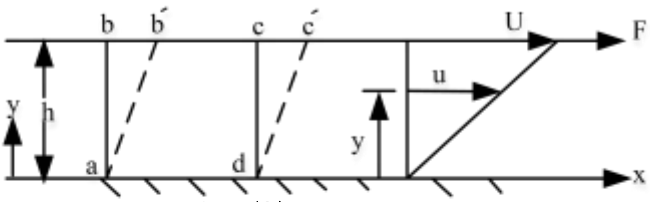


Fig. (3)

The fluid in the immediate contact with a solid boundary has the same velocity as the boundary; i.e., there is no slip at the boundary. The fluid in the area (abcd) flows to the new position (ab'c'd), each fluid particle moving parallel to the plate at the velocity (u) varies uniformly from zero at the stationary plate to (U) at the upper plate. Experiments show that, other quantities being held constant, (F) id directly proportional to (A) and (U) and is inversely proportional to thickness (h). In equation form;

$$F \propto \frac{AU}{h}$$

Thus;

$$F = \mu \, \frac{AU}{h} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (1.1)$$

Where; μ = Proportionality constant, viscosity of the fluid

A = Surface area parallel to the force F

Defining the shear stress (τ = F/A), then;

$$\tau = \mu \, \frac{U}{h} \qquad \ldots\ldots\ (1.2)$$

The ratio (U/h) is the angular velocity of line (ab), or it is the rate of angular deformation of the fluid, i.e., the rate at which the angle (bad) decreases. The angular velocity may be written as (du/dy) as;

$$\frac{du}{dy} = \frac{u_2 - u_1}{y_2 - y_1} = \frac{U - 0}{h - 0} = \frac{U}{h} \begin{pmatrix} 1 \ at \ the \ lower \ plate \\ 2 \ at \ the \ upper \ plate \end{pmatrix}$$

(du / dy) is more general and may be visualized as the rate at which one layer moves relative to an adjacent layer. Thus, **equ. (1.2)** may be written as;

$$\tau = \mu \, \frac{du}{dy} \qquad \textbf{(Newton's Law of Viscosity)} \qquad \ldots\ldots\ldots\ldots\ (1.3)$$

Materials other than fluids, such as solids and plastics, cannot satisfy the definition of fluid and **equ. (1.3)**, since they do not deform continuously unless their "yield shear stress" is applied.

Fluids may be classified as "Newtonian" or ” Non - Newtonian". In Newtonian fluid, there is a linear relation between the applied shear stress (τ) and the resulting angular deformation (du/dy) (i.e., μ is constant in equ. (1.3), see **Fig. (4)**. In Non - Newtonian fluids, the relation is non - liner. An “ideal plastic" has a definite yield stress and a constant linear relation of (τ) to (du/dy). A “thixotropic" substance, such as printer's ink, has a viscosity that is dependent upon the immediate prior angular deformation of the substance and has a tendency to take a set when at rest. Gases and thin liquids tend to be Newtonian fluids, while thick, long - chained hydrocarbons may be non - Newtonian fluids.

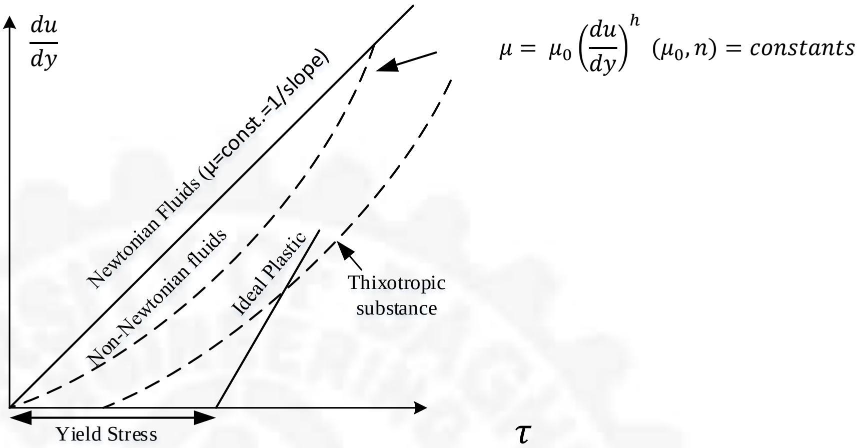


Fig. (4): Rheological Diagram

1.5 Basic Definitions and Concepts

1- Density(ρ)(rho)

It is the mass (m) of a fluid per unit volume (∀);

$$\rho = \frac{m}{\forall} \quad \frac{kg}{m^3}, \frac{slug}{ft^3} \qquad \text{………………. (1.4)}$$

For water at standard pressure (760 mmHg) and 4 °C : $\rho_w = 1000\ kg/m^3$

For air at standard pressure (760 mmHg) and 20 °C : $\rho_{air} = 1.2\ kg/m^3$

2- Specific Volume (v_s)

It is the volume occupied by unit mass of fluid; i.e., it is the reciprocal of the density (ρ);

$$v_s = \frac{\forall}{m} = \frac{1}{\rho} \quad \frac{m^3}{kg}, \frac{ft^3}{slug} \qquad \text{………………… (1.5)}$$

3- Specific Weight (γ) (gama)

It is the force of gravity (weight) (W) of a fluid per unit volume (∀). It is also called the unit gravity force.

$$\gamma = \frac{W}{\forall} = \frac{mg}{\forall} \rightarrow \gamma = \rho g \quad \frac{N}{m^3}, \frac{lbf}{ft^3} \qquad \text{……………….(1.6)}$$

(γ) Changes with location depending upon gravity (g).

γ_{water} =9806 N/m^3 at 5 °C at sea level

γ_{air} = 11.8 N/m^3 at 20 °C and p_{atmstd} = 101325 Pa

4- Specific Gravity (s)

It is the ratio of specific weight of a substance to that of water; it is also the ratio of the mass of a substance to the mass of an equivalent volume of water at standard condition. It may also be expressed as a ratio of its density to that of water.

s = $\frac{\gamma}{\gamma_w} = \frac{\rho}{\rho_w} = \frac{m}{m_w}$ (dimensionless) ………………..... (1.7)

The specific gravity is also called the "relative density”. In equ. (1.7), (γ_w) is taken as ($\gamma_w = 9810$ N/m^3) at standard reference temperature (4 °C). For gases, air or oxygen are usually taken as the references fluid instead of water.

5- Pressure (p)

It is the normal force (F) pushing against a plane area divided by the area (A), see **Fig. (5)**;

$p = \frac{F}{A} \quad F \perp \perp A \left(\frac{N}{m^2} = \text{Pa} \ , \frac{lbf}{ft^2} (psf) \ , \frac{lbf}{in^2} (psi) \right)$ .... (1.8)

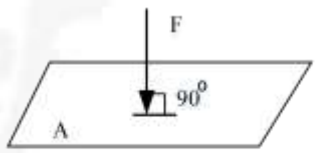


Fig. (5)

If a fluid exerts a pressure against the walls of a container, the container will exert a reaction on the fluid which will be compressive. Liquids can sustain very High compressive pressure, but they are very week in tension. The pressure intensity is a scalar quantity, has a magnitude only. Pressure may be express in terms of an equivalent height (h) of fluid column;

p = γ h ……………………… (1.9)

6- Shear Stress (τ) (taw)

It is the shear force (F) acts upon the surface (parallel) area (A), see **Fig. (6)**

$\tau = \frac{F}{A}$; $(F//A)$ $\left(\frac{N}{m^2}, \frac{lbf}{ft^2}, \frac{lbf}{in^2}\right)$ …………….... (1.10)

Unlike pressure, shear stress may have two components, see **Fig. (6);**

$\tau_{zx} = \frac{F_x}{A}$ ……………...... (1.10a) (z ⊥ xy - plane)

$\tau_{zy} = \frac{F_y}{A}$ …………………..... (1.10b)

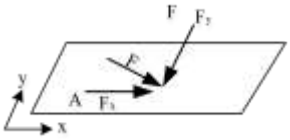


Fig. (6)

7. Viscosity

It is the property of a fluid by virtue of which it offers resistance to shear. There are two types of viscosity; "Dynamic (or Absolute) viscosity (μ) , and the "Kinematic Viscosity". Their definitions are;

$\mu = \frac{\tau}{du/dy}$ .... (1.11) (from Newton's law of viscosity, **equ. (1.3)**)

$\nu = \frac{\mu}{\rho}$ .... (1.12)

Units of Viscosity

SI Units: μ : $\frac{N.s}{m^2}(Pa.s), \frac{kg}{m.s}$

Poise P = $\frac{dyne.s}{cm^2}$, $\frac{g}{cm.s}$ $\left(1\frac{N.s}{m^2} = 10\ P\right)$

$\nu: \frac{m^2}{s}, Stoke = \frac{cm^2}{s}$

U.S. Units: μ : $\frac{lbf\ .sec}{ft^2}, \frac{slug}{ft.sec.}$ $(1\ \frac{slug}{ft.sec.} = 47.9\ \frac{kg}{m.s})$

$\nu: \frac{ft^2}{sec}$

Causes of Viscosity:

1. Cohesion forces between molecules, (effective in liquids).
2. Exchange of momentum by molecules in the layers, (effective in gases).

Notes:

1. Viscosity of gases increases with temperature, but viscosity of liquids decreases with temperature, see **Fig. (C.1)** and **(C.2)** p.p (536-537) in your textbook.

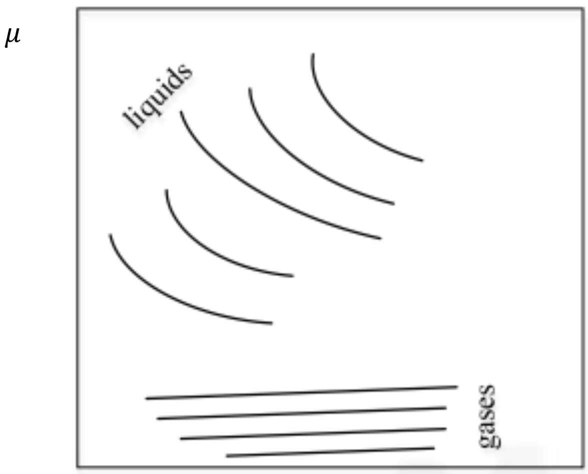


Fig. (c.1)
p.p536 in textbook

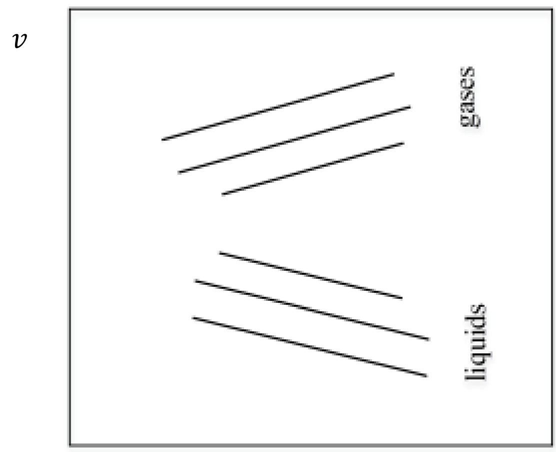


Fig. (c.2)
p.p.537 in text book

2. For ordinary pressure, viscosity is independent of pressure.
3. Kinematic viscosity represents the momentum diffusion coefficient; its value for gases is larger than that for liquids.

8- Specific Heat (c)

It is the property that decries the capacity of a substance to store thermal energy. It is the amount of heat that must be transmitted to a unit mass of a substance to raise its temperature by one degree.

For gases, (c) depends on the process accompanied the change in temperature; accordingly, there are two types;

c_p = Specific heat at constant pressure.

c_v = Specific heat at constant volume.

$\frac{c_p}{c_v} = k$ Specifics' heat ratio ....…………....... (1.13)

$c_p - c_v = R$ R = Gas constant .... (1.14)

Examples: Water: $c_w = 4187 \quad J/kg\ k$

Air: $c_p = 1004.5 \quad \frac{J}{kgK}$, $R = 287 \quad \frac{J}{kgK}$, $c_v = 718 \quad \frac{J}{kgK}$, $k = 1.4$

9- Vapor Pressure (p_v)

It is the pressure at which a liquid will boil. Vapor pressure is a function of temperature (it increases with temperature). When the liquid pressure is less than the vapor pressure, liquid flashes into vapor. This is the phenomenon of "cavitation" which results in the “erosion" of the metal parts.

Ex. Water at (20 °C) has p_v = 2.447 kPa

At (100 °C) has p_v = 101.3 kPa

Mercury at (20 °C) has p_v = 0.173 kPa

10- Equation of State: Perfect Gas

The perfect gas is defined as a substance that satisfies the “perfect gas law"; which has the following forms;

$p\forall=mRT$ …………………… (1.15a)

$p\forall_s=RT$ …………………… (1.15 b)

$p=\rho\, RT$ …………………… (1.15c)

$p\forall=n\bar{R}T$ …………………… (1.15d)

Where;

$R=\frac{\bar{R}}{M_w}$ …………………………….. (1.15e)

And $M_w=\frac{m}{n}$ ……………………… (1.15f)

p= Absolute Pressure (Pa)

T=Absolute lute Temperature (K)

R= Gas constant (J/kgK)

$\bar{R}$=Universal Gas Constant =8312 J/mol K

Mw=Molecular Weight (kg/ mol)

n=Number of Moles

Real gases below critical pressure and above critical temperature tend to obey the perfect gas law. The perfect law encompasses both Charles' law (which states that for constant pressure ($\forall\alpha T$) and Boyle's law (which states that for constant temperature : ραp).

11- Bulk Modulus of Elasticity (k)

It expresses the compressibility of a fluid (liquids and gasses). It represents the change of volume (d∀) caused by change in pressure (dp). It is defined as;

$k = -\frac{dp}{\frac{d\forall}{\forall}} = \frac{dp}{\frac{d\rho}{\rho}}$ (Pa) ……………………………………… (1.16)

Where:

dp=Change in pressure (Pa)=p_2-p_1

d∀=Change in volume (m3) =$\forall_2 - \forall_1$

dρ=Charge in density ($\frac{kg}{m^3}$) =$\rho_2 - \rho_1$

∀=Original volume (m^3) =$\forall_1$

ρ=Original density (kg/m^3) =ρ_1

If dp>0 (p_2>p_1), then:

d∀< 0 ($\forall_2 < \forall_1$)

$d\rho > 0(\rho_2 > \rho_1)$

For perfect gases;

P= ρRT→ $\frac{dp}{d\rho}$=RT→K=$\rho\frac{dp}{d\rho}$=ρRT→K=p

Ex. K_{water}=2.2 Gpa

K air=0.0001 Gpa

12- Surface Tension (σ) (sigma)

At the interface between a liquid and a gas, or two immiscible liquids, a film or special layer seems to form on the liquid, apparently owing to attraction of liquid molecules below the surface. It is a simple experiment to place a small needle on a water surface and observe that it is supported there by the film.

The formation of this film may be visualized on the basis of "surface energy" or work per unit area required to bring the molecules to the surface. The "surface tension σ" is then the stretching force required to form the film, obtained by dividing the surface energy term by unit length of the film in equilibrium. The surface tension of water varies from about (0.074$\frac{N}{m}$) at (20ºC) to (0.059 $\frac{N}{m}$) at (100ºC).

Capillarity

Capillarity attraction is caused by surface tension and by the relative value of "adhesion"(Which is the attraction between liquid particles and solid) to "cohesion" (which is the molecular attraction between liquid particles), see **Fig. (7).**

The capillarity rise (h) is usually calculated by applying equilibrium equation to the capillary tube shown in **Fig.(8)**, and as follows;

$\uparrow \sum Fy = 0$

$F_\sigma \cos\theta = w$

$2\pi r\sigma\cos\theta = \pi r^2 h\gamma$

Thus;

$h = \frac{2\sigma\cos\theta}{\gamma r} = \frac{4\sigma\cos\theta}{\gamma d}$ ……………. (1.17)

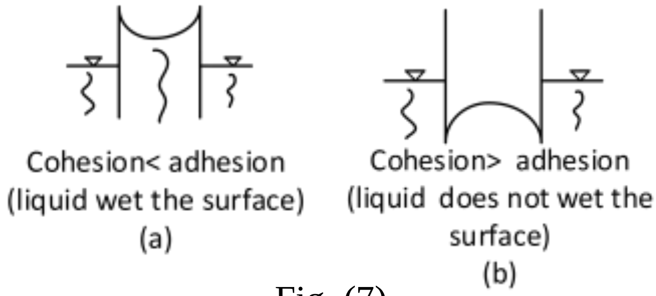
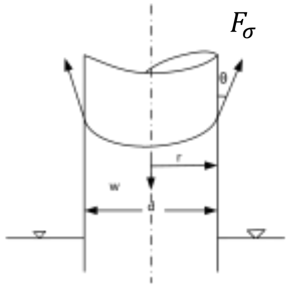


Fig. (7)



Where;

h = Capillary rise (m)

θ =Wetting angle [for clean tubes,

= (0^o) for water and (140^o) for mercury]

Note:

1- For d>12 mm, capillary effects are negligible.

2- Equ (1.17) is plotted in Fig. (1.4) in your textbook.

Some Applications of Surface Tension

The action of surface tension is to increase the pressure within droplet, bubble and liquid jet. To calculate the pressure sustained in these cases, a force balance is made, and as follows;

1- Droplet

For a section of half of spherical droplet, see **Fig. (9)**;

$F_P = F_\sigma$

$P * \pi r^2 = 2\pi r\sigma$

Thus: $p = \frac{2\sigma}{r}$ ……………………………. (1.18)

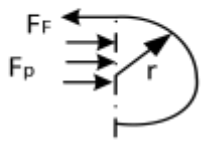


Fig. (9)

2- Bubble

For a section of half of a spherical bubble, see **Fig (10);**

$F_P = F_{\sigma_1} + F_{\sigma_2}$ $\quad F_{\sigma_1} = F_{\sigma_2} = 2\pi r\sigma$

$p * \pi r^2 = 2 * 2\pi r\sigma$

Thus;

$p = \frac{4\sigma}{r}$ ………………….. (1.19)

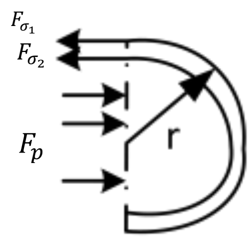


Fig. (10)

3- Liquid Jet

For a section of a half of a cylindrical liquid jet, see **Fig. (11)** ;

$F_P = F_{\sigma_1} + F_{\sigma_2}$

p*L2r=σL + σL=2 σL

Thus $p=\frac{\sigma}{r}$ …………………. (1.20)

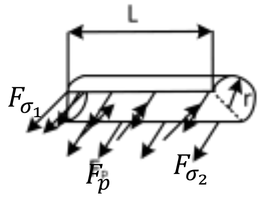


Fig. (11)

4- Measurements of (σ): Ring Pulled out of a Liquid

For a ring wetted by a liquid, see **Fig. (12);**

$F = W + F_{\sigma_i} + F_{\sigma_o}$

$F = W + \pi\sigma(D_i + D_o)$………………….. (1.21)

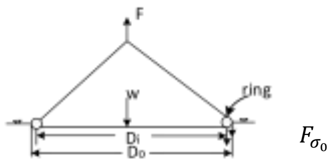


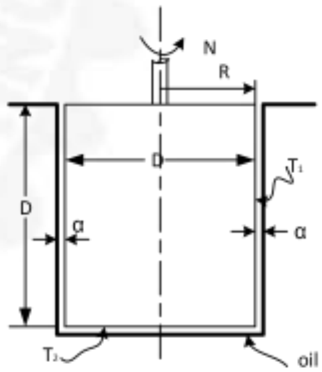
Fig. (12)

Equ. (1.21) is usually used to measure (σ). The weight (W) is usually negelcted.

Examples

Example (1.1): The weight of the cylinder shown in the figure is (W) and its radius of gyration is (K_G). The cylinder rotates at angular speed (N). Develop an expression for viscosity of oil (μ) required to stop the cylinder in (10) seconds.

Sol.:



$T = T_1 + T_2$

$T_1 = F*R = \mu\frac{UA}{h} * R = \mu\frac{\frac{D}{2}N*\pi DD}{a} * \frac{D}{2}$

$\therefore T_1 = \frac{\pi\mu N D^4}{4a}$

$dT_2 = \mu\frac{rN*2\pi r dr}{a} * r = \frac{2\pi\mu N}{a} r^3 dr$

$\therefore T_2 = \int_0^{D/2} dT_2 = \frac{\pi\mu N D^4}{32a}$

$\therefore T = \mu[\frac{\pi\mu N D^4}{4a} + \frac{\pi\mu N D^4}{32a}] = \mu\frac{9\pi N D^4}{32a}$

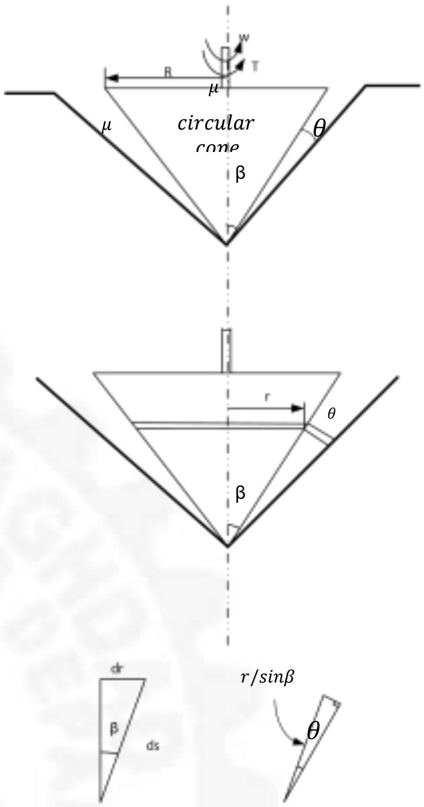
$$\sum M = I\alpha = \frac{w}{g}K_G^2\frac{N}{t}$$

$$\mu\frac{9\pi N D^4}{32a} = \frac{w}{g}K_G^2\frac{N}{10}$$

$\therefore \mu = \frac{32}{90\pi}\frac{wK_G^2 a}{gD^4}$

Example (1.2): Derive an expression for the torque (T) required to rotate the circular cone shown in the figure at a rate of (ω), in terms of the related variables (μ, ω,θ and R).

Sol.:



dF= $\mu\frac{UdA}{h}$

$=\frac{\mu*r\omega*2\pi rds}{\frac{r}{sin\beta}sin\theta}$

$=\frac{\mu r\omega 2\pi rdr/sin\beta}{(r/sin\beta)sin\theta}$

dF=$\frac{2\pi\mu\omega}{sin\theta}rdr$

dT=dF*r

$=\frac{2\pi\mu\omega}{sin\theta}r^2dr$

T=$\int_0^R dT$

$=\frac{2\pi\omega\mu R^3}{3sin\theta}$

$\therefore$T=$\frac{2\pi}{3}\frac{\omega\mu R^3}{sin\theta}$

Example (1.3): A reservoir of glycerin has a mass of (1200 kg) and have a volume of (0.952 m^3). Find the glycerin weight, density, specific weight and specific gravity.

Sol.:

W=mg=1200*9.8→W=11770 N

$\rho=\frac{m}{\forall}=\frac{1200}{0.952}\rightarrow\rho$=1261 kg/$m^3$

$\left.\begin{array}{l}\gamma=\frac{w}{\forall}=\frac{11770}{0.952}\\ \gamma=\rho g=1261*9.8\end{array}\right\}$ γ = 12360

$s=\frac{\gamma}{\gamma_w}=\frac{12360}{9810}\rightarrow s=1.26$

Example (1.4): Convert (15.14 Poise) to kinematic viscosity in (ft^2/sec) if the liquid has a specific gravity of (0.964).

Sol.:

ρ=sρ_w=0.964*1000→ ρ=964 kg/m^3

μ =15.14P $*\frac{1\ Pa.s}{10\ P} \rightarrow \mu = 1.514\ \ Pa.s$

$$v = \frac{\mu}{\rho} = \frac{1.514}{964}\frac{m^2}{s} * \frac{ft^2}{(0.3048)^2} \rightarrow v = 0.0169\frac{ft^2}{sec}$$

Example (1.5): The bulk modules of elasticity of water is (2.2 GPa), what pressure is required to reduce its volume by (0.5 percent).

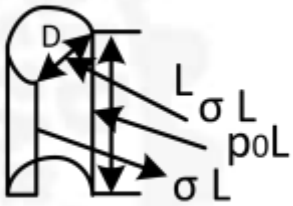
Sol.:

k=$-\frac{dp}{\frac{d\forall}{\forall}}$

2.2*10^9=$-\frac{102-0}{-0.005} \rightarrow p_2 = 11$ MPa

Example (1.6): A small circular jet of mercury (0.1 mm) in diameter issues from an opening. What is the pressure difference between the inside and outside of the jet? Use (σ = 0.514N/m)

Sol.:



$\sum F = 0$

p*DL=2σL

Thus

p=$\frac{2\sigma}{D} = \frac{2*0.514}{0.1*10^{-3}} \rightarrow$ p = 10280 Pa

Problems

The problems number listed in the table below refer to the problems in the "textbook", Chapter "1";

Article No.	Related Problems
1.4	1,2,3,5,14,15,16,17,22
1.5	7,9,11,12,28,39,42,43,46,47

Chapter -2-

Fluid Statics

2.1 Introduction

In static fluids, no relative motion between the fluids particles exist, therefore no velocity gradients in the fluid exist, and hence no "shear stresses" exist. Only "normal stresses (pressure)" exist. In this chapter, the pressure distribution in a static fluid and its effects on surfaces and bodies submerged or floating in it will be investigated.

2.2 Pressure Variation in a Static Fluid

The pressure (p) in a static fluid may change with space coordinates (x, y, z) and with direction (θ).Thus, to find the differential change in pressure (dp) in a static fluid, we may write;

p= p (x, y, z, θ)

$$dp = \frac{\partial p}{\partial x}\,dx + \frac{\partial p}{\partial y}\,dy + \frac{\partial p}{\partial z}\,dz + \frac{\partial p}{\partial \theta}\,d\theta \quad \ldots\ldots\ldots\ (2.1)$$

To derive the differential equation for the pressure (p), we have to derive an expressions For the partial derivatives $\left(\frac{\partial p}{\partial x}, \frac{\partial p}{\partial y}, \frac{\partial p}{\partial z}\ and\ \frac{\partial p}{\partial \theta}\right)$ in equation (2.1), Which represents the changes in pressure with (x, y, z and θ), see **Fig. (1).**

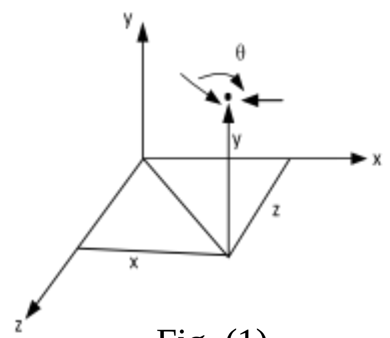


Fig. (1)

2.2.1 Pressure Variation with Direction

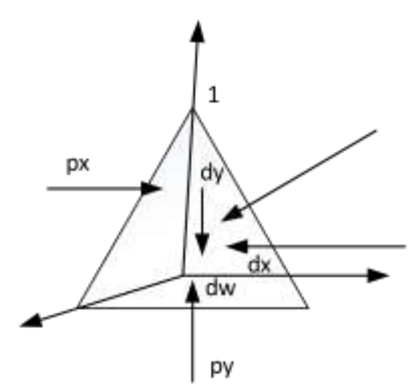
The pressure at a point in a static fluid acts with the same magnitude in all directions. This physical fact will be proved mathematically by taking a fluid element of tetrahedron shape, see **Fig. (2).** we will apply Newton's 2nd law in the three directions, knowing that;

$$\delta A = \delta A\,x + \delta Ay + \delta A_z$$

$$= l\delta A \;\rightarrow m\delta A \;\rightarrow n\delta A$$

Where l, m, n = direction cosines

$$\underset{\longrightarrow}{+}\sum F_x = ma_x = 0\ [since\ a_x = 0]$$



$p_x \perp \perp$ yz − plane (area δA_x)
$p_y \perp \perp$ xz − plane (area δA_y)
$p_z \perp \perp$ xy − plane (area δA_z)
$p \perp \perp$ 123 − plane (area δA)

Fig. (2)

$p_x \delta A_x - p\delta Al = 0$

$p_x l\delta A - p\delta Al = 0 \;\rightarrow\; p_x = p$

$+\swarrow \sum F_z = ma_z = 0$ [$since\ a_z = 0$]

$p_z \delta A_z - p\ \delta An = 0$

$p_z n\delta A - p\delta An = 0 \;\rightarrow\; p_z = p$

$+\uparrow \sum F_y = ma_y = 0$ [$since\ a_y = 0$]

$p_y \delta A_y - p\ \delta Am - dw = 0$

H.O.T (Higher Order Term)

$$p_y m\delta A - p\delta Am - \gamma \cancelto{0}{\frac{dxdydz}{2}} = 0 \;\rightarrow p_y = p$$

Thus; $p_x = p_y = p_z = p$ independent of direction

If we take the limit as $[(d_x, d_y, d_z) \rightarrow 0]$, the element approaches a "point", thus, the above result applies for a point in a static fluid, that is the pressure is independent of direction, and hence;

$$\frac{\partial p}{\partial \theta} = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots (2.2)$$

2.2.2 Pressure Variation in Space

To find the pressure variation in a static fluid with space coordinates (x, y, z), a cubical fluid element will be studied, see **Fig. (3).** The pressure (p) at the center of the element is assumed to change with the three coordinates and we want to find the rate of changes $\left(\frac{\partial p}{\partial x}, \frac{\partial p}{\partial y}\ and\ \frac{\partial p}{\partial z}\right)$. The Newton's 2nd law is applied for the element of **Fig. (3).**

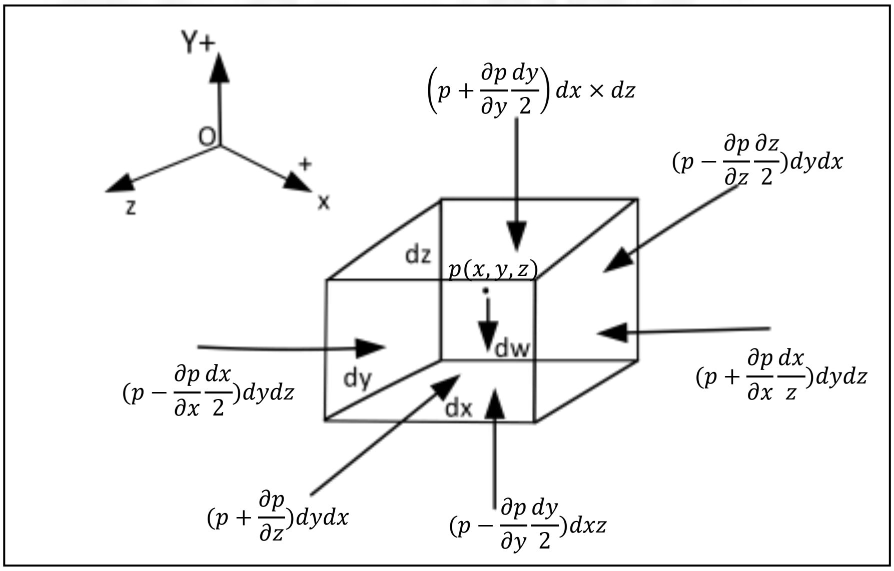


Fig. (3)

$\overset{+}{\rightarrow}\sum F_x = ma_x = 0\ [since\ a_x = 0]$

$$\left(p - \frac{\partial p}{\partial x}\frac{dx}{2}\right) dydz - \left(p + \frac{\partial p}{\partial x}\frac{dx}{2}\right) dydz = 0$$

$$-\frac{\partial p}{\partial x}\, dx\, dy\, dz = 0$$

Or; $\frac{\partial p}{\partial x}\, d\forall = 0$

Since $d\forall \neq 0 \rightarrow \frac{\partial p}{\partial x} = 0$ …………………….. (2.3)

$+\swarrow\sum F_z = ma_z = 0\ [since\ a_z = 0]$

$$\left(p - \frac{\partial p}{\partial z}\frac{dz}{2}\right) dy\, dx - \left(p + \frac{\partial p}{\partial z}\frac{dz}{2}\right) dy\, dx = 0$$

$-\frac{\partial p}{\partial z}\, dx\, dy\, dz = 0 \rightarrow \frac{\partial p}{\partial z} = 0$ ………………………. (2.4)

Equations (2.3) and (2.4) indicate that no changes in pressure occur in the horizontal xz- plane, i. e., all points in the same horizontal plane in a static fluid have the same pressures.

$+\uparrow\sum F_y = ma_y = 0\ [since\ a_y = 0]$

$$\left(p - \frac{\partial p}{\partial y}\frac{dy}{2}\right) dxdz - \left(p + \frac{\partial p}{\partial y}\frac{dy}{2}\right) dx\, dz - \gamma dxdydz = 0$$

$$-\frac{\partial p}{\partial y} dxdydz - \gamma dxdydz = 0$$

Or; $\frac{\partial p}{\partial y} = -\gamma$ ……………………………………. (2.5)

Equation (2.5) indicates that the pressure in a static fluid decreases as (y), the vertical coordinate, increases. This is a logical result physically, since the increase in (y) means decease in the fluid column height which causes the pressure.

2.2.3 Basic Hydrostatic Equation

This equation is obtained by substituting equations (2.2), (2.3), (2.4) and (2.5) into equation (2.1). Thus;

$$dp = \underbrace{\frac{\partial p}{\partial x}}_{\substack{0 \\ \text{Equ. (2.3)}}} dx + \underbrace{\frac{\partial p}{\partial y}}_{\substack{-\gamma\gamma \\ \text{Equ. (2.5)}}} dy + \underbrace{\frac{\partial p}{\partial z}}_{\substack{0 \\ \text{Equ. (2.4)}}} dz + \underbrace{\frac{\partial p}{\partial \theta}}_{\substack{0 \\ \text{Equ. (2.2)}}} d\theta$$

Hence: $dp = -\gamma\, dy$

Or; $\frac{dp}{dy} = -\gamma$ Basic Hydrostatic Equation …………………. (2.6)

This is the basic differential equation whose integration gives the pressure distribution equation in a static fluid. The integration depends on how the specific weight ($\gamma = \rho g$) vary with the vertical coordinate (y). Accordingly, the distribution for incompressible and compressible fluids is different.

2.2.4 Pressure Variation for Incompressible Fluids

For incompressible fluids, (ρ=constant), and hence ($\gamma = \rho g$ = constant), and the integration of equ. (2.6) yields;

$$\int dp = -\int \gamma dy$$

$p = -\gamma y + \text{const.}$

Or; $p + \gamma y = \text{const.}$

The (const.) of integration is the same for any two points. (1) and (2), thus;

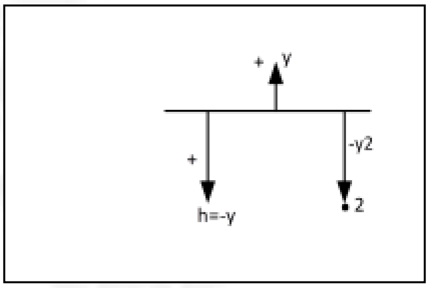


Fig. (4)

$$p_1 + \underbrace{\cancel{\gamma y_1}}_{0\ (y_1=0)} = p_2 + \gamma y_2$$

$p_2 = p_1 - \gamma y_2$

If we use the coordinate (h = -y) measured vertically downward, i. e., (h = -y_2), then see **Fig (4);**

$p_2 = p_1 + \gamma h$ …………………………… (2.7)

As mentioned previously, the pressure increases as we move downward (+h) and decreases as we move upward (-h).

2.2.5 Pressure Variation for Compressible Fluids

For compressible fluids, the density (ρ) changes with temperature (T) as the perfect gas law (p = ρ RT) indicates. And since the temperature (T) varies with altitude (y), the density (ρ) also changes with (y), and accordingly ($\gamma = \rho g$) changes with (y) also. Thus;

$\rho = \frac{p}{RT}$ ………………….. (2.8)

$$\gamma = \rho g = \frac{pg}{RT}$$

Hence, equ. (2.6) becomes;

$$\frac{dp}{dy} = -\frac{pg}{RT} \text{ .... } (2.9)$$

To integrate this equation, we must know how (T) varies with (y). For this purpose, the atmosphere is divided into two layers;

I - Troposphere: (from sea level up to 10769m)

$$T = T_0 + \beta(y - y_0) \text{ .... } (2.10)$$

Where: T_0= Reference temperature at seal eve ($y_0 = 0$) = 288k

β = Lapes rate = -0.00651 ˚C/m = -0.00357 ˚F/ft

Thus, equ. (2.9) becomes;

$$\frac{dp}{p} = -\frac{g}{R}\frac{dy}{T_o + \beta(y - y_o)}$$

Integrate from ($y = y_o$)where($p = p_o$), to ($y = y$) where($p = p$). Thus:

$$\frac{p}{p_o} = \left[\frac{T_o+\beta(y-y_o)}{T_o}\right]^{-\frac{g}{R\beta}} \text{ .... } (2.11)$$

II - Stratosphere: (from y = 10769 m up to 32 km)

$$T = constant = 218\,K \text{ (at y} = 10769 \text{ from equ. 2.10) .... } (2.12)$$

Thus: $\frac{dp}{p} = -\frac{g}{RT}\,dy$

And hence: $\frac{p}{p_o} = e^{-\frac{g(y-y_o)}{RT}}$ .... (2.13)

(p_o) is the pressure at ($y = y_o = 10769$)

2.3 Pressure Measurements

2.3.1 Absolute and Gage Pressures

The absolute pressure (p_{abs}) is the pressure measured relative to a complete vacuum (zero absolute pressure), whereas the gage pressure (p_{gage}) is the pressure measured relative to the local atmospheric pressure ($p_{atm)loc}$).The local atmospheric pressure may be larger than, smaller than are equal to standard atmospheric pressure ($p_{atm)std}$) which is a constant value, see **Fig. (5).** Thus;

$p_{abs} = p_{atm)loc} + p_{gage}$ .... (2.14)

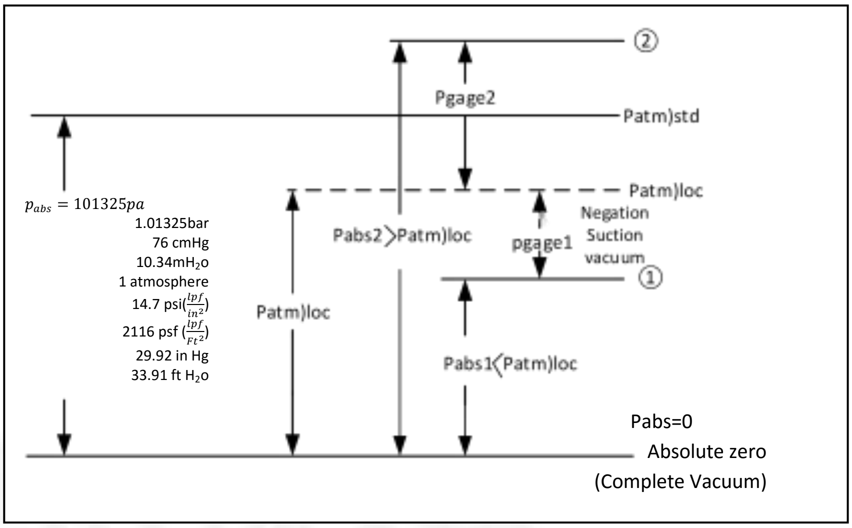


Fig. (5)

2.3.2 Atmospheric Pressure Measurement

The "barometer" is usually used to measure the atmospheric pressure. It consists of a glass or Perspex tube with one open and immersed in a bath of mercury, see **Fig. (6)**. the pressure at the free surface is (p_{atm}), which is equal to the pressure at point 2 (p_2) (see equs. (2.3), and (2.4)). Thus, using equ. (2.7) ($p_2 = p_1 + \gamma h$);

$p_{atm} = p_{vapor} + \gamma h$ .... (2.15)

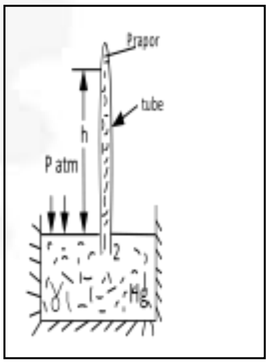


Fig. (6)

The vapor pressure of mercury is very small and can be neglected, hence;

$p_{atm} = \gamma h$ .... (2.16)

And;

$h_{atm} = \frac{p_{atm}}{\gamma} = h$ .... (2.17)

If ($p_{atm} = p_{atm)std}$), then (h = 76 cm). The mercury is used because its specific weight is large and thus (h) will be small and reasonable. Also the vapor pressure is very small and can be neglected.

"**Aneroid Barometer**" is also used to measure the atmospheric pressure, see **Fig. (7)**. It measures the difference in pressure between the atmospheric pressure (p_{atm}) and an evacuated cylinder by means of a sensitive elastic diaphragm and linkage system.

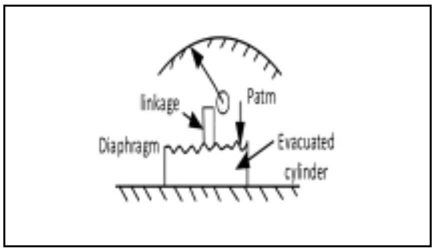


Fig. (7)

2.3.3 Gage Pressure Measurements

2.3.3.1 Bourdon Gage

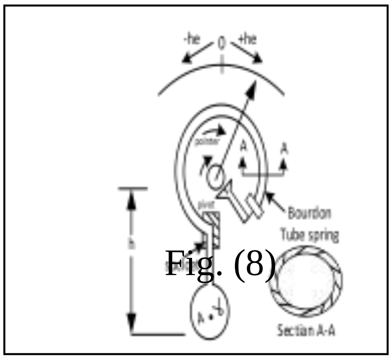


Fig. (8)

The "bourdon gage" shown in **Fig. (8)** is a typical of the devices used for measuring gage pressure. The pressure element is a hollow, curved, flat metallic tube closed at one end; the other end is connected to the pressure to be measured. When the internal pressure is increased, the tube tends to straighten, pulling on a linkage to which is attached a pointer and causing the pointer to move. The dial reads zero when the inside and outside of the tube are at the same pressure, regardless of its particular value. Usually, the gage measures pressure relative to the pressure of the medium surrounding. The tube, which is the local atmosphere. According to equ. (2.7); the pressure recorded by the gage of **Fig. (8)** is;

$p_A = gage\ reading + \gamma h$ ……………………... (2.18)

$p_{A_{Vacumm}} = gage\ reading + \gamma h$ ……………………... (2.19)

2.3.3.2 Pressure Transducers

The devices work on the principle of conversion pressure energy to mechanical energy, then to electrical signals, sees **Fig. (9)**.

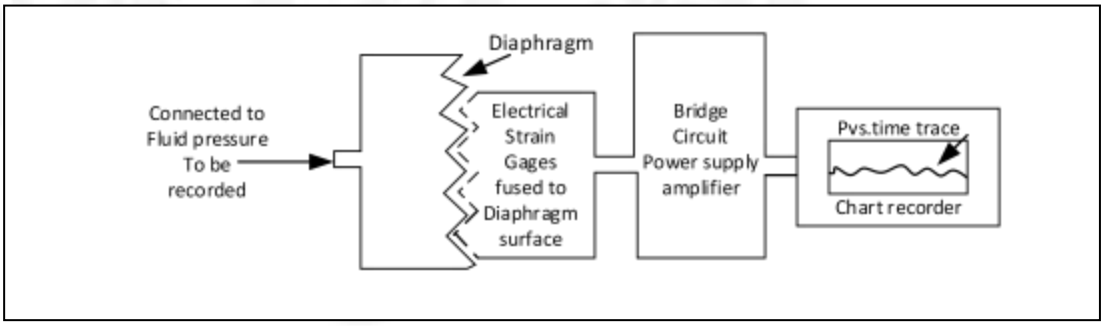


Fig. (9)

2.3.3.3 Manometers

Manometers are devices that employ liquid columns for determining difference in pressure. They are usually made of glass (or PVC) tubes filled of one or more liquids with different specific gravities and that are not miscible in each other. Each manometer has tow ends. The general procedure that should be followed in working any manometer problem is as follows;

1. Start at one end (or any meniscus if the circuit is continuous) and write the pressure there in an appropriate unit or in an appropriate symbol if it is unknown.
2. Add to this the change in pressure, in the same unit, from one meniscus to the next (plus if the next meniscus is lower, minus if higher).
3. Continue until the other end of the gage (or the starting meniscus) is reached and equate the expression to the pressure at that point, known or unknown.

2.3.3.3.1 Piezometer

This type of the manometers, illustrated in **Fig. (10)**, is used to measure the pressure in a liquid when it is above zero gage. Using equ. (2.7);

$$p_{Aabs} = p_{atm} + \gamma h \text{ (Pa)}$$
$$= p_{atm} + s\gamma_w h \text{ (Pa)} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2.20)$$
$$p_{Agage} = \gamma h = s\gamma_w h \text{ (Pa)} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2.21)$$

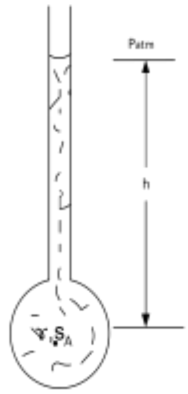


Fig. (10)

Piezometer would not work for negative gage pressure, because air would flow into the container through the tube. It is also impractical for measuring large pressures at A, since the vertical tube would need to be very long.

For measurement of small negative or positive gage pressures, the tube may take the form shown in **Fig. (11)**. For this arrangement;

$$\left.\begin{aligned} p_{Aabs} &= p_{atm} - \gamma h \text{ (Pa)} \\ &= p_{atm} - s\gamma_w h \text{ (Pa)} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (2.22)$$
$$p_{Agage} = -\gamma h = -s\gamma_w h \text{ (Pa)} \quad \ldots\ldots (2.23)$$
$$h_{Agage} = -hs\ (mH_2O)(or\ cm, mm, \ldots.H_2O)$$

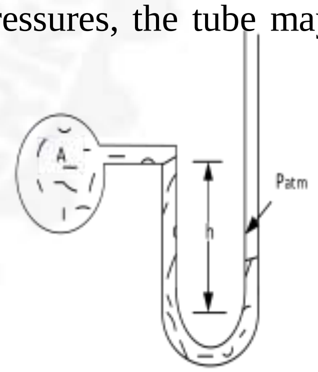


Fig. (11)

For greater negative or positive gage pressure, a second liquid of greater (s) is employed, see **Fig. (12).**

$$\left.\begin{aligned} p_{Aabs} &= p_{atm} + \gamma_2 h_2 - \gamma_1 h_1 \\ &= p_{atm} + s_2\gamma_w h_2 - s_1\gamma_w h_1 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (2.24)$$
$$\left.\begin{aligned} p_{Agage} &= \gamma_2 h_2 - \gamma_1 h_1 = s_2\gamma_w h_2 - s_1\gamma_w h_1 \\ h_{Agage} &= h_2 s_2 - h_1 s_1 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (2.25)$$

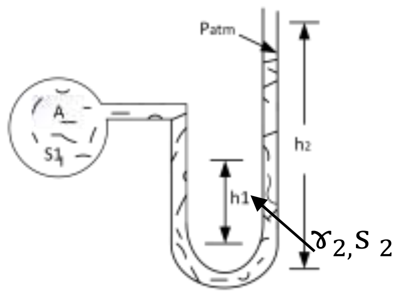


Fig. (12)

2.3.3.3.2Differential Manometer

This type of manometers (**Fig. (13)**) determines the difference in pressure at two points A and B. the equation of this monometer is:

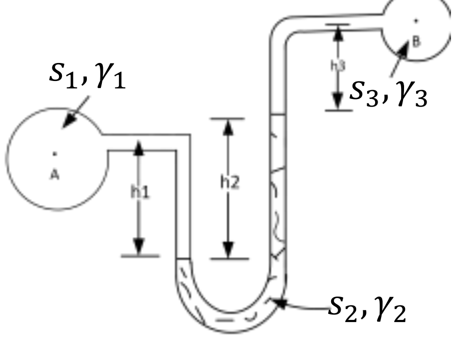


Fig. (13)

$$p_A + \gamma_1 h_1 - \gamma_2 h_2 - \gamma_3 h_3 = p_B$$

Or;

$$p_A - p_B = -\gamma_1 h_1 + \gamma_2 h_2 + \gamma_3 h_3 \text{ (Pa)} \quad \ldots. (2.26)$$

And;

$$h_A - h_B = -h_1 s_1 + h_2 s_2 + h_3 s_3 \text{ (Units of length of } H_2O) \ldots. (2.27)$$

Equation (2.26) represents the difference in both absolute and gage pressures at A and B.

2.3.3.3.3 Micromanometers

This type of manometers are used for measuring small pressure differences, see **Fig . (14)**. The small difference (Δy) at C.S.A.(A) is magnified to a large difference (R/2) at smaller C.S.A.(a).

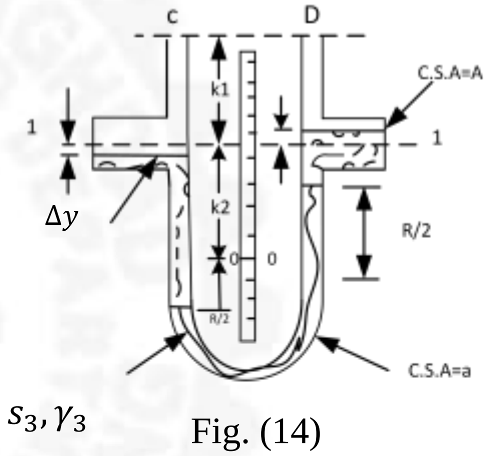


Fig. (14)

The manometer equation is;

$$p_c + (k_1 + \Delta y)\gamma_1 + \left(k_2 - \Delta y + \frac{R}{2}\right)\gamma_2 - R\gamma_3 - \left(k_2 - \frac{R}{2} + \Delta y\right)\gamma_2 - (k_1 - \Delta y)\gamma_1 = p_D$$

But; $A\,\Delta y = \frac{R}{2}\, a$

Thus, we can show that:

$$p_c - p_D = R\left[\gamma_3 - \gamma_2\left(1 - \frac{a}{A}\right) - \gamma_1 \frac{a}{A}\right] = R * constant \quad \ldots\ldots\ldots (2.28)$$

2.3.3.3.4 Inclined Manometer

The inclined manometer (**Fig. (15)**) is frequently used for measuring small difference in gage pressure. It is adjusted to read zero, by moving the inclined scale. Since the inclined tube requires a greater displacement of the meniscus for given pressure difference than a vertical tube,

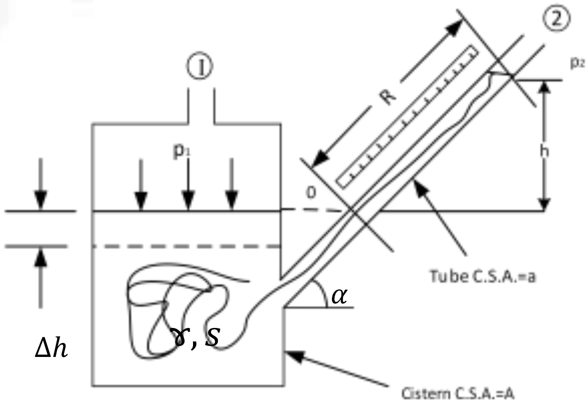


Fig. (15)

it offers greater accuracy in reading the scale. The equations of the manometer are;

$$p_1 - \gamma(h + \Delta h) = p_2$$

Thus;

$$p_1 - p_2 = s\gamma_w\,(h + \Delta h)$$

but h=R sin α

and $A\Delta h = Ra \;\rightarrow\; \Delta h = R\frac{a}{A}$

Thus;

$$p_1 - p_2 = Rs\gamma_w\left(\sin\alpha + \frac{a}{A} = R * const.\right) \ldots\ldots\ldots\ldots\ldots\ldots (2.29)$$

The device is usually designed so that the angle (α) can be changed to more than one value.

Note:

The value of pressure (p) can be converted to columns of any fluid by the following relation;

$$p = \gamma_1 h_1 = \gamma_2 h_2 = \gamma_3 h_3 = \ldots \gamma_m h_m$$

Thus;

$$\left.\begin{aligned} h_1 &= \frac{p}{\gamma_1} \text{ Unit length of fluid 1} \\ h_2 &= \frac{p}{\gamma_2} \text{ Unit length of fluid 2} \\ h_3 &= \frac{p}{\gamma_3} \text{ Unit length of fluid 3} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.30)$$

2.4 Hydrostatic Forces on Submerged Surface

Hydrostatic forces excreted by a static fluid are always perpendicular to the surface on which they act.

2.4.1 Horizontal Surfaces

A plane surface in a horizontal position in a fluid at rest is subjected to a constant pressure ($p = \gamma h$), see **Fig.(16)**. To find the resultant hydrostatic fore (F), consider an element (dA), on which an element force (dF) acts;

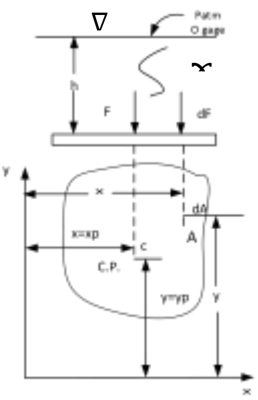


Fig.(16)

$$dF = p\,dA = \gamma h\,dA$$

Thus; $F = \int dF = \int_A \gamma h\,dA = \gamma h \int_A dA$

Hence;

$F = \gamma hA = pA$ ... ... (2.31)

Center of Pressure C.P. (x_p, y_p)

It is the point through which the resultant hydrostatic force (F) passes. To find the coordinates of center of pressure (x_p, y_p), take moments about x & y coordinates;

$F*x_p = \int dF * x$

$\gamma hA * x_p = \int \gamma hx dA = \gamma h \int x dA$

Hence; $x_p = \frac{1}{A}\int x dA \rightarrow x_p = \bar{x}$

$F * y_p = \int dF * y$

$\gamma hA * y_p = \int \gamma hy dA = \gamma h \int y dA$

Hence; $y_p = \frac{1}{A}\int y dA \rightarrow y_p = \bar{y}$

Hence; C.P. = C

$(x_p, y_p) = (\bar{x}, \bar{y})$ [for horizontal surfaces only] …………...(2.32)

2.4.2 Inclined Surfaces

For inclined plane surfaces, the pressure ($p = \gamma h$) varies anlog the surface, since (h) is not constant, see **Fig. (17)**. For element (dA), the force acting on it (dF) is;

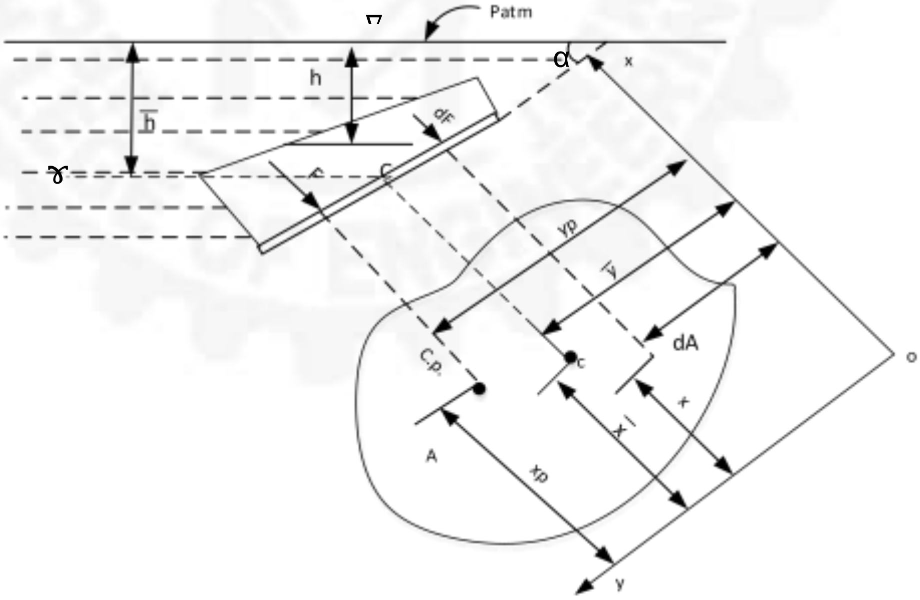


Fig. (17)

$h = y \sin\alpha$

$\bar{h} = \bar{y} \sin\alpha$

$p = \gamma \gamma h$

$p_c = \gamma \gamma \bar{h}$

If ($\alpha = 90°$)(vertical surfacse) → h=y ; $\bar{h} = \bar{y}$

dF= pdA= $\gamma hdA = \gamma y \sin\alpha dA$

F=$\int dF = \int \gamma y \sin\alpha dA = \gamma \sin\alpha \int ydA = \gamma \sin\alpha * A\bar{y} = \gamma\bar{y}\sin\alpha A$

Hence;

$F = \gamma \bar{h} A = p_c A$ ……………..... (2.33)

To find the center of pressure (x_p, y_p);

$F * y_p = \int dF * y$

$\gamma\bar{y}\sin\alpha A y_p = \int \gamma\, y \sin\alpha\, y dA$

Thus; $y_p = \frac{1}{Ay}\int y^2\, dA = \frac{I_{ox}}{A\bar{y}} = \frac{I_{cx} + A\bar{y}^2}{A\bar{y}}$

Where; I_{ox} = second moment of area (A) about horizontal axis (ox).

From the parallel axis theorem;

$I_{ox} = I_{cx} + A\bar{y}^2$

Where; I_{cx} = Second moment of area (A) about horizontal centroidal axis (cx).

$\bar{y}$ = Distance between the two axes (ox) & (cx)

Thus;

$y_p = \bar{y} + \frac{I_{cx}}{A\bar{y}}$ …………………….. (2.34)

$F * x_p = \int dF * x$

$\gamma\bar{y}\sin\alpha A x_p = \int \gamma\, y \sin\alpha\, x dA$

Thus; $x_p = \frac{1}{Ay}\int xy\, dA = \frac{I_{oxy}}{A\bar{y}} = \frac{I_{cxy} + A\bar{x}\bar{y}}{A\bar{y}}$

Where;

I_{oxy}= Product moment of area (A) about axes (ox & oy)

I_{cxy}= Product moment of area (A) about centroidal axes (cx & cy)

$I_{oxy} = I_{cxy} + A\bar{x}\bar{y}$ (parallel axis theorem)

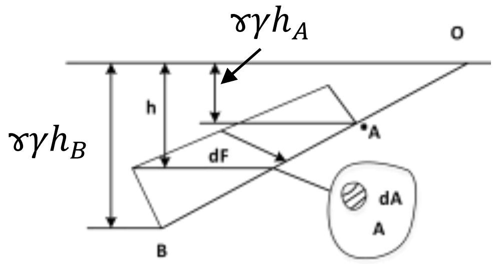
Thus;

$x_p = \bar{x} + \frac{I_{cx}}{A\bar{y}}$ ....………………..... (2.35)

When x <u>or</u> y <u>or</u> both are axes of symmetry, then ($I_{cxy} = 0$) & ($x_p = \bar{x}$)

The Pressure Prism

It is a prismatic volume with its base the given surface area and with altitude at any point of the base given by (p = γh), see Fig. **(18)**



dF = pdA = $\gamma h dA = \gamma d\forall$

F= $\int dF = \int \gamma d\forall \ \rightarrow F = \gamma\forall$ ……………..... (2.36)

Thus, the volume of the pressure prism equals the magnitude of the resultant force acting on one side of the surface.

Fig. (18)

To find cent of pressure;

F * x_P= $\int$ dFx

$$\gamma\forall x_p = \int \gamma x d\forall \ \rightarrow x_p = \frac{\int x d\forall}{\forall} = \frac{\bar{x}\forall}{\forall} \rightarrow x_p = \bar{x}$$

Similarly, we can show that; $y_p = \bar{y}; z_p = \bar{z}$

i.e. C.P. = C of volume ∀

$x_p, y_p, z_p = (\bar{x}, \bar{y}, \bar{z})_\forall$ …….……………... (2.37)

2.4.3 Curved Surfaces

When the elemental force (dF = pdA) vary in direction, as in curved surfaces, they must be added as vector quantities; i.e., their components (horizontal and vertical) are added vectorially. In **Fig. (19)**, the force (dF) on an element (ds) of the curved surface is replaced by its two components, horizontal (dF_H) and vertical (dF_V).

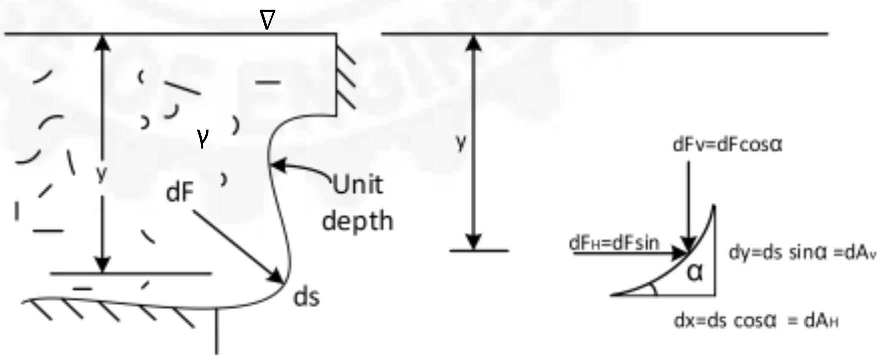


Fig. (19)

Horizontal Component (F_H)

The horizontal component of pressure force on a curved surface is equal to the pressure force exerted on a projection of the curved surface. The vertical plane of projection is normal to the direction of the component

$dF = pdA = \gamma y ds.1$

$dF_H = dF \sin\alpha = \gamma y ds \sin\alpha = \gamma y dy = \gamma y dA_v$

Thus; $F_H = \int d\,F_H = \int \gamma y dA_v \rightarrow F_H = \gamma A_v \bar{y}_v = p_{cv}\,A_v$ .... (2.38)

Where;

A_v = Vertical projection of the curved surface on a vertical plane

$\bar{y}_v$ =Centroid of (A_v) distance to the free surface

$F_H * y_{PH} = \int dF_H y \quad \rightarrow y_{PH} = \bar{y}_v + \frac{I_{cxv}}{A_v \bar{y}_v}$ .... (2.39)

$F_H * y_{PH} = \int dF_H y \quad \rightarrow x_{PH} = \bar{x}_v + \frac{I_{cxyv}}{A_v \bar{y}_v}$ .... (2.40)

Equations (2.38), (2.39) and (2.40) are similar to equations (2.33), (2.34) and (2.35) for inclined surfaces, with (α = 90°), i.e., vertical surface, which is the vertical projection (A_v) of the curved surface.

Vertical Component (F_v)

The vertical component of pressure force on a curved surface is equal to the weight of liquid vertically above the curved surface and extending up to the free surface;

$dF_v = dF \cos\alpha = \gamma yds \cos\alpha = \gamma ydx = \gamma\, d\forall$

Thus; $F_v = \int dF_v = \gamma \int d\forall \rightarrow F_v = \gamma\forall$ .... (2.41)

Where;

$\forall$ = Volum of the liquid enclosed between the curved surface and the free surface

$F_v * x_{Pv} = \int dF_v x \rightarrow x_{pv} = \bar{x}_\forall$

$F_v * y_{Pv} = \int dF_v y \rightarrow y_{pv} = \bar{y}_\forall$

$F_v * z_{Pv} = \int dF_v z \rightarrow z_{pv} = \bar{z}_\forall$

i.e.;$(C.P.)_{F_v} = (C)_\forall$

$(x_p, y_p, z_p)_{F_v} = (\bar{x}, \bar{y}, \bar{z})_\forall$ .... (2.42)

Note;

When the liquid is below the curved surfaces, see **Fig. (20)**, and the pressure magnitude is known at some point o (p_0), an "imaginary" or equivalent free surface (I.S.)

can be constructed ($\frac{p}{\gamma} = h$) above (o). The direction of force (F_s) is reversed to obtain the real fore (F). The resultant of (F_H) and (F_v) is found as;

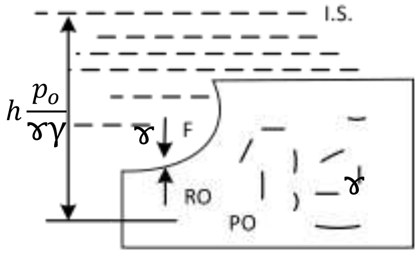


Fig. (20)

$$F = \sqrt{F_H^2 + F_v^2}$$

$$\theta = tan^{-1}\frac{F_v}{F_H} \qquad \text{.......................}(2.43)$$

2.5 Buoyancy

The buoyant force (F_B) is the resultant force exerted on a body by a static fluid in which it is submerged or floating. Its magnitude is equal to the weight of the displaced volume. It acts vertically upward through the center of buoyancy (B) which is the centroid (C) of the displaced volume. To prove the above statements mathematically, consider the body shown in **Fig. (21)**. the body has 6 curved surfaces, on which 4 horizontal forces (F_1,F_2, F_3 andF_4) and 2 vertical forces (F_5 andF_6) are acting. To find the resultant of these forces;

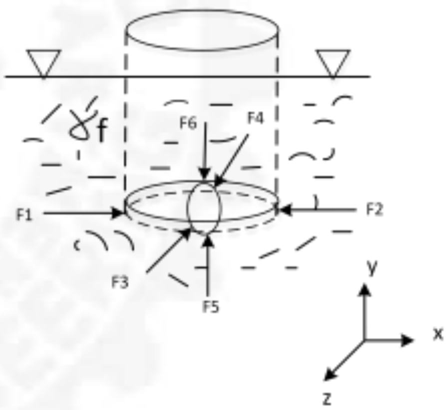


Fig. (21)

$$R_x = \sum F_x = F_1 - F_2$$

$$= \gamma\bar{y}_{v_1}A_{v_1} - \gamma\bar{y}_{v_2}A_{v_2}$$

$$= 0 \text{ (since } \bar{y}_{v_1} = \bar{y}_{v_2} \& A_{v_1} = A_{v_2})$$

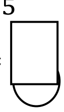
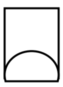
$$R_z = \sum F_z = F_3 - F_4$$

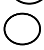
$$= \gamma\bar{y}_{v_3}A_{v_3} - \gamma\bar{y}_{v_4}A_{v_4}$$

$$= 0 \text{ (since } y_{v_3} = y_{v_{24}} \& A_{v_3} = A_{v_4})$$

$$\uparrow R_y = \sum F_y = F_5 - F_6$$

$$= \gamma_f\forall_5 - \gamma_f\forall_6$$

$= \gamma_f *$  $- \gamma_f *$ 

$= \gamma_f *$ 

$$= \gamma_f\forall_{displaced} \uparrow$$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} = Ry = \gamma\forall_{disp.} = F_B$$

Thus : $F_B = \gamma_f\forall_{disp.}$ .... (2.44)

To find the center of buoyancy (B), take moments;

$$F_B * X_B = \int d\, F_B x$$

$$\gamma_f \forall_{disp.} x_B = \int \gamma_f d\forall_{disp.} x \rightarrow x_B = \frac{\int x d\forall}{\forall} = \bar{x}$$

Similarly, we can show that $y_B = \bar{y}$ & $z_B = \bar{z}$

Hence; $(B)_{F_B} = (C)_{\forall_{disp.}}$

$(x_B, y_B, z_B)_{F_B} = (\bar{x}, \bar{y}, \bar{z})_{\forall_{disp.}}$ ……………………................ (2.45)

Notes:

1- When the body is submerged in two fluids, then the buoyant force (F_B) is the sum of the two buoyant forces exerted by the two fluids, see **Fig. (22)**

$$F_B = F_{B1} + F_{B2}$$

$$= \gamma_{f1}\forall_1 + \gamma_{f2}\forall_2$$

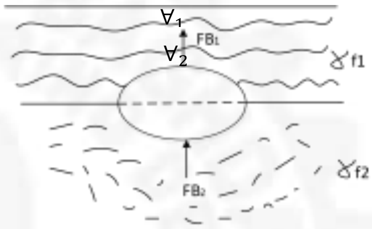


Fig. (22)

2- When the body is floating, only the submerged volume is used in calculating (F_B), see **Fig. (23)**

$$F_B = \gamma_f \forall_{disp.}$$

$$\sum F_y = 0 \rightarrow W = F_B$$

$$\gamma_b \forall = \gamma_f \forall_{disp.}$$

Thus; $\frac{\forall_{disp.}}{\forall} = \frac{\gamma_b}{\gamma_f}$

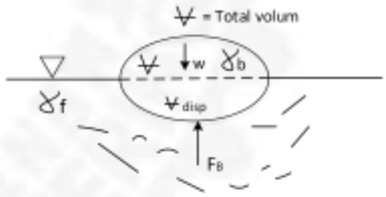


Fig. (23)

$\forall$ = Total volume of body

$\forall_{disp.}$= Displaced volume

γ_b = Body specific weight

3- To find the volume ($\forall$) and weight (W) of an oval - shaped body, it is weighted in two fluids (F_1&F_2), see **Fig. (24)**

$$W = F_1 + F_{B1} = F_2 + F_{B2} \rightarrow \forall = \frac{F_1 - F_2}{\gamma_2 - \gamma_1}$$

$$= F_1 + \gamma_1 \forall = F_2 + \gamma_2 \forall \quad \rightarrow W = \frac{F_1\gamma_2 - F_2\gamma_1}{\gamma_2 - \gamma_1}$$

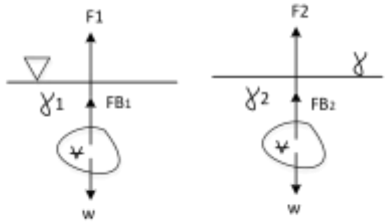


Fig. (24)

F_1 =Weight of body in fluid 1

F_2 =Weight of body in fluid 2

2.5.1 Hydrometers

They are devices that use the principle of buoyant force to determine the specific gravity (or relative density) s of liquids. **Fig. (25)** shows a hydrometer in two liquids. It has a stem of prismatic cross section of area (a).

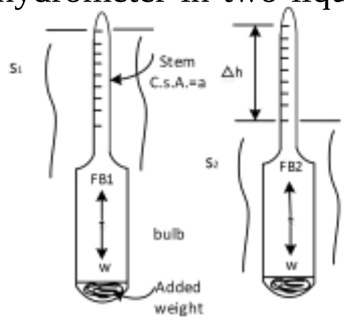


Fig. (25)

In fluid 1 : W = $\gamma_1 \forall_1 = s_1 \gamma_w \forall_1$

In fluid 2 : W = $\gamma_2 \forall_2 = s_2 \gamma_w (\forall_1 - a\Delta h)$

Hence, we can show that.

$$\Delta h = \frac{\forall_1}{a} \frac{s_2 - s_1}{s_2} \qquad \text{………………….. (2.46)}$$

Where ($\forall_1$ = Submerged volume in fluid 1)

Note:

1. Equation (2.46) is used to divide the scale of the stem after specifying the geometry and weight of the hydrometer.
2. Hydrometer is made usually from a glass or Perspex tubes. If the weight of these tubes do not satisfy the hydrometer equation (W = F_B), then additional weight, such as mud, lead, is added inside the bulb.
3. For each hydrometer, there are minimum and maximum values of (s) which can be measured by its scale.

2.6 Stability of Submerged and Floating Bodies

2.6.1 Submerged Bodies

A completely submerged (immersed) object has a "rotational stability" when a restoring couple is set up by any small angular displacement. This occurs only when the center of gravity (G) is below the center of buoyancy (B), see **Fig. (26)**

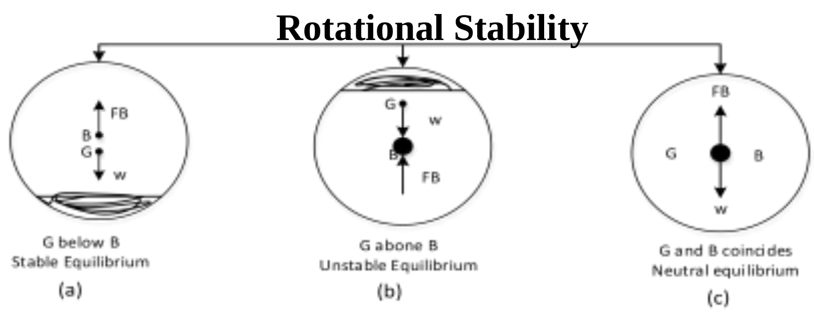


Fig. (26)

2.6.2 Floating Bodies

I- Liner Stability; A floating body has a linear stability when a small liner displacement in any direction set up restoring force tending to return it to its original position. A small upward displacement decreases the volume of liquid displaced, resulting in an unbalanced downward force, and small downward displacement results in greater buoyant force, which causes an unbalanced upward force.

II- Rotational Stability; Any floating object with G below B floats in stable equilibrium. Certain floating objects, however, are in stable equilibrium even when G above B. This may occurs when the line of buoyant force after the heel intersects the center line above G, at a point called the "Metacenter M", **see Fig. (27)**, which is a prismatic body (all parallel cross – sections are identical). When the body is displaced by angle (α) clockwise, a counter clockwise restoring couple is set up, whose value is;

Restoring Couple = W * MG sin α ....… (2.27)

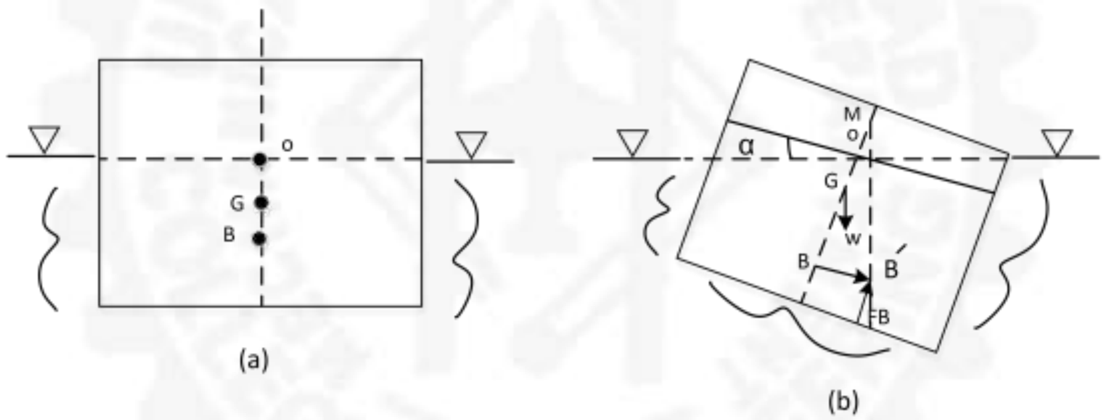


Fig. (27)

The distance (MG) is called the "Metacentric Height", and it is direct measure of the stability of the body. The following criteria are used for this purpose;

$$\left.\begin{array}{l} MG > 0 \ \{M \text{ above } G\} \rightarrow \text{Stable Equilibrium} \\ MG < 0 \ \{M \text{ below } G\} \rightarrow \text{Unstable Equilibrium} \\ MG = 0 \ \{M \text{ and } G \text{ coincides}\} \rightarrow \text{Neutral Equilibrium} \end{array}\right\} \quad \ldots\ldots\ldots \ (2.48)$$

For a general floating body of variable cross- section, such as a ship, a convenient formula can be developed for determination of the metacetic height (MG). This will be done in the next article.

2.6.3 Metacenter

The (Metacenter M) is the intersection of the buoyant force after the angular displacement and the original center line; or, it is the intersection of the lines of action of the buoyant forces before and after the heel (angular displacement).

In what follows, we will derive a formula for determining the Metacentric height (MG). This will be done for a ship, which is a non – prismatic body (parallel cross – sections are not identical), **see Fig. (28)**. The results are, however, general and can be used for bodies with prismatic cross – sections.

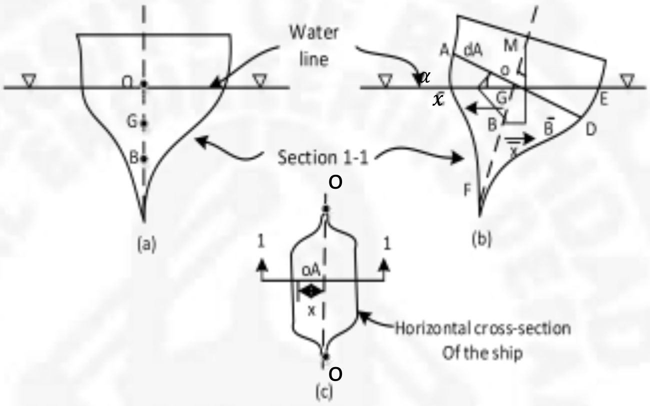


Fig. (28).

The Metacentric height (MG) is found as;

$$MG = MB \pm BG \qquad \ldots\ldots\ldots\ldots\ldots (2.49)$$

(-) sign is used when G above B

(+) sign is used when G below B

$$\text{Now; } MB = \frac{\bar{x}}{\tan\alpha} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.50)$$

Where; $\bar{x}$= Centroidal distance of the displaced volume after the heel (B')

To specify ($\bar{x}$), we take moments of volume about the line (0-0);

$$(\bar{x}\forall)_{\mathrm{CoEDF}} = (\bar{x}\forall)_{\mathrm{oED}} + (\bar{x}\forall)_{\mathrm{oDF}} - (\bar{x}\forall)_{\mathrm{oCF}}$$

$$= (\bar{x}\forall)_{\mathrm{oED}} + \cancel{(\bar{x}\forall)_{\mathrm{oDF}}} - [\cancel{(\bar{x}\forall)_{\mathrm{oAF}}} - (\bar{x}\forall)_{\mathrm{oAc}}]$$

$$= (\bar{x}\forall)_{\mathrm{oED}} + (\bar{x}\forall)_{\mathrm{oAc}}$$

$$= \int_{\mathrm{oED}} x\,d\forall + \int_{\mathrm{oAc}} x\,d\forall$$

$=\int_{waterline} xd\,\forall$

But $d\forall = x\tan\alpha\, dA$

Thus; $\bar{x}\forall = \int_{waterline} x^2 \tan\alpha\, dA$

And, hence;

$x = \frac{\tan\alpha}{\forall}\int x^2\, dA = \tan\alpha \frac{I_{oo}}{\forall}$ .... (2-51)

Where: $I_{oo} = \int x^2 dA =$ Second moment of area of water line surface about longitudinal axis (o-o)

$\forall$ = Displaced (submerged) volume

Thus, (2.51) in (2.50) gives;

$MB = \frac{I_{oo}}{\forall}$ .... (2.52)

And hence (2.52) in (2.49) gives

$MB = \frac{I_{oo}}{\forall} \pm BG$ .... (2.53)

After calculating (MG) from equ. (2.53), the criteria specified in equ. (2.48) is used to specify the state of equilibrium of the body.

2.7 Relative Equilibrium

In this application, the fluid is in motion such that no layer moves relative to an adjacent layer; as a result, the shear stresses throughout the fluid are zero (absent), which is the same as in fluid statics. Therefore, the laws of fluid static are applied for this type of motion. Examples are;

1. A fluid moves with uniform velocity
2. A fluid moves with uniform linear acceleration
3. A fluid with uniform rotation

2.7.1 Uniform Linear Acceleration

A liquid in an open vessel is given a uniform linear acceleration ($a = a_x i + a_y j$), see **Fig. (29)**. After some time the liquid adjusts to the acceleration so that it moves as a solid; i.e, the distance between any two fluid particles remains fixed, and hence no shear stresses occur. We will apply Newton's 2nd law to find the pressure gradients and distribution

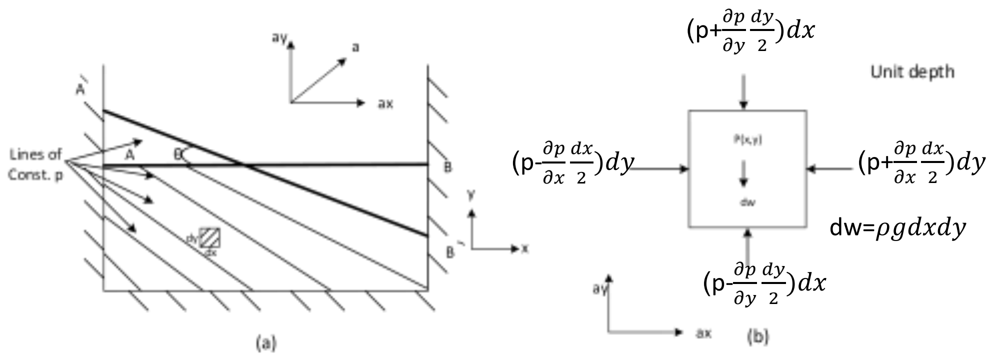


Fig. (29)

$\rightarrow \sum F_x = ma_x$

$(p-\frac{\partial p}{\partial x}\frac{dx}{2})dy-(p+\frac{\partial p}{\partial x}\frac{dx}{2})dy=\rho dxdya_x \rightarrow \frac{\partial p}{\partial x}=-\rho a_x$ …(2.54)

$\sum F_y = m\, a_y$

$(p-\frac{\partial p}{\partial y}\frac{dy}{2})dx-(p+\frac{\partial p}{\partial y}\frac{dy}{2})dx-\rho gdxdy=\rho gdxdya_y$

Thus;

$$\frac{\partial p}{\partial y}=-\rho(a_y+g)$$ …………………………. (2.55)

Equations (2.54) and (2.55) will reduce to equations (2.3) and (2.5) if (a_x=0 and a_y=0) respectively. Now, to find the change in pressure;

$dp = \frac{\partial p}{\partial x}dx+\frac{\partial p}{\partial y}dy$

$dp = -\rho a_x dx-\rho(a_y+g)dy$ …………………………. (2.56)

The angle of inclination (θ) for the lines of constant pressure can be found from equ. (2.56) by setting (dp=0); thus;

$0 = -\rho a_x dx-\rho(a_y+g)dy$

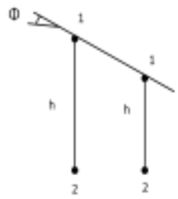
Hence;

$$\frac{dy}{dx}=tan\theta=-\frac{a_x}{a_y+g}$$ ………………….. (2.57)

Equation (2.56) can be integrated with B.C. (p=p_0 at x=0, y=0) to obtain;

$p = p_0-\rho a_x x-\rho(a_y+g)y$ ………………….. (2.58)

Equation (2.55) can be integrated in analogous way to equ. (2.7), to obtain the following formula;



$p_2 = p_1 + \rho(a_y + g)h$ ………………………… (2.59)

Again, for (a_y=0), equ. (2.59) reduces to equ. (2.7) For static fluids

Note; For open tanks ($p_0 = p_{atm}$) and point (o) is at the middle (center) of the width of the tank. For closed tanks ($p_0 \neq p_{atm}$) and point (o) is not at the middle (center) of the width.

1.7.2 Uniform Rotation

Rotation of a fluid, moving as a solid, about an axis is called" Forced Vortex Motion". Every particle of the fluid has the same angular velocity. This motion is to be distinguished from" Free-Vortex Motion", in which each particle moves in a circular path with speed varying inversely as the distance from the center. A liquid in a container, when rotated about a vertical axis at constant angular velocity (ω), see **Fig (30)**, moves like solid after some time interval. No shear stresses exist in the liquid, and the only acceleration that occurs is directed radially inward towards the axis of rotation ($a_n=\omega^2 r$ and $a_t = 0$).Thus, analogous to equations (2.54) and (2.55), we can wirte;

$$\frac{\partial p}{\partial r} = -\rho a_r = -\rho(-\omega^2 r) = \rho\omega^2 r \quad \text{………….. (2.60)}$$

$$\frac{\partial p}{\partial y} = -\rho(a_y + g) \quad \text{…………… (2.61)}$$

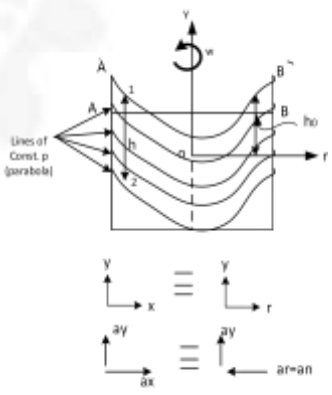


Fig. (30)

The pressure distribution analogous to equ. (2.58) will be;

$p = p_0 + \frac{1}{2}\rho\omega^2 r^2 - \rho(a_y + g)y$ ……………..(2.62)

For points along the horizontal plane (y=0), equ.(2.62) gives;

$$\frac{p - p_0}{\gamma} = h_0 = \frac{\omega^2 r^2}{2g} \text{……………..(2.63)}$$

Which gives the distance to the free surface.

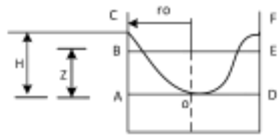
In a similar way, equ. (2.61) can be integrated to obtain;

$p_2 = p_1 + \rho(a_y + g)h$ ……………………… (2.64)

Where: h= Vertical downward distance from the free surface

Note:

1. Lines of constant pressure are of parabolic type.
2. Volume of the parabola = $\frac{1}{2}$ (Volume of cylinder with same base and height)
3. $\underset{ABEDO}{\forall_{cyl.}} = \underset{oABCFED}{\forall_{parabola}}$



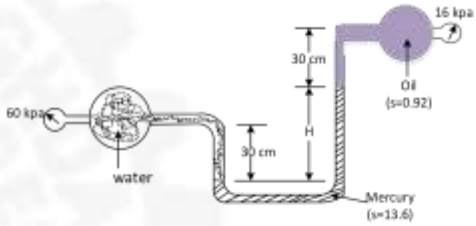
$$\pi r_0^2 * z = \frac{1}{2}\pi r_0^2 * H$$

Thus; $z = \frac{H}{2}$

Examples

Example (2.1): For the system shown in the figure, calculate the manometer reading (H).

Sol.:



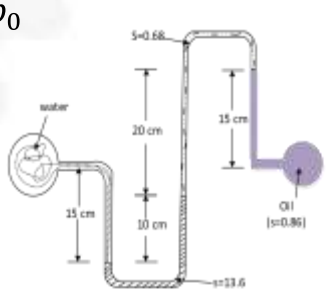
$60000+0.2\gamma_w - 13.6\ \gamma_w * \text{H} - 0.92\gamma_w * 0.3 = 16000$

$\text{H} = \frac{60000+0.2\gamma_w - 0.0276\ \gamma_w - 16000}{13.6\ \gamma_w}$

$\therefore H = 0.3242$ m $= 32.42$ cm

Example (2.2): Determine the pressure difference between the water pipe and the oil pipe shown in the figure, in Pascals and in meters of water.

Sol.:



$$p_w + 0.15\gamma_w - 13.6\gamma_w * 0.1 - 0.68\gamma_w * 0.2 + 0.86\ \gamma_w * 0.15 = p_0$$

$$p_w - p_0 = \gamma_w(-0.15 + 13.6 + 0.136 - 0.129)$$

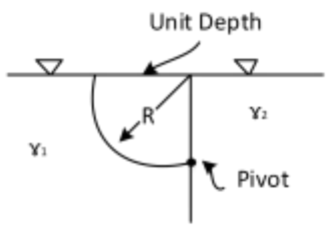
$$\therefore p_w - p_0 = 11938.77\text{Pa}$$

$$h_w - h_0 = \frac{p_w - p_0}{\gamma_w}$$

$$h_w - h_0 = 1.217 \text{ m } H_2O$$

Example (2.3): If the weightless quarter cylindrical gate shown in the figure is in equilibrium, what is the ratio between (γ_1) and (γ_2)?

Sol.:



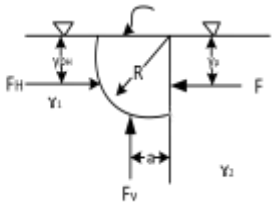
$F = \gamma\bar{h}A = \gamma_2 \frac{R}{2} R = \gamma_2 \frac{R^2}{2}$

$$y_p = \bar{y} + \frac{I}{A\bar{y}} = \frac{2}{3}R$$

$F_H = \gamma\, \bar{y}_v A_v = \gamma_1 \frac{R^2}{2}$

$y_{pH} = \bar{y}_v + \frac{I}{A_v \bar{y}_v} = \frac{2}{3} R$

$F_v = \gamma_1 \forall = \gamma_1 \frac{\pi}{4} R^2$



$a = \frac{4R}{3\pi}$

$\sum M_{pivot} = 0$

$F*(R - y_p) = F_H * (R - y_{p_H}) + F_v * a$

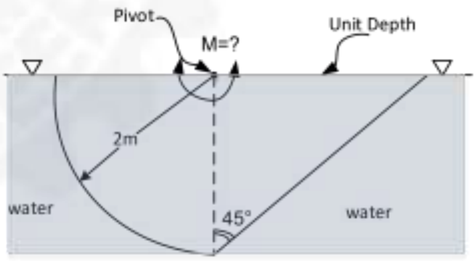
$\gamma_2 \frac{R^3}{6} = \gamma_1 \frac{R^3}{6} + \gamma_1 \frac{R^3}{3}$

$\gamma_2 \frac{R^3}{6} = \gamma_1 \frac{R^3}{2}$

$\therefore \frac{\gamma_1}{\gamma_2} = \frac{1}{3}$

Example (2.4): For the two-dimensional weightless solid body shown in the figure, determine the magnitude and direction of the moment (M) applied at the pivot required to hold the body in the position shown in the figure.

<ins>**Sol.:**</ins>



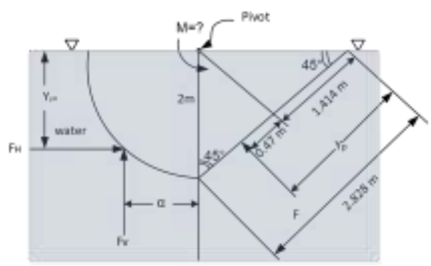
$F_H = \gamma y_v A_v = 9810*1*2 = 19620$ N

$y_{p_H} = \bar{y}_v + \frac{I}{A_v y_v} = 1.332$ m

$F_v = \gamma \forall = \gamma \frac{\pi}{4} r^2 = 30819$ N

$a = \frac{4r}{3\pi} = 0.849$ m

$F = \gamma \bar{h} A = 9810*1*2.828 = 27746.9$ N



$y_p = 1.414 + \frac{1*(2.828)^3/12}{2.828*1.414} = 1.885$ m

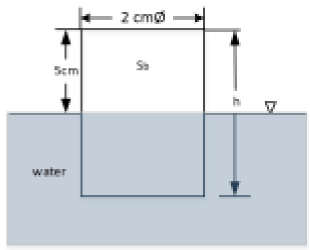
$+\sum M_{pivot} = 0$

$F*0.47 + F_v * a - F_H * y_p - M = 0$

$\therefore M = 13078$ N.m

Example (2.5): A (2cm) diameter cylinder of wood (s=0.5) floats in water with (5 cm) above the water surface. Determine the depth of submergence of this cylinder when placed in glycerin (s=1.25). Will it float in stable, unstable or neutral equilibrium in this case?

<u>**Sol.:**</u>

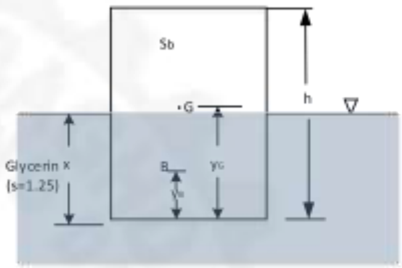


<u>In water:</u> $W=F_B$

$s_b \gamma_w A h = \gamma_w A(h-0.5)$

Thus ; h=10 cm

<u>In glycerin:</u>



$s_b \gamma_w A h = s_g \gamma_w A x$

$\therefore$ x=4 cm

y_G =5cm , y_B=2cm

$BG = 5 - 2 = 3\ cm$

$I_{00}=\frac{\pi}{64}D^4 = \frac{\pi}{4}cm^4$

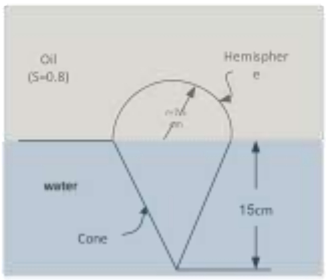
$\forall=\frac{\pi}{4}D^2 * x = 4\pi cm^3$

$MG=\frac{I_{00}}{\forall} - BG = \frac{\frac{\pi}{4}}{4\pi} - 3 = -2.937$cm

$\because MG < 0 \rightarrow \therefore$Unstable Equilibrium

Example (2. 6): Calculate the weight and specific gravity of the object shown in the figure to float at the water-oil interface as shown.

<u>**Sol.:**</u>



$W=F_{B_{water}} + F_{B_{oil}}$

$=\gamma_w * \frac{1}{3}\pi(0.075)^2 * 0.15 + 0.8\gamma_w * \frac{2}{3}\pi(0.075)^3$

$=\gamma_w[8.836 * 10^{-4} + 0.8 * 8.836 * 10^{-4}]$

=9810*0.00159

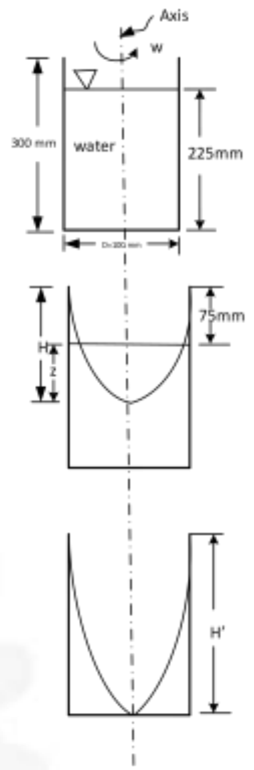
$\therefore$W=15.6 N

$s=\frac{W/\forall}{\gamma_w} = \frac{15.6/(2*8.836*10^{-4})}{9810} = \frac{156000}{173362}$

$\therefore$s=0.899

Example (2.7): The cylindrical vessel shown in the figure is rotated about its vertical longitudinal axis. Calculate:

1. The angular velocity at which water will start to spill over the sides.
2. The angular velocity at which the water depth at the center is zero, and the volume of water lost for this case.



Sol.:

(a) $\frac{\pi}{4}D^2z = \frac{1}{2}\frac{\pi}{4}D^2H$

$\therefore$ H = 2z = 150mm

H = $\frac{\omega^2 r^2}{2g}$ → 0.15= $\frac{\omega^2 * (\frac{D}{2})^2}{2g}$

$\therefore \omega$ = 34.4 rad/sec

(b) H'=$\frac{\omega^2 * (\frac{D}{2})^2}{2g}$ → 0.3=$\frac{\omega^2 * (0.1)^2}{2*9.81}$ → ω = 48.5 rad/sec

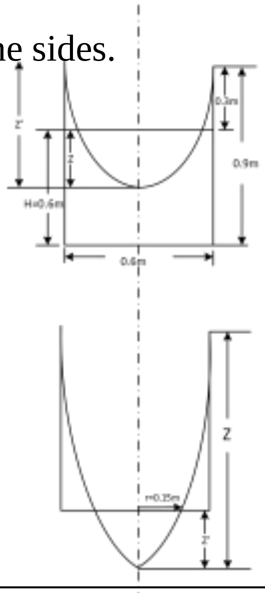
$\forall' = \frac{1}{2}\frac{\pi}{4}D^2H' = 1.178*10^{-3}\ m^3$

$\forall_{lost} = \frac{\pi}{4}D^2 * 0.225 - \forall'$

$\forall_{lost} = 0.59 * 10^{-3} m^3 \therefore$

Example (2.8): An open cylindrical tank (0.9m) high and (0.6m) in diameter is two – thirds filled with water when it is stationary. The tank is rotated about its vertical axis, calculate:

1. The maximum angular velocity at which no water is to spill over the sides.
2. The angular velocity at which the bottom of the tank is free of water for a radius of (150mm).



Sol.:

a- H = $\frac{2}{3}$*0.9 = 0.6m

$\frac{\pi}{4}D^2z = \frac{1}{2}\frac{\pi}{4}D^2z'$

$\therefore z' = 0.6$m

$$z' = \frac{\omega^2 r^2}{2g} \rightarrow \quad 0.6 = \frac{\omega^2 (0.3)^2}{2 * 9.81}$$

$\therefore \omega$ =11.43 rad/sec

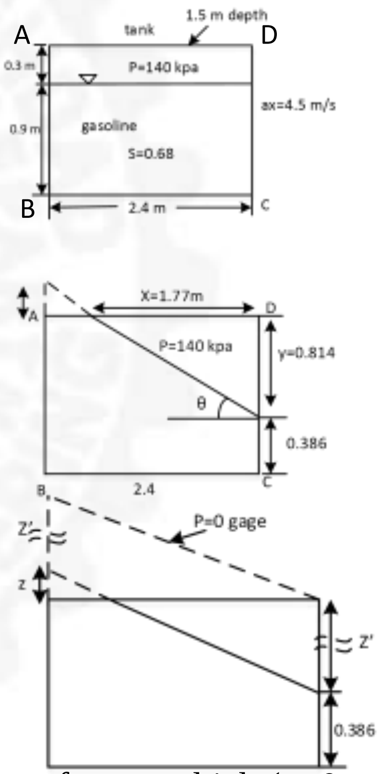
b- $\quad$ z-z' = 0.9

$$\frac{\omega^2 (0.3)^2}{2g} - \frac{\omega^2 (0.15)^2}{2g} = 0.9$$

W = 16.16 rad/sec

Example (2.9): For the closed rectangular tank shown in the figure, calculate the pressure at points (A) and (B), and the forces on sides (AB) and (CD).

Sol.:



$\tan\theta = -\frac{ax}{ay+g} = -\frac{4.5}{0+9.8} \rightarrow \theta = -24.66^\circ$

$\tan\theta = \frac{y}{x} \rightarrow y = 0.46x$ ……………….(1)

$\frac{yx}{2} = 2.4 * 0.3 = 0.72$ …………………….. (2)

(1)& (2) gives:

y = 0.814 m and x =1.77 m

$p_2 = p_1 + (a_y + g)$ h

=140000 + 0.68*1000(0 + 9.8) h

Thus; p_2 =140000 + 6670.8h

z = 2.4 tanθ ⟶ z = 0.288 m

p_A = 140000 + 6670.8*z → p_A = 141.92 kPa

p_B = 140000 + 6670.8*(1.2 + z) → p_A = 149.93 kPa

To calculate the hydrostatic forces, we have to imagine a free surface at which (p=0 gage) above the existing surface at which (p=140 kPa):

Z' = $\frac{140000}{0.68*9810}$ → z' = 21m

$F_{AB}=\gamma \bar{h} A$=0.68*9810*(0.6+z+z')*1.2*1.5→F_{AB}=262.6 kN

$F_{CD}=\gamma \bar{h} A$=0.68*9810*(0.386+z'-0.6)*1.2*1.5→F_{CD}=249.6 kN

Problems

The problems number listed in the table below refers to the problems in the "textbook", chapter"2".

Article No.	Related Problems
2.2+2.3	4,5,7,10,13,14,16,20,21,22,24,26,29,30,31,32,33,34
2.4	36,37,39,40,44,45,46,53,59,60,61,62,63,65,68,69,71,78,79,82,86,89,91,101,106
2.5+2.6	92,93,96,97,98,100,102,103,105
2.7	110,112,114,115,116,117,118,122,123,124,126

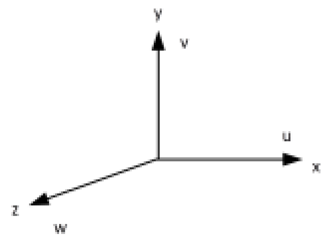
Chapter -3-

Fluid Flow Concepts

3.1 Definitions and Concepts

Velocity ($\vec{V}$): It is the time rate of change of displacement of fluid particles. It is a vector quantity:

$$\left.\begin{aligned} \vec{V} &= \vec{V}(x,y,z,t) \\ \vec{V} &= ui + vj + wk \\ u &= dx/dt \\ v &= dy/dt \\ w &= dz/dt \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots \ (3.1)$$



Acceleration ($\vec{a}$): It is the time rate of change of velocity vector.

$$\vec{a} = \vec{a}(x,y,z,t)$$

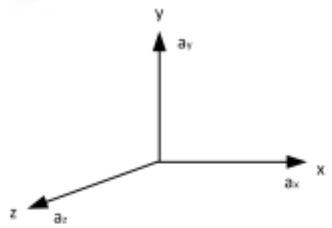
$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{d}{dt}\vec{V}(x,y,z,t) = \frac{\partial \vec{V}}{\partial t} + \frac{\partial \vec{V}}{\partial x}\frac{dx}{dt} + \frac{\partial \vec{V}}{\partial y}\frac{dy}{dt} + \frac{\partial \vec{V}}{\partial z}\frac{dz}{dt}$$

Thus;

$$a = \frac{\partial \vec{V}}{\partial t} + u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}$$

And;

$$\left.\begin{aligned} a_x &= \frac{du}{dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} \\ a_y &= \frac{dv}{dt} = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} \\ a_z &= \underbrace{\frac{dw}{dt} = \frac{\partial w}{\partial t}}_{\text{Total Acceleration}} + \underbrace{u\frac{\partial w}{\partial x}}_{\text{Local Acceleration}} + \underbrace{v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}}_{\text{Convective Acceleration}} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots(3.2)$$



Where; $\vec{a} = a_x i + a_y j + a_z k$

Streamline (S.L.): It is an imaginary line or curve drawn in the fluid flow such that the tangent drawn at any point of it indicates the direction of velocity ($\vec{V}$) at that point. Since the

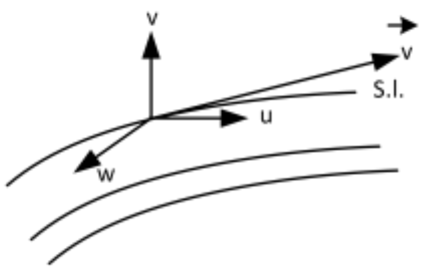


Fig. (1)

velocity vector has a zero component normal to streamline, there can be no flow across a streamline at any point, see **Fig.(1).** Streamlines indicate the direction of motion in various sections of fluid flow.

Types and Classification of Flow

1- Internal and External Flow

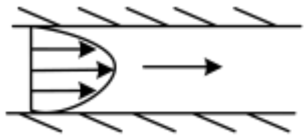


Fig. (2)

Internal Flow; is bounded by a wall (surface) around all the circumference of flow. Examples are pipe or duct flows, flows between turbine or compressor or pump blades see **Fig (2)**.

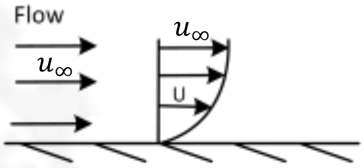


Fig. (3)

External Flow; is bounded by a wall (surface) from one side and free at other sides. Examples are flow over a flat plate, over airfoil, over a car, over airplane fuselage, see **Fig. (3).**

2- Steady and Unsteady Flow

Steady Flow; none of the flow and fluid variables (velocity, acceleration, density.....) vary with time.

Unsteady Flow; any one of the variables change with time

$$\left.\begin{aligned} &\text{Steady Flow:} \quad \frac{\partial(\ )}{\partial t} = 0 \\ &\text{Unsteady Flow:} \quad \frac{\partial(\ )}{\partial t} \neq 0 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.3)$$

3- Uniform and Non – Uniform Flow

Uniform Flow; velocity vector ($\vec{V}$)remains the same at all sections of the flow.

Non - Uniform Flow; velocity vector ($\vec{V}$) changes from section to sections of the flow.

$$\left.\begin{aligned} &\text{Uniform flow;} \quad \frac{\partial \vec{V}}{\partial s} = 0 \\ &\text{Non - Uniform flow;} \quad \frac{\partial \vec{V}}{\partial s} \neq 0 \end{aligned}\right\} \quad \text{s=space} \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.4)$$

4- Laminar and Turbulent Flow;

Laminar Flow; fluid particles move in smooth paths in layers or laminas with one layer sliding over an adjacent layer. The paths of individual particles do not cross or interact see **Fig.(4)**. Laminar flows occur at low velocities (low Reynolds number Re) , where (Re $= \frac{\rho v \ l}{\mu}$) ; l = characteristic length.

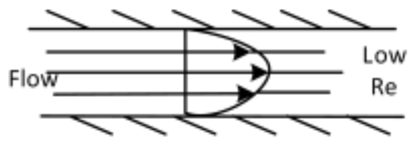


Fig. (4)

Turbulent Flow; paths of various particles are irregular and random with no systematic pattern of flow see Fig.(5). Turbulent flow occurs at high velocities (high Re). In turbulent flow, the velocity (u) fluctuates with time. The time average velocity ($\bar{u}$) can be found as, see **Fig. (6)**;

$$\left.\begin{aligned} \bar{u} &= \frac{1}{T}\int_0^T u dt \\ u &= \bar{u} + u' \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots(3.5)$$

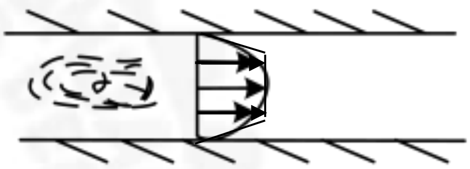


Fig. (5)

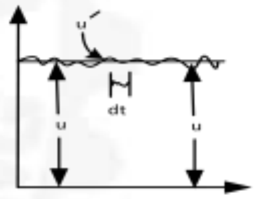


Fig. (6)

where;

u = Instantaneous velocity

u' = Fluctuation from the mean value

T= Time period

Note: For pipe flow; Re < 2000 Laminar Flow

Re > 4000 Turbulent Flow

5- One, Two - and Three - Dimensional Flow

One - Dimensional (1-D) Flow; the variation of fluid and flow parameters transverse to the main flow are absent. These parameters remain constant at any cross - section normal to the main flow see **Fig.(7)**. Mathematically;

$$\vec{V} = \vec{V}(x,t) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (3.6)$$

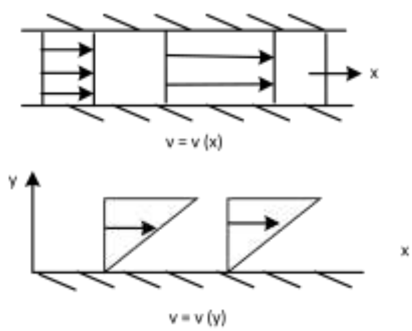


Fig. (7)

Two Dimensional (2- D) Flow; the variation in the flow and fluid parameters takes place in x- and y- directions only see Fig.(8). This means that all the particles move in parallel planes (x-y planes) along identical paths in each plane. there is no variation normal to this plan. Mathematically;

$$\vec{V} = \vec{V}(x, y, t) \qquad ....(3.7)$$

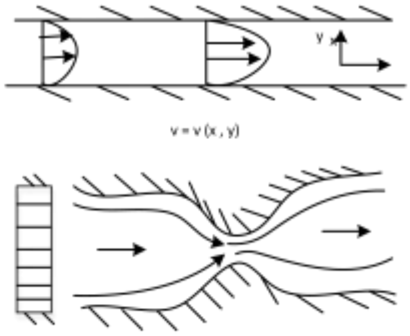


Fig. (8)

Three - Dimensional (3-D) Flow; the flow and fluid parameters have variation in all the three directions, See Fig.(9).Mathematically;

$$\vec{V} = \vec{V}(x, y, z, t) \qquad ....(3.8)$$

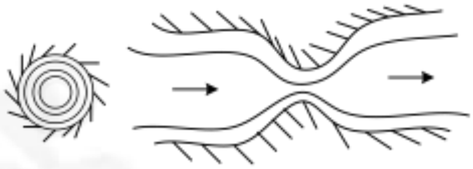


Fig. (9)

6- Viscous (Real) and Non- Viscous (Ideal) Flow

Viscous (Real) Flow; effects of viscosity exist and cause reduction of velocity inside the boundary-layer (b.l.)

Non - Viscous (Inviscid, Ideal); effects of viscosity are absent outside the (b.l.), see **Fig.(10).**

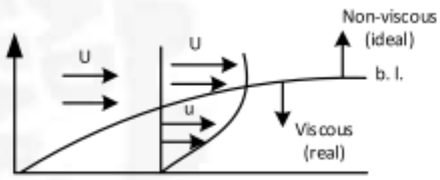


Fig. (10)

7- Incompressible and Compressible Flow

Incompressible Flow; the flow in which the density (ρ) is assumed constant (= constant). Examples are flow of liquids and gases with low velocities (M ≤ 0.3).

Compressible Flow; the flow in which the density (ρ) is not constant, but varies with pressure and temperature. Examples are gas flow and special types of liquid flow (such as water hammer phenomena).

Volume Flow Rate (Q) and Mass Flow Rate ($\dot{m}$)

Volume Flow Rate(Q); is the volume rate of fluid passing a section in a certain fluid flow. Thus (see **Fig. (11)**);

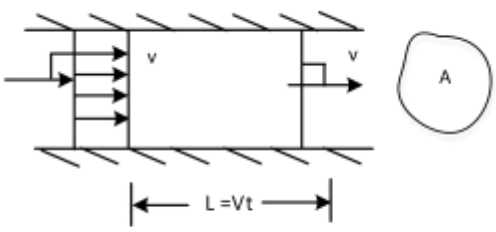


Fig. (11)

$Q = \frac{\forall}{t} = \frac{L.A}{t} = \frac{Vt.A}{t} = V.A$

Or

Q= A.V = AV A⊥V $\frac{m^3}{s}$ ....(3.9)

Where;

A=Cross - sectional area of the flow.

V= Average velocity of the flow.

A.V =AV cos α = AV cos(0) =AV

Mass Flow Rate ($\dot{m}$) ; is the mass rate of fluid passing a section in a certain fluid flow;

$\dot{m} = \frac{m}{t} = \frac{\rho\forall}{t} = \rho Q$

Or;

$\dot{m} = \rho AV = \rho Q$ kg/s …………....(3.10)

If the velocity (V) is variable across the section, then;

Q= $\int_A V.dA$ …………..... (3.11)

$\dot{m}$= $\int_A \rho V.dA$ …………..... (3.12)

EX: For laminar flow through pipes, the velocity distribution across the section is given by;

$\frac{V}{V_{max}} = 1 - \left(\frac{r}{R}\right)^2$, see **Fig. (12)**

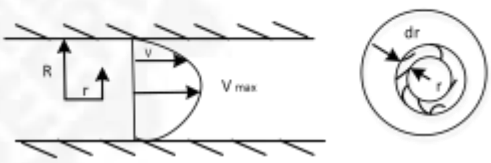


Fig. (12)

Thus;

Q = $\int_A V.dA = \int_0^R V_{max}\left(1 - \left(\frac{r}{R}\right)^2\right) \times 2\pi r dr$

Hence; Q = $\pi R^2 \frac{V_{max}}{2}$ = AV, where V= average velocity

The average velocity (V) can be calculated as;

V = $\frac{1}{A}\int V.dA = \frac{1}{\pi R^2}\int V_{max}\left(1 - \left(\frac{r}{R}\right)^2\right) \times 2\pi r dr \rightarrow V = \frac{V_{max}}{2}$

$\dot{m} = \int_A \rho V.dA \rightarrow \dot{m} = \rho\pi R^2 \frac{V_{max}}{2} = \rho AV$

System (sys)

It is a quantity of matter of fixed mass and identity. the system boundaries may be fixed or movable. System exchanges energy only with the surrounding

Control Volume (C.V.)

It is a fixed region in the space bounded by the control surface (C.S.). The control volume (C.V.) can exchange both mass and energy with the surrounding.

Note; System approach is usually used in solid mechanics, where the body is clearly identified and can be followed during its motion. In fluid mechanics, a "system" of fluid cannot be easily followed during its motion, since its boundaries are not clear. Instead, a “control volume" approach is used, where a fixed volume specified in the fluid is considered and the changes in this c.v. due to flow of fluid system through it is studied.

3.2 General Control Volume Equation

The general control volume equation, which will be derived in this article, relates the changes in a general property of the "**System**" **(Mass, Energy, Momentum)** to that occurred in a “Control Volume" through which the system flows, see **Fig. (13).**

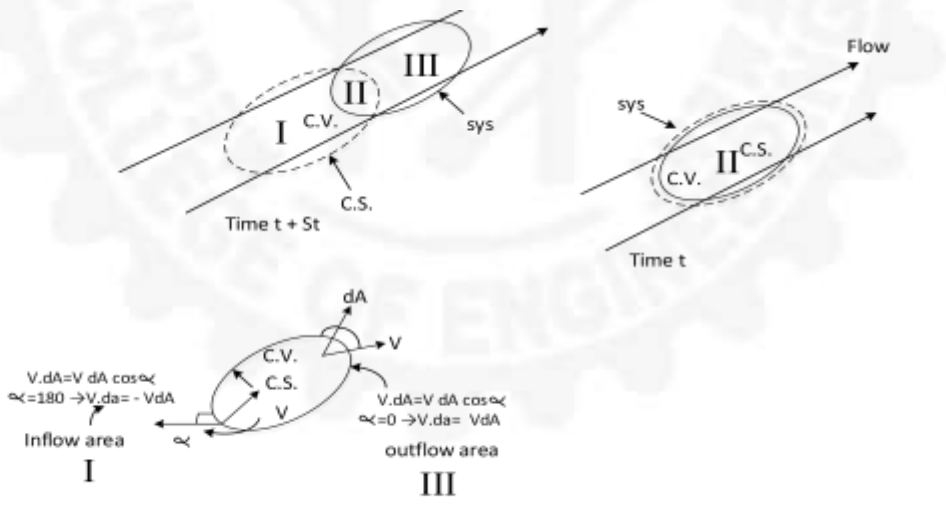


Fig. (13)

Let;

N = Total amount of a general property within the system (mass, momentum, energy)

And;

η = The amount of (N) per unit mass (N/m)

Thus;

$$N = \eta\ m = \eta\ \rho\forall = \int_{\forall} \eta\rho d\forall = \int_{\forall} \eta\rho L.dA$$

And;

$$\dot{N} = \int_{A} \eta\rho \dot{L}.dA = \int_{A} \eta\rho V.dA$$

Now, the rate of change of (N) within the system is;

$$\frac{dN_{sys}}{dt} = \lim_{\delta t\to 0} \frac{N_{sys)t+\delta t} - N_{sys)t}}{\delta t} = \lim_{\delta t\to 0} \frac{\left(\int_{II}\eta\rho d\forall + \int_{III}\eta\rho d\forall\right)_{t+\delta t} - \left(\int_{II}\text{Ʒ}\eta\rho d\forall\right)_{t}}{\delta t}$$

By adding and subtracting the term $\left(\int_{I}\eta\rho d\forall\right)_{t+\delta t}$;

$$\frac{dN_{sys}}{dt} = \lim_{\delta t\to 0} \frac{\left(\int_{II}\eta\rho d\forall + \int_{I}\eta\rho d\forall\right)_{t+\delta t} - \left(\int_{II}\eta\rho d\forall\right)_{t}}{\delta t}$$

$$+ \lim_{\delta t\to 0} \frac{\left(\int_{III}\eta\rho d\forall - \int_{I}\eta\rho d\forall\right)_{t+\delta t}}{\delta t}$$

$$= \lim_{\delta t\to 0} \frac{N_{C.V.)t+\delta t} - N_{C.V.)t}}{\delta t} + \lim_{\delta t\to 0} \frac{(N_{III} - N_{I})_{t+\delta t}}{\delta t}$$

$$= \frac{\partial N_{C.V.}}{\partial t} + \dot{N}_{III} - \dot{N}_{I}$$

$$= \frac{\partial}{\partial t}\int_{C.V.}\eta\rho d\forall + \int_{outflow\ area}\eta\rho\, V.dA - \int_{inflow\ area}\eta\rho\, V.dA$$

$$= \frac{\partial}{\partial t}\int_{C.V.}\eta\rho d\forall + \int_{C.S.}\eta\rho\, V.dA$$

Hence, the final form of the "General Control Volume Equation" is;

$$\frac{dN_{sys}}{dt} = \frac{\partial}{\partial t}\int_{C.V.}\eta\rho d\forall + \int_{C.S.}\eta\rho\, V.dA \qquad \text{......................} \ (3.13)$$

Equation (3.13) will be used to derive *FOU*R laws of conservation that are used to analyze any fluid flow problem. These are:

1- Law of Conservation of Mass: Continuity Equation (C.E.)

2- Law of Conservation of Energy: Energy Equation (E.E.)

3- Law of Conservation of Liner Momentum: Momentum Equation (M.E.)

4- Law of Conservation of Angular Momentum: Angular Momentum Equation.

3.3 Law of Conservation of Mass: Continuity Equation (C.E.)

The property (N) is the mass (m) of the system. Thus;

N= m → $\eta = \frac{N}{m} = 1$

$\frac{dN_{sys}}{dt} = \frac{dm}{d} = 0$ [mass cannot be created nor can be destroyed]

Thus; from equ.(3.13);

$0 = \frac{\partial}{\partial t}\int_{C.V.} \rho d\forall + \int_{C.S.} \rho\, V.dA$ General C.E. ....……….... (3.14)

For the flow control volume shown in **Fig.(14);**

$$\int_{C.S.} \rho\, V.dA = (\rho AV\ \cos\alpha)_{out} + (\rho AV\ \cos\alpha)_{in}$$

$$= (\rho AV\,)_{out} - (\rho AV\,)_{in}$$

$$= \dot{m}_{out} - \dot{m}_{in}$$

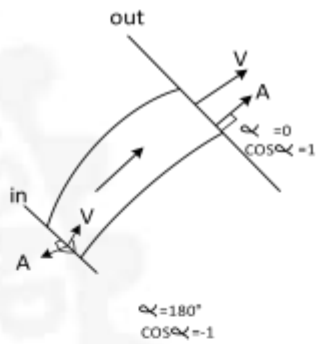


Fig. (14)

Hence, equ.(3.14) can be written as;

$\frac{\partial m_{C.V.}}{\partial t} + \dot{m}_{out} - \dot{m}_{in} = 0$ ....……….. (3.15)

For steady flow, ($\frac{\partial m_{C.V.}}{\partial t} = 0$), thus;

$$\dot{m}_{in} = \dot{m}_{out}$$

$\sum_{\substack{all \\ inlets}} \rho A\,V = \sum_{\substack{all \\ outels}} \rho AV$ C.E. for steady flow …………………..(3.16)

For incompressible flow ($\rho_{in} = \rho_{out}$), hence;

$$Q_{in} = Q_{out}$$

$\sum_{\substack{all \\ inlets}} AV = \sum_{\substack{all \\ outels}} AV$ C.E. for steady incompressible flow ………..(3.17)

3.3.1 Continuity Equation at a Point

To derive the continuity equation at a point, not for a C.V. (equ.3.15), we will apply this equation for an infinitesimal cubic element of fluid, see **Fig.(15).**

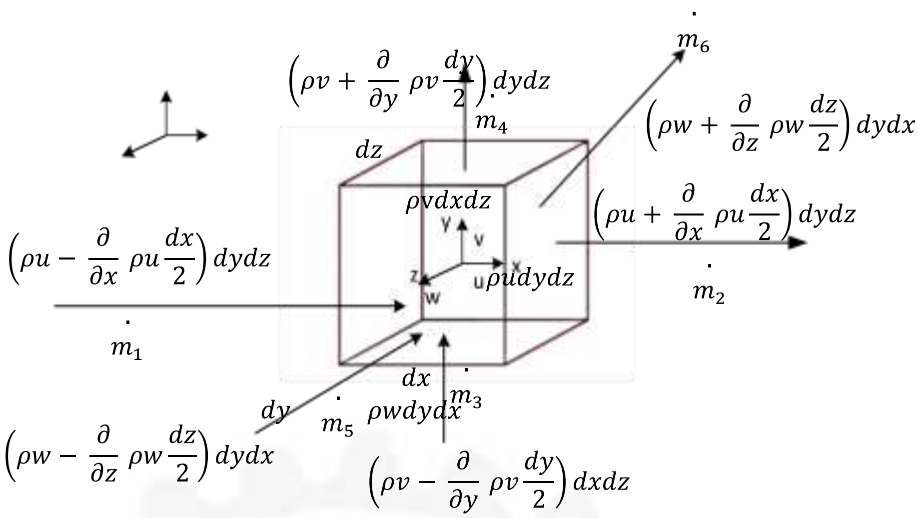


Fig. (15)

Using equ.(3.15) ;

$$\frac{\partial \rho dxdydz}{\partial t} + (\dot{m}_2 + \dot{m}_4 + \dot{m}_6)_{out} - (\dot{m}_1 + \dot{m}_3 + \dot{m}_5)_{in}$$

Substituting for each term and simplify, it is obtained;

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0 \qquad \text{General C.E. at a point} \ldots\ldots\ldots(3.18)$$

For incompressible flow ($\rho = constant$), i.e;

$\frac{\partial \rho}{\partial t} = 0; \frac{\partial \rho}{\partial x} = 0, \frac{\partial \rho}{\partial y} = 0$ and $\frac{\partial \rho}{\partial z} = 0$. Thus, equ.(3.18) becomes;

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad \text{C.E. at a point for incompressible flow} \ldots\ldots\ldots(3.19)$$

3.4 Law of Conservation of Energy: Energy Equation (E.E.)

The property (N) is the energy (E) of the system, which has three forms; internal energy (Eu), potential energy ($E_{P.E}$) and the kinetic energy ($E_{K.E}$). Thus;

$$\left.\begin{aligned} N = E_{sys} &= E_u + E_{P.E.} + E_{k.E.} \\ &= mu + mgz + \frac{1}{2}mv^2 \end{aligned}\right\} \qquad \ldots\ldots\ldots(3.20)$$

And;

$\eta = \frac{N}{m} = e = u + gz + \frac{V^2}{2}$.......................................(3.21)

Thus; equ.(3.13) becomes;

$$\frac{dE_{sys}}{dt} = \frac{\partial}{\partial t}\int_{c.v.} e\rho d\forall + \int_{c.s.} e\rho V.dA$$

1st law of thermodynamics:

Q - W = dE_{sys} ………………………………………. (3.22)

Where: (Q=Heat) and (W=Work). Hence;

$\frac{dQ}{dt} - \frac{dw}{dt} = \frac{dE_{sys}}{dt} = \frac{\partial}{\partial t}\int_{c.v.} e\rho d\forall + \int_{c.s.} e\rho V.dA$ ……………(3.23)

Now, the work (W) has two components; shaft work (Ws) and flow work ($W_f = \int_{c.s.} pd\forall$); i.e.;

W = Ws + W_F

= Ws+$\int_{c.s.} pd\forall$

= Ws+$\int_{c.s.} pL.dA$

$\frac{dw}{dt} = \frac{dW_s}{dt}$+$\int_{c.s.} pV.dA$

Hence, equ.(3.23) becomes;

$\frac{dQ}{dt} - \frac{dW_s}{dt} = \frac{\partial}{\partial t}\int e\rho d\forall + \int_{c.s.}(\frac{p}{\rho}$+e) $\rho V.dA$ General E.E. …………(3.24)

For steady flow,($\frac{\partial}{\partial t}\int_{c.v.} e\rho d\forall = 0$); thus;

$\frac{dQ}{dt} - \frac{dW_s}{dt} = \int_{c.s.}(\frac{p}{\rho} + e)\rho V.dA$ ……………………………..(3.25)

Using (3.21); and for one inlet an one outlet ($\dot{m}_{in} = \dot{m}_{out} = \dot{m} = \rho V.dA$)

$\dot{Q} - \dot{W_s} = \dot{m}\left[(\frac{p}{\rho} + u + gz + \frac{V^2}{2})_{out} - (\frac{p}{\rho} + u + gz + \frac{V^2}{2})_{in}\right]$ ……….(3.26)

Now;

$$\left.\begin{aligned}\frac{Q^{\cdot}}{m^{\cdot}} &= q \quad \text{Heat per unit mass} \\ \frac{W_s^{\cdot}}{m^{\cdot}} &= w_s = g(h_T - h_p) \text{ work per unit mass}\end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.27)$$

Where h_T = Turbine head (m)

hp = Pump head (m)

Substitute (3.27) in (3.26) gives;

$$q - w_s = \left(\frac{p}{\rho} + u + gz + \frac{V^2}{2}\right)_{out} - \left(\frac{p}{\rho} + u + gz + \frac{V^2}{2}\right)_{in} \ldots\ldots\ldots\ldots\ldots\ldots(3.28)$$

Equ.(3.28) is the ***Steady State Steady Flow Energy Equation (S.S.S.F.E.E)***. In differential from, it can be written as;

$$dq - dw_s = d\frac{p}{\rho} + du + gdz + d\frac{V^2}{2}$$

$$= \frac{1}{\rho}dp + pd\frac{1}{\rho} + du + gdz + vdv$$

$$dq - dw_s = du + pd\forall + \frac{dp}{\rho} + vdv + gdz \ldots\ldots\ldots\ldots\ldots\ldots(3.29)$$

From thermodynamics;

$$du + pd\forall = Tds \ldots\ldots\ldots\ldots\ldots\ldots(3.30)$$

and;

$$Tds - dq = d(gh_L) \ldots\ldots\ldots\ldots\ldots(3.31)$$

Where;

s=Entropy (J/kgK)

h_L=Losses head (m)

Thus; equ.(3.29) becomes;

$$\frac{dp}{\rho} + vdv + gdz + dw_s + d(gh_L) = 0 \ldots\ldots\ldots\ldots(3.32)$$

Integrate equ.(3.32) between sections (1) and (2) for incompressible fluids ($\rho = const.$), using equ.(3.27); it is obtained (after dividing by g);

$$\underbrace{\left(\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1\right)}_{\text{Available Energy at section (1)}} + \underbrace{hp}_{\text{Energy Added}} = \underbrace{\left(\frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2\right)}_{\text{Available Energy at section (2)}} + \underbrace{h_T}_{\text{Energy Subtracted}} + \underbrace{h_{L_{1-2}}}_{\text{Energy Lost}} \qquad \ldots\ldots\ldots.. (3.33)$$

Equation (3.33) in the ***S.S.S.F.E.E.*** for incompressible fluids. It is applied between two sections (1) and (2) along, the fluid flow, see **Fig.(16).**

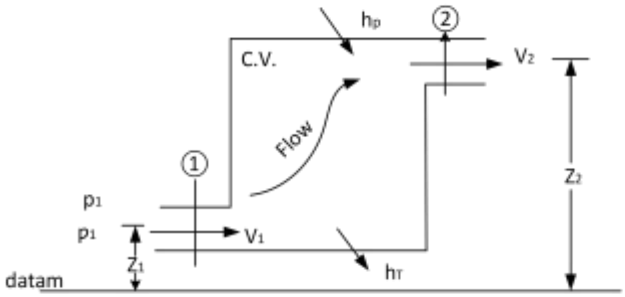


Fig. (16)

3.4.1 Bernoulli's Equation (B.E.)

It is a special from of the E.E (3.33) along a streamline without losses (h_L=0) and with no shaft work (h_p=0,h_T=0). Thus, B.E. will be;

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2 \qquad \text{B.E.} \quad \ldots\ldots\ldots\ldots\ldots\ldots(3.34)$$

B.E. is applied for steady frictionless flow of incompressible fluid along a stream line

<u>Notes for Application of equations (3.33) and (3.34) (E.E &B.E.)</u>

1- Available energy of the fluid has three forms;

$\frac{p}{\gamma}$ = Pressure (flow) energy per unit weight (m)

$\frac{V^2}{2g}$ = Kinetic energy per unit weight (m)

z = Potential energy per unit weight (m)

2- Each term of the E.E. and B.E. represents energy per unit weight (head h (m)).

3- The energy of the fluid can be represented by four forms;

a- Total energy E = $p\forall+\frac{1}{2}mv^2+\rho\forall gz$ (J)

b- Energy per unit weight h = $\frac{p}{\gamma}+\frac{V^2}{2g}+z$ (m) (N. m/N)

c- Energy per unit volume $\frac{E}{\forall}=p+\frac{1}{2}\rho v^2+\rho gz$ (N/m^2)

d- Energy per unit mass $\frac{E}{m}=\frac{p}{\rho}+\frac{V^2}{g}+gz$ (J/kg)

4- In equs. (3.33) and (3.34), section (1) is upstream and section (2) is downstream.

5- (hp) in the E.E. represents the energy per unit weight added to the fluid by a pump, compressor, fan, blower. (h_T) represents the energy per unit weight subtracted from the fluid by a turbine or windmill.

6- To calculate the power (P);

$$P = \frac{Energy}{time} = \frac{Energy}{weight}\cdot\frac{weight}{time} = h * \frac{mg}{t} = \dot{m}gh = \rho Q g h = \gamma Q h$$

Thus; P = $\dot{m}$gh = $\gamma Q h$

7- (V) in the E.E. and B.E. is the average velocity at the section.

8- (hp) in the E.E represents the output power (OP) of the pump. To calculate the input (shaft) power (IP);

$$\eta_p = \frac{OP}{IP} = \frac{\gamma Q hp}{IP} \quad \text{……………………(3.36)}$$

η_p = Pump efficieny

9- (h_T) in the E.E. represents the Input Power (IP) to turbine. To calculate the output power (OP);

$$\eta_T = \frac{OP}{IP} = \frac{OP}{Q h_T} \quad \text{……………………..(3.37)}$$

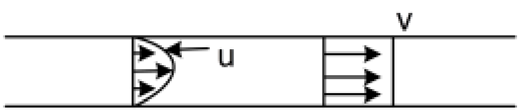
η_T = Turbine efficiency

10- <u>Kinetic Energy Correction Factor (α):</u>

The K.E. term ($\frac{V^2}{2g}$) in the E.E. does not represent the average of the K.E. across the section. Instead, we should calculate $(\frac{u^2}{2g})_{avg.}$ across the section, where (u) is the variable velocity across the section. To account for that, we should use ($\alpha\frac{V^2}{2g}$) in the E.E, where α is the K.E correction factor, calculated from;

$$\alpha \dot{m} g \frac{V^2}{2g} = \int g \frac{u^2}{2g} d\dot{m}$$

$$\alpha \frac{V^2}{2g}\rho A V = \int \frac{u^2}{V} g \rho u dA$$



Hence;

$$\alpha = \frac{1}{A}\int ((\frac{u}{V})^3\, dA \quad \text{…….………….(3.38)}$$

For laminar flow in pipe ($\frac{u}{U_{max}} = 1 - (\frac{r}{R})^2$), $\alpha = 2$

For turbulent flow in pipe $\alpha = 1.01 - 1.1 \approx 1$

3.5 Law of Conservation of Linear Momentum: Momentum Equation (M.E)

This law represents the application of the Newton's 2nd law for a system. The property (N) is the linear momentum of the System (mv); that is;

$$\sum F = ma = m\frac{dv}{dt} = \frac{d}{dt}(mv)_{sys} \qquad \text{..................................} (3.39)$$

The R.H.S. of this equation is obtained from the general control volume equation (3.13). Thus, with;

N = (mv) linear momentum

$$\eta = \frac{N}{m} = \frac{mv}{m} = \text{v}$$

Equ.(3.13) becomes;

$$\frac{d(mv)_{sys}}{dt} = \frac{\partial}{\partial t}\int_{c.v.}\rho \text{v} d\forall + \int_{c.s.}\rho \text{vv}.dA \qquad \text{.................................}(3.40)$$

Substitute (3.40) in(3.39), the general momentum equation (M.E) is obtained;

$$\left.\begin{aligned} \sum F &= \frac{\partial}{\partial t}\int_{c.v.}\rho \text{v} d\forall + \int_{c.s.}\rho \text{vv}.dA \\ \sum F &= \frac{\partial(\text{mv})_{c.v}}{\partial t}(m\cdot\text{v})_{out} - (m\cdot\text{v})_{in} \end{aligned}\right\} \qquad \text{.........................}(3.41)$$

Since the force (F) and the momentum $(m\cdot\text{v})$ are vector quantites, equ.(3.41) may be written in three directions as;

$$\sum F_x = \frac{\partial(\text{mv})_{xc.v}}{\partial t} + (m\cdot\text{v})_{xout} - (m\cdot\text{v})_{in} \qquad \text{..............}(3.41a)$$

$$\sum F_y = \frac{\partial(\text{mv})_{yc.v}}{\partial t} + (m\cdot\text{v})_{yout} - (m\cdot\text{v})_{in} \qquad \text{..............}(3.41b)$$

$$\sum F_y = \frac{\partial(\text{mv})_{zc.v}}{\partial t} + (m\cdot\text{v})_{Zout} - (m\cdot\text{v})_{in} \qquad \text{..............}(3.41c)$$

For steady flow,($\frac{\partial(\text{mv})_{c.v.}}{\partial t} = 0$), and the general M.E. reduces to ;

$$\sum F = (m\cdot\text{v})_{out} - (m\cdot\text{v})_{in} \qquad \text{M.E for steady flow} \qquad \text{..................} (3.42)$$

Also,

$$\sum F_x = (m\cdot\text{v})_{xout} - (m\cdot\text{v})_{xin} \qquad \text{..............}(3.42a)$$

$$\sum F_y = (m\cdot\text{v})_{yout} - (m\cdot\text{v})_{yin} \qquad \text{..............}(3.42b)$$

$$\sum F_z = (m\cdot\text{v})_{zout} - (m\cdot\text{v})_{zin} \qquad \text{..............}(3.42c)$$

For problems with one inlet and one outlet, $(m\cdot\text{v}_{in} = m\cdot\text{v}_{out} = m\cdot)$.Thus, equs.(3.41) and (3.42) become;

$\sum F = \frac{\partial (\mathrm{mv})_{c.v.}}{\partial t} + m^{\cdot}(V_{out} - V_{in})$ ……………(3.43)

$\sum F = m^{\cdot}(V_{out} - V_{in})$ ………………(3.44)

This can be written in three directions as;

$\sum F_x = \frac{\partial (\mathrm{mv})_{xc.v}}{\partial t} + m^{\cdot}(V_{xout} - V_{xin})$ ……………(3.43a)

$\sum F_y = \frac{\partial (\mathrm{mv})_{yc.v}}{\partial t} + m^{\cdot}(V_{yout} - V_{yin})$ ……………(3.43b)

$\sum F_z = \frac{\partial (\mathrm{mv})_{zc.v}}{\partial t} + m^{\cdot}(V_{zout} - V_{zin})$ ……………(3.43c)

And;

$\sum F_x = m^{\cdot}(V_{xout} - V_{xin})$ ……………(3.44a)

$\sum F_y = m^{\cdot}(V_{yout} - V_{yin})$ ……………(3.44b)

$\sum F_z = m^{\cdot}(V_{zout} - V_{zin})$ ……………(3.44b)

3.5.1 Applications of Momentum Equation

3.5.1.1 Fixed Vanes (Blades)

Fixed vanes (or blades) are usually used to deflect a jet of fluid to a certain direction (θ), see **Fig.(17)**.

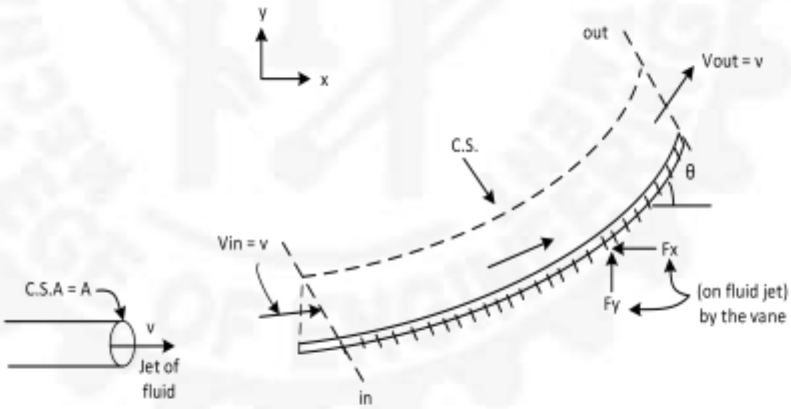


Fig. (17)

The momentum equation is used to derive expressions for the two Forces (F_x) and (F_y) required to hold the blade stationary. The following assumptions for the problem are used;

1- Steady flow.
2- Frictionless flow.
3- Incompressible fluid.
4- Change in P.E. is neglected (Zin=Zout)

5- Atmospheric pressure at inlet and outlet

Applying the B.E. between the inlet and outlet gives;

$$\frac{P_{in}}{\gamma}+\frac{V_{in}^2}{2g}+Z_{in}=\frac{P_{out}}{\gamma}+\frac{V_{out}^2}{2g}+Z_{out}\rightarrow V_{out}=V_{in}=V$$

The momentum equations (3.44a) and (3.44b) will be applied;

$$\overset{+}{\rightarrow}\sum F_x = m^{\cdot}(V_{xout}-V_{xin})$$

$-F_x = \rho AV(V\cos\theta - V) \rightarrow F_x = \rho AV^2(1-\cos\theta)$ $\longleftarrow$ on fluid $\longrightarrow$ on vane

$$+\uparrow\sum Fy = m^{\cdot}(V_{y\,out}-V_{y\,in})$$

Fy$==\rho AV(V\sin\theta - 0) \rightarrow Fy = \rho AV^2\sin\theta$ $\uparrow$ on fluid $\downarrow$ on vane

The resultant force (F) is calculated as;

F$=\sqrt{F_x^2+F_y^2}$ with $\alpha = \tan^{-1}\frac{Fy}{Fx}$

3.5.1.2 Moving Vanes (Blades)

3.5.1.2.1 Single Moving Vane

The vane here is moving with a speed (u), and the fluid jet is approached with velocity (V_1). To make this problem similar to that of fixed vanes, we use the principle of relative motion, that is, assuming the vane fixed and the fluid is approached with relative velocity (V_1-u), see **Fig. (18)**

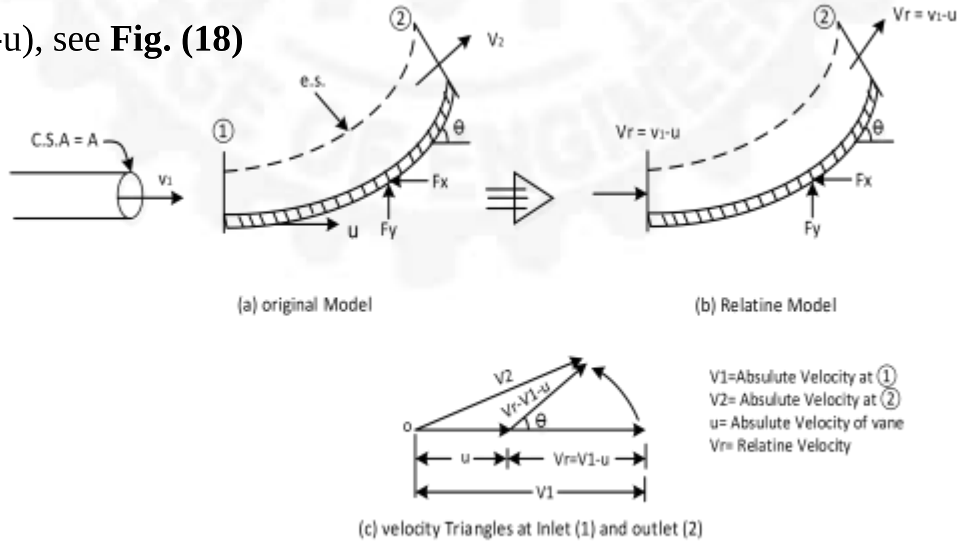


Fig. (18)

The M.E is used to find (F_x) and (F_y) required to obtain the desired motion (with velocity u). The M.E.(3.44) is applied on the relative model, thus;

$\sum F = \dot{m_r}(V_{rout} - V_{rin}) = \rho Q_r(V_{rout} - V_{rin})$..................(3.45)

With; $\dot{m_r} = \rho Q_r = \rho A V_r$

$Q_r = AV_r$

$V_r = V_1 - u$

Thus, we can show that;

$F_x = \rho A V_r^2(1 - cos\theta) = \rho A(V_1 - u)^2(1 - cos\theta)$ .... (4.46)

Fy= $\rho A V_r^2 sin\theta = \rho A(V_1 - u)^2 \sin\theta$ .... (3.47)

Power Delivered by the vane =$F_x u = \gamma Q_r \frac{V_1^2 - V_2^2}{2g}$ .... (3.48)

K.E. remaining in the jet = $\gamma Q_r \frac{V_2^2}{2g}$ .... (3.49)

3.5.1.2.2 Series of Moving Vanes

A series of vanes are fixed on a wheel, and thus a rotational motion is obtained. Compared to the single moving vane, where part of the mass flow rate ($\dot{m_r}$) can reach the moving vane, for a series of moving vanes the entire mass flow rate ($\dot{m_{bas}}$) will be used, thus, the M.E. will be;

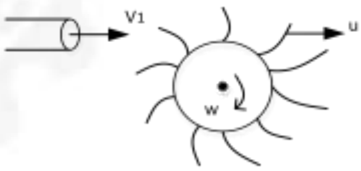
$\sum F = \dot{m_{abs}}(V_{rout} - V_{rin}) = \rho Q_{abs}(V_{rout} - V_{rin})$ ....(3.50)

Where;

$\dot{m_{bas}} = \rho Q_{abs} = \rho A V_1$

$Q_{abs} = AV_1$

$V_r = V_1 - u$



And, thus;

$F_x = \rho A V_1 v_r(1\text{-}\cos\theta) = \rho A V_1(V_1 - u)(1 - \cos\theta)$ ....(3.51)

$F_y = \rho A V_1 v_r \sin\theta = \rho A V(V_1 - u)\sin\theta$ ....(3.52)

Power delivered by the vanes= $F_x = \gamma Q_{abs} \frac{V_1^2 - V_2^2}{2g}$ ....(3.53)

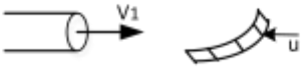
K.E. remaining in the jet= $\gamma Q_{abs} \frac{V_2^2}{2g}$ ....(3.54)

Note:

1- The most important application for a series of moving vanes are the turbine wheels, pumps, compressor, fans,

2- If the blade moves towards the jet, then:

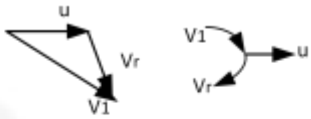
$V_r = V_1 + u$



Power$= \gamma Q_{abs} \frac{V_2^2 - V_1^2}{2g}$ $v_2 > v_1$

3- If (V_1) and (u) are not in-live, (Vr) is found by adding them vectorially;

$V_r = V_1 \nrightarrow u$



3.5.1.3 Forces on Pipe Bends

The momentum equation is used to calculate the forces (F_x) and (F_y) required to hold the vertical pipe bend shown in **Fig.(19).** Equs. (3.44a) and (3.44b) is applied here;

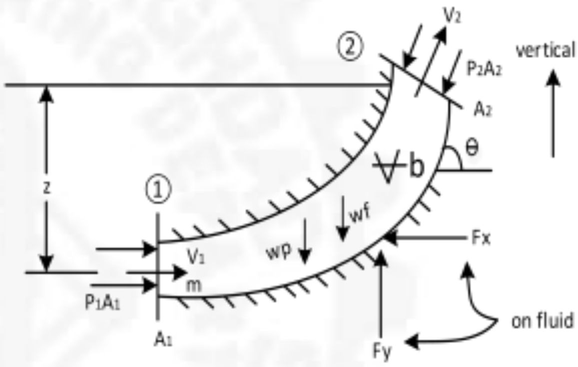


Fig. (19)

$$\overset{+}{\rightarrow}\sum F_x = m^{\cdot}(V_{xout} - V_{xin})$$

$$p_1A_1 - p_2A_2cos\theta - F_x = m^{\cdot}(v_2 cos\theta - 0) \rightarrow F_x = (\quad)^{\leftarrow onfluid}_{\rightarrow on\ bend}$$

$$\overset{+}{\uparrow}\sum F_y = m^{\cdot}(V_{yout} - V_{yin})$$

$$F_y - w_b - w_f - p_2A_2sin\theta = m^{\cdot}(v_2 sin\theta - 0) \rightarrow F_y = (\quad)^{\uparrow onfluid}_{\downarrow on\ bend}$$

Where W_b=Bend Weight

W_f=Fluid Weight$=\gamma_f \forall_b$ $(\forall_b$=bend volume)

Notes:

1- Before applying the M.E, the C.E. and E.E. are usually applied between sections(1) and (2) to find the unknowns that are not given with the data, such as $(V_2, p_2, \ldots)$.

2- If the plane of the bend is horizontal, then;

a- $(z_1 = z_2)$ and (z) will not be used in the E.E.

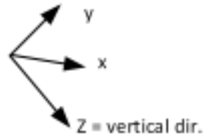
b- (Fx & Fy) will be in the horizontal plane

c- (W_b & W_f) will not appear in the y-M.E

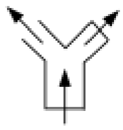
d- There will be a M.E. in the z-direction (equ.3.44c), with;

$\sum F_z = 0$ [no flow in the z-direction]

$F_z - w_b + w_f = 0 \longrightarrow F_z = w_b + w_f$



3- If there is more than inlet or outlet (such as in Y-bend), we must use equs.(3.42a,3-42b and 3-42c).



3.5.1.4 Theory of Propellers

Propellers are used to generate thrust against the drag exerted by the fluid on a body moving through it. The thrust is generated by increasing the momentum of the fluid passing through it, see **Fig.(20)**.

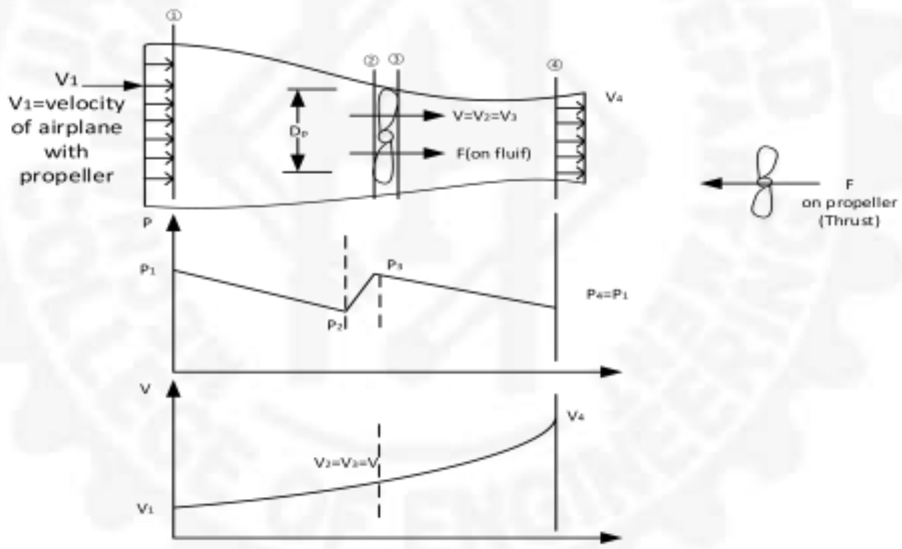


Fig.(20)

Applying M.E., equ.(3.44a);

F=$\dot{m}$(v_4-v_1)=ρQ(v_4-v_1)=ρAV(v_4-v_1)=$(p_3 - p_2)A$ …………(3.55)

Where A=$\frac{\pi}{4}D_p^2$ Area of propeller

Thus; $p_3 - p_2 = \rho$v(v_4-v_1) ……………………(3.56)

Applying B.E. from 1→ 2& from 3→ 4;

$$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2 \qquad Z_1=Z_2$$
$$p_3 + \frac{1}{2}\rho V_3^2 = p_4 + \frac{1}{2}\rho V_4^2 \qquad Z_3=Z_4$$

Since $p_1=p_4$, hence;

$p_3-p_2=\frac{1}{2}\rho(V_4^2 - V_1^2)$ ……………………..(3.57)

Equating equations (3.56) and (3.57), and simplification yields;

$V = \frac{V_1+V_4}{2}$ ……………………..(3.58)

Output Power $OP=FV_1=\rho Q(V_4 - V_1)V_1$ ……………………..(3.59)

Input Power $IP=\frac{1}{2}\rho Q(V_4^2 - V_1^2)$…………………(3.60)

K.E remaining in the slip stream$=\frac{1}{2}\rho Q(V_4 - V_1)^2$ ……………(3.61)

IP=OP+K.E. remaining in slip stream

The theoretical mechanical efficiency (e_t) is;

$e_t = \frac{OP}{IP} = \frac{\rho Q(V_4-V_1)v_1}{\frac{1}{2}\rho Q(V_4^2-V_1^2)} \rightarrow e_t = \frac{2V_1}{V_1+V_4} = \frac{V_1}{V}$………………(3.62)

If $\Delta V = V_4 - V_1$, then;

$e_t = \frac{1}{1+\frac{\Delta V}{2V_1}}$……………………………..(3.63)

(e_t) is maximum when (ΔV) is minimum, which occurs when (ρQ)is large i.e., accelerating larger Q to a smaller (V_4) for a given (V_1)

3.5.1.5 Jet Propulsion

I- Jet Engines

The thrust of the jet engine (F_{th}) is generated by expanding the gases produced from burning a fuel with air in the combustion chamber, to a higher velocity (v_2>v_1), see **Fig.(21)** Applying M.E,equ.(3.42a) gives;

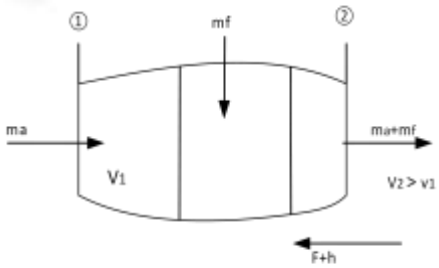


Fig. (21)

$F_{th} =(\dot{m}_a + \dot{m}_f)v_2 - \dot{m}_a v_1$

or; $F_{th}=\dot{m}_a[(1 + f)v_2 - v_1]$ …………………(3.64)

Where $f = \frac{\dot{m}_f}{\dot{m}_a}$ fuel-air ratio

II- Boats

The pump of the boat accelerates the jet of water from (v_1) to (v_2), see **Fig(22)**. Applying M.E. gives;

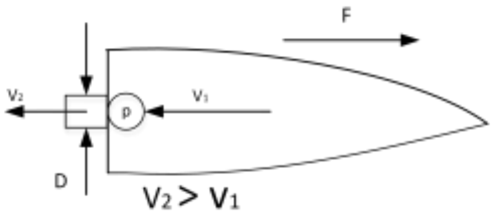


Fig. (22)

$F=\rho Q(v_2 - v_1)$ ……………………..(3.65)

$$Q = \frac{\pi}{4} D^2 v_2$$

3.5.1.6 Rocket Mechanics

The rocket motor carries its fuel and oxidizer with it, and so, there is no mass flow enters the motor, only exit mass flow rate exists (gases produced by burring the fuel and oxidizer at a rate of $m^{\cdot}$), and hence producing the thrust of the rocket motor ($m^{\cdot} v_r$). The mass of the rocket is changing with time, therefore we have to apply the general M.E (equ.3.41b), see **Fig (23)**;

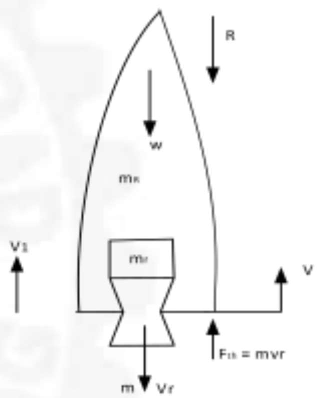


Fig. (23)

R = Air resistance

m_R = Mass of rocket body

mf = Fuel mass

$m^{\cdot}$ = Rate of fuel buring

V_r= Exit gas velocity relative to rocket

V_1= absolute velocity of the rocket

V= rocket velocity w.r.t. a reference moving with the rocket

V=0 but $\frac{dV}{dt} \neq 0$; $\frac{dV}{dt} = \frac{dV_1}{dt}$ = Rocket acceleration

Total mass m = $m_R + m_f = m_R + (m_{f_i} - m^{\cdot} t)$

$=(m_R + m_f)- m^{\cdot} t$

$=m_i - m^{\cdot} t$ ……………………….(3.66)

Where;

m_{f_i} = Initial mass of the fuel

m_i= Initial mass of the rocket

M.E.;

$$\uparrow \sum Fy = \frac{\partial (mv)_{yc.v.}}{\partial t} + (m^{\cdot} v)_{yout} - (m^{\cdot} v)_{yin}$$

$$-R-mg=\frac{\partial (mv)_{c.v.}}{\partial t}+(-m{\cdot}V_r)=m\frac{\partial V}{\partial t}+V\frac{\partial m}{\partial t}-m{\cdot}V_r$$

Thus;

$$\frac{dV}{dt}=a=\frac{m{\cdot}V_r-R-(m_i-m{\cdot}t)g}{m_i-m{\cdot}t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.67)$$

$$e_t=\frac{OP}{IP}=\frac{F_{th}V_1}{\frac{1}{2}m{\cdot}V_r^2+\frac{1}{2}m{\cdot}V_1^2}=\frac{m{\cdot}V_rV_1}{\frac{1}{2}m{\cdot}V_r^2+\frac{1}{2}m{\cdot}V_1^2}=\frac{2V_r/V_1}{1+(\frac{V_r}{V_1})^2} \quad \ldots\ldots\ldots\ldots\ldots(3.68)$$

Where

$F_{th}=m{\cdot}V_r$ thrust of the rocket ……………….(3.69)

3.5.2 Euler's Equation of Motion

This equation is applied for frictionless flow along a streamline. To derive this equation, we will apply Newton's 2nd law the fluid element shown in Fig.(24).

$$\uparrow \sum F_s=ma_s=m\frac{dv}{dt}$$

Since V=V(s,t), then;

$$a=\frac{dv}{dt}=\frac{\partial v}{\partial t}+\frac{\partial v}{\partial s}\frac{\partial s}{\partial t}=\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial s} \quad \ldots\ldots\ldots(3.70)$$

Thus;

$$pdA-(p+\frac{\partial p}{\partial s}ds)dA-\rho gdsdAcos\theta=$$

$$\rho dsdA(\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial s})$$

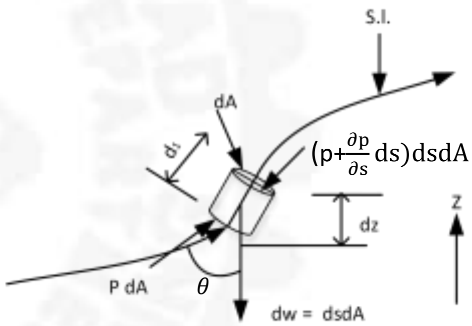


Fig.(24)

$$cos\theta=-\frac{dz}{ds}$$

Simplification yields;

$$\frac{1}{\rho}\frac{\partial p}{\partial s}+v\frac{\partial v}{\partial s}+g\frac{dz}{ds}+\frac{\partial v}{\partial t}=0$$

Euler's Equation

1- Friction flow ………………(3.71)
2- Along a s.l.

3.5.3 Bernoull's Equation

Euler's equation (3.71) for steady ($\frac{\partial v}{\partial t}=0$) incompressible fluid ($\rho$ =const.) yields;

$$\frac{dp}{\rho}+vdv+gdz=0$$

Integrating this equation;

$$\frac{p}{\rho}+\frac{v^2}{2}+gz=const.$$

Or; Bernoulli's Equation

$$\left.\begin{array}{l} \frac{p}{\gamma}+\frac{v^2}{2g}+z=const. \\ \frac{p_1}{\gamma}+\frac{v_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{v_2^2}{2g}+z_2 \end{array}\right\}$$

1- Steady flow
2- Incompressible fluid …………..(3.72)
3- Frictionless flow
4- Along the same s.l.

3.6 Some Applications of Continuity, Momentum and Energy Equations

3.6.1 Losses due to Sudden Expansion

The head loss due to sudden expansion (he) from a C.S.A. (A1) to a larger C.S.A (A_2) is found by applying the three continuity, momentum and energy equations. The loss in energy occurs due to formation of a separated region after the sudden expansion, see **Fig.(25)** This region is a low pressure region accompanied by the formation of vortices. The following assumptions are made to analyze the problem;

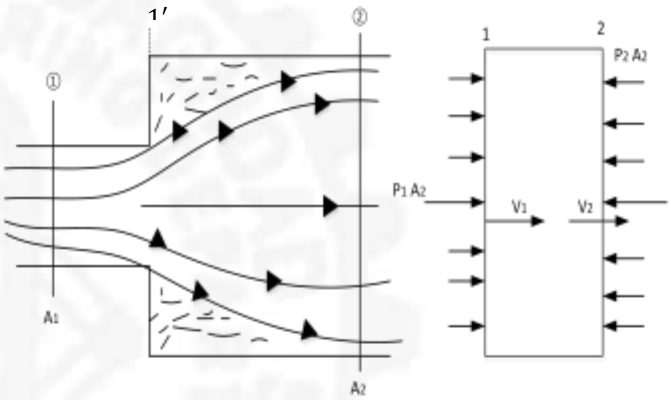


Fig.(25)

1- Steady incompressible turbulent flow.
2- Uniform velocity over the cross – sections.
3- Shear forces on the wall are negligible.
4- At the sudden expansion section (1'), the lateral acceleration of the fluid particles in the eddy along the surface is small, and so, hydrostatic pressure variation occurs across the section ($p=p_1$).

<u>C.E:</u> $A_1V_1=A_2V_2$ …………………………(a)

<u>E.E:</u> $\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+he$……………………… (b)

M.E: $p_1A_2-p_2A_2=\dot{m}(V_2-V_1)=\rho A_2V_2^2-\rho A_1V_1^2$………………...(c)

Solution of equs. (a), (b) and (c) simultaneously for (he) gives;

$$he=\frac{(V_1-V_2)^2}{2g}=\frac{V_1^2}{2g}\left(1-\frac{A_1}{A_2}\right)^2 \text{…………………(3.73)}$$

For circular pipes;

$$he=\left[1-\left(\frac{D_1}{D_2}\right)^2\right]^2\frac{V_1^2}{2g} \text{…………………(3.73a)}$$

3.6.2 Hydraulic Jump

A rapidly flowing stream of liquid in an open channel suddenly changes to a slowly flowing stream with a larger C.S.A. and a sudden rise in elevation of liquid surface. This phenomenon is called" hydraulic jump". It converts the K.E to P.E. and losses or irreversibilities (rough surface of the jump), see **Fig.(26).** The following assumptions are used;

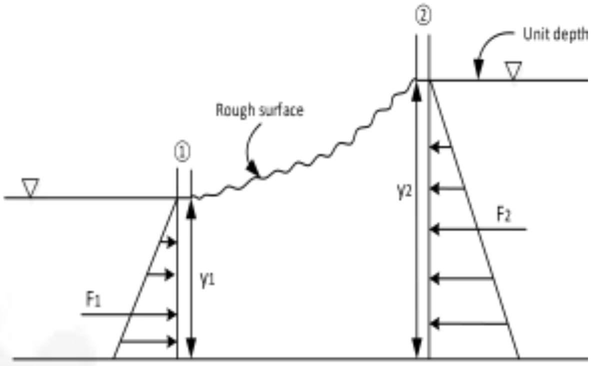


Fig.(26)

1- Steady incompressible flow.
2- Uniform velocity across the C.S.A. A_1&A_2.
3- Shear forces at the wall are negligible.
4- Hydrostatic pressure distribution across the sections (1) &(2).

The continuity, momentum and energy equations are applied to the problem to find the head loss (hj) due to hydraulic jump, and the new elevation (y_2).

C.E.: $A_1V_1=A_2V_2$

$A_1y_1.1 =A_2y_2 .1$ ………………..(a)

E.E: $\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+hj$

$y_1+\frac{V_1^2}{2g}=y_2+\frac{V_2^2}{2g}+hj$ ……………….(b)

M.E: $\sum F=\dot{m}(V_2-V_1)$

$F_1-F_2=\rho A_2V_2^2-\rho A_1V_1^2$

$\gamma\frac{y_1}{2}y_1-\gamma\frac{y_2}{2}y_2=\rho Ay_2V_2^2-\rho y_1V_1^2$…………………(c)

From (a) and (c);

$$y_2=-\frac{y_1}{2}+\sqrt{(\frac{y_1}{2})^2+\frac{2V_1^2y_1}{g}} \quad …………………(3.74)$$

From (b);

$$h_j=\frac{(y_2-y_1)^3}{4y_1y_2} \quad ……………(3.75)$$

3.7 Law of Conservation of Angular Momentum: Moment of Momentum Equation

The property (N) of the system here is the moment of linear momentum (mrxv). This law represents the application of the Newton's 2nd law of motion for a system;

$\sum M = \frac{d}{dt}(\text{mrxv})_{sys}$

rxF=$\frac{d}{dt}(mrxv)_{sys}$ …………………..(3.76)

The R.H.S of this equation is obtained from the general control volume equation (3.13). Thus; with;

$N_{sys} = \text{mrxv}$

$\eta = \frac{N}{m} = \text{rxv}$

Equ.(3.13) becomes;

$\frac{d(mrxv)_{sys}}{dt} = \frac{\partial}{\partial t}\int_{c.v.} \rho rxv d\forall + \int_{c.s.}(rxv)(v.dA)$ ……..(3.77)

Hence, equ. (3.76) becomes;

$$\underbrace{\text{r x F}}_{\text{Torque on C.V.}} = \underbrace{\frac{\partial}{\partial t}\int_{c.v.} \rho rxv d\forall}_{\text{Rate of Change of Moment of Momentum in C.V.}} + \underbrace{\int_{c.s.}(rxv)(v.dA)}_{\text{Net Efflux of Moment of Momentum from C.V.}} \quad \text{.....……..(3.78)}$$

Equ. (3.78) is the General Moment of Momentum Equation. The most important application of this equation is for turbo machines, in which the normal (n) and tangential (t) coordinates are used, see **Fig.(27)**. The C.V. is the annular flow bounded by the root circle (r_1) and the tip circle (r_2) of the blades. The flow is assumed steady. The torque (T) on the (C.V.) is produced from the tangential components of forces ($F_t r$). Thus, with;

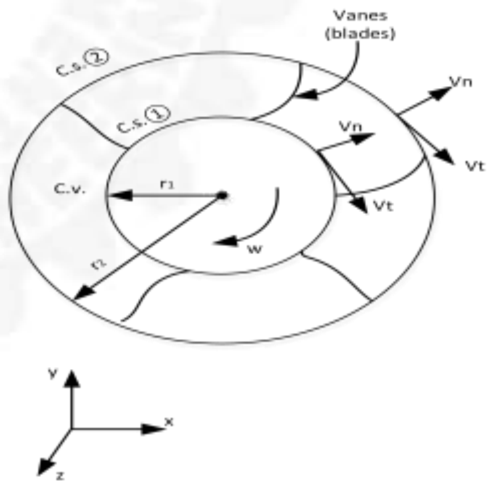


Fig.(27)

rxv=rV_t

v.dA=V_ndA=Q=Q_2=Q_1

Equ.(3.78) reduces to;

$F_t r = \int_{c.s.} \rho r V_t V_n dA$

$= \int_{A_2} \rho r_2 V_{t_2} V_{n_2} dA - \int_{A_1} \rho r_1 V_{t_1} V_{n_1} dA$

Or;

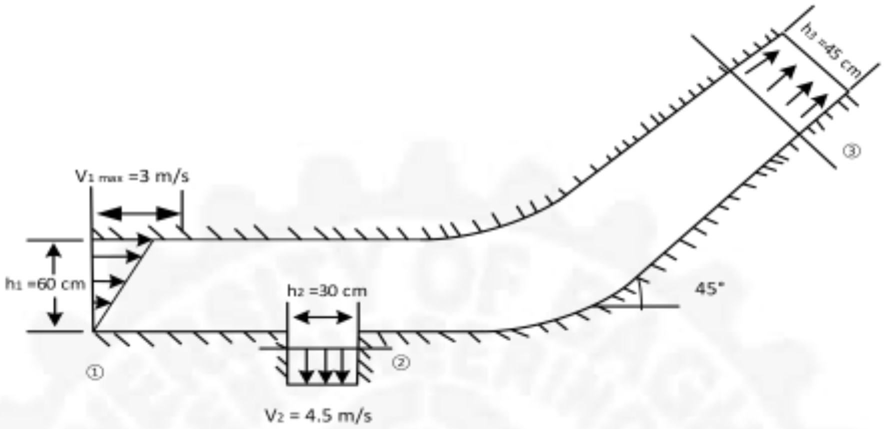
T=$\rho Q[(rv_t)_2 - (rv_t)_1]$ Euler's Equation for Turbomachines ….(3.79)

For problems with more than one inlet and outlet, this equation becomes;

T=$(\rho Q r_2 v_t)_2 - (\rho Q r v_t)_1$ ....… (3.80)

Examples

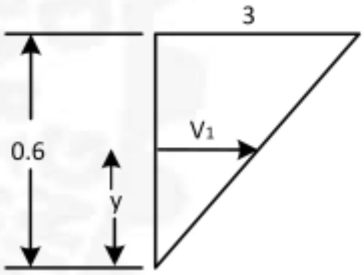
Example (3.1): A two-dimensional reducing bend has a linear velocity profile at section (1). The flow is uniform at sections (2) and (3). The fluid is incompressible and the flow is steady. Find the magnitude of the uniform velocity at section (3).



Sol.:

$$\frac{V_1}{y}=\frac{3}{0.6}\rightarrow V_1=5y$$

C.E.: $Q_1=Q_2+Q_3$



$$\int_0^{0.6} 5y\,dy = 4.5*0.3+0.45*V_3$$

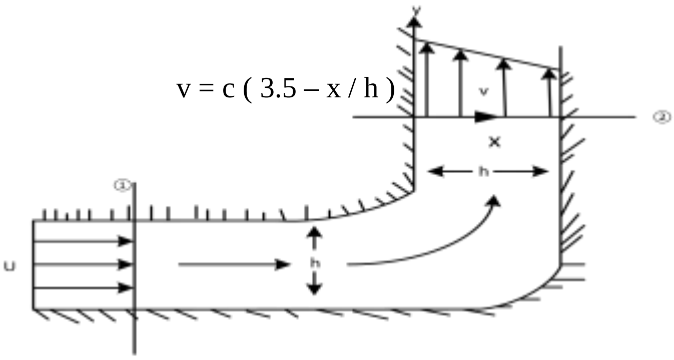
$$\left[\frac{5y^2}{2}\right]_0^{0.6}=1.35+0.45V_3$$

$$\frac{5*0.36}{2}=1.35+0.45V_3$$

$$V_3=\frac{0.9-1.35}{0.45}$$

$\therefore V_3 = -1\,m/s$ (-ve sign means inflow at 3)

Example (3.2): Water enters a two-dimensional channel of constant width (h), with uniform velocity (U). The channel makes a (90^o) bend that distorts the flow to produce the velocity profile shown at the exit. Evaluate the constant (c) in terms of (U).



Sol.:

C.E.: $A_1V_1=A_2V_2$

$Uh_1=\int_0^h v dx = c\int_0^h v(3.5-\frac{x}{h})dx.1$

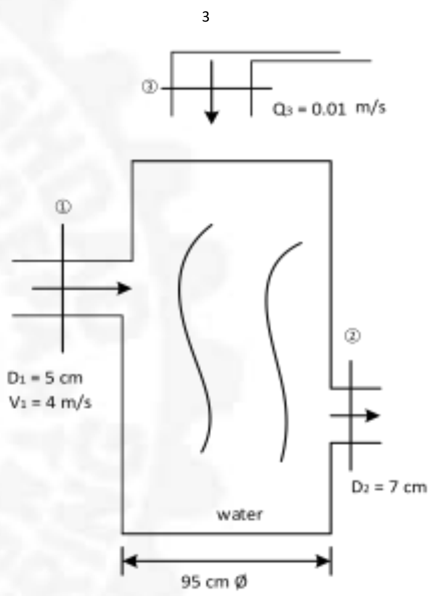
$Uh=c\left[3.5x-\frac{x^2}{2h}\right]_0^h$

$Uh=c\left[3.5h-\frac{h}{2}\right]$

Uh=3ch

$\therefore$ c=$\frac{U}{3}$ m/s

Example (3.3): For the water tank shown in the figure, calculate the exit velocity (V_2) required to lower the water level in the tank by (10 cm) in (100 seconds).



Sol.:

$Q_1=A_1V_1=\frac{\pi}{4}D_1^2V_1=\frac{\pi}{4}(0.05)^2*4 \rightarrow Q_1=0.00785\frac{m^3}{2}$

General C.E.:

$$\frac{\partial m_{c.v}}{\partial t}+m_{out}^{\cdot}-m_{in}^{\cdot}=0$$

$$m_{c.v}=\rho A h=\rho*\frac{\pi}{4}(0.95)^2*h$$

Thus;

$$\rho*\frac{\pi}{4}(0.95)^2\frac{dh}{dt}+\rho Q_2-\rho Q_1-\rho Q_3=0$$

But; $\frac{dh}{dt}=\frac{-0.1}{100}=-0.001$ m/s (level decreasing)

Thus; $Q_2=(Q_1+Q_3)-(-0.001)\frac{\pi}{4}(0.95)^2$

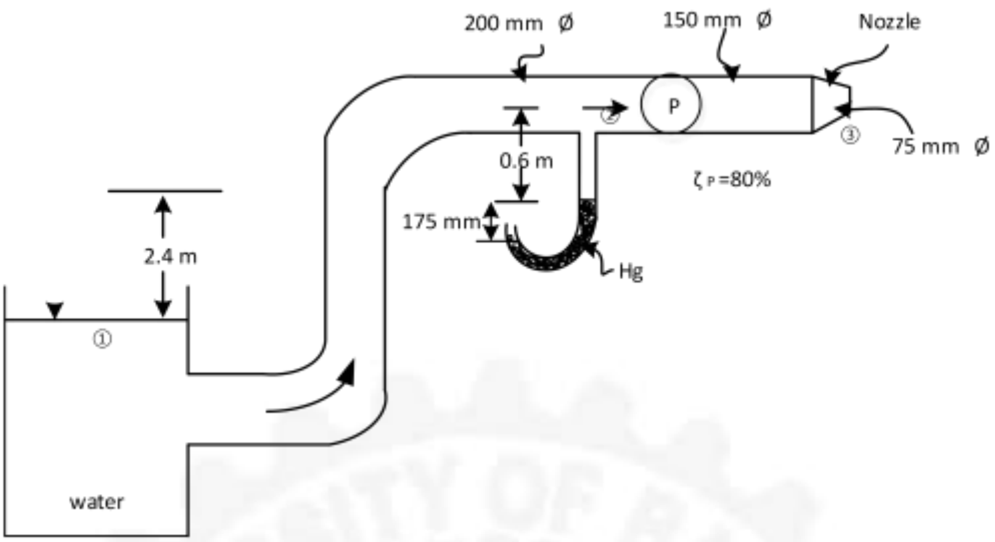
$=(0.00785+0.01)+0.001\frac{\pi}{4}(0.95)^2$

$Q_2=0.7267$ m^3/s

$$Q_2=A_2V_2=\frac{\pi}{4}D_2^2V_2$$

$0.7267=\frac{\pi}{4}(0.07)^2V_2 \rightarrow V_2=188.8\frac{m}{s}$

Example (3.4): Neglecting the losses, calculate the required pump shat horsepower.



Sol.:

B.E. 1-2: $0+0+0=\frac{p_2}{\gamma_w}+\frac{V_2^2}{2g}+2.4$ ....(1)

Man. Equ.: $P_2+0.6\gamma_w+0.175*13.6\gamma_w=0$

$\frac{p_2}{\gamma_w} = -0.6 - 0.175 * 13.6 = -2.98,$

Sub. in (1) → V_2= 3.372 m/s

C.E.: $A_2V_2=A_3V_3$

V_3= 23.98 m/s

E.E. 1-3: $0+0+0+hp=0+\frac{(23.98)^2}{2*9.81}+2.4+0$

∴hp=31.74 m

$$\xi_p = \frac{\gamma Q hp}{IP}$$

$\therefore IP = 36.636$ kW =49.1 h.p.

Example (3.5): A horizontal axisymmetric jet of air ($\rho = 1.22$ kg/m^3) with (10mm) diameter strikes a stationary vertical disk of (200mm) diameter. The jet speed is (50 m/s) at the nozzle exit. A manometer is connected to the center of the disk. Neglecting the losses and the difference in potential head, calculate:

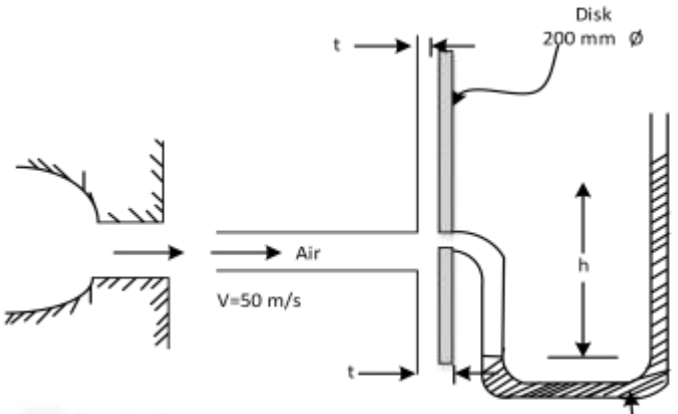
a- The deflection (h) of the manometer.

b- The force exerted by the jet on the disk.

c- The thickness (t) of the air jet at the exit.

Sol.:

B.E. 1-2



$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+Z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+Z_2$$

$$p_2=\gamma_{air}\frac{V_1^2}{2g}=1.22*9.81*\frac{(50)^2}{2*9.81}$$

$$p_2=1525\text{ Pa}$$

Man.equ.:

$$p_2+\gamma_{air}h_{air}-s\gamma_w h=0$$

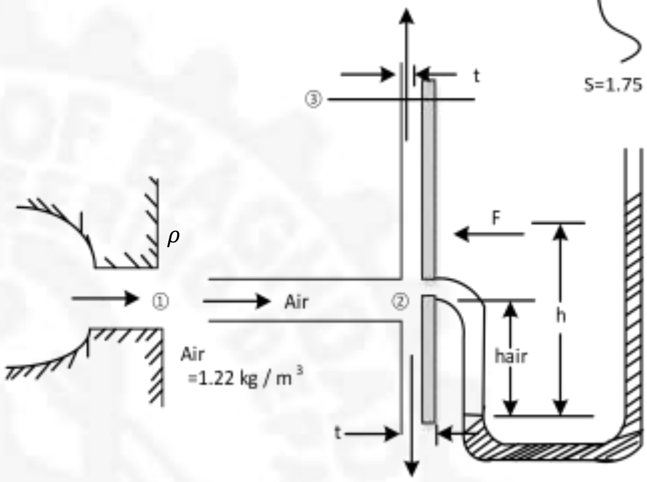
$\therefore h=8.88$ cm =0.0888m

$$\rightarrow\sum F_x=\dot{m}(V_{xout}-V_{xin})$$

-F=1.22*$\frac{\pi}{4}(0.01)^2*50[0-50]$

$\therefore F=0.239N\leftarrow$ on fluid

$F=0.239N\rightarrow$ on disk



B.E. 1-3:

V_1=V_3

Q_{in}= Q_{out}

$$\frac{\pi}{4}d^2V_1=\pi D\,tV_3$$

$\therefore$t=0.125 mm

Example (3.6): At what speed (u) should the vane shown in the figure travel for maximum Power? What should be the angle (θ) for maximum power?

Sol.:

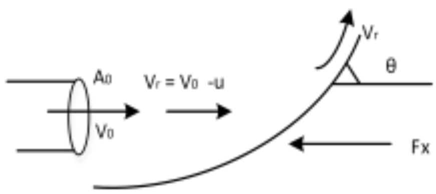
$$\overset{+}{\rightarrow}\sum F_x=\dot{m}_r(V_{rout}-V_{rin})$$

-F_x=$\rho A_0V_r^2(V_r\cos\theta\text{-}V_r)$

Thus;

F_x=$\rho A_0V_r^2(1\text{-}\cos\theta)$

or



F_x=$\rho A_0(V_0-u)^2(1\text{-}\cos\theta)$

Power P=F_xu= $\rho A_0(V_0-u)^2$(1-cosθ)u

For maximum Power

$\frac{\partial p}{\partial u}=\rho A_0$(1-cos$\theta$)$[(V_0-u)^2.1+u*2(V_0-u)*-1]$=0 u=$\frac{V_0}{3}$

$\frac{\partial p}{\partial \theta}=\rho A_0\big[(V_0-u)^2 u[-(-sin\theta)]\big]$=0 $\rightarrow$ $sin\theta$ =0 $\theta=0$ min. power

$\theta=180$ max.power

Example(3.7): An air plane traveling (400 km/h) through still air ($\gamma=12$ N/m^3), discharge (1000m3/s) through its two (2.25m) diameter propellers . Dertermine;

a- The theoretical efficiency b- the thrust c- The pressure difference across the propeller, and d- The theoretical power required

Sol.

a- V_1=400 $\frac{km}{h}*\frac{1000m}{1km}*\frac{1h}{3600s}\rightarrow V_1=111.1$ m/s

V=$\frac{Q}{A_r}=\frac{1000}{\frac{\pi}{4}(2.25)^2}\rightarrow$ V $=125.8$ m/s

$e_t=\frac{V_1}{V}=\frac{111.1}{125.8}\rightarrow e_t=88.3\%$

b- F=$\rho Q(V_4-V_1)=\frac{\gamma}{g}Q(V_4-V_1)$

V=$\frac{V_1+V_4}{2}\rightarrow V_4=2V-V_1\rightarrow V_4=140.5\ m/s$

Thus; F=$\frac{12}{9.8}*1000*(140.5-111.1)\rightarrow$ F $=36$kN

c- F=$(p_3-p_2)A\rightarrow p_3-p_2=\frac{F}{A}=\frac{36000}{\frac{\pi}{4}(2.25)^2}\rightarrow p_3-p_2$=4.53 kpa

d- e_t=$\frac{OP}{IP}\Rightarrow IP=\frac{OP}{e_t}=\frac{FV_t}{e_t}=\frac{36000*111.1}{0.883}\rightarrow IP=4.53$ kW

Example (3.8): Determine the burring time for a rocket that initially has a gravity force of (4.903 MN), of which (70 percent) is propellant. It consumes fuel at constant rate, and its initial thrust is (10 percent) greater than its gravity force. The exhaust gases velocity is (Vr=3300 m/s). Considering (g) as constant at (9.8 m/s^2), and the flight to be vertical without air resistance, find the speed of the rocket at burnout time, its height above the ground, and the maximum height it will attain.

Sol.:

F_{th}=1.1Wi=1.1*4.903→ F_{th}=5.393 MN

$F_{th}=m^{\cdot}V_r \rightarrow m^{\cdot} = \frac{F_{th}}{V_r} = \frac{5.393*10^6}{3300} \rightarrow m^{\cdot} = 1634.3$ kg/s

Mass of propellant = $\frac{0.7*4.903}{9.8} \rightarrow m_{fuel} = 350214.3\ kg$

Hence;

The burring time=$\frac{m_{fuel}}{m^{\cdot}}=\frac{350214.3}{1634.3} \rightarrow$ buringtime=214.35 s

$a=\frac{\partial v}{\partial t} = \frac{m^{\cdot}V_r - R - (m_i - m^{\cdot}t)g}{m_i - m^{\cdot}t}$

$m_i = \frac{W_i}{g}=\frac{4.903*10^6}{9.8} \rightarrow m_i = 500306.1$ kg

Thus;

$a=\frac{dV}{dt} = \frac{1634.3*3300-9.8[500306.1-1634.3\ t]}{500306.1-1634.3\ t}$

Simplification gives;

$$\frac{dv}{dt} = -9.8 + \frac{3298.16}{305.94 - t}$$

Thus;

$V= -9.8t - 3298.16\ \ln(305.94 - t) + const.$

When t=0, V=0→ $const. = 3298.16 ln(305.94)$

Thus;

$V= -9.8 - 3298.16\ \ln(1 - \frac{t}{305.94})$

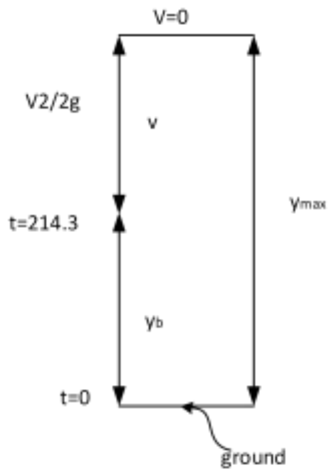
at burnout, t=214.3, thus;

$V= -9.8 * 214.3 - 3298.16\ \ln(1 - \frac{214.3}{305.94}) \rightarrow V = 1873.24 \frac{m}{s}$

The height at t= 214.3; (y_b) is

$y_b=\int_{t=0}^{t=214.3} V dt = \int_0^{214.3} \left[-9.8t - 3298.16\ \ln(1 - \frac{t}{305.94})\right] dt \rightarrow$

$y_b = 117.2$ km



The rocket will glide ($V^2/2g$) higher after burnout, thus

$y_{max}= y_b+\frac{V^2}{2g}$

$=117.2*10^3+\frac{1873.24^2}{2*9.8} \rightarrow y_{max} = 296.25\ km$

Problems

The problems number listed in the table below refer to the problems in the "textbook", chapter"3"

Article No.	Related problems
3.3	6,7,8,10,12,13
3.4 + 3.4.1	17,18,20,21,22,24,26,27,28,30,31,32,33,34,35,36,38,39,41,42,46,47,51,52,53,54,55,58,59,60,61,63,64
3.5 + 3.5.1.3	70,76,77,78,82,84,85,87,88,89,125,137,138,139
3.5.1.1 + 3.5.1.2	100,111,112,113,114,116,118,120,121,122,123
3.5.1.4 + 3.5.1.5	91,92,93,95,97,99
3.5.1.6	101,102,105,106,107,108
3.6	127,128,130,133,134,136
3.7	140,141,142,143,144,145,146

Chapter -4-

Dimensional Analysis and Similitude

4.1 Introduction

The solution of most engineering problems involving fluid mechanics rely on data acquired by experimental means. The full-scale structure employed in the actual engineering design is called the ***"PROTOTYPE"***, while the replica of the structure on which tests are made is called the ***"MODEL"***, which is usually made much smaller than the prototype for economic reasons.

The process of manufacturing the model, and the conversion of the experimental results obtained from the model tests, both are based on laws and relations obtained from “Dimensional Analysis” and “Similitude”, sees **Fig. (1)**.

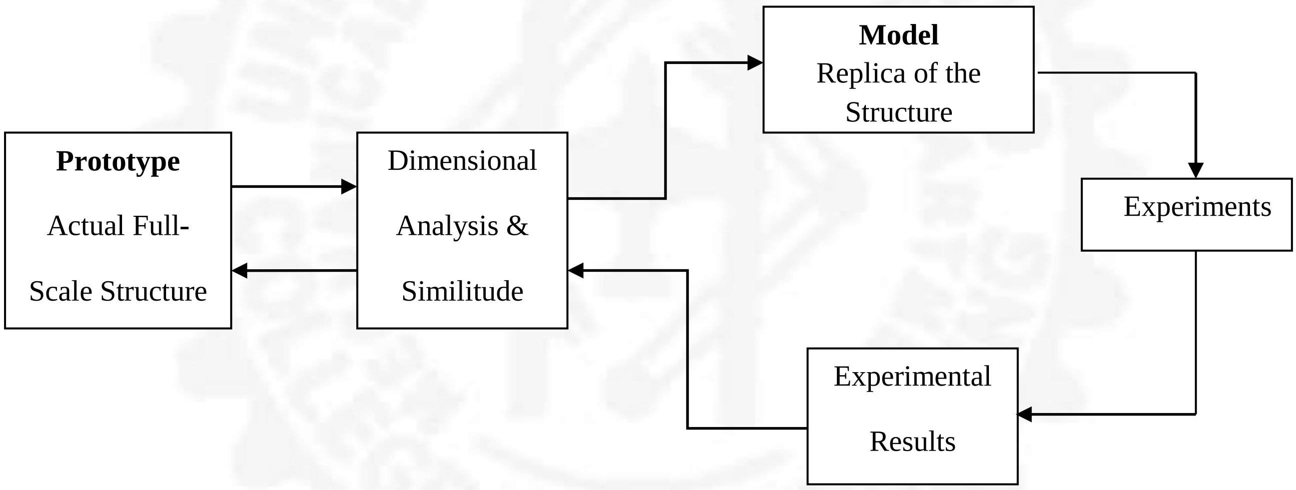


Fig. (1)

4.1.1 Dimensions and Units

"Dimensions" are physical variables that specify the behavior and the nature of a certain system, whereas the "Units" are used to specify the amount of these dimensions.

In fluid mechanics problems, the "fundamental" dimensions are three; these are Mass (M), Length (T) and Time (T). The dimensions of any physical variable include these three dimensions (one, two or all). In heat transfer and thermodynamic problems, a fourth dimension for temperature (θ) is added. **Table (1)** includes the dimensions of the most important variables.

Table (1)

Quantity	Dimension	Quantity	Dimension
Mass	M	Dynamic Viscosity (μ)	$ML^{-1}T^{-1}$
Length	L	Kinematic Viscosity (ν)	L^2T^{-1}
Time	T	Density	ML^{-3}
Force	MLT^{-2}	Pressure	$ML^{-1}T^{-2}$
Area	L^2	Specific Weight	$ML^{-2}T^{-2}$
Volume	L^3	Surface Tension	MT^{-2}
Velocity	LT^{-1}	Power	ML^2T^{-3}
Discharge	L^3T^{-1}	Work	ML^2T^{-2}
Angle	$M^\circ L^\circ T^\circ$	Angular Velocity	T^{-1}

Notes:

1- Any analytically derived equation should be dimensionally homogeneous, i.e., the sum of exponents for each dimension in the R. H.S. of the equation should be equal to that in the L. H. S. of the equation. Also, the dimensions of each term in any equation should be the same.

Examples:

1) $Q = AV$

$L^3T^{-1} = L^2 . LT^{-1}$

L: 3 = 2 + 1 = 3

T: -1 = -1

2) $P = \gamma h$

$ML^{-1}T^{-2} = ML^{-2}T^{-2} .L$

M: 1 = 1

L: -1 = -2 + 1 = -1

T: -2 = -2

Empirical equations need not be dimensionally homogeneous, for example, the Chezy formula;

$V = c\sqrt{R.S.}$ c = constant

$\mathrm{LT}^{-1} = c\,\sqrt{L.\,M^{\circ}L^{\circ}T^{\circ}}$

$\mathrm{LT}^{-1} = cL^{1/2}$

The equality of dimensions is absorbed in the constant of the equation (c).

2- In fluid mechanics problems, the maximum number of dimensions in any physical quantity, or group of physical quantities is three dimensions (m=3), i. e., (m) could be (0, 1, 2 or 3 (max)).

3- Physical variables in fluid mechanics are usually classified in to three groups;

1) Geometry Variables (L, d, A, V, Angle,)
2) Fluid Properties Variables (ρ, μ, ν, k, σ,)
3) Kinematics and Dynamic Aspects of Flow Variables (p, v, a, g, F, ω, P, W,)

4- Any variable which has no dimension is called "Dimensionless" variable; with dimensions (M°L°T°).

4.2 Dimensional Analysis

The basic objective in dimensional analysis is to reduce the number of separate variables involved in a problem to a smaller number of independent dimensionless groups of variables called “Dimensionless Parameters”. As the functional relationship between the variables can be analytically established (by using dimensional analysis), the experimental work is reduced to finding the value of the constants. Dimensional analysis can be gainfully used to plan the experiment and reduce the number of variables to be tested; and the results can be presented meaningfully.

Consider, for example, the motion of a sphere in a fluid. The drag force (F_D) excreted by the fluid on the sphere is known to depend on the sphere diameter (D), velocity (V), fluid density (ρ) and viscosity (μ), see **Fig. (2)**. If we want to study the effect of the independent variable (P, M, V, D) on the dependent variable (F_D), we have to investigate the effect of each of the four variables, separately and keep the other three constants. Thus, if, for example, we made (5) experiments for each variable, we need (5^4 = 625) experiment to study these effects and relations, which is a very complex and laborious process, since it involves the use of different fluids, different diameters and different velocities. Besides; the results presentation is a very complex and cannot be representative. To solve this problem, we use the principle of dimensional analysis, by which we replace the original relation;

$F_D = f(V,\ D,\ \rho,\ \mu)$.................. (4.1)

to the following relation:

$$\left.\begin{aligned} &\frac{F_D}{\rho v^2 D^2} = f\left(\frac{VD\rho}{\mu}\right) \\ &\text{Or; } C_D = f\ (Re) \end{aligned}\right\} \qquad \text{.. (4.2)}$$

C_D = Drag Coefficient , Re = Reynolds Number

We should note the following;

1- Number of variables in the original problem (equ. 4.1) is (n = 5).
2- Number of "dimensionless" variables in the transformed problem (equ. 4.2) is (2), which is the difference between the number of variables in the problem (n = 5) and the number of dimensions of the problem (m = 3).
3- The number of experiments in the original problem is (5^4 = 625), whereas in the transformed problem it is (5^1 =5).
4- We note that (ρ, V, D) are repeated with (C_D) and (Re) in (equ. 4.2). These are called the repeated variables of the problem, and their number equals to the number of dimensions of the problem (m = 3).
5- During the (5) experiments, we have to change (Re) five times and measure the drag force (F_D) and then draw the relation between them. The change of (Re) can be done by changing one of the four independent variables (ρ, V, D, μ) and keep the other three constants.
6- The obtained results (C_D = f (Re) relation) can be used for different ranges of (P, V, D, M) that used in the experiments.
7- The process of finding the dimensionless parameters (C_D & Re) from the original variables (F_D, ρ, V, D, μ) is made by using the "Buckingham π - Theorem", which will be described in the next article.

4.2.1The Buckingham π – Theorem

"If a physical phenomenon involves (n) quantities, and if these quantities involve (m) dimensions, then the quantities can be arranged into (n-m) dimensionless parameter".

Let (A_1, A_2… A_n) be the (n) quantities involved in the problem. A functional relationship should exist such that;

$F\ (A_1, A_2, \ldots, A_n) = 0$ ……………………….. (4.3)

If the number of dimensions in the above (n) quantities is (m), then there exists (n-m) dimensionless parameters called (π) such that;

$f\ (\pi_1, \pi_2, \ldots \pi_{n-m}) = 0$ ....…………… (4.4)

Where each (π) term is a non-dimensional (dimensionless) product involving the above quantities.

The method of determining the π-parameters is to select (m) of the (A) quantities with different dimensions that containing among them the (m) dimensions and use them as "Repeating Variables" together with one of the other (A) quantities for each (π). Thus;

$$\left.\begin{aligned} \pi_1 &= A_1^{x_1} A_2^{y_1} A_3^{z_1} A_4 \\ \pi_2 &= A_1^{x_2} A_2^{y_2} A_3^{z_2} A_5 \\ &\vdots \\ \pi_{n-m} &= A_1^{x_{n-m}} A_2^{y_{n-m}} A_3^{z_{n-m}} A_n \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(4.5)}$$

(A_1, A_2, A_3) are the repeating variables

The exponents are to be determined such that (π) is dimensionless.

The Procedure for the Application of the π-Theorem:

1- Select the (n) variables involved in the problem.
2- Write the functional relationship F (A_1, A_2… An) =0.
3- Identify the dependent variable of the problem.
4- Select the repeating variables;
   a) They must contain the dimensions of the problem, collectively.
   b) Each repeating variable must have different dimensions from the others.
   c) They must be physically the most significant variables.
      Ex.
   d) They must not contain the dependent variable.
   e) They must contain least dimensions (Ex. (v, g) choose v).
   f) They should not form a π- term by themselves.

g) It is essential that no one of the repeating variables be derivable from the others.

h) A variable of minor importance should not be selected as repeating variables.

5- Write the π-parameters in terms of the unknown exponents;

$$\pi_1 = A_1^{x_1} A_2^{y_1} A_3^{z_1} A_4$$

$$\pi_2 = A_1^{x_2} A_2^{y_2} A_3^{z_2} A_5$$

$$\vdots$$

$$\pi_{n-m} = A_1^{x_{n-m}} A_2^{y_{n-m}} A_3^{z_{n-m}} A_n$$

6- For each π -expression, writ the equations of the exponents so that the sum of the exponents of each dimension will be zero.

7- Solve the equations simultaneously.

8- Substitute back into the π - expressions of step (5-).

9- Establish the functional relationship [f $(\pi_1, \pi_2, \dots, \pi_{n-m})$ =0] or solve for one of the (π's) explicitly; for example;

$$\pi_2 = f(\pi_1, \pi_3, \dots \dots, \pi_{n-m})$$

10- Recombine, if desired to alter the forms of the π - parameters, keeping the same number of π's $[(\pi_1, \pi_2, \pi_3) \rightarrow (c\pi_1, \frac{\pi_2}{c}, \frac{\pi_1 \pi_2}{\pi_3})$, c = constant].

4.2.2 Common Dimensionless Numbers

In a hypothetical fluid flow problem, the pressure difference (ΔP) between two points in the flow field depends on the geometry of the flow (l, l_1, l_2), fluid properties (ρ, μ, σ, K) and the kinematics of the flow (V and g). Thus;

$$F(\Delta p, l, l_1, l_2, \rho, \mu, \sigma, K, V, g) = 0$$

$$\left.\begin{aligned} n &= 10 \\ m &= 3 \end{aligned}\right\} \longrightarrow \text{No. of } \pi\text{'s} = 10 - 3 = 7$$

Dependent Variable = Δp

Repeating Variables = ρ, V, l

$$\pi_1 = \rho^{x1} V^{y1} l^{z1} \Delta P \longrightarrow \pi_1 = \frac{\Delta p}{\rho V^2}$$

Or: $\pi_1 = \frac{\Delta p}{\frac{1}{2}\rho V^2}$, which is called the (**Pressure Coefficient Cp**) i.e.: $C_p = \frac{\Delta p}{\frac{1}{2}\rho V^2} = \frac{\text{Pressure Force Fp}}{\text{Inertia Force Fi}}$ ………………… (4.6)

$F_p = \Delta p \times A \propto \Delta p \times l^2$

$F_i = ma \propto \rho l^3 \times \frac{l}{t^2} \propto \rho l^2 (\frac{l}{t})^2 \propto \rho l^2 v^2 \}$ $\longrightarrow$ $C_p = \frac{F_p}{F_i} = \frac{\Delta p}{\frac{1}{2}\rho V^2}$

Euler number (E) is usually defined as;

$E = \frac{V}{\sqrt{\Delta p/\rho}} = \sqrt{\frac{1}{\frac{1}{2}Cp}}$ ……………….. (4.7)

$\pi_2 = \rho^{x2} V^{y2} l^{z2} \mu$ $\longrightarrow$ $\pi_2 = \frac{\mu}{\rho l v}$

Or: $\pi_2 = \frac{\rho l v}{\mu}$, which is called the (**Reynolds Number Re**)

i.e.: $R_e = \frac{\rho l v}{\mu} = \frac{\text{Inertia Force Fi}}{\text{Viscous Force Fv}}$ .....…………………. (4.8)

Viscous Force $Fv = \tau A \propto \mu \frac{dv}{dy} A \propto \mu \frac{v}{l} l^2 \propto \mu l v$

Thus; Re=$\frac{Fi}{Fv} = \frac{\rho l^2 v^2}{\mu l v} = \frac{\rho l v}{\mu}$

At high (Re), the effect of viscosity is small and the (Re) effect is negligible because the intertie effect is large. High (Re) means either (v) is large or (μ) is small or (l) is large.

$\pi_3 = \rho^{x_3} v^{y_3} l^{z_3} g \rightarrow \pi_3 = \frac{gl}{v^2}$

Or;

$\pi_3 = \frac{v}{\sqrt{gl}}$, which is called the (**Froud Number Fr**)

i.e.;

$Fr = \frac{v}{\sqrt{gl}} = \frac{\text{Inertia Force} \quad \text{Fi}}{\text{Gravity Force} \quad \text{Fg}}$ ….. . … … ….. . (4.9)

Gravity Force Fg = weight = ɤγ∀ ∝ $\rho g\, l^3$

Thus;

$Fr = \sqrt{\frac{Fi}{Fg}} \sqrt{\frac{\rho\, l^2 v^2}{\rho g\, l^3}} = \frac{v}{\sqrt{gl}}$

(Fr) is used when there is density stratification, such as between salt water and fresh water.

$$\pi_4 = \rho^{x_4} v^{y_4} l^{z_4}\, \sigma \;\rightarrow\; \pi_4 = \frac{\sigma}{\rho l v^2}$$

Or;

$$\pi_4 = \frac{\rho l v^2}{\sigma}$$ which is called the (**Weber Number W**)

i.e.;

$$W = \frac{\rho l v^2}{\sigma} = \frac{\text{Inertia Force} \quad \text{Fi}}{\text{Surface Tension Force} \;\; \text{F}\sigma} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(4.10)}$$

$$\text{Surface Tension Force } \text{F}\sigma = \sigma l \;\rightarrow \text{W} = \frac{\text{Fi}}{\text{F}\sigma} = \frac{\rho l^2 v^2}{\sigma l} = \frac{\rho l v^2}{\sigma}$$

Weber number (W) is used when surface tension effects are predominant, such as in capillary tubes and capillary waves in channels.

$$\pi_5 = \rho^{x_5} v^{y_5} l^{z_5}\, k \;\rightarrow\; \pi_5 = \frac{k}{\rho v^2}$$

Or;

$$\pi_5 = \frac{v}{\sqrt{k/\rho}}$$ which is called (**Mach Number M**)

i.e.;

$$M = \frac{v}{\sqrt{k/\rho}} = \frac{V}{C} = \frac{\mathit{Inertia\ Force} \quad Fi}{\mathit{Elastic\ or\ Compressible\ Force} \;\; Fc} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(4.11)}$$

Where: C= speed of sound in fluid = $\sqrt{k/\rho}$

$$\text{Fc} = \Delta\, p_c l^2 = \rho v c\, l^2 \;\rightarrow M = \frac{Fi}{Fc} = \frac{\rho l^2 v^2}{\rho v c l^2} = \frac{v}{c} = \frac{v}{\sqrt{k/\rho}}$$

Mach number effects are important when (M > 0.3 or 0.4). (M) is a measure of the ratio of K.E. of the flow to internal energy of the fluid.

$$\pi_6 = \rho^{x_6} v^{y_6} l^{z_6} l_1 \;\rightarrow\; \pi_6 = \frac{l_1}{l} \;\; or;\; \pi_6 = \frac{l}{l_1} \ldots \text{(4.12)}$$

$$\pi_7 = \rho^{x_7} v^{y_7} l^{z_7} l_2 \;\rightarrow\; \pi_6 = \frac{l_2}{l} \;\; or;\; \pi_7 = \frac{l}{l_2} \ldots \text{(4.13)}$$

π_6 *and* π_7 are geometry parameters.

Hence;

$$f\left(c_p, Re, Fr, W, M, \frac{l}{l_1}, \frac{l}{l_2}\right) = 0$$

Or;

$$\underbrace{c_p}_{\text{Pressure Effect}} = f\Big(\underbrace{Re}_{\text{Viscous Effect}}, \underbrace{Fr}_{\text{Gravity Effect}}, \underbrace{W}_{\text{Surface Tension Effect}}, \underbrace{M}_{\text{Compressibility Effect}}, \underbrace{\frac{l}{l_1}, \frac{l}{l_2}}_{\text{Geometry Effect}}\Big) \quad \text{.......................................} \quad (4.14)$$

Equation (4.14) includes all effects which could be involved in a certain fluid mechanics problem. Usually, not all these effects exist with the same degree of importance, but one of them (or two) usually overcome the other effects. The inertia effect, of course, exists always in all applications, since it is responsible of the fluid flow. The pressure effect is the dependent parameter of the problem.

4.3 Similitude

It is the theory and art of predicting the prototype performance from model tests and observations. It involves the application of dimensionless numbers, such as (Re, Fr, W, and M) to predicate prototype performance from model tests. The similitude involves three types;

I- Geometrical Similitude

The shape of the model is similar to the prototype. The ratio between all corresponding lengths (including roughness elements) between model (m) and prototype) (p) are equal. The ratio (λ_l) is called the "**Scale Ratio**" or "**Model Ratio**". (λ is read as "lambda")

$$\lambda_l = \frac{l_p}{l_m} \quad \text{Length Ratio} \quad \text{...........................} \quad (4.15)$$

$$\lambda_A = \frac{A_p}{A_m} = \frac{l_p^2}{l_m^2} = \lambda_l^2 \quad \text{Area Ratio} \quad \text{...........................} \quad (4.16)$$

$$\lambda_\forall = \frac{\forall_p}{\forall_m} = \frac{l_p^3}{l_m^3} = \lambda_l^3 \quad \text{Volume Ratio} \quad \text{..........................} \quad (4.17)$$

II- Kinematic Similitude

a- The paths of homologous moving particles are geometrically similar.

b- The ratios of velocities and accelerations at homologous particles are equal.

Velocity Ratio $\lambda_v = \frac{v_p}{v_m} = \frac{l_p / t_p}{l_m / t_m} = \frac{\lambda_l}{\lambda_t}$ .... (4.18)

Acceleration Ratio $\lambda_a = \frac{a_p}{a_m} = \frac{l_p / t_p^2}{l_m / t_m^2} = \frac{\lambda_l}{\lambda_t^2}$ .... (4.19)

Discharge Ratio $\lambda_Q = \frac{Q_p}{Q_m} = \frac{l_p^3 / t_p}{l_m^3 / t_m} = \frac{\lambda_l^3}{\lambda_t}$ .... (4.20)

III- Dynamic Similitude

This implies that in geometrically and kinematically similar systems at homologous points the ratio of corresponding forces between the systems are same. This implies that the dimensionless numbers derived in article (4.2.2) are the same between model (m) and prototype (p);

$$\left(Re, Fr, W, M, \frac{l}{l_1}, \frac{l}{l_2}\right)_m = \left(Re, Fr, W, M, \frac{l}{l_1}, \frac{l}{l_2}\right)_p$$

Hence from equ. (4.14);

$$cp_m = cp_p$$

$$\left(\frac{\Delta pA}{\frac{1}{2}\rho v^2 A}\right)_m = \left(\frac{\Delta pA}{\frac{1}{2}\rho v^2 A}\right)_p$$

Or;

$$\left(\frac{F}{\rho v^2 A}\right)_m = \left(\frac{F}{\rho v^2 A}\right)_p$$

$$\frac{F_p}{F_m} = \frac{\rho_p}{\rho_m}\frac{v_p^2}{v_m^2}\frac{A_p}{A_m}$$

Hence; $\lambda_F = \lambda_\rho \lambda_v^2 \lambda_l^2$ .... (4.21)

Equ. (4.21) is a general formula. To find the force ratio between model and prototype we need to calculate the velocity ratio (λv), which is found by equating (Re, Fr, W, M) between (m) and (p). But since these numbers have different forms, each one give different value for (λv), so, we cannot solve the problem by equating all these numbers. Instead, we choose one of them which have greater effect than the others on the problem. Most engineering applications for incompressible fluids, the (Re) and (Fr) are the most important numbers. Each number has certain applications, as will be explained soon.

4.3.1 Re - Criterion

This criterion is applied for engineering applications in which only inertia and viscous effects are important. These include the following;

1- Flow in closed conduits (internal flow).
2- Flow around immersed bodies.
3- Flow in wind and water tunnels.

The problem here is solved by equating (Re); thus;

$$Re_m = Re_p$$

$$\left(\frac{\rho l v}{\mu}\right)_m = \left(\frac{\rho l v}{\mu}\right)_p \rightarrow \frac{v_p}{v_m} = \frac{\mu_p}{\mu_m}\frac{l_m}{l_p}\frac{\rho_m}{\rho_p}$$

Or;

$$\lambda v = \frac{\lambda\mu}{\lambda\rho\lambda l} = \frac{\lambda v}{\lambda l} \qquad \text{........................ (4.22)}$$

Substitute (4.22) into (4.21) gives;

$$\lambda_F = \lambda\rho\,\frac{\lambda_\mu^2}{\lambda_\rho^2\lambda_l^2}\,\lambda_l^2 = \lambda\rho\frac{\lambda_v^2}{\lambda_l^2}\,\lambda_l^2$$

Or;

$$\lambda_F\,\frac{\lambda_\mu^2}{\lambda\rho} = \lambda\rho\,\lambda_v^2 \qquad \text{........................... (4.23)}$$

It is clear from equ. (4.23) that when the same fluid is used for the model and prototype ($\lambda\rho = 1$, $\lambda v = 1$) , the force ratio is ($\lambda_F = 1$), i.e. ($F_p = F_m$). this result is for problems using Re-criterion only.

4.3.2 Fr - Criterion

This criterion is applied for engineering applications in which only inertia and gravity effects are important. These include the following;

1- Open channel flow.
2- Dams and spillways.
3- Hydraulic jump.
4- Ships and boats (neglecting viscous effects).

Now;

$$Fr_m = Fr_p$$

$$\left(\frac{v}{\sqrt{gl}}\right)_m = \left(\frac{v}{\sqrt{gl}}\right)_p \rightarrow \frac{v_p}{v_m} = \sqrt{\frac{g_p l_p}{g_m l_m}}$$

Or;

$$\lambda_v = \sqrt{\lambda_g}\,\sqrt{\lambda_l} \quad \text{..} \ (4.24)$$

Substitute (4.24) into (4.21) gives;

$$\lambda_F = \lambda_\rho \lambda_g \lambda_l \lambda_l^2 \rightarrow \lambda_F = \lambda_\rho \lambda_g \lambda_l^3 \quad \text{.............................} \ (4.25)$$

4.4 Ships Models Tests

The ships are usually exposed to two components of drag. These are viscous (frictional) drag (F_{Dv}), which is a (Re- criterion), and the ware drag (F_{Dw}), which is a (Fr-criterion). Thus, the total drag (F_{DT}) is:

$$\left.\begin{aligned} &F_{DT} = F_{DV} + F_{DW} \\ &\text{And;} \\ &F_{DTm} = F_{DVm} + F_{DWm} \\ &F_{DTp} = \underbrace{F_{DVp}}_{\text{Re-Criterion}} + \underbrace{F_{DWp}}_{\text{Fr-Criterion}} \end{aligned}\right\} \quad \text{.......................................} \ (4.26)$$

To find the velocity ratio (λ_v), we use equs. (4.22) and (4.24). Thus;

Re – Criterion: $\lambda v = \frac{\lambda\mu}{\lambda\rho\lambda l} = \frac{\lambda v}{\lambda l} \longrightarrow V_m > V_p$ (since $\lambda l > 1$)

Fr – Criterion: $\lambda_v = \sqrt{\lambda_g}\,\sqrt{\lambda_l} \longrightarrow V_m < V_p$

So, we cannot solve the problem by equating both (Re + Fr), since each number gives different value. To solve the problem, we test the model according to (Fr- criterion) and use equ. (4.24) for (λv), and calculate the viscous drag (F_{Dv}) from existing relations. The ship model is tested in the laboratory and (F_{DTm}) is measured, with the velocity calculated from equ. (4.24). The following procedure is used to calculate the total drag on the prototype and the power required to drive it;

1- Make a model test according to Fr- criterion, equ. (4.24), and measure (F_{DTm}) in the laboratory.
2- Calculate (F_{DVm}) from analytical or empirical relation.
3- Calculate $F_{DWm} = F_{DTm} - F_{DVm}$.
4- Using Fr- criterion, equ. (4.25), calculate (F_{DWp}), i.e., $F_{DWp} = F_{DWm}\, \lambda_\rho \lambda_g \lambda_l^3$.
5- Calculate (F_{DVp}) from analytical or empirical relations.
6- Calculate ($F_{DTp} = F_{DVp} + F_{DWp}$).
7- Calculate the required (Shaft Power = $F_{DTp} V_p$ / η_p), where (η_p) is the propeller efficiency.

Note:

1. All problems of ships models tests which consider the viscous (F_{DV}) and gravity (F_{DW}) effects are solved by following the procedure described above (steps 1 – 7). The simple difference is in the relation used to calculate (F_{DV}), steps 2 and 5.
2. If we neglect the viscous effects (F_{DV}) and consider the gravity effects (F_{DW}) only, then the problem is solved according to Fr – Criterion (Article 4.3.2).

Examples

Example (4.1): The drag force (F_D) on a sphere moving in a fluid depends on the sphere diameter (D) and velocity (V), and the fluid density (P) and viscosity (M). Find the functional relationship for the drag force (F_D).

<u>Sol.</u>

$F\,(F_D, V, D, \rho, \mu) = 0$

$F_D \rightarrow MLT^{-2}$, $V \rightarrow LT^{-1}$, $D \rightarrow L$, $\rho \rightarrow ML^{-3}$, $\mu \rightarrow ML^{-1}T^{-1}$

$$\left.\begin{array}{l} n = 5 \\ m = 3 \end{array}\right\} \longrightarrow \text{No. of } \pi\text{'s} = n - m = 5 - 3 = 2$$

Dependent variable = F_D

Repeating variable = m = 3 = (ρ, V, D)

ρ → Properties; V → Kinematics & Dynamic Aspects; D → Geometry

$\pi_1 : \rho^{x1}\, V^{y1}\, D^{z1}\, F_D$

$\pi_2 : \rho^{x2}\, V^{y2}\, D^{z2}\mu$

$\pi_1 : M^o\, L^o\, T^o = (ML^{-3})^{x1}\, (LT^{-1})^{y1}\, (L)^{z1}\, MLT^{-2}$

$$\left.\begin{array}{l} M: 0 = x_1 + 1 \\ L: 0 = -3\,x_1 + y_1 + z_1 + 1 \\ T: 0 = -y_1 - 2 \end{array}\right\} \longrightarrow \begin{array}{l} x_1 = -1 \\ y_1 = -2 \\ z_1 = -2 \end{array}$$

Thus;

$$\pi_1 = \rho^{-1}\, V^{-2}\, D^{-2}\, F_D \longrightarrow \pi_1 = \frac{F_D}{\rho V^2 D^2} = C_D \quad \text{Drag Coefficient}$$

$\pi_2 : M^o\, L^o\, T^o = (ML^{-3})^{x2}\, (LT^{-1})^{y2}\, (L)^{z2}\, ML^{-1}T^{-1}$

$$\left.\begin{array}{l} M: 0 = x_2 + 1 \\ L: 0 = -3\,x_2 + y_2 + z_2 - 1 \\ T: 0 = -y_1 - 1 \end{array}\right\} \longrightarrow \begin{array}{l} x_2 = -1 \\ y_2 = -1 \\ z_2 = -1 \end{array}$$

Thus;

$$\pi_2 = \rho^{-1}\, V^{-1}\, D^{-1}\mu \qquad \pi_2 = \frac{\mu}{\rho V D}$$

Or; $\pi_2 = \frac{1}{\pi_2} = \frac{\rho VD}{\mu} \longrightarrow \quad \pi_2 = \frac{\rho VD}{\mu} = Re$ Reynolds Number

Thus;

f (π_1, π_2) = 0 $\longrightarrow$ f (C_D, Re) = 0

C_D = f (Re) The function (f) is found from experiments

Example (4.2): A(1/5) scale model automobile is tested in a wind tunnel with the same air properties as the prototype. The air velocity in the tunnel is (350 km/h), and the measured model drag is (350 N). Determine the drag of the prototype automobile and the power required to overcome this drag.

Sol.:

Use Re-criterion (equs. 4.22 and 44.23).

$Re_m = Re_p$

Equ. (4.22): $\lambda v = \frac{\lambda\mu}{\lambda\rho\lambda l}$

Since the same air is used ($\lambda\mu = 1$ & $\lambda\rho = 1$), thus;

$$\lambda v = \frac{1}{\lambda_l} \rightarrow v_p = v_m \frac{1}{\lambda l} = 350 * \frac{1}{5} \rightarrow v_p = 70\ km/h$$

Equ. (4.23):

$$\lambda_F \frac{\lambda_\mu^2}{\lambda\rho} = 1 \rightarrow F_p = F_m = 350\ N$$

Power = $F_p v_p = 350 * 70 * \frac{1000}{3600} \rightarrow$ Power $= 6805.6$ W

Example (4.3): The wave resistance of a model of a ship at (1:25) scale is (7 N) at a model speed of (1.5 m/s). What are the corresponding velocity and wave resistance of the prototype ? Assume the model is tested in fresh water (ρ = 1000 kg/m^3) and the ship will operate in the ocean where (s = 1.03) for the water.

Sol.:

Use Fr-criterion (equs. 4.24 and 44.25).

$Fr_m = Fr_p$

Equ. (4.24): $\lambda_v = \sqrt{\lambda_g}\sqrt{\lambda_l} \rightarrow V_p = 1.5\sqrt{1}\sqrt{25} \rightarrow V_p = 7.5$ m/s

Equ. (4.25): $\lambda_F = \lambda_\rho\lambda_g\lambda_l^3 \rightarrow F_p = 7 * 1.03 * 1 * 25^3 \rightarrow F_p = 112656$ N

Example (4.4): A (10 m) model of an ocean tanker (500 m) long is dragged through fresh water at (3 m/s) with a total measured resistance of (103 N). The surface drag coefficient (c_f) for model and prototype are (0.245) and (0.0147) respectively, in the equation ($F = c_f A V^2$). The wetted surface area of the model is (20 m^2). Find the total drag on the tanker and the power required at the propeller shaft assuming an efficiency of (90 %) for the propeller.

Sol.:

$F_{DVm} = c_{fm} A_m V_m^2 = 0.245 * 20 * 3^2 \rightarrow F_{DVm} = 44.1$ N

$F_{DWm} = F_{DTm} - F_{DVm} = 103 - 44.1 \rightarrow F_{DWm} = 58.9$ N

Using Fr-criterion;

$V_p = V_m\sqrt{\lambda_g}\sqrt{\lambda_l} = 3\sqrt{1}\sqrt{500/10} \rightarrow V_p = 21.2$ m/s

$F_{DWp} = F_{DWp}\,\lambda_\rho\lambda_g\lambda_l^3 \rightarrow F_{DWp} = 58.9 * 1 * 1 * (500/10)^3 \rightarrow F_{DWp} = 7362500$ N

$\lambda_A = \frac{A_p}{A_m} = \frac{l_p^2}{l_m^2} = \lambda_l^2 = (\frac{500}{10})^2 \rightarrow A_p = 20 * 50^2 \rightarrow A_p = 50000\ m^2$

$F_{DVp} = c_{fp} A_p V_p^2 = 0.0147 * 50000 * 21.2^2 \rightarrow F_{DVp} = 330338.4$ N

$F_{DTp} = F_{DVp} + F_{DWp} \rightarrow F_{DTp} = 7692838.4$ N

Shaft Power = $F_{DTp} * V_p / \eta_p \rightarrow$ Shaft Power = 181.2 MW = $243 * 10^4$ hp

Problems

The problems number listed in the table below refer to the problems in the "textbook", Chapter "4";

Article No.	Related Problems
4.1	5,6
4.2	2,8,10,11,12,13,17,20
4.3	18,21,23,26,33,35

Chapter -5-

Viscous Fluid Flow

5.1 Introduction

In viscous flow, the viscous (frictional) effects are very important and must be considered. Real (viscous) fluid flow may be either laminar or turbulent. In laminar flow (low Re) the influence of viscosity is predominant. In turbulent flow (high Re) the inertial effects are predominant.

5.2 Equations of Motion for Viscous Flow

These equations must include the following forces:

1. Pressure forces.
2. Inertia forces.
3. Viscous (shear) forces.
4. Body forces.
5. External forces.

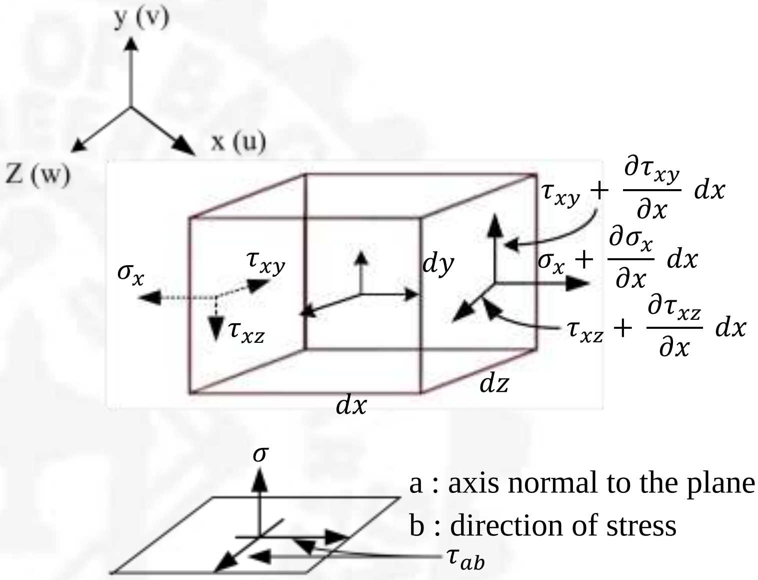


Fig. (5.1)

<u>For each plane:</u>

(1) Normal stress (σ).

(2) Shear stresses (τ_{ab}).

Thus;

6 surfaces → 12 Shear forces

6 Normal forces

Also, there is; 3 Body forces (B_x, B_y, B_z) per unit mass

3 External forces (Fe_x, Fe_y, Fe_z)) per unit mass

Hence; the total forces = 24 forces

Now; u=u (x, y, z, t), v = v (x, y, z, t) w = w (x, y, z, t)

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial u}{\partial z}\frac{\partial z}{\partial t} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} \quad ..(5.1)$$

$$a_y = \frac{dv}{dt} = \frac{\partial v}{\partial t} + \frac{\partial v}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial v}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial v}{\partial z}\frac{\partial z}{\partial t} = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} \quad ..(5.2)$$

$$a_z = \frac{dw}{dt} = \frac{\partial w}{\partial t} + \frac{\partial w}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial t} = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} \quad ..(5.3)$$

Stoke's law;

$$\sigma_x = -p - \frac{2}{3}\mu\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) + 2\,\mu\,\frac{\partial u}{\partial x} \qquad \ldots\ldots\ldots(5.4)$$

$$\sigma_y = -p - \frac{2}{3}\mu\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) + 2\,\mu\,\frac{\partial v}{\partial y} \qquad \ldots\ldots\ldots(5.5)$$

$$\sigma_z = -p - \frac{2}{3}\mu\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) + 2\,\mu\,\frac{\partial w}{\partial z} \qquad \ldots\ldots\ldots(5.6)$$

$$\tau_{xy} = \tau_{yx} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \qquad \ldots\ldots\ldots(5.7)$$

$$\tau_{yz} = \tau_{zy} = \mu\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) \qquad \ldots\ldots\ldots(5.8)$$

$$\tau_{xz} = \tau_{zx} = \mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \qquad \ldots\ldots\ldots(5.9)$$

For incompressible fluids, the continuity equation reduces to;

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad \ldots\ldots\ldots(5.10)$$

Now, the equations of motion for viscous flow are the Newton's 2nd law. Thus;

$$\sum F_x = \mathrm{ma_x}; \qquad \sum F_y = \mathrm{ma_y}; \qquad and \sum F_z = \mathrm{ma_z} \qquad \ldots\ldots\ldots \quad (5.11)$$

Applying (5.11) on the cubical element (Fig.(5.1)), and using the relations (5.1) to (5.10), simplification and re-arranging yields the following equations of motion for viscous incompressible fluids. They are known as " **Navier Stokes**" equations;

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{p}\frac{\partial p}{\partial x} + B_x + Fe_x + \frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)..(5.12)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{1}{p}\frac{\partial p}{\partial y} + B_y + Fe_y + \frac{\mu}{\rho}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)..(5.13)$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1}{p}\frac{\partial p}{\partial z} + B_z + Fe_z + \frac{\mu}{\rho}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)..(5.14)$$

Local Acclamation — Convective Acceleration — Pressure Forces — Body Forces — External Forces — Viscous forces

Inertia forces

5.3 Laminar Flow Between Parallel Plates: "Couette Flow"

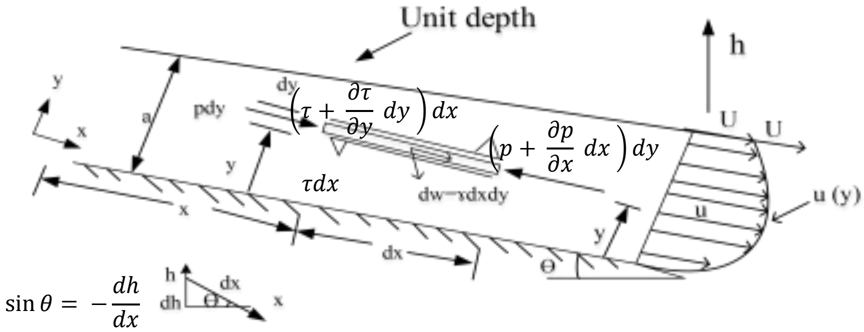


Fig. (5.2)

Assumptions:

1. Steady incompressible flow ($\frac{\partial u}{\partial t} = 0, \rho = constant$)
2. One – dimensional flow (v=0, w=0) and ($\frac{\partial}{\partial z} = 0$)
3. Fully – developed flow ($\frac{\partial u}{\partial x} = 0, u = u(y) only$)
4. Newtonian fluid ($\mu = constant = \frac{\tau}{du/dy}$)
5. Parallel flow ($\frac{\partial p}{\partial y} = 0$)

Applying the Newton's law for the element of **Fig.(5.2)**;

$$\sum F_x = ma_x = 0 \ (since\ a_x = 0\ from\ \mathbf{equ.(5.1)})$$

Thus;

$$pdy - \left(p + \frac{dp}{dx}\,dx\right)dy - \tau dx + \left(\tau + \frac{d\tau}{dy}\,dy\right)dx + \gamma dx\,dy\,\sin\theta = 0$$

Using (sin θ = - dh/dx) and simplification yields;

$$\frac{d\tau}{dy} = \frac{d}{dx}\,(p + \gamma h) \qquad \text{…………....(5.15)}$$

but;

$\tau = \mu \frac{du}{dy}$ (Newton's law of viscosity). Thus;

$$\mu \frac{d^2u}{dy^2} = \frac{d}{dx}\,(p + \gamma h) \qquad \text{…………....(5.16)}$$

Integrate **equ. (5.16)** twice to obtain;

$$u = \frac{y^2}{2\mu}\frac{d}{dx}(p+\gamma h) + \frac{A}{\mu}y + B \qquad \text{...........(5.17)}$$

B.Cs. ; at y=0 u=0 →B=0

at y=a u=U →A=$\frac{\mu U}{a} - \frac{a}{2}\frac{d}{dx}(p+\gamma h)$

Thus; equ. (5.17) gives;

$$u = \frac{Uy}{a} - \frac{(ay-y^2)}{2\mu}\frac{d}{dx}(p+\gamma h) \qquad \text{...........(5.18)}$$

The volume flow rate (Q) across the section is ;

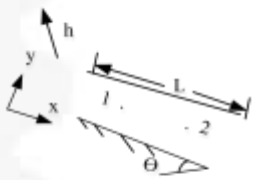
$$Q = \int_0^a u\,dy.1 \;\rightarrow Q = \frac{Ua}{2} - \frac{a^3}{12\mu}\frac{d}{dx}(p+\gamma h) \qquad \text{...........(5.19)}$$

The shear stress $\tau = \mu\frac{du}{dy} \rightarrow \tau = \frac{\mu U}{a} - \frac{(a-2y)}{2}\frac{d}{dx}(p+\gamma h)$ ....(5.20)

The term $\frac{d}{dx}(p+\gamma h)$ in the above equation is constant for each flow. It can be calculated by one of the following two methods;

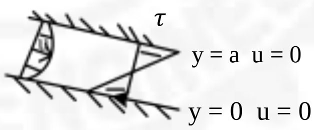
1. $\frac{d}{dx}(p+\gamma h) = \frac{(p+\gamma h)_2-(p+\gamma h)_1}{L} = (-ve\ value)\ (p+\gamma h)_1 > (p+\gamma h)_2$

2. $\frac{d}{dx}(p+\gamma h) = \frac{dp}{dx} + \gamma\frac{dh}{dx} = \frac{p_2-p_1}{L} - \gamma\sin\theta = (-ve\ value)$



Note; There are many other cases for **equ.(5.17)** with different boundary conditions; these are :

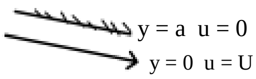
1. Fixed plates (U=0)



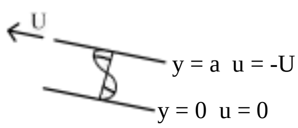
$$u = \frac{y^2-ay}{2\mu}\frac{d}{dx}(p+\gamma h)$$

$$Q = -\frac{a^3}{12\mu}\frac{d}{dx}(p+\gamma h)$$

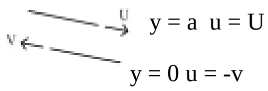
2. Lower plate moving.



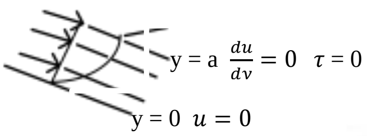
3. Upper plate moves upward when the shaded areas are equal, then (Q=0)



4. Both plates move in opposite directions.



5. Free surface flow



5.4 Losses in Laminar Flow

For steady laminar flow, the reduction in $(p + \gamma h)$ represents the work done on fluid per unit volume. The work done is converted into irreversibilities (losses) by the action of viscous shear. Thus, the net work done and the loss of potential energy represents the losses per unit time due to irreversibilities.

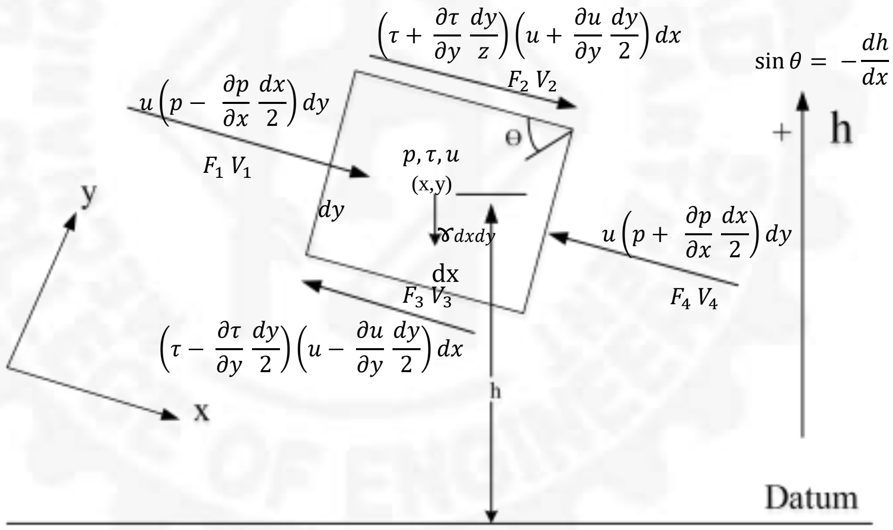


Fig. (5.3): Work Done and Loss of P.E. for a Fluid Particle

Net Power = $F_3V_3 + F_2V_2 - F_1V_1 - F_2V_2 - \gamma dxdy \sin\theta\, u$ ………….. (5.21)

Substitute for each term in equ. (5.21), simplifying and neglecting higher order terms and dividing by the element volume, and using equ.(5.15), it is obtained;

$$\frac{Net\ Power}{Unit\ Volume} = -\tau\frac{du}{dy} = -\mu\left(\frac{du}{dy}\right)^2 = -\frac{\tau^2}{\mu} \qquad \text{……………. (5.22)}$$

For flow between two fixed parallel plates;

$u = \frac{y^2-ay}{2\mu}\frac{d}{dx}(p+\gamma h)$ and $Q = -\frac{a^3}{12\mu}\frac{d}{dx}(p+\gamma h)$

Thus;

$$Net\ Power = -\int_0^a \mu(\frac{du}{dy})^2\,dy = -\frac{a^3}{12\mu}\left[\frac{d}{dx}(p+\gamma h)\right]^2 = Q\,\frac{d}{dx}(p+\gamma h)$$

Hence;

$$Net\ Power\ Input\ = Q\frac{d}{dx}(p+\gamma h) \qquad \text{.......... (5.23)}$$

The power done by the fluid between (1) and (2) is;

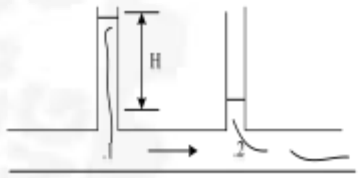
$$Net\ Power_{1-2} = \int_{x_2}^{x_2} Q\,\frac{d}{dx}(p+\gamma h)\,dx = Q\frac{d}{dx}(p+\gamma h)*L = Q\frac{\Delta(p+\gamma h)}{L}L$$

Thus;

$$Net\ Power_{1-2} = Q\Delta(p+\gamma h) = losses_{\ 1-2} \qquad \text{. ....(5.24)}$$

For horizontal flow, $(h_1 = h_2)$;

$$Net\ Power_{1-2} = Q\Delta p = Q\gamma H \qquad \text{........(5.25)}$$



Note:

Equations (5.24) and (5.25) can be derived using the energy equation from (1)to (2);

$$\frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2 + h_l \rightarrow h_l = \frac{p_1-p_2}{\gamma} + (z_1 - z_2)$$

Thus;

$$Net\ Power = losses_{\ 1-2} = \gamma Q h_l = Q[p_1 - p_2 + \gamma\,(z_1 - z_2)]$$

$$= Q\Delta(p+\gamma h) \qquad \text{......................... (5.24)}$$

For horizontal flows; $h_l = \frac{p_1-p_2}{\gamma}$ $\quad (z_1 = z_2)$

Thus;

$$Net\ Power = losses_{\ 1-2} = Q\Delta p = \gamma Q H \qquad \text{........... (5.25)}$$

5.5 Laminar Flow through Circular Tubes and Annuli

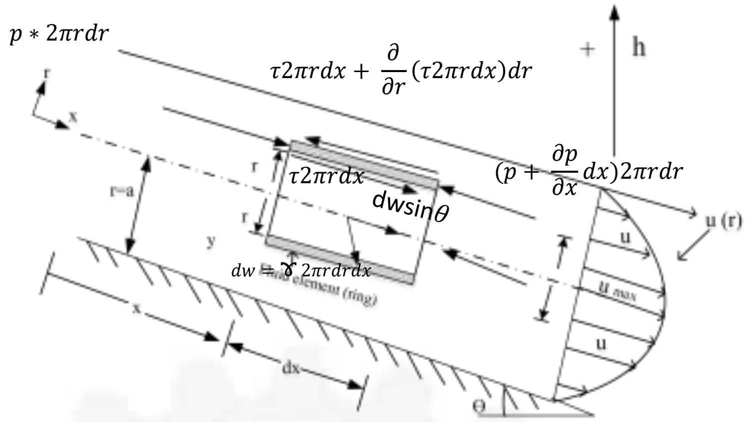


Fig. (5.4)

The same assumptions used in article (5.3) for flow between parallel plates are used and applied here. The difference is that the y-axis is replaced by the r- axis.

$$\sum F_x = \mathrm{m}a_x = 0 \; (since\ a_x = 0 \; (\text{from } \mathbf{equ.(5.1)}))$$

$$\mathrm{p} * 2\pi rdr - \left(p + \frac{\partial p}{\partial x}dx\right)2\pi rdr + \tau * 2\pi rdx - \left(\tau.2\pi rdx + \frac{d}{dr}\,(\tau * 2\pi rdx)dr\right) + \gamma * 2\pi rdrdx \sin\theta = 0$$

Thus;

$$\frac{d}{dx}(p + \gamma \mathrm{h}) + \frac{1}{r}\frac{d}{dr}\,(\tau r) = 0 \qquad \text{.................. (5.26)}$$

Integrate once;

$$\frac{r^2}{2}\frac{d}{dx}(p + \gamma \mathrm{h}) + \tau r = A \qquad \text{.................... (5.27)}$$

But;

$$\tau = \mu\frac{du}{dy} = -\mu\frac{du}{dr}$$

Hence, (5.27) becomes;

$$\frac{du}{dr} = \frac{r}{2\mu}\frac{d}{dx}(p + \gamma \mathrm{h}) - \frac{A}{\mu r}$$

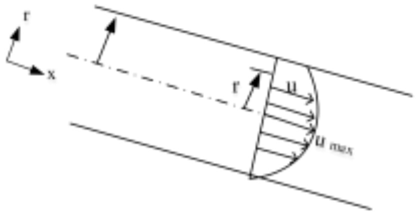
Integrate;

$$u = \frac{r^2}{4\mu}\frac{d}{dx}(p + \gamma \mathrm{h}) - \frac{A}{\mu r}\,ln\,(r) + B \qquad \text{................... (5.28)}$$

Equation (5.28) is applied for both circular tubes and annuli the difference is in the boundary conditions.

5.5.1. Circular Tubes: Hagen - Poiseuille Equation

The boundary conditions for equation (5.28) are;



$$r = 0 \;\; \frac{du}{dr} = 0 \;\rightarrow A = 0$$

$$r = a \;\; u = 0 \;\;\rightarrow B = -\frac{a^2}{4\mu}\frac{d}{dx}(p + \gamma h)$$

Thus:

$$u = -\frac{a^2 - r^2}{4\mu}\frac{d}{dx}(p + \gamma h) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(5.29)}$$

The maximum velocity (u_{max}) is at (r=0);

$$u_{max} = -\frac{a^2}{4\mu}\frac{d}{dx}(p + \gamma h) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\text{(5.30)}$$

The average velocity (V) is;

$$V = \frac{1}{A}\int u dA = \frac{1}{\pi a^2}\int_0^a -\frac{a^2 - r^2}{4\mu}\frac{d}{dx}(p + \gamma h) * 2\pi r dr$$

Hence;

$$V = -\frac{a^2}{8\mu}\frac{d}{dx}(p + \gamma h) = \frac{u_{max}}{2} \qquad \ldots\ldots\ldots\text{(5.31)}$$

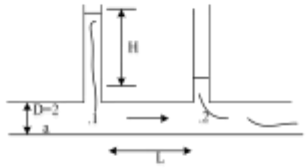
Thus; equ. (5.29) can be written as ;

$$u = u_{max}\left(1 - \frac{r^2}{a^2}\right) = 2v\left(1 - \frac{r^2}{a^2}\right) \qquad \ldots\ldots\ldots\text{(5.32)}$$

Now, the volume flow rate (Q) is;

$$Q = \int_0^a u * 2\pi r dr \rightarrow Q = -\frac{\pi a^2}{8\mu}\frac{d}{dx}(p + \gamma h) = \pi a^2 v = \pi a^2 \frac{u_{max}}{2} \qquad \ldots\text{(5.33)}$$

For horizontal tubes; h= constant;



$$\frac{d}{dx}(p + \gamma h) = \frac{dp}{dx} + \gamma\frac{dh}{dx} = \frac{p_2 - p_1}{L} = -\frac{p_1 - p_2}{L} = -\frac{\Delta p}{L}$$

Hence $Q = \frac{\Delta p\, \pi D^4}{128\, \mu L}$ Hagen - Poiseuilli equation .... (5.34) (horizontal tubes)

Thus;

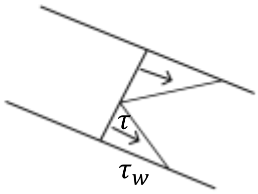
$$\Delta p = \frac{128\,\mu L Q}{\pi D^4} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\text{(5.35)}$$

And ;

$$v = \frac{\Delta p D^2}{32\mu L} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(5.36)}$$

for horizontal tubes

The shear stress (τ) is calculated as ;



$$\tau = \mu \frac{du}{dy} = -\mu \frac{du}{dr} = -\frac{r}{2}\frac{d}{dx}(p + \gamma h) \qquad \text{...........(5.37)}$$

Notes:

1. The term $\frac{d}{dx}(p + \gamma h)$ is calculated as in article (5.3) for flow between parallel plates.

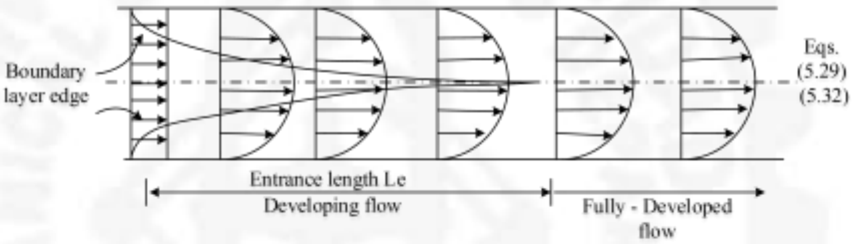
2. For flow through circular pipes and tubes;

Re < 2000 laminar flow.

2000 < Re< 4000 transition flow.

Re> 4000 turbulent flow.

3. Equations (5.29) and (5.32) are for fully - developed flow, for which ($\frac{du}{dx} = 0$), which occurred far from the entrance of the pipe.



$$\frac{Le}{D} = 0.058 \quad Re$$

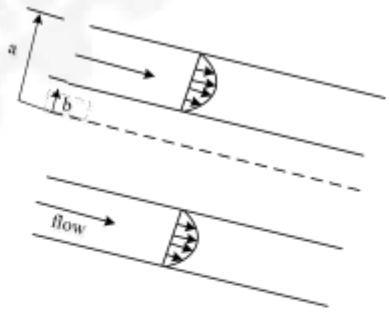
5.5.2 Annuli

The boundary conditions for **equ.(5.28)** are;

At r = a u = 0

r = b u = 0

Thus, we can show that;



$$u = -\frac{1}{4\mu}\frac{d}{dx}(p + \gamma h)\left[a^2 - r^2 + \frac{a^2 - b^2}{lnblna}\ln\frac{a}{r}\right] \qquad \text{............ (5.38)}$$

$$Q = \int_b^a u\, 2\pi r dr = -\frac{\pi}{8\mu}\frac{d}{dx}(p + \gamma h)\left[a^4 - b^4 + \frac{(a^2 - b^2)^2}{lnblna}\right] \qquad \text{.......... (5.39)}$$

5.6 Boundary –Layer Flow

Boundary –layer (b.l.) is a very thin layer of fluid adjacent to the surface, in which the viscosity effects are important. The flow inside the boundary –layer is viscous (real) flow, while outside the boundary-layer (main stream) the flow is inviscid (frictionless, ideal).

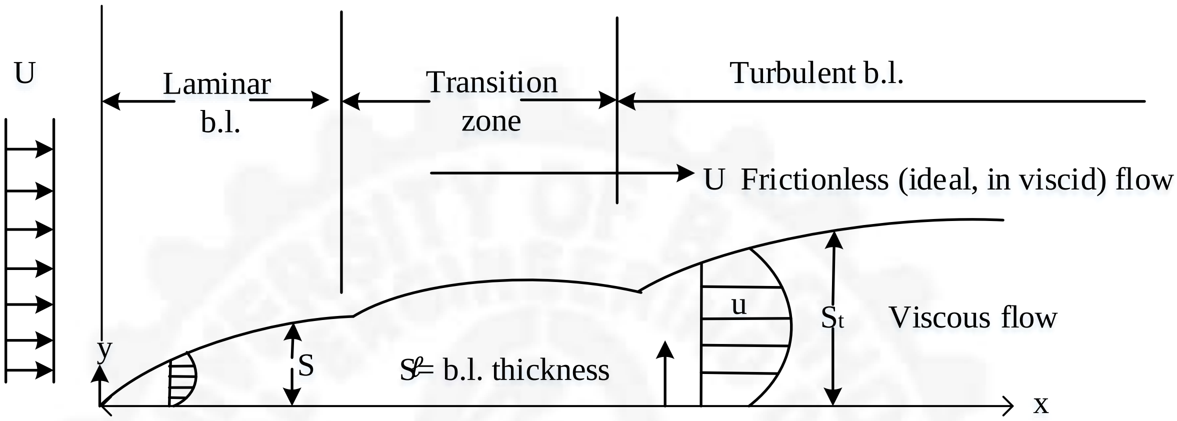


Fig.(5.5): Growth of b.l. along a Smooth Flat Plate

To derive the equations of boundary –layer, we will apply the M.E to a control volume of the b.l over a surface.

$\sum F_x = \frac{\partial}{\partial t}(mu)_{c.v.} + (\dot{m}u)_{out} - (\dot{m}u)_{in}$

For steady flow of incompressible fluid along one side of a smooth plate, the only force acting is due to drag or shear at the plate, since the pressure is constant across the b.l. and around the periphery of the c.v. of **Fig.(5.6)**. Thus, for unit depth of the plate;

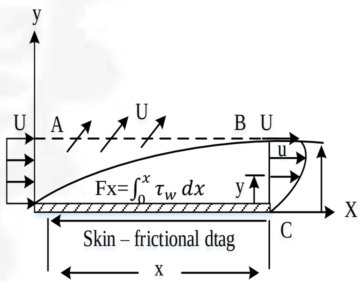


Fig.(5.6)

$\sum F_x = -F_x = -\int \tau_w \, dx = (\dot{m}u)_{BC_x} + (\dot{m}u)_{AB_x} - (\dot{m}u)_{AD_x}$.....(5.40)

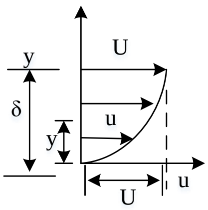
C.S.	$\dot{m}$	$(\dot{m}u)$
AD	$\rho U\delta$	$\rho U^2\delta$
BC	$\rho\int_0^\delta u dy$	$\rho\int_0^\delta u^2 dy$
AB	$\rho U\delta - \int_0^\delta \rho u dy$	$\rho U^2\delta - \rho U\int_0^\delta u dy$

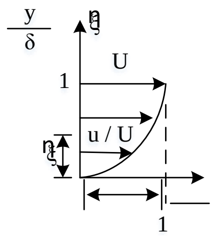
Hence, **equ.(5-40)** gives;

$F_x=\int_0^x \tau_w\, dx = \rho \int_0^\delta u(U-u)dy = \rho U^2 \int_0^\delta \frac{u}{U}(1-\frac{u}{U})dy$............ (5.41)

The velocity profile inside the b.l.$(\frac{u}{U})$is usually given as;

$(\frac{u}{U}) = F(\frac{y}{\delta})=F(\eta)$





Where $\eta = \frac{y}{\delta} \Rightarrow y = \delta\eta$ &$dy= \delta d\eta$

$y=0 \quad \eta = 0$

$y= \delta \quad \eta = 1$

Thus**, equ.(5.41)** becomes;

$Fx=\int_0^x \tau_w\, dx = \rho U^2 \delta \int_0^1 \frac{u}{U}(1-\frac{u}{U})d\eta$

$$\frac{dF_x}{d_x} = \frac{d}{d_x}\int_0^x \tau_w\, dx = \rho U^2 \frac{d\delta}{dx}\int_0^1 \frac{u}{U}(1-\frac{u}{U})d\eta$$

Hence;

$\tau_w = \rho U^2 \frac{d\delta}{dx}\int_0^1 \frac{u}{U}(1-\frac{u}{U})d\eta$.................. (5.42)

Equation (5.42) is the M.E for 2-D steady flow along a flat plate (b.l. equation). The problem of the b.l. is to find;

1. Boundary –layer thickness along the plate $\delta(x)$
2. The skin-frictional force $(F_x = \int_0^x \tau_w\, dx)$

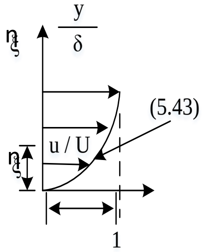
Equation (5-42) is a general equation applicable for both laminar and turbulent boundary layers. The only difference is in the form of the velocity profile $(\frac{u}{U})$.

5.6.1 Laminar Boundary – Layer

The velocity profile inside the laminar b.l. is given by the following formula;

$\frac{u}{U} = F(\eta) = \frac{3}{2}\eta - \frac{\eta^3}{2} \quad 0\le y \le \delta =1 \quad y> \delta$ .... (5.43)

(5.43) in (5.42) gives;



$\tau_w = \rho U^2 \frac{d\delta}{dx}\int_0^1 (\frac{3}{2}\eta - \frac{\xi^3}{2})(1-\frac{3}{2}\eta+\frac{\eta^3}{2})d\eta=0.139\, \rho U^2 \frac{d\delta}{dx}$....... (5.44)

At the wall;

$$\tau_w = \mu \frac{\partial u}{\partial y}|_{y=0} = \mu \frac{\partial u}{\partial \eta}\frac{\partial \eta}{dy}|_{\eta=0} = \frac{\mu}{\delta}\frac{\partial}{\partial \eta}(\frac{3}{2}\eta - \frac{\eta^3}{2})$$

Thus;

$\tau_w = \frac{3}{2}\,\mu\frac{U}{\delta}$ ………… (5.45)

Equation (5.44) and (5.45) gives;

$\delta d\delta = 10.78\,\frac{\mu}{\rho U}dx$

Integrate with B.C ($x=0, \delta_l = 0$);

$\frac{\delta_l}{x} = \frac{4.65}{\sqrt{Re_x}}$ Where: $Re_x = \frac{\rho u x}{\mu} = \frac{ux}{v}$………………………………… (5.46)

Substitute (5.46) into (5.45) gives;

$\tau_w = 0.322\sqrt{\frac{\mu\rho U^3}{x}}$……………………………….. (5.47)

The skin-frictional drag force ($F_{D_{f_l}}$) is;

$F_{D_{f_l}} = \int_0^L \tau_w dx = 0.644\sqrt{\mu\rho U^3 L}$……………….. (5.48)

The skin-frictional drag coefficient (C_{D_f}) is;

$C_{D_{f_l}} = \frac{F_{D_{F_l}}}{\frac{1}{2}\rho U^2 L.1} = \frac{1.328}{\sqrt{Re_L}}$ Where: $Re_L = \frac{\rho u L}{u} = \frac{uL}{v}$………………………………… (5.49)

Note:

For Re<5*10^5 Laminar b.l.

$5*10^5 < \text{Re} < 1*10^6$ Transition

Re$> 1*10^6$ Turbulent b.l.

5.6.2 Turbulent Boundary –Layer

The velocity profile is given by the 1/7th power law;

$\frac{u}{U} = (\frac{y}{\delta})^{1/7} = \eta^{1/7}$………………… (5.50)

(5.50) into (5.42) gives;

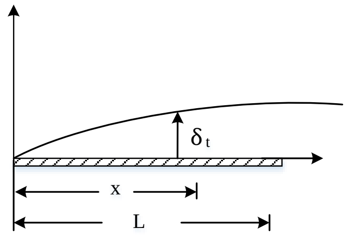
$\tau_w = \frac{7}{72}\rho U^2\frac{d\delta}{dx}$ ………………….. (5.51)

Equ.(5.50) cannot be used to find ($\tau_w = \mu\frac{\partial u}{\partial y}|_{y=0}$), since it has no derivative at (y=0).

Therefore, an empirical formula for (τ_w) is used;

$\tau_w = 0.0228\,\rho U^2(\frac{v}{U\delta})^{1/4}$ ………….. (5.52)

Equate (5.51) and (5.52), and integrate with B.C.(x=0,$\delta = 0$), i.e., we assume turbulent b.l. from (x=0); we get;



$\frac{\delta_t}{x} = \frac{0.37}{Re_x^{1/5}}$…………………….. (5.53)

From (5.52); $\tau_w = 0.029\rho U^2 (\frac{\nu}{U})^{1/5}$……………… (5.54)

And $F_{D_{f_l}} = \int_0^L \tau_w dx = \frac{0.036\rho U^2 L}{Re_L^{1/5}}$………… (5.55)

&

$F_{D_{f_t}} = \frac{0.072}{Re_L^{1/5}}$ ……………………….. (5.56)

<u>Note:</u> If we take the laminar part of the b.l. into consideration; then:

$C_{D_{f_t}} = \frac{0.074}{Re_L^{1/5}} - \frac{1700}{Re_L}$…………………… (5.57)

Boundary-Layer Displacement Thickness (δ^*)

It is the distance that the main stream (inviscid flow) is displaced due to the existence of the boundary-layer;

$\rho U \delta^* = \int_0^\delta \rho(U - u)\,\mathrm{dy}$

Thus;

$\rho U \delta^* = \int_0^\delta (1 - \frac{u}{U})\,\mathrm{dy}$

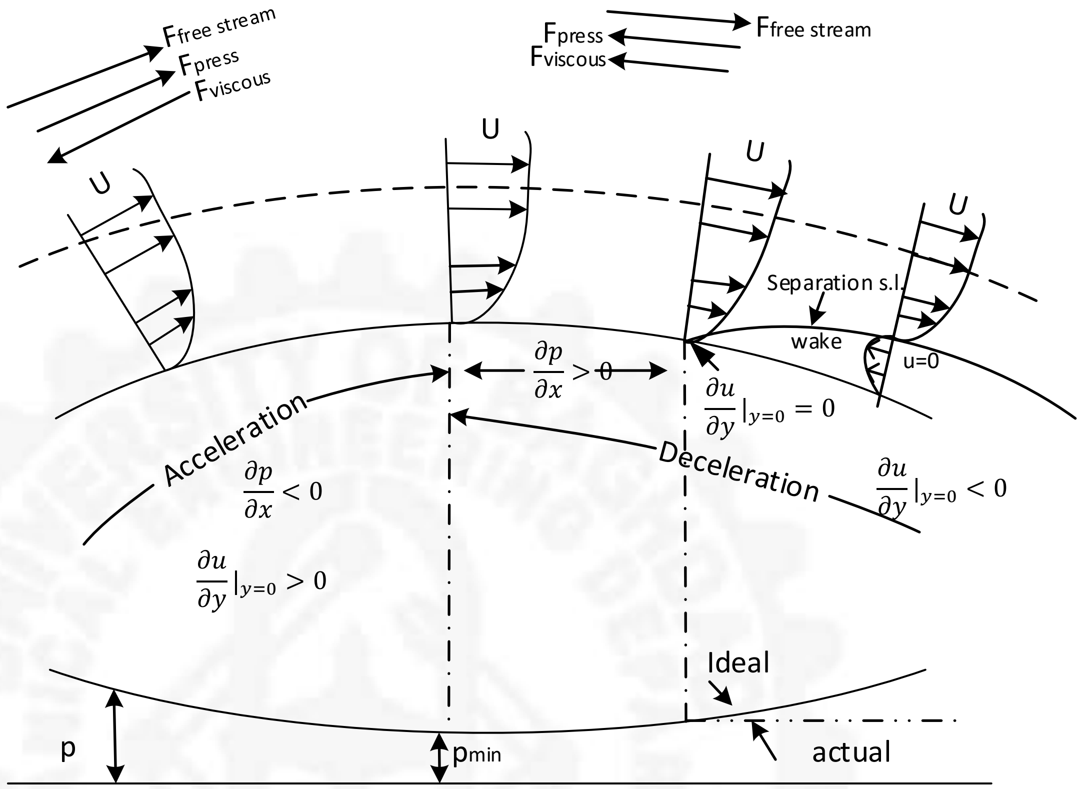
$= \delta \int_0^1 (1 - \frac{u}{U})\,\mathrm{d\eta}$ ……………. (5.58)

5.6.3 Boundary-Layer Separation

The fluid inside the b.l. is affected by three forces;

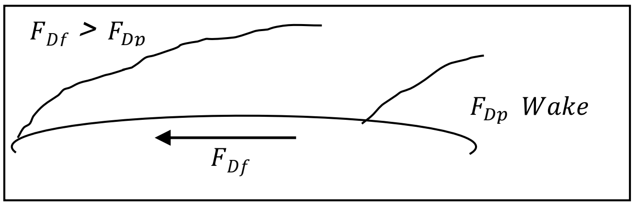
1. The forward pull due to the momentum of the outer free moving main stream (with velocity U).
2. The retarding viscous forces on the solid boundary (frictional effects).
3. The pressure forces due to pressure gradient along the boundary, which may be either negative (or favourable $\frac{\partial p}{\partial x} < 0$), or positive (adverse $\frac{\partial p}{\partial x} > 0$) or zero pressure gradient ($\frac{\partial p}{\partial x} = 0$).

Separation of the boundary –layer occurs when the resultant of the above three forces become against the flow direction, and thus pushes the flow far away from the surface and causes separation of the boundary-layer, see **Fig.(5.7).**



Notes:

1. Separation occurs when $(\frac{\partial p}{\partial x} > 0)$ only. When $(\frac{\partial p}{\partial x} = 0$ flat surfaces) or when $(\frac{\partial p}{\partial x} < 0)$, no separation can occur.
2. The onset of separation is the point at which $(\frac{\partial u}{\partial y}|_{y=0} = 0)$
3. The region between the separation s.ℓ. and the boundary is called "wake" of the body.
4. The "wake" of the body caused additional "pressure or form drag" in addition to the "skin-frictional drag"



$F_D = F_{D_f} + F_{D_p}$ $\quad$ F_D = Total Drag

5. In the design, we have to delay the separation point in order to reduce the wake size.
6. The separation point depends on;

   a- The body shape.

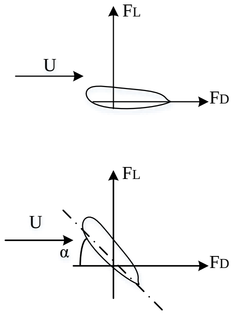
b- The roughness of the surface.

c- The turbulence level in the free steam.

d- The Reynolds number.

e- Flow conditions.

5.7 Drag and Lift



The drag force (FD) is the force component parallel to the relative approach velocity excerted on the body by the moving fluid. The lift force (F_L) is the fluid force component on a body at right angle to the relative approach velocity

$F_D = F_{D_f} + F_{D_P}$ …………………………………… (5.59)

Total (profile)

Pressure (form) drag (wake effect)

Skin (frictional) Drag (viscous effects)

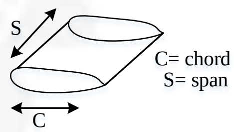
$F_{Df_{lam.}} < F_{Df_{turb.}}$

$F_{Dp_{lam.}} > F_{Dp_{turb.}}$

$F_{D\ lam.} > F_{Dp_{turb.}}$

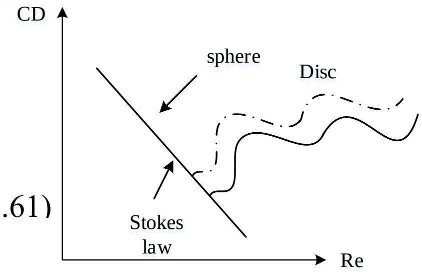
The drag coefficient (C_D) is defined as;

$C_D = \frac{F_D}{\frac{1}{2}\rho U^2 A}$ ……………………………… (5.60)



Where;

A=projected area perpendicular to the flow (⊥U), except for airfoils, where A=c*s



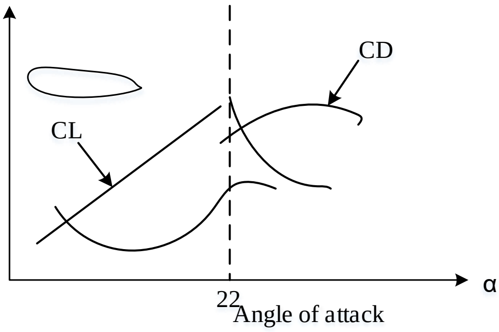
Generally;

$$\left.\begin{aligned} C_D &= C_D(\text{Re,Shape}) \\ &= C_D(\text{M,Shape})\text{for compressible flow} \end{aligned}\right\} \quad \text{……….. (5.61)}$$

See **Fig(5.21)** and **Table (5.1)** in your textbook

The lift coefficient (C_L) is defined as;

$C_L = \frac{F_L}{\frac{1}{2}\rho U^2 A}$ ………………… (5.62)



See **Fig.(5.23)** & **Table (5.1)** in your textbook.

Stoke's Law

The flow of viscous incompressible fluid around a sphere with very small Re ($Re=\frac{UD}{\nu} < 1$) has been studied by Stoke. He found that;

$F_D=6\pi\mu Ur = 3\pi\mu DU$ ………….. (5.63)

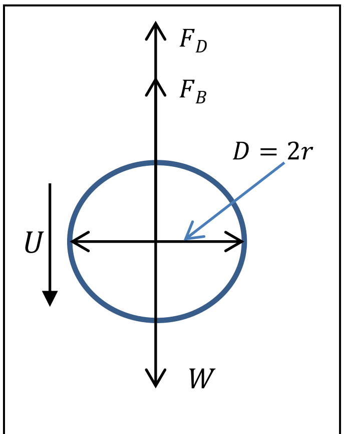


Fig. (**5.8**)

To find the terminal velocity (U) of the sphere (at which a=0), see **Fig.(5.8)**;

$$\overset{\uparrow}{+}\sum Fy = ma_y = 0$$

$$F_D + F_B = W$$

$$6\pi\mu ru + \gamma_f \frac{4}{3}\pi r^3 = \gamma_s \frac{4}{3}\pi r^3$$

Thus;

$$U = \frac{2}{9}\frac{r^2}{\mu}(\gamma_s - \gamma_f) \qquad \text{……………………} \ (5.64)$$

To calculate the drag coefficient (C_D) for this case;

$$C_D = \frac{F_D}{\frac{1}{2}\rho U^2 A} = \frac{3\pi\mu u D}{\frac{1}{2}\rho U^2 \frac{\pi}{4} D^2} = \frac{24\mu}{\rho UD} \rightarrow C_D = \frac{24}{Re}$$

Thus;

$$C_D = \frac{24}{Re} \qquad \text{…………………..} \ (5.65)$$

5.8 Resistance to Flow in Open and Closed Conduits

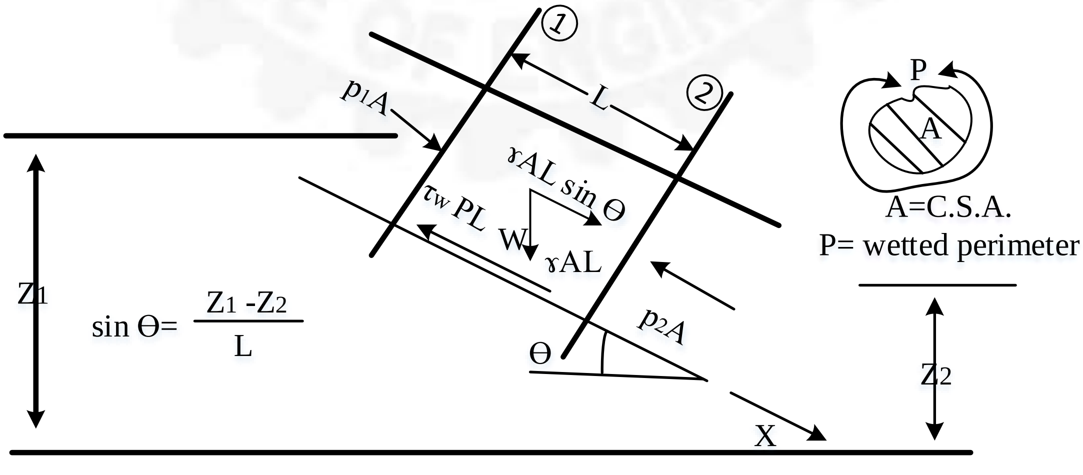


Fig.(5.9): Uniform flow in Conduits

Consider the steady uniform incompressible flow in conduit of constant C.S.A. (A) and wetted perimeter (p), see. **Fig.(5.9)**. We wish to calculate the head loss due to friction h_f between sections (1) and (2).

E.E. 1-2: $\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+h_f$ …………………. (5.66)

C.E.: $A_1V_1=A_2V_2$

Since $A_1=A_2=A \rightarrow v_1=v_2=v$ ……………… (5.67)

M.E.: $\sum F_x=ma_x=0$[since a_x=0 (steady and v=const.)]

$p_1A-p_2A+\gamma ALsin\theta-\tau_w PL=0$

Thus; $\frac{p_1-p_2}{\gamma}+(z_1-z_2)=\frac{\tau_w PL}{A\gamma}$ …………….. (5.68)

The wall shear stress (τ_w) is usually defined as;

$\tau_w=c_f*\frac{1}{2}\rho v^2$ …………….. (5.69)

Where c_f =Fanning friction factor (found experimentally, or theoretically)

Using (5.66), (5.68) and (5.69), we can show that;

$h_f=c_f\frac{L}{R_h}\frac{v^2}{2g}$……………… (5.70)

Where $R_h=\frac{A}{P}$ hydraulic radius

For circular pipe,$R_h=\frac{\frac{\pi}{4}D^2}{\pi D}=\frac{D}{4}$

Equation (5.70) is a general relation applicable for open and closed conduits, laminar and turbulent flow, and for any shape of uniform cross-section.

The head loss per unit weight per unit length (S) is defined as;

S=$\frac{h_f}{L}=\frac{c_f}{R_h}\frac{V^2}{2g}$ ………………… (5.71)

Thus;

V=$\sqrt{\frac{2g}{c_f}}\sqrt{R_h s}=c\sqrt{R_h s}$………… (5.72)

Where;

C = $\sqrt{2g/c_f}$; Equation (5.72) is called Chezy formula.

5.8.1 Open Channels Flow

For incompressible, steady flow at constant depth in prismatic (constant C.S.A) open channels, "Manning" formula is widely used, in which;

$C=\frac{c_m}{n}R_h^{1/6}$ ……………………….. (5.73)

From (5.72);

$V=\frac{c_m}{n}R_h^{2/3}S^{1/2}$ ……………………………… (5.74)

Where;

C_m=(1) in SI units and (1.49) in U.S units

S= slope of the bottom of the channel=slope of the water surface

n= Absolute roughness coefficient, depends on roughness of the surface (see **Table (5.2)** in your text)

The discharge $Q=Av=\frac{c_m}{n}AR_h^{2/3}S^{1/2}$…………………….. (5.75)

Note: For open channel; ($p_1=p_2$), thus;

$$\frac{d}{dx}(p+\gamma h)=\frac{dp}{dx}+\gamma\frac{dh}{dx}=-\gamma sin\theta=-\gamma\frac{z_1-z_2}{L}$$

$S=\frac{h_f}{L}=\frac{\frac{d}{dx}(p+\gamma h)}{\gamma}=-\frac{z_1-z_2}{L}=$ slope of the channel

5.8.2 Steady Incompressible Flow through Pipes

For pipes,

$c_f=\frac{f}{4}$ ………………………….. (5.76)

Where;

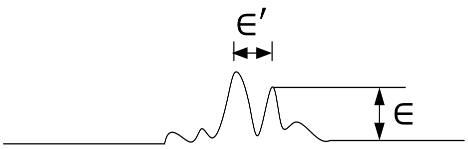
f=Darcy friction factor

Knowing that ($R_h=\frac{D}{4}$), **equ.(5.70)** becomes;

$h_f=f\frac{L}{D}\frac{v^2}{2g}$……………... (5.77) Darcy-Weisbach equation

All quantities in **equ.(5.77)** except (f) can be measured experimentally. The Friction factor (f) is known to depend on the following parameters;

$f=f(V,D,\rho,\mu,\in,\in',m)$ …………………(5.78)



where;

$\in$=measure of the size of the roughness projection (m)

$\in'$=measure of the arrangement or spacing of the roughing

m= from factor, depends on the shape of the individual roughness elements, dimensionless

Since (f) is a dimensionless factor, it can be shown (using π-theorem);

$$f=f\left(\frac{vD\rho}{\mu}, \frac{\in}{D}, \frac{\in'}{D}, m\right)$$

$$=f\left(Re, \frac{\in}{D}, \frac{\in'}{D}, m\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots(5.79)$$

The term $(\frac{\in}{D})$ is called the "relative roughness". The experiments of Nikaradse show that for one value of $(\frac{\in}{D})$, the (f ~Re) curve is smoothly connected regardless of the actual pipe diameters (D). The experiments proved that for one type of roughness, the following relation is applied;

$$f=f(R_e, \frac{\in}{D}) \ldots\ldots\ldots\ldots(5.80)$$

Equation (5.80) is plotted as "Moody Diagram", see **Fig (5.10)** and **(5.32)** in your textbook

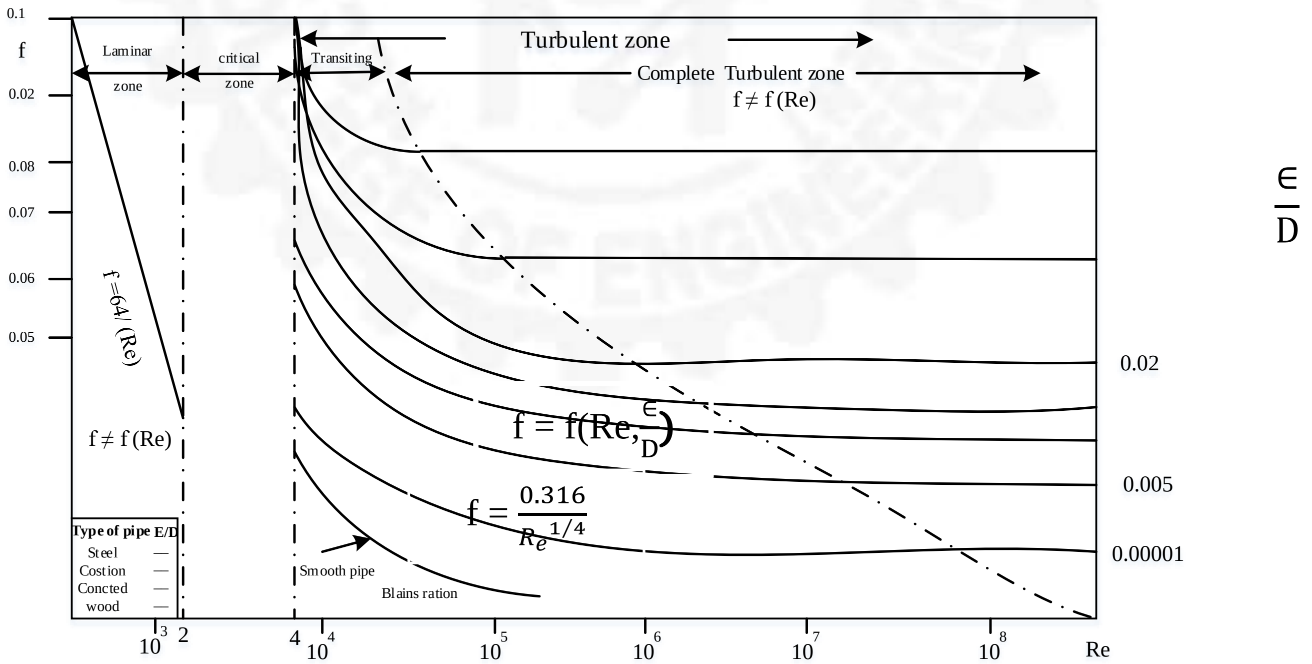


Fig.(5.10): Moody Diagram

For laminar flow, equation (5.35) is;

$\Delta p = \frac{32\mu L v}{D^2}$ ………. (5.35)

But $\Delta p = \gamma h_f \left[E.E: \frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2 + h_f \Rightarrow p_1 - p_2 = \Delta p = \gamma h_f\right]$

Thus; with (5.77);

$\frac{32\mu L v}{D^2} = \rho g f \frac{L}{D}\frac{v^2}{2g} \Rightarrow f = 64\frac{\mu}{\rho v D} = \frac{64}{Re}$……………….. (5.81)

Thus, for laminar flow, (f) is independent of (∈).

For turbulent flow in smooth pipe, Blasius relation is usually used;

$f = \frac{0.316}{Re^{1/4}}$ ……….. (5.82)

Equation (5.82) is valid up to (Re=100000).

5.8.3 Simple Pipe Problems

They are the problems in which the pipe friction (h_f) is the only loss exist. In these problems, the variables are usually 6 (Q, L, D, h_f, v,∈). Three of these six variables are generally given or may be determined, these are (L, v, ∈). Thus, three variables remained. Accordingly three types of problems exist, which may be solved by using;

1- Darcy-Weisbach equation 2.C.E. 3. Moody Diagram

Type "I": (L,v, ∈, Q, D), to find : h_f

Procedure;

1. Calculate $V = \frac{4Q}{\pi D^2}$
2. Calculate $Re = \frac{vD\rho}{\mu} = \frac{vD}{\nu}$
3. Calculate $\frac{\epsilon}{D}$
4. By using (Re, $\frac{\epsilon}{D}$), calculated (f) from Moody Diagram
5. Calculate $h_f = f\frac{L}{D}\frac{v^2}{2g}$

Type "II" Given:(L,v, ∈, h_f, D), to find : Q

Procedure:

1. Assume $f = f_1$

2. Calculate (V_1) from $h_f = f_1 \frac{L}{D}\frac{v^2}{2g}$
3. Calculate $Re_1 = \frac{v_1 D}{\nu}$
4. Calculate $\frac{\epsilon}{D}$
5. Using $(Re_1, \frac{\epsilon}{D})$ calculate new value (f_2) from Moody Diagram.
6. Compare (f_1) with (f_2) and continue until convergence is attaint
7. Calculate $Q = \frac{\pi}{4} D^2 V_1$

Type"III" Given (L, ν, ϵ, h_f, Q) to find (D)

Procedure:

1. Assume $f = f_1$
2. Calculate (D_1) from $h_f = \frac{f_1 L}{D_1}\frac{v_1^2}{2g} = f_1 \frac{L}{D_1^5}\frac{8Q^2}{\tau^2 g} \Rightarrow D_1^5 = C_1 f_1 \Rightarrow D_1$
3. Calculate $Re_1 = \frac{v_1 D_1}{\nu} = \frac{4Q}{\pi D_1 \nu}$
4. Calculate$(\frac{\epsilon}{D_1})$
5. Using $(Re_1, \frac{\epsilon}{D_1})$ calculate (f_2) from Moody Diagram
6. Compare (f1) with (f2) until convergence
7. $D = D_1$

Note: In place of Moody Diagram, the following explicit form of (f) may be used;

$$f = \frac{1.325}{\left[ln(\frac{\epsilon}{3.7D}) + (\frac{5.74}{Re^{0.9}})\right]^2} \qquad 10^{-6} < \frac{\epsilon}{D} \leq 10^{-2} \qquad \text{..........(5.83)}$$

$$5000 \leq Re \leq 10^8$$

Also, Colebrook formula may be used;

$$\frac{1}{\sqrt{f}} = -0.86 ln\left[(\frac{\epsilon}{3.7D}) + (\frac{2.51}{Re\sqrt{f}})\right] \text{....................(5.84)}$$

Equation (5.84) is for commercial pipes in transtion zone

5.8.4 Secondary (Minor) Losses

They are the losses occurred in pipelines due to bends, elbows, joins, valves,…etc. They are called secondary losses, while the frictional losses (h_f) are called "primary losses". Thus;

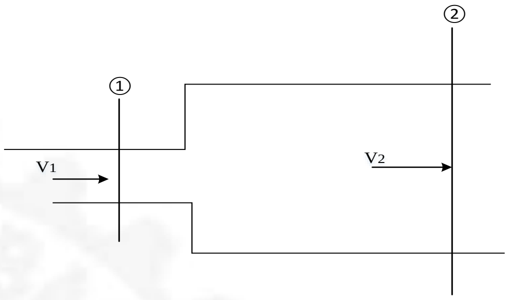
Total losses=Primary (Frictional) Losses +Secondary (Minor) Losses

h_l= $h_f + h_{minor}$

$=f\frac{L}{D}\frac{v^2}{2g} + k\,\frac{v^2}{2g}$…………………. (5.85)

Where (K) is the loss factor for the minor losses. It is found experimentally, except losses due to sudden expansion or contraction.

1. Loss due to sudden Expansion (he)

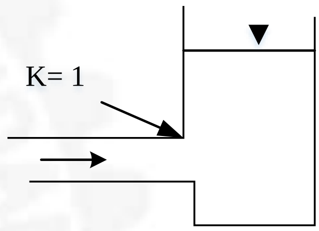


he=$\frac{(v_1-v_2)^2}{2g} = \left[1-(\frac{D_1}{D_2})^2\right]^2 \frac{v_1^2}{2g} = k\,\frac{V^2}{2g}$ ……..(5.86)

Thus;

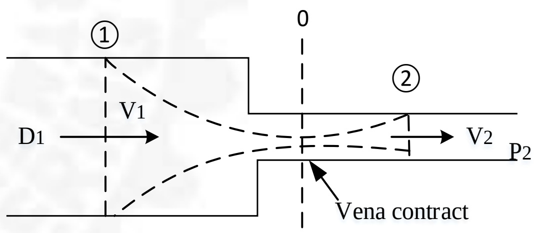
$k_e = \left[1-(\frac{D_1}{D_2})^2\right]^2$ ……………. (5.87)

If the expansion is form pipe to reservoir ($\frac{D_1}{D_2}$~0), then ($k_e = 1$) and the complete kinetic energy ($V^2/2g$) is converted into thermal energy.



2. Losses due to sudden Contraction (hc)

The head loss due to sudden contraction (hc) may be found using equ.(5.86). The proces of converting pressure $_0$head to velocity head ((1) → (0)) is very efficient and hence the head loss from (1) to (0) is small compared to the loss between (0) and (2). Thus;



hc=$h_{1-0} + h_{0-2} = \frac{(V_0-V_2)^2}{2g}$…………………(5.88)

C.E. A_0V_0=A_2V_2

or; $C_cA_2V_0 = A_2V_2 \Rightarrow V_0 = \frac{V_2}{C_c}$……… (5.89)

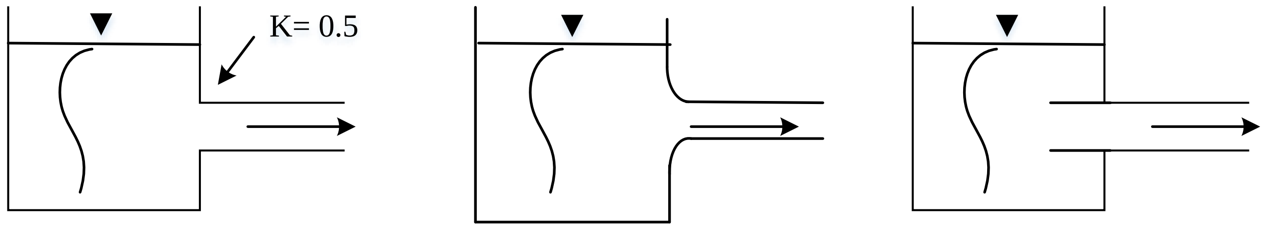
Where; C_c =Contraction Coefficient (found experimentally)

Thus;

h_c=$(\frac{1}{C_c}-1)^2\frac{V_2^2}{2g}$……………… (5.90)

And hence; $k_c = (\frac{1}{C_c}-1)^2$

3. Head loss at Pipe Entrance



Square intel
K= 0.5

Round intel
K=0.01 – 0.05

Re-entrant intel
K=0.8 – 1

Note: see table (5.3) in your textbook for (k) values.

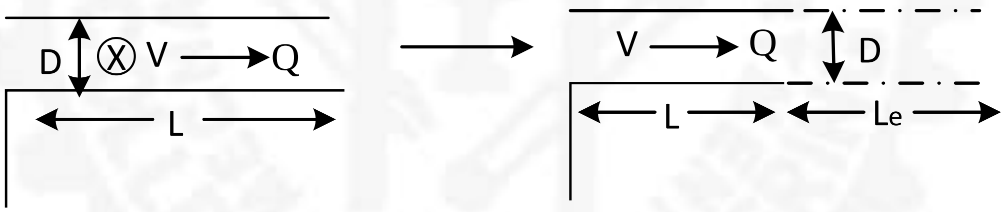
Equivalent Length (Le)

Minor losses may be expressed in terms of equivalents length (Le) of pipes that have the same head loss for the same discharge thus;

$$k\frac{V^2}{2g} = f\frac{Le}{D}\frac{V^2}{2g} \Rightarrow Le = \frac{kD}{f} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5.88)$$

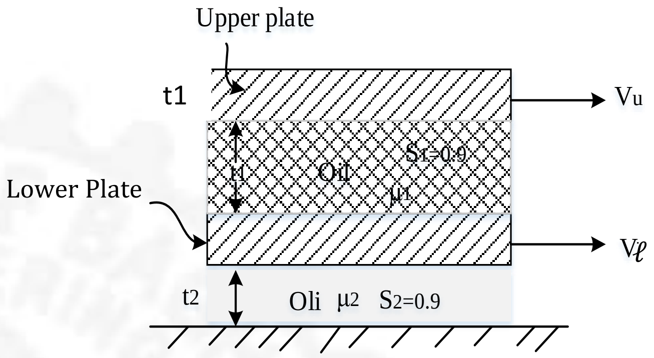
minor losses may be neglected when they are only (5%) or less than (hf). In general, minor losses may be neglected when there is (1000D) length between each minor loss.

Note;



Examples

Example (5.1): The upper plate shown in the figure is moving to the right with (Vu=80 cm/s) and the lower plate is free to move laterally under the action of the viscous forces applied to it. After steady –state conditions have been established, what velocity (Vℓ) will the lower plate have? Take:(t_1=2mm, t_2=1mm, $\mu_1 = 0.1$ Pa.s, $\mu_2 = 0.04$ Pa.s). Assume constant pressure for the two plates.



Sol.:

Since p= constant & θ = 0, then:

$$u = \frac{Uy}{a} \ \& \ \tau = \frac{\mu U}{a}$$

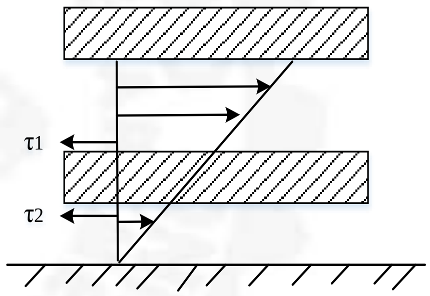
$$\tau_1 = \tau_2$$

$$\mu_1 \frac{V_u - V_l}{t_1} = \mu_2 \frac{V_l}{t_2}$$

$$V_l \left[\frac{\mu_2}{t_2} + \frac{\mu_1}{t_1}\right] = V_u \frac{\mu_1}{t_1}$$

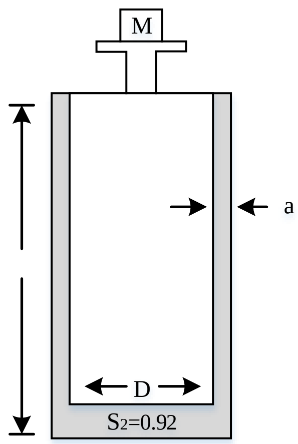
$$V_l = V_u \left[\frac{\frac{\mu_1}{t_1}}{\frac{\mu_2}{t_2} + \frac{\mu_1}{t_1}}\right]$$

$$= 0.8\left[\frac{\frac{0.1}{2*10^{-3}}}{\frac{0.04}{1*10^{-3}} + \frac{0.1}{2*10^{-3}}}\right]$$



$\therefore V_l = 0.444$ cm/s

Example (5.2): In the piston-cylinder apparatus for pressure gage tester shown in the figure, the piston is loaded to develop pressure of known magnitude. Calculate the mass (M) required to produce (1.5 MPa(gage) in the cylinder, when the piston moves with (0.02mm/s). Calculate also the leakage flow rate of oil for these conditions. Assume steady uniform flow and neglect the weight of the piston.



Sol.:

$$\sum F_y = may = 0$$

$$p*\frac{\pi}{4}D^2 + F_v = Mg$$

$$p*\frac{\pi}{4}D^2 + \tau|_{y=a} * \pi DL = M_g \ldots\ldots\ldots(1)$$

$$\tau|_{y=a} = \frac{\mu U}{a} - \frac{(a-2*a)}{2}\left[\frac{dp}{dx} - \gamma\right]$$

$$= \frac{0.5*0.02*10^{-3}}{0.005*10^{-3}} + \frac{0.005*10^{-3}}{2}\left[\frac{1.5*10^{-6}}{25*10^{-3}} - 0.92*9810\right]$$

$\tau|_{y=a} = 152P_a$ sub. In (1)

$$1.5*10^6*\frac{\pi}{4}*(0.006)^2 + 152*\pi*0.006*0.025 = M*9.81$$

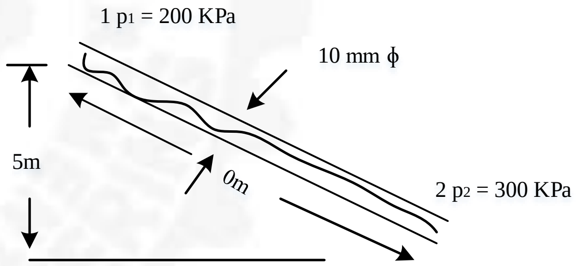
$$\therefore M = 4.335kg$$

$$Q = \left(\frac{U_a}{2} - \frac{a^3}{12\mu}\left[\frac{dp}{dx} - \gamma\right]\right) * \pi D$$

$$= \left(\frac{0.02*10^{-3}*0.005*10^{-3}}{2} - \frac{(0.005*10^{-3})^3}{12*0.5}[59990974.8]\right) * \pi * 0.006$$

$$\therefore Q = -2.26*10^{-11}\,m^3/s = 2.26*10^{-11}\,m^3/s \uparrow = 0.0226\,m^3/s$$

Example (5.3): Determine the direction of flow through the tube shown in the figure, in which (γ = 8000N/m^3) and ($\mu = 0.04$Pa.s). Find the quantity flowing in liters per second and calculate the Reynolds number for the flow.



Sol.:

Let the elevation datum be taken at section2;

$p_1 + \gamma h_1 = 200000 + 8000*5 = 240$ kPa

$p_2 + \gamma h_2 = 300000 + 8000*0 = 300000$ kPa

Since $(p_2 + \gamma h_2) > (p_1 + \gamma h_1)$

$\therefore$ Flow from (2) to (1)

$$\frac{d}{dx}(p+\gamma h) = \frac{(p+\gamma h)_2 - (p+\gamma h)_1}{l} = \frac{300000-240000}{10} \Rightarrow \frac{d}{dx}(p+\gamma h) = 6000\frac{N}{m^3}$$

$$Q = -\frac{\pi a^4}{8\mu}\frac{d}{dx}(p+\gamma h)$$

$$= -\frac{\pi(0.005)^4}{8*0.04} * 6000 \Rightarrow Q = -0.00036\frac{m^3}{s}$$

-ve flow upward

$= -0.0368$L/s

$$V = \frac{Q}{A} = \frac{0.0000368}{\frac{\pi}{4}(0.01)^2} \Rightarrow V = 0.4686\,m/s$$

$$Re = \frac{VD\rho}{\mu} = \frac{0.4686*0.01*\frac{8000}{9.8}}{0.04} \Rightarrow \qquad Re = 95.6$$

Since (Re<2000), the flow is laminar.

Example (5.4): For the hypothetical boundary –layer on the flat plate shown in the figure, calculate:

a. The skin-frictional drag force on the top side per meter of width.

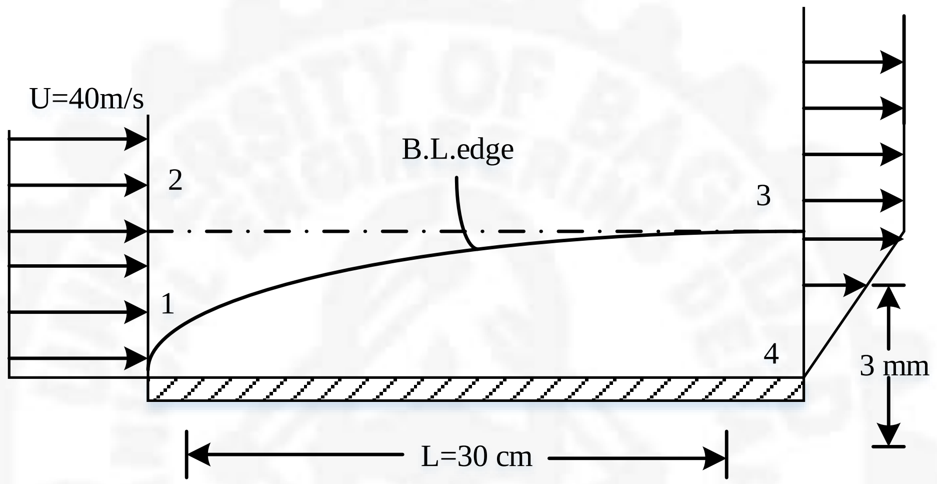
b. The drag coefficient of the top side.

c. The wall shear stress on the plate at the trailing edge.

d. The displacement thickness $(\delta_1)$of the boundary –layer at the trailing edge.

e. The displacement thickness $(\delta_1)$of the boundary –layer at the trailing edge.

Given constants:ρ = 1.2 kg/m^3 ,μ=1.8*10^{-5} Pa.s

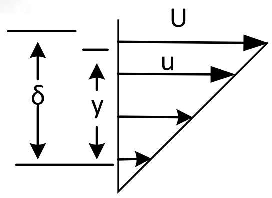


Sol.:

a- At section (3):

$\frac{u}{U} = \frac{y}{\delta}$

M.E.: -Fx=$(\dot{m}u)_{34} + (\dot{m}u)_{23} - (\dot{m}u)_{12}$



C.S.	$\dot{m}$	$(\dot{m}u)$
12	$\rho U\delta$	$\rho U^2\delta$
34	$\rho\int_0^\delta udy$	$\rho\int_0^\delta u^2dy$
23	$\rho U\delta$-$\rho\int_0^\delta udy$	$\rho U^2\delta$- $\rho U\int_0^\delta udy$

-Fx = $\rho\int_0^\delta udy + \rho U^2\delta - \rho U\int_0^\delta udy - \rho U^2\delta$

Fx=$\rho U\int_0^\delta udy - \rho\int_0^\delta u^2dy$

$= \rho U^2\frac{1}{\delta}\int_0^\delta ydy - \rho U^2\frac{1}{\delta^2}\int_0^\delta y^2dy$

=0.5 $\rho U^2\delta - \frac{1}{3}\rho U^2\delta$

= $0.1667\ \rho U^2\delta = 0.1667 * (40)^2 * 1.2 * 0.003$

$\therefore Fx$ =0.96 N→on plate

b- $C_D = \frac{F_D}{\frac{1}{2}\rho U^2 A}$

$= \frac{0.96}{0.5 * 1.2 * (40)^2 * 0.3 * 1}$

$\therefore C_D$ =0.00333

c- $\tau_w = \mu \frac{\partial u}{\partial y}_{y=0} = \frac{\mu U}{\delta} = \frac{1.8*10^{-5}*40}{0.003}$

$\therefore \tau_w$ =0.24 Pa

d- $\delta_1 = \int_0^\delta \left(1-\frac{u}{U}\right)dy = \int_0^\delta \left(1-\frac{y}{\delta}\right)dy$

$= \left[y - \frac{y^2}{2\delta}\right]_0^\delta = \frac{\delta}{2}$

$\therefore \delta_1$ =1.5 mm

Example (5.5): A smooth flat plate (3m) wide and (30m) long is towed through still water at (20℃) with a speed of (6m/s). Determine the drag on one side of the plate and the drag on the first (3m) of the plate.

Sol.:

For water at (20℃), from Table (c.1): (ρ= 998.2 kg/m^3) and (ν = 1.007 * 10^{-6} $\frac{m^2}{s}$). For the whole plate; $R_e = \frac{VL}{\nu} = \frac{6\times30}{1.007\times10^{-6}}$ → Re = 1.787 *10^8 ; turbulent

$C_D = \frac{0.072}{Re^{0.2}}$ → C_D= 0.0016

Drag force $F_D = C_D * \frac{1}{2} PU^2 * L * b$

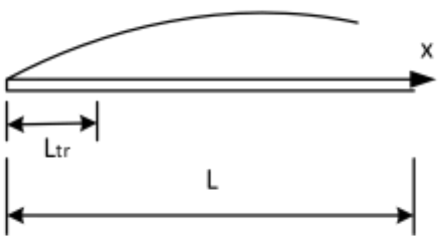
$= 0.0016 * \frac{1}{2} * 998.2 * 6^2 * 30 * 3$ → F_D = 2587.3 N

Let (Re_{tr} = 5 * 10^5) at which transition to turbulent b.l. occurs;

Thus:

$Re_{tr} = \frac{VL_{tr}}{\nu}$

$5 * 10^5 = \frac{6*L_{tr}}{1.007*10^{-1}}$ → L_{tr} = 0.084 m



For the first (3m);

$Re = \frac{VL}{v} = \frac{6*3}{1.007*10^{-6}}$ ⟶ Re = 1.787 * 10^7: turbulent

$C_D = \frac{0.072}{Re^{0.2}}$ ⟶ $\underline{C_D = 0.00255}$

$F_D = C_D * \frac{1}{2} PU^2 L * b$

$= 0.00255 * \frac{1}{2} * 998.2 * 6^2\ 3 * 3$ ⟶ $F_D = 412.7$ N

Note: $L_{tr} << L$, so it can be neglected, and me assume the b.l. turbulent right from the leading edge (x = O).

Example (5.6): How many (30m) diameter parachutes (C_D = 1.2) should be used to drop a bulldozer weighing (45kN) at a terminal speed of (10m/s) through still air at (100kPa) and (20℃).

Sol.:

$P_{air} = \frac{p}{RT} = \frac{100000}{287*(20+273)}$ ⟶ $\rho_{air} = 1.19$ kg/m^3

$\sum F_y = 0$

$W = F_D + F_B$

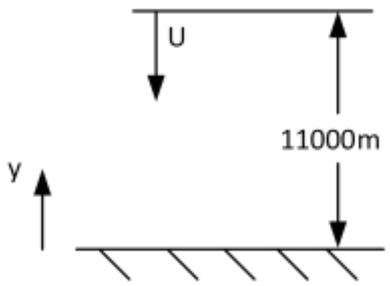
$w = n[C_D * \frac{1}{2}\rho U^2 * \frac{\pi}{4} D^2 + \rho g * \frac{\pi}{6} D^3]$

$45000 = n[1.2 * \frac{1}{2} 1.19 * 10^2 * \frac{\pi}{4} 30^2 + 1.19 * 9.8 * \frac{\pi}{6} 30^3]$ ⟶ $\underline{n = 0.4}$

$\underline{n = 1 \text{ parachute}}$

Example (5.7): A jet aircraft discharges solid particles of matter (d = 10 mm) and (s = 2.5) at the base of stratosphere at (11000m). Assume the viscosity (μ) of air to vary with (y) from see level as ($\mu = 1.78*10^{-5} - 3.06*10^{-10}y$). Estimate the time for these particles to reach the see level. Neglect air currents and wind effects.

Sol.:



Using Stokes law; $U = \frac{2}{9}\frac{r^2}{m}(\gamma_s - \gamma_f)$; and assuming $\gamma_f << \gamma_s$

Thus;

$U = -\frac{dy}{\partial t} = \frac{2}{9}\frac{r^2 \gamma_s}{\mu}$

Thus;

$$\int_{t=0}^{t} dt = -\int_{11000}^{0} \frac{9}{2}\,\frac{1}{r^2 \gamma_s}\,\mu\,dy$$

$$t = \frac{9}{2*(5*10^{-6})^2*2.5*9810} \int_{11000}^{0} (1.78*10^{-5} - 3.06*10^{-10} y)dy$$

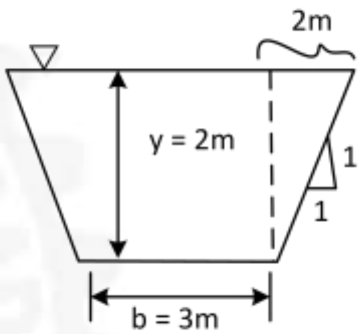
Thus; t = 1301189 seconds = 15.06 day

Example (5.8): Determine the discharge for a trapezoidal channel (see the figure),with a bottom width (b = 3m) and side slopes (1 on 1). The depth is (2m), and the slope of the bottom is (0.0009). the channel has a finished concrete linins.

Sol:

From table (5.2) p.p. (230); text book, $\longrightarrow$ n = 0.012

$A = 3*2 + 2*[\frac{2\times2}{2}]$ $\longrightarrow$ A = 10 m^2



$P = 3 + 2*2\sqrt{2} = 8.66\ m$

$R_h = \frac{A}{P} = \frac{10}{8.66}$ $\longrightarrow$ R_h= 1.1547

Thus;

$$Q = \frac{C_m}{n} A R h^{2/3} S^{1/2}$$

$= \frac{1}{0.012} * 10 * (1.1547)^{2/3} (0.0009)^{1/2}$ $\longrightarrow$ Q = 27.52 m^3/s

Example (5.9): Determine the head loss for flow of (140 L/s) of oil (ν= 0.00001 m^2/s) through (400m) of (200mm) diameter cast iron pipe.

Sol:

$Re = \frac{4Q}{\pi D \nu} = \frac{4*0.14}{\pi*0.2*0.00001}$ $\longrightarrow$ Re = 89127

From Moody diagram, Fig (5.32) p.p. 237 in your textbook, (ϵ = 0.25mm)

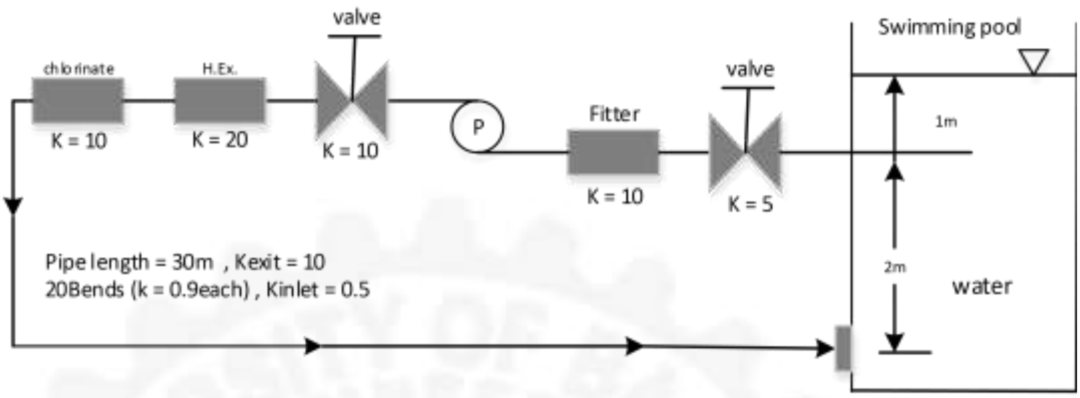
Thus; $\frac{\epsilon}{D} = \frac{0.25}{200} = 0.00125$

With (Re = 89127) and ($\frac{\epsilon}{D}$ = 0.00125), from Moody diagram f = 0.023

$$h_f = f\,\frac{L}{D}\,\frac{V^2}{2g} = f\,\frac{L}{D^5}\,\frac{8Q^2}{\pi^2 g}$$

$= 0.023 * \frac{400}{(0.2)^5}\,\frac{8*(0.14)^2}{\pi^2*2.8}$ $\longrightarrow$ $h_f = 46.58\ \frac{N.M}{N}$

Example (5.10): Estimate the cost per month required to treat and circulate the water of the swimming pool shown in the figure. The circulation rate is (0.1 m^3/s) through (100mm) diameter smooth pipe. The pump efficiency is (80%) and (1kwh = 65 fils). Include all losses and also take ($V = 10^{-6} m^2/s$).



Sol.:

$V = \frac{Q}{A} = \frac{0.1}{\frac{\pi}{4}(0.1)^2}$

$\therefore V = 12.73\ m/s$

$Re = \frac{VD}{\nu} = \frac{12.73*0.1}{10^{-6}}$

$\therefore Re = 12.73 * 10^5$

$f = \frac{0.316}{Re^{1/4}} = \frac{0.316}{(1273000)^{0.25}}$

$\therefore f = 0.009407$

Apply the E.E. between two points on the free surface of the pool:

0 + 0 + 0 + hp = 0 + 0 + 0 + 0 + hℓ

hp = hℓ $= h_f + \sum k\frac{V^2}{2g} = [f\frac{L}{D} + \sum k]\frac{V^2}{2g}$

$= [0.009407*\frac{30}{0.1} + 0.5 + 5 + 10 + 10 + 20 + 10 + 20*0.9 + 10]*\frac{(12.73)^2}{2*9.81}$

$\therefore hp = 712.98m$

IP $= \frac{\gamma Q hp}{\eta p} = \frac{9810*0.1*712.98}{0.8}$

$\therefore IP = 874291.73W = 874.29kW$

No. of kwh = IP * t = 874.29 * 30 * 24

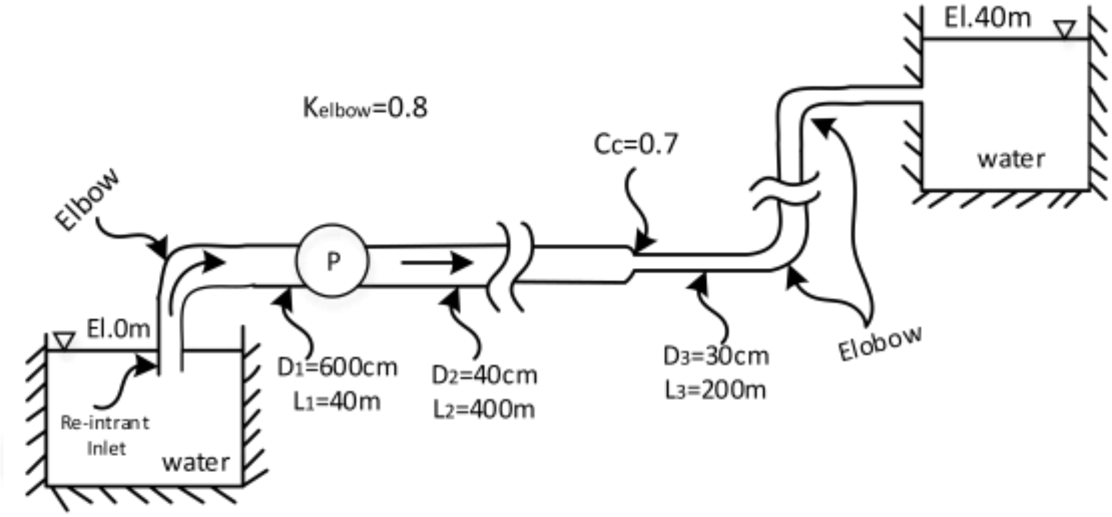
∴ No. of kwh = 629488.8 kWh

Cost = 629488.8 * 0.065

∴ Cost = 40916.8 I.D.

Example (5.11): The pump of the piping system shown in the figure is used to supply ($1m^3/s$) of water to the uphill station. Take ($f = 0.014$) and include all minor losses. Calculate:

a. The required pump shaft power knowing that its efficiency is (80%).
b. The operational cost (in I.D.) of pumping ($10000m^3$) of water to the upper reservoir, knowing that: (1kwh = 300fils).



Sol.:

a- $V = \frac{Q}{A} = \frac{4Q}{\pi D^2}$

$\therefore V_1 = 3.536\ m/s$, $V_2 = 7.96$ m/s , $V_3 = 14.15$ m/s

$hf = f\ \frac{L}{D} = \frac{V^2}{2g}$

$\therefore hf_1 = 0.1487\ m$, $hf_2 = 45.21\ m$, $hf_3 = 95.2\ m$

E. E.:

$$hp = 40 + hf_1 + hf_2 + hf_3 + k_{inlet}\frac{V1^2}{2g} + k_{exit}\frac{V3^2}{2g} + 2k_{elbow}\frac{V3^2}{2g} + k_{elbow}\frac{V3^2}{2g} + (\frac{1}{C_c} - 1)^2\frac{V3^2}{2g}$$

$$= 40 + 0.1487 + 45.21 + 95.2 + 1 * \frac{(3.536)^2}{2*9.81} + 1 * \frac{(14.15)^2}{2*9.81} + 2 * 0.8 * \frac{(14.15)^2}{2*9.81} + 0.8 * \frac{(3.536)^2}{2*9.81} + (\frac{1}{0.7} - 1)^2 * \frac{(14.15)^2}{2*9.81}$$

$\therefore hp = 210.11m$

$$Ip = \frac{op}{\eta_p} = \frac{\gamma Q hp}{\eta_p} = \frac{9810*1*210.11}{0.8}$$

$\therefore IP = shaft\ power = 2.6\ MW$

b- $t = \frac{\forall}{Q} = \frac{10000}{1}$

∴ t = 10000 s = 2.78 hr

No. of kwh = IP * t = 2600 * 2.78

∴ No. of kwh = 7228 kwh

$Cost =$ 7228 * 0.3

∴ Cost = 2168.4 I.D

Problems:

The problems number listed in the table below refer to the problems in the "text book" , chapter "5".

Article No.	Related Problems
5.3	1,2,3,6,7,9,10,11,12,13,14,16,17
5.5	20,21,22,24,27,28,29,31,33
5.6	43,44,45,46,47,48
5.7	42,49,51,52,54,55,56,57,62,63,64
5.8.1	65,66,68,69,70,72,75,77,78,81,82
5.8.2 + 5.8.3 + 5.8.4	25,83,84,87,88,89,90,95,96,98,99, 100,101,103,105,106,108,109,110,112,114, 115,116,117,118,119,120,121,123,124,125,129

Chapter -6-

Fluid and Flow Measurements

The chapter includes the measurements method used for the most important fluid and flow parameters, such as density, pressure, velocity, discharge, viscosity. All the described methods involve the application of the basic fundamental principles covered in the previous chapters.

6.1 Density Measurements

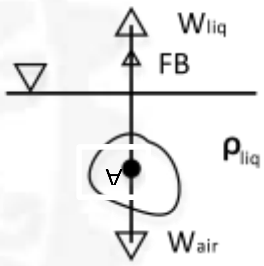
The most important methods are;

1. Weighing method; weighing a known volume (∀)of the liquid;

$\rho = \frac{m}{\forall} = \frac{w/g}{\forall}$ ………………. (6.1)

(w and ∀) measured

2. Buoyancy Principle: A solid object of known volume (∀) is weighed in air (w_{air}) and in the liquid (w_{liq}) whose density (ρ) is to be determined;

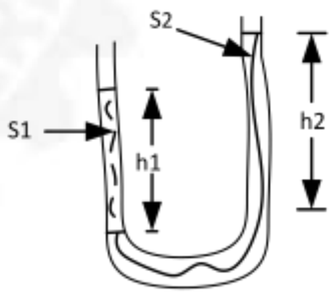


$w_{air} = w_{liq} + F_B$

$w_{air} = w_{liq} + \rho_{liq} g \forall$ ………………….. (6.2)

($w_{air}, w_{liq}, \forall$) measured

3. Hydrostatic Principle: Placing two immiscible liquids in a U-tub, one of known density (S_1), the other of unkown (S_2).



$S_1 \gamma_w h_1 = S_2 \gamma_w h_2$ ……………… (6.3)

(h_1, h_2) measured⇒ $S_2 =$ ✓

$\rho_2 = S_2 \rho_w$

6.2 Pressure Measurements

At first, we must signify between three types of pressure. These are total (stagnation) pressure (p_o), static pressure (p) and the dynamic pressure (p_d). The relation between these three types is;

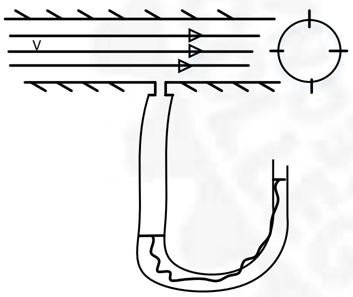
$p_o = p + pd = p + \frac{1}{2}\rho V^2$……………….(6.4)

The static pressure (p) is the pressure of the fluid "sensed "or" feeled" by an observer that is moving with the fluid with the same velocity (v) in magnitude and direction, i.e, the observer must not disturb the flow. The total (stagnation) pressure (p_o) is the pressure of the fluid when it is stopped and comes at rest (v=0). If you swim in a river moving with a velocity (v), and you leave your body to move "freely" without resisting the flow, then the pressure you "sensed" or "feeled" is the static pressure of the flow. If you stopped yourself in the river, then the flow becomes "stagnant" at your body (v=0), and hence the pressure you "sensed" or "feeled" is the total (stagnation) pressure. The difference between the two pressures is the dynamic pressure ($p_d=\frac{1}{2}\rho v^2$), which is a result of the kinetic energy of the fluid.

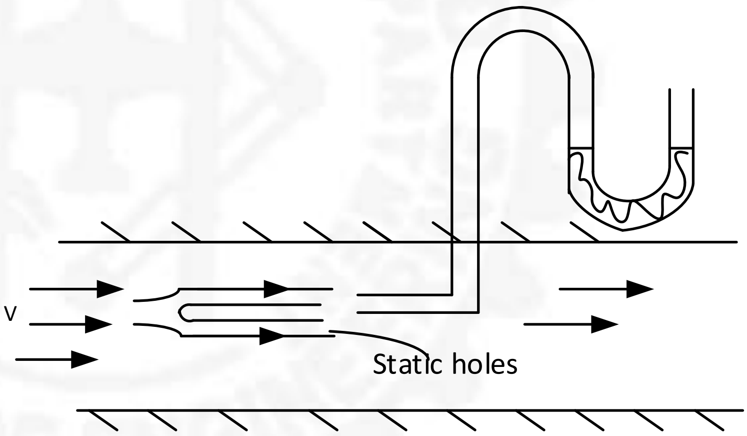
Pressure measurements include the measurements of "static" and "total" pressure only. The dynamic pressure is calculated as the difference between the two.

6.2.1 Static Pressure Measurements

The static pressure may be measured either by a "piezometer" opening or by a "static tube"

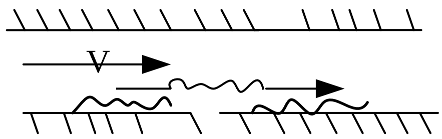


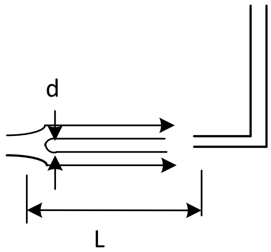
Piezometer openings



Static Tube

<u>Piezometer Opening;</u> a number of openings (holes) are made on the periphery of the tube, pipe or duct. These holes are connected through a "ring" around them with one opening connected to the manometer. When the inner surface of the tube is rough, this method is not effective, since the flow will be disturbed. In this case, the static tube is usually used.





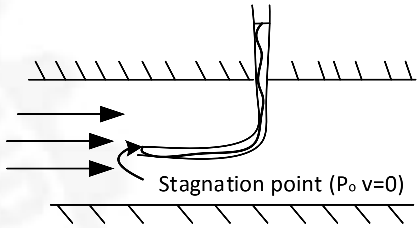
Static Tube; it is a blunt tube with closed nose and a number of openings around the periphery at a section far away from the nose (L > 14d); to ensure that the flow becomes paralell after the nose curvature.

Note: The discrepancy (Δh) between the true and the measured static pressure normally varies as the square of the velocity of flow, i.e. ;

$$\Delta h = C\frac{V^2}{2g} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6.5)$$

Where (C) is a correction factor determined by calibration

6.2.2 Total (Stagnation) Pressure Measurements



This pressure is measured by using the "Pitot tube", which is an open-nose tube. The flow at the open-end comes at rest. This point is called "Stagnation Point", at which ($p=p_0$) and ($V=0$).

6.3 Velocity Measurements

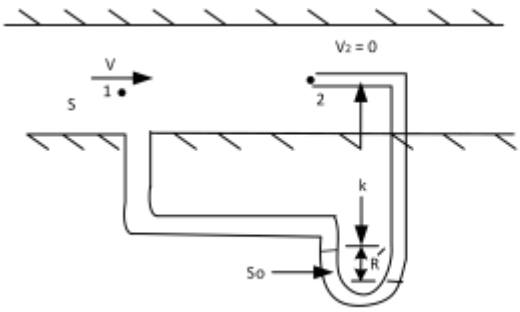
The velocity is measured is measured through the measurement of the dynamic pressure ($p_d=\frac{1}{2}\rho V^2$). As was mentioned earlier, this pressure is calculated as the difference between the measured total (stagnation) and the static pressures. Thus;

$$p_d = p_0 - p$$

$$\frac{1}{2}\rho V^2 = p_0 - p \Rightarrow V = \sqrt{2\frac{p_0 - p}{\rho}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6.6)$$

The "Pitot tube" and the " piezometer opening" or the static tube" are connected to the same manometer to measure the velocity directly, as follows;

1. Combined Pitot Tube and Piezometer Openings



B.E. 1-2 $\Rightarrow V_t = \sqrt{2\frac{p_2 - p_1}{s\rho_w}}$ ……………. (a)

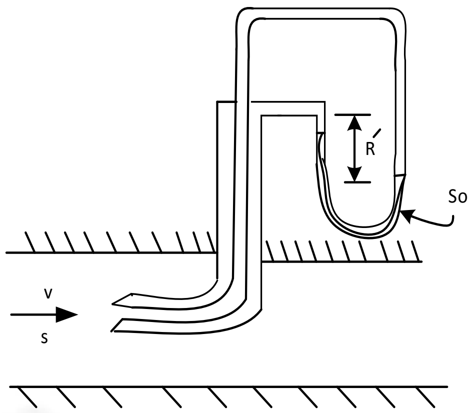
Manometer equ.:

$$\frac{p_2 - p_1}{s\rho_w} = R'\left(\frac{s_0}{s} - 1\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (b)$$

Thus;

$$V_t = \sqrt{2gR'(\frac{S_0}{s} - 1)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (6.7)$$

2. <u>Pitot -Static Tube ;</u> which combines the "Pitot" and "static" tubes. The same relation for the velocity (6.7) is applied here. A correction factor (C) must be introduced into equation (6.7) to take into account the losses and errors. Thus, the actual velocity (Va) is



$$V_a = C\sqrt{2gR'(\frac{S_0}{s} - 1)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (6.8)$$

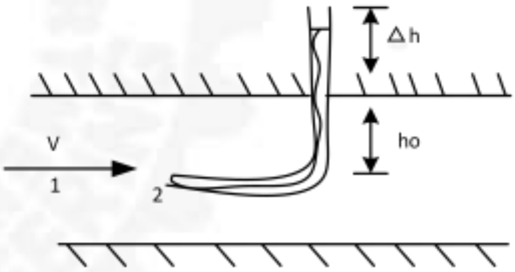
(C) is found by calibration

<u>Note:</u>

The " Pitot Tube" alone may be used to measure the velocity, as follows;

B.E. 1-2: $\frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2$

$z_1 = z_2; V_2 = 0$,thus



$$V_1 = V_t = \sqrt{2\frac{p_2 - p_1}{\rho}}$$

but $p_2 = \gamma(h_o + \Delta h)$

$p_1 = \gamma h_o$

Hence;

$$V = \sqrt{2\frac{\gamma\Delta h}{\rho}}$$

or ;

$$V_t = \sqrt{2g\Delta h} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (6.9)$$

A correction factor (c) may also be used with this equation to calculate the actual velocity;

$$V_a = C\,V_t = C\sqrt{2g\Delta h}$$

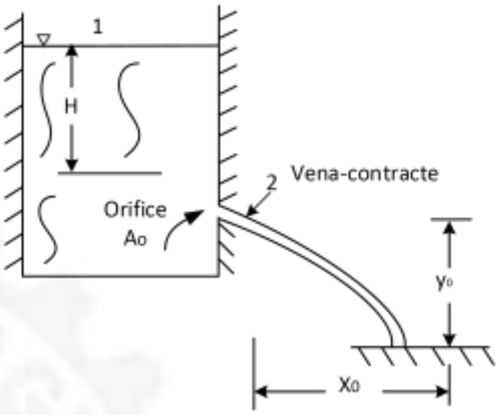
6.4. Discharge Measurements

6.4.1 Orifice Meters

There are two types for this meter. One is used for reservoirs and the other for pipes.

6.4.1.1 Orifice in Reservoirs

B.E. 1-2: $\frac{p_1}{\gamma} + \frac{V_{1t}^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_{2t}^2}{2g} + z_2$



Thus;

$V_{2t} = \sqrt{2gH}$ ………………….. (6.10)

The actual velocity (V_{2a}) is found by defining the velocity coefficient (C_v) as;

$C_v = \frac{V_{2a}}{V_{2t}}$ ………………… (6.11)

Hence;

$V_{2a} = C_v\sqrt{2gH}$ ……………. (6.12)

The actual discharge (Q_a) is given by;

$Q_a = V_{2a}A_2$ ………. (6.13)

Where;

A_2= jet area at the vena-contract. (A_2) is related to the orifice area (A_0) through the coefficient of contraction (Cc) by;

$C_c = \frac{A_2}{A_0}$ ……………….. (6.14)

Thus; $Q_a = C_v\sqrt{2gH}C_cA_0 = C_vC_cA_0\sqrt{2gH}$

or;

$Q_a = C_dA_0\sqrt{2gH} = C_dQ_{th}$ ……………….. (6.15)

Where;

$C_d = C_vC_c$ ……………… (6.16) discharge coefficient

$C_d = \frac{Q_a}{Q_{th}}$ ……………. (6.17)

Where; $Q_{th} = A_0\sqrt{2gH}$ and $Q_a = \frac{\forall}{t}$ ………………….(6.18)

(C_d) may be found from equation (6.15) by measuring A_0, H and $Q_a = \forall/t$. Thus, determination of either (C_c) or (C_v) permits the determination of the other from equation (6.16). Several methods exists;

1. **<u>Trajectory Method;</u>** by measuring (x_0,y_0), the actual velocity (V_{2a}) can be determind if the air resistance is neglected.

$X_0 = V_{2a}t \Rightarrow t = X_0/V_{2a}$

$-y_0 = \overset{0}{\cancel{V_{ix}}}t - \frac{1}{2}gt^2 = -\frac{1}{2}g(\frac{x_0}{V_{2a}})^2 \Rightarrow V_{2a} = \frac{x_0}{\sqrt{2y_0/g}}$ …… (6.19)

But $V_{2t} = \sqrt{2gH}$

Hence $C_v = \frac{V_{2a}}{V_{2t}} \Rightarrow C_c = \frac{cd}{cv} =$

2. Direct measurement of (V_{2a}) with a Pitot or Pitot –static tube placed at the vena-contract (point 2)
3. Direct measurement of jet diameter by outside calipers
4. **Momentum Method**

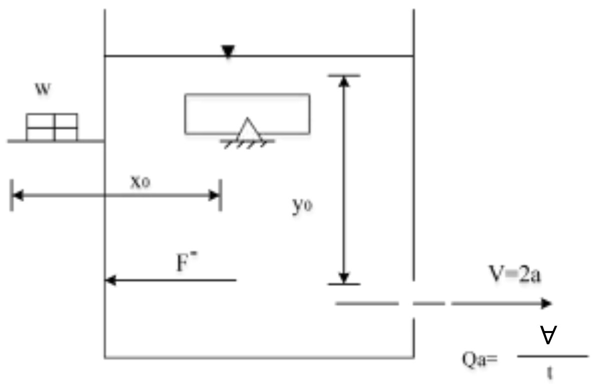
$\sum F_x = m^{\cdot}(V_{xout} - V_{xin})$

$F' = \rho Q_a(V_{2a} - 0)$

But $F' * y_0 = w \times x_0$

Thus;

$V_{2a} = \frac{Wx_0}{\rho Q_a y_0}$…………………..(6.20)



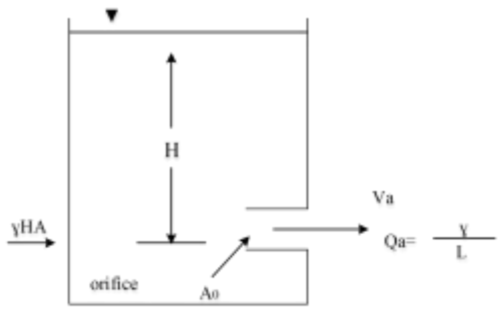
<u>Losses in Orifice</u> ; To calculate the losses in orifice, apply the E.E. between (1) and (2), which gives;

$$h_\ell = H - \frac{V_{2a}^2}{2g} = H(1 - \frac{V_{2a}^2}{2gH}) = \frac{V_{2a}^2}{2g}(\frac{2gH}{V_{2a}^2} - 1)$$

Hence;

$h_\ell = H(1 - Cv^2) = \frac{V_{2a}^2}{2g}(\frac{1}{Cv^2} - 1)$……………….(6.21)

Borda Mouthpiece



$\sum F_x = m^{\cdot}(V_{xout} - V_{xin})$

$\gamma A_0 H = \rho Q_a V_a$

but; $Q_a = c_d A_0 \sqrt{2gH}$

hence;

$\gamma H A_0 = \rho c_d A_0 \sqrt{2gH} c_v \sqrt{2gH}$

Thus;

$1 = 2C_d C_v = 2C_v^2 C_c$……………. (6.22)

Unsteady Orific Flow from Reservoirs Time of Emptying Reservoirs

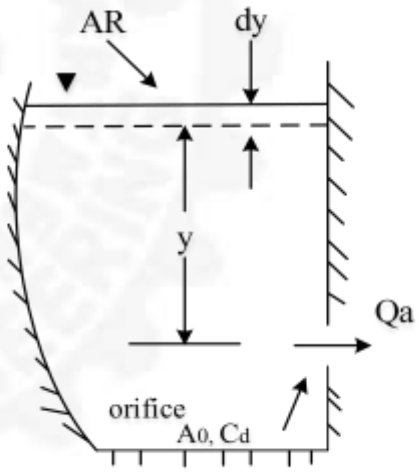
The determination of the time to lower the reservoir surface a given distance is an unsteady flow case of some practical interest. If the reservoir Surface drops slowly enough, the error from using B.E. is negligible.

C.E. $\frac{\partial m_{c.v}}{\partial t} + m^{\cdot}_{out} - \overset{0}{\cancel{m^{\cdot}_{in}}} = 0$

Thus;

$$\frac{\partial m_{c.v}}{\partial t} = -m^{\cdot}_{out} = -\rho Q_a$$

$$\rho A_R \frac{dy}{dt} = -\rho Q_a$$



or;

$$-Q_a dt = A_R dy$$

but

$$Q_a = c_d A_0 \sqrt{2gy}$$

hence;

$$A_R dy = -c_d A_0 \sqrt{2gy} dt$$

$$\int_{t_1}^{t_2} dt = -\frac{1}{\sqrt{2g} c_d A_0} \int_{y_1}^{y_2} A_R \, y^{-1/2} dy$$

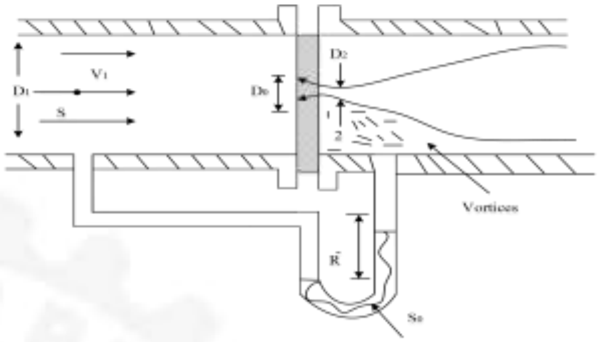
Where (A_R) is a known function of (y). If (A_R= constant), then

$t_2 - t_1 = \frac{2A_R}{c_d A_0 \sqrt{2g}} (\sqrt{y_1} - \sqrt{y_2})$ ………………. (6.23)

6.4.1.2 Orifice in Pipes

It is used to measure the discharge in pipes at law Re (low velocities). It is simply a plate with central hole (orifice). To calculate the discharge (Qa);

B.E. 1-2: $\frac{p_1}{\gamma} + \frac{V_{1t}^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_{2t}^2}{2g} + z_2$ ………… (1)



C.E. $A_1V_1=A_2V_2=Cc\ A_0\ V_2$

$\frac{\pi}{4}D_1^2V_1 = C_c\frac{\pi}{4}D_0^2V_2$ ……………. (2)

(1) and (2) gives;

$$V_{2t} = \sqrt{\frac{2g\frac{p_1-p_2}{\gamma}}{1-c_c^2(\frac{D_0}{D_1})^4}} \quad …………………….. (3)$$

Manometer Equation gives; $\frac{p_1-p_2}{\gamma} = R'(\frac{S_0}{S} - 1)$ ………….. (4)

Thus;

$$V_{2t} = \sqrt{\frac{2gR'(\frac{S_0}{S}-1)}{1-c_c^2(\frac{D_0}{D_1})^4}} \quad ………………… (5)$$

And;

$$V_{2a} = c_v V_{2t} = c_v\sqrt{\frac{2gR'(\frac{S_0}{S}-1)}{1-c_c^2(\frac{D_0}{D_1})^4}} \quad ………………… (6)$$

The actual discharge (Qa) is;

$Q_a = V_{2a}c_cA_0 = c_vV_{2t}c_cA_0 = c_dV_{2t}A_0$

Hence;

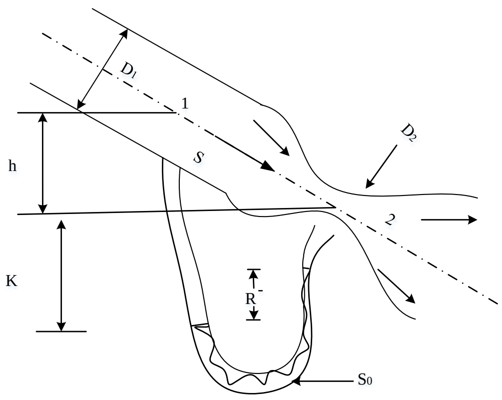
$$Q_a = c_dA_0\sqrt{\frac{2gR'(\frac{S_0}{S}-1)}{1-c_c^2(\frac{D_0}{D_1})^4}} \quad …………………… (6.24)$$

Note;

The orifice in a pipe causes vortices to be formed behind the orifice plate. These vortices represent losses. The losses increase as the velocity increases. Therefore the orifice in a pipe is used for low Re. At higher Re, Venturi meter is used.

6.4.2 Venturi Meter

It is used to measure the discharge in pipes at high velocity (high Re). The contraction coefficient is unity, thus;



Cc=1

$c_d = c_v$ ………………… (6.25)

B.E. 1-2: $\frac{p_1}{s\gamma_\omega} + \frac{V_{1t}^2}{2g} + h = \frac{p_2}{s\gamma_\omega} + \frac{V_{2t}^2}{2g}$ …………….. (1)

C.E. $\frac{\pi}{4}D_1^2V_1 = \frac{\pi}{4}D_2^2V_2$ …………… (2)

(1) and (2) gives;

$$V_{2t} = \sqrt{\frac{2g\left[h+\frac{p_1-p_2}{s\gamma_\omega}\right]}{1-(\frac{D_2}{D_1})^4}}$$ ……………….. (3)

Manometer equ.: $\frac{p_1-p_2}{s\gamma_\omega} = R'(\frac{s_0}{s} - 1) - h$ …………… (4)

Thus;

$$V_{2t} = \sqrt{\frac{2gR'(\frac{s_0}{s}-1)}{1-(\frac{D_2}{D_1})^4}}$$ …………………….(5)

$Q_a = V_{2a}A_2 = c_vV_{2t}A_2$

Thus;

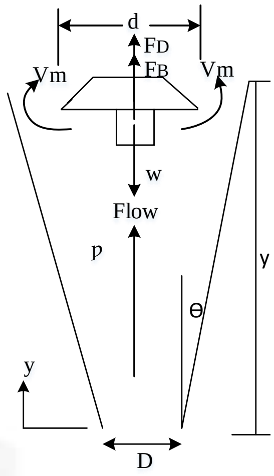
$$Q_a = c_vA_2\sqrt{\frac{2gR'(\frac{s_0}{s}-1)}{\left[1-(\frac{D_2}{D_1})^4\right]}}$$ …………………….. (6.26)

Thus; (Q_a) depends on (R') regardless of the orientation of the venturi meter, whether it is horizontal, vertical or inclined.

In the venture meter, no vortices is formed and the losses in energy is reduced, therefore it can be used for high velocities.

6.4.3 Rotameter

This device is used to measure the flow rate based on the princi ple that the drag force on a body depends on the velocity over it. A "notched bob" is placed in a conical container through which theflow passes over the bob. Applying force balance on the bob;



$F_D + F_B = W$

$$c_D * \frac{1}{2}\rho V_m^2 * \frac{\pi}{4} d^2 + \rho g \forall_b = \rho_b g \forall_b \text{ .... (1)}$$

Where;

c_D=Drag Coefficient

V_m=Mean velocity in the annular space

ρ = Fluid density

ρ_b = Body density

$\forall_b$= Bob volume

Thus; A_b =Bob frontal area $=\frac{\pi}{4} d^2$

$$V_m = \sqrt{\frac{1}{c_d}\frac{2g\forall_b}{A_b}\left(\frac{\rho_b}{\rho} - 1\right)} \qquad \text{..................... (6.27)}$$

and;

$$Q = AV_m = A\sqrt{\frac{1}{c_d}\frac{2g\forall_b}{A_b}\left(\frac{\rho_b}{\rho} - 1\right)} \qquad \text{...................... (6.28)}$$

Where;

$$A = \frac{\pi}{4}[(D + ay)^2 - d^2] \qquad \text{......................... (6.29)}$$

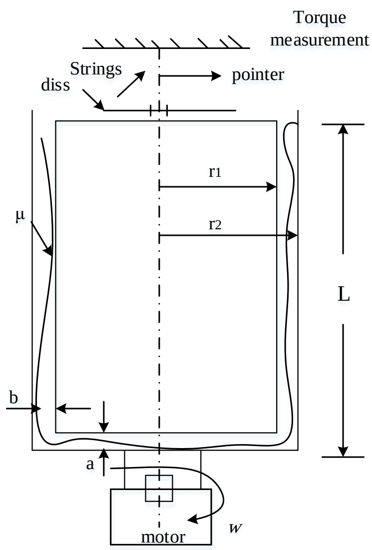
A=Annular area

a= constant indicating the tube taper=2y tanθ

6.5 Viscosity Measurements

6.5.1 Rotating Concentric Cylinders

This method is based on Newton's law of viscosity;



$$\tau = \frac{F}{A} = \mu\frac{du}{dy}$$

When the outer cylinder is rotated, the motion is transmitted to the inner cylinder by the action of viscosity (μ) of the fluid, thus excerting a torque (T) on it. This torque (T) is measured by measuring the deflection angle (θ) rotated by the inner cylinder,

T=Kθ …………………… (6.30)

Where K= stringe constant (N.m/deg)

This torque (T) equals the sum of the torques excerted on the cylinder walls (Tc) and bottom (Td). Hence

T=Tc+Td

$$K\theta=\mu\frac{r_2\omega}{b}2\pi r_1 L r_1+\int_0^{r_1}\int_0^{2\pi}\mu\frac{r\omega}{a}r\,d\theta\,dr*r$$

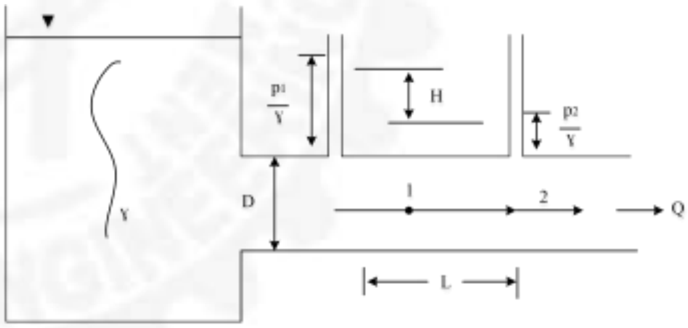
Or

$$K\theta=\mu\left[\frac{2\pi r_1^2 r_2\omega L}{b}+\frac{\pi\omega r_1^4}{2a}\right] \quad \text{……………………… (6.31)}$$

Thus, to measure the viscosity (μ), we need only to measure the deflection angle (θ),all other parameters are known (specification of the device)

6.5.2 Capillary Flow Method

This method is based on the application of the Hagen-Poiseullif equation (5.34) for steady, laminar incompressible flow.



$$Q=\frac{\Delta p\pi D^4}{128\mu L} \quad \text{…………………….. (6.32)}$$

$$\Delta p=p_1-p_2=\gamma H$$

$Q=\forall/t$

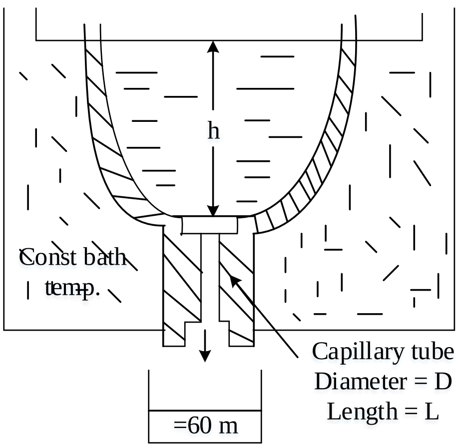
(Q,H) measured

(γ, L, D) known

Thus, from equation (6.32), viscosity (μ) is measured.

6.5.3 Saybolt Viscometer

This method is also based on capillary flow and Hagen-Poiseuille equation. It is used to measure the kinematic viscosity (υ). The time (t) necessary to drain ($\forall$= 60ml) is recorded. (t) in seconds is the saybolt reading.



$Q=\frac{\Delta p\pi D^4}{128\mu L}=\frac{\Delta p\pi D^4}{128v\rho L}$ $v\rho$

But $Q=\frac{\forall}{t}$ and $\Delta p=\rho gh$

Thus;

$v=\frac{gh\pi D^4}{128\forall L}t=c_1t$ ………………..(6.33)

Where h=head in the reservoirA correction term is added to equ.(6.33) totake into consideration that the flow is not fully developed. Thus;

$v=c_1t+\frac{c_1}{t}$ ……….(6.34) (C_1,C_2)= constants of the device

Examples

Example(6.1): A (75mm) diameter orifice under a haed of (4.88 m) discharges (907.6 kg) of water in (32.6s). The trajectory was determined by measuring (X_0=4.76m) for a drop of (1.22 m).Determine C_v,C_c,C_d, the head loss per unit gravity force, and the power loss.

Sol.:

$V_{2t} = \sqrt{2gH} = \sqrt{2 * 9.8 * 4.88} \Rightarrow V_{2t} = 9.783$ m/s

$t = \sqrt{2y_0/g} = \sqrt{\frac{2*1.22}{9.8}} \Rightarrow t = 0.499$s

$V_{2a} = \frac{x_0}{t} = \frac{4.76}{0.499} \Rightarrow V_{2a} = 9.539$ m/s

$$c_v = \frac{V_{2a}}{V_{2t}} = \frac{9.539}{9.783} \Rightarrow c_v = 0.975$$

$$Q_a = \frac{\forall}{t} = \frac{m/\rho}{t} = \frac{907.6/1000}{32.6} \Rightarrow Q_a = 0.0278\ \mathrm{m^3/s}$$

$$Q_t = A_0 V_{2t} = A_0\sqrt{2gH} = \frac{\pi}{4}(0.075)^2 * 9.783 \Rightarrow Q_t = 0.04322\frac{m^3}{s}$$

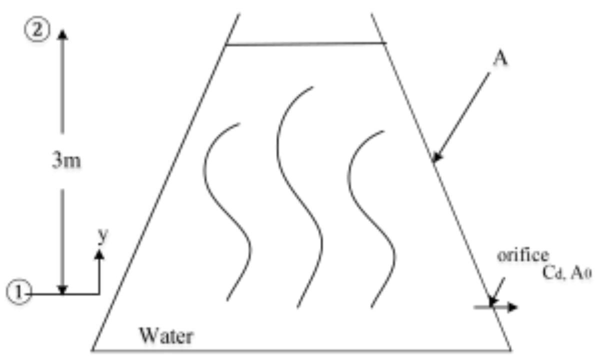
$$c_d = \frac{Q_a}{Q_t} = \frac{0.0278}{0.04322} \Rightarrow c_d = 0.643$$

$$c_c = \frac{c_d}{c_v} = \frac{0.643}{0.975} \Rightarrow c_c = 0.659$$

The head loss=$H(1-c_v^2)$ $=4.88(1-0.975^2) \Rightarrow$ loss=$0.241\frac{N.m}{N}$

Power loss=γQ* Loss =9810*0.0278*0.241$\Rightarrow$ Power loss=65.7 W

Example (6.2): A tank has a horizontal C.S.A. of ($2m^2$) at the elevation of the orifice, and the area varies linearly with elevation so that it is ($1m^2$) at a horizontal cross-section (3m) above the orifice. For a (100mm) diameter orifice, (cd=0.65), compute the time, in seconds, to lower the surface from 2.5 to 1 m above the orifce.



Sol.:

A=a+by

$$\left.\begin{aligned} y_1 &= 0 \;\; A_1 = 2 \\ y_2 &= 3 \; A_2 = 1 \end{aligned}\right\} \Rightarrow \begin{aligned} &a=2 \\ &b=-\tfrac{1}{3} \end{aligned}$$

Thus;

A=2$-\frac{1}{3}$y……………………….(1)

C.E:

$$\frac{\partial m}{\partial t} + \dot{m}_{out} - \cancel{\dot{m}_{in}} = 0$$

$$\rho A\frac{dy}{dt} + \rho c_d A_0\sqrt{2gy}=0$$

Hence;

$$t=-\frac{1}{c_d A_0\sqrt{2g}}\int_{y_1}^{y_2} Ay^{-1/2}dy$$

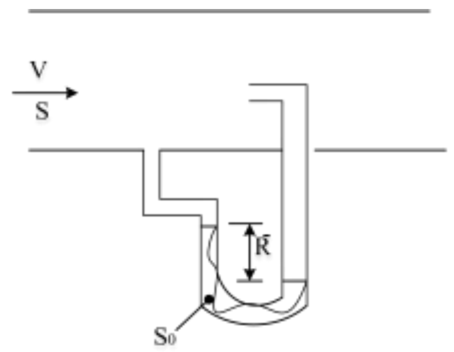
Using equ.(1);

$$t=-\frac{1}{0.65*\frac{\pi}{4}(0.1)^2\sqrt{2*9.81}}\int_{2.5}^{1}(2-\frac{y}{3})y^{-1/2}dy$$

Which gives; t =73.8 s

Example (6.3): In the figure shown air is flowing, for which (p=101 kpa and T=5 °C) and mercury is in the monometer. For (R'=200 mm), calculate the velocity (V)

Sol.:



$$V=\sqrt{2gR'(\frac{s_0}{s}-1)}$$

$$\rho=\frac{p}{RT}=\frac{101000}{287*278} \Rightarrow \rho = 1.266 \text{ kg/m}^3$$

$$\frac{s_0}{s}=\frac{13.6*1000}{1.266} \Rightarrow \frac{s_0}{s}=10742$$

Thus; V=$\sqrt{2*9.8*0.2(1074-1)}$ ⇒ V=205 m/s

Problems

The problem number listed in the table below refer to the problems in the "text book", (chapter(8)).

Article No.	Related Problems
6.2	1, 2
6.3	3, 4, 5, 6, 8, 10
6.4.1	16, 18, 19, 20, 21, 22,23,24, 25,26, 27, 28, 29, 30, 31, 32, 34, 3 37, 38
6.4.2	39, 40, 41, 42
6.5	58, 59, 60

Chapter -7-
Closed Conduits Flow Networks

7.1 Introduction

The analysis in this chapter is limited to incompressible fluid (ρ =constant), i.e., liquids and gases with very small velocities. Isothermal conditions is assumed to eliminate thermodynamic effects.

The network considered here consists mainly from (pipes, pumps, fittings, valves, reservoirs). The analysis will be based on the fundamental principles and equations of chapter "3" (Continuity and energy equations) and chapter"5" (viscous flow through pipes, and calculation of the losses for the flow).

7.2 Pipe Friction Formula

In the network analysis, we need a formula that relates the head loss (h_f) to the discharge (Q) of the flow. (h_f) here represents the frictional losses if we neglect the minor losses, and it represents the total losses (frictional +minor) if the minor losses is to be considered, in this case the length used represents the actual pipe length plus the equivalent length (Le) for the minor losses. We will list here some of the conventional formula used for this purpose;

Exponential Pipe –Friction Formula

$$\frac{h_f}{L} = \frac{RQ^n}{D^m} \qquad \text{.................................... (7.1)}$$

Where; $\frac{h_f}{L}$ = Head loss per unit length of the pipe

Q = discharge (m^3/s)

D = inside diameter (m)

n, m ,R= constants

Hazen-Williams Formula

$$\left.\begin{aligned} \frac{h_f}{L} &= \frac{RQ^n}{D^m} \\ R &= \frac{10.675}{C^n} \\ n = 1.852\ &;\ m = 4.8704 \end{aligned}\right\} \qquad \text{...........................(7.2)}$$

C= constant depends on pipe roughness

Darcy- Weisbach Equation

$$h_f = f\frac{L}{D}\frac{v^2}{2g} = f\frac{L}{D^5}\frac{8Q^2}{\pi^2 g} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7.3)$$

Equations (7.2) and (7.3) gives;

$$f = \frac{1014.2}{C^{1.852}D^{0.0184}}Re^{-0.148} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7.4)$$

Conventional Formula

$$h_f = rQ^n \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7.5)$$

r=constant depends on pipe

n= exponent (usually 2)

7.3 Concept of Hydraulic and Energy Grade Lines

The Hydraulic Grade Line (HGL) is a line showing the sum of pressure and potential energies per unit weight $(\frac{p}{\gamma} + z)$, while the energy grade line (EGL) represents the total energy of the fluid (pressure, potential and kinetic energies per unit weight). Both lines sloping downward with flow direction due to the losses in energy, except across a pump, where the energy increases by (hp). Thus;

$$HGL = \frac{p}{\gamma} + z \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7.6)$$

$$EGL = \frac{p}{\gamma} + z + \frac{V^2}{2g} = HGL + \frac{V^2}{2g} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7.7)$$

$$\text{Hydraulic Gradient} = \frac{d}{dx}\left(\frac{p}{\gamma} + z\right) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7.8)$$

$$\text{Energy Gradient} = \frac{d}{dx}\left(\frac{p}{\gamma} + z + \frac{V^2}{2g}\right) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7.9)$$

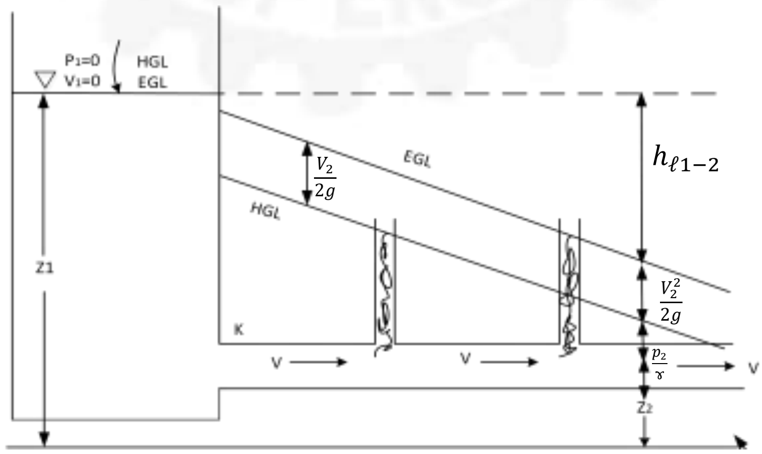


Fig. (7.1)

Notes:

1. EGL is positioned ($v^2/2g$) above HGL. Thus, if ($v=0$), as in a large reservoir, they coincide.
2. Due to losses, EGL always slopes downward in the direction of the flow. The only exception is when there is a pump:

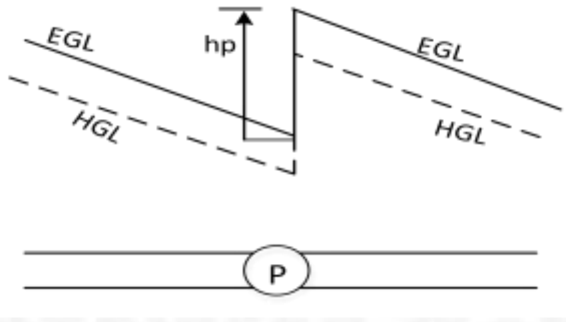


Fig. (7.2)

3. If there is a turbine, there will be abrupt drop in EGL and HGL. The kinetic energy can be converted to pressure energy by gradual expansion

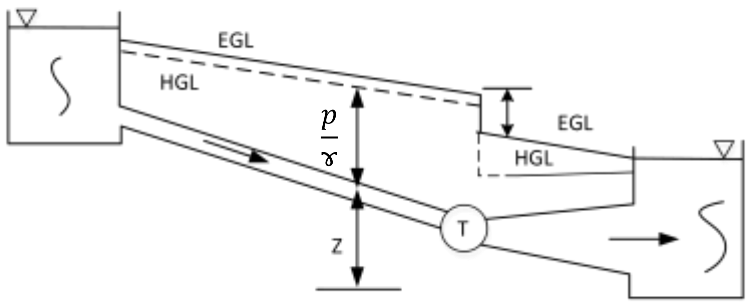


Fig. (7.3)

If the outlet to a reservoir is an abrupt expansion, all the k.E. is lost, i.e., E.G.L. drops an amount $\frac{v^2}{2g}$ at the outlet. If the flow passage changes in diameter, then the slope and the distance between the EGL and HGL changes.

$D_1 > D_2$

$\frac{V_1^2}{Z_g} < \frac{V_1^2}{Z_g}$

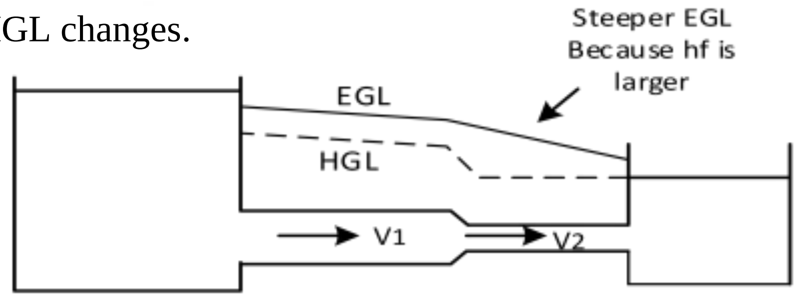


Fig. (7.4)

4. For negative $(\frac{p}{\gamma})$, such as in a siphon;

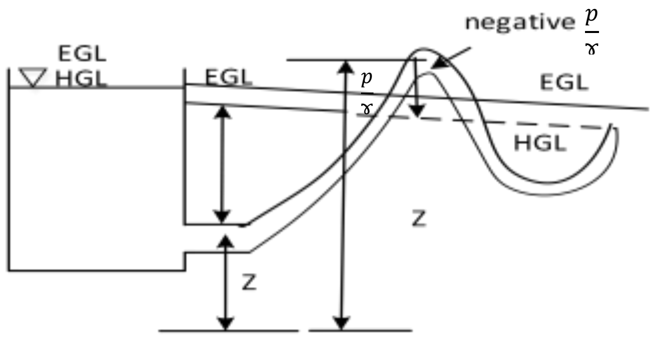


Fig. (7.5)

5. For long pipes, $(\frac{V^2}{2g})$ is small compared to $(h_f = f\frac{L}{D}\frac{V^2}{2g})$ and may be neglected. Thus, the EGL and HGL coincides.

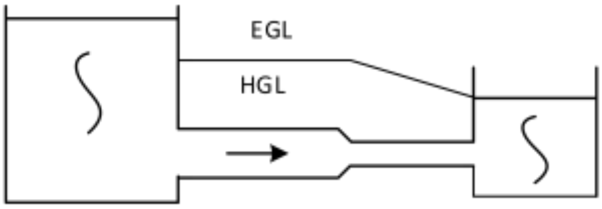


Fig. (7.6)

7.4 Combination of Pipes

7.4.1 Pipes in Series

$Q_1 = Q_2 = Q_3 = \ldots\ldots\ldots = Q_n$

$$h_\ell = h_{f1} + h_{f2} + \cdots + h_{fn} + \sum k\frac{V^2}{2g} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7.10)$$

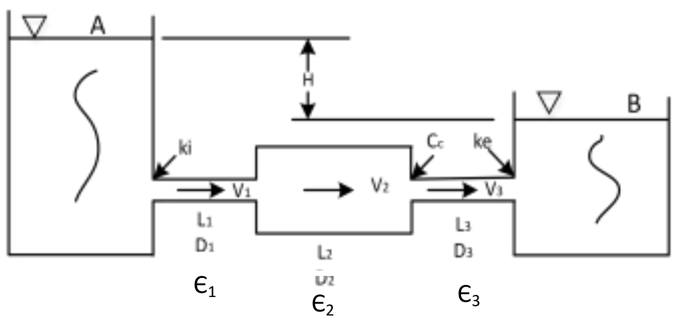


Fig. (7.7)

E.E.A-B

$$H = k_i\frac{V_1^2}{2g} + f_1\frac{L_1}{D_1}\frac{V_1^2}{2g} + \frac{(v_1 - v_2)^2}{2g} + f_2\frac{L_2}{D_2}\frac{V_2^2}{2g} + (\frac{1}{C_c} - 1)^2\frac{V_3^2}{2g} + f_3\frac{L_3}{D_3}\frac{V_3^2}{2g} + k_e\frac{V_3^2}{2g}$$

Equivalent Pipes

Two pipes systems are said to be equivalent when the same head loss produce the same discharge in both systems: Thus; for the two pipes to be equivalent;

$h_{f1} = h_{f2}$ and $Q_1 = Q_2$

Hence;

$$f_1 \frac{L_1}{D_1^5}\frac{8Q_1^2}{\pi^2 g} = f_2 \frac{L_2}{D_2^5}\frac{8Q^2}{\pi^2 g}$$

And;

$$L_{2eq} = L_1 \frac{f_1}{f_2}(\frac{D_2}{D_1})^5 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7.11)$$

Ex.: (D_2=250 mm, L_2=300 m, f_2=0.018) and ($D_1 = 150$mm, f_1=0.02). We want to find the equivalent length of pipe (2) in terms of pipe (1). Using (7.11);

$$L_{2eq} = 300\frac{0.02}{0.018}(\frac{150}{250})^5 \Rightarrow L_{2eq} = 25.9\text{m}$$

Thus; a (25.9 m) of (150 mm) pipe is equivalent to (300 m) of (250 mm) pipe for the assumed conditions.

7.4.2 Pipes in Parallel

$$\left.\begin{aligned} & Q = Q_1 + Q_2 + Q_3 + \ldots\ldots + Q_n \\ & h_{f1} = h_{f2} = h_{f3} = \cdots = h_{fn} = (\frac{p_A}{\gamma} + z_A) - (\frac{p_B}{\gamma} + z_B) \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots(7.12)$$

Two types of problems exist;

Type (I)

Given (P_A, Z_A, P_B, Z_B), to find Q?

Procedure:

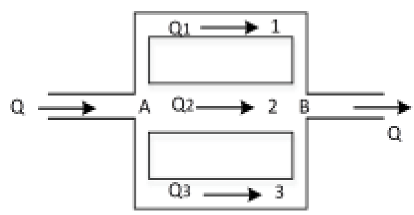


Fig. (7.8)

1. $h_f = (\frac{p_A}{\gamma} + z_A) - (\frac{p_B}{\gamma} + z_B) = h_{f1} = h_{f2} = h_{f3}$
2. Using the simple pipe problems method, with h_{f1} known, we can find (Q_1, Q_2, Q_3) from (h_{f1}, h_{f2}, h_{f3}) by using the relations in article (7.2) [$h_f \sim Q$]
3. Using C.E.; $Q = Q_1 + Q_2 + Q_3$

<u>Type (II)</u>

Given (Q) and (D, ∈, μ for each pipe), to find (Q_1, Q_2, Q_3 and h_{f1}, h_{f2}, h_{f3})

Procedure

1. Assume a discharge (Q_1') through pipe (1).
2. Calculate (h_{f1}') using (Q_1').

3. Using $(h'_{f2} = h'_{f1})$, calculate (Q'_2) and (Q'_3).
4. Check C.E., If $(Q' = Q'_1 + Q'_2 + Q'_3)$ is equal to the given (Q), then the solution is over. If not,assume;

   $Q''_1 = \frac{Q'_1}{Q'}Q \quad Q''_2 = \frac{Q'_2}{Q} \quad Q''_3 = \frac{Q'_3}{Q'}$

   You can use (Q''_1) and repeat steps (2-4) to check.
5. Calculate $(h_{f1}, h_{f2}, h_{f3})$from (Q''_1, Q''_2, Q''_3) and check that (h_{f1}, h_{f2}, h_{f3}).
6. $Q_1=Q''_1$; $Q_2=Q''_2$; $Q_3=Q''_3$

7.4.3 Branching Pipes

The following principles have to be applied in solving any problem involving branching pipes;

1. A the junction (J), C.E. must be satisfied, i.e; Inflow=Outflow
2. The Darcy-Weisbach or any pipe friction formula should be satisfied for each pipe.

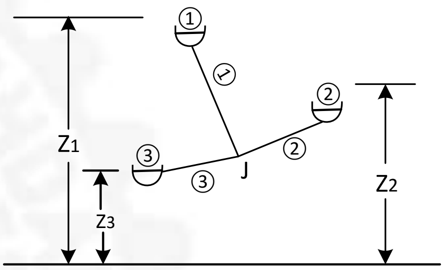


Fig. (7.9)

3. There can be only one value of piezometric head $(\frac{p}{\gamma} + z)$ at any point including the junction (j).
4. The flow will occur in the direction of falling piezometric head in a uniform pipe length $[(\frac{p}{\gamma} + z)_{\text{high}} \xrightarrow{\text{Flow}} (\frac{p}{\gamma} + z)_{\text{low}}]$

Three types of problems exist for this type of flow. (D, ϵ, μ and $h_f \sim Q$. Pipe friction formula) are given.

__Type "I"__

Given $\underset{3}{Z_1}, \underset{25}{Z_2}, \underset{20}{Z_3}, Q_1$ (assumed) Find Q_2, Q_3

__Procedure:__

1. Apply E.E. 1-J: $(\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + Z_1) = (\frac{p}{\gamma} + z)_j + \frac{V_j^2}{2g} + h_{f1}$ .Here we assume that point (1) is at the pipe (1) inlet, and we neglect the head of water in the tank; i.e.; $p_1 = 0$ and $(v_1 = v_j)$. Thus; $(\frac{p}{\gamma} + z)_J = Z_1 - h_{f1} = 27$ (assumed)

2. Since $(\frac{p}{\gamma}+z)_J = 27 > Z_2, Z_3$,then flow is from J→ (2)& (3)
3. E.E. J→ 2 $\Rightarrow h_{f2} = (\frac{p}{\gamma}+z)_J - Z_2 \Rightarrow Q_2 =$
4. E.E J→ 3 $\Rightarrow h_{f3} = \left(\frac{p}{\gamma}+z\right)_J - Z_3 \Rightarrow Q_3 =$

   Or; C.E at J: Q_1=$Q_2 + Q_3 \Rightarrow Q_3 = Q_1 - Q_2 =$

Type "II" Given Z_2, Z_3, Q_1 Find: Z_1, Q_2, Q_3

Type "III" Given Z_1, Z_2, Z_3 Find: Q_1, Q_2, Q_3

The last two types of problems are solved by trial and error since $(\frac{p}{\gamma}+z)_J$ cannot be calculated directly.

Procedure:

1. Assume $(\frac{p}{\gamma}+z)_J = (\ )$. The assumed value could be either larger than z_2 (flow from(1)→(2)&(3)) or smaller than Z_2 but larger than Z_3 (flow (1)+ flow (2) → (3))
2. Apply E.E.1→ J $\Rightarrow h_{f1} = (\ ) \Rightarrow Q_1 =$
3. Apply E.E. J→ 2 (or 2-J) $\Rightarrow h_{f2}$=( ) $\Rightarrow Q_2 =$
4. Apply E.E J→ 3 $\Rightarrow h_{f3}$ =( ) $\Rightarrow Q_3$
5. Check C.E at the Junction (J).If $(Q_{in} = Q_{out})$, then the solution is over. If not assume another value of $(\frac{p}{\gamma}+z)_J$ and repeat the procedure from step (2) until convergence, the new assume value should satisfy;

   If $(Q_{in} > Q_{out})_J$. Then assume higher value for $(\frac{p}{\gamma}+z)_J$.

   If$(Q_{in} < Q_{out})_J$ then assume lower value for $(\frac{p}{\gamma}+z)_J$

7.4.4 Pipe Networks

The flow in any network must satisfy the following three basic conditions;

1. At each junction, the C.E. must be satisfied, i.e.:

   Inflow = Out flow
2. The flow in each pipe must satisfy the pipe friction law

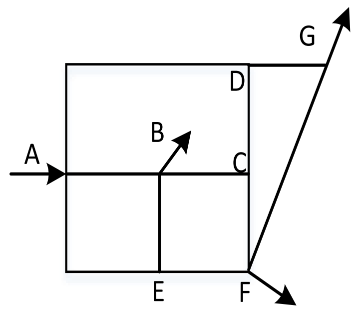


Fig. (7.10)

for flow in a single pipe.

3. The algebraic sum of the head loss around any closed loop must be zero (because we return to the same point).

"Hardy Cross Method" for pipe network is a systematic method for computation. It consists from the following steps;

1. Assume the most suitable distribution of flow that satisfy the C.E.
2. Compute ($h_L = rQ^n$) for each pipe (h_L = +ve clockwise and -ve counterclockwise).
3. Check condition (3) i.e., $\sum h_L = \sum rQ^n = 0$ for each loop.
4. If condition (3) is not satisfied, adjust the flow in each pipe by an amount (ΔQ) for each loop, i.e.;

   $Q = Q_o + \Delta Q$

   and $\Delta Q = -\frac{\sum rQ_o^n}{\sum |rnQ_o^{n-1}|} = -\frac{\sum h_L}{n\sum\left|\frac{hL}{Qo}\right|}$ .... (7.13)

   The numerator in **equ. (7.13)** is summed algebraically, while the denominator is summed arithmetically. Hence, if ($\Delta Q = -ve$) it must be subtracted from clockwise (Qo) and added to counterclockwise ones, and vice versa.
5. The above procedcare is repeated until ($\Delta Q = 0$).

7.5: Pumping Stations

7.5.1: Single Pump in Pipe Line

Consider the single pump in the pipe line system shown in the figure. The pump must fulfill the following requirements;

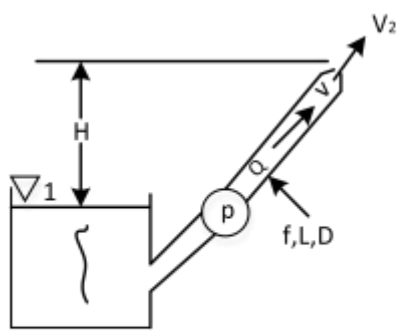


Fig.(7.11)

1. Raise the fluid to a static Head (H).
2. Maintain the flow rate (Q).
3. Overcome the losses in the pipe system.
4. Produce the k. E. with (V_2) at the pipe exit.

To calculate the pump head (hp) required for these condition, apply the E. E. from (1-2), which gives;

$$hp = H + \frac{V_2^2}{zg} + \frac{V^2}{zg}\left(f\frac{L}{D} + \sum K_{minor}\right)$$

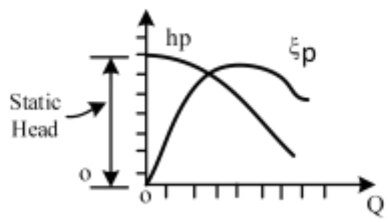
or; by using (V=Q/A), we can show that:

$$hp = H + const.* Q^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots (7.14) \qquad \text{“System Curve”}$$

Equ. (7.14) is called the “System Curve” , which gives the head (hp) that must be supplied to maintain the flow.

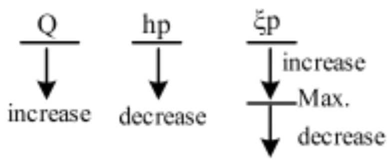
To select a pump that fulfills the system requirement equ. (7.14), we have to know the “pump characteristics”. For each pump, there is a “Pump Characteristics Curves” that gives the relation between the pump head (hp), pump efficiency (η_p or e_p) and. The discharge (Q), i. e. , two curves (hp~Q and η_p~Q). These characteristics are usually given in one of the following three forms;

1. <u>Graphs</u>:



(7.12)

2. <u>Tables:</u>



3. Equations:

$$hp = a + bQ^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots (7.15)$$

$$\eta p = cQ + dQ^2 \qquad \ldots\ldots\ldots\ldots\ldots... (7.16)$$

where (a, b, c, d) are constants (could be + ve or – ve)

Now, if we select a pump whose characteristics are given (either by curves, tables or equations) and placed it in the system of Fig. (7.11), the point of intersection of the system curve (7.14) and the (hp~Q) curve of the pump is called the “Operation Point”.

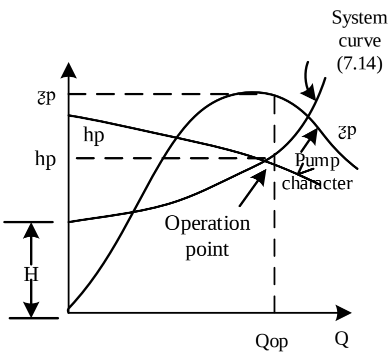


Fig. (7.13)

The designer always try to make the operation point close the point of maximum efficiency.

If the pump characteristics equations (7.15 & 7.16) are used, equs. (7.14) and (7.15) are solved simultaneously to obtain (Q_{op}), i. e:

$H + const\ Q_{op}^2 = a + bQ_{op}^2 \ \rightarrow\ Q_{op} = (\quad)$

and then equ. (7.16) is used to obtain (ηp).

The table may also be used by assuming a certain value of (Q), calculate (hp) from equ. (7.14) and compare it with (hp) from table until convergence.

If there is no point of intersection between the system curve and the pump characteristics (H > static head of pump), then either we select other pump with the larger static head > H), or we can use the principle of combination of pumps (similar or dis -similar ones).

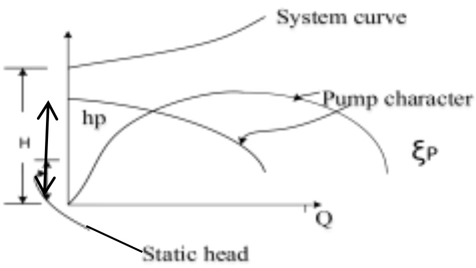


Fig. (7.14)

7.5.2 Pumps in Series

For two (or n) pumps connected in series which is used to obtain "High Head " and "Low Discharge"; see Fig. (7.15);

$Q_1 = Q_2 = --- = Q_n = Q_e$ ………….. (7.17)

$h_{pe} = h_{p1} + h_{p2} + --- + h_{pn}$ ……….. (7.18)

$Ip_e = Ip_1 + Ip_2$

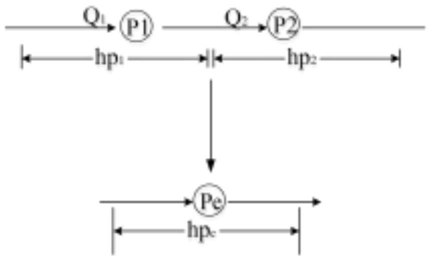


Fig. (7.15)

$$\frac{\gamma\, Qe\, hpe}{\eta\, pe} = \frac{\gamma\, Q1\, hp1}{\eta\, p1} + \frac{\gamma\, Q2\, hp2}{\eta\, p2}$$

Thus,

$$\eta_{pe} = \frac{hp1 + hp2 + -- + hpn}{\frac{hp1}{\eta p1} + \frac{hp2}{\eta p2} + --- + \frac{hpn}{\eta pn}} \quad ----- (7.19)$$

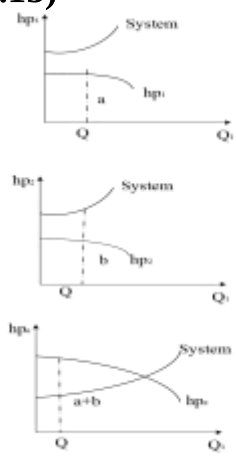


Fig. (7.16)

If twin (2), or (n) "identical" pumps are connected in series, then

$Q_1 = Q_2 = ---- = Q_n$ (identical pump)…….. (7.20)

$hp_1 = hp_2 = -- = hp_n$

Thus; from equs. (7.17) to (7.19);

$Qe = Q_1 = Q_2 = --- = Q_n$ ………(7.21)

$hp_e = nhp_1 = nhp_2 = --- = nph_n$ ………(7.22)

$\eta pe = \eta p_1 = \eta p_2 = ---\eta p_n$ ……..(7.23)

7.5.3 Pumps in Parallel

This connection is used to produce "High Discharge" and "low Head". For two pumps connected in parallel;

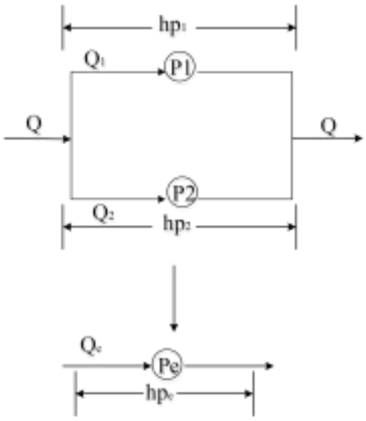


Fig. (7.17)

$Qe = Q_1 + Q_2 + --- + Q_n$ ……………(7.24)

$hp_e = hp_1 = hp_2 = --- = hp_n$ ……………..(7.25)

$IPe = IP_1 + IP_2$

$$\frac{\gamma\, Qe\, hpe}{\xi\, pe} = \frac{\gamma\, Q1\, hp1}{\xi\, p1} + \frac{\gamma\, Q2\, hp2}{\xi\, p2}$$

Thus;

$$\xi_{pe} = \frac{Q1+Q2+--+Qn}{\frac{Q1}{\xi p1}+\frac{Q2}{\xi p2}+---+\frac{Qn}{\xi pn}} \quad \text{……………. (7.26)}$$

If twin (2) or (n) identical pumps are connected in parallel, then;

$$\left.\begin{array}{l} Q_1 = Q_2 = ---- = Q_n \\ hp_1 = hp_2 = -- = hp_n \end{array}\right\} \text{(identical pumps) ………… (7.27)}$$

Thus; from equs. (7.24) to (7.26);

$Qe = nQ_1 = nQ_2 = --- = nQ_n$ …..……(7.28)

$hp_e = hp_1 = hp_2 = --- = hp_n$ ……………..(7.29)

$\eta pe = \eta p_1 = \eta p_2 = ---\eta p_n$ ……….(7. 30)

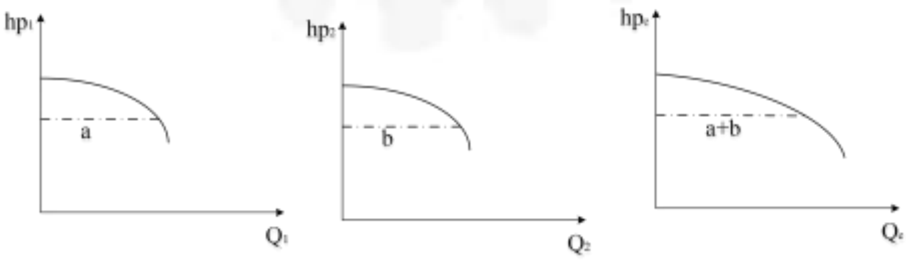


Fig. (7.18)

Note; "High discharge" is required when we want to reduce the time of operation of a certain flow system (Ex. Filling a tank).

Summary of Pumps Connections

If the pump characteristics are given by table or equation form, then the following table summarizes the rules of connecting (n) identical pumps.

Table			Equation	Connection
Q	hp	η_p		
- - - $\downarrow$ Increase	- - - $\downarrow$ Decrease	- Increase $\downarrow$ max $\downarrow$ Decrease	$hp = a+bQ^2$ $\eta_p = cQ+dQ^2$	Single Pump
Same Value	Multiply Each Value by(n)	Same Value	$hp_e = n(a+bQ^2)$ $\xi_{p_e} = CQe+dQe^2$	(n)Identical Pumps Connected In series
Multiply Each Value by(n)	Same Value	Same Value	$hp_e = a+b\left(\frac{Qe}{n}\right)^2$ ʒ$p_e = c\left(\frac{Qe}{n}\right) + d\left(\frac{Qe}{n}\right)^2$	(n) Identical Pumps Connected in Parallel

7.6 Conduits with Non-Circular Cross- Section

For ducts with non-circular cross -sections, the diameter (D) is replaced with ($D_h=4R_h$), where);

D_h = Hydraulic Diameter $=4\,R_h = 4\frac{A}{P}$

R_h = Hydraulic Radius $= \frac{\text{C.S.A}}{\text{wetted perimeter}} = \frac{A}{P}$ …………… (7.31)

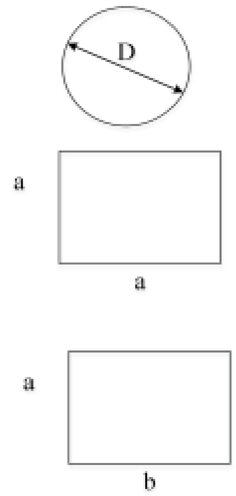
Thus;

$h_f = f\frac{L}{D_h}\frac{V^2}{2g}$ …………..(7.32)

$Re = \frac{\rho v\, D_h}{\mu} = \frac{V D_h}{v}$ …………… (7.33)

$\frac{\epsilon}{D} = \frac{\epsilon}{Dh}$ ……………(7.34) (in Moody Diagram)

EX:

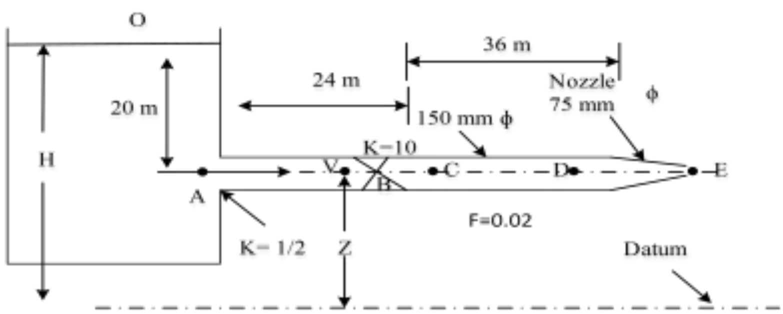


1. Circular pipe $D_h = 4\frac{A}{P} = 4\frac{\frac{\pi}{4}D^2}{\pi D} = D$
2. Square Duct $D_h = 4\frac{a^2}{4a} = a$
3. Rectangular Duct $D_h = 4\frac{ab}{2(a+b)} = 2\frac{ab}{a+b}$

Examples

Example (7.1): determine the elevation of hydraulic and energy grade lines at points A, B, C, D, and E shown in the figure Take (z=3m), and losses due to nozzle as $(0.1\frac{VE^2}{zg})$.

Sol.



E.E. 0-E

$0+0+23 = 0+\frac{VE^2}{2g} + 3 + \frac{1}{2}\frac{V^2}{2g} + 0.02\frac{60}{0.15}$

$\frac{V^2}{2g} + 10\frac{V^2}{2g} + 0.1\frac{VE^2}{2g}$ -------- 1

C.E.: $\frac{\pi}{4}(0.15)^2\, v = \frac{\pi}{4}(0.075)^2\, v_E$

Thus; $v_E = 4\,v$ ----2

1 & 2 gives; $\frac{V^2}{2g} = 0.554$ and $\frac{VE^2}{2g} = 16\frac{V^2}{zg}$

E.E. O-A

$$23 = \left(\frac{p}{\gamma} + \frac{V^2}{2g} + z\right)_A + 0.5\frac{V^2}{2g}$$

Thus; H.G.L. $_A = \left(\frac{p}{\gamma} + z\right)_A = 23 - 1.5\frac{V^2}{2g} = 23 - 1.5 * 0.554 \rightarrow$ H.G.L. $_A = 22.17$m

E.G.L. $_A = \left(\frac{p}{\gamma} + z + \frac{V^2}{2g}\right)_A = 23 - 0.5\frac{V^2}{2g} = 23 - 0.5 * 0.554 \rightarrow$ E.G.L. $_A = 22.72$m

E.E.O-B

$$23 = \left(\frac{p}{\gamma} + \frac{V^2}{2g} + z\right)_B + 0.5\frac{V^2}{2g} + 0.02\frac{24}{0.15}\frac{V^2}{2g} =$$

Thus; H.G.L.$_B$ = 23-(1.5 +0.02 $\frac{24}{0.15}$) $\frac{V^2}{2g}$ → H.G.L.$_B$=20.4 m

E.G.L.$_B$ =H.G.L.$_B$+ $\frac{V^2}{2g}$ = 20.4 +0.55 → E.G.L.$_B$=20.95 m

Across the valve, the H.G.L. drops by (10 $\frac{V^2}{2g}$), or (5.54m). Hence at (c);

H.G.L.$_C$=H.G.L.$_B$-5.54 → H.G.L.$_C$=14.86m

E.G.L.$_C$=E.G.L.$_B$-5.54 → E.G.L.$_C$=15.41m

<u>E.E.O-D</u>

$$23=\left(\frac{p}{\gamma}+\frac{V^2}{2g}+z\right)_D+\left(10.5+0.02\frac{60}{0.15}\right)\frac{V^2}{2g}$$

Thus; H.G.L.$_D$=23-19.5*0.554 → H.G.L.$_D$=12.2m

E.G.L.$_D$=H.G.L.$_D$+$\frac{V^2}{2g}$ → E.G.L.$_D$=12.75m

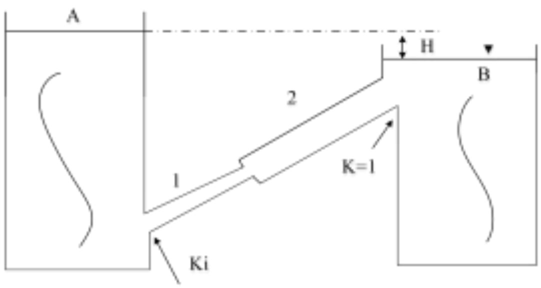
<u>At point E</u>

H.G.L.$_E$=$\frac{p_E}{\gamma}$ +Z_E =0+3 → H.G.L.$_E$=3m

E.G.L.$_E$=$\frac{p_E}{\gamma}$ + $\frac{V^2}{2g}$ + Z_E =0+16 $\frac{V^2}{2g}$ +3 → E.G.L.$_E$=11.86m

<u>Example:(7.2):</u> in the figure shown;

K_i=0.5, L_1=300m, D_1 =600mm, ϵ_1=2mm, L_2 =240m, D_2=1m, ϵ_2=0.3mm, $\nu=3*10^{-6}\frac{m^2}{s}$ and H=6m.



Determine the discharge through the system.

<u>Sol</u>.:

Applying E.E. from A to B, using

C.E($v_2=v_1(\frac{D1}{D2})^2=v_1*0.6^2$);

$$6=\frac{V1^2}{2g}\left[0.5+f_1\frac{300}{0.6}+(1-0.6^2)^2+f_2\frac{240}{1}0.6^4+0.6^4\right]$$

Thus;

$$6=\frac{V1^2}{2g}(1.0392+500f_1+31.104f_2) \qquad \ldots\ldots\ldots\ldots\ldots\ldots 1$$

For ($\frac{\epsilon 1}{D1}$=0.0033) and ($\frac{\epsilon 2}{D2}$=0.0003) and the Moody diagram, values of (f) are assumed for the fully turbulent range;

i.e.; f_1=0.026 & f_2= 0.015

Thus; from equ. 1 we obtain; v_1=2.848 m/s

and v_2= 1.025 m/s

$Re_1 = \frac{v1\ D1}{\nu} = \frac{2.848*0.6}{3*10^{-6}} \rightarrow Re_1=569600$

$Re_2 = \frac{v2\ D2}{\nu} = \frac{1.025*1}{3*10^{-6}} \rightarrow Re_2=341667$

Again, from moody diagram, with these (Re_1, Re_2) and ($\frac{\epsilon 1}{D1}$, $\frac{\epsilon 2}{D2}$) we obtain;

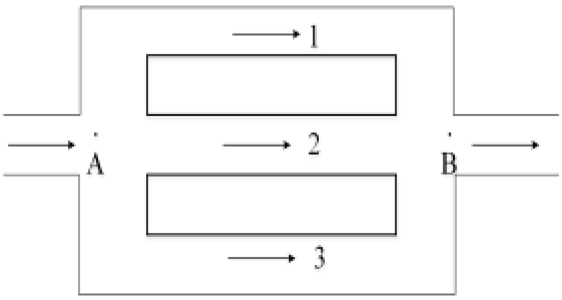
f_1=0.0265 and f_2= 0.0168

From equ. 1 , we obtain v_1=2.819 m/s

Thus, $Q=A_1V_1= \frac{\pi}{4}\ D_1^2 v_1 \rightarrow Q=0.797\ m^3/s$

Ex: (7.3) In the figure shown, L_1=900m, D_1=300mm, ϵ_1=0.3 mm, L_2=600m, D_2=200mm, ϵ_2=0.03mm, L_3=1200m, D_3=400mm, ϵ_3=0.24mm, ρ=1028kg/m^3, ν=2.8*10^{-6}m^2/s, p_A=560kpa, Z_A=30m, Z_B=24m.

For a total flow of (340 L/s), determine the flow through each pipe and the pressure at (B).



Sol.

Assume Q'_1 =85 L/s $\rightarrow v_1' = \frac{4\ Q'_1}{\pi D_1} \rightarrow v_1'$=1.2 m/s

$$\left.\begin{aligned} Re'_1 &= \frac{V'_1\ D_1}{\nu} = 129000 \\ \frac{\epsilon_1}{D_1} &= 0.001 \end{aligned}\right\} \rightarrow \text{Moody Diagram} \rightarrow f_1' = 0.022$$

$hf'_1 = f_1' \frac{L_1}{D_1} \frac{V'1}{2g} = 0.022 \frac{900}{0.3} \frac{1.2^2}{2\times 9.8} \rightarrow hf_1' = 4.85m$

<u>For Pipe 2</u>

$hf_2' = hf_1' = 4.85m = f_2' \frac{L2}{D2} \frac{V_2'}{2g}$

assuming f'_2=0.02 (fully turbulent, $\frac{c2}{D2}$ =0.00015), thus. V_2' =1.26 m/s,

$Re'_2=\frac{V'_2 D_2}{\upsilon}$ =91400, from Moody Diagram, f'_2=0.019 and V'_2=1.291 m/s

$Q'_2=\frac{\pi}{4} D'_2 V'_2$ → Q'_2=40.6 L/s

<u>For Pipe 3</u>

$hf'_3 = hf'_1$ =4.85m = $f'_3 \frac{1200}{0.4} \frac{V'_3}{2g}$

with ($\frac{c3}{D3}$ =0.0006) assume f'_3=0.02, then V'_3=1.259 m/s, Re'_3=180000, and hence;

f'_3=0.02 → Q'_3=158.2 L/s

The total discharge $Q'=\Sigma Q'= Q'_1+ Q'_2+ Q'_3$ → Q'=283.8 L/s

Hence; $Q_1= \frac{Q'_1}{Q'} Q$; $Q_2= \frac{Q'_2}{Q'} Q$; $Q_3= \frac{Q'_3}{Q'} Q$

Thus; Q_1= 101.3 L/s Q_2 =48.64 L/s Q_3 =189.53 L/s

The values of (hf):

$v_1=\frac{4 Q_1}{\pi D_1}$ =1.436 m/s Re_1=155000 f_1=0.021 → hf_1 =6.62 m

v_2=1.548 m/s Re_2=109500 f_2=0.019 → hf_2 =6.62m

V_3=1.512 m/s Re_3=216000 f_3=0.019 → hf_3 =6.64m

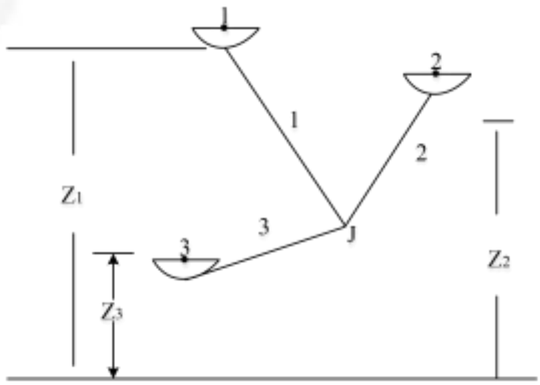
Thus, ($hf_1 = hf_2 = hf_3$) and the solution is over. To find (p_B);

<u>E.E.A-B</u> $\frac{P_A}{\gamma} + Z_A = \frac{P_B}{\gamma} + Z_B + hf$

$\frac{P_B}{\gamma} = \frac{560000}{1028*9.8} +30 -24 -6.64 = 54.8$

Hence P_B =54.8*1028*9.8 → P_B =552.5 kPa

<u>Examle (7.4)</u>: In the figure shown; find the dis charges for water at (20° c) with the following pipes data and reservoir elevations:



L_1=3000 m D_1=1m $\frac{\epsilon_1}{D_1}$ =0.0002

L_2 = 600m D_1= 0.45m $\frac{\epsilon_2}{D_2}$ = 0.002

L_3 = 1000m D_3 =0.6 m $\frac{\epsilon_3}{D_3}$=0.001

Z_1=30m Z_2=18m Z_3=9m

<u>Sol.:</u>

Assume $(\frac{p}{\gamma} +Z)_J$ =23 m

<u>E.E. 1-J</u> → $30=23+hf_1$ → $hf_1=7=f_1\frac{1000}{1}\frac{V1^2}{2g}$ $f_1=0.014$ $V_1=1.75$ m/s

$Q_1=1.38m^3/s$

<u>E.E. J-2</u> → $5=f_2\ \frac{600}{0.45}\ \frac{V2^2}{2g}$ $f_2=0.024$ $V_2=1.75$ m/s $Q_2=0.278m^3/s$

<u>E.E. J-3</u> → $14=f_3\ \frac{1000}{0.6}\ \frac{V3^2}{2g}$ $f_3=0.02$ $V_3=2.87$ m/s $Q_3=0.811m^3/s$

So that; (Inflow $=Q_1=1.38\ m^3/s$) is greater than the out flow ($Q_2+Q_3= 0.278 + 0.811= 1.089$) by amount ($0.291m^3/s$).

Assume $((\frac{p}{\gamma}+Z)_J=24.6$ m; and similarly);

<u>E.E. 1-J</u> $5.4= f_1\ \frac{3000}{1}\ \frac{V1^2}{2g}$ $f_1=0.015$ $V_1=1.534$ m/s $Q_2=1.205m^3/s$

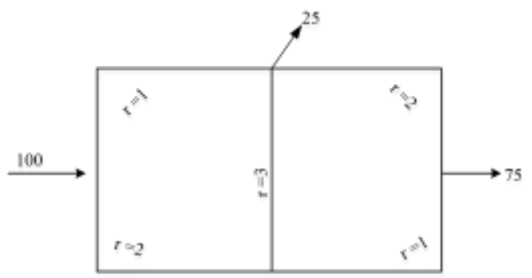
<u>E.E. J-2</u> $6.6= f_2\ \frac{600}{0.45}\ \frac{V2^2}{2g}$ $f_2=0.024$ $V_2=2.011$ m/s $Q_2=0.32m^3/s$

<u>E.E. J-3</u> $15.6= f_3\ \frac{1000}{0.6}\ \frac{V3^2}{2g}$ $f_3=0.02$ $V_3=3.029$ m/s $Q_3=0.856m^3/s$

Inflow is still greater by ($0.029\ m^3/s$). By extrapolating linearly,

$(\frac{p}{\gamma}+Z)_J=24.8m$ → $Q_1=1.183m^3/s$ $Q_2=0.325m^3/s$ $Q_3=0.856m^3/s$

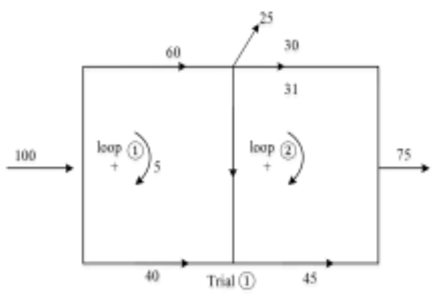
<u>Example (7.5):</u> Calculate the discharge through each pipe of the network shown in the figure. Take (n=2).



<u>Sol.:</u>

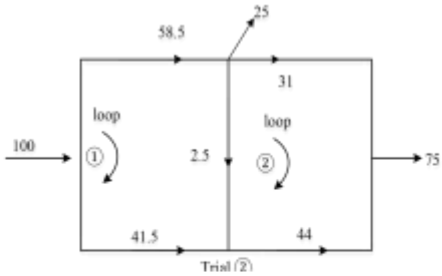
<u>First Trail</u>

Loop 1		Loop 2	
$r\ Q_0^n$	$n\ rQ_0^{n-1}$	$r\ Q_0^n$	$n\ rQ_0^{n-1}$
$1*60^2=3600$	$2*1*60=120$	$2*30^2=1800$	$2*2*30=120$
$3*5^2=75$	$2*3*5=30$	$3*5^2=-75$	$2*3*5=30$
$2*40^2=-3200$	$2*2*40=160$	$1*45^2=-2025$	$2*1*45=90$
$\Sigma\ r\ Q_0^n=475$	$\Sigma\lvert n\ rQ_0^{n-1}\rvert=310$	$\Sigma rQ_0^n=-300$	$\Sigma\lvert n\ rQ_0^{n-1}\rvert=240$
$\Delta Q=-\frac{475}{310}\approx -1.5$		$\Delta Q=-\frac{-300}{240}\approx 1$	



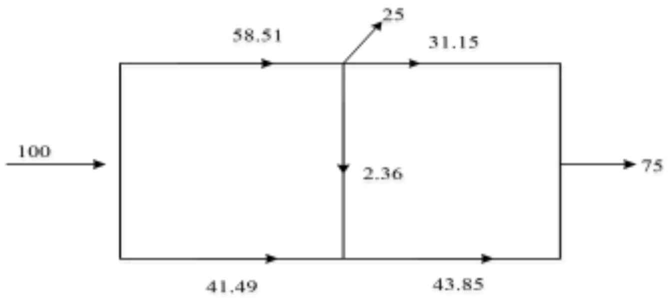
Trial 2 $Q=Q_0+\Delta Q$

Loop 1		Loop2	
$r\,Q_0^{\,n}$	$n\,rQ_0^{\,n-1}$	$r\,Q_0^{\,n}$	$n\,rQ_0^{\,n-1}$
$1*58.5^2=3422.25$	117	$2*31^2=1922$	124
$3*2.5^2=18.75$	15	$3*2.5^2=-19$	15
$2*41.5^2=-3444.5$	166	$1*44^2=-1936$	88
$\Sigma\, r\,Q_0^{\,n}=-3.5$	$\Sigma\lvert n\,rQ_0^{\,n-1}\rvert=298$	$\Sigma\, r\,Q_0^{\,n}=-33$	$\Sigma\lvert\, n\,rQ_0^{\,n-1}\rvert=227$
$\Delta Q=-\frac{-3.5}{298}=0.012$		$\Delta Q=-\frac{-33}{227}=0.145$	

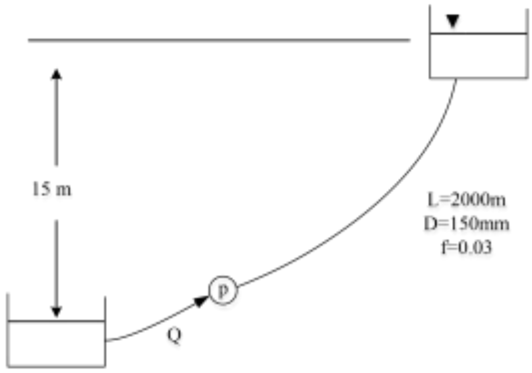


$2.5=5+(-1.5)-1$

Thus $\Delta Q\approx 0$ for all loops, and hence the final distribution will be;



Example (7.6): A pump characteristics are given by; ($h_p=31.12000Q^2$) and (ʒ$_p$ $=-190000Q^2+8590Q$), where (h_p) is in (m) and (Q) in (m^3/s) and (ʒ$_p$) is the efficiency (%).



a. Find the characteristics of three identical pumps connected in parallel
b. For the piping system shown in the figure, estimate the difference in operational cost between one pump and three pumps in parallel to supply ($1000m^3$) of water to the upper reservoir if the cost of (1KWh) is (65 fils). Neglect minor losses
c. Repeat (a) and (b) for three identical pumps in series.

Sol.:

(a) $Qe=3Q_1=3Q_2=3Q_3=3Q$

$$hp_e=hp_1=hp_2=hp_3 \quad\rightarrow\quad hp_e=31-12000\left(\frac{Qe}{3}\right)^2$$

$$ʒp_e = ʒp_1 = ʒp_2 = ʒp_3 \quad\rightarrow\quad ʒp_e=-190000\left(\frac{Qe}{3}\right)^2+8590\left(\frac{Qe}{3}\right)$$

(b) E.E → $h_p = 15 + h_f = 15 + f\frac{L}{D^5}\frac{8Q^2}{\pi^2 g}$

Or ; $h_p = 15 + 65352Q^2$------- (1)

One pump

$31 - 12000Q^2 = 15 + 65352Q^2$ → $Q = 0.01438\,m^3/s$

$h_p = 31 - 12000(0.01438)^2$ → $h_p = 28.51m$

$\eta_p = -190000(0.01438)^2 + 8590(0.01438)$ → $\eta_p = 84.23\%$

$Ip = \frac{\gamma Q hp}{\eta p}$ → $Ip = 4774.5W$

$t = \frac{\forall}{Q} = \frac{1000}{0.01438}$ → $t = 695410.3\ s$

no. of KWh= $Ip*t = \frac{4774.5}{1000} * \frac{695410.3}{3600}$ → no. of KWh=922.3

cost. =922.3*0.065 → cost =59.95 ID

Three Pumps

$Qe = 3Q_1 = 3*0.01438$ → $Qe = 0.04314\,m^3/s$

$h_{pe} = h_{p1} = h_{p2} = h_{p3}$ → $h_{pe} = 28.51m$

$\eta_{pe} = \eta_{p1} = \eta_{p2} = \eta_{p3}$ → $\eta_{pe} = 84.23\%$

$t = \frac{\forall}{Qe} = \frac{1000}{0.04314}$ → $t = 231803.4\ s$

$Ip = \frac{\gamma Q hp}{\eta p}$ → $Ip = 14324.5\ W$

no.of KWh= Ip*t → no. of KWh=922.3

cost. =922.3*0.065 → cost =59.95 ID

Thus, there is no difference in cost. Only the time will be reduced

(c) $Qe = Q_1 = Q_2 = Q_3 = Q$

$hp_e = 3Q_1 = 3Q_2 = 3Q_3$ → $hp_e = 93 - 36000\ Q^2$

$\eta p_e = \eta p_1 = \eta p_2 = \eta p_3$ → $\eta p_e = -190000Q^2 + 8590Q$

One pump; the same as in (b)

Three Pumps in Series

$Qe = Q_1 = Q_2 = Q_3 = Q$ → $Qe = 0.01438\ m^3/s$

$hp_e = 3\ hp = 3*28.51$ → $hp_e = 85.53$

$\eta p_e = \eta p_1 = \eta p_2 = \eta p_3$ → $\eta p_e = 84.23\%$

$Ip=\frac{\gamma Q\,hpe}{\eta pe}$ → $Ip=14324.5$ W

$t=\frac{\forall}{Q}=\frac{1000}{0.01438}$ $t=695410.3$ s

no. of KWh= Ip*t → no. of KWh=2767

Cost. =2767*0.065 → cost. =179.86 ID

Cost is higher than for single pump.

Example(7.7): Determine the head loss, in millimeters of water, required for flow of (300 m^3/min) of air at (20˚c) and (100 kpa) through a rectangular galvanized-iron section (700 mm) wide, (350mm) high, and (70m) long.

Sol.:

$R_h=\frac{A}{P}=\frac{0.7*0.35}{2(0.7+0.35)}$ → $R_h=0.117$ m

$D_h=2\,R_h$ → $D_h=0.468$m

$\frac{\epsilon}{D}=\frac{\epsilon}{D_h}=\frac{0.00015}{0.468}$ → $\frac{\epsilon}{D}=0.00032$

$V=\frac{Q}{A}=\frac{300/60}{0.7*0.35}$ → $v=20.41$ m/s

$\rho=\frac{p}{RT}=\frac{100000}{287*293}$ → $\rho=1.189$ kg/m^3

From Fig. (c.1); $\mu=2.2*10^{-5}$ Pa. s

$Re=\frac{VD_h\rho}{\mu}=\frac{20.41*0.468*1.189}{2.2*10^{-5}}$ → $Re=516200$

From Moody Diagram; f=0.0165

Thus;

$h_f=f\,\frac{L}{D_h}\,\frac{V^2}{2g}$

$=0.0165\,\frac{70}{0.468}\,\frac{20.41^2}{2*9.8}$ → $h_f=52.42$ m air

$\gamma_{air}=\rho_{air}\,g=1.189*9.8$ → $\gamma_{air}=11.66$ N/m^3

Thus;

$h_f(mH_2O)=h_f*\frac{\gamma_{air}}{\gamma_w}$

$=52.42*\frac{11.66}{9810}$ → $h_f=0.0623\,mH_2O=62.33\,mmH_2O$

Problems

The problems number listed in the table below refer to the problems in the "text book", chapter (10)

Article No.	Related Problems
7.3	1,2,3,5,6,7,8
7.4	10,11,15,17,19,28,31,32,38,40,41
7.5	20,22,23,24,25,26,27,33,34,35,36,37,39
7.6	44, 45,46,47

Appendix –A-

COURSE FOLIO

Course Title – Symbol: **Fluid Mechanics / I – ME202**

Instructor:

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Course Description (or Catalog Data)

ME202 - Fluid Mechanics / I

This course introduces the description of phenomena associated with fluid flow. Topics covered: physical properties of fluids; fluid statics; principles of conservation of mass, energy and momentum; control volume technique; Bernoulli equation; dimensional analysis and similitude; viscous flow in pipes and channels; laminar and turbulent flow;

boundary layer theory; drag and lift; Moody diagram; pipe problems; flow and fluid measurements; analysis of pipes and pumps networks. Physical understanding of fluid flows and applications to practical problems will be stressed. The course is designed to provide a background to higher level courses involving fluid flow and heat transfer. The course is taught through 5 hrs per week, 3 theories, 1 tutorial, and 1 experimental.

Prerequisites: ME101 & ME102 Courses

Goals / Objectives

1. Introduce basic definitions and introductory concepts of fluid mechanics.
2. Introduce the description of pressure distribution in a static fluid and its effects on submerged surfaces and bodies.
3. Introduce the description of phenomena associated with fluid flow phenomena.
4. Explain and derive the conservation laws that govern fluid motion (continuity, energy, and momentum equations).
5. Introduce the principles of “Dimensional Analysis” and “Similitude” and their application to fluid mechanics problems.
6. Introduce the principles of viscous flow, boundary layer, drag and lift, primary and secondary losses in pipe flow.
7. Enable the student to analyze and design pipes network and pumps connection.
8. Enable the student to measure the fluid properties and flow parameters, and to design and conduct experiments of fluid mechanics.
9. Provide a strong physical and analytical understanding of fluid flows in order to function in the capacity of mechanical engineer in an engineering company dealing with fluid machinery.
10. Provide a background to higher level courses involving fluid flow and head transfer.

Student Learning Outcomes

At the end of the class, the student will be able to:

a. Define Fluids and Fluid Mechanics and distinguish between incompressible and compressible fluids, and understand and define the basic fluid properties; especially density and viscosity, and apply Newton's law of viscosity.
b. Calculate; the pressure in static fluid, hydrostatic forces on submerged surfaces, buoyancy forces, stability of submerged and floating bodies, Metacenter, and forces on accelerated fluids.
c. Be familiar with continuity, energy, and momentum equations, and their applications to fluid flow problems.
d. Understand and apply the principles of dimensional analysis and similitude to fluid mechanics problems.
e. Formulate and solve incompressible laminar flows for simple parallel flows in Cartesian and polar coordinates.
f. Analyze boundary layer flows over flat plate.
g. Estimate drag and lift forces in laminar and turbulent flows for different immersed bodies.
h. Calculate frictional losses in pipe problems for both laminar and turbulent flows, by using Moody Diagram.
i. Calculate secondary (minor) losses for various pipes fittings and connections.
j. Know how to measure flow properties (pressure, velocity, discharge) and fluid properties (density and viscosity).
k. Be able to analyze and design pipes network and connection, and pumping stations and connection.
l. Be able to apply modern knowledge and to apply mathematics, science, engineering and technology to fluid mechanics problems and applications.
m. Design and conduct experiments of fluid mechanics, as well as analyze, interpret data and apply the experimental results for the services.

n. Work in groups and function on multi-disciplinary teams.

o. Identify, formulate and solve engineering fluid mechanics problems.

p. Understand professional, social and ethical responsibilities.

q. Communicate effectively.

r. Use the techniques, skills, and modern engineering tools necessary for engineering practice in fluid mechanics applications.

Course Schedule

Week	Theoretical Content	Hours / Week		
		Theo.	Tut.	Exp.
1	Introductory Concepts To Fluid Mechanics	3	1	1
2	Introductory Concepts To Fluid Mechanics	3	1	1
3	Fluid Statics : Pressure Distribution In Static Fluids	3	1	1
4	Pressure Measurements	3	1	1
5	Forces On Immersed Surfaces	3	1	1
6	Forces On Immersed Surfaces	3	1	1
7	Buoyancy And Floatation	3	1	1
8	Buoyancy And Floatation	3	1	1
9	Buoyancy And Floatation	3	1	1
10	Accelerated Fluid And Relative Motion	3	1	1
11	Introduction To Fluid Motion	3	1	1
12	Continuity Equation	3	1	1
13	Energy Equation	3	1	1
14	Momentum Equation	3	1	1
15	Momentum Equation	3	1	1
16	Dimensional Analysis And Similitude	3	1	1

17	Dimensional Analysis And Similitude	3	1	1
18	Dimensional Analysis And Similitude	3	1	1
19	Laminar Viscous Flow Between Parallel Plates	3	1	1
20	Laminar Viscous Flow Through Circular Tubes	3	1	1
21	Boundary Layer Theory, Drag & Lift	3	1	1
22	Losses In Pipes : Moody Diagram	3	1	1
23	Losses In Pipes : Moody Diagram	3	1	1
24	Losses In Pipes : Moody Diagram	3	1	1
25	Measurements Of Fluid Flow	3	1	1
26	Measurements Of Fluid Flow	3	1	1
27	Measurements Of Fluid Flow	3	1	1
28	Analysis Of Piping And Pumping Networks	3	1	1
29	Analysis Of Piping And Pumping Networks	3	1	1
30	Analysis Of Piping And Pumping Networks	3	1	1

Textbook and References

1. **"Fluid Mechanics"; by Victor L. Streeter and E. Benjamin Wylie, First SI Metric Edition, M G. GNW Hill , 1988.**
2. **"Fundamental of Fluid Mechanics"; by Bruce E. Munson, Theodore H. Okiishi, and Wade W. Huesch, Benjamin Wylie, Sixth Edition, 2009**
3. **"Fluid Mechanics : Fundamentals and Applications"; by Yunus A. Çengel and John M. Cimbala, M G. GNW Hill Higher Education, 2006**
4. **"Introductory Fluid Mechanics" ; by Joseph Katz, Cambridge University Press, 2010**

5. **"Elementary Fluid Mechanics", by John K. Vennard and Robert L. Streat, 5th ed., John Wiley and Sons, 1976.**
6. **"Engineering Fluid Mechanics by John A. Robert and Clayton T. Crow, 2nd ed., Houghton Mifflin Coo, 1988.**

Grading Units

Academic System	☐ **Semester System**		**√ Annual**
Course Assessment for Annual System (%100)	**Quest**	**Laboratory Work**	**Final Examination**
	30 % (Tests, , Quizzes, and Extracurricular Activities)	****	**70%**
Additional Information	**1- The laboratory experiments are included in the general course (Laboratories / 2).** **2- Two - Three tests are made; and not Less than (20-25) Quizzes are usually made.**		

Typical Grading

Excellent	90-100%
Very Good	80-89%
Good	70-79%
Fair	60-69%
Pass	50-59%
Poor	<50%

Grading Policy

1. Q uizzes:
   - There will be a (20 – 25) closed books and notes quizzes during the academic year.
   - The quizzes will count 20% of the total course grade.
2. Tests, 2-3 Nos. and will count 10% of the total course grade.
3. Extracurricular Activities, this is optional and will count extra marks (1 – 5 %) for the student, depending on the type of activity.
4. Final Exam:

- The final exam will be comprehensive, closed books and notes, and will take place on (Saturday-8th) , (January), 2013 from 9:00 AM - 12:00 PM in rooms (M12 + M13)
- The final exam will count 70% of the total course grade

Assessment Plan

1. Reinforcement is done through tests, quizzes, extracurricular activities and student engagement during lectures as shown in the table below.

Course Outcomes	Strategies/Actions	Assessment Methods
a) Define Fluids and Fluid Mechanics and distinguish between incompressible and compressible fluids, and understand and define the basic fluid properties; especially density and viscosity, and apply Newton's law of viscosity.	Lecture plan and in-class activities: each class will commence with a summary of the previous lecture, questions will be asked and the responses will be used to evaluate the students' understanding of the topics covered.	• In-class questions and discussion • Quizzes • Lab. Experiments
b) Calculate; the pressure in static fluid, hydrostatic forces on submerged surfaces, buoyancy forces, stability of submerged and floating bodies, Metacenter, and forces on accelerated fluids.	Lecture plan and in-class activities: each class will commence with a summary of the previous lecture, questions will be asked and the responses will be used to evaluate the students' understanding of the topics covered.	• In-class questions and discussion • Quizzes • Lab. Experiments
c) Be familiar with continuity, energy, and momentum equations, and their applications to fluid flow problems.	Lecture plan and in-class activities: each class will commence with a summary of the previous lecture, questions will be asked and the responses will be used to evaluate the students' understanding of the topics covered.	• In-class questions and discussion • Quizzes • Lab. Experiments
d) Understand and apply the principles of	Lecture plan and in-class activities: each class will	• In-class questions and discussion

dimensional analysis and similitude to fluid mechanics problems.	commence with a summary of the previous lecture, questions will be asked and the responses will be used to evaluate the students' understanding of the topics covered.	• Quizzes • Lab. Experiments
e) Formulate and solve incompressible laminar flows for simple parallel flows in Cartesian and polar coordinates.	Lecture plan and in-class activities: each class will commence with a summary of the previous lecture, questions will be asked and the responses will be used to evaluate the students' understanding of the topics covered.	• In-class questions and discussion • Quizzes • Lab. Experiments
f) Analyze boundary layer flows over flat plate.	Lecture plan and in-class activities: each class will commence with a summary of the previous lecture, questions will be asked and the responses will be used to evaluate the students' understanding of the topics covered.	• In-class questions and discussion • Quizzes • Lab. Experiments
g) Estimate drag and lift forces in laminar and turbulent flows for different immersed bodies.	Lecture plan and in-class activities: each class will commence with a summary of the previous lecture, questions will be asked and the responses will be used to evaluate the students' understanding of the topics covered.	• In-class questions and discussion • Quizzes • Lab. Experiments
h) Calculate frictional losses in pipe problems for both laminar and turbulent flows, by using Moody Diagram.	Lecture plan and in-class activities: each class will commence with a summary of the previous lecture, questions will be asked and the responses will be used to evaluate the students' understanding of the topics covered.	• In-class questions and discussion • Quizzes • Lab. Experiments

i) Calculate secondary (minor) losses for various pipes fittings and connections .	Lecture plan and in-class activities: each class will commence with a summary of the previous lecture, questions will be asked and the responses will be used to evaluate the students' understanding of the topics covered.	• In-class questions and discusion • Quizzes • Lab. Experiments
j) Know how to measure flow properties (pressure, velocity, discharge) and fluid properties (density and viscosity).	Lecture plan and in-class activities: each class will commence with a summary of the previous lecture, questions will be asked and the responses will be used to evaluate the students' understanding of the topics covered.	• In-class questions and discussion • Quizzes • Lab. Experiments
k) Be able to analyze and design pipes network and connection, and pumping stations and connection.	Lecture plan and in-class activities: each class will commence with a summary of the previous lecture, questions will be asked and the responses will be used to evaluate the students' understanding of the topics covered.	• In-class questions and discussion • Quizzes • Lab. Experiments
l) Be able to apply modern knowledge and to apply mathematics, science, engineering and technology to fluid mechanics problems and applications.	• Lectures • Experiments • Tests and Exams • Extracurricular activities	• In-class questions and discussion • Quizzes • Lab. Experiments • Extracurricular activities
m) Design and conduct experiments of fluid mechanics, as well as analyze, interpret data and apply the experimental results for the services.	Conducting experiments in Fluid Mechanics Lab. and technical writing of reports including discussions and conclusions	• In-class questions and discussion • Quizzes • Lab. Experiments
n) Work in groups and function on multi-disciplinary teams.	• Conducting experiments in groups • Tutorial hour work • Extracurricular	Monitoring, Guiding, Evaluation, Quizzes …..

	activities	
o) Identify, formulate and solve engineering fluid mechanics problems.	Homework , Tutorial hour, Solved Examples …..	• In-class questions and discussion • Quizzes • Tests
p) Understand professional, social and ethical responsibilities.	In- and Out-Class oral conservations, lectures….	• Discussions and conservations • Extracurricular activities
q) Communicate effectively.	In- and Out-Class oral conservations, lectures….	• Discussions and conservations • Extracurricular activities
r) Use the techniques, skills, and modern engineering tools necessary for engineering practice in fluid mechanics applications.	In- and Out-Class oral conservations, lectures….	• Discussions and conservations • Extracurricular activities
s) Be able to analyze and design pipes network and connection, and pumping stations and connection.	Lecture plan and in-class activities: each class will commence with a summary of the previous lecture, questions will be asked and the responses will be used to evaluate the students' understanding of the topics covered.	• In-class questions and discussion • Quizzes • Lab. Experiments
t) Be able to apply modern knowledge and to apply mathematics, science, engineering and technology to fluid mechanics problems and applications.	• Lectures • Experiments • Tests and Exams • Extracurricular activities	• In-class questions and discussion • Quizzes • Lab. Experiments • Extracurricular activities
u) Design and conduct experiments of fluid mechanics, as well as analyze, interpret data and apply the experimental results for the services.	Conducting experiments in Fluid Mechanics Lab. and technical writing of reports including discussions and conclusions	• In-class questions and discussion • Quizzes • Lab. Experiments
v) Work in groups and function on multi-disciplinary teams.	• Conducting experiments in groups • Tutorial hour work	Monitoring, Guiding, Evaluation, Quizzes …..

	• Extracurricular activities	
w) Identify, formulate and solve engineering fluid mechanics problems.	Homework , Tutorial hour, Solved Examples …..	• In-class questions and discussion • Quizzes • Tests
x) Understand professional, social and ethical responsibilities.	In- and Out-Class oral conservations, lectures….	• Discussions and conservations • Extracurricular activities
y) Communicate effectively.	In- and Out-Class oral conservations, lectures….	• Discussions and conservations • Extracurricular activities
z) Use the techniques, skills, and modern engineering tools necessary for engineering practice in fluid mechanics applications.	In- and Out-Class oral conservations, lectures….	• Discussions and conservations • Extracurricular activities

2. Lists the responses obtained from student survey conducted at the end of academic year. A student's opinion questionnaire was made for a selected specimen of the students. The results of this questionnaire are shown in Table (1) and figure (1) for the students opinion about curriculum , and in Table (2) and figure (2) for the students opinion about faculty member.

Table (1): Students Opinion Questionnaire about Curriculum
Academic Year 2010 – 2011

Code No. & Curriculum Name: **:** **ME 202 FLUID MECHANICS / I**
Year: **2nd Year**

Faculty Member's Name: **Prof. Dr. Ihsan Y. Hussain**

Dear Students: For the development of the educational process at the university, we hope to express your opinion by answering accurately with mark √ in the place which reflects your opinion taking into consideration the accuracy and objectivity.

Score		1	2	3	4	5
No.	**Question**	**Strongly Agree**	**Agree**	**I Don't Know**	**Disagree**	**I Don't Agree At All**
1	Overall, this Curriculum subject is good and useful	9	7	0	0	0
2	Lecture time is sufficient to cover the contents of the article	7	6	0	3	0
3	The content of article commensurate with the objective of Curriculum	6	9	1	0	0
4	Subject content is an interdependent information	5	7	1	3	0
5	Textbooks and references are available and meaningful	3	7	1	5	0
6	available of References helpful for stimulate and thinking	2	8	2	1	3
7	The book is free of grammatical errors Printing	3	6	5	2	0
8	Contents of the book are of outdated information	2	6	7	0	1
9	The book contains a variety of examples and exercises	7	7	1	1	0
10	The evaluation of the subject system is appropriate (test method)	6	7	1	0	2
11	Exams reflect the content of the subject	6	6	1	3	0
12	Number of exams be exhaustive of the content subject	6	8	2	0	0
13	Examinations and assignments helped to absorb the subject	5	8	0	0	3
14	Examinations and exercises are in line with the objectives of the subject	5	8	1	2	0
15	Examinations and exercises help to think of more conservation	5	9	0	2	0
16	Number of exams and the their recurrence are appropriate	6	6	2	1	1
17	The case of equipped lecture halls satisfactory	2	3	0	5	6
18	Capabilities and laboratories are appropriate and effective	1		1	6	8

Table (2): Students Opinion Questionnaire about Faculty Member

Academic Year 2010 - 2011

Code No. & Curriculum Name: **ME 202 - FLUID MECHANICS (1)**

Year: **2nd Year** ***Faculty Member's Name:*** **Dr. Ihsan Y. Hussain**

Is the plan of teaching the subject was distributed from the beginning of the year? Yes 13 **No** I don't know 3

Is the faculty member is committed to the specific office hours of the subject? Yes 12 No 1 I don't know 3

If the answer is (No) explained that____________

Dear Students: For the development of the educational process at the university we hope to express your opinion by answering accurately with mark √ in the place which reflects your opinion taking into consideration the accuracy and objectivity.

Score		1	2	3	4	5
No.	**Question**	**Strongly Agree**	**Agree**	**I Don't Know**	**Disagree**	**I Don't Agree At All**
1	Has the ability to communicate scientific material in a smooth and easy manner	2	11	1	1	1
2	Keep to use the tools and techniques of modern education	1	2	2	9	2
3	Illustrates the theoretical aspects in the subject with examples from the reality	3	11	0	2	0
4	Gives the scientific material in a manner covering the time of the lecture	3	8	1	4	0
5	Committed to the dates of lectures	12	4	0	0	0
6	Improve in the management ranks and give equal opportunities to students in dialogue and discussion	1	12	1	1	1
7	Motivates students and encourages them to think and research	0	10	1	4	1
8	Respects the different views of the students	1	13	1	1	0
9	Through self-learning encourages students to search for what is new and modern	1	7	2	5	1
10	Accept criticism and suggestions with an open mind	0	7	7	1	1
11	Be objective and fair in his / her evaluation of students	2	10	3	1	0
12	Uses a variety of methods to assess the performance of students (such as reports, research, and quizzes),	4	4	1	7	1
13	Follow up activities and duties to put the evaluation weights	0	2	5	9	0
14	Has the ability to discuss all issues of the subject	10	4	1	0	1
15	Working to increase the knowledge of the outcome requested	2	10	1	2	1

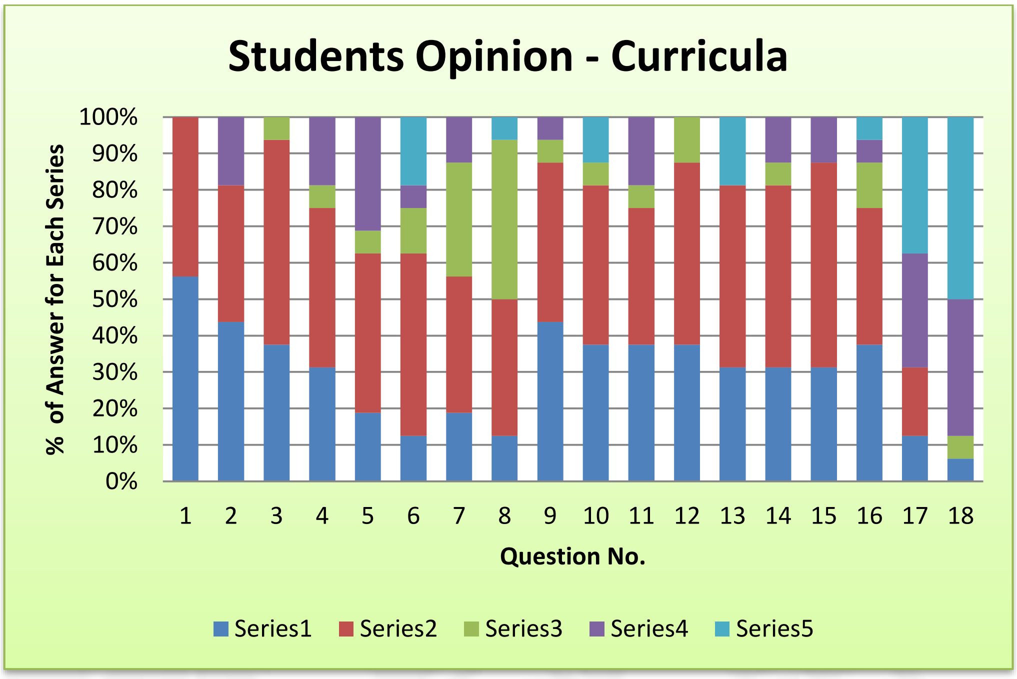

Figure (1): Students Opinion Questionnaire about Fluid Mechanics (1) Curriculum

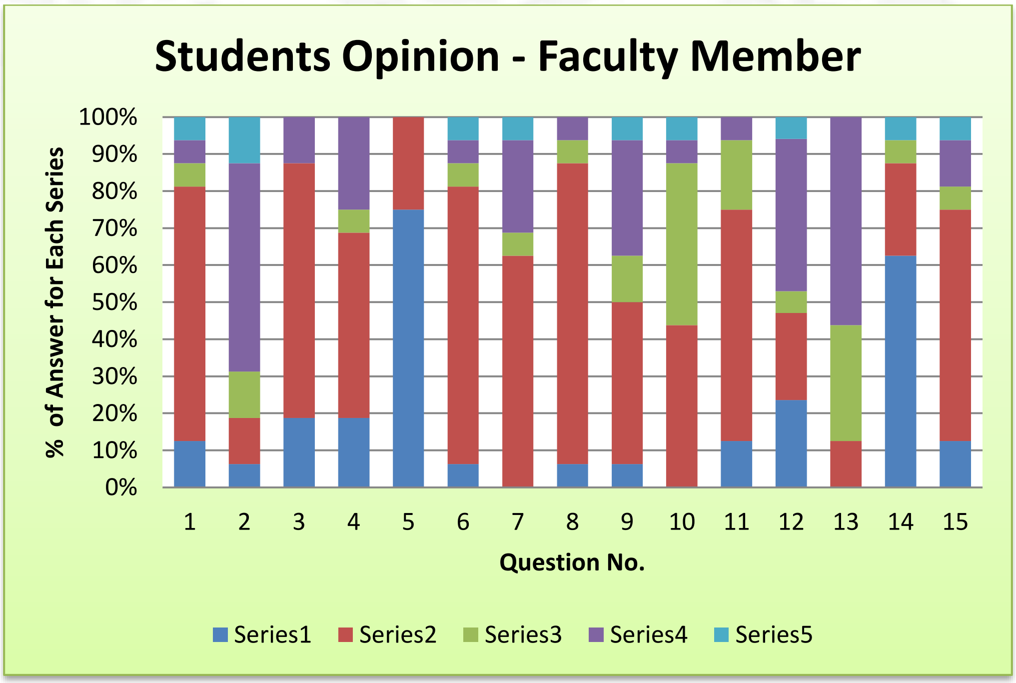

Figure (2): Students Opinion Questionnaire about Fluid Mechanics (1) Instructor

3. Students rating performance: Table (3) shows the results of quizzes and tests related to some of the course outcomes.

Table (3): Students Rating Related to some of the Course LOs

LO	Theoretical Content	Mark (10 %)		
		Avg.	St. Dev.	% Success
a	Introductory Concepts To Fluid Mechanics	4.7		48.8 %
	Introductory Concepts To Fluid Mechanics	5.1		58.5 %
b	Fluid Statics : Pressure Distribution In Static Fluids & Pressure Measurements	3.95		33.3 %
	Forces On Immersed Surfaces	3.8		31.1 %
	Buoyancy And Floatation	2.1		13 %
	Accelerated Fluid And Relative Motion	3.1		17.5 %
c	Continuity Equation	6.7		84.4 %
	Energy Equation	3.05		29.8 %
	Momentum Equation	1		4 %
d	Dimensional Analysis And Similitude			
	Dimensional Analysis And Similitude			
	Dimensional Analysis And Similitude			
a, b, c, d	Test / I	3.15		30.1 %
e	Laminar Viscous Flow Between Parallel Plates	5		68.9 %
	Laminar Viscous Flow Through Circular Tubes	3.9		40 %
f	Boundary Layer Theory	5		52.9 %
g	Drag & Lift	5.35		58.5 %
h & i	Losses In Pipes : Moody Diagram			
	Losses In Pipes : Moody Diagram			

	Losses In Pipes : Moody Diagram			
j	**Measurements Of Fluid Flow**			
	Measurements Of Fluid Flow			
	Measurements Of Fluid Flow			
k	**Analysis Of Piping And Pumping Networks**			
	Analysis Of Piping And Pumping Networks			
	Analysis Of Piping And Pumping Networks			

Results of Tables 1, 2, and 3 are used in **Table 4** to assess changes/improvements from the previous evaluation as well as changes/improvements that will be made to the course to improve student achievement of the learning outcomes.

Table (4): Changes/Improvements Term

Assessment of changes/improvements made in previous term	• Good improvement in experimental part • Good response from students for the extracurricular activities • Good response for the case of the subject submitted to the students, which includes lectures and modern references
Changes/improvements that will be made next time the subject is offered	• Use the tools and techniques of modern education • Use of Interactive Class • Using scientific films and videos in teaching the subject • Increase the number of extracurricular activities • Decrease the number of quizzes • Improve Laboratory equipment