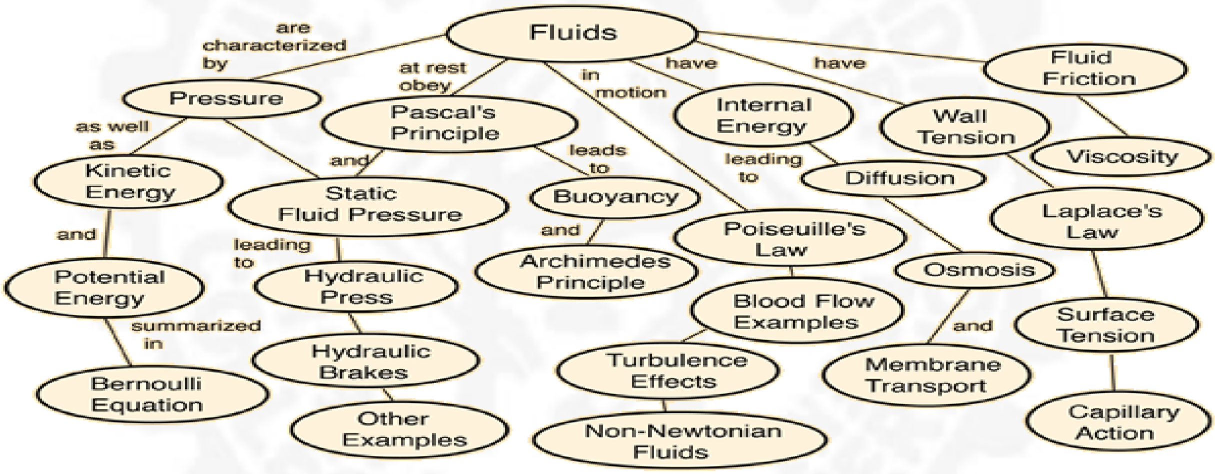
Ministry of Higher Education & Scientific Research

University of Baghdad – College of Engineering

Mechanical Engineering Department

Incompressible Fluid Mechanics

Sheets of Problems



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Preface

The present sheets of problems are to accompanied the book ***Incompressible Fluid Mechanics*** issued by the author. The book is a handout lectures for the B.Sc. Course ***ME202 : Fluid Mechanics / I***. The course is designed for B.Sc. Sophomore Students in the Mechanical Engineering Discipline. The time schedule needed to cover the course material is 32 weeks, 3 hrs. per week. The course had been taught by the author (course tutor) for more than 25 years. A short c.v. for the author is given below;

- ***Prof. Dr. Ihsan Y. Hussain*** **/ Baghdad - 1964**
- **B.Sc. (1986), M.Sc. (1989), & Ph.D. (1997) in Mechanical Engineering from the Mech. Engr. Dept. – University of Baghdad**
- **Professor of Mechanical Engineering – Thermo-Fluids**
- **Teaching Undergraduate Courses and Laboratories in Various Iraqi Universities (Baghdad, Al-Kufa, Babylon, Al - Nahrain ….) in Various Subjects of Mechanical Engineering**
- **Teaching Advanced Graduate Courses (M.Sc. and Ph.D.) in various Iraqi Universities (Baghdad, Technology, Babylon, Al-Kufa, Al-Mustansyrya, Al-Nahrain…) in the Areas of (Fluid Mechanics, Heat Transfer, CFD, Porous Media, Gas Dynamics, Viscous Flow, FEM, BEM**
- **Lines of Research Covers the Following Fields ;**
  - **Aerodynamics**
  - **Convection Heat Transfer (Forced, Free, and Mixed)**
  - **Porous Media (Flow and Heat Transfer)**
  - **Electronic Equipment Cooling**
  - **Heat Transfer in Manufacturing Processes (Welding, Rolling, … etc.)**
  - **Inverse Conduction**
  - **Turbomachinery (Pumps, Turbines, and Compressors)**
  - **Heat Exchangers**
  - **Jet Engines**
  - **Phase-Change Heat Transfer**
  - **Boundary Layers (Hydrodynamic and Thermal)**
- **Head of the Mech. Engr. Dept. / College of Engineering - University of Baghdad (December / 2007 – October / 2011)**
- **Member of Iraqi Engineering Union (Official No. 45836).**
- **ASHRAE Member (8161964)**
- **Head of (Quality Improvement Council of Engineering Education in Iraq QICEEI)**
- **Supervised (41) M.Sc. Thesis and (20) Ph.D. Dissertations**
- **Publication of more than (70) Papers in the Various Fields Mentioned above**
- **Member in the Evaluation and Examining Committees of more than (300) M.Sc. and Ph.D. Students in their Theses and Dissertations**
- **Member of the Editing Committee of a number of Scientific Journals**
- **Evaluation of more than (700) Papers for Various Journals and Conferences**
- **Working within the Mechanical Groups in the Consulting Engineering Bureau, College of Engineering, University of Baghdad, and in the Dept. of Mech. Engr. , in the Mechanical Design of Various Projects**
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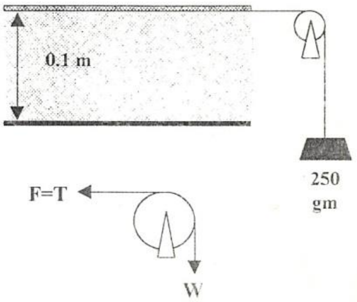
"Table of Contents"

Sheet No. 1
Introductory Concepts

(1) The space between the lower fixed plate and the upper moving one shown in the figure is filled with oil of specific gravity (0.85) and kinematic viscosity of (4 Stokes). The contact area of the upper plate with oil is (0.7 m^2). Determine the velocity of the upper plate?



Sol.:

$$\mu = \nu \times \rho = \frac{4}{10000} \times 0.85 \times 1000 \Rightarrow \mu = 0.340 \text{ Pa.s}$$

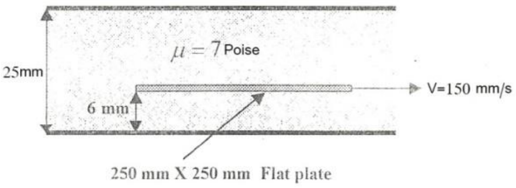
Pully:

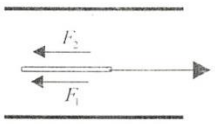
$$\sum \mathrm{M} = 0 \Rightarrow \mathrm{T} = \mathrm{W}$$

$$\mathrm{F} = \mathrm{m} \times \mathrm{g} = \frac{250}{1000} \times 9.81 \Rightarrow \mathrm{F} = 2.45 \text{ N}$$

$$\mathrm{F} = \mu \frac{VA}{h} \Rightarrow \mathbf{V = 0.96\ m/s}$$

(2) For the (250 mm x 250 mm) plate shown in the figure, determine the force exerted by the liquid on the plate ?





Sol.:

$$\mathrm{F} = \mathrm{F}_1 + \mathrm{F}_2$$

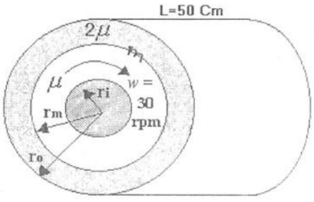
$$= \mu \frac{VA}{h_1} + \mu \frac{VA}{h_2}$$

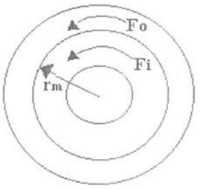
$$= \mu V A \left[\frac{1}{h_1} + \frac{1}{h_2}\right]$$

$$= 0.7 \times 0.15 \times 0.25 \times 0.25 \left[\frac{1}{0.006} + \frac{1}{0.019}\right]$$

F = 1.44N

(3) In the figure shown below, the torque used to rotate the cylinder (m) between the inner and the outer cylinders is (8 N.m) . Calculate the viscosity (μ) ?





Sol.:

$$\mathrm{T} = \mathrm{F_i r_m} + \mathrm{F_O r_m} = (\mathrm{F_i} + \mathrm{F_O})\, \mathrm{r_m}$$

$$= \left[\mu \frac{r_m w \times 2\pi r_m l}{r_m - r_i} + 2\mu \frac{r_m w \times 2\pi r_m l}{r_0 - r_m}\right]$$

$$= \mu 2\, \pi r_m{}^3 wl \left[\frac{1}{r_m - r_i} + \frac{2}{r_0 - r_m}\right]$$

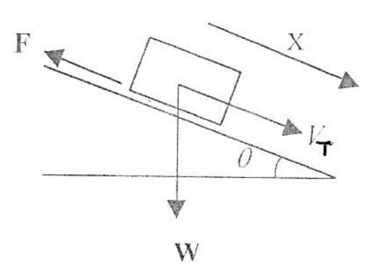
$$8 = \mu 2\, \pi \times 0.152^3 \times 30 \times \frac{2\pi}{60} \times 0.5 \times \left[\frac{1}{0.152 - 0.15} + \frac{2}{0.156 - 0.152}\right]$$

$$\mathbf{\mu = 0.2308\ Pa.s}$$

(4) An (8 Kg) box slides down a (20^0) inclined plane while lubricated by a (2 mm) thick film of oil having a kinematic viscosity of (5.176 Stoke) and a specific gravity of (0.85). If the box has a constant area of (0.2 m^2) , what is its terminal speed ?

Sol.:

$$\mu = \nu\rho = \nu S \rho_w \Rightarrow \mu = 0.44 \text{ Pa.s}$$



$$\sum F_x = ma_x = 0 \Rightarrow W \times \sin\theta - F = 0$$

$$W \times \sin\theta - \mu \frac{V_T \times A}{h} = 0$$

$$V_T = \frac{W \times \sin\theta \times h}{\mu \times A} = \frac{mg \times \sin\theta \times h}{\mu \times A}$$

$$\mathbf{V_T = 0.61\ m/s}$$

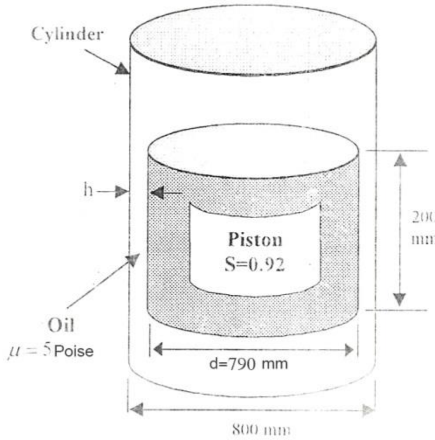
(5) For the piston shown in the figure , determine :

a-The constant downward velocity of the piston due its weight.

b- The tension force required to pull the piston upward with the same velocity.

Sol.:

a) $F = \mu \frac{V \times A}{h} = \mu \frac{V\pi dL}{h}$ ……(1)



$$\sum F_y = ma_y = 0 \Rightarrow F = W = S\gamma_w \times \frac{\pi}{4} d^2 L$$

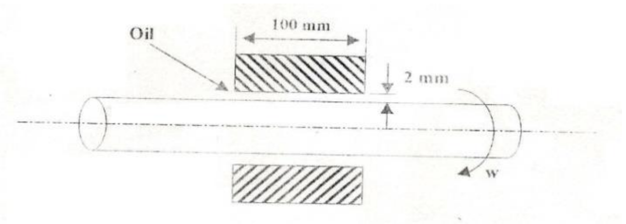
$$\Rightarrow F = 898.26N$$

From (1) ⇒ **V = 7.18 m/s**

b) $\sum F_y = ma_y = 0$

T – F - W = 0 ⇒ T = F + W ⇒ **T = 1796.52N**

(6) A shaft of diameter (0.35 m) rotates at (200 rpm) inside a sleeve as shown in the figure. The kinematic viscosity of the oil is (10 Stokes) and its specific gravity (0.8). Calculate the power lost in friction?



Sol.:

$$\mu = \nu \times \rho = \nu \times s \times \rho_w$$

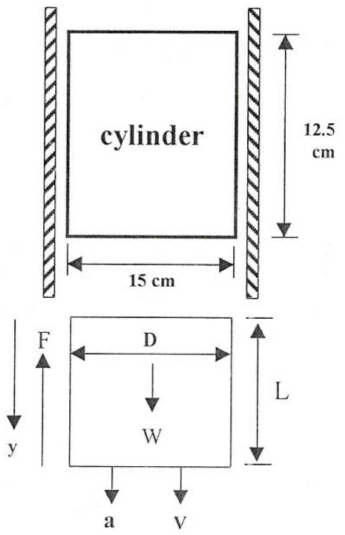
$$= \frac{10}{10000} \times 0.8 \times 1000 \Rightarrow \mu = 0.8 \mathrm{Pa.s}$$

$$\mathrm{T} = \mathrm{F} \times \frac{d}{2} = \mu \frac{uAd}{2h} = \mu \frac{d/2\omega\pi dl}{h} \times \frac{d}{2} = \frac{\pi}{2}\mu \frac{d^3\omega l}{h}$$

$$= 0.8 \times \frac{\pi}{4} \times \frac{(0.35)^3 \times 0.1 \times 200 \times (2\pi/60)}{0.002} \Rightarrow \mathrm{T} = 28.21\mathrm{N.m}$$

$\mathrm{P} = \mathrm{T} \times \omega = 28.21 \times 200 \times (2\pi/60) \Rightarrow$ **P=590.8 W**

(7) A cylinder of weight (90 N) slides in a lubricated pipe as shown in the figure. The clearance between the cylinder and pipe is (0.025 mm). If the cylinder is observed to decelerates at a rate of (0.61m /s^2) when the speed is (6.1 m/s). What is the viscosity of the oil?



Sol.:

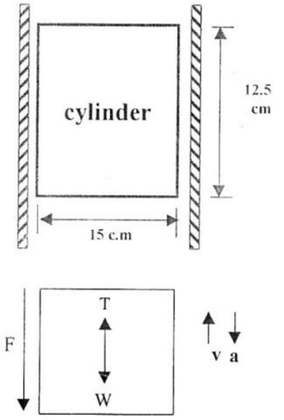
$$\downarrow \sum \mathrm{F_Y} = \mathrm{m} \times a_Y \Rightarrow W - F = \frac{W}{g} \times (-a)$$

$$\mathrm{W} - \mu \frac{V\pi Dl}{h} = -\frac{W}{g} a \Rightarrow W(1 + \frac{a}{g}) = \mu \frac{V\pi dl}{h}$$

$$\therefore \mu = \frac{Wh}{\pi DlV}(1 + \frac{a}{g}) = \frac{90 \times 0.025 \times 10^{-3}(1 + \frac{0.61}{9.8})}{\pi \times 0.15 \times 0.125 \times 6.1}$$

Thus: $\boldsymbol{\mu = 6.6516 \times 10^{-3}\ \mathrm{Pa.s}}$

(8) A cylinder of weight (90 N) slides in a lubricated pipe as shown in the figure . The clearance between the cylinder and pipe is (0.025 mm). If the viscosity of the oil is (6.6 x 10^{-3} Pa.s) , calculate the tension force required to raise the cylinder at a speed of (6.12 m/s) with a deceleration of (0.61 m/s^2)?



Sol.:

$$\uparrow \sum \mathrm{F_y} = \mathrm{m} \times a_y$$

$$\mathrm{T-W-F}=\frac{W}{g}\times(-a)$$

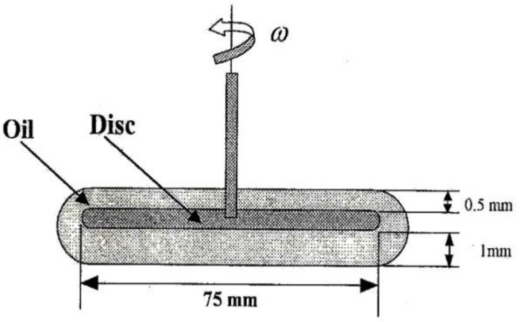
$$\mathrm{T-\mu\frac{V\pi D\mathit{l}}{h}}-W=-\frac{W}{g}a$$

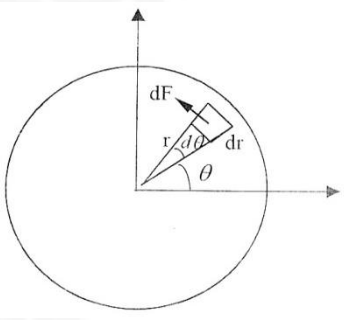
$$\mathrm{T=\mu\frac{V\pi D\mathit{l}}{h}}+W(1-\frac{a}{g})$$

T = 95. 58+84. 39

T = 180. 25 N

(9) In some electrical measurement devices , the motion of a pointer mechanism is dampened by having a circular disc turn (with the pointer) in a container of oil (see the figure) . What is the damping torque for (ω= 0.2 r ad/s) , if the oil has a viscosity of (8 x 10^{-3}N.S/m^2). Neglect effects on the outer edge of the rotating disc ?





<u>Sol.:</u>

dT = r (dF$_1$ + dF$_2$)

$$=\mu\times r\times\omega\times rd\theta dr\times r\left[\frac{1}{h_1}+\frac{1}{h_2}\right]$$

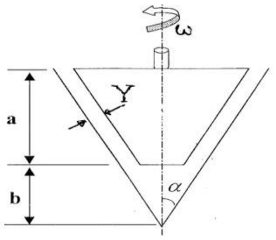
$$=8\times10^{-3}\times r\times0.2\times rd\theta dr\times r\left[\frac{1}{0.5/1000}+\frac{1}{1/1000}\right]$$

$$=4.8r^3d\theta dr$$

$$\mathrm{T}=\int_{\mathrm{r}=0}^{0.0375}\int_{\theta=0}^{2\pi}4.8\mathrm{r}^3\,\mathrm{d\theta dr}=4.8\times2\pi\left[\frac{r^4}{4}\right]_0^{0.0375}$$

Thus; $\mathbf{T=1.49\times10^{-5}\ N.m}$

(10) In the figure shown below , oil of viscosity (μ)fills the small gap of thickness (Y). Neglecting the fluid stress exerted on the circular bottom , show that the torque (T) required to rotate the truncated cone at a constant speed (ω) is given by :



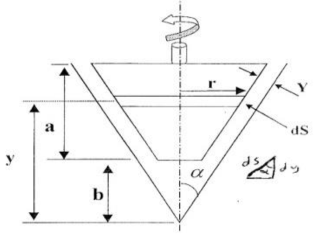
$$\mathbf{T}=\left(\frac{\pi\mu\omega tan^3\alpha}{2Ycos}\right)\times[(a+b)^4-a^4]$$

<u>Sol.:</u>

$$\mathrm{dF}=\mu\frac{\mathrm{u}}{\mathrm{Y}}\mathrm{dA}=\mu\frac{\mathrm{r}\omega}{\mathrm{Y}}2\pi\mathrm{rdS}=\frac{2\pi\mu\omega}{Y}r^2\mathrm{dS}$$

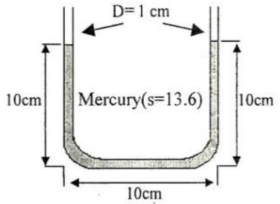
$$\mathrm{dT}=\mathrm{dF}\times r=\frac{2\pi\mu\omega}{Y}r^3\mathrm{dS}=\frac{2\pi\mu\omega}{Y}(ytan\alpha)^3\frac{dy}{cos\alpha}$$

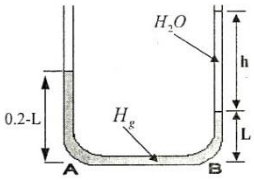
$$\mathrm{T}=\int_{\mathrm{b}}^{\mathrm{a+b}}\mathrm{dT}\Rightarrow\ \mathbf{T}=\left(\frac{\pi\mu\omega tan^3\alpha}{2Ycos\alpha}\right)\times[(a+b)^4-a^4]$$



Sheet No. 2
Pressure Measurements

(1) In the U- tube shown in the figure, if (10 cm^3) of water is poured into the right hand leg, what will the free surface height in each leg be?





Sol.:

$\forall_{H_2O} = h \times \frac{\pi}{4} d^2$

$10 = h. \times \frac{\pi}{4} 1^2 \Rightarrow h = 12.73 \text{ cm}$

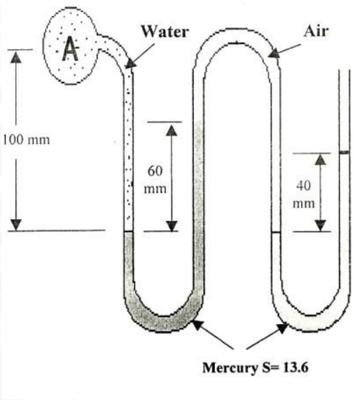
$p_A = p_B$

$13.6 \times \gamma_W (0.2 - L) = \gamma_W \times 0.1273 + 13.6 \times \gamma_W \times L \Rightarrow L = 9.53\text{cm}$

Thus: **L.H.S.= 20 - 9.53 = 10.47cm** **R.H.S.= 12.73 + 9.53 = 22.26cm**

(2) If (p_{atm}=100 kPa), find the absolute pressure at point (A) in (Pa) and in (mm Hg)?

Sol.:



$p_A + 0.1 \times \gamma_W - 0.06 \times 13.6 \times \gamma_W + \cancelto{0}{\gamma_{air} \times h_{air}}$

$-0.04 \times 13.6 \times \gamma_W = 100000$

$\mathbf{p_A)_{abs} = 112360.6 \ Pa}$

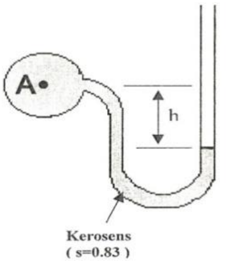
$$p_A)_{abs} = \frac{112360.6}{13.6 \times 9810} \times 1000$$

$\mathbf{p_A)_{abs} = 842.2 \ mmHg}$

(3) If (P_A = 30kPa suction) and the barometer reading is (29 in Hg), determine:

a) The reading (h) b) The absolute pressure at point (A) in (m H_2O)

Sol.:



a) $p_A + s \times \gamma_W \times h = 0$

$-30 \times 10^3 + 0.83 \times 9810 \times h = 0 \Rightarrow \mathbf{h = 3.68 \ m}$

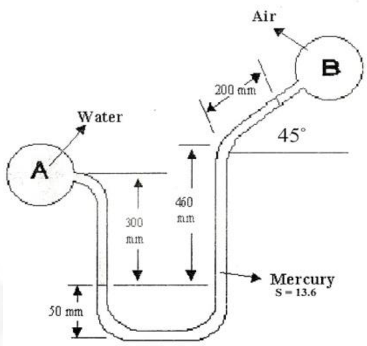
b) $p_A)_{abs} = p_{atm} + p_{gage}$

$29 \times \frac{101325}{29.92} + (-30000) \Rightarrow p_A)_{abs} = 68209.4 \text{ Pa}$

$h_A)_{abs} = \frac{p_A)_{abs}}{\gamma_W} \Rightarrow \mathbf{h_A)_{abs} = 6.95\ mH_2O}$

(4) Find the pressure difference (P_A-P_B) in (Pa) and in millimeters of Mercury?

Sol.:



$p_A + 0.3 \times 9810 - 13.6 \times 9810 \times (0.46 + 0.2\sin 45) = p_B$

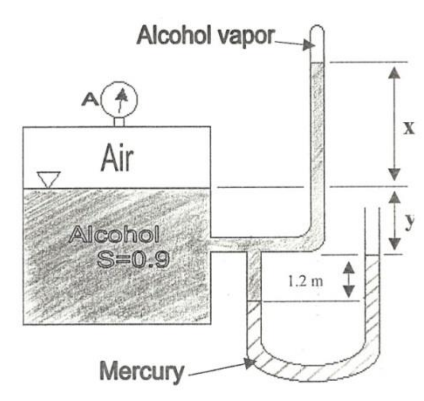
∴ $\mathbf{p_A - p_B = 77296.2}$ **Pa**

$h_A\text{-}h_B = \frac{77296.2}{13.6 \times 9810} \times 1000$

$\mathbf{h_A\text{-}h_B = 579.36}$ **mmHg**

(5) If (p_{atm} = 100kPa) , the gage reading at (A) is (34 kPa) , and the vapor pressure of the alcohol is (11.56 kPa) absolute. Compute (x) and (y)?

Sol.:



$p_A - X * S * \gamma_W = p_V$

$(34000 + 10000) - X * 0.9 * 9810 = 11560$

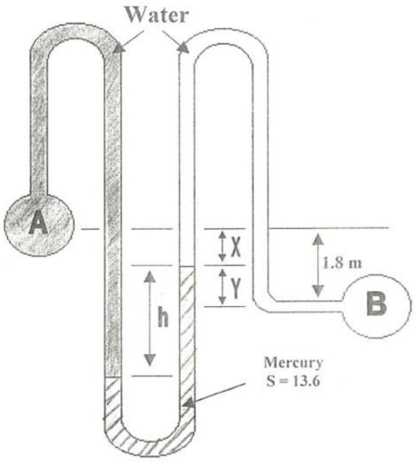
X = 13.87 m

$p_A + y * S * \gamma_W + 1.2 * S * \gamma_W - 13.6 * 1.2 * \gamma_W = 0 \Rightarrow$

$\boldsymbol{y = 13.08}$ **m**

(6) Vessels (A) and (B) contains water under pressures of (300 KPa) and (150 kPa) respectively . What is the deflection of the mercury (h) in the differential gage?

Sol.:



$p_A + (h + x)\gamma_W - S\gamma_W h + \gamma_W y = p_B$

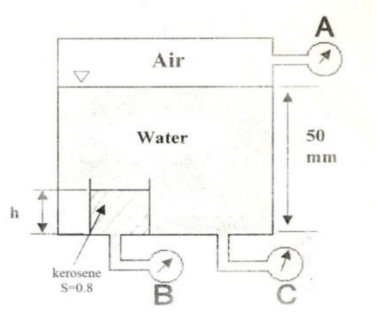
$p_A + h\gamma_W + x\gamma_W - S\gamma_W h + \gamma_W y = p_B$

$p_A + h(\gamma_W - S\gamma_W) + \gamma_W(x + y) = p_B$

But: x + y = 1.8 , thus ;

$h = \frac{P_B - P_A - 1.8\gamma_W}{\gamma_W(1-S)} \Rightarrow \mathbf{h = 1.356\ m}$

(7) Gage (B) reads 13.8 kPa and gage (c) reads 13.82 kPa. Find the reading of gage (A) and the Kerosene height (h)?

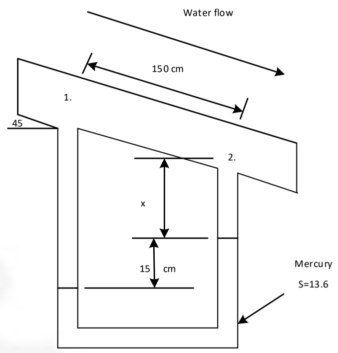


Sol.:

$p_A + 9810\times 0.05 = p_C \Rightarrow \mathbf{p_{Agage} = 13.329\ kPa}$

$p_A + (0.05 - h) \times \gamma_W + S\gamma_W h = p_B \Rightarrow \mathbf{h = 9.93\ mm}$

(8) Find the pressure difference (p_1-p_2) of the setup shown in the figure?

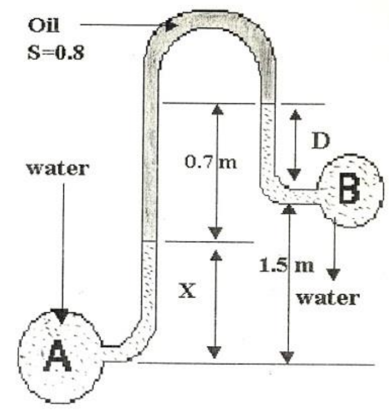


Sol.:

$p_1 + (1.5\sin 45 + X + 0.15)\,\gamma_w - 0.15 \times 13.6\gamma_w - x\gamma_w = p_2$

$\mathbf{p_1 - p_2 = 8135.8\ Pa}$

(9) Calculate the pressure difference between (A) and (B) for the setup shown in the figure?



Sol.:

$p_A - X\gamma_W - 0.7 \times S\gamma_W + D\gamma_W = p_B$

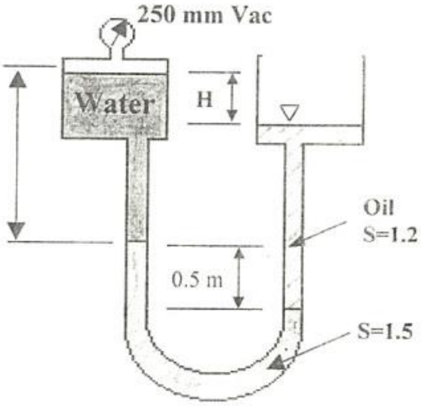
D = X + 0.7 - 1.5 = X - 0.8

Thus;

$p_A - p_B = 0.7S\gamma_W + 0.8\gamma_W = \gamma_W(0.7 \times 0.8 + 0.8)$

$\mathbf{p_A - p_B = 13.34\ kPa}$

(10) Determine the difference in height (H) of the Fig.?



Sol.:

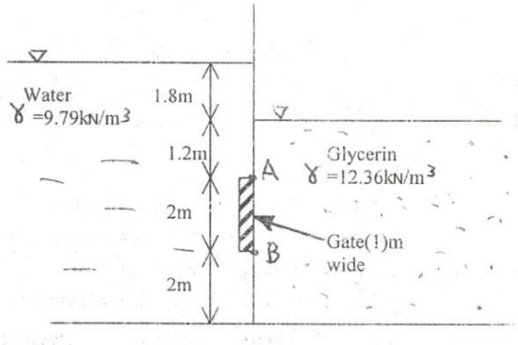
$-0.25 + 6 + 0.5 \times 1.5 - 0.5 \times 1.2 - (6 - H) \times 1.2 = 0$

Thus;

H = 1.08 m

Sheet No. 3
Hydrostatic Forces

(1) Find the resultant hydrostatic force on the rectangular gate (Ab)shown in the figure and its line of action.



<u>Sol.:</u>

$$F_1 = \gamma_1 \bar{h}_1 A_1 = 9790 * (1.8 + 1.2 + 1) * 2 * 1$$

$$\Longrightarrow \quad F_1 = 78.32\ kN$$

$$y_{P1} = \bar{y}_1 + \frac{\bar{I}_{CX}}{A_{\bar{y}_1}} 4 + \frac{1.2^3/12}{2 * 1 * 4} \Longrightarrow y_{P1} = 4.0833\ m$$

$$F_2 = \gamma_2 \bar{h}_2 A_2 = 12360 * (1.2 + 1) * 2.1 \ \Longrightarrow F_2 = \ 54.38\ \mathrm{kN}$$

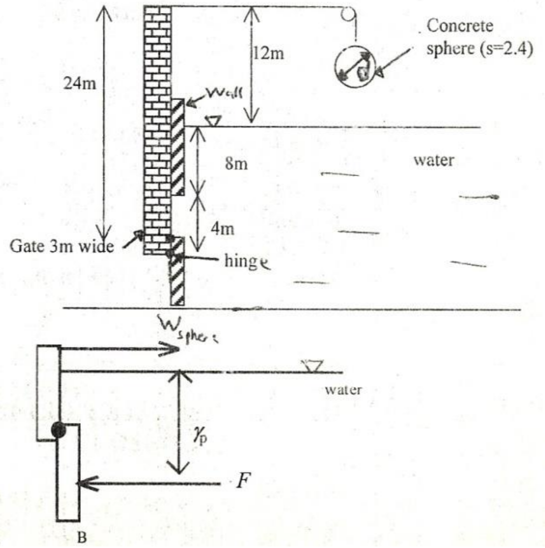
$$y_{P2} = \bar{y}_2 + \frac{\bar{I}_{CX}}{A_2 \bar{y}_2} = \frac{1.2^3/12}{2 * 1 * 2.2} \Longrightarrow y_{P2} = 2.35\ m$$

$$F_{RES} = F_1 - F_2 \ \Longrightarrow \mathbf{F_{RES} = 23.94\ kN}$$

$$\sum M_b : F_{RES} * a = F_1(5 - y_{P1}) - F_2(3.2 - y_{P2}) \Longrightarrow \mathbf{a = 1.068\ m}$$

(2) What dimater (d) of the concrete is just sufficient to keep the gate closed?

<u>Sol.:</u>



$$F = \gamma \bar{h} A = 9810 * (8 + 2) * 4 * 3 = 1177.2\ kN$$

$$y_P = 10 + \frac{3 * 4^3\ /12}{4 * 3 * 10} \Longrightarrow y_P = 10.133\ m$$

$$\sum M_B = 0$$

$$W_{SPHER} \ * (12 + 8 + 4) - 1177.2 * (12 - y_P) = 0$$

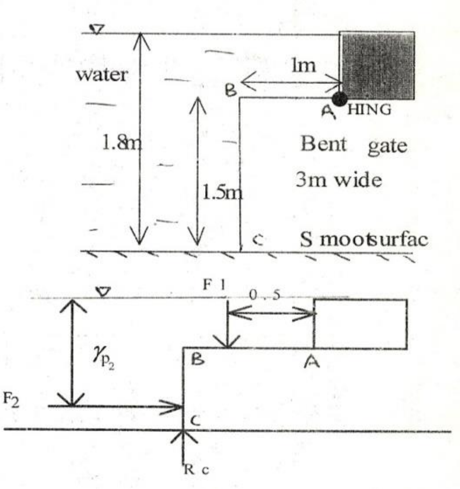
$$W_{SPHER} \ = 91.576\ kN$$

$$W_{SPHER} \ = \gamma_{SPHER} \ \times V_{SPHER}$$

$$91.576 * 10^3 = 2.4 * 9810 * \frac{\pi}{6} d^3 \Longrightarrow$$

$$\mathbf{d = 1.95\ m}$$

(3): Find the reaction at c, neglect friction.



Sol.:

$$F_1 = \gamma h A = 9810 * 0.3 * 1 * 3 = 8829\ N$$

$$F_2 = \gamma \bar{h}_2 A = 9810 * 1.05 * 1.5 * 3 = 46352.25\ N$$

$$y_{P2} = \bar{y}_2 + \frac{\bar{I}_{CX}}{A_{\bar{y}_2}} = 1.05 + \frac{3 * 1.5^3/12}{3 * 1.5 * 10.5} = 1.228m$$

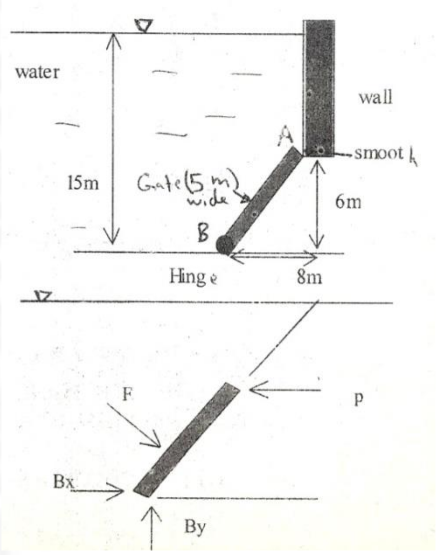
$$\sum M_A = 0 : \quad F_1 * 0.5 = F_2(y_{P2} - 0.3)$$

Thus; $\mathbf{R_C = 47455.87\ N}$

(4) For the gate shown in the figure; determine;

a) The hydrostatic force on the gate and its center of pressure.

b) The reaction force at A and B, neglect friction and the gate weight



Sol.:

(a) $\mathrm{F_1 = \gamma \bar{h} A = 9810 * (9 + 3) * 10 * 5 \Rightarrow}$

$$\mathbf{F = 5.886\ MN}$$

$$y_P = \bar{y} + \frac{\bar{I}_{CX}}{A_{y_2}} = 20 + \frac{5*10^3/12}{10*5*20} \Rightarrow \mathbf{y_P = 20.416\ m}$$

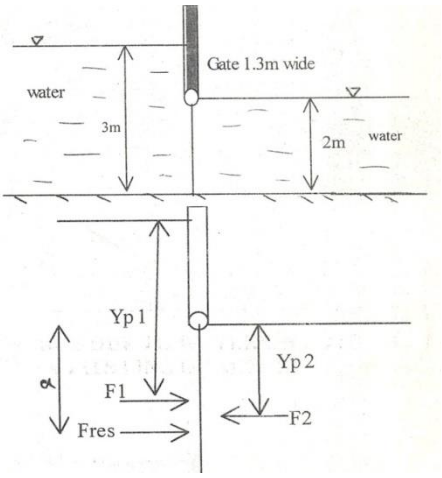
(b) $\sum M_B = 0$

$$P * 6 = F * (25 - Y_P) \Rightarrow \mathbf{P = 4.497\ MN}$$

$$\mp \sum F_X = 0 \Rightarrow B_X + F \sin\theta - P = 0 \Rightarrow \mathbf{B_X = 0.965\ MN}$$

$$\mp \sum F_Y = 0 \Rightarrow B_Y + F \cos\theta = 0 \Rightarrow \mathbf{B_Y = 4.709\ MN}$$

(5) Determine the resultant force due to water on both side of the gate, including its line of action



Sol.:

$$F_1 = \gamma \bar{h}_1 A = 9810 * 2 * 21.3 = 51.1\ kN$$

$$\mathrm{y_{P1} = \bar{y}_1 + \frac{\bar{I}_{CX}}{Ay_2} = 2 + \frac{1.3 * 2^3/12}{1.3 * 2 * 2} = 2.166\ m}$$

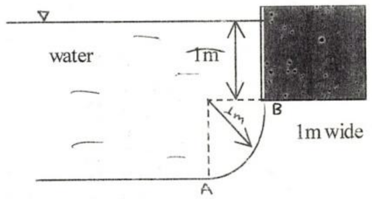
$$\mathrm{F_2 = \gamma \bar{h}_2 A = 9810 * 1 * 1.3 * 2 = 25.50\ kN}$$

$$y_{P2}=\bar{y}_2+\frac{\bar{I}_{CX}}{A\bar{y}_2}=1+\frac{1.3*2^3/12}{1.3*2*1}=1.333\text{ m}$$

$$F_{NET}=F_1-F_2\ \Longrightarrow \mathbf{F_{NET}=25.6\ kN}$$

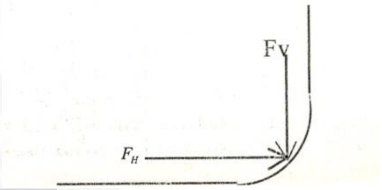
$$\sum M_{HINGE}=0:F_{NET}*a=F_1(y_{P1}-1)-F_2y_{P2}\Longrightarrow \mathbf{a=1.0189m}$$

(6) Find the horizontal and vertical components of the hydrostatic force on the curved surface (Ab) including their lines of action.



Sol.:

$$F_H=\gamma\bar{Y}_VA_V=9810*1.5*1*1\Longrightarrow \mathbf{F_H=14715\ N}$$

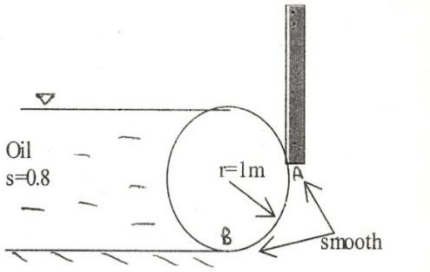


$$y_{PH}=y_V+\frac{\bar{I}_{CXV}}{A_V\bar{y}_V}=1.5+\frac{1*1^3/12}{1*1*1.5}=1.555\ m$$

$$F_V=\gamma\forall=9810*1*\left[\frac{\pi}{4}1^2+1*1\right]\Longrightarrow \mathbf{F_V=17514.7\ N}$$

$$\left(\frac{\pi}{4}1^2+1*1\right)*a=\frac{\pi}{4}1^2\left(1*\frac{4*1}{3\pi}\right)+1*1*0.5\Longrightarrow \mathbf{a=0.513\ m}$$

(7) The (2m) dimeter cylinder weight (25kN) and is (1.6m) long. Determine the reaction at (A)and(B)neglect friction.

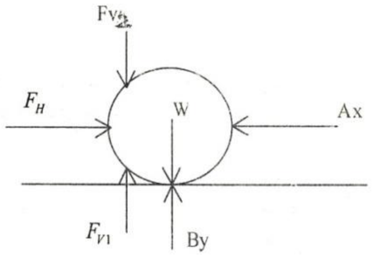


Sol.:

$$F_H=\gamma\bar{y}_VA_V=0.8*9810*1*2*1.6=25113.6\ N$$

$$F_{V_1}=\gamma\forall_1=0.8*9810*1.6\left[\frac{\pi}{4}1^2+1*1\right]=22418.88\ N$$

$$F_{V_2}=\gamma\forall_2=0.8*9810*1.6\left[1*1-\frac{\pi}{4}1^2\right]=2694.1\text{ N}$$



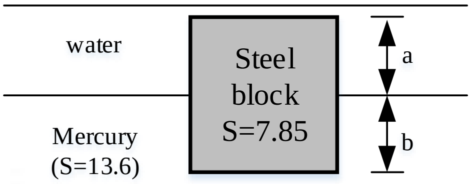
$$\sum F_X=0\Longrightarrow \boldsymbol{A_X=F_H=25113\ N}$$

$$\sum F_Y=0\Longrightarrow F_{V_2}+W=F_{V_1}+B_y$$

$$\Longrightarrow \boldsymbol{B_y=5275.7\ N}$$

Sheet No. 4
Buoyancy and Floatation

(1) A block of steel (S =7.85) floats at a mercury water interface as shown in the figure .Find the ratio ($\frac{a}{b}$) for this condition.



Sol.:

$$W = F_{B_{TOT}} = F_{B_w} + F_{B_{Hg}}$$

$$7.85\,\gamma_w A(a+b) = \gamma_w Aa + 13.6\gamma_w Ab$$

$$7.85\,(a+b) = a + 13.6\,b \;\Longrightarrow\; \boldsymbol{\frac{a}{b} = 0.839}$$

(2) An object weighs (55N) when submerged in oil (S=0.8) and a force of (45N) is required to hold it submerged in mercury (S=13.6). Determine its weight, volume, specific weigh and specific gravity.

Sol.:

Oil: $\mathrm{W} = 55 + 0.8\,\gamma_{\mathrm{w}}\forall$

Hg: $\mathrm{W} = \mathrm{F_B} - 45 = 13.6\,\gamma_{\mathrm{w}}\forall - 45$

Thus; $\boldsymbol{\forall = 7.96 * 10^{-4}\ \mathrm{m}^3}$

$\mathrm{W} = 55 + 0.8\,\gamma_{\mathrm{w}}\forall \Longrightarrow \boldsymbol{\mathrm{W} = 61.25\ \mathrm{N}}$

$\gamma = \frac{\mathrm{W}}{\mathrm{V}} \Longrightarrow \boldsymbol{\gamma = 76910.4}$

$\mathrm{s} = \frac{\gamma}{\gamma_{\mathrm{W}}} \Longrightarrow \boldsymbol{\mathrm{s} = 7.84}$

(3) A hollow cube (1m) on each side weighs (2.4 kN). The cube is tied to a solid concrete block weighing (10 kN) and has (S=2.4). Will these two objects tied together float or sink in water?

Sol.:

$$\mathrm{W_{Tot}} = \mathrm{W_{cube}} + \mathrm{W_{conc}} = 2.4\ \mathrm{kN} + 10\ \mathrm{kN} \Longrightarrow \boldsymbol{\mathrm{W_{Tot}} = 12.4\ \mathrm{kN}}$$

$$\mathrm{F_{B_{TOT}}} = \mathrm{F_{B_{cub}}} + \mathrm{F_{B_{conc}}} = \gamma_{\mathrm{w}} * 1 + \gamma_{\mathrm{w}} * \frac{\mathrm{W}}{\mathrm{S}\gamma_{\mathrm{w}}} = \gamma_{\mathrm{w}} + \frac{\mathrm{w}}{\mathrm{S}} = 9810 + \frac{10000}{2.4}$$

$$\Longrightarrow \boldsymbol{\mathrm{F_{B_{TOT}}} = 13.976\ \mathrm{kN}}$$

Since $F_{B_{TOT}} > W_{TOT}$ **both objects will float in water**

(4) How many kilograms of concert ($\gamma = 25\ \mathrm{kN} / \mathrm{m}^3$) must be attached to a beam having a volume of ($0.1m^3$) and (S=0.65) to cause both to sink in water.

Sol.:

$$W \; > \; F_B$$

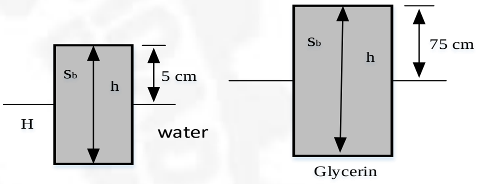
$$S\gamma_w\, \forall_{beam} + \gamma_{con}\forall_{con} > \gamma_w\,(\forall_{beam} + \forall_{con})$$

$$\forall_c > 0.0226\, m^3$$

$$m = \frac{\gamma}{g}\forall = 57.36\ Kg \Rightarrow \boldsymbol{m > 57.36\ \mathrm{kg}}$$

(5) A cylinder of wood floats in water with (5 cm) above the water surface. When placed in glycerin of (S=1.35), the cylinder floats with (7.5 cm) above the liquid surface. Determine the specific gravity of the wood and the cylinder height.

Sol.:



In water: $W = F_{BW}$

In Glycerin: $W = F_{BG}$

Thus; $F_{BW} = F_{BG}$

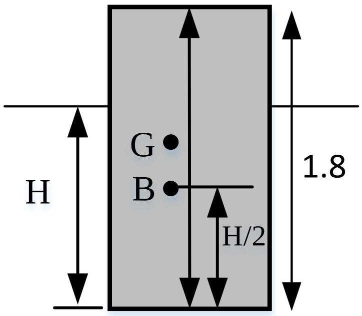
$$\gamma_w A(h - 0.05) = 1.35\, \gamma_w A(h - 0.075) \Longrightarrow \mathbf{h = 14.64\ cm}$$

$$W = F_{BW}$$

$$S_b\gamma_w Ah = \gamma_w A(h - 0.05) \;\Longrightarrow\; \mathbf{S_b = 0.658}$$

(6) A cylinder (1.35 m) diameter and (1.8 m) high has a specific gravity of (0.291). Will the cylinder float in stable equilibrium in water?

Sol.:



$$S\gamma_w A\, 1.8 = \gamma_w AH \;\Longrightarrow H = 1.8\, S = 0.5238\ \mathrm{m}$$

$$B_G = 0.9 - \frac{0.5238}{2} = 0.6361$$

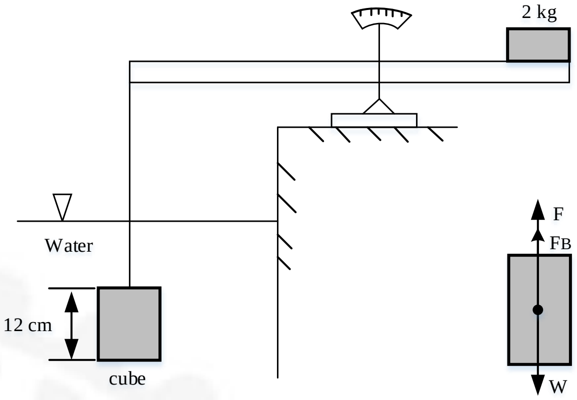
$$\frac{\mathrm{I_{00}}}{\forall} = \frac{\frac{\pi}{64}\ \mathrm{D}^4}{\frac{\pi}{4}\ \mathrm{D^2H}} = 0.21746$$

$$\mathrm{M_G} = \frac{\mathrm{I_{00}}}{\forall} - \mathrm{BG} = 0.21746 - 0.6281 = -0.42$$

$\mathbf{M_G < 0} \;\rightarrow$ **Unstable**

(7) The solid cube (12cm) on a side shown in the figure is balanced by a (2 kg – mass) on the beam scale when the cube is immersed in water. What is the specific weight and specific gravity of the cube material?

Sol.:



$F = mg = 2 * 9.81 = 19.62\text{N}$

$\sum F = 0 \quad \Rightarrow \quad W = F_B + F$

$= 9810 * 0.12 + 19.62 = 16.95 + 19.62$

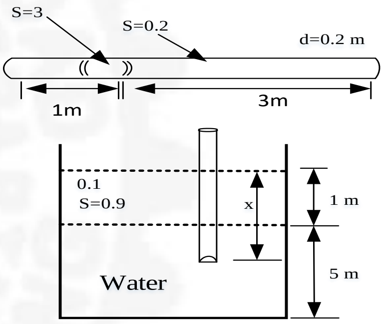
Thus; $W = 36.57\ N$

$\gamma = \frac{w}{\forall} = \frac{36.57}{0.12^3} \Longrightarrow \boldsymbol{\gamma = 21163.2\ \text{N/m}^3}$

$\text{s} = \frac{\gamma}{\gamma_w} \Longrightarrow \mathbf{s = 2.157}$

(8) Determine the depth of submergence of the compound cylinder in the water – oil tank shown in the figure?

Sol.:



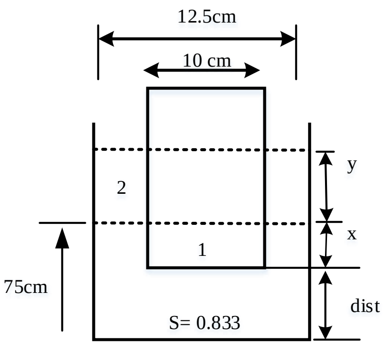
$W_{tot} = F_{B_{tot}}$

$3\,\gamma_w A * 1 + 0.2\,\gamma_w A * 3 = 0.9\,\gamma_w A * 1 + \gamma_w A(x - 1)$

Thus; $\mathbf{x = 3.7\ m}$

(9) A (10 cm) diameter cylinder of height (9m) and weight (3.8 N) floats in liquid (S=0.833) contained in cylindrical tank having a diameter of (12.5 cm). Before immersion the liquid was (7.5cm) deep. Determine the distance between the bottom of the cylinder and the bottom of the tank.

Sol.:



$\forall_1 = \forall_2 \Rightarrow \quad \frac{\pi}{4}\,(0.1)^2 X = \frac{\pi}{4}(0.125^2 - 0.1^2)y$

$x = 0.5625\,y \quad \ldots .(1)$

$W = F_B \Rightarrow \ 3.8 = 0.833\,\gamma_w \frac{\pi}{4}\ 0.1^2(x + y)$

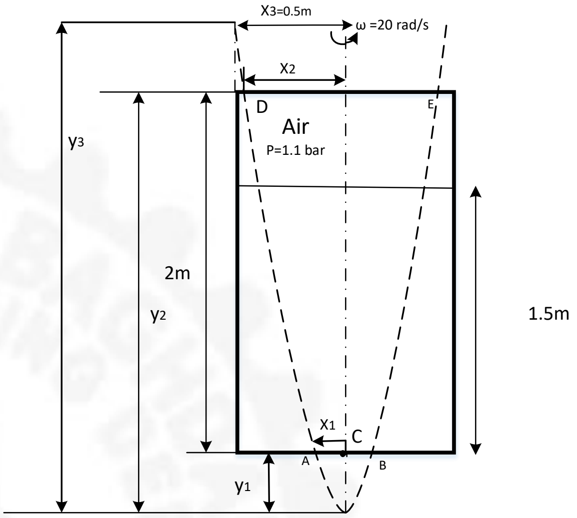
$x + y = 0.059 \quad \ldots .(2)$

$(1)\&\,(2) \rightarrow \quad y = 3.79\ cm \quad \& \quad x = 2.18\ cm$

$dist = 7.5 - x = 7.5 - 2.13 \ \Rightarrow\ \mathbf{dist = 5.37\ cm}$

Sheet No. 5
Relative Equilibrium

(1) The closed cylindrical Tank shown in the figure is rotated with (20 rad/sec). Calculate the bottom area which will be exposed (free of water), and calculate the pressure at point C?



Sol.:

To locate the parabola;

$$y_3 = \frac{20^2 \times 0.5^2}{2\times 9.8} \Rightarrow y_3 = 5.1\text{m}$$

$$y_1 = \frac{20^2}{2\times 9.81} \times X_1^2 \qquad\qquad y_2 = \frac{20^2}{2\times 9.81} \times X_2^2$$

$y_2 - y_1 = 2$ ……………..(1)

Also $\forall_{air}$=Const.

$\forall_{ODE} - \forall_{OAB} = \forall_{air}$

$$\frac{1}{2}\pi X_2^2 y_2 - \frac{1}{2}\pi X_1^2 y_1 = \frac{\pi}{4} 1^2 \times 0.5$$

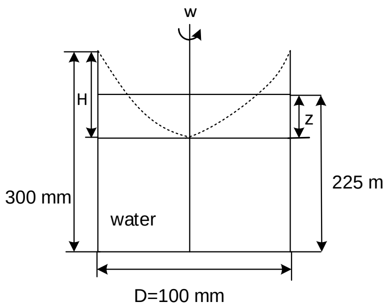
$$\frac{1}{2}\pi X_2^2 \frac{20^2}{2\times 9.81} \times X_2^2 = \frac{1}{2}\pi X_1^2 \frac{20^2}{2\times 9.81} \times X_1^2 = \frac{\pi}{4} 1^2$$ ……………..(2)

(1)&(2) $\Rightarrow X_1 = 0.116\text{ m} \Rightarrow A_{exp} = \pi X_1^2 \Rightarrow A_{exp} = 0.042\text{ m}^2$

$$y_1 = \frac{20^2 \times 0.116^2}{2} = 0.275$$

Pc = Pair-$\gamma_w y_1$ ⇒ $\mathbf{p_C = 0.727}$ **bar**

(2)The Cylindrical vessel shown in the figure is rotated about vertical longitudinal axis. Calculate;



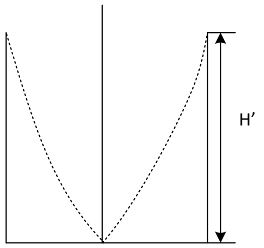
- **a) The angular velocity at which water will start to spill over the sides.**
- **b) The angular velocity, at which the water depth at the center is zero, and the volume of water lost for this case.**

Sol.:

a) $\frac{\pi}{4}D^2z = \frac{1}{2}\frac{\pi}{4}D^2H \Rightarrow H = 2Z = 150\text{ mm}$

$$H = \frac{\omega^2 r^2}{2g} \Rightarrow 0.15 = \frac{\omega^2(\frac{D}{2})^2}{2g} \Rightarrow \boldsymbol{\omega = 34.4\ rad/s}$$

b) $H' = \frac{\omega^2(\frac{D}{2})^2}{2g} \Rightarrow 0.3 = \frac{\omega^2(\frac{D}{2})^2}{2g} \Rightarrow \boldsymbol{\omega = 48.5\ rad/s}$



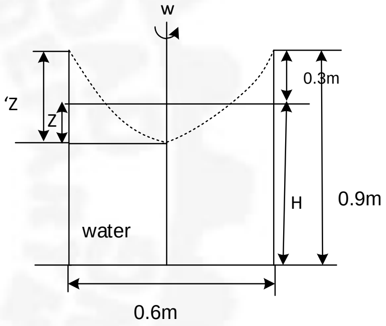
$$\forall = \frac{1}{2}\frac{\pi}{4}D^2H' = 1.178 \times 10^{-3}m^3$$

$$\forall_{lost} = \frac{\pi}{4}D^2 \times 0.225 - V' \Rightarrow \boldsymbol{\forall_{lost} = 0.59 \times 10^{-3}m^3}$$

(3) An open cylindrical tank (0.9m) high and (0.6m) diameter is two – thirds filled with water when stationary. The tank is rotated about its vertical axis. Calculate;

a) The maximum angular velocity at which no water is to spill over the sides.

b) The angular velocity at which the bottom of the tank is free of water for a radius of 150 mm.

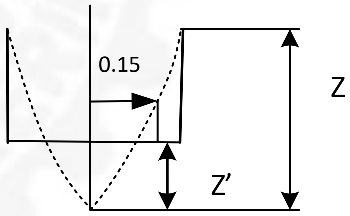


<u>Sol.:</u>

a) $H = \frac{2}{3} \times 0.9 = 0.6\text{m}$

$$\frac{\pi}{4}D^2Z = \frac{1}{2}\frac{\pi}{4}D^2Z' \Rightarrow Z' = 0.6$$

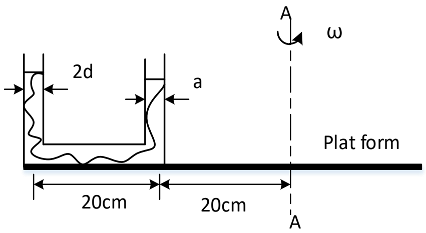
$$Z' = \frac{\omega^2 r^2}{2g} = \frac{w^2 0.3}{2g} \Rightarrow \underline{\omega = 11.43\ rad/s}$$



b) $Z - Z' = 0.9$

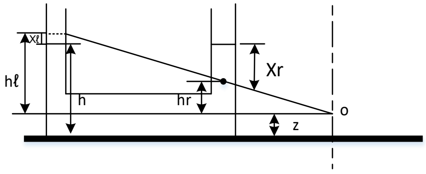
$$\frac{\omega^2(0.3)^2}{2g} - \frac{\omega^2(0.15)^2}{2g} = 0.9 \Rightarrow \underline{\omega = 16.16\ rad/s}$$

(4) The U-tube shown in the figure is rotated about axis (A-A) at (4 rad/s). What will be the increase in the elevation of the liquid in the larger leg of the tube after rotation?



<u>Sol.:</u>

$$h_r = \frac{\omega^2 r_r^2}{2g} = h - z - X_r$$



$$h_\ell = \frac{\omega^2 r_\ell^2}{2g} = h - z + X_\ell$$

Subtract

$$\frac{\omega^2(r_\ell^2-r_r^2)}{2g}=X_r+X_\ell\text{..........}(1)$$

$$\frac{\pi}{4}d^2x_r=\frac{\pi}{4}(2d)^2X_\ell$$

$$X_r=4X_\ell\text{ ....}(2)$$

(2) in (1): $\frac{4}{2g}(0.4^2-0.2^2)=5X_\ell \quad\Rightarrow\quad \mathbf{X_\ell=0.0196\ m{=}19.6mm}$

(5) When the U-tube shown is not rotated the water stands in the tube as shown. If the tube is rotated about the eccentric axis at rate of (8 red/s), what are the new levels of water in the left and right legs of the U-tube?

Sol.:

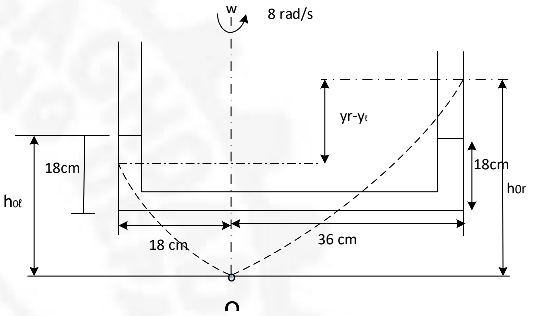
$$h_{0r}-h_{0\ell}=y_r-y_\ell$$

$$\frac{\omega^2 0.36^2}{2\times9.8}-\frac{\omega^2 0.18^2}{2\times9.8}=y_r-y_\ell\text{.........}(1)$$

Also,

$$y_r+y_\ell=0.36\text{............}(2)$$

(1)&(2) gives ; $\mathbf{y_\ell=2.1\ cm}$; $\mathbf{y_r=33.9\ cm}$



(6) In the figure, (ax=9.806 m/s^2) and (a$_y$=0) Find the pressure at (A,B and C).

Sol.:

$$\tan\theta=\frac{-a_x}{ay+g}=-1$$

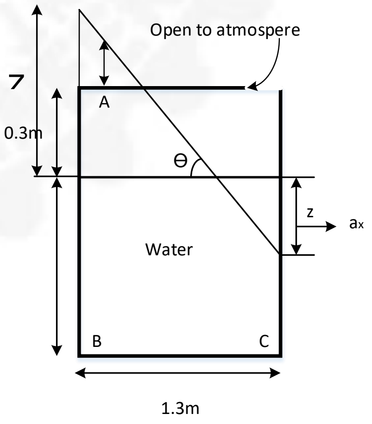
z=0.65 tanθ=0.65

$p=p_0+\rho(ay+g)h=p_0+\gamma h$

$\mathbf{p_A=0.35\ \gamma_w Pa}$

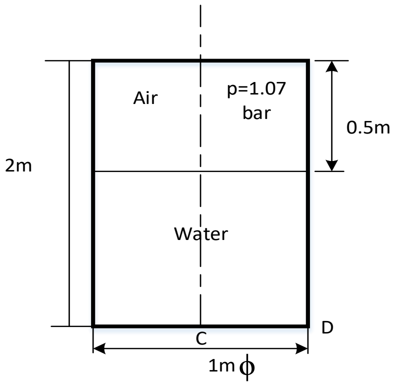
$\mathbf{p_B=1.65\ \gamma_w Pa}$

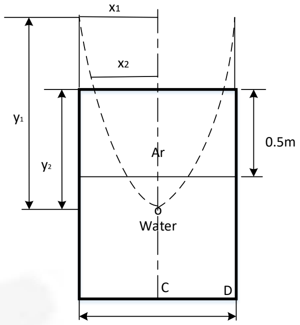
$\mathbf{p_C=0.35\ \gamma_w Pa}$



(7) The closed tank shown in the figure is rotated with (12 rad/s) about its axis. Find the pressure at (C) and (D).

Sol.:





$\forall_{air} =$ const.

$\frac{1}{2}\pi x_2^2 y_2 = \frac{\pi}{4} 1^2 0.5$………..(1)

Also; $y_2 = \frac{12^2 x^2}{2 \times 9.81}$

Thus;

(1) &(2): y_2=1.36cm; x_2=0.43 cm

OC =2-1.36=0.64m

$y_1 = \frac{12^2 0.5^2}{2 \times 9.8} \Rightarrow y_1 = 1.83\text{m}$

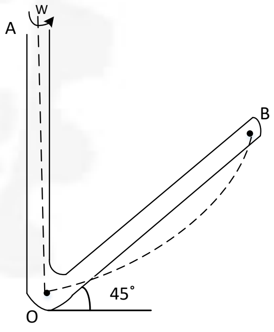
$p_2 = p_1 + \rho_w(ay + g)h = p_{air} + \gamma_w h$

$= 1.07 \times 10^5 + 9810h$

pc $=1.07 \times 10^5 + 9810 \times 0.64 \Rightarrow$ $\mathbf{pc \ = \ 1.13 \ bar}$

$p_D = 1.07 \times 10^5 + 9810 \times (1.83 + 0.64) \Rightarrow$ $\mathbf{p_D = 1.31 \times 10^5 \ Pa}$

(8) The tube (AOB) shown in the figure is completely filled with water to a height of (230 mm) above (O). Calculate.



a) The angular velocity (ω) so that (p_B=p_O).

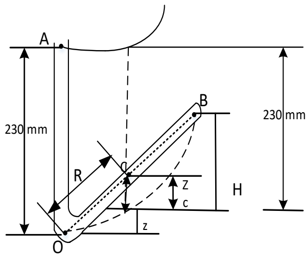
b) The minimum pressure in (OB) and where it occurs.

Sol.:

a) $H = \frac{\omega^2 r_B^2}{2g}$

$0.3 \sin 45 = \frac{\omega^2 (0.3 cos 45)^2}{2g} \Rightarrow$ $\mathbf{\omega = 9.61 \ rad/s}$

b) $Z = \frac{\omega^2 r^2}{2g} = \frac{\omega R^2 cos^2\theta}{2g}$



The minimum pressure occurs at point (C) where (Zc is max.)

$Z_c = R \sin\theta - Z = R \sin\theta - \frac{\omega^2 (R cos\theta)^2}{2g}$

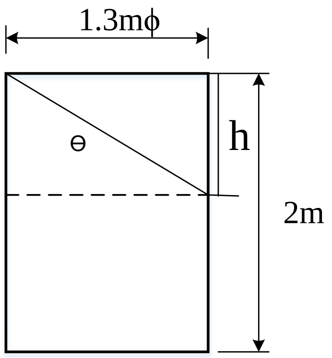
$\frac{dZ_c}{dR} = \emptyset = \sin\theta - \frac{\omega^2 R cos^2\theta}{g} \Rightarrow R = 0.15\text{m}$

Then; $p_c = \gamma h_c = 9.81 \times (0.23 - Z_c)$

Thus; $\mathbf{p_c = 1736.3 \ Pa}$

(9) A circular Cross –sectional tank of (2m) depth and (1.3m) diameter is filled with liquid and accelerated uniformly in a horizontal direction. If one – third of the liquid spills out, determine the acceleration.

Sol.:



$\forall_{\text{spilld}} = \frac{1}{3}\forall$

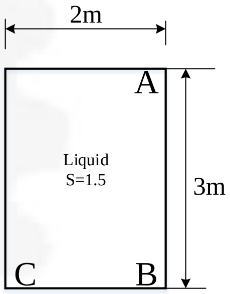
$\frac{h}{2}\frac{\pi}{4}D^2 = \frac{1}{3}\frac{\pi}{4}D2$

$h = \frac{2}{3}2 = 1.333\text{m}$

$\tan\theta = \frac{h}{D} = -\frac{ax}{ay+g} \Rightarrow \mathbf{a_x = 10.05\ m/s^2}$

(10)The closed tank shown which is full of liquid is accelerated downward at ($\frac{2}{3}$ g m/s^2) and to the right at(9 m/s^2). Determine (p_C-p_A) and (p_B-p_A).

Sol.:



$a_y = -\frac{2}{3}g \quad ; \quad a_x = g$

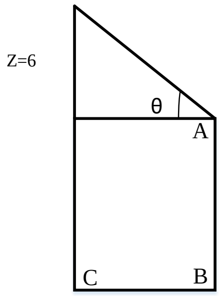
$p_1 - p_2 = -\rho a_x(x_1 - x_2) - \rho(ay + g)(y_1 - y_2)$

$p_C - p_A$: $x_1 - x_2 = -2$ & $y_1 - y_2 = -3$

∴ **$p_C - p_A = 44.15$ kPa**

$p_B - p_A$: $x_1 - x_2 = 0$ & $y_1 - y_2 = -3$

∴ **$p_B - p_A = 14.7$ kPa**



Or;

$\tan\theta = \frac{a_x}{ay+g} = -3 \Rightarrow z = 6$

$p_C - p_A = [p_0 + \rho(a_y + g)h_c] - [p_0 + \rho(a_y + g)h_A]$

$= +s\rho_w(2ay + g)9$

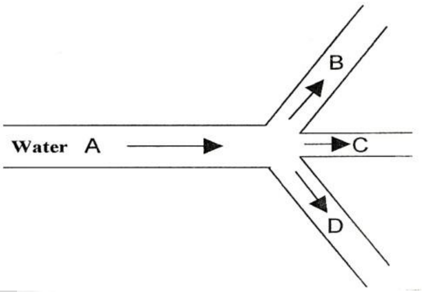
= 44100Pa

$p_B - p_A = s\rho_w(-\frac{2}{3}g + g) \times 3$

=14700 Pa

Sheet No. 6
Continuity Equation

(1) In the figure shown, if (Q_A=15ℓ/s, DA=100mm,D_D=50mm, D_B=Dc=25mm, Q_B=3Q_D, Vc=4m/s), find (Q_B, Qc, Q_D, V_A, V_D, V_B)?



Sol.:

$Qc = (\frac{\pi}{4})(0.025)^2 \times 4 \Rightarrow \boldsymbol{Q_c = 1.96 \times 10^{-3} m^3/s}$

$Q_A = Q_B + Q_c + Q_D$

$Q_A = 3Q_D + Q_c + Q_D$

$15\times 10^{-3} = 4Q_D + 1.96 \times 10^{-3} \Rightarrow \boldsymbol{Q_D = 3.26 \times 10^{-3} m^3/s}$

$Q_B = 3\ Q_D \Rightarrow \boldsymbol{Q_B = 9.78 \times 10^{-3} m^3/s}$

$V_A = Q_A / A_A \Rightarrow$ **V_A = 1.91 m/s**

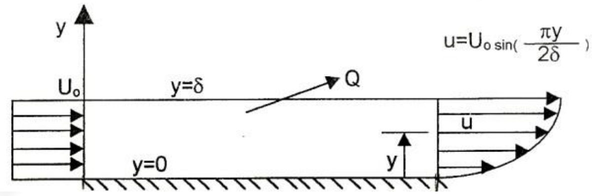
$V_D = Q_D / A_D \Rightarrow$ **V_D = 1.66 m/s**

$V_B = Q_B / A_B \Rightarrow$ **V_B = 19.92 m/s**

(2) Consider the steady incompressible flow past the plate shown in the figure. Determine the volume flow rate (Q) across the upper surface per unit depth in terms of the inlet velocity (U_0) and the boundary layer thickness (δ)?

Sol.:

C.E for steady incompressible flow;



$Q_{in} = Q_{out}$

$Q_1 = Q_2 + Q$

$U_0 * \delta * 1 = \int_0^{\delta} U_0 \sin(\pi y/2\delta) dy + Q$

$Q = U_0\delta - U_0(2\delta/\pi)[-\cos(\pi y/2\delta)]_0^{\delta}$

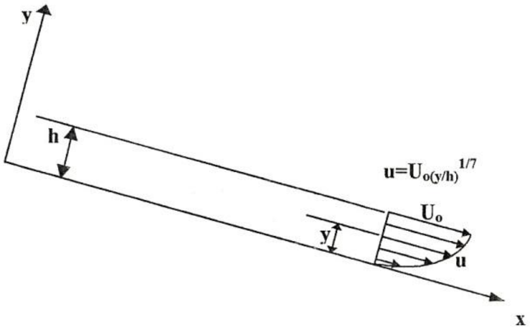
$Q = U_0\delta - U_0(2\delta/\pi)[+0 - (+1)]$

$Q = U_0\delta - (2U_0\delta/\pi)$

$Q = U_0\delta(1 - 2/\pi)$

$\boldsymbol{Q = 0.363\ U_0\delta}$

(3) Water flows steadily down a spillway as shown. If (U_0=1.5m/s), (h=2m) and the width is (20m), how many hours will it take (10^6m^3) of water to pass this section of the spillway?



<u>Sol.:</u>

$Q = \int udA = \int_0^h U_0(y/h)^{1/7}\, 20dy$

$Q = (20U_0/h^{1/7})\left[(y)^{8/7}/8/7\right]_0^h$

$Q = 7/8*.20U_0(h^{7/8}/h^{1/7}) = 52.5\ m^3/s$

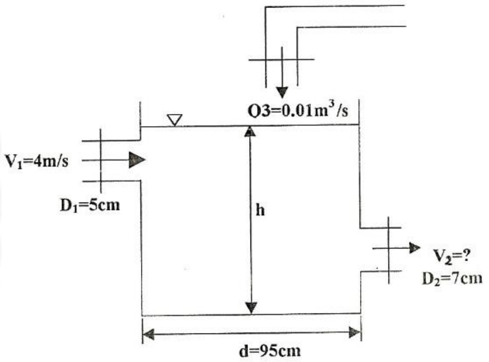
t = ∀/Q=106/52.5 ⇒ **t = 19050s = 5.29 hrs.**

(4) In the open tank shown in the figure;

a) If the water level (h) is constant, determine the exit velocity V_2?

b) If (V_2=6m/s), find (dh/dt)?

<u>Sol.:</u>



a) $Q_1 + Q_3 = Q_2$

$(\pi/4)(0.05)^2 * 4 + 0.01 = Q_2$

$Q_2 = 0.01785\ m^3/s$

$V_2 = Q_2/A_2$ ⇒ **$V_2 = 4.64$ m/s**

b) $(dm_{c.v}/dt) + \dot{m}_{out} - \dot{m}_{out} = 0$

$(d/dt)(\rho\pi/4)(d^2h) + \rho[Q_2 - (Q_1 + Q_3)] = 0$

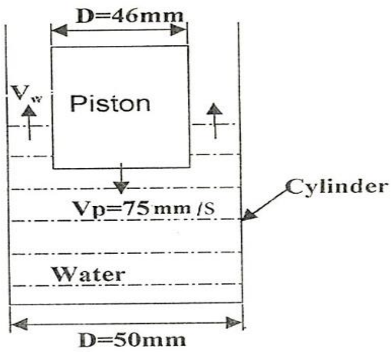
$(\pi/4)d^2(\mathrm{dh/dt}) + Q_2 = Q_1 + Q_3$

$(\pi/4)(0.95)^2(\mathrm{dh/dt}) + (\pi/4)(0.07)^2 * 6 = (\pi/4)(0.05)^2.4 + 0.01$

$\mathbf{(dh/dt) = -0.0074m/s}$ **falling**

(5) Determine the velocity of the water flow through the annular gap between the piston and cylinder shown in the figure?

<u>Sol.:</u>



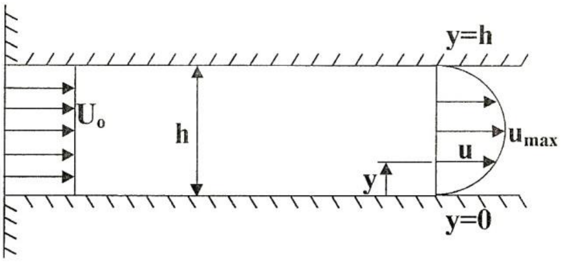
C.E. $Q_{in} = Q_{out}$

$V_p(\pi/4)D_p^2 = V_w(\pi/4)(D_c^2 - D_p^2)$

$75*(46)^2 = Vw(50^2-46^2)$ ⇒ $\mathbf{V_w = 0.413m/s}$

(6) The flow in the inlet between parallel plates in the figure is uniform ($u=U_0=4$cm/s), while downstream the flow develops into the parabolic laminor profile [$u=ay(h-y)$], where (a) is a constant. If (h=1cm) and the fluid is water, for steady flow what is the value of (umax)?

Sol.:



$Q_{in}=Q_{out}$

$hU_0b = \int udA = \int_0^h ay(h-y)bdy$

$hU_0b = ab[(y^2h/2)-(y^3/3)]_0^h$

$hU_0 = ah^3/6$

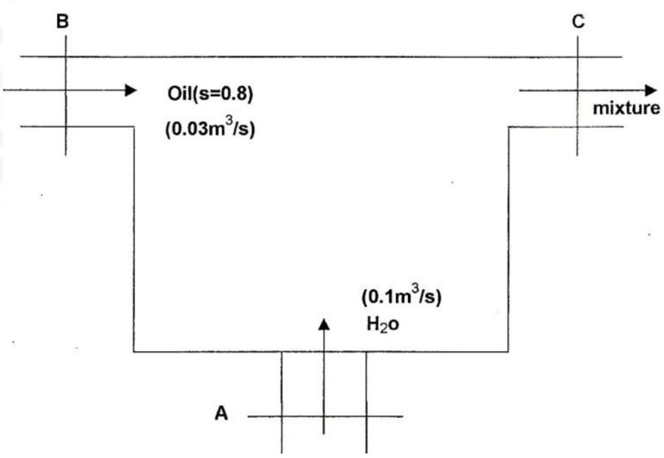
$a = (6\,U_0/h^2) = 2400$

Thus; $u = 2400y\,(h-y)$

$u_{max} = u|_{y=h/2} \;\Rightarrow\; \mathbf{u_{max} = 6\ cm/s}$

(7) Water and oil are forced to flow steadily through the device shown in the figure. If the liquids are incompressible and form a homogeneous mixture of oil globules in water, what are the average velocity and density of the mixture leaving through pipe (C) having a(0.3m) diameter. Assume no chemical reaction between oil and water and their mixture is incompressible?

Sol.:



$Q_{in} = Q_{out}$

$Q_A + Q_B = Q_C \Rightarrow Q_C = 0.13\text{m}^3/\text{s}$

$Q_C = A_C V_C = \left(\frac{\pi}{4}\right)(0.3)^2 V_C$

Thus; $\mathbf{V_C = 1.84\ m/s}$

$\dot{m}_{in} = \dot{m}_{out}$

$\dot{m}_A + \dot{m}_B = \dot{m}_c$

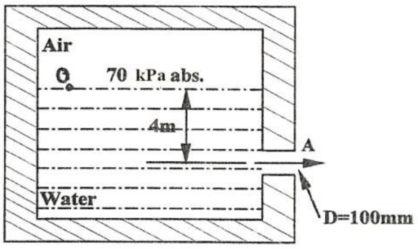
$\rho_A Q_A + \rho_B Q_B = \rho_c Q_c$

$1000*0.1+0.8*1000*0.03 = \rho_c * 0.13$

Thus; $\boldsymbol{\rho_c = 953\ \text{kg/m}^3}$

Sheet No. 7
Energy and Bernoulli's Equations

(1) For losses of (0.1N.m/N), determine the mass flow rate of water at (A) in the figure below. Barometer reading is (750mmHg)?



Sol.:

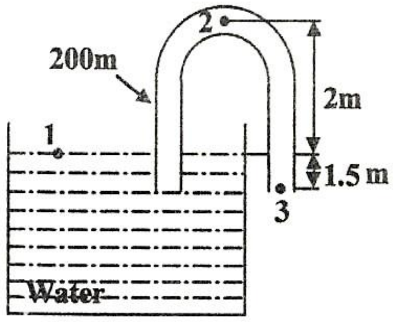
$p_{atm} = 0.75 * 13.6 * 9810 \rightarrow p_{atm} = 100062$ Pa

E.E. o → A :

$(70000/9810) +0+4 = (100062/9810) +(V_A^2 /2g) +0+0.1 \rightarrow V_A = 4.05$m/s

$\dot{m} = \rho AV = 1000 * (\pi/4)0.1^2 * 4.05 \rightarrow$ **$\dot{m}$ = 31.78 kg/s**

(2): The siphon shown in the figure is filled with water and discharging (150 ℓ/s). Find the power required to overcome the losses from point (1) to point (3). If two-thirds of the losses head occur between points (1) and (2), find the absolute pressure at point (2). Take (P_{atm}=100 kPa)



Sol.:

$V = Q/A = (0.15) / ((\pi/4)\ 0.2^2) \rightarrow V = 4.775$m/s

E.E 1-3 :

$0+0+1.5 = 0+(4.775)^2/2 * 9.81+0+h_L \rightarrow h_\ell = 0.337$m

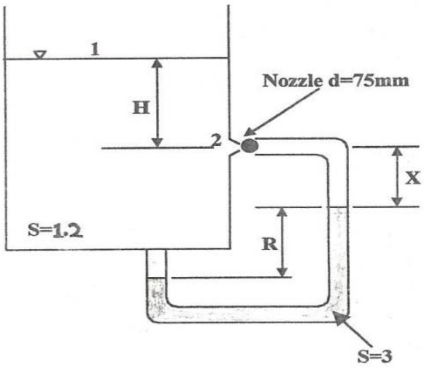
Power = 9810 * 0.15 * 0.337 → **Power = 495.7 W**

E.E. 1-2 :

$0+0+0 = (p_2/9810) +(4.775)^2/2 * 9.81) +2+(2/3) *0.337 \rightarrow p_2 = -33.2$ kPa

$p_{2abs} = 100-33.2 \rightarrow$ **p_{2abs}=66.8 kPa**

(3): For losses of (0.1H) through the nozzle shown in the figure, what is the gage difference (R) in terms of (H)?



Sol.:

$0+0+H = (p_2/1.2\gamma_w)\ 0+0+0.1H$

Thus; $0.9\ H = (p_2/1.2\gamma_w)$ … (1)

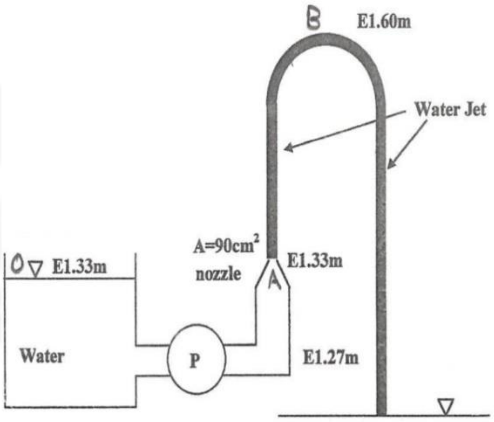
Man. equ. : $p_2 + 1.2\,X\gamma_w + 3R\gamma_w - 1.2\gamma_w\,(R+X+H) = 0$

$(P_2 / 1.2\gamma_w) = R+H-(3/1.2)\,R = H-1.5R$ ....(2)

(2) in (1) → $0.9H = H - 1.5R \rightarrow 1.5R = o.1H \rightarrow R = (0.1/1.5)\,H \rightarrow$ **R = 0.0667 H**

(4) Determine the shaft power required for the pump to produce the flow shown in the figure. The pump efficiency is (80%) and the losses may be neglected?

Sol.:



E.E. O-B:

$0+0+33+h_p = 0+0+60+0+0 \Rightarrow h_p = 27\text{m}$

E.E O-A :

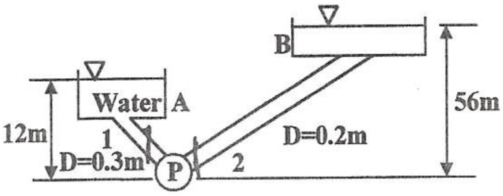
$0+0+33+27 = 0+(V_A^2/2g) +33+0+0 \Rightarrow V_A = 23\text{m/s}$

$Q=A_AV_A = 0.009 * 23 \Rightarrow Q = 0.207\ \text{m}^3/\text{s}$

$I_P = (\gamma\, Q h_p / \eta P) \rightarrow$ **I_P = 68.55 kW**

(5) For the system shown in the figure, the pump (P) must delivers (0.16 m^3) of water from tank (A) to tank (B). If the head lost from (A) to the pump is (2.5m) and from pump to (B) is (6.6m), calculate the shaft power required for the pump and the pressures at the pump inlet and pump outlet, knowing that the pump efficiency is (80%)?

Sol.:



E.E A-B:

$0+0+0+h_p = 0+0+44+0+(2.5+6.6) \rightarrow h_p = 53.1\ \text{m}$

$IP = (\gamma Q h_p/0.8)$ **IP = 104.2 kW**

$V_1 = (Q/A) = (0.16/\,(\pi/4)0.3^2)\ V_1 = 2.263\ \text{m/s}$

E.E. A-1:

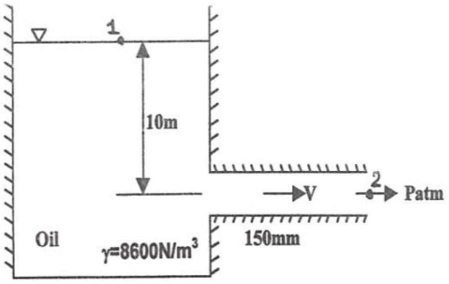
$0+0+12+0 = (p_1/\,\gamma w)=(2.263^2/2g) +0+2.5 \rightarrow$ **p_1 = 90.63 kPa**

$V_2 = (0.16/\,(\pi/4)0.2^2)\ V_2 = 5.1\ \text{m/s}$

E.E 2-B:

$(P_2/\gamma) + (5.1^2/2g) +0 = 0+0+56+6.6 \rightarrow$ **p_2 = 60.123 kPa**

(6) For (100 ℓ/s) flow in the figure below, determine the power required to overcome the losses through the system?



Sol.:

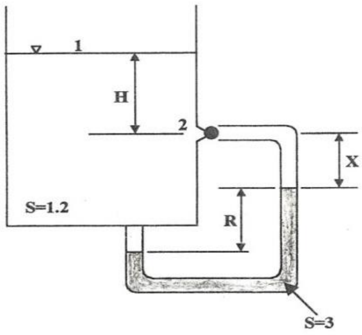
$V = (Q/A) = (0.1) / ((\pi/4)\ 0.15^2 \rightarrow V = 5.66m/s$

E.E. 1-2;

$0+0+10 = 0+(5.66^2/2*9.81) +0+h_\ell \rightarrow h_\ell = 8.366m$

$P = \gamma Q h_\ell = 8600*0.1*8.366 \rightarrow$ **P =7194.9W**

(7) Neglecting losses, calculate (H) in terms of (R) in the figure below?



Sol.:

B.E 1-2: $0+0+H = (p_2/\gamma)+0+0 \rightarrow H = (p_2/1.2\gamma w)$ … (1)

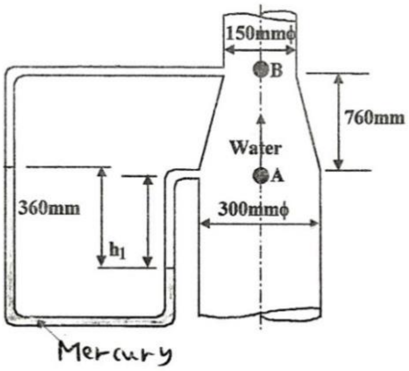
Man. equ.: $p_2+1.2X\gamma w+3R\gamma w-1.2\gamma w\ (R+X+H) = 0$

$p_2+3R\gamma w-1.2R\gamma w-1.2\gamma wH = 0$

$(p_2/1.2\gamma w) = R+H-(3/1.2)\ R$ …. (2)

(2) in (1): $H = R\ (1-3/1.2) +H \rightarrow$ **R=0 For all H**

(8) Determine the mass flow rate of water through the pipe shown in the figure if no energy lost between (A) and (B)?



Sol.:

B.E. A→ B:

$(p_A/\gamma w)+(V_A^2/2g) +Z_A = (p_B/ \gamma w)+(V_B^2/2g) +Z_B$

$(V_B^2/2g) = ((P_A-P_B)/ \gamma w)-0.76+(V_A^2/2g)$ …. (1)

C.E. $(\pi/4)0.3^2V_A = (\pi/4)0.15^2V_B \rightarrow V_A=V_B0.25$ … (2)

Man .equ.:

$p_A+h_1\ \gamma w$ - $0.36 * 13.6\ \gamma w$ $-(0.76+h_1-0.36)\ \gamma w = p_B$

$((p_A-p_B)/ \gamma w) = 0.36(-1+13.6)-0.76=3.776$ … (3)

(2) & (3) in (1) $\rightarrow (V_B^2 -(0.25V_B)^2)/2g=3.776-0.76 \rightarrow V_B = 9.738m/s$

$Q = (\pi/4)0.15^2 * 9.738 \rightarrow Q=0.172\ m^3/s$

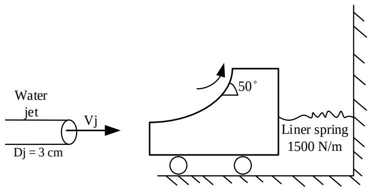
$\dot{m}=\rho Q \rightarrow$ **ṁ = 172 kg/s**

Sheet No. 8
Momentum Equation

(1): For the system shown in the figure, determine;

a) The water jet velocity (V_j) required to deflect the spring by (10 cm).

b) The force on the wheels caused by the jet



Sol.:

a) Fspring = KΔX → 1500 * 0.1 → Fspring = 150 N

Jet: $\sum F_x = \dot{m}(V_{x\,out} - V_{x\,in})$

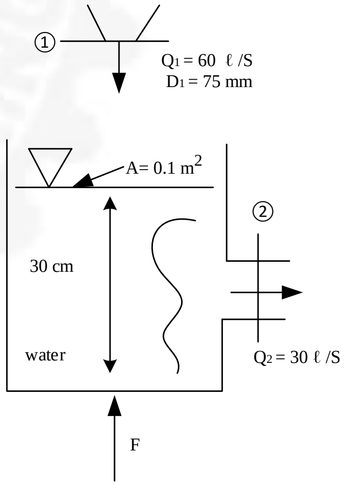
$$-F_x = \rho A_j V_j (V_j \cos 50 - V_j)$$

$$-150 = \rho \frac{\pi}{4}(0.03)^2 {V_j}^2 (\cos 50 - 1) \Longrightarrow \mathbf{V_j = 24.4\ m/s}$$

b) $\uparrow \sum F_y = \dot{m}(V_{y\,out} - V_{y\,in})$

$$F_y = \rho A_j V_j \left(V_j \sin 50 - 0\right) \longrightarrow F_y = 322.4\ N \uparrow on\ jet \longrightarrow \mathbf{F_y = 322.4\ N \downarrow on\ wheel}$$

(2) The water tank shown in the figure has gravity force of (90N) and water depth of (30cm). Find the vertical force required to support the tank.



Sol.:

$V_1 = Q_1 \backslash A_1 \quad \longrightarrow \quad V_1 = 13.6\text{m/s}$

$$\text{M.E:} \sum \text{Fy} = \frac{\partial}{\partial t}(\text{mv})_{\text{yc.v}} + (\dot{\text{m}}\text{v})_{\text{yout}} - (\dot{\text{m}}\text{v})_{\text{yin}}$$

$$\text{F} - \text{W}_{\text{tank}} - \text{W}_{\text{water}} = \frac{\partial}{\partial t}(\text{mv}_{\text{rise}}) + 0 + \rho Q_1 V_1$$

$$F - W_{tank} - W_{water} = V_{rise}\frac{\partial m}{\partial t} + m\cancel{\frac{\partial v_{rise}}{\partial t}} + \rho Q_1 V_1 \quad \ldots 1$$

$$\text{C.E:} \frac{\partial m}{\partial t} + \dot{m}_{out} - \dot{m}_{in} = o \rightarrow \frac{\partial m}{\partial t} = \rho(Q_1 - Q_2) \qquad \ldots ..2$$

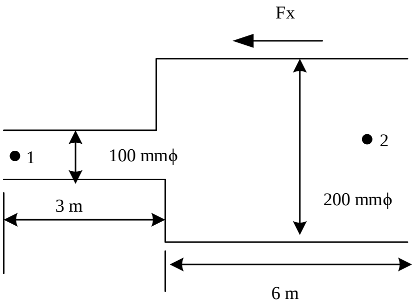
$$\rho A \frac{\partial h}{\partial t} = \rho(Q_1 - Q_2) \Rightarrow \frac{\partial h}{\partial\, t} = V_{rise} = \frac{Q_1 - Q_2}{A} \ \ldots\ldots\ldots 3$$

3 and 2 in1;

$$\text{F} - 90 - 0.1 \times 0.3 \times 9810 =$$

$$\frac{(60-30)*10^{-3}}{0.1} * 1000 * (60-30) * 10^{-3} + 1000 * 60 * 10^{-3} * 13.6 \rightarrow \mathbf{F = 915\ N}$$

(3) Water fills the piping system shown in the figure. At one instant $p_1 = 70\ kPa, p_2 = 0, V_1 = 3\ m/s$) and the flow rate is increasing by(3.2 l/s^2). Find the force (F_x) required to hold the piping system stationary.



Sol.:

$C.E: A_1 V_1 = A_2 V_2 \Rightarrow V_2 = 0.25\, V_1 = 0.75\ {}^{m}/_{s}$

$M.E.: \overset{+}{\rightarrow}\ \sum F_x = \frac{\partial}{\partial t}(mv) + \dot{m}\,(Vont - Vin)$

$p_1 A_1 - F_x = \frac{\partial}{\partial t}(\rho\forall V) + \rho A_1\, V_1(V2 - V1)$

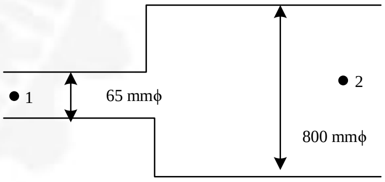
$$= \rho\forall\frac{\partial V}{\partial t}\ \ \rho A_1V_1(V_2 - V_1) = \rho\forall\frac{\partial}{\partial t}\left(\frac{Q}{A}\right) + \rho A_1V_1(V_2 - V_1)$$

$$= \rho L\frac{\partial Q}{\partial t} + \rho A_1\, V_1(V_{2-}V_1)$$

$$70*10^3*\frac{\pi}{4}0.1^2 - F_X = 1000*9*.0032 + 1000*\frac{\pi}{4}0.1^2*3(0.75-3) \Rightarrow$$

$\mathbf{F_X = 574N \leftarrow}$

(4) Water flows through the piping system with sudden expansion shown in the figure. If ($p_1 = 70\ kPa, V_1 = 6m/s$), calculate;



- **a) The power required to overcome the losses.**
- **b) The pressure difference in (cm H_2O).**

Sol.:

a) He $= \frac{V1^2}{2g}\left[1 - \left(\frac{D1}{D2}\right)^2\right]^2 \Rightarrow he = 0.212m$

Q= $A_1V_1 \Rightarrow Q = 2\, m^{3/S}$

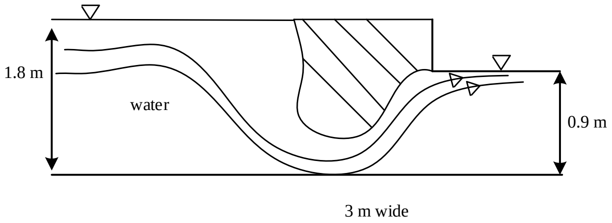
Power = $\gamma Qhe \Rightarrow$ **Power = 4.14 kW**

b) $C.E.:\ A_1\, V_1 = A_2V_2 \Rightarrow V_2 = 3.96\, m/s$

$$E.E.1-2:\ \frac{P1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + h\ell$$

$$\frac{p_2 - p_1}{\gamma} = \frac{V_1^2 - V_2^2}{2g} - he \Rightarrow \frac{p_2 - p_1}{\gamma_w} = \mathbf{0.825\ mH_{20} = 82.5\ cmH_{20}}$$

(5) The water passage shown in the figure is (3m) wide normal to the figure. Determine the horizontal force acting on the shaded structure. Assume frictionless steady flow.



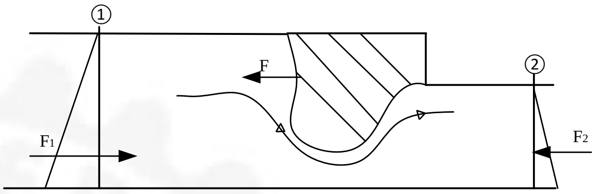
Sol.:

$$\cancel{\frac{p_1}{2\gamma}}+\frac{V_1^2}{2g}+z_1=\cancel{\frac{p_2}{\gamma}}+\frac{V_2^2}{2g}+z_2$$

$$\frac{V_1^2}{2g}+1.8=\frac{V_2^2}{2g}+0.9\ldots\ldots1$$

C.E.: $1.8*3*V_1\ =0.9*3*V_2$

$$\Rightarrow\ \ V_2=2V_1\ \ldots\ldots2$$



$1\&2\Rightarrow V_1\ =2.42m/s\ \&\ V_2\ =4.84m/s$

$F_1\ =\ \gamma\bar{h}_1A_1\ =9810*0.9*1.8*3\Rightarrow F_1\ =47676.5N$

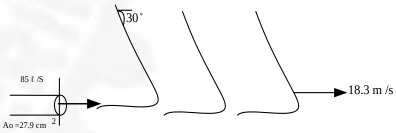
$F_2\ =\gamma\bar{h}_2\ A_2\ =9810*045*0.9*3\Rightarrow F_2\ =11919.15N$

$$\sum F_x\ =\dot{m}(V_{xout}-V_{xin})$$

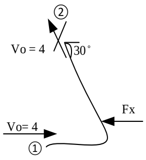
$F_1-F_2-F\ =\rho A_1V_1(V_2-V_1)\Rightarrow$ $\mathbf{F=4132.9N\rightarrow}$ **on structure**

(6) For the series of moving Vanes shown in the figure, determine the power that can be obtained from the vanes and the energy remaining in the jet.

Sol.:



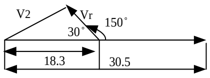
$$V_o\ =\frac{q}{A_o}=\frac{0.085}{0.00279}\Rightarrow V_o\ =30.5m/s$$



$$\sum F_x\ =\dot{m}_{abs}(\ vr_{xout}\ -\ vr_{xin})$$

$-Fx\ =\rho A_0V_0[-(V_O-u)cos30-(V_0-u)]$

$\Rightarrow Fx\ =1935N$



$Power\ =F_xu1935*18.3\Rightarrow$ $\mathbf{Power=35.41\ kW}$

$$Power=\ \gamma Q_{abs}\frac{V_1^2-V_2^2}{2g}=9810*0.085\frac{30.5^2-V_2^2}{2\times9.8}\Rightarrow V_2\ =9.89m/s$$

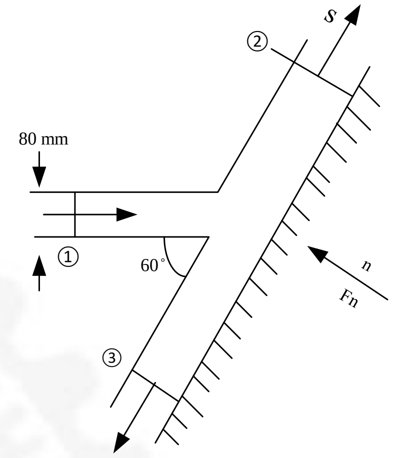
K.E. remaining in the jet= $\gamma Q_{abs}\frac{V_2^2}{2g}\Rightarrow$ $\mathbf{K.E.=4.16kW}$

Or; $V_2x\ =18.3-12.5\ cos\ 30=7.17m/s\ \&\ V_{2y}\ =12.5\ sin\ 30=6.25m/s$

$Thus;\ V_2\ =9.89m/s$

(7) A two- dimensional jet of water impinges on a plane wall at an angle of (60°) the jet is (80mm) thickness and its velocity is (10m/s). Calculate the normal force exerted on the wall per unit width and the thickness of each of the two sheets of water flowing over the wall. Refer to the diagram and neglect the effects of gravity and viscosity.

Sol.:



$C.E.: \ Q_{in} = Q_{out} \Rightarrow A_1V_1 = A_2V_2 + A_3V_3 \dots\dots 1$

$B.E\ 1-2: \Longrightarrow V_2 = V_1$

$B.E\ 1-3: \Longrightarrow V_3 = V_1$

$\therefore V_{1=}V_2 = V_3 = V$

Thus: $A_1 = A_2 + A_3 \dots\dots 2$

M.E.: $\sum F_S = (\dot{m}v)_{s_{out}} - (\dot{m}v)_{s_{in}}$

$0 = (\rho A_2 V - \rho A_3 V) - \rho A_1 v cos60$

$A_2 - A_3 + A_1 cos60 \dots\dots 3$

2 & 3 $\Longrightarrow A_{3=}\frac{A1}{2}(1 - Cos60) \Longrightarrow A_{3=} 20mm^2 \longrightarrow$ **$t_3 = 20$mm**

$A_2 = \frac{3}{4}A_1 \Longrightarrow A_2 = 60mm^2 \Longrightarrow$ **$t_2 = 60$ mm**

$\sum Fn = (\dot{m}v)_{n_{out}} - (\dot{m}v)_{n_{in}}$

$-Fn = o - \rho A_1 V^2 \sin 60 \Longrightarrow$ **Fn = 692.82N** ↘

(8) In the figure below, (Q_1= 0.45$Q_\circ$), what is the value of the plate argle(θ)?

Sol.:

$C.E.: \ Q_\circ = Q_1 + Q_2 = 0.45Q_{0+}Q2 \Longrightarrow Q_2 = 0.55Q_0$

M.E.: $\sum Fs = (\dot{m}v)_{s_{out}} - (\dot{m}v)_{s_{in}}$

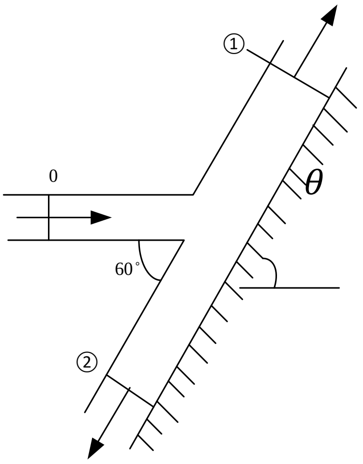
$0 = (\rho Q_1 V_0 - \rho Q_2\ V_\circ) - \rho Q_0 V_O\ Cos\theta$

$Q_1 - Q_2 = Q_0\ Cos\theta$

Or;

$cos\theta = \frac{Q_1-Q_2}{Q_0} = \frac{-0.1Q_0}{Q_0}$

$cos\theta = -0.1 \Longrightarrow \boldsymbol{\theta = 95.74^0}$



(9) At what speed (u) should the vane shown in the figure travel for maximum power? What should be the angle (θ) for maximum power?

<u>Sol.:</u>

$$\sum Fx = \dot{m}r(Vr_{xout} - Vr_{xin})$$

$$-Fx = \rho A_0(V_0 - u)[(V_0 - u)cos\theta - (V_0 - u)]$$

$$Fx = \rho A_0(V_0 - u)^2(1 - \cos\theta)$$

Power= P= $F_x * u = \rho A_0(V_0 {-} u)^2\, u(1 - cos\theta)$

For maximum power:

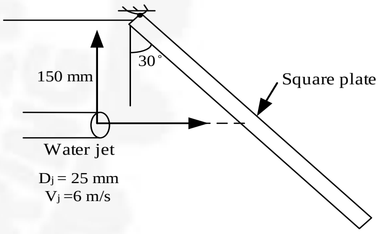
$$\frac{dP}{du} = o = \ \rho A_0(\,1 - coso)\{(V_0 - u)^2\,.1 + u \times 2(\,V_0 - u) \times -1\} \Longrightarrow \ \mathbf{u = V_0/3}$$

$$\frac{dP}{d\theta} = 0 = \ \rho A_0(V_0 - u)^2\, u[-(-sin\theta)] \Longrightarrow sin\theta = 0 \quad \Longrightarrow \quad \theta = 0 \text{ Min power}$$

$$\boldsymbol{\theta = 180^0} \textbf{ For max power}$$

(10) A square plate of uniform thickness and (30cm) side length hangs vertically from hinges at its top edge. When horizontal jet strikes the plate as shown in the figure, the plate comes to rest at an angle of (30°) with the vertical as shown. Calculate the mass of the plate.

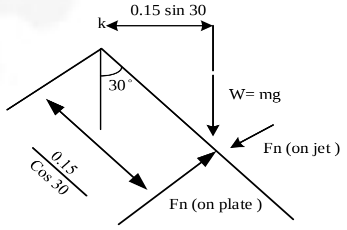
<u>Sol.:</u>



$$\sum F_n = (\dot{m}y)_{nout} - (\dot{m}v)_{nin}$$

$$-F_n = 0 - \rho Aj\ vj(vj\ cos30) \Longrightarrow F_n = 15.3N \swarrow on\ jet$$

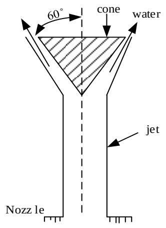
$$= 15.3\ N\ \ \nearrow on\ plate$$



$$F_n * \frac{0.15}{\cos 30} = m * 9.8 * 0.15\ \ sin30$$

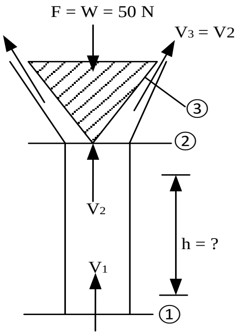
$$\Longrightarrow \ \mathbf{m = 3.6\ kg}$$

(11) As shown in the figure, a cone is held stationary in the vertical direction under the effect of a jet of water striking it from below. The cone weight (50 N). The initial speed of the jet is (15m/s) and the initial jet diameter is (3cm). Find the height to which the cone remain stationary above the nozzle, water density is (1000 kg/m^3).



<u>Sol.:</u>

V_1=15 m/s, d_1= 3cm, F= w = 50N



$B.E.: \ 2 \Longrightarrow 3 \Longrightarrow V3 = V2: C.E.: \ \dot{m}1 = \dot{m}_2 = \rho A_1 V_1$

$M.E.\uparrow \{Fy = \ \dot{m}(V_{yout} - V_{yin})$

$-50 = 1000 * \frac{\pi}{4}(0.03)^2 * 15 * (V_2 cos\, 60 - V_2) \Longrightarrow V_2 = 9.43 m/s$

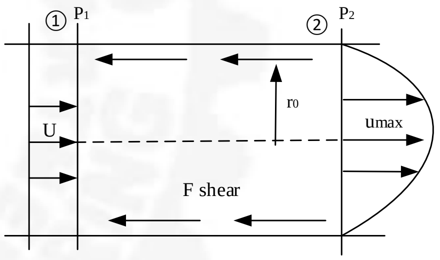
$B.E. 1 - 2: \ \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$

$0 + \frac{15^2}{2*9.8} + 0 = 0 + \frac{9.43^2}{2*9.8} + h \Longrightarrow \mathbf{h = 6.94\ m}$

(12) For laminar flow in pipe, the velocity distribution changes from uniform at section 1. to parabolic at section 2. as showed in the figure. At the fully developed section, the velocity distribution is given as ($u = u_{max}(1 - \left(\frac{r}{ro}\right)^2)$). Find the resisting shear force if p1 and p2 are the pressures and ρ is the fluid density.

Sol.:

$\mathrm{M.E.:} \Longrightarrow \sum F \ = (\dot{m}v)_{out} - (\dot{m}v)_{in}$



$(p_1 - p_2)\pi {r_0}^2 - Fsh = \int \rho u_2^2 dA - \rho U^2 \pi\, {r_0}^2$

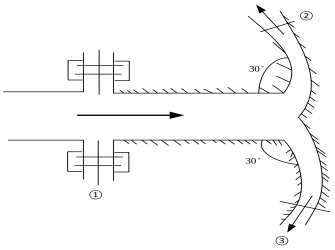
$= \int_0^{r_0} \rho u_{max}^2 \ (1 - \frac{r^2}{{r_0}^2})^2\, 2\pi\, rdr - \rho U^2 \pi {r_0}^2$

$= \rho u_{max}^2\, {r_0}^2 \frac{\pi}{3} - \ \rho U^2\, \pi\, {r_0}^2$

$= \rho\pi {r_0}^2 (\frac{u_{maxs}}{3} - U^2)$

$\mathbf{F_{shear} = (p_1 - \ p_2)\pi {r_0}^2 - \ \rho\pi {r_0}^2\, (\frac{U_{max}}{3} - \ U^2)}$

(13) Water exits to the atmosphere ($p_{atm} = 101kPa$) through a split nozzle as show in the figure. Duct areas are ($A_1 = 0.0\,1m^2$) and ($A_2 = \ A_3 = 0.005m^2$) The flow rates are($Q_2 = \ Q_3 = 150\ m^3/h$), and the inlet pressure is ($p_1 = 140\ kPa\ abs.$).Compute the force on the flange bolts at section 1.



Sol.:

$C.E.: \ Q_1 = Q_2 + Q_3 \Longrightarrow Q_1 = 300m^3/h$

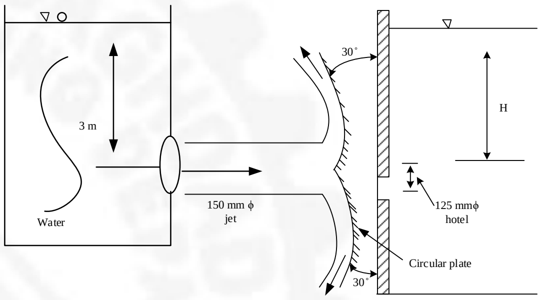
$V = \frac{Q}{A} \Longrightarrow V_1 = 8.333m/\,s, V_2 = V_3 = 8.333m/s$

$M.E.: \sum Fx = (\dot{m}v)_{xout} - (\dot{m}v)_{xin}$

$-F_{bolt} + p_1A_1 = \{-\rho Q_2V_2cos30 - \rho Q_3V_3cos30\} - \rho Q_1V_1 \ (140 - 101) * 10^3 \Longrightarrow$

$\mathbf{F_{bolt} = 1683N} \longrightarrow$

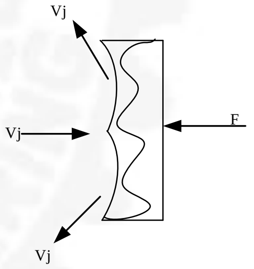
(14) The circular plate shown in the figure covers the (125mm) diameter hole. What is the maximum height (H) which can be maintained without leakage? Assume steady frictionless flow.



Sol.:

$B.E.: \ O - jet. \Longrightarrow 0 + 0 + 3 = 0 + \frac{Vj^2}{2g} + 0$

$\Longrightarrow V_j = 7.67m/s$



$M.E.: \sum Fx = \dot{m}(V_{xout} - V_{xin})$

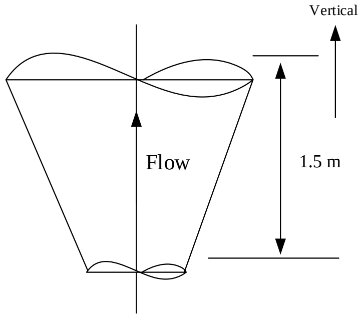
$-F = \rho \frac{\pi}{4} D_j^2 \ V_j\left(-V_j \ sin30 - V_j\right) \Longrightarrow$

$F = 1557.8\text{N}$

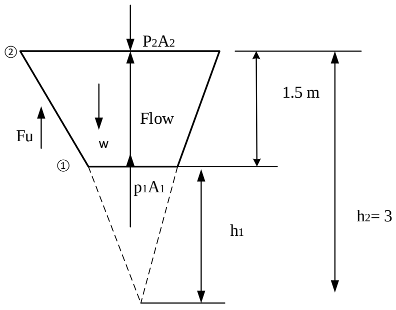
$F = \gamma \ \bar{h}A$

$1557.8 = 9810 * H * \frac{\pi}{4} 0.125^2 \ \Longrightarrow \ \mathbf{H = 12.94m}$

(15) A conical enlargement in a vertical pipe is (1.5m) long and enlarges the pipe diameter from (0.3m) at the lower end to (0.6m) at the larger end, see the figure. Calculate the magnitude and direction of the vertical force on the enlargement if the flow velocity and pressure of water at the smaller end are (3.96m/s) and (207kPa) respectively. Assume that the energy loss in the enlargement is (0.09N.m/N). ($\forall_{cone} = \frac{1}{3}$ ***base*** $*$ ***height*** **)**



Sol.:



$$\frac{0.3}{h_1} = \frac{0.6}{h_1+105} \Longrightarrow h_1 = 1.5m$$

$$W = ɣ∀= 9810\{\frac{1}{3}\pi(0.3)^2 * 3 - \frac{1}{3}\pi(0.15)^2 1.5\}$$

$$Thus;\ W = 3120.4N$$

$$C.E:\ \ A_1V_1 = A_2V_2 \Longrightarrow V_2 = 0.99m/s$$

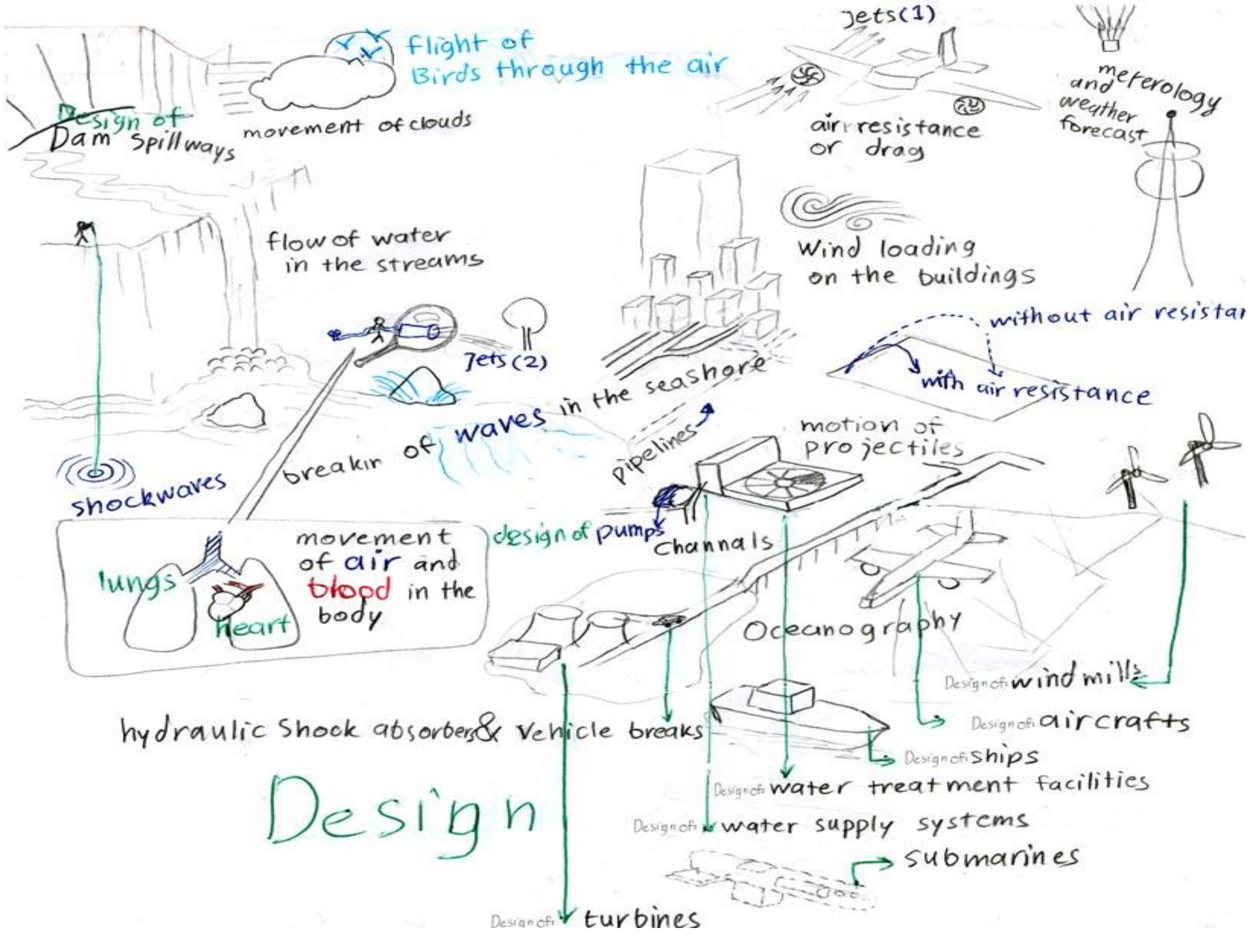
$$E.E.1-2:\quad \frac{P_1}{ɤ} + \frac{V_1^2}{2_g} + z_1 = \frac{P_2}{ɣ} + \frac{V_2^2}{3} + z_2 + h_\ell$$

$$\Longrightarrow p_2 = 198.75\ kP_a$$

$$M.E.:\ \sum F_y = m(V_{yout} - V_{yin})$$

$$F_V + p_1A_1 - p_2A_2 - W = \rho\frac{\pi}{4}d_1^2\ V_1\{V_2 - V_1\} \Longrightarrow F_V = 43.85kN \uparrow on\ fluid$$

$$\mathbf{F_V = 43.85kN \downarrow on\ wall}$$



Sheet No. 9
Dimensional Analysis and Similitude

(1) Assuming that the thrust (F) of a screw propeller is dependent upon the diameter(d), speed of advance (V), fluid density(ρ), revolutions per second(N), and coefficient of viscosity (μ), show that thrust can be expressed by the equation $\left(F = \rho d^2 V^2 f\left\{\frac{\mu}{\rho d v}, \frac{dN}{V}\right\}\right)$.

Sol.:

$$F = f(d, V, N, \mu, \rho) \Longrightarrow F(F, d, V, N, \mu, \rho) = 0$$

$$n = 6 \quad , m = 3 \ \Longrightarrow \text{No. oF } \ \pi' s = 3$$

Dependent variable= F: repeated variables = ρ, N, d

$$\pi_1 = \rho^{x1}\, N^{y1}\, d^{z1}\, F$$

$$M^{\circ} L^{\circ} T^{\circ} = (ML^{-3})^{X1}(T^{-1})^{y1}(L)^{Z1} MLT^{-2} \Longrightarrow \pi_1 = \frac{F}{\rho N^2 d4}$$

$$\pi_2 = \ \rho^{x2} N^{y2} d^{z2}\ \mu$$

$$M^{\circ} L^{\circ} T^{\circ} = (ML^{-3})^{X2})(T^{-1})^{y2}(L)^{Z2} ML^{-1}T^{-1} \Longrightarrow \pi_2 = \frac{\mu}{\rho d^2 N}$$

$$\pi_3 = \rho^{x3} N^{y3} d^{z3}$$

$$M^{\circ} L^{\circ} T^{\circ} = (ML^{-3})^{X3}(T^{-1})^{y3}(d)^{Z3} LT^{-1} \Longrightarrow \pi_3 = \frac{V}{Nd}$$

$$\pi_1^{'} = \frac{\pi_1}{\pi_3^{2}} \Longrightarrow \pi_1^{'} = \frac{F}{\rho d^2 V^2}$$

$$\pi_2^{'} = \frac{\pi_2}{\pi_3} \Longrightarrow \pi_2^{'} = \frac{\mu}{\rho d V}$$

$$\pi_3^{'} = \frac{1}{\pi_3} \Longrightarrow \pi_3^{'} = \frac{dN}{V}$$

$$\therefore f'\left(\frac{F}{\rho d^2 V^2}, \frac{\mu}{\rho d V}, \frac{dN}{V}\right) = 0$$

$$or: \frac{F}{\rho d^2 V^2} = f\left(\frac{\mu}{\rho d V}, \frac{dN}{V}\right)$$

Thus: $\mathbf{F = \rho d^2 V^2\ f\left(\frac{\mu}{\rho d V}, \frac{dN}{V}\right)}$

(2) The wave resistance of a model of a ship at (1:25) scale is (7N) at a model speed of (1.5m/s). What are the corresponding velocity and wave resistance of the prototype? Assume the model is tested in fresh water ($\boldsymbol{\rho = 1000 kg/m^3}$) and the ship will operate in the ocean where the specific gravity of water is (1.03).

Sol.:

Use Fr- criterion;

$\mathrm{Fr}_m = \mathrm{Fr}_p$

$$\frac{\mathrm{V_m}}{\sqrt{g_m \ell_m}} = \frac{\mathrm{V_p}}{\sqrt{g_p \ell_p}} \Rightarrow V_p = V_m \sqrt{\cancelto{1}{\frac{g_p}{g_m}}} \sqrt{\frac{\ell_p}{\ell_m}} = 1.5 \times 1 \times \sqrt{25} \Rightarrow \mathbf{V_p = 7.5\ m/s}$$

$$\frac{F_m}{\rho_m V_m^2 1_m^2} = \frac{F_P}{\rho_p V_{P2} \ell_\rho^2} \Longrightarrow F_p = F_m \frac{\rho_p}{\rho_m} \frac{V_p^2}{V_m^2} \frac{\ell_p^2}{1_m^2} = 7 * 1.03 * 25 * 25^2 \Rightarrow \mathbf{F_p = 112656\ N}$$

(3) The size of droplets(d) produced by a liquid spray nozzle is thought to depend on the nozzle diameter (D), jet velocity (V), and the properties of the liquid ($\boldsymbol{\rho, \mu, \sigma}$). Determine the functional relationship for the droplet size (d).

Sol.:

$d = d(D, V, \rho, \mu, \sigma) \Longrightarrow F(\, d, D, V, \rho, \mu, \sigma) = 0$

$n = 6, \qquad m = 3 \Longrightarrow No.of\ \pi's = 3$

dependent Variable= d ; repeated variables= D,V,ρ

$$\pi_1 = \rho^{x1} D^{y1} V^{z1}\, d \Longrightarrow M^{\circ} L^{\circ} T^{\circ} = (ML^{-3})^{X1} (L)^{y1} (LT^{-1})^{z1} L \Longrightarrow \pi_1 = \frac{d}{D}$$

$$\pi_2 = \rho^{x2} D^{y2} V^{z2} \mu \Longrightarrow M^{\circ} L^{\circ} T^{\circ} = (ML^{-3})^{x2} (L)^{y2} (LT^{-1})^{Z2}\, ML^{-1} T^{-1} \Longrightarrow \pi_2 = \frac{\mu}{\rho V D}$$

$$\pi_3 = \rho^{x3} D^{y3} V^{z3} \sigma M^{\circ} L^{\circ} T^{\circ} = (ML^{-3})^{x3} (L)^{y3} (LT^{-1})^{Z3} MT^{-2} \Longrightarrow \pi_3 = \frac{\sigma}{\rho D V^2}$$

Thus;

$$f(\pi_1, \pi_2, \pi_3) = 0 \Longrightarrow f\left(\frac{d}{D}, \frac{\mu}{\rho V D}, \frac{\sigma}{\rho D V^2}\right) = 0$$

Or;

$$\frac{d}{D} = f'\left(\frac{\mu}{\rho V D}, \frac{\sigma}{\rho D V_2}\right) \Rightarrow \mathbf{d = D\, f'\left(\frac{1}{Re}, \frac{1}{W}\right)}$$

(4) A (1/5) scale model automobile tested in a wind tunnel with the same air properties as the prototype. The air velocity in the tunnel is (350 km/ h) and the measured model drag is (350N). Determine the drag of the prototype automobile and the power required to overcome this drag.

<u>Sol.:</u>

Use R-e criterion;

$Re_m = Re_p$

$$(\frac{V\ell\rho}{\mu})_m = (\frac{V\ell\rho}{\mu})_\rho \Longrightarrow V_\rho = V_m \cancelto{1}{\frac{\mu_p}{\mu_m}} . \frac{\ell_m}{\ell_p} . \cancelto{1}{\frac{\rho_m}{\rho_P}} = 350 \times 1 \times \frac{1}{5} \times 1 \Longrightarrow V_P\text{=70 km/h}$$

$$\frac{\mathrm{F_m}}{\rho_m V_m^2 \ell_m^2} = \frac{\mathrm{F_p}}{\rho_P V_p^2 \ell_p^2} \Longrightarrow F_P = Fm \cancelto{1}{\frac{\rho_p}{\rho_m}} \frac{V_P^2}{V_m^2} \frac{\ell_\rho^2}{\ell_m^2} = \cancelto{1}{\frac{\rho_P}{\rho_m}} \cancelto{1}{\frac{\mu_P^2}{\mu_m^2}} \frac{\ell_m^2}{\ell_p^2} \cancelto{1}{\frac{\rho_m^2}{\rho_p^2}} \frac{\ell_P^2}{\ell_m^2} * Fm = Fm$$

$\therefore$ $\boldsymbol{F_P = F_m = 350\ N}$

$\mathrm{Power} = \mathrm{F_P} * \mathrm{V_P} \Longrightarrow$ $\mathbf{Power = 6805.6W}$

(5) The pressure difference (Δp) between two points in a pipe due to turbulent flow depends on (V,D, ρ, μ, L , ϵ), where (L) is the distance between the two points and (ϵ) is the roughness elements height. Using Buckingham theorem, determine the functional relationship for (Δp).

<u>Sol.:</u>

$\Delta P = \Delta P(\ V, D, \rho, \mu, L, \epsilon) \Longrightarrow F(\Delta P, V, D, \rho, \mu. L. \epsilon) = 0$

$n = 7, m = 3 \Longrightarrow \mathrm{no.\,of}\ \pi's = 4$

Dependent variable = Δp: repeated variables = ρ, V, D

$$\pi_1 = \rho^{x1}\, V^{y1} D^{x1}\, \Delta p \Longrightarrow\ \pi_1 = \frac{\Delta p}{\rho V^2}$$

$$\pi_2 = \rho^{x2} V^{y2} D^{z2} \mu \Longrightarrow \pi_2 = \frac{\mu}{\rho V D}$$

$$\pi_3 = \ \rho^{x3} V^{y3} D^{z1} \epsilon \Longrightarrow \pi_3 = \frac{\epsilon}{D}$$

$$\pi_4 = \rho^{x4} V^{y4} D^{z4} L \Longrightarrow \pi_4 = \frac{L}{d}$$

$$\therefore \text{f}\left(\frac{\Delta p}{\rho V^2}, \frac{\mu}{\rho v d}, \frac{\epsilon}{D}, \frac{L}{D}\right) = 0$$

$$Or;\ \boldsymbol{\Delta p = \rho V^2 f'\left(\frac{\mu}{\rho VD}, \frac{\epsilon}{D}, \frac{L}{D}\right) = \rho V^2 f'(\frac{1}{Re}, \frac{\epsilon}{D}, \frac{L}{D})}$$

(6) A completely submerged body is to be propelled through water at aspeed of (6m/s). In order to estimate the power required to overcome the frictional drag a (1:10) scale model of the body is mounted in a closed circuit wind tunnel in which the air density can be varied. The model is tested at the tunnel speed of (75m/s) under dynamically similar conditions and the air temperature is maintained at (15°C). Determine the pressure of the air in the working section of the tunnel. If the drag on the model under these conditions was (2.5 kgf), determine the power required to overcome the drag on the full scale body. Take (Rair= 287J/kgK, μ_{water}= 0.012Poise, μ_{air}= 0.000186 Poise; 1Poise= 1gm/cm.s)

Sol.:

$$Re_m = Re_p$$

$$\left(\frac{\rho_m V_m \ell_m}{\mu_m}\right)_{air} = (\frac{\rho_P V_P \ell_P}{\mu_P})_{water} \Longrightarrow \rho_m = \rho_P * \frac{V_P}{V_m} \frac{\ell_P}{\ell_m} \frac{\mu_m}{\mu_P}$$

$$= 1000 * \frac{6}{75} * 10 * \frac{0.000186}{0.012}$$

$$\therefore \rho_m = 12.375 kg/m^3$$

$$P_{air} = \rho_{air} R_{air} T = 12.375 * 287 * 288 \Longrightarrow P_{air} = 1.025 \text{ Mpa}$$

$$\frac{F_m}{\rho_m V_m^2 \ell_m^2} = \frac{F_p}{\rho_P V_p^2 \ell_p^2} \Longrightarrow F_p = 129.03 \text{kgf} = 1264.52 \text{ N}$$

$$\text{Power} = F_\rho V_\rho \Longrightarrow \mathbf{Power = 7587.1W}$$

(7) The power input (P) to a centrifugal pump is assumed to be a function of volume flow rate (Q), impeller diameter (D), rotational speed (ω), the density (ρ) *and the* viscosity (μ) of the fluid. Determine the functional relationship for the power.

Sol.:

$$\mathrm{P} = \mathrm{P}(\mathrm{Q}, \mathrm{D}, \omega, \rho, \mu)$$

Or; $\mathrm{F}(\mathrm{P}, \mathrm{Q}, \mathrm{D}, \omega . \rho, \mu) = 0$

$\mathrm{n} = 6, \mathrm{m} = 3 \Longrightarrow \mathrm{no.\,of}\ \pi' \mathrm{s} = 3$

Dependent Variable= P, repeated variables=ρ, ω, D

$$\pi_1 = \mathrm{p}^{\mathrm{x}1} \omega^{\mathrm{y}1} \mathrm{D}^{\mathrm{z}1} \mathrm{P}$$

$$\mathrm{M}^{\circ}\mathrm{L}^{\circ}\mathrm{T}^{\circ} = (\mathrm{ML}^{-3})^{x_1} (\mathrm{T}^{-1})^{y_1} (\mathrm{L})^{z_1} \mathrm{ML}^2\mathrm{T}^{-3} \Longrightarrow \pi_1 = \frac{\mathrm{P}}{\rho \omega^3 \mathrm{D}^5}$$

$$\pi_2 = \rho^{x_2} \omega^{y_2} \mathrm{D}^{\mathrm{z}_2} \mathrm{Q}$$

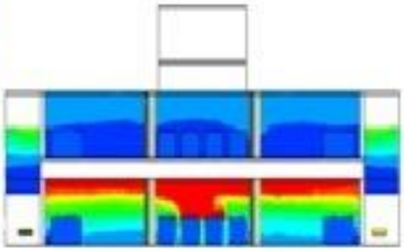
$$\mathrm{M}^{\circ}\mathrm{L}^{\circ}\mathrm{T}^{\circ} = (\mathrm{ML}^{-3})^{x_2} (\mathrm{T}^{-1})^{y_2} (\mathrm{L})^{\mathrm{z}_2} \mathrm{L}^3\mathrm{T}^{-1} \Longrightarrow \pi_2 = \frac{\mathrm{Q}}{\omega \mathrm{D}^3}$$

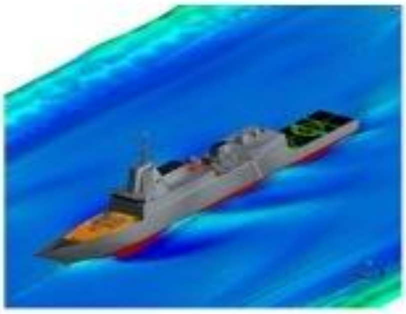
$$\pi_3 = \rho^{x_3} \omega^{y_3} D^{z_3} \mu$$

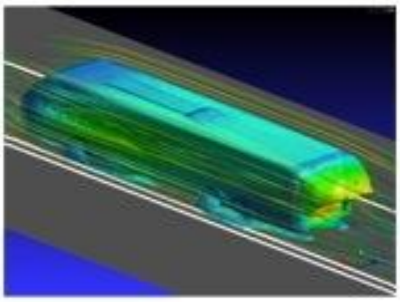
$$M^{\circ}L^{\circ}T^{\circ} = (\mu L^{-3})^{x_3} (T^{-1})^{y_3} (L)^{z_3}\ \mathrm{ML}^{-1}\mathrm{T}^{-1} \Longrightarrow \pi_3 = \frac{\mu}{\rho \omega D^2}$$

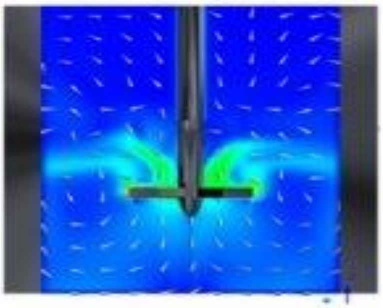
$$\therefore f\left(\frac{P}{\rho \omega^3 D^5}, \frac{Q}{\omega D^3}, \frac{\mu}{\rho \omega D^2}\right) = 0$$

Or; $\boldsymbol{\mathrm{P} = \rho \omega^3 \mathrm{D}^5\ \mathrm{f}'\left(\frac{\mathrm{Q}}{\omega \mathrm{D}^3}, \frac{\mu}{\rho \omega \mathrm{D}^2}\right)}$









Sheet No. 10
Viscous Laminar Flows

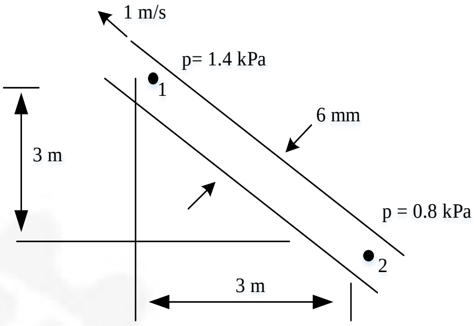
(1) For the flow between the include flat plates show in the figure, determine:

a) The discharge per unit depth.

b) The shear stress exerted on the upper plate.

c) The maximum velocity and where it occurs.

Take ($\rho = 850\ kg/m^3, \mu = 0.08 Pa.s$).



Sol.:

a) $\frac{d}{dx}(p+\gamma h) = \frac{(p+\gamma h)_2-(p+\gamma h)_1}{\ell} =$

$\frac{(800+0)-(1400+850\times9.8\times3)}{3\sqrt{2}} = -6035\ N/m^3$

$Q = \frac{Ua}{2} - \frac{a^3}{12\mu}\frac{d}{dx}(p+\gamma h) = \frac{-1\times0.006}{2} - \frac{-6035\times(0.006)^3}{12\times0.08} \Longrightarrow \mathbf{Q = -0.00264 m^3/s} \nwarrow$

b) $\tau = \mu\frac{\partial u}{\partial y}|_{y=a} = \frac{\mu U}{a} - \frac{1}{2}\frac{d}{dx}(p+\gamma h)[a-2\times a] \Longrightarrow \tau = -31.44\ Pa$

$= 31.44\text{Pa} \searrow onfluid \Longrightarrow$ $\boldsymbol{\tau = 31.44}$**Pa ↖ on plate**

c) $u = \frac{Uy}{a} - \frac{1}{2\mu}\frac{d}{dx}(p+\ \gamma h)[ay-y^2]$

Thus; $u = 59.64\ y - 37718\, y^2$

$\frac{\partial u}{\partial y} = 59.646 - 2*37718y = 0 \Longrightarrow \boldsymbol{y = 0.00079\ m\ for\ u_{max}}$

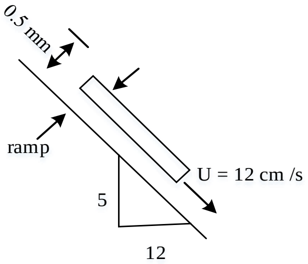
$u_{max} = 59.646\times0.00079 - 37718\times(0.00079)^2 \Longrightarrow \mathbf{u_{max} = 0.0236m/s}$

(2) A board (1m*1m) that weighs (15N) slides down the inclined ramp shown in the figure. Assuming laminar flow of oil (s= 0.8) with constant pressure, determine:

a) The dynamic viscosity of oil.

b) The shear stress at the ramp.

c) The discharge.



Sol.:

a) $\frac{d}{dx}(p+\gamma h)=\frac{dp}{dx}+\gamma\frac{dh}{\gamma h}=0-\gamma\sin\theta=\ -0.8*9810*\frac{5}{13}=-3018.46\frac{N}{m^3}$

$at\ y=a\ \tau=\frac{F_{sh}}{A}=\frac{w\sin\theta}{A}=\frac{15*\frac{5}{13}}{1*1}\Longrightarrow\tau_{|y=a}5.77Pa$

$\tau_{|_{y=a}}=\frac{\mu U}{a}-\frac{1}{2}\frac{d}{dx}(p+\gamma h)[a-2*\ a]\Longrightarrow$ $\boldsymbol{\mu=0.027\mathrm{Pa.s}}$

b) $\tau_{|_{y=0}}=\frac{\mu U}{a}-\frac{1}{2}\frac{d}{dx}(p+\gamma h)[a]\Longrightarrow$ $\boldsymbol{\tau_{|\mathrm{ramp}}=7.28\ \mathrm{Pa}}$

c) $Q=\frac{Ua}{2}-\frac{a^3}{12\mu}\ \boldsymbol{\frac{d}{dx}}\ (\boldsymbol{p}+\gamma\boldsymbol{h})\Longrightarrow$ $\boldsymbol{Q=3.11*10^{-5}\mathrm{m}^3/\mathrm{s}}$

(3) Oil having (s=0.8) and ($\boldsymbol{\mu=0.02\ \mathrm{Pa.s}}$)flows downward between two vertical fixed plate spaced (10mm) apart. If the discharge per meter of width is (0.01$\boldsymbol{m^3}$/s), determine:

- **a) The pressure gradient (dp/ dx).**
- **b) The shear stress at each plate.**

Assume laminar flow.

<u>Sol.:</u>

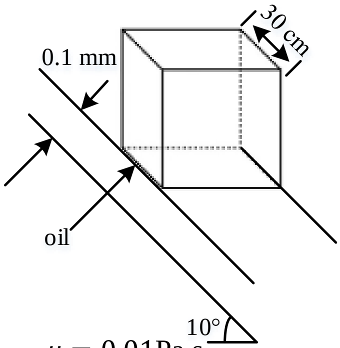
a) $Q=\cancelto{0}{\frac{Ua}{2}}-\frac{a^3}{12\mu}(\frac{dp}{dx}+\gamma\frac{dh}{dx})$

$0.01=\ -\frac{(0.01)^3}{12*0.02}(\frac{dp}{dx}-0.8*9810*\cancelto{1}{\sin 90})\Longrightarrow\quad\frac{dp}{dx}=5448\mathrm{Pa/m}$

b) $\tau=\frac{\mu U}{\sigma}-\frac{1}{2}\left\{\frac{dp}{dx}-\gamma\right\}(a-2y)\Longrightarrow$ $\boldsymbol{\tau_{|y=0}=12\ \mathrm{Pa}\ \&\ \tau_{|y=a}=-12\mathrm{Pa}}$

(4) The cube shown in the figure weighs (200N) and is allowed to slide down the inclined surface as shown. Assuming laminar flow with constant pressure, determine:

- **a) The terminal speed (U) of the cube.**
- **b) The shear stress at the inclined plate.**
- **c) The discharge.**



$\mu=0.01\mathrm{Pa.s}$

$\gamma=7407\ \frac{N}{m^3}$

<u>Sol.:</u>

a) $\frac{d}{dx}(p+\gamma h)=\cancelto{0}{\frac{dp}{dx}}-\gamma\ sin\theta=\ -7407\sin 10=-1286.2N/m^3$

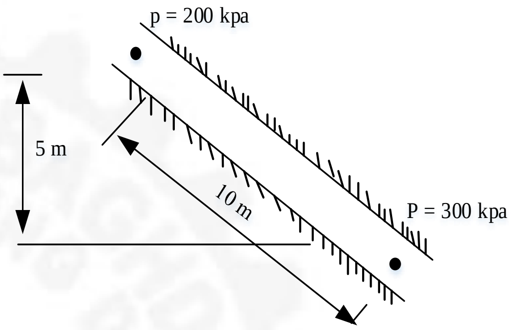
$$\tau|_{y=a} = \frac{Fsh}{A} = \frac{wsin\theta}{A} = \frac{200sin10}{0.3*0.3} \Rightarrow \tau|_{y=a} = 385.88\text{Pa}$$

$$\tau|_{y=a} = \frac{\mu u}{a} - \frac{1}{2}\frac{d}{dx}(p+\gamma h)\{a - 2*a\} \Rightarrow \mathbf{U = 3.86\ m/s}$$

b) $\tau|_{y=0} = \frac{\mu u}{a} - \frac{1}{2}\frac{d}{dx}(p+\gamma h)[a] \Rightarrow \boldsymbol{\tau|_{y=a} = 386\ \mathbf{Pa}}$

c) $Q = [\frac{Ua}{2} - \frac{a^3}{12\mu}\frac{d}{dx}(p+ \gamma h)] * 0.3 \Rightarrow \mathbf{Q = 5.8 * 10^{-5}\ m^3/s}$

(5) For the laminar flow through the (10mm) diameter inclined tube shown in the figure, determine:



- **a) The direction of the flow.**
- **b) The discharge.**
- **c) The Reynolds number for the flow.**

Take ($\boldsymbol{\gamma = \gamma 000N/m^3}$) and ($\boldsymbol{\mu = 0.04Pa.s}$).

Sol.:

a) $(p+\gamma h)_{upper} = 200000 + 8000 * 5 = 240\text{ kPa}$

$(p+\gamma h)_{lower} = 300000 + 0 = 300\ kPa$

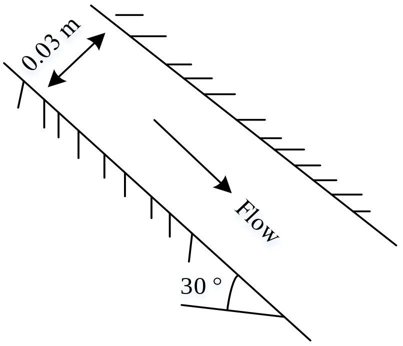
Thus, **the flow is upward ↖**

b) $Q = -\frac{\pi a^4}{8\mu}\frac{d}{dx}(p+\gamma h) = -\frac{\pi(0.005)^4}{8*0.04}\frac{(300-240)*10^3}{10} \Rightarrow \mathbf{Q = -0.0368\ l/s}$ ↖

c) $V = \frac{Q}{A} = \frac{Q}{\frac{\pi}{4}d^2} \Rightarrow V = 0.4686m/s$

$Re = \frac{Vd\rho}{\mu} = \frac{vd\gamma}{\mu g} \Rightarrow \mathbf{Re = 95.6}$

(6) Oil($\boldsymbol{\mu = 0.478\ Pa.s, v = 5.3 * 10^{-4}\, m^2/s}$ flows downward at a rate of (0.222 ℓ /s) in the pipe shown in the figure. Determine;



- **a) The pressure gradient (dp/ dx)**
- **b) The wall shear stress.**
- **c) The maximum velocity.**

Assume laminar flow.

Sol.:

a. $\rho = \frac{\mu}{v} \Longrightarrow \rho = 901.88 kg/m^3$

$\gamma = \rho g \Longrightarrow \gamma = 8838.5 N/m^3$

$$\frac{d}{dx}(p + \gamma h) = \frac{dp}{dx} - \gamma\, sin\theta$$

$Q = -\frac{\pi a4}{8\mu}[\frac{dp}{dx} - \gamma\, sin30\,] \Longrightarrow \mathbf{\frac{dp}{dx} = -918.5\ N/m^3}$

b. $\tau = -\frac{r}{2}\frac{d}{dx}(p + \gamma h) = -\frac{r}{2}\,[\frac{dp}{dx} - \gamma\, sin\theta]$

$\tau_w = \tau_{|r=\frac{d}{2}} = \frac{0.03}{4}[-918.5 - 8838.5 - 5\sin 30] \Longrightarrow \mathbf{\tau_w = 40Pa}$

C. $u_{max} = -\frac{a^2}{4\mu}\frac{d}{dx}(p + \gamma h) \Longrightarrow \mathbf{u_{max} = 0.63 m/s}$

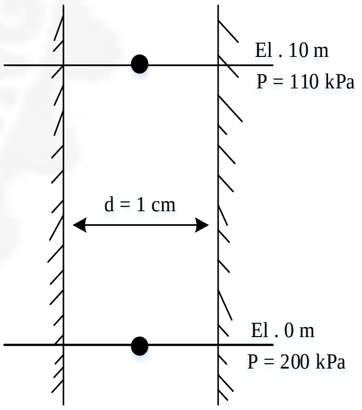
Or; $u_{max} = 2V = 2\frac{Q}{\pi a^2} = \frac{Q}{\frac{\pi}{4}d^2} \Longrightarrow \mathbf{u_{max} = 0.63 m/s}$

(7) Liquid in the pipe shown in the figure has $(\gamma = \mathbf{10 kN/m^3})$ ***and*** $(\boldsymbol{\mu} = \mathbf{3.125 * 10^{-3}\ Pa.s})$.

a) Is the liquid stationary, moving upward or moving upward or moving downward in the pipe?

b) Calculate($\mathbf{u_{max}}$**, V,Q and** $\boldsymbol{\tau_w}$**)**

Assume laminar flow.



Sol.:

a) $(p + \gamma h)_{upper} = 110000 + 10000 * 10 = 210\ \text{kPa}$

$(p + \gamma h)_{lower} = 200000 + 0 = 200\ \text{kPa}$

$sine(p + \gamma h)_{upper} > (p + \gamma h)_{lower} \Longrightarrow$ **flow is downward ↓**

b) $\frac{d}{dx}(p + \gamma h) = \frac{200*10^3 - 210*10^3}{10} = -1000 N/m^3$

$u_{max} = -\frac{a2}{4\mu}\frac{d}{dx}(p + \gamma h) \Longrightarrow \mathbf{u_{max} = 2 m/s}$

$V = \frac{u_{max}}{2} = -\frac{a2}{4\mu}\frac{d}{dx}(p + \gamma h) \Longrightarrow \mathbf{V = 1 m/s}$

$Q = \frac{\pi}{4}d^2 V \Longrightarrow \mathbf{Q = 7.85 * 10^{-5} m^3/s}$

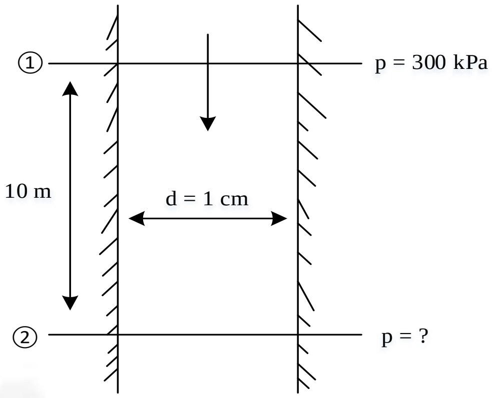
$\tau_w = \tau_{|r=\frac{d}{2}} = -\frac{d}{4}\frac{d}{dx}(p + \gamma h) \Longrightarrow \mathbf{\tau_w = 2.5\ Pa}$

(8) Water flows downward in the vertical pipe show in the figure with maximum velocity of (2m/ s). Using (μ= 0.1 Pa.s) and assuming laminar flow, determine;

a) The discharge.

b) The pressure at section2.

c) The wall shear stress.



Sol.:

a) $u_{max} = 2V \;\Longrightarrow V = 1m/s$

$Q = \frac{\pi}{4}d^2v \Longrightarrow \;\; \boldsymbol{Q = 7.85 * 10^{-5}\ \mathrm{m^3/s}}$

b) $Q = -\frac{\pi a4}{8\mu}[\frac{dp}{dx} - \gamma\ sin90]$

$7.85 * 10^{-5} = -\ \frac{\pi(0.005)^4}{8*0.1}\left[\frac{p_2 - 300*10^3}{10} - 9810\right] \Longrightarrow \mathbf{p_2 = 78\ kPa}$

c) $\tau_w = -\ \frac{D}{4}\frac{d}{dx}(p + \gamma h) = -\ \frac{D}{4}\ \frac{(\boldsymbol{p}+\gamma\boldsymbol{h})_2 - (\boldsymbol{p}+\gamma\boldsymbol{h})_1}{\mathbf{10}} \Longrightarrow \boldsymbol{\tau_w = \ 80\ \mathrm{Pa}}$

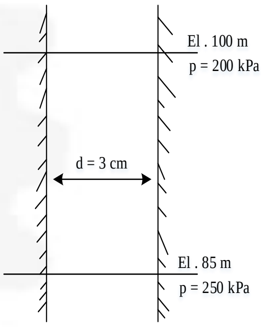
(9) Oil (s=0.9,$\mu = 0.05$ Pa.s) flows steadily in the pipe shown in the figure. Assuming laminar flow, determine;

a) The flow direction, up or down?

b) The velocity at the center of the pipe.

c) The discharge (Q)

d) The velocity and shear stress at (6mm) from the center.



Sol.:

a) $(p + \gamma h)_{upper}\ = 200000 + 0.9 * 9810 * 100 = 1082kN/m^3$

$(p + \gamma h)_{lower}\ = 250000 + 0.9 * 9810 * 85 = 999.7kN/m^3$

$sine\ (p + \gamma h)_{upper}\ > \ (p + \gamma h)_{lower} \Longrightarrow$ **flow is downward ↓**

b) $\frac{d}{dx}((p + \gamma h) = \frac{(p+\gamma h)_{lower-}\ (p+\gamma h)_{upper}}{\ell} = \frac{(999.7-1082)*10^3}{15} = -5486.66N/m^3$

$at\ the\ center\ u = u_{max} = -\frac{a^2}{4\mu}\frac{d}{dx}(p + \gamma h) \Longrightarrow \mathbf{u_{max} = 6.17m/s}$

c) $V = \frac{U_{max}}{2} = 3.85m/s \;\Longrightarrow Q = \frac{\pi}{4}\ d^2V \;\Longrightarrow\; \mathbf{Q = 2.18 * 10^{-3}m^3/s}$

d) $u = -\ \frac{a^2 - r^2}{4\mu}\ \frac{d}{dx}\ (p + \gamma h) = u_{max}\left(1 - \frac{r^2}{a^2}\right)$

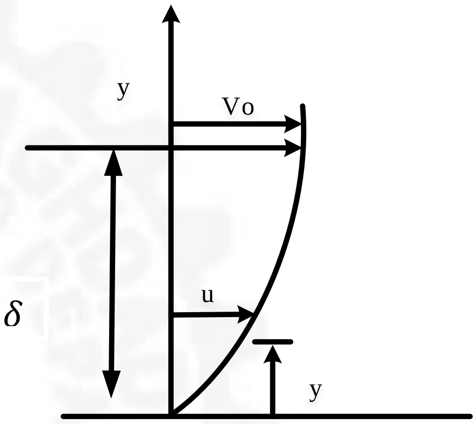
$Thus\ at\ r = 6mm;\ \ u = 6.17\left(1 - \frac{0.006^2}{0.015^2}\right) \quad \Longrightarrow \mathbf{u = 5.183\ m/s}$

$$\tau = -\mu\frac{du}{dr} = -\frac{r}{2}\frac{d}{dx}(p + \gamma h)$$

$Thus\ at\ r = 6mm;\ \ \tau = -\frac{0.006}{2} * -5486.66 \;\Longrightarrow \boldsymbol{\tau = 16.46\ \mathrm{Pa}}$

Sheet No. 11
Boundary Layer, Drag, and Lift

(1)For a laminar flow established over a horizontal flat plate, at a given location along the plate, the thickness of the boundary layer is found to be (δ).The assumed velocity profile at this location on the plate is given as [u=$\alpha y+\beta y^3$(for 0≤ $y \le \delta$)] and the free stream velocity outside the boundary layer is (V_0). Determine the constant (α) and (β), and the shear stress at the surface of the plate, if the fluid coefficient of viscosity is(μ).



Sol.:

u = $\alpha y+\beta y^3$

at y = δ u=V_0 ⇒ $V_0 = \alpha\delta + \beta\delta^3$………(1)

at y = δ $\frac{\partial u}{\partial y} = 0 \Rightarrow 0 = \alpha + 3\beta\delta^2$ ………(2)

(1) and (2) gives;

$\boldsymbol{\alpha = \frac{3}{2}\frac{V_0}{\delta}}$ $\boldsymbol{\beta = -\frac{1}{2}\frac{V_0}{\delta^3}}$

u = $\frac{3}{2}\frac{V_0}{\delta}y - \frac{V_0}{2\delta^3}y^3$

$\tau_w = \mu\frac{du}{dy}\Big|_{y=0} \Rightarrow \boldsymbol{\tau_w = \frac{3}{2}\frac{\mu V_0}{\delta}}$

(2) Estimate the skin frictional drag on a flat plate (10m) long and (2m) wide with a velocity of (130 km/h) traveling through air at (90 kPa abs.) and (25°C).Take (μ =2× 10^{-5}Pa.s) and (R_{air}=287 J/kgK).

Sol.:

$\rho = \frac{P}{RT} = \frac{90000}{287\times298} \Rightarrow \rho = 1.05 \text{ kg/m}^3$

$\mathrm{Re_L} = \frac{VL\rho}{\mu} = \frac{130\times\frac{1000}{3600}\times10\times1.05}{2\times10^{-5}} \Rightarrow \mathrm{Re_L} = 0.2\times10^8 > 10^6 \Rightarrow$ Turbulent b.l.

$\mathrm{C_D} = \frac{0.072}{\mathrm{Re_L}^{1/5}} = \frac{0.072}{(0.2\times10^8)^{1/5}} \Rightarrow \mathrm{C_D} = 2.5\times10^{-3}$

Drag = $\mathrm{C_D} \times \frac{1}{2}\rho V^2 A = 2.5\times10^{-3}\times\frac{1}{2}\times1.05\times\left(130\times\frac{1000}{3600}\right)^2\times10\times2$

⇒ **Drag = 34.24 N**

(3) How many (30m) diameter parachutes (C_D=1.2) should be used to drop a bulldozer weighing (45kN) at a terminal speed of (10m/s) through still air at (100 kPa) and (20°C). Take (Rair=287 $\frac{J}{kgK}$)

Sol.:

$\rho_{air} = \frac{P}{RT} = \frac{100000}{287\times293} \Rightarrow \rho_{air} = 1.19$ kg/m^3

$\sum Fy = 0$ (a_y=0)

$F_D+F_B=W$

$n\left[C_D \times \frac{1}{2}\rho_{air}V^2\frac{\pi}{4}D^2 + \rho_{air}g \times \frac{\pi}{6}D^3\right]$=W

$n\left[1.2 \times \frac{1}{2} \times 1.19 \times 10^2\frac{\pi}{4}(30)^2 + 1.19 \times 9.8 \times \frac{\pi}{6}(30)^3\right] = 45$

Thus; **n = 0.9 ⇒ n = 1**

Pro.(4):A spherical balloon (6.3m) in diameter contains helium and ascends through air at (100 kPa) and (5°c). Balloon and payload weigh (1332.5 N). What is the terminal velocity of ascension? Take (C_D=0.21) and (R_{helium}= 2077 J/kgK, Rair=287 J/kgK)

Sol.:

$\rho = \frac{p}{RT} \Rightarrow \rho_{air} = 1.253\frac{kg}{m^3}\&\rho_{helium} = 0.1732$ kg/m^3

$\sum Fy = may = 0$ (ay=0)

$F_B = F_D + W_{helium} + W_{payload}$

$\rho_{air}g\frac{\pi}{6}D^3 = C_D \times \frac{1}{2}\rho_{air}V^2\frac{\pi}{4}D^2 + \rho_{helium}g \times \frac{\pi}{6}D^3 + 1332.5 \Rightarrow$ **V =3m/s**

(5) Using the velocity distribution $\frac{u}{U} = sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)$, determine the equation for growth of the laminar boundary layer and for the drag coefficient along a smooth flat plate in two –dimensional flow.

Sol.:

$\tau_w = \rho U^2\frac{\partial\delta}{\partial x}\int_0^1\frac{u}{U}(1-\frac{u}{U})d\eta$ ……..(5.22)

1. $\frac{u}{U} = sin(\frac{\pi}{2}\eta)$ ……(1) where $\eta = \frac{y}{\delta}$

2. (1) into (5.22) $\Rightarrow \tau_w = \rho U^2 \frac{\partial\delta}{\partial x}\int_0^1 sin(\frac{\pi}{2}\eta)\left[1-sin(\frac{\pi}{2}\eta)\right]d\eta = \rho U^2 \frac{\partial\delta}{\partial x}\{\frac{2}{\pi}(1-\frac{\pi}{4})\}$ ………(2)

3. $\tau_w = \mu\frac{\partial u}{\partial y}|_{y=0} = \mu\frac{\partial u}{\partial\eta}\frac{\partial\eta}{\partial y}|_{\eta=0} = \mu\frac{1}{\delta}\frac{\pi}{2}cos(\frac{\pi}{2}\eta)|_{\eta=0} \Rightarrow \tau_w = \mu\frac{U\pi}{2\delta}$ …..(3)

4. equate (2) &(3)$\Rightarrow \rho U^2\frac{\partial\delta}{\partial x}\{\frac{2}{\pi}(1-\frac{4}{\pi})\}=\mu\frac{U\pi}{2\delta} \Rightarrow \delta d\delta = 11.5\frac{\mu}{\rho U}dx$ ...(4)

5. Integrate (4)$\Rightarrow \frac{\delta^2}{2} = 11.5\frac{\mu}{\rho U}x + \text{const.}$

At x=0 $\delta = 0 \Rightarrow$const.=0 $\Rightarrow \delta = 4.8\sqrt{\frac{\mu x}{\rho U}} = 4.8X\sqrt{\frac{\mu}{\rho UX}}$

Thus;

$\boldsymbol{\frac{\delta}{x} = \frac{4.8}{\sqrt{Re_x}}}$ ….(5)

6- Substitute (5) into (3) $\Rightarrow \tau_w = \frac{\mu U\pi}{2}\frac{\sqrt{Re_x}}{4.8X}$

Thus;

$\tau_w = 0.327\sqrt{\frac{\mu\rho U^3}{x}}$ ……(6)

7-Skinfriction drag= $\int_0^L \tau_w\,dx = \int_0^L 0.327\sqrt{\frac{\mu\rho U^3}{x}}dx$

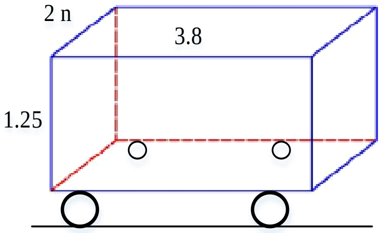
Thus;

Drag = $0.654\sqrt{\mu\rho U^3 L}$ …..(7)

8- Calculate

$C_D = \frac{D_{res}}{\frac{1}{2}\rho U^2 L*1} \Rightarrow C_D = \frac{0.654\sqrt{\mu\rho U^3 L}}{\frac{1}{2}\rho U^2 L*1} = \frac{0.654U\sqrt{\mu\rho UL}}{\frac{1}{2}\rho U^2 L} \Rightarrow$ $\boldsymbol{C_D = \frac{1.308}{\sqrt{Re_L}}}$ ….(8)

(6) Estimate the drag force on a car (3.8m long*2m wide* 1.25 m high) moving at (100 km/h) in still air, if the wheel rolling frictional force of the car is (136N).Estimate the engine power. For frontal area (C_D=0.4), $C_{Dl} = \frac{1.328}{\sqrt{Re_L}}$, $C_{D_t} = \frac{0.072}{{Re_L}^{1/5}}$, $\nu_{air} = 10^{-5}$m²/s, $\rho_{air} = 1.2$ kg/m³ .



Sol.:

$$Re_L=\frac{UL\rho}{\mu}=\frac{UL}{\nu}=\frac{100\times\frac{1000}{3600}\times 3.8}{10^{-5}}\Rightarrow Re_L=1.06\times 10^{7}$$

$Re_L > 1\times 10^{6}$ → turbulent; $C_D=\frac{0.072}{(1.06\times 10^{7})^{0.2}}=2.833\times 10^{-3}$

$$F_{D_f}=C_D\times\frac{1}{2}\rho U^2[1.25\times 3.8\times 2+3.8\times 2]\Rightarrow F_{D_f}=22.43\text{N}$$

$$F_{D_P}=C_D\times\frac{1}{2}\rho U^2\times 2\times 1.25=0.4\times\frac{1}{2}\rho U^2\times 2\times 1.25\Rightarrow F_{D_P}=462.96\text{N}$$

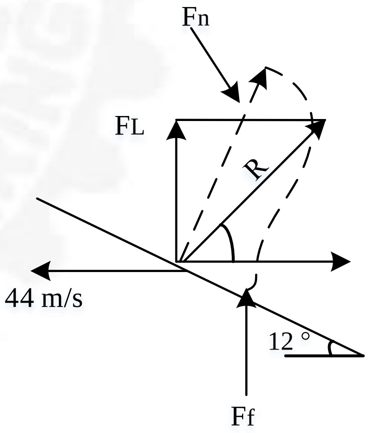
$$F_{D_{total}}=22.43+462.96+136\Rightarrow F_{D_{total}}=621.4\text{ N}$$

Power = $F_{D_{total}}\times U\Rightarrow$ **Power = 17.26 kW**

(7) A (3m× 4m) plate moves at (44 m/s) in still air at an angle of (12°) with the horizontal.Using a coefficient of drad (C_D=0.17) and a coefficient of lift (C_L=0.68), determine :

- **a) The resultant force exerted by the air on the plate.**
- **b) The frictional force.**
- **c) The horsepower required to keep the plate moving. Use ($\gamma = 11.4$ N/m^3)**

Sol.:



a) $F_D=C_D\times\frac{1}{2}\rho v^2A$

$$=0.17\times\frac{1}{2}\times\frac{11.4}{9.8}44^2\times 3\times 4\Rightarrow F_D=2297.12\text{N}$$

$F_L=C_L\times\frac{1}{2}\rho v^2A$

$$=0.68\times\frac{1}{2}\times\frac{11.4}{9.8}44^2\times 3\times 4\Rightarrow F_L=9213.2\text{N}$$

$$R=\sqrt{{F_D}^2+{F_L}^2}\Rightarrow \mathbf{R=9495.3\text{ N}}$$

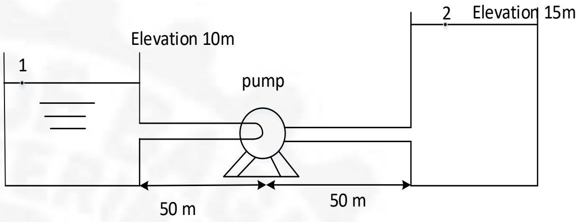
$$\theta_x=tan^{-1}\frac{F_L}{F_D}\Rightarrow \boldsymbol{\theta_x=89.97^\circ}$$

b) $F_f=R\cos(\theta_x+12)\Rightarrow$ **F_f = 331.382N**

c) Power = $F_D\times V$=2297.12× 44 ⇒ **Power = 101073.28 W = 135.5 hp**

Sheet No. 12
Pipe Problems

(1) The flow of (0.1 m^3/s) of water is to be maintained in the system shown in the figure. Find the power that must be added to the water by the pump. The pipe diameter is (15 cm) and the friction factor (f=0.02). The entrance and exit loss coefficients for the two reservoirs are (0.1) and (1) respectively. The water density is (1000 kg/m^3).



Sol.:

$$V=\frac{Q}{A}=\frac{0.1}{\frac{\pi}{4}(0.15)^2}$$

$$\therefore V = 5.66 \text{ m/s}$$

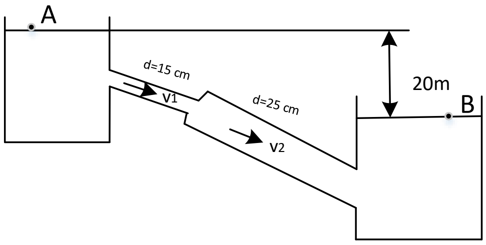
E.E. 1-2:

$$\cancelto{0}{\frac{p_1}{\gamma}}+\cancelto{0}{\frac{V_1^2}{2g}}+z_1+hp=\cancelto{0}{\frac{p_1}{\gamma}}+\cancelto{0}{\frac{V_2^2}{2g}}+z_2+\cancelto{0}{h_T}+h_\ell$$

$$0+0+10+\text{hp} = 0+0+15+0+\frac{5.66}{2g}\left[0.1+0.02\frac{100}{0.15}+1\right] \Rightarrow \text{h}_\text{p} = 28$$

$$\text{Power} = \gamma Q\text{h}_\text{p} = \rho g Q \text{h}_\text{p} = 1000\times 9.8\times 0.1\times 28.6 \Rightarrow \textbf{Power = 28047.6 W}$$

(2) Two large reservoirs are connected by a pipeline which is (15cm) diameter for the first (10m) and(25cm) diameter for the remaining (30m).The entrance and exist loss coefficients are (0.5) and (1) respectively, and the change of section is sudden. Calculate the rate of flow if (f=0.02) for both pipes and water density is (1000 kg/m^3).



Sol.:

E.E. A-B:

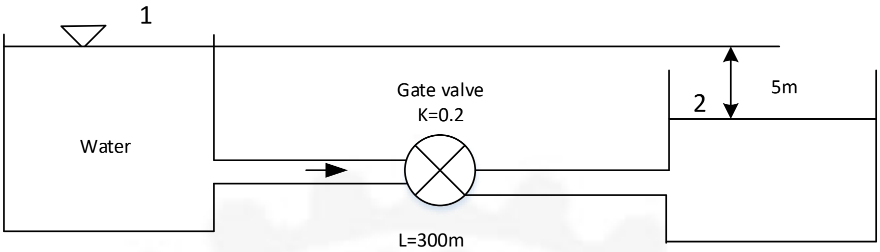
$$\frac{\text{p}_\text{A}}{\gamma}+\frac{\text{V}_\text{A}^2}{2\text{g}}+\text{z}_\text{A}+\text{hp}=\frac{\text{p}_\text{B}}{\gamma}+\frac{\text{V}_\text{B}^2}{2\text{g}}+\text{z}_\text{A}+h_T+h_\ell$$

$$0+0+20+0 = 0+0+0+\frac{Q^2}{\frac{\pi}{4}(0.15)^2 2g}\left[0.5+\left[1-\left(\frac{0.15}{0.25}\right)^2\right]+0.02\frac{10}{0.15}\right]$$

$$+\frac{Q^2}{\frac{\pi}{4}(0.25)^2 2g}\left[0.02\frac{30}{0.25}+1\right] \Rightarrow \boldsymbol{Q=0.2136\frac{m^3}{s}}$$

(3) Determine the size of the steel pipe ($\epsilon = 4.6 \times 10^{-5}$m) needed for a discharge of (2m^3/s) in the flow system shown in the figure. Use the following explicit formula for (f) and include all minor losses:

$$f=0.25\left[log\left(\frac{\epsilon}{3.7D}+\frac{5.74}{Re^{0.9}}\right)\right]^{-2}$$



Sol.:

E.E.1-2;

$$0+0+5+0 = 0+0+0+0+h_f+ki\frac{V^2}{2g}+k_v\frac{V^2}{2g}+k_e\frac{V^2}{2g}$$

$$5 = \frac{V^2}{2g}\left(f\frac{L}{D}+k_i+k_v+k_e\right)$$

$$5 = \frac{Q^2}{\left[\frac{\pi}{4}(D)^2\right]^2\times 2\times g}\left(f\frac{300}{0.79}+0.5+0.2+1\right)$$

Thus;

$$5 = \frac{0.33}{D^5}\left[300\frac{f}{D}+1.7\right] \;....(1)$$

Also;

$$f = 0.25\left[log\left(\frac{\epsilon}{3.7D}+\frac{5.74}{Re^{0.9}}\right)\right]^{-2} \;....(2)$$

$$Re = \frac{VD}{\upsilon} = \frac{4Q}{\pi D\upsilon} \;.....(3)$$

Assume $D_1 = 1$m

From 3→ $Re_1 = 2.55\times 10^6$

From 2→ $f_1 = 0.0116$

From1→ $D_2 = 0.79m \neq D_1$

Repeat the procedure, we get; **D = 0.79 m**

(4) Solve problem (3) above for unknown discharge (Q) with diameter given as (0.79m).

Slo.:

E.E.1-2;

$$5 = \frac{Q^2}{\left[\frac{\pi}{4}(0.79)^2\right]^2 \times 2 \times 9.8}\left(f\,\frac{300}{0.79} + 0.5 + 0.2 + 1\right)$$

Or; $5 = \frac{Q^2}{4.95}(375\mathrm{f} + 1.7)$ ….(1)

Also; f = $0.25\left[log(\frac{\epsilon}{3.7D} + \frac{5.74}{Re^{0.9}}\right]^{-2}$ ….(2)

Re = $\frac{4Q}{\pi D v}$ …….(3)

Assume $f_1 = 0.012$

From 1→ Q_1=1.998 m^3/s

From 3→ Re_1=3.18× 10^6 m^3/s

From 2→ $f_2 = 0.0117 \neq f_1$ repeat the procedure

<u>Alterrate solution;</u>

Assume Q_1=2m^3/s

From 1→ $f_1 = 0.012$

From 3→ $Re_1 = 3.22 \times 10^6$

From 2→ $f_2 = 0.012 = f_1 = \surd \Rightarrow$ Q = 2 m^3/s

<u>Alterrate solution;</u>

Assume Q_1 = 2m^3/s

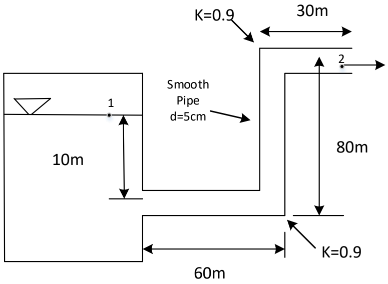
From 3→ $Re_1 = 3.22 \times 10^6$

From 2→ $f_1 = \surd$

From 1→ $Q_2 = \surd$

Compare Q_1 and Q_2

(5) What gage pressure (p) is needed to produce a flow rate of (50 m^3/h) of water ($\rho = 998$ kg/m^3,$\mu = 0.00102$ Pa.s) through the smooth pipe shown in the figure. Include all minor losses.



<u>Sol.:</u>

E.E.1-2;

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + hp = \frac{p_1}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_T + h_\ell$$

$$\frac{p}{\gamma}+0+10+0=0+\frac{V_2^2}{2g}+80+0+f\,\frac{170}{0.05}\frac{V^2}{2g}+0.5\frac{V^2}{2g}+1.8\frac{V^2}{2g} \quad \text{.....(1)}$$

$$V=\frac{Q}{A}=\frac{50/3600}{\frac{\pi}{4}0.05^2}\Rightarrow V=7.074\ \text{m/s}$$

$$R_e=\frac{Vd\rho}{\mu}=\frac{7.074\times0.05\times998}{0.00102}\Rightarrow R_e=3.46\times10^5$$

$$f=0.316/(3.46\times10^5)^{0.25}\Rightarrow f=0.01303$$

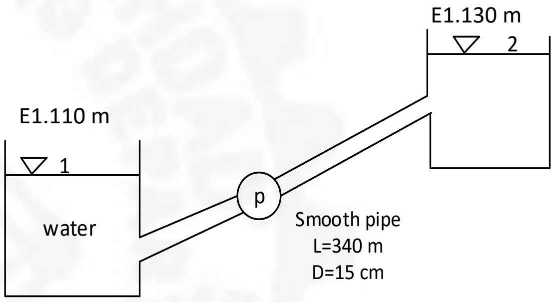
Thus from (1) we get; **p = 1.83 MPa**

(6) For the system shown in the figure, calculate;

a) The power required to pump (10ℓ/s) if the pump efficiency is (70%).

b) The operational cost of pumping per month in ID if (1kWh=65 fils)

Include minor losses and assume smooth pipe. Take ($\rho = 998\frac{kg}{m^3}$) and $\mu = 1.02\times10^{-3}$ Pa.s)



Sol.:

a) $$V=\frac{Q}{A}=\frac{10\times10^{-3}}{\frac{\pi}{4}0.15^2}\Rightarrow V=0.566\ \text{m/s}$$

$$\text{Re}=\frac{\text{v}D\rho}{\mu}=\frac{0.566\times0.15\times998}{1.02\times10^{-3}}\Rightarrow \text{Re}=83068.82$$

$$f=\frac{0.316}{(83068.82)^{0.25}}\Rightarrow f=0.0186$$

E.E.1-2;

$$0+0+110+hp=0+0+130+0+\frac{0.566^2}{2\times9.8}\left(1+0.0186\frac{340}{0.15}+1\right)\Rightarrow hp=2.072\text{m}$$

$$IP=\frac{OP}{\eta_p}=\frac{\gamma Qhp}{\eta_p}=\frac{998\times9.8\times0.01\times20.72}{0.7}\Rightarrow \textbf{IP = 2903.56 W}$$

b) No. of kWh = IP×time =2903.56× $\frac{1\text{kw}}{1000}\times(30\times24)h$ ⇒ No. of kWh =2090.56 kWh

Cost = 2090.56× 65 × $\frac{1\text{ID}}{1000}$ ⇒ **Cost=135.886 ID**

(7) For the system of problem (6) above, determine the cost of pumping (10^8m^3) of water to the upper reservoir.

Sol.:

The same as in problem (6), we find; IP=2903.56 W

$$\text{Time}=\frac{\forall}{Q}=\frac{10^8}{0.01}\Rightarrow \text{time}=1\times10^{10}s=2.77\times10^6\text{hr}$$

$$\text{Cost}=\left(2903.56\times\frac{1}{1000}\times2.77\times10^6\right)\text{kWh}\times\left(\frac{65}{1000}\right)\frac{\text{ID}}{\text{kWh}}\Rightarrow \textbf{Cost = 524243ID}$$

(8) Water is pumped at the rate of (10ℓ/s) through a (150 mm) diameter pipeline to a water tank at a height of (20 m) from the level of water in the sump. If the length of the pumping system is (340m) and the pump efficiency is (70%), calculate the operational cost of pumping per year in ID if the cost of (1kwh) is (350 fils).Take (f=0.025) and neglect minor losses.

<u>Sol.:</u>

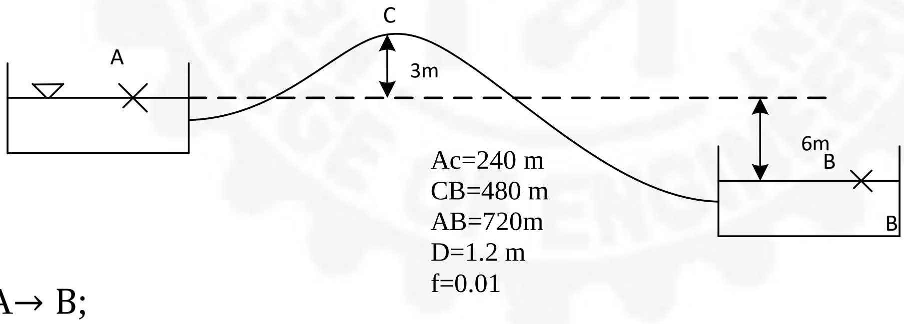
$$\text{E.E.} \Rightarrow \text{hp} = \text{H} + \text{hf} = 20 + 0.025\,\frac{340}{0.15}\,\frac{(0.01)^2}{\left[\frac{\pi}{4}(0.15)^2\right]^2 \times 2 \times 9.8} \Rightarrow \text{hp} = 20.926\text{m}$$

$$\text{IP} = \frac{\text{OP}}{\xi_\text{p}} = \frac{\gamma Q hp}{\xi_\text{p}} \Rightarrow \text{IP} = 2.93\text{kW}$$

$$\text{No. of kWh} = \text{Ip} \times \text{time} = 2.93 \times 365 \times 24 \Rightarrow \text{No. of kWh} = 25689.6$$

$$\text{Cost} = 25689.6 \times \frac{350}{1000} \Rightarrow \textbf{Cost = 8990 ID}$$

(9) A pipe line connecting two reservoirs having a difference of level of (6m) is (720 m) long, and rises to a hight of (3m) above the upper reservoir at a distance of (240 m) from the entrance before falling to the lower reservoir. If the pipe is (1.2m) in diameter and (f=0.04), what will be the discharge and the pressure at the highest point of the pipe line.



<u>Sol.:</u>

E.E.A→ B;

$$0{+}0{+}6{+}0{=}0{+}0{+}0{+}0{+}0.04\frac{720}{1.2}\frac{V^2}{2\times 9.8} \Rightarrow V = 2.22 \text{ m/s}$$

$$\text{Q} = \frac{\pi}{4}1.2^2 \times 2.22 \Rightarrow Q = 2.51\text{m}^3/\text{s}$$

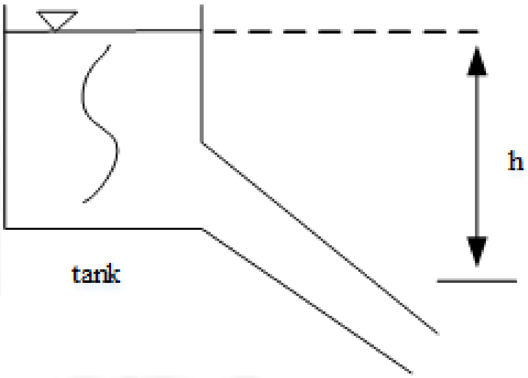
E.E.A→ C

$$0{+}0{+}0{+}0 = \frac{\text{pc}}{9810} + \frac{2.22^2}{2\times 9.8} + 3 + 0 + 0.04\,\frac{240}{1.2}\,\frac{2.22^2}{2\times 9.8} \Rightarrow \boldsymbol{p_c = -51.6\ \text{kPa}}$$

Sheet No. 13
Fluid and Flow Measurements

(1) A cylindrical tank of (1.5m) diameter is emptied through a (5cm) diameter smooth pipe as shown in the figure. The shock loss coefficient at entry to the pipe is (0.5), and all other losses are neglected. Find the time required for the water head above the pipe outlet to decrease from (5m) to (2m).

Sol.:

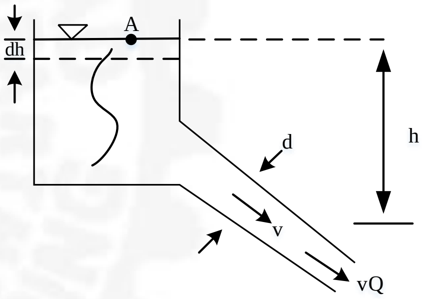


E.E.A to B;

$$\overset{0}{\cancel{\frac{p_A}{\gamma}}}+\overset{0}{\cancel{\frac{V_A{}^2}{2g}}}+Z_A=\overset{0}{\cancel{\frac{p_B}{\gamma}}}+\frac{V_B{}^2}{2g}+\overset{0}{\cancel{Z_B}}+h_\ell$$

$$0+0+h=0+\frac{V^2}{2g}+0+0.5\frac{V^2}{2g}$$

Thus; $V=\sqrt{\frac{2g}{1.5}}\sqrt{h}$



$$Q=\frac{\pi}{4}d^2V=\frac{\pi}{4}\,d^2\sqrt{\frac{2g}{1.5}}\sqrt{h}$$

Now; the water level falls (dh) in time (dt), hence;

Qdt = -Adh (A=C.S.A of tank= Const.)

Thus;

$$dt=-\frac{A}{Q}dh=-\frac{A\sqrt{1.5}}{\frac{\pi}{4}d^2\sqrt{2g}}\,h^{-1/2}\,dh$$

Integrating:

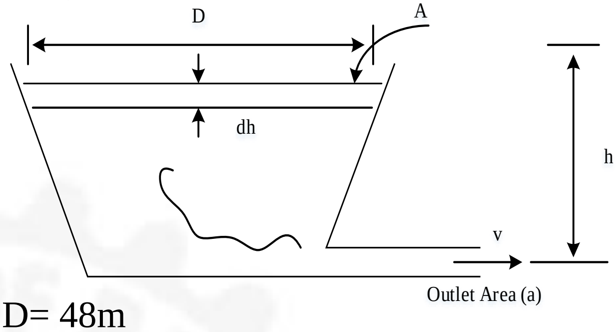
$$\int_{\circ}^{t}dt=-\frac{A\sqrt{1.5}}{\frac{\pi}{4}d^2\sqrt{29}}\int_{H_1}^{H_2}h^{-1/2}dh$$

Hence; $t=\frac{8A\sqrt{1.5}}{\pi\,d^2\sqrt{2g}}\,(\,H_1^{1/2}-H_2^{1/2})$

Putting; $A=\frac{\pi}{4}(1.5)^2=1.767m^2$, $d=0.05$, $H_1=5m$, $H_2=2m$

Thus ; $\mathbf{t=409.25\ s}$

(2) A reservoir is circular in plane and, when full, the diameter of the water surface is (60m). When the water level falls (1.2m) the diameter of the surface is (48m). Discharge takes place through a (0.6m) diameter outlet, (3m) below high water level, which can be treated as an orifice with a coefficient of discharge of (0.8). Determine the time required to lower the water level (1.2m) if the reservoir is full at the start.



Sol.:

$-Adh = Qdt \ldots 1$

Now;

$Q = C_d a\sqrt{2gh} \ldots\ldots .2$

When; h=3m , D=60m : and when h=1.8m , D= 48m

Thus; $D = a + bh \Longrightarrow a = 30 \; and \; b = 10$

$D = 30 + 10h \ldots\ldots\ldots 3$

Substitute 3&2 in 1;

$$dt = -\frac{A}{Q}dh = -\frac{\frac{\pi}{4}(30+10h)^2}{0.8 * \frac{\pi}{4}(0.6)^2\sqrt{2g}}\, h^{-\frac{1}{2}}\, dh$$

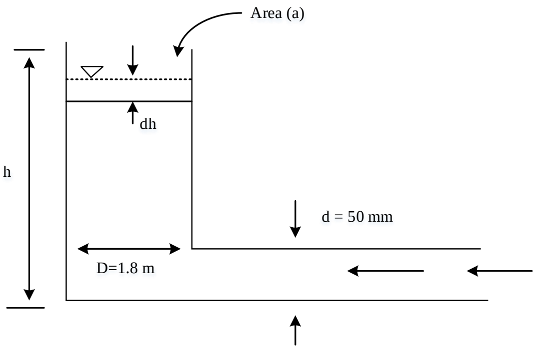
Hence; $dt = -(707h^{-\frac{1}{2}} + 471h^{\frac{1}{2}} + 78.7h^{\frac{3}{2}})dh$

Integrating from $H_1 = 3m \; to \; H_2 = 3 - 1.2 = 1.8m;$

Time for level to fall; $t = -\int_3^{1.8}(707h^{-\frac{1}{3}} + 471h^{1/2} + 78.7h^{3/2})dh$

$$t = -\left[1414\, h^{\frac{1}{2}} + 3/4h^{3/2} + 31.5\, h^{\frac{5}{2}}\right]_3^{1.8} \Rightarrow \mathbf{t = 1778\,s = 29\,min.38\,s}$$

(3) A circular take (1.8m) in diameter and open to the atmosphere at the top is supplied through a horizontal pipe (30m) long and (50mm) in diameter entering the base of the tank. A pump feeds the pipe and maintains a constant gage pressure of (45kPa) at the entry to the pipe. Find the time required to raise the level of water in the tank from (0.9m) to (1.8m) above the pipe inlet if (f=0.04).



Sol.:

E.E from pipe inlet to the free surface;

$$\frac{45000}{9810}+\frac{V^2}{2g}+0=0+0+h+f\,\frac{L}{D}\,\frac{V^2}{2g}$$

$$\frac{V^2}{2g}\left(1-f\,\frac{L}{D}\right)=H-4.58$$

$$\frac{Q^2}{\left(\frac{\pi}{4}d^2\right)^2 2g}\left(1-0.04\,\frac{30}{0.05}\right)=h-4.58$$

Or; $Q=[(6-1.3h)*10^{-3}]^{\frac{1}{2}}=[0.006-0.013h]^{1/2}$

Now; $Qdt=+Adh$

Thus; $dt=\frac{Adh}{(0.006-0.0013h)^{1/2}}$

$Let;\ 0.006-0.0013h=x \Longrightarrow dx=-0.0013dh$

$Thus;\ \ dt=\dfrac{-A\times^{-\frac{1}{2}}\ dx}{0.0013}$

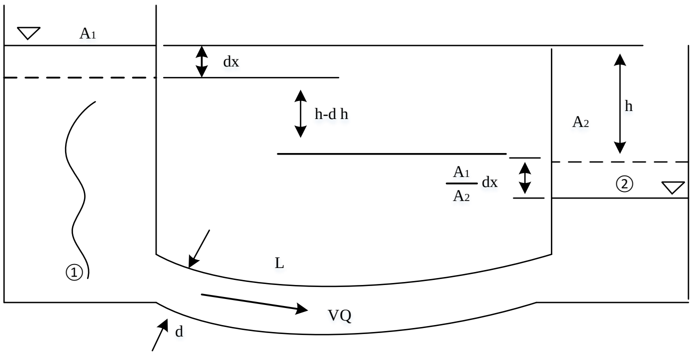
$Integrating,\ \ t=-\dfrac{2A}{0.0013}\,[\,x_2^{\frac{1}{2}}-x_1^{\frac{1}{2}}\,]$

$put;\ A=\dfrac{\pi}{4}1.8^2=2.54m^2{:}\,h_1=0.9m\ \Longrightarrow x_1=0.0048m$

$h_2=1.8m \Longrightarrow\ x_2=0.00366m$

$Thus{:}\ t=-\frac{2*2.54}{0.0013}\,[(\,0.0036)^{\frac{1}{2}}-(0.0048)^{\frac{1}{2}}]\rightarrow \mathbf{t=34.32\ s}$

(4) Two tanks of constant cross- sectional areas (A_1) *and* (A_2) respectively are connected by a pipe of diameter (d) and length (L) and has friction factor (f). Neglecting shock losses at inlet and outlet from the pipe, find an expression for the time taken for the difference of level in the two tanks to change from (H_1) to (H_2). If ($2.8m^3$) of water to pass from on tank to the other through a pipe (25mm) in diameter and (150m) long with (f= 0.04).



<u>Sol.:</u>

E.E between free surfaces: $h = h_f = f\ \frac{L}{d}\ \frac{V^2}{2g}$

Or; $V = \sqrt{\frac{2gd}{fL}}\ h^{1/2}$

If the level in tank (1) falls (dx) in time (dt), then rise in level in tank (2) will be $\frac{A_1}{A_2}\ dx$

Change in head producing the flow = $dh = dx + \frac{A_1}{A_2}\ dx = dx\left(1 + \frac{A_1}{A_2}\right)$

Also; $-A_1\ dx = \frac{\pi}{4}\ d^2 V\ dt$

Thus: $dt = -\ \frac{4A_1}{\pi d^2 V}\ dx$

Substituting for V and dx in terms of (h);

$$dt = -\ \frac{4A_1A_2}{\pi d^2(\ A_1 + A_2)}\sqrt{\frac{fL}{2gd}}\ h^{-1/2}\ dh$$

$Integrate\ frome\ H_1\ to\ H_2\ \Longrightarrow \mathbf{t} = \frac{\mathbf{8\ A_1A_2}}{\mathbf{\pi d^2(A_1{+}A_2)}}\sqrt{\frac{\mathbf{fL}}{\mathbf{2gd}}}\ [\ \mathbf{H_1^{1/2} - H_2^{1/2}}]$

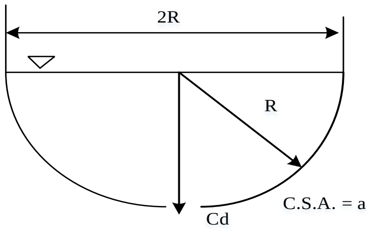
When $(2.8m^3)$ leave tank (1) $\Longrightarrow dx = \frac{2.8}{8.4} = 0.334\ m$

$Thus\ \ dh = dx\left(1 + \frac{A_1}{A_2}\right) = 0.334\left(1 + \frac{8.4}{4.8}\right) \Longrightarrow dh = 0.944$

$H_1 = 1.8\ m \Longrightarrow\ \ H_2 = H_1 - dh = 1.8 - 0.944 = H_2 = 0.856$

$Thus\ \ \ t = \frac{8{*}8.4{*}4.6}{\pi{*}0.025^2(8.4{+}4.6)}\sqrt{\frac{0.04{*}150}{29{*}0.025}}\ [(1.8)^{\frac{1}{2}} - (0.856)^{\frac{1}{2}}] \Longrightarrow \mathbf{t = 17750\ s}$

(5) Derive an expression for the time of emptying a hemi- spherical vessel through an orifice opening of the bottom, when it is filled up to the top with a liquid.



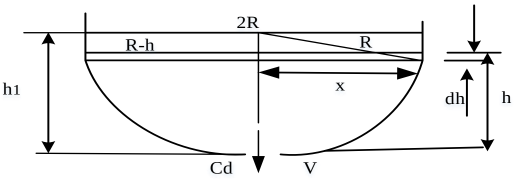
Sol.:

$B.E.:\ \longrightarrow V = \sqrt{2gh}$

$C.E.:\ \frac{\partial mc.v.}{\partial t} + \dot{m}_{out} - \dot{m}_{in} = 0$

$\dot{m_{out}} =\ \rho a\sqrt{2gh\ cd}$

$\frac{\partial m\ c.v.}{\partial t} = \frac{\rho\pi \times^2\ dh}{dt}$



$$but;\ \times^2 = R^{\ 2} - (R-h)^2 = 2Rh - h^2$$

$$\therefore\ \frac{\partial m\ c.v.}{\partial t} = \frac{\rho\pi(\ 2Rh - h^2)dh}{dt}$$

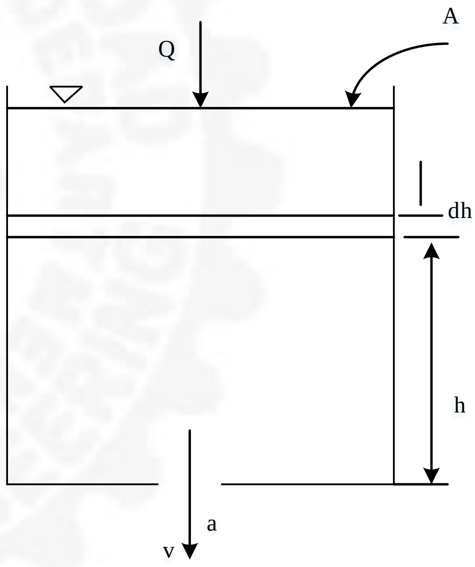
Thus; $\frac{\rho\,\pi(\,2Rh - h^2)dh}{dt} = \ \rho\, a\sqrt{2gh}\,cd$

And; $\int_o^t dt \ = \ -\int_{h_1}^{h_2} \frac{\pi(\,2Rh-h^2)dh}{c_d\, a\,\sqrt{2g}\,\sqrt{h}}$

Hence; $t = \frac{\pi}{c_d\ a\,\sqrt{2g}}[\,\frac{4}{3}R\left(\,h_1^{3/2} - h_2^{3/2}\right) - \frac{2}{5}\left(h_1^{5/2} - h_2^{5/2}\right)]$

$When\ \ h_1 = R\ and\ h_2 = 0\ \Longrightarrow$ $\mathbf{t = \frac{14\,\pi\,R^{\frac{5}{2}}}{15\,a\sqrt{2g}\ c_d}}$ **time for emptying the tank**

(6) Derive an expression for the time required to raise the water level in a tank of uniform C.S.A (A) from (h_1) to (h_2) when an orifice of C.S.A (a) at the bottom is discharged g to atmosphere while there is a constant inflow at a rate of (Q) in to the tank.



Sol.:

$$B.E. \rightarrow V = \ \sqrt{2gh}$$

$$C.E. \frac{\partial\ mc.v.}{\partial t} + \dot{m_{out}} - \dot{m_{in}} = 0$$

$$\frac{\rho A dh}{dt} + \ \rho a\,\sqrt{2gh} - \ \rho Q = 0$$

$$\frac{Adh}{dt} = Q - a\,\sqrt{2gh} = Q - K\,\sqrt{h}\ \ (\,Where\ a\,\sqrt{zg} = k)$$

Thus; $\int_0^t dt = \ \int_{h_1}^{h_2} \frac{Adh}{Q-k\,\sqrt{h}}\ \Longrightarrow t = \int_{h_1}^{h_2} \frac{Adh}{Q-k\sqrt{h}}$

$$let;\ Q - k\,\sqrt{h} = \ \ Z\ \Longrightarrow dZ = \ -\frac{kdh}{2\sqrt{h}} or\ dh = \frac{-2\sqrt{h}}{K} dz = \ -\ \frac{2(Q - Z)}{K^2} dZ$$

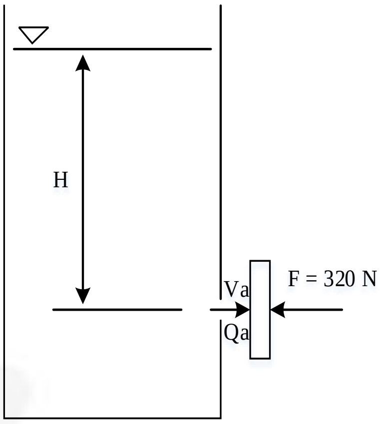
Hence; $t = -\ \frac{2A}{k^2}\int \frac{(Q-z)}{Z}\ dZ = \ -\ \frac{2A}{k^2}\ \ \int(\frac{Q}{Z} -\ \ 1)\ dZ$

Thus ; $t = -\ \frac{2A}{k^2}\ [Q\ell n(z) - z] = \ -\ \frac{2A}{k^2}[Q\ell n(Q - k\,\sqrt{h}) - (Q - k\sqrt{h})]_{h_1}^{h_2}$

Or; $\mathbf{t = \frac{2A}{k^2}[\,Q\ell\,n\,\left(\frac{Q-K\,\sqrt{h_1}}{Q-K\,\sqrt{h_2}}\right) + K\left(\sqrt{h_1} -\ \sqrt{h_2}\right)]\ \ k = a\,\sqrt{2g}}$

The liquid level in the tank will become steady when Q = a $\sqrt{2gh}$

(7) A (100mm) diameter orifice discharges (44.6 ℓ/s) of water under a head of (2.75 m). A flat plate held normal to the jet at the vena-contracta requires a force of (320 N) to resist impact of the jet. Find (C_D, C_v andC_c).



<u>Sol.:</u>

$[\, F = \dot{m}(V_{xout} - V_{xin}\,)$

$-F = \rho Q_a(O - V_a) \Rightarrow F = \rho Q_a V_a$

$320 = 1000 * 0.0446 * V_a \Rightarrow V_a = 7.175\ m/s$

B.E.: $V_t = \sqrt{2gH} = \sqrt{2 * 9.8 * 2.75} \Rightarrow V_t = 7.342\ m/s$

$C_V = \frac{Va}{Vt} \Rightarrow \mathbf{C_V = 0.977}$

$Q_t = A_t V_t = \frac{\pi}{4}\ 0.1^2 * 7.342 \Rightarrow Q_t = 0.0577 m^3/s$

$C_d = \frac{Q_a}{Q_t} \Rightarrow \mathbf{C_d = 0.773}$

$\mathrm{C_d = C_C\, C_V}$

$\therefore C_C = \frac{C_d}{C_v} \Rightarrow \mathbf{C_C = 0.791}$

(8) A (75mm) diameter orifice discharges (1.812m^3) of liquid (s = 1.07) in (82.2s) under a (2.75m) head. The velocity at the vena- contracta is determined by a Pitot - static tube with coefficient (1). The manometer liquid is acetylene tetrabromide (s= 2.96) and the gage difference is ($R' = 1.02m$). Determine (C_v, C_c and C_d).

<u>Sol.:</u>

$$V_a = C\sqrt{2gR'(\frac{so}{s} - 1)} = 1\sqrt{2 * 9.8 * 1.02(\frac{2.96}{1.07} - 1)} \Rightarrow Va = 5.94\ m/s$$

$B.E.;\ V_t = \sqrt{2gH} = \sqrt{2 * 9.8 * 2.75} \Rightarrow V_t = 7.342 m/s$

$C_v = \frac{Va}{V_t} \Rightarrow \mathbf{C_V = 0.809}$

$Q_a = \frac{\forall}{t} = \frac{1.812}{82.2} \Rightarrow Q_a = 0.02204\ m^3/s$

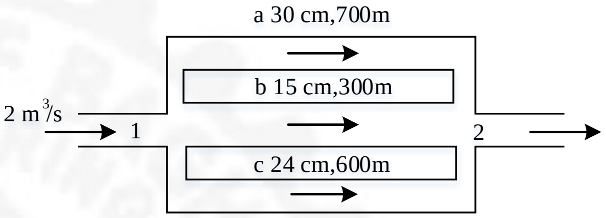
$Q_t = V_t A_\circ = 7.342 * \frac{\pi}{4}(0.075)^2 \Rightarrow Q_t = 0.0324 m^3/s$

$C_d = \frac{Q_a}{Q_t} \Rightarrow \mathbf{C_d = 0.679}$

$C_c = \frac{C_d}{C_v} \Rightarrow \mathbf{C_C = 0.839}$

Sheet No. 14
Pipes and Pumps Networks Analysis

(1) 30 cm, 15cm, and 24cm diameter steel pipes of 700m, 300m, and 600m in length respectively are connected in parallel as shown in the figure. The volume water flow rate through the system is ($2m^3$/s) and the corresponding friction factors are f_a = 0.016, f_b= 0.017, and f_c= 0.019 respectively. Because of the length of the pipes, neglect all other losses in the system. If the pressure at branch (1) is (4000kPa) gage, calculate the pressure in branch (2), and the water speed in each pipe. Water density is (1000kg/m^3)



<u>Sol.:</u>

$$h_{fa} = h_{fb} \Longrightarrow fa\ \frac{L_a}{D_a^5}\frac{8Q_a^2}{\pi^2 g} = f_b \frac{L_b}{D_b^5}\frac{8Q_b^2}{\pi^2 g} \quad \Longrightarrow Q_a = 3.817\ \mathrm{Q}_b \ldots (1)$$

$$h_{fc} \Longrightarrow\ h_{fb} \Longrightarrow f_c \frac{L_c}{D_c^5}\frac{8Q_c^2}{\pi^2 g} = f_b \frac{L_b}{D_b^5}\frac{8Q_b^2}{z^2 g} \Longrightarrow Q_c\ = 2.166\ \mathrm{Q}_b \ldots (2)$$

$$Q_a + Q_b + Q_c = 2$$

$$3.817\ Q_b + Q_b + 2.166\ Q_b = 2\ \Longrightarrow Q_b\ = 0.286 m^3/s$$

$$Q_a\ = 1.093 m^3/s$$

$$Q_c\ = 0.62 m^3/s$$

$$\mathrm{E.E.}\ 1-2: \frac{p_1}{\gamma} + \frac{Vb^2}{2g} + Z_1\ = \frac{p_2}{\gamma} + \frac{V_b^2}{2g} + Z_2 + h_{fb}$$

$$\frac{4000*10^3}{9810}\ = \frac{p_2}{9910} + 0.017\ \frac{300}{(0.15)^5}\ \frac{8(0.286)^2}{\pi^2 g} \Rightarrow \mathbf{p_2 = -457.37\ kPa}$$

$$V_a\ = \frac{Q_a}{\frac{\pi}{4}D_a^2} = \frac{1.093}{\frac{\pi}{4}(0.3)^2}\ \Longrightarrow \mathbf{V_a = 15.4\ 6m/s}$$

$$V_b\ = \frac{Q_b}{\frac{\pi}{4}D_b^2} = \frac{0.286}{\frac{\pi}{4}(0.15)^2}\ \Longrightarrow\ \mathbf{V_b = 16.18\ m/s}$$

$$V_c\ = \frac{Q_c}{\frac{\pi}{4}D_c^2} = \frac{0.62}{\frac{\pi}{4}(0.24)^2}\ \Longrightarrow \mathbf{V_c = 13.7\ m/s}$$

(2) Two reservoirs are connected by three pipes laid in parallel, their diameters are respectively d, 2d and 3d and they are all the same length L. Assuming f to be the same for all pipes, What will be the discharge though the larger pipe if that through the smallest is (0.03 m^3/s).

Sol.:

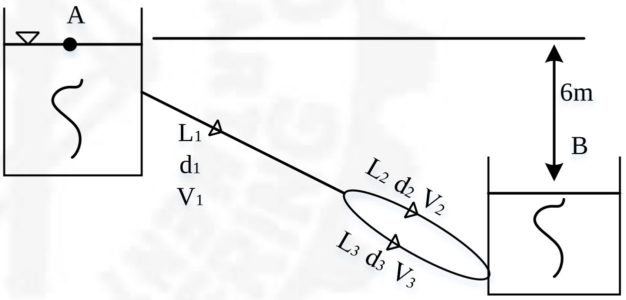
$$h_{f1} = h_{f2} = h_{f3}$$

$$\frac{fL}{d_1^5}\frac{8Q_1^2}{\pi^2 g} = \frac{fL}{d_2^5}\frac{8Q_2^2}{\pi^2 g} = \frac{fL}{d_3^5}\frac{8Q^2}{\pi^2 g}$$

With $d_1 = d,\ \ d_2 = 2d,\quad d_3 = 3d,\quad Q_1 = 0.03$

$$\frac{(0.03)^2}{d^5} = \frac{Q_2^2}{32\,d^5} = \frac{Q_3^2}{243\,d^5} \Longrightarrow \mathbf{Q_2 = 0.1697\ m^3/s;\quad Q_3 = 0.467 m^3/s}$$

(3) For the pipe line system show in the figure, (f= 0.04) for all pipes. Determine the total discharge.



Sol.:

$$h_{f2} = h_{f3}$$

$$f\,\frac{L_2}{d_2}\frac{V_2^2}{2g} = f\,\frac{L_3}{d_3}\frac{V_3^2}{2g} \Longrightarrow V_2 = V_3$$

C.E.: $Q_1 = Q_2 + Q_3$

$$\frac{\pi}{4}d_1^2 V_1 = \frac{\pi}{4}d_2^2 V_2 + \frac{\pi}{4}d_3^2 V_3$$

Thus: $V_2 = V_3 = 2V_1$

E.E. A-B:

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B + h_{f1} + h_{f2}$$

$$0 + 0 + 6 = 0 + 0 + 0 + f\,\frac{L_1}{d_1}\frac{V_1^2}{2g} + f\,\frac{L_2}{d_2}\frac{V_2^2}{2g}$$

$$6 = 0.04\,\frac{3000}{0.6}\,\frac{V_1^2}{2*9.8} + 0.04\,\frac{3000}{0.3}\,\frac{(2V_1)^2}{2*9.8} \Longrightarrow V_1 = 0.256\,m/s$$

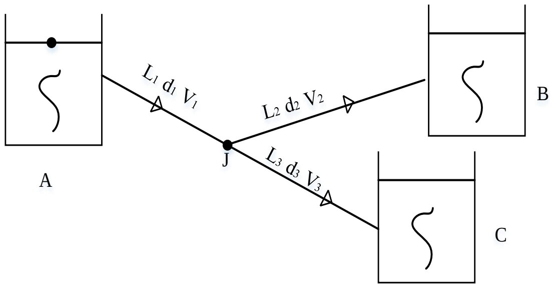
$$Q = \frac{\pi}{4}d_1^2\,V_1 \ \Longrightarrow\ \mathbf{Q = 0.0725\ m^3/s}$$

(4) For the branching pipe line system show in the figure, (f= 0.04) for all pipe. Determine the discharge entering reservoirs B&C.

$z_A = 60m\,,\ L_1 = 1500m\ ,\ d_1 = 300mm$

$z_B = 30m \quad L_2 = l_3 = 1500m$

$z_c = 15m \quad D_2 = d_3 = 300mm$



Sol.:

E.E.A to B:

$$60 = 30 + h_{f1} + h_{f2} = 30 + f\frac{L_1}{d_1}\frac{V_1^2}{2g} + f\frac{l_2}{d}\frac{V_2^2}{2g} \Rightarrow 10.2V_2^2 = 30 \ldots (1)$$

E.E.A to C:

$$60 = 15 + h_{f1} + h_{f3} \Rightarrow 10.2\,V_1^2 + 10.2\,V_3^2 = 45 \ldots (2)$$

C.E.: $\frac{\pi}{4}\,d_1^2 V_1 = \frac{\pi}{4}\,d_2^2 V_2 + \frac{\pi}{4}\,d_3^2 V_3 \Rightarrow V_1 = V_2 + V_3 \ldots (3)$

$$from\ (1);\ \ V_2 = \sqrt{2.94 - V_1^2}$$

$$from\ (2);\ \ V_3 = \sqrt{4.42 - V_1^2}$$

$$substitute\ in\ (3) \Rightarrow V_1 - \sqrt{2.94 - V_1^2} - \sqrt{4.42 - V_1^2} = 0 \ldots (4)$$

Solve (4) by trial of error or graphically. If the square roots are to be real, V_1 cannot excued the value given by $V_1^2 = .294\ or\ (V_1 = 1.71)$.

Call the L.H.S. of equ. (4) (R) and choose a value of V_1 less than (1.71) and calculate (R):

V_1	R
1.6	-0.36
1.7	+0.38

Thus the required value of V_1 lies about midway between (1.6) and (1.7). The solution is:

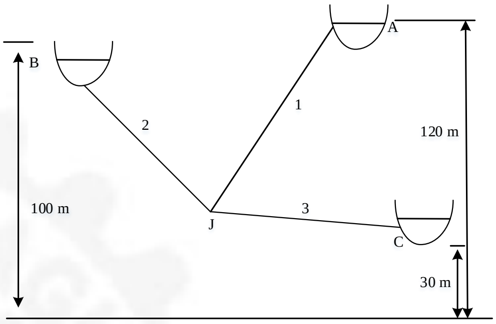
$V_1 = 1.663m/s \Rightarrow V_2 = 0.4\ m/s\ \ \&\ V_3 = 1.28\ m/s$

$Q_B = \frac{\pi}{4}d_2^2 V_2 \Rightarrow \mathbf{Q_B = 0.028m^3/s}$

$Q_c = \frac{\pi}{4}d_3^2 V_3 \Rightarrow \mathbf{Q_c = 0.0907m^3/s}$

(5) Consider the three interconnected reservoirs shown in the figure. Determine the discharge through the pipes.

Pipe	L(m)	D(m)	f
1	2000	1	0.013
2	2300	0.6	0.02
3	2500	1.2	0.023



Sol.:

Assume $(\frac{p}{\gamma}+z)_j = 80$

$$\text{E.E.}\, A \longrightarrow j \Longrightarrow 120 = 80 + f_1 \frac{L_1}{D_1^5}\frac{8Q_1^2}{\pi^2 g} \Longrightarrow Q_1 = 4.315 m^3/s$$

$$\text{E.E.}\, B \longrightarrow j \Longrightarrow 100 = 80 + f_2 \frac{L_2}{D_2^5}\frac{8Q_2^2}{\pi^2 g} \Longrightarrow Q_2 = 0.639 m^3/s$$

$$\text{E.E}\, j \longrightarrow C \Longrightarrow 80 = 30 + f_3 \frac{L_3}{D_3^5}\frac{8Q_3^2}{\pi^2 g} \Longrightarrow Q_3 = 5.117 m^3/s$$

$$Q_1 + Q_2 = 4.955 < Q_3 \Longrightarrow Q_{out} > Q_{in}\ Assume \left(\frac{f}{\gamma} + z\right)_j < 80$$

Assume $(\frac{p}{\gamma} + z)_j = 78m$

$$E.E.\ A \longrightarrow j \Longrightarrow 120 = 78 + f_1 \frac{L_1}{D_1^5}\frac{8Q_1^2}{\pi^2 g} \Longrightarrow Q_1 = 4.4216 m^3/s$$

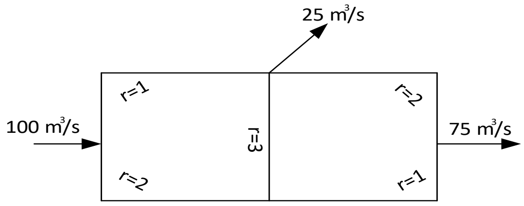
$$E.E.\ B \longrightarrow j \Longrightarrow 100 = 78 + f_2 \frac{L_2}{D_2^5}\frac{8Q_2^2}{\pi^2 g} \Longrightarrow Q_2 = 0.6709 m^3/s$$

$$E.E.\ j \longrightarrow C \Longrightarrow 78 = 30 + f_3 \frac{L_3}{D_3^5}\frac{8Q_3^2}{\pi^2 g} \Longrightarrow Q_3 = 5.0139\, m^3/s$$

$$Q_1 + Q_2 = 5.093 \frac{m^3}{s} \simeq Q_3(5.013)$$

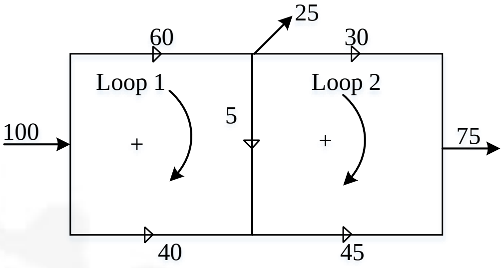
Thus; $\mathbf{Q_1 = 4.4216\ m^3/s}$ **,** $\mathbf{Q_2 = 0.6709\ m^3/s}$ **,** $\mathbf{Q_3 = 5.013\ m^3/s}$

(6) Using Hardy Cross method for pipe networks, calculate the discharge through each pipe of the network shown in the figure. Take (n=2)



Sol.:

First Trial



Trial (1)

Loope (1)

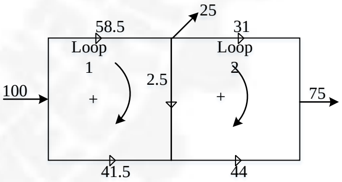
$rQ_\circ^n$	$nrQ_\circ^{n-1}$
$1\times 60^2 = 3600$	$2\times 1\times 60 = 120$
$3\times 5^2 = 75$	$2\times 3\times 5 = 30$
$2\times 40^2 = 3200$	$2\times 2\times 40 = 160$
$\sum rQ_\circ^n = 475$	$\sum \lvert nrQ_\circ^{n-1}\rvert = 310$

$$\Delta Q = -\frac{475}{310} \approx -1.5$$

Loope (2)

$rQ_\circ^n$	$nrQ_\circ^{n-1}$
$2\times 30^2 = 1800$	$2\times 2\times 30 = 120$
$3\times 5^2 = -75$	$2\times 3\times 5 = 30$
$1\times 45^{2=-2025}$	$2\times 1\times 45 = 90$
$\sum rQ_\circ^n = -300$	$\sum \lvert nrQ_\circ^{n-1}\rvert = 240$

$$\Delta Q = -\frac{-300}{240} \approx 1$$



Trial (2)

Trial (2)

Loope (1) $Q = Q_\circ + \Delta Q$

$rQ_\circ^n$ $\quad nrQ_\circ^{n-1}$

$1 * 58.5^2 = 3422.25 \longrightarrow 117$

$3 * 2.5^2 = 18.75 \longrightarrow 15$

$2 * 41.5^2 = -3444.5 \longrightarrow 166$

$\sum rQ_\circ^n = -3.5 \quad \sum \lvert nrQ_\circ^n\rvert = 298$

$\Delta Q = -\frac{-3.5}{298}$=0.012

Loope (2)

$rQ_\circ^n$ $\quad nrQ_\circ^{n-1}$

$2\times 31^2 = 1922 \longrightarrow 124$

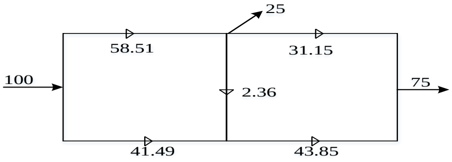
$3\times 2.5^2 = -19 \longrightarrow 15$

$1\times 44^2 = -1936 \longrightarrow 88$

$\sum rQ_\circ^n = -3.3 \quad \sum \lvert nrQ_\circ^n\rvert = 227$

$\Delta Q = -\frac{-33}{227}$=0.145

Thus; ΔQ≃ 0 for all loops and the final distribution will be →



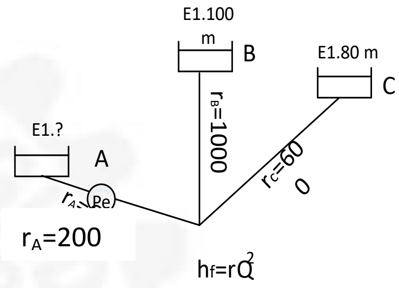
(7) Two identical pumps in series are used to pump water from reservoir (A) in the piping system show in the figure. The characteristics of each pump are (hp = 60-110Q^2) and $\eta_P = -200Q^2 + 225Q$) where (hp) is in (m) and (Q) in (m^3/s). The discharge to reservoir (B) is (0.1m^3/s)

(a) Find the characteristics of the equivalent pump.

(b) Find the elevation of reservoir (A) and the station input power.

(c) Estimate the operation cost of pumping (1000m^3) of water to reservoir (c), if the cost of (1kwh) is (65 fils). Neglect minor lessees.

<u>Sol.:</u>



(a) $\boldsymbol{Q_e = Q_1 = Q_2}$

$$\boldsymbol{h_{pe} = h_{p1} + h_{p2} = 2h_p \Rightarrow h_{pe} = 120 - 220\,Q^2}$$

$$\boldsymbol{\eta_{pe} = \eta_{p1} = \eta_{p2} = \eta_p \Rightarrow \eta_{pe} = -200Q^2 + 225Q}$$

$$(b)E.E.J \to B \Rightarrow (\frac{p}{\gamma} + z)_J = 100 + h_{fB} = 100 + r_B Q_B^2 = 100 + 1000(0.1)^2$$

$$\Rightarrow (\frac{p}{\gamma} + z)_J = 110m$$

$$E.E.:\quad J \longrightarrow c: (\frac{p}{\gamma} + z)_J = 80 + r_c\,Q_c^2 \Rightarrow Q_c = 0.2236\,m^3/s$$

$$C.E.:\quad Q_A = Q_B + Q_c \ \Rightarrow Q_A = 0.3236m^3/s$$

$$h_p = h_{pe} = 120 - 220Q_A^2 \Rightarrow \ \ hp = 96.96m$$

$$E.E.A \longrightarrow J \qquad z_A + h_p = \left(\frac{p}{\gamma} + z\right)_J + r_A Q_A^2 \ \Rightarrow \mathbf{Z_A = 33.97m}$$

$$\eta_p = \eta_{pe} = -220\,Q_A^2 + 225\,Q_A \Rightarrow \eta_p = 51.86\%$$

$$IP = \frac{OP}{\eta_p} = \frac{\gamma Q_A h_P}{\eta_p} \ \Rightarrow \mathbf{IP = 593.45\ kW}$$

$$\text{(C) } t = \frac{\forall}{Q_c} = \frac{1000}{0.2236} \Rightarrow t = 4472.3\ s = 1.24\ h$$

$$no.of\ kWh = IP * t = 593.45 * 1.24 \Rightarrow no.of\ kWh = 737.24$$

$$Cost = 737.24 * \frac{65}{1000} \Rightarrow \mathbf{cost = 47.92\ ID}$$

(8) A pump characteristics are given by $(\boldsymbol{h_p = 31 - 1200Q^2})$ *and* $(\boldsymbol{\eta_p = -190000\,Q^2 + 8590Q})$ where $(\boldsymbol{h_p})$is in (m) and (Q) in $\boldsymbol{m^3/s}$ and$(\boldsymbol{\eta_p})$ is the efficiency (%).

a) Find the characteristic of three identical pumps connected in parallel.

b) For the piping system shown in the figure, estimate the difference in operational cost between one pump operation and three pumps in parallel operation to supply $(\boldsymbol{10000m^3})$ of water to the upper reservoir if the cost of (1 kWh) is (65 fils). Neglect minor losses.

Sol.:

(a) $\boldsymbol{Q_e = 3Q_1 = 3Q_2 = 3Q_3 = 3Q}$

$\boldsymbol{h_{pe} = h_{p1} = h_{p2} = h_{p3} \Rightarrow h_{pe} = 31 - 12000(\frac{Q_e}{3})^2}$

$\boldsymbol{\eta_{pe} = \eta_{p1} = \eta_{p2} = \eta_{p3} \Rightarrow \eta_{pe} = -190000(\frac{Q_e}{3})^2 + 8590(\frac{Q_e}{3})}$

(b) E.E. $\longrightarrow hp = 15 + h_f = 15 + f\frac{l}{D^5}\frac{8Q^2}{\pi^2 g}$

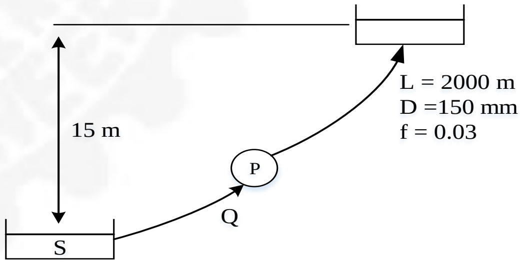
Or: $hp = 15 + 653352Q^2$ ... (1)

One pump: $31 - 12000Q^2 = 15 + 65352Q^2 \Longrightarrow Q = 0.01438m^3/s$

$hp = 31 - 12000(0.014388)^2 \Longrightarrow hp = 28.51m$

$\eta_p = -190000(0.01438)^2 + 8590(0.01438) \Longrightarrow \eta_p = 84.23\%$

$IP = \frac{\gamma Q hp}{\eta_p} \Longrightarrow IP = 4774.54W$



$t = \frac{\forall}{Q} = \frac{10000}{0.01438} \Longrightarrow t = 695410.3\ s$

$no.of\ kWh = IP * t \Longrightarrow no.of\ kWh = 922.3$

$cost. = 922.3 * 0.065 \Longrightarrow$ $\mathbf{cost = 59.95\ ID}$

Three pumps:

$Q_e = 3\ Q_1 = 3 * 0.01438 \Longrightarrow Q_e = 0.04314m^3/s$

$h_{pe} = h_{p1} = h_{p2} = h_p \Longrightarrow h_p = 28.51m$

$\eta p_e = \eta p_1 = \eta p_2 = \eta p = 84.23\%$

$t = \frac{\forall}{Q_e} \Longrightarrow t = 231803.43s$

$IP = \frac{\gamma Q_e h_{pe}}{\eta p_e} \Longrightarrow IP = 14324.5W$

$no.of\ kWh = IP * t \Longrightarrow no.of\ kmh = 922.3$

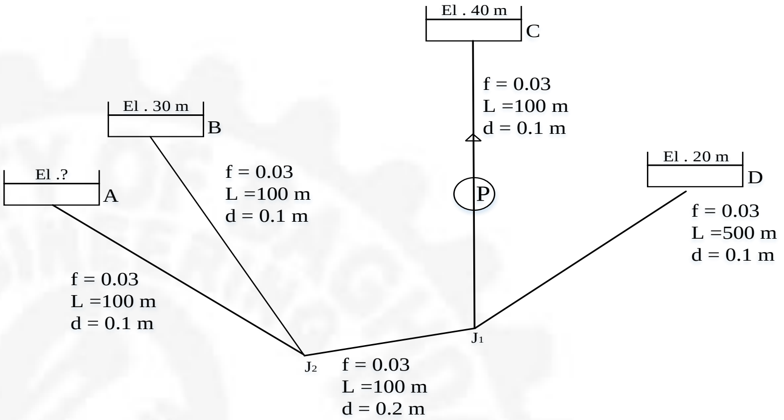
$cost = 922.3 * 0.065 \Longrightarrow$ $\mathbf{cost = 59.95\ ID}$ **i.e no difference**

(9) For the branching pipe system shown in the figure;

- **a. If (Q_C = 20 l/s), (h_p = 25 m), find the elevation of (A) and the discharges (Q_A, Q_B, Q_D).**
- **b. If two identical pumps (characteristics shown in table below) are placed in parallel in the line (c) and if $(Q_c = 20\ell/s)$, estimate the operational cost of pumping $(10^4\ m^3)$ of water if $(1kwh = 0.065\ ID)$.**

<u>Sol.:</u>

Q(ℓ/s)	$h_p(m)$	$\eta p\%$
0	28.5	0
10	28	42
20	25	75
30	18	78
34	12.5	65
35	5	28



(a) E. E. $J_1 - C$: $(\frac{P}{\gamma} + z)_{J_1} + h_p = z_c + h_{fc}$

$$(\frac{p}{\gamma} * z)_{J_1} = 40 + f_c \frac{L_c}{d_c^5} \frac{8Q_c^2}{\pi^2 g} \Longrightarrow (\frac{p}{\gamma} + z)_{J_1} = 24.92m$$

$E.E.J_1 \longrightarrow D \Longrightarrow 24.92 = 20 + f_D \frac{L_D}{d_D^5} \frac{8Q_D^2}{\pi^2 g} \Longrightarrow \mathbf{Q_D = 1.437\ell/s}$

$C.E.at\ J_1: Q_{J_1J_2} = Q_c + Q_d \Longrightarrow Q_{J_1J_2} = 21.437\ell/s$

$E.E.J_2 \rightarrow J_1: (\frac{P}{\gamma} + \mathrm{z})_{J_2} = (\frac{P}{\gamma} + \mathrm{z})_{J_1} + h_{f_{J_2J_1}} \Longrightarrow (\frac{P}{\gamma} + \mathrm{z})_{J_2} = 25.3m$

$E.E.\ B \longrightarrow J_2:\ 30 = 25.3 + f_B \frac{L_B}{d_B^5} \frac{8Q_B^2}{\pi^2 g} \Longrightarrow \mathbf{Q_B = 3.8\ \ell/s}$

$C.E.at\ J_2: \quad Q_A + Q_B = Q_{J_1J_2} \Longrightarrow \mathbf{Q_A = 7.64\ \ell/s}$

$E.E.A \longrightarrow j_2:\ Z_A = (\frac{p}{\gamma} + z)_{J_2} + h_{fA} \Longrightarrow \mathbf{z_A = 26.7m}$

(b) Equivalent pump characteristics:

$$h_{pe} = h_{p1} = h_{p2}$$
$$Q_e = 2Q_1 = 2Q_2 = 2Q \Rightarrow$$
$$\eta p_e = \eta p_1 = \eta p_2$$

Q(ℓ/s)	h_{PE}(m)	ηp_e%
0	28.5	0
20	28	42
40	25	75
60	18	78
68	12.5	65
70	5	28

Table

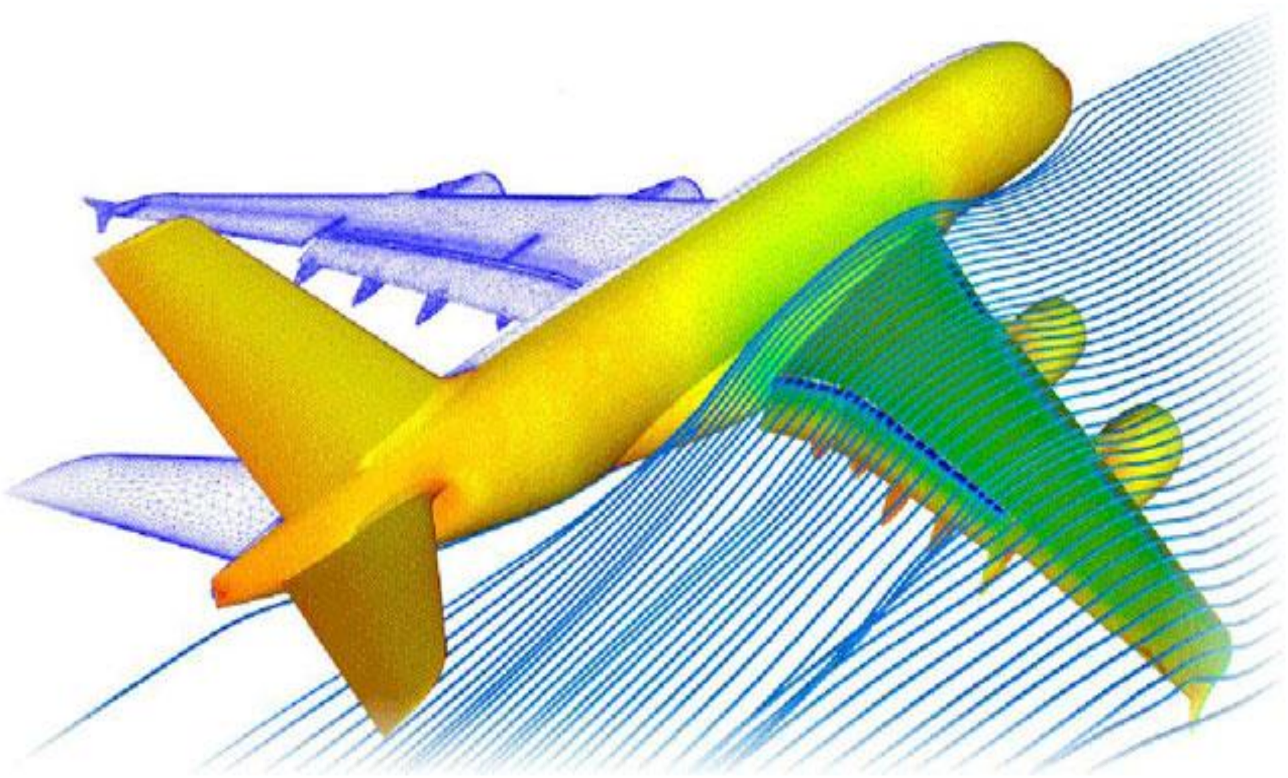
$$Thus; with\ Q_C = 20\ l/s \Longrightarrow hp = 28m \quad \& \quad \eta p_e = 42\%$$

$$\text{IP}=\frac{\gamma Q_e h_{pe}}{\eta p_e} \Longrightarrow \text{Ip}=13.08 \text{ kW}$$

$$t = \frac{\forall}{Q} = \frac{10^4}{0.02} \Longrightarrow t = 138.8h$$

$$no.of\ kWh = \text{Ip} * t \Longrightarrow no.of\ kWh = 1816.67$$

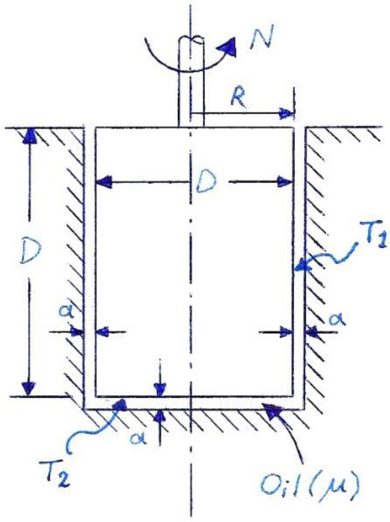
$$cost = 1816.67 * 0.065 \Longrightarrow \quad \mathbf{cost = 118.08\ ID}$$



Sheet No. 15
Miscellaneous Problems

(1) The weight of the cylinder shown in the figure is (W) and its radius of gyration is (k_G). The cylinder rotates at angular speed (N). Develop an expression for viscosity of oil (μ) required to stop the cylinder in (10) seconds.

Sol.:



$$T = T_1 + T_2$$

$$T_1 = F * R = \mu\frac{UA}{h} * R = \mu\frac{\frac{D}{2}N * \pi DD}{a} * \frac{D}{2}$$

$$\therefore T_1 = \frac{\pi\mu ND^4}{4a}$$

$$dT_2 = \mu\frac{rN * 2\pi rdr}{a} * r = \frac{2\pi\mu N}{a}r^3dr$$

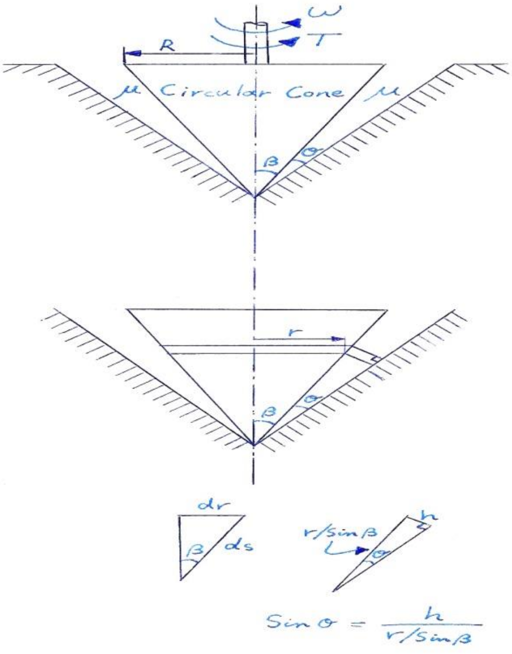
$$\therefore T_2 = \int_0^{D/2} dT_2 = \frac{\pi\mu ND^4}{32a}$$

$$\therefore T = \mu\left[\frac{\pi\mu ND^4}{4a} + \frac{\pi\mu ND^4}{32a}\right] = \mu\frac{9\pi ND^4}{32a}$$

$$\sum M = I\alpha = \frac{W}{g}K_G^2\,\frac{N}{t}$$

$$\mu\frac{a\pi ND^4}{32a} = \frac{W}{g}K_G^2\,\frac{N}{10} \quad\Longrightarrow\quad \mathbf{\mu = \frac{32}{90\pi}\,\frac{wK_G^2\,a}{gD^4}}$$

(2) Derive an expression for the torque (T) required to rotate the circular cone shown in the figure at a rate of (ω), in terms of the related variables (μ, ω, θ *and* R).



Sol.:

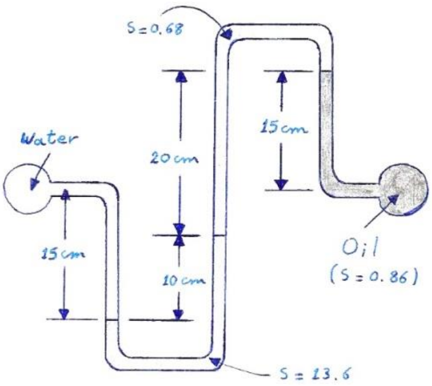
$$dF = \mu\frac{UdA}{h} = \frac{\mu{*}r\omega{*}2\pi rds}{\frac{r}{\sin\beta}sin\theta} = \frac{\mu r\omega 2\pi r\ dr/sin\beta}{\left(\frac{r}{sin\beta}\right)sin\theta}$$

$$dF = \frac{2\pi\mu\omega}{sin\ \theta}\ rdr$$

$$dT = dF * R = \frac{2\pi\mu\omega}{sin\ \theta}\ r^2dr$$

$$T = \int_{\circ}^{R} dT = \frac{2\pi\omega\mu R^3}{3sin\theta} \quad\Longrightarrow\quad \mathbf{T = \frac{2\pi}{3}\,\frac{\omega\mu R^3}{sin\theta}}$$

(3) Determine the pressure different between the water pipe and the oil pipe shown in the figure, in Pascals and in meters of water.



Sol.:

$$p_w + 0.15\gamma_w - 13.6\gamma_w * 0.1 - 0.68\gamma_w * 0.2 +$$

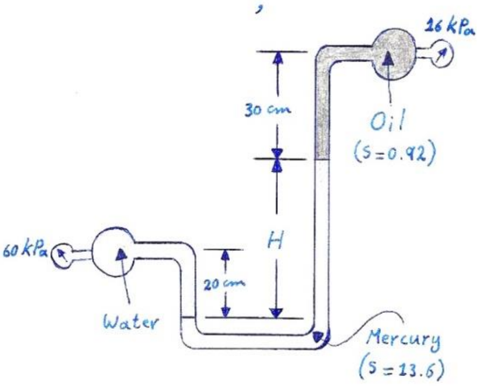
$$0.86\gamma_w * 0.15 = p_0$$

$$p_w - p_0 = \gamma_w(-0.15 + 13.6 + 0.136 - 0.129)$$

$$\therefore p_w - p_0 = 11938.77 \text{ Pa}$$

$$h_w - h_\circ = \frac{P_w - P_0}{\gamma_w} \Longrightarrow \therefore \mathbf{h_w - h_\circ = 1.217 \; m \; H_2O}$$

(4) For the system shown in the figure, calculate the manometer reading (H).



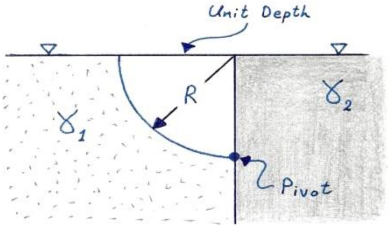
Sol.:

$$60000 + 0.2\gamma_w - 13.6\gamma_w * H - 0.92\gamma_w * 0.3 = 16000$$

$$H = \frac{60000 + 0.2\gamma_w - 0.276\gamma_w - 16000}{13.6\gamma_w}$$

$$\therefore \mathbf{H = 0.3242m = 32.42 \; cm}$$

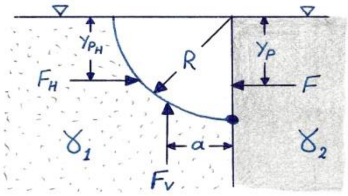
(5) If the weightless quarter- cylindrical gate shown in the figure is in equilibrium, what is the ratio between (γ_1) and (γ_2)?



Sol.:

$$F = \gamma \bar{h} A = \gamma_2 \frac{R}{2} R = \gamma_2 \frac{R^2}{2}$$

$$y_p = \bar{y} + \frac{I}{A\bar{y}} = \frac{2}{3} R$$



$$F_H = \gamma \bar{y}_v A_v = \gamma_1 \frac{R^2}{2}$$

$$y_{pH} = \bar{y}_v + \frac{I}{A_V \bar{y}_v} = \frac{2}{3} R$$

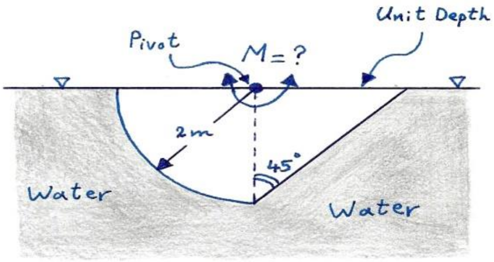
$$F_V = \gamma_1 \forall = \gamma_1 \frac{\pi}{4} R^2 \quad ; \quad a = \frac{4R}{3\pi}$$

$$\curvearrowleft + \sum M_{pivot} = 0: F * (R - y_p) = F_{H*}(R - y_{PH}) + F_V * a$$

$$\gamma_2 \frac{R^3}{6} = \gamma_1 \frac{R^3}{6} + \gamma_1 \frac{R^3}{3} \Longrightarrow \gamma_2 \frac{R^3}{6} = \gamma_1 \frac{R^3}{2}$$

$$\therefore \boldsymbol{\frac{\gamma_1}{\gamma_2} = \frac{1}{3}}$$

(6) For the two dimensional weightless solid body shown in the figure, determine the magnitude and direction of the moment (M) applied at the pivot required to hold the body in the position shown in the figure.



Sol.:

$$F_H = ɣ\,\bar{y_v}\,A_V = 9810 * 1 * 2 = 19620\,N$$

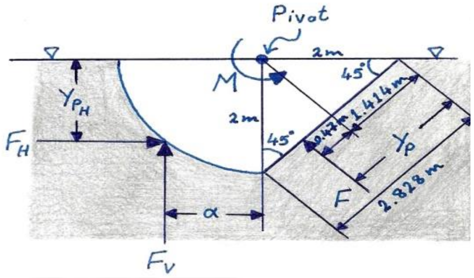
$$y_{pH} = \bar{y_v} + \frac{I}{A_V\ \bar{y_v}} = 1.332m$$

$$F_V = ɣ\forall = \ ɣ * \frac{\pi}{4} r^2 = 30819\,N$$

$$a = \frac{4r}{3\pi} = 0.849m$$

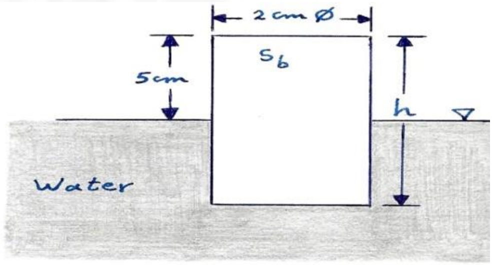
$$F = ɣ\bar{h}A = 9810 * 1 * 2.828 = 27746.9N$$

$$y_P = 1.414 + \frac{1 * (2.828)^3/12}{2.828 * 1.414} = 1.885m$$



$$\overset{+}{\circlearrowleft}\sum M_{Pivot} = 0\ :\ F * 0.47 + F_V * a - F_H * y_p - M = 0 \Longrightarrow\ \ \mathbf{M = 13078\,N.m} \circlearrowleft$$

(7) A (2cm) diameter cylinder of wood (s=0.5) floats in water with (5cm) above the water surface. Determine the depth of submergence of this cylinder when placed in glycerin (s=1.25).Will it float in stable, unstable or neutral equilibrium in this case?



Sol.:

In water: $W = F_B$

$$S_b\,ɣ_w\ Ah = ɣ_w A(h - 0.05) \Longrightarrow h = 10\ cm$$

In glycerin: $W = F_B$

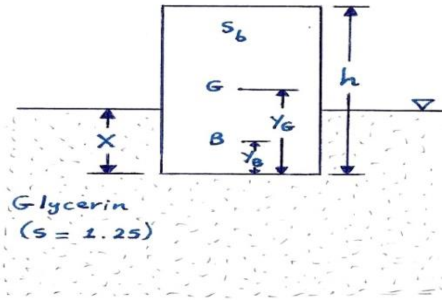
$$S_b\,ɣ_w\ Ah = S_g\,ɣ_w\,A \times \Longrightarrow \quad \mathbf{\times = 4cm}$$

$$y_G = 5cm\ , \qquad y_B = 2cm$$

$$BG = 5 - 2 = 3cm$$

$$I_{\circ\circ} = \frac{\pi}{64} D^4 = \frac{\pi}{4} cm^4$$

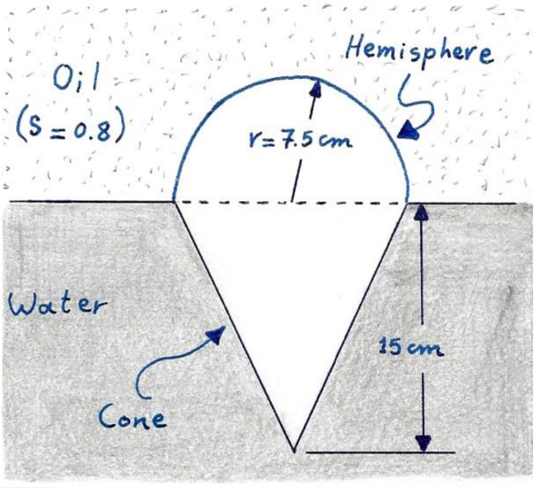
$$\forall = \frac{\pi}{4} D^2 * \times = 4\pi\ cm^3$$



$$MG = \frac{I_{\circ\circ}}{\forall} - BG = \frac{\frac{\pi}{4}}{4\pi} - 3 = -2.937$$

$\therefore$ **MG $< 0 \Longrightarrow$ Unstable**

(8) Calculate the weight and specific gravity of the object shown in the figure to float at the water- oil interface as shown.



Sol.:

$$W = F_{Bwater} + F_{Boil}$$

$$= \gamma_w * \frac{1}{3}\pi(0.075)^2 * 0.15 + 0.8\gamma_w * \frac{2}{3}\pi(0.075)^3$$

$$= \gamma_w[8.836 * 10^{-4} + 0.8 * 8.836 * 10^{-4}]$$

$$= 9810 * 0.00159 \Longrightarrow \mathbf{W = 15.6\ N}$$

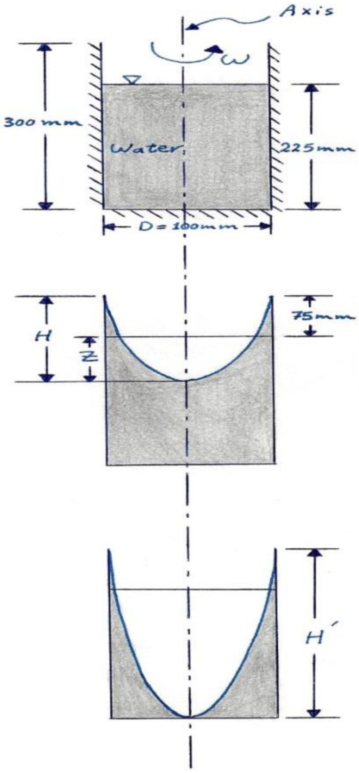
$$s = \frac{W/\forall}{\gamma_w} = \frac{15.6/(2*8.836*10^{-4})}{9810} = \frac{156000}{173362}$$

Thus; $\mathbf{s = 0.899}$

(9) The cylindrical vessel shown in the figure is rotated about its vertical longitudinal axis. Calculate:

- **a) The angular velocity at which water will start to spill over the sides.**
- **b) The angular velocity at which the water depth at the center is zero, and the volume of water lost for this case.**

Sol.:



a) $\frac{\pi}{4}D^2 z = \frac{1}{2}\frac{\pi}{4}D^2 H \Longrightarrow\ H = 2Z = 150mm$

$$H = \frac{\omega^2 r^2}{2g} \Longrightarrow 0.1 = \frac{\omega^2 * (\frac{D}{2})^2}{2g} \Longrightarrow \boldsymbol{\omega = 34.4\ rad/s}$$

b) $\acute{H} = \frac{\omega^2 * (\frac{D}{2})^2}{2g} \Longrightarrow 0.3 = \frac{\omega^2 * (\frac{0.1}{2})^2}{2*9.81} \Longrightarrow \boldsymbol{\omega = 48.5 rad/s}$

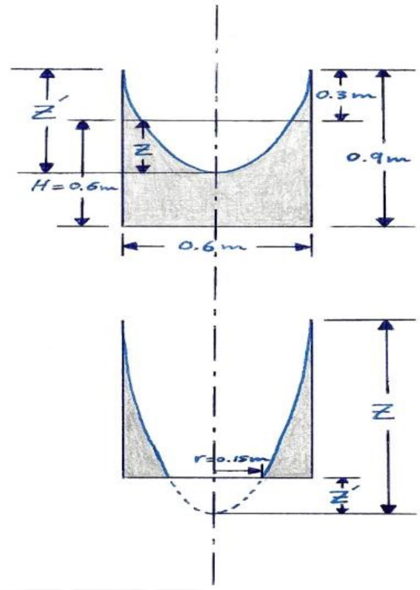
$$\acute{\forall} = \frac{1}{2}\frac{\pi}{4}D^2\acute{H} == 1.178 * 10^{-3} m^3$$

$$\forall_{lost} = \frac{\pi}{4}D^2 * 0.225 - \acute{\forall}$$

$$\mathbf{\forall_{lost} = 0.59 * 10^{-3} m^3}$$

(10) An open cylindrical tank (0.9m) high and (0.6m) in diameter is two- third filled with water when it is stationary. The tank is rotated about its vertical axis, calculate:

- **a) The maximum angular velocity at which no water is to spill over the sides.**
- **b) The angular velocity at which the bottom of the tank is free of water for a radius of (150mm).**



Sol.:

a) $H=\frac{2}{3}*0.9=0.6m$

$\frac{\pi}{4}D^2Z=\frac{1}{2}\,\frac{\pi}{4}D^2\acute{Z}\Longrightarrow\ \acute{Z}=0.6m$

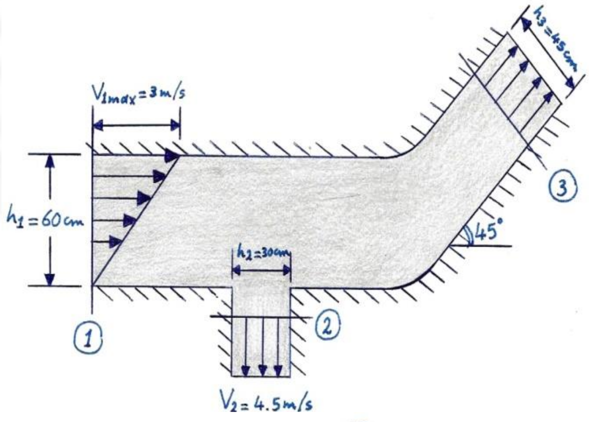
$Z=\frac{\omega^2r^2}{2g}\Longrightarrow 0.6=\frac{\omega^2*(0.3)^2}{2*9.81}\quad\Longrightarrow\ \boldsymbol{\omega=11.43\ \text{rad/s}}$

b) $Z-\acute{Z}=0.9$

$\frac{\omega^2*(0.3)^2}{2g}-\frac{\omega^2*(0.15)^2}{2g}=0.9\ \Longrightarrow\ \boldsymbol{\omega=16.16\ \text{rad/s}}$

(11) A two-dimensional reducing bend has a linear velocity profile at section (1). The flow is uniform at sections (2) and (3). The fluid is incompressible and the flow is steady. Find the magnitude of the uniform velocity at section (3).

Sol.:



$\frac{V_1}{y}\ \frac{3}{0.6}\Longrightarrow V_1=5y$

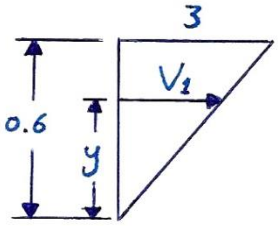
$C.E: Q_1=Q_2+Q_3$

$\int_{\circ}^{0.6}5y\,dy=4.5*0.3+0.45*V_3$

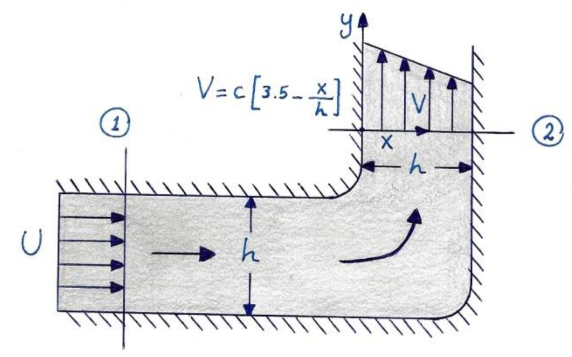
$[\frac{5y^2}{2}]_0^{0.6}=1.35+0.45V_3$

$\frac{5*0.36}{2}=1.35+0.45V_3$

$V_3=\frac{0.9-1.35}{0.45}\quad\Longrightarrow\ \boldsymbol{V_3=-1\text{m/s}}$



(12) Water enters a two- dimensional channel of constant width (h), with uniform velocity (V). The channel makes $a(90^\circ)$ bend that distorts the flow to produce the velocity profile shown at the exit. Evaluate the constant (c) in terms of (U).



<u>Sol.:</u>

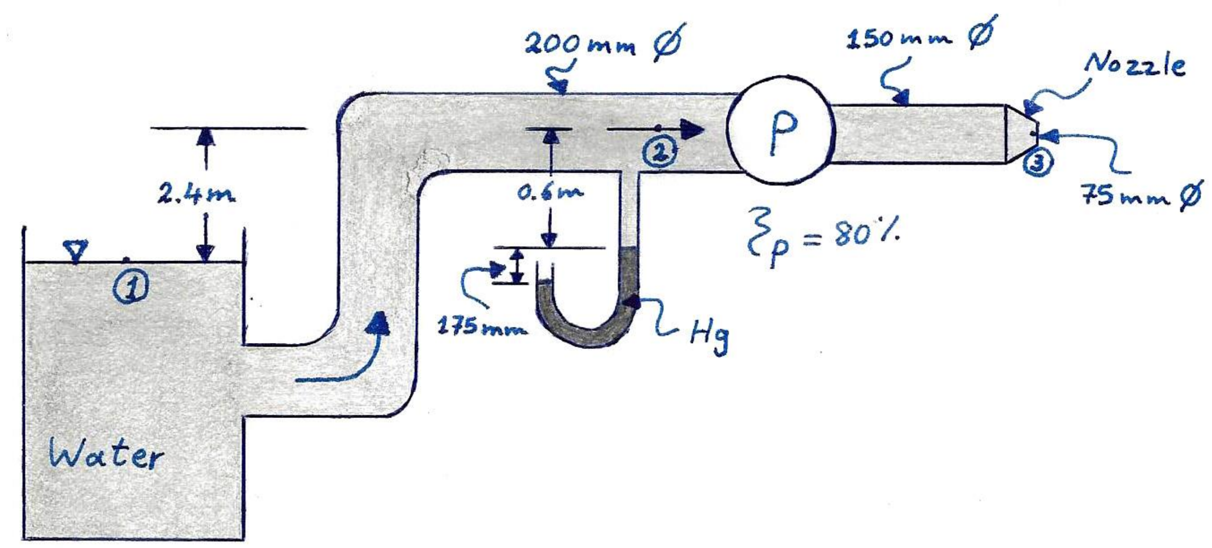
C.E. : $A_1V_1 = A_2V_2$

$Uh_1 = \int_0^h V dx = c\int_0^h (3.5 - \frac{x}{h})dx\,.1$

$Uh = c[3.5 \times - \frac{x^2}{2h}]_\circ^h = C[3.5h - \frac{h}{2}]$

Thus; $Uh = 3ch \Longrightarrow$ $\mathbf{c = \frac{U}{3} m/s}$

(13) Neglecting the losses, calculate the required pump shaft horsepower.



<u>Sol.:</u>

B.E.: 1-2: $0 + 0 + 0 = \frac{P_2}{\gamma w} + \frac{V_2^2}{2g} + 2.4$ ... (1)

Man. equ.: $p_2 + 0.6\gamma w + 0.175 * 13.6\gamma w = 0$

$\frac{p_2}{\gamma w} = -0.6 - 0.175 * 13.6 = -2.98$

$sub.in\ (1) \Longrightarrow V_2 = 3.372\, m/s$

$C.E: A_2V_2 = A_3V_3 \Longrightarrow\ V_3 = 23.98\, m/s$

E.E. 1-3: $0 + 0 + 0 + h_p = 0 + \frac{(23.98)^2}{2*9.81} + 2.4 + 0 \Longrightarrow\ h_p = 31.74m$

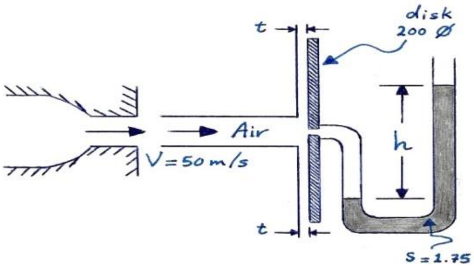
$\eta p = \frac{\gamma Q h_p}{IP} \Longrightarrow$ $\mathbf{IP = 36.636kW = 49.1\ hp}$

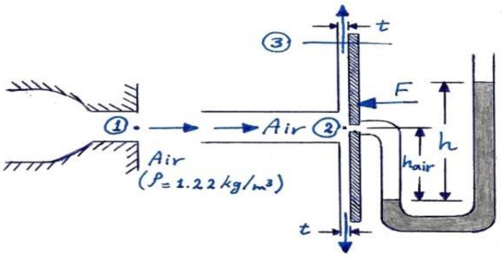
(14) A horizontal axisymmetric jet of air ($\rho = \frac{1.22kg}{m^3}$) with (10mm) diameter strikes a stationary vertical disk of (200mm) diameter. The jet is (50m/s) at the nozzle exit. A manometer is connected to the center of the disk. Neglecting the losses and the difference in potential head, calculate:

a) The deflection (h) of the manometer.

b) The force exerted by the jet on the disk.

c) The thickness (t) of the air jet at the exit.





<u>Sol.:</u>

B.E. 1-2: $\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$

Thus; $p_2 = \gamma_{air} \frac{V_1^2}{2g} = 1.22 * 9.81 * \frac{(50)^2}{2*9.81}$

$\Longrightarrow \;\; p_2 = 1525\,Pa$

$Man.equ.: p_2 + \gamma_{air}\;\; hair - s\gamma_w h = 0 \Longrightarrow$ $\mathbf{h = 8.88m = 0.0888cm}$

$\rightarrow \sum F_x = \dot{m}(V_{xout} - V_{xin})$

$-F = 1.22 * \frac{\pi}{4}(0.01)^2 * 50[0 - 50] \Longrightarrow \; F = 0.239N \;\leftarrow on\;fluid$

$\Longrightarrow$ $\mathbf{F = 0.239N \;\rightarrow}$ **on disk**

B.E.1-3 $\Longrightarrow \;\; V_1 = V_3$

$Q_{in} = Q_{out}$

$\frac{\pi}{4}d^2V_1 = \pi\,Dt\,V_3 \Longrightarrow$ $\mathbf{t = 0.115mm}$

(15) The wave resistance of a model of a ship at (1:25) scale is (7N), at a model speed of (1.5m/s). What are the corresponding velocity and wave resistance of the prototype? The model is tested in fresh water $\rho = 1000kg/m^3$) and the prototype operates in ocean (s=1.03).

<u>Sol.:</u>

Using Fr- Criteria:

$$F_{rm} = F_{rp} \;\; \Longrightarrow \frac{V_m}{\sqrt{g_m \ell_m}} = \frac{V_p}{\sqrt{g_p \ell_p}}$$

$V_P = V_m\sqrt{\frac{\ell_p}{\ell_m}} = 1.5 * \sqrt{25} \Longrightarrow$ $\mathbf{V_p = 7.5m/s}$

$$C_{pm} = C_{pp}$$

$$\frac{F_m}{\rho_m V_m^2 \ell_m^2} = \frac{F_p}{\rho_p V_P^2 \ell_P^2}$$

$$F_p = F_m \frac{\rho_p}{\rho_m} \frac{V_P^2}{V_m^2} \frac{\ell_P^2}{\ell_m^2} = 7 * 1.03 * \frac{(7.5)^2}{(1.5)^2} * (25)^2 \Longrightarrow \mathbf{F_p = 112656.25N}$$

(16) A (1/5) scale model automobile is tested in a wind tunnel with the same air properties as the prototype. The air velocity in the tunnel is (350km/h) and the measured model drag is (350 N). Determine the drag of the prototype automobile and the power required to overcome this drag. Assume complete dynamic similarity between the model and the prototype.

Sol.:

Using Re- criteria:

$$Re_m = Re_p \Longrightarrow (\frac{V\ell\rho}{\mu})_m = (\frac{V\ell\rho}{\mu})_p$$

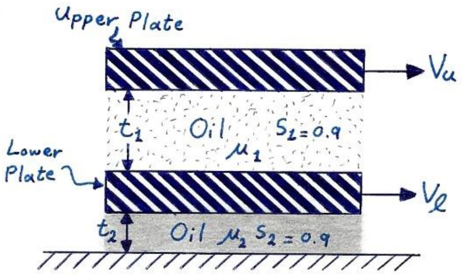
$$V_p = V_m \frac{\mu_p}{\mu_m} \frac{\ell_m}{\ell_p} \frac{\rho_m}{\rho_p} = 350 * \frac{1}{5} \Longrightarrow V_p = 70km/h$$

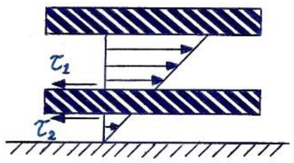
$$C_{p_m} = C_{p_p} \Longrightarrow \frac{F_m}{\rho_m V_m^2 \ell_m^2} = \frac{F_p}{\rho_p V_p^2 \ell_p^2}$$

$$F_p = F_m \frac{\rho_p}{\rho_m} \frac{V_P^2}{V_m^2} \frac{\ell_p^2}{\ell_m^2} = 350 * \frac{(70)^2}{(350)^2} * (5)^2 \Longrightarrow \mathbf{F_P = 350\ N}$$

$$Power = F_p V_p = 350 * 70 * \frac{1000}{3600} \Longrightarrow \mathbf{Power = 6805.6\ W}$$

(17) The upper plate shown in the figure is moving to the right with $V_a = 80m/s$ and the lower plate is free to move laterally under the action of the viscous forces applied to it. After steady- state conditions have been established, what velocity (Vℓ) will the lower plate have? Assume constant pressure for the two plates, and use $(t_1 = 2mm,\ t_2 = 1mm, \mu_1 = 0.1\text{Pa.s},\ \mu_2 = 0.04\text{Pa.s})$





Sol.:

Since p= constant and Q=0, then;

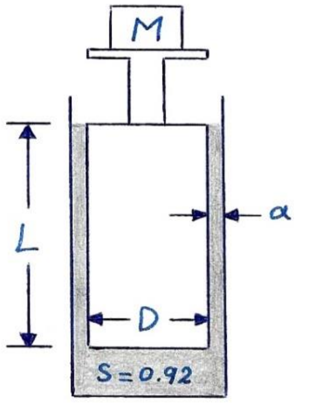
$$u = \frac{Uy}{a} \quad \& \quad \tau = \frac{\mu U}{a}$$

$$\tau_1 = \tau_2 \implies \mu_1 \frac{V_u - V_\ell}{t_1} = \mu_2 \frac{V_\ell}{t_2} \implies V_\ell \left[\frac{\mu_2}{t_2} + \frac{\mu_1}{t_1}\right] = V_u \frac{\mu_1}{t_1}$$

Thus; $V_\ell = V_u \left[\frac{\frac{\mu_1}{t_1}}{\frac{\mu_2}{t_2} + \frac{\mu_1}{t_1}}\right] = 0.8\left[\frac{\frac{0.1}{2*10^{-3}}}{\frac{0.04}{1*10^{-3}} + \frac{0.1}{2*10^{-3}}}\right] \implies \mathbf{V_\ell = 0.4444 m/s}$

(18) In the piston- cylinder apparatus for pressure gage tester shown in the figure, the piston is loaded to develop a pressure of know magnitude. Calculate the mass (M) required to produce (1.5 MP$a(gage)$) in the cylinder, when the piston moves with (0.02mm.s). Calculate also the leakage flow rate of oil for these conditions. Assume steady uniform flow and neglect the weight of the piston.

Sol.:



$D = 6mm, \mu = 0.5\ Pa.s$
$L = 25mm, a = 0.005mm$

$$\sum Fy = may = 0$$

$$p * \frac{\pi}{4} D^2 + F_V = Mg$$

$$p * \frac{\pi}{4} D^2 + \tau|_{Y=a} * \pi DL = Mg \ldots (1)$$

$$\tau|_{Y=a} = \frac{\mu U}{a} - \frac{(a-2*a)}{2}\left[\frac{dp}{dx} - \gamma\right]$$

$$= \frac{0.5 * 0.02 * 10^{-3}}{0.005 * 10^{-3}} + \frac{0.005 * 10^{-3}}{2}\left[\frac{1.5 * 10^6}{25 * 10^{-3}} - 0.92 * 9810\right]$$

$$\tau|_{y=a} = 152 Pa$$

$sub.in \ \ (1);$

$1.5 * 10^6 * \frac{\pi}{4} * (0.006)^2 + 152 * \pi * 0.006 * 0.025 = M * 9.81 \implies \mathbf{M = 4.335kg}$

$$Q = \left(\frac{Ua}{2} - \frac{a^3}{12\mu}\left[\frac{dp}{dx} - \gamma\right]\right) * \pi D$$

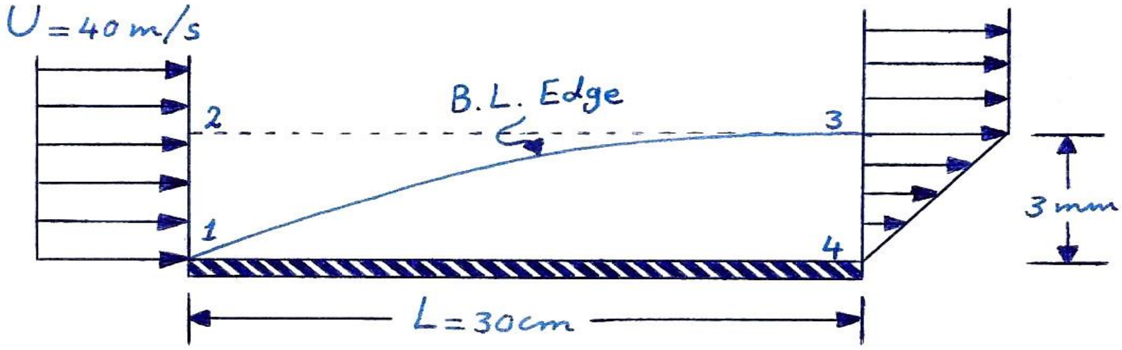
$$= \left(\frac{0.02 * 10^{-3} * 0.005 * 10^{-3}}{2} - \frac{(0.005 * 10^{-3})^3}{12 * 0.5}[59990974.8]\right) * \pi * 0.006$$

$\mathbf{Q = -2.26 * 10^{-11} m^3/s = 2.26 * 10^{-11} m^3/s \uparrow = 0.0226\ mm^3/s}$

(19) For the hypothetical boundary- layer on the flat plate shown in the figure, calculate:

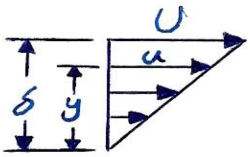
- **a) the skin – frictional drag force on the top side per meter of width.**
- **b) The drag coefficient of the top side.**
- **c) The wall shear stress on the plate at the trailing edge.**
- **d) The displacement thickness $(\boldsymbol{s_1})$ of the boundary- layer at the trailing edge.**

Given constants: $\boldsymbol{\rho = 1.2\frac{kg}{m^3}, \mu = 1.8 * 10^{-5} Pa.s}$



Sol.:

a) at section (3):



$$\frac{u}{U} = \frac{y}{\delta}$$

$$M.E.: \; -Fx = (\dot{m}u)_{34} + (\dot{m}u)_{23} - (\dot{m}u)_{12}$$

C.S.	$\dot{m}$	$(\dot{m}u)$
12	$\rho U\delta$	$\rho U^2\delta$
34	$\rho\int_0^\delta udy$	$\rho\int_0^\delta u^2dy$
23	$\rho U\delta - \rho\int_0^\delta udy$	$\rho U^2\delta - \rho\int_0^\delta udy$

$$\therefore -Fx = \rho\int_0^\delta u^2dy + \rho U^2\delta - \rho U\int_0^\delta udy - \rho U^2\delta$$

$$F_x = \rho u\int_0^\delta udy - \rho\int_0^\delta u^2dy = \rho U^2\frac{1}{\delta}\int_0^\delta ydy - \rho U^2\frac{1}{\delta^2}\int_0^\delta y^2dy$$

$$= 0.5\rho U^2\delta - \frac{1}{3}\rho U^2\delta$$

$$0.1667\rho U^2\delta = 0.1667 * (40)^2 * 1.2 * 0.003$$

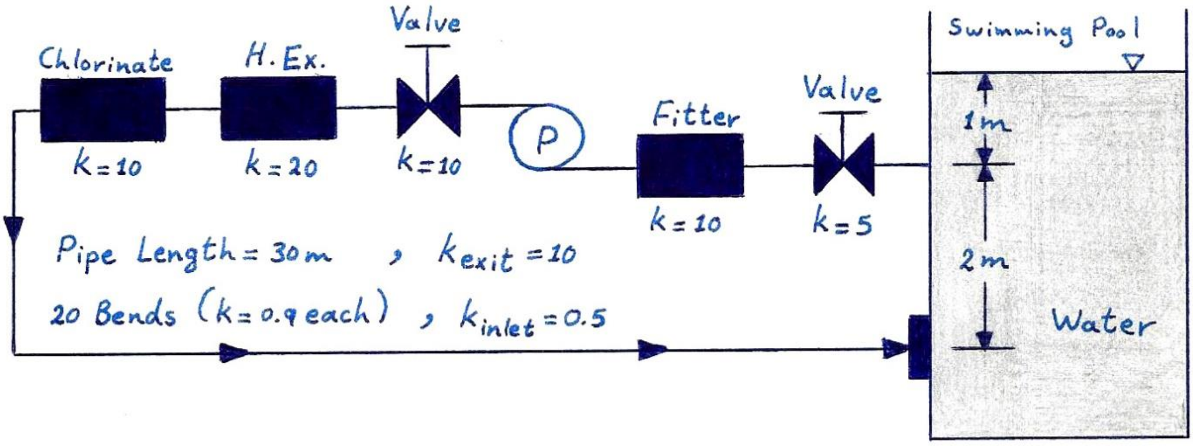
Thus; $\mathbf{F_X = 0.96N}$ **→ on plate**

b) $C_D = \frac{F_D}{\frac{1}{2}\rho U^2 A} = \frac{0.96}{0.5*1.2*(40)^2*0.3*1} \Longrightarrow$ $\mathbf{C_D = 0.0033}$

c) $\tau_w = \mu\frac{\partial u}{\partial y}|_{y=o} = \frac{\mu U}{\delta} = \frac{1.8*10^{-5}*40}{0.003} \Longrightarrow$ $\boldsymbol{\tau_w = 0.24\mathrm{Pa}}$

d) $\delta_1 = \int_\circ^\delta (1-\frac{u}{U})dy = \int_\circ^\delta (1-\frac{y}{\delta})dy = [y-\frac{y^2}{2\delta}]_0^\delta = \frac{\delta}{2} \Longrightarrow$ $\boldsymbol{\delta_1 = 1.5\mathrm{mm}}$

(20) Estimate the cost per month required to treat and circulate the water of the swimming pool shown in the figure. The circulation rate is $(0.1m^3/s)$ through (100mm) diameter smooth pipe. The pump efficiency is (80%) and (1kWh= 65fils). Include all losses and also take$(v = 10^{-10}\ m^2/s)$.



Sol.:

$$V = \frac{Q}{A} = \frac{0.1}{\frac{\pi}{4}(0.1)^2} \Longrightarrow V = 12.73m/s$$

$$Re = \frac{VD}{v} = \frac{12.73*0.1}{10^{-6}} \Longrightarrow Re = 12.73*10^5$$

$$f = \frac{0.316}{Re^{1/4}} = \frac{0.316}{(1273000)^{0.25}} \Longrightarrow f = 0.009407$$

Apply the E.E between two points on the free surface of the pool:

$$\therefore 0+0+0+hp = 0+5+5+5+h\ell$$

$$hp = h\ell = h_f + \sum k\frac{V^2}{2g} = [f\frac{L}{D} + \sum K]\frac{V^2}{2g}$$

$$= \left[0.009407 * \frac{30}{0.1} + 0.5 + 5 + 10 + 10 + 20 + 10 + 20*0.9 + 10\right] * \frac{(12.73)^2}{2*9.81}$$

$\therefore hp = 712.98m$

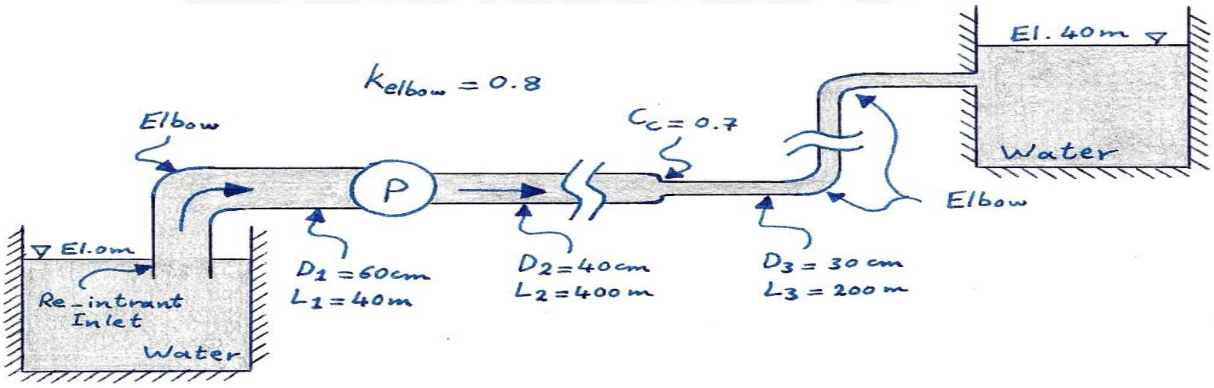
$IP = \frac{\gamma Qhp}{\eta p} = \frac{9810*0.1*712.98}{0.8} \Longrightarrow IP = 874291.73\,W \;= 874.29\,kW$

$No.of\ kWh = IP * t = 874.29 * 30 * 24 \Longrightarrow No.of\ kWh = 629488.8\,kWh$

$Cost = 629488 * 0.065 \Longrightarrow$ $\mathbf{Cost = 40916.8\ ID}$

(21) The pump of the piping system shown in the figure is used to supply $(1m^3/s)$ of water to the uphill station. Take (f= 0.014) and include all minor losses. Calculate:

- **a) The required pump shaft power knowing that its efficiency is (80%).**
- **b) The operational cost (in I.D.) of pumping ($10000m^3$) of water to the upper reservoir, knowing that: (1kWh= 300 files).**



Sol.:

a) $V = \frac{Q}{A} = \frac{4Q}{\pi D^2} \Longrightarrow V_1 = 3.536\frac{m}{s}, V_2 = 7.96, V_3 = 14.15\frac{m}{s}$

$hf = f\frac{L}{D}\frac{V^2}{2g} \Longrightarrow hf_1 = 0.1487m, hf_2 = 45.21m, hf_3 = 95.2m$

E.E.: $hp = 40 + h_{f1} + h_{f2} + h_{f3} + K_{inlet}\frac{V_1^2}{2g} + K_{exit}\frac{V_3^2}{2g} + 2k_{elbow}\frac{V_3^2}{2g} + k_{elbow}\frac{V_1^2}{2g} + (\frac{1}{Cc} - 1)^2\frac{V_3^2}{2g} = 40 + 0.1487 + 45.21 + 95.2 + 1 * \frac{(3.536)^2}{2*9.81} + 1 * \frac{(14.15)^2}{2*9.81} + 2 * 0.8 * \frac{(14.15)^2}{2*9.81} + 0.8 * \frac{(3.536)^2}{2*9.81} + (\frac{1}{0.7} - 1)^2 * \frac{(14.15)^2}{2*9.81}$

$\therefore hp = 210.11m$

$IP = \frac{0P}{\eta P} = \frac{\gamma Qhp}{\eta P} = \frac{9810*1*210.11}{0.8} \Longrightarrow$ $\mathbf{IP = Shaft\ Power = 2.6MW}$

b) $t = \frac{\forall}{Q} = \frac{10000}{1} \Longrightarrow t = 10000s = 2.78\,hr$

$No.of\ kWh = IP * t = 2600 * 2.78 \Longrightarrow No.of\ kWh = 7228kWh$

$Cost = 7228 * 0.3 \Longrightarrow$ $\mathbf{Cost = 2168.4\ ID}$