# Performance Evaluation and Comparison Between LDPC and Turbo Coded MC-CDMA

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## ABSTRACT

This work presents a comparison between the Convolutional Encoding CE, Parallel Turbo code and Low density Parity Check (LDPC) coding schemes with a MultiUser Single Output MUSO Multi-Carrier Code Division Multiple Access (MC-CDMA) system over multipath fading channels. The decoding technique used in the simulation was iterative decoding since it gives maximum efficiency at higher iterations. Modulation schemes used is Quadrature Amplitude Modulation QAM. An 8 pilot carrier were used to compensate channel effect with Least Square Estimation method. The channel model used is Long Term Evolution (LTE) channel with Technical Specification TS 25.101v2.10 and 5 MHz bandwidth bandwidth including the channels of indoor to outdoor/ pedestrian channel and Vehicular channel. The results showed that the performance of the proposed system was better when the LDPC was used as a coding technique.

# تقييم و مقارنة كفاءة LDPC and Turbo Coded MC-CDMA

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## الخلاصة

يقدم هذا العمل مقارنة ل Convolutional Encoding CE, Parallel Turbo code and Low density Parity Check (LDPC) coding schemes مع نظام (MC-CDMA) متعدد المستخدمين ذو الخرج المفرد خلال قناة الخفوت متعدد المسارات. تقنية فتح الشفرات المستخدمة في البحث هي فك الشفرات التكراري لأنه يعطي الكفاءة القصوى مع عشرة تكرارات . نوعيات التضمين المتبعة هي Quadrature Amplitude Modulation QAM. تم استخدام 8 pilot carriers لمعادلة تأثيرالقناة مع Least Square Estimation method.القناة المستخدمة في النموذج هي قناة الجيل الثالث ( LTE) ذات المواصفات التقنية TS 25.101v2.10مع عرض قناة مقداره 5MHz وتشمل القنوات داخل وخارج ألابنيه والقنوات المتحركة. بينت النتائج ان كفاءة النظام المقترح افضل مع تقنية تشفير الLDPC.

## KEY WORDS

MC-CDMA, PARALLEL TURBO CODE, LDPC, SUM-PRODUCT DECODING ALGORITHM, CONVOLUTIONAL CODING

## Introduction

The enormous growth of interest for multicarrier (MC) systems can be ascribed to its high bandwidth efficiency and its immunity to channel dispersion. Recently, different combinations of orthogonal frequency division multiplexing (OFDM) and code division multiple access (CDMA) have been investigated in the context of high data rate communication over dispersive channels [Rodriguez 2003, Mottier 2002, Phan2003]. One of these systems is multicarrier CDMA (MC-CDMA), which has been proposed for downlink communication in mobile radio. In MC-CDMA the data symbols are multiplied with a higher rate chip sequence and then modulated on orthogonal carriers.

The MC-CDMA proved to be a suitable technique for the downlink transmission. Uplink transmission, due to the more complex propagation conditions, introduces additional problems which result in harder applicability of MC-CDMA in uplink [Ivan 2002]. MC-CDMA is very suitable for the downlink. The time and frequency synchronism between the users in the downlink allows a simple realisation of efficient channel estimation at the mobile station required for coherent detection [Ivan 2002].

It was in 1993, an epoch of CDMA application, that three types of new multiple access schemes based on a combination of code division and OFDM techniques were proposed, such as "multicarrier (MC) CDMA, multicarrier DS-CDMA, and multitone (MT) CDMA. These schemes were developed by different researchers, namely. MC-CDMA by N. Yee. J. P. Linnartz and G. Fettweis For more details about MC-CDMA refer to [Husam 2010, Aqiel2011 and Shinsuke 1997].

Wireless mobile communication systems present several design challenges resulting from the mobility of users throughout the system and the time-varying channel (Multipath fading). There has been an increasing demand for efficient and reliable digital communication systems. To tackle these problems effectively, an efficient design of forward error coding (FEC) scheme is required for providing high coding gain. To obtain high coding gains with moderate decoding complexity, concatenation of codes with iterative decoding algorithms has proved to be an attractive scheme [Husam 2010]. From these codes are the TURBO CODE and the Low Density Parity Check LDPC codes.

In the next sections a brief review of both the Parallel Concatenated Convolutional (TURBO CODE) codes and Low-Density Parity-Check (LDPC) codes are given. The system proposed and the simulation results were introduced then. Finally, conclusions of the work were given.

## Parallel Concatenated Convolutional (TURBO CODE) Encoding

The convolutional turbo coder consists of a parallel concatenation of recursive systematic convolutional RSC encoders separated by a pseudo-random interleaver [Ramasmay 2006, Husam 2010]. The main aim of RSC is to produce more high weight codes even though input contains more number of zeros [Shanmugam 2005]. A natural rate for such a code is 1/3 (one systematic bit and two parity bits for one data bit). The rate can be relatively easily increased by puncturing the parity bits but reducing the rate below 1/3 is more difficult and may involve repetition of some bits [Ramasmay 2006]. The structure of such a Turbo coder is shown in Figure (3a).

One important feature of turbo codes is the iterative decoding which uses a soft-in/soft-out (SISO) like the Max-Log- Maximum A Posteriori (MLMAP) algorithm is a good compromise between performance and complexity [Vogt 1999]. It is very simple and, with the correction operation, also very effective [Robertson 1995]. Compared to the Maximum A Posteriori (MAP)/Log-MAP algorithm no SNR-information is necessary and the critical path within the add-compare-select (ACS) unit is shorter because of the maximum operation without the correction term [Robertson 1997].

Like other methods max-log-APP algorithm calculates approximate log-likelihood ratios LLR's for each input sample as an estimate of which possible information bit was transmitted at each sample time[Robertson 1995].They are calculated according to [Robertson 1995, Robertson 1997, Husam 2010]

$$L_i = \max_m [A_i^m + D_i^{0,m} + B_{i+1}^{f(0,m)}] - \max_m [A_i^m + D_i^{1,m} + B_{i+1}^{f(1,m)}] \quad [1]$$

where i is the sample time index, m $\in\{0, \ldots, N_s-1\}$ is the present state, $N_s$ is the number of encoder states, f(d, m) is the next state given present state m and input bit d $\in \{0,1\}$, $A_i^m$ is the forward state metric for state m at time i, $B_i^m$ is the reverse or backward state metric for state m at time i, and $D_i^{d,m}$ is the branch metric at time i given present state m and input bit d$\in\{0,1\}$. More formally, the state and branch metrics are given by [Robertson 1995, Robertson 1997, Husam 2010]

$$A_i^m = \max[A_{i-1}^{b(0,m)} + D_{i-1}^{0,b(0,m)}, A_{i-1}^{b(1,m)} + D_{i-1}^{1.b(1,m)}] \quad [2]$$

$$B_i^m = \max[D_i^{0,m} + B_{i+1}^{f(0,m)}, D_i^{1.m} + B_{i+1}^{f(1,m)}] \quad [3]$$

$$D_i^{d,m} = \frac{1}{2}(x_i d' + y_i c'^{d,m}) \quad [4]$$

where b(d,m) is the previous state given present state m and previous input bit d$\in\{0,1\}$, $x_i$ is the $i^{th}$ systematic sample, $y_i$ is the $i^{th}$ parity sample, d is a systematic bit, $c^{d,m}$ is the corresponding coded bit given state m and bit d, $d'^{d,m}$ =1-2d , and $c'^{d,m} = 1 - 2c^{d,m}$. The state metrics provide a measure of the probability that state m is the correct one at time i, while the branch metrics are a measure of the probability that each possible combination of encoder outputs is the correct one given the channel outputs $x_i$ and $y_i$.

The Max-Log-APP algorithm is sub-optimum due to the approximations involved. However, most of the performance loss associated with this sub optimality can be recovered by applying a simple scale factor correction to the output of the constituent decoder. The so-called extrinsic information may be approximated as [Robertson 1995, Robertson 1997, Husam 2010]

$$L_{ex}^n = sf \cdot (L_{out}^n - L_{in}^n) \quad [5]$$

where $n \in \{1,2\}$ denotes one of the constituent decoders, $L_{out}^n$ represents the set of LLRs produced by the max-log-MAP decoder, $L_{in}^n$ represents the set of input LLRs, and *sf* is an appropriate scale factor. The turbo concatenated decoder architecture is shown in Figure (3b).

## LDPC CODING

LDPC codes are linear block codes specified by a very sparse (containing mostly 0's and only a small number of 1's) random parity-check matrix, but are not systematic. The parity-check matrix of an LDPC is an $M \times N$ matrix **A** , where $M$ is the number of parity bits, and $N$ is the transmitted block length ($N = K + M$ , with $K$ as the source block length). The matrix **A** is specified by a fixed column weight $j$ and a fixed row weight $k = j\ N / M$ (in the MacKay's and Neal's codes $k$ is as uniform as possible (MacKay 1999, Aqiel 2011), and code rate $R = K / N$. LDPC codes can be decoded using probability propagation algorithm known as the

sum-product or belief propagation algorithm [Kschischang 2001], which is represented by a factor graph [Tanner Graph] that contains two types of nodes: the "bit nodes" corresponding to a column of the parity-check matrix, which also corresponds to a bit in codeword and the "check nodes" corresponding to a row of the parity-check matrix, which represents a parity-check equation.

## SUM-PRODUCT DECODING ALGORITHM

The decoding problem is to find the most probable vector x such that **A**x mod 2 = 0 , with the likelihood of x given by $x_n\Pi_n f_n$, where $f_n^0 = 1 - f_n^1$ and $f^1_n = 1/(1 + \exp(-2y_n / \sigma^2))$ for AWGN channel or $f^1_n = (y_n / \sigma^2) \exp[-y^2_n / 2\sigma^2]$ for Rayleigh channel, and $y_n$ , $\sigma^2$ represent the received bit and noise variance, respectively. We denote the set of bits, n , that participate in check m as $N(m) \equiv \{n : A_{mn} = 1\}$ , where $A_{mn}$ represents the element of the mth row and nth column in the parity-check matrix. Similarly, we define the set of checks m in which bit n participates as $M(n) \equiv \{m : A_{mn} = 1\}$ . We denote a set N (m) with bit n excluded as N (m) \ n. The algorithm has two alternating parts, in which quantities $q_{mn}$ and $r_{mn}$ associated with each non-zero element in the matrix A are iteratively update. The quantity x $q_{mn}$ is meant to be the probability that bit n of x is x, given the information obtained via checks other than check m. The quantity $r_{mn}$ is meant to be the probability of check m being satisfied if bit n of is *x* considered fixed at x and the other bits have a separable distribution given by the probabilities $\{q_{mn'} : n' ? N(m) \setminus n\}$ . The aposteriori probabilities for a bit are calculated by gathering all the extrinsic information from the check nodes that connect to it, which can be obtained by the following iterative sum-product procedure [Luis 2006, Aqiel 2011].

Step 1: Initialization The variables $q^0_{mn}$ and $q^1_{mn}$ , which are the probabilities sent from the nth bit node to the $_m th$ check node along a connecting edge of a factor graph, are initialized to the values $f^0_n$ and $f^1_n$ ,respectively.

Step 2: Horizontal Step (bit node to check node) We define $\Delta q_{mn} \equiv q^0_{mn} - q^1_{mn}$ and compute eq.(6) and eq. (7) for each m, n and x = 0,1:

$$q^0_{mn} = \prod_{n' \in N(m)\setminus n} q^0_{mn'} \qquad (6)$$

$$r^0_{mn} = \{ 1+ (-1)^0 \Delta_{mn}\}/2$$

$$r^0_{mn} = \{1+\Delta_{mn}\}/2 \qquad (7)$$

$$r^1_{mn} = \{ 1- \Delta_{mn} \}/2$$

Where, $r_{mn}$ represents the probability information sent from the $m_{th}$ check node to the nth bit node.

Step 3: Vertical Step (check node to bit node)

For each n , m and x = 0,1 we update eq.(8):

$$q^0_{mn} = \propto_{mn} f^0_n \prod_{m' \in M(n)\setminus m} r^0_{m'n}$$

$$q^0_{mn} = \alpha_{mn} f^o_n \prod_{m' \in M(n)\setminus m} r^0_{m'n} \qquad (8)$$

Where, $\alpha_{mn}$ is a normalization factor chosen such that $q^0_{mn}$ $q^0_{mn} + q^1_{mn} = 1$. We can also update the aposteriori probabilities $q^0_n$ $q^0_n$ and $q^1_n$ $q^0_n$, given by eq. (9):

$$q^0_{mn} = \alpha_{mn} f^o_n \prod_{m' \in M(n)\setminus m} r^0_{mn} \qquad (9)$$

Where, $\alpha_n$ is a normalization factor chosen such that $q_n^o + q_n^1 = 1$.

Step 4: Check stop criterion soft decision is made on the $q_n^1$ $q_n^1$ . The resulting decoded vector xˆ is checked against the parity-check matrix A. If Axˆ = 0,the decoder stops and outputs ˆx. Otherwise, it repeats the procedure from the Step 2. The sum-product algorithm sets a maximum number of iterations: if the number of iterations reaches that maximum, the decoder stops and outputs ˆx as the results of the decoding.

## The Proposed System and Results

The proposed system is a MUSO MC-CDMA system. A complete block diagram of the proposed system is shown in Figure (4). The simulation was done using MATLAB R2010a package. A 20 Mbps was transmitted using the system. The channel is ITU LTE Vehicular channel. First the incoming data encoded using the Convolution code CE or TURBO CODE code or LDPC code as illustrated in the mentioned figure. The main parameters of the system are listed in table (1).

The parallel Turbo coder with both the upper and lower coder of a generator polynomial of [1 0 1 1; 1 1 0 1] ) polynomial generators and a constraint length of (4). With the Max-Log-MAP decoding algorithm which is an iterative decoding algorithm. The random interleaver length was 1024 in both cases

The LDPC specifications used are irregular [16384] parity check matrix of rate ½ .The decoding algorithm is Sum-Product Decoding Algorithm, which is the soft decision type of message passing.

The performance of both Turbo code and LDPC code systems depend upon the number of iteration of the decoder. Since variations of the multipath fading channel affect the performance of the system, knowledge of the channel is crucial for accurate signal demodulation. Pilot-symbol aided-modulation (PSAM) is one of the well known techniques to estimate the channel state at pilot symbol positions. The method of estimation was the least square LS method.

**Table 1 Simulation parameters for the indoor to outdoor/pedestrian environment**

| | |
|---|---|
| No. of active users | 4 |
| Total Number of users | 48 |
| Spreading code | Walsh Hadmard |
| Bandwidth | 5 MHz |
| Spreading factor | 48 |
| FFT size | 256 |
| Effective symbol Duration | 5.5556e-005 |
| Guard time duration | ¼FFTlength, 1.1111e-005 |
| No of paths | 8 |
| Pilot carriers | 8 |
| Channel estimation | LS |
| Doppler velocity | 60Km/h |
| Modulation technique | QAM with M = 4,16and 64 |
| Convolution code generator polynomial | [53, 75] octal |
| CC decoding algorithm | Viterbi |

Figures (5, 6 and 7) show the performance of the system (BER versus SNR) over LTE channel for uncoded data, convolution, TURBO CODE and LDPC coded data data with QAM with M= 4, 16 and 64 respectively with AWGN channel. It can be seen that the LDPC coded MC-CDMA system behaves better than any one of the others uses the TURBO CODE or CE or uncoded data.

## Conclusions:

One can observes from the results that the LDPC gives a better BER for the Rayleigh channel for low SNR and the difference in dBs increases for higher values of SNR.

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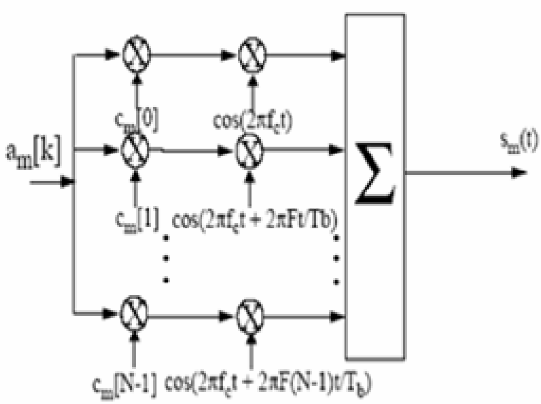


Fig.(1): MC-CDMA transmitter.

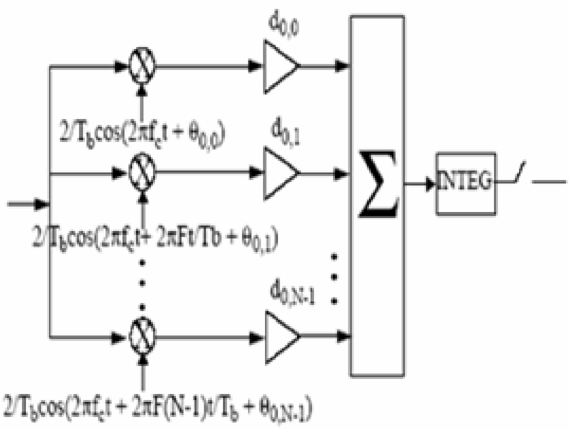


Fig.(2): MC-CDMA receiver.

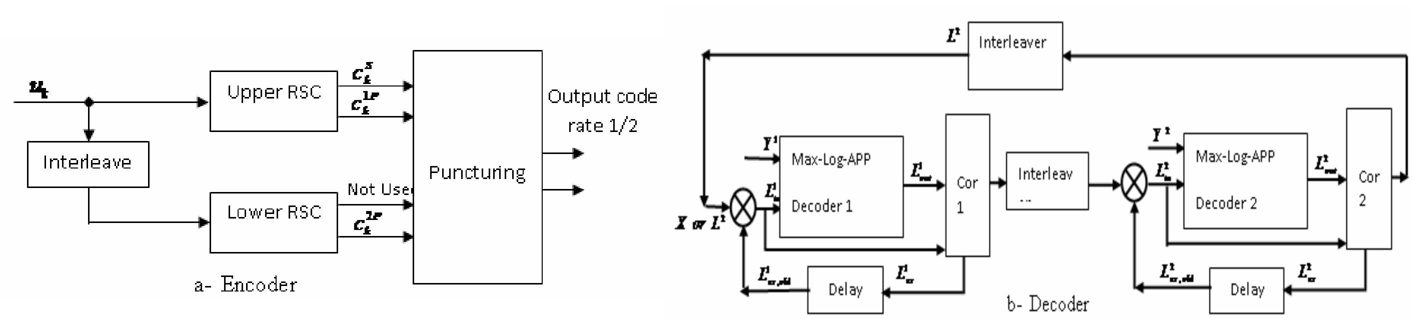


**Fig. (3) Parallel concatenation convolutional code (TURBO CODE) a- Encoder b- Decoder**

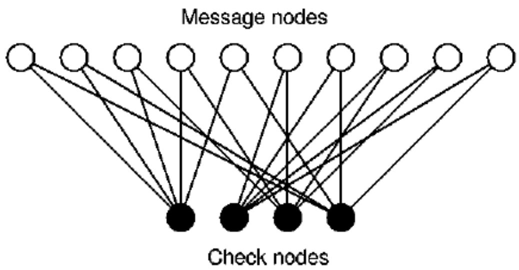


**Fig. (4): Tanner Graph**

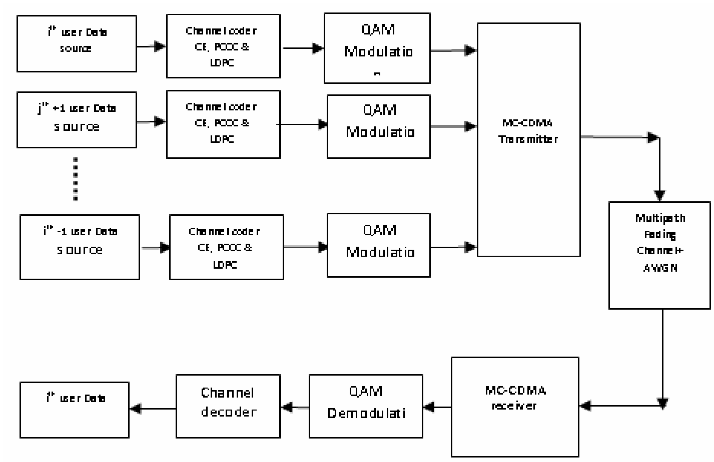


**Fig. (4) Block Diagram of the Proposed MUSO Coded MC-CDMA System**

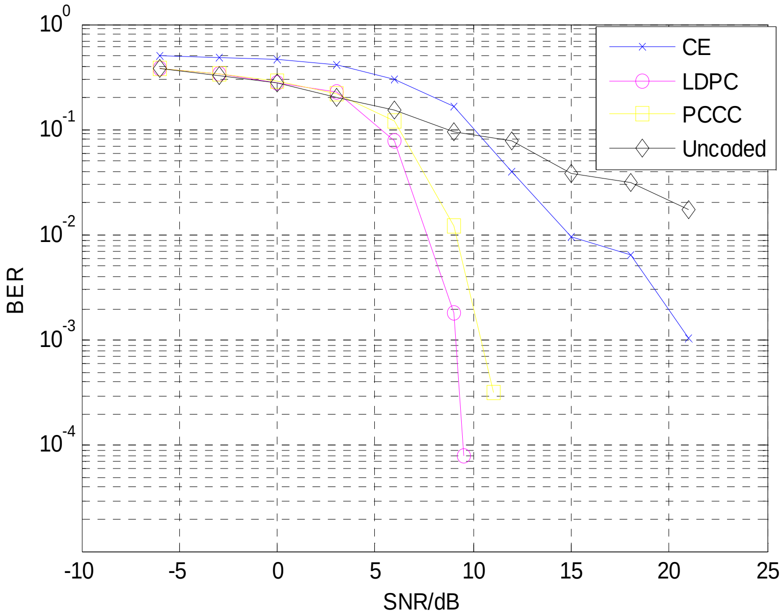


**Fig. (5): Coded MC-CDMA performance for Rayleigh Fading channel with 4QAM modulation.**

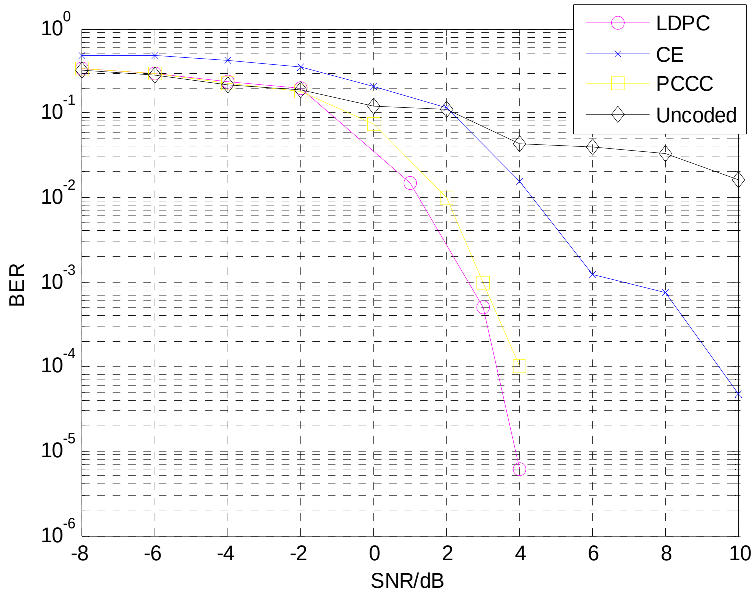


**Fig. (6): Coded MC-CDMA performance for Rayleigh Fading channel with 16QAM modulation.**

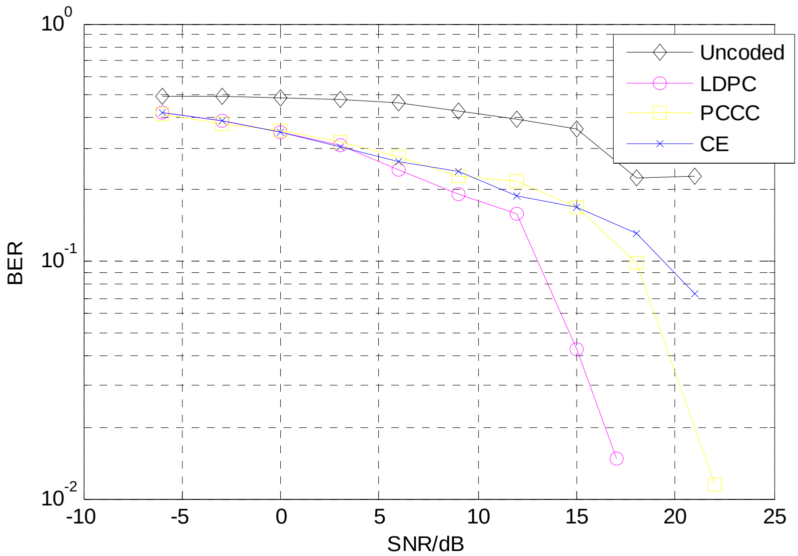


**Fig. (7): Coded MC-CDMA performance for Rayleigh Fading channel with 64 QAM modulation.**