DESIGN OF A VARIABLE GAIN NONLINEAR FUZZY CONTROLLER AND PERFORMANCE ENHANCEMENT DUE TO GAIN VARIATION

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ABSTRACT
In this paper, variable gain nonlinear PD and PI fuzzy logic controllers are designed and the effect of the variable gain characteristic of these controllers is analyzed to show its contribution in enhancing the performance of the closed loop system over a conventional linear PID controller. Simulation results and time domain performance characteristics show how these fuzzy controllers outperform the conventional PID controller when used to control a nonlinear plant and a plant that has time delay.

الخلاصة
في هذا البحث تم تصميم مسيطرين ضبابيين لا خطيين ذوا ربح متغير الاول (تناسبي – تكاملي) والثاني (تناسبي – تفاضلي) وتم تحليل تأثير خاصية الربح المتغير واسهامه في تحسين عمل النظام الكلي مقارنة بمسيطر تقليدي (تناسبي – تكاملي – تفاضلي). النتائج التمثيلية وخصائص العمل في مجال الزمن أوضحت كيف يتفوق هذان المسيطران الضبابيان على المسيطر التقليدي عند إستخدامهم للسيطرة على نظامين الاول نظام لا خطي والثاني نظام يمتلك زمن تأخير.

KEYWORDS
Variable gain nonlinear fuzzy controller, plant with time delay, nonlinear plant.

INTRODUCTION

Revealing explicit structure of fuzzy controllers is important primarily because it provides insightful information about what a fuzzy controller is, how it works, and how it relates to and differs from a classical controller. A fuzzy controller is not fuzzy anymore (i.e., not a black-box controller anymore) once its explicit structure is disclosed and it just becomes a conventional nonlinear controller (Ying 2000 -1) (Ying 2000 -2).

A fuzzy controller is called a linear (or nonlinear) fuzzy controller if its output is a linear (or nonlinear) function of its inputs. In most cases, whether or not a fuzzy controller is linear cannot be judged directly from its configuration. The explicit structure of the fuzzy controller must be derived to accurately determine its type (Ying 2000 -2).

There are three sources of nonlinearity in a fuzzy controller. First, the rule base. The position, shape, and number of membership functions on the premise side, as well as nonlinear input scaling, cause nonlinear characteristics. Even the rules themselves can express a nonlinear control strategy. Second, the inference engine. If the connectives are implemented as min and max respectively, they are nonlinear. The same applies to min-activation and max-accumulation. Third, the defuzzification method. Several defuzzification methods are nonlinear (Driankov, Hellendoorn, and Reinfrank, 1996).

Utilizing the nonlinear characteristics of a fuzzy controller is an important issue. The Type ACS201 P1.0 Intelligent Fuzzy Logic Control (IFLC) software product is part of the Advanced Control Solutions (ACS) from Fisher-Rosemount Systems, Inc. The IFLC product uses fuzzy logic algorithms to improve control loop performance. With autotuning functionality, the IFLC provides superior performance for a variety of applications. The nonlinearity built into the IFLC reduces overshoot and settling time, achieving tighter control of the process loop. Specifically, the fuzzy logic controller treats small control errors differently from large control errors and penalizes large overshoots more severely. It also severely penalizes large changes in the error, helping to reduce oscillation (www.emersonprocess.com).

Ultimately, the goal of tuning is to shape the nonlinearity that is implemented by the fuzzy controller. This nonlinearity, sometimes called the "control surface," is affected by all the main fuzzy controller parameters (Passino and Yurkovich 1998).

The fuzzy PD and PI controllers to be introduced in this paper are natural extensions of their conventional versions, which preserve the linear structures of the PID controllers, with simple and conventional analytical formulas as the final results of the design. Thus, they can directly replace the conventional PID controllers in any operating control systems (plants, processes). The main difference is that these fuzzy controllers are designed by employing fuzzy logic control principles and techniques to obtain new controllers that possess analytical formulas very similar to the conventional digital PID controllers. After the design is completed, all the fuzzy logic IF-THEN rules, membership functions, defuzzification formulas, etc. will not be needed any more in applications: what one can see is a conventional controller with a few simple formulas similar to the familiar PID controllers. Thus, in operations the controllers do not use any "look-up" table at any step, and so can be operated in real time. A control engineer who doesn't have any knowledge about fuzzy logic and/or fuzzy control systems can use them just like the conventional ones, particularly for higher-order, time-delayed, and nonlinear systems, and for those systems that have only vague mathematical models or contain significant uncertainties. The key reason, which is the price to pay, for such success is that these fuzzy controllers are slightly more complicated than the conventional ones, in the sense that they have variable control gains in their linear structures. These variable gains are nonlinear functions of the errors and changing rates of the error signals. The main contribution of these variable gains in improving the control performance is that they are self-tuned gains and can adapt to the rapid changes of the errors and the (changing) rates of the error signals caused by the time-delayed effects, nonlinearities, and uncertainties of the underlying system (plant, process) (Chen and Pham 2001).

Several researches had been made that utilizes the variable gain aspect of fuzzy controllers such as (Ying 2000 -1), which has proved that that the analytical structure of a (tow-input two-output) fuzzy PI/PD controllers is the sum of two nonlinear PI/PD controllers whose gains continuously change with system outputs. In (Brdys and Littler 2002), a

variable gain PI fuzzy controller was proposed to control a nonlinear servo system where traditional methods for controlling this system using linear control techniques are inadequate because of hard nonlinearities in the dynamics. The variable gain characteristics would compensate these nonlinearities. (Bonfe and Mainardi 2004) presents a variable gain fuzzy PID controller to control a robot arm and the experiments show the ability of this controller to greatly reduce the overshoot. (Miloudi, Draou, and AlRadadi 2002) presents an original variable gain PI controller for speed control of an induction machine drive. (Dash, Morris, and Mishra 2004) presents the design of a nonlinear variable-gain fuzzy controller for a flexible ac transmission systems device to enhance the transient stability performance of power systems.

NONLINEAR VARIABLE GAIN CONTROLLER

A variable gain PI controller (VGPI) is a generalization of a classical PI controller where the proportional and integrator gains vary along a tuning curve (Miloudi, Draou, and AlRadadi 2002). A variable gain PD controller has the same relation with the classical PD controller. In a linear PD controller, the control variable is given by

$$u = k_e e + k_{\dot{e}} \frac{de}{dt}$$

where k_e and $k_{\dot{e}}$ are constants representing the proportional and derivative gains, respectively. For convenience, let $\frac{de}{dt}$ and $k_{\dot{e}}$ be denoted by r and k_r, respectively. These gains can be considered as the sensitivity of the control variable u to e and r, respectively (i.e., $k_e = \frac{\partial u}{\partial e}$ and $k_r = \frac{\partial u}{\partial r}$) (Haines and Hittle 2006). The purpose is to design a nonlinear controller so that $\frac{\partial u}{\partial e}$ is not constant but an increasing function of e in the region $\{e \geq 0\}$ and a decreasing function of e in the region $\{e \leq 0\}$. This means that $\frac{\partial^2 u}{\partial e^2}$ must be nonnegative in the region $\{e \geq 0\}$ and nonpositive in the region $\{e \leq 0\}$. The same thing must hold for $\frac{\partial u}{\partial r}$ and r.

$$\frac{\partial u}{\partial e} = k_e + \frac{\partial k_e}{\partial e} e + \frac{\partial k_r}{\partial e} r \tag{1}$$

$$\frac{\partial u}{\partial r} = \frac{\partial k_e}{\partial r} e + k_r + \frac{\partial k_r}{\partial r} r \tag{2}$$

$$\frac{\partial^2 u}{\partial e^2} = 2\frac{\partial k_e}{\partial e} + \frac{\partial^2 k_e}{\partial e^2} e + \frac{\partial^2 k_r}{\partial e^2} r \tag{3}$$

$$\frac{\partial^2 u}{\partial r^2} = 2\frac{\partial k_r}{\partial r} + \frac{\partial^2 k_r}{\partial r^2} r + \frac{\partial^2 k_e}{\partial r^2} e \tag{4}$$

DESIGN OF A NONLINEAR VARIABLE GAIN FUZZY LOGIC CONTROLLER

A block diagram of a PD fuzzy controller is shown in Fig. 1 (Passino and Yurkovich 1998). The proposed fuzzy controller uses two identical input fuzzy sets, namely Positive $(\widetilde{P})$ and Negative $(\widetilde{N})$. The membership functions of these fuzzy sets are

$$\mu_{\widetilde{P}}(x) = \begin{cases} 0 & \text{when } x < -1 \\ \frac{1}{2}(x+1) & \text{when } -1 \leq x \leq 1 \\ 1 & \text{when } x > 1 \end{cases},$$

$$\mu_{\widetilde{N}}(x) = \begin{cases} 1 & \text{when } x < -1 \\ \frac{1}{2}(-x+1) & \text{when } -1 \leq x \leq 1 \\ 0 & \text{when } x > 1 \end{cases}$$

where x is the input variable $k_1 e$ or $k_2 \frac{de}{dt}$, where k_1 and k_2 are scaling factors.

Three output fuzzy sets, namely Positive $(\widetilde{P})$, Zero $(\widetilde{Z})$, and Negative $(\widetilde{N})$ are used. They are of singleton type and there nonzero values are at 1, 0, and -1, respectively. The input and output fuzzy sets are shown in Fig.2

The fuzzy controller uses the following four fuzzy rules:

IF e is Positive AND r is Positive THEN u is Positive
IF e is Positive AND r is Negative THEN u is Zero
IF e is Negative AND r is Positive THEN u is Zero
IF e is Negative AND r is Negative THEN u is Negative

Using the Zadeh fuzzy AND operator, the Lukasiewicz fuzzy OR operator, and the center-average defuzzification technique, the analytical structure of the fuzzy controller can be derived. To focus the analysis of the fuzzy controller to the region near the equilibrium point $(k_1 e, k_2 r) = (0,0)$, only the square region $[-1,1]\times[-1,1]$ of the $k_1 e - k_2 r$ phase plane will be considered. The controller output is

$$u = \begin{cases} \dfrac{1}{4-2k_1|e|}(k_1 e + k_2 r) & \text{When} \quad k_2|r| \le k_1|e| \\ \dfrac{1}{4-2k_2|r|}(k_1 e + k_2 r) & \text{When} \quad k_1|e| \le k_2|r| \end{cases}$$

The following analysis shows that for this fuzzy controller the partial derivatives in eq (3) and eq(4) are nonnegative in the region $k_2|r| \le k_1|e|$. A similar analysis can be carried out for the region $k_1|e| \le k_2|r|$.

$k_e = \dfrac{k_1}{4-2k_1|e|}$ and $k_r = \dfrac{k_2}{4-2k_1|e|}$ where k_e and k_r are positive in $[-1,1]\times[-1,1]$

$$\frac{\partial k_e}{\partial e} = \frac{2k_1^2 \operatorname{sgn}(e)}{(4-2k_1|e|)^2}, \quad \frac{\partial k_r}{\partial e} = \frac{2k_1 k_2 \operatorname{sgn}(e)}{(4-2k_1|e|)^2},$$

$$\frac{\partial^2 k_e}{\partial e^2} = \frac{8k_1^3 \operatorname{sgn}^2(e)(4-2k_1|e|)}{(4-2k_1|e|)^4}$$

$$\frac{\partial^2 k_r}{\partial e^2} = \frac{8k_1^2 k_2 \operatorname{sgn}^2(e)(4-2k_1|e|)}{(4-2k_1|e|)^4}$$

$$\frac{\partial k_e}{\partial r} = \frac{\partial k_r}{\partial r} = \frac{\partial^2 k_e}{\partial r^2} = \frac{\partial^2 k_r}{\partial r^2} = 0$$

Substituting the above expressions of the partial derivatives in eq(1) through eq(4) yields

$$\frac{\partial u}{\partial e} = \frac{k_1}{4-2k_1|e|} + \frac{2k_1 \operatorname{sgn}(e)}{(4-2k_1|e|)^2}(k_1 e + k_2 r)$$

$$\frac{\partial u}{\partial r} = \frac{k_2}{4-2k_1|e|}$$

$$\frac{\partial^2 u}{\partial e^2} = 4\frac{k_1^2 \operatorname{sgn}(e)}{(4-2k_1|e|)^2} + \frac{8k_1^2 \operatorname{sgn}^2(e)(4-2k_1|e|)}{(4-2k_1|e|)^4} \times (k_1 e + k_2 r) \tag{5}$$

$$\frac{\partial^2 u}{\partial r^2} = 0 \tag{6}$$

For $e > 0$, $\operatorname{sgn}(e) = 1 > 0$, $|e| = e$, and

$k_2|r| \le k_1|e| \Rightarrow k_2|r| \le k_1 e \Rightarrow -k_1 e \le k_2 r \le k_1 e$

$\Rightarrow 0 \le k_1 e + k_2 r \le 2k_1 e$

Since each term of the left hand side of eq.(5) is nonnegative, this implies that $\frac{\partial^2 u}{\partial e^2}$ is nonnegative. It is clear from eq.(6) that $\frac{\partial^2 u}{\partial r^2} = 0$ implies that $\frac{\partial^2 u}{\partial r^2} \ge 0$, i.e., $\frac{\partial^2 u}{\partial r^2}$ is nonnegative.

For $e < 0$, $\operatorname{sgn}(e) = -1 < 0$, $|e| = -e$, and

$k_2|r| \le k_1|e| \Rightarrow k_2|r| \le -k_1 e \Rightarrow k_1 e \le k_2 r \le -k_1 e$

$\Rightarrow 2k_1 e \le k_1 e + k_2 r \le 0$

Since each term of the left hand side of eq.(5) is nonpositive, this implies that $\frac{\partial^2 u}{\partial e^2}$ is nonpositive. It is clear from eq.(6) that $\frac{\partial^2 u}{\partial r^2} = 0$ implies that $\frac{\partial^2 u}{\partial r^2} \le 0$, i.e., $\frac{\partial^2 u}{\partial r^2}$ is nonpositive.

Fig. 3 and **Fig. 4** show the graphs of $\frac{\partial u}{\partial e}$ and $\frac{\partial u}{\partial r}$ as functions of $k_1 e$ and $k_2 r$, respectively (without loss of generality, k_1 and $k2$ were assumed to be 1). These surfaces are valid only above the region $k_2|r| \le k_1|e|$. It is obvious from **Fig. 3** that the surface of $\frac{\partial u}{\partial e}$ is discontinuous at $(k_1 e, k_2 r) = (0,0)$ because of the discontinuity of $\mathrm{sgn}(e)$ at $e = 0$.

A block diagram of a PI fuzzy controller is shown in **Fig. 5** (Passino and Yurkovich 1998) (Reznik 1997).

The structure of the PI fuzzy controller is the same as that of the PD fuzzy controller except that the derivative input $\frac{de}{dt}$ in the PD fuzzy controller is replaced by the integral input $\int_0^t edt$ in the PI fuzzy controller. Therefore, if $\frac{de}{dt}$ is replaced by $\int_0^t edt$ in the above design and analysis, the result is a PI fuzzy controller.

SIMULATION RESULTS

Since fuzzy control has the potential to outperform linear control when nonlinear systems or systems that have time delay are involved (Ying 2000 –1), the above PD and PI fuzzy controllers are used to control such systems.

Controlling a Nonlinear Plant

Most real plants have nonlinear dynamics. A common nonlinear functions that appear in process dynamic models are enthalpy as a function of temperature, fluid flow as a function of pressure drop, radiation heat transfer rate as a function of temperature,…etc (Smith and Corripio 2005). The nonlinear plant that is used in this simulation is a nonlinear mass-spring-damper system shown in **Fig. 6** (Lam, Leung, and Tam 2001). The dynamic of this plant is given by

$$\ddot{x}(t) = -\dot{x}(t) - 0.01x(t) - 0.1x(t)^3 + (1.4387 - 0.13\dot{x}(t)^2)u(t)$$

Fig. 7 (**Fig. 8**) shows a step response comparison of this plant when a nonlinear PD (PI) fuzzy controller and a conventional PID controller are used. **Table 1** shows the time domain performance characteristics for these responses. The PI fuzzy controller has a large overshoot since it lacks the derivative term in its structure. However, the PD fuzzy controller still outperforms the PID controller.

Controlling a Plant with Time Delay

Many industrial processes can be approximated by first-order dynamics and a time delay (Astrom 1997) (Landau, and Zito 2006) (Chen and Pham 2001) (Ogata 1010) (Fadali 2009). The steering control of a moon vehicle is an example of a plant that has a time delay (Dorf and Bishop 2008). The dynamics of the plant is given by

$$0.2\dot{\theta}(t) + \theta(t) = 2u(t - 0.1)$$

Fig. 9 (**Fig. 10**) shows a step response comparison of this plant when a nonlinear PD (PI) fuzzy controller and a conventional PID controller are used. **Table 2** shows the time domain performance characteristics for these responses. It is obvious that the two fuzzy controllers outperform the PID controller.

CONCLUSIONS

Fuzzy control and conventional PID control produce the same control performance for linear systems. Therefore, using fuzzy control should be avoided in such cases since a fuzzy controller has many more design parameters than a PID controller which has only three design parameters and its design and implementation is effective and efficient. However, PID control may not generate satisfactory control performance if the plant is nonlinear, time varying, or has time delay. In such cases, fuzzy control can outperform PID control, especially due to its variable gain characteristics. Simulation results show two cases were fuzzy control gives better performance characteristics than PID control.

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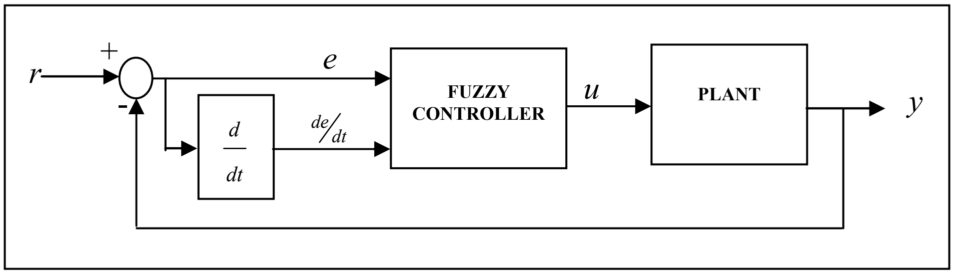

Fig. 1: A block diagram of a PD fuzzy controller.

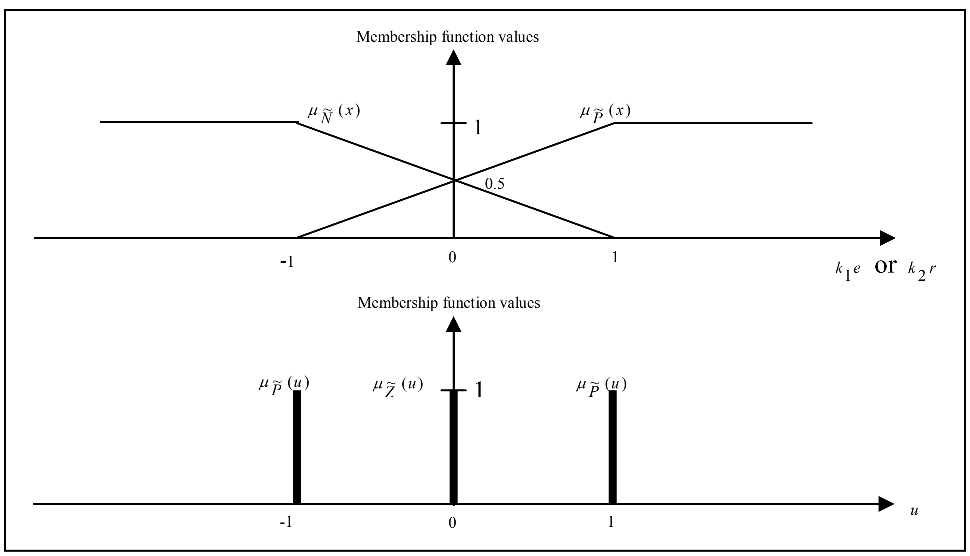

Fig. 2: The input and output fuzzy sets.

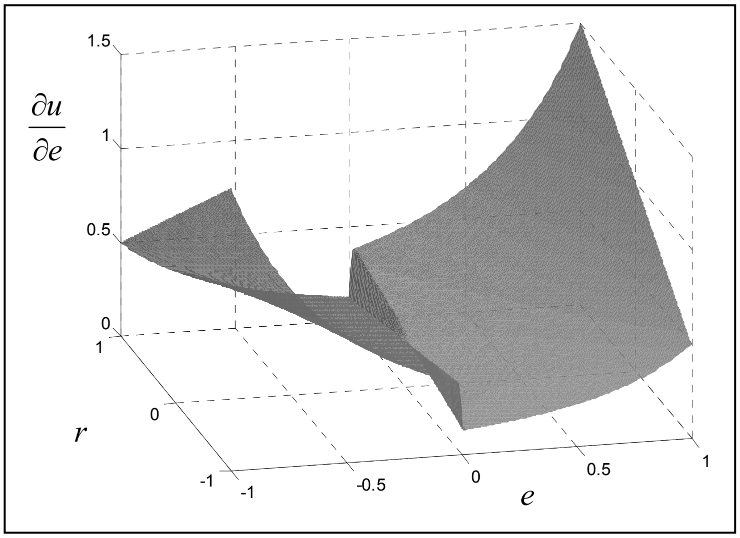

Fig. 3: Graph of $\frac{\partial u}{\partial e}$ as functions of $k_1 e$ and $k_2 r$.

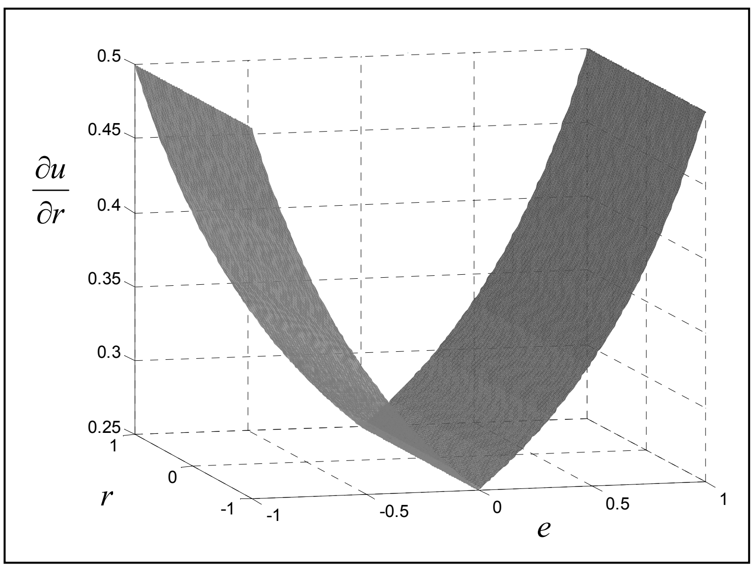

Fig. 4 Graph of $\frac{\partial u}{\partial r}$ as functions of $k_1 e$ and $k_2 r$.

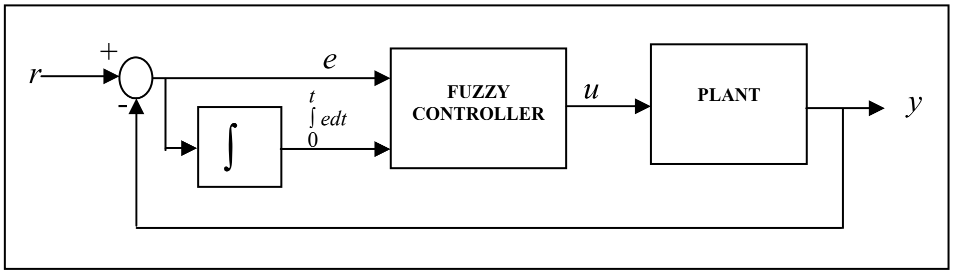

Fig. 5: A block diagram of a PI fuzzy controller (version 1)

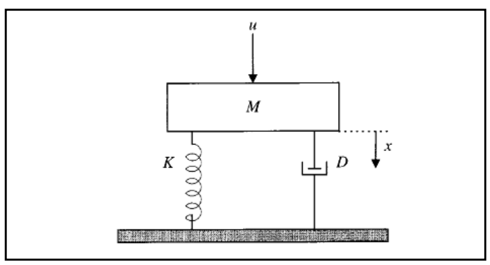

Fig. 6: A nonlinear mass-spring-damper system

Table 1: Time domain performance characteristics for the nonlinear plant

Type of controller / Performance characteristics	PD fuzzy Fig. 7	PI fuzzy Fig. 8	PID
Rise time (sec)	0.92	0.48	1
Percentage overshoot	0	56.3944	6.7093
Settling time (2%) (sec)	2.4	11.76	3.38

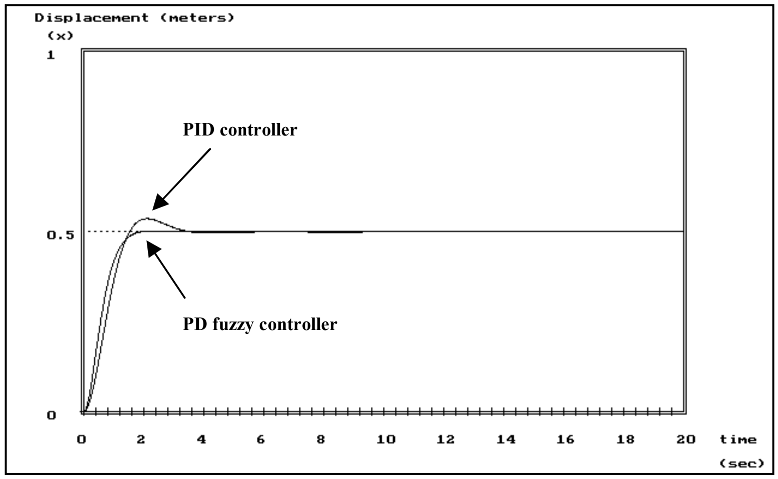

Fig. 7: Step response of the nonlinear plant using a nonlinear PD fuzzy controller and a conventional PID controller.

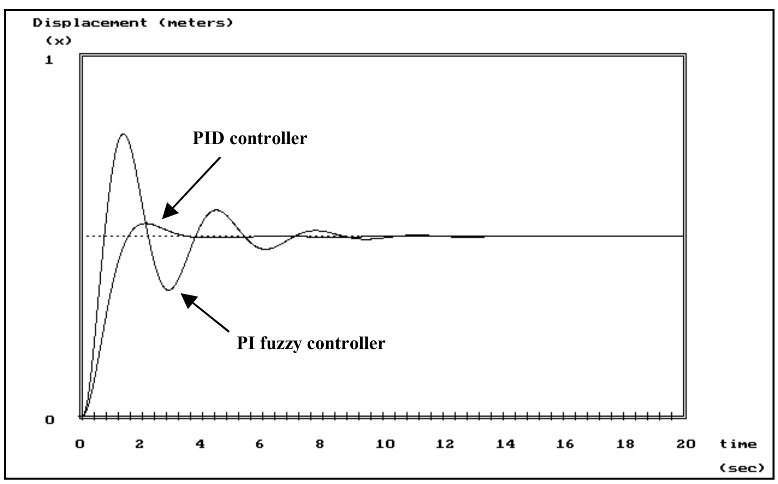

Fig. 8: Step response of the nonlinear plant using a nonlinear PI fuzzy controller and a conventional PID controller.

Table 2: Time domain performance characteristics for the plant with time delay

Type of controller / Performance characteristics	PD fuzzy Fig. 9	PI fuzzy Fig. 10	PID
Rise time (sec)	0.46	0.38	1.08
Percentage overshoot	0	0	0.2505
Settling time (2%) (sec)	0.74	0.96	1.92

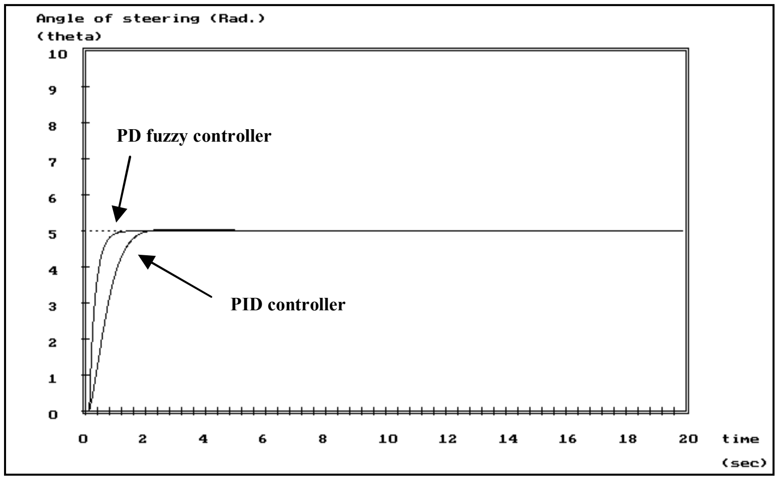

Fig. 9: Step response of the plant with time delay using a nonlinear PD fuzzy controller and a conventional PID controller.

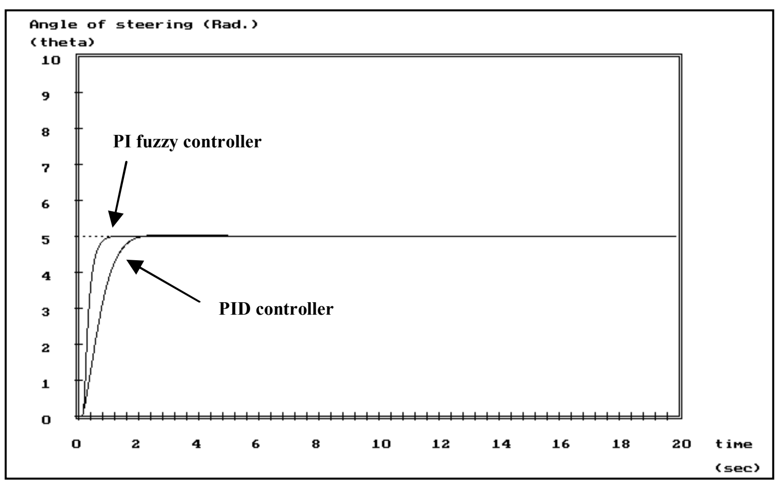

Fig. 10: Step response of the plant with time delay using a nonlinear PD fuzzy controller and a conventional PID controller.