

University of Baghdad
Fourth Year
College of Engineering

Water Resources
Engineering Dept.

First Semester

Week No.	Subject (details)
1-2-3	Introduction.....Types of bridges
4-5	Concrete design of slab bridge
6-7-8	Concrete design of Dick girder bridge(Monthly Exam)
9-10-11-12	Beam on Rigid and Elastic foundation
13-14-15	Concrete design of aqueduct(2 nd Exam)

Second Semester

Week No.	Subject (details)
1-2-3-4	Concrete design of box culvert
5-6-7	Concrete design of circular tanks
8-9-10-11	Concrete design rectangular tanks(3 rd Exam)
12-13-14-15	Concrete design of Retaining walls(4 th Exam)

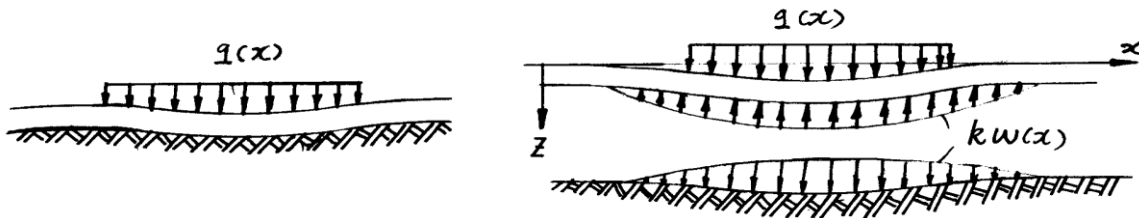
RIGID AND ELASTIC FOUNDATIONS

* Bending stress of beams on rigid foundations and solutions

$$\sigma = \frac{F}{A} \pm \frac{M \cdot c}{I}$$

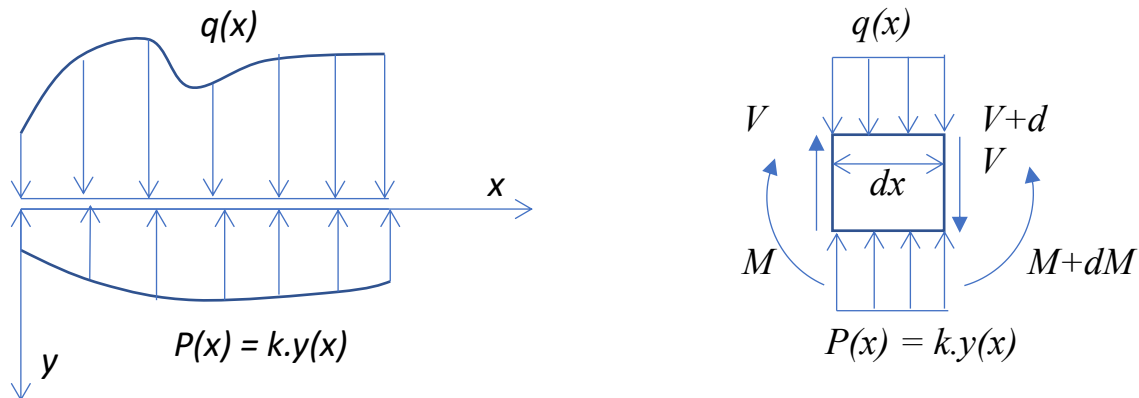
** Bending stress of beams on rigid foundations and solutions

*** Introduction and Foundation Models ---- Winkler Foundation



$p = k \cdot y(x)$ k_0 : is the foundation modulus (unit: N/m²/m)

For beams with width b , we use $p = k \cdot y(x) = k_0 b \cdot y(x)$



Governing Equations for Uniform Straight Beams on Elastic Foundations

$$\frac{d^4 y}{dx^4} + \frac{kb}{EI} y = \frac{q(x)}{EI}$$

الاشتقاق والحل مطلوب

$$\frac{d^4 y}{dx^4} + 4\beta^4 y = \frac{q(x)}{EI} \quad \beta = \sqrt[4]{\frac{kb}{4EI}}$$

$$y_h = e^{-\beta x} [c_1 \cos(\beta x) + c_2 \sin(\beta x)] + e^{\beta x} [c_3 \cos(\beta x) + c_4 \sin(\beta x)]$$

$$y_h' = \beta e^{-\beta x} [(c_2 - c_1) \cos(\beta x) - (c_1 + c_2) \sin(\beta x)] + \beta e^{\beta x} [(c_4 - c_3) \cos(\beta x) + (c_3 + c_4) \sin(\beta x)]$$

$$y_h'' = 2\beta^2 e^{-\beta x} [-c_2 \cos(\beta x) + c_1 \sin(\beta x)] + 2\beta^2 e^{\beta x} [c_4 \cos(\beta x) - c_3 \sin(\beta x)]$$

$$y_h''' = 2\beta^3 e^{-\beta x} [(c_1 + c_2) \cos(\beta x) + (c_2 - c_1) \sin(\beta x)] + 2\beta^3 e^{\beta x} [(c_4 - c_3) \cos(\beta x) - (c_3 + c_4) \sin(\beta x)]$$

or

$$y_h = A_1 \cosh(\beta x) \cos(\beta x) + A_2 \cosh(\beta x) \sin(\beta x) + A_3 \sinh(\beta x) \cos(\beta x) + A_4 \sinh(\beta x) \sin(\beta x)$$

$$y_h' = \beta [(A_2 + A_3) \cosh(\beta x) \cos(\beta x) + (A_4 - A_1) \cosh(\beta x) \sin(\beta x) + (A_1 + A_4) \sinh(\beta x) \cos(\beta x) + (A_2 - A_3) \sinh(\beta x) \sin(\beta x)]$$

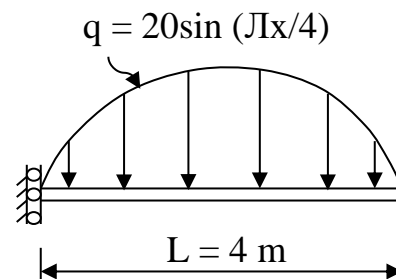
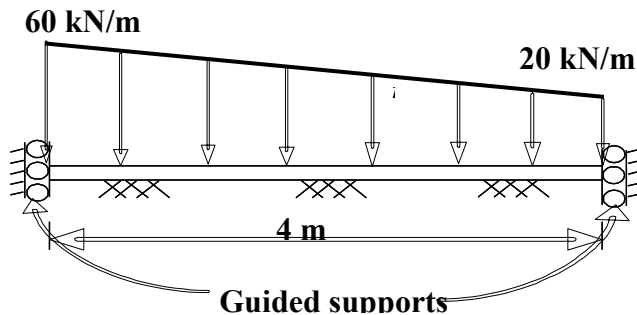
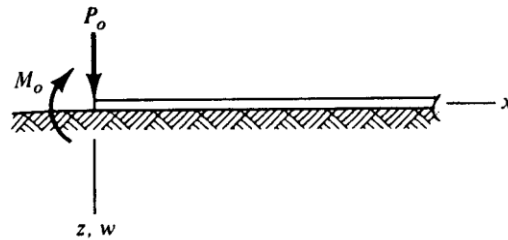
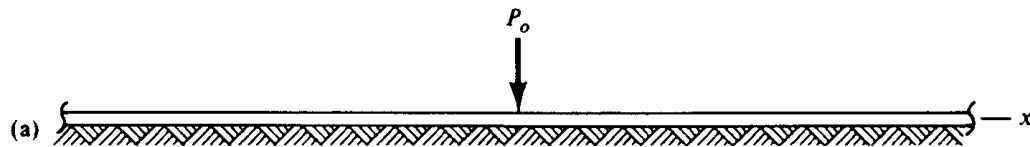
$$y_h'' = 2\beta^2 [A_4 \cosh(\beta x) \cos(\beta x) - A_3 \cosh(\beta x) \sin(\beta x) + A_2 \sinh(\beta x) \cos(\beta x) - A_1 \sinh(\beta x) \sin(\beta x)]$$

$$y_h''' = 2\beta^3 [(A_2 - A_3) \cosh(\beta x) \cos(\beta x) - (A_4 + A_1) \cosh(\beta x) \sin(\beta x) + (A_4 - A_1) \sinh(\beta x) \cos(\beta x) - (A_2 + A_3) \sinh(\beta x) \sin(\beta x)]$$

$$y = y_h + y_p$$

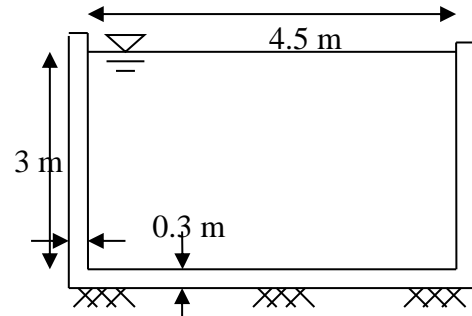
Examples: For the beams on elastic foundation shown in the Figures,

1. State the homogeneous solution.
2. Find the particular solution.
3. State only the boundary conditions needed



Example (4): Design and Draw the pressure distribution in the subsoil; shear force, and bending moment diagram of the base for the concrete aqueduct shown in fig having the following data:

$E_{conc.} = 25 \times 10^6 \text{ kn/m}^2$	$\gamma_{conc.} = 24 \text{ kn/m}^3$
$\gamma_{water.} = 10 \text{ Kn/m}^3$	$K_{soil} = 14000 \text{ Kn/m}^3$
$f_c' = 28 \text{ Mpa}$	$F_y = 410 \text{ Mpa}$



Solution:

$$\beta = 0.4994 \quad 1/m \quad \beta L = 2.2474958 \quad mid$$

$$EI = 56250 \text{ Kn.m}^2$$

B.C. :-

Put $A_2 = A_3 = 0$ from symmetry

$$\text{when } x = L/2 = 2.25 \text{ m} \quad y''' = -23/EI \quad -0.000409 \quad y'' = 45/EI \quad 0.0008$$

$$\beta L/2 = 1.1237 \quad 2\beta^2 = 0.4988877 \quad 2\beta^3 = 0.2491662$$

$$\cosh(\beta L/2) \cos(\beta L/2) = 0.735227346 \quad \sinh((\beta L/2) \sin(\beta L/2) = 1.24046228$$

$$\cosh(\beta L/2) \sin(\beta L/2) = 1.533576952 \quad \sinh((\beta L/2) \cos(\beta L/2) = 0.59470233$$

$$y''/2\beta^2 = A4 \cosh(\beta x) \cos(\beta x) + A1 \sinh(\beta x) \sin(\beta x)$$

0.001603567	=	0.735227346	×A4	-	1.240462276	×A1	eq(1)
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$$y'''/2\beta^3 = A4 (\sinh(\beta x) \cos(\beta x) - \cosh(\beta x) \sin(\beta x)) - A1 (\sinh(\beta x) \cos(\beta x) + \cosh(\beta x) \sin(\beta x))$$

-0.001641029	=	-0.93887462	×A4	-	2.128279283	×A1	eq(2)
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from eq(1) and eq (2) we get	A1 =	-1.10E-04	& A4 =	2.00E-03
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$$P(x) = K y = K(A1 \cosh(\beta x) \cos(\beta x) + \sinh(\beta x) \sin(\beta x) + 37.2/K)$$

$$V(x) = 2\beta^3 (-(A1 + A4) \cosh(\beta x) \sin(\beta x) + (A4 - A1) \sinh(\beta x) \cos(\beta x))$$

$$M(x) = 2\beta^2 (A4 \cosh(\beta x) \cos(\beta x) - A1 \sinh(\beta x) \sin(\beta x))$$

<u>x</u>	<u>β</u>	<u>A1</u>	<u>A4</u>	<u>p(x)</u>	<u>v(x)</u>	<u>M(x)</u>
-2.25	0.4994	-0.00010955	0.00199621	70.739475	23	45
-2	0.4994	-0.00010955	0.00199621	63.496692	15.53589738	49.77926
-1.75	0.4994	-0.00010955	0.00199621	57.026121	9.787231956	52.91094
-1.5	0.4994	-0.00010955	0.00199621	51.377	5.554329151	54.79921
-1.25	0.4994	-0.00010955	0.00199621	46.579161	2.627753394	55.79698
-1	0.4994	-0.00010955	0.00199621	42.648512	0.792457395	56.20404
-0.75	0.4994	-0.00010955	0.00199621	39.591705	-0.16933773	56.26601
-0.5	0.4994	-0.00010955	0.00199621	37.409951	-0.47632285	56.17394
-0.25	0.4994	-0.00010955	0.00199621	36.101994	-0.3471303	56.06419
0	0.4994	-0.00010955	0.00199621	35.666233	0	56.01853
0.25	0.4994	-0.00010955	0.00199621	36.101994	0.347130298	56.06419
0.5	0.4994	-0.00010955	0.00199621	37.409951	0.476322849	56.17394
0.75	0.4994	-0.00010955	0.00199621	39.591705	0.16933773	56.26601
1	0.4994	-0.00010955	0.00199621	42.648512	-0.79245739	56.20404
1.25	0.4994	-0.00010955	0.00199621	46.579161	-2.62775339	55.79698
1.5	0.4994	-0.00010955	0.00199621	51.377	-5.55432915	54.79921
1.75	0.4994	-0.00010955	0.00199621	57.026121	-9.78723196	52.91094
2	0.4994	-0.00010955	0.00199621	63.496692	-15.5358974	49.77926
2.25	0.4994	-0.00010955	0.00199621	70.739475	-23	45

Example: Design the simply supported reinforced concrete slab bridge having Cross-section shown in the figure and the following data:

Width = 8 m	Span c/c = 5.4 m	Clear span = 5 m	Asphalt Wt = 1.5 kn/m ²
Truck: MS18	$f_c' = 25$ Mpa	$f_y = 300$ Mpa	Hand rail Wt = 1 Kn/m

Solution:

-Slab design:

$$t_{\min.} = \frac{l}{20} = \frac{5}{20} = 0.25 \text{ mm} \quad \text{use } t = 0.3 \text{ mm}$$

$$\text{Span } l = \min.(\text{Span c/c}, \text{Clear span} + t) = \min.(5.4, 5.3) = 5.3 \text{ m}$$

Dead moment:

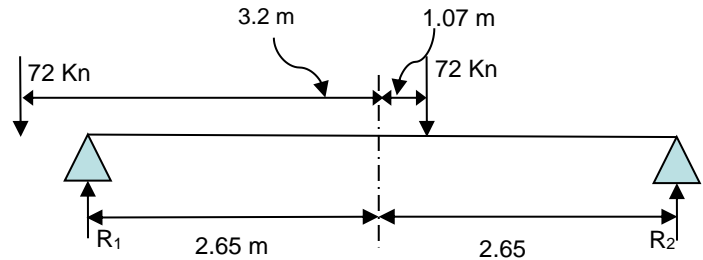
$$w_d = \gamma_{\text{conc.}} \times t + \text{asphalt } w_t = 24 \times 0.3 + 1.5 = 8.7 \text{ Kn/m}^2$$

$$M_d = \frac{w_d \times (l)^2}{8} = \frac{8.7 \times (5.3)^2}{8} = 30.55 \text{ Kn.m/m}$$

Live moment:

1- احتمال دخول ثلاث عجلات غير وارد لان طول الجسر 5.3 م و المسافة الكلية بين العجلات هو (4.27+4.27) م

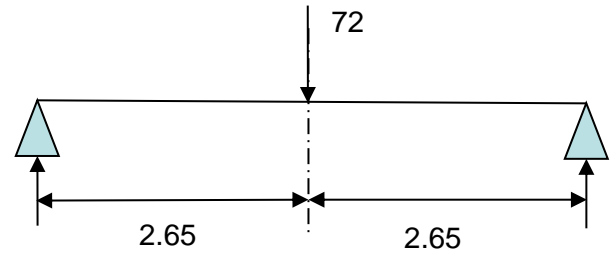
2- احتمال دخول عجلتين ايضا غير وارد بعد الحسابات



3- احتمال دخول عجلة واحدة تكون بالمنتصف

$$\begin{aligned}
 E &= 1.22 + 0.06S \leq 2.14m \\
 &= 1.22 + 0.06 \times 5.3 = 1.538m < 2.14m \\
 \therefore E &= 1.538m
 \end{aligned}$$

$$M_L = \frac{\frac{P \times L}{4}}{E} = \frac{\frac{72 \times 5.3}{4}}{1.538} = 62.03 \text{ Kn.m/m}$$



Impact moment :

$$I = \frac{15.24}{I + 38.1} \leq 30\% = \frac{15.24}{5.3 + 38.1} = 0.352 > 0.3$$

use $I = 0.3$

$$M_I = I \times M_L = 0.3 \times 62.03 = 18.61 \text{ Kn.m/m}$$

$$\text{Total moment } M_t = M_d + M_l + M_I = 30.55 + 62.03 + 18.6 = 111.2 \text{ Kn.m/m}$$

$$n = \frac{E_s}{E_c} = \frac{200000}{4730\sqrt{f_c}} = \frac{200000}{4730\sqrt{25}} = 8.46, \quad r = \frac{f_s}{f_c \text{ max.}} = \frac{140}{0.4 \times 25} = 14$$

$$K = \frac{n}{n + r} = \frac{8.46}{8.46 + 14} = 0.377, \quad J = 1 - \frac{K}{3} = 1 - \frac{0.377}{3} = 0.874$$

$$d_{req.} = \sqrt{\frac{2 \times M_t}{f_c \text{ max.} \times K \times J \times b}} = \sqrt{\frac{2 \times (111.2 \times 10^6)}{10 \times 0.377 \times 0.874 \times 1000}} = 259.8 \text{ mm},$$

$$d_{ava.} = t - \text{cover} - \phi/2 = 300 - 25 - 25/2 = 262.5 \text{ mm} > d_{req.} \quad \text{ok}$$

$$A_{st.} = \frac{M_t}{f_s \times J \times d} = \frac{111.2 \times 10^6}{140 \times 0.874 \times 262.5} = 3462 \text{ mm}^2 / m$$

$$s = 1000 \times \frac{A_{bar}}{A_{st.}} = 1000 \times \frac{491}{3462} = 141 \text{ mm}, \quad \text{use } \phi 25 @ 140 \text{ mm top \& bottom, } A_{st} = 1000 \times \frac{491}{140} = 3507 \text{ mm}$$

Secondary reinforcement:

$$A_{sd.} = \frac{1}{\sqrt{3.28s_c}} \times A_{st.} \leq 50\% \times A_{st.} = \frac{1}{\sqrt{3.28 \times 5.3}} \times A_{st.} \leq 0.5 \times A_{st.} = 0.24 \times A_{st.} \leq 0.5 \times A_{st.}$$

$$A_{sd} = 0.5 \times 3507 = 842 \text{ mm}^2 / m$$

$$s = 1000 \times \frac{A_{bar}}{A_{st.}} = 1000 \times \frac{201}{842} = 239 \text{ mm}$$

use $\phi 16 @ 230 \text{ mm}$ top & bottom

Curb design: Dead moment:

$$W_d = \gamma_{conc.} \times b \times h + w_{handrail} = 24 \times 0.6 \times 0.5 + 1 = 8.6 \text{ Kn/m}$$

$$M_d = \frac{w_d \times l^2}{8} = \frac{8.6 \times (5.3)^2}{8} = 30.2 \text{ Kn.m}$$

Live moment: $M_L = 0.1 \times P \times S = 0.1 \times 72 \times 5.3 = 38.16 \text{ Kn.m}$

Total moment $M_t = M_d + M_L = 30.2 + 38.16 = 68.36 \text{ Kn.m}$

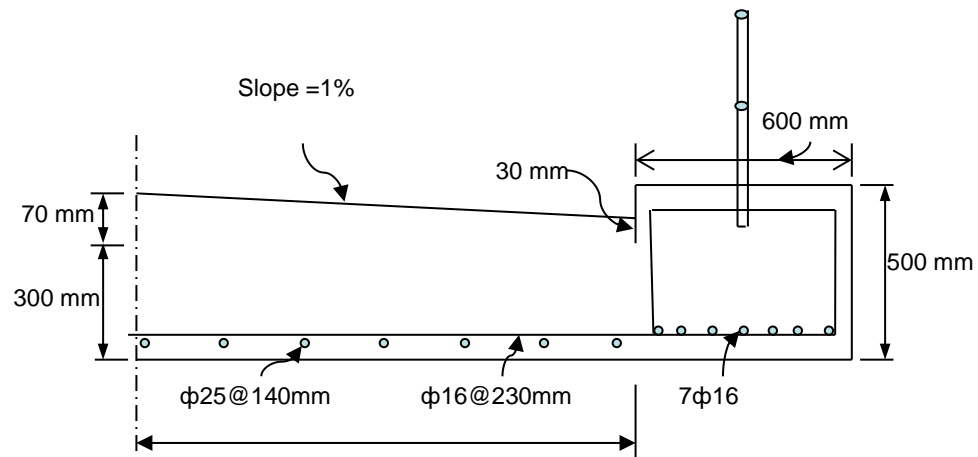
Design: $n = 8.46, \quad r = 14, \quad k = 0.377, \quad j = 0.874$

$$d_{req.} = \sqrt{\frac{2 \times M_t}{f_c \max. \times K \times J \times b}} = \sqrt{\frac{2 \times (68.36 \times 10^6)}{10 \times 0.377 \times 0.874 \times 1000}} = 262.975 \text{ mm},$$

$$d_{ava.} = t - \text{cover} - d_s - \phi / 2 = 500 - 50 - 16 - 16/2 = 426 \text{ mm} > d_{req.} \quad \text{ok}$$

$$A_{st.} = \frac{M_t}{f_s \times J \times d} = \frac{68.36 \times 10^6}{140 \times 0.874 \times 426} = 1311 \text{ mm}^2$$

$$\text{NO. of } \Phi 16 = \frac{A_s}{A_{bar}} = \frac{1311}{201} = 6.5 \quad \text{use } 7\phi 16$$



Example: Design the simply supported reinforced concrete girder-deck bridge having Cross-section shown in the figure and the following data:

Width = 8.7 m

Span c/c = 15.6 m

Clear span = 15 m

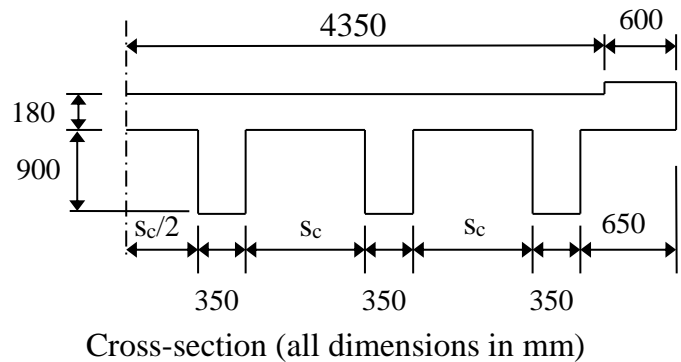
Truck: MS18

$f_c' = 25 \text{ Mpa}$ & $f_y = 400 \text{ Mpa}$

Asphalt $W_t = 1.5 \text{ kn/m}^2$

Hand rail $W_t = 1 \text{ Kn/m}$

Curb height = 150 mm



Solution:

-Slab design:

$$2.5 \times s_c = \{8700/2 + 600 - 650 - 3 \times 350\} \rightarrow s_c = 1300 \text{ mm}$$

$$t_{\min.} = \frac{l}{28} = \frac{1300}{28} = 46.4 \text{ mm} < t = 180 \text{ mm} \quad \text{ok}$$

$$w_d = \gamma_{\text{conc.}} \times t + \text{asphalt } w_t = 24 \times 0.18 + 1.5 = 5.82 \text{ Kn/m}^2$$

$$M_d = \frac{w_d \times (s)^2}{10} = \frac{5.82 \times (1.3)^2}{10} = 0.984 \text{ Kn.m/m}$$

$$M_l = 0.8 \times \frac{3.28 \times s_c + 2}{32} \times p = 0.8 \times \frac{3.28 \times 1.3 + 2}{32} \times 72 = 11.275 \text{ Kn.m/m}$$

$$I = \frac{15.24}{l + 38.1} \leq 30\% = \frac{15.24}{1.3 + 38.1} = 0.387 > 0.3$$

use $I = 0.3$

$$M_l = I \times M_l = 0.3 \times 11.275 = 3.383 \text{ Kn.m/m}$$

Total moment $M_t = M_d + M_l + M_l = 0.984 + 11.275 + 3.383 = 15.642 \text{ Kn.m/m}$

$$n = \frac{E_s}{E_c} = \frac{200000}{4730\sqrt{f_c}} = \frac{200000}{4730\sqrt{25}} = 8.46, \quad r = \frac{f_s}{f_c \text{ max.}} = \frac{170}{0.4 \times 25} = 17$$

$$K = \frac{n}{n + r} = \frac{8.46}{8.46 + 17} = 0.332, \quad J = 1 - \frac{K}{3} = 1 - \frac{0.332}{3} = 0.889$$

$$d_{req.} = \sqrt{\frac{2 \times M_t}{f_c \text{ max.} \times K \times J \times b}} = \sqrt{\frac{2 \times (15.642 \times 10^6)}{10 \times 0.332 \times 0.889 \times 1000}} = 102.94 \text{ mm},$$

$$d_{ava.} = t - \text{cover} - \phi / 2 = 180 - 25 - 16 / 2 = 147 \text{ mm} > d_{req.} \quad \text{ok}$$

$$A_{st.} = \frac{M_t}{f_s \times J \times d} = \frac{15.642 \times 10^6}{170 \times 0.889 \times 147} = 704.1 \text{ mm}^2 / m$$

$$s = 1000 \times \frac{A_{bar}}{A_{st.}} = 1000 \times \frac{201}{704.1} = 285.5 \text{ mm}, \quad \text{use } \phi 16 @ 280 \text{ mm top \& bottom, } A_{st} = 1000 \times \frac{201}{280} = 717.8 \text{ mm}$$

Secondary reinforcement:

$$A_{sd.} = \frac{2.2}{\sqrt{3.28s_c}} \times A_{st.} \leq 67\% \times A_{st.} = \frac{2.2}{\sqrt{3.28 \times 1.3}} \times A_{st.} \leq 0.67 \times A_{st.} = 1.06 \times A_{st.} \leq 0.67 \times A_{st.}$$

$$A_{sd} = 0.67 \times 717.8 = 481 \text{ mm}^2 / m$$

$$s = 1000 \times \frac{A_{bar}}{A_{st.}} = 1000 \times \frac{113}{481} = 235 \text{ mm} \quad \text{use } \phi 12 @ 230 \text{ mm top \& bottom}$$

-Interior Girder:

$$h = 900 + 180 = 1080 \text{ mm} = 1.08 \text{ m}$$

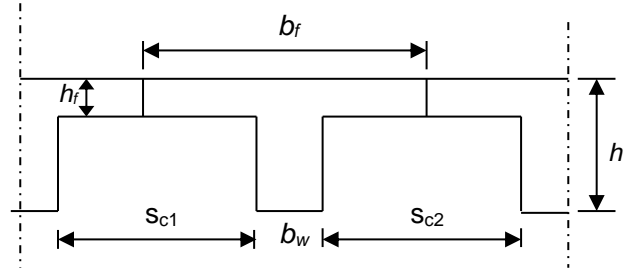
$$\text{span } l = \min.[l_{c/c}, l_c + h]$$

$$= \min.[15.6, 15 + 1.08] = 15.6 \text{ m}$$

$$b_f = \min.\left\{\frac{l}{4}, 16 \times h_f + b_w, \frac{s_{c1} + s_{c2}}{2} + b_w\right\}$$

$$= \min.\left\{\frac{15.6}{4}, 16 \times 0.18 + 0.35, \frac{1.3 + 1.3}{2} + 0.35\right\}$$

$$= \min.\{3.9, 3.23, 1.65\} = 1.65 \text{ m}$$



Dead moment:

$$W_d = W_{slab} \times b_f + \gamma_{conc.} \times b_w \times (h - h_f)$$

$$= 5.82 \times 1.65 + 24 \times 0.35 \times 0.9$$

$$= 17.163 \text{ Kn/m}$$

$$M_d = \frac{W_d \times (l)^2}{8} = \frac{17.163 \times (15.6)^2}{8} = 522.1 \text{ Kn.m}$$

Live moment:

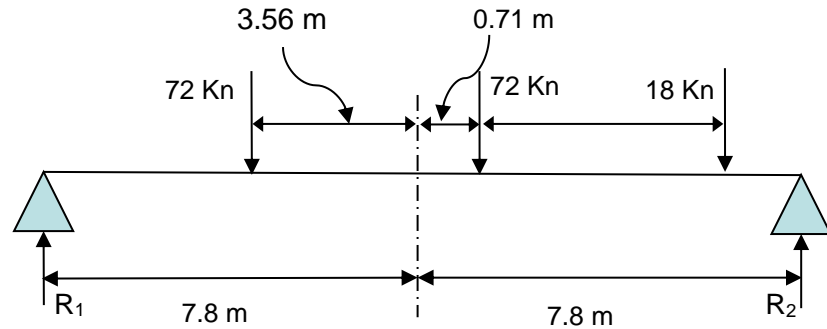
$$\sum F_y = R_y$$

$$72 + 72 + 18 = R \Rightarrow R_y = 162 \text{ Kn}$$

$$\sum M_{72} = R_y \times X$$

$$72 \times 4.27 - 18 \times 4.27 = 162 \times X$$

$$X = 1.42 \Rightarrow X/2 = 0.71 \text{ m}$$



$$\sum M_{R2} = 0$$

$$R_1 = \frac{162 \times (7.8 + 0.71)}{15.6} = 88.373 \text{ Kn}$$

$$M_{\max.} = R_1 \times (7.8 + 0.71) - 72 \times 4.27$$

$$M_{\max.} = 444.615 \text{ Kn.m}$$

$$f_{\text{int.}} = 0.66 \times s \quad \& \quad s = s_c + b_w = 1.3 + 0.35 = 1.65 \text{ m}$$

$$= 0.66 \times 1.65 = 1.089$$

$$\therefore M_l = f_{\text{int.}} \times M_{\text{max.}} = 1.089 \times 444.615 = 484.186 \text{ Kn.m}$$

Impact moment:

$$I = \frac{15.28}{l + 38.1} = \frac{15.28}{15.6 + 38.1} = 0.284 < 0.3$$

$$\therefore I = 0.284$$

$$M_l = I \times M_l = 0.284 \times 484.186 = 137.51 \text{ Kn.m}$$

$$M_t = M_d + M_l + M_l = 522.1 + 484.186 + 137.51 = 1143.8 \text{ Kn.m}$$

Use three layers and $\Phi 36$

$$d = h - \text{cover} - d_s - \phi - 40 - \frac{\phi}{2} = 1080 - 50 - 14 - 36 - 40 - \frac{36}{2} = 922 \text{ mm}$$

$$A_s = \frac{M_t}{f_s \times (d - \frac{h_f}{2})} = \frac{1143.8 \times 10^6}{170 \times (922 - \frac{180}{2})} = 8086.8 \text{ mm}^2$$

$$\text{No. of } \phi 36 = \frac{A_s}{A_{\text{bar}}} = \frac{8086.8}{1018} = 7.9 \Rightarrow \text{use } 8\phi 36$$

$$b_{\text{req}} = 2 \times \text{cover} + 2 \times d_s + n \times \Phi + (n-1)s = 2 \times 50 + 2 \times 14 + 3 \times 36 + 2 \times 36$$

$$= 308 \text{ mm} < b_w = 350 \text{ mm ok}$$

Shear design:

$$V_d = \frac{W_d \times l}{2} = \frac{17.163 \times 15.6}{2} = 155.87 \text{ Kn}$$

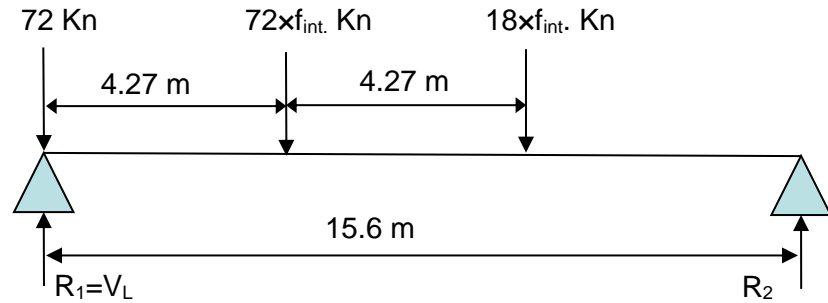
ملاحظة : اكبر قوة قص تكون عادة في المساند وتوضع اقل عجلة على المسند و لا تضرب بمعامل التوزيع

$$\sum M_{R2} = 0$$

$$V_L = \frac{72 \times (15.6) + 72 \times f_{\text{int.}} \times (15.6 - 4.27) + 18 \times f_{\text{int.}} \times (15.6 - 2 \times 4.27)}{15.6} = 137.82 \text{ Kn}$$

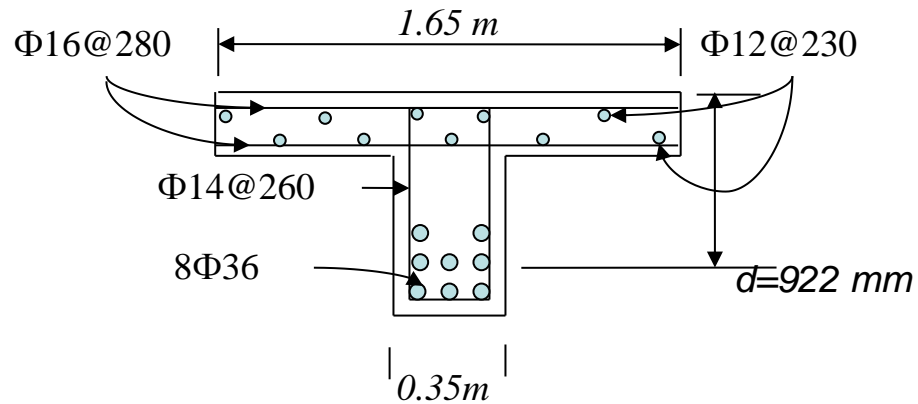
$$\begin{aligned} V_t &= V_d + V_L \times (1 + I) \\ &= 133.87 + 137.82 \times (1.284) \\ &= 310.83 \text{ Kn} \end{aligned}$$

$$\begin{aligned} V_c &= 0.079 \sqrt{f'_c} \times b_w \times d \\ &= 0.79 \sqrt{25} \times 350 \times 922 \times 10^{-3} \\ &= 127.5 \text{ Kn} \quad \& \quad 3V_c = 382.4 \text{ Kn} \end{aligned}$$



$$V_c < V_t < 3V_c$$

$$\begin{aligned} \therefore \text{Spacing} &= \min. \left\{ \frac{d}{2}, 600, \frac{3A_v f_y}{b_w}, \frac{A_v f_s d}{V - V_c} \right\} & A_v &= 2 \times \frac{\pi}{4} \times 14^2 = 307.9 \text{ mm}^2 \\ &= \min. \left\{ \frac{922}{2}, 600, \frac{3 \times 307.9 \times 400}{350_w}, \frac{307.9 \times 170 \times 922}{(310.83 - 127.5) \times 10^3} \right\} \\ &= \min. \{461, 600, 1056, 263\} & \therefore \text{use } &\phi 14 @ 260 \end{aligned}$$

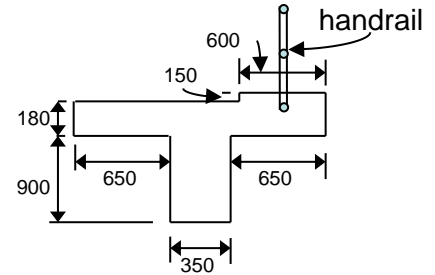


-Exterior Girder:

$$W_d = \gamma_{\text{conc.}} \times (0.9 \times 0.35 + 0.18 \times 1.65 + 0.15 \times 0.6) + W_{\text{asphalte.}} \times b_f + W_{\text{handrail}}$$

$$= 20.323 \text{ Kn/m}$$

$$M_d = \frac{W_d \times (l)^2}{8} = \frac{20.323 \times (15.6)^2}{8} = 618.226 \text{ Kn.m}$$



$$\sum M_{R1} = 0$$

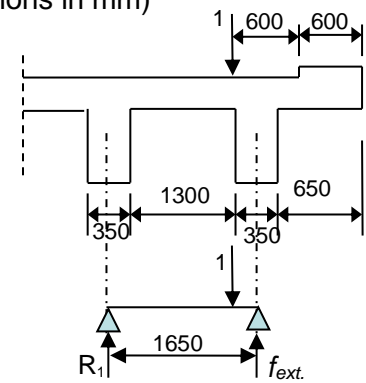
$$f_{\text{ext.}} = \frac{1 \times 1.275}{1.65} = 0.773$$

$$\therefore M_l = f_{\text{ext.}} \times M_{\text{max.}} = 0.773 \times 444.615 = 343.7 \text{ Kn.m}$$

$$M_t = M_d + M_l + M_l = 618.226 + 343.7 \times (1 + 1)$$

$$= 1059.5 \text{ Kn.m} < M_t \text{ interior}$$

(all dimensions in mm)



يوصي ال (code) بان تسليح (exterior girder) اكبر او يساوي تسليح (interior girder)

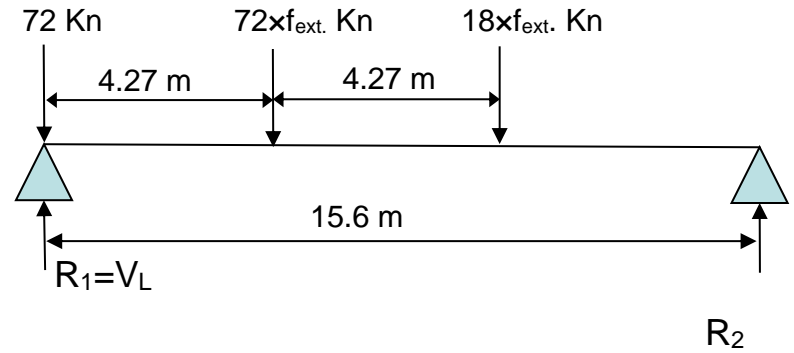
لذلك يعاد نفس تسليح ال (interior girder).

Shear design:

$$V_d = \frac{W_d \times l}{2} = \frac{20.323 \times 15.6}{2} = 158.5 \text{ Kn}$$

$$\sum M_{R2=0} = \frac{72 \times (15.6) + 72 \times f_{ext.} \times (15.6 - 4.27) + 18 \times f_{ext.} \times (15.6 - 2 \times 4.27)}{15.6} = 118.72 \text{ Kn}$$

$$\begin{aligned} V_t &= V_d + V_L \times (1 + I) \\ &= 158.5 + 118.72 \times (1.284) \\ &= 310.96 \text{ Kn} \approx V_t \text{ interior} \end{aligned}$$



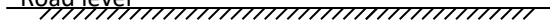
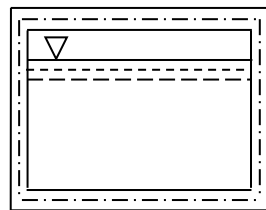
نستعمل نفس التسليح للعزم والقص بالنسبة لل (exterior girder) و ال (interior girder).

Box Culverts

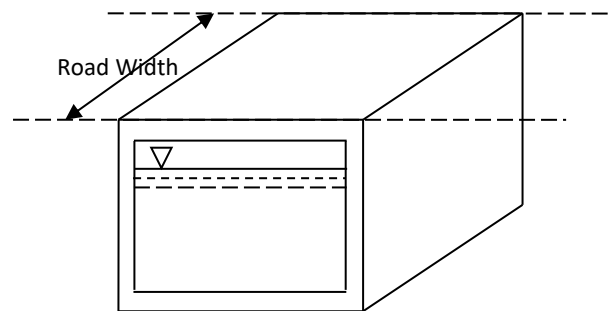
These are provided for conveying water to serve the following requirements:

- To serve as means for a cross drain.
- To provide a supporting slab for a roadway, under which the cross-drainage flows.

Road level

Box Culvert



Box Culvert

The culvert should be designed to remain safe for the following cases:

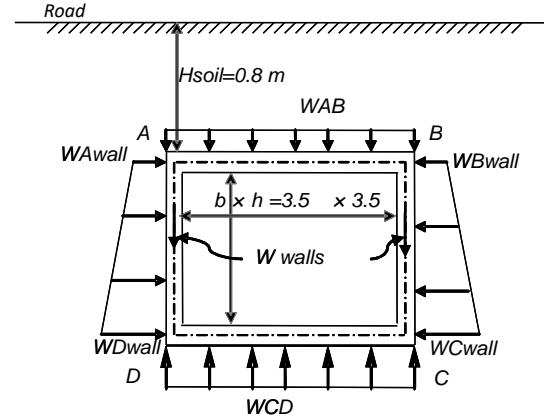
- **Case I.** When the top slab carries the dead and the live load and the culvert is empty.
- **Case II.** When the top slab carries the dead and the live load and the culvert is full of water.
- **Case III.** When the sides of the culvert do not carry the live load and the culvert is full.

Example: Design a box culvert having inside dimensions $b \times h = 3.5\text{m} \times 3.5\text{m}$. The live load on the culvert is 50 Kn/m^2 . The soil at the site weight 18 Kn/m^3 having an angle of repose of 30° . The culvert is 0.8 m below the road.

Solution:

Case I. When the top slab carries the dead and the live load and the culvert is empty.

Loads:



$$\text{total load on top slab} = \gamma_{conc.} \times t_{AB} + \gamma_{soil} \times h_{soil} + W_{live}$$

$$\text{Weight of vertical wall} = \gamma_{conc.} \times t \times h_{c/c}$$

$$+ \uparrow \sum F_y = 0$$

$$\text{Reaction of the bottom slab } (W_{CD}) = \frac{(2 \times W_{wall} + W_{AB} \times b_{c/c})}{b_{c/c}}$$

At any depth from the level of the road, lateral pressure

$$W_{B(wall)} = W_{A(wall)} = (W_L + \gamma_{soil} \times (H_{soil} + t / 2)) \times K_a$$

$$W_{C(wall)} = W_{D(wall)} = (W_L + \gamma_{soil} \times (H_{soil} + t / 2 + h_{c/c})) \times K_a$$

$$\begin{aligned}
 WL &= 50 \text{ Kn/m}^2 & H_{\text{soil}} &= 0.8 \text{ m} \\
 \gamma_{\text{conc.}} &= 24 \text{ Kn/m}^3 & b &= 3.5 \text{ m} \\
 \gamma_{\text{sol}} &= 18 \text{ Kn/m}^3 & h &= 3.5 \text{ m} \\
 \gamma_{\text{water}} &= 10 \text{ Kn/m}^3 & \text{use } t &= 0.3 \text{ m} \\
 Ka &= 0.333 & h_{c/c} = b_{c/c} &= 3.8 \text{ m} \\
 WAB &= 71.6 \text{ Kn/m}^2 \\
 W_{\text{wall}} &= 27.36 \text{ Kn/m}^2 \\
 WA_{\text{wall}} = WB_{\text{wall}} &= 22.37 \text{ Kn/m}^2 \\
 WC_{\text{wall}} = WD_{\text{wall}} &= 45.17 \text{ Kn/m}^2 \\
 WCD &= 86 \text{ Kn/m}^2
 \end{aligned}$$

joint	A		B		C		D	
member	AD	AB	BA	BC	CB	CD	DC	DA
D.F.	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
F.E.M.	-37.9	86.16	-86.2	37.89	-43.38	103.5	-103.5	43
Bal.	-24.1	-24.1	24.13	24.13	-30.06	-30.06	30.06	30
CO.	15.03	12.07	-12.1	-15.03	12.07	15.03	-15.03	-12
Bal.	-13.5	-13.5	13.55	13.55	-13.55	-13.55	13.55	14
M total	-60.5	60.54	-60.5	60.54	-74.91	74.91	-74.91	75

$$\begin{aligned}
 RA = RB &= 136.04 \text{ Kn/m} & \text{on the top slab AB} \\
 MAB \text{ midspan} &= 68.69 \text{ Kn.m/m} & \text{cut of points @ } x = 0.545 \text{ m} \\
 & & & 3.255 \text{ m} \\
 RC = RD &= 163.40 \text{ Kn/m} & \text{on the bottom slab CD} \\
 MCD \text{ midspan} &= -80.32 \text{ Kn.m/m} & \text{cut of points @ } x = 0.508 \text{ m} \\
 & & & 3.292 \text{ m} \\
 RB = RA &= 53.16 \text{ Kn/m} & \text{on the side walls} \\
 RC = RD &= 75.16 \text{ Kn/m} & \text{on the side walls}
 \end{aligned}$$

$$V_{BC} = RB - W_B \times X - 3 \times X^2$$

$$V_{BC} = 54.71 - 22.37X - 3X^2 \quad \text{put } V=0 \text{ find } X = 1.94 \text{ m}$$

$$M_{BC} = -M_{b\text{wall}} + R_{B\text{wall}} \times X - W_{B\text{WALL}} \times X^2 / 2 - (W_C - W_B) \times X^3 / (6 \times h_{c/c})$$

$$M_{BC} = -63.5 + 54.71X - 11.185X^2 - X^3 \quad \text{for } X = 1.94 \text{ M} = -6.742 \text{ Kn.m/m}$$

Case II:

$$\begin{aligned}
 WL &= 50 \text{ Kn/m}^2 & H_{\text{soil}} &= 0.8 \text{ m} \\
 \gamma_{\text{conc.}} &= 24 \text{ Kn/m}^3 & b &= 3.5 \text{ m} \\
 \gamma_{\text{siol}} &= 18 \text{ Kn/m}^3 & h &= 3.5 \text{ m} \\
 \gamma_{\text{water}} &= 10 \text{ Kn/m}^3 & \text{use } t &= 0.3 \text{ m} \\
 Ka &= 0.333 & h_{c/c} &= \\
 WAB &= 71.6 \text{ Kn/m}^2 & b_{c/c} &= 3.8 \text{ m} \\
 W_{\text{wall}} &= 27.36 \text{ Kn/m}^2 \\
 WA_{\text{wall}} = WB_{\text{wall}} &= 22.37 \text{ Kn/m}^2 \\
 WC_{\text{wall}} = WD_{\text{wall}} &= 7.167 \text{ Kn/m}^2 \\
 WCD &= 86 \text{ Kn/m}^2
 \end{aligned}$$

joint	A		B		C		D	
member	AD	AB	BA	BC	CB	CD	DC	DA
D.F.	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
F.E.M.	-19.6	86.16	-86.2	19.6	-15.94	103.5	-103.5	16
Bal.	-33.3	-33.3	33.28	33.28	-43.77	-43.77	43.77	44
CO.	21.89	16.64	-16.6	-21.89	16.64	21.89	-21.89	-17
Bal.	-19.3	-19.3	19.26	19.26	-19.26	-19.26	19.26	19
M total	-50.3	50.26	-50.3	50.25	-62.34	62.34	-62.34	62

$$\begin{aligned}
 &\text{on the top slab AB} \\
 RA = RB &= 136.04 \text{ Kn/m} \\
 MAB \text{ midspan} &= 78.98 \text{ Kn.m/m} & \text{cut of points @ } x &= 0.464 \text{ m} \\
 & & & 3.336 \text{ m} \\
 &\text{on the bottom slab CD} \\
 RC = RD &= 163.40 \text{ Kn/m} \\
 MCD \text{ midspan} &= -92.89 \text{ Kn.m/m} & \text{cut of points @ } x &= 0.389 \text{ m} \\
 & & & 3.411 \text{ m} \\
 &\text{on the side walls} \\
 RB = RA &= 29.69 \text{ Kn/m} \\
 &\text{on the side walls} \\
 RC = RD &= 26.42 \text{ Kn/m}
 \end{aligned}$$

$$V_{BC} = R_c - W_c \times X - 2 \times X^2$$

put

$$V = 0$$

find

$$V_{BC} = -26.42 + 7.167X + 2X^2$$

$$X =$$

$$2.087 \quad m$$

$$M_{BC} = -M_{cwall} + R_{cwall} \times X - W_{cWALL} \times X^2 / 2 - (W_B - W_C) \times X^3 / (6 \times hc/c)$$

$$\text{at } x = 2.087 \text{ m } M = -29.38$$

$$M_{BC} = -57.09 + 23.66X - 3.584X^2 - 0.666X^3$$

$$Kn.m/m$$

Case III:

WL =	50	Kn/m ²	H _{soil} =	0.8	m
γ _{conc.} =	24	Kn/m ³	b =	3.5	m
γ _{siol} =	18	Kn/m ³	h =	3.5	m
γ _{water} =	10	Kn/m ³	use t =	0.3	m
			hc/c =		
Ka =	0.333		bc/c =	3.8	m
WAB =	71.6	Kn/m ²			
W _{wall}	27.36	Kn/m ²			
WA _{wall} = WB _{wall} =	5.7	Kn/m ²			
WC _{wall} = WD _{wall} =	-9.5	Kn/m ²			
WCD =	86	Kn/m ²			

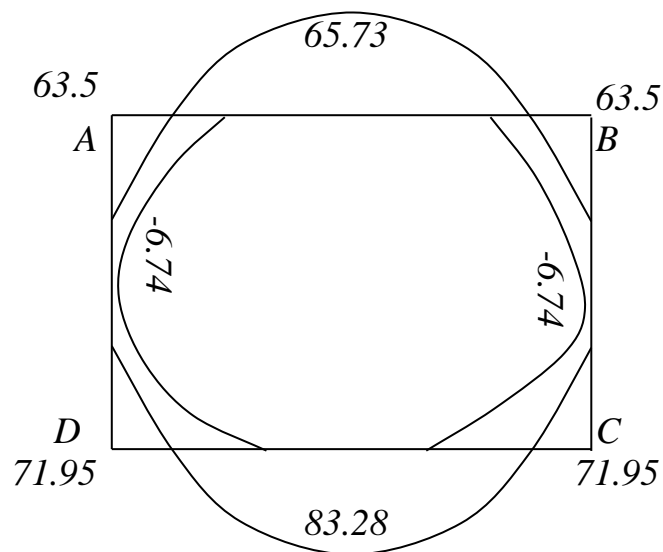
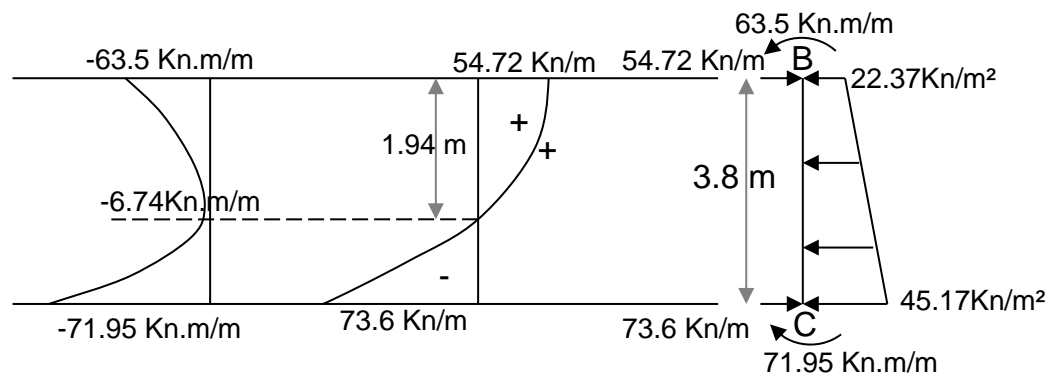
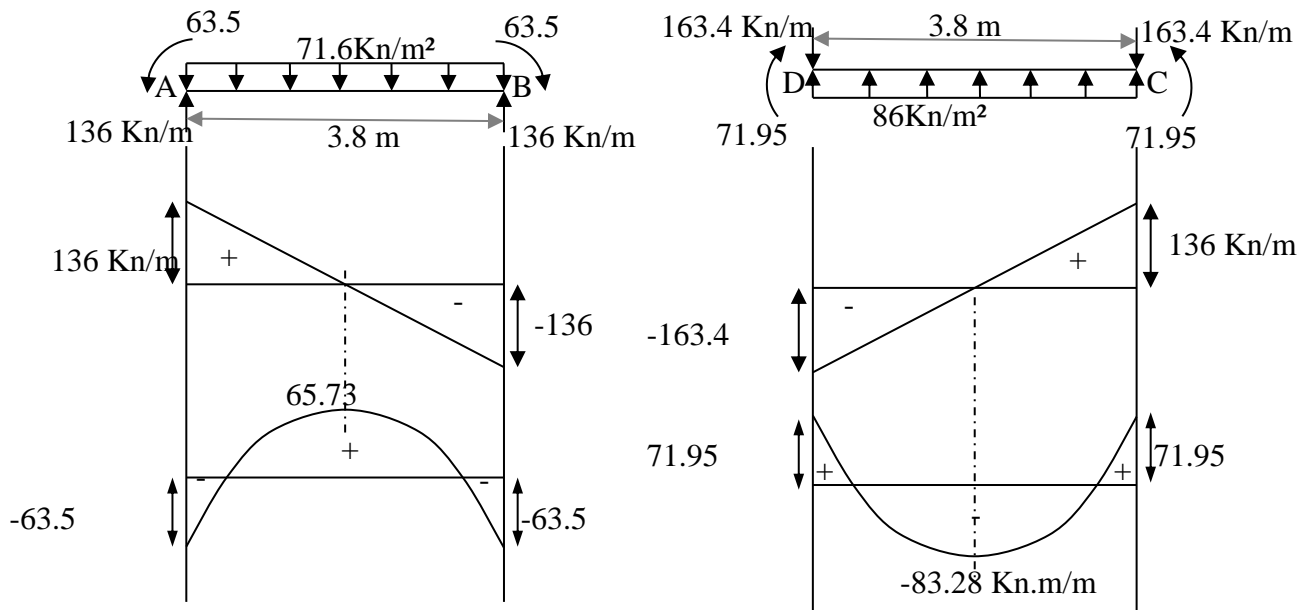
joint	A		B		C		D	
member	AD	AB	BA	BC	CB	CD	DC	DA
D.F.	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
F.E.M.	0.457	86.16	-86.2	-0.457	4.12	103.5	-103.5	-4.1
Bal.	-43.3	-43.3	43.31	43.31	-53.8	-53.8	53.8	54
CO.	26.9	21.65	-21.7	-26.9	21.65	26.9	-26.9	-22
Bal.	-24.3	-24.3	24.28	24.28	-24.28	-24.28	24.28	24
M total	-40.2	40.23	-40.2	40.23	-52.31	52.31	-52.31	52

on the

top

$$R_A = R_B = 136.04 \quad Kn/m \quad \text{slab AB}$$

$$\begin{aligned}
 MAB \text{ midspan} &= 89.01 \text{ Kn.m/m} & \text{cut of points @ } x &= m \\
 RC = RD &= 163.40 \text{ Kn/m} & \text{on the bottom slab CD} & \\
 MCD \text{ midspan} &= -102.9 \text{ Kn.m/m} & \text{cut of points @ } x &= m \\
 RB = RA &= -1.98 \text{ Kn/m} & \text{on the side walls on the side} & \\
 RC = RD &= -5.24 \text{ Kn/m} & \text{on the side walls} & \\
 V_{BC} &= R_c - W_c \times X - 2 \times X^2 & & \\
 & & \text{put } V=0 & \\
 & & \text{find } X= & \\
 V_{BC} &= -26.42 + 7.167X + 2X^2 & & 2.087 \text{ m} \\
 M_{BC} &= -M_{cwall} + R_{cwall} \times X - W_{cWALL} \times X^2 / 2 - (W_B - W_C) \times X^3 / (6 \times hc/c) & & \\
 & & \text{at } x=2.087 \text{ m } M &= -29.38 \\
 M_{BC} &= -57.09 + 23.66X - 3.584X^2 - 0.666X^3 & & \text{Kn.m/m}
 \end{aligned}$$



B.M.D. (Kn.m/m)

Circular Tanks

Example: Find the maximum reinforcement for the circular tank having the following data:

Base : fixed	Tensial strength of concrete = 2 Mpa		H= 6 m
$f'_c=20$ Mpa	f_s (hoop)=95Mpa	f_s (vertical)=135Mpa	D(inside) =16 m
$\gamma_w=10$ kn/m ³	$C=300 \times 10^{-6}$	$n=10$	$E_s=200$ Gpa

Solution:

Assume $t=250$ mm

$$\frac{H^2}{D.t} = \frac{(6)^2}{16 \times 0.25} = 9$$

$$T = c_1 \cdot \gamma H R \quad (c_1 \text{ from table A 3.1}) \quad c_1 = 0.5915$$

$$M = c_2 \cdot \gamma H^3 \quad (c_2 \text{ from table A 3.3}) \quad c_2 = 0.00335$$

$$V = c_3 \cdot \gamma H^2 \quad (c_3 \text{ from table A 3.12}) \quad c_3 = 0.166$$

$$T_{\max.} = 0.5915(480) = 283.92 \text{ kn/m} \quad @0.6H$$

$$M_{\max.} + ve = 0.00335(2160) = 7.236 \text{ kn.m/m} \quad @0.7H$$

$$M_{\max.} - ve = 0.0134(2160) = 28.944 \text{ kn.m/m} \quad @\text{the base}$$

$$V_{\max.} = 0.166(360) = 59.76 \text{ kn/m}$$

Design of hoop reinforcement (horizontal reinforcement)

$$A_s)_{hoop} = \frac{T_{\max.}}{f_s)_{hoop}} = \frac{283.92 \times 10^3}{95} = 2988.63 \text{ mm}^2 / m > A_s)_{\min} = 0.0025 A_g$$

$$= 0.0025(1000 \times 250)$$

$$= 625 \text{ mm}^2 / m$$

$$\text{use } \phi 16 \text{ m} s = 1000 \times \frac{\pi/4 (16)^2}{2988.63} = 134.5 \text{ mm} < s_{\max.} = \min.(3t, 500) = 500 \text{ mm}$$

use $\phi 16 @ 125 \text{ mm c/c}$ each face

check stress in concrete

$$f_{ct} = \frac{c \times E_s \times A_s)_{hoop} + T}{A_g + nA_s)_{hoop}} = \frac{300 \times 10^{-6} \times 200000 \times 2988.63 + 283.92 \times 10^3}{1000 \times 250 + 10 \times 2988.63} = 1.655 \text{ Mpa} < 2 \text{ Mpa}$$

Design of flexure reinforcement (vertical reinforcement)

$$r = \frac{fs}{fc} = \frac{135}{0.45 \times 20} = 15$$

$$k = \frac{n}{n+r} = \frac{10}{10+15} = 2/5, \quad j = 1 - \frac{k}{3} = 1 - \frac{2/5}{3} = 13/15$$

use $\phi 20$

$$d = 250 - 25 - \frac{20}{2} = 215 \text{ mm}$$

$$d_{req.} = \sqrt{\frac{2M}{fc.k.j.b}} = \sqrt{\frac{2 \times (28.944 \times 10^6)}{9 \times (\frac{2}{5}) \times (\frac{13}{15}) \times (1000)}} = 136.5 \text{ mm} < d \text{ o.k.}$$

$$As_1 = \frac{M}{fs.j.d} = \frac{28.944 \times 10^6}{135 \times (\frac{13}{15}) \times 215} = 1150.626 \text{ mm}^2 / m$$

$$As_2 = \frac{M}{fs.j.d} = \frac{7.236 \times 10^6}{135 \times (\frac{13}{15}) \times 215} = 287.66 \text{ mm}^2 / m$$

$$A_{s,\min} = 0.0015 A_g = 0.0015 (1000 \times 250) = 375 \text{ mm}^2 / m$$

$$\text{use } As_2 = A_{s,\min} = 375 \text{ mm}^2 / m$$

$$s_1 = 1000 \times \frac{314}{115.626} = 272.9 \text{ mm} < S_{\max.} = \min(3t, 500) = 500 \text{ mm}$$

use $\phi 20 @ 250 \text{ mm c/c}$ (inner face)

$$s_1 = 1000 \times \frac{113}{375} = 301.33 \text{ mm} < S_{\max.} = \min(3t, 500) = 500 \text{ mm}$$

use $\phi 12 @ 300 \text{ mm c/c}$ (outer face)

Check Shear:

$$V_{\text{base}} = 59.76 \text{ kn}$$

$$V_c = 0.09\sqrt{f'_c}b.d = 0.09\sqrt{20} \times 1000 \times 215 \times 10^{-3} = 86.5358 \text{ kn/m}$$

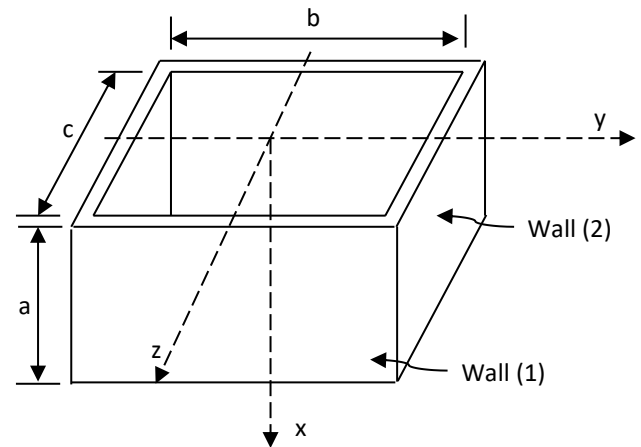
$$V_{\text{base}} < V_c \text{ o.k}$$

H.W. Re-design the tank of previous example assuming hinge base

rectangular tank

Example : Design the critical sections for the rectangular tank shown in figure , use the following data:

$a = 6\text{ m}$	$b = 15\text{ m}$	$c = 7.5\text{ m}$
$f_y = 300\text{ Mpa}$	$f'_c = 20\text{ Mpa}$	$n = 9$
top : free	bottom:hinged	$\gamma_w = 10\text{ kn/m}^3$



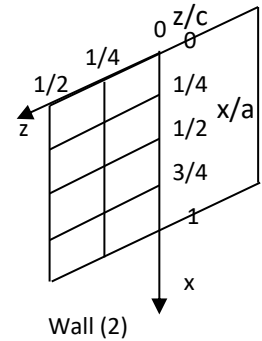
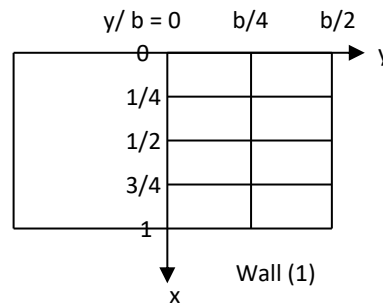
Solution :

$$\frac{b}{a} = \frac{15}{6} = 2.5$$

$$\frac{c}{a} = \frac{7.5}{6} = 1.25$$

Moment Coefficient: top: free

Bottom: hinge



	$y=0$		$y=b/4$		$y=b/2$		$z=c/4$		$z=0$	
$x/a =$	M_x	M_y	M_x	M_y	M_x	M_y	M_x	M_z	M_x	M_z
0	0	0.069	0	0.035	0	-0.092	0	-0.030	0	-0.010
1/4	0.025	0.059	0.015	0.034	-0.018	-0.089	-0.005	-0.024	-0.002	-0.003
1/2	0.045	0.048	0.031	0.031	-0.016	-0.082	0.003	-0.012	0.008	0.007
3/4	0.044	0.029	0.034	0.02	-0.012	-0.059	0.011	-0.002	0.018	0.008

$$M = C_1 \times \gamma_w \times a^3 = C_1 \times 10 \times 6^3 = C_1 \times (2160)$$

Shear Coefficient: *top: free*

Bottom: hinge

$b/a =$	1	2	3
<i>top of fixed side edge</i>	0.01	0.1	0.165
<i>mid-point of fixed side edge</i>	0.258	0.375	0.406
<i>lower third-point of fixed side edge</i>	0.311	0.406	0.416
<i>lower quarter point</i>	0.315	0.39	0.398

$$V = C_2 \times \gamma_w a^2 = C_2 \times 10 \times 6^2 = C_2 \times 360$$

To find wall thickness:

For moment:

$$n = 9 : r = \frac{f_s}{0.45 f_c'} = \frac{140}{0.45 \times 20} = 15.556 : k = \frac{9}{9 + 15.556} = 0.3667:$$

$$j = 1 - \frac{k}{3} = 1 - \frac{0.3667}{3} = 0.877$$

$$M_{\max.} = -0.092 \times 2160 = 198.75 \text{ kn.m/m } (M_y \text{ @ } x/a = 0 \text{ \& } y = b/2)$$

$$d_{\text{req.}} = \sqrt{\frac{2M}{f_c \cdot k \cdot j \cdot b}} = \sqrt{\frac{2 \times 198.75 \times 10^6}{9 \times 0.3667 \times 0.877 \times 1000}} = 370.5 \text{ mm}$$

For shear:

Maximum coeff. (@ lower third)

$$V_{\max.} = \left(\frac{0.406 + 0.416}{2} \right) \times 360 = 147.96 \text{ kn/m}$$

$$V_c = \frac{\sqrt{f'_c}}{11} \times b \times d_{req.} \rightarrow 147.96 \times 1000 = \frac{\sqrt{20}}{11} \times 1000 \times d_{req.}$$

$$d_{req.} = 363.93 \text{ mm}$$

Use $d = 400 \text{ mm} \rightarrow h = d + 50 = 450 \text{ mm}$

Flexural design:

A) Design for horizontal reinforcement (due to M & p)

1- For negative moment @ corner (junction between wall(1) & wall (2)). (due to M_y & P_2)

Shear force on wall (1) = axial force on wall (2)

$$V_1 = P_2 \left[\frac{b}{a} = 2.5 \right] \text{ for shear calculation}$$

@ Top: $\left(\frac{x}{a} = 0, y = \frac{b}{2} \right)$

$$V_1 = P_2 = \frac{0.1 + 0.165}{2} \times 360 = 47.7 \text{ kn/m}$$

$$M_y = 0.092 \times 2160 = 198.75 \text{ kn.m/m}$$

@ Mid-point $\left(\frac{x}{a} = \frac{1}{2}, y = \frac{b}{2} \right)$

$$V_1 = P_2 = \frac{0.375 + 0.406}{2} \times 360 = 140.58 \text{ kn/m}$$

$$M_y = 0.082 \times 2160 = 177.12 \text{ kn.m/m}$$

@ Lower-quarter-point $\left(\frac{x}{a} = \frac{3}{4}, y = \frac{b}{2} \right)$

$$V_1 = P_2 = \frac{0.39 + 0.398}{2} \times 360 = 141.84 \text{ kn/m}$$

$$M_y = 0.059 \times 2160 = 127.44 \text{ kn.m/m}$$

Design:

$$d'' = d - \frac{h}{2} = 400 - \frac{450}{2} = 175 \text{ mm}, \quad M_s = M + P \times d'', \quad A_{st} = \frac{M_s}{f_s j d} - \frac{P}{f_s},$$

use $\phi 25$, $A_{bar} = 490 \text{ mm}^2$,

$$A_{st \text{ min.}} = 0.0025 A_g = 0.0025 \times b \times t = 0.0025 \times 1000 \times 450 = 1125 \text{ mm}^2 / \text{m}$$

location	M (kn.m/m)	p (kn)	M _s (kn.m/m)	A _{st} (mm ² /m)	S (mm)
top $\left(\frac{x}{a} = 0, y = \frac{b}{2}\right)$	198.75	47.7	190.4025	3876.9+340.7 = 4217.6	116.3 use Φ25@110
Mid $\left(\frac{x}{a} = \frac{1}{2}, y = \frac{b}{2}\right)$	177.12	140.58	152.5185	3105.5+1004.1 =4109.7	119.23 use Φ25@110
Lower $\left(\frac{x}{a} = \frac{3}{4}, y = \frac{b}{2}\right)$	127.44	141.84	102.618	2089.5+1013.1 =3102.6	157.93 use Φ25@150

2. For positive moment @ wall (1) (due to M_y & P₁=V₂)

Shear force on wall (2) = axial force on wall (1)

$$V_2 = P_1 \left[\frac{b}{a} = 1.25 \right] \text{ for shear calculation}$$

@ Top: $\left(\frac{x}{a} = 0, y = 0\right)$ $V_2 = P_1 = (0.01 + \frac{0.1 - 0.01}{4}) \times 360 = 11.7 \text{ kn/m}$
 $M_y = 0.069 \times 2160 = 149.04 \text{ kn.m/m}$

@ Mid-point $\left(\frac{x}{a} = \frac{1}{2}, y = 0\right)$
 $V_2 = P_1 = (0.258 + \frac{0.375 - 0.258}{4}) \times 360 = 103.41 \text{ kn/m}$
 $M_y = 0.048 \times 2160 = 103.68 \text{ kn.m/m}$

@ Lower-quarter-point $\left(\frac{x}{a} = \frac{3}{4}, y = 0\right)$

$$V_2 = P_1 = \left(0.319 + \frac{0.39 - 0.315}{4}\right) \times 360 = 120.15 \text{ kn/m}$$

$$M_y = 0.029 \times 2160 = 62.64 \text{ kn.m/m}$$

location	M (kn.m/m)	p (kn)	M _s (kn.m/m)	A _{st} . (mm ² /m)	S (mm)
top $\left(\frac{x}{a} = 0, y = 0\right)$	149.04	11.7	146.99	2993+83.6 = 3076.6	159.3 use Φ25@150
Mid $\left(\frac{x}{a} = \frac{1}{2}, y = 0\right)$	103.68	103.41	85.58	1742.6+738.6 = 2481.3	197.5 use Φ25@190
Lower $\left(\frac{x}{a} = \frac{3}{4}, y = 0\right)$	62.64	120.15	41.61	847.3+858.2 = 1705.5	287.3 use Φ25@280

3. For positive moment @ wall (2) (due to M_z & P₂=V₁)

Shear force on wall (1) = axial force on wall (2)

$$V_1 = P_2 \left[\frac{b}{a} = 2.5 \right] \text{ for shear calculation}$$

@ Top: $\left(\frac{x}{a} = 0, z = 0\right)$ $V_1 = P_2 = \frac{0.1 + 0.165}{2} \times 360 = 47.7 \text{ kn/m}$
 $M_z = 0.01 \times 2160 = 21.6 \text{ kn.m/m}$

@ Mid-point $\left(\frac{x}{a} = \frac{1}{2}, z = 0\right)$ $V_1 = P_2 = \frac{0.375 + 0.406}{2} \times 360 = 140.58 \text{ kn/m}$
 $M_z = 0.007 \times 2160 = 15.2 \text{ kn.m/m}$

@ Lower-quarter-point $\left(\frac{x}{a} = \frac{3}{4}, z = 0\right)$

$$V_1 = P_2 = \frac{0.39 + 0.398}{2} \times 360 = 141.84 \text{ kn/m}$$

$$M_z = 0.008 \times 2160 = 17.28 \text{ kn.m/m}$$

location	M (kn.m/m)	p (kn)	M _s (kn.m/m)	A _{st.} (mm ² /m)	S (mm)
top $\left(\frac{x}{a} = 0, z = 0\right)$	21.6	47.7	13.25	269.8+340.7 = 610.5 use 1125	435.5 use Φ25@430
Mid $\left(\frac{x}{a} = \frac{1}{2}, z = 0\right)$	15.2	140.58	9.4	191.4+1004 = 1195	409 use Φ25@400
Lower $\left(\frac{x}{a} = \frac{3}{4}, z = 0\right)$	17.28	141.84	7.5	152.7+1013 = 1165	420.3 use Φ25@420

$$S_{\max.} = \min[3t, 500] = 500 \text{ mm}$$

B) Design for vertical reinforcement (due to M_x)

Use Φ16, $A_{bar.} = \frac{\pi}{4} \times 16^2 = 201 \text{ mm}^2$, $A_{st.min.} = 0.0015A_g = 0.0015 \times 1000 \times 450 = 675 \text{ mm}^2 / m$

	location	Coefficient	M (kn.m/m)	A _{st.} (mm ² /m)	S (mm)
wall (1) long wall	$\left(\frac{x}{a} = \frac{1}{2}, y = 0\right)$	+0.045	97.2	1979.15	101.5 use Φ16@100
wall (1) long wall	$\left(\frac{x}{a} = \frac{1}{4}, y = \frac{b}{2}\right)$	-0.018	-38.88	791.66	253.9 use Φ16@250
wall (2) short wall	$\left(\frac{x}{a} = \frac{3}{4}, z = 0\right)$	0.018	38.88	791.66	253.9 use Φ16@250
wall (2) short wall	$\left(\frac{x}{a} = \frac{1}{4}, z = \frac{c}{4}\right)$	-0.005	-10.8	220 < 675 use 675	297.78

					<i>use</i> $\Phi 16@290$
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SOIL PHYSICS

Class Notes

By

Assistant Professor

Maysam Th. Al-Hadidi

These notes are photocopies of the Textbook

“Environmental Soil Physics”

by Daniel Hillel,

Academic Press, 1998



INTRODUCTION:

Soil: is the weathered and fragmented outer layer of the earth's terrestrial surface. It is formed initially through disintegration, decomposition and decomposition rocks by physical, chemical, and biological processes.

Soil science: is the study of the soil in all its ramified and facets: as a central link in the biosphere, as a medium for the production of agricultural commodities, and as a raw material for industry and construction.

As such, it shares interests with geology, sedimentology, terrestrial ecology, and geobotany as well as with such applied sciences as agronomy and engineering.

Soil science itself is commonly divided into several subdivisions, including:

- 1- pedology (soil formation and classification).
- 2- Soil chemistry.
- 3- Soil mineralogy.
- 4- Soil biology.
- 5- Soil fertility.
- 6- soil mechanics

SOIL PHYSICS:

Soil physics is one of the major subdivisions of soil science. It seeks to define, measure, and predict the physical properties and behavior of the soil, both in its natural state and under the influence of human activity.

As physics deals in general with the forms and interactions of matter and energy, so soil physics deals specifically with the state and movement of matter and with the fluxes and transformations of energy in the soil. On the one hand, the fundamental study of soil physics aims at understanding the mechanisms governing such processes as terrestrial energy exchange, the cycles of water and of transportable materials, and the growth of plants in the field.

Increasingly, the main concern of soil physics has shifted from the laboratory to the field and from a restricted one-dimensional view to an expansive three-dimensional view interfacing with the domains of sister disciplines such as meteorology and climatology, hydrology, ecology, and geochemistry.



THE SOIL PROFILE:

The most obvious part of any soil is its surface zone. Through it, matter and energy are transported between the soil and the atmosphere. The surface may be smooth or pitted, granular or crusted and cracked, level or sloping, vegetated or fallow. Such conditions affect the processes of radiation and heat exchange, water and solute movement.

To describe the latter, we must examine the soil in depth. We can do this by digging a trench and sectioning the soil from the surface downward, thus revealing what is commonly termed the *soil profile*.

The soil profile typically consists of a succession of more-or-less distinct strata. These may result from the pattern of deposition, or sedimentation, as can be observed in wind-deposited (*aeolian*) soils and particularly in water deposited (*alluvial*) soils.

The top layer, or *A horizon*, is the zone of major biological activity and is therefore generally enriched with organic matter and darker in color than the underlying soil. Underneath the A horizon is the *B horizon*, where some of the materials (e.g., clay or carbonates) that are leached from the A horizon by percolating water tend to accumulate.

Underlying the B horizon is the *C horizon*, which is the soil's parent material. In a soil formed of bedrock in situ (called a *residual soil*), the C horizon consists of the weathered and fragmented rock material.

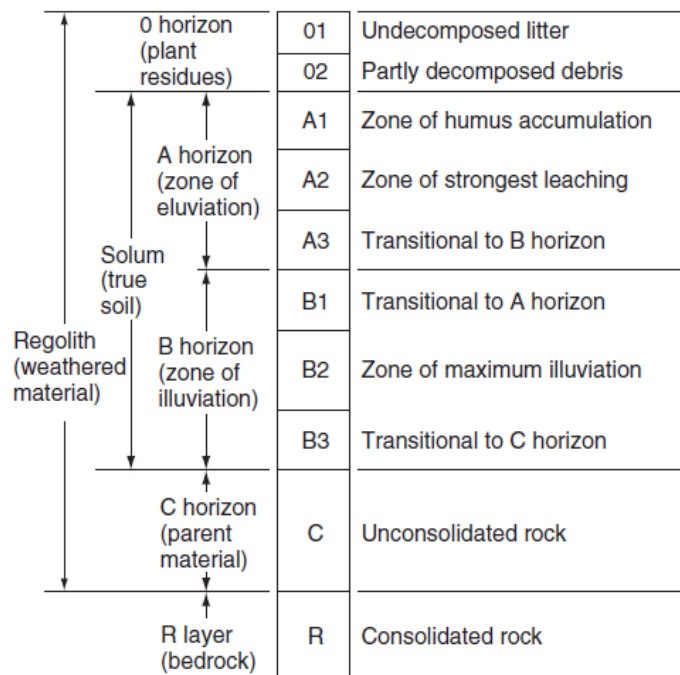


Fig. 1.2. Descriptive terminology for soil profile horizons.



VOLUME AND MASS RELATIONSHIPS OF SOIL CONSTITUENTS:

Let us consider the volume and mass relationships among the three phases of the soil, and define some basic parameters that can help to characterize the soil physically. Figure 1.4 is a schematic depiction of a hypothetical soil in which the three phases have been separated and stacked one atop the other for the purpose of showing their relative volumes and masses.

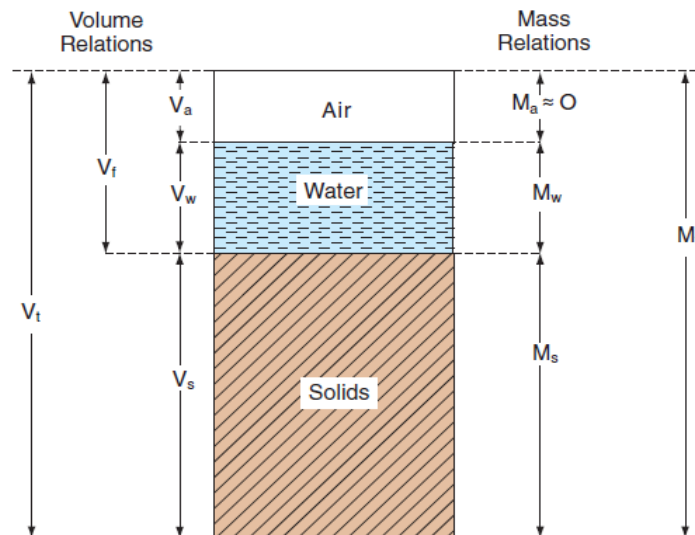


Fig. 1.4. Schematic diagram of the soil as a three-phase system.

In the figure, the masses of the phases are indicated on the right-hand side: the mass of air M_a , which is negligible compared to the masses of solids and water; the mass of water M_w ; the mass of solids M_s ; and the total mass M_t .

(These masses can also be represented in terms of their weights, being the product of each mass and gravitational acceleration.) The volumes of the same components are indicated on the left-hand side of the diagram: volume of air V_a , volume of water V_w , volume of pores $V_f = V_a + V_w$, volume of solids V_s , and the total volume of the representative sample V_t .

On the basis of this diagram, we can now define terms that are generally used to express the quantitative interrelations of the three primary soil phases.

Density of Solids (Mean Particle Density) ρ_s

$$\rho_s = M_s / V_s$$

In most mineral soils, the mean mass per unit volume of solids is about 2600–2700 kg/m³.

Sometimes the density is expressed in terms of the *specific gravity* which is the ratio of the density of any material to that of water at 4°C and at atmospheric pressure.

The latter density is about 1000 kg/m³, so the specific gravity of the solid phase in a typical mineral soil is about 2.65, a value that is numerically (though not dimensionally) equal to the density expressed in (g/cm³) units.



Dry Bulk Density ρ_b

$$\rho_b = M_s / V_t = M_s / (V_s + V_a + V_w)$$

The dry bulk density expresses the ratio of the mass of solids to the total soil volume (solids and pores together). Obviously, ρ_b is always smaller than ρ_s .

The bulk specific gravity of sandy soils with a relatively low volume of pores may be as high as 1.6, whereas that of aggregated loams and clay soils may be below 1.2. Whereas the mean particle density is typically constant, the bulk density is highly labile.

It is affected by:

- 1- The structure of the soil.
- 2- Its looseness or degree of compaction.
- 3- Its swelling and shrinkage characteristics, which depend on clay content and water content.

Total (Wet) Bulk Density ρ_t

$$\rho_t = M_t / V_t = (M_s + M_w) / (V_s + V_a + V_w)$$

This is an expression of the total mass of a moist soil per unit volume. As such, this parameter depends more strongly than does the dry bulk density on soil wetness or water content.

Dry Specific Volume v_b

$$v_b = V_t / M_s = 1 / \rho_b$$

The volume of a unit mass of a dry soil (the reciprocal of the dry bulk density) serves as another useful index of the degree of looseness or compaction of a soil body.

Porosity f

$$f = V_f / V_t = (V_a + V_w) / (V_s + V_a + V_w)$$

Porosity is an index of the relative pore space in a soil. Its value generally ranges from 0.3 to 0.6 (30–60%). Coarse-textured soils tend to be less porous than fine-textured soils, though the mean size of individual pores is greater in the former. In clayey soils, the porosity is highly variable because the soil alternately swells, shrinks, aggregates, disperses, compacts, and cracks.

Void Ratio e

$$e = V_f / V_s = (V_a + V_w) / (V_t - V_f)$$

The void ratio is also an index of the fractional pore space, but it relates that space to the volume of solids rather than to the total volume of the soil. As such, it ranges between 0.3 and 2.

Void ratio is the index generally preferred by soil engineers, while porosity is more frequently used by agronomists.



Soil Wetness (Water Content)

The water content of a soil can be expressed in various ways: relative to the mass of solids, or to the total mass, or to the volume of solids, or to the total volume, or to the volume of pores. The various indexes are defined as follows.

Mass Wetness w

$$w = M_w / M_s$$

This is the mass of water relative to the mass of dry soil particles. The standard definition of *dry soil* refers to a mass of soil dried to equilibrium (in practice, over a 24-hour period) in an oven at 105°C, though a clay soil may still contain an appreciable amount of water at that state. Mass wetness is sometimes expressed as a decimal fraction but more often as a percentage.

The water content at saturation (when all pores are filled with water) is also higher in clayey than in sandy soils. In different soils, w can range between 25% and 60%, depending on bulk density. In the special case of organic soils, such as peat or muck soils, the saturation water content on the mass basis may exceed 100%.

Volume Wetness θ

$$\theta = V_w / V_t = V_w / (V_s + V_f)$$

$$\text{Or } \theta = \rho_b * w$$

The volume wetness (often termed *volumetric water content*) is generally computed as a percentage of the total soil volume. At saturation, therefore, it is equal to the porosity.

The use of θ rather than w to express water content is often more convenient because it is more directly applicable to the computation of fluxes and water volumes added to soil by rain or irrigation and to quantities extracted from the soil by evaporation and transpiration. The volume ratio is also equivalent to the depth ratio of soil water, that is, the depth of water per unit depth of soil.

Water Volume Ratio v_w

$$v_w = V_w / V_s$$

For swelling soils, in which porosity changes markedly with wetness, it may be preferable to refer the volume of water present in a sample to the invariant volume of particles rather than to total volume. At saturation, v_w is equal to the void ratio e .

Degree of Saturation s

$$s = V_w / V_f = V_w / (V_a + V_w)$$

This index expresses the water volume present in the soil relative to the pore volume. Index s ranges from zero in a completely dry soil to unity (100%) in a saturated soil. Complete saturation, however, is hardly ever attainable in field conditions, since some air is nearly always present.



Air-Filled Porosity (Fractional Air Content) f_a

$$f_a = V_a/V_t = V_a/(V_s + V_a + V_w)$$

This is a measure of the relative content of air in the soil and as such is an important criterion of soil aeration. It is related negatively to the degree of saturation s (i.e., $f_a = 1 - s$). The relative volume of air in the soil may also be expressed as a fraction, a , of the pore volume (rather than of the total soil volume). Thus, $a = V_a/V_f = V_a/(V_a + V_w)$

Sample Problem

A sample of moist soil with a wet mass of 1.0 kg and a volume of 0.64 liters ($6.4 \times 10^{-4} \text{ m}^3$) was dried in the oven and found to have a dry mass of 0.8 kg. Assuming the typical value of particle density for a mineral soil (2650 kg/m^3), calculate the bulk density ρ_b , porosity f , void ratio e , mass wetness w_m , volume wetness θ , water volume ratio v_w , degree of saturation s , and air-filled porosity f_a .

Solution:

$$\text{Bulk density: } \rho_b = M_s/V_t = 0.8 \text{ kg}/6.4 \times 10^{-4} \text{ m}^3 = 1250 \text{ kg/m}^3$$

$$\text{Porosity: } f = 1 - \rho_b/\rho_s = 1 - 1250/2650 = 1 - 0.472 = 0.528$$

$$\text{Alternatively, } f = V_f/V_t = (V_t - V_s)/V_t$$

$$\text{and since } V_s = M_s/\rho_s = 0.8 \text{ kg}/2650 \text{ kg/m}^3 = 3.02 \times 10^{-4} \text{ m}^3$$

$$\text{hence } f = (6.4 - 3.02) \times 10^{-4} \text{ m}^3 / 6.4 \times 10^{-4} \text{ m}^3 = 0.528 = 52.8\%$$

$$\text{Void ratio: } e = V_t/V_s = (V_t - V_s)/V_s = (6.4 - 3.02) \times 10^{-4} \text{ m}^3 / 3.02 \times 10^{-4} \text{ m}^3 = 1.12$$

$$\text{Mass wetness: } w = M_w/M_s = (M_t - M_s)/M_s = (1.0 - 0.8) \text{ kg}/0.8 \text{ kg} = 0.25 = 25\%$$

$$\text{Volume wetness: } \theta = V_w/V_t = 2.0 \times 10^{-4} \text{ m}^3 / 6.4 \times 10^{-4} \text{ m}^3 = 0.3125 = 31.25\%$$

(Note: $V_w = M_w/\rho_w$, wherein $\rho_w \approx 1000 \text{ kg/m}^3$)

$$\text{Alternatively, } \theta = w\rho_b/\rho_w = 0.25(1250 \text{ kg/m}^3/1000 \text{ kg/m}^3) = 0.3125$$

$$\text{Water volume ratio: } v_w = V_w/V_s = 2.0 \times 10^{-4} \text{ m}^3 / 3.02 \times 10^{-4} \text{ m}^3 = 0.662$$

$$\text{Degree of saturation: } s = V_w/(V_t - V_s) = 2.0 \times 10^{-4} \text{ m}^3 / (6.4 - 3.02) \times 10^{-4} \text{ m}^3 = 0.592$$

$$\text{Air-filled porosity: } f_a = V_a/V_t = (6.4 - 2.0 - 3.02) \times 10^{-4} \text{ m}^3 / 6.4 \times 10^{-4} \text{ m}^3 = 0.216$$

Sample Problem

What is the equivalent depth of water contained in a soil profile 1 m deep if the mass wetness of the upper 0.4 m is 15% and that of the lower 0.6 m is 25%? Assume a bulk density of 1200 kg/m^3 in the upper layer and 1400 in the lower layer. How much water does the soil contain, in cubic meters per hectare of land?

$$\theta = w(\rho_b/\rho_w), \text{ where } \rho_w = 1000 \text{ kg/m}^3$$

$$\text{Volume wetness in the upper layer: } \theta_1 = 0.15(1200/1000) = 0.18$$

$$\text{Equivalent depth in upper 0.4 m} = 0.18 \times 0.4 \text{ m} = 0.072 \text{ m} = 72 \text{ mm}$$

$$\text{Volume wetness in lower layer: } \theta_2 = 0.25(1400/1000) = 0.35$$

$$\text{Equivalent depth in lower 0.6 m} = 0.35 \times 0.6 \text{ m} = 0.21 \text{ m} = 210 \text{ mm}$$



Total depth of water in 1-m profile = $0.072 \text{ m} + 0.210 \text{ m} = 0.282 \text{ m}$

Volume of water contained in 1-m profile per hectare ($1 \text{ ha} = 10^4 \text{ m}^2$) = $0.282 \text{ m} \times 10000 \text{ m}^2 = 2820 \text{ m}^3$



PARTICLE SIZES, SHAPES, AND SPECIFIC SURFACE

PARTICLE SIZE ANALYSIS:

It is possible to classify or group soil particles according to their sizes and to characterize the soil as a whole in terms of the relative proportions of those size groups. The groups may differ from one another in mineral composition as well as in particle size.

TEXTURAL FRACTIONS

The traditional method of characterizing particle sizes in soils is to divide the array of particle diameters into three conveniently separable size ranges known as *textural fractions* or *separates*, namely, *sand*, *silt*, and *clay*. The actual procedure of separating out these fractions and of measuring their proportions is called *mechanical analysis*, for which standard techniques have been devised. The results of this analysis yield the *mechanical composition* of the soil, a term often used interchangeably with *soil texture*.

Unfortunately, there is as yet no universally accepted scheme for classifying particle sizes. For instance, the classification standardized in America by the U.S. Department of Agriculture differs from that of the International Soil Science Society (ISSS), as well as from those promulgated by the American Society for Testing Materials (ASTM), the Massachusetts Institute of Technology (MIT), and various national institutes abroad. The classification followed by soil engineers often differs from that of agricultural soil scientists.

The same terms are used to designate differing size ranges, an inconsistency that can be confusing indeed. Several of the often-used particle size classification schemes are compared in Fig. 3.1.

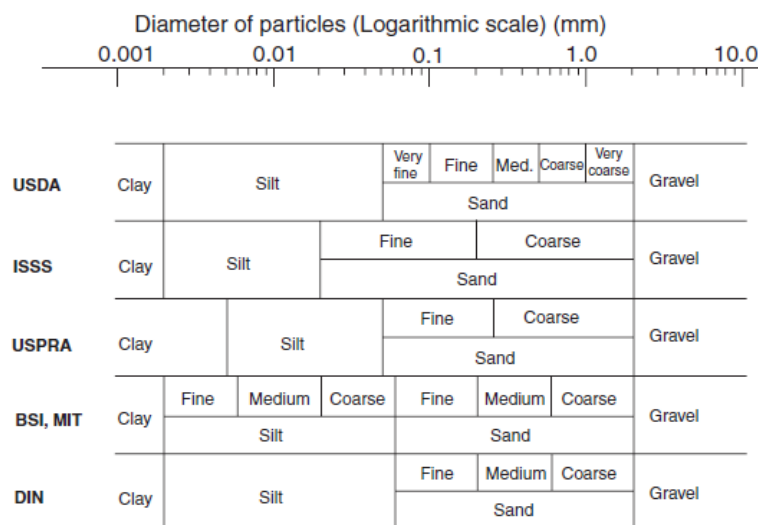


Fig. 3.1. Several conventional schemes for the classification of soil fractions according to particle diameter ranges; U.S. Department of Agriculture (USDA); International Soil Science Society (ISSS); U.S. Public Roads Administration (USPRA); British Standards Institute (BSI); Massachusetts Institute of Technology (MIT); German Standards (DIN).



An essential criterion for determining soil texture is the upper limit of particle size that is to be included in the definition of *soil material*. Some soils contain large rocks that obviously do not behave like soil, but, if numerous, might affect the behavior of the soil in bulk. The conventional definition of soil material includes particles smaller than 2 mm in diameter. Larger particles are generally referred to as *gravel*, and still larger rock fragments, several centimeters in diameter, are variously called *stones*, *cobbles*, or — if very large — *boulders*.

The largest particles that are generally recognized as soil material are designated *sand*, defined as particles ranging in diameter from 2000 μm (2 mm) to 50 μm (USDA classification) or to 20 μm (ISSS classification). The sand fraction is often further divided into subfractions such as coarse, medium, and fine sand.

The next fraction is silt, which consists of particles intermediate in size between sand and clay. Mineralogically and physically, silt particles generally resemble sand particles. However, since the silt are smaller, particles have a greater surface area per unit mass, and are often coated with strongly adherent clay, silt may exhibit, to a limited degree, some of the physicochemical characteristics generally attributed to clay.

The clay fraction, with particles ranging from 2 μm downwards, is the colloidal fraction. Clay particles are characteristically plate like or needlelike in shape and generally belong to a group of minerals known as the *alumino-silicates*. These are secondary minerals, formed in the soil itself in the course of its evolution from the primary minerals that were contained in the original rock. In some cases, however, the clay fraction may include particles (such as iron oxide and calcium carbonate) that do not belong to the alumino-silicate clay mineral category.

TEXTURAL CLASSES

The overall textural designation of a soil, called *textural class*, is conventionally based on the mass ratios of the three fractions. Soils with different proportions of sand, silt, and clay are assigned to different classes, as shown in the triangular diagram of Fig. 3.3. To illustrate the use of the textural triangle, let us assume that a soil is composed of 50% sand, 20% silt, and 30% clay. Note that the lower left apex of the triangle represents 100% sand and the right side of the triangle represents 0% sand. Now find the point of 50% sand on the bottom edge of the triangle and follow the diagonally leftward line rising from that point and parallel to the zero line for sand. Next, identify the 20% line for silt, which is parallel to the zero line for silt, namely, the left edge of the triangle. Where the two lines intersect, they meet the 30% line for clay. The soil in this example happens to fit the category of “sandy clay loam.”

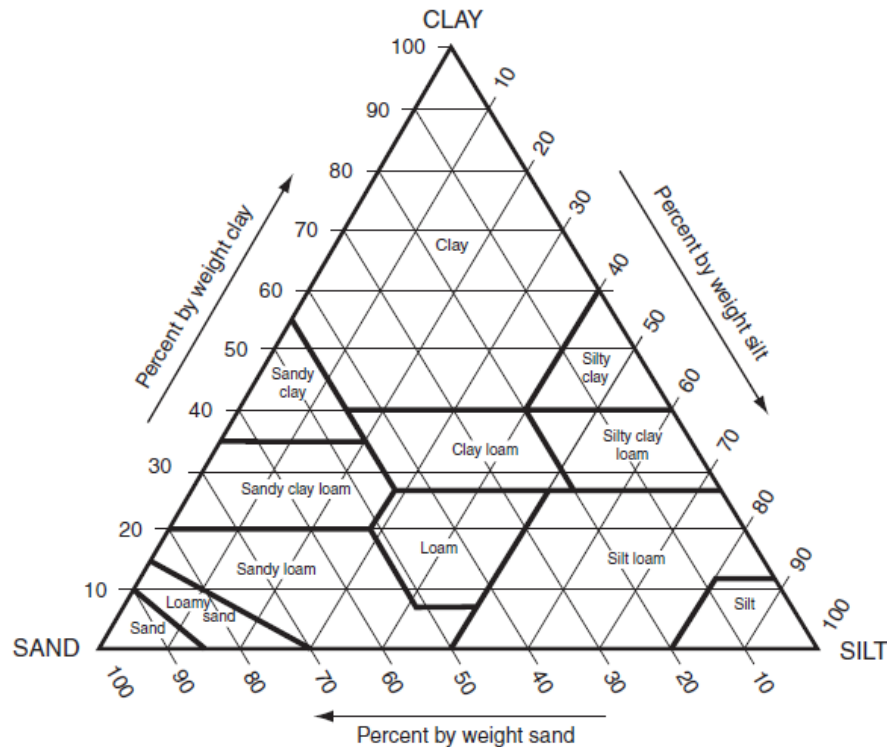


Fig. 3.3. Textural triangle, showing the percentages of clay (below 0.002 mm), silt (0.002–0.05 mm), and sand (0.05–2.0 mm) in the conventional soil textural classes.

Characterization of particle size:

Actual separation of particles into size groups can be carried out by passing the suspension through graded sieves, down to a particle diameter of approximately 0.05 mm. To separate and classify still finer particles, the method of sedimentation is usually used, based on measuring the relative settling velocity of particles of various sizes in an aqueous suspension.

The primary soil particles, often naturally aggregated, must be separated and made discrete by removal of cementing agents (such as organic matter, calcium carbonate, and iron oxides) and by deflocculating the clay. Removal of organic matter is usually achieved by oxidation with hydrogen peroxide, and calcium carbonate can be dissolved by addition of hydrochloric acid. Deflocculation is carried out by means of a chemical dispersing agent (such as sodium metaphosphate) and by mechanical agitation (shaking, stirring, or ultrasonic vibration). The function of the dispersing agent is to replace the cations adsorbed to the clay, particularly divalent or trivalent cations, with sodium. This has the effect of increasing the hydration of the clay micelles, thus causing them to repel each other rather than coalesce, as they do in the flocculated state.

According to *Stokes' law*, the velocity of a spherical particle settling under the influence of gravity in a fluid of a given density and viscosity is proportional to the square of the particle's radius.



We shall now derive this law:

$$F_g = m_s g = \rho_s v_s g = \rho_s \frac{4}{3} \pi R^3 g$$

The buoyancy force

$$F_b = m_f g = \rho_f \frac{4}{3} \pi R^3 g$$

The viscous resistance force

$$F_r = 6\pi\eta ru$$

Where η is the viscosity of the fluid, r is the radius of the particle, and u is its velocity.

Net force down = weight of particle - buoyancy force up

$$F_g = \left(\frac{4}{3}\right)\pi r^3(\rho_s - \rho_f)g$$

Setting the two forces equal, we obtain Stokes' law:

$$u_t = (2/9)(r^2g/\eta)(\rho_s - \rho_f) = (d^2g/18\eta)(\rho_s - \rho_f)$$

The use of Stokes' law to measure particle sizes is based on certain simplifying Assumptions:

1. The particles are large enough to be unaffected by random motion of the fluid molecules.
2. The particles are rigid, spherical, and smooth.
3. All particles have the same density.
4. The suspension is dilute enough so particles settle independent.
5. Fluid flow around the particles is laminar (slow enough to avoid onset of turbulence).

Sample Problem

Using Stokes' law, calculate the time needed for all sand particles (diameter $>50 \mu\text{m}$) to settle out of a depth of 0.2 m in an aqueous suspension at 30°C . How long would it take for all silt particles to settle out? How long for "coarse" clay ($>1 \mu\text{m}$)?

We use

$$t = 18h\eta/d^2g(\rho_s - \rho_f).$$

Substituting the appropriate values for depth h (0.20 m), viscosity η (0.0008 kg/m sec), particle diameter d (50 μm , 2 μm , and 1 μm for the lower limits of sand, silt, and coarse clay, respectively), gravitational acceleration g (9.81 m/sec²), average particle density ρ_s ($2.65 \times 10^3 \text{ kg/m}^3$), and water density (10^3 kg/m^3), we obtain the following answers.

(a) For all sand to settle out, leaving only silt and clay in suspension:

$$t = \frac{18 \times 0.2 \times (8 \times 10^{-4})}{(50 \times 10^{-6})^2 \times 9.81 \times (2.65 - 1.0) \times 10^3} = 71 \text{ sec}$$



(b) For all silt to settle out, leaving only clay in suspension:

$$t = \frac{18 \times 0.2 \times (8 \times 10^{-4})}{(2 \times 10^{-6})^2 \times 9.81 \times 1.65 \times 10^3} = 44,500 \text{ sec} = 12.36 \text{ hr}$$

(c) For all coarse clay to settle out, leaving only fine clay in suspension:

$$t = \frac{18 \times 0.2 \times (8 \times 10^{-4})}{(1 \times 10^{-6})^2 \times 9.81 \times 1.65 \times 10^3} = 178,800 \text{ sec} = 49.44 \text{ hr}$$

PARTICLE SIZE DISTRIBUTION

Any attempt to divide into discrete fractions what is usually a continuous array of particle sizes is necessarily arbitrary, and the further classification of soils into distinct textural classes is doubly so. Although this approach is widely followed and in some ways useful, greater information is to be gained from measuring and displaying the complete array and distribution of particle sizes. Figure 3.4 presents typical *particle size distribution curves*.

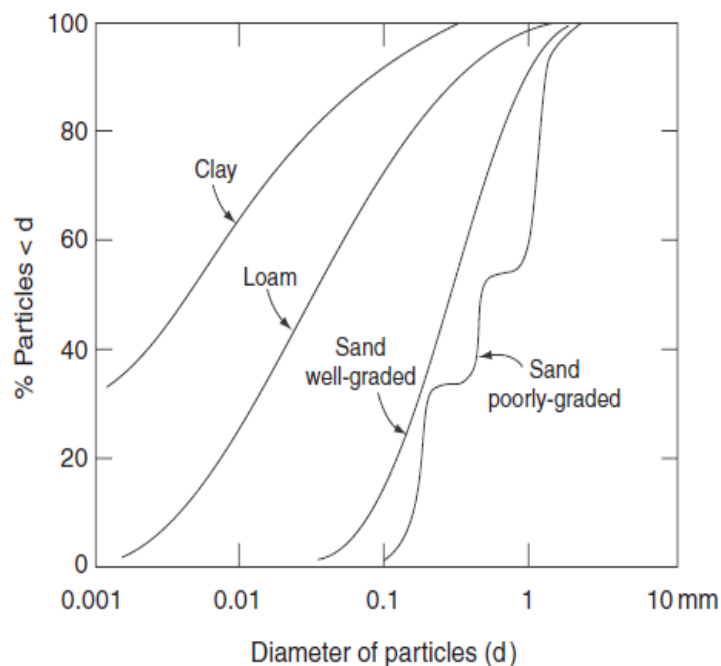


Fig. 3.4. Particle size distribution curves for various types of soil material (schematic).

STRUCTURE OF CLAY MINERALS:

The most prevalent minerals in the clay fraction of temperate region soils are silicate clays, whereas in tropical regions hydrated oxides of iron and aluminum are more prevalent.

The most common mineral is *kaolinite*. Other minerals in the same group are *halloysite* and *dickite*.

Clay particles are generally platy shape



SPECIFIC SURFACE

The specific surface of soil material can be defined as the surface area a_s of particles per unit mass (a_m) or per unit volume of particles (a_v) or per unit bulk volume of soil (a_b):

$$a_m = A_s/M_s$$

$$a_v = A_s/V_s$$

$$a_b = A_s/V_t$$

where M_s is the mass of particles of volume V_s contained in a bulk volume V_t of soil.

Specific surface has traditionally been expressed in terms of square meters per gram (a_m) or square meters per cubic centimeter (a_v). To convert from m^2/gm to m^2/kg , one need only multiply by 10^3 . To convert from m^2/cm^3 to m^2/m^3 , one needs to multiply by 10^6 .

Specific surface obviously depends on the sizes of the soil particles. It also depends on their shapes. Flattened or elongated particles expose greater surface per unit mass or volume than do equidimensional (cubical or spherical) particles. Since clay particles are generally platy, they contribute more to the overall specific surface of a soil than is indicated by their small size alone.

MEASURING SPECIFIC SURFACE BY ADSORPTION

The usual procedure for determining surface area is to measure the amount of gas or liquid needed to form a *monomolecular layer* over the entire surface in a process of adsorption. The standard method is to use an inert gas such as nitrogen.

Sample Problem

Estimate the approximate specific surface (m^2/g) of a soil composed of 10% coarse sand (average diameter 1 mm), 20% fine sand (average diameter 0.1 mm), 30% silt (average diameter 0.02 mm), 20% kaolinite clay (average platelet thickness 4×10^{-8} m), 10% illite clay (average thickness 0.50×10^{-8} m), and 10% montmorillonite (average thickness 10^{-9} m). For the sand and silt fractions, we use Eq.

$$a_m = (6/2.65)[(0.1/0.1) + (0.2/0.01) + (0.3/0.002)] = 0.0387 \text{ m}^2/\text{g} = 38.7 \text{ m}^2/\text{kg}$$

For the clay fraction, we use Eq. in summation form to include the partial specific surface values for kaolinite, illite, and montmorillonite, respectively:

$$\begin{aligned} a_m &= (0.2 \times 0.75)/(400 \times 10^{-8}) + 0.1 \times 0.75/(50 \times 10^{-8}) + 0.1 \times 0.75/(10 \times 10^{-8}) \\ &= 3.78 \text{ m}^2/\text{g} \text{ (kaol.)} + 15.09 \text{ m}^2/\text{g} \text{ (ill.)} + 75.45 \text{ m}^2/\text{g} \text{ (mont.)} \\ &= 94.32 \text{ m}^2/\text{g} = 94,320 \text{ m}^2/\text{kg} \end{aligned}$$

Total for the soil $= 0.0387 + 94.32 = 94.36 \text{ m}^2/\text{g} = 94,360 \text{ m}^2/\text{kg}$

Note: The clay fraction, only 40% of the soil mass, accounts for 99.96% of the specific surface. Montmorillonite alone (10% of the mass) accounts for nearly 80% of this soil's specific surface.



SOIL STRUCTURE AND AGGREGATION

SOIL STRUCTURE:

Is the arrangement or organization of the particles in the soil.

Since soil particles differ in shape, size, and orientation and can be variously associated and interlinked, the mass of them can form complex and irregular patterns that are difficult to characterize in exact geometric terms.

A further complication is the inherently unstable nature of soil structure and its nonuniformity in space. Soil structure is affected by changes in climate, biological activity, and soil management practices, and it is vulnerable to destructive forces of a mechanical and physicochemical nature.

For these various reasons, we have no truly objective or universally applicable way to measure soil structure.

TYPES OF SOIL STRUCTURE:

In general, we can recognize three broad categories of soil structure:

- 1- *Single grained*: particles are entirely unattached to one another, the structure is completely loose, and such soils were labeled *as structure less*.
- 2- *Massive*: the soil is tightly packed in large cohesive blocks, as is sometimes the case with dried clay.
- 3- *Aggregated*: Between these two extremes, we can recognize an intermediate condition in which the soil particles are associated in quasi-stable small clods known as *aggregates*.

STRUCTURE OF GRANULAR SOILS:

The structure of most coarse-textured soils is single grained, because there is little tendency for the grains to adhere and form aggregates.

The arrangement and internal mode of packing of the grains depends on their distribution of sizes and shapes as well as the manner in which the material had been deposited or formed in place.

STRUCTURE OF AGGREGATED SOILS:

In soils with an appreciable content of clay, the primary particles tend, under favorable circumstances, to group themselves into composite structural units known as *aggregates*.

The visible aggregates, which are generally of the order of several millimeters to several centimeters in diameter, are often called *peds* or *macroaggregates*. These are usually assemblages of smaller groupings, or *microaggregates*, which themselves are associations of the ultimate structural units, that is, the flocs, clusters, or packets of clay particles.



CHARACTERIZATION OF SOIL STRUCTURE:

The structure of the soil can be studied directly by microscopic observation of thin slices under polarized light. The arrangements of minute clay particles can be examined by means of electron microscopy, using either transmission or scanning techniques. The structure of single-grained soils, as well as of aggregated soils, can be expressed quantitatively in terms of the total porosity and of the pore size distribution.

Total porosity f of a soil sample is usually computed from the measured bulk density ρ_b , using the following equation:

$$f = 1 - \rho_b / \rho_s$$

Where : ρ_s is the average particle density.

Bulk density is generally measured by means of a *core sampler* designed to extract “undisturbed” samples of known volume from various depths in the profile.

It is sometimes possible to divide pore size distribution into two distinguishable ranges:

- 1- *macropores*: are mostly the interaggregate spaces, which serve as the principal avenues for the infiltration and drainage of water and for aeration.
- 2- *micropores*: are the intraaggregate capillaries responsible for the retention of water and solutes.

SHAPES OF AGGREGATES:

The shapes of aggregates can be classified as follows:

1. *Platy*: Horizontally layered, thin and flat aggregates resembling wafers. Such structures occur, for example, in recently deposited clay soils.
2. *Prismatic* or *columnar*: Vertically oriented pillars, up to 15 cm in diameter. Such structures are common in the B horizon of clayey soils, particularly in semiarid regions. Where the tops are flat, these vertical aggregates are called *prismatic*, and where rounded, *columnar*.
3. *Blocky*: Cubelike blocks of soil, up to 10 cm in size, sometimes angular with well-defined planar faces. These structures occur most commonly in the upper part of the B horizon.
4. *Spherical*: Rounded aggregates, generally not much larger than 2 cm in diameter, often found in a loose condition in the A horizon. Such units are called *granules* and, where particularly porous, *crumbs*.

AGGREGATE SIZE DISTRIBUTION:

Aggregate size distribution is an important determinant of the soil's pore size distribution and has a bearing on the erodibility of the soil surface, particularly by



wind. In the field, adjacent aggregates often adhere to one another, though of course not as tenaciously as do the particles within each aggregate.

Separating and classifying soil aggregates necessarily involves a disruption of the original, in situ structural arrangement. The application of too great a force may break up the aggregates themselves. Hence the determination of aggregate size distribution depends on the mechanical means employed to separate the aggregates.

Various indexes have been proposed for the distribution of aggregate sizes. If a single characteristic parameter is desired (so as to allow correlation with such factors as erosion, infiltration, evaporation, and aeration), a method must be adopted for assigning a appropriate weighting factor to each size range of aggregates. One of the most widely used indexes is the *mean weight diameter*, based on weighting the masses of aggregates of the various size classes according to their respective sizes. The mean weight diameter X is thus defined by the following equation:

$$X = \sum_{i=1}^n x_i w_i$$

Here x_i is the mean diameter of any particular size range of aggregates separated by sieving and w_i is the weight of the aggregates in that size range as a fraction of the total dry weight of the sample.

An alternative index of aggregate size distribution is the *geometric mean diameter* Y , calculated according to the following equation:

$$Y = \exp \left[\frac{\left(\sum_{i=1}^n w_i \log x_i \right)}{\left(\sum_{i=1}^n w_i \right)} \right]$$

where w_i is the weight of aggregates in a size class of average diameter x_i and the denominator $\sum w_i$ (for i values from 1 to n) is the total weight of the sample.

Sample Problem

Calculate the mean weight diameters of the assemblages of aggregates given in Table 5.3. The percentages refer to the mass fractions of dry soil in each diameter range. First we determine the mean diameters of the seven aggregate diameter ranges:

Range: 0–0.5, 0.5–1, 1–2, 2–5, 5–10, 10–20, 20–50 mm

Mean: 0.25, 0.75, 1.5, 3.5, 7.5, 15, 35 mm

Recall that the mean weight diameter X is defined by Eq.

$$X = \sum_{i=1}^n x_i w_i$$

Hence, for the dry-sieved virgin soil,



$$Y = (0.25 \times 0.1) + (0.75 \times 0.1) + (1.5 \times 0.15) + (3.5 \times 0.15) + (7.5 \times 0.2) + (15 \times 0.2) + (35 \times 0.1) = 8.85 \text{ mm}$$

For the dry-sieved cultivated soil,

$$X = (0.25 \times 0.25) + (0.75 \times 0.25) + (1.5 \times 0.15) + (3.5 \times 0.15) + (7.5 \times 0.1) + (15 \times 0.07) + (35 \times 0.03) = 4.30 \text{ mm}$$

For the wet-sieved virgin soil,

$$X = (0.25 \times 0.3) + (0.75 \times 0.15) + (1.5 \times 0.15) + (3.5 \times 0.15) + (7.5 \times 0.15) + (15 \times 0.05) + (35 \times 0.05) = 4.56 \text{ mm}$$

For the wet-sieved cultivated soil,

$$X = (0.25 \times 0.5) + (0.75 \times 0.25) + (1.5 \times 0.15) + (3.5 \times 0.05) + (7.5 \times 0.04) + (15 \times 0.01) + (35 \times 0.0) = 1.16 \text{ mm}$$

Note: Wet sieving reduced the mean weight diameter from 8.85 to 4.56 mm in the virgin soil and from 4.30 to 1.16 mm in the cultivated soil. This indicates the degree of instability of the various aggregates under the slaking effect of immersion in water. The influence of cultivation is generally to reduce the water stability of soil aggregates and hence to render the soil more vulnerable to crusting and erosion processes.

TABLE 5.3 Aggregate Data for Sample Problem

Aggregate diameter range (mm)	Dry sieving		Wet sieving	
	Virgin soil (%)	Cultivated soil (%)	Virgin soil (%)	Cultivated soil (%)
0.0–0.5	10	25	30	50
0.5–1.0	10	25	15	25
1–2	15	15	15	15
2–5	15	15	15	5
5–10	20	10	15	4
10–20	20	7	5	1
20–50	10	3	5	0

AGGREGATE STABILITY:

The ability of soil aggregates to withstand disruptive forces, whether such forces are imposed mechanically (as during tillage or trampling) or by the action of water (causing swelling, slaking, and dispersion of clay).

it expresses the resistance of aggregates to breakdown when subjected to potentially disruptive processes. The reaction of a soil to forces acting on it depends not only on the soil itself but also on the nature of the forces and the manner they are applied.



HYDROLOGY CIRCLE IN FIELD:

To understand changes in soil moisture we must start with an understanding of the sources and losses of water.

Sources of water to the biosphere from the atmosphere include precipitation.

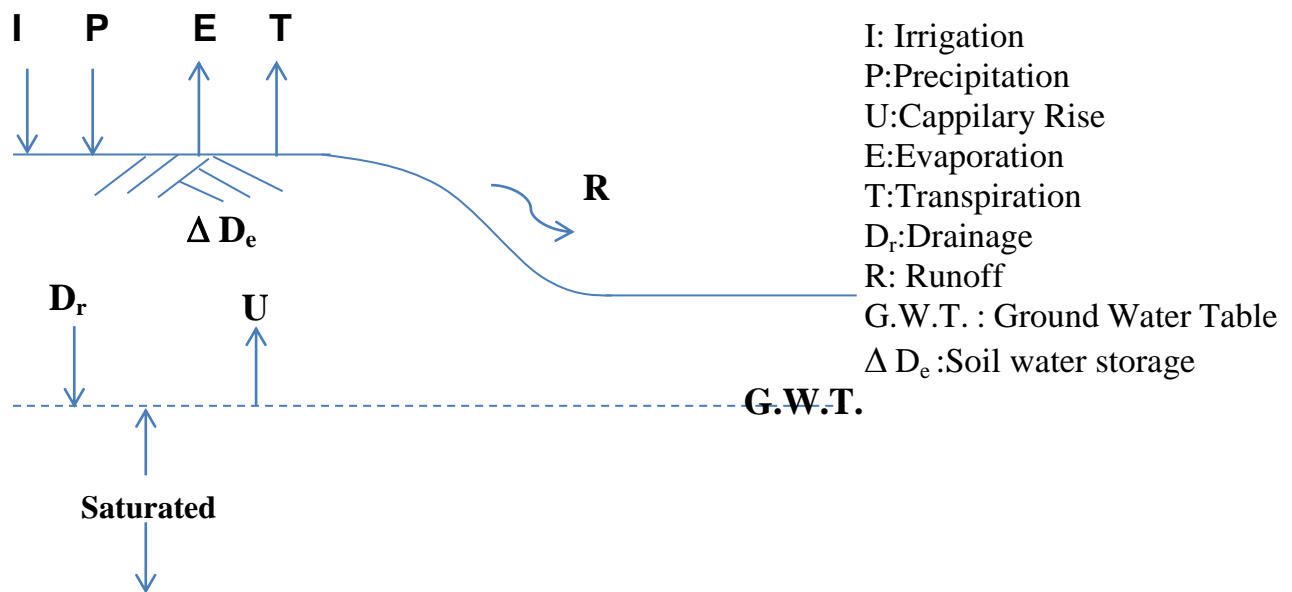
Sources of water from the soil include capillary rise from the water table, direct axis of the water table by roots and hydraulic lift of moisture by roots.

And the other source of water to the soil by irrigation.

Losses of water from the biosphere include losses to the atmosphere and to deep soil layers.

Losses to the atmosphere include direct evaporation of surface water, evaporation from the soil matrix, plant transpiration. Losses through the soil column involve transport via percolation and saturated flow and unsaturated flow.

Flow through the soil column is vertical and lateral, a distinctly three-dimensional process.



Sources = losses + soil water storage

$$I + P + U = E + T + D_r + R + \Delta D_e$$

Soil Water

Soil Water is classified according to how "tightly" it is being held in the Soil.

The following below the most important water content which are defined according to the type of the water:

- 1- **Field capacity (fc):** Free water or gravitational water will drain from a soil until the soil water potential reaches **-1/3 bar**.



Gravitational water is not considered available to plants because it is in the soil only a short time and reduces oxygen levels to the point where the plant will not be absorbing water anyway.

The time dependence of soil wetness in the upper zone is illustrated in Fig. 16.5 for a sandy soil, in which the unsaturated conductivity falls off rapidly with increasing suction, and for a clayey soil, in which the decrease of conductivity is more gradual and hence redistribution tends to persist longer.

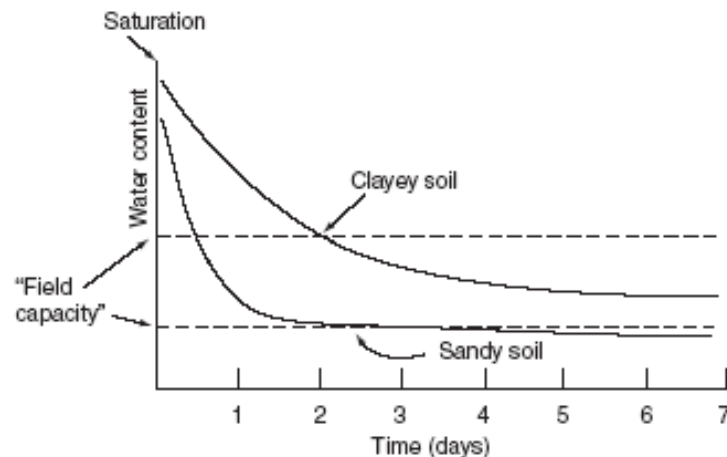


Fig. 16.5. The monotonic decrease of soil wetness with time in the initially wetted zone of a clayey and a sandy soil during redistribution. The dashed lines show the wetness values remaining in each soil after two days.

- 2- **Permanent Wilting point (pwp):** As the soil continues to dry--or water is used by plants--more and more energy is needed by the plants to remove the water. Eventually a point is reached where the plant can no longer remove water. This is called the **wilt point** and occurs at -15 bars water potential for most plants.
- 3- **Available water:** From - 1/3 to -15 bars is the zone of **available water**.

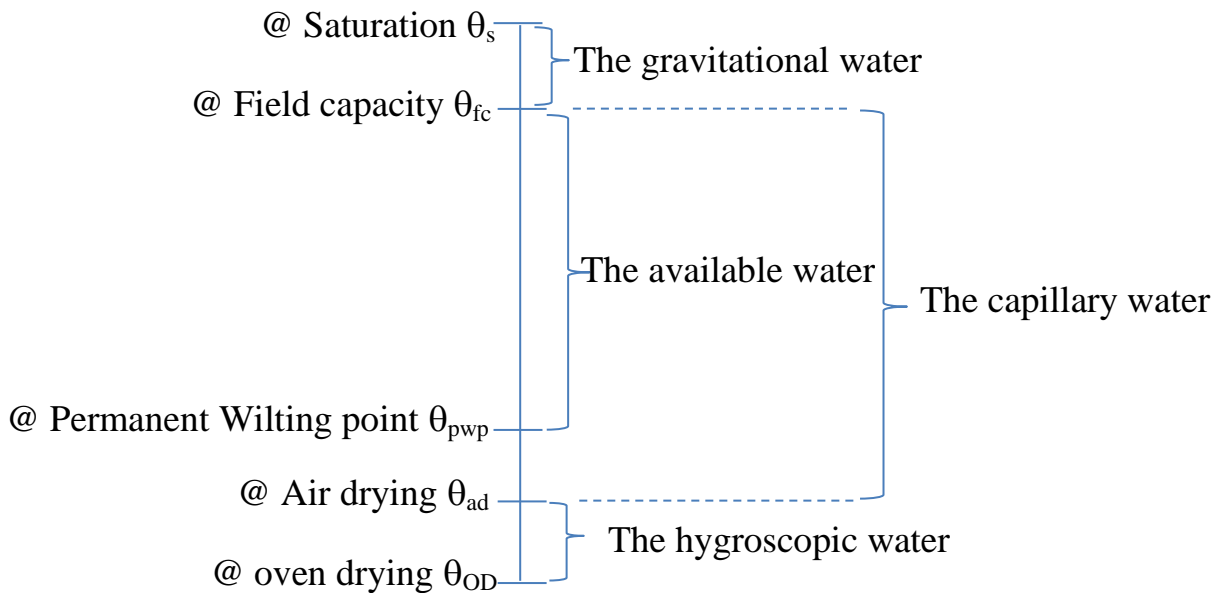
If the soil dries to an **air dry** state, the potential is -31 bars. (This assumes that the air has 100% relative humidity.) Plants cannot exert enough tension to pull water away from the soil. Tension is used to express water potential with positive numbers. So a tension of +15 bars equals a potential of -15 bars.

Additional drying requires putting the soil in an oven to drive off the tightly held water. Water is held in the soil like a series of beads, the farther the beads are from the soil particle, the weaker they are held by cohesion. The investigation in the laboratory will be using the soil placed in the drying cans last week. The sample in the lab was somewhere between field capacity and the wilt point (that is, between - 1/3 and -15 bars).

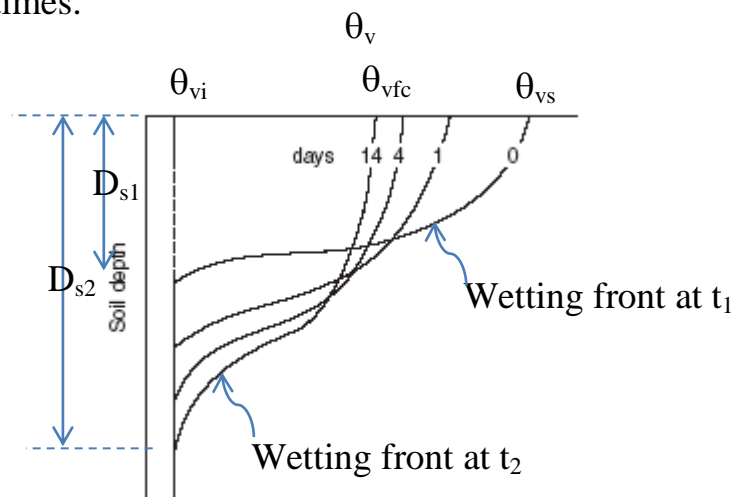
You let the sample air dry, and later it was placed in the oven and dried, then weighed to obtain the **oven dry** weight. When working with determining % water by weight or volume, we always use the oven dry weight as the standard value with which to compare.



Figure below show the graduated of the water content from the saturation to the total drying.



The figure below show the variation of volumetric water content, (θ_v) with depth and with different times.



$$D_e = \Delta\theta * D_s = (\theta_{vfc} - \theta_{vi}) * D_s$$

Where:

D_s : soil layer depth

MEASUREMENT OF SOIL WETNESS

The most important methods are:

- 1- Gravimetric method: The traditional (gravimetric) method of measuring mass wetness consists of removing a sample by augering into the soil and



then determining its moist and dry weights. The *moist weight* is determined by weighing the sample as it is at the time of sampling; the *dry weight* is obtained after drying the sample to a constant weight in an oven.

- 2- Neutron moderation method: The method is practically independent of temperature and pressure. Its main disadvantages, are the high initial cost of the instrument, low degree of spatial resolution, difficulty of measuring moisture in the soil surface zone, and the health hazard associated with exposure to neutron and gamma radiation. The instrument, known as a *neutron moisture meter* consists of two main components: (a) a *probe* (containing a *source of fast neutrons* and a *detector of slow neutrons*), which is lowered into an *access tube* inserted vertically into the soil, and (b) a *scaler* or *rate meter* (usually battery powered and portable) to monitor the flux of the slow neutrons that are scattered and attenuated in the soil.

Sample Problem

You are required to know how to calculate the amount of water in a soil given various water potentials. This will help you understand that it is not the total amount of water in a soil that determines if water is available to plants, but is the plant available water.

DATA:

Soil Core Volume = **250** cc (for each soil core below)

Weight of soil core at -1/3 bar (field capacity) = **420** g (July 4, 19??)

Weight of soil core at -15 bar (wilt point) = **350** g

Weight of soil core at present field condition = **395** g (on July 10)

Weight of Oven dry soil core = **300** g

Questions:

1) What is the Bulk Density?

Answer: B.D. = $300 \text{ g} / 250 \text{ cc} = \mathbf{1.2 \text{ g/cc}}$ (remember, always use oven dry weight)

2) What is the % water by weight at field capacity?

FC - Oven dry

Answer: $420 \text{ g} - 300 \text{ g} = 120 \text{ g}$ of water

$120 \text{ g water} / 300 \text{ g soil} = 0.4$ (*use oven dry weight)

$0.4 \times 100 = \mathbf{40\% \text{ water by weight at field capacity}}$

3) What is the % water by volume at field capacity?

FC - Soil Volume =

Answer: $420 \text{ g} - 300 \text{ g} = 120 \text{ g}$ of water

$120 \text{ g water} / 250 \text{ cc soil} = 0.48$

$0.48 \times 100 = \mathbf{48\% \text{ water by volume at field capacity}}$

(Another way to calculate this is: $\text{BD} \times \% \text{ water wt.} = \% \text{ water by volume}$):

$1.2 \times 40\% \text{ water by weight} = 48\% \text{ water by volume}$



4) What is the total possible % Available Water-holding Capacity (AWC) by volume?

$$(AWC = FC - WP)$$

$$FC - WP / \text{Soil Volume} =$$

$$\text{Answer: } (420-350) / 250 = 0.28 \times 100 = \mathbf{28\%}$$

$$\text{available water } [(FC-WP) / \text{vol.}]$$

5) How many inches of AWC are in the upper 5 ft. of soil?

$$\text{inches of soil} \times \% \text{ AWC} =$$

$$\text{Answer: } 5\text{ft.} \times (12"/\text{ft.}) \times 0.28 = \mathbf{16.8" \text{ of AWC in upper 5 ft. of soil.}}$$

6) How many inches of available water are left in the soil at present field condition? (ΔD_e)

Field Cond. = 395 and Wilt Pt. = 350; therefore:

$$(395-350)/250 = 45/250 = .18 (\% \text{AWC by Vol.})$$

and

$$0.18 \times 60" (\text{of soil}) = \mathbf{10.8" \text{ of water available in upper 5 feet.}}$$

In other words, the soil has lost 6" of water (16.8-10.8) since it was at field capacity.

7) What is the depth of wetting for a 1.5 inch rainfall event (D_e) for the soil at Field Condition?

Answer : Field Capacity - Field Cond./ vol $\times 100 = \% \text{ water by vol between field condition and Field Capacity. or } 420 - 395/250 = 25/250 = 0.1 \text{ or } 10\%$

thus 1.5 inches of rain will infiltrate ($1.5 / 0.1$) = **15 inches.** of soil because

$$\text{Inches of water} = \% \text{ Water vol} \times \text{Soil Depth. or } 1.5 = 0.1 \times ? \text{ inches of soil}$$

Sample Problem

The data in Table below were obtained before and after irrigation. From these data, calculate the mass and volume wetness values of each layer before and after the irrigation, and determine the amount of water (in millimeters) added to each layer and to the **profile** as a whole.

Data for Sample Problem

Sampling time	Sample number	Depth (m)	Bulk density (kg/m ³)	Wet sample + container (kg)	Dry sample + container (kg)	Container (kg)
Before irrigation	1	0.0-0.4	1.2×10^3	0.160	0.150	0.05
	2	0.6-1.0	1.5×10^3	0.146	0.130	0.05
After irrigation	3	0.0-0.4	1.2×10^3	0.230	0.200	0.05
	4	0.6-1.0	1.5×10^3	0.206	0.170	0.05



Solution:

Using eq. $w = M_w/M_s$

we obtained the following mass wetness values:

$$w_1 = (0.160 - 0.150)/(0.150 - 0.050) = 0.1$$

$$w_2 = (0.146 - 0.130)/(0.130 - 0.050) = 0.2$$

$$w_3 = (0.230 - 0.200)/(0.200 - 0.050) = 0.2$$

$$w_4 = (0.206 - 0.170)/(0.170 - 0.050) = 0.3$$

Using eq. $\theta = \rho_b * w$

we obtain the following volume wetness values:

$$\theta_1 = 1.2 \times 0.1 = 0.12$$

$$\theta_2 = 1.5 \times 0.2 = 0.30$$

$$\theta_3 = 1.2 \times 0.2 = 0.24$$

$$\theta_4 = 1.5 \times 0.3 = 0.45$$

Using Eq. $D_e = \theta * D_s$

we obtain the following water depths per layer:

$$D_{e1} = 0.12 \times 400 \text{ mm} = 48 \text{ mm}$$

$$D_{e2} = 0.30 \times 600 \text{ mm} = 180 \text{ mm}$$

$$D_{e3} = 0.24 \times 400 \text{ mm} = 96 \text{ mm}$$

$$D_{e4} = 0.45 \times 600 \text{ mm} = 270 \text{ mm}$$

$$\text{Depth of water in profile before irrigation} = 48 + 180 = 228 \text{ mm.}$$

$$\text{Depth of water in profile after irrigation} = 96 + 270 = 366 \text{ mm.}$$

$$\text{Depth of water added to top layer} = 96 - 48 = 48 \text{ mm.}$$

$$\text{Depth of water added to bottom layer} = 270 - 180 = 90 \text{ mm.}$$

$$\text{Depth of water added to entire profile} = 48 + 90 = 138 \text{ mm.}$$



THE LIQUID PHASE

The variable amount of water contained in a unit mass or volume of soil and the energy state of water in the soil are important factors affecting the growth of plants. Numerous other soil properties depend on water content.

Among these are mechanical properties, such as consistency, plasticity, strength, compatibility, penetrability, stickiness, and traffic ability. In clayey soils, swelling and shrinking associated with addition or extraction of water change the bulk density, porosity, and pore size distribution. Soil water content also governs the air content and gas exchange of the soil, thus affecting the respiration of roots, the activity of microorganisms, and the chemical state of the soil.

Capillary and Soil Water:

The movement of water up a wick typifies the phenomenon of capillarity. Two forces cause capillarity:

- (1) The attraction of water for the solid (adhesion or adsorption).
- (2) The surface tension of water which is due largely to the attraction of water molecules for each other (cohesion). This phenomenon can be described in terms of energy. Energy per unit area has the same dimensions as force per unit length.

Its units

- 1- Force/length = dyn/cm = $\text{gm. cm. sec}^2/\text{cm} = \text{gm. sec}^2$ or
- 2- Energy/area = $\text{erg/cm}^2 = \text{gm.cm.sec}^{-2}.\text{cm}^{-1}/\text{cm}^2 = \text{gm.sec}^{-2}.\text{cm}^{-2}$.

The height of rise in a capillary tube is also inversely proportional to the tube radius. Capillary rise is also inversely proportional to the density of the liquid, and is directly proportional to the liquid's surface tension and the degree of its adhesive attraction to the soil surface.

Capillary forces are at work in all moist soils. However, the rate of movement and rise in height are less than one would expect because soil pores are not straight and there can be air entrapment slowing down water movement. In fine textured soils, capillary rise is high but rate of flow is slow because of frictional forces in tiny pores. In coarse-textured (sandy) soils, pores are large presenting little resistance to capillary rise but limiting the rise in capillary water.

CONTACT ANGLE OF WATER ON SOLID SURFACES



If we place a drop of liquid on a dry solid surface, the liquid will usually displace the gas that covered the surface of the solid and it will spread over that surface to a certain extent. Where its spreading ceases and the edge of the drop comes to rest, it will form a typical angle with the surface of the solid. This angle, termed *contact angle*, is illustrated in Fig. 2.6.

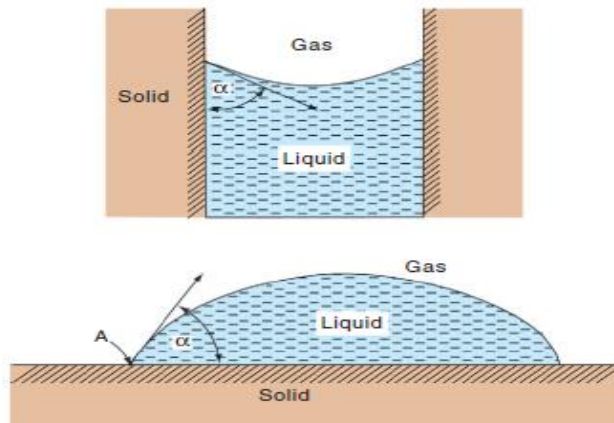


Fig. 2.6. The contact angle of a meniscus in a capillary tube and a drop resting on the surface of a solid.

We now consider what factors determine the magnitude of the angle α . We can expect that angle to be acute if the adhesive affinity between the solid and liquid is strong relative to the cohesive forces inside the liquid itself and to the affinity between the gas and the solid. We can then say that the liquid “wets” the solid. A contact angle of zero would mean the complete flattening of the drop and the perfect wetting of the solid surface by the liquid. On the other hand, a contact angle of 180° would imply a complete non wetting or rejection of the liquid by the gas-covered solid. In that case the drop would retain its spherical shape without spreading over the surface at all (assuming no gravity effect). Surfaces on which water exhibits an obtuse contact angle are called *water repellent* or *hydrophobic* (Greek: “water-hating”).

The contact angle of a given liquid on any particular solid is generally characteristic of their interaction under given physical conditions. This angle, however, may be different in the case of a liquid that is advancing over the surface than in the case of the same liquid receding over the surface. This phenomenon, where it occurs, is called *contact angle hysteresis*. The wetting angle of pure water on clean and smooth mineral surfaces is generally zero, but where the surface is rough or coated with adsorbed surfactants of a hydrophobic nature, the contact angle, and especially the wetting angle, can be considerably greater than zero and may even exceed 90° . This phenomenon is illustrated in given Fig. 2.7.

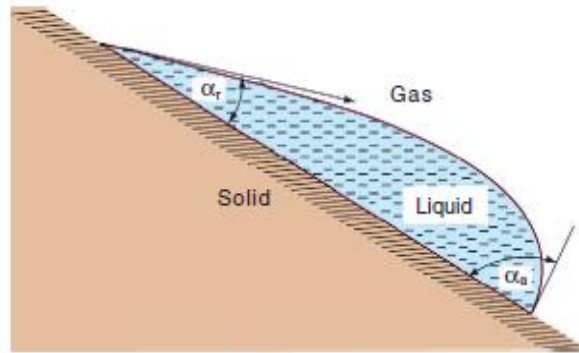


Fig. 2.7. Hypothetical representation of a drop resting on an inclined surface. The contact angle α_a at the advancing edge of the drop is shown to be greater than the corresponding angle α_r at the receding edge.

THE PHENOMENON OF CAPILLARITY

A capillary tube dipped in a body of free water will form a meniscus as the result of the contact angle of water with the walls of the tube. The curvature of this meniscus will be greater (i.e., the radius of curvature will be smaller) the narrower the tube. The occurrence of curvature causes a pressure difference to develop across the liquid–gas interface.

A liquid with an acute contact angle (e.g., water on glass) will form a concave meniscus, and therefore the liquid pressure under the meniscus P_1 will be smaller than the atmospheric pressure P_0 (Fig. 2.8). Hence, water inside the tube will be driven up the tube from its initial location (shown as a dashed curve in Fig. 2.8) by the greater pressure of the free water (i.e., water at atmospheric pressure, under a horizontal air–water interface) at the same level.

The upward movement will stop when the pressure difference between the water inside the tube and the water under the flat surface outside the tube is countered by the hydrostatic pressure exerted by the water column in the capillary tube. If the capillary tube is cylindrical and if the contact angle of the liquid on the walls of the tube is zero, the meniscus will be a hemisphere (and in a two-dimensional drawing can be represented as a semicircle) with its radius of curvature equal to the radius of the capillary tube. If, however, the liquid contacts the tube at an angle greater than zero but smaller than 90° , then the diameter of the tube ($2r$) is the length of a chord cutting a section of a circle with an angle of $(\pi - 2\alpha)$ as shown in Fig. 2.9. Thus,

$$R = r / \cos \alpha$$

Here R is the radius of curvature of the meniscus, r is the radius of the capillary, and α is the contact angle.

The pressure difference ΔP between the capillary water (under the meniscus) and the atmosphere, therefore, is



$$\Delta P = (2\gamma \cos \alpha)/r$$

Recalling that hydrostatic pressure is proportional to the depth d below the free water surface (i.e., $P = \rho g d$, where ρ is liquid density and g is gravitational acceleration), we can infer that the hydrostatic tension (negative pressure) in a capillary tube is proportional to the height h above the free water surface. Hence the height of capillary rise is

$$h_c = (2\gamma \cos \alpha)/g(\rho_l - \rho_g)\gamma$$

Where ρ_g is the density of the gas (generally neglected), ρ_l is the density of the liquid, g is the acceleration of gravity, r is the capillary radius, α is the contact angle, and γ is the surface tension.

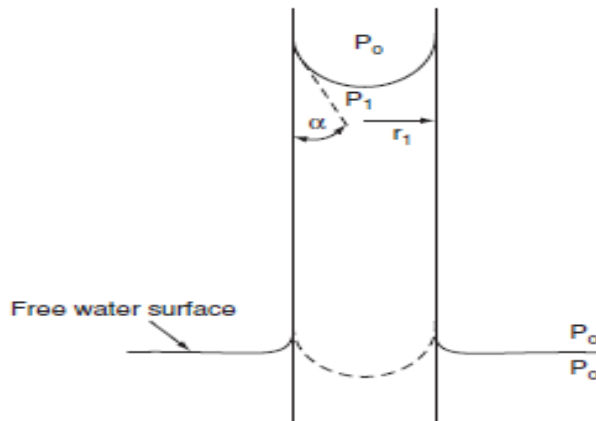


Fig. 2.8 . capillary rise

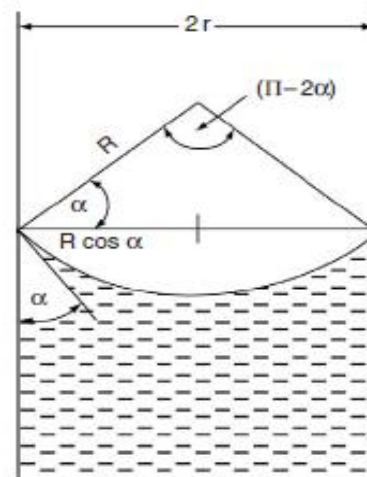


Fig. 2.9 . the geometric relationship of the radius of curvature R to the radius of the capillary r and the contact angle α

Sample Problems

Calculate the equilibrium capillary rise of water and mercury at 20°C in glass cylindrical capillary tubes of the following diameters: (a) 2 mm; (b) 0.5 mm; (c) 0.1 mm. Disregard the density of atmosphere.

$$\Delta h = (2\gamma \cos \alpha) / (\rho g r)$$

For water: $\gamma = 7.27 \times 10^{-2} \text{ kg/sec}^2 (= \text{N/m})$; $\alpha = 0$; $\rho = 998 \text{ kg/m}^3$

(a) $r = 10^{-3} \text{ m}$:

$$\Delta h = (2 \times 7.27 \times 10^{-2} \text{ kg/sec}^2) / (998 \text{ kg/m}^3 \times 9.81 \text{ m/sec}^2 \times 10^{-3} \text{ m}) = 1.48 \times 10^{-2} \text{ m} = 1.48 \text{ cm}$$

(b) $r = 2.5 \times 10^{-4} \text{ m}$:

$$\Delta h = (2 \times 7.27 \times 10^{-2} \text{ kg/sec}^2) / (998 \text{ kg/m}^3 \times 9.81 \text{ m/sec}^2 \times 2.5 \times 10^{-4} \text{ m}) = 5.92 \times 10^{-2} \text{ m} = 5.92 \text{ cm}$$

(c) $r = 5.0 \times 10^{-5} \text{ m}$:



$$\Delta h = (2 \times 7.27 \times 10^{-2} \text{ kg/sec}^2) / (998 \text{ kg/m}^3 \times 9.81 \text{ m/sec}^2 \times 5.0 \times 10^{-5} \text{ m}) = 29.6 \times 10^{-2} \text{ m} = 29.6 \text{ cm}$$

For mercury: $\gamma = 0.43 \text{ kg/sec}^2$; $\alpha = 180^\circ$; $\rho = 1,360 \text{ kg/m}^3$

(a) $r = 10^{-3} \text{ m}$:

$$\Delta h = [2 \times 0.43 \text{ kg/sec}^2 \times (-1)] / (1360 \text{ kg/m}^3 \times 9.81 \text{ m/sec}^2 \times 10^{-3} \text{ m}) = -0.64 \times 10^{-2} \text{ m} = -0.64 \text{ cm (capillary depression)}$$

(b) $r = 2.5 \times 10^{-4} \text{ m}$:

$$\Delta h = [2 \times 0.43 \text{ kg/sec}^2 \times (-1)] / (1360 \text{ kg/m}^3 \times 9.81 \text{ m/sec}^2 \times 2.5 \times 10^{-4} \text{ m}) = -2.58 \times 10^{-2} \text{ m} = -2.58 \text{ cm}$$

(c) $r = 5.0 \times 10^{-5} \text{ m}$:

$$\Delta h = [2 \times 0.43 \text{ kg/sec}^2 \times (-1)] / (1360 \text{ kg/m}^3 \times 9.81 \text{ m/sec}^2 \times 5.0 \times 10^{-5} \text{ m}) = -12.9 \times 10^{-2} \text{ m} = -12.9 \text{ cm}$$

Energy status of soil water:

The difference in energy level of water from one site or one condition (e.g., wet soil) to another (e.g., dry soil) determines the direction and rate of water movement in soils and plants. In a wet soil, most of the water is retained in large pores or thick water films around particles. Therefore, most of the water molecules in wet soil are not very close to a particle surface and so are not held very tightly by the soil solids (soil matrix). In this condition, the water molecules have considerable freedom of movement, so their energy level is near that of water molecules in a pool of pure water outside the soil. In a drier soil, however, the water that remains is located in small pores and thin water films, and is therefore held tightly by the soil solids. Thus the water molecules in a drier soil have little freedom of movement, and the energy level of the water is much lower than that of water in wet soil. If wet and dry soils are brought in touch with each other, water will move from the wet soil (higher energy state) to the drier soil (lower energy).

Usually the energy status of soil water in a particular location in the profile is compared to that of pure water at a standard pressure and temperature, unaffected by the soil and located at some reference elevation. *The difference in energy levels between this pure water in the reference state and that of soil water is termed **Soil Water Potential**.* Thus the term potential implies a difference in energy status. Knowing the potential energy in soil can help us estimate how much work the plant must expend to extract a unit amount of water. Potential energy rather than kinetic energy is used to describe soil water flow because kinetic energy is considered negligible by virtue of slow movement of soil water.

Expression of water potential

The soil-water potential is expressible in at least 3 ways:

1. Energy per unit mass: This is often taken to be the fundamental expression of potential (J/kg) with dimensions of L^2T^{-2}
2. Energy per unit volume: This is unit of pressure (Pressure = force/area = force



$\times \text{distance/area} \times \text{distance} = \text{energy/volume}$). (Units N/m^2 , kPa .) = $\text{ML}^{-1}\text{T}^{-2}$

3. Energy per unit weight (hydraulic head): Since we can express soil water in units of hydrostatic pressure, it means we can also express it in terms of an equivalent head of water, which is height of a liquid column corresponding to the given pressure. The sum of matric and gravitational (elevation) heads is generally called the hydraulic head (or hydraulic potential) and is useful in evaluating directions and magnitudes of the water-moving forces throughout the soil profile.

Components of soil water potential

The International Soil Science Society defines total potential of soil water as “the amount of work that must be done per unit quantity of pure water in order to transport reversibly and isothermally an infinitesimal quantity of water from a pool of pure water at a specified elevation at atmospheric pressure to the soil water(at the point under consideration”. Soil water is subject to a number of force fields, which cause its potential to differ from that of pure, free water. Each of these forces is a component of the total soil water potential and they result from the attraction of the solid matrix for water, as well as from the presences of solutes and the action of external gas pressure and gravitation.

Thus, the total potential is the summation of the component potentials.

The components of soil water potential for shallow soils are:

Gravitational potential: The force of gravity exerted on a water column produces the gravitation potential. The gravitation potential is related to the work done to transport water from one pool to another, as when lifting a column of water up the xylem of a tree:

$$\psi_z = \rho g z$$

Pressure potential:

Water potential exerted by the pressure or weight of water

$$\psi_p = \frac{P}{\rho_w}$$

Matric potential: It is water potential due to attraction between water and soils.

Adhesive and cohesive forces bind water to soil particles (Campbell and Norman, 1999).

These interactions reduce the potential of water, giving it a negative sign.

$$\psi_m \sim a w^{-b}$$

Osmotic potential: Osmotic potential arises from the dilution of solutes in water, eg salts, sugars etc. For the osmotic potential to drive water flow, a semi-permeable membrane must separate two bodies of water, such as cells, and pools of water.

$$\psi_s = -c \phi v R T$$



c is the concentration, ϕ is the osmotic coefficient and v is the number of ions per mole (Campbell and Norman 1998).

The total water potential is thus:

$$\psi_t = \psi_p + \psi_m + \psi_z + \psi_s$$

We know the mass of water is $1000 \text{ kg} = \text{m}^3$ and water is incompressible, we can substitute mass for density, effectively $1 \text{ J/kg} = 1 \text{ kPa}$

When $\Delta\psi_z = 0$ the ψ_t named as ψ_w and $\psi_w = \psi_p + \psi_m + \psi_s$

Soil-Water-Plant Relations:

Osmotic and matric potential are important for plant-water. Gravimetric potential is negligible as the suction needed to raise water; typically 1 m is less than 0.1 bars.

Organisms, cells:

$$\psi = \psi_s + \psi_p$$

Inside cells osmotic potentials and pressure potential are most important

Unsaturated Flow:

$$\psi = \psi_z + \psi_m$$

Gravitational and matrix potential are dominant

Saturated Flow:

$$\psi = \psi_p + \psi_z$$

Pressure and gravity are the main components driving water flow in saturated soils. The pressure term includes overburden effects. At a point a point beneath the water table, the pressure potential is equal and opposite to the gravitational potential.

$$0 = \psi_p + \psi_z$$

$$\psi_p = -\psi_z$$

Osmotic potential is important only if there are solutes in the water.

Flow in the field and named as ψ_h

$$\psi_h = \psi_p + \psi_m + \psi_z$$

This terminology considers mixed flow in the saturated and unsaturated zones. The pressure term is zero above the water table. The matrix potential is zero below the water table.

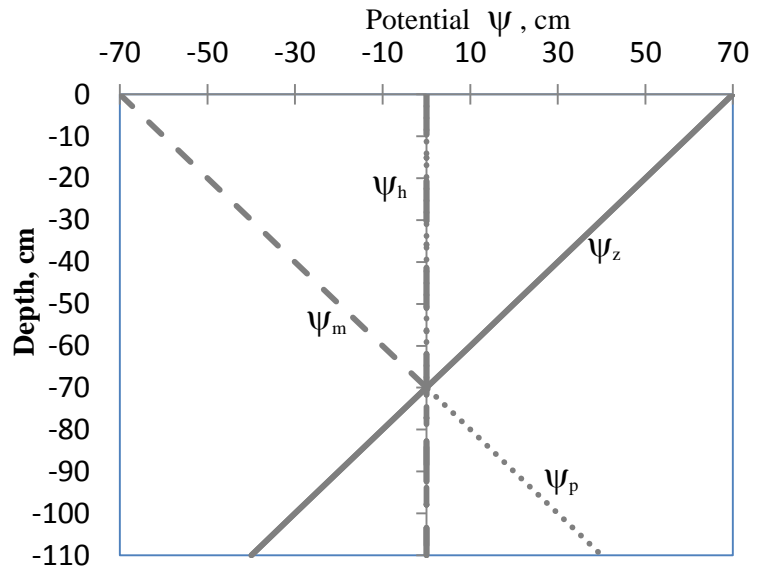
Sample Problems

You have soil in equilibrium condition with water table level at (-70 cm), if the reference plane at (-70 cm) , find (ψ_p , ψ_m , ψ_z and ψ_h) along the depth of the soil till (-110) cm depth .

The solution:



ψ_p	ψ_m	ψ_z	ψ_h	Depth (cm)
0	-70	70	0	0
0	-60	60	0	-10
0	-50	50	0	-20
0	-40	40	0	-30
0	-30	30	0	-40
0	-20	20	0	-50
0	-10	10	0	-60
0	0	0	0	-70
10	0	-10	0	-80
20	0	-20	0	-90
30	0	-30	0	-100
40	0	-40	0	-110

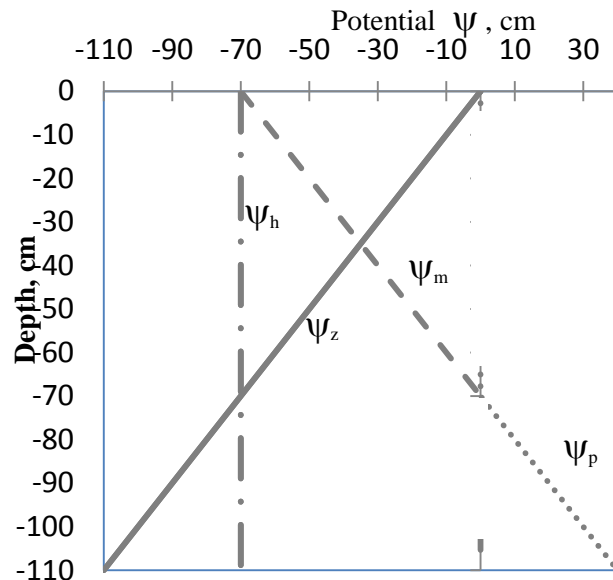


Sample Problems

You have the same soil in example above but the reference plane at the surface of the soil, find (ψ_p , ψ_m , ψ_z and ψ_h) along the depth of the soil till (-110)cm depth.

The solution:

ψ_p	ψ_m	ψ_z	ψ_h	Depth (cm)
0	-70	0	-70	0
0	-60	-10	-70	-10
0	-50	-20	-70	-20
0	-40	-30	-70	-30
0	-30	-40	-70	-40
0	-20	-50	-70	-50
0	-10	-60	-70	-60
0	0	-70	-70	-70
10	0	-80	-70	-80
20	0	-90	-70	-90
30	0	-100	-70	-100
40	0	-110	-70	-110



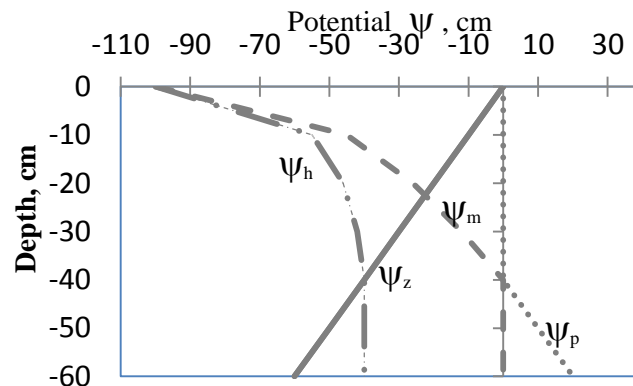
Sample Problems

You have soil in with water table at 40 cm below the surface and the evaporation happen at the surface, the reference plane at the surface of the soil, find (ψ_p , ψ_z and ψ_h) along the depth of the soil till (-60)cm depth. if ψ_m is given.



The solution:

ψ_p	ψ_m given	ψ_z	ψ_h	Depth (cm)
0	-100	0	-100	0
0	-45	-10	-55	-10
0	-26	-20	-46	-20
0	-12	-30	-42	-30
0	0	-40	-40	-40
10	0	-50	-40	-50
20	0	-60	-40	-60



Soil Water Content and Potential (characteristic curves)

Quantity of water in soil is expressed by gravimetric or volumetric water content. However, water can also be characterized by describing its free energy per unit mass, which is termed potential. The tenacity with which water is held in soil solid is characterized by matric or pressure potential. When volumetric water content and matric potential are plotted graphically, the relationship is termed **Soil Moisture Characteristic Curve**.

When all soil pores are filled with water, the soil is at its maximum retentive capacity called saturation. In the field, the lowest wetness you can observe is called air-dryness and in the laboratory it is called oven-dry condition.

As yet, no universally applicable theory exists for the prediction of the matric suction versus wetness relationship from basic soil properties (i.e., texture and structure). The adsorption and pore geometry effects are generally too complex to be described by a simple model. Several empirical equations have been proposed that describe the soil-moisture characteristic for some soils and within limited suction ranges. One such equation was advanced by Visser (1966):

$$\psi = a(f - \theta)^b / \theta^c$$

Here ψ is matric suction, f is porosity, θ is volumetric wetness, and a , b , c are constants. Use of this equation is hampered by the difficulty of evaluating its constants. Visser found that b varied from 0 to 10, a from 0 to 3, and f from 0.4 to 0.6.

Alternative equations to describe the relationship between wetness and matric suction have been proposed by Laliberte (1969), White et al. (1970), Su and



Brooks (1975), and van Genuchten (1978). An equation presented by Brooks and Corey (1966a) is

$$(\theta - \theta_r)/(\theta_m - \theta_r) = (\psi_r/\psi)^\lambda$$

For suction values greater than the air-entry suction ψ_e . The exponent λ has been termed the pore size distribution index. In this equation, θ is the volume wetness (a function of the suction ψ), θ_m is the maximum wetness (saturation), and θ_r is the “residual” wetness remaining at high suction in the small pores that do not form a continuous network (the intra-aggregate pores).

The amount of water retained at low values of matric suction (say, between 0 and 100 kPa) depends on capillarity and the pore size distribution. Hence it is strongly affected by the soil’s structure. At higher suctions, water retention is due increasingly to adsorption, so it is influenced less by the structure and more by the soil’s texture and specific surface. The greater the clay content, in general, the greater the water retention at any particular suction and the more gradual the slope of the curve. In a sandy soil, most of the pores are relatively large, and once these large pores are emptied at a given suction, only a small amount of water remains. In a clayey soil, more of the water is adsorbed, so increasing the matric suction causes a more gradual decrease in wetness (Fig. 6.8).

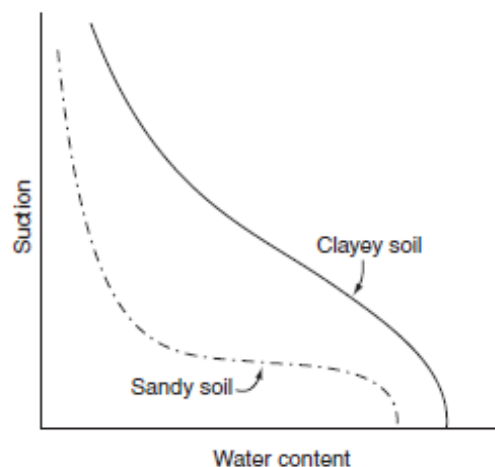


Fig. 6.8. The effect of texture on soil-water retention.

The Effect of Compaction on Soil

Because soil structure influences the shape of the soil-moisture characteristic curve primarily in the low-suction range, we may expect that the effect of compaction (which destroys the aggregated structure) is to reduce the total porosity and especially the volume of the large inter aggregate pores. As a result of compaction, the saturation water content as well as the initial decrease of water content with the application of low suction are diminished.

On the other hand, the volume of intermediate-size pores is likely to be greater in a compact soil (because some of the originally large pores have been squeezed into



intermediate size by compaction), while the micropores remain unaffected, and thus the curves for the compacted and uncompacted soil tend to converge in the high-suction range (Fig. 6.9).

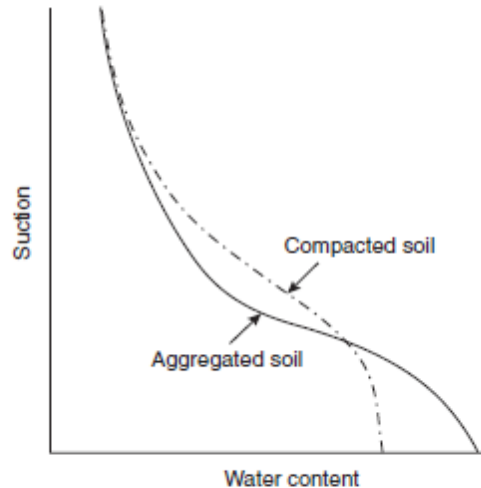


Fig. 6.9. The effect of soil structure on soil-water retention.

The slope of the soil-moisture characteristic curve, which is the change of water content per unit change of matric potential, is generally termed the *differential* (or *specific*) *water capacity* c_θ

$$c_\theta = d\theta/d\phi_p \quad \text{or} \quad c_\theta = -d\theta/d\psi$$

The c_θ term is analogous to the well-known *differential heat capacity*, which is the change in the heat content of a body per unit change in the thermal potential (temperature). However, while the differential heat capacity is fairly constant with temperature for many materials, the differential water capacity in soils is strongly dependent on the matric potential.

Measurement of Soil Water Potential

Total water potential is often thought of as the sum of matric and osmotic (solute) potentials and is a very useful index for characterizing the energy status of soil water with respect to plant water uptake.

Tensiometer: A device to measure the matric potential of soil moisture in situ, consisting of a porous (ceramic) cup filled with water, with a manometer to monitor the pressure of the water in the cup at equilibrium with soil moisture. As soil moisture diminishes, its matric tension increases, hence it draws water from the cup, which in turn registers a subatmospheric pressure, called tension.

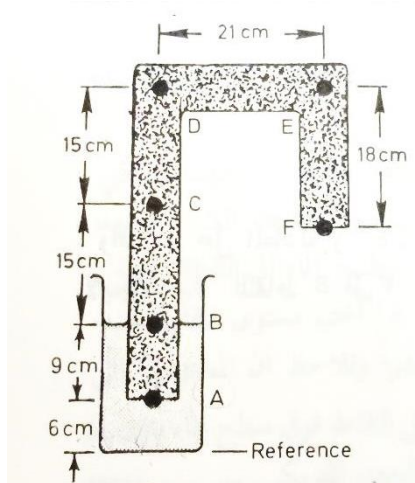
Tension-plate assembly: A device for equilibrating an initially saturated soil sample with a known matric tension value, applicable to the tension range of 0 to 1 bar.

Calculate potential in soil columns

Sample Problems



Given soil column as shown below, the level of water in ban keep constant and the evaporation is prevent in its open end for a period enough to reach to calibration, so there is no net flow in the soil column.



Find (ψ_h and its components ψ_p , ψ_z and ψ_m) for point A to F in soil column.

Solution:

At point B (water surface) $\psi_p = 0$ and $\psi_m = 0$

And ψ_z = the vertical distance from the reference so
 $\psi_z = 6 + 9 = 15 \text{ cm}$

$\therefore \psi_h = \psi_p + \psi_z + \psi_m = 0 + 15 + 0 = 15 \text{ cm}.$

We know the ψ_h equal in all points within the soil
so we could write $\psi_{hB} = \psi_{hA} = \psi_{hC} = \psi_{hD} = \psi_{hE} = \psi_{hF}$

And the ψ_z for the point is the vertical distance so:

$$\psi_{zA} = 6 \text{ cm}$$

$$\psi_{zB} = 6 + 9 = 15 \text{ cm}$$

$$\psi_{zC} = 6 + 9 + 15 = 30 \text{ cm}$$

$$\psi_{zD} = 6 + 9 + 15 + 15 = 45 \text{ cm}$$

$$\psi_{zE} = 6 + 9 + 15 + 15 = 45 \text{ cm}$$

$$\psi_{zF} = 6 + 9 + 15 + 15 - 18 = 27 \text{ cm}$$

ψ_p at the water surface and above water its equal to 0 so

$$\psi_{pB} = \psi_{pC} = \psi_{pD} = \psi_{pE} = \psi_{pF} = 0$$

And $\psi_m = \psi_h - \psi_z - \psi_p$

$$\psi_{mB} = 15 - 15 - 0 = 0 \text{ cm}$$

$$\psi_{mC} = 15 - 30 - 0 = -15 \text{ cm}$$

$$\psi_{mD} = 15 - 45 - 0 = -30 \text{ cm}$$

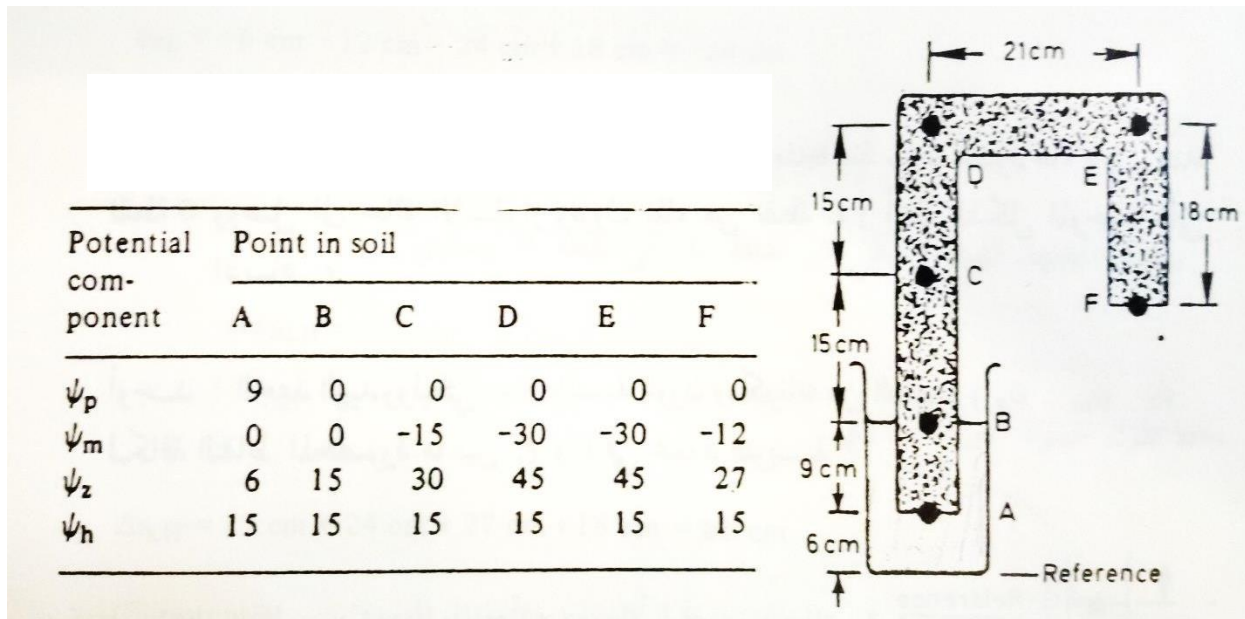
$$\psi_{mE} = 15 - 45 - 0 = -30 \text{ cm}$$

$$\psi_{mF} = 15 - 27 - 0 = -12 \text{ cm}$$

Point A below water surface so $\psi_{mA} = 0$

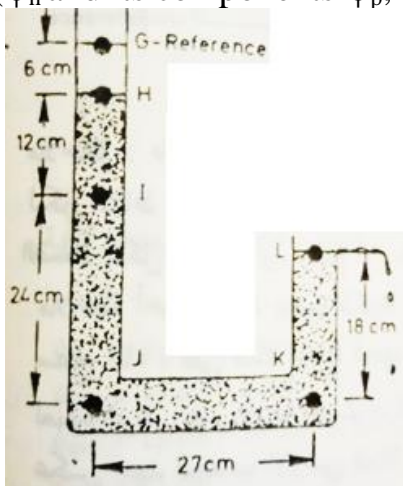
$$\text{And } \psi_{pA} = \psi_{hA} - \psi_{zA} - \psi_{mA} = 15 - 6 - 0 = 9 \text{ cm}$$

The final results in table below:



Sample Problems

Given soil column as shown below, column of water applied above this column in point G as shown and reach to equilibrium with dropping water from point L. Find (ψ_h and its components ψ_p , ψ_z and ψ_m) for point G to L in soil column.



Solution:

The water in flowing state so there is ψ_h between the two ends of the tube.

From the figure H equal to 6 cm (the column of water above the soil) and below the reference so $\psi_{zH} = -6$ cm, Point H below water so $\psi_{mH} = 0$ cm, and $\psi_{pH} = 6$ cm (because the column of water above this point equal to 6 cm). $\therefore \psi_h = \psi_p + \psi_z + \psi_m =$

as a result $\psi_{hH} = \psi_{pH} + \psi_{zH} + \psi_{mH} = 6 + (-6) + 0 = 0$ cm at point L and because dropping water $\psi_{mL} = 0$ cm, and no column of water above point L so $\psi_{pL} = 0$ cm, $\psi_{zL} = (-6) + (-12) + (-24) + (18) = (-24)$ cm

And $\psi_{hL} = \psi_{pL} + \psi_{zL} + \psi_{mL} = 0 + (-24) + 0 = (-24)$ cm

The difference in hydraulic potential between point L and point H will be:

$$\Delta\psi_{hLH} = -24 - 0 = -24 \text{ cm}$$

And the potential gradient along the soil column $(\nabla\psi_h) = \Delta\psi_{hLH} / \Delta s_{LH} = -24 / 81 =$
 $\therefore (\nabla\psi_h) = -0.296$

$$\Delta\psi_{hHI} = \nabla\psi_h * \Delta s_{HI} = -0.296 * 12 = -3.56 \text{ cm}$$

And in same way:

$$\Delta\psi_{hIJ} = \nabla\psi_h * \Delta s_{IJ} = -0.296 * 24 = -7.11 \text{ cm}, \Delta\psi_{hJK} = -8.00 \text{ cm}, \Delta\psi_{hKL} = -5.33 \text{ cm}$$



Now we could find the hydraulic potential at each point as below:

$$\begin{aligned}\psi_{hH} &= 0 \text{ cm}, \\ \psi_{hI} &= \psi_{hH} + \Delta\psi_{hHI} = 0 + (-3.56) = -3.56 \text{ cm}, \\ \psi_{hJ} &= \psi_{hI} + \Delta\psi_{hIJ} = (-3.56) + (-7.11) = -10.67 \text{ cm}, \\ \psi_{hK} &= \psi_{hJ} + \Delta\psi_{hJK} = (-10.67) + (-8.00) = -18.67 \text{ cm}, \\ \psi_{hL} &= \psi_{hK} + \Delta\psi_{hKL} = (-18.67) + (-5.33) = -24 \text{ cm}\end{aligned}$$

The next step is to find the gravitational potential ψ_z as below:

$$\begin{aligned}\psi_{zH} &= -6 \text{ cm}, \\ \psi_{zI} &= -6 + (-12) = -18 \text{ cm}, \\ \psi_{zJ} &= -6 + (-12) + (-24) = -42 \text{ cm}, \\ \psi_{zK} &= -6 + (-12) + (-24) = -42 \text{ cm}, \\ \psi_{zL} &= -6 + (-12) + (-24) + 18 = -24 \text{ cm}\end{aligned}$$

As shown all point below water surface so $\psi_m = 0$ for all point that's mean

$$\psi_{mH} = \psi_{mI} = \psi_{mJ} = \psi_{mK} = \psi_{mL} = 0 \text{ cm}$$

Now the pressure potential ψ_p could be calculated easily as below:

$$\begin{aligned}\psi_p &= \psi_h - \psi_z - \psi_m \\ \psi_{pH} &= 0 - (-6) - 0 = 6 \text{ cm}, \\ \psi_{pI} &= (-3.56) - (-18) - 0 = 14.44 \text{ cm}, \\ \psi_{pJ} &= (-10.67) - (-42) - 0 = 31.33 \text{ cm}, \\ \psi_{pK} &= (-18.67) - (-42) - 0 = 23.33 \text{ cm}, \\ \psi_{pL} &= (-24) - (-24) - 0 = 0 \text{ cm},\end{aligned}$$

The final results shown in table below:

Point in soil						Potential component
L	K	J	I	H	G	
0	23.33	31.33	14.44	6	0	ψ_p
0	0	0	0	0	0	ψ_m
-24	-42	-42	-18	-6	0	ψ_z
-24	-18.67	-10.67	-3.56	0	0	ψ_h



WATER FLOW IN SOIL

The flow in soil if it saturated or unsaturated divided into:

- 1- Steady state water flow: in this type of flow the flow properties (θ , ψ_h) not change with time but could change with location.
- 2- Transient state water flow.

Water Flow in Saturated Soil

The discharge Q , that is, the volume flowing through a section of length L per unit time, can now be evaluated. The volume of a paraboloid of revolution is $(1/2)$ (base. height). Hence

$$Q = (1/2)\pi R^2 u_{\max} = \pi R^4 \Delta p / 8\eta L$$

This equation, known as *Poiseuille's law*, indicates that the volume flow rate is proportional to the pressure drop per unit distance ($\Delta p/L$) and the fourth power of the radius of the tube.

Laminar flow prevails only at relatively low flow velocities and in narrow tubes. As the radius of the tube and the flow velocity are increased, the point is reached at which the mean flow velocity is no longer proportional to the pressure drop, and the parallel *laminar flow* changes into a *turbulent flow* with fluctuating eddies. Conveniently, however, laminar flow is the rule rather than the exception in most water flow processes taking place in soils, because of the narrowness of soil pores.

DARCY'S LAW



We now examine the flow of water in a macroscopically uniform, saturated soil body, and attempt to describe the quantitative relations connecting the rate of flow, the dimensions of the body, and the hydraulic conditions at the inflow and outflow boundaries. Figure 7.3 shows a horizontal column of soil through which a steady flow of water is occurring from left to right, from an upper reservoir to a lower one, in each of which the water level is maintained constant. Experience shows that the discharge rate Q , being the volume V flowing through the column per unit time, is directly proportional to the crosssectional area and to the hydraulic head drop ΔH and inversely proportional to the length of the column L :

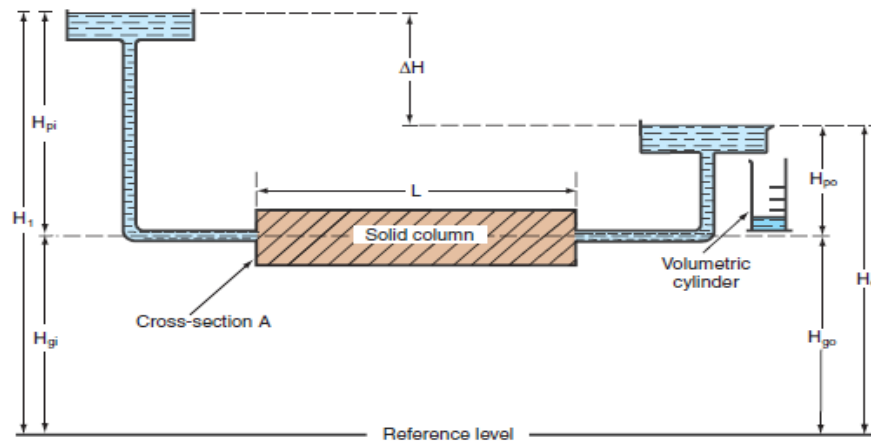


Fig. 7.3. Flow in a horizontal saturated column.

$$Q = V/t \propto A \Delta H/L$$

$$\Delta H = H_1 - H_0$$

The head drop per unit distance in the direction of flow ($\Delta H/L$) is the *hydraulic gradient*, which is, in fact, the driving force. The specific discharge rate Q/A (i.e., the volume of water flowing through a unit cross-sectional area per unit time t) is called the *flux density* (or simply the flux) and is indicated by q . Thus, the flux is proportional to the hydraulic gradient:

$$q = Q/A = V/At \propto \Delta H/L$$

The proportionality factor K is termed the *hydraulic conductivity*:

$$q = K \Delta H/L$$

This equation is known as *Darcy's law*, after Henry Darcy, the French engineer who discovered it over a century ago in the course of his classic investigation of seepage rates through sand filters in the city of Dijon.

Sample Problem

The water in an irrigation hose is at a hydrostatic pressure of 100 kPa (1 bar). Five dripirrigation emitters are inserted into the wall of the hose. Calculate the drip rate (L/hr) from the emitters if each contained a coiled capillary tube 1 m long and the



capillary diameters are 0.2, 0.4, 0.6, 0.8, and 1.0 mm. Assuming laminar flow, what fraction of the total discharge is due to the single largest emitter?

We use Poiseuille's law to calculate the discharge: $Q = \frac{\pi R^4 \Delta P}{8 \eta L}$. Substituting the values for π (3.14), the pressure differential ΔP ($10^5 \text{ N/m}^2 = 10^5 \text{ kg/m sec}^2$), the viscosity η (10^{-3} kg/m sec , at 20°C), the capillary tube length (1 m), and the appropriate tube radial (0.1, 0.2, 0.3, 0.4, and 0.05 mm), we obtain

$$Q_1 = \frac{3.14 \times (10^{-4} \text{ m})^4 \times 10^5 \text{ kg/m sec}^2}{8 \times 10^{-3} \text{ kg/m sec} \times 1 \text{ m}} = 3.9 \times 10^{-9} \text{ m}^3/\text{sec} \\ = 0.014 \text{ L/hr}$$

$$Q_2 = \frac{3.14 \times (2 \times 10^{-4} \text{ m})^4 \times 10^5 \text{ kg/m sec}^2}{8 \times 10^{-3} \text{ kg/m sec} \times 1 \text{ m}} = 2^4 Q_1 = 16 \times 1.4 \times 10^{-2} \\ = 0.226 \text{ L/hr}$$

$$Q_3 = 3^4 Q_1 = 81 Q_1 = 1.14 \text{ L/hr}$$

$$Q_4 = 4^4 Q_1 = 256 Q_1 = 3.61 \text{ L/hr}$$

$$Q_5 = 5^4 Q_1 = 625 Q_1 = 8.81 \text{ L/hr}$$

Total discharge from all five emitters:

$$Q_{\text{total}} = 0.014 + 0.226 + 1.14 + 3.61 + 8.81 = 13.8 \text{ L/hr}$$

Fractional contribution of the single largest emitter:

$$Q_5/Q_{\text{total}} = 8.81/13.8 = 0.639 = 63.8\%$$

The single largest emitter thus accounts for nearly two-thirds of the total discharge, while the smallest emitter accounts for only 0.1% (though its diameter is only one-fifth that of the largest emitter).

Note: Modern drip-irrigation emitters generally depend on partially turbulent (rather than completely laminar) flow, to reduce sensitivity to pressure fluctuations and vulnerability to clogging by particles.

Sample Problem

Let us suppose we were given the task of purifying Dijon's water, with its 10,000 denizens. Since they drink mostly wine, their daily water requirements are modest, say, no more than 20 liters per day per person, on average. Let us suppose further that we knew (with the benefits of hindsight) that a column thickness of 0.30 m was adequate for filtration and that the hydraulic conductivity of the available sand was $2 \times 10^{-5} \text{ m/sec}$. Could we calculate the area of the filter bed needed under a hydrostatic pressure head of 0.7 m? Consider the flow to be vertically downward to a fixed drainage plane.

We begin by calculating the discharge Q needed:



$$Q = \frac{10^4 \text{ persons} \times 20 \text{ L/person day} \times 10^{-3} \text{ m}^3/\text{L}}{8.64 \times 10^4 \text{ sec/day}} = 2.31 \times 10^{-3} \text{ m}^3/\text{sec}$$

We recall Darcy's law:

$$Q = AK \Delta H/L$$

Hence, the area needed is:

$$A = QL/K \Delta H$$

The hydraulic head drop ΔH equals the sum of the pressure head and gravitational head drops:

$$\Delta H = 0.7 + 0.3 = 1.0 \text{ m}$$

Substituting these values for L (0.3 m), ΔH (1 m), and K (2×10^{-5} m/sec), we obtain

$$A = \frac{2.31 \times 10^{-3} \text{ m}^3/\text{sec} \times 0.3 \text{ m}}{2 \times 10^{-5} \text{ m/sec} \times 1 \text{ m}} = 34.7 \text{ m}^2$$

Note: Since populations and per capita water use tend to increase and filter beds tend to clog, it might be wise to apply a factor of safety to our calculations and increase the filtration capacity several fold (especially to accommodate peak demand hours).

Incidentally, per capita water use in the U.S. (with running toilets, showers, and dishwashers) ranges from 100 to 400 L/day.

GRAVITATIONAL, PRESSURE, AND TOTAL HYDRAULIC HEADS:

The water entering the column of Fig. 7.3 is under a pressure, which is the sum of the hydrostatic pressure and the atmospheric pressure acting on the surface of the water in the reservoir. Since the atmospheric pressure is the same at both ends of the system, we can disregard it and consider only the hydrostatic pressure. Accordingly, the water pressure at the inflow boundary $P_i = \rho_w g H_{pi}$. Since ρ_w and g are both nearly constant, we can express this pressure in terms of the pressure head H_{pi} .

Water flow in a horizontal column occurs in response to a pressure head gradient. Flow in a vertical column may be caused by gravitation as well as pressure. The *gravitational head* H_g at any point is determined by the height of the point relative to some reference plane, while the pressure head is determined by the height of the water column resting on that point. The total hydraulic head H is the sum of these two heads:

$$H = H_p + H_g$$

To apply Darcy's law to vertical flow, we must consider the total hydraulic head at the inflow and at the outflow boundaries (H_i and H_o , respectively):



$$H_i = H_{pi} + H_{gi} \text{ and } H_o = H_{po} + H_{go}$$

Darcy's law thus becomes

$$q = K[(H_{pi} + H_{gi}) - (H_{po} + H_{go})]/L$$

The gravitational head is often designated as z , which is the vertical distance in the rectangular coordinate system x, y, z . It is convenient to set the reference level as the point $z = 0$ at the bottom of a vertical column or at the center of a horizontal column. However, the exact elevation of this hypothetical level is unimportant, since the absolute values of the hydraulic heads determined in reference to it are immaterial and only their differences from one point in the soil to another affect flow.

The pressure and gravity heads can be represented graphically in a simple way. To illustrate this, we shall immerse and equilibrate a vertical soil column in a water reservoir so that the upper surface of the column will be level with the water surface, as shown in Fig. 7.4. The coordinates of Fig. 7.4 are arranged so that the height above the bottom of the column is indicated by the vertical axis z ; and the pressure, gravity, and hydraulic heads are indicated on the horizontal axis. The gravity head is determined with respect to the reference level $z = 0$, and it increases with height at the ratio of 1:1. The pressure head is determined with reference to the free-water surface, at which the hydrostatic pressure is zero.

Accordingly, the hydrostatic pressure head at the top of the column is zero and at the bottom of the column is equal to L , the column length. Just as the gravity head diminishes from top to bottom, so the pressure head increases.

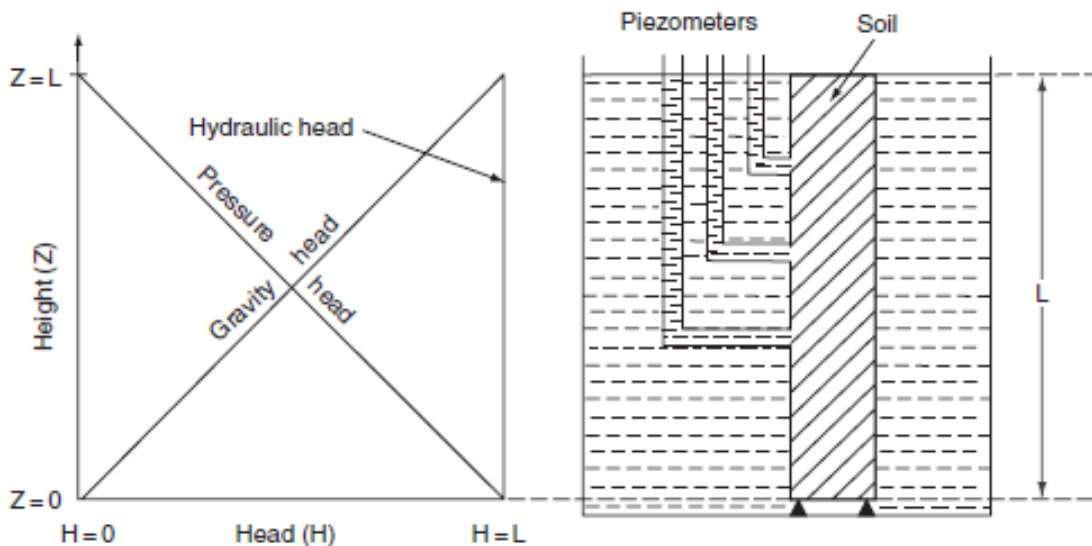


Fig. 7.4. Distribution of pressure, gravity, and total hydraulic heads in a vertical column immersed in water, at equilibrium.

Consequently their sum, which is the hydraulic head, remains constant all along the column. This is a state of equilibrium, at which no flow occurs.



This statement deserves to be further elaborated. The water pressure is not equal along the column, being greater at the bottom than at the top of the column.

Why, then, will the water not flow from a zone of higher pressure to one of lower pressure? If the pressure gradient were the only force causing flow (as it is, in fact, in a horizontal column), the water would tend to flow upward.

However, opposing the pressure gradient is a gravitational gradient of equal magnitude, resulting from the fact that the water at the top is at a higher gravitational potential than that at the bottom. In the illustration given, the two opposing gradients in effect cancel each other, so the total hydraulic head is constant throughout the column, as indicated by the standpipes (*piezometers*) connected to the column at the left.

As we already pointed out, it is convenient to set the reference level at the bottom of the column so that the gravitational potential can always be positive. On the other hand, the pressure head of water can be either positive or negative: It is positive under a free-water surface (i.e., a water table) and negative above it.

FLOW IN A VERTICAL COLUMN

Figure 7.5 shows a uniform, saturated vertical column, the upper surface of which is ponded under a constant head of water H_1 , and the bottom surface of which is set in a lower, constant-level reservoir. Flow is thus taking place from the higher to the lower reservoir through a column of length L .

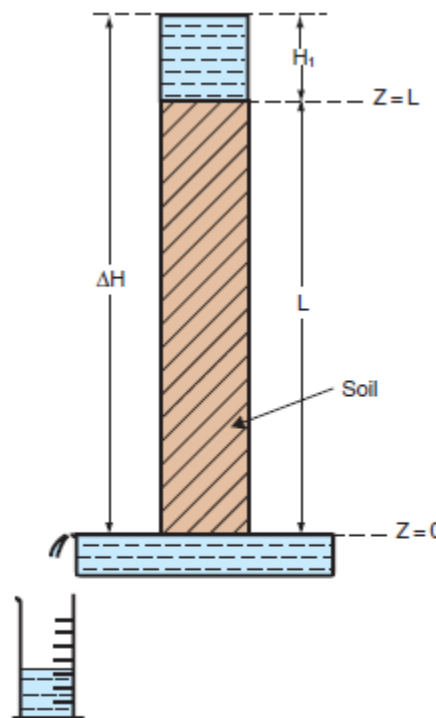


Fig. 7.5. Downward flow of water in a vertical saturated column.



In order to calculate the flux according to Darcy's law, we must know the hydraulic head gradient, which is the ratio of the hydraulic head drop (between the inflow and outflow boundaries) to the column length, as shown here:

		Pressure head		Gravity head
Hydraulic head at inflow boundary H_i	=	H_1	+	L
Hydraulic head at outflow boundary H_o	=	0	+	0
Hydraulic head difference $\Delta H = H_i - H_o$	=	H_1	+	L

The Darcy equation for this case is

$$q = K \Delta H / L = K(H_1 + L) / L = KH_1 / L + K$$

Comparison of this case with the horizontal one show that the rate of downward flow of water in a vertical column is greater than in a horizontal column by the magnitude of the hydraulic conductivity. It is also apparent that, if the ponding depth H_1 is negligible, the flux is equal to the hydraulic conductivity.

This is due to the fact that, in the absence of a pressure gradient, the only driving force is the gravitational head gradient, which, in a vertical column, has the value of unity (since this head varies with height at the ratio of 1:1).

We now examine the case of upward flow in a vertical column, as shown in Fig. 7.6. In this case, the direction of flow is opposite to the direction of the gravitational gradient, and the hydraulic gradient becomes

		Pressure head		Gravity head
Hydraulic head at inflow boundary H_i	=	H_1	+	0
Hydraulic head at outflow boundary H_o	=	0	+	L
Hydraulic head difference $\Delta H = H_i - H_o$	=	H_1	-	L

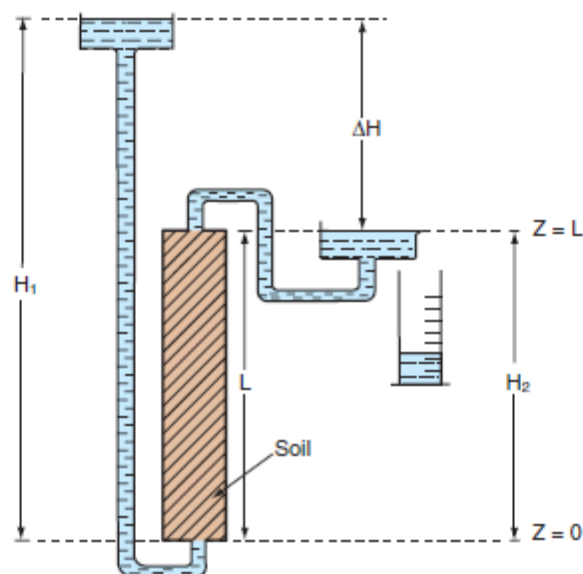


Fig. 7.6. Steady upward flow in a saturated vertical column.



The Darcy equation is thus:

$$q = K(H_1 - H_2)/L = KH_1/L - KH_2/L = K \Delta H/L$$

FLOW IN A COMPOSITE COLUMN

Figure 7.7 depicts steady flow through a soil column consisting of two distinct layers, each homogeneous within itself and differing from the other layer in thickness and hydraulic conductivity. Layer 1 is at the inlet and layer 2 is at the outlet side of the column. The hydraulic head values at the inlet surface, at the interlayer boundary, and at the outlet end are designated H_1 , H_2 , and H_3 , respectively. At steady flow, the flux through both layers must be equal:

$$q = K_1 (H_1 - H_2)/L_1 = K_2 (H_2 - H_3)/L_2$$

Where q is the flux, K_1 and L_1 are the conductivity and thickness (respectively) of the first layer, and K_2 and L_2 are the same for the second layer. Here we have disregarded any possible contact resistance between the layers.

Thus,

$$H_2 = H_1 - qL_1/K_1 \quad \text{and} \quad qL_2/K_2 = H_2 - H_3$$

Therefore,

$$qL_2/K_2 = H_1 - qL_1/K_1 - H_3 \quad \text{and} \quad q = (H_1 - H_3) / (L_2/K_2 + L_1/K_1)$$

The reciprocal of the conductivity has been called the *hydraulic resistivity*, and the ratio of the thickness to the conductivity ($R_h = L/K$) has been called the hydraulic resistance per unit area. Hence,

$$q = \Delta H / (R_{h1} + R_{h2})$$

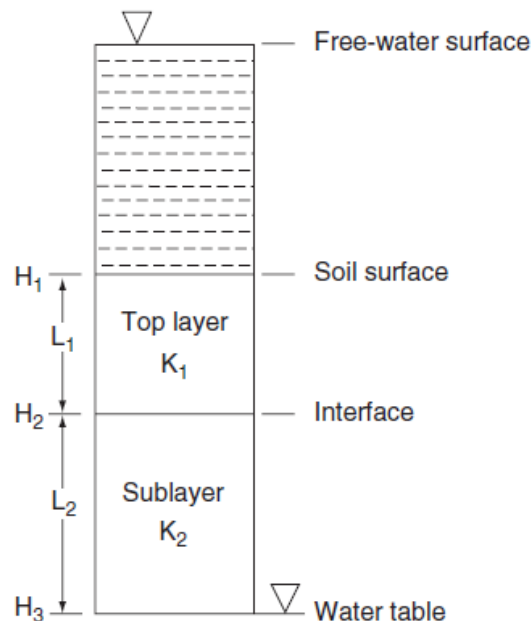


Fig. 7.7. Downward flow through a composite column.



where ΔH is the total hydraulic head drop across the entire system and R_{h1} , R_{h2} are the hydraulic resistances of layers 1 and 2.

Sample Problem

Let us consider two cases of steady downward percolation through a two-layer soil profile, the top of which is submerged under a 1-m head of water and the bottom of which is defined by a water table. Each of the two layers is 0.50 m thick. In the one case, the conductivity of the top layer is 10^{-6} m/sec and that of the sub layer is 10^{-7} m/sec. In the second case, the same layers are reversed.

To calculate the flux and the hydraulic and pressure heads at the interface between the layers, we use the Ohm's law analogy for steady flow through two resistors in series:

$$q = \Delta H / (R_1 + R_2)$$

Where q is the flux, ΔH is the total hydraulic head drop across the profile, and R_1 , R_2 are the hydraulic resistances of the top layer and sub layer, respectively.

Each resistance is proportional directly to the layer's thickness and inversely to its conductivity

(i.e., $R = L/K$). The pressure head at the soil surface is 1 m, and the gravity head (with reference to the soil's bottom) is also 1 m. Both the pressure and gravity heads at the bottom boundary are zero. Hence,

$$q = (1 \text{ m} + 1 \text{ m}) / (0.5 \text{ m} / 10^{-6} \text{ m/sec} + 0.5 \text{ m} / 10^{-7} \text{ m/sec}) = 3.64 \times 10^{-7} \text{ m/sec}$$

We can now apply Darcy's equation to the top layer alone to obtain the hydraulic head at the interlayer interface:

$$q = K_1 \Delta H_1 / L_1 = K_1 (H_{\text{surface}} - H_{\text{interface}}) / L_1$$

Hence:

$$H_{\text{interface}} = H_{\text{surface}} - qL_1/K_1$$

$$= 2 \text{ m} - (3.64 \times 10^{-7} \text{ m/sec}) (0.5 \text{ m}) / 10^{-6} \text{ m/sec} = 1.818 \text{ m}$$

Since the gravity head at the interface is 0.5 m (above our reference datum at the bottom of the profile), the pressure head H_p is

$$H_p = H - H_g = 1.818 - 0.50 = 1.318 \text{ m}$$

We now reverse the order of the layers so that the less conductive layer overlies the more conductive. The total head drop remains the same and so does the total resistance.



Therefore the flux remains the same (assuming that both layers are still saturated and the conductivity of each does not change). Applying Darcy's equation to the top layer, we have, as previously,

$$H_{\text{interface}} = 2.0 - (3.64 \times 10^{-7}) (0.5) / 10^{-7} = 0.18 \text{ m}$$

In this case the pressure head at the interface is

$$H_p = H - H_g = 0.18 - 0.50 = -0.32 \text{ m}$$

Note: Comparison between the interface pressures of the two cases illustrates an important principle regarding flow in layered profiles. With the more conductive layer on top, flow is impeded at the interface and there is a pressure buildup, which in our case increased the pressure head from 1 m at the surface to 1.218 m at the interface. The opposite occurs when the upper layer is less conductive. In this case the pressure is dissipated through the top layer, often to the extent that a negative pressure develops at the interface.

HYDRAULIC CONDUCTIVITY

The hydraulic conductivity K is the ratio of the flux to the potential gradient.

The hydraulic conductivity K is not a property of the soil alone. Rather, it depends jointly on the attributes of the soil and of the fluid. The soil characteristics that affect K are the total porosity, the distribution of pore sizes, and tortuosity — in short, the soil's pore geometry. The fluid attributes that affect conductivity are density and viscosity.

It is possible in theory, and sometimes in practice, to separate K into two factors: *intrinsic permeability* of the soil k and *fluidity* of the permeating liquid (or gas) f :

$$K = kf$$

If K is given in terms of m/sec (LT^{-1}), then k is in units of m^2 (L^2) and f is in units of $\text{m}^{-1} \text{sec}^{-1}$ ($L^{-1}T^{-1}$).

Fluidity is related directly to density and inversely to viscosity:

$$f = \rho g / \eta$$

Hence,

$$k = K \eta / \rho g$$

Where η is the dynamic viscosity (N sec/m^2 , or Pa sec), ρ is the fluid density (kg/m^3), and g is the gravitational acceleration (m/sec^2).

While fluidity varies with temperature and composition of the fluid, permeability is ideally a property of the porous medium's pore geometry alone, provided the fluid and the solid matrix do not interact in such a way as to change the properties of either. In a stable porous body, the same permeability will be obtained with



different fluids (water, air, or oil). However, in many cases water does interact with the solid matrix to modify its permeability, so hydraulic conductivity cannot be resolved into separate and independent properties of water and of soil.

LIMITATIONS OF DARCY'S LAW

Darcy's law is not universally valid for all conditions of liquid flow in porous media. It has long been recognized that the linearity of the flux versus hydraulic gradient relationship fails at high flow velocities, where inertial forces are no longer negligible compared to viscous forces. Darcy's law applies only as long as flow is laminar (i.e., nonturbulent movement of adjacent layers of the fluid relative to one another) and where soil–water interaction does not result in a change of fluidity or of permeability with a change in gradient.

Laminar flow prevails in silts and finer materials for most commonly occurring hydraulic gradients found in nature. In coarse sands and gravels, however, hydraulic gradients much in excess of unity may result in nonlaminar flow conditions, so in such cases Darcy's law may not be applicable.

The quantitative criterion for the onset of turbulent flow is the *Reynolds number* N_{Re} :

$$N_{Re} = \frac{d u \rho}{\eta}$$

where u is the mean flow velocity, d is the effective pore diameter, ρ is the liquid density, and η is its viscosity.

For porous media, therefore, it is safe to assume that flux remains linear with hydraulic gradient only as long as N_{Re} is smaller than unity.

As flow velocity increases, especially in systems of large pores, the occurrence of turbulent eddies or nonlinear laminar flow results in “waste” of effective energy; that is, some energy is dissipated by the internal churning of the liquid so that the hydraulic potential gradient becomes less effective in inducing flow. This is illustrated in Fig. 7.11.

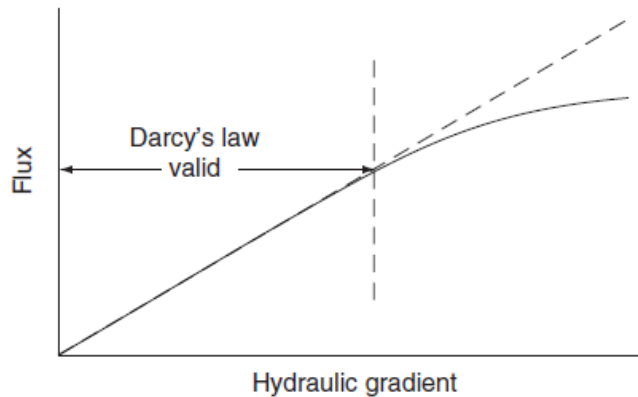


Fig. 7.11. The deviation from Darcy's law at high flux, where flow becomes turbulent.

Deviations from Darcy's law may also occur at the opposite end of the flow-velocity range, namely, at low gradients and in narrow pores. Some investigators have claimed that, in clayey soils, small hydraulic gradients may cause no flow or only low flow rates that are less than proportional to the gradient.

A possible reason for this anomaly is that the water in close proximity to the particle surfaces and subject to their adsorptive force fields may be more rigid than ordinary bulk water and may exhibit the properties of a *Bingham liquid* (having a *yield value*) rather than a *Newtonian liquid* (Hillel, 1980a).

The adsorbed (or "bound") water may even have a quasi-crystalline structure similar to that of ice. Some soils may exhibit a *threshold gradient* (Miller, R. J. and Low, 1963), below which the flux is either zero (the water remaining apparently immobile) or at least lower than predicted by the Darcy relation, and only at gradients exceeding the threshold value does the flux become proportional to the gradient (Fig. 7.12). These phenomena and their possible explanations, though highly interesting, are generally of little importance in practice, so Darcy's law can be employed in the vast majority of cases pertaining to the flow of water in soil.

Another possible cause for apparent flow anomalies in clay soils is their tendency to swell or compress (Smiles, 1976). As commonly formulated, Darcy's law applies to flow relative to a geometrically fixed porous matrix, and it may seem to fail when the particles composing the matrix are themselves moving relative to a fixed frame of reference.

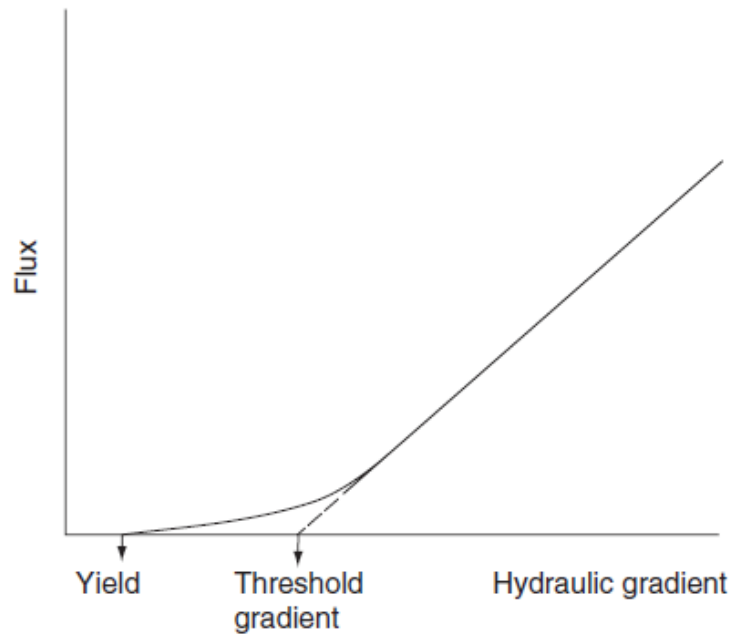


Fig. 7.12. Possible deviations from Darcy's law at low gradients.

Water flow in unsaturated soil:

Perhaps the most important difference between unsaturated and saturated flow is in the hydraulic conductivity. When the soil is saturated, all of the pores are water filled and conducting. The water phase is then continuous and the conductivity is maximal. When the soil desaturates, some of the pores become air filled, so the



conductive portion of the soil's cross-sectional area diminishes. Furthermore, as suction develops, the first pores to empty are the largest ones, which are the most conductive (remember Poiseuille's law!), thus relegating flow to the smaller pores. At the same time, the large empty pores must be circumvented, so, with progressive desaturation, tortuosity increases. In coarse-textured soils, water may be confined almost entirely to the capillary wedges at the contact points of the particles, thus forming separate and discontinuous pockets of water. In aggregated soils, too, the large interaggregate spaces that confer high conductivity at saturation become (when emptied) barriers to liquid flow from one aggregate to another.

For all these reasons, the transition from saturation to unsaturation generally entails a steep drop in hydraulic conductivity, which may decrease by several orders of magnitude (sometimes down to one-millionth of its value at saturation) as suction increases from 0 to 1 MPa. At still higher suctions, or lower wetness values, the conductivity may be so low that very steep suction gradients, or very long times, are required for any appreciable flow to occur at all.

Darcy's law, though originally conceived for flow in saturated porous media, has been extended to unsaturated flow, with the provision that the conductivity is now a function of the matric suction head [i.e., $K = K(\psi)$]:

$$q = -K(\psi) \nabla H$$

where ∇H is the hydraulic head gradient, which may include both suction and gravitational components. This equation, along with its alternative formulations, is known as *Richards' equation*.

This Equation as written here fails to take into account the hysteresis of soil-water characteristics. In practice, the hysteresis problem can sometimes be evaded by limiting the use of this equation to cases in which the suction (or wetness) change is monotonic — either increasing or decreasing continuously. However, in processes involving successive wetting and drying phases, the

$K(\psi)$ function may be highly hysteretic. However, the relation of conductivity to volumetric wetness $K(\theta)$ or to degree of saturation $K(S)$ is affected by hysteresis to a lesser extent than is the $K(\psi)$ function. Darcy's law for unsaturated soil can thus be written

$$q = -K(\theta) \nabla H$$

which, however, still leaves us the problem of dealing with the hysteresis between ψ and θ .

To account for transient flow processes, we introduce the continuity principle:

$$\partial \theta / \partial t = -\nabla \cdot q$$

Thus,

$$\partial \theta / \partial t = \nabla \cdot [K(\psi) \nabla H]$$



The hydraulic head is, in general, the sum of the pressure head (or its negative, the suction head ψ) and the gravitational head (or elevation z), we can write

$$\partial\theta/\partial t = \nabla \cdot [K(\psi)\nabla(\psi - z)]$$

Since ∇z is zero for horizontal flow and unity for vertical flow, we can rewrite Eg. (8.6) as follows:

$$\frac{\partial\theta}{\partial t} = -\nabla \cdot (K(\psi)\nabla\psi) + \frac{\partial K}{\partial z} \quad (8.12a)$$

or

$$\frac{\partial\theta}{\partial t} = -\frac{\partial}{\partial x} \left(K \frac{\partial\psi}{\partial x} \right) - \frac{\partial}{\partial y} \left(K \frac{\partial\psi}{\partial y} \right) - \frac{\partial}{\partial z} \left(K \frac{\partial\psi}{\partial z} \right) + \frac{\partial K}{\partial z} \quad (8.13)$$

Processes may occur in which ∇z (the gravity gradient) is negligible compared to the strong matric suction gradient $\nabla\psi$. in such cases,

$$\partial\theta/\partial t = \nabla \cdot [K(\psi)\nabla\psi]$$

or, in a one-dimensional horizontal system,

$$\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial x} \left[K(\psi) \frac{\partial\psi}{\partial x} \right]$$

The matric suction gradient $\delta\psi/\delta x$ can be expanded by the chain rule:

$$\frac{\partial\psi}{\partial x} = \frac{d\psi}{d\theta} \frac{\partial\theta}{\partial x}$$

Here $\partial\theta/\partial x$ is the wetness gradient and $\partial\psi/\partial\theta$ is the reciprocal of *specific water capacity*, $C(\theta)$:

$$C(\theta) = d\theta/d\psi$$

which is the slope of the soil-moisture characteristic curve at a particular value of wetness θ .

We can now rewrite the Darcy equation as follows:

$$q = K(\theta) \frac{\partial\psi}{\partial x} = -\frac{K(\theta)}{C(\theta)} \frac{\partial\theta}{\partial x}$$

To cast this equation into a form analogous to Fick's law of diffusion, a function is introduced called the *diffusivity*, D :

$$D(\theta) = K(\theta)/C(\theta) = K(\theta)(d\psi/d\theta)$$

D is thus defined as the ratio of the hydraulic conductivity K to the specific water capacity C , and since both are functions of soil wetness, so D must be also. To avoid confusion between the classical concept of diffusivity pertaining to the diffusive transfer of components in the gaseous and liquid phases and this borrowed application of the same term to describe convective flow, we propose to



qualify it with the adjective *hydraulic*. Here, therefore, we shall employ the term *hydraulic diffusivity* when referring to D of Eq. (8.19). We can now rewrite Eq. (8.10):

$$q = -D(\theta)\nabla\theta \quad (8.20)$$

or, in one dimension,

$$q = -D(\theta)(\partial\theta/\partial x) \quad (8.21)$$

Which is mathematically identical to Fick's first law of diffusion. Hydraulic diffusivity can thus be viewed as the ratio of the flux (in the absence of gravity and of hysteresis effects) to the soil-water content (wetness) gradient. As such, D has dimensions of length squared per unit time (L^2T^{-1}), since K has the dimensions of volume per unit area per time (LT^{-1}) and the specific water capacity C has dimensions of volume of water per unit volume of soil per unit change in matric suction head (L^{-1}). In the use of Eq. (8.21), the gradient of wetness is taken to represent, implicitly, a gradient of matric potential, which is the true driving force.

Introducing the hydraulic diffusivity into Eq. (8.15), for one-dimensional flow in the absence of gravity, we obtain

$$\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial x} \left[D(\theta) \frac{\partial\theta}{\partial x} \right] \quad (8.22)$$

which has only one dependent variable (θ) rather than the two (θ and ψ) of Eq. (9.15).

Soil Hysteresis: is the relation between matric potential and soil wetness.

It can be obtained in two ways:

(1) In desorption.

(2) In sorption.

(1) In desorption, by taking an initially saturated sample and applying increasing suction, in stepwise manner, to gradually dry the soil while taking successive measurements of wetness versus suction.

(2) In sorption, by gradually wetting up an initially dry soil sample while reducing the suction incrementally.

Each of these two methods yields a continuous curve, but the two curves will in general not be identical. The equilibrium soil wetness at a given suction is greater in desorption (drying) than in sorption (wetting).

This dependence of the equilibrium content and state of soil water upon the direction of the process leading up to it is called *hysteresis*



Figure 6.10 shows a typical soil-moisture characteristic curve and illustrates the hysteresis effect in the relationship between soil wetness and matric suction at equilibrium.

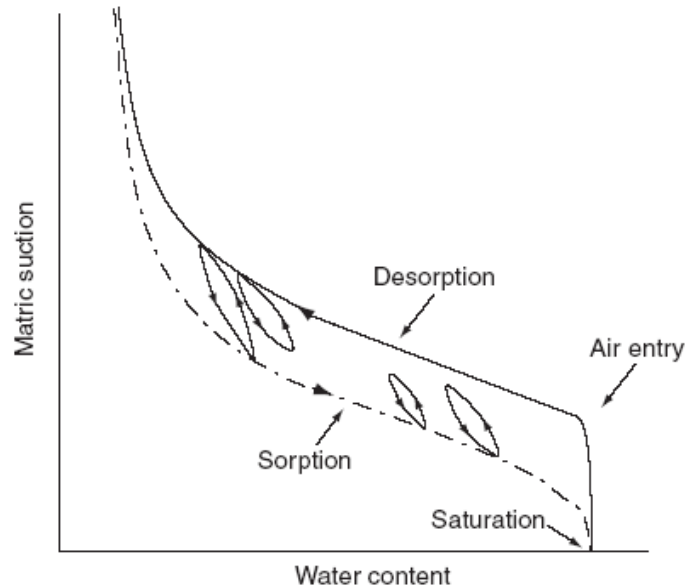


Fig. 6.10. Suction vs. water content curves in sorption and desorption. The intermediate loops are scanning curves, representing complete or partial transitions between the main branches.

The hysteresis effect may be attributed to several causes:

1. The geometric nonuniformity of the individual pores
2. The contact-angle effect by which the contact angle is greater and hence the radius of curvature greater in the case of an advancing meniscus than in the case of a receding one.
3. Entrapped air, which further decreases the water content of newly wetted soil.
4. Swelling, shrinking, or aging phenomena, which result in differential changes of soil structure, depending on the wetting and drying history of the sample.

WATER ENTRY INTO SOIL INFILTRATION

When water is supplied from above to the soil surface, whether by precipitation or irrigation, it typically penetrates the surface and is absorbed into successively



deeper layers of the profile. At times, however, a portion of the arriving water may fail to penetrate but instead will tend to accrue at the surface or flow over it. The penetrated water is itself later partitioned between the amount that returns to the atmosphere by direct evaporation from the soil or by the extraction and transpiration of plants and the amount that continues to seep downward and eventually recharges the groundwater reservoir.

Infiltration is the term applied to the process of water entry into the soil, generally by downward flow through all or part of the soil surface.

The rate of this process, relative to the rate of water supply, determines how much water will enter the root zone and how much, if any, will run off. Hence the rate of infiltration affects not only the water economy of terrestrial plants but also the amount of overland flow and its attendant dangers of soil erosion and stream flooding.

Knowledge of the infiltration process as it is affected both by the soil's properties and transient conditions and by the mode of water supply is therefore a prerequisite for efficient soil and water management.

INFILTRATION CAPACITY OR INFILTRABILITY

The *infiltration rate* is defined as the volume flux of water flowing into the profile per unit of soil surface area. For the special condition in which the rainfall rate exceeds the ability of the soil to absorb water, infiltration proceeds at a maximal rate, which Horton (1940) called the soil's *infiltration capacity*.

Hillel (1971) suggested the term *infiltrability* to designate the infiltration flux resulting when water at atmospheric pressure is made freely available at the soil surface. This one-word replacement avoids the extensity–intensity confusion in the term *infiltration “capacity”* and permits use of the term *infiltration “rate”* in the literal sense to represent the surface flux in any set of circumstances, whatever the rate or pressure at which the water is supplied to the soil. The infiltration rate can be expected to exceed infiltrability whenever water is ponded over the soil to a depth sufficient to cause the hydrostatic pressure at the surface to be significantly greater than atmospheric pressure.

On the other hand, if water is applied slowly or at subatmospheric pressure, the actual infiltration rate may be smaller than the infiltrability. As long as the rate of water delivery to the surface is smaller than the soil's infiltrability, water infiltrates as fast as it arrives and the process is *supply controlled* (or *flux controlled*). However, once the delivery rate exceeds the soil's infiltrability, the latter determines the actual infiltration rate, and thus the process becomes *soil controlled*. The soil may limit the rate of infiltration either at the surface or within the profile. Thus the process may be either *surface controlled* or *profile controlled*.



Soil infiltrability and its variation with time are known to depend on the initial wetness and suction as well as on the texture, structure, and uniformity (or layering sequence) of the profile. In general, soil infiltrability is relatively high in the early stages of infiltration, particularly where the soil is initially quite dry, but it tends to decrease and eventually to approach asymptotically a constant rate that we call the *steady-state infiltrability*.

The figures below show the relation between cumulative infiltration and infiltration rate with time (Fig. 14.2).

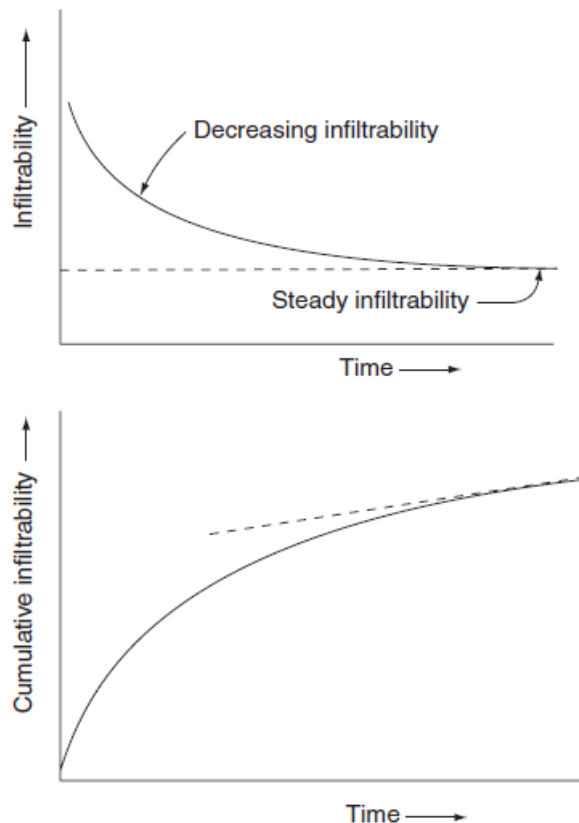


Fig. 14.2. Time dependence of infiltrability and of cumulative infiltration under shallow ponding.

The decrease of infiltrability from an initially high rate can in some cases result from gradual deterioration of soil structure and the partial sealing of the profile by the formation of a surface crust. It can also result from the detachment and migration of pore-blocking particles, from swelling of clay, as well as from entrapment of air bubbles or the bulk compression of the air originally present in the soil if it is prevented from escaping during its displacement by incoming water. Primarily, however, the decline of infiltrability results from the decrease in the *matric suction gradient*, which occurs inevitably as infiltration proceeds. If the surface of an initially dry soil is suddenly saturated (e.g., by ponding), the difference of hydraulic potential between the saturated surface and the relatively dry soil just below creates a steep matric suction gradient. As the wetted zone



deepens, the same difference of potential acting over a greater distance expresses itself as a diminishing gradient. And as the wetted part of the profile deepens, the suction gradient eventually becomes vanishingly small. In a horizontal column, the infiltration rate eventually tends to zero. In downward flow into a vertical column under continuous ponding, the infiltration rate tends asymptotically to a steady, gravity-induced rate that approximates the soil's saturated hydraulic conductivity if the profile is homogeneous and structurally stable (Fig. 14.2).

Relatively dry soil just below creates a steep matric suction gradient. As the wetted zone deepens, the same difference of potential acting over a greater distance expresses itself as a diminishing gradient. And as the wetted part of the profile deepens, the suction gradient eventually becomes vanishingly small. In a horizontal column, the infiltration rate eventually tends to zero. In downward flow into a vertical column under continuous ponding, the infiltration rate tends asymptotically to a steady, gravity-induced rate that approximates the soil's saturated hydraulic conductivity if the profile is homogeneous and structurally stable (Fig. 14.2).

PROFILE MOISTURE DISTRIBUTION DURING INFILTRATION:

An examination of an initially dry, texturally uniform soil profile at any moment during infiltration under ponding generally shows the three zones as below:

- 1- *Saturated Zone*: surface zone to be saturated to a depth of several millimeters or centimeters.
- 2- *Transmission zone*: is a less-than-saturated, lengthening zone of apparently uniform wetness.
- 3- *Wetting zone*: in this zone soil wetness increases with time at each point, but at any given time wetness decreases with depth at a steepening gradient, down to a *wetting front*.

At the wetting front, the moisture gradient is so steep that there appears to be a sharp boundary between the moistened soil above and the initially dry soil beneath.

The typical moisture profile during infiltration is shown schematically in Fig. 14.3. Repeated examinations of the moisture profile of a stable soil during infiltration generally reveal that the transmission zone lengthens (deepens) continuously and that the wetting zone and the wetting front likewise move downward continuously, with the latter becoming less steep as it moves deeper into the profile.

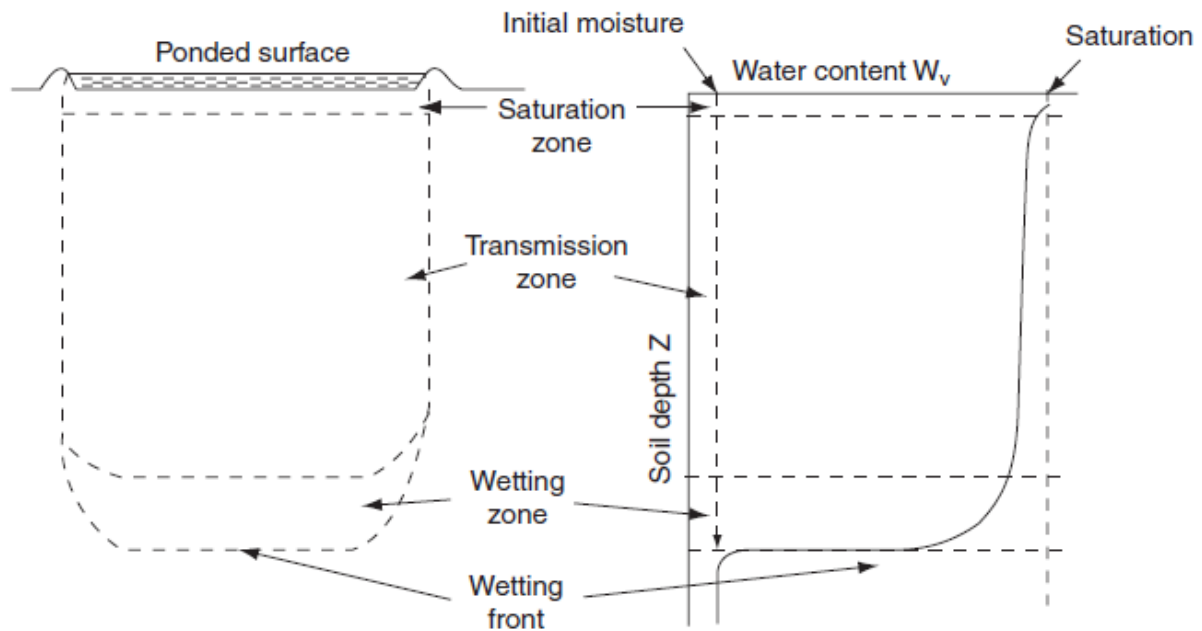


Fig. 14.3. The infiltration moisture profile. At left, a schematic section of the profile; at right, the curve of water content versus depth. The common occurrence of a saturation zone as distinct from the transmission zone may result from the structural instability of the surface zone.

THE EFFECT OF INITIAL WATER STATE ON INFILTRATION RATE:

Figure (Fig. 14.6) show the Infiltration rate with time in an initially dry and in an initially moist soil.

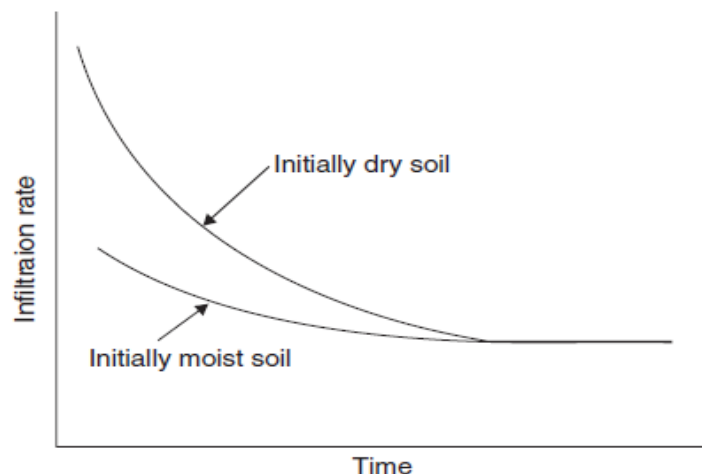


Fig. 14.6. Infiltrability as function of time in an initially dry and in an initially moist soil.

When infiltration takes place into an initially wet soil, the suction gradients are weak from the start and become negligible much sooner.

Models of Infiltration Capacity:

1- Green and Ampt (1911):

Earliest equation was proposed by Green and Ampt (1911) in which the infiltration capacity is:

$$i = i_c + \frac{b}{I}$$



where i and I are the infiltration rate and cumulative infiltration, respectively, i_c is analogous to the saturated hydraulic conductivity (note: at large times, $I \rightarrow \infty$ and $i = i_c$), and b is an empirical parameter. Note, time does not explicitly appear in the Green-Ampt equation. The model can be re-formulated so that:

$\frac{dI}{dt} = i_c + \frac{b}{I}$, and hence time becomes explicit. Naturally, by re-arranging the equation $\frac{dI}{i_c + \frac{b}{I}} = dt$, and integrating both sides:

$$t = \frac{I}{i_c} - \frac{b}{i_c^2} \log(i_c I + b)$$

2- Kostiakov (1932):

The next equation was proposed by Kostiakov, and is given by:

$$i = B t^{-n}$$

Where B and n are constants. The Kostiakov model is flawed at the two end-members:

$$\begin{aligned} t \rightarrow \infty, i &\rightarrow 0, \\ t \rightarrow 0, i &\rightarrow \infty. \end{aligned}$$

The cumulative infiltration is given as:

$$I = \frac{B}{1-n} t^{1-n}$$

3- Horton (1940):

The Horton model is a 3 parameter model derived to alleviate the problems in the asymptotic limits of the Kostiakov model.

$$i = i_c + (i_o - i_c) e^{-\beta t}$$

i_c = saturated hydraulic conductivity

i_o = initial infiltration rate (or i at $t=0$)

β = constant that varies with soil type and soil cover.

- 4- **Philip (1957):** Using an analytical solution to Richard's equation for soil moisture redistribution: $\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x} \left[K(\theta) \frac{\partial H}{\partial x} \right]$, where θ is the volumetric soil moisture, K is the hydraulic conductivity function, and H is the total energy, Philip (1957) demonstrated that:

$$i = K_s + \frac{1}{2} S_p t^{-1/2}$$

Where i is the infiltration capacity, K_s is the saturated hydraulic conductivity, and S_p is the sorption coefficient.

Note, for short times, $t^{-1/2}$ is very large and sorption controls the infiltration capacity



(i.e. $i \approx \frac{1}{2} \frac{S_p}{\sqrt{t}}$). For large times, $i \rightarrow K_{sat}$.

The cumulative infiltration capacity is given by:

$$I = K_s t + S_p t^{1/2}$$

Example:

The initial infiltration capacity of a watershed is estimated as 1.5 in/hr, and the time constant taken to be 0.35 hr^{-1} . The equilibrium capacity is estimated as 0.2 in/hr.

- What are the values of (f) at $t = 10 \text{ min}$, 30 min , 1 hr , 2 hr , and 6 hr .
- what is the total volume of infiltration over the 6 hour time period?

From the Horton equation, we have:

$$i = i_c + (i_o - i_c)e^{-\beta t}$$

i_c = saturated hydraulic conductivity

i_o = initial infiltration rate (or i at $t=0$)

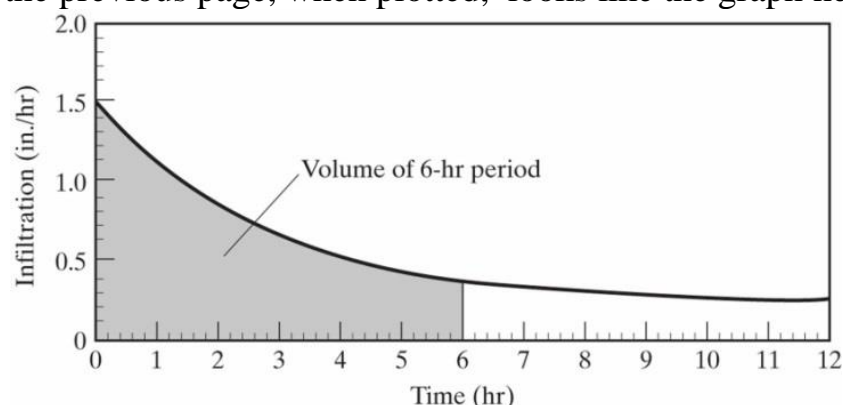
β = constant that varies with soil type and soil cover.

$$f = 0.2 + 1.3(e^{-0.35t})$$

- Substituting in values of t yields:

t	f
1/6	1.43
1/2	1.29
1	1.12
2	0.85
6	0.36

- The table on the previous page, when plotted, looks like the graph here.



The *volume* of water can be found by taking the definite integral (use font SymbolMT J) under the curve from 0 to 6 hours. Here the integration is easy, and turns out like this:

$$\int_0^6 (0.2 + 1.3e^{-0.35t}) dt = \left[0.2t + \frac{1.3}{-0.35} e^{-0.35t} \right]_0^6$$

Evaluating the right side for $t = 6$ and then subtracting the values for $t=0$ yields an answer of 4.46 inches over the watershed.



EVAPORATION FROM BARE SOIL

EVAPORATION PROCESSES:

Evaporation is the primary process of water transfer in the hydrological cycle. The water is transformed into vapour and transported to the sky.

The evaporation plus transpiration from a vegetated surface with unlimited water supply is known as *potential evaporation* or *potential evapotranspiration* (PE) and it constitutes the maximum possible rate due to the prevailing meteorological conditions. Thus PE is the maximum value of the actual evaporation (E_t): $PE = E_t$ when water supply is unlimited.

Actual evaporation is the amount of water which is evaporated a normal day which means that if for instance the soil runs out of water, the actual evaporation is the amount of water which has been evaporated, and not the amount of water which could have been evaporated if the soil had had an infinite amount of water to evaporate.

Because of the variability of region and seasons, water managers who are responsible for planning and adjudicating the distribution of water resources need to have a thorough understanding of the evapotranspiration process and knowledge about the spatial and temporal rates of evapotranspiration.

PHYSICAL CONDITIONS

Three conditions are necessary for evaporation to occur and persist. First, there must be a continual supply of heat to meet the latent heat requirement (about 2.5×10^6 J/kg, or 590 cal/g of water evaporated at 15°C). This heat can come from the body itself, thus causing it to cool, or — as is more common — it can come from the outside in the form of radiated or advected energy. Second, the vapor pressure in the atmosphere over the evaporating body must remain lower than the vapor pressure at the surface of that body (i.e., there must be a vapor pressure gradient between the body and the atmosphere), and the vapor must be transported away by diffusion or convection or both. These two conditions — supply of energy and removal of vapor — are generally external to the evaporating body and are influenced by meteorological factors, such as air temperature, humidity, wind velocity, and radiation, which together determine the *atmospheric evaporativity* (the maximal flux at which the atmosphere can vaporize water from a free-water surface).

The third condition for evaporation to be sustained is that there be a continual supply of water from or through the interior of the body to the site of evaporation. This condition depends on the content and potential of water in the body as well as on its conductive properties, which together determine the maximal rate at which water can be transmitted to the evaporation site (usually, the surface). Accordingly, the actual evaporation rate is determined either by the *evaporativity* of the atmosphere or by the soil's own ability to deliver water (sometimes called the



evaporability of soil moisture), whichever is the lesser (and hence the limiting factor).

If the top layer of soil is initially quite wet, as it typically is at the end of an infiltration episode, the process of evaporation will generally reduce soil wetness and thus increase matric suction at the surface. This, in turn, will cause soil moisture to be drawn upward from the layers below, provided they are sufficiently moist.

Calculation of actual evaporation

There are a few formulas which are more common than others. In this chapter will we just write about the Penman-Monteith formula because it's the most known and most used formula when it's about calculating evapotranspiration. The origin of the equation is the Penman equation and later Monteith developed the formula even more.

Penman-Monteith

The most known formula for evapotranspiration is the Penman-Monteith formula,

$$E_T = \frac{\Delta R_n + (\rho^* c_p) \frac{e_a - e_d}{r_a}}{\lambda (\Delta + \gamma^* (1 + \frac{r_s}{r_a}))}$$

where R_n = net radiation (W/m^2)
 ρ = density of air
 c_p = specific heat of air
 r_s = net resistance to diffusion through the surfaces of the leaves and soil (s/m)

r_a = net resistance to diffusion through the air from surfaces to height of measuring instruments (s/m).

γ = hygrometric constant

$\Delta = de/dT$

e_a = saturated vapour pressure at air temperature

e_d = mean vapour pressure

Estimating actual evapotranspiration from potential

The calculation of potential evaporation (PE) from readily available meteorological data is seen to be much simpler operation than the computation or measurement of actual evapotranspiration (E_t) from a vegetated surface. However, water loss from a catchment area does not always proceed at the potential rate, since this is dependent on a continuous water supply. When the vegetation is unable to abstract



water from the soil, then the actual evaporation becomes less than potential. Thus the relationship between E_t and PE depends upon the soil moisture content.

- Here is an example of the relationship between PE and E_t , according to Bergström, 1992:

$$\begin{aligned} E_t &= PE && \text{when } h \geq h_{FC} \\ E_t &= PE \left[\frac{h - h_{WP}}{h_{FC} - h_{WP}} \right] && \text{when } h_{WP} < h < h_{FC} \\ E_t &= 0 && \text{when } h \leq h_{WP} \end{aligned}$$

Where:

h is the amount of soil moisture (mm).

h_{FC} is the amount of soil moisture corresponding to field capacity (mm).

h_{WP} is the amount of soil moisture corresponding to the wilting point (mm).

Field Capacity, Wilting Point and Available Water in a Soil Column

Field Capacity (FC or θ_{fc})—Soil water content where gravity drainage becomes negligible—Soil is not saturated but still a very wet condition.

The field capacity is a function of soil texture and soil profile properties.

For sands, $\theta_{fc} \approx (0.1)$ while for clays $\theta_{fc} \approx (0.3)$.

Permanent Wilting Point (WP or θ_w)—Soil water content beyond which plants cannot recover from water stress (dead)—Still some water in the soil but not enough to be of use to plants

The wilting point soil is a function of soil texture, soil profile properties and vegetation characteristics. For sands, $\theta_w \approx 0.05$ while for clays $\theta_w \approx 0.25$

Available water for plant use

$$\theta_a = \theta_{fc} - \theta_w$$

SOIL PHYSICS

Class Notes

By

Assistant Professor

Maysam Th. Al-Hadidi

These notes are photocopies of the Textbook
“Environmental Soil Physics”

by Daniel Hillel,
Academic Press, 1998



INTRODUCTION:

Soil: is the weathered and fragmented outer layer of the earth's terrestrial surface. It is formed initially through disintegration, decomposition and decomposition rocks by physical, chemical, and biological processes.

Soil science: is the study of the soil in all its ramified and facets: as a central link in the biosphere, as a medium for the production of agricultural commodities, and as a raw material for industry and construction.

As such, it shares interests with geology, sedimentology, terrestrial ecology, and geobotany as well as with such applied sciences as agronomy and engineering.

Soil science itself is commonly divided into several subdivisions, including:

- 1- pedology (soil formation and classification).
- 2- Soil chemistry.
- 3- Soil mineralogy.
- 4- Soil biology.
- 5- Soil fertility.
- 6- soil mechanics

SOIL PHYSICS:

Soil physics is one of the major subdivisions of soil science. It seeks to define, measure, and predict the physical properties and behavior of the soil, both in its natural state and under the influence of human activity.

As physics deals in general with the forms and interactions of matter and energy, so soil physics deals specifically with the state and movement of matter and with the fluxes and transformations of energy in the soil. On the one hand, the fundamental study of soil physics aims at understanding the mechanisms governing such processes as terrestrial energy exchange, the cycles of water and of transportable materials, and the growth of plants in the field.

Increasingly, the main concern of soil physics has shifted from the laboratory to the field and from a restricted one-dimensional view to an expansive three-dimensional view interfacing with the domains of sister disciplines such as meteorology and climatology, hydrology, ecology, and geochemistry.



THE SOIL PROFILE:

The most obvious part of any soil is its surface zone. Through it, matter and energy are transported between the soil and the atmosphere. The surface may be smooth or pitted, granular or crusted and cracked, level or sloping, vegetated or fallow. Such conditions affect the processes of radiation and heat exchange, water and solute movement.

To describe the latter, we must examine the soil in depth. We can do this by digging a trench and sectioning the soil from the surface downward, thus revealing what is commonly termed the *soil profile*.

The soil profile typically consists of a succession of more-or-less distinct strata. These may result from the pattern of deposition, or sedimentation, as can be observed in wind-deposited (*aeolian*) soils and particularly in water deposited (*alluvial*) soils.

The top layer, or *A horizon*, is the zone of major biological activity and is therefore generally enriched with organic matter and darker in color than the underlying soil. Underneath the A horizon is the *B horizon*, where some of the materials (e.g., clay or carbonates) that are leached from the A horizon by percolating water tend to accumulate.

Underlying the B horizon is the *C horizon*, which is the soil's parent material. In a soil formed of bedrock in situ (called a *residual soil*), the C horizon consists of the weathered and fragmented rock material.

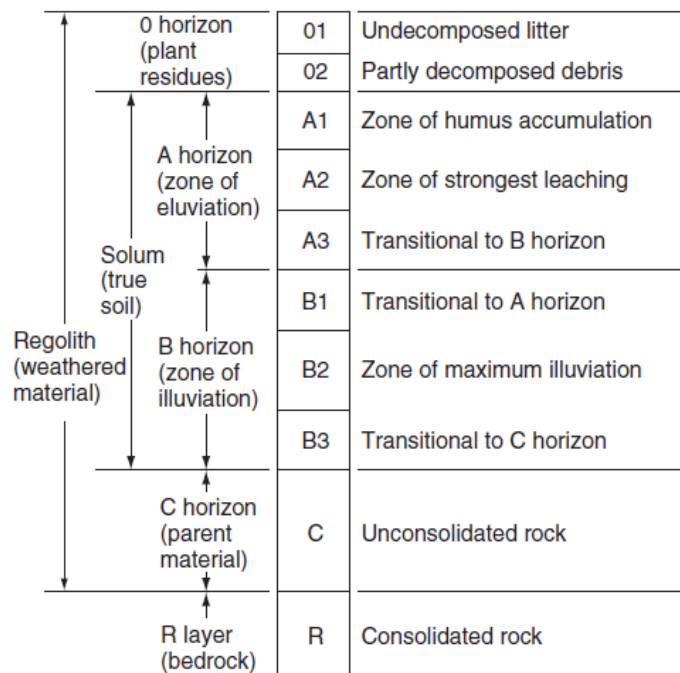


Fig. 1.2. Descriptive terminology for soil profile horizons.



VOLUME AND MASS RELATIONSHIPS OF SOIL CONSTITUENTS:

Let us consider the volume and mass relationships among the three phases of the soil, and define some basic parameters that can help to characterize the soil physically. Figure 1.4 is a schematic depiction of a hypothetical soil in which the three phases have been separated and stacked one atop the other for the purpose of showing their relative volumes and masses.

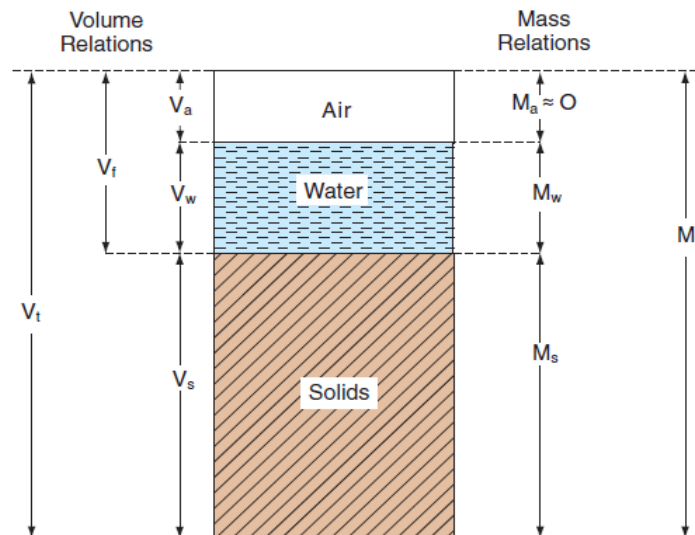


Fig. 1.4. Schematic diagram of the soil as a three-phase system.

In the figure, the masses of the phases are indicated on the right-hand side: the mass of air M_a , which is negligible compared to the masses of solids and water; the mass of water M_w ; the mass of solids M_s ; and the total mass M_t .

(These masses can also be represented in terms of their weights, being the product of each mass and gravitational acceleration.) The volumes of the same components are indicated on the left-hand side of the diagram: volume of air V_a , volume of water V_w , volume of pores $V_f = V_a + V_w$, volume of solids V_s , and the total volume of the representative sample V_t .

On the basis of this diagram, we can now define terms that are generally used to express the quantitative interrelations of the three primary soil phases.

Density of Solids (Mean Particle Density) ρ_s

$$\rho_s = M_s / V_s$$

In most mineral soils, the mean mass per unit volume of solids is about 2600–2700 kg/m³.

Sometimes the density is expressed in terms of the *specific gravity* which is the ratio of the density of any material to that of water at 4°C and at atmospheric pressure.

The latter density is about 1000 kg/m³, so the specific gravity of the solid phase in a typical mineral soil is about 2.65, a value that is numerically (though not dimensionally) equal to the density expressed in (g/cm³) units.



Dry Bulk Density ρ_b

$$\rho_b = M_s / V_t = M_s / (V_s + V_a + V_w)$$

The dry bulk density expresses the ratio of the mass of solids to the total soil volume (solids and pores together). Obviously, ρ_b is always smaller than ρ_s .

The bulk specific gravity of sandy soils with a relatively low volume of pores may be as high as 1.6, whereas that of aggregated loams and clay soils may be below 1.2. Whereas the mean particle density is typically constant, the bulk density is highly labile.

It is affected by:

- 1- The structure of the soil.
- 2- Its looseness or degree of compaction.
- 3- Its swelling and shrinkage characteristics, which depend on clay content and water content.

Total (Wet) Bulk Density ρ_t

$$\rho_t = M_t / V_t = (M_s + M_w) / (V_s + V_a + V_w)$$

This is an expression of the total mass of a moist soil per unit volume. As such, this parameter depends more strongly than does the dry bulk density on soil wetness or water content.

Dry Specific Volume v_b

$$v_b = V_t / M_s = 1 / \rho_b$$

The volume of a unit mass of a dry soil (the reciprocal of the dry bulk density) serves as another useful index of the degree of looseness or compaction of a soil body.

Porosity f

$$f = V_f / V_t = (V_a + V_w) / (V_s + V_a + V_w)$$

Porosity is an index of the relative pore space in a soil. Its value generally ranges from 0.3 to 0.6 (30–60%). Coarse-textured soils tend to be less porous than fine-textured soils, though the mean size of individual pores is greater in the former. In clayey soils, the porosity is highly variable because the soil alternately swells, shrinks, aggregates, disperses, compacts, and cracks.

Void Ratio e

$$e = V_f / V_s = (V_a + V_w) / (V_t - V_f)$$

The void ratio is also an index of the fractional pore space, but it relates that space to the volume of solids rather than to the total volume of the soil. As such, it ranges between 0.3 and 2.

Void ratio is the index generally preferred by soil engineers, while porosity is more frequently used by agronomists.



Soil Wetness (Water Content)

The water content of a soil can be expressed in various ways: relative to the mass of solids, or to the total mass, or to the volume of solids, or to the total volume, or to the volume of pores. The various indexes are defined as follows.

Mass Wetness w

$$w = M_w / M_s$$

This is the mass of water relative to the mass of dry soil particles. The standard definition of *dry soil* refers to a mass of soil dried to equilibrium (in practice, over a 24-hour period) in an oven at 105°C, though a clay soil may still contain an appreciable amount of water at that state. Mass wetness is sometimes expressed as a decimal fraction but more often as a percentage.

The water content at saturation (when all pores are filled with water) is also higher in clayey than in sandy soils. In different soils, w can range between 25% and 60%, depending on bulk density. In the special case of organic soils, such as peat or muck soils, the saturation water content on the mass basis may exceed 100%.

Volume Wetness θ

$$\theta = V_w / V_t = V_w / (V_s + V_f)$$

$$\text{Or } \theta = \rho_b * w$$

The volume wetness (often termed *volumetric water content*) is generally computed as a percentage of the total soil volume. At saturation, therefore, it is equal to the porosity.

The use of w rather than θ to express water content is often more convenient because it is more directly applicable to the computation of fluxes and water volumes added to soil by rain or irrigation and to quantities extracted from the soil by evaporation and transpiration. The volume ratio is also equivalent to the depth ratio of soil water, that is, the depth of water per unit depth of soil.

Water Volume Ratio v_w

$$v_w = V_w / V_s$$

For swelling soils, in which porosity changes markedly with wetness, it may be preferable to refer the volume of water present in a sample to the invariant volume of particles rather than to total volume. At saturation, v_w is equal to the void ratio e .

Degree of Saturation s

$$s = V_w / V_f = V_w / (V_a + V_w)$$

This index expresses the water volume present in the soil relative to the pore volume. Index s ranges from zero in a completely dry soil to unity (100%) in a saturated soil. Complete saturation, however, is hardly ever attainable in field conditions, since some air is nearly always present.



Air-Filled Porosity (Fractional Air Content) f_a

$$f_a = V_a/V_t = V_a/(V_s + V_a + V_w)$$

This is a measure of the relative content of air in the soil and as such is an important criterion of soil aeration. It is related negatively to the degree of saturation s (i.e., $f_a = 1 - s$). The relative volume of air in the soil may also be expressed as a fraction, a , of the pore volume (rather than of the total soil volume). Thus, $a = V_a/V_f = V_a/(V_a + V_w)$

Sample Problem

A sample of moist soil with a wet mass of 1.0 kg and a volume of 0.64 liters ($6.4 \times 10^{-4} \text{ m}^3$) was dried in the oven and found to have a dry mass of 0.8 kg. Assuming the typical value of particle density for a mineral soil (2650 kg/m^3), calculate the bulk density ρ_b , porosity f , void ratio e , mass wetness w_m , volume wetness θ , water volume ratio v_w , degree of saturation s , and air-filled porosity f_a .

Solution:

$$\text{Bulk density: } \rho_b = M_s/V_t = 0.8 \text{ kg}/6.4 \times 10^{-4} \text{ m}^3 = 1250 \text{ kg/m}^3$$

$$\text{Porosity: } f = 1 - \rho_b/\rho_s = 1 - 1250/2650 = 1 - 0.472 = 0.528$$

$$\text{Alternatively, } f = V_f/V_t = (V_t - V_s)/V_t$$

$$\text{and since } V_s = M_s/\rho_s = 0.8 \text{ kg}/2650 \text{ kg/m}^3 = 3.02 \times 10^{-4} \text{ m}^3$$

$$\text{hence } f = (6.4 - 3.02) \times 10^{-4} \text{ m}^3 / 6.4 \times 10^{-4} \text{ m}^3 = 0.528 = 52.8\%$$

$$\text{Void ratio: } e = V_t/V_s = (V_t - V_s)/V_s = (6.4 - 3.02) \times 10^{-4} \text{ m}^3 / 3.02 \times 10^{-4} \text{ m}^3 = 1.12$$

$$\text{Mass wetness: } w = M_w/M_s = (M_t - M_s)/M_s = (1.0 - 0.8) \text{ kg}/0.8 \text{ kg} = 0.25 = 25\%$$

$$\text{Volume wetness: } \theta = V_w/V_t = 2.0 \times 10^{-4} \text{ m}^3 / 6.4 \times 10^{-4} \text{ m}^3 = 0.3125 = 31.25\%$$

(Note: $V_w = M_w/\rho_w$, wherein $\rho_w \approx 1000 \text{ kg/m}^3$)

$$\text{Alternatively, } \theta = w\rho_b/\rho_w = 0.25(1250 \text{ kg/m}^3/1000 \text{ kg/m}^3) = 0.3125$$

$$\text{Water volume ratio: } v_w = V_w/V_s = 2.0 \times 10^{-4} \text{ m}^3 / 3.02 \times 10^{-4} \text{ m}^3 = 0.662$$

$$\text{Degree of saturation: } s = V_w/(V_t - V_s) = 2.0 \times 10^{-4} \text{ m}^3 / (6.4 - 3.02) \times 10^{-4} \text{ m}^3 = 0.592$$

$$\text{Air-filled porosity: } f_a = V_a/V_t = (6.4 - 2.0 - 3.02) \times 10^{-4} \text{ m}^3 / 6.4 \times 10^{-4} \text{ m}^3 = 0.216$$

Sample Problem

What is the equivalent depth of water contained in a soil profile 1 m deep if the mass wetness of the upper 0.4 m is 15% and that of the lower 0.6 m is 25%? Assume a bulk density of 1200 kg/m^3 in the upper layer and 1400 in the lower layer. How much water does the soil contain, in cubic meters per hectare of land?

$$\theta = w(\rho_b/\rho_w), \text{ where } \rho_w = 1000 \text{ kg/m}^3$$

$$\text{Volume wetness in the upper layer: } \theta_1 = 0.15(1200/1000) = 0.18$$

$$\text{Equivalent depth in upper 0.4 m} = 0.18 \times 0.4 \text{ m} = 0.072 \text{ m} = 72 \text{ mm}$$

$$\text{Volume wetness in lower layer: } \theta_2 = 0.25(1400/1000) = 0.35$$

$$\text{Equivalent depth in lower 0.6 m} = 0.35 \times 0.6 \text{ m} = 0.21 \text{ m} = 210 \text{ mm}$$



Total depth of water in 1-m profile = $0.072 \text{ m} + 0.210 \text{ m} = 0.282 \text{ m}$

Volume of water contained in 1-m profile per hectare ($1 \text{ ha} = 10^4 \text{ m}^2$) = $0.282 \text{ m} \times 10000 \text{ m}^2 = 2820 \text{ m}^3$



PARTICLE SIZES, SHAPES, AND SPECIFIC SURFACE

PARTICLE SIZE ANALYSIS:

It is possible to classify or group soil particles according to their sizes and to characterize the soil as a whole in terms of the relative proportions of those size groups. The groups may differ from one another in mineral composition as well as in particle size.

TEXTURAL FRACTIONS

The traditional method of characterizing particle sizes in soils is to divide the array of particle diameters into three conveniently separable size ranges known as *textural fractions* or *separates*, namely, *sand*, *silt*, and *clay*. The actual procedure of separating out these fractions and of measuring their proportions is called *mechanical analysis*, for which standard techniques have been devised. The results of this analysis yield the *mechanical composition* of the soil, a term often used interchangeably with *soil texture*.

Unfortunately, there is as yet no universally accepted scheme for classifying particle sizes. For instance, the classification standardized in America by the U.S. Department of Agriculture differs from that of the International Soil Science Society (ISSS), as well as from those promulgated by the American Society for Testing Materials (ASTM), the Massachusetts Institute of Technology (MIT), and various national institutes abroad. The classification followed by soil engineers often differs from that of agricultural soil scientists.

The same terms are used to designate differing size ranges, an inconsistency that can be confusing indeed. Several of the often-used particle size classification schemes are compared in Fig. 3.1.

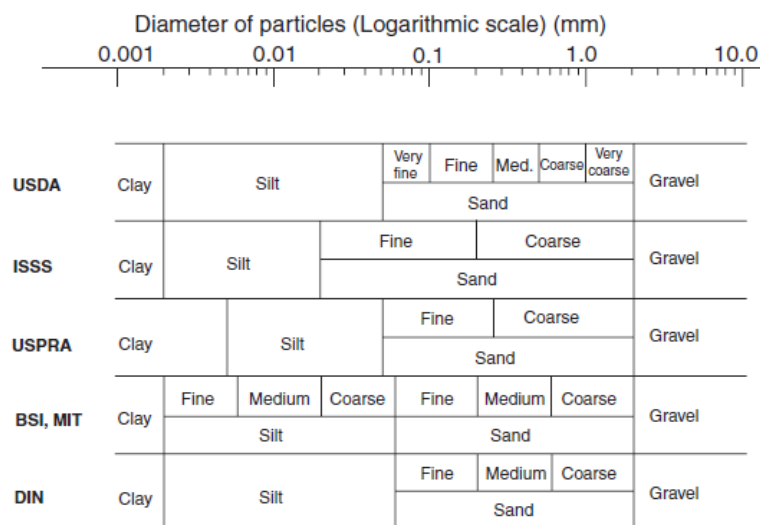


Fig. 3.1. Several conventional schemes for the classification of soil fractions according to particle diameter ranges; U.S. Department of Agriculture (USDA); International Soil Science Society (ISSS); U.S. Public Roads Administration (USPRA); British Standards Institute (BSI); Massachusetts Institute of Technology (MIT); German Standards (DIN).



An essential criterion for determining soil texture is the upper limit of particle size that is to be included in the definition of *soil material*. Some soils contain large rocks that obviously do not behave like soil, but, if numerous, might affect the behavior of the soil in bulk. The conventional definition of soil material includes particles smaller than 2 mm in diameter. Larger particles are generally referred to as *gravel*, and still larger rock fragments, several centimeters in diameter, are variously called *stones*, *cobbles*, or — if very large — *boulders*.

The largest particles that are generally recognized as soil material are designated *sand*, defined as particles ranging in diameter from 2000 μm (2 mm) to 50 μm (USDA classification) or to 20 μm (ISSS classification). The sand fraction is often further divided into subfractions such as coarse, medium, and fine sand.

The next fraction is silt, which consists of particles intermediate in size between sand and clay. Mineralogically and physically, silt particles generally resemble sand particles. However, since the silt are smaller, particles have a greater surface area per unit mass, and are often coated with strongly adherent clay, silt may exhibit, to a limited degree, some of the physicochemical characteristics generally attributed to clay.

The clay fraction, with particles ranging from 2 μm downwards, is the colloidal fraction. Clay particles are characteristically plate like or needlelike in shape and generally belong to a group of minerals known as the *alumino-silicates*. These are secondary minerals, formed in the soil itself in the course of its evolution from the primary minerals that were contained in the original rock. In some cases, however, the clay fraction may include particles (such as iron oxide and calcium carbonate) that do not belong to the alumino-silicate clay mineral category.

TEXTURAL CLASSES

The overall textural designation of a soil, called *textural class*, is conventionally based on the mass ratios of the three fractions. Soils with different proportions of sand, silt, and clay are assigned to different classes, as shown in the triangular diagram of Fig. 3.3. To illustrate the use of the textural triangle, let us assume that a soil is composed of 50% sand, 20% silt, and 30% clay. Note that the lower left apex of the triangle represents 100% sand and the right side of the triangle represents 0% sand. Now find the point of 50% sand on the bottom edge of the triangle and follow the diagonally leftward line rising from that point and parallel to the zero line for sand. Next, identify the 20% line for silt, which is parallel to the zero line for silt, namely, the left edge of the triangle. Where the two lines intersect, they meet the 30% line for clay. The soil in this example happens to fit the category of “sandy clay loam.”

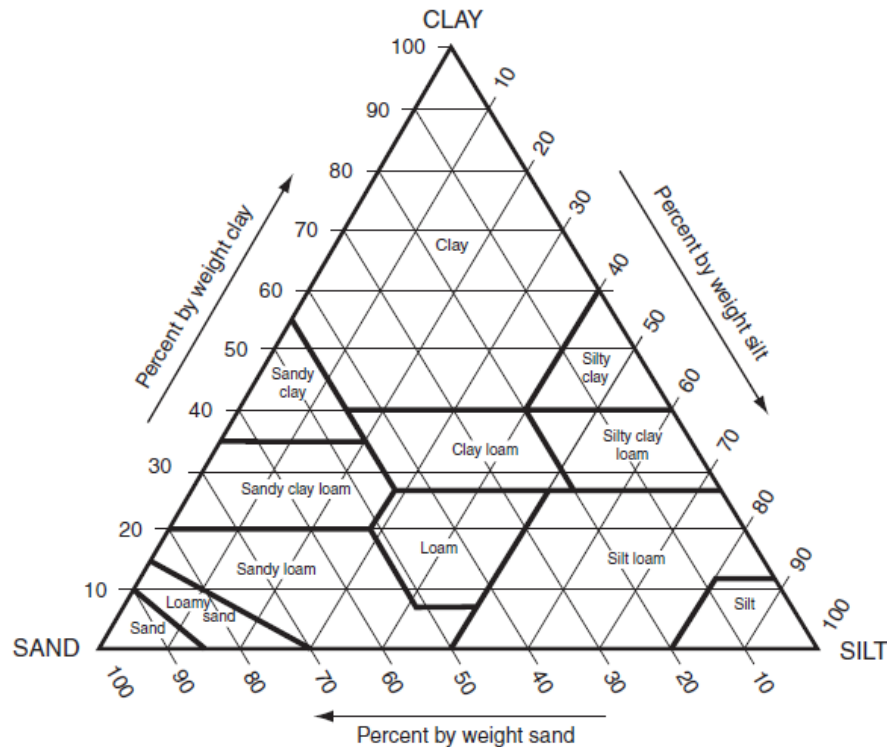


Fig. 3.3. Textural triangle, showing the percentages of clay (below 0.002 mm), silt (0.002–0.05 mm), and sand (0.05–2.0 mm) in the conventional soil textural classes.

Characterization of particle size:

Actual separation of particles into size groups can be carried out by passing the suspension through graded sieves, down to a particle diameter of approximately 0.05 mm. To separate and classify still finer particles, the method of sedimentation is usually used, based on measuring the relative settling velocity of particles of various sizes in an aqueous suspension.

The primary soil particles, often naturally aggregated, must be separated and made discrete by removal of cementing agents (such as organic matter, calcium carbonate, and iron oxides) and by deflocculating the clay. Removal of organic matter is usually achieved by oxidation with hydrogen peroxide, and calcium carbonate can be dissolved by addition of hydrochloric acid. Deflocculation is carried out by means of a chemical dispersing agent (such as sodium metaphosphate) and by mechanical agitation (shaking, stirring, or ultrasonic vibration). The function of the dispersing agent is to replace the cations adsorbed to the clay, particularly divalent or trivalent cations, with sodium. This has the effect of increasing the hydration of the clay micelles, thus causing them to repel each other rather than coalesce, as they do in the flocculated state.

According to *Stokes' law*, the velocity of a spherical particle settling under the influence of gravity in a fluid of a given density and viscosity is proportional to the square of the particle's radius.



We shall now derive this law:

$$F_g = m_s g = \rho_s v_s g = \rho_s \frac{4}{3} \pi R^3 g$$

The buoyancy force

$$F_b = m_f g = \rho_f \frac{4}{3} \pi R^3 g$$

The viscous resistance force

$$F_r = 6\pi\eta ru$$

Where η is the viscosity of the fluid, r is the radius of the particle, and u is its velocity.

Net force down = weight of particle - buoyancy force up

$$F_g = \left(\frac{4}{3}\right)\pi r^3(\rho_s - \rho_f)g$$

Setting the two forces equal, we obtain Stokes' law:

$$u_t = (2/9)(r^2g/\eta)(\rho_s - \rho_f) = (d^2g/18\eta)(\rho_s - \rho_f)$$

The use of Stokes' law to measure particle sizes is based on certain simplifying Assumptions:

1. The particles are large enough to be unaffected by random motion of the fluid molecules.
2. The particles are rigid, spherical, and smooth.
3. All particles have the same density.
4. The suspension is dilute enough so particles settle independent.
5. Fluid flow around the particles is laminar (slow enough to avoid onset of turbulence).

Sample Problem

Using Stokes' law, calculate the time needed for all sand particles (diameter $>50 \mu\text{m}$) to settle out of a depth of 0.2 m in an aqueous suspension at 30°C . How long would it take for all silt particles to settle out? How long for "coarse" clay ($>1 \mu\text{m}$)?

We use

$$t = 18h\eta/d^2g(\rho_s - \rho_f).$$

Substituting the appropriate values for depth h (0.20 m), viscosity η (0.0008 kg/m sec), particle diameter d (50 μm , 2 μm , and 1 μm for the lower limits of sand, silt, and coarse clay, respectively), gravitational acceleration g (9.81 m/sec²), average particle density ρ_s ($2.65 \times 10^3 \text{ kg/m}^3$), and water density (10^3 kg/m^3), we obtain the following answers.

(a) For all sand to settle out, leaving only silt and clay in suspension:

$$t = \frac{18 \times 0.2 \times (8 \times 10^{-4})}{(50 \times 10^{-6})^2 \times 9.81 \times (2.65 - 1.0) \times 10^3} = 71 \text{ sec}$$



(b) For all silt to settle out, leaving only clay in suspension:

$$t = \frac{18 \times 0.2 \times (8 \times 10^{-4})}{(2 \times 10^{-6})^2 \times 9.81 \times 1.65 \times 10^3} = 44,500 \text{ sec} = 12.36 \text{ hr}$$

(c) For all coarse clay to settle out, leaving only fine clay in suspension:

$$t = \frac{18 \times 0.2 \times (8 \times 10^{-4})}{(1 \times 10^{-6})^2 \times 9.81 \times 1.65 \times 10^3} = 178,800 \text{ sec} = 49.44 \text{ hr}$$

PARTICLE SIZE DISTRIBUTION

Any attempt to divide into discrete fractions what is usually a continuous array of particle sizes is necessarily arbitrary, and the further classification of soils into distinct textural classes is doubly so. Although this approach is widely followed and in some ways useful, greater information is to be gained from measuring and displaying the complete array and distribution of particle sizes. Figure 3.4 presents typical *particle size distribution curves*.

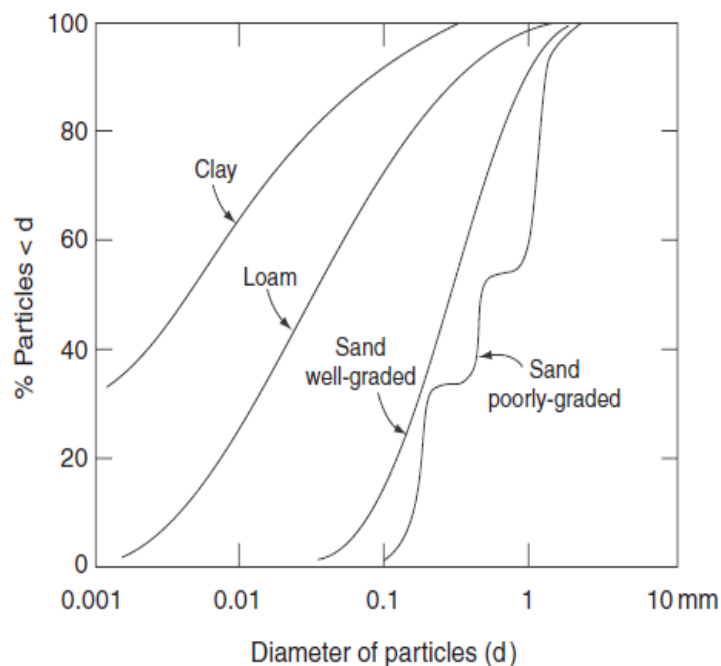


Fig. 3.4. Particle size distribution curves for various types of soil material (schematic).

STRUCTURE OF CLAY MINERALS:

The most prevalent minerals in the clay fraction of temperate region soils are silicate clays, whereas in tropical regions hydrated oxides of iron and aluminum are more prevalent.

The most common mineral is *kaolinite*. Other minerals in the same group are *halloysite* and *dickite*.

Clay particles are generally platy shape



SPECIFIC SURFACE

The specific surface of soil material can be defined as the surface area a_s of particles per unit mass (a_m) or per unit volume of particles (a_v) or per unit bulk volume of soil (a_b):

$$a_m = A_s/M_s$$

$$a_v = A_s/V_s$$

$$a_b = A_s/V_t$$

where M_s is the mass of particles of volume V_s contained in a bulk volume V_t of soil.

Specific surface has traditionally been expressed in terms of square meters per gram (a_m) or square meters per cubic centimeter (a_v). To convert from m^2/gm to m^2/kg , one need only multiply by 10^3 . To convert from m^2/cm^3 to m^2/m^3 , one needs to multiply by 10^6 .

Specific surface obviously depends on the sizes of the soil particles. It also depends on their shapes. Flattened or elongated particles expose greater surface per unit mass or volume than do equidimensional (cubical or spherical) particles. Since clay particles are generally platy, they contribute more to the overall specific surface of a soil than is indicated by their small size alone.

MEASURING SPECIFIC SURFACE BY ADSORPTION

The usual procedure for determining surface area is to measure the amount of gas or liquid needed to form a *monomolecular layer* over the entire surface in a process of adsorption. The standard method is to use an inert gas such as nitrogen.

Sample Problem

Estimate the approximate specific surface (m^2/g) of a soil composed of 10% coarse sand (average diameter 1 mm), 20% fine sand (average diameter 0.1 mm), 30% silt (average diameter 0.02 mm), 20% kaolinite clay (average platelet thickness 4×10^{-8} m), 10% illite clay (average thickness 0.50×10^{-8} m), and 10% montmorillonite (average thickness 10^{-9} m). For the sand and silt fractions, we use Eq.

$$a_m = (6/2.65)[(0.1/0.1) + (0.2/0.01) + (0.3/0.002)] = 0.0387 \text{ m}^2/\text{g} = 38.7 \text{ m}^2/\text{kg}$$

For the clay fraction, we use Eq. in summation form to include the partial specific surface values for kaolinite, illite, and montmorillonite, respectively:

$$\begin{aligned} a_m &= (0.2 \times 0.75)/(400 \times 10^{-8}) + 0.1 \times 0.75/(50 \times 10^{-8}) + 0.1 \times 0.75/(10 \times 10^{-8}) \\ &= 3.78 \text{ m}^2/\text{g} \text{ (kaol.)} + 15.09 \text{ m}^2/\text{g} \text{ (ill.)} + 75.45 \text{ m}^2/\text{g} \text{ (mont.)} \\ &= 94.32 \text{ m}^2/\text{g} = 94,320 \text{ m}^2/\text{kg} \end{aligned}$$

Total for the soil $= 0.0387 + 94.32 = 94.36 \text{ m}^2/\text{g} = 94,360 \text{ m}^2/\text{kg}$

Note: The clay fraction, only 40% of the soil mass, accounts for 99.96% of the specific surface. Montmorillonite alone (10% of the mass) accounts for nearly 80% of this soil's specific surface.



SOIL STRUCTURE AND AGGREGATION

SOIL STRUCTURE:

Is the arrangement or organization of the particles in the soil.

Since soil particles differ in shape, size, and orientation and can be variously associated and interlinked, the mass of them can form complex and irregular patterns that are difficult to characterize in exact geometric terms.

A further complication is the inherently unstable nature of soil structure and its nonuniformity in space. Soil structure is affected by changes in climate, biological activity, and soil management practices, and it is vulnerable to destructive forces of a mechanical and physicochemical nature.

For these various reasons, we have no truly objective or universally applicable way to measure soil structure.

TYPES OF SOIL STRUCTURE:

In general, we can recognize three broad categories of soil structure:

- 1- *Single grained*: particles are entirely unattached to one another, the structure is completely loose, and such soils were labeled *as structure less*.
- 2- *Massive*: the soil is tightly packed in large cohesive blocks, as is sometimes the case with dried clay.
- 3- *Aggregated*: Between these two extremes, we can recognize an intermediate condition in which the soil particles are associated in quasi-stable small clods known as *aggregates*.

STRUCTURE OF GRANULAR SOILS:

The structure of most coarse-textured soils is single grained, because there is little tendency for the grains to adhere and form aggregates.

The arrangement and internal mode of packing of the grains depends on their distribution of sizes and shapes as well as the manner in which the material had been deposited or formed in place.

STRUCTURE OF AGGREGATED SOILS:

In soils with an appreciable content of clay, the primary particles tend, under favorable circumstances, to group themselves into composite structural units known as *aggregates*.

The visible aggregates, which are generally of the order of several millimeters to several centimeters in diameter, are often called *peds* or *macroaggregates*. These are usually assemblages of smaller groupings, or *microaggregates*, which themselves are associations of the ultimate structural units, that is, the flocs, clusters, or packets of clay particles.



CHARACTERIZATION OF SOIL STRUCTURE:

The structure of the soil can be studied directly by microscopic observation of thin slices under polarized light. The arrangements of minute clay particles can be examined by means of electron microscopy, using either transmission or scanning techniques. The structure of single-grained soils, as well as of aggregated soils, can be expressed quantitatively in terms of the total porosity and of the pore size distribution.

Total porosity f of a soil sample is usually computed from the measured bulk density ρ_b , using the following equation:

$$f = 1 - \rho_b / \rho_s$$

Where : ρ_s is the average particle density.

Bulk density is generally measured by means of a *core sampler* designed to extract “undisturbed” samples of known volume from various depths in the profile.

It is sometimes possible to divide pore size distribution into two distinguishable ranges:

- 1- *macropores*: are mostly the interaggregate spaces, which serve as the principal avenues for the infiltration and drainage of water and for aeration.
- 2- *micropores*: are the intraaggregate capillaries responsible for the retention of water and solutes.

SHAPES OF AGGREGATES:

The shapes of aggregates can be classified as follows:

1. *Platy*: Horizontally layered, thin and flat aggregates resembling wafers. Such structures occur, for example, in recently deposited clay soils.
2. *Prismatic* or *columnar*: Vertically oriented pillars, up to 15 cm in diameter. Such structures are common in the B horizon of clayey soils, particularly in semiarid regions. Where the tops are flat, these vertical aggregates are called *prismatic*, and where rounded, *columnar*.
3. *Blocky*: Cubelike blocks of soil, up to 10 cm in size, sometimes angular with well-defined planar faces. These structures occur most commonly in the upper part of the B horizon.
4. *Spherical*: Rounded aggregates, generally not much larger than 2 cm in diameter, often found in a loose condition in the A horizon. Such units are called *granules* and, where particularly porous, *crumbs*.

AGGREGATE SIZE DISTRIBUTION:

Aggregate size distribution is an important determinant of the soil's pore size distribution and has a bearing on the erodibility of the soil surface, particularly by



wind. In the field, adjacent aggregates often adhere to one another, though of course not as tenaciously as do the particles within each aggregate.

Separating and classifying soil aggregates necessarily involves a disruption of the original, in situ structural arrangement. The application of too great a force may break up the aggregates themselves. Hence the determination of aggregate size distribution depends on the mechanical means employed to separate the aggregates.

Various indexes have been proposed for the distribution of aggregate sizes. If a single characteristic parameter is desired (so as to allow correlation with such factors as erosion, infiltration, evaporation, and aeration), a method must be adopted for assigning a appropriate weighting factor to each size range of aggregates. One of the most widely used indexes is the *mean weight diameter*, based on weighting the masses of aggregates of the various size classes according to their respective sizes. The mean weight diameter X is thus defined by the following equation:

$$X = \sum_{i=1}^n x_i w_i$$

Here x_i is the mean diameter of any particular size range of aggregates separated by sieving and w_i is the weight of the aggregates in that size range as a fraction of the total dry weight of the sample.

An alternative index of aggregate size distribution is the *geometric mean diameter* Y , calculated according to the following equation:

$$Y = \exp \left[\frac{\sum_{i=1}^n w_i \log x_i}{\sum_{i=1}^n w_i} \right]$$

where w_i is the weight of aggregates in a size class of average diameter x_i and the denominator $\sum w_i$ (for i values from 1 to n) is the total weight of the sample.

Sample Problem

Calculate the mean weight diameters of the assemblages of aggregates given in Table 5.3. The percentages refer to the mass fractions of dry soil in each diameter range. First we determine the mean diameters of the seven aggregate diameter ranges:

Range: 0–0.5, 0.5–1, 1–2, 2–5, 5–10, 10–20, 20–50 mm

Mean: 0.25, 0.75, 1.5, 3.5, 7.5, 15, 35 mm

Recall that the mean weight diameter X is defined by Eq.

$$X = \sum_{i=1}^n x_i w_i$$

Hence, for the dry-sieved virgin soil,



$$Y = (0.25 \times 0.1) + (0.75 \times 0.1) + (1.5 \times 0.15) + (3.5 \times 0.15) + (7.5 \times 0.2) + (15 \times 0.2) + (35 \times 0.1) = 8.85 \text{ mm}$$

For the dry-sieved cultivated soil,

$$X = (0.25 \times 0.25) + (0.75 \times 0.25) + (1.5 \times 0.15) + (3.5 \times 0.15) + (7.5 \times 0.1) + (15 \times 0.07) + (35 \times 0.03) = 4.30 \text{ mm}$$

For the wet-sieved virgin soil,

$$X = (0.25 \times 0.3) + (0.75 \times 0.15) + (1.5 \times 0.15) + (3.5 \times 0.15) + (7.5 \times 0.15) + (15 \times 0.05) + (35 \times 0.05) = 4.56 \text{ mm}$$

For the wet-sieved cultivated soil,

$$X = (0.25 \times 0.5) + (0.75 \times 0.25) + (1.5 \times 0.15) + (3.5 \times 0.05) + (7.5 \times 0.04) + (15 \times 0.01) + (35 \times 0.0) = 1.16 \text{ mm}$$

Note: Wet sieving reduced the mean weight diameter from 8.85 to 4.56 mm in the virgin soil and from 4.30 to 1.16 mm in the cultivated soil. This indicates the degree of instability of the various aggregates under the slaking effect of immersion in water. The influence of cultivation is generally to reduce the water stability of soil aggregates and hence to render the soil more vulnerable to crusting and erosion processes.

TABLE 5.3 Aggregate Data for Sample Problem

Aggregate diameter range (mm)	Dry sieving		Wet sieving	
	Virgin soil (%)	Cultivated soil (%)	Virgin soil (%)	Cultivated soil (%)
0.0–0.5	10	25	30	50
0.5–1.0	10	25	15	25
1–2	15	15	15	15
2–5	15	15	15	5
5–10	20	10	15	4
10–20	20	7	5	1
20–50	10	3	5	0

AGGREGATE STABILITY:

The ability of soil aggregates to withstand disruptive forces, whether such forces are imposed mechanically (as during tillage or trampling) or by the action of water (causing swelling, slaking, and dispersion of clay).

it expresses the resistance of aggregates to breakdown when subjected to potentially disruptive processes. The reaction of a soil to forces acting on it depends not only on the soil itself but also on the nature of the forces and the manner they are applied.



HYDROLOGY CIRCLE IN FIELD:

To understand changes in soil moisture we must start with an understanding of the sources and losses of water.

Sources of water to the biosphere from the atmosphere include precipitation.

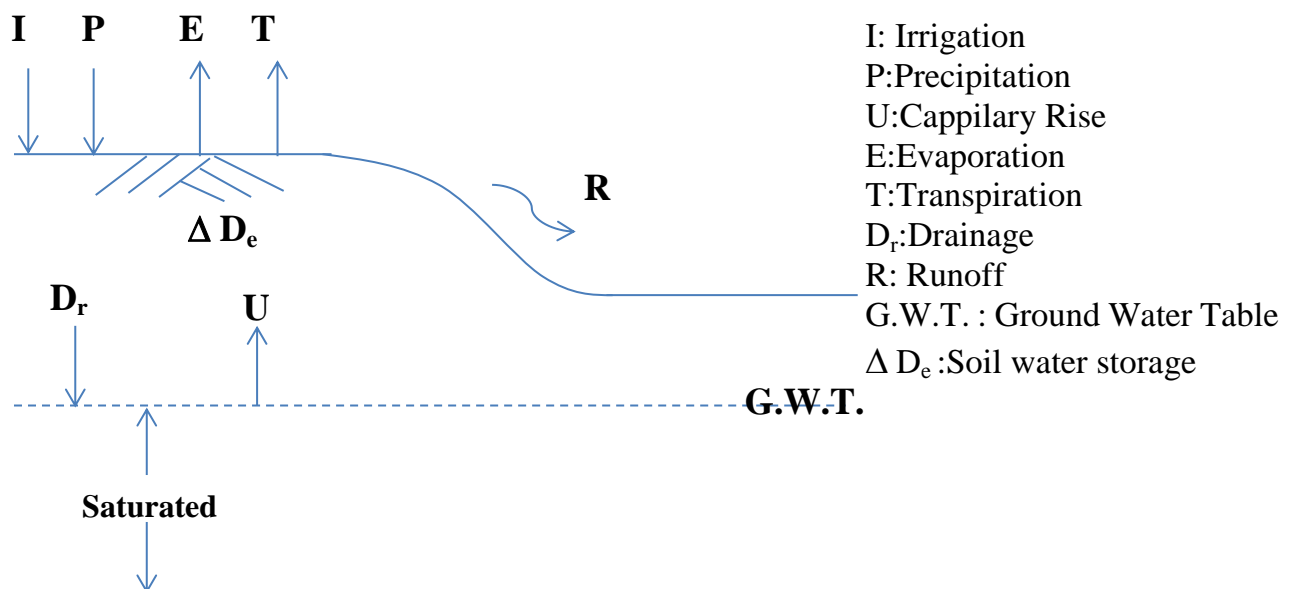
Sources of water from the soil include capillary rise from the water table, direct axis of the water table by roots and hydraulic lift of moisture by roots.

And the other source of water to the soil by irrigation.

Losses of water from the biosphere include losses to the atmosphere and to deep soil layers.

Losses to the atmosphere include direct evaporation of surface water, evaporation from the soil matrix, plant transpiration. Losses through the soil column involve transport via percolation and saturated flow and unsaturated flow.

Flow through the soil column is vertical and lateral, a distinctly three-dimensional process.



Sources = losses + soil water storage

$$I + P + U = E + T + D_r + R + \Delta D_e$$

Soil Water

Soil Water is classified according to how "tightly" it is being held in the Soil.

The following below the most important water content which are defined according to the type of the water:

- 1- **Field capacity (fc):** Free water or gravitational water will drain from a soil until the soil water potential reaches **-1/3 bar**.

Gravitational water is not considered available to plants because it is in the soil only a short time and reduces oxygen levels to the point where the plant will not be absorbing water anyway.



The time dependence of soil wetness in the upper zone is illustrated in Fig. 16.5 for a sandy soil, in which the unsaturated conductivity falls off rapidly with increasing suction, and for a clayey soil, in which the decrease of conductivity is more gradual and hence redistribution tends to persist longer.

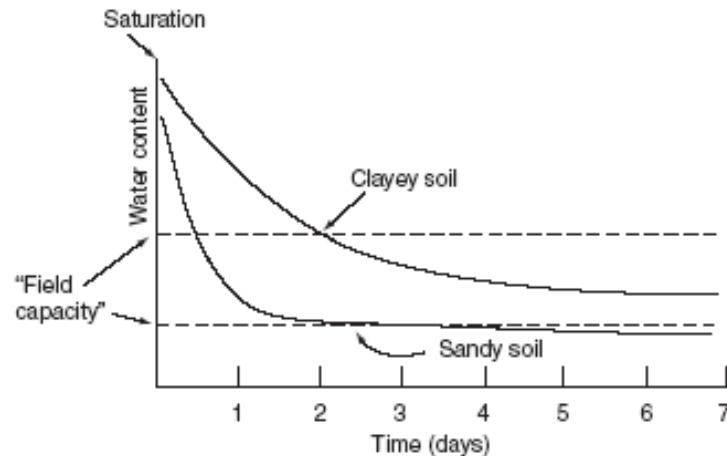


Fig. 16.5. The monotonic decrease of soil wetness with time in the initially wetted zone of a clayey and a sandy soil during redistribution. The dashed lines show the wetness values remaining in each soil after two days.

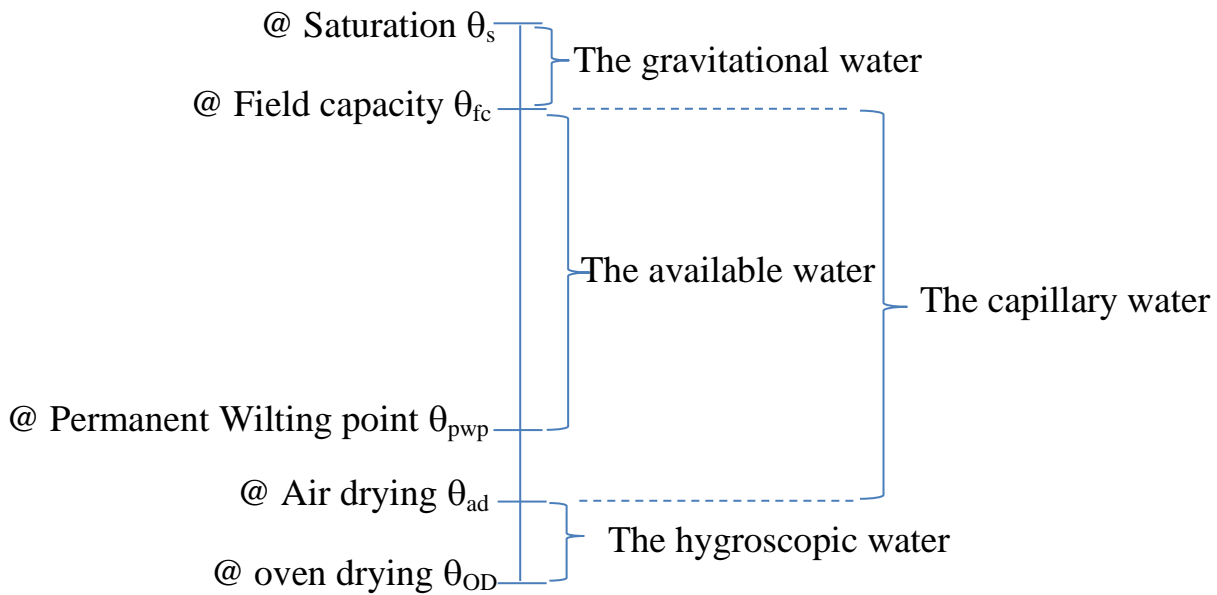
- 2- **Permanent Wilting point (pwp):** As the soil continues to dry--or water is used by plants--more and more energy is needed by the plants to remove the water. Eventually a point is reached where the plant can no longer remove water. This is called the **wilt point** and occurs at -15 bars water potential for most plants.
- 3- **Available water:** From - 1/3 to -15 bars is the zone of **available water**.

If the soil dries to an **air dry** state, the potential is -31 bars. (This assumes that the air has 100% relative humidity.) Plants cannot exert enough tension to pull water away from the soil. Tension is used to express water potential with positive numbers. So a tension of +15 bars equals a potential of -15 bars.

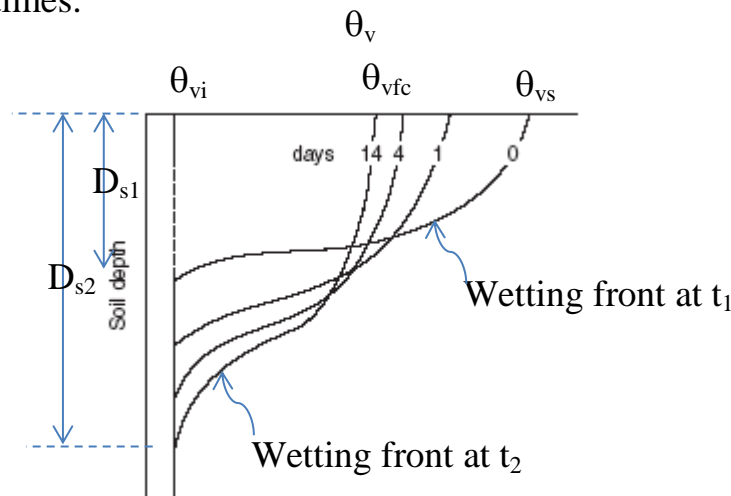
Additional drying requires putting the soil in an oven to drive off the tightly held water. Water is held in the soil like a series of beads, the farther the beads are from the soil particle, the weaker they are held by cohesion. The investigation in the laboratory will be using the soil placed in the drying cans last week. The sample in the lab was somewhere between field capacity and the wilt point (that is, between - 1/3 and -15 bars).

You let the sample air dry, and later it was placed in the oven and dried, then weighed to obtain the **oven dry** weight. When working with determining % water by weight or volume, we always use the oven dry weight as the standard value with which to compare.

Figure below show the graduated of the water content from the saturation to the total drying.



The figure below show the variation of volumetric water content, (θ_v) with depth and with different times.



$$D_e = \Delta\theta * D_s = (\theta_{vfc} - \theta_{vi}) * D_s$$

Where:

D_s : soil layer depth

MEASUREMENT OF SOIL WETNESS

The most important methods are:

- 1- Gravimetric method: The traditional (gravimetric) method of measuring mass wetness consists of removing a sample by augering into the soil and then determining its moist and dry weights. The *moist weight* is determined by weighing the sample as it is at the time of sampling; the *dry weight* is obtained after drying the sample to a constant weight in an oven.



- 2- Neutron moderation method: The method is practically independent of temperature and pressure. Its main disadvantages, are the high initial cost of the instrument, low degree of spatial resolution, difficulty of measuring moisture in the soil surface zone, and the health hazard associated with exposure to neutron and gamma radiation. The instrument, known as a *neutron moisture meter* consists of two main components: (a) a *probe* (containing a *source of fast neutrons* and a *detector of slow neutrons*), which is lowered into an *access tube* inserted vertically into the soil, and (b) a *scaler* or *rate meter* (usually battery powered and portable) to monitor the flux of the slow neutrons that are scattered and attenuated in the soil.

Sample Problem

You are required to know how to calculate the amount of water in a soil given various water potentials. This will help you understand that it is not the total amount of water in a soil that determines if water is available to plants, but is the plant available water.

DATA:

Soil Core Volume = **250** cc (for each soil core below)

Weight of soil core at -1/3 bar (field capacity) = **420** g (July 4, 19??)

Weight of soil core at -15 bar (wilt point) = **350** g

Weight of soil core at present field condition = **395** g (on July 10)

Weight of Oven dry soil core = **300** g

Questions:

1) What is the Bulk Density?

Answer: B.D. = $300 \text{ g} / 250 \text{ cc} = \mathbf{1.2 \text{ g/cc}}$ (remember, always use oven dry weight)

2) What is the % water by weight at field capacity?

FC - Oven dry

Answer: $420 \text{ g} - 300 \text{ g} = 120 \text{ g}$ of water

$120 \text{ g water} / 300 \text{ g soil} = 0.4$ (*use oven dry weight)

$0.4 \times 100 = \mathbf{40\% \text{ water by weight at field capacity}}$

3) What is the % water by volume at field capacity?

FC - Soil Volume =

Answer: $420 \text{ g} - 300 \text{ g} = 120 \text{ g}$ of water

$120 \text{ g water} / 250 \text{ cc soil} = 0.48$

$0.48 \times 100 = \mathbf{48\% \text{ water by volume at field capacity}}$

(Another way to calculate this is: $\text{BD} \times \% \text{ water wt.} = \% \text{ water by volume}$):

$1.2 \times 40\% \text{ water by weight} = 48\% \text{ water by volume}$

4) What is the total possible % Available Water-holding Capacity (AWC) by volume?

($\text{AWC} = \text{FC} - \text{WP}$)



FC - WP / Soil Volume =

Answer: $(420-350) / 250 = 0.28 \times 100 = \mathbf{28\%}$

available water [(FC-WP) / vol.]

5) How many inches of AWC are in the upper 5 ft. of soil?

inches of soil x % AWC =

Answer: $5\text{ft.} \times (12"/\text{ft.}) \times 0.28 = \mathbf{16.8" \text{ of AWC in upper 5 ft. of soil.}}$

6) How many inches of available water are left in the soil at present field condition? (ΔD_e)

Field Cond. = 395 and Wilt Pt. = 350; therefore:

$(395-350)/250 = 45/250 = .18$ (%AWC by Vol.)

and

$0.18 \times 60"$ (of soil) = **10.8" of water available in upper 5 feet.**

In other words, the soil has lost 6" of water (16.8-10.8) since it was at field capacity.

7) What is the depth of wetting for a 1.5 inch rainfall event (D_e) for the soil at Field Condition?

Answer : Field Capacity - Field Cond./ vol x 100 = % water by vol between field condition and Field Capacity. or $420 - 395/250 = 25/250 = 0.1$ or 10%

thus 1.5 inches of rain will infiltrate $(1.5 / 0.1) = \mathbf{15 \text{ inches.}}$ of soil because

Inches of water = % Water vol x Soil Depth. or $1.5 = 0.1 \times ? \text{ inches of soil}$

Sample Problem

The data in Table below were obtained before and after irrigation. From these data, calculate the mass and volume wetness values of each layer before and after the irrigation, and determine the amount of water (in millimeters) added to each layer and to the **profile** as a whole.

Data for Sample Problem

Sampling time	Sample number	Depth (m)	Bulk density (kg/m ³)	Wet sample + container (kg)	Dry sample + container (kg)	Container (kg)
Before irrigation	1	0.0-0.4	1.2×10^3	0.160	0.150	0.05
	2	0.6-1.0	1.5×10^3	0.146	0.130	0.05
After irrigation	3	0.0-0.4	1.2×10^3	0.230	0.200	0.05
	4	0.6-1.0	1.5×10^3	0.206	0.170	0.05

Solution:

Using eq. $w = M_w/M_s$

we obtained the following mass wetness values:



$$w_1 = (0.160 - 0.150)/(0.150 - 0.050) = 0.1$$

$$w_2 = (0.146 - 0.130)/(0.130 - 0.050) = 0.2$$

$$w_3 = (0.230 - 0.200)/(0.200 - 0.050) = 0.2$$

$$w_4 = (0.206 - 0.170)/(0.170 - 0.050) = 0.3$$

Using eq. $\theta = \rho_b * w$

we obtain the following volume wetness values:

$$\theta_1 = 1.2 \times 0.1 = 0.12$$

$$\theta_2 = 1.5 \times 0.2 = 0.30$$

$$\theta_3 = 1.2 \times 0.2 = 0.24$$

$$\theta_4 = 1.5 \times 0.3 = 0.45$$

Using Eq. $D_e = \theta * D_s$

we obtain the following water depths per layer:

$$D_{e1} = 0.12 \times 400 \text{ mm} = 48 \text{ mm}$$

$$D_{e2} = 0.30 \times 600 \text{ mm} = 180 \text{ mm}$$

$$D_{e3} = 0.24 \times 400 \text{ mm} = 96 \text{ mm}$$

$$D_{e4} = 0.45 \times 600 \text{ mm} = 270 \text{ mm}$$

$$\text{Depth of water in profile before irrigation} = 48 + 180 = 228 \text{ mm.}$$

$$\text{Depth of water in profile after irrigation} = 96 + 270 = 366 \text{ mm.}$$

$$\text{Depth of water added to top layer} = 96 - 48 = 48 \text{ mm.}$$

$$\text{Depth of water added to bottom layer} = 270 - 180 = 90 \text{ mm.}$$

$$\text{Depth of water added to entire profile} = 48 + 90 = 138 \text{ mm.}$$



THE LIQUID PHASE

The variable amount of water contained in a unit mass or volume of soil and the energy state of water in the soil are important factors affecting the growth of plants. Numerous other soil properties depend on water content.

Among these are mechanical properties, such as consistency, plasticity, strength, compatibility, penetrability, stickiness, and traffic ability. In clayey soils, swelling and shrinking associated with addition or extraction of water change the bulk density, porosity, and pore size distribution. Soil water content also governs the air content and gas exchange of the soil, thus affecting the respiration of roots, the activity of microorganisms, and the chemical state of the soil.

Capillary and Soil Water:

The movement of water up a wick typifies the phenomenon of capillarity. Two forces cause capillarity:

- (1) The attraction of water for the solid (adhesion or adsorption).
- (2) The surface tension of water which is due largely to the attraction of water molecules for each other (cohesion). This phenomenon can be described in terms of energy. Energy per unit area has the same dimensions as force per unit length.

Its units

- 1- Force/length = dyn/cm = $gm. cm. sec^2/cm = gm. sec^2$ or
- 2- Energy/area = $erg/cm^2 = gm. cm. sec^{-2}. cm^{-1}/cm^2 = gm. sec^{-2}. cm^{-2}$.

The height of rise in a capillary tube is also inversely proportional to the tube radius. Capillary rise is also inversely proportional to the density of the liquid, and is directly proportional to the liquid's surface tension and the degree of its adhesive attraction to the soil surface.

Capillary forces are at work in all moist soils. However, the rate of movement and rise in height are less than one would expect because soil pores are not straight and there can be air entrapment slowing down water movement. In fine textured soils, capillary rise is high but rate of flow is slow because of frictional forces in tiny pores. In coarse-textured (sandy) soils, pores are large presenting little resistance to capillary rise but limiting the rise in capillary water.

CONTACT ANGLE OF WATER ON SOLID SURFACES

If we place a drop of liquid on a dry solid surface, the liquid will usually displace the gas that covered the surface of the solid and it will spread over that surface to a certain extent. Where its spreading ceases and the edge of the drop comes to rest, it will form a typical angle with the surface of the solid. This angle, termed *contact angle*, is illustrated in Fig. 2.6.

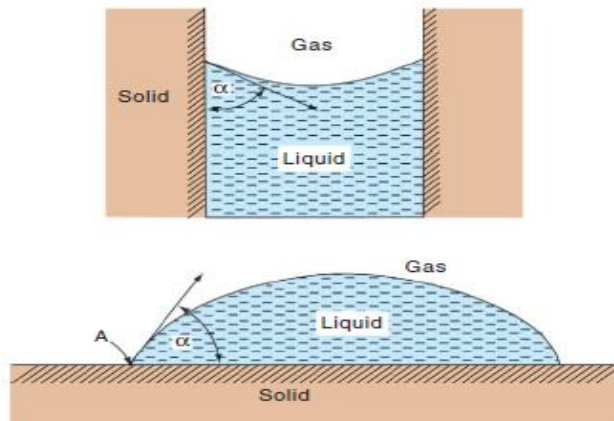


Fig. 2.6. The contact angle of a meniscus in a capillary tube and a drop resting on the surface of a solid.

We now consider what factors determine the magnitude of the angle α . We can expect that angle to be acute if the adhesive affinity between the solid and liquid is strong relative to the cohesive forces inside the liquid itself and to the affinity between the gas and the solid. We can then say that the liquid “wets” the solid. A contact angle of zero would mean the complete flattening of the drop and the perfect wetting of the solid surface by the liquid. On the other hand, a contact angle of 180° would imply a complete non wetting or rejection of the liquid by the gas-covered solid. In that case the drop would retain its spherical shape without spreading over the surface at all (assuming no gravity effect). Surfaces on which water exhibits an obtuse contact angle are called *water repellent* or *hydrophobic* (Greek: “water-hating”).

The contact angle of a given liquid on any particular solid is generally characteristic of their interaction under given physical conditions. This angle, however, may be different in the case of a liquid that is advancing over the surface than in the case of the same liquid receding over the surface. This phenomenon, where it occurs, is called *contact angle hysteresis*. The wetting angle of pure water on clean and smooth mineral surfaces is generally zero, but where the surface is rough or coated with adsorbed surfactants of a hydrophobic nature, the contact angle, and especially the wetting angle, can be considerably greater than zero and may even exceed 90° . This phenomenon is illustrated in given Fig. 2.7.

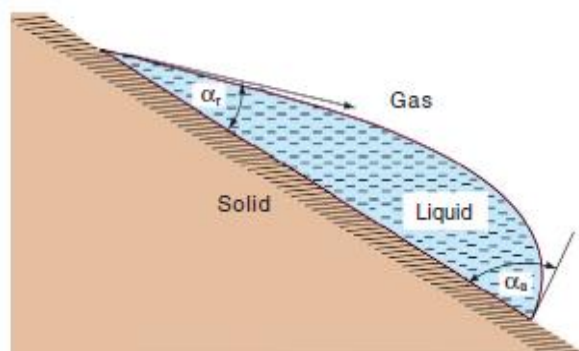


Fig. 2.7. Hypothetical representation of a drop resting on an inclined surface. The contact angle α_a at the advancing edge of the drop is shown to be greater than the corresponding angle α_r at the receding edge.



THE PHENOMENON OF CAPILLARITY

A capillary tube dipped in a body of free water will form a meniscus as the result of the contact angle of water with the walls of the tube. The curvature of this meniscus will be greater (i.e., the radius of curvature will be smaller) the narrower the tube. The occurrence of curvature causes a pressure difference to develop across the liquid–gas interface.

A liquid with an acute contact angle (e.g., water on glass) will form a concave meniscus, and therefore the liquid pressure under the meniscus P_1 will be smaller than the atmospheric pressure P_0 (Fig. 2.8). Hence, water inside the tube will be driven up the tube from its initial location (shown as a dashed curve in Fig. 2.8) by the greater pressure of the free water (i.e., water at atmospheric pressure, under a horizontal air–water interface) at the same level.

The upward movement will stop when the pressure difference between the water inside the tube and the water under the flat surface outside the tube is countered by the hydrostatic pressure exerted by the water column in the capillary tube. If the capillary tube is cylindrical and if the contact angle of the liquid on the walls of the tube is zero, the meniscus will be a hemisphere (and in a two-dimensional drawing can be represented as a semicircle) with its radius of curvature equal to the radius of the capillary tube. If, however, the liquid contacts the tube at an angle greater than zero but smaller than 90° , then the diameter of the tube ($2r$) is the length of a chord cutting a section of a circle with an angle of $(\pi - 2\alpha)$ as shown in Fig. 2.9. Thus,

$$R = r / \cos \alpha$$

Here R is the radius of curvature of the meniscus, r is the radius of the capillary, and α is the contact angle.

The pressure difference ΔP between the capillary water (under the meniscus) and the atmosphere, therefore, is

$$\Delta P = (2\gamma \cos \alpha) / r$$

Recalling that hydrostatic pressure is proportional to the depth d below the free water surface (i.e., $P = \rho g d$, where ρ is liquid density and g is gravitational acceleration), we can infer that the hydrostatic tension (negative pressure) in a capillary tube is proportional to the height h above the free water surface. Hence the height of capillary rise is

$$h_c = (2\gamma \cos \alpha) / g(\rho_l - \rho_g)$$

Where ρ_g is the density of the gas (generally neglected), ρ_l is the density of the liquid, g is the acceleration of gravity, r is the capillary radius, α is the contact angle, and γ is the surface tension.

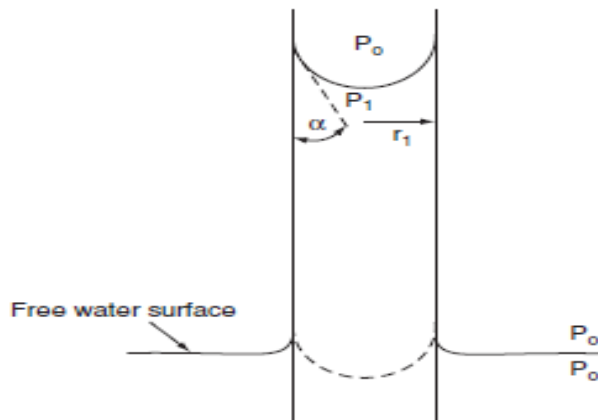


Fig. 2.8 . capillary rise

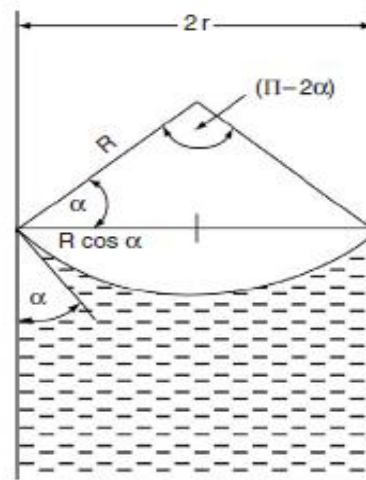


Fig. 2.9 . the geometric relationship of the radius of curvature R to the radius of the capillary r and the contact angle α

Sample Problems

Calculate the equilibrium capillary rise of water and mercury at 20°C in glass cylindrical capillary tubes of the following diameters: (a) 2 mm; (b) 0.5 mm; (c) 0.1 mm. Disregard the density of atmosphere.

$$\Delta h = (2\gamma \cos \alpha) / (\rho g r)$$

For water: $\gamma = 7.27 \times 10^{-2} \text{ kg/sec}^2 (= \text{N/m})$; $\alpha = 0$; $\rho = 998 \text{ kg/m}^3$

(a) $r = 10^{-3} \text{ m}$:

$$\Delta h = (2 \times 7.27 \times 10^{-2} \text{ kg/sec}^2) / (998 \text{ kg/m}^3 \times 9.81 \text{ m/sec}^2 \times 10^{-3} \text{ m}) = 1.48 \times 10^{-2} \text{ m} = 1.48 \text{ cm}$$

(b) $r = 2.5 \times 10^{-4} \text{ m}$:

$$\Delta h = (2 \times 7.27 \times 10^{-2} \text{ kg/sec}^2) / (998 \text{ kg/m}^3 \times 9.81 \text{ m/sec}^2 \times 2.5 \times 10^{-4} \text{ m}) = 5.92 \times 10^{-2} \text{ m} = 5.92 \text{ cm}$$

(c) $r = 5.0 \times 10^{-5} \text{ m}$:

$$\Delta h = (2 \times 7.27 \times 10^{-2} \text{ kg/sec}^2) / (998 \text{ kg/m}^3 \times 9.81 \text{ m/sec}^2 \times 5.0 \times 10^{-5} \text{ m}) = 29.6 \times 10^{-2} \text{ m} = 29.6 \text{ cm}$$

For mercury: $\gamma = 0.43 \text{ kg/sec}^2$; $\alpha = 180^\circ$; $\rho = 1,360 \text{ kg/m}^3$

(a) $r = 10^{-3} \text{ m}$:

$$\Delta h = [2 \times 0.43 \text{ kg/sec}^2 \times (-1)] / (1360 \text{ kg/m}^3 \times 9.81 \text{ m/sec}^2 \times 10^{-3} \text{ m}) = -0.64 \times 10^{-2} \text{ m} = -0.64 \text{ cm (capillary depression)}$$

(b) $r = 2.5 \times 10^{-4} \text{ m}$:

$$\Delta h = [2 \times 0.43 \text{ kg/sec}^2 \times (-1)] / (1360 \text{ kg/m}^3 \times 9.81 \text{ m/sec}^2 \times 2.5 \times 10^{-4} \text{ m}) = -2.58 \times 10^{-2} \text{ m} = -2.58 \text{ cm}$$

(c) $r = 5.0 \times 10^{-5} \text{ m}$:

$$\Delta h = [2 \times 0.43 \text{ kg/sec}^2 \times (-1)] / (1360 \text{ kg/m}^3 \times 9.81 \text{ m/sec}^2 \times 5.0 \times 10^{-5} \text{ m}) = -12.9 \times 10^{-2} \text{ m} = -12.9 \text{ cm}$$



Energy status of soil water:

The difference in energy level of water from one site or one condition (e.g., wet soil) to another (e.g., dry soil) determines the direction and rate of water movement in soils and plants. In a wet soil, most of the water is retained in large pores or thick water films around particles. Therefore, most of the water molecules in wet soil are not very close to a particle surface and so are not held very tightly by the soil solids (soil matrix). In this condition, the water molecules have considerable freedom of movement, so their energy level is near that of water molecules in a pool of pure water outside the soil. In a drier soil, however, the water that remains is located in small pores and thin water films, and is therefore held tightly by the soil solids. Thus the water molecules in a drier soil have little freedom of movement, and the energy level of the water is much lower than that of water in wet soil. If wet and dry soils are brought in touch with each other, water will move from the wet soil (higher energy state) to the drier soil (lower energy).

Usually the energy status of soil water in a particular location in the profile is compared to that of pure water at a standard pressure and temperature, unaffected by the soil and located at some reference elevation. *The difference in energy* levels between this pure water in the reference state and that of soil water is termed **Soil Water Potential**. Thus the term potential implies a difference in energy status. Knowing the potential energy in soil can help us estimate how much work the plant must expend to extract a unit amount of water. Potential energy rather than kinetic energy is used to describe soil water flow because kinetic energy is considered negligible by virtue of slow movement of soil water.

Expression of water potential

The soil-water potential is expressible in at least 3 ways:

1. Energy per unit mass: This is often taken to be the fundamental expression of potential (J/kg) with dimensions of L^2T^{-2}
2. Energy per unit volume: This is unit of pressure (Pressure = force/area = force x distance/area x distance = energy/volume). (Units N/m^2 , kPa.) = $ML^{-1}T^{-2}$
3. Energy per unit weight (hydraulic head): Since we can express soil water in units of hydrostatic pressure, it means we can also express it in terms of an equivalent head of water, which is height of a liquid column corresponding to the given pressure. The sum of matric and gravitational (elevation) heads is generally called the hydraulic head (or hydraulic potential) and is useful in evaluating directions and magnitudes of the water-moving forces throughout the soil profile.

Components of soil water potential

The International Soil Science Society defines total potential of soil water as “the amount of work that must be done per unit quantity of pure water in order to transport reversibly and isothermally an infinitesimal quantity of water from a pool of pure water at a specified elevation at atmospheric pressure to the soil water(at



the point under consideration". Soil water is subject to a number of force fields, which cause its potential to differ from that of pure, free water. Each of these forces is a component of the total soil water potential and they result from the attraction of the solid matrix for water, as well as from the presences of solutes and the action of external gas pressure and gravitation.

Thus, the total potential is the summation of the component potentials.

The components of soil water potential for shallow soils are:

Gravitational potential: The force of gravity exerted on a water column produces the gravitation potential. The gravitation potential is related to the work done to transport water from one pool to another, as when lifting a column of water up the xylem of a tree:

$$\psi_z = \rho g z$$

Pressure potential:

Water potential exerted by the pressure or weight of water

$$\psi_p = \frac{P}{\rho_w}$$

Matric potential: It is water potential due to attraction between water and soils.

Adhesive and cohesive forces bind water to soil particles (Campbell and Norman, 1999).

These interactions reduce the potential of water, giving it a negative sign.

$$\psi_m \sim a w^{-b}$$

Osmotic potential: Osmotic potential arises from the dilution of solutes in water, eg salts, sugars etc. For the osmotic potential to drive water flow, a semi-permeable membrane must separate two bodies of water, such as cells, and pools of water.

$$\psi_s = -c \phi v R T$$

c is the concentration, ϕ is the osmotic coefficient and v is the number of ions per mole (Campbell and Norman 1998).

The total water potential is thus:

$$\psi_t = \psi_p + \psi_m + \psi_z + \psi_s$$

We know the mass of water is $1000 \text{ kg} = \text{m}^3$ and water is incompressible, we can substitute mass for density, effectively $1 \text{ J/kg} = 1 \text{ kPa}$

When $\Delta\psi_z = 0$ the ψ_t named as ψ_w and $\psi_w = \psi_p + \psi_m + \psi_s$

Soil-Water-Plant Relations:

Osmotic and matric potential are important for plant-water. Gravimetric potential is negligible as the suction needed to raise water; typically 1 m is less than 0.1 bars.

Organisms, cells:



$$\psi = \psi_s + \psi_p$$

Inside cells osmotic potentials and pressure potential are most important

Unsaturated Flow:

$$\psi = \psi_z + \psi_m$$

Gravitational and matrix potential are dominant

Saturated Flow:

$$\psi = \psi_p + \psi_z$$

Pressure and gravity are the main components driving water flow in saturated soils. The pressure term includes overburden effects. At a point beneath the water table, the pressure potential is equal and opposite to the gravitational potential.

$$0 = \psi_p + \psi_z$$

$$\psi_p = -\psi_z$$

Osmotic potential is important only if there are solutes in the water.

Flow in the field and named as ψ_h

$$\psi_h = \psi_p + \psi_m + \psi_z$$

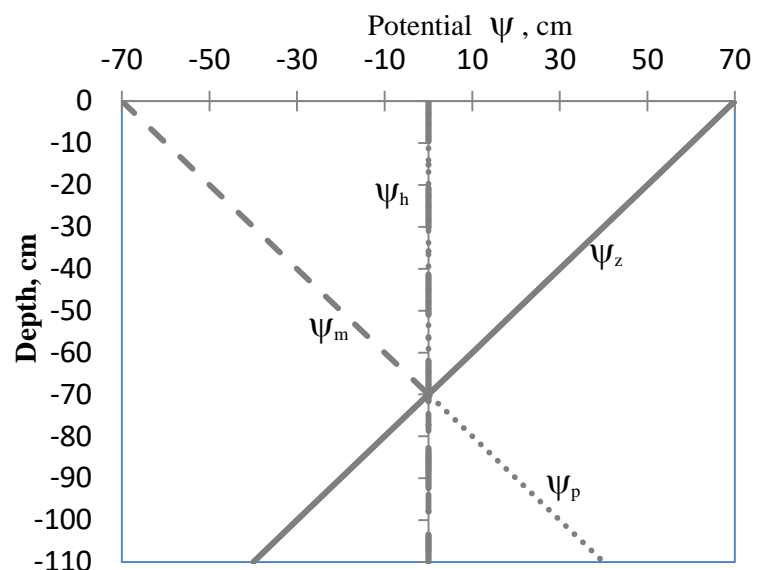
This terminology considers mixed flow in the saturated and unsaturated zones. The pressure term is zero above the water table. The matrix potential is zero below the water table.

Sample Problems

You have soil in equilibrium condition with water table level at (-70 cm), if the reference plane at (-70 cm), find (ψ_p , ψ_m , ψ_z and ψ_h) along the depth of the soil till (-110) cm depth .

The solution:

ψ_p	ψ_m	ψ_z	ψ_h	Depth (cm)
0	-70	70	0	0
0	-60	60	0	-10
0	-50	50	0	-20
0	-40	40	0	-30
0	-30	30	0	-40
0	-20	20	0	-50
0	-10	10	0	-60
0	0	0	0	-70
10	0	-10	0	-80
20	0	-20	0	-90
30	0	-30	0	-100
40	0	-40	0	-110



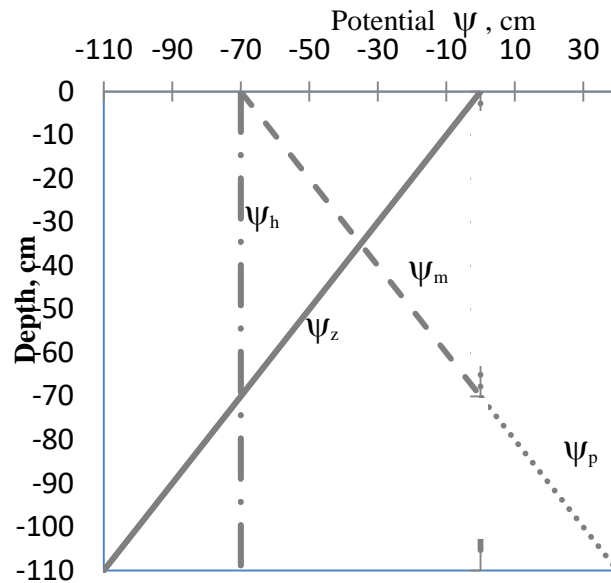


Sample Problems

You have the same soil in example above but the reference plane at the surface of the soil, find (ψ_p , ψ_m , ψ_z and ψ_h) along the depth of the soil till (-110)cm depth .

The solution:

ψ_p	ψ_m	ψ_z	ψ_h	Depth (cm)
0	-70	0	-70	0
0	-60	-10	-70	-10
0	-50	-20	-70	-20
0	-40	-30	-70	-30
0	-30	-40	-70	-40
0	-20	-50	-70	-50
0	-10	-60	-70	-60
0	0	-70	-70	-70
10	0	-80	-70	-80
20	0	-90	-70	-90
30	0	-100	-70	-100
40	0	-110	-70	-110

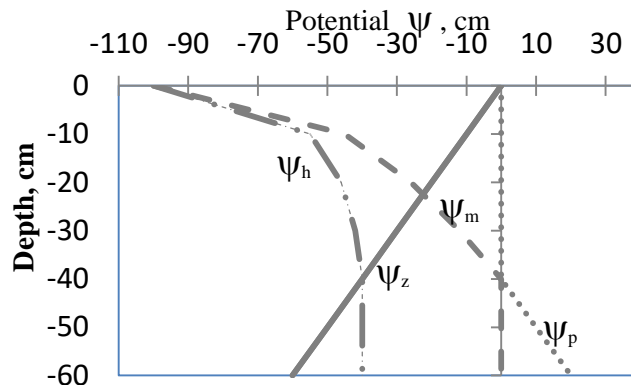


Sample Problems

You have soil in with water table at 40 cm below the surface and the evaporation happen at the surface , the reference plane at the surface of the soil , find (ψ_p , ψ_z and ψ_h) along the depth of the soil till (-60)cm depth .if ψ_m is given .

The solution:

ψ_p	ψ_m given	ψ_z	ψ_h	Depth (cm)
0	-100	0	-100	0
0	-45	-10	-55	-10
0	-26	-20	-46	-20
0	-12	-30	-42	-30
0	0	-40	-40	-40
10	0	-50	-40	-50
20	0	-60	-40	-60





Soil Water Content and Potential (characteristic curves)

Quantity of water in soil is expressed by gravimetric or volumetric water content. However, water can also be characterized by describing its free energy per unit mass, which is termed potential. The tenacity with which water is held in soil solid is characterized by matric or pressure potential. When volumetric water content and matric potential are plotted graphically, the relationship is termed **Soil Moisture Characteristic Curve**.

When all soil pores are filled with water, the soil is at its maximum retentive capacity called saturation. In the field, the lowest wetness you can observe is called air-dryness and in the laboratory it is called oven-dry condition.

As yet, no universally applicable theory exists for the prediction of the matric suction versus wetness relationship from basic soil properties (i.e., texture and structure). The adsorption and pore geometry effects are generally too complex to be described by a simple model. Several empirical equations have been proposed that describe the soil-moisture characteristic for some soils and within limited suction ranges. One such equation was advanced by Visser (1966):

$$\psi = a(f - \theta)^b/\theta^c$$

Here ψ is matric suction, f is porosity, θ is volumetric wetness, and a , b , c are constants. Use of this equation is hampered by the difficulty of evaluating its constants. Visser found that b varied from 0 to 10, a from 0 to 3, and f from 0.4 to 0.6.

Alternative equations to describe the relationship between wetness and matric suction have been proposed by Laliberte (1969), White et al. (1970), Su and Brooks (1975), and van Genuchten (1978). An equation presented by Brooks and Corey (1966a) is

$$(\theta - \theta_r)/(\theta_m - \theta_r) = (\psi_c/\psi)^\lambda$$

For suction values greater than the air-entry suction ψ_e . The exponent λ has been termed the pore size distribution index. In this equation, θ is the volume wetness (a function of the suction ψ), θ_m is the maximum wetness (saturation), and θ_r is the “residual” wetness remaining at high suction in the small pores that do not form a continuous network (the intra-aggregate pores).

The amount of water retained at low values of matric suction (say, between 0 and 100 kPa) depends on capillarity and the pore size distribution. Hence it is strongly affected by the soil’s structure. At higher suctions, water retention is due increasingly to adsorption, so it is influenced less by the structure and more by the soil’s texture and specific surface. The greater the clay content, in general, the greater the water retention at any particular suction and the more gradual the slope of the curve. In a sandy soil, most of the pores are relatively large, and once these



large pores are emptied at a given suction, only a small amount of water remains. In a clayey soil, more of the water is adsorbed, so increasing the matric suction causes a more gradual decrease in wetness (Fig. 6.8).

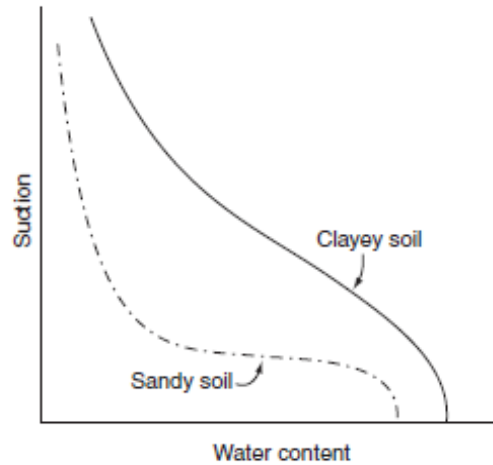


Fig. 6.8. The effect of texture on soil-water retention.

The Effect of Compaction on Soil

Because soil structure influences the shape of the soil-moisture characteristic curve primarily in the low-suction range, we may expect that the effect of compaction (which destroys the aggregated structure) is to reduce the total porosity and especially the volume of the large inter aggregate pores. As a result of compaction, the saturation water content as well as the initial decrease of water content with the application of low suction are diminished.

On the other hand, the volume of intermediate-size pores is likely to be greater in a compact soil (because some of the originally large pores have been squeezed into intermediate size by compaction), while the micropores remain unaffected, and thus the curves for the compacted and uncompact soil tend to converge in the high-suction range (Fig. 6.9).

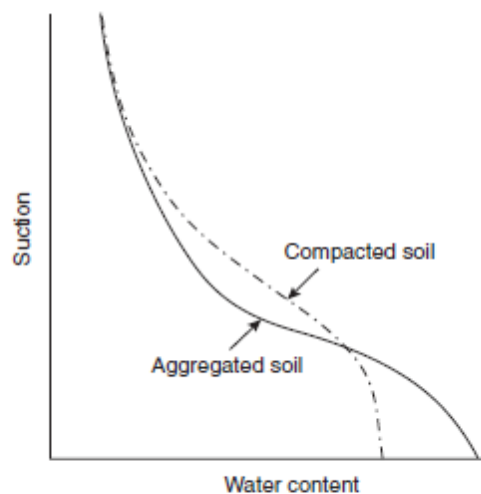


Fig. 6.9. The effect of soil structure on soil-water retention.



The slope of the soil-moisture characteristic curve, which is the change of water content per unit change of matric potential, is generally termed the *differential* (or *specific*) *water capacity* c_θ

$$c_\theta = d\theta/d\phi_p \quad \text{or} \quad c_\theta = -d\theta/d\psi$$

The c_θ term is analogous to the well-known *differential heat capacity*, which is the change in the heat content of a body per unit change in the thermal potential (temperature). However, while the differential heat capacity is fairly constant with temperature for many materials, the differential water capacity in soils is strongly dependent on the matric potential.

Measurement of Soil Water Potential

Total water potential is often thought of as the sum of matric and osmotic (solute) potentials and is a very useful index for characterizing the energy status of soil water with respect to plant water uptake.

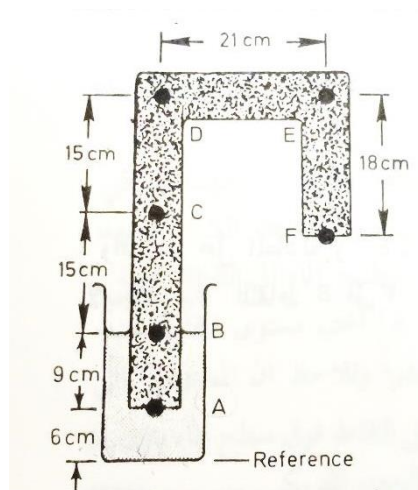
Tensiometer: A device to measure the matric potential of soil moisture in situ, consisting of a porous (ceramic) cup filled with water, with a manometer to monitor the pressure of the water in the cup at equilibrium with soil moisture. As soil moisture diminishes, its matric tension increases, hence it draws water from the cup, which in turn registers a subatmospheric pressure, called tension.

Tension-plate assembly: A device for equilibrating an initially saturated soil sample with a known matric tension value, applicable to the tension range of 0 to 1 bar.

Calculate potential in soil columns

Sample Problems

Given soil column as shown below, the level of water in ban keep constant and the evaporation is prevent in its open end for a period enough to reach to calibration, so there is no net flow in the soil column.



Find (ψ_h and its components ψ_p , ψ_z and ψ_m) for point A to F in soil column.

Solution:

At point B (water surface) $\psi_p = 0$ and $\psi_m = 0$

And ψ_z = the vertical distance from the reference so
 $\psi_z = 6 + 9 = 15 \text{ cm}$

$\therefore \psi_h = \psi_p + \psi_z + \psi_m = 0 + 15 + 0 = 15 \text{ cm}$.

We know the ψ_h equal in all points within the soil so we could write $\psi_{hB} = \psi_{hA} = \psi_{hC} = \psi_{hD} = \psi_{hE} = \psi_{hF}$



And the ψ_z for the point is the vertical distance so:

$$\begin{aligned}\psi_{zA} &= 6 \text{ cm} \\ \psi_{zB} &= 6 + 9 = 15 \text{ cm} \\ \psi_{zC} &= 6 + 9 + 15 = 30 \text{ cm} \\ \psi_{zD} &= 6 + 9 + 15 + 15 = 45 \text{ cm} \\ \psi_{zE} &= 6 + 9 + 15 + 15 = 45 \text{ cm} \\ \psi_{zF} &= 6 + 9 + 15 + 15 - 18 = 27 \text{ cm}\end{aligned}$$

ψ_p at the water surface and above water its equal to 0 so

$$\psi_{pB} = \psi_{pC} = \psi_{pD} = \psi_{pE} = \psi_{pF} = 0$$

And $\psi_m = \psi_h - \psi_z - \psi_p$

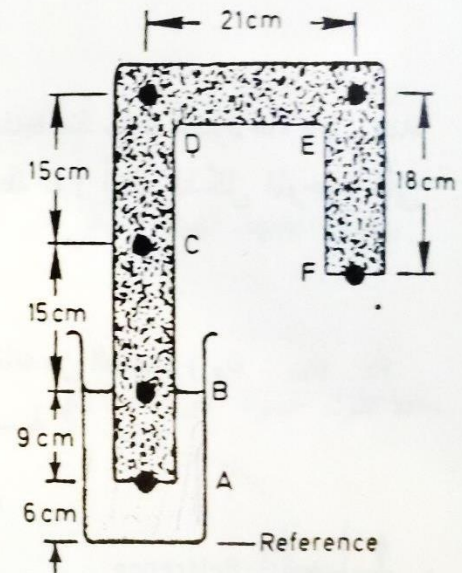
$$\begin{aligned}\psi_{mB} &= 15 - 15 - 0 = 0 \text{ cm} \\ \psi_{mC} &= 15 - 30 - 0 = -15 \text{ cm} \\ \psi_{mD} &= 15 - 45 - 0 = -30 \text{ cm} \\ \psi_{mE} &= 15 - 45 - 0 = -30 \text{ cm} \\ \psi_{mF} &= 15 - 27 - 0 = -12 \text{ cm}\end{aligned}$$

Point A below water surface so $\psi_{mA} = 0$

And $\psi_{pA} = \psi_{hA} - \psi_{zA} - \psi_{mA} = 15 - 6 - 0 = 9 \text{ cm}$

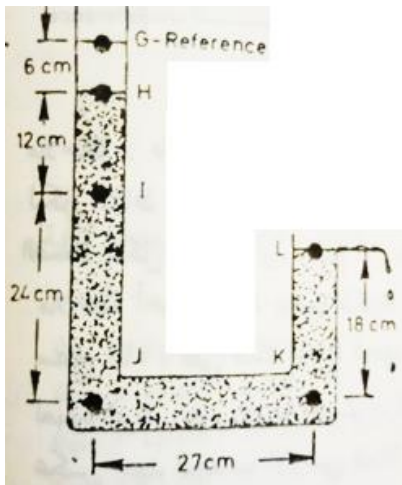
The final results in table below:

Potential com- ponent	Point in soil					
	A	B	C	D	E	F
ψ_p	9	0	0	0	0	0
ψ_m	0	0	-15	-30	-30	-12
ψ_z	6	15	30	45	45	27
ψ_h	15	15	15	15	15	15



Sample Problems

Given soil column as shown below, column of water applied above this column in point G as shown and reach to equilibrium with dropping water from point L. Find (ψ_h and its components ψ_p , ψ_z and ψ_m) for point G to L in soil column.



Solution:

The water in flowing state so there is ψ_h between the two ends of the tube.

From the figure H equal to 6 cm (the column of water above the soil) and below the reference so $\psi_{zH} = -6$ cm, Point H below water so $\psi_{mH} = 0$ cm, and $\psi_{pH} = 6$ cm (because the column of water above this point equal to 6 cm). $\therefore \psi_h = \psi_p + \psi_z + \psi_m =$

as a result $\psi_{hH} = \psi_{pH} + \psi_{zH} + \psi_{mH} = 6 + (-6) + 0 = 0$ cm at point L and because dropping water $\psi_{mL} = 0$ cm, and no column of water above point L so $\psi_{pL} = 0$ cm, $\psi_{zL} = (-6) + (-12) + (-24) + (18) = (-24)$ cm

And $\psi_{hL} = \psi_{pL} + \psi_{zL} + \psi_{mL} = 0 + (-24) + 0 = (-24)$ cm

The difference in hydraulic potential between point L and point H will be:

$$\Delta\psi_{hLH} = -24 - 0 = -24 \text{ cm}$$

And the potential gradient along the soil column $(\nabla\psi_h) = \Delta\psi_{hLH} / \Delta s_{LH} = -24 / 81 =$
 $\therefore (\nabla\psi_h) = -0.296$

$$\Delta\psi_{hHI} = \nabla\psi_h * \Delta s_{HI} = -0.296 * 12 = -3.56 \text{ cm}$$

And in same way:

$$\Delta\psi_{hIJ} = \nabla\psi_h * \Delta s_{IJ} = -0.296 * 24 = -7.11 \text{ cm}, \Delta\psi_{hJK} = -8.00 \text{ cm}, \Delta\psi_{hKL} = -5.33 \text{ cm}$$

Now we could find the hydraulic potential at each point as below:

$$\psi_{hH} = 0 \text{ cm},$$

$$\psi_{hI} = \psi_{hH} + \Delta\psi_{hHI} = 0 + (-3.56) = -3.56 \text{ cm},$$

$$\psi_{hJ} = \psi_{hI} + \Delta\psi_{hIJ} = (-3.56) + (-7.11) = -10.67 \text{ cm},$$

$$\psi_{hK} = \psi_{hJ} + \Delta\psi_{hJK} = (-10.67) + (-8.00) = -18.67 \text{ cm},$$

$$\psi_{hL} = \psi_{hK} + \Delta\psi_{hKL} = (-18.67) + (-5.33) = -24 \text{ cm}$$

The next step is to find the gravitational potential ψ_z as below:

$$\psi_{zH} = -6 \text{ cm},$$

$$\psi_{zI} = -6 + (-12) = -18 \text{ cm},$$

$$\psi_{zJ} = -6 + (-12) + (-24) = -42 \text{ cm},$$

$$\psi_{zK} = -6 + (-12) + (-24) = -42 \text{ cm},$$

$$\psi_{zL} = -6 + (-12) + (-24) + 18 = -24 \text{ cm}$$

As shown all point below water surface so $\psi_m = 0$ for all point that's mean

$$\psi_{mH} = \psi_{mI} = \psi_{mJ} = \psi_{mK} = \psi_{mL} = 0 \text{ cm}$$

Now the pressure potential ψ_p could be calculated easily as below:



$$\psi_p = \psi_h - \psi_z - \psi_m$$

$$\psi_{pH} = 0 - (-6) - 0 = 6 \text{ cm},$$

$$\psi_{pI} = (-3.56) - (-18) - 0 = 14.44 \text{ cm},$$

$$\psi_{pJ} = (-10.67) - (-42) - 0 = 31.33 \text{ cm},$$

$$\psi_{pK} = (-18.67) - (-42) - 0 = 23.33 \text{ cm},$$

$$\psi_{pL} = (-24) - (-24) - 0 = 0 \text{ cm},$$

The final results shown in table below:

Point in soil						Potential component
L	K	J	I	H	G	
0	23.33	31.33	14.44	6	0	ψ_p
0	0	0	0	0	0	ψ_m
-24	-42	-42	-18	-6	0	ψ_z
-24	-18.67	-10.67	-3.56	0	0	ψ_h



WATER FLOW IN SOIL

The flow in soil if it saturated or unsaturated divided into:

- 1- Steady state water flow: in this type of flow the flow properties (θ , ψ_h) not change with time but could change with location.
- 2- Transient state water flow.

Water Flow in Saturated Soil

The discharge Q , that is, the volume flowing through a section of length L per unit time, can now be evaluated. The volume of a paraboloid of revolution is $(1/2)$ (base. height). Hence

$$Q = (1/2)\pi R^2 u_{\max} = \pi R^4 \Delta p / 8\eta L$$

This equation, known as *Poiseuille's law*, indicates that the volume flow rate is proportional to the pressure drop per unit distance ($\Delta p/L$) and the fourth power of the radius of the tube.

Laminar flow prevails only at relatively low flow velocities and in narrow tubes. As the radius of the tube and the flow velocity are increased, the point is reached at which the mean flow velocity is no longer proportional to the pressure drop, and the parallel *laminar flow* changes into a *turbulent flow* with fluctuating eddies. Conveniently, however, laminar flow is the rule rather than the exception in most water flow processes taking place in soils, because of the narrowness of soil pores.

DARCY'S LAW

We now examine the flow of water in a macroscopically uniform, saturated soil body, and attempt to describe the quantitative relations connecting the rate of flow, the dimensions of the body, and the hydraulic conditions at the inflow and outflow boundaries. Figure 7.3 shows a horizontal column of soil through which a steady flow of water is occurring from left to right, from an upper reservoir to a lower one, in each of which the water level is maintained constant. Experience shows that the discharge rate Q , being the volume V flowing through the column per unit time, is directly proportional to the crosssectional area and to the hydraulic head drop ΔH and inversely proportional to the length of the column L :

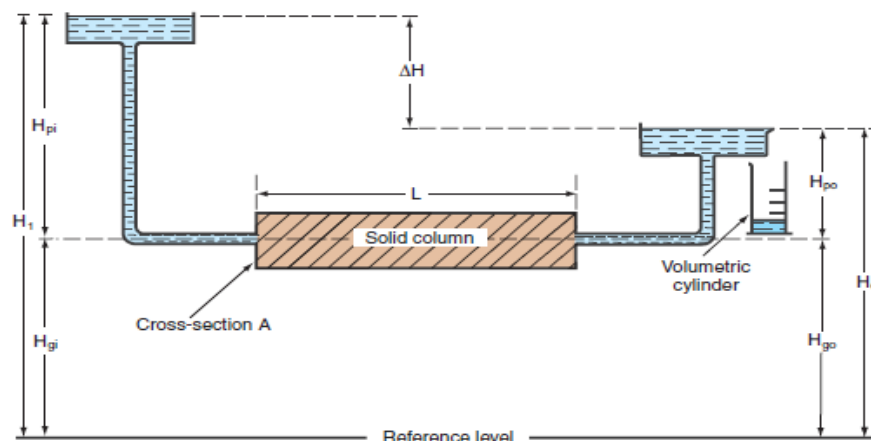


Fig. 7.3. Flow in a horizontal saturated column.



$$Q = V/t \propto A \Delta H/L$$

$$\Delta H = H_i - H_o$$

The head drop per unit distance in the direction of flow ($\Delta H/L$) is the *hydraulic gradient*, which is, in fact, the driving force. The specific discharge rate Q/A (i.e., the volume of water flowing through a unit cross-sectional area per unit time t) is called the *flux density* (or simply the flux) and is indicated by q . Thus, the flux is proportional to the hydraulic gradient:

$$q = Q/A = V/At \propto \Delta H/L$$

The proportionality factor K is termed the *hydraulic conductivity*:

$$q = K \Delta H/L$$

This equation is known as *Darcy's law*, after Henry Darcy, the French engineer who discovered it over a century ago in the course of his classic investigation of seepage rates through sand filters in the city of Dijon.

Sample Problem

The water in an irrigation hose is at a hydrostatic pressure of 100 kPa (1 bar). Five dripirrigation emitters are inserted into the wall of the hose. Calculate the drip rate (L/hr) from the emitters if each contained a coiled capillary tube 1 m long and the capillary diameters are 0.2, 0.4, 0.6, 0.8, and 1.0 mm. Assuming laminar flow, what fraction of the total discharge is due to the single largest emitter?

We use Poiseuille's law to calculate the discharge: $Q = \pi R^4 \Delta P / 8 \eta L$. Substituting the values for π (3.14), the pressure differential ΔP ($10^5 \text{ N/m}^2 = 10^5 \text{ kg/m sec}^2$), the viscosity η (10^{-3} kg/m sec , at 20°C), the capillary tube length (1 m), and the appropriate tube radial (0.1, 0.2, 0.3, 0.4, and 0.05 mm), we obtain

$$Q_1 = \frac{3.14 \times (10^{-4} \text{ m})^4 \times 10^5 \text{ kg/m sec}^2}{8 \times 10^{-3} \text{ kg/m sec} \times 1 \text{ m}} = 3.9 \times 10^{-9} \text{ m}^3/\text{sec}$$

$$= 0.014 \text{ L/hr}$$

$$Q_2 = \frac{3.14 \times (2 \times 10^{-4} \text{ m})^4 \times 10^5 \text{ kg/m sec}^2}{8 \times 10^{-3} \text{ kg/m sec} \times 1 \text{ m}} = 2^4 Q_1 = 16 \times 1.4 \times 10^{-2}$$

$$= 0.226 \text{ L/hr}$$

$$Q_3 = 3^4 Q_1 = 81 Q_1 = 1.14 \text{ L/hr}$$

$$Q_4 = 4^4 Q_1 = 256 Q_1 = 3.61 \text{ L/hr}$$

$$Q_5 = 5^4 Q_1 = 625 Q_1 = 8.81 \text{ L/hr}$$

Total discharge from all five emitters:

$$Q_{\text{total}} = 0.014 + 0.226 + 1.14 + 3.61 + 8.81 = 13.8 \text{ L/hr}$$

Fractional contribution of the single largest emitter:

$$Q_5/Q_{\text{total}} = 8.81/13.8 = 0.639 = 63.8\%$$



The single largest emitter thus accounts for nearly two-thirds of the total discharge, while the smallest emitter accounts for only 0.1% (though its diameter is only one-fifth that of the largest emitter).

Note: Modern drip-irrigation emitters generally depend on partially turbulent (rather than completely laminar) flow, to reduce sensitivity to pressure fluctuations and vulnerability to clogging by particles.

Sample Problem

Let us suppose we were given the task of purifying Dijon's water, with its 10,000 denizens. Since they drink mostly wine, their daily water requirements are modest, say, no more than 20 liters per day per person, on average. Let us suppose further that we knew (with the benefits of hindsight) that a column thickness of 0.30 m was adequate for filtration and that the hydraulic conductivity of the available sand was 2×10^{-5} m/sec. Could we calculate the area of the filter bed needed under a hydrostatic pressure head of 0.7 m? Consider the flow to be vertically downward to a fixed drainage plane.

We begin by calculating the discharge Q needed:

$$Q = \frac{10^4 \text{ persons} \times 20 \text{ L/person day} \times 10^{-3} \text{ m}^3/\text{L}}{8.64 \times 10^4 \text{ sec/day}} = 2.31 \times 10^{-3} \text{ m}^3/\text{sec}$$

We recall Darcy's law:

$$Q = AK \Delta H/L$$

Hence, the area needed is:

$$A = QL/K \Delta H$$

The hydraulic head drop ΔH equals the sum of the pressure head and gravitational head drops:

$$\Delta H = 0.7 + 0.3 = 1.0 \text{ m}$$

Substituting these values for L (0.3 m), ΔH (1 m), and K (2×10^{-5} m/sec), we obtain

$$A = \frac{2.31 \times 10^{-3} \text{ m}^3/\text{sec} \times 0.3 \text{ m}}{2 \times 10^{-5} \text{ m/sec} \times 1 \text{ m}} = 34.7 \text{ m}^2$$

Note: Since populations and per capita water use tend to increase and filter beds tend to clog, it might be wise to apply a factor of safety to our calculations and increase the filtration capacity several fold (especially to accommodate peak demand hours).

Incidentally, per capita water use in the U.S. (with running toilets, showers, and dishwashers) ranges from 100 to 400 L/day.



GRAVITATIONAL, PRESSURE, AND TOTAL HYDRAULIC HEADS:

The water entering the column of Fig. 7.3 is under a pressure, which is the sum of the hydrostatic pressure and the atmospheric pressure acting on the surface of the water in the reservoir. Since the atmospheric pressure is the same at both ends of the system, we can disregard it and consider only the hydrostatic pressure. Accordingly, the water pressure at the inflow boundary $P_i = \rho_w g H_{pi}$. Since ρ_w and g are both nearly constant, we can express this pressure in terms of the pressure head H_{pi} .

Water flow in a horizontal column occurs in response to a pressure head gradient. Flow in a vertical column may be caused by gravitation as well as pressure. The *gravitational head* H_g at any point is determined by the height of the point relative to some reference plane, while the pressure head is determined by the height of the water column resting on that point. The total hydraulic head H is the sum of these two heads:

$$H = H_p + H_g$$

To apply Darcy's law to vertical flow, we must consider the total hydraulic head at the inflow and at the outflow boundaries (H_i and H_o , respectively):

$$H_i = H_{pi} + H_{gi} \text{ and } H_o = H_{po} + H_{go}$$

Darcy's law thus becomes

$$q = K[(H_{pi} + H_{gi}) - (H_{po} + H_{go})]/L$$

The gravitational head is often designated as z , which is the vertical distance in the rectangular coordinate system x, y, z . It is convenient to set the reference level as the point $z = 0$ at the bottom of a vertical column or at the center of a horizontal column. However, the exact elevation of this hypothetical level is unimportant, since the absolute values of the hydraulic heads determined in reference to it are immaterial and only their differences from one point in the soil to another affect flow.

The pressure and gravity heads can be represented graphically in a simple way. To illustrate this, we shall immerse and equilibrate a vertical soil column in a water reservoir so that the upper surface of the column will be level with the water surface, as shown in Fig. 7.4. The coordinates of Fig. 7.4 are arranged so that the height above the bottom of the column is indicated by the vertical axis z ; and the pressure, gravity, and hydraulic heads are indicated on the horizontal axis. The gravity head is determined with respect to the reference level $z = 0$, and it increases with height at the ratio of 1:1. The pressure head is determined with reference to the free-water surface, at which the hydrostatic pressure is zero.

Accordingly, the hydrostatic pressure head at the top of the column is zero and at the bottom of the column is equal to L , the column length. Just as the gravity head diminishes from top to bottom, so the pressure head increases.

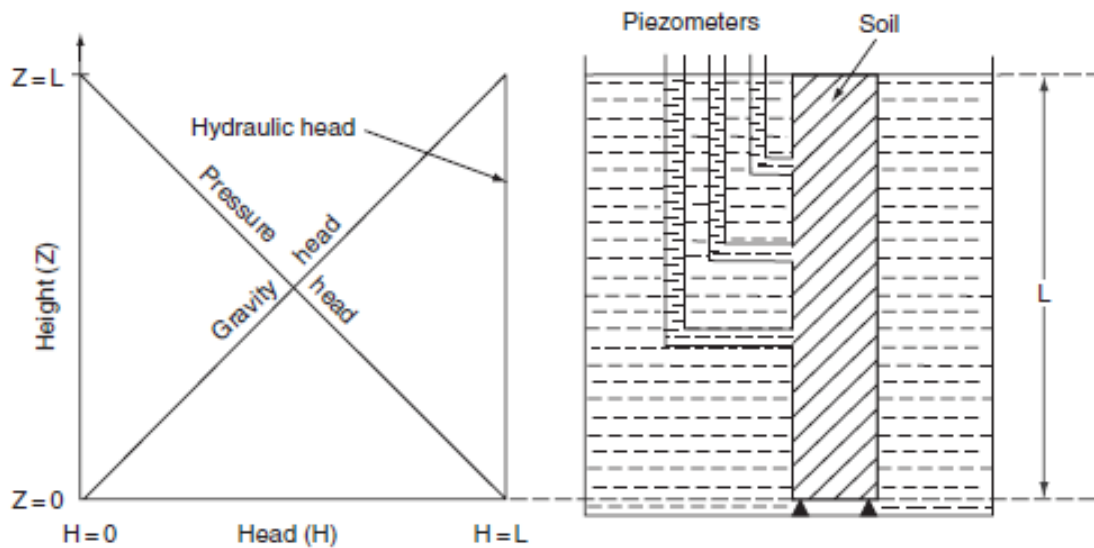


Fig. 7.4. Distribution of pressure, gravity, and total hydraulic heads in a vertical column immersed in water, at equilibrium.

Consequently their sum, which is the hydraulic head, remains constant all along the column. This is a state of equilibrium, at which no flow occurs.

This statement deserves to be further elaborated. The water pressure is not equal along the column, being greater at the bottom than at the top of the column.

Why, then, will the water not flow from a zone of higher pressure to one of lower pressure? If the pressure gradient were the only force causing flow (as it is, in fact, in a horizontal column), the water would tend to flow upward.

However, opposing the pressure gradient is a gravitational gradient of equal magnitude, resulting from the fact that the water at the top is at a higher gravitational potential than that at the bottom. In the illustration given, the two opposing gradients in effect cancel each other, so the total hydraulic head is constant throughout the column, as indicated by the standpipes (*piezometers*) connected to the column at the left.

As we already pointed out, it is convenient to set the reference level at the bottom of the column so that the gravitational potential can always be positive. On the other hand, the pressure head of water can be either positive or negative: It is positive under a free-water surface (i.e., a water table) and negative above it.

FLOW IN A VERTICAL COLUMN

Figure 7.5 shows a uniform, saturated vertical column, the upper surface of which is ponded under a constant head of water H_1 , and the bottom surface of which is set in a lower, constant-level reservoir. Flow is thus taking place from the higher to the lower reservoir through a column of length L .

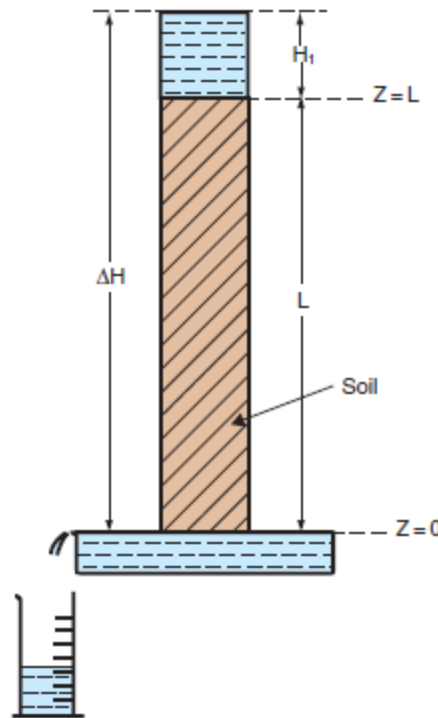


Fig. 7.5. Downward flow of water in a vertical saturated column.

In order to calculate the flux according to Darcy's law, we must know the hydraulic head gradient, which is the ratio of the hydraulic head drop (between the inflow and outflow boundaries) to the column length, as shown here:

		Pressure head	Gravity head
Hydraulic head at inflow boundary H_i	=	H_1	+ L
Hydraulic head at outflow boundary H_o	=	0	+ 0
Hydraulic head difference $\Delta H = H_i - H_o$	=	H_1	+ L

The Darcy equation for this case is

$$q = K \Delta H / L = K(H_1 + L) / L = KH_1 / L + K$$

Comparison of this case with the horizontal one show that the rate of downward flow of water in a vertical column is greater than in a horizontal column by the magnitude of the hydraulic conductivity. It is also apparent that, if the ponding depth H_1 is negligible, the flux is equal to the hydraulic conductivity.

This is due to the fact that, in the absence of a pressure gradient, the only driving force is the gravitational head gradient, which, in a vertical column, has the value of unity (since this head varies with height at the ratio of 1:1).

We now examine the case of upward flow in a vertical column, as shown in Fig. 7.6. In this case, the direction of flow is opposite to the direction of the gravitational gradient, and the hydraulic gradient becomes



		Pressure head	Gravity head
Hydraulic head at inflow boundary H_i	=	H_1	+ 0
Hydraulic head at outflow boundary H_o	=	0	+ L
Hydraulic head difference $\Delta H = H_i - H_o$	=	H_1	- L

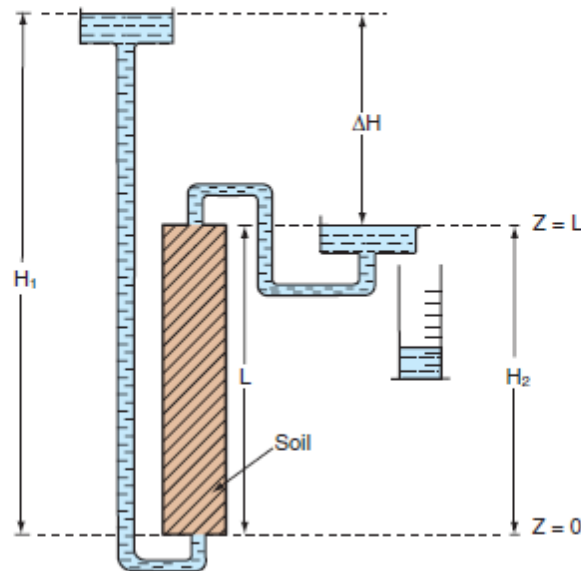


Fig. 7.6. Steady upward flow in a saturated vertical column.

The Darcy equation is thus:

$$q = K(H_1 - L)/L = KH_1/L - K = K \Delta H/L$$

FLOW IN A COMPOSITE COLUMN

Figure 7.7 depicts steady flow through a soil column consisting of two distinct layers, each homogeneous within itself and differing from the other layer in thickness and hydraulic conductivity. Layer 1 is at the inlet and layer 2 is at the outlet side of the column. The hydraulic head values at the inlet surface, at the interlayer boundary, and at the outlet end are designated H_1 , H_2 , and H_3 , respectively. At steady flow, the flux through both layers must be equal:

$$q = K_1 (H_1 - H_2)/L_1 = K_2 (H_2 - H_3)/L_2$$

Where q is the flux, K_1 and L_1 are the conductivity and thickness (respectively) of the first layer, and K_2 and L_2 are the same for the second layer. Here we have disregarded any possible contact resistance between the layers.

Thus,

$$H_2 = H_1 - qL_1/K_1 \quad \text{and} \quad qL_2/K_2 = H_2 - H_3$$

Therefore,

$$qL_2/K_2 = H_1 - qL_1/K_1 - H_3 \quad \text{and} \quad q = (H_1 - H_3) / (L_2/K_2 + L_1/K_1)$$



The reciprocal of the conductivity has been called the *hydraulic resistivity*, and the ratio of the thickness to the conductivity ($R_h = L/K$) has been called the hydraulic resistance per unit area. Hence,

$$q = \Delta H / (R_{h1} + R_{h2})$$

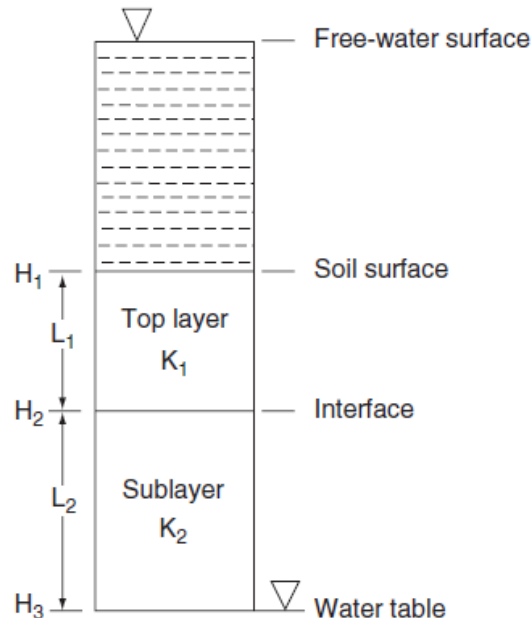


Fig. 7.7. Downward flow through a composite column.

where ΔH is the total hydraulic head drop across the entire system and R_{h1} , R_{h2} are the hydraulic resistances of layers 1 and 2.

Sample Problem

Let us consider two cases of steady downward percolation through a two-layer soil profile, the top of which is submerged under a 1-m head of water and the bottom of which is defined by a water table. Each of the two layers is 0.50 m thick. In the one case, the conductivity of the top layer is 10^{-6} m/sec and that of the sub layer is 10^{-7} m/sec. In the second case, the same layers are reversed.

To calculate the flux and the hydraulic and pressure heads at the interface between the layers, we use the Ohm's law analogy for steady flow through two resistors in series:

$$q = \Delta H / (R_1 + R_2)$$

Where q is the flux, ΔH is the total hydraulic head drop across the profile, and R_1 , R_2 are the hydraulic resistances of the top layer and sub layer, respectively.

Each resistance is proportional directly to the layer's thickness and inversely to its conductivity



(i.e., $R = L/K$). The pressure head at the soil surface is 1 m, and the gravity head (with reference to the soil's bottom) is also 1 m. Both the pressure and gravity heads at the bottom boundary are zero. Hence,

$$q = (1 \text{ m} + 1 \text{ m}) / (0.5 \text{ m}/10^{-6} \text{ m/sec} + 0.5 \text{ m}/10^{-7} \text{ m/sec}) = 3.64 \times 10^{-7} \text{ m/sec}$$

We can now apply Darcy's equation to the top layer alone to obtain the hydraulic head at the interlayer interface:

$$q = K_1 \Delta H_1 / L_1 = K_1 (H_{\text{surface}} - H_{\text{interface}}) / L_1$$

Hence:

$$H_{\text{interface}} = H_{\text{surface}} - qL_1/K_1$$

$$= 2 \text{ m} - (3.64 \times 10^{-7} \text{ m/sec}) (0.5 \text{ m}) / 10^{-6} \text{ m/sec} = 1.818 \text{ m}$$

Since the gravity head at the interface is 0.5 m (above our reference datum at the bottom of the profile), the pressure head H_p is

$$H_p = H - H_g = 1.818 - 0.50 = 1.318 \text{ m}$$

We now reverse the order of the layers so that the less conductive layer overlies the more conductive. The total head drop remains the same and so does the total resistance.

Therefore the flux remains the same (assuming that both layers are still saturated and the conductivity of each does not change). Applying Darcy's equation to the top layer, we have, as previously,

$$H_{\text{interface}} = 2.0 - (3.64 \times 10^{-7}) (0.5) / 10^{-7} = 0.18 \text{ m}$$

In this case the pressure head at the interface is

$$H_p = H - H_g = 0.18 - 0.50 = -0.32 \text{ m}$$

Note: Comparison between the interface pressures of the two cases illustrates an important principle regarding flow in layered profiles. With the more conductive layer on top, flow is impeded at the interface and there is a pressure buildup, which in our case increased the pressure head from 1 m at the surface to 1.218 m at the interface. The opposite occurs when the upper layer is less conductive. In this case the pressure is dissipated through the top layer, often to the extent that a negative pressure develops at the interface.



HYDRAULIC CONDUCTIVITY

The hydraulic conductivity K is the ratio of the flux to the potential gradient.

The hydraulic conductivity K is not a property of the soil alone. Rather, it depends jointly on the attributes of the soil and of the fluid. The soil characteristics that affect K are the total porosity, the distribution of pore sizes, and tortuosity — in short, the soil's pore geometry. The fluid attributes that affect conductivity are density and viscosity.

It is possible in theory, and sometimes in practice, to separate K into two factors: *intrinsic permeability* of the soil k and *fluidity* of the permeating liquid (or gas) f :

$$K = kf$$

If K is given in terms of m/sec (LT^{-1}), then k is in units of m^2 (L^2) and f is in units of $m^{-1} sec^{-1}$ ($L^{-1}T^{-1}$).

Fluidity is related directly to density and inversely to viscosity:

$$f = \rho g / \eta$$

Hence,

$$k = K\eta / \rho g$$

Where η is the dynamic viscosity (N sec/m², or Pa sec), ρ is the fluid density (kg/m³), and g is the gravitational acceleration (m/sec²).

While fluidity varies with temperature and composition of the fluid, permeability is ideally a property of the porous medium's pore geometry alone, provided the fluid and the solid matrix do not interact in such a way as to change the properties of either. In a stable porous body, the same permeability will be obtained with different fluids (water, air, or oil). However, in many cases water does interact with the solid matrix to modify its permeability, so hydraulic conductivity cannot be resolved into separate and independent properties of water and of soil.

LIMITATIONS OF DARCY'S LAW

Darcy's law is not universally valid for all conditions of liquid flow in porous media. It has long been recognized that the linearity of the flux versus hydraulic gradient relationship fails at high flow velocities, where inertial forces are no longer negligible compared to viscous forces. Darcy's law applies only as long as flow is laminar (i.e., nonturbulent movement of adjacent layers of the fluid relative to one another) and where soil–water interaction does not result in a change of fluidity or of permeability with a change in gradient.

Laminar flow prevails in silts and finer materials for most commonly occurring hydraulic gradients found in nature. In coarse sands and gravels, however, hydraulic gradients much in excess of unity may result in nonlaminar flow conditions, so in such cases Darcy's law may not be applicable.

The quantitative criterion for the onset of turbulent flow is the *Reynolds number* NRe :



$$N_{Re} = \frac{d u \rho}{\eta}$$

where u is the mean flow velocity, d is the effective pore diameter, ρ is the liquid density, and η is its viscosity.

For porous media, therefore, it is safe to assume that flux remains linear with hydraulic gradient only as long as N_{Re} is smaller than unity.

As flow velocity increases, especially in systems of large pores, the occurrence of turbulent eddies or nonlinear laminar flow results in “waste” of effective energy; that is, some energy is dissipated by the internal churning of the liquid so that the hydraulic potential gradient becomes less effective in inducing flow. This is illustrated in Fig. 7.11.

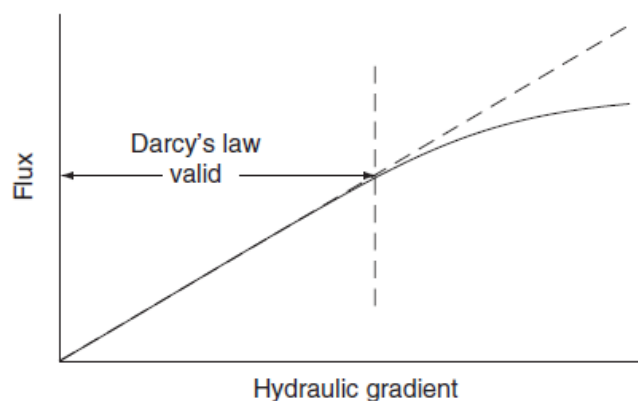


Fig. 7.11. The deviation from Darcy's law at high flux, where flow becomes turbulent.

Deviations from Darcy's law may also occur at the opposite end of the flow-velocity range, namely, at low gradients and in narrow pores. Some investigators have claimed that, in clayey soils, small hydraulic gradients may cause no flow or only low flow rates that are less than proportional to the gradient.

A possible reason for this anomaly is that the water in close proximity to the particle surfaces and subject to their adsorptive force fields may be more rigid than ordinary bulk water and may exhibit the properties of a *Bingham liquid* (having a *yield value*) rather than a *Newtonian liquid* (Hillel, 1980a).

The adsorbed (or “bound”) water may even have a quasi-crystalline structure similar to that of ice. Some soils may exhibit a *threshold gradient* (Miller, R. J. and Low, 1963), below which the flux is either zero (the water remaining apparently immobile) or at least lower than predicted by the Darcy relation, and only at gradients exceeding the threshold value does the flux become proportional to the gradient (Fig. 7.12). These phenomena and their possible explanations, though highly interesting, are generally of little importance in practice, so Darcy's law can be employed in the vast majority of cases pertaining to the flow of water in soil.



Another possible cause for apparent flow anomalies in clay soils is their tendency to swell or compress (Smiles, 1976). As commonly formulated, Darcy's law applies to flow relative to a geometrically fixed porous matrix, and it may seem to fail when the particles composing the matrix are themselves moving relative to a fixed frame of reference.

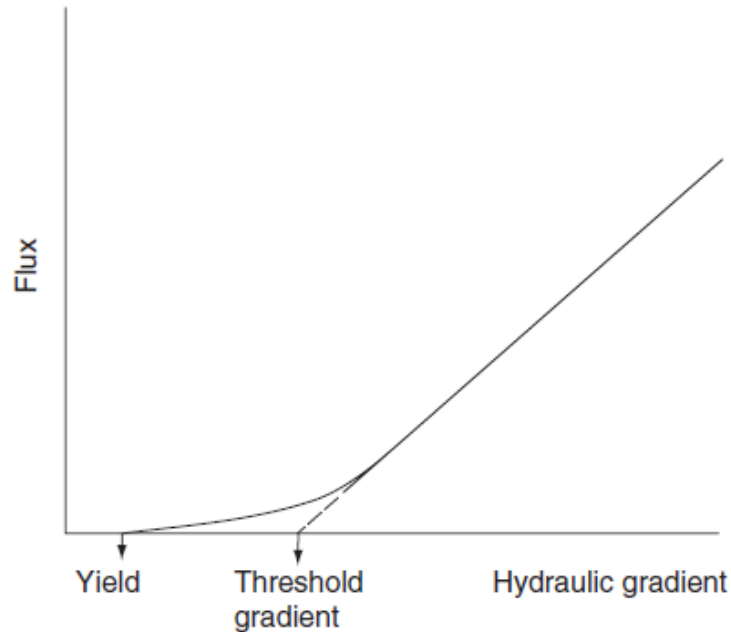


Fig. 7.12. Possible deviations from Darcy's law at low gradients.



Water flow in unsaturated soil:

Perhaps the most important difference between unsaturated and saturated flow is in the hydraulic conductivity. When the soil is saturated, all of the pores are water filled and conducting. The water phase is then continuous and the conductivity is maximal. When the soil desaturates, some of the pores become air filled, so the conductive portion of the soil's cross-sectional area diminishes. Furthermore, as suction develops, the first pores to empty are the largest ones, which are the most conductive (remember Poiseuille's law!), thus relegating flow to the smaller pores. At the same time, the large empty pores must be circumvented, so, with progressive desaturation, tortuosity increases. In coarse-textured soils, water may be confined almost entirely to the capillary wedges at the contact points of the particles, thus forming separate and discontinuous pockets of water. In aggregated soils, too, the large interaggregate spaces that confer high conductivity at saturation become (when emptied) barriers to liquid flow from one aggregate to another.

For all these reasons, the transition from saturation to unsaturation generally entails a steep drop in hydraulic conductivity, which may decrease by several orders of magnitude (sometimes down to one-millionth of its value at saturation) as suction increases from 0 to 1 MPa. At still higher suctions, or lower wetness values, the conductivity may be so low that very steep suction gradients, or very long times, are required for any appreciable flow to occur at all.

Darcy's law, though originally conceived for flow in saturated porous media, has been extended to unsaturated flow, with the provision that the conductivity is now a function of the matric suction head [i.e., $K = K(\psi)$]:

$$q = -K(\psi) \nabla H$$

where ∇H is the hydraulic head gradient, which may include both suction and gravitational components. This equation, along with its alternative formulations, is known as *Richards' equation*.

This Equation as written here fails to take into account the hysteresis of soil-water characteristics. In practice, the hysteresis problem can sometimes be evaded by limiting the use of this equation to cases in which the suction (or wetness) change is monotonic — either increasing or decreasing continuously. However, in processes involving successive wetting and drying phases, the

$K(\psi)$ function may be highly hysteretic. However, the relation of conductivity to volumetric wetness $K(\theta)$ or to degree of saturation $K(S)$ is affected by hysteresis to a lesser extent than is the $K(\psi)$ function. Darcy's law for unsaturated soil can thus be written

$$q = -K(\theta) \nabla H$$

which, however, still leaves us the problem of dealing with the hysteresis between ψ and θ .



To account for transient flow processes, we introduce the continuity principle:

$$\partial\theta/\partial t = -\nabla \cdot q$$

Thus,

$$\partial\theta/\partial t = \nabla \cdot [K(\psi)\nabla H]$$

The hydraulic head is, in general, the sum of the pressure head (or its negative, the suction head ψ) and the gravitational head (or elevation z), we can write

$$\partial\theta/\partial t = \nabla \cdot [K(\psi)\nabla(\psi - z)]$$

Since ∇z is zero for horizontal flow and unity for vertical flow, we can rewrite Eq. (8.6) as follows:

$$\frac{\partial\theta}{\partial t} = -\nabla \cdot (K(\psi)\nabla\psi) + \frac{\partial K}{\partial z} \quad (8.12a)$$

or

$$\frac{\partial\theta}{\partial t} = -\frac{\partial}{\partial x} \left(K \frac{\partial\psi}{\partial x} \right) - \frac{\partial}{\partial y} \left(K \frac{\partial\psi}{\partial y} \right) - \frac{\partial}{\partial z} \left(K \frac{\partial\psi}{\partial z} \right) + \frac{\partial K}{\partial z} \quad (8.13)$$

Processes may occur in which ∇z (the gravity gradient) is negligible compared to the strong matric suction gradient $\nabla\psi$. in such cases,

$$\partial\theta/\partial t = \nabla \cdot [K(\psi)\nabla\psi]$$

or, in a one-dimensional horizontal system,

$$\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial x} \left[K(\psi) \frac{\partial\psi}{\partial x} \right]$$

The matric suction gradient $\delta\psi/\delta x$ can be expanded by the chain rule:

$$\frac{\partial\psi}{\partial x} = \frac{d\psi}{d\theta} \frac{\partial\theta}{\partial x}$$

Here $\partial\theta/\partial x$ is the wetness gradient and $\partial\psi/\partial\theta$ is the reciprocal of *specific water capacity*, $C(\theta)$:

$$C(\theta) = d\theta/d\psi$$

which is the slope of the soil-moisture characteristic curve at a particular value of wetness θ .

We can now rewrite the Darcy equation as follows:

$$q = K(\theta) \frac{\partial\psi}{\partial x} = -\frac{K(\theta)}{C(\theta)} \frac{\partial\theta}{\partial x}$$

To cast this equation into a form analogous to Fick's law of diffusion, a function is introduced called the *diffusivity*, D :

$$D(\theta) = K(\theta)/C(\theta) = K(\theta)(d\psi/d\theta)$$



D is thus defined as the ratio of the hydraulic conductivity K to the specific water capacity C , and since both are functions of soil wetness, so D must be also. To avoid confusion between the classical concept of diffusivity pertaining to the diffusive transfer of components in the gaseous and liquid phases and this borrowed application of the same term to describe convective flow, we propose to qualify it with the adjective *hydraulic*. Here, therefore, we shall employ the term *hydraulic diffusivity* when referring to D of Eq. (8.19). We can now rewrite Eq. (8.10):

$$q = -D(\theta)\nabla\theta \quad (8.20)$$

or, in one dimension,

$$q = -D(\theta)(\partial\theta/\partial x) \quad (8.21)$$

Which is mathematically identical to Fick's first law of diffusion. Hydraulic diffusivity can thus be viewed as the ratio of the flux (in the absence of gravity and of hysteresis effects) to the soil-water content (wetness) gradient. As such, D has dimensions of length squared per unit time (L^2T^{-1}), since K has the dimensions of volume per unit area per time (LT^{-1}) and the specific water capacity C has dimensions of volume of water per unit volume of soil per unit change in matric suction head (L^{-1}). In the use of Eq. (8.21), the gradient of wetness is taken to represent, implicitly, a gradient of matric potential, which is the true driving force.

Introducing the hydraulic diffusivity into Eq. (8.15), for one-dimensional flow in the absence of gravity, we obtain

$$\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial x} \left[D(\theta) \frac{\partial\theta}{\partial x} \right] \quad (8.22)$$

which has only one dependent variable (θ) rather than the two (θ and ψ) of Eq. (9.15).

Soil Hysteresis: is the relation between matric potential and soil wetness.

It can be obtained in two ways:

(1) In desorption.

(2) In sorption.

(1) In desorption, by taking an initially saturated sample and applying increasing suction, in stepwise manner, to gradually dry the soil while taking successive measurements of wetness versus suction.

(2) In sorption, by gradually wetting up an initially dry soil sample while reducing the suction incrementally.



Each of these two methods yields a continuous curve, but the two curves will in general not be identical. The equilibrium soil wetness at a given suction is greater in desorption (drying) than in sorption (wetting).

This dependence of the equilibrium content and state of soil water upon the direction of the process leading up to it is called *hysteresis*

Figure 6.10 shows a typical soil-moisture characteristic curve and illustrates the hysteresis effect in the relationship between soil wetness and matric suction at equilibrium.

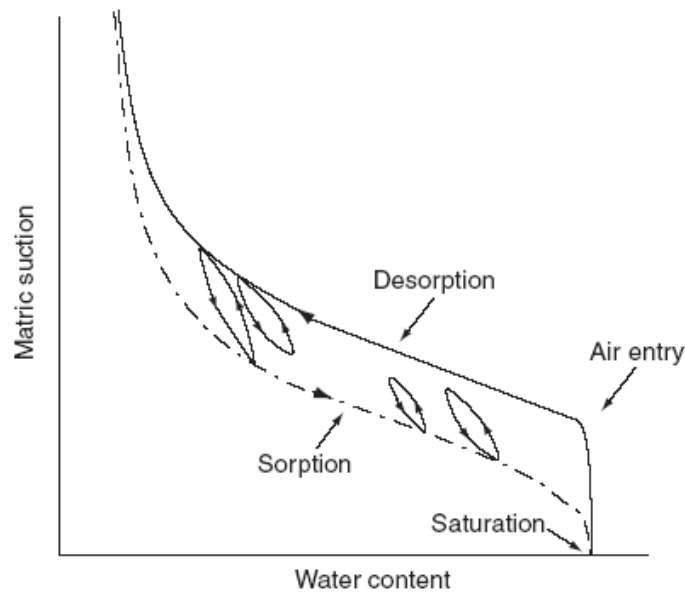


Fig. 6.10. Suction vs. water content curves in sorption and desorption. The intermediate loops are scanning curves, representing complete or partial transitions between the main branches.

The hysteresis effect may be attributed to several causes:

1. The geometric nonuniformity of the individual pores
2. The contact-angle effect by which the contact angle is greater and hence the radius of curvature greater in the case of an advancing meniscus than in the case of a receding one.
3. Entrapped air, which further decreases the water content of newly wetted soil.
4. Swelling, shrinking, or aging phenomena, which result in differential changes of soil structure, depending on the wetting and drying history of the sample.



WATER ENTRY INTO SOIL INFILTRATION

When water is supplied from above to the soil surface, whether by precipitation or irrigation, it typically penetrates the surface and is absorbed into successively deeper layers of the profile. At times, however, a portion of the arriving water may fail to penetrate but instead will tend to accrue at the surface or flow over it. The penetrated water is itself later partitioned between the amount that returns to the atmosphere by direct evaporation from the soil or by the extraction and transpiration of plants and the amount that continues to seep downward and eventually recharges the groundwater reservoir.

Infiltration is the term applied to the process of water entry into the soil, generally by downward flow through all or part of the soil surface.

The rate of this process, relative to the rate of water supply, determines how much water will enter the root zone and how much, if any, will run off. Hence the rate of infiltration affects not only the water economy of terrestrial plants but also the amount of overland flow and its attendant dangers of soil erosion and stream flooding.

Knowledge of the infiltration process as it is affected both by the soil's properties and transient conditions and by the mode of water supply is therefore a prerequisite for efficient soil and water management.

INFILTRATION CAPACITY OR INFILTRABILITY

The *infiltration rate* is defined as the volume flux of water flowing into the profile per unit of soil surface area. For the special condition in which the rainfall rate exceeds the ability of the soil to absorb water, infiltration proceeds at a maximal rate, which Horton (1940) called the soil's *infiltration capacity*.

Hillel (1971) suggested the term *infiltrability* to designate the infiltration flux resulting when water at atmospheric pressure is made freely available at the soil surface. This one-word replacement avoids the extensity–intensity confusion in the term *infiltration “capacity”* and permits use of the term *infiltration “rate”* in the literal sense to represent the surface flux in any set of circumstances, whatever the rate or pressure at which the water is supplied to the soil. The infiltration rate can be expected to exceed infiltrability whenever water is ponded over the soil to a depth sufficient to cause the hydrostatic pressure at the surface to be significantly greater than atmospheric pressure.

On the other hand, if water is applied slowly or at subatmospheric pressure, the actual infiltration rate may be smaller than the infiltrability. As long as the rate of water delivery to the surface is smaller than the soil's infiltrability, water infiltrates as fast as it arrives and the process is *supply controlled* (or *flux controlled*).



However, once the delivery rate exceeds the soil's infiltrability, the latter determines the actual infiltration rate, and thus the process becomes *soil controlled*. The soil may limit the rate of infiltration either at the surface or within the profile. Thus the process may be either *surface controlled* or *profile controlled*.

Soil infiltrability and its variation with time are known to depend on the initial wetness and suction as well as on the texture, structure, and uniformity (or layering sequence) of the profile. In general, soil infiltrability is relatively high in the early stages of infiltration, particularly where the soil is initially quite dry, but it tends to decrease and eventually to approach asymptotically a constant rate that we call the *steady-state infiltrability*.

The figures below show the relation between cumulative infiltration and infiltration rate with time (Fig. 14.2).

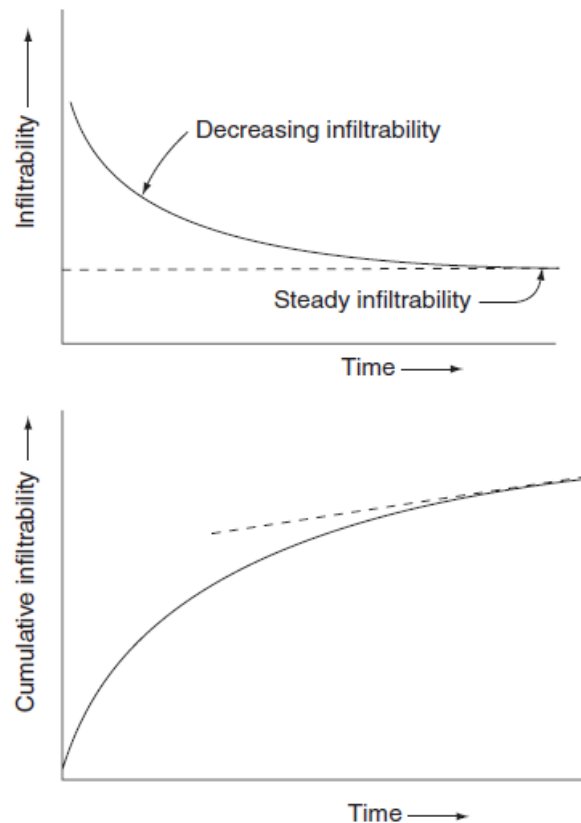


Fig. 14.2. Time dependence of infiltrability and of cumulative infiltration under shallow ponding.

The decrease of infiltrability from an initially high rate can in some cases result from gradual deterioration of soil structure and the partial sealing of the profile by the formation of a surface crust. It can also result from the detachment and migration of pore-blocking particles, from swelling of clay, as well as from entrapment of air bubbles or the bulk compression of the air originally present in the soil if it is prevented from escaping during its displacement by incoming water. Primarily, however, the decline of infiltrability results from the decrease in the



matric suction gradient, which occurs inevitably as infiltration proceeds. If the surface of an initially dry soil is suddenly saturated (e.g., by ponding), the difference of hydraulic potential between the saturated surface and the relatively dry soil just below creates a steep matric suction gradient. As the wetted zone deepens, the same difference of potential acting over a greater distance expresses itself as a diminishing gradient. And as the wetted part of the profile deepens, the suction gradient eventually becomes vanishingly small. In a horizontal column, the infiltration rate eventually tends to zero. In downward flow into a vertical column under continuous ponding, the infiltration rate tends asymptotically to a steady, gravity-induced rate that approximates the soil's saturated hydraulic conductivity if the profile is homogeneous and structurally stable (Fig. 14.2).

Relatively dry soil just below creates a steep matric suction gradient. As the wetted zone deepens, the same difference of potential acting over a greater distance expresses itself as a diminishing gradient. And as the wetted part of the profile deepens, the suction gradient eventually becomes vanishingly small. In a horizontal column, the infiltration rate eventually tends to zero. In downward flow into a vertical column under continuous ponding, the infiltration rate tends asymptotically to a steady, gravity-induced rate that approximates the soil's saturated hydraulic conductivity if the profile is homogeneous and structurally stable (Fig. 14.2).

PROFILE MOISTURE DISTRIBUTION DURING INFILTRATION:

An examination of an initially dry, texturally uniform soil profile at any moment during infiltration under ponding generally shows the three zones as below:

- 1- *Saturated Zone*: surface zone to be saturated to a depth of several millimeters or centimeters.
- 2- *Transmission zone*: is a less-than-saturated, lengthening zone of apparently uniform wetness.
- 3- *Wetting zone*: in this zone soil wetness increases with time at each point, but at any given time wetness decreases with depth at a steepening gradient, down to a *wetting front*.

At the wetting front, the moisture gradient is so steep that there appears to be a sharp boundary between the moistened soil above and the initially dry soil beneath.

The typical moisture profile during infiltration is shown schematically in Fig. 14.3. Repeated examinations of the moisture profile of a stable soil during infiltration generally reveal that the transmission zone lengthens (deepens) continuously and that the wetting zone and the wetting front likewise move downward continuously, with the latter becoming less steep as it moves deeper into the profile.

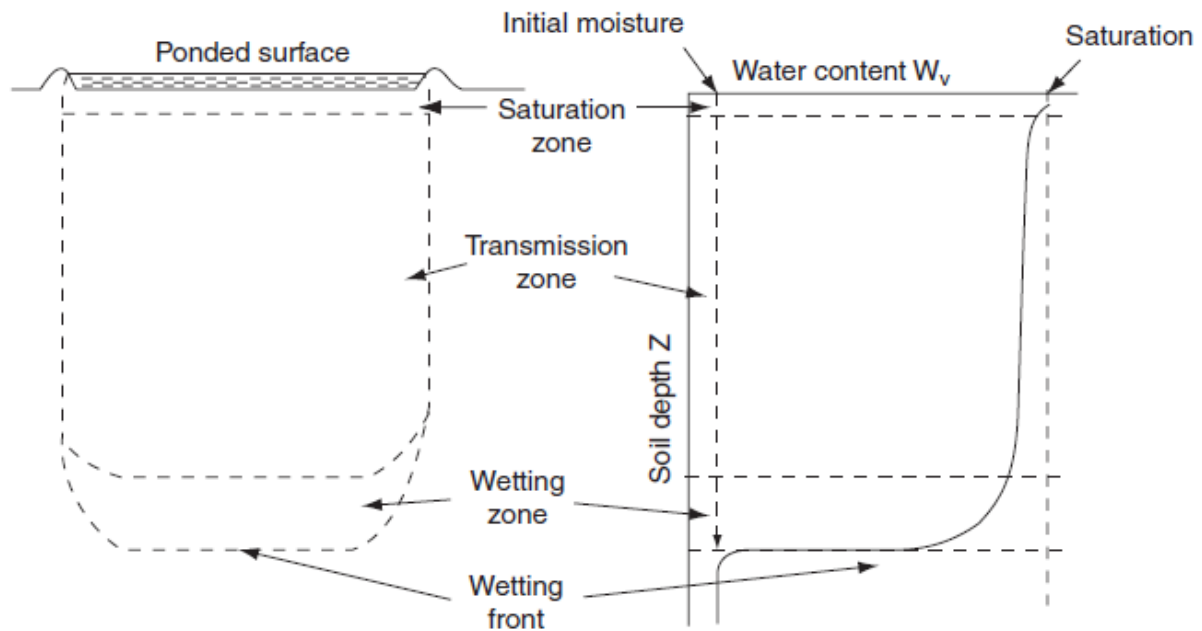


Fig. 14.3. The infiltration moisture profile. At left, a schematic section of the profile; at right, the curve of water content versus depth. The common occurrence of a saturation zone as distinct from the transmission zone may result from the structural instability of the surface zone.

THE EFFECT OF INITIAL WATER STATE ON INFILTRATION RATE:

Figure (Fig. 14.6) show the Infiltration rate with time in an initially dry and in an initially moist soil.

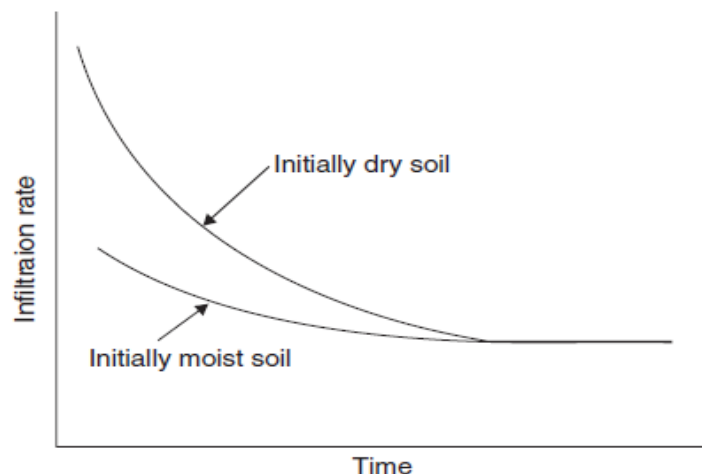


Fig. 14.6. Infiltrability as function of time in an initially dry and in an initially moist soil.

When infiltration takes place into an initially wet soil, the suction gradients are weak from the start and become negligible much sooner.

Models of Infiltration Capacity:

1- Green and Ampt (1911):

Earliest equation was proposed by Green and Ampt (1911) in which the infiltration capacity is:

$$i = i_c + \frac{b}{I}$$



where i and I are the infiltration rate and cumulative infiltration, respectively, i_c is analogous to the saturated hydraulic conductivity (note: at large times, $I \rightarrow \infty$ and $i = i_c$), and b is an empirical parameter. Note, time does not explicitly appear in the Green-Ampt equation. The model can be re-formulated so that:

$\frac{dI}{dt} = i_c + \frac{b}{I}$, and hence time becomes explicit. Naturally, by re-arranging the equation $\frac{dI}{i_c + \frac{b}{I}} = dt$, and integrating both sides:

$$t = \frac{I}{i_c} - \frac{b}{i_c^2} \log(i_c I + b)$$

2- Kostiakov (1932):

The next equation was proposed by Kostiakov, and is given by:

$$i = B t^{-n}$$

Where B and n are constants. The Kostiakov model is flawed at the two end-members:

$$\begin{aligned} t \rightarrow \infty, i &\rightarrow 0, \\ t \rightarrow 0, i &\rightarrow \infty. \end{aligned}$$

The cumulative infiltration is given as:

$$I = \frac{B}{1-n} t^{1-n}$$

3- Horton (1940):

The Horton model is a 3 parameter model derived to alleviate the problems in the asymptotic limits of the Kostiakov model.

$$i = i_c + (i_o - i_c) e^{-\beta t}$$

i_c = saturated hydraulic conductivity

i_o = initial infiltration rate (or i at $t=0$)

β = constant that varies with soil type and soil cover.

- 4- **Philip (1957):** Using an analytical solution to Richard's equation for soil moisture redistribution: $\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x} \left[K(\theta) \frac{\partial H}{\partial x} \right]$, where θ is the volumetric soil moisture, K is the hydraulic conductivity function, and H is the total energy, Philip (1957) demonstrated that:

$$i = K_s + \frac{1}{2} S_p t^{-1/2}$$

Where i is the infiltration capacity, K_s is the saturated hydraulic conductivity, and S_p is the sorption coefficient.

Note, for short times, $t^{-1/2}$ is very large and sorption controls the infiltration capacity



(i.e. $i \approx \frac{1}{2} \frac{S_p}{\sqrt{t}}$). For large times, $i \rightarrow K_{sat}$.

The cumulative infiltration capacity is given by:

$$I = K_s t + S_p t^{1/2}$$

Example:

The initial infiltration capacity of a watershed is estimated as 1.5 in/hr, and the time constant taken to be 0.35 hr⁻¹. The equilibrium capacity is estimated as 0.2 in/hr.

- What are the values of (f) at $t = 10$ min, 30 min, 1 hr, 2 hr, and 6 hr.
- what is the total volume of infiltration over the 6 hour time period?

From the Horton equation, we have:

$$i = i_c + (i_o - i_c)e^{-\beta t}$$

i_c = saturated hydraulic conductivity

i_o = initial infiltration rate (or i at $t=0$)

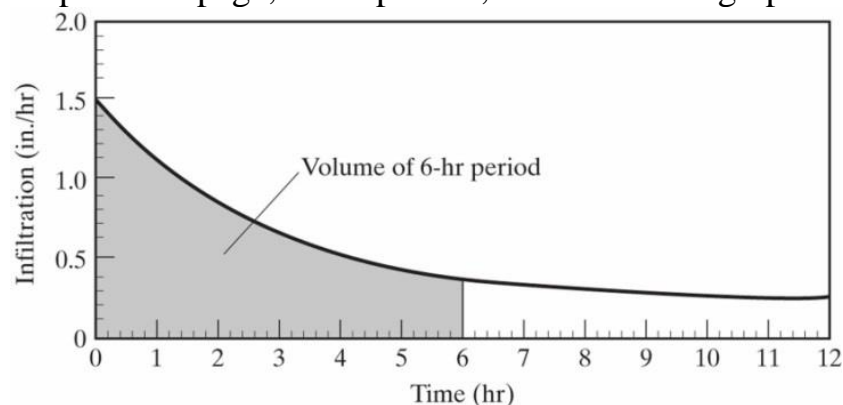
β = constant that varies with soil type and soil cover.

$$f = 0.2 + 1.3(e^{-0.35t})$$

- Substituting in values of t yields:

t	f
1/6	1.43
1/2	1.29
1	1.12
2	0.85
6	0.36

- The table on the previous page, when plotted, looks like the graph here.



The *volume* of water can be found by taking the definite integral (use font SymbolMT ⌋) under the curve from 0 to 6 hours. Here the integration is easy, and turns out like this:

$$\int_0^6 (0.2 + 1.3e^{-0.35t}) dt = \left[0.2t + \frac{1.3}{-0.35} e^{-0.35t} \right]_0^6$$

Evaluating the right side for $t = 6$ and then subtracting the values for $t=0$ yields an answer of 4.46 inches over the watershed.



EVAPORATION FROM BARE SOIL

EVAPORATION PROCESSES:

Evaporation is the primary process of water transfer in the hydrological cycle. The water is transformed into vapour and transported to the sky.

The evaporation plus transpiration from a vegetated surface with unlimited water supply is known as *potential evaporation* or *potential evapotranspiration* (PE) and it constitutes the maximum possible rate due to the prevailing meteorological conditions. Thus PE is the maximum value of the actual evaporation (E_t): $PE = E_t$ when water supply is unlimited.

Actual evaporation is the amount of water which is evaporated a normal day which means that if for instance the soil runs out of water, the actual evaporation is the amount of water which has been evaporated, and not the amount of water which could have been evaporated if the soil had had an infinite amount of water to evaporate.

Because of the variability of region and seasons, water managers who are responsible for planning and adjudicating the distribution of water resources need to have a thorough understanding of the evapotranspiration process and knowledge about the spatial and temporal rates of evapotranspiration.

PHYSICAL CONDITIONS

Three conditions are necessary for evaporation to occur and persist. First, there must be a continual supply of heat to meet the latent heat requirement (about 2.5×10^6 J/kg, or 590 cal/g of water evaporated at 15°C). This heat can come from the body itself, thus causing it to cool, or — as is more common — it can come from the outside in the form of radiated or advected energy. Second, the vapor pressure in the atmosphere over the evaporating body must remain lower than the vapor pressure at the surface of that body (i.e., there must be a vapor pressure gradient between the body and the atmosphere), and the vapor must be transported away by diffusion or convection or both. These two conditions — supply of energy and removal of vapor — are generally external to the evaporating body and are influenced by meteorological factors, such as air temperature, humidity, wind velocity, and radiation, which together determine the *atmospheric evaporativity* (the maximal flux at which the atmosphere can vaporize water from a free-water surface).

The third condition for evaporation to be sustained is that there be a continual supply of water from or through the interior of the body to the site of evaporation. This condition depends on the content and potential of water in the body as well as on its conductive properties, which together determine the maximal rate at which water can be transmitted to the evaporation site (usually, the surface). Accordingly, the actual evaporation rate is determined either by the *evaporativity* of the atmosphere or by the soil's own ability to deliver water (sometimes called the



evaporability of soil moisture), whichever is the lesser (and hence the limiting factor).

If the top layer of soil is initially quite wet, as it typically is at the end of an infiltration episode, the process of evaporation will generally reduce soil wetness and thus increase matric suction at the surface. This, in turn, will cause soil moisture to be drawn upward from the layers below, provided they are sufficiently moist.

Calculation of actual evaporation

There are a few formulas which are more common than others. In this chapter will we just write about the Penman-Monteith formula because it's the most known and most used formula when it's about calculating evapotranspiration. The origin of the equation is the Penman equation and later Monteith developed the formula even more.

Penman-Monteith

The most known formula for evapotranspiration is the Penman-Monteith formula,

$$E_T = \frac{\Delta R_n + (\rho (e_a - e_d) \frac{c_p}{r_a})}{\lambda (\Delta + \gamma (1 + \frac{r_s}{r_a}))}$$

where R_n = net radiation (W/m^2)
 ρ = density of air
 c_p = specific heat of air
 r_s = net resistance to diffusion through the surfaces of the leaves and soil (s/m)

r_a = net resistance to diffusion through the air from surfaces to height of measuring instruments (s/m).

γ = hygrometric constant

$\Delta = de/dT$

e_a = saturated vapour pressure at air temperature

e_d = mean vapour pressure

Estimating actual evapotranspiration from potential

The calculation of potential evaporation (PE) from readily available meteorological data is seen to be much simpler operation than the computation or measurement of actual evapotranspiration (Et) from a vegetated surface. However, water loss from a catchment area does not always proceed at the potential rate, since this is dependent on a continuous water supply. When the vegetation is unable to abstract



water from the soil, then the actual evaporation becomes less than potential. Thus the relationship between E_t and PE depends upon the soil moisture content.

- Here is an example of the relationship between PE and E_t , according to Bergström, 1992:

$$\begin{aligned} E_t &= PE && \text{when } h \geq h_{FC} \\ E_t &= PE \left[\frac{h - h_{WP}}{h_{FC} - h_{WP}} \right] && \text{when } h_{WP} < h < h_{FC} \\ E_t &= 0 && \text{when } h \leq h_{WP} \end{aligned}$$

Where:

h is the amount of soil moisture (mm).

h_{FC} is the amount of soil moisture corresponding to field capacity (mm).

h_{WP} is the amount of soil moisture corresponding to the wilting point (mm).

Field Capacity, Wilting Point and Available Water in a Soil Column

Field Capacity (FC or θ_{fc})—Soil water content where gravity drainage becomes negligible—Soil is not saturated but still a very wet condition.

The field capacity is a function of soil texture and soil profile properties.

For sands, $\theta_{fc} \approx (0.1)$ while for clays $\theta_{fc} \approx (0.3)$.

Permanent Wilting Point (WP or θ_w)—Soil water content beyond which plants cannot recover from water stress (dead)—Still some water in the soil but not enough to be of use to plants

The wilting point soil is a function of soil texture, soil profile properties and vegetation characteristics. For sands, $\theta_w \approx 0.05$ while for clays $\theta_w \approx 0.25$

Available water for plant use

$$\theta_a = \theta_{fc} - \theta_w$$