UNIVERSITY OF BAGHDAD
COLLEGE OF ENGINEERING
CIVIL ENGINEERING DEPARTMENT

DESIGN OF STEEL STRUCTURES

ACCORDING TO AISC (13TH EDITION)

JUNIOR COURSE 2018-2019

Instructor

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SYLLABUS

1. INTRODUCTION
2. SPECIFICATIONS, LOADS, AND METHODS OF DESIGN
3. ANALYSIS OF TENSION MEMBERS
4. DESIGN OF TENSION MEMBERS
5. INTRODUCTION TO AXIALLY LOADED COMPRESSION MEMBERS
6. DESIGN OF AXIALLY LOADED COMPRESSION MEMBERS AND COLUMN BASE PLATES
7. ANALYSIS AND DESIGN OF BEAMS
8. BENDING AND AXIAL FORCE (BEAM-COLUMNS)
9. COVER-PLATED BEAMS AND BUILT-UP GIRDERS (PLATE GIRDERS)
10. BOLTED CONNECTIONS
11. WELDED CONNECTIONS
12. BUILDING CONNECTIONS
13. ECCENTRICALLY LOADED BOLTED AND WELDED CONNECTIONS
14. DESIGN OF STEEL BUILDINGS

RECOMMENDED TEXTBOOK

McCormac, J. C. & Csernak, S. F. (2012). Structural Steel Design. Pearson Prentice Hall.

With

American Institute of Steel Construction. (2005). 13th Edition, Steel construction Manual. American Institute of Steel Construction.

REFERENCES

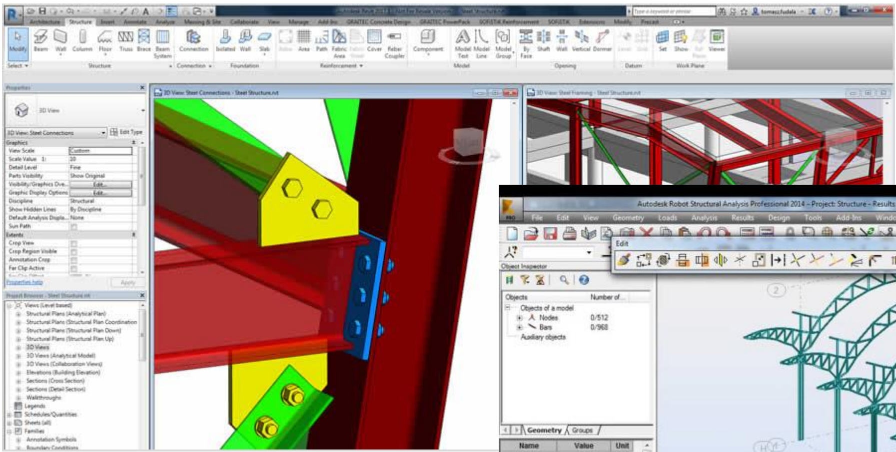
Salmon, G. & Johnson, J. E. (2008). Steel Structures: Design and Behavior, 4th. Edition. Editorial Harper Collins.

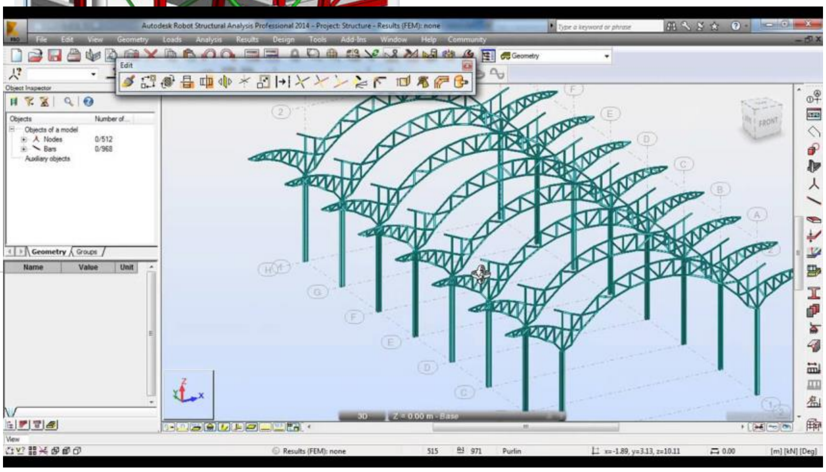
Segui, W. T. (2012). Steel Design. Cengage Learning.

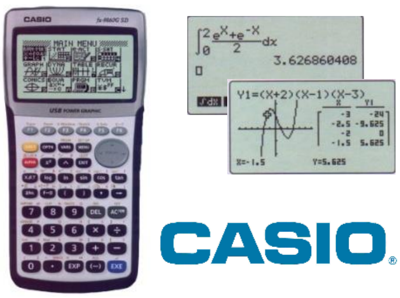
Subramanian, N. (2011). Steel Structures-Design and Practice. Oxford University Press.

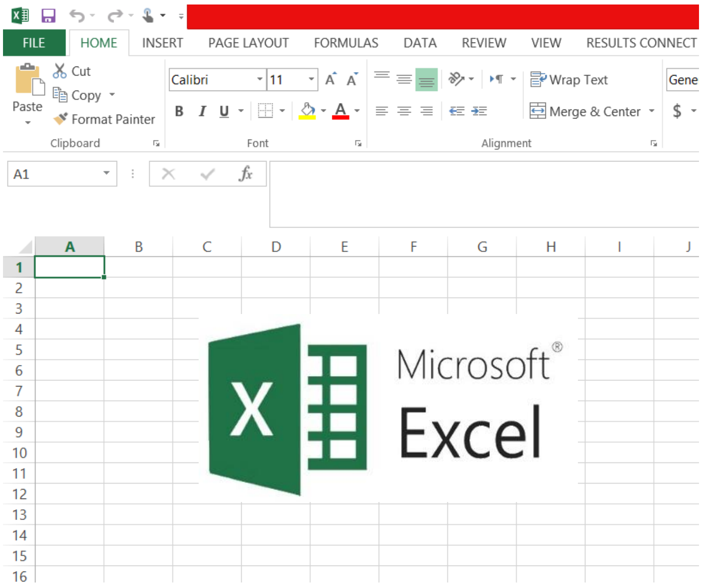
SOFTWARE AND TOOLS

1. **REVIT®-ROBOT®** environment is the powerful modeling, analysis and design tool. It is capable of analyzing any structure exposed to static loading, a dynamic response, soil-structure interaction, wind, earthquake, and moving loads.
2. **MATLAB** and **MICROSOFT® EXCEL** are required to perform general algorithms for the modeling and analysis of structural systems.
3. Scientific Calculator is required for arithmetic manipulations.





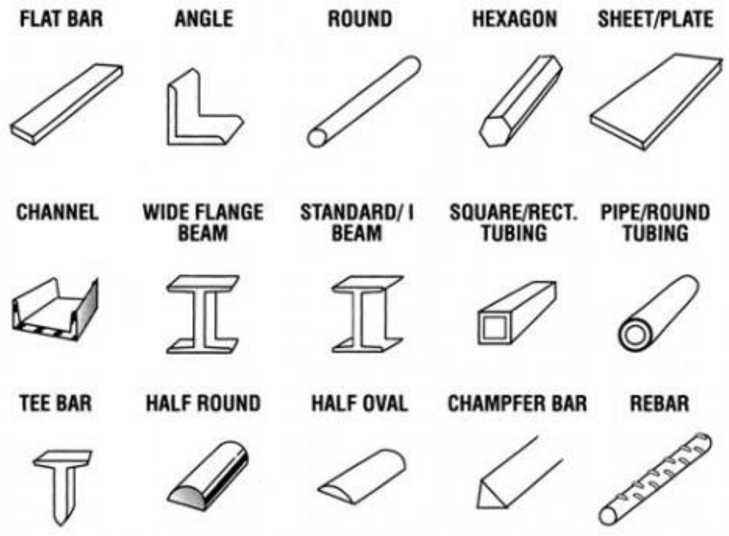




ANALYSIS OF TENSION MEMBERS 3

3.1 TYPES OF TENSION MEMBERS

Tension members are structural elements that are subjected to axial tensile forces. They are used in various types of structures and include truss members, bracing for buildings and bridges, cables in suspended roof systems, and cables in suspension and cable-stayed bridges. Any cross-sectional configuration may be used, because for any given material, the only determinant of the strength of a tension member is the cross-sectional area.



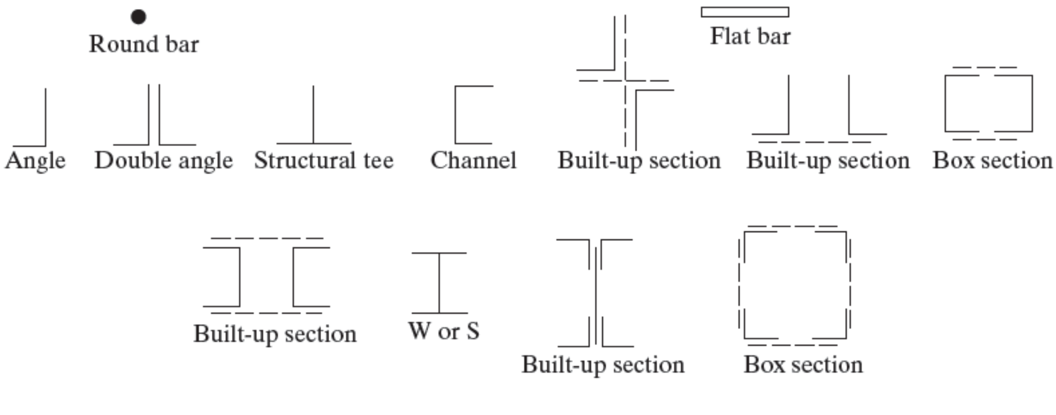


Figure 3-1: Types of Tension Members

3.2 NOMINAL STRENGTHS OF TENSION MEMBERS

The AISC Specification (**AISC D2, Page 26**) states that the nominal strength of a tension member, is to be the smaller of the values obtained by substituting into the following two expressions:

For the limit state of yielding in the gross section (which is intended to prevent excessive elongation of the member),

$$P_n = F_y A_g \qquad \text{(AISC Equation D2-1)}$$

$$\phi_t P_n = \phi_t F_y A_g = \text{design tensile strength by LRFD } (\phi_t = 0.9)$$

$$\frac{P_n}{\Omega_t} = \frac{F_y A_g}{\Omega_t} = \text{allowable tensile strength for ASD } (\Omega_t = 1.67)$$

For tensile rupture in the net section, as where bolt or rivet holes are present,

$$P_n = F_u A_e \qquad \text{(AISC Equation D2-2)}$$

$$\phi_t P_n = \phi_t F_u A_e = \text{design tensile rupture strength for LRFD } (\phi_t = 0.75)$$

$$\frac{P_n}{\Omega_t} = \frac{F_u A_e}{\Omega_t} \text{ allowable tensile rupture strength for ASD } (\Omega_t = 2.00)$$

3.3 NET AREA

- **AREA DETERMINATION,** AISC Chapter D, Page 27

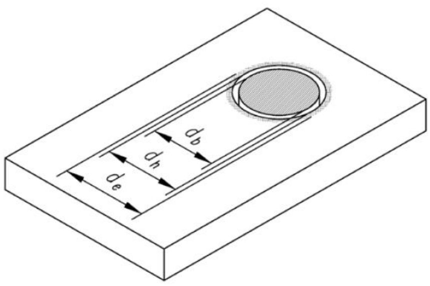
1. **Gross Area**, AISC Chapter D, Page 27

   The gross area, A_g, of a member is the total cross-sectional area.

2. **Net Area**, AISC Chapter D, Page 27

$$A_n = A_g - A_{Holes}$$

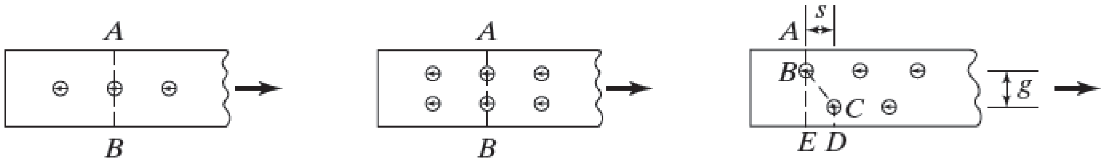
$$d_e = d_b + \frac{1}{8}"$$



(for Detailing $d_e = d_b + \frac{1}{16}$")

For a chain of holes extending across a part in any diagonal or zigzag line, the net width of the part shall be obtained by deducting from the gross width the sum of the diameters or slot dimensions as provided in Section J3.2, of all holes in the chain, and adding, for each *gage* space in the chain, the quantity $s^2/4g$

$$A_n = A_g - A_{Holes} + \sum_{i=1}^{N} \frac{S_i^2}{4g_i} t, \quad N: Number\ of\ zigzag\ lines$$



In determining the net area across plug or *slot welds*, the *weld metal* shall not be considered as adding to the net area.

User Note: Section J4.1(b) limits A_n to a maximum of $0.85A_g$ for splice plates with holes.

$$A_e = A_n \leq 0.85A_g$$

3.4 EFFECTIVE NET AREA

When a member other than a flat plate or bar is loaded in axial tension until failure occurs across its net section, its actual tensile failure stress will probably be less than the coupon tensile strength of the steel, unless all of the various elements which make up the section are connected so that stress is transferred uniformly across the section.

According to AISC Chapter D, Page 28

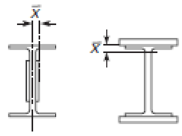
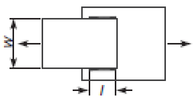
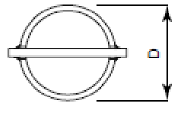
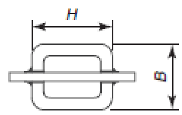
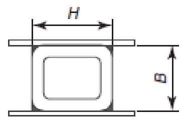
3. Effective Net Area

The effective area of tension members shall be determined as follows:

$$A_e = A_n U \quad \text{(D3-1)}$$

where U, the shear lag factor, is determined as shown in Table D3.1.

TABLE D3.1
Shear Lag Factors for Connections to Tension Members

Case	Description of Element		Shear Lag Factor, U	Example
1	All tension members where the tension load is transmitted directly to each of cross-sectional elements by fasteners or welds. (except as in Cases 3, 4, 5 and 6)		$U = 1.0$	—
2	All tension members, except plates and HSS, where the tension load is transmitted to some but not all of the cross-sectional elements by fasteners or longitudinal welds (Alternatively, for W, M, S and HP, Case 7 may be used.)		$U = 1 - \bar{x}/l$	
3	All tension members where the tension load is transmitted by transverse welds to some but not all of the cross-sectional elements.		$U = 1.0$ and A_n = area of the directly connected elements	—
4	Plates where the tension load is transmitted by longitudinal welds only.		$l \geq 2w \ldots U = 1.0$ $2w > l \geq 1.5w \ldots U = 0.87$ $1.5w > l \geq w \ldots U = 0.75$	
5	Round HSS with a single concentric gusset plate		$l \geq 1.3D \ldots U = 1.0$ $D \leq l < 1.3D \ldots U = 1 - \bar{x}/l$ $\bar{x} = D/\pi$	
6	Rectangular HSS	with a single concentric gusset plate	$l \geq H \ldots U = 1 - \bar{x}/l$ $\bar{x} = \dfrac{B^2 + 2BH}{4(B+H)}$	
		with two side gusset plates	$l \geq H \ldots U = 1 - \bar{x}/l$ $\bar{x} = \dfrac{B^2}{4(B+H)}$	
7	W, M, S or HP Shapes or Tees cut from these shapes. (If U is calculated per Case 2, the larger value is permitted to be used)	with flange connected with 3 or more fasteners per line in direction of loading	$b_f \geq 2/3d \ldots U = 0.90$ $b_f < 2/3d \ldots U = 0.85$	—
		with web connected with 4 or more fasteners in the direction of loading	$U = 0.70$	—
8	Single angles (If U is calculated per Case 2, the larger value is permitted to be used)	with 4 or more fasteners per line in direction of loading	$U = 0.80$	—
		with 2 or 3 fasteners per line in the direction of loading	$U = 0.60$	—

l = length of connection, in. (mm); w = plate width, in. (mm); $\bar{x}$ = connection eccentricity, in. (mm); B = overall width of rectangular HSS member, measured 90 degrees to the plane of the connection, in. (mm); H = overall height of rectangular HSS member, measured in the plane of the connection, in. (mm)

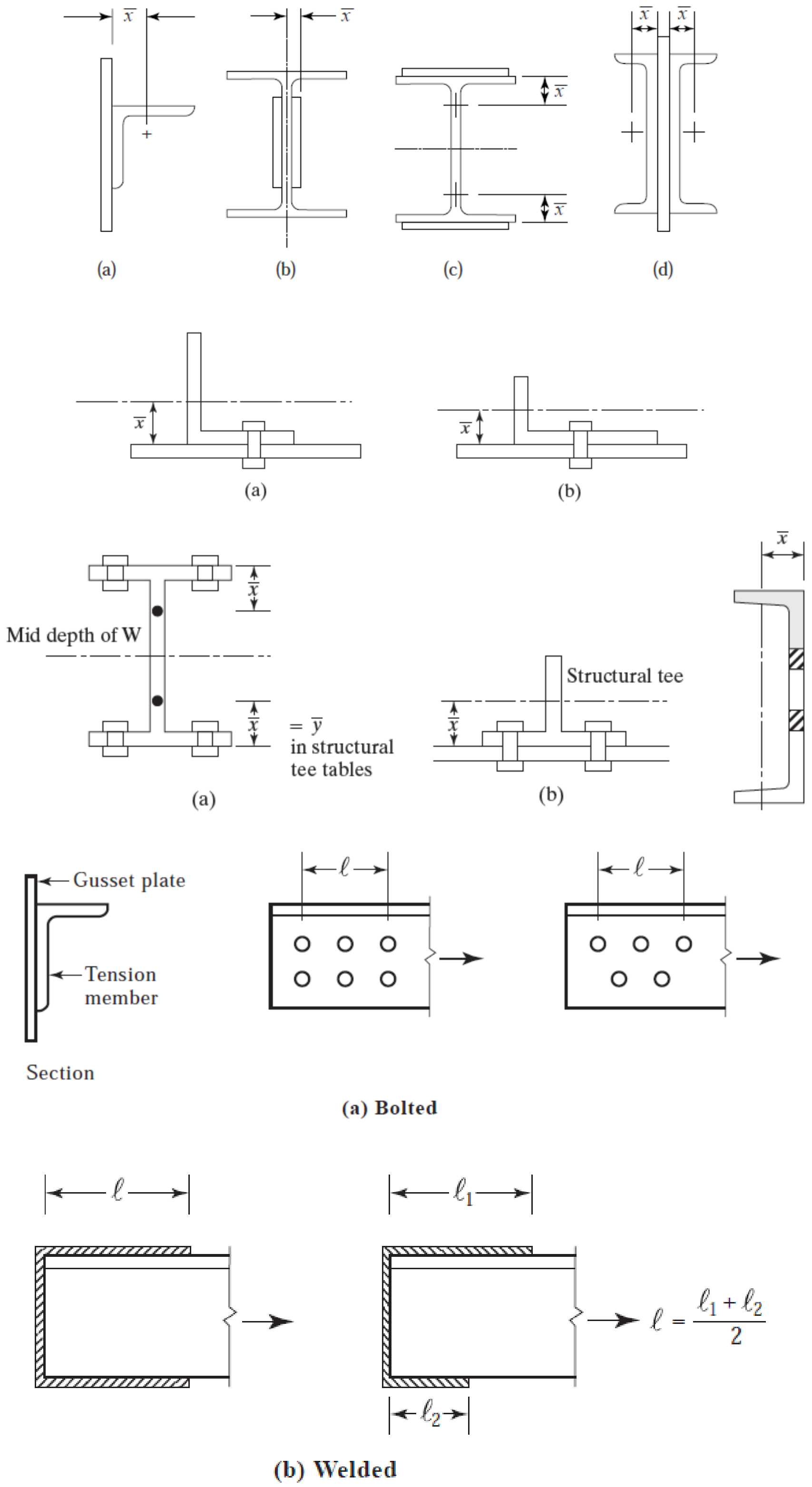


Figure 3-2: Connection eccentricity $\bar{x}$ for various cases

1. Bolted Members

Should a tension load be transmitted by bolts, the gross area is reduced to the net area A_n of the member, and U is computed as follows:

$$U = 1 - \frac{\bar{x}}{L}$$

2. Welded Members

When tension loads are transferred by welds, the rules from **AISC Table D-3.1, Page 29**, that are to be used to determine values for A and U (A_e as for bolted connections $= AU$) are as follows:

- Should the load be transmitted only by longitudinal welds to other than a plate member, or by longitudinal welds in combination with transverse welds, A is to equal the gross area of the member A_g (**Table 3.2, Case 2**).
- Should a tension load be transmitted only by transverse welds, A is to equal the area of the directly connected elements and U is to equal 1.0 (**Table 3.2, Case 3**).
- Tests have shown that when flat plates or bars connected by longitudinal fillet welds are used as tension members, they may fail prematurely by shear lag at the corners if the welds are too far apart. Therefore, the AISC Specification states that when such situations are encountered, the length of the welds may not be less than the width of the plates or bars. The letter A represents the area of the plate, and UA is the effective net area. For such situations, the values of U to be used (**Table 3.2, Case** 4) are as follows:

When $l \geq 2w$ $\quad U = 1.0$
When $2w > l \geq 1.5w$ $\quad U = 0.87$
When $1.5w > l \geq w$ $\quad U = 0.75$

Here, l = weld length, in
w = plate width (distance between welds), in

3.5 BLOCK SHEAR

The LRFD design strength and the ASD allowable strengths of tension members are not always controlled by tension yielding, tension rupture, or by the strength of the bolts or welds with which they are connected. They may instead be controlled by block shear strength, as described in this section. The failure of a member may occur along a path involving tension on one plane and shear on a perpendicular plane, as shown in Fig.3.16,where several possible block shear failures are illustrated. For these situations, it is possible for a "**BLOCK**" of steel to tear out.

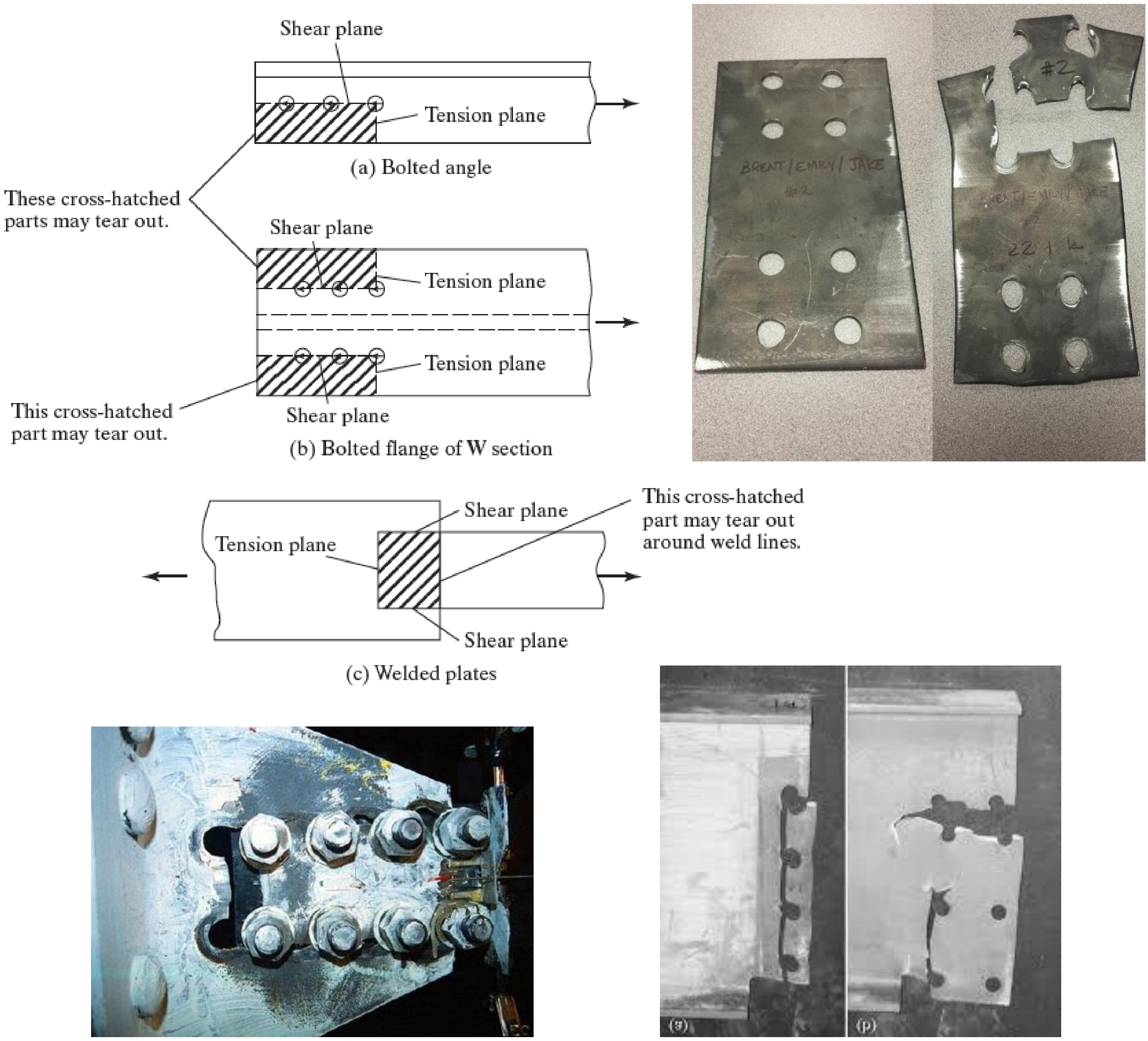


Figure 3-3: Block shear

SUMMARY: ANALYSIS OF TENSION MEMBERS

The Steel Construction Manual **AISC Chapter D, Page 26** limit states that will be considered are:

- **SLENDERNESS LIMITATIONS,** AISC Chapter D, Page 26

D1. SLENDERNESS LIMITATIONS

There is no maximum slenderness limit for design of members in tension.

User Note: For members designed on the basis of tension, the slenderness ratio L/r preferably should not exceed 300. This suggestion does not apply to rods or hangers in tension.

- **TENSILE STRENGTH,** AISC Chapter D, Page 26

D2. TENSILE STRENGTH

The *design tensile strength*, $\phi_t P_n$, and the *allowable tensile strength*, P_n/Ω_t, of tension members, shall be the lower value obtained according to the *limit states*

- **TENSILE YIELDING,** AISC Chapter D, Page 26

(a) For tensile yielding in the gross section:

$$P_n = F_y A_g \quad \text{(D2-1)}$$

$$\phi_t = 0.90 \text{ (LRFD)} \qquad \Omega_t = 1.67 \text{ (ASD)}$$

- **TENSILE RUPTURE,** AISC Chapter D, Page 27

(b) For tensile rupture in the net section:

$$P_n = F_u A_e \quad \text{(D2-2)}$$

$$\phi_t = 0.75 \text{ (LRFD)} \qquad \Omega_t = 2.00 \text{ (ASD)}$$

- **AREA DETERMINATION,** AISC Chapter D, Page 27

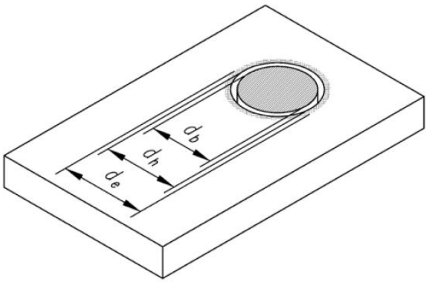
3. **Gross Area**, AISC Chapter D, Page 27

The gross area, A_g, of a member is the total cross-sectional area.

4. Net Area, AISC Chapter D, Page 27

$$A_n = A_g - A_{Holes}$$

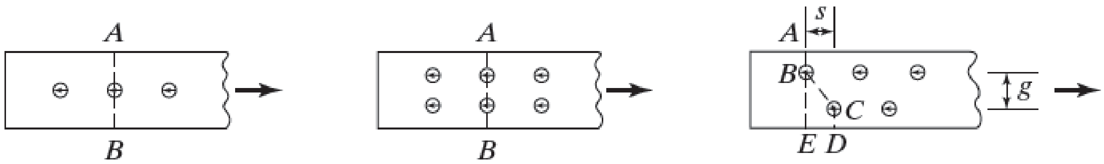
$$d_e = d_b + \frac{1}{8}"$$



Or

For a chain of holes extending across a part in any diagonal or zigzag line, the net width of the part shall be obtained by deducting from the gross width the sum of the diameters or slot dimensions as provided in Section J3.2, of all holes in the chain, and adding, for each *gage* space in the chain, the quantity $s^2/4g$

$$A_n = A_g - A_{Holes} + \sum_{i=1}^{N} \frac{S_i^2}{4g_i} t, \quad N: Number\ of\ zigzag\ lines$$



In determining the net area across plug or *slot welds*, the *weld metal* shall not be considered as adding to the net area.

User Note: Section J4.1(b) limits A_n to a maximum of $0.85A_g$ for splice plates with holes.

$$A_e = A_n \leq 0.85A_g$$

5. Effective Net Area, AISC Chapter D, Page 28

3. Effective Net Area

The effective area of tension members shall be determined as follows:

$$A_e = A_n U \quad \text{(D3-1)}$$

where U, the shear lag factor, is determined as shown in Table D3.1.

TABLE D3.1
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Case	Description of Element		Shear Lag Factor, U	Example
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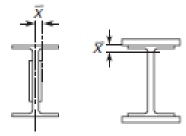
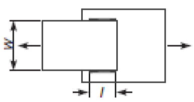
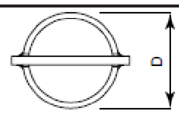
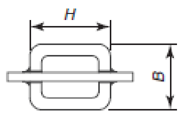
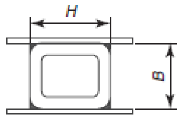
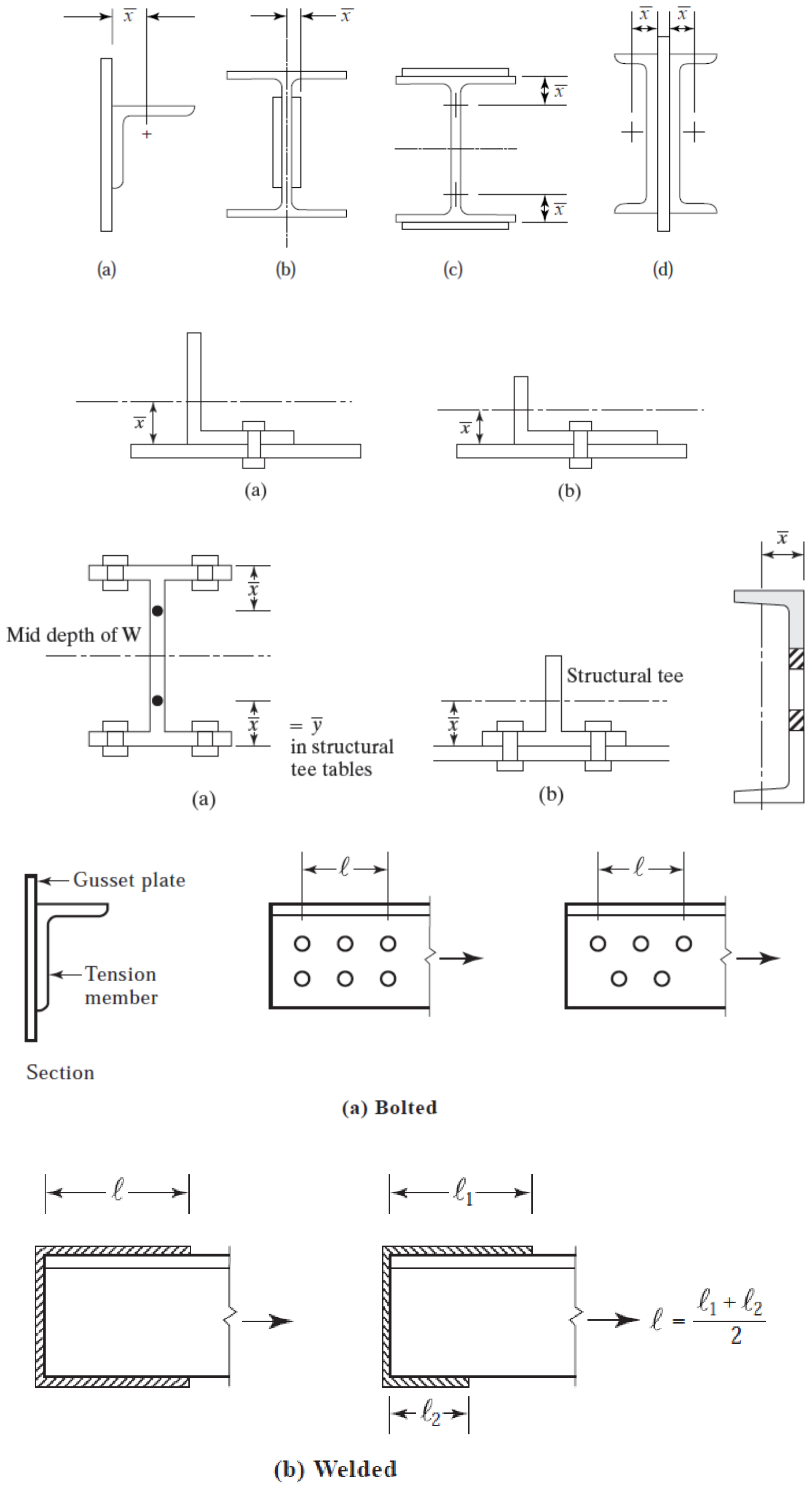
2	All tension members, except plates and HSS, where the tension load is transmitted to some but not all of the cross-sectional elements by fasteners or longitudinal welds (Alternatively, for W, M, S and HP, Case 7 may be used.)		$U = 1 - \bar{x}/l$	
3	All tension members where the tension load is transmitted by transverse welds to some but not all of the cross-sectional elements.		$U = 1.0$ and A_n = area of the directly connected elements	—
4	Plates where the tension load is transmitted by longitudinal welds only.		$l \geq 2w \ldots U = 1.0$ $2w > l \geq 1.5w \ldots U = 0.87$ $1.5w > l \geq w \ldots U = 0.75$	
5	Round HSS with a single concentric gusset plate		$l \geq 1.3D \ldots U = 1.0$ $D \leq l < 1.3D \ldots U = 1 - \bar{x}/l$ $\bar{x} = D/\pi$	
6	Rectangular HSS	with a single concentric gusset plate	$l \geq H \ldots U = 1 - \bar{x}/l$ $\bar{x} = \dfrac{B^2 + 2BH}{4(B+H)}$	
		with two side gusset plates	$l \geq H \ldots U = 1 - \bar{x}/l$ $\bar{x} = \dfrac{B^2}{4(B+H)}$	
7	W, M, S or HP Shapes or Tees cut from these shapes. (If U is calculated per Case 2, the larger value is permitted to be used)	with flange connected with 3 or more fasteners per line in direction of loading	$b_f \geq 2/3d \ldots U = 0.90$ $b_f < 2/3d \ldots U = 0.85$	—
		with web connected with 4 or more fasteners in the direction of loading	$U = 0.70$	—
8	Single angles (If U is calculated per Case 2, the larger value is permitted to be used)	with 4 or more fasteners per line in direction of loading	$U = 0.80$	—
		with 2 or 3 fasteners per line in the direction of loading	$U = 0.60$	—

l = length of connection, in. (mm); w = plate width, in. (mm); $\bar{x}$ = connection eccentricity, in. (mm); B = overall width of rectangular HSS member, measured 90 degrees to the plane of the connection, in. (mm); H = overall height of rectangular HSS member, measured in the plane of the connection, in. (mm)



Connection eccentricity $\bar{x}$ for various cases

- **BLOCK SHEAR STRENGTH,** AISC Chapter J, Page 112

3. Block Shear Strength

The *available strength* for the *limit state* of *block shear rupture* along a shear failure path or path(s) and a perpendicular tension failure path shall be taken as

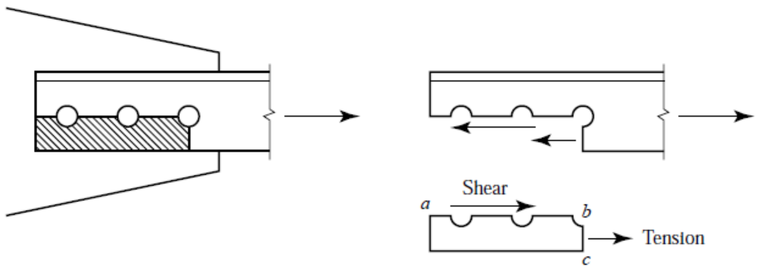
$$R_n = 0.6F_u A_{nv} + U_{bs} F_u A_{nt} \leq 0.6F_y A_{gv} + U_{bs} F_u A_{nt} \quad \text{(J4-5)}$$

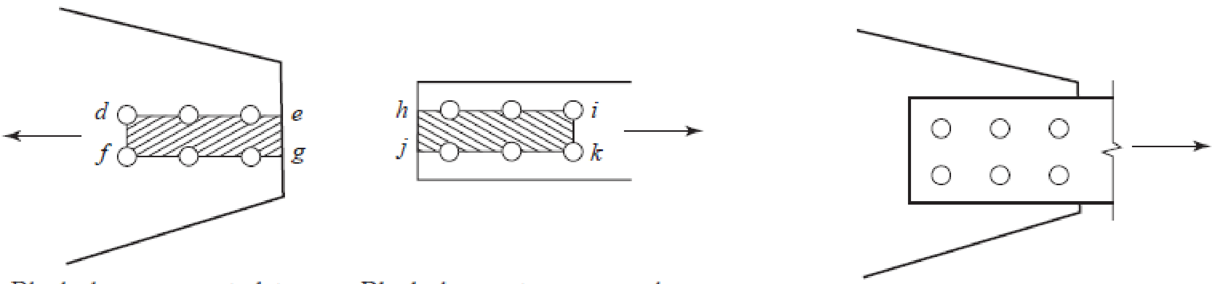
$$\phi = 0.75\,\text{(LRFD)} \qquad \Omega = 2.00\,\text{(ASD)}$$

where

A_{gv} = gross area subject to shear, in.^{2} (mm^2)
A_{nt} = *net area* subject to tension, in.^{2} (mm^2)
A_{nv} = net area subject to shear, in.^{2} (mm^2)

Where the tension *stress* is uniform, $U_{bs} = 1$; where the tension stress is non-uniform, $U_{bs} = 0.5$.

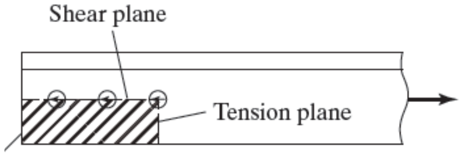




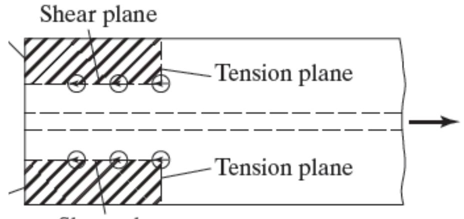
Block shear in gusset plate

Block shear in tension member

Plate tension member



(a) Bolted angle



(b) Bolted flange of W section

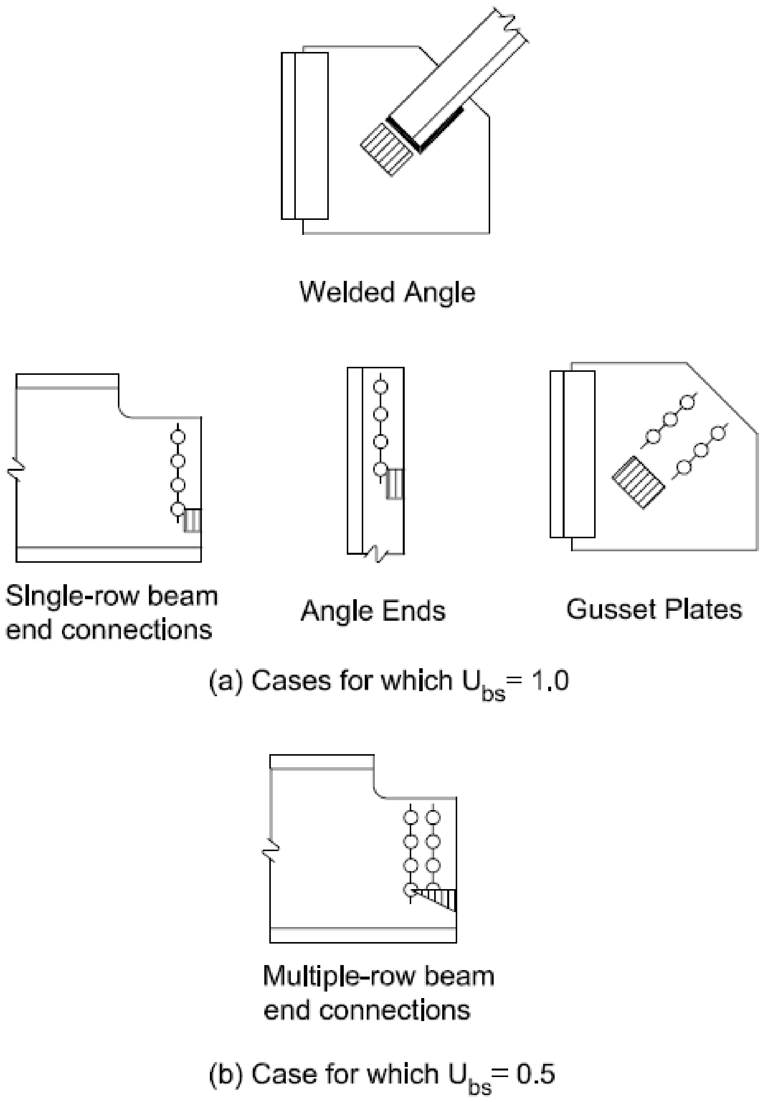
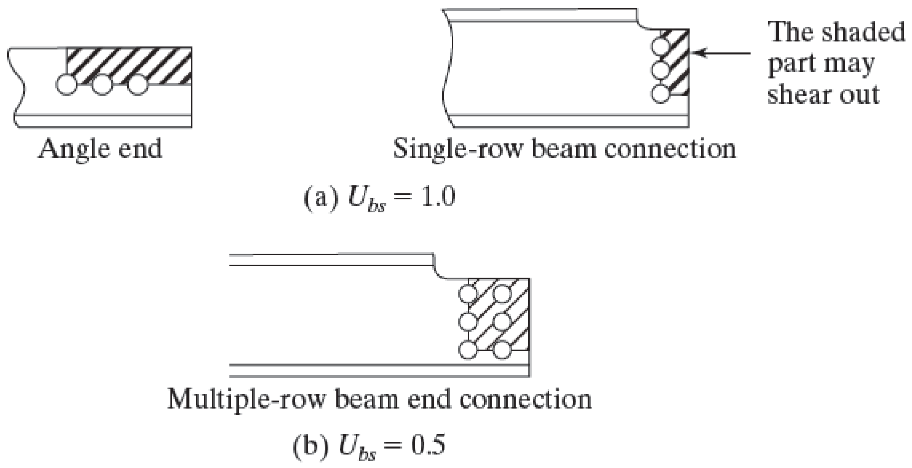


Fig. C-J4.2., **AISC Chapter Comm. J4, Page 352,** Block Shear Tensile Stress Distributions.



Example 3.1 Analysis of Tension Members

Determine the net area of the 3/8 × 8-in plate shown in Fig. 3.4 The plate is connected at its end with two lines of 3/4-in bolts.

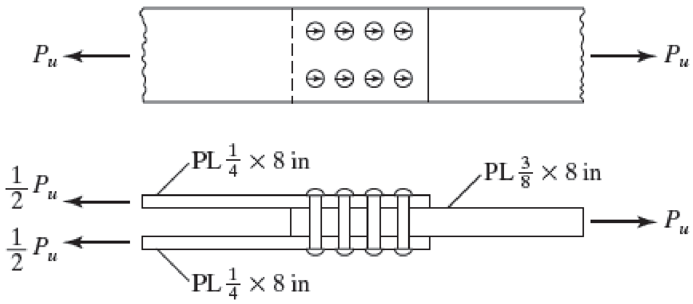


Figure 3-4

Solution

$$A_n = \left(\frac{3}{8}\text{ in}\right)(8\text{ in}) - 2\left(\frac{3}{4}\text{ in} + \frac{1}{8}\text{ in}\right)\left(\frac{3}{8}\text{ in}\right) = 2.34\text{ in}^2\ (1510\text{ mm}^2)$$

Ans.

Example 3.2 Analysis of Tension Members

Determine the critical net area of the 1/2-in-thick plate shown in Fig. 3.5, using the AISC Specification (Section B4.3b). The holes are punched for 3/4-in bolts.

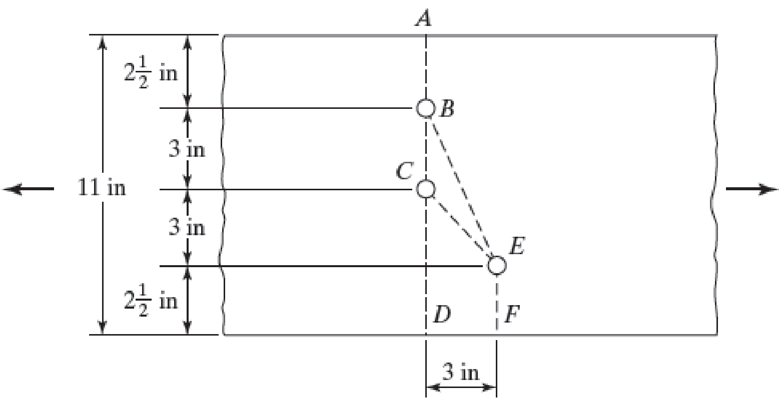


Figure 3-5

Solution

The critical section could possibly be *ABCD*, *ABCEF*, or *ABEF*. Hole diameters to be subtracted are 3/4 + 1/8 = 7/8 in. The net areas for each case are as follows:

$$ABCD = (11 \text{ in})\left(\frac{1}{2}\text{ in}\right) - 2\left(\frac{7}{8}\text{ in}\right)\left(\frac{1}{2}\text{ in}\right) = 4.63 \text{ in}^2$$

$$ABCEF = (11 \text{ in})\left(\frac{1}{2}\text{ in}\right) - 3\left(\frac{7}{8}\text{ in}\right)\left(\frac{1}{2}\text{ in}\right) + \frac{(3\text{ in})^2}{4(3\text{ in})}\left(\frac{1}{2}\text{ in}\right) = 4.56 \text{ in}^2 \longleftarrow$$

$$ABEF = (11 \text{ in})\left(\frac{1}{2}\text{ in}\right) - 2\left(\frac{7}{8}\text{ in}\right)\left(\frac{1}{2}\text{ in}\right) + \frac{(3\text{ in})^2}{4(6\text{ in})}\left(\frac{1}{2}\text{ in}\right) = 4.81 \text{ in}^2$$

Ans. 4.56 in^2

Example 3.3 — Analysis of Tension Members

For the two lines of bolt holes shown in Fig. 3.6, determine the pitch that will give a net area *DEFG* equal to the one along *ABC*. The problem may also be stated as follows: Determine the pitch that will give a net area equal to the gross area less one bolt hole. The holes are punched for 3/4-in bolts.

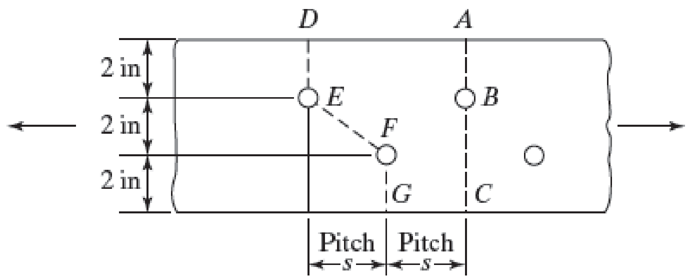


Figure 3-6

Solution

The hole diameters to be subtracted are 3/4 in + 1/8 in = 7/8 in.

$$ABC = 6\text{ in} - (1)\left(\frac{7}{8}\text{ in}\right) = 5.13\text{ in}$$

$$DEFG = 6\text{ in} - 2\left(\frac{7}{8}\text{ in}\right) + \frac{s^2}{4(2\text{ in})} = 4.25\text{ in} + \frac{s^2}{8\text{ in}}$$

$$ABC = DEFG$$

$$5.13 = 4.25 + \frac{s^2}{8} \qquad s = 2.65\text{ in}$$

Example 3.4 — Analysis of Tension Members

Determine the net area of the W12 × 16 (A_g = 4.71 in^2) shown in Fig. 3.7, assuming that the holes are for 1-in bolts.

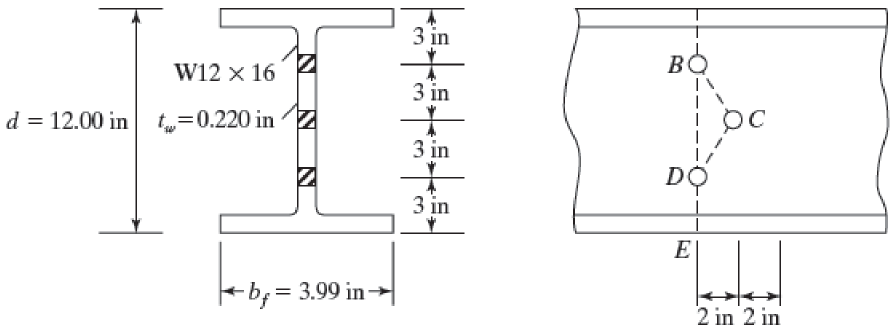


Figure 3-7

Solution

Net areas: hole ϕ is 1 in + $\frac{1}{8}$ in = $1\frac{1}{8}$ in

$$ABDE = 4.71\text{ in}^2 - 2\left(1\frac{1}{8}\text{ in}\right)(0.220\text{ in}) = 4.21\text{ in}^2$$

$$ABCDE = 4.72\text{ in}^2 - 3\left(1\frac{1}{8}\text{ in}\right)(0.220\text{ in}) + (2)\frac{(2\text{ in})^2}{4(3\text{ in})}(0.220\text{ in}) = 4.11\text{ in}^2 \leftarrow$$

Example 3.5 — Analysis of Tension Members

Determine the net area along route $ABCDEF$ for the C15 × 33.9 (A_g = 10.00 in^2) shown in Fig. 3.8. Holes are for $\frac{3}{4}$-in bolts.

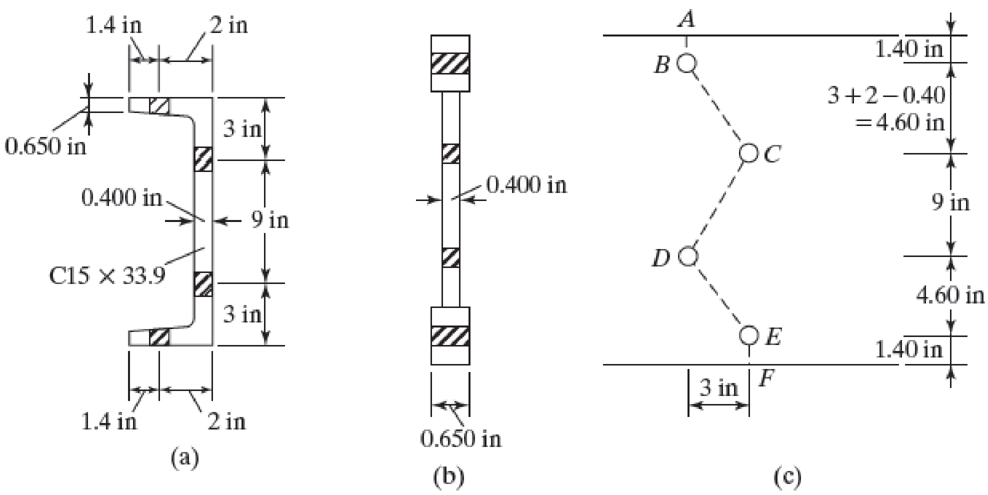


Figure 3-8

Solution

Approximate net A along

$$ABCDEF = 10.00\text{ in}^2 - 2\left(\frac{7}{8}\text{ in}\right)(0.650\text{ in}) - 2\left(\frac{7}{8}\text{ in}\right)(0.400\text{ in}) + \frac{(3\text{ in})^2}{4(9\text{ in})}(0.400\text{ in}) + (2)\frac{(3\text{ in})^2}{(4)(4.60\text{ in})}\left(\frac{0.650\text{ in} + 0.400\text{ in}}{2}\right)$$

$$= 8.78\text{ in}^2$$

Ans. 8.78 in^2

Example 3.6 Analysis of Tension Members

Determine the LRFD design tensile strength and the ASD allowable design tensile strength for a W10 × 45 with two lines of $\frac{3}{4}$-in diameter bolts in each flange using A572 Grade 50 steel, with $F_y = 50$ ksi and $F_u = 65$ ksi, and the AISC Specification. There are assumed to be at least three bolts in each line 4 in on center, and the bolts are not staggered with respect to each other.

Solution

Using a W10 × 45 ($A_g = 13.3\text{ in}^2$, $d = 10.10$ in, $b_f = 8.02$ in, $t_f = 0.620$ in)

Nominal or available tensile strength of section $P_n = F_y A_g = (50\text{ ksi})(13.3\text{ in}^2) = 665\text{ k}$

(a) Gross section yielding

LRFD with $\phi_t = 0.9$	ASD with $\Omega_t = 1.67$
$\phi_t P_n = (0.9)(665\text{ k}) = 598.5\text{ k}$	$\frac{P_n}{\Omega t} = \frac{665\text{ k}}{1.67} = 398.2\text{ k}$

(b) Tensile rupture strength

$$A_n = 13.3 \text{ in}^2 - (4)\left(\frac{3}{4}\text{ in } + \frac{1}{8}\text{ in}\right)(0.620 \text{ in}) = 11.13 \text{ in}^2$$

Referring to tables in Manual for one-half of a W10 × 45 (or, that is, a WT5 × 22.5), we find that

$$\bar{x} = 0.907 \text{ in } (\bar{y} \text{ from AISC Manual Table 1-8})$$

Length of connection, L = 2 (4 in) = 8 in

From Table **D3.1**(Case 2), $U = 1 - \frac{\bar{x}}{L} = 1 - \frac{0.907 \text{ in}}{8 \text{ in}} = 0.89$

But $b_f = 8.02 \text{ in} > \frac{2}{3}d = \left(\frac{2}{3}\right)(10.1) = 6.73 \text{ in}$

$\therefore$ U from Table 3.2 (Case 7) is 0.90 ←

$$A_e = UA_n = (0.90)(11.13 \text{ in}^2) = 10.02 \text{ in}^2$$
$$P_n = F_uA_e = (65 \text{ ksi})(10.02 \text{ in}^2) = 651.3 \text{ k}$$

LRFD with $\phi_t = 0.75$	ASD with $\Omega_t = 2.00$
$\phi_t P_n = (0.75)(651.3 \text{ k}) = 488.5 \text{ k}$ ←	$\frac{P_n}{\Omega_t} = \frac{651.3 \text{ k}}{2.00} = 325.6 \text{ k}$ ←

$LRFD = 488.5\ k$ (Rupture controls)

$ASD = 325.6\ k$ (Rupture controls)

Ans.

Example 3.7 — Analysis of Tension Members

Determine the LRFD design tensile strength and the ASD allowable tensile strength for an A36 (F_y = 36 ksi and F_u = 58 ksi) L6 × 6 × 3/8 in that is connected at its ends with one line of four 7/8-in-diameter bolts in standard holes 3 in on center in one leg of the angle.

Solution

Using an L6 × 6 × $\frac{3}{8}$ ($A_g = 4.38\text{ in}^2, \bar{y} = \bar{x} = 1.62\text{ in}$) nominal or available

tensile strength of the angle

$$P_n = F_y A_g = (36\text{ ksi})(4.38\text{ in}^2) = 157.7\text{ k}$$

(a) Gross section yielding

LRFD with $\phi_t = 0.9$	ASD with $\Omega_t = 1.67$
$\phi_t P_n = (0.9)(157.7\,k) = 141.9\text{ k} \leftarrow$	$\frac{P_n}{\Omega_t} = \frac{157.7\text{ k}}{1.67} = 94.4\text{ k} \leftarrow$

(b) Tensile rupture strength

$$A_n = 4.38\text{ in}^2 - (1)\left(\frac{7}{8}\text{in} + \frac{1}{8}\text{in}\right)\left(\frac{3}{8}\text{in}\right) = 4.00\text{ in}^2$$

$$\text{Length of connection, } L = (3)(3\text{ in}) = 9\text{ in}$$

$$U = 1 - \frac{\bar{x}}{L} = 1 - \frac{1.62\text{ in}}{9\text{ in}} = 0.82$$

From Table D3.1, Case 8, for 4 or more fasteners in the direction of loading, $U = 0.80$. Use calculated $U = 0.82$.

$$A_e = A_n U = (4.00\text{ in}^2)(0.82) = 3.28\text{ in}^2 \qquad P_n = F_u A_e = (58\,ksi)(3.28\,in^2) = 190.2\,k$$

LRFD with $\phi_t = 0.75$	ASD with $\Omega_t = 2.00$
$\phi_t P_n = (0.75)(190.2\text{ k}) = 142.6\text{ k}$	$\frac{P_n}{\Omega_t} = \frac{190.2\text{ k}}{2.00} = 95.1\text{ k}$

Ans. LRFD = 141.9 k (Yielding controls) ASD = 94.4 k (Yielding controls)

Example 3.8 — Analysis of Tension Members

The $1 \times 6\ in$ plate shown in the figure is connected to a $1 \times 10\ in$ plate with longitudinal fillet welds to transfer a tensile load. Determine the LRFD design tensile.

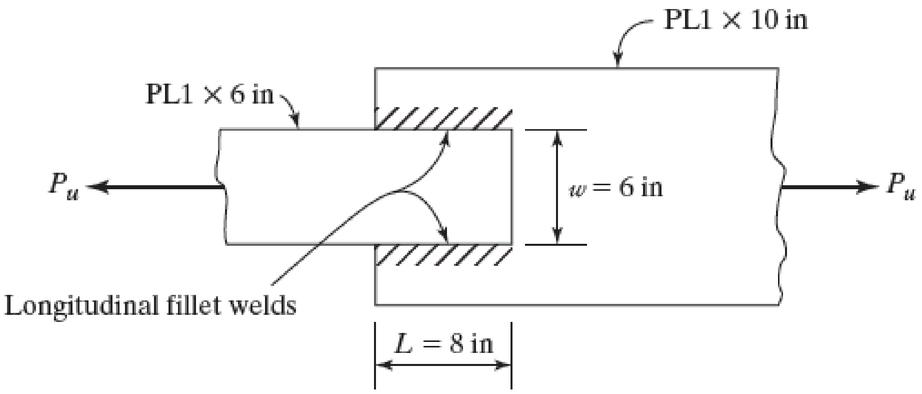


Figure 3-9

Solution

Considering the nominal or available tensile strength of the smaller PL $1 \times 6\ in$

$$P_n = F_y A_g = (50 \text{ ksi})(1 \text{ in} \times 6 \text{ in}) = 300 \text{ k}$$

(a) Gross section yielding

LRFD with $\phi_t = 0.9$	ASD with $\Omega_t = 1.67$
$\phi_t P_n = (0.9)(300 \text{ k}) = 270 \text{ k}$	$\frac{P_n}{\Omega_t} = \frac{300 \text{ k}}{1.67} = 179.6 \text{ k}$

(b) Tensile rupture strength

$$1.5w = 1.5 \times 6 \text{ in} = 9 \text{ in} > L = 8 \text{ in} > w = 6 \text{ in}$$

$\therefore U = 0.75$ from Table **D3.1** Case 4

$$A_e = A_n U = (6.0 \text{ in}^2)(0.75) = 4.50 \text{ in}^2$$

$$P_n = F_u A_e = (65 \text{ ksi})(4.50 \text{ in}^2) = 292.5 \text{ k}$$

LRFD with $\phi_t = 0.75$	ASD with $\Omega_t = 2.00$
$\phi_t P_n = (0.75)(292.5 \text{ k}) = 219.4 \text{ k} \leftarrow$	$\frac{P_n}{\Omega_t} = \frac{292.5 \text{ k}}{2.00} = 146.2 \text{ k} \leftarrow$

Ans. LRFD = 219.4 k (Rupture controls) ASD = 146.2 k (Rupture controls)

Example 3.9 — Analysis of Tension Members

Compute the LRFD design tensile strength and the ASD allowable tensile strength of the angle shown in in the figure. It is welded on the end (transverse) and sides (longitudinal) of the 8 *in* leg only. $F_y = 50\ ksi$ and $F_u = 70\ ksi$.

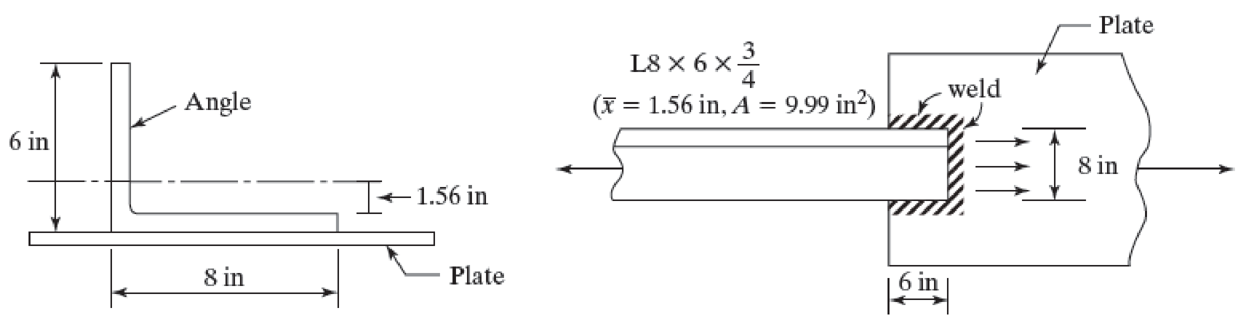


Figure 3-10

Solution

Nominal or available tensile strength of the angle $= P_n = F_y A_g = (50\ \text{ksi})(9.99\ \text{in}^2) = 499.5\ \text{k}$

(a) Gross section yielding

LRFD with $\phi_t = 0.9$	ASD with $\Omega_t = 1.67$
$\phi_t P_n = (0.9)(499.5\ \text{k}) = 449.5\ \text{k}$	$\frac{P_n}{\Omega_t} = \frac{499.5\ \text{k}}{1.67} = 299.1\ \text{k}$

(b) Tensile rupture strength (As only one leg of L is connected, a reduced effective area needs to be computed.) Use Table D3.1 (Case 2)

$$U = 1 - \frac{\bar{x}}{L} = 1 - \frac{1.56\ \text{in}}{6\ \text{in}} = 0.74$$

$$A_e = A_g U = (9.99\ \text{in}^2)(0.74) = 7.39\ \text{in}^2$$

$$P_n = F_u A_e = (70\ \text{ksi})(7.39\ \text{in}^2) = 517.3\ \text{k}$$

LRFD with $\phi_t = 0.75$	ASD with $\Omega_t = 2.00$
$\phi_t P_n = (0.75)(517.3\ \text{k}) = 388.0\ \text{k} \leftarrow$	$P_n = \frac{517.3\ \text{k}}{2.00} = 258.6\ \text{k} \leftarrow$

Ans. LRFD = 388.0 k (Rupture controls) ASD = 258.6 k (Rupture controls)

Example 3.10 — Analysis of Tension Members

A tension member of a $W10 \times 45$ with two lines of $3/4\ in$ diameter bolts in each flange using A572 Grade 50 steel, and the AISC Specification. There are assumed to be at least three bolts in each line 4 in on center, and the bolts are not staggered with respect to each other. It is assumed to be connected at its ends with two $3/8\ \times 12\ in$ plates, as shown in the figure. If two lines of $3/4\ in$ bolts are used in each plate, determine the LRFD design tensile force and the ASD allowable tensile force that the two plates can transfer.

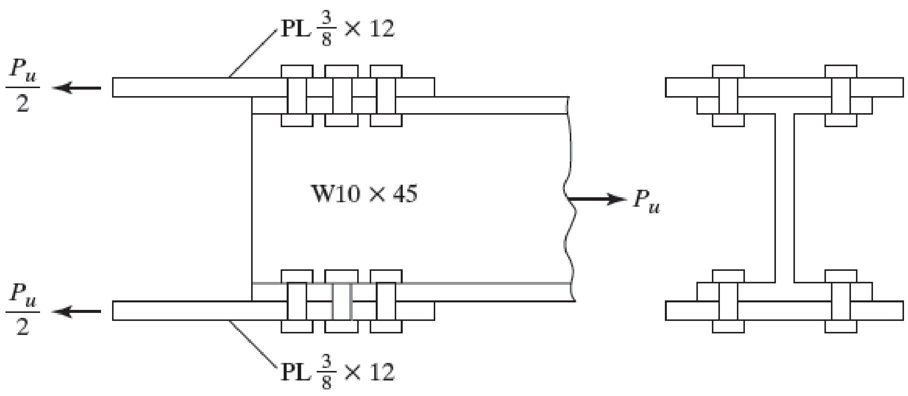


Figure 3-11

Solution

Nominal strength of plates

$$R_n = F_yA_g = (50\text{ ksi})\left(2 \times \frac{3}{8}\text{ in} \times 12\text{ in}\right) = 450\text{ k}$$

(a) Tensile yielding of connecting elements

LRFD with $\phi = 0.90$	ASD with $\Omega = 1.67$
$\phi R_n = (0.90)(450\text{ k}) = 405\text{ k}$	$\frac{R_n}{\Omega} = \frac{450\text{ k}}{1.67} = 269.5\text{ k}$

(b) Tensile rupture of connecting elements

$$A_n \text{ of 2 plates} = 2\left[\left(\frac{3}{8}\text{ in} \times 12\text{ in}\right) - 2\left(\frac{3}{4}\text{ in} + \frac{1}{8}\text{ in}\right)\left(\frac{3}{8}\text{ in}\right)\right] = 7.69\text{ in}^2$$

$$0.85A_g = (0.85)\left(2 \times \frac{3}{8}\text{ in} \times 12\text{ in}\right) = 7.65\text{ in}^2 \leftarrow$$

$$R_n = F_uA_e = (65\text{ ksi})(7.65\text{ in}^2) = 497.2\text{ k}$$

LRFD with $\phi = 0.75$	ASD with $\Omega = 2.00$
$\phi R_n = (0.75)(497.2\text{ k}) = 372.9\text{ k} \leftarrow$	$\frac{R_n}{\Omega} = \frac{497.2\text{ k}}{2.00} = 248.6\text{ k} \leftarrow$

Ans. LRFD = 372.9 k (Rupture controls) ASD = 248.6 k (Rupture controls)

Example 3.11 Analysis of Tension Members, Block Shear

The A572 Grade 50 tension member shown in **Error! Reference source not found.** is connected with three $3/4\ in$ bolts. Determine the LRFD block shear rupture strength and the ASD allowable block-shear rupture strength of the member. Also calculate the LRFD design tensile strength and the ASD allowable tensile strength of the member.

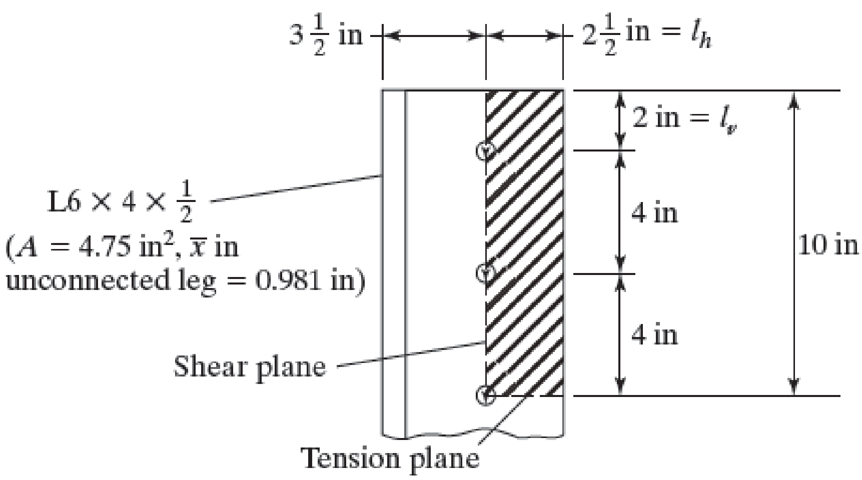


Figure 3-12

Solution

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt} \le 0.6F_yA_{gv} + U_{bs}F_uA_{nt} \quad \text{(J4-5)}$$

$$\phi = 0.75\,(\text{LRFD}) \qquad \Omega = 2.00\,(\text{ASD})$$

where

A_{gv} = gross area subject to shear, in.^{2} (mm^2)
A_{nt} = *net area* subject to tension, in.^{2} (mm^2)
A_{nv} = net area subject to shear, in.^{2} (mm^2)

$$A_{gv} = (10\text{ in})\left(\frac{1}{2}\text{in}\right) = 5.0\text{ in}^2$$

$$A_{nv} = \left[10\text{ in} - (2.5)\left(\frac{3}{4}\text{in} + \frac{1}{8}\text{in}\right)\right]\left(\frac{1}{2}\text{in}\right) = 3.91\text{ in}^2$$

$$A_{nt} = \left[2.5\text{ in} - \left(\frac{1}{2}\right)\left(\frac{3}{4}\text{in} + \frac{1}{8}\text{in}\right)\right]\left(\frac{1}{2}\text{in}\right) = 1.03\text{ in}^2$$

$$U_{bs} = 1.0$$

$$R_n = (0.6)(65\text{ ksi})(3.91\text{ in}^2) + (1.0)(65\text{ ksi})(1.03\text{ in}^2) = 219.44\text{ k}$$

$$\le (0.6)(50\text{ ksi})(5.0\text{ in}^2) + (1.0)(65\text{ ksi})(1.03\text{ in}^2) = 216.95\text{ k}$$

$$219.44\text{ k} > 216.95\text{ k}$$

$$\therefore\ R_n = 216.95\text{ k}$$

(a) Block shear strength

LRFD with $\phi = 0.75$	ASD with $\Omega = 2.00$
$\phi R_n = (0.75)(216.95\text{ k}) = 162.7\text{ k} \leftarrow$	$\frac{R_n}{\Omega} = \frac{216.95\text{ k}}{2.00} = 108.5\text{ k} \leftarrow$

(b) Nominal or available tensile strength of angle

$$P_n = F_yA_g = (50\text{ ksi})(4.75\text{ in}^2) = 237.5\text{ k}$$

Gross section yielding

LRFD with $\phi_t = 0.9$	ASD with $\Omega_t = 1.67$
$\phi_tP_n = (0.9)(237.5\text{ k}) = 213.7\text{ k}$	$\frac{P_n}{\Omega_t} = \frac{237.5\text{ k}}{1.67} = 142.2\text{ k}$

(c) Tensile rupture strength

$$A_n = 4.75 \text{ in}^2 - \left(\frac{3}{4}\text{in} + \frac{1}{8}\text{in}\right)\left(\frac{1}{2}\text{in}\right) = 4.31 \text{ in}^2$$

$$L \text{ for bolts} = (2)(4 \text{ in}) = 8 \text{ in}$$

$$U = 1 - \frac{\bar{x}}{L} = 1 - \frac{0.981 \text{ in}}{8 \text{ in}} = 0.88$$

$$A_e = UA_n = (0.88)(4.31 \text{ in}^2) = 3.79 \text{ in}^2$$

$$P_n = F_u A_e = (65 \text{ ksi})(3.79 \text{ in}^2) = 246.4 \text{ k}$$

LRFD with $\phi_t = 0.75$	ASD with $\Omega_t = 2.00$
$\phi_t P_n = (0.75)(246.4 \text{ k}) = 184.8 \text{ k}$	$\frac{P_n}{\Omega_t} = \frac{246.4 \text{ k}}{2.00} = 123.2 \text{ k}$

Ans. LRFD = 162.7 k (Block shear controls) ASD = 108.5 k (Block shear controls)

Example 3.12 Analysis of Tension Members

Determine the LRFD design strength and the ASD allowable strength of the A36 plates shown in Figure 3-13. Include block shear strength in the calculations.

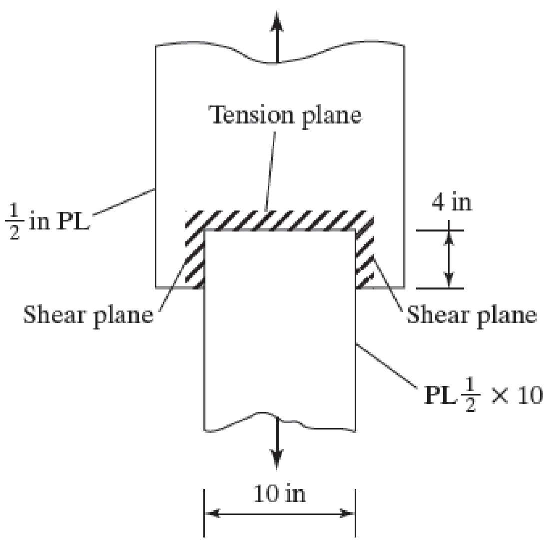


Figure 3-13

Solution

(a) Gross section yielding

$$P_n = F_y A_g = (36\ ksi)\left(\frac{1}{2}\ in\ \times 10\ in\right) = 180\ k$$

LRFD with $\phi_t = 0.9$	ASD with $\Omega_t = 1.67$
$\phi_t P_n = (0.9)(180\text{ k}) = 162\text{ k} \leftarrow$	$\frac{P_n}{\Omega_t} = \frac{180\text{ k}}{1.67} = 107.8\text{ k} \leftarrow$

(b) Tensile rupture strength

$$U = 1.0\ (\text{Table } \mathbf{D3.1}\ \text{Case } 1)$$

$$A_e = (1.0)\left(\frac{1}{2}\text{in} \times 10\text{ in}\right) = 5.0\text{ in}^2$$

$$P_n = F_u A_e = (58\text{ ksi})(5.0\text{ in}^2) = 290\text{ k}$$

LRFD with $\phi_t = 0.75$	ASD with $\Omega_t = 2.00$
$\phi_t P_n = (0.75)(290\text{ k}) = 217.5\text{ k}$	$\frac{P_n}{\Omega_t} = \frac{290\text{ k}}{2.00} = 145\text{ k}$

(c) Block shear strength

$$A_{gv} = \left(\frac{1}{2}\text{ in}\right)(2 \times 4\text{ in}) = 4.00\text{ in}^2$$

$$A_{nv} = 4.00\text{ in}^2$$

$$A_{nt} = \left(\frac{1}{2}\text{ in}\right)(10\text{ in}) = 5.0\text{ in}^2$$

$$U_{bs} = 1.0$$

$$R_n = (0.6)(58\text{ ksi})(4.0\text{ in}^2) + (1.00)(58\text{ ksi})(5.0\text{ in}^2) = 429.2\text{ k}$$

$$\leq (0.6)(36\text{ ksi})(4.0\text{ in}^2) + (1.00)(58\text{ ksi})(5.0\text{ in}^2) = 376.4\text{ k}$$

$$429.2\text{ k} > 376.4\text{ k}$$

$$\therefore\ R_n = 376.4\text{ k}$$

LRFD with $\phi = 0.75$	ASD with $\Omega = 2.00$
$\phi R_n = (0.75)(376.4\text{ k}) = 282.3\text{ k}$	$\dfrac{R_n}{\Omega} = \dfrac{376.4\text{ k}}{2.00} = 188.2\text{ k}$

Ans. LRFD = 162 k (Yielding controls) ASD = 107.8 k (Yielding controls)

Example 3.13 Analysis of Tension Members

Determine the LRFD tensile design strength and the ASD tensile strength of the $W12 \times 30$ ($F_y = 50\ ksi, F_u = 65\ ksi$) shown in Figure 3-14 if $3 \times 7/8\ in$ bolts are used in each flange in the connection. Include block shear calculations for the flanges.

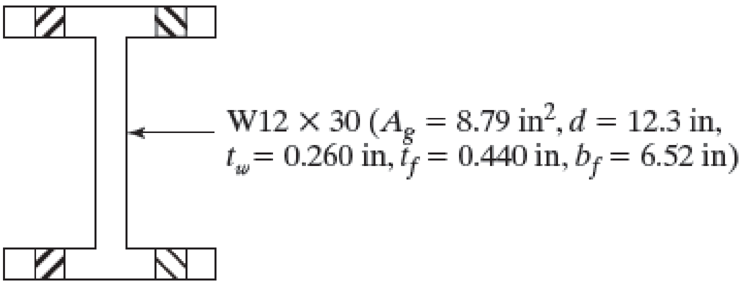


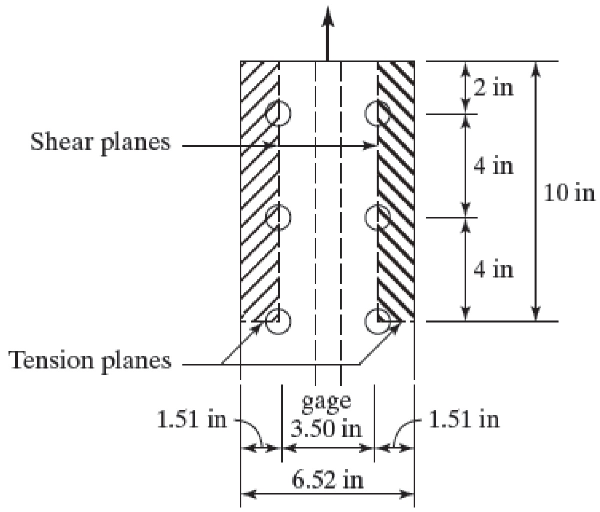
Figure 3-14

Solution

(a) Gross section yielding

$$P_n = F_y A_g = (50)(8.79) = 439.5\text{ k}$$

LRFD with $\phi_t = 0.9$	ASD with $\Omega_t = 1.67$
$\phi_t P_n = (0.9)(439.5) = 395.5\text{ k}$	$\dfrac{P_n}{\Omega_t} = \dfrac{439.5}{1.67} = 263.2\text{ k}$



(b) Tensile rupture strength

$$A_n = 8.79 \text{ in}^2 - (4)\left(\frac{7}{8}\text{ in} + \frac{1}{8}\text{ in}\right)(0.440 \text{ in}) = 7.03 \text{ in}^2$$

$$\bar{x} = \bar{y} \text{ in table } = 1.27 \text{ in for } \text{WT6} \times 15$$

$$U = 1 - \frac{\bar{x}}{L} = 1 - \frac{1.27 \text{ in}}{2 \times 4 \text{ in}} = 0.84$$

$$b_f = 6.52 \text{ in} < \frac{2}{3} \times 12.3 = 8.20 \text{ in}$$

$\therefore$ Use $U = 0.85$ for Case 7 in Table **D3.1**

$$A_e = UA_n = (0.85)(7.03 \text{ in}^2) = 5.98 \text{ in}^2$$

$$P_n = F_uA_e = (65 \text{ ksi})(5.98 \text{ in}^2) = 388.7 \text{ k}$$

LRFD with $\phi_t = 0.75$	ASD with $\Omega_t = 2.00$
$\phi_t P_n = (0.75)(388.7 \text{ k}) = 291.5 \text{ k} \leftarrow$	$\frac{P_n}{\Omega_t} = \frac{388.7 \text{ k}}{2.00} = 194.3 \text{ k} \leftarrow$

(c) Block shear strength considering both flanges

$$A_{gv} = (4)(10 \text{ in})(0.440 \text{ in}) = 17.60 \text{ in}^2$$

$$A_{nv} = (4)\left[10 \text{ in} - (2.5)\left(\frac{7}{8}\text{ in} + \frac{1}{8}\text{ in}\right)\right]0.440 \text{ in} = 13.20 \text{ in}^2$$

$$A_{nt} = (4)\left[1.51 \text{ in} - \left(\frac{1}{2}\right)\left(\frac{7}{8}\text{ in} + \frac{1}{8}\text{ in}\right)\right]0.440 \text{ in} = 1.78 \text{ in}^2$$

$$R_n = (0.6)(65 \text{ ksi})(13.20 \text{ in}^2) + (1.00)(65 \text{ ksi})(1.78 \text{ in}^2) = 630.5 \text{ k}$$

$$\leq (0.6)(50 \text{ ksi})(17.60 \text{ in}^2) + (1.00)(65 \text{ ksi})(1.78 \text{ in}^2) = 643.7 \text{ k}$$

$$630.5 \text{ k} < 643.7 \text{ k}$$

$$\therefore R_n = 630.5 \text{ k}$$

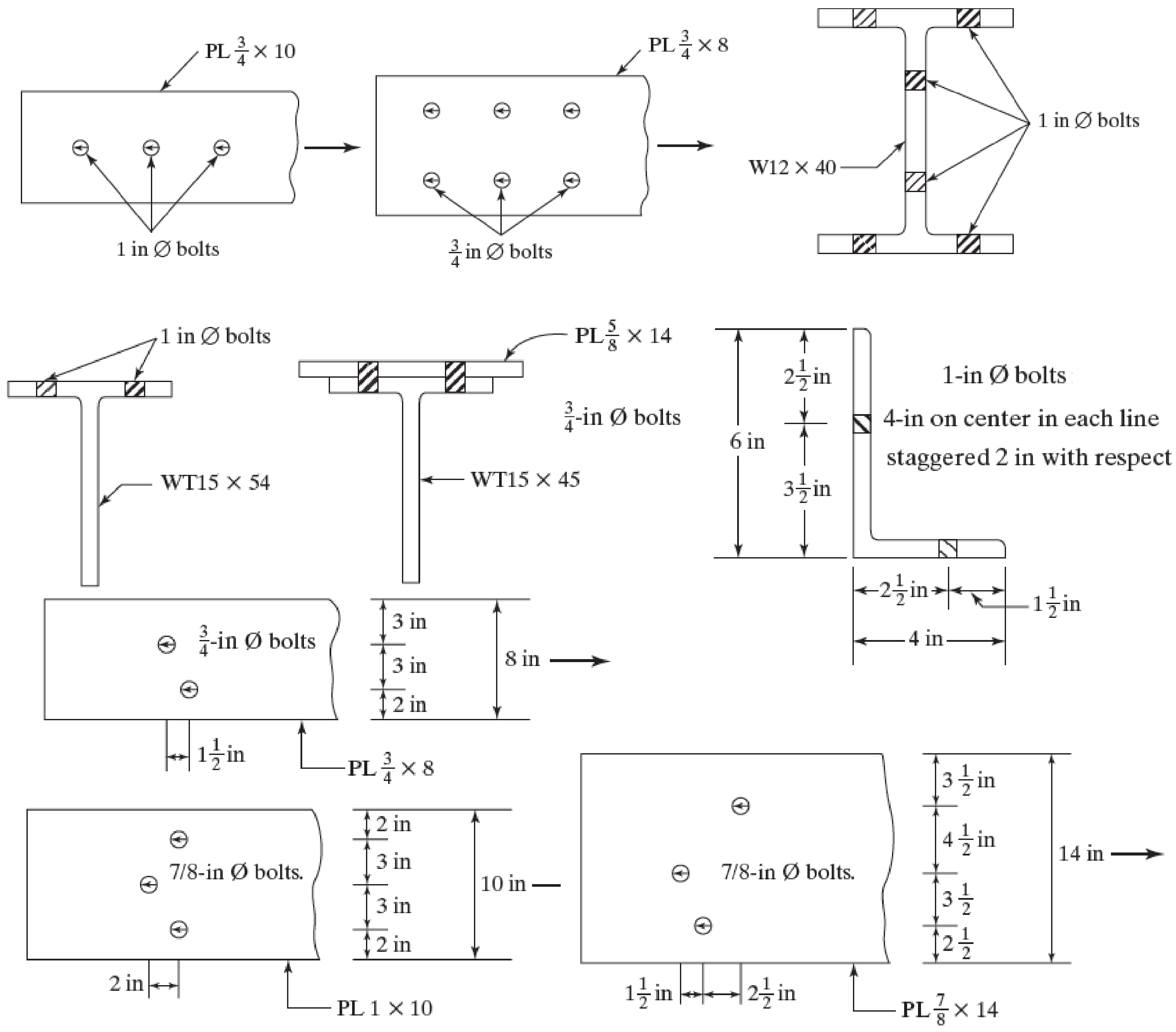
LRFD with $\phi = 0.75$	ASD with $\Omega = 2.00$
$\phi R_n = (0.75)(630.5 \text{ k}) = 472.9 \text{ k}$	$\frac{R_n}{\Omega} = \frac{630.5 \text{ k}}{2.00} = 315.2 \text{ k}$

Ans. LRFD = 291.5 k (Rupture controls) ASD = 194.3 k (Rupture controls)

3.6 HOMEWORKS

Homework 3-1 — Analysis of Tension Members

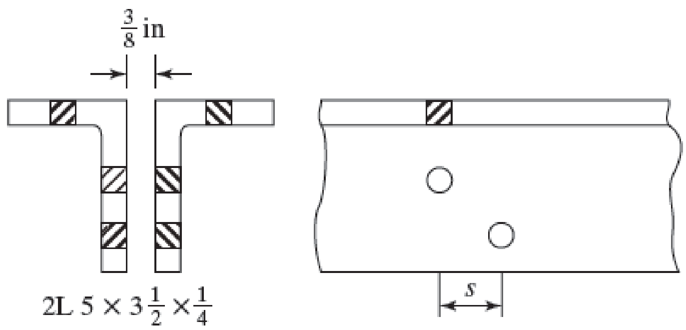
Compute the net area of each of the given members.



Homework 3-2

Analysis of Tension Members

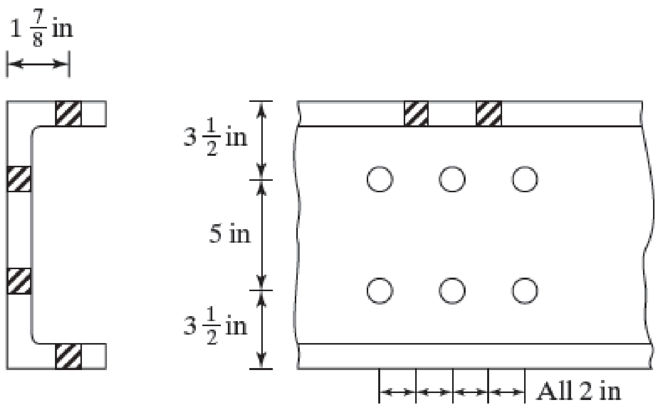
Determine the smallest net area of the tension member shown in figure. The holes are for $3/4-in$ Ø bolts at the usual gage locations. The stagger is $1\ 1/2\ in$.



Homework 3-3

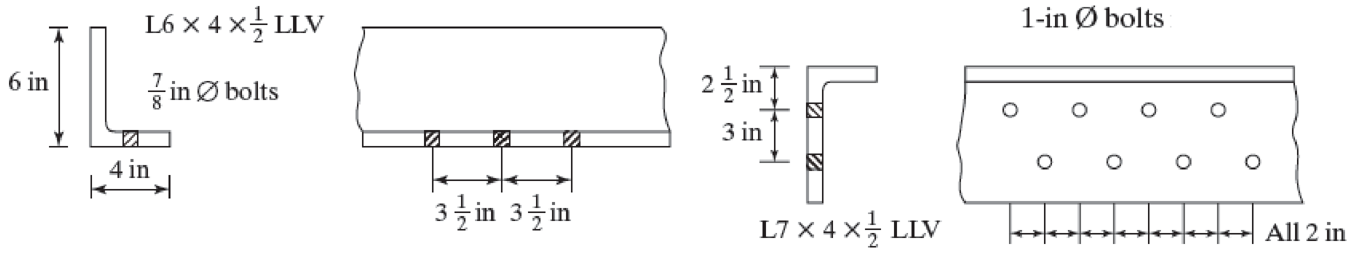
Analysis of Tension Members

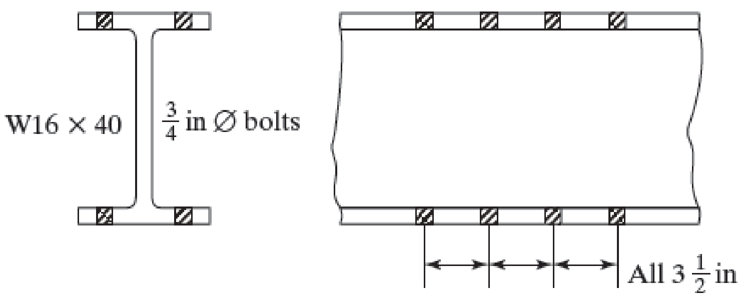
Determine the effective net cross-sectional area of the shown in the figure. Holes are $for\ 3/4\ in$ Ø bolts.



Homework 3-4 — Analysis of Tension Members

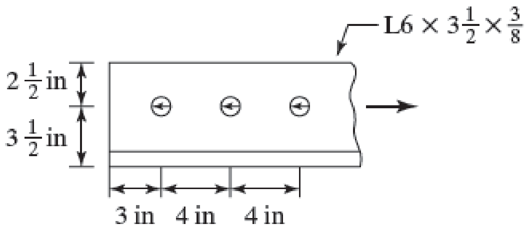
Determine the effective net areas of the sections shown by using the U values given in Table D3.1 (AISC).



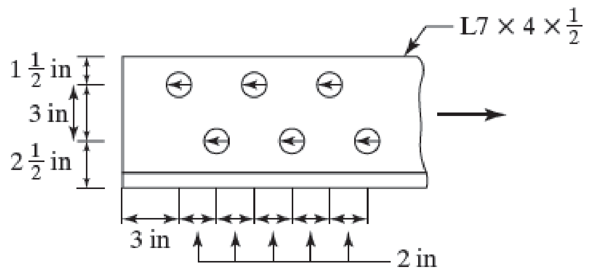


Homework 3-5 — Analysis of Tension Members

Determine the LRFD design strength and the ASD allowable strength of sections given. Neglect block shear.

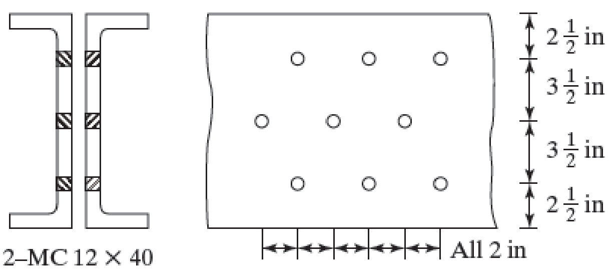


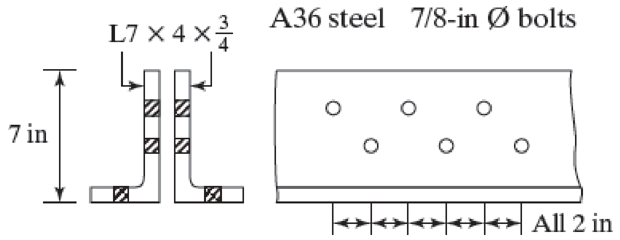
A36 steel and 7/8-in Ø bolts

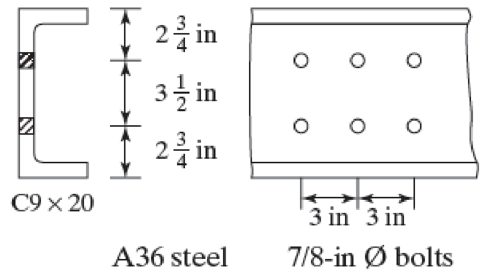


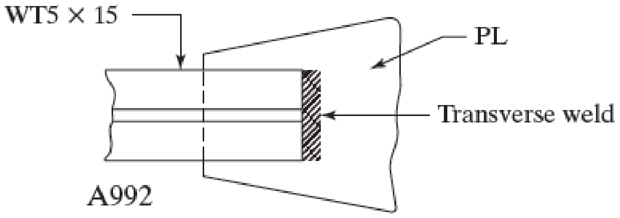
A36 steel and 3/4-in Ø bolts

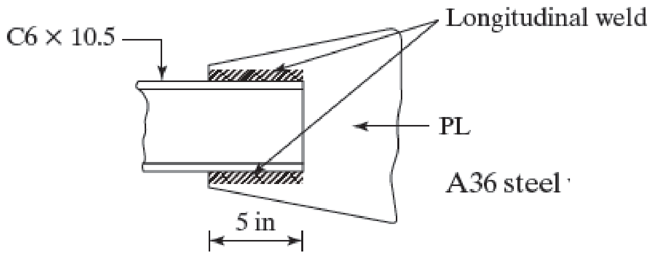
A36 steel and 7/8-in Ø bolts

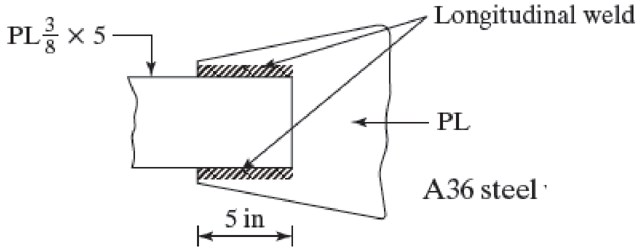






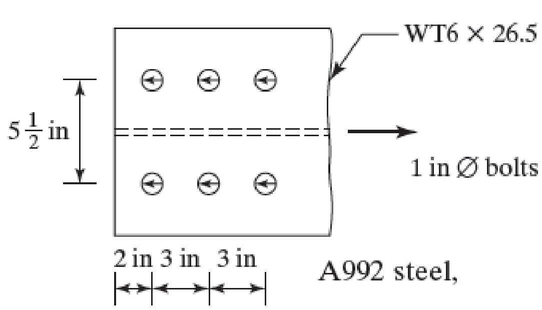


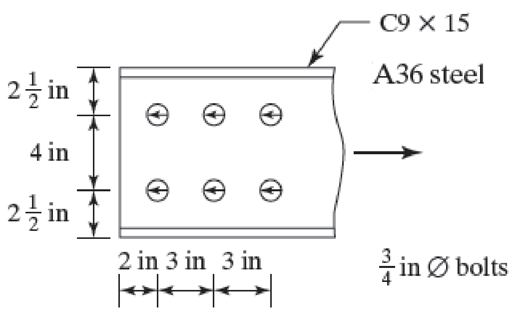




Homework 3-6 — Analysis of Tension Members

Determine the LRFD design strength and the ASD allowable strength of the sections given, including block shear.





3.7 BOLTED AND WELDED CONNECTIONS, AISC CHAPTER J

For bolted and welded connections, the Steel Construction Manual **AISC Chapter J,** limit states that will be considered are:

- **SHEARING STRENGTH OF BOLTS,** AISC Chapter J, Page 108

6. Tension and Shear Strength of Bolts and Threaded Parts

The *design tension* or *shear strength*, ϕR_n, and the *allowable tension* or *shear strength*, R_n/Ω, of a snug-tightened or pretensioned high-strength bolt or threaded part shall be determined according to the *limit states* of *tensile rupture* and *shear rupture* as follows:

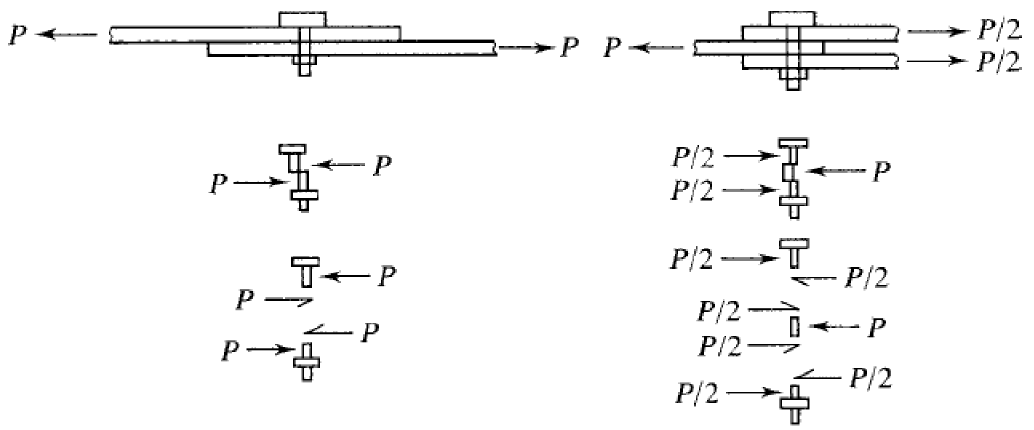
$$R_n = F_n A_b \qquad \text{(J3-1)}$$

$$\phi = 0.75 \text{ (LRFD)} \qquad \Omega = 2.00 \text{ (ASD)}$$

where

F_n = nominal tensile stress F_{nt}, or shear stress, F_{nv} from Table J3.2, ksi (MPa)

A_b = nominal unthreaded body area of bolt or threaded part (for upset rods, see footnote d, Table J3.2), in.^{2} (mm^2)



(a) **Single Shear** (b) **Double Shear**

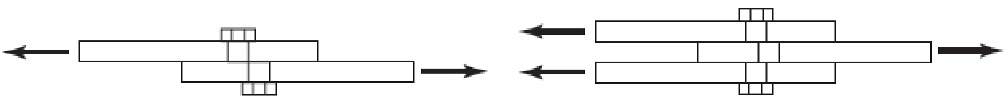


TABLE J3.2
Nominal Stress of Fasteners and Threaded Parts, ksi (MPa)

Description of Fasteners	Nominal Tensile Stress, F_{nt}, ksi (MPa)	Nominal Shear Stress in Bearing-Type Connections, F_{nv}, ksi (MPa)
A307 bolts	45 (310) [a][b]	24 (165) [b][c][f]
A325 or A325M bolts, when threads are not excluded from shear planes	90 (620) [e]	48 (330) [f]
A325 or A325M bolts, when threads are excluded from shear planes	90 (620) [e]	60 (414) [f]
A490 or A490M bolts, when threads are not excluded from shear planes	113 (780) [e]	60 (414) [f]
A490 or A490M bolts, when threads are excluded from shear planes	113 (780) [e]	75 (520) [f]
Threaded parts meeting the requirements of Section A3.4, when threads are not excluded from shear planes	$0.75\ F_u$ [a][d]	$0.40F_u$
Threaded parts meeting the requirements of Section A3.4, when threads are excluded from shear planes	$0.75\ F_u$ [a][d]	$0.50F_u$

[a] Subject to the requirements of Appendix 3.
[b] For A307 bolts the tabulated values shall be reduced by 1 percent for each $^1/_{16}$ in. (2 mm) over 5 diameters of length in the grip.
[c] Threads permitted in shear planes.
[d] The nominal tensile strength of the threaded portion of an upset rod, based upon the cross-sectional area at its major thread diameter, A_D, which shall be larger than the nominal body area of the rod before upsetting times F_y.
[e] For A325 or A325M and A490 or A490M bolts subject to tensile fatigue loading, see Appendix 3.
[f] When bearing-type connections used to splice tension members have a fastener pattern whose length, measured parallel to the line of force, exceeds 50 in. (1270 mm), tabulated values shall be reduced by 20 percent.

- **BEARING STRENGTH OF BOLTS,** AISC Chapter J, Page 111

10. Bearing Strength at Bolt Holes

The *available bearing strength*, ϕR_n and R_n/Ω, at bolt holes shall be determined for the *limit state* of *bearing* as follows:

$$\phi = 0.75 \text{ (LRFD)} \qquad \Omega = 2.00 \text{ (ASD)}$$

(a) For a bolt in a *connection* with standard, oversized, and short-slotted holes, independent of the direction of loading, or a long-slotted hole with the slot parallel to the direction of the bearing *force*:

(i) When deformation at the bolt hole at *service load* is a design consideration

$$R_n = 1.2\, L_c t F_u \le 2.4\, d t F_u \qquad \text{(J3-6a)}$$

$Deformation \le 0.25\ in$

(ii) When deformation at the bolt hole at service load is not a design consideration

$$R_n = 1.5\, L_c t F_u \le 3.0\, d t F_u \qquad \text{(J3-6b)}$$

$Deformation > 0.25\ in$

(b) For a bolt in a connection with long-slotted holes with the slot perpendicular to the direction of force:

$$R_n = 1.0\, L_c t F_u \le 2.0\, d t F_u \qquad \text{(J3-6c)}$$

(c) For connections made using bolts that pass completely through an unstiffened box member or *HSS*, see Section J7 and Equation J7-1,

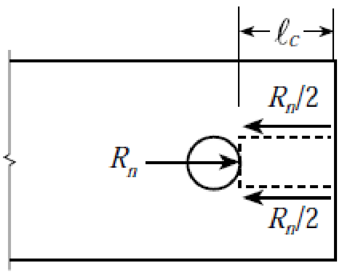
where

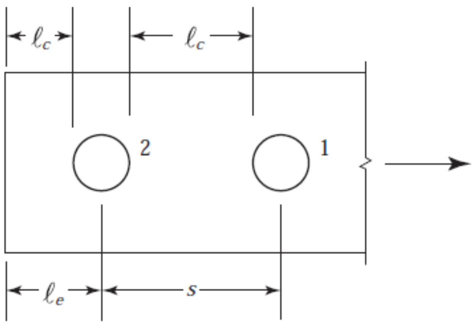
d = nominal bolt diameter, in. (mm)

F_u = *specified minimum tensile strength* of the connected material, ksi (MPa)

L_c = clear distance, in the direction of the force, between the edge of the hole and the edge of the adjacent hole or edge of the material, in. (mm)

t = thickness of connected material, in. (mm)





$$L_c = \min\ [L_{c1}, L_{c2}, L_{c3}, \cdots]$$

- **STRENGTH OF FILLET WELDED CONNECTIONS,** AISC Chapter J2, Page 98

4. Strength

The *design strength*, ϕR_n and the *allowable strength*, R_n/Ω, of welds shall be the lower value of the base material and the *weld metal* strength determined according to the *limit states* of *tensile rupture*, *shear rupture* or *yielding* as follows:

For the base metal

$$R_n = F_{BM} A_{BM} \tag{J2-2}$$

For the weld metal

$$R_n = F_w A_w \tag{J2-3}$$

where

F_{BM} = *nominal strength* of the base metal per unit area, ksi (MPa)
F_w = nominal strength of the weld metal per unit area, ksi (MPa)
A_{BM} = cross-sectional area of the base metal, in.^{2} (mm^2)
A_w = effective area of the weld, in.^{2} (mm^2)

The values of ϕ, Ω, F_{BM}, and F_w and limitations thereon are given in Table J2.5.

Alternatively, for *fillet welds* loaded in-plane the *design strength*, ϕR_n and the *allowable strength*, R_n/Ω, of welds is permitted to be determined as follows:

$$\phi = 0.75 \text{ (LRFD)} \qquad \Omega = 2.00 \text{ (ASD)}$$

(a) For a linear weld group loaded in-plane through the center of gravity

$$R_n = F_w A_w \tag{J2-4}$$

$$R_n = (0.6\, F_{EXX})(0.707\, w)(L)$$

F_{nw} = (nominal strength of base metal $0.60\, F_{EXX}$)
A_{we} = (throat)(weld length) = $(0.707\, w)(L)$
F_{EXX} = electrode classification number, ksi (MPa)
A_w = effective area of the weld, in.^{2} (mm^2)

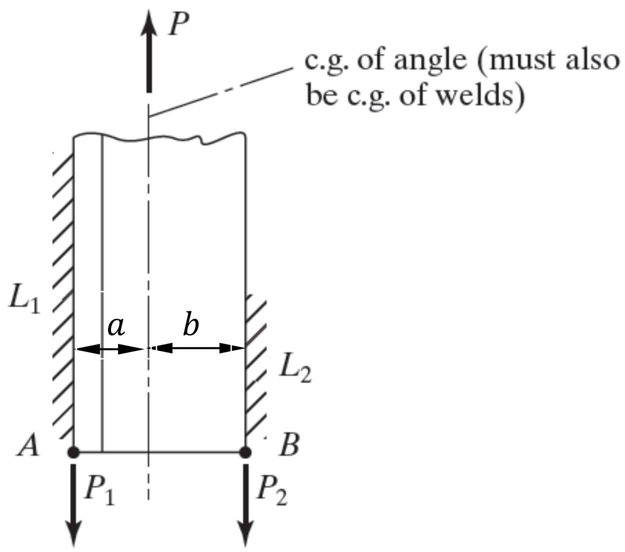
$$\beta = 1.2 - 0.002(L/w) \leq 1.0 \quad \text{(J2-1)}$$

where

L = actual length of end-loaded weld, in. (mm)
w = weld leg size, in. (mm)

When the length of the weld exceeds 300 times the leg size, the value of β shall be taken as 0.60.

$$or\ \frac{L}{w} < 100$$



$$L = L_1 + L_2$$

$$L_1 = \frac{b}{a+b}L, \qquad L_1 = \frac{a}{a+b}L, \quad L_1 > L_2, \ \ b > a$$

- **STRENGTH OF WELDED CONNECTIONS WITH BOTH LONGITUDINAL AND TRANSVERSE FILLET WELDS,** AISC Chapter J2, Page 101

(c) For *fillet weld* groups concentrically loaded and consisting of elements that are oriented both longitudinally and transversely to the direction of applied *load*, the combined strength, R_n, of the fillet weld group shall be determined as the greater of

$$R_n = R_{wl} + R_{wt} \quad \text{(J2-9a)}$$

or

$$R_n = 0.85R_{wl} + 1.5R_{wt} \quad \text{(J2-9b)}$$

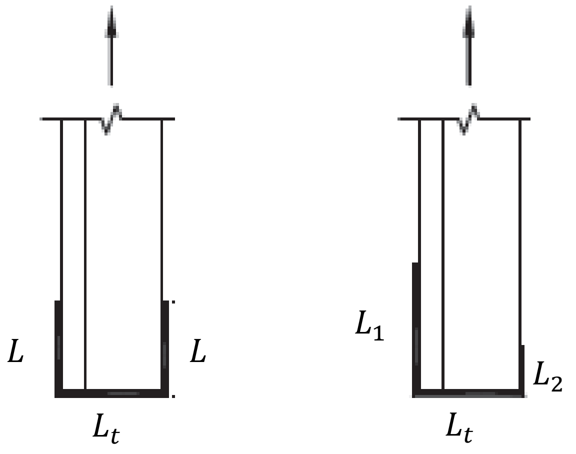
where

R_{wl} = the total nominal strength of longitudinally loaded fillet welds, as determined in accordance with Table J2.5, kips (N)

R_{wt} = the total nominal strength of transversely loaded fillet welds, as determined in accordance with Table J2.5 without the alternate in Section J2.4(a), kips (N)

$$R_{wl} = R_n \text{ for side welds} = (0.6\ F_{EXX})(0.707\ w)(L_l)$$

$$R_{wt} = R_n \text{ for transverse end weld} = (0.6\ F_{EXX})(0.707\ w)(L_t)$$



$L_l = 2L$ $\qquad$ $L_l = L_1 + L_2$

- **THE MAXIMUM/MINIMUM SIZE OF A FILLET WELD,** AISC Chapter J2, Page 96

TABLE J2.4
Minimum Size of Fillet Welds

Material Thickness of Thinner Part Joined, in. (mm)	Minimum Size of Fillet Weld,[a] in. (mm)
To 1/4 (6) inclusive	1/8 (3)
Over 1/4 (6) to 1/2 (13)	3/16 (5)
Over 1/2 (13) to 3/4 (19)	1/4 (6)
Over 3/4 (19)	5/16 (8)

[a] Leg dimension of fillet welds. Single pass welds must be used.
Note: See Section J2.2b for maximum size of fillet welds.

The maximum size of fillet welds of connected parts shall be:

(a) Along edges of material less than 1/4-in. (6 mm) thick, not greater than the thickness of the material.

(b) Along edges of material 1/4 in. (6 mm) or more in thickness, not greater than the thickness of the material minus 1/16 in. (2 mm), unless the weld is especially designated on the drawings to be built out to obtain full-throat thickness. In the as-welded condition, the distance between the edge of the base metal and the toe of the weld is permitted to be less than 1/16 in. (2 mm) provided the weld size is clearly verifiable.

$$\text{Maximim size of a fillet weld} \leq \text{Material thickness} \qquad for \quad \text{Material thickness} < \frac{1}{4}"$$

$$\text{Maximim size of a fillet weld} \leq \text{Material thickness} - \frac{1}{6}" \quad for \quad \text{Material thickness} \geq \frac{1}{4}"$$

DESIGN OF TENSION MEMBERS 4

4.1 SELECTION OF SECTIONS

Although the designer has considerable freedom in the selection, the resulting members should have the following properties:

1. Compactness.
2. Dimensions that fit into the structure with reasonable relation to the dimensions of the other members of the structure.
3. Connections to as many parts of the sections as possible to minimize shear lag.

Specifications usually recommend that slenderness ratios be kept below certain maximum values in order that some minimum compressive strengths be provided in the members. For tension members other than rods, the AISC Specification does not provide a maximum slenderness ratio for tension members, but **Section D.1** of the specification suggests that a maximum value of **300** be used.

It should be noted that **out-of-straightness does not affect the strength of tension members very much**, *because the tension loads tend to straighten the members*. (The same statement cannot be made for compression members.) For this reason, the AISC Specification is a little more liberal in its consideration of tension members, including those subject to some compressive forces due to transient loads such as wind or earthquake.

The recommended **maximum slenderness ratio of 300** is not applicable to tension rods. Maximum $\boldsymbol{L/r}$ values for rods are left to the designer's judgment. If a maximum value of 300 were specified for them, they would seldom be used, because of their extremely small radii of gyration, and thus very high slenderness ratios.

The design of steel members is, in effect, **a trial-and-error process**, although tables such as those given in the Steel Manual often enable us to directly select a desirable section. For a tension member, one can **estimate the area required**, **select a section from the AISC Manual** providing the corresponding area, and **check the section's strength**.

SUMMARY: DESIGN OF TENSION MEMBERS

The Steel Construction Manual **AISC Chapter D, Page 26** limit states that will be considered are:

- **LOAD COMBINATIONS,** AISC Chapter 2, Pages 2-8 and 2-9

For LRFD	For ASD
$P_u = 1.4D$ $P_u = 1.2D + 1.6L$	$P_a = D + L$

- **TENSILE YIELDING,** AISC Chapter D, Page 26

To satisfy the first of these expressions, the minimum gross area must be at least equal to

$$\min A_g = \frac{P_u}{\phi_t F_y}.$$

(a) For tensile yielding in the gross section: $\phi_t = 0.90$ (LRFD)

- **TENSILE RUPTURE,** AISC Chapter D, Page 27

To satisfy the second expression, the minimum value of A_e must be at least

$$\min A_e = \frac{P_u}{\phi_t F_u}.$$

And since $A_e = UA_n$ for a bolted member, the minimum value of A_n is

$$\min A_n = \frac{\min A_e}{U} = \frac{P_u}{\phi_t F_u U}.$$

Then the minimum A_g is

$$= \min A_n + \text{estimated area of holes}$$
$$= \frac{P_u}{\phi_t F_u U} + \text{estimated area of holes}$$

(b) For tensile rupture in the net section: $\phi_t = 0.75$ (LRFD)

Assume U, to be checked later

If the **ASD** equations are used for tension member design, the allowable strength is the lesser of

$$\frac{F_y A_g}{\Omega_t} \quad or \quad \frac{F_y U A_n}{\Omega_t}$$

From these expressions, the minimum gross areas required are as follows:

$$\min A_g = \frac{\Omega_t P_a}{F_y}$$

$$\min A_g = \frac{\Omega_t P_a}{F_u U} + \text{estimated area of holes}$$

- **CHECK SLENDERNESS LIMITATIONS,** AISC Chapter D, Page 26

$$\min r = \frac{L}{300}$$

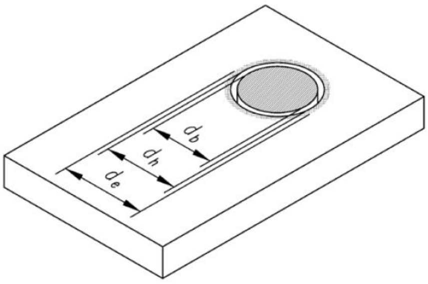
- **SELECT A TRIAL SECTION**

Select a Lightest Available Section with a largest Radius of Gyration

1. **Check the Gross Area**, AISC Chapter D, Page 27

   The gross area, A_g, of a member is the total cross-sectional area.

2. **Check the Net Area**, AISC Chapter D, Page 27



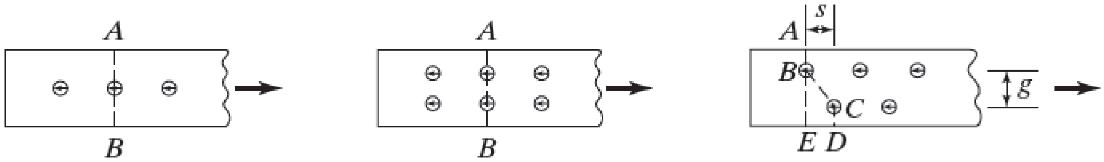
$$A_n = A_g - A_{Holes}$$

$$d_e = d_b + \frac{1}{8}"$$

Or

For a chain of holes extending across a part in any diagonal or zigzag line, the net width of the part shall be obtained by deducting from the gross width the sum of the diameters or slot dimensions as provided in Section J3.2, of all holes in the chain, and adding, for each *gage* space in the chain, the quantity $s^2/4g$

$$A_n = A_g - A_{Holes} + \sum_{i=1}^{N} \frac{S_i^2}{4g_i} t, \quad N: Number\ of\ zigzag\ lines$$



In determining the net area across plug or *slot welds*, the *weld metal* shall not be considered as adding to the net area.

User Note: Section J4.1(b) limits A_n to a maximum of $0.85A_g$ for splice plates with holes.

$$A_e = A_n \leq 0.85A_g$$

3. **Check the Effective Net Area**, AISC Chapter D, Page 28

3. Effective Net Area

The effective area of tension members shall be determined as follows:

$$A_e = A_n U \qquad \text{(D3-1)}$$

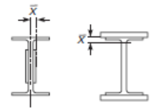
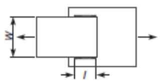
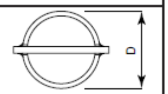
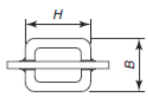
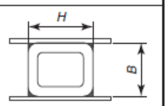
where U, the shear lag factor, is determined as shown in Table D3.1.

NOTE: **FULL LOAD COMBINATIONS**, **AISC Chapter 2, Pages 2-8 and 2-9**

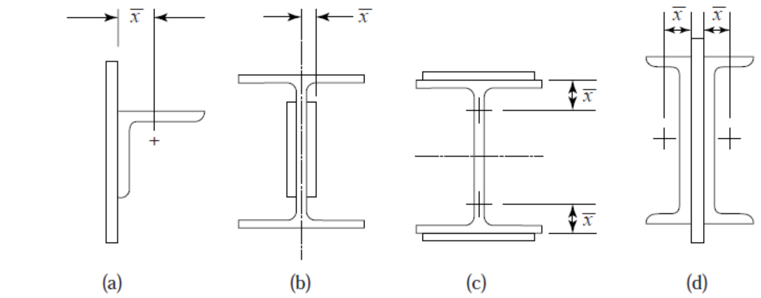
For LRFD	For ASD
$1.4D$	D
$1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R)$	$D + L$
$1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (0.5L \text{ or } 0.8W)$	$D + (L_r \text{ or } S \text{ or } R)$
$1.2D + 1.6W + 0.5L + 0.5(L_r \text{ or } S \text{ or } R)$	$D + 0.75L + 0.75(L_r \text{ or } S \text{ or } R)$
$1.2D \pm 1.0E + 0.5L + 0.2S$	$D \pm (W \text{ or } 0.7E)$
$0.9D \pm (1.6W \text{ or } 1.0E)$	$D + 0.75(W \text{ or } 0.7E) + 0.75L + 0.75(L_r \text{ or } S \text{ or } R)$
	$0.6D \pm (W \text{ or } 0.7E)$

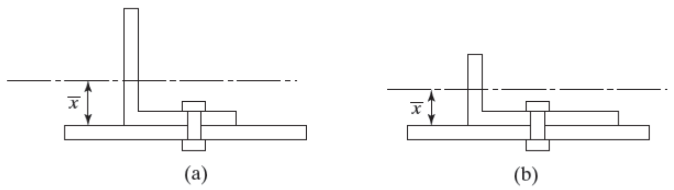
D = dead load
L = live load due to occupancy
L_r = roof live load
S = snow load
R = nominal load due to initial rainwater or ice exclusive of the ponding contribution
W = wind load
E = earthquake load

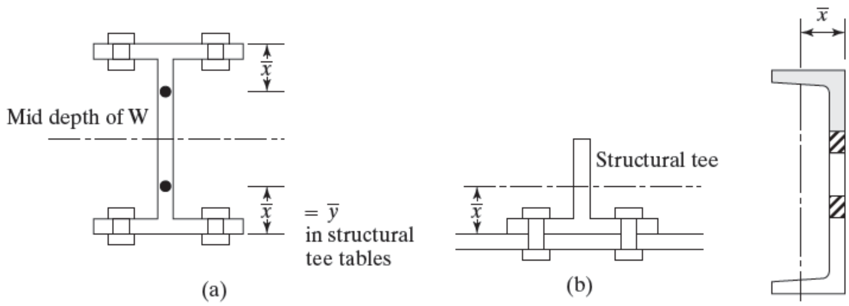
TABLE D3.1
Shear Lag Factors for Connections to Tension Members

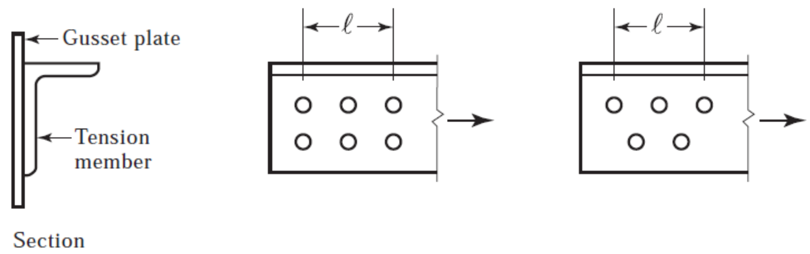
Case	Description of Element		Shear Lag Factor, U	Example
1	All tension members where the tension load is transmitted directly to each of cross-sectional elements by fasteners or welds. (except as in Cases 3, 4, 5 and 6)		$U = 1.0$	—
2	All tension members, except plates and HSS, where the tension load is transmitted to some but not all of the cross-sectional elements by fasteners or longitudinal welds (Alternatively, for W, M, S and HP, Case 7 may be used.)		$U = 1 - \bar{x}/l$	
3	All tension members where the tension load is transmitted by transverse welds to some but not all of the cross-sectional elements.		$U = 1.0$ and A_n = area of the directly connected elements	—
4	Plates where the tension load is transmitted by longitudinal welds only.		$l \geq 2w \ldots U = 1.0$ $2w > l \geq 1.5w \ldots U = 0.87$ $1.5w > l \geq w \ldots U = 0.75$	
5	Round HSS with a single concentric gusset plate		$l \geq 1.3D \ldots U = 1.0$ $D \leq l < 1.3D \ldots U = 1 - \bar{x}/l$ $\bar{x} = D/\pi$	
6	Rectangular HSS	with a single concentric gusset plate	$l \geq H \ldots U = 1 - \bar{x}/l$ $\bar{x} = \dfrac{B^2 + 2BH}{4(B+H)}$	
		with two side gusset plates	$l \geq H \ldots U = 1 - \bar{x}/l$ $\bar{x} = \dfrac{B^2}{4(B+H)}$	
7	W, M, S or HP Shapes or Tees cut from these shapes. (If U is calculated per Case 2, the larger value is permitted to be used)	with flange connected with 3 or more fasteners per line in direction of loading	$b_f \geq 2/3d \ldots U = 0.90$ $b_f < 2/3d \ldots U = 0.85$	—
		with web connected with 4 or more fasteners in the direction of loading	$U = 0.70$	—
8	Single angles (If U is calculated per Case 2, the larger value is permitted to be used)	with 4 or more fasteners per line in direction of loading	$U = 0.80$	—
		with 2 or 3 fasteners per line in the direction of loading	$U = 0.60$	—

l = length of connection, in. (mm); w = plate width, in. (mm); $\bar{x}$ = connection eccentricity, in. (mm); B = overall width of rectangular HSS member, measured 90 degrees to the plane of the connection, in. (mm); H = overall height of rectangular HSS member, measured in the plane of the connection, in. (mm)

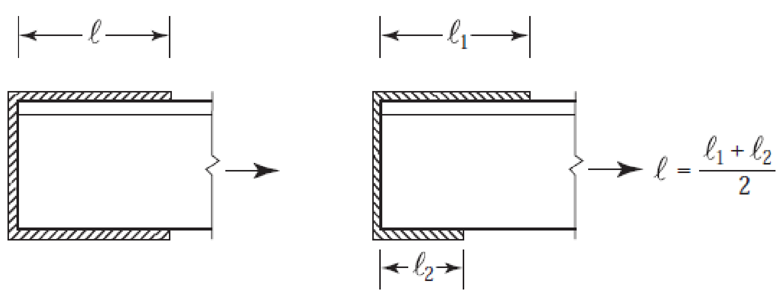








(a) Bolted



(b) Welded

Connection eccentricity $\bar{x}$ for various cases

Example 4.1 — Design of Tension Members

Select a 30-ft-long W12 section of A992 steel to support a tensile service dead load $P_D = 130$ k and a tensile service live load $P_L = 110$ k. As shown in Fig. 4.1, the member is to have two lines of bolts in each flange for 7/8-in bolts (at least three in a line 4 in on center).

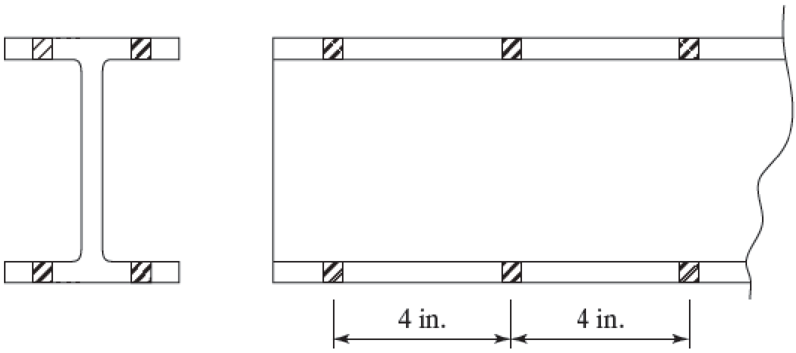


Figure 4-1

Solution

(a) Considering the necessary load combinations

LRFD	ASD
$P_u = 1.4D = (1.4)(130\text{ k}) = 182\text{ k}$ $P_u = 1.2D + 1.6L = (1.2)(130\text{ k}) + (1.6)(110\text{ k}) = 332\text{ k}$	$P_a = D + L = 130\text{ k} + 110\text{ k}$ $= 240\text{ k}$

(b) Computing the minimum A_g required, using LRFD Equations

$$1.\ \min A_g = \frac{P_u}{\phi_t F_y} = \frac{332\text{ k}}{(0.90)(50\text{ ksi})} = 7.38\text{ in}^2$$

$$2.\ \min A_g = \frac{P_u}{\phi_t F_u U} + \text{estimated hole areas}$$

Assume that $U = 0.85$ from Table **D3.1** Case 7, and assume that flange thickness is about 0.380 in after looking at W12 sections in the LRFD Manual which have areas of 7.38 in^2 or more. $U = 0.85$ was assumed since b_f appears to be less than 2/3 d.

$$\min A_g = \frac{332 \text{ k}}{(0.75)(65 \text{ ksi})(0.85)} + (4)\left(\frac{7}{8}\text{ in} + \frac{1}{8}\text{ in}\right)(0.380 \text{ in}) = 9.53 \text{ in}^2 \leftarrow$$

(c) Preferable minimum r

$$\min r = \frac{L}{300} = \frac{(12 \text{ in/ft})(30 \text{ ft})}{300} = 1.2 \text{ in}$$

Try W12 × 35 ($A_g = 10.3$ in^2, $d = 12.50$ in, $b_f = 6.56$ in,
$t_f = 0.520$ in, $r_{\min} = r_y = 1.54$ in)

Checking

(a) Gross section yielding

$$P_n = F_y A_g = (50 \text{ ksi})(10.3 \text{ in}^2) = 515 \text{ k}$$

LRFD with $\phi_t = 0.9$	ASD with $\Omega_t = 1.67$
$\phi_t P_n = (0.9)(515 \text{ k}) = 463.5 \text{ k} > 332 \text{ k}$ **OK**	$\frac{P_n}{\Omega_t} = \frac{515 \text{ k}}{1.67} = 308.4 > 240 \text{ k}$ **OK**

(b) Tensile rupture strength

From Table **D3.1** Case 2

$\bar{x}$ for half of W12 × 35 or, that is, a WT6 × 17.5 = 1.30 in

$$L = (2)(4 \text{ in}) = 8 \text{ in}$$

$$U = \left(1 - \frac{\bar{x}}{L}\right) = \left(1 - \frac{1.30 \text{ in}}{8 \text{ in}}\right) = 0.84$$

From Table D3.1 Case 7

$U = 0.85$, since $b_f = 6.56 \text{ in} < \frac{2}{3}d = \left(\frac{2}{3}\right)(12.50 \text{ in}) = 8.33 \text{ in},$

$$A_n = 10.3 \text{ in}^2 - (4)\left(\frac{7}{8}\text{ in} + \frac{1}{8}\text{ in}\right)(0.520 \text{ in}) = 8.22 \text{ in}^2$$

$$A_e = (0.85)(8.22 \text{ in}^2) = 6.99 \text{ in}^2$$

$$P_n = F_u A_e = (65 \text{ ksi})(6.99 \text{ in}^2) = 454.2 \text{ k}$$

LRFD with $\phi_t = 0.75$	ASD with $\Omega_t = 2.00$
$\phi_t P_n = (0.75)(454.2 \text{ k}) = 340.7 \text{ k} > 332 \text{ k}$ **OK**	$\frac{P_n}{\Omega_t} = \frac{454.2 \text{ k}}{2.00} = 227.1 \text{ k} < 240 \text{ k}$ **N.G.**

(c) Slenderness ratio

$$\frac{L_y}{r_y} = \frac{12 \text{ in/ft} \times 30 \text{ ft}}{1.54 \text{ in}} = 234 < 300, \textbf{OK} \qquad \text{OK}$$

Ans. By LRFD, use W12 × 35. By ASD, use next larger section W12 × 40.

Example 4.2 Design of Tension Members

Design a 9-ft single-angle tension member to support a dead tensile working load of 30 k and a live tensile working load of 40 k. The member is to be connected to one leg only with 7/8-in bolts (at least four in a line 3 in on center). Assume that only one bolt is to be located at any one cross section. Use A36 steel with $F_y = 36$ ksi and $F_u = 58$ ksi.

Solution

LRFD	ASD
$P_u = (1.2)(30) + (1.6)(40) = 100 \text{ k}$	$P_a = 30 + 40 = 70 \text{ k}$

1. $$\min A_g \text{ required} = \frac{P_u}{\phi_t F_y} = \frac{100}{(0.9)(36)} = 3.09 \text{ in}^2$$

2. Assume that $U = 0.80$, Table **D3.1** (Case 8)

$$\min A_n \text{ required} = \frac{P_u}{\phi_t F_u U} = \frac{100 \text{ k}}{(0.75)(58 \text{ ksi})(0.80)} = 2.87 \text{ in}^2$$

$$\min A_g \text{ required} = 2.87 \text{ in}^2 + \text{bolt hole area} = 2.87 \text{ in}^2 + \left(\frac{7}{8}\text{ in} + \frac{1}{8}\text{ in}\right)(t)$$

3. $$\text{Min } r \text{ required} = \frac{(12 \text{ in/ft})(9 \text{ ft})}{300} = 0.36 \text{ in}$$

Angle $t_{(in)}$	Area of one 1-in bolt hole (in^2)	Gross area required = larger of $P_u/\phi_t F_y$ or $P_u/\phi_t F_u U$ + est. hole area (in^2)	Lightest angles available, their areas (in^2) and least radii of gyration (in)
5/16	0.312	3.18	$6 \times 6 \times \frac{5}{16} (A = 3.67, r_z = 1.19)$
3/8	0.375	3.25	$6 \times 3\frac{1}{2} \times \frac{3}{8} (A = 3.44, r_z = 0.763)$
7/16	0.438	3.30	$4 \times 4 \times \frac{7}{16} (A = 3.30, r_z = 0.777) \leftarrow$ $5 \times 3 \times \frac{7}{16} (A = 3.31, r_z = 0.644)$
1/2	0.500	3.37	$4 \times 3\frac{1}{2} \times \frac{1}{2} (A = 3.50, r_z = 0.716)$
5/8	0.625	3.50	$4 \times 3 \times \frac{5}{8} (A = 3.99, r_z = 0.631)$
	Try L4 $\times$ 4 $\times \frac{7}{16}$ ($\bar{x}$ = 1.15 in)		

Checking

(a) Gross section yielding

$$P_n = F_y A_g = (36 \text{ ksi})(3.30 \text{ in}^2) = 118.8 \text{ k}$$

LRFD with $\phi_t = 0.9$	ASD with $\Omega_t = 1.67$
$\phi_t P_n = (0.9)(118.8 \text{ k}) = 106.9 \text{ k} > 100 \text{ k OK}$	$\frac{P_n}{\Omega_t} = \frac{118.8 \text{ k}}{1.67} = 71.1 \text{ k} > 70 \text{ k OK}$

(b) Tensile rupture strength

$$A_n = 3.30 \text{ in}^2 - (1)\left(\frac{7}{16} \text{ in}\right) = 2.86 \text{ in}^2$$

$$U = 1 - \frac{\bar{x}}{L} = 1 - \frac{1.15 \text{ in}}{(3)(3 \text{ in})} = 0.87 \leftarrow$$

U from Table **D3.1**(Case 8) $= 0.80$

$A_e = U A_n = (0.87)(2.86 \text{ in}^2) = 2.49 \text{ in}^2$

$P_n = F_u A_e = (58 \text{ ksi})(2.49 \text{ in}^2) = 144.4 \text{ k}$

LRFD with $\phi_t = 0.75$	ASD with $\Omega_t = 2.00$
$\phi_t P_n = (0.75)(144.4 \text{ k}) = 108.3 \text{ k} > 100 \text{ k OK}$	$\frac{P_n}{\Omega_t} = \frac{144.4 \text{ k}}{2.00} = 72.2 \text{ k} > 70 \text{ k OK}$

Ans. By LRFD, use L4 × 4 × $\frac{7}{16}$. By ASD, select L4 × 4 × $\frac{7}{16}$.

ANALYSIS OF AXIALLY LOADED COMPRESSION MEMBERS

5

5.1 INTRODUCTION

There are three general modes by which axially loaded columns can fail. These are **flexural buckling**, **local buckling**, and **torsional buckling**. These modes of buckling are briefly defined as follows:

1. **Flexural buckling** (also called Euler buckling) is the primary type of buckling. Members are subject to flexure, or bending, when they become unstable.

2. **Local buckling** occurs when some part or parts of the cross section of a column are so thin that they buckle locally in compression before the other modes of buckling can occur. The susceptibility of a column to local buckling is measured by the **width-thickness ratios** ($e.g., b/t_f\ ,\ h/t_w$) of the parts of its cross section.

3. **Flexural torsional buckling** may occur in columns that have certain crosssectional configurations. These columns fail by **twisting** (**torsion**) or by a combination of **torsional and flexural buckling**.

5.2 SECTIONS USED FOR COLUMNS

Theoretically, **an endless number of shapes can be selected to safely resist a compressive load in a given structure**. From a practical viewpoint, however, the number of possible solutions is severely limited by such considerations as sections available, connection problems, and type of structure in which the section is to be used.

The sections used for compression members usually are similar to those used for tension members, with certain exceptions. The exceptions are caused by the fact that the strengths of compression members vary in some inverse relation to the slenderness ratios, and stiff members are required. Individual rods, bars, and plates usually are too slender to make satisfactory compression members, unless they are very short and lightly loaded.

Single-angle members (a) are satisfactory for use as bracing and compression members in light trusses. Equal-leg angles may be more economical than unequal-leg angles, because their least r values are greater for the same area of steel.

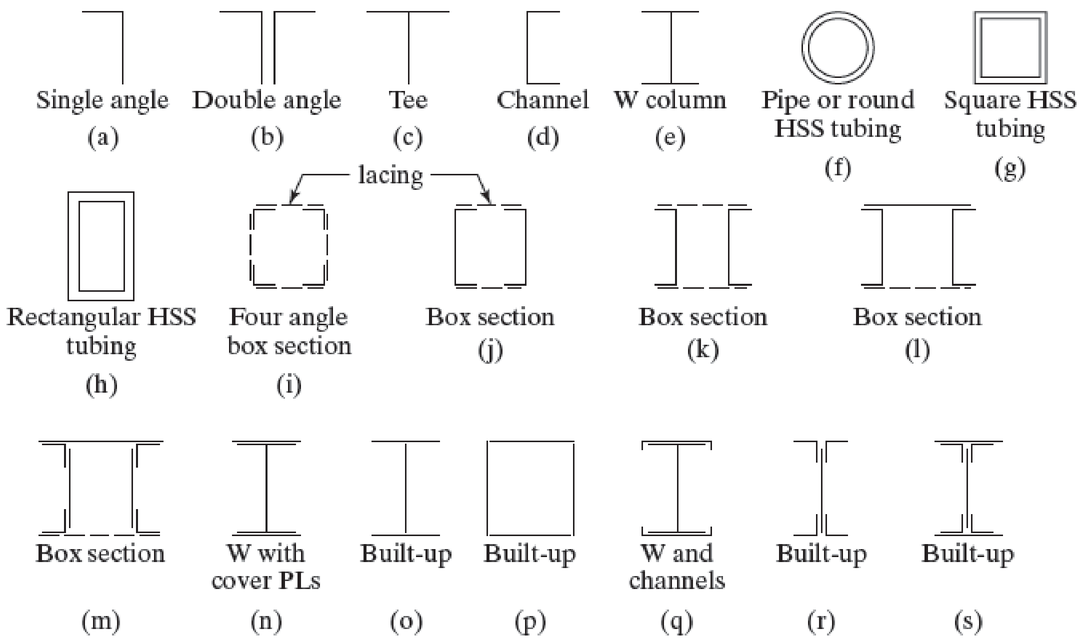


Figure 5-1

Types of compression members.

The top chord members of bolted roof trusses might consist of a pair of angles back to back (b).

There will often be a space between them for the insertion of a gusset or connection plate at the joints necessary for connections to other members. An examination of this section will show that it is probably desirable to use unequal-leg angles with the long legs back to back to give a better balance between the r values about the x and y axes.

If roof trusses are welded, gusset plates may be unnecessary, and structural tees (c) might be used for the top chord compression members because the web members can be welded directly to the stems of the tees.

Single channels (d) are not satisfactory for the average compression member because of their almost negligible r values about their web axes. They can be used if some method of providing extra lateral support in the weak direction is available.

The W shapes (e) are the most common shapes used for building columns and for the compression members of highway bridges. Their r values, although far from being equal about the two axes,are much more nearly balanced than are the same values for channels.

Hollow structural sections (square, rectangular, or round) and steel pipe are very valuable sections for buildings, bridges, and other structures. These clean, neatlooking sections are easily fabricated and erected. For small and medium loads, the round sections (f) are quite satisfactory. They are often used as columns in long series of windows, as short columns in warehouses, as columns for the roofs of covered walkways, in the basements and garages of residences, and in other applications. Round columns have the advantage of being equally rigid in all directions and are usually very economical, unless moments are too large for the sizes available. The AISC Manual furnishes the sizes of these sections and classifies them as being either round HSS sections or standard, extra strong, or double extra strong steel pipe.

Square and rectangular tubing (g) and (h) are being used more each year. For many years, only a few steel mills manufactured steel tubing for structural purposes. Perhaps the major reason tubing was not used to a great extent is the difficulty of making connections with rivets or bolts. This problem has been fairly well eliminated, however, by the advent of modern welding.

The use of tubing for structural purposes by architects and engineers in the years to come will probably be greatly increased for several reasons:

1. The most efficient compression member is one that has a constant radius of gyration about its centroid, a property available in round HSS tubing and pipe sections. Square tubing is the next-most-efficient compression member.
2. Four-sided and round sections are much easier to paint than are the six-sided open W, S, and M sections. Furthermore, the rounded corners make it easier to apply paint or other coatings uniformly around the sections.
3. They have less surface area to paint or fireproof.
4. They have excellent torsional resistance.
5. The surfaces of tubing are quite attractive.
6. When exposed, the round sections have wind resistance of only about two-thirds of that of flat surfaces of the same width.
7. If cleanliness is important, hollow structural tubing is ideal, a s it doesn’t have the problem of dirt collecting between the flanges of open structural shapes.

5.3 DEVELOPMENT OF COLUMN FORMULAS

The use of columns dates to before the dawn of history, but it was not until 1729 that a paper was published on the subject, by Pieter van Musschenbroek, a Dutch mathematician. He presented an empirical column formula for estimating the strength of rectangular columns.

The testing of columns with various slenderness ratios results in a scattered range of values, such as those shown by the broad band of dots in Figure 5-2. The dots will not fall on a smooth curve, even if all of the testing is done in the same laboratory, because of the difficulty of exactly centering the loads, lack of perfect uniformity of the materials, varying dimensions of the sections, residual stresses, end restraint variations, and other such issues. The usual practice is to attempt to develop formulas that give results representative of an approximate average of the test results. The student should also realize that laboratory conditions are not field conditions, and column tests probably give the limiting values of column strengths.

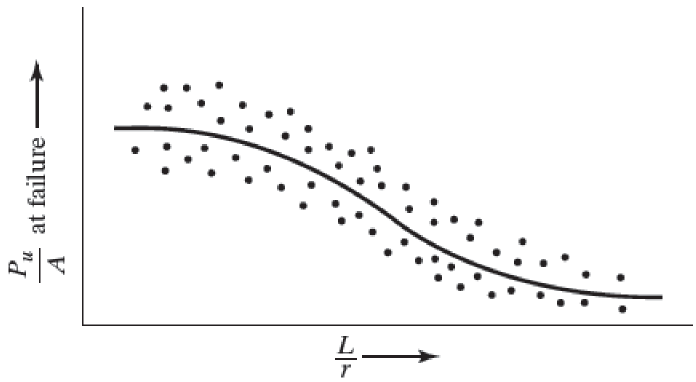


Figure 5-2

Test result curve.

Test result curve

5.4 THE EULER FORMULA

For a column to buckle elastically, it will have to be long and slender. Its buckling load P can be computed with the Euler formula that follows:

$$P = \frac{\pi^2 EI}{L^2}.$$

This formula usually is written in a slightly different form that involves the column's slenderness ratio. Since $r = \sqrt{I/A}$, we can say that $I = Ar^2$. Substituting this value into the Euler formula and dividing both sides by the cross-sectional area, the Euler buckling stress is obtained:

$$\frac{P}{A} = \frac{\pi^2 E}{(L/r)^2} = F_e.$$

Example 5.1 — Analysis of Axially Loaded Compression Members

(a) A W10 × 22 is used as a 15-ft long pin-connected column. Using the Euler expression, determine the column's critical or buckling load. Assume that the steel has a proportional limit of 36 ksi.

(b) Repeat part (a) if the length is changed to 8 ft.

Solution

(a) Using a 15-ft long W10 × 22 ($A = 6.49\ \text{in}^2$, $r_x = 4.27$ in, $r_y = 1.33$ in)

Minimum $r = r_y = 1.33$ in

$$\frac{L}{r} = \frac{(12\ \text{in/ft})(15\ \text{ft})}{1.33\ \text{in}} = 135.34$$

$$\text{Elastic or buckling stress } F_e = \frac{(\pi^2)(29 \times 10^3\ \text{ksi})}{(135.34)^2}$$

$$= 15.63\ \text{ksi} < \text{the proportional limit of 36 ksi}$$

OK column is in elastic range

Elastic or buckling load $= (15.63\ \text{ksi})(6.49\ \text{in}^2) = 101.4$ k

(b) Using an 8-ft long W10 × 22,

$$\frac{L}{r} = \frac{(12\ \text{in/ft})(8\ \text{ft})}{1.33\ \text{in}} = 72.18$$

$$\text{Elastic or buckling stress } F_e = \frac{(\pi^2)(29 \times 10^3\ \text{ksi})}{(72.18)^2} = 54.94\ \text{ksi} > 36\ \text{ksi}$$

∴ column is in inelastic range and Euler equation is not applicable.

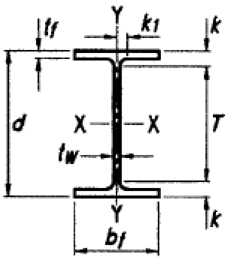


Table 1-1 (continued)
W Shapes
Dimensions

Shape	Area, A	Depth, d		Web Thickness, t_w		Web $\frac{t_w}{2}$	Flange Width, b_f		Flange Thickness, t_f		Distance k_{des}	Distance k_{det}	k_1	T	Workable Gage
	in.²	in.		in.		in.	in.		in.		in.	in.	in.	in.	in.
W10×30	8.84	10.5	10 1/2	0.300	5/16	3/16	5.81	5 3/4	0.510	1/2	0.810	1 1/8	11/16	8 1/4	2 3/4[g]
×26	7.61	10.3	10 3/8	0.260	1/4	1/8	5.77	5 3/4	0.440	7/16	0.740	1 1/16	11/16	↓	↓
×22[c]	6.49	10.2	10 1/8	0.240	1/4	1/8	5.75	5 3/4	0.360	3/8	0.660	15/16	5/8	▼	▼

SUMMARY: ANALYSIS OF COMPRESSION MEMBERS

The Steel Construction Manual **AISC Chapter E, Page 32** limit states that will be considered are:

- **SLENDERNESS OF COMPRESSION ELEMENTS,** AISC Chapter **B4** Table B4.1, Page 16

$$\boldsymbol{\lambda} = \frac{\boldsymbol{b}}{\boldsymbol{t_f}} < \boldsymbol{\lambda_r} \;\; and \;\; \boldsymbol{\lambda} = \frac{\boldsymbol{h}}{\boldsymbol{t_w}} < \boldsymbol{\lambda_r}, \qquad b = \frac{b_f}{2}, \qquad h = d - 2k$$

TABLE B4.1
Limiting Width-Thickness Ratios for Compression Elements

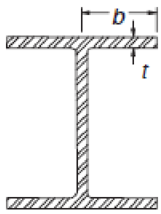
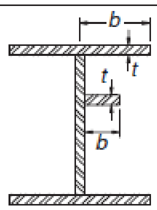
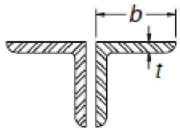
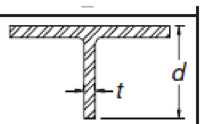
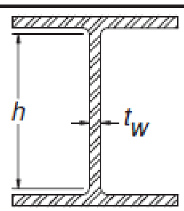
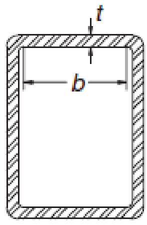
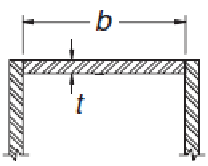
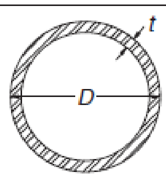
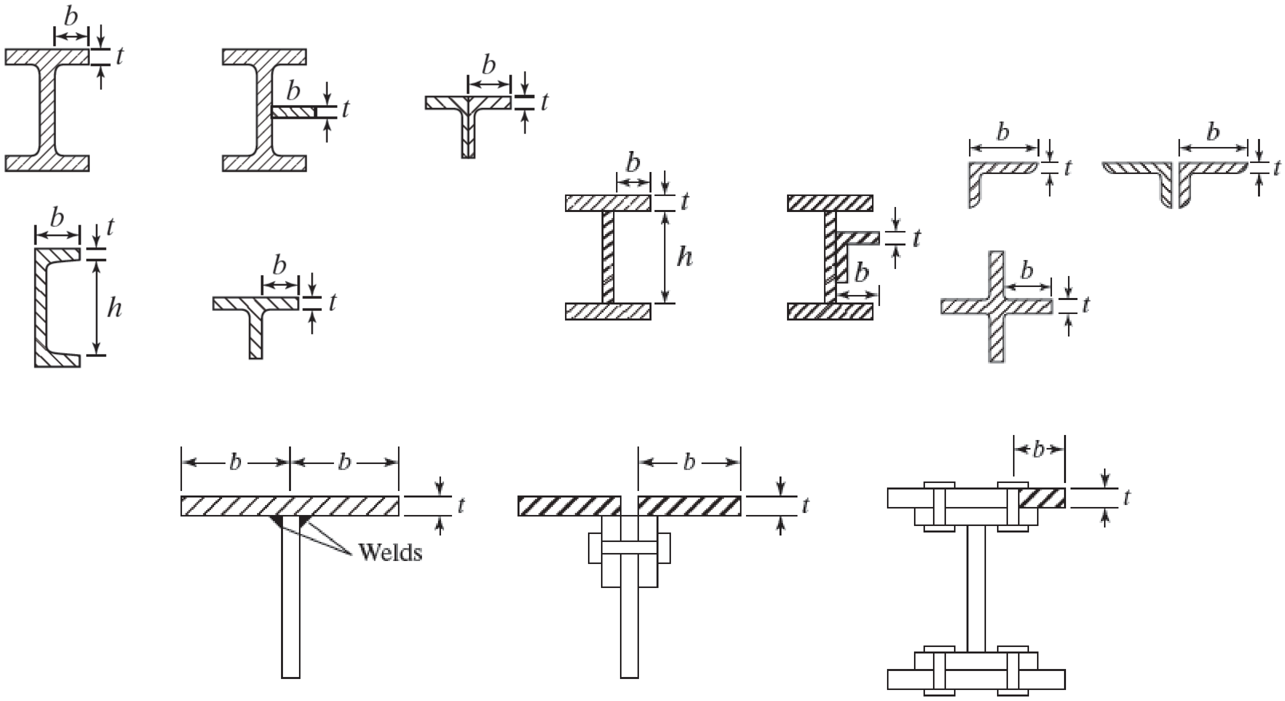
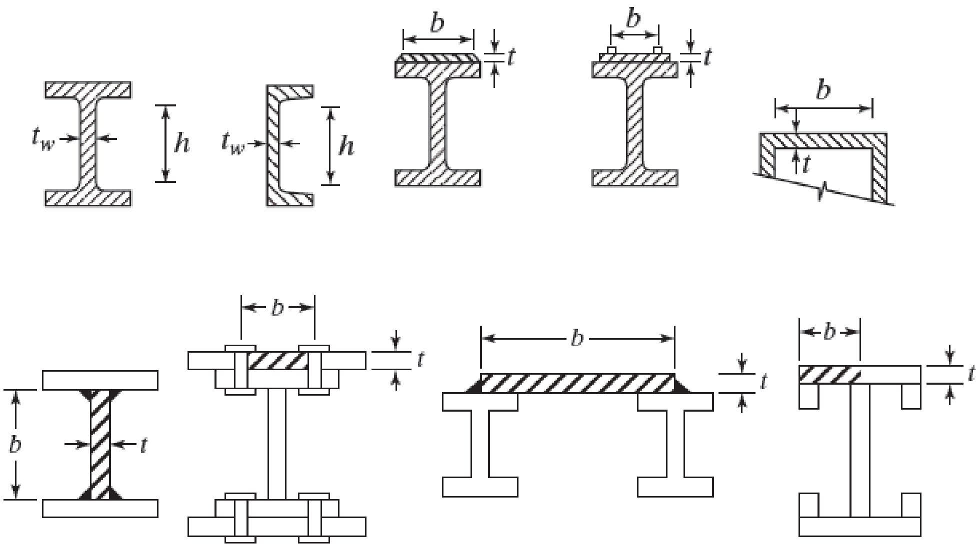
	Case	Description of Element	Width Thickness Ratio	Limiting Width-Thickness Ratios: λ_p (compact)	Limiting Width-Thickness Ratios: λ_r (noncompact)	Example
Unstiffened Elements	3	Uniform compression in flanges of rolled I-shaped sections, plates projecting from rolled I-shaped sections; outstanding legs of pairs of angles in continuous contact and flanges of channels	b/t	NA	$0.56\sqrt{E/F_y}$	
	4	Uniform compression in flanges of built-up I-shaped sections and plates or angle legs projecting from built-up I-shaped sections	b/t	NA	$0.64\sqrt{k_c E/F_y}$[a]	
	5	Uniform compression in legs of single angles, legs of double angles with separators, and all other unstiffened elements	b/t	NA	$0.45\sqrt{E/F_y}$	

TABLE B4.1 (cont.)
Limiting Width-Thickness Ratios for Compression Elements

	Case	Description of Element	Width Thick-ness Ratio	Limiting Width-Thickness Ratios: λ_p (compact)	Limiting Width-Thickness Ratios: λ_r (noncompact)	Example
	8	Uniform compression in stems of tees	d/t	NA	$0.75\sqrt{E/F_y}$	
Elements	10	Uniform compression in webs of doubly symmetric I-shaped sections	h/t_w	NA	$1.49\sqrt{E/F_y}$	
	12	Uniform compression in flanges of rectangular box and hollow structural sections of uniform thickness subject to bending or compression; flange cover plates and diaphragm plates between lines of fasteners or welds	b/t	$1.12\sqrt{E/F_y}$	$1.40\sqrt{E/F_y}$	
	14	Uniform compression in all other stiffened elements	b/t	NA	$1.49\sqrt{E/F_y}$	
	15	Circular hollow sections In uniform compression In flexure	 D/t D/t	 NA $0.07E/F_y$	 $0.11E/F_y$ $0.31E/F_y$	



(a) Unstiffened elements

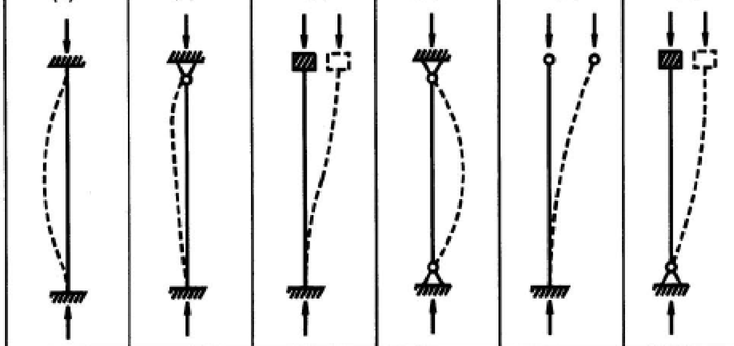


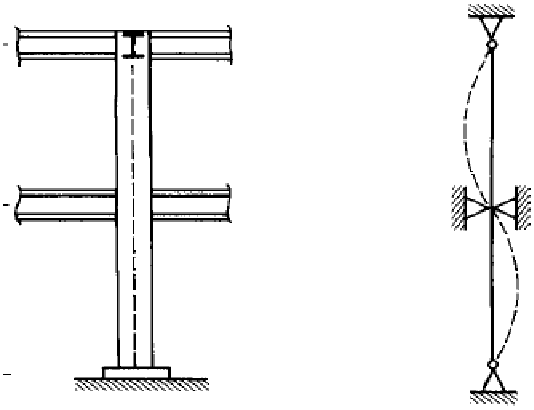
(b) Stiffened elements

- **EFFECTIVE LENGTH FACTOR (K),** AISC Chapter E, Page 26

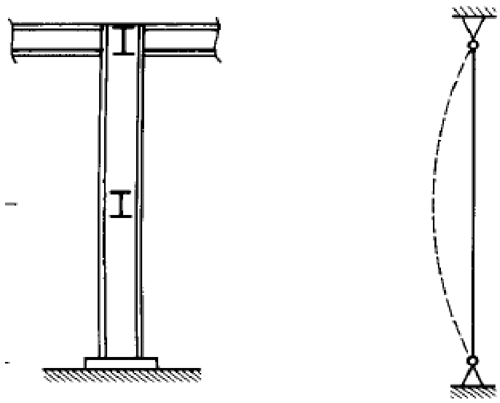
1. **Simple Members**, AISC Chapter Comm. C2, Page 240

TABLE C-C2.2
Approximate Values of Effective Length Factor, *K*

Buckled shape of column is shown by dashed line.	(a)	(b)	(c)	(d)	(e)	(f)
						
Theoretical *K* value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design value when ideal conditions are approximated	0.65	0.80	1.2	1.0	2.10	2.0
End condition code	 *Rotation fixed and translation fixed*; *Rotation free and translation fixed*; *Rotation fixed and translation free*; *Rotation free and translation free*					



Minor Axis Buckling



Major Axis Buckling

2. **Braced Frames (sidesway inhibited)**, AISC Chapter Comm. C2, Page 241

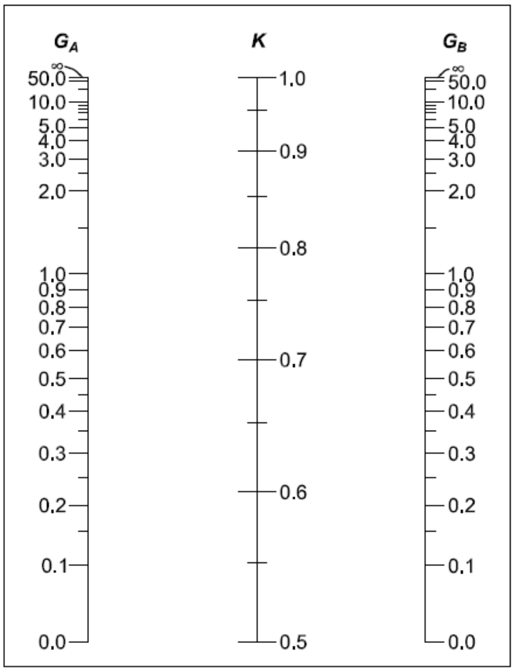
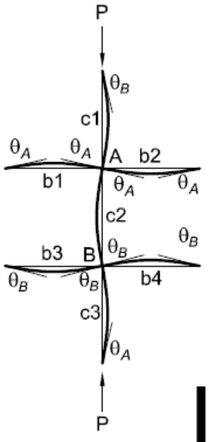
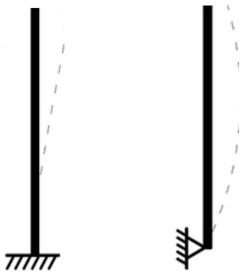


Fig. C-C2.3. Alignment chart—sidesway inhibited (braced frame).





$$G = \frac{\Sigma(E_c I_c / L_c)}{\Sigma(E_g I_g / L_g)} = \frac{\Sigma(EI/L)_c}{\Sigma(EI/L)_g}$$

$G_{Fixed} = 1$ $\quad G_{Pinned} = 10$

$\Sigma E_c I_c / L_c$ = sum of the stiffnesses of all columns at the end of the column under consideration.

$\Sigma E_g I_g / L_g$ = sum of the stiffnesses of all girders at the end of the column under consideration.

$E_c = E_g = E$, the modulus of elasticity of structural steel.

1. If the far end of a girder is fixed, multiply the $(EI/L)_g$ of the member by 2.0.

2. If the far end of the girder is pinned, multiply the $(EI/L)_g$ of the member by 1.5.

3. Moment Frames (sidesway uninhibited), AISC Chapter Comm. C2, Page 242

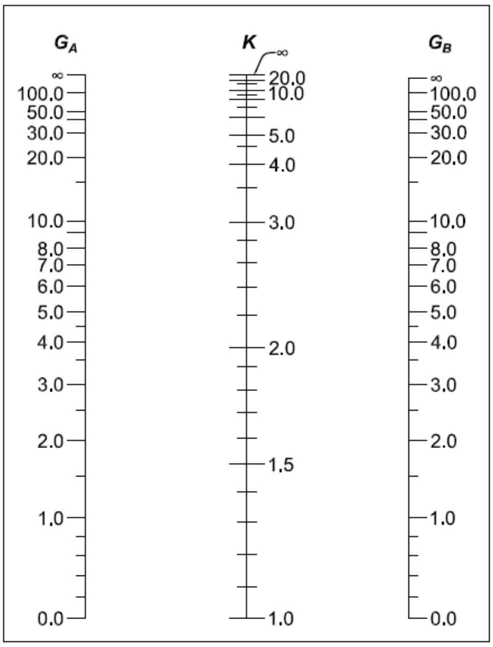
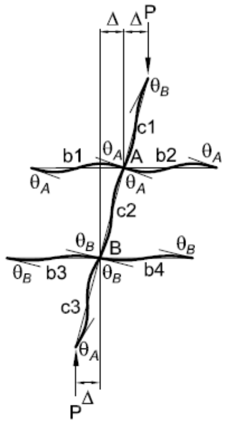
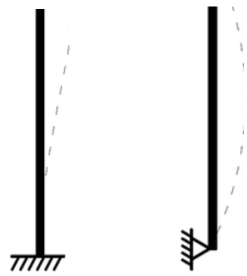


Fig. C-C2.4. Alignment chart—sidesway uninhibited (moment frame).



$$G = \frac{\Sigma(E_c I_c / L_c)}{\Sigma(E_g I_g / L_g)} = \frac{\Sigma(EI/L)_c}{\Sigma(EI/L)_g}$$



$G_{Fixed} = 1$ $G_{Pinned} = 10$

$\Sigma E_c I_c / L_c$ = sum of the stiffnesses of all columns at the end of the column under consideration.

$\Sigma E_g I_g / L_g$ = sum of the stiffnesses of all girders at the end of the column under consideration.

$E_c = E_g = E$, the modulus of elasticity of structural steel.

1. If the far end of a girder is fixed, multiply the $(EI/L)_g$ of the member by 2/3.

2. If the far end of the girder is pinned, multiply the $(EI/L)_g$ of the member by 0.5.

- **SLENDERNESS LIMITATIONS,** AISC Chapter E2, Page 32

E2. SLENDERNESS LIMITATIONS AND EFFECTIVE LENGTH

The effective length factor, K, for calculation of column slenderness, KL/r, shall be determined in accordance with Chapter C,

User Note: For members designed on the basis of compression, the slenderness ratio KL/r preferably should not exceed 200.

- **NOMINAL COMPRESSIVE STRENGTH,** AISC Chapter E3, Page 33

1. By using **AISC Equations E3-1 to E3-4,** Page 33

$$\phi_c P_n = \phi_c F_{cr} A_g = \text{LRFD compression strength } (\phi_c = 0.90)$$

$$\frac{P_n}{\Omega_c} = \frac{F_{cr} A_g}{\Omega_c} = \text{ASD allowable compression strength } (\Omega_c = 1.67)$$

The *nominal compressive strength*, P_n, shall be determined based on the *limit state* of *flexural buckling*.

$$P_n = F_{cr} A_g \quad \text{(E3-1)}$$

The *flexural buckling stress*, F_{cr}, is determined as follows:

(a) When $\frac{KL}{r} \leq 4.71\sqrt{\frac{E}{F_y}}$ (or $F_e \geq 0.44F_y$)

$$F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right] F_y \quad \text{(E3-2)}$$

(b) When $\frac{KL}{r} > 4.71\sqrt{\frac{E}{F_y}}$ (or $F_e < 0.44F_y$)

$$F_{cr} = 0.877 F_e \quad \text{(E3-3)}$$

where

F_e = elastic critical buckling stress determined according to Equation E3-4, Section E4, or the provisions of Section C2, as applicable, ksi (MPa)

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} \quad \text{(E3-4)}$$

2. By using **AISC Table 4-22,** Page 4-318

Table 4–22
Available Critical Stress for Compression Members

	F_y = 35ksi			F_y = 36ksi			F_y = 42ksi			F_y = 46ksi			F_y = 50ksi	
$\frac{Kl}{r}$	F_{cr}/Ω_c ksi ASD	$\phi_c F_{cr}$ ksi LRFD	$\frac{Kl}{r}$	F_{cr}/Ω_c ksi ASD	$\phi_c F_{cr}$ ksi LRFD	$\frac{Kl}{r}$	F_{cr}/Ω_c ksi ASD	$\phi_c F_{cr}$ ksi LRFD	$\frac{Kl}{r}$	F_{cr}/Ω_c ksi ASD	$\phi_c F_{cr}$ ksi LRFD	$\frac{Kl}{r}$	F_{cr}/Ω_c ksi ASD	$\phi_c F_{cr}$ ksi LRFD
1	**21.0**	31.5	1	**21.6**	32.4	1	**25.1**	37.8	1	**27.5**	41.4	1	**29.9**	45.0
2	**21.0**	31.5	2	**21.6**	32.4	2	**25.1**	37.8	2	**27.5**	41.4	2	**29.9**	45.0
3	**20.9**	31.5	3	**21.5**	32.4	3	**25.1**	37.8	3	**27.5**	41.4	3	**29.9**	45.0
4	**20.9**	31.5	4	**21.5**	32.4	4	**25.1**	37.8	4	**27.5**	41.4	4	**29.9**	44.9
5	**20.9**	31.5	5	**21.5**	32.4	5	**25.1**	37.7	5	**27.5**	41.3	5	**29.9**	44.9
6	**20.9**	31.4	6	**21.5**	32.3	6	**25.1**	37.7	6	**27.5**	41.3	6	**29.9**	44.9
7	**20.9**	31.4	7	**21.5**	32.3	7	**25.1**	37.7	7	**27.5**	41.3	7	**29.8**	44.8
8	**20.9**	31.4	8	**21.5**	32.3	8	**25.1**	37.7	8	**27.4**	41.2	8	**29.8**	44.8
9	**20.9**	31.4	9	**21.5**	32.3	9	**25.0**	37.6	9	**27.4**	41.2	9	**29.8**	44.7
10	**20.9**	31.3	10	**21.4**	32.2	10	**25.0**	37.6	10	**27.4**	41.1	10	**29.7**	44.7
11	**20.8**	31.3	11	**21.4**	32.2	11	**25.0**	37.5	11	**27.3**	41.1	11	**29.7**	44.6
12	**20.8**	31.3	12	**21.4**	32.2	12	**24.9**	37.5	12	**27.3**	41.0	12	**29.6**	44.5
13	**20.8**	31.2	13	**21.4**	32.1	13	**24.9**	37.4	13	**27.2**	40.9	13	**29.6**	44.4
14	**20.7**	31.2	14	**21.3**	32.1	14	**24.8**	37.3	14	**27.2**	40.9	14	**29.5**	44.4
15	**20.7**	31.1	15	**21.3**	32.0	15	**24.8**	37.3	15	**27.1**	40.8	15	**29.5**	44.3
16	**20.7**	31.1	16	**21.3**	32.0	16	**24.8**	37.2	16	**27.1**	40.7	16	**29.4**	44.2
17	**20.7**	31.0	17	**21.2**	31.9	17	**24.7**	37.1	17	**27.0**	40.6	17	**29.3**	44.1
18	**20.6**	31.0	18	**21.2**	31.9	18	**24.7**	37.1	18	**27.0**	40.5	18	**29.2**	43.9
19	**20.6**	30.9	19	**21.2**	31.8	19	**24.6**	37.0	19	**26.9**	40.4	19	**29.2**	43.8
20	**20.5**	30.9	20	**21.1**	31.7	20	**24.5**	36.9	20	**26.8**	40.3	20	**29.1**	43.7
21	**20.5**	30.8	21	**21.1**	31.7	21	**24.5**	36.8	21	**26.7**	40.2	21	**29.0**	43.6
22	**20.4**	30.7	22	**21.0**	31.6	22	**24.4**	36.7	22	**26.7**	40.1	22	**28.9**	43.4
23	**20.4**	30.7	23	**21.0**	31.5	23	**24.3**	36.6	23	**26.6**	40.0	23	**28.8**	43.3
24	**20.3**	30.6	24	**20.9**	31.4	24	**24.3**	36.5	24	**26.5**	39.8	24	**28.7**	43.1

$$\phi_c P_n = \phi_c F_{cr} A_g = \text{LRFD compression strength } (\phi_c = 0.90)$$

$$\frac{P_n}{\Omega_c} = \frac{F_{cr} A_g}{\Omega_c} = \text{ASD allowable compression strength } (\Omega_c = 1.67)$$

3. By using **AISC Table 4-1 to Table 4-11,** Page 4-10 to Page 4-157

W14

Table 4-1
Available Strength in Axial Compression, kips
W Shapes

$F_y = 50$ ksi

Shape		W14×											
Wt/ft		730[h]		665[h]		605[h]		550[h]		500[h]		455[h]	
Design		P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
…f gyration r_y	0	6440	9670	5870	8820	5330	8010	4850	7290	4400	6610	4010	6030
	11	6070	9130	5530	8310	5010	7530	4550	6840	4120	6200	3750	5640
	12	6010	9030	5470	8220	4950	7440	4500	6760	4070	6120	3710	5570
	13	5940	8920	5400	8110	4890	7350	4440	6670	4020	6040	3660	5500
	14	5860	8810	5330	8010	4820	7250	4380	6580	3960	5950	3600	5420
	15	5780	8690	5250	7890	4750	7140	4310	6480	3900	5860	3550	5330

Property	730[h]	665[h]	605[h]	550[h]	500[h]	455[h]
L_r (ft)	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
A_g (in.^{2})	215	196	178	162	147	134
I_x (in.^{4})	14300	12400	10800	9430	8210	7190
I_y (in.^{4})	4720	4170	3680	3250	2880	2560
r_y (in.)	4.69	4.62	4.55	4.49	4.43	4.38
Ratio r_x/r_y	1.74	1.73	1.71	1.70	1.69	1.67
$P_{ex}(KL^2)/10^4$ (k-in.^{2})	409000	355000	309000	270000	235000	206000
$P_{ey}(KL^2)/10^4$ (k-in.^{2})	135000	119000	105000	93000	82400	73300

ASD	LRFD
$\Omega_c = 1.67$	$\phi_c = 0.90$

[h] Flange thickness is greater than 2 in. Special requirements may apply per AISC Specification Section A3.1c.

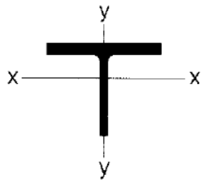
$$(KL)_{Gov.} = \max\ [(KL)_y,\ \ (KL)_{y\,eq}]$$

From **AISC Table 4-1,** Page 4-4

$$(KL)_{y\,eq} = \frac{(KL)_x}{\frac{r_x}{r_y}}$$

F_y = 50 ksi

Table 4–7
Available Strength in Axial Compression, kips
WT Shapes

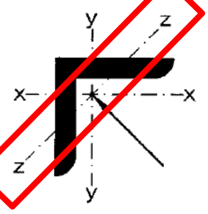


WT18

Shape			WT18×								
Wt/ft			151^c		141^c		131^c		123.5^c		115.5^c	
Design			P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD
...d axis	X-X Axis	0	1210	1820	1050	1580	921	1380	813	1220	708	1060
		10	1170	1760	1020	1530	894	1340	791	1190	690	1040
		12	1150	1730	1010	1510	883	1330	782	1180	682	1030
		14	1130	1700	991	1490	869	1310	771	1160	673	1010
		16	1110	1670	972	1460	854	1280	758	1140	663	997
		18	1090	1630	952	1430	837	1260	744	1120	652	980
		20	1060	1590	930	1400	819	1230	729	1100	639	961
		22	1030	1550	906	1360	799	1200	712	1070	626	940
		24	999	1500	880	1320	778	1170	695	1040	611	919
		26	967	1450	853	1280	756	1140	676	1020	596	896

F_y = 36 ksi

Table 4–11
Available Strength in Axial Compression, kips
Concentrically Loaded Single Angles

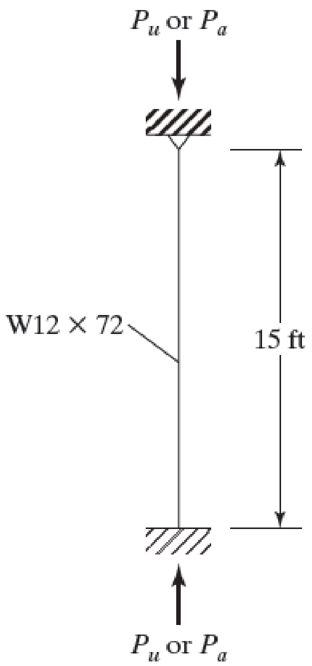


L8

Shape		L8×8×									
		1 1/8		1		7/8		3/4		5/8		9/16^c	
Wt/ft		57.2		51.3		45.3		39.2		33.0		29.8	
Design		P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD
...of gyration, r_z	0	360	541	323	486	285	428	246	369	207	311	179	270
	1	359	539	322	484	284	426	245	368	206	310	179	269
	2	356	534	319	480	281	422	243	365	204	307	177	267
	3	350	526	314	473	277	416	239	359	201	302	175	263
	4	342	515	308	462	271	407	234	352	197	296	171	258
	5	333	500	299	450	263	396	228	342	192	288	167	251
	6	322	484	289	435	255	383	220	331	185	278	162	243
	7	309	464	278	417	245	368	211	318	178	268	156	234

Example 5.2 Analysis of Axially Loaded Compression Members

Determine the LRFD design strength and the ASD allowable strength for the column shown in the figure, if a 50-ksi steel is used.



Solution

(a) Using a W12 × 72 ($A = 21.1\text{ in}^2$, $r_x = 5.31$ in, $r_y = 3.04$ in, $d = 12.3$ in, $b_f = 12.00$ in, $t_f = 0.670$ in, $k = 1.27$ in, $t_w = 0.430$ in)

$$\frac{b}{t} = \frac{12.00/2}{0.670} = 8.96 < 0.56\sqrt{\frac{E}{F_y}} = 0.56\sqrt{\frac{29{,}000}{50}} = 13.49$$

∴ Nonslender unstiffened flange element

$$\frac{h}{t_w} = \frac{d - 2\text{ k}}{t_w} = \frac{12.3 - 2(1.27)}{0.430} = 22.70 < 1.49\sqrt{\frac{E}{F_y}} = 1.49\sqrt{\frac{29{,}000}{50}} = 35.88$$

∴ Nonslender stiffened web element

$K = 0.80$

Obviously, $(KL/r)_y > (KL/r)_x$ and thus controls

$$\left(\frac{KL}{r}\right)_y = \frac{(0.80)(12 \times 15)\text{ in}}{3.04\text{ in}} = 47.37$$

By straight-line interpolation, $\phi_c F_{cr} = 38.19$ ksi, and $\frac{F_{cr}}{\Omega_c} = 25.43$ ksi from Table 4-22 in the Manual using $F_y = 50$ ksi steel.

LRFD	ASD
$\phi_c P_n = \phi_c F_{cr} A_g = (38.19)(21.1) = 805.8\ \text{k}$	$\frac{P_n}{\Omega_c} = \frac{F_{cr} A_g}{\Omega_c} = (25.43)(21.1) = 536.6\ \text{k}$

(b) Entering Table 4-1 in the Manual with $KL\ (0.8)(15) = 12$ ft

LRFD	ASD
$\phi_t P_n = 807\ \text{k}$	$\frac{P_n}{\Omega_c} = 537\ \text{k}$

(c) Elastic critical buckling stress

$$\left(\frac{KL}{r}\right)_y = 47.37 \qquad \text{from part (a)}$$

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} = \frac{(\pi^2)(29{,}000)}{(47.37)^2} = 127.55\ \text{ksi} \qquad \text{(AISC Equation E3-4)}$$

Flexural buckling stress F_{cr}

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000\ \text{ksi}}{50\ \text{ksi}}} = 113.43 > \left(\frac{KL}{r}\right)_y = 47.37$$

$$\therefore\ F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right]F_y = \left[0.658^{\frac{50}{127.55}}\right]50 = 42.43\ ksi \qquad \text{(AISC Equation E3-2)}$$

LRFD $\phi_c = 0.90$	ASD $\Omega_c = 1.67$
$\phi_c F_{cr} = (0.90)(42.43) = 38.19\ \text{ksi}$	$\frac{F_{cr}}{\Omega_c} = \frac{42.43}{1.67} = 25.41\ \text{ksi}$
$\phi_c P_n = \phi_c F_{cr} A = (38.19)(21.1)$	$\frac{P_n}{\Omega_c} = \frac{F_{cr}}{\Omega_c} A = (25.41)(21.1)$
$= 805.8\ \text{k}$	$= 536.2\ \text{k}$

Example 5.3 — Analysis of Axially Loaded Compression Members

An HSS $16 \times 16 \times \frac{1}{2}$ with $F_y = 46$ ksi is used for an 18-ft-long column with simple end supports.

(a) Determine $\phi_c P_n$ and $\frac{P_n}{\Omega_c}$ with the appropriate AISC equations.

(b) Repeat part (a), using Table 4-4 in the AISC Manual.

Solution

(a) Using an HSS

$$16 \times 16 \times \frac{1}{2}(A = 28.3 \text{ in}^2, t_{\text{wall}} = 0.465 \text{ in}, r_x = r_y = 6.31 \text{ in})$$

Calculate $\frac{b}{t}$

b is approximated as the tube size $-2 \times t_{\text{wall}}$

$$\frac{b}{t} = \frac{16 - 2(0.465)}{0.465} = 32.41 < 1.40\sqrt{\frac{E}{F_y}} = 1.40\sqrt{\frac{29{,}000}{46}}$$

$$= 35.15 \quad \therefore \text{ Section has no slender elements}$$

Calculate $\frac{KL}{r}$ and F_{cr}

$K = 1.0$

$$\left(\frac{KL}{r}\right)_x = \left(\frac{KL}{r}\right)_y = \frac{(1.0)(12 \times 18) \text{ in}}{6.31 \text{ in}} = 34.23$$

$$< 4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{46}} = 118.26$$

$\therefore$ Use AISC Equation E3-2 for F_{cr}

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} = \frac{(\pi^2)(29{,}000)}{(34.23)^2} = 244.28 \text{ ksi}$$

$$F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right]F_y = \left[0.658^{\frac{46}{244.28}}\right]46$$

$$= 42.51 \text{ ksi}$$

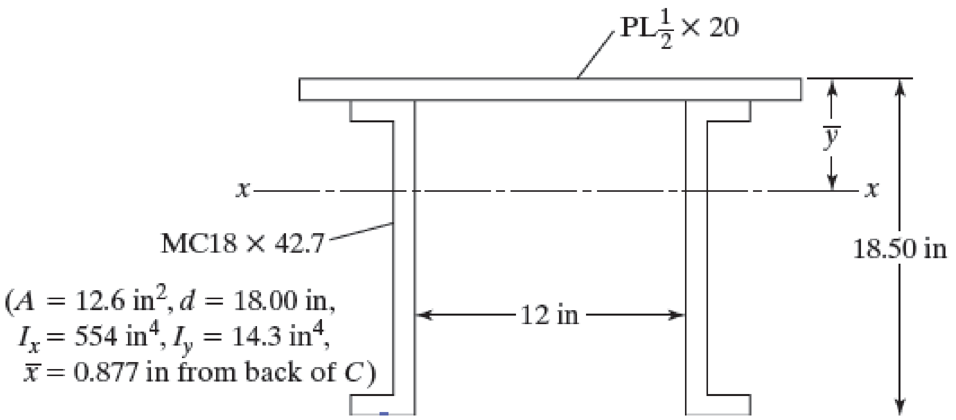
LRFD $\phi_c = 0.90$	ASD $\Omega_c = 1.67$
$\phi_c F_{cr} = (0.90)(42.51) = 38.26$ ksi	$\dfrac{F_{cr}}{\Omega_c} = \dfrac{42.51}{1.67} = 25.46$ ksi
$\phi_c P_n = \phi_c F_{cr} A = (38.26)(28.3)$	$\dfrac{P_n}{\Omega_c} = \dfrac{F_{cr}}{\Omega_c} A = (25.46)(28.3)$
$= 1082$ k	$= 720$ k

(b) From the Manual, Table 4-4

LRFD	ASD
$\phi_c P_n = 1080$ k	$\dfrac{P_n}{\Omega_c} = 720$ k

Example 5.4 — Analysis of Axially Loaded Compression Members

Determine the LRFD design strength and the ASD allowable strength for the axially loaded column shown, if KL=19 ft and 50-ksi steel is used.



Solution

$$A_g = (20)\left(\tfrac{1}{2}\right) + (2)(12.6) = 35.2 \text{ in}^2$$

$$\bar{y} \text{ from top} = \frac{(10)(0.25) + (2)(12.6)(9.50)}{35.2} = 6.87 \text{ in}$$

$$I_x = (2)(554) + (2)(12.6)(9.50 - 6.87)^2 + \left(\frac{1}{12}\right)(20)\left(\frac{1}{2}\right)^3 + (10)(6.87 - 0.25)^2$$

$$= 1721 \text{ in}^4$$

$$I_y = (2)(14.3) + (2)(12.6)(6.877)^2 + \left(\tfrac{1}{12}\right)\left(\tfrac{1}{2}\right)(20)^3 = 1554 \text{ in}^4$$

$$r_x = \sqrt{\frac{1721}{35.2}} = 6.99 \text{ in}$$

$$r_y = \sqrt{\frac{1554}{35.2}} = 6.64 \text{ in}$$

$$\left(\frac{KL}{r}\right)_x = \frac{(12)(19)}{6.99} = 32.62$$

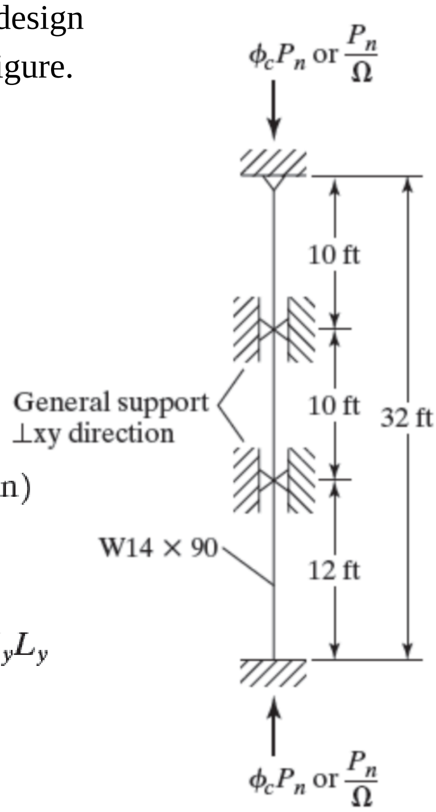
$$\left(\frac{KL}{r}\right)_y = \frac{(12)(19)}{6.64} = 34.34 \leftarrow$$

From the Manual, Table 4-22, we read for $\frac{KL}{r} = 34.34$ that $\phi_c F_{cr} = 41.33$ ksi and $\frac{F_{cr}}{\Omega_c} = 27.47$ ksi, for 50 ksi steel.

LRFD	ASD
$\phi_c P_n = \phi_c F_{cr} A_g = (41.33)(35.2) = 1455 \text{ k}$	$\frac{P_n}{\Omega_c} = \frac{F_{cr} A_g}{\Omega_c} = (27.47)(35.2) = 967 \text{ k}$

Example 5.5 — Analysis of Axially Loaded Compression Members

Determine the LRFD design strength and the ASD allowable design strength for the 50 ksi axially loaded $W14 \times 90$ shown in the figure.



Solution

(a) Using W14 × 90 ($A = 26.5\ \text{in}^2$, $r_x = 6.14$ in, $r_y = 3.70$ in)
Determining effective lengths

$$K_xL_x = (0.80)(32) = 25.6\ \text{ft}$$
$$K_yL_y = (1.0)(10) = 10\ \text{ft} \leftarrow \text{governs for } K_yL_y$$
$$K_yL_y = (0.80)(12) = 9.6\ \text{ft}$$

Computing slenderness ratios

$$\left(\frac{KL}{r}\right)_x = \frac{(12)(25.6)}{6.14} = 50.03 \leftarrow$$
$$\left(\frac{KL}{r}\right)_y = \frac{(12)(10)}{3.70} = 32.43$$

$$\left.\begin{aligned} \phi_c F_{cr} &= 37.49\ \text{ksi} \\ \frac{F_{cr}}{\Omega_c} &= 24.90\ \text{ksi} \end{aligned}\right\} \text{from Manual, Table 4–22, } F_y = 50\ \text{ksi}$$

LRFD	ASD
$\phi_c P_n = \phi_c F_{cr} A_g = (37.49)(26.5)$ $= 993\ \text{k}$	$\frac{P_n}{\Omega_c} = \frac{F_{cr}A_g}{\Omega_c} = (24.90)(26.5) = 660\ \text{k}$

(b) Noting from part (a) solution that there are two different KL values

$$K_xL_x = 25.6 \text{ ft}$$
$$K_yL_y = 10 \text{ ft}$$

We would like to know which of these two values is going to control. This can easily be learned by determining a value of K_xL_x that is equivalent to K_yL_y. The slenderness ratio in the x direction is equated to an equivalent value in the y direction as follows:

$$\frac{K_xL_x}{r_x} = \text{Equivalent}\,\frac{K_yL_y}{r_y}$$

$$\text{Equivalent } K_yL_y = r_y\frac{K_xL_x}{r_x} = \frac{K_xL_x}{\dfrac{r_x}{r_y}}$$

Thus, the controlling K_yL_y for use in the tables is the larger of the real $K_yL_y = 10$ ft, or the equivalent K_yL_y.

$$\frac{r_x}{r_y} \text{ for W14} \times 90 \text{ (from the bottom of Table 4–1 of the Manual)} = 1.66$$

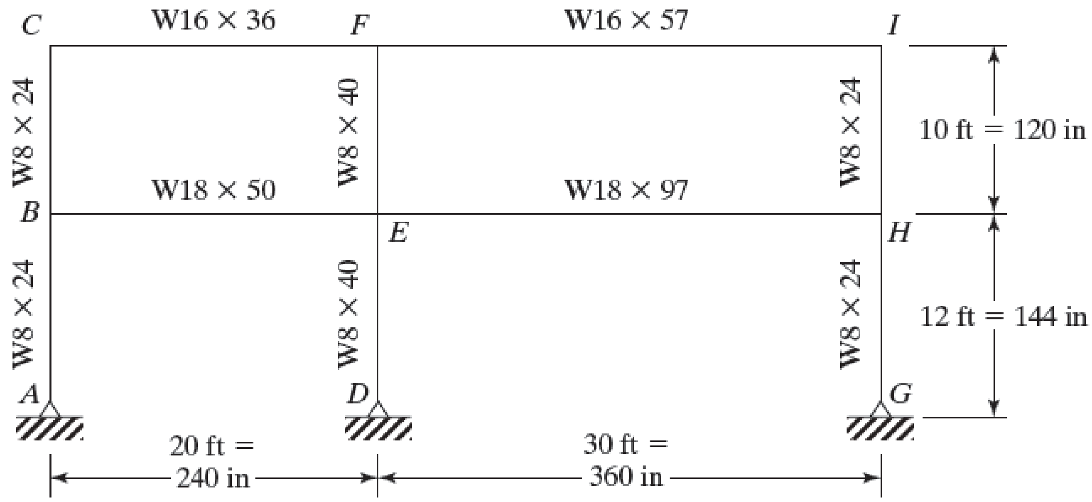
$$\text{Equivalent } K_yL_y = \frac{25.6}{1.66} = 15.42 \text{ ft} > K_yL_y \text{ of 10 ft}$$

From column tables with $K_yL_y = 15.42$ ft, we find by interpolation that

$\phi_cP_n = 991$ k and $\dfrac{P_n}{\Omega_c} = 660$ k.

Example 5.6 — Analysis of Axially Loaded Compression Members

Determine the effective length factor for each of the columns of the frame shown in the figure if the frame is not braced against sidesway.



Solution

Stiffness factors: E is assumed to be 29,000 ksi for all members

	Member	Shape	I	L	I/L
Columns	AB	W8 × 24	82.7	144	0.574
	BC	W8 × 24	82.7	120	0.689
	DE	W8 × 40	146	144	1.014
	EF	W8 × 40	146	120	1.217
	GH	W8 × 24	82.7	144	0.574
	HI	W8 × 24	82.7	120	0.689
Girders	BE	W18 × 50	800	240	3.333
	CF	W16 × 36	448	240	1.867
	EH	W18 × 97	1750	360	4.861
	FI	W16 × 57	758	360	2.106

G factors for each joint:

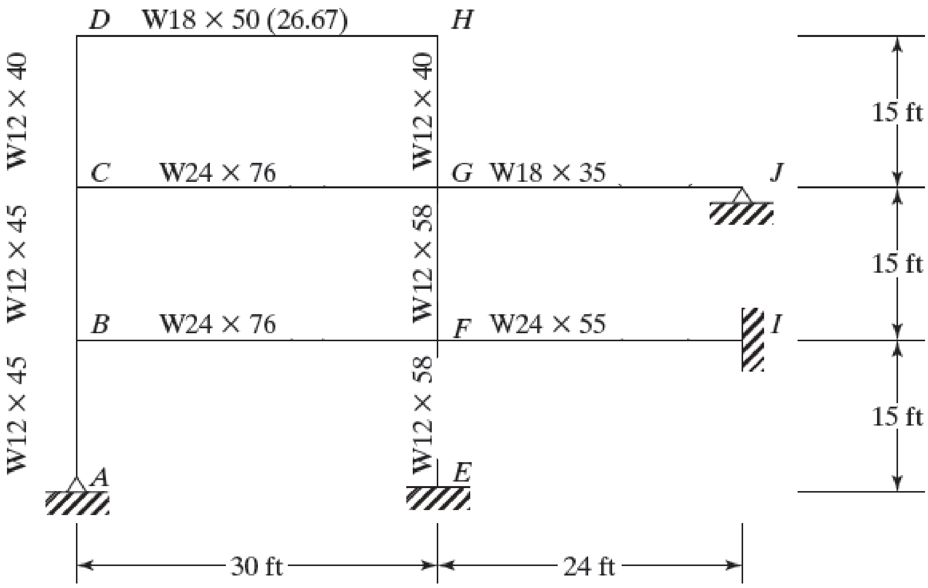
Joint	$\Sigma(I_c/L_c)/\Sigma(I_g/L_g)$	G
A	Pinned Column, $G = 10$	10.0
B	$\frac{0.574 + 0.689}{3.333}$	0.379
C	$\frac{0.689}{1.867}$	0.369
D	Pinned Column, $G = 10$	10.0
E	$\frac{1.014 + 1.217}{(3.333 + 4.861)}$	0.272
F	$\frac{1.217}{(1.867 + 2.106)}$	0.306
G	Pinned Column, $G = 10$	10.0
H	$\frac{0.574 + 0.689}{4.861}$	0.260
I	$\frac{0.689}{2.106}$	0.327

Column K factors from chart

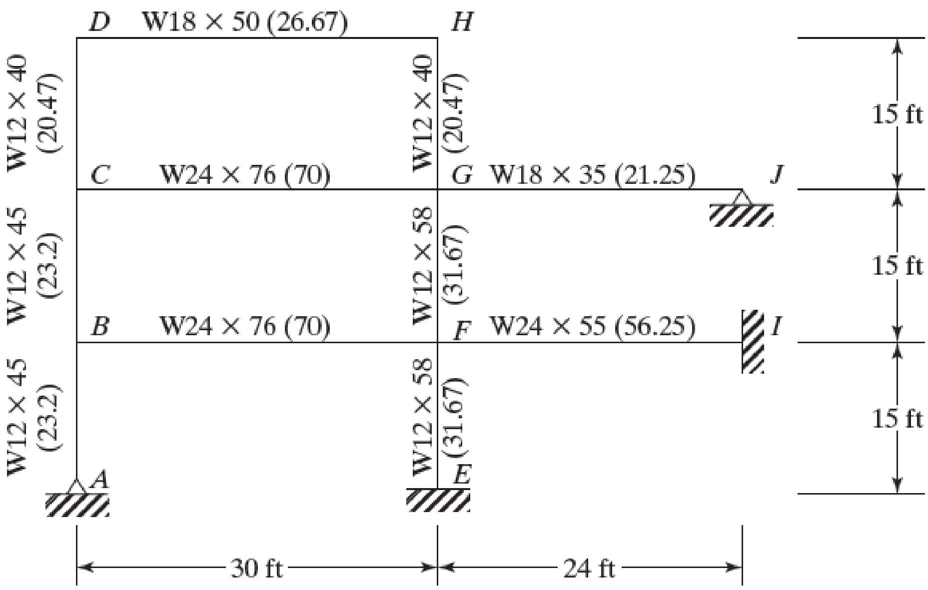
Column	G_A	G_B	K*
AB	10.0	0.379	1.76
BC	0.379	0.369	1.12
DE	10.0	0.272	1.74
EF	0.272	0.306	1.10
GH	10.0	0.260	1.73
HI	0.260	0.327	1.10

Example 5.7 — Analysis of Axially Loaded Compression Members

Determine K factors for each of the columns of the frame shown in the figure.



Solution



1. For member *FI*, the *I*/*L* value is multiplied by 2.0, because its far end is fixed and there is no sidesway on that level.
2. For member, *GJ*, *I*/*L* is multiplied by 1.5, because its far end is pinned and there is no sidesway on that level.

Condition at Far End of Girder	Sidesway Prevented, Multiply by:	Sidesway Uninhibited, Multiply by:
Pinned	1.5	0.5
Fixed against rotation	2.0	0.67

$G_A = 10 \qquad G_E = 1.0$

$$G_B = \frac{23.2 + 23.2}{70} = 0.663 \qquad G_F = \frac{31.67 + 31.67}{70 + (2.0)(56.25)} = 0.347$$

$$G_C = \frac{23.2 + 20.47}{70} = 0.624 \qquad G_G = \frac{31.67 + 20.47}{70 + (1.5)(21.25)} = 0.512$$

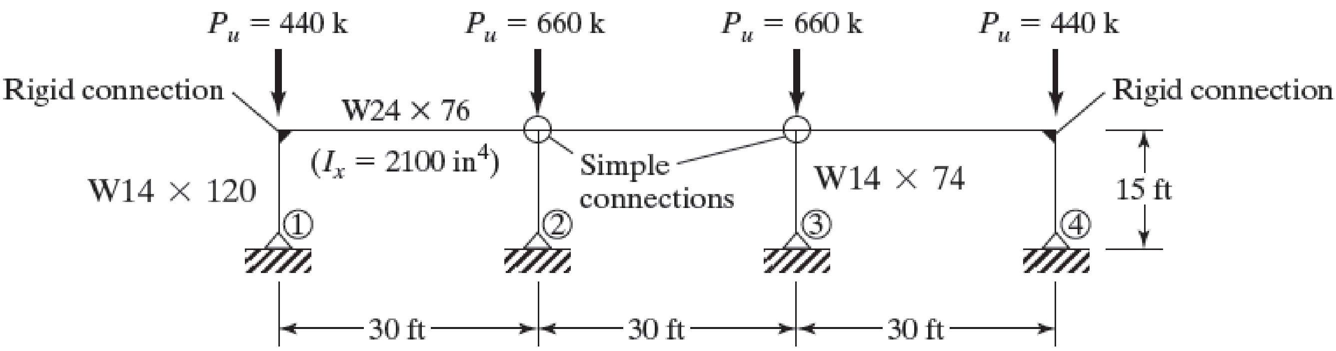
$$G_D = \frac{20.47}{26.67} = 0.768 \qquad G_H = \frac{20.47}{26.67} = 0.768$$

Finally, the K factors are selected from the appropriate alignment chart

Column	*G* Factors	Chart used	*K* Factors
AB	10 and 0.663	no sidesway	0.83
BC	0.663 and 0.624	no sidesway	0.72
CD	0.624 and 0.768	has sidesway	1.23
EF	1.0 and 0.347	no sidesway	0.71
FG	0.347 and 0.512	no sidesway	0.67
GH	0.512 and 0.768	has sidesway	1.21

Example 5.8 — Analysis of Axially Loaded Compression Members

For the frame shown in the figure, which consists of 50 ksi steel, beams are rigidly connected to the exterior columns, while all other connections are simple. The columns are braced top and bottom against sidesway, out of the plane of the frame, so that in that direction. Sidesway is possible in the plane of the frame. Using the LRFD method, check the interior columns assuming that $K_x = K_y = 1.0$ and check the exterior columns with K_x as determined from the alignment chart and $P_u = 1100\ k$ (With this approach to column buckling, the interior columns could carry no load at all, since they appear to be unstable under sidesway conditions.) The end columns are assumed to have no bending moment at the top of the member.



Solution

For Interior Columns

Assume $K_x = K_y = 1.0$, $KL = (1.0)(15) = 15$ ft, $P_u = 660$ k.

W14 × 74; $\phi P_n = 667\ \text{k} > P_u = 660\ \text{k}$

For Exterior Columns

W14 × 120 ($A = 35.3\ \text{in}^2$, $I_x = 1380\ \text{in}^4$, $r_x = 6.24$ in, $r_y = 3.74$ in).

$$G_{top} = \frac{1380/15}{2100/30 \times 0.5} = 2.63$$

(noting that girder stiffness is multiplied by 0.5, since sidesway is permitted and far end of girder is hinged).

$$G_{\text{bottom}} = 10$$

From the chart

$K_x = 2.22$

$$\frac{K_x L_x}{r_x} = \frac{(2.22)(12 \times 15)}{6.24} = 64.04$$

$$\phi_c F_{cr} = 33.38 \text{ ksi}$$

$$\phi_c P_n = (33.38)(35.3) = 1178 \text{ k} > P_u = 1100 \text{ k}$$

Out of plane: $K_y = 1.0$, $P_u = 440$ k

$$\frac{K_y L_y}{r_y} = \frac{1.0\,(12 \times 15)}{3.74} = 48.13$$

$$\phi F_{cr} = 37.96 \text{ ksi}$$

$$\phi_c P_n = (37.96)\,(35.3) = 1340 \text{ k} > P_u = 440 \text{ k}$$

DESIGN OF AXIALLY LOADED COMPRESSION MEMBERS AND BASE PLATES 6

6.1 INTRODUCTION

The design of columns by formulas involves a **trial-and-error** process. The LRFD design stress $\emptyset_c F_{cr}$ and the ASD allowable stress F_{cr}/Ω_c are not known until a column size is selected, and vice versa.

A column size may be assumed, the $\boldsymbol{r}$ values for that section obtained from the Manual or calculated, and the design stress found by substituting into the appropriate column formula. It may then be necessary to try a larger or smaller section.

The effective slenderness ratio ($\boldsymbol{KL/r}$) for the average column of **10-** to **15**-ft (**3-** to **4.6-**m) length will generally fall between about **40** and **60**. For a particular column, a $\boldsymbol{KL/r}$ somewhere in this approximate range is assumed and substituted into the appropriate column equation to obtain the design stress. (To do this, you will first note that the AISC for $\boldsymbol{KL/r}$ values from **0** to **200** has substituted into the equations, with the results shown, in AISC Table 4-22.This greatly expedites our calculations.)

To estimate the effective slenderness ratio for a particular column, the designer may estimate a value a little higher than 40 to 60 if the column is appreciably longer than the 10- to 15-ft range, and vice versa.

A **very heavy factored column load**—say, in the **750-** to **1000-**k range or higher—will require a rather large column for which the radii of gyration will be larger, and the designer may estimate a little smaller value of KL/r.

For **lightly loaded bracing members**, the designer may estimate high slenderness ratios of **100** or more.

Average Column Length	Average Factored Column Load	Estimated KL/r
10- to 15-ft	Less than 750 k	40 to 60
10- to 15-ft	750- to 1000-k range or higher	Less than 40
10- to 15-ft	lightly loaded bracing members	More than 100

DESIGN OF COMPRESSION MEMBERS

The Steel Construction Manual **AISC Chapter E, Page 32** limit states that will be considered are:

I. By using AISC Table 4-22, Trial and Error Procedure, Page 4-318

- **LOAD COMBINATIONS,** AISC Chapter 2, Pages 2-8 and 2-9

For LRFD	For ASD
$P_u = 1.4D$ $P_u = 1.2D + 1.6L$	$P_a = D + L$

- **By using** AISC Table 4-22, **Page 4-318**

$$\text{Assume } \left(\frac{KL}{r}\right) = 50 \text{ to be checked later}$$

Table 4–22 (continued)
Available Critical Stress for Compression Members

Kl/r	$F_y = 35$ksi F_{cr}/Ω_c ksi ASD	$F_y = 35$ksi $\phi_c F_{cr}$ ksi LRFD	Kl/r	$F_y = 36$ksi F_{cr}/Ω_c ksi ASD	$F_y = 36$ksi $\phi_c F_{cr}$ ksi LRFD	Kl/r	$F_y = 42$ksi F_{cr}/Ω_c ksi ASD	$F_y = 42$ksi $\phi_c F_{cr}$ ksi LRFD	Kl/r	$F_y = 46$ksi F_{cr}/Ω_c ksi ASD	$F_y = 46$ksi $\phi_c F_{cr}$ ksi LRFD	Kl/r	$F_y = 50$ksi F_{cr}/Ω_c ksi ASD	$F_y = 50$ksi $\phi_c F_{cr}$ ksi LRFD
48	18.6	28.0	48	19.1	28.7	48	21.8	32.8	48	23.6	35.4	48	25.3	38.0
49	18.5	27.9	49	19.0	28.5	49	21.7	32.6	49	23.4	35.2	49	25.1	37.7
50	18.4	27.7	50	18.9	28.4	50	21.6	32.4	50	23.3	35.0	50	24.9	37.5
51	18.3	27.6	51	18.8	28.3	51	21.4	32.2	51	23.1	34.8	51	24.8	37.2
52	18.3	27.4	52	18.7	28.1	52	21.3	32.0	52	23.0	34.5	52	24.6	36.9
53	18.2	27.3	53	18.6	28.0	53	21.2	31.8	53	22.8	34.3	53	24.4	36.7
54	18.1	27.1	54	18.5	27.8	54	21.0	31.6	54	22.6	34.0	54	24.2	36.4
55	18.0	27.0	55	18.4	27.6	55	20.9	31.4	55	22.5	33.8	55	24.0	36.1
56	17.9	26.8	56	18.3	27.5	56	20.7	31.2	56	22.3	33.5	56	23.8	35.8

- **CALCULATE THE AREA REQUIRED** , AISC Chapter E3, Page 33

$$A\ \text{Reqd} = \frac{P_u}{\phi_c F_{cr}} \qquad A\ \text{Reqd} = \frac{P_a}{F_{cr}/\Omega}$$

LRFD compression strength ($\phi_c = 0.90$)

ASD allowable compression strength ($\Omega_c = 1.67$)

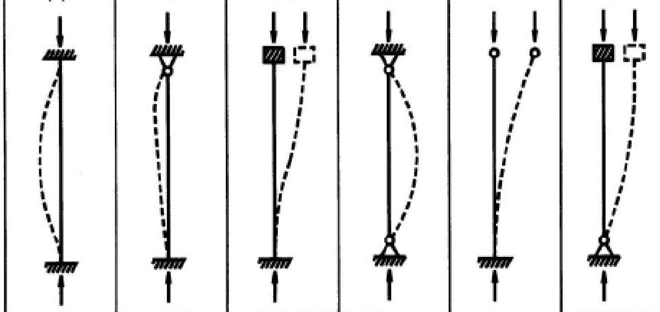
- **SELECT A TRIAL SECTION**

Select a Lightest Available Section with a largest Radius of Gyration

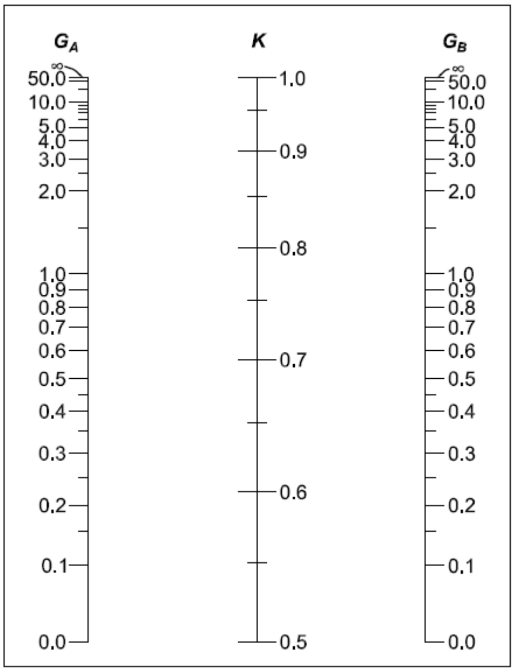
- **CALCULATE THE EFFECTIVE LENGTH FACTOR (K),** AISC Chapter E, Page 26

1. **Simple Members**, AISC Chapter Comm. C2, Page 240

TABLE C-C2.2
Approximate Values of Effective Length Factor, *K*

	(a)	(b)	(c)	(d)	(e)	(f)
Buckled shape of column is shown by dashed line.						
Theoretical *K* value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design value when ideal conditions are approximated	0.65	0.80	1.2	1.0	2.10	2.0
End condition code	*Rotation fixed and translation fixed* *Rotation free and translation fixed* *Rotation fixed and translation free* *Rotation free and translation free*					

2. Braced Frames (Sidesway Inhibited), AISC Chapter Comm. C2, Page 241



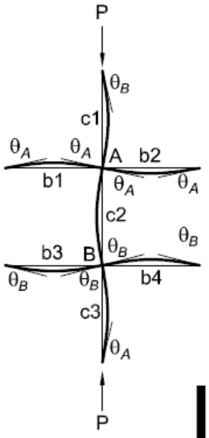
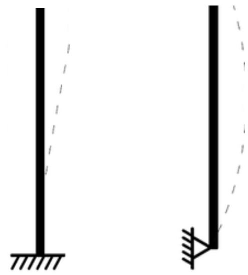


Fig. C-C2.3. Alignment chart—sidesway inhibited (braced frame).



$$G = \frac{\Sigma(E_c I_c / L_c)}{\Sigma\left(E_g I_g / L_g\right)} = \frac{\Sigma(EI/L)_c}{\Sigma(EI/L)_g}$$

$G_{Fixed} = 1 \qquad G_{Pinned} = 10$

$\Sigma E_c I_c / L_c =$ sum of the stiffnesses of all columns at the end of the column under consideration.

$\Sigma E_g I_g / L_g =$ sum of the stiffnesses of all girders at the end of the column under consideration.

$E_c = E_g = E$, the modulus of elasticity of structural steel.

1. If the far end of a girder is fixed, multiply the $(EI/L)_g$ of the member by 2.0.

2. If the far end of the girder is pinned, multiply the $(EI/L)_g$ of the member by 1.5.

3. Moment Frames (Sidesway Uninhibited), AISC Chapter Comm. C2, Page 242

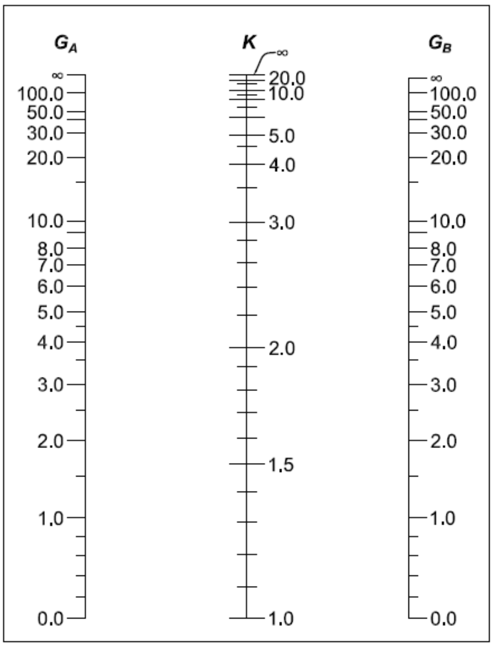
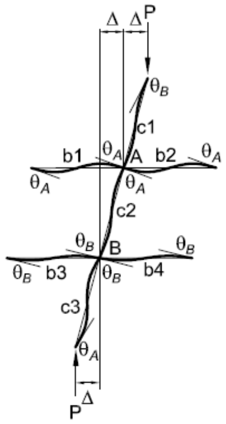


Fig. C-C2.4. Alignment chart—sidesway uninhibited (moment frame).



$$G = \frac{\Sigma(E_c I_c / L_c)}{\Sigma(E_g I_g / L_g)} = \frac{\Sigma(EI/L)_c}{\Sigma(EI/L)_g}$$

$G_{Fixed} = 1$ $\quad G_{Pinned} = 10$

$\Sigma E_c I_c / L_c$ = sum of the stiffnesses of all columns at the end of the column under consideration.

$\Sigma E_g I_g / L_g$ = sum of the stiffnesses of all girders at the end of the column under consideration.

$E_c = E_g = E$, the modulus of elasticity of structural steel.

1. If the far end of a girder is fixed, multiply the $(EI/L)_g$ of the member by 2/3.

2. If the far end of the girder is pinned, multiply the $(EI/L)_g$ of the member by 0.5.

- **CHECK THE SECTION,** by using AISC Table 4-22, **Page 4-318**

Table 4–22
Available Critical Stress for Compression Members

	F_y = 35ksi			F_y = 36ksi			F_y = 42ksi			F_y = 46ksi			F_y = 50ksi	
$\frac{Kl}{r}$	F_{cr}/Ω_c ksi ASD	$\phi_c F_{cr}$ ksi LRFD	$\frac{Kl}{r}$	F_{cr}/Ω_c ksi ASD	$\phi_c F_{cr}$ ksi LRFD	$\frac{Kl}{r}$	F_{cr}/Ω_c ksi ASD	$\phi_c F_{cr}$ ksi LRFD	$\frac{Kl}{r}$	F_{cr}/Ω_c ksi ASD	$\phi_c F_{cr}$ ksi LRFD	$\frac{Kl}{r}$	F_{cr}/Ω_c ksi ASD	$\phi_c F_{cr}$ ksi LRFD
1	21.0	31.5	1	21.6	32.4	1	25.1	37.8	1	27.5	41.4	1	29.9	45.0
2	21.0	31.5	2	21.6	32.4	2	25.1	37.8	2	27.5	41.4	2	29.9	45.0
3	20.9	31.5	3	21.5	32.4	3	25.1	37.8	3	27.5	41.4	3	29.9	45.0
4	20.9	31.5	4	21.5	32.4	4	25.1	37.8	4	27.5	41.4	4	29.9	44.9
5	20.9	31.5	5	21.5	32.4	5	25.1	37.7	5	27.5	41.3	5	29.9	44.9
6	20.9	31.4	6	21.5	32.3	6	25.1	37.7	6	27.5	41.3	6	29.9	44.9
7	20.9	31.4	7	21.5	32.3	7	25.1	37.7	7	27.5	41.3	7	29.8	44.8
8	20.9	31.4	8	21.5	32.3	8	25.1	37.7	8	27.4	41.2	8	29.8	44.8
9	20.9	31.4	9	21.5	32.3	9	25.0	37.6	9	27.4	41.2	9	29.8	44.7
10	20.9	31.3	10	21.4	32.2	10	25.0	37.6	10	27.4	41.1	10	29.7	44.7
11	20.8	31.3	11	21.4	32.2	11	25.0	37.5	11	27.3	41.1	11	29.7	44.6
12	20.8	31.3	12	21.4	32.2	12	24.9	37.5	12	27.3	41.0	12	29.6	44.5
13	20.8	31.2	13	21.4	32.1	13	24.9	37.4	13	27.2	40.9	13	29.6	44.4
14	20.7	31.2	14	21.3	32.1	14	24.8	37.3	14	27.2	40.9	14	29.5	44.4
15	20.7	31.1	15	21.3	32.0	15	24.8	37.3	15	27.1	40.8	15	29.5	44.3
16	20.7	31.1	16	21.3	32.0	16	24.8	37.2	16	27.1	40.7	16	29.4	44.2
17	20.7	31.0	17	21.2	31.9	17	24.7	37.1	17	27.0	40.6	17	29.3	44.1
18	20.6	31.0	18	21.2	31.9	18	24.7	37.1	18	27.0	40.5	18	29.2	43.9
19	20.6	30.9	19	21.2	31.8	19	24.6	37.0	19	26.9	40.4	19	29.2	43.8
20	20.5	30.9	20	21.1	31.7	20	24.5	36.9	20	26.8	40.3	20	29.1	43.7
21	20.5	30.8	21	21.1	31.7	21	24.5	36.8	21	26.7	40.2	21	29.0	43.6
22	20.4	30.7	22	21.0	31.6	22	24.4	36.7	22	26.7	40.1	22	28.9	43.4
23	20.4	30.7	23	21.0	31.5	23	24.3	36.6	23	26.6	40.0	23	28.8	43.3
24	20.3	30.6	24	20.9	31.4	24	24.3	36.5	24	26.5	39.8	24	28.7	43.1

$$\phi_c P_n = \phi_c F_{cr} A_g = \text{LRFD compression strength } (\phi_c = 0.90)$$

$$\frac{P_n}{\Omega_c} = \frac{F_{cr} A_g}{\Omega_c} = \text{ASD allowable compression strength } (\Omega_c = 1.67)$$

If $\phi_c P_n < P_u$ or $\frac{P_n}{\Omega} < P_a \Rightarrow$ **Try the next section, Repeat the Procedure**

II. By using **AISC Table 4-1 to Table 4-11,** Page 4-10 to Page 4-157

- **LOAD COMBINATIONS,** AISC Chapter 2, Pages 2-8 and 2-9

For LRFD	For ASD
$P_u = 1.4D$ $P_u = 1.2D + 1.6L$	$P_a = D + L$

- **By using** AISC Table 4-1 to Table 4-11

Assume $(KL)_y$ to be checked later

- **SELECT A TRIAL SECTION,** by using AISC Table 4-1 to Table 4-11

Select a Lightest Available Section

Table 4–1
Available Strength in Axial Compression, kips
W Shapes

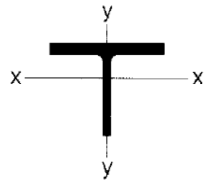
W14

F_y = 50 ksi

Shape		W14×											
Wt/ft		730[h]		665[h]		605[h]		550[h]		500[h]		455[h]	
Design		P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
f gyration, r_y	0	6440	9670	5870	8820	5330	8010	4850	7290	4400	6610	4010	6030
	11	6070	9130	5530	8310	5010	7530	4550	6840	4120	6200	3750	5640
	12	6010	9030	5470	8220	4950	7440	4500	6760	4070	6120	3710	5570
	13	5940	8920	5400	8110	4890	7350	4440	6670	4020	6040	3660	5500
	14	5860	8810	5330	8010	4820	7250	4380	6580	3960	5950	3600	5420
	15	5780	8690	5250	7890	4750	7140	4310	6480	3900	5860	3550	5330

F_y = 50 ksi

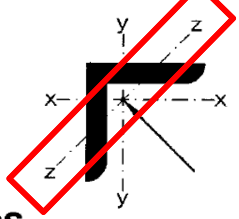
Table 4–7
Available Strength in Axial Compression, kips
WT Shapes



Shape		WT18×									
Wt/ft		151[c]		141[c]		131[c]		123.5[c]		115.5[c]	
Design		P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
d axis, X-X Axis	0	1210	1820	1050	1580	921	1380	813	1220	708	1060
	10	1170	1760	1020	1530	894	1340	791	1190	690	1040
	12	1150	1730	1010	1510	883	1330	782	1180	682	1030
	14	1130	1700	991	1490	869	1310	771	1160	673	1010
	16	1110	1670	972	1460	854	1280	758	1140	663	997
	18	1090	1630	952	1430	837	1260	744	1120	652	980
	20	1060	1590	930	1400	819	1230	729	1100	639	961
	22	1030	1550	906	1360	799	1200	712	1070	626	940
	24	999	1500	880	1320	778	1170	695	1040	611	919
	26	967	1450	853	1280	756	1140	676	1020	596	896

F_y = 36 ksi

Table 4–11
Available Strength in Axial Compression, kips
Concentrically Loaded Single Angles

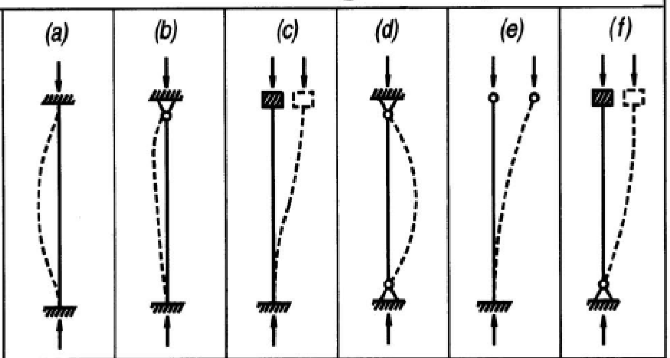


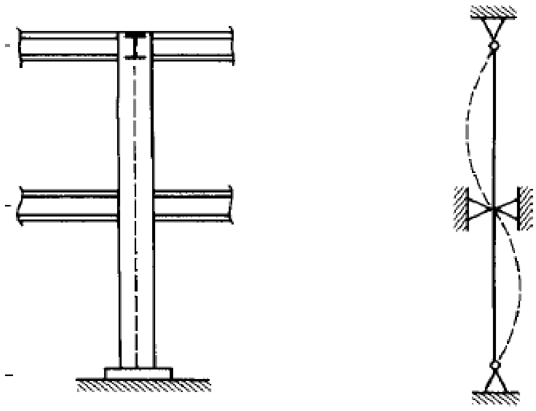
Shape		L8×8×											
		1 1/8		1		7/8		3/4		5/8		9/16[c]	
Wt/ft		57.2		51.3		45.3		39.2		32.0		29.8	
Design		P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
of gyration, r_z	0	360	541	323	486	285	428	246	369	207	311	179	270
	1	359	539	322	484	284	426	245	368	206	310	179	269
	2	356	534	319	480	281	422	243	365	204	307	177	267
	3	350	526	314	473	277	416	239	359	201	302	175	263
	4	342	515	308	462	271	407	234	352	197	296	171	258
	5	333	500	299	450	263	396	228	342	192	288	167	251
	6	322	484	289	435	255	383	220	331	185	278	162	243
	7	309	464	278	417	245	368	211	318	178	268	156	234

- **CALCULATE THE EFFECTIVE LENGTH FACTOR (K),** AISC Chapter E, Page 26

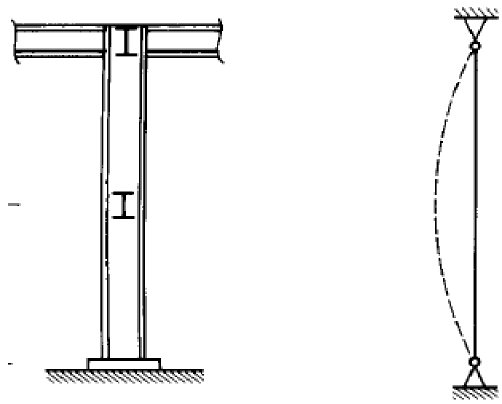
1. **Simple Members**, AISC Chapter Comm. C2, Page 240

TABLE C-C2.2
Approximate Values of Effective Length Factor, *K*

	(a)	(b)	(c)	(d)	(e)	(f)
Buckled shape of column is shown by dashed line.						
Theoretical *K* value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design value when ideal conditions are approximated	0.65	0.80	1.2	1.0	2.10	2.0
End condition code	*Rotation fixed and translation fixed* *Rotation free and translation fixed* *Rotation fixed and translation free* *Rotation free and translation free*					



Minor Axis Buckling



Major Axis Buckling

2. **Braced Frames (Sidesway Inhibited)**, AISC Chapter Comm. C2, Page 241

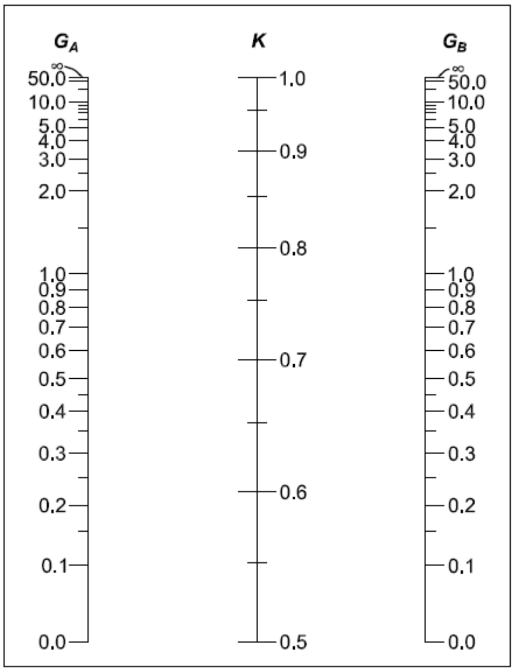
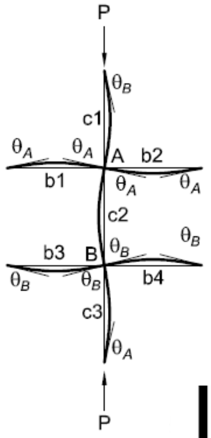


Fig. C-C2.3. Alignment chart—sidesway inhibited (braced frame).



$$G = \frac{\Sigma(E_c I_c / L_c)}{\Sigma(E_g I_g / L_g)} = \frac{\Sigma(EI/L)_c}{\Sigma(EI/L)_g}$$

$G_{Fixed} = 1$ $G_{Pinned} = 10$

$\Sigma E_c I_c / L_c$ = sum of the stiffnesses of all columns at the end of the column under consideration.

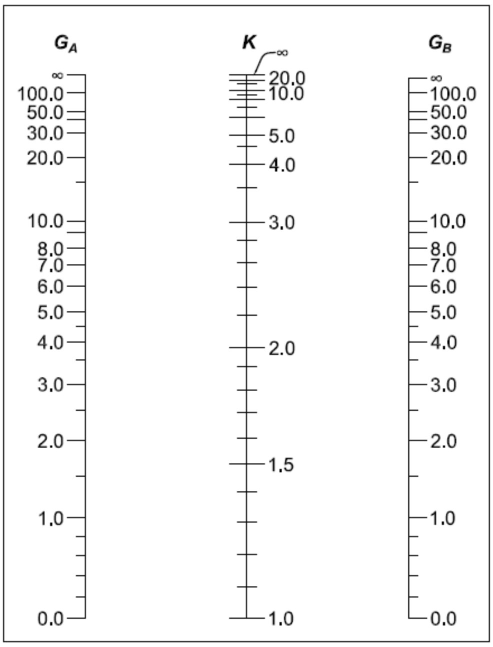
$\Sigma E_g I_g / L_g$ = sum of the stiffnesses of all girders at the end of the column under consideration.

$E_c = E_g = E$, the modulus of elasticity of structural steel.

1. If the far end of a girder is fixed, multiply the $(EI/L)_g$ of the member by 2.0.

2. If the far end of the girder is pinned, multiply the $(EI/L)_g$ of the member by 1.5.

3. **Moment Frames (Sidesway Uninhibited)**, AISC Chapter Comm. C2, Page 242



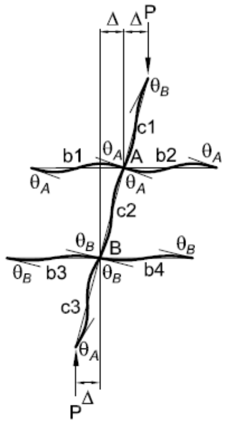


Fig. C-C2.4. Alignment chart—sidesway uninhibited (moment frame).

$$G = \frac{\Sigma(E_c I_c/L_c)}{\Sigma(E_g I_g/L_g)} = \frac{\Sigma(EI/L)_c}{\Sigma(EI/L)_g}$$

$G_{Fixed} = 1$ $G_{Pinned} = 10$

$\Sigma E_c I_c/L_c$ = sum of the stiffnesses of all columns at the end of the column under consideration.

$\Sigma E_g I_g/L_g$ = sum of the stiffnesses of all girders at the end of the column under consideration.

$E_c = E_g = E$, the modulus of elasticity of structural steel.

1. If the far end of a girder is fixed, multiply the $(EI/L)_g$ of the member by 2/3.

2. If the far end of the girder is pinned, multiply the $(EI/L)_g$ of the member by 0.5.

- **CHECK THE EFFECTIVE LENGTH**

$$(KL)_{y\,eq} = \frac{(KL)_x}{\frac{r_x}{r_y}}$$

$$(KL)_y$$

$$(KL)_{Gov.} = \max\ [(KL)_y,\ (KL)_{y\,eq}]$$

If $(KL)_{Gov.} > (KL)_{assumed}$ $\Rightarrow$ Try the next section, Repeat the Procedure

Whenever possible, the designer should provide extra support for the weak direction of a column. Otherwise, the member is inefficient: It has an excess of strength in one direction. When K_xL and K_yL are different, K_yL will control unless r_x/r_y is smaller than K_xL/K_yL. When the two ratios are equal, the column has equal strength in both directions. For most of the W-shapes in the column load tables, r_x/r_y ranges between 1.6 and 1.8, but it is as high as 3.1 for some shapes.

Example 6.1 — Design of Axially Loaded Compression Members

Using $F_y = 50$ ksi, select the lightest W14 available for the service column loads $P_D = 130$ k and $P_L = 210$ k. $KL = 10$ ft.

Solution

LRFD	ASD
$P_u = (1.2)(130\text{ k}) + (1.6)(210\text{ k}) = 492\text{ k}$	$P_a = 130\text{ k} + 210\text{ k} = 340\text{ k}$
Assume $\dfrac{KL}{r} = 50$	Assume $\dfrac{KL}{r} = 50$
Using $F_y = 50$ ksi steel	Using $F_y = 50$ ksi steel
$\phi_c F_{cr}$ from AISC Table 4-22 $= 37.5$ ksi	$\dfrac{F_{cr}}{\Omega_c} = 24.9$ ksi (AISC Table 4-22)
$A\text{ Reqd} = \dfrac{P_u}{\phi_c F_{cr}} = \dfrac{492\text{ k}}{37.5\text{ ksi}} = 13.12\text{ in}^2$	$A\text{ Reqd} = \dfrac{P_a}{F_{cr}/\Omega} = \dfrac{340\text{ k}}{24.9\text{ ksi}} = 13.65\text{ in}^2$
Try W14 × 48 ($A = 14.1\text{ in}^2$, $r_x = 5.85$ in, $r_y = 1.91$ in)	Try W14 × 48 ($A = 14.1\text{ in}^2$, $r_x = 5.85$ in, $r_y = 1.91$ in)
$\left(\dfrac{KL}{r}\right)_y = \dfrac{(12\text{ in/ft})(10\text{ ft})}{1.91\text{ in}} = 62.83$	$\left(\dfrac{KL}{r}\right)_y = \dfrac{(12\text{ in/ft})(10\text{ ft})}{1.91\text{ in}} = 62.83$
$\phi_c F_{cr} = 33.75$ ksi from AISC Table 4-22	
$\phi_c P_n = (33.75\text{ ksi})(14.1\text{ in}^2) = 476\text{ k} < 492\text{ k}$ N.G.	$\dfrac{F_{cr}}{\Omega_c} = 22.43$ ksi from AISC Table 4-22

LRFD	ASD
Try next larger section W14 × 53 ($A = 15.6\ in^2$, $r_y = 1.92\ in$)	$\frac{P_n}{\Omega_c} = (22.43\ ksi)(14.1\ in^2) = 316\ k < 340\ k$ N.G.
$\left(\frac{KL}{r}\right)_y = \frac{(12\ in/ft)(10\ ft)}{1.92\ in} = 62.5$	Try next larger section W14 × 53 ($A = 15.6\ in^2$, $r_y = 1.92\ in$).
$\phi_c F_{cr} = 33.85\ ksi$	$\left(\frac{KL}{r}\right)_y = \frac{(12\ in/ft)(10\ ft)}{1.92\ in} = 62.5$
$\phi_c P_n = (33.85\ ksi)(15.6\ in^2)$	$\frac{F_{cr}}{\Omega_c} = 22.5\ ksi$
$= 528\ k > 492\ k$ **OK**	$\frac{P_n}{\Omega_c} = (22.5\ ksi)(15.6\ in^2) = 351\ k > 340\ k$ **OK**
Use W14 × 53.	**Use W14 × 53.**

Example 6.2 — Design of Axially Loaded Compression Members

Use the AISC column tables (both LRFD and ASD) for the designs to follow.

(a) Select the lightest W section available for the loads, steel, and KL of Example 6-1. $F_y = 50$ ksi.

(b) Select the lightest satisfactory rectangular or square HSS sections for the situation in part (a). $F_y = 46$ ksi.

(c) Select the lightest satisfactory round HSS section, $F_y = 42$ ksi for the situation in part (a).

(d) Select the lightest satisfactory pipe section, $F_y = 35$ ksi, for the situation in part (a).

Solution

Entering Tables with $K_y L_y = 10$ ft, $P_u = 492$ k for LRFD and $P_a = 340$ k for ASD from Example 6-1 solution.

LRFD	ASD
(a) W8 × 48 ($\phi_c P_n = 497\ \text{k} > 492\ \text{k}$) from Table 4-1	(a) W10 × 49 $\left(\dfrac{P_n}{\Omega_c} = 366\ \text{k} > 340\ \text{k}\right)$ from Table 4-1
(b) Rectangular HSS HSS 12 × 8 × $\frac{3}{8}$ @ 47.8 #/ft ($\phi_c P_n = 499\ \text{k} > 492\ \text{k}$) from Table 4-3 Square HSS HSS 10 × 10 × $\frac{3}{8}$ @ 47.8 #/ft ($\phi_c P_n = 513\ \text{k} > 492\ \text{k}$) from Table 4-4	(b) Rectangular HSS HSS 12 × 10 × $\frac{3}{8}$ @ 52.9 #/ft $\left(\dfrac{P_n}{\Omega_c} = 379\ \text{k} > 340\ \text{k}\right)$ from Table 4-3 Square HSS ** HSS 12 × 12 × $\frac{5}{16}$ @ 48.8 #/ft $\left(\dfrac{P_n}{\Omega_c} = 340\ \text{k} = 340\ \text{k}\right)$ from Table 4-4
(c) Round HSS 16.000 × 0.312 @ 52.3 #/ft ($\phi_c P_n = 529\ \text{k} > 492\ \text{k}$) from Table 4-5	(c) Round HSS 16.000 × 0.312 @ 52.3 #/ft $\left(\dfrac{P_n}{\Omega_c} = 352\ \text{k} > 340\ \text{k}\right)$ from Table 4-5
(d) XS Pipe 12 @ 65.5 #/ft ($\phi_c P_n = 530\ \text{k} > 492\ \text{k}$) from Table 4-6	(d) XS Pipe 12 @ 65.5 #/ft $\left(\dfrac{P_n}{\Omega_c} = 353\ \text{k} > 340\ \text{k}\right)$ from Table 4-6

Example 6.3 — Design of Axially Loaded Compression Members

Select the lightest available W12 section, using both the LRFD and ASD methods for the following conditions: $F_y = 50$ ksi, $P_D = 250$ k, $P_L = 400$ k, $K_xL_x = 26$ ft and $K_yL_y = 13$ ft.

(a) By trial and error
(b) Using AISC tables

Solution

(a) Using trial and error to select a section, using the LRFD expressions, and then checking the section with both the LRFD and ASD methods

LRFD	ASD
$P_u = (1.2)(250\text{ k}) + (1.6)(400\text{ k}) = 940\text{ k}$	$P = 250\text{ k} + 400\text{ k} = 650\text{ k}$
Assume $\frac{KL}{r} = 50$	Assume $\frac{KL}{r} = 50$
Using $F_y = 50$ ksi steel $\phi_c F_{cr} = 37.5$ ksi (AISC Table 4-22)	Using $F_y = 50$ ksi steel $\frac{F_{cr}}{\Omega_c} = 24.9$ ksi (AISC Table 4-22)
$A \text{ Reqd} = \frac{940\text{ k}}{37.5\text{ ksi}} = 25.07\text{ in}^2$	$A \text{ Reqd} = \frac{650\text{ k}}{24.9\text{ ksi}} = 26.10\text{ in}^2$
Try W12 × 87 ($A = 25.6\text{ in}^2, r_x = 5.38\text{ in}, r_y = 3.07\text{ in}$)	Try W12 × 87 ($A = 25.6\text{ in}^2, r_x = 5.38\text{ in}, r_y = 3.07\text{ in}$)
$\left(\frac{KL}{r}\right)_x = \frac{(12\text{ in/ft})(26\text{ ft})}{5.38\text{ in}} = 57.99 \leftarrow \therefore \left(\frac{KL}{r}\right)_x$ controls	$\left(\frac{KL}{r}\right)_x = \frac{(12\text{ in/ft})(26\text{ ft})}{5.38\text{ in}} = 57.99 \leftarrow \therefore \left(\frac{KL}{r}\right)_x$ controls
$\left(\frac{KL}{r}\right)_y = \frac{(12\text{ in/ft})(13\text{ ft})}{3.07\text{ in}} = 50.81$	$\left(\frac{KL}{r}\right)_y = \frac{(12\text{ in/ft})(13\text{ ft})}{3.07\text{ in}} = 50.81$
$\phi_c F_{cr} = 35.2$ ksi (Table 4-22)	$\frac{F_{cr}}{\Omega_c} = 23.4$ ksi (Table 4-22)
$\phi_c P_n = (35.2\text{ ksi})(25.6\text{ in}^2)$ $= 901\text{ k} < 940\text{ k}$ **N.G.**	$\frac{P_n}{\Omega_c} = (23.4\text{ ksi})(25.6\text{ in}^2)$ $= 599\text{ k} < 650\text{ k}$ **N.G.**

A subsequent check of the next-larger W12 section, a W12 × 96, shows that it will work for both the LRFD and ASD procedures.

(b) Using AISC Tables. Assuming K_yL_y controls

Enter Table 4-1 with $K_yL_y = 13$ ft, $F_y = 50$ ksi and $P_u = 940$ k

LRFD

Try W12 × 87 $\left(\frac{r_x}{r_y} = 1.75\right)$; $\phi P_n = 954$ k

$$\text{Equivalent } K_yL_y = \frac{K_xL_x}{\frac{r_x}{r_y}}$$

$$= \frac{26}{1.75} = 14.86 > K_yL_y \text{ of 13 ft.} \quad \therefore \quad K_xL_x \text{ controls}$$

Use $K_yL_y = 14.86$ ft and reenter tables

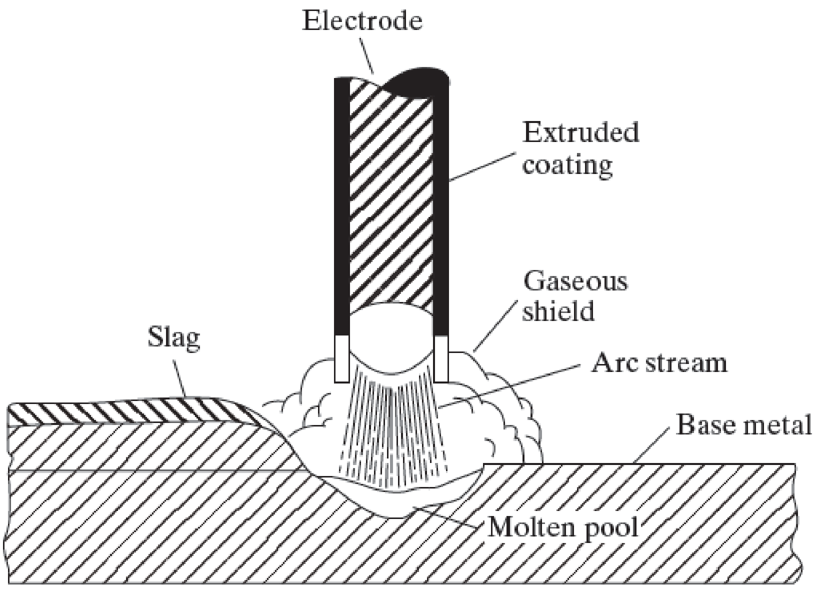
LRFD	ASD
Use W12 × 96 $\phi_c P_n = 994\text{ k} > 940\text{ k}$ **OK**	**Use W12 × 96** $\frac{P_n}{\Omega_c} = 662\text{ k} > 650\text{ k}$ **OK**

WELDED CONNECTIONS 11

11.1 TYPES OF WELDING

The processes that are listed in AWS (The American Welding Society) Specification are:

1. Shielded Metal Arc Welding (SMAW),
2. Submerged Arc Welding (SAW),
3. Gas Metal Arc Welding (GMAW), and
4. Flux-Cored Arc Welding (FCAW). The SMAW process is the usual process applied for hand welding, while the other three are typically automatic or semiautomatic.



Elements of the shielded metal arc welding process (SMAW).

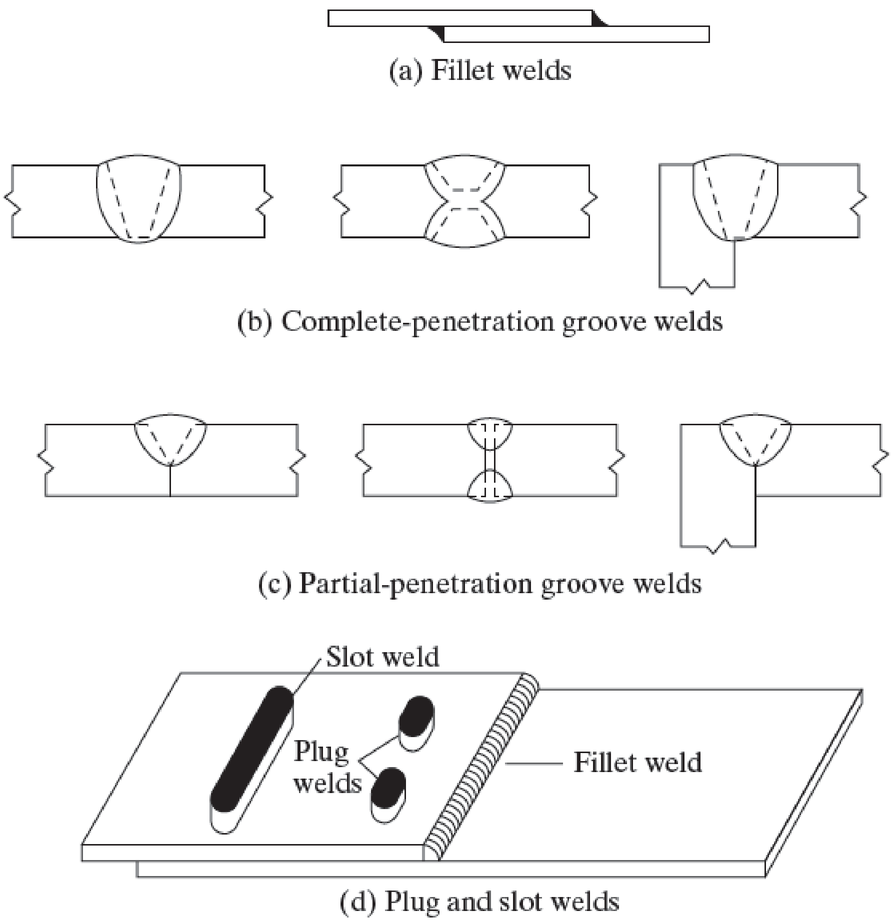
11.2CLASSIFICATION OF WELDS

11.2.1TYPE OF WELD

The two main types of welds are the

- **Fillet Welds.**
- **Groove Welds**.

In addition, there are **plug** and **slot** welds, which are not as common in structural work



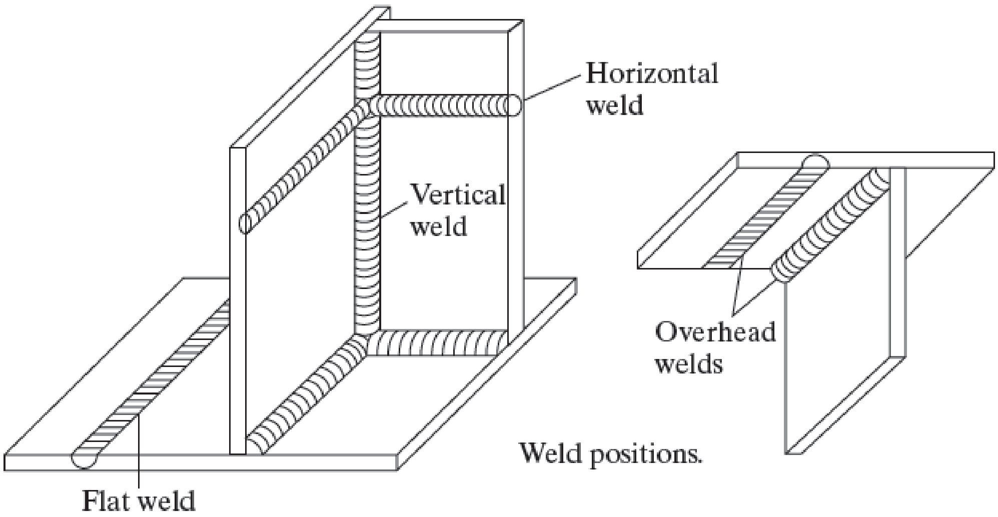
Four types of structural welds.

11.2.2POSITION

Welds are referred to as:

- **Flat.**
- **Horizontal.**
- **Vertical.**
- **Overhead.**

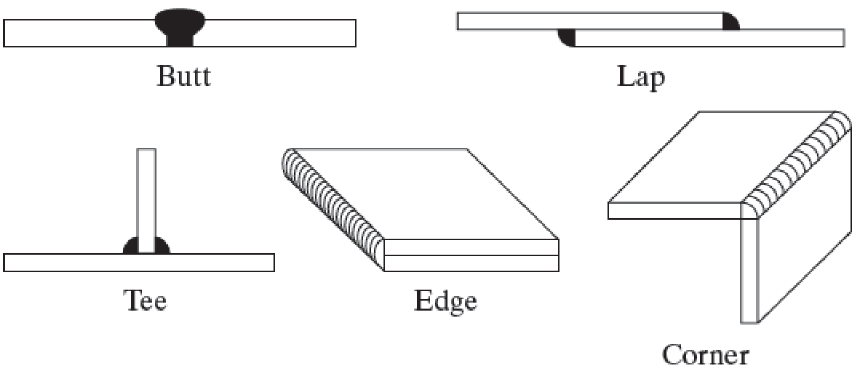
listed in order of their economy, with the flat welds being the most economical and the overhead welds being the most expensive.



Weld positions.

11.2.3TYPE OF JOINT

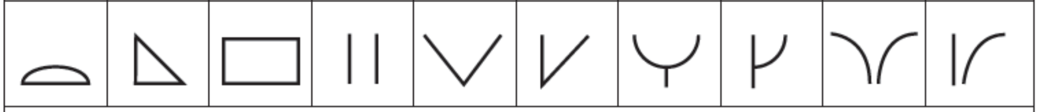
Welds can be further classified according to the type of joint used: **butt, lap, tee, edge, corner**, etc.



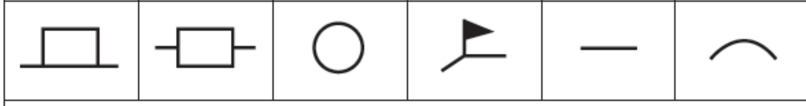
11.3WELDING SYMB

Prequalified Welded Joints

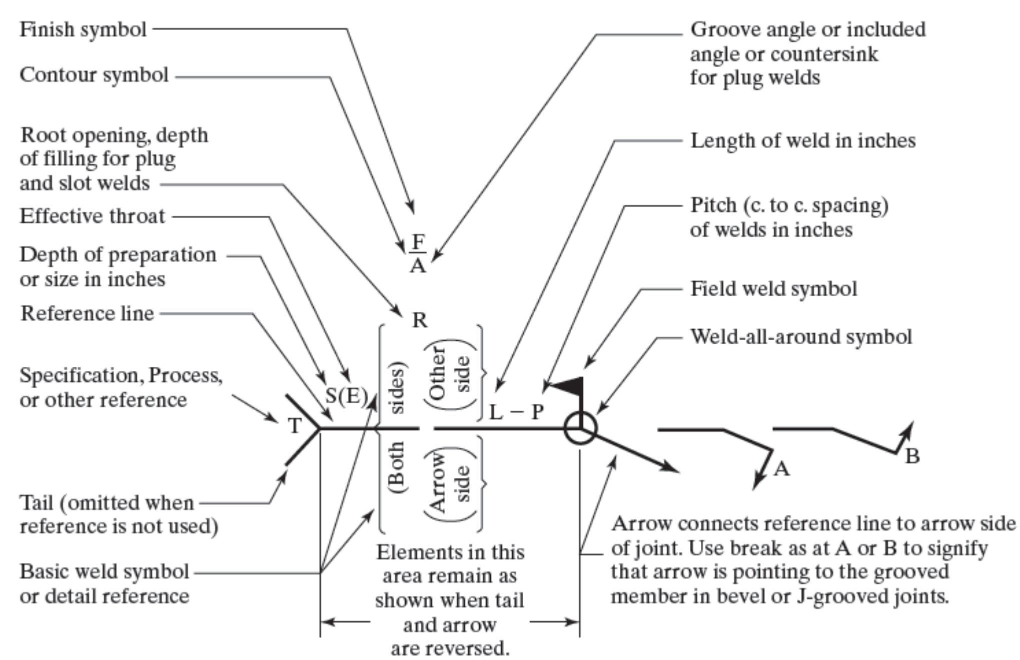
Basic Weld Symbols									
			Groove or Butt						
Back	Fillet	Plug or Slot	Square	V	Bevel	U	J	Flare V	Flare Bevel



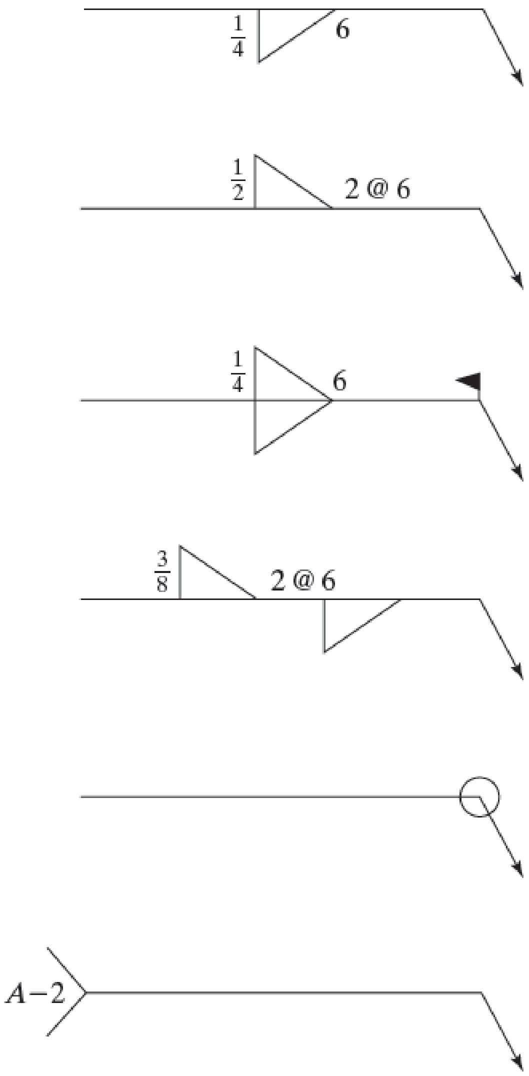
Supplementary Weld symbols						
				Contour		For other basic and supplementary weld symbols, see AWS A2.4
Backing	Spacer	Weld All Around	Field Weld	Flush	Convex	



Standard Location of Elements of a Welding Symbol



Note:
Size, weld symbol, length of weld, and spacing must read in that order, from left to right, along the reference line. Neither orientation of reference nor location of the arrow alters this rule.
The perpendicular leg of , weld symbols must be at left.
Dimensions of fillet welds must be shown on both the arrow side and the other side.
Symbols apply between abrupt changes in direction of welding unless governed by the "all around" symbol or otherwise dimensioned.
These symbols do not explicitly provide for the case that frequently occurs in structural work, where duplicate material (such as stiffeners) occurs on the far side of a web or gusset plate. The fabricating industry has adopted this convention: that when the billing of the detail material discloses the existence of a member on the far side as well as on the near side, the welding shown for the near side shall be duplicated on the far side.



Fillet weld on near side (arrow side). Size of weld ($\frac{1}{4}$ in) is given to left of weld symbol and length (6 in) to right of symbol.

$\frac{1}{2}$-in fillet weld on far side, 2-in-long intermittent welds 6 in on centers.

$\frac{1}{4}$-in fillet field weld on both sides and 6 in long. As welds same on both sides not necessary; but permissible to give sizes on both sides of line. The flag indicates it to be a field weld.

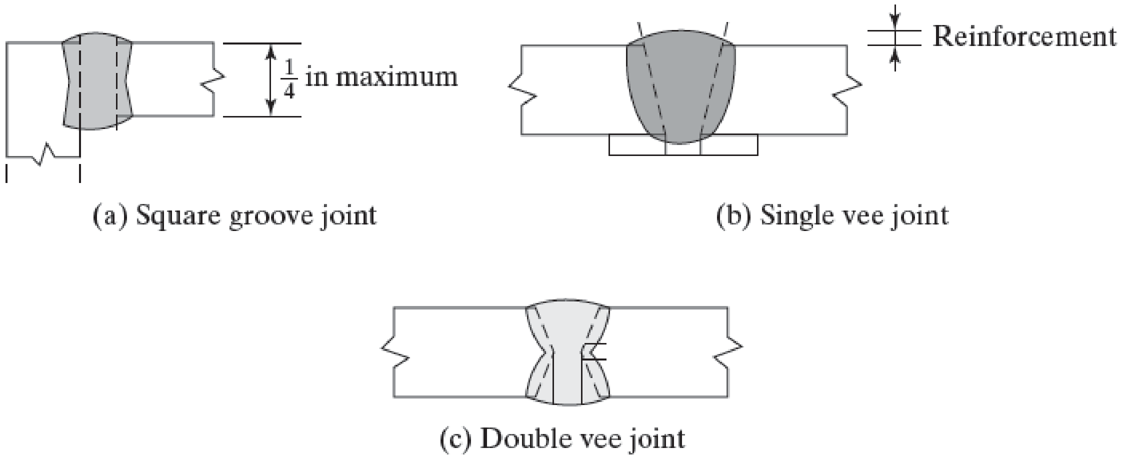
A staggered intermittent $\frac{3}{8}$-in fillet weld 2 in long, 6 in on centers, both sides.

Weld all around joint symbol.

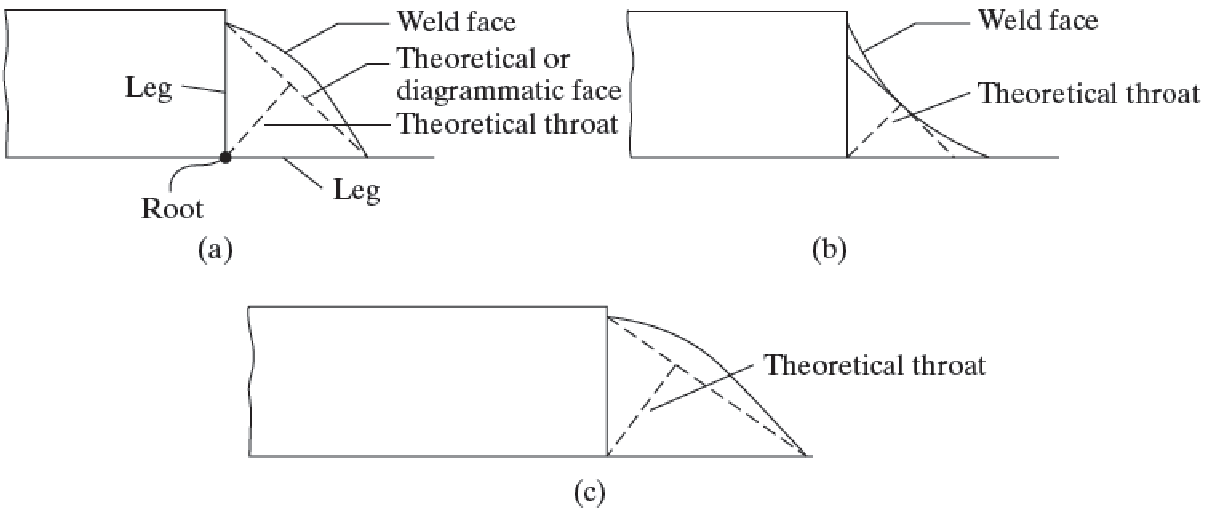
The tail used to indicate reference to a certain specification or process.

Sample weld symbols.

11.4 GROOVE WELDS



11.5 FILLET WELDS



(a) Convex surface. (b) Concave surface. (c) Unequal leg fillet weld.

11.6STRENGTH OF WELDS

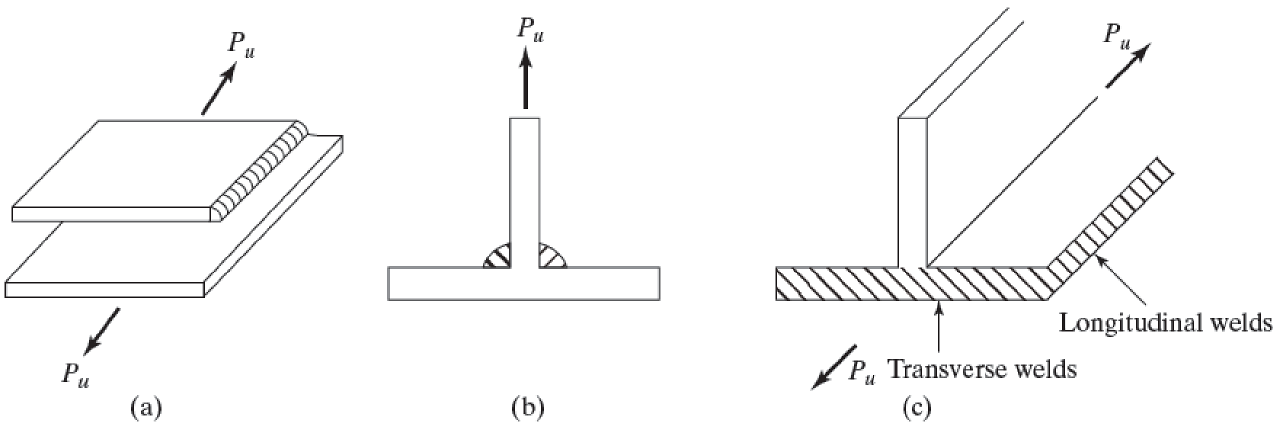
The stress in a fillet weld is usually said to equal the load divided by the effective throat area of the weld, with no consideration given to the direction of the load.

Tests have shown, however, that **transversely loaded fillet welds are appreciably stronger than ones loaded parallel to the weld's axis**.

Transverse fillet welds are **stronger** for two reasons:

First, they are more uniformly stressed over their entire lengths, while longitudinal fillet welds are stressed unevenly, due to varying deformations along their lengths;

second, tests show that failure occurs at angles other than 45°,giving them larger effective throat areas.



(a) Longitudinal fillet weld. (b) Transverse fillet weld. (c) Transverse and longitudinal welds.

The method of determining the strength of fillet welds along their longitudinal axes, regardless of the load directions, is usually used to simplify computations. It is rather common for designers to determine the strength of all fillet welds by assuming that the loads are applied in the longitudinal direction.

11.7 AISC REQUIREMENTS

Table J2.5 of the AISC Specification, provides nominal strengths for various types of welds, including fillet welds, plug and slot welds, and complete-penetration and partial-penetration groove welds.

The design strength $\emptyset R_n$ of a particular weld and the allowable strength R_n/Ω of welded joints shall be the lower value of the base material strength determined according to the limit states of tensile rupture and shear rupture, and the weld metal strength determined according to the limit state of rupture by the expressions to follow:

For the base metal, the nominal strength is

$$R_n = F_{nBM}A_{BM} \qquad \text{(AISC Equation J2-2)}$$

For the weld metal, the nominal strength is

$$R_n = F_{nw}A_{we} \qquad \text{(AISC Equation J2-3)}$$

Available Strength of Welded Joints, ksi (MPa)					
Load Type and Direction Relative to Weld Axis	**Pertinent Metal**	**ϕ and Ω**	**Nominal Strength (F_{nBM} or F_{nw}) ksi (MPa)**	**Effective Area (A_{BM} or A_{we}) in^2 (mm^2)**	**Required Filler Metal Strength Level[a][b]**
COMPLETE-JOINT-PENETRATION GROOVE WELDS					
Tension Normal to weld axis	Strength of the joint is controlled by the base metal.				Matching filler metal shall be used. For T and corner joints with backing left in place, notch tough filler metal is required. See Section J2.6.
Compression Normal to weld axis	Strength of the joint is controlled by the base metal.				Filler metal with a strength level equal to or one strength level less than matching filler metal is permitted.

<table>
<tr><td>Tension or Compression
Parallel to weld axis</td><td colspan="4">Tension or compression in parts joined parallel to a weld need not be considered in design of welds joining the parts.</td><td>Filler metal with a strength level equal to or less than matching filler metal is permitted.</td></tr>
<tr><td>Shear</td><td colspan="4">Strength of the joint is controlled by the base metal.</td><td>Matching filler metal shall be used. [c]</td></tr>
<tr><td colspan="6">PARTIAL-JOINT-PENETRATION GROOVE WELDS INCLUDING FLARE VEE GROOVE AND FLARE BEVEL GROOVE WELDS</td></tr>
<tr><td rowspan="2">Tension
Normal to weld axis</td><td>Base</td><td>$\phi = 0.75$
$\Omega = 2.00$</td><td>F_u</td><td>Effective Area</td><td rowspan="10">Filler metal with a strength level equal to or less than matching filler metal is permitted.</td></tr>
<tr><td>Weld</td><td>$\phi = 0.80$
$\Omega = 1.88$</td><td>$0.60F_{EXX}$</td><td>See J2.1a</td></tr>
<tr><td>Compression
Column to Base Plate and column splices designed per J1.4(a)</td><td colspan="4">Compressive stress need not be considered in design of welds joining the parts.</td></tr>
<tr><td rowspan="2">Compression
Connections of members designed to bear other than columns as described in J1.4(b)</td><td>Base</td><td>$\phi = 0.90$
$\Omega = 1.67$</td><td>F_y</td><td>See J4</td></tr>
<tr><td>Weld</td><td>$\phi = 0.80$
$\Omega = 1.88$</td><td>$0.60F_{EXX}$</td><td>See J2.1a</td></tr>
<tr><td rowspan="2">Compression
Connections not finished-to-bear</td><td>Base</td><td>$\phi = 0.90$
$\Omega = 1.67$</td><td>F_y</td><td>See J4</td></tr>
<tr><td>Weld</td><td>$\phi = 0.80$
$\Omega = 1.88$</td><td>$0.90F_{EXX}$</td><td>See J2.1a</td></tr>
<tr><td>Tension or Compression
Parallel to weld axis</td><td colspan="4">Tension or compression in parts joined parallel to a weld need not be considered in design of welds joining the parts.</td></tr>
<tr><td rowspan="2">Shear</td><td>Base</td><td colspan="3">Governed by J4</td></tr>
<tr><td>Weld</td><td>$\phi = 0.75$
$\Omega = 2.00$</td><td>$0.60F_{EXX}$</td><td>See J2.1a</td></tr>
</table>

TABLE 14.1 Continued

Load Type and Direction Relative to Weld Axis	Pertinent Metal	ϕ and Ω	Nominal Strength (F_{nBM} or F_{nw}) ksi (MPa)	Effective Area (A_{BM} or A_{we}) in^2 (mm^2)	Required Filler Metal Strength Level[a][b]
FILLET WELDS INCLUDING FILLETS IN HOLES AND SLOTS AND SKEWED T-JOINTS					
Shear	Base	Governed by J4			Filler metal with a strength level equal to or less than matching filler metal is permitted.
	Weld	$\phi = 0.75$ $\Omega = 2.00$	$0.60F_{EXX}^{[d]}$	See J2.2a	
Tension or Compression Parallel to weld axis	Tension or compression in parts joined parallel to a weld need not be considered in design of welds joining the parts.				
PLUG AND SLOT WELDS					
Shear Parallel to faying surface on the effective area	Base	Governed by J4			Filler metal with a strength level equal to or less than matching filler metal is permitted.
	Weld	$\phi = 0.75$ $\Omega = 2.00$	$0.60F_{EXX}$	J2.3a	

[a] For matching weld metal see AWS D1.1, Section 3.3.
[b] Filler metal with a strength level one strength level greater than matching is permitted.
[c] Filler metals with a strength level less than matching may be used for groove welds between the webs and flanges of built-up sections transferring shear loads, or in applications where high restraint is a concern. In these applications, the weld joint shall be detailed and the weld shall be designed using the thickness of the material as the effective throat, $\phi = 0.80$, $\Omega = 1.88$ and $0.60F_{EXX}$ as the nominal strength.
[d] Alternatively, the provisions of J2.4(a) are permitted, provided the deformation compatibility of the various weld elements is considered. Alternatively, Sections J2.4(b) and (c) are special applications of J2.4(a) that provide for deformation compatibility.

Source: AISC Specification, Table J2.5, p. 16.1–114 and 16.1–115, June 22, 2010.

In the preceding equations,

F_{nBM} = the nominal stress of the base metal, ksi
F_{nw} = the nominal stress of the weld metal, ksi
A_{BM} = effective area of the base metal, in^2
A_{we} = effective area of the weld, in^2

The filler metal electrodes for shielded arc welding are listed as E60XX, E70XX, etc. In this classification,

- The letter **E** represents an **electrode**,
- The first set of digits (60, 70, 80, 90, 100, or 110) indicates the **minimum tensile strength** of the weld, in **ksi**.

In addition to the nominal stresses given in **Table J2.5 of the AISC Specification**, there are several other provisions applying to welding given in Section J2.2b of the LRFD Specification. Among the more important are the following:

1. The minimum length of a fillet weld may not be less than four times the nominal leg size of the weld. Should its length actually be less than this value, the weld size considered effective must be reduced to one-quarter of the weld length.
2. The **maximum size of a fillet weld** along edges of material less than **1/4 in thick** equals the **material thickness**. For **thicker material**, it may not be larger than the material thickness **less 1/16 in**, unless the weld is specially built out to give a fullthroat thickness.
3. The minimum permissible size fillet welds of the AISC Specification are given in **Table J2.4 of the AISC Specification**. They vary from 1/8 in for 1/4 in or thinner material up to 5/16 in for material over 3/4 in in thickness. The smallest practical weld size is about 1/8 in, and the most economical size is probably about 1/4 or 5/16 in. The 5/16-in weld is about the largest size that can be made in one pass with the shielded metal arc welded process (SMAW); with the submerged arc process (SAW),1/2 in is the largest size.

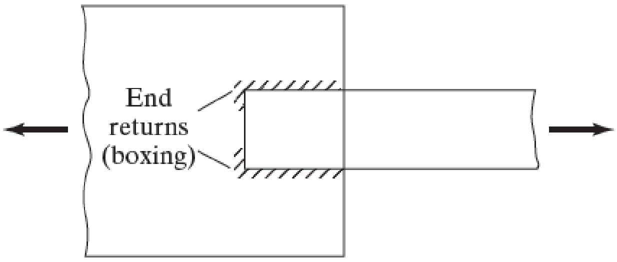
Minimum Size of Fillet Welds

Material Thickness of Thinner Part Joined, in (mm)	Minimum Size of Fillet Weld,[a] in (mm)
To $\frac{1}{4}$ (6) inclusive	$\frac{1}{8}$ (3)
Over $\frac{1}{4}$ (6) to $\frac{1}{2}$ (13)	$\frac{3}{16}$ (5)
Over $\frac{1}{2}$ (13) to $\frac{3}{4}$ (19)	$\frac{1}{4}$ (6)
Over $\frac{3}{4}$ (19)	$\frac{5}{16}$ (8)

[a] Leg dimension of fillet welds. Single pass welds must be used. See Section J2.2b of the LRFD Specification for maximum size of fillet welds.

Source: AISC Specification, Table J2.4, p. 16.1–111, June 22, 2010.

4. Sometimes, end returns or boxing is used at the end of fillet welds. In the past, such practices were recommended to provide better fatigue resistance and to make sure that weld thicknesses were maintained over their full lengths.



5. When longitudinal fillet welds are used for the connection of plates or bars, their length may not be less than the perpendicular distance between them, because of **shear lag**.
6. For lap joints, the minimum amount of lap permitted is equal to five times the thickness of the thinner part joined,but may not be less than 1 in (**AISC J2.2b**). The purpose of this minimum lap is to keep the joint from rotating excessively.
7. Should the actual length (l) of an end-loaded fillet weld be greater than 100 times its leg size (w),the **AISC Specification (J2.2b)** states that, due to stress variations along the weld, it is necessary to determine a smaller or effective length for strength determination. This is done by multiplying l by the term , as given in the following equation in which w is the weld leg size:

$$\beta = 1.2 - 0.002\,(l/w) \leq 1.0 \qquad \text{(AISC Equation J2-1)}$$

If the actual weld length is greater than 300 w,t he effective length shall be taken as 180 w.

Example 11.1 — Welded Connections

a. Determine the design strength of a 1-in length of a 1/4-in fillet weld formed by the shielded metal arc process (SMAW) and E70 electrodes with a minimum tensile strength $F_{EXX} = 70\ ksi$. Assume that load is to be applied parallel to the weld length.

b. Repeat part (a) if the weld is 20 in long.

c. Repeat part (a) if the weld is 30 in long.

Solution

a. $R_n = F_{nw}A_{we}$

$= (\text{nominal strength of base metal } 0.60\ F_{EXX})(\text{throat } t)(\text{weld length})$

$= (0.60 \times 70\text{ ksi})\left(\frac{1}{4}\text{ in} \times 0.707 \times 1.0\right) = 7.42\text{ k/in}$

LRFD $\phi = 0.75$	ASD $\Omega = 2.00$
$\phi R_n = (0.75)(7.42) = \mathbf{5.56\ k/in}$	$\frac{R_n}{\Omega} = \frac{7.42}{2.00} = \mathbf{3.71\ k/in}$

b. Length, $l = 20$ in

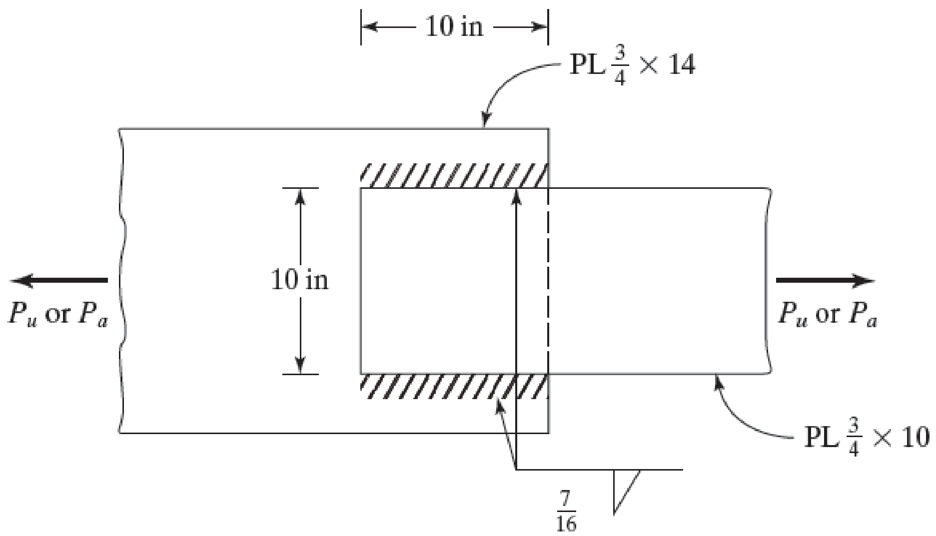
LRFD	ASD
$\frac{l}{w} = \frac{20}{\frac{1}{4}} = 80 < 100$ $\therefore \beta = 1.0$ $\phi R_n L = (5.56)(20) = \mathbf{111.2\ k}$	$\frac{L}{w} = \frac{20}{\frac{1}{4}} = 80 < 100$ $\therefore \beta = 1.0$ $\frac{R_n}{\Omega}L = (3.71)(20) = \mathbf{74.2\ k}$

c. Length, $l = 30$ in

LRFD	ASD
$\frac{l}{w} = \frac{30}{\frac{1}{4}} = 120 > 100$ $\therefore \beta = 1.2 - (0.002)(120) = 0.96$ $\phi R_n \beta L = (5.56)(0.96)(30) = \mathbf{160.1\ k}$	$\frac{L}{w} = \frac{30}{\frac{1}{4}} = 120 > 100$ $\therefore \beta = 1.2 - (0.002)(120) = 0.96$ $\frac{R_n}{\Omega}\beta L = (3.71)(0.96)(30) = \mathbf{106.8\ k}$

Example 11.2 — Welded Connections

What is the design strength of the connection shown in Fig. 14.12 if the plates consist of A572 Grade 50 steel (F_u = 65 ksi)? E70 electrodes were used, and the 7/16-in fillet welds were made by the SMAW process.



Solution

$$\text{Weld strength} = F_{we}A_{we} = (0.60 \times 70 \text{ ksi})\left(\frac{7}{16}\text{ in} \times 0.707 \times 20 \text{ in}\right) = 259.8 \text{ k}$$

$$\text{Checking the length to weld size ratio } \frac{L}{w} = \frac{10 \text{ in}}{{}^{7}/_{16} \text{ in}} = 22.86 < 100$$

$\therefore$ No reduction in weld strength is required as $\beta = 1.0$.

LRFD $\phi = 0.75$	ASD $\Omega = 2.00$	
$\phi R_n = (0.75)(259.8) = 194.9$ k	$\frac{R_n}{\Omega} = \frac{259.8}{2.00} = 129.9$ k	← controls

Check tensile yielding for $\frac{3}{4} \times 10\ PL$

$$R_n = F_y A_g = (50\text{ ksi})\left(\frac{3}{4}\text{ in} \times 10\text{ in}\right) = 375\text{ k}$$

LRFD $\phi_t = 0.90$	ASD $\Omega_t = 1.67$
$\phi_t R_n = (0.90)(375) = 337.5\text{ k}$	$\frac{R_n}{\Omega_t} = \frac{375}{1.67} = 224.6\text{ k}$

Check tensile rupture strength for $\frac{3}{4} \times 10$ PL

$$A_e = A_g U$$

since the weld length, $l = 10$ in, is equal to the distance between the welds, $U = 0.75$ (see Case 4, AISC Table D3.1)

$$A_e = \frac{3}{4}\text{ in} \times 10\text{ in} \times 0.75 = 5.62\text{ in}^2$$

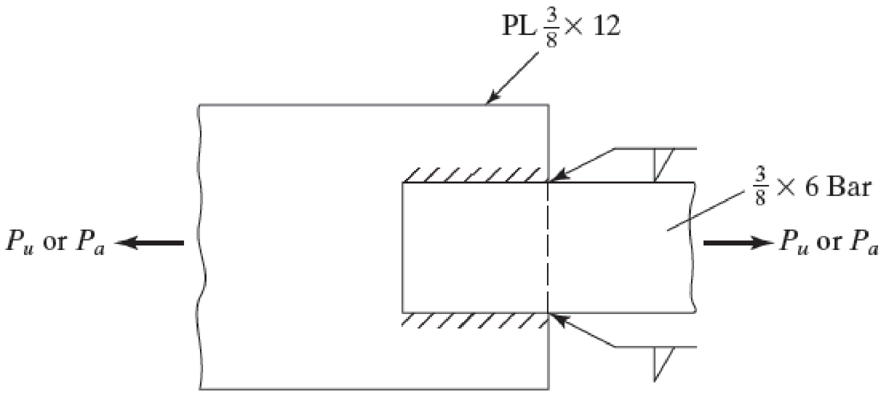
$$R_n = F_u A_e = (65\text{ ksi})(5.62\text{ in}^2) = 365.3\text{ k}$$

LRFD $\phi_t = 0.75$	ASD $\Omega_t = 2.00$
$\phi R_n = (0.75)(365.3) = 274.0\text{ k}$	$\frac{R_n}{\Omega_t} = \frac{365.3}{2.00} = 182.7\text{ k}$

LRFD *Ans* = 194.9 k **ASD *Ans* = 129.9 k**

Example 11.3 — Welded Connections

Using 50 ksi steel and E70 electrodes, design SMAW fillet welds to resist a full-capacity load on the 3/8 × 6-in member shown



Solution

Tensile yield strength of gross section of $\frac{3}{8} \times 6$ bar

$$R_n = F_y A_g = (50\ \text{ksi})\left(\frac{3}{8}\ \text{in} \times 6\ \text{in}\right) = 112.5\ \text{k}$$

LRFD $\phi_t = 0.90$	ASD $\Omega_t = 1.67$	
$\phi_t R_n = (0.90)(112.5) = 101.2\ \text{k}$	$\dfrac{R_n}{\Omega_t} = \dfrac{112.5}{1.67} = 67.4\ \text{k}$	← controls

Tensile rupture strength of $\frac{3}{8} \times 6$ bar, assume $U = 1.0$ (conservative)

$$A_e = \frac{3}{8}\ \text{in} \times 6\ \text{in} \times 1.0 = 2.25\ \text{in}^2$$

$$R_n = F_u A_e = (65\ \text{ksi})(2.25\ \text{in}^2) = 146.2\ \text{k}$$

LRFD $\phi_t = 0.75$	$\Omega_t = 2.00$
$\phi_t R_n = (0.75)(146.2) = 109.6\text{ k}$	$\dfrac{R_n}{\Omega_t} = \dfrac{146.2}{2.00} = 73.1\text{ k}$

∴ Tensile capacity of bar is controlled by yielding.

Design of weld

$$\text{Maximum weld size} = \frac{3}{8} - \frac{1}{16} = \frac{5}{16}\text{ in}$$

$$\text{Minimum weld size} = \frac{3}{16}\text{ in (Table 14.2)}$$

Use $\frac{5}{16}$ weld (maximum size with one pass)

$$R_n \text{ of weld per in} = F_w A_{we} = (0.60 \times 70\text{ ksi})\left(\frac{5}{16}\text{ in} \times 0.707\right)$$

$$= 9.28\text{ k/in}$$

LRFD $\phi = 0.75$	ASD $\Omega = 2.00$
$\phi R_n = (0.75)(9.28) = 6.96\text{ k/in}$	$\dfrac{R_n}{\Omega} = \dfrac{9.28}{2.00} = 4.64\text{ k/in}$
Weld length reqd $= \dfrac{101.2}{6.96}$	Weld length reqd $= \dfrac{67.4}{4.64}$
$= 14.54$ in or $7\frac{1}{2}$ in each side	$= 14.53$ in or $7\frac{1}{2}$ in each side
$\dfrac{L}{w} = \dfrac{7.5}{\frac{5}{16}} = 24 < 100$ OK $\beta = 1.00$	$\dfrac{L}{w} = \dfrac{7.5}{\frac{5}{16}} = 24 < 100$ OK $\beta = 1.00$

Use $7\frac{1}{2}$-in welds each side. **Use $7\frac{1}{2}$-in welds each side.**

11.8 DESIGN OF CONNECTIONS FOR MEMBERS WITH BOTH LONGITUDINAL AND TRANSVERSE FILLET WELDS

The AISC in its **Section J2.4c** states that the total nominal strength of a connection with side and transverse welds is to equal the **larger of the values** obtained with the following two equations:

$$R_n = R_{wl} + R_{wt} \qquad \text{(J2-9a)}$$

or

$$R_n = 0.85 R_{wl} + 1.5 R_{wt} \qquad \text{(J2-9b)}$$

where

R_{wl} = the total nominal strength of longitudinally loaded fillet welds, as determined in accordance with Table J2.5, kips (N)

R_{wt} = the total nominal strength of transversely loaded fillet welds, as determined in accordance with Table J2.5 without the alternate in Section J2.4(a), kips (N)

$$\phi = 0.75 \text{ (LRFD)} \qquad \Omega = 2.00 \text{ (ASD)}$$

$$R_n = F_w A_w \qquad \text{(J2-4)}$$

where

$$F_w = 0.60 F_{EXX} \left(1.0 + 0.50 \sin^{1.5} \theta\right) \qquad \text{(J2-5)}$$

and

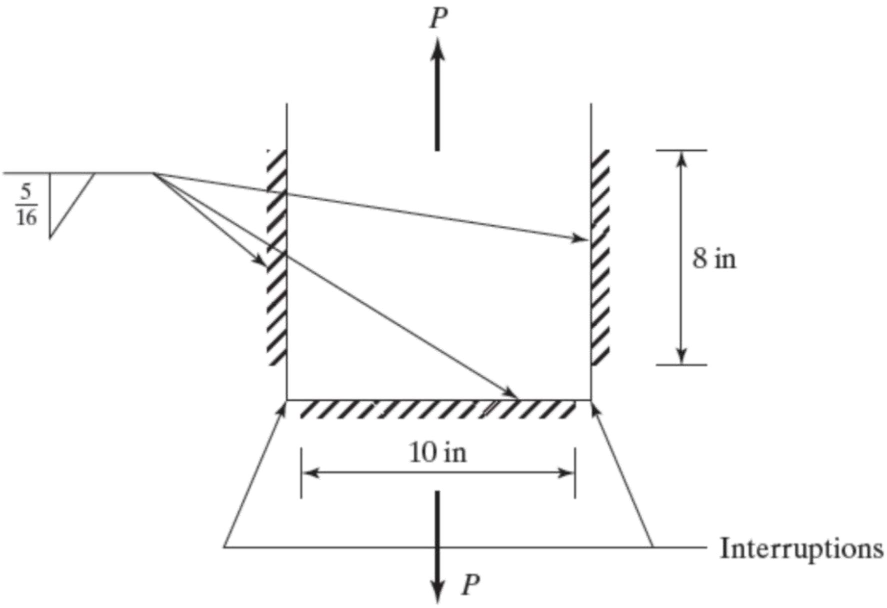
F_{EXX} = electrode classification number, ksi (MPa)

θ = angle of loading measured from the weld longitudinal axis, degrees

A_w = effective area of the weld, in.^{2} (mm^2)

Example 11.4 Welded Connections

Determine the total LRFD design strength and the total ASD allowable strength of the 5/16-in E70 fillet welds shown



Solution

$$\text{Effective throat } t = (0.707)\left(\frac{5}{16}\text{ in}\right) = 0.221\text{ in}$$

$$R_{wl} = R_n \text{ for side welds} = F_{nw}A_{we} = (0.60 \times 70\text{ ksi})(2 \times 8\text{ in} \times 0.221\text{ in})$$
$$= 148.5\text{ k}$$
$$R_{wt} = R_n \text{ for transverse end weld} = F_{nw}A_{we}$$
$$= (0.60 \times 70\text{ ksi})(10\text{ in} \times 0.221\text{ in}) = 92.8\text{ k}$$

Applying AISC Equations J2-10a and J2-10b

$$R_n = R_{nwl} + R_{nwt} = 148.5\text{ k} + 92.8\text{ k} = 241.3\text{ k}$$
$$R_n = 0.85R_{nwl} + 1.5R_{nwt} = (0.85)(148.5\text{ k}) + (1.5)(92.8\text{ k}) = 265.4\text{ k} \leftarrow \text{controls}$$

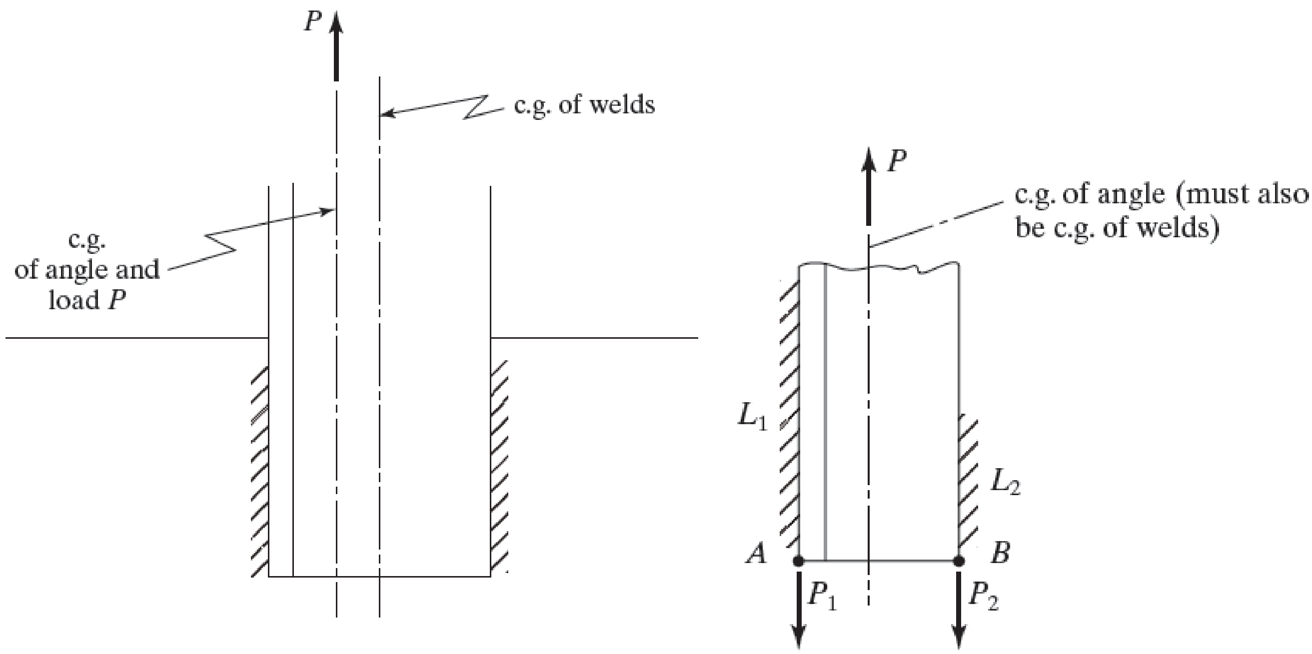
LRFD $\phi = 0.75$	ASD $\Omega = 2.00$
$\phi R_n = (0.75)(265.4) = \mathbf{199\ k}$	$\frac{R_n}{\Omega} = \frac{265.4}{2.00} = \mathbf{132.7\ k}$

11.10 DESIGN OF FILLET WELDS FOR TRUSS MEMBERS

Should the members of a welded truss consist of single angles, double angles, or similar shapes and be subjected to static axial loads only, the **AISC Specification (J1.7)** permits the connections to be designed by the procedures described in the preceding section.

> **7. Placement of Welds and Bolts**
>
> Groups of welds or bolts at the ends of any member which transmit axial *force* into that member shall be sized so that the center of gravity of the group coincides with the center of gravity of the member, unless provision is made for the eccentricity. The foregoing provision is not applicable to end connections of statically loaded single angle, double angle, and similar members.

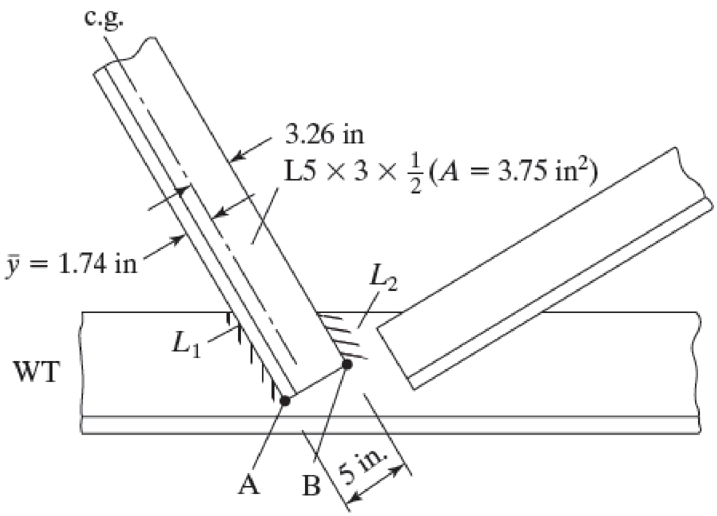
The designers can select the weld size, calculate the total length of the weld required, and place the welds around the member ends as they see fit. (It would not make sense, of course, to place the weld all on one side of a member, such as for the angle of shown, because of the rotation possibility.)



Eccentrically loaded welds.

Example 11.5 — Welded Connections

Use $F_y = 50$ ksi and $F_u = 65$ ksi, E70 electrodes, and the SMAW process to design side fillet welds for the full capacity of the 5 × 3 × 1/2-in angle tension member shown. Assume that the member is subjected to repeated stress variations, making any connection eccentricity undesirable. Check block shear strength of the member. Assume that the WT chord member has adequate strength to develop the weld strengths and that the thickness of its web is 1/2 in. Assume that $U = 0.87$.



Solution

Tensile yielding on gross section

$$P_n = F_y A_g = (50 \text{ ksi})(3.75 \text{ in}^2) = 187.5 \text{ k}$$

Tensile rupture on net section

$$A_e = UA_g = (0.87)(3.75 \text{ in}^2) = 3.26 \text{ in}^2$$

$$P_n = F_u A_e = (65 \text{ ksi})(3.26 \text{ in}^2) = 211.9 \text{ k}$$

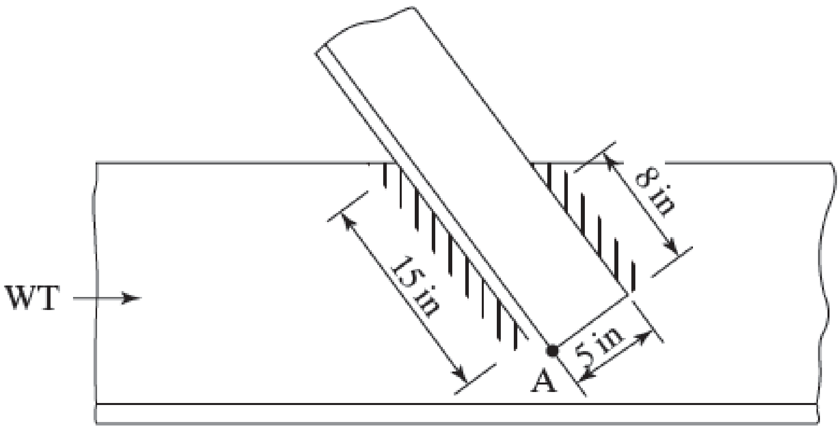
LRFD	ASD
For tensile yielding ($\phi_t = 0.90$)	For tensile yielding ($\Omega_t = 1.67$)
$\phi_t P_n = (0.9)(187.5) = 168.7\text{ k}$	$\dfrac{P_n}{\Omega_t} = \dfrac{187.5}{1.67} = 112.3\text{ k}$
For tensile rupture ($\phi_t = 0.75$)	For tensile rupture ($\Omega_t = 2.00$)
$\phi_t P_n = (0.75)(211.9) = 158.9\text{ k} \leftarrow$	$\dfrac{P_n}{\Omega_t} = \dfrac{211.9}{2.00} = 105.9\text{ k} \leftarrow$

$$\text{Maximum weld size} = \frac{1}{2} - \frac{1}{16} = \frac{7}{16}\text{ in}$$

Use $\frac{5}{16}$-in weld (largest that can be made in single pass)

$$\text{Effective throat } t \text{ of weld} = (0.707)\left(\frac{5}{16}\text{ in}\right) = 0.221\text{ in}$$

LRFD	ASD
Design strength/in of $\frac{5}{16}$-in welds ($\phi = 0.75$)	Allowable strength/in of $\frac{5}{16}$-in welds ($\Omega = 2.00$)
$= (0.75)(0.60 \times 70)(0.221)(1)$	$= \dfrac{(0.60 \times 70)(0.221)(1)}{2.00}$
$= 6.96\text{ k/in}$	$= 4.64\text{ k/in}$
Weld length reqd $= \dfrac{158.9}{6.96}$	Weld length reqd $= \dfrac{105.9}{4.64}$
$= 22.83\text{ in}$	$= 22.82\text{ in}$
Taking moments about point A in Fig. 14.18	Taking moments about point A in Fig. 14.18
$(158.9)(1.74) - 5.00P_2 = 0$	$(105.9)(1.74) - 5.00P_2 = 0$
$P_2 = 55.3\text{ k}$	$P_2 = 36.85\text{ k}$
$L_2 = \dfrac{55.3\text{ k}}{6.96\text{ k/in}} = 7.95\text{ in}$ (**say, 8 in**)	$L_2 = \dfrac{36.85\text{ k}}{4.64\text{ k/in}} = 7.94\text{ in}$ (**say, 8 in**)
$L_1 = 22.83 - 7.95 = 14.88\text{ in}$ (**say, 15 in**)	$L_1 = 22.82 - 7.94 = 14.88\text{ in}$ (**say, 15 in**)



Checking block shearing strength, assuming dimensions previously described

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt} \le 0.6F_yA_{gv} + U_{bs}F_uA_{nt}$$
$$= (0.6)(65)(15 + 8)\left(\frac{1}{2}\right) + (1.00)(65)\left(5 \times \frac{1}{2}\right)$$
$$\le (0.6)(50)(15 + 8)\left(\frac{1}{2}\right) + (1.00)(65)\left(5 \times \frac{1}{2}\right)$$
$$= 611 \text{ k} > 507.5 \text{ k}$$
$$\therefore R_n = 507.5 \text{ k}$$

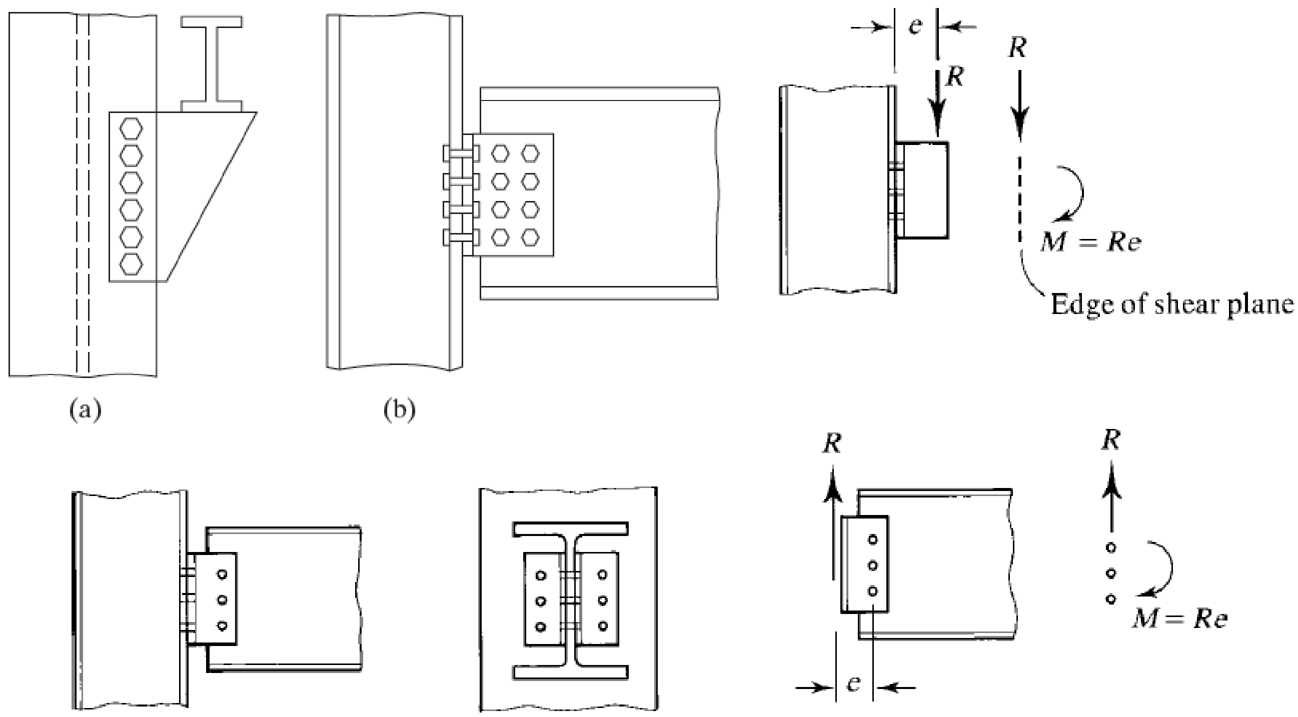
LRFD $\phi = 0.75$	ASD $\Omega = 2.00$
$\phi R_n = (0.75)(507.5) = 380.6 \text{ k} > 158.9 \text{ k}$ **OK**	$\frac{R_n}{\Omega} = \frac{507.5}{2.00} = 253.8 \text{ k} > 105.9\ k$ **OK**

ECCENTRICALLY LOADED BOLTED AND WELDED CONNECTIONS 13

13.1 ECCENTRICALLY LOADED BOLTED CONNECTIONS

Eccentrically loaded bolt groups are subjected to shears and bending moments. This situations are much more common than most people suspect. For instance, in a truss it is desirable to have the center of gravity of a member lined up exactly with the center of gravity of the bolts at its end connections. This feat is not quite as easy to accomplish as it may seem, and connections are often subjected to moments.

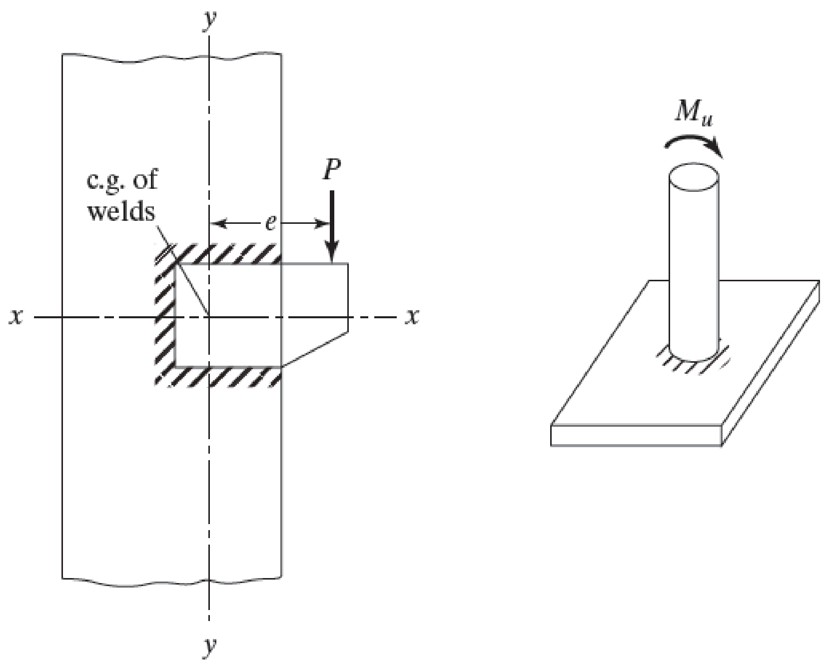
Eccentricity is quite obvious in the figure shown, where a beam is connected to a column with a plate. In part (b) of the figure, another beam is connected to a column with a pair of web angles. It is obvious that this connection must resist some moment, because the center of gravity of the load from the beam does not coincide with the reaction from the column.



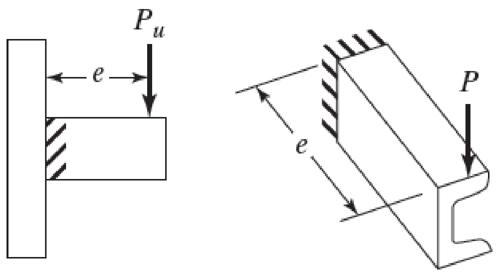
13.2 ECCENTRICALLY LOADED WELDED CONNECTIONS

Fillet welds are frequently loaded with eccentrically applied loads, with the result that the welds are subjected to either shear and torsion or to shear and bending. A figure below is presented to show the difference between the two situations. Shear and torsion, shown in part (a) of the figure, are the subject of this section, while shear and bending, shown in part (b) of the figure.

As is the case for eccentrically loaded bolt groups, the AISC Specification provides the design strength of welds, but does not specify a method of analysis for eccentrically loaded welds. It's left to the designer to decide which method to use.



(a)



(b)

ECCENTRICALLY LOADED BOLTED CONNECTIONS

$$M = P_y\, e_x - P_x\, e_y$$

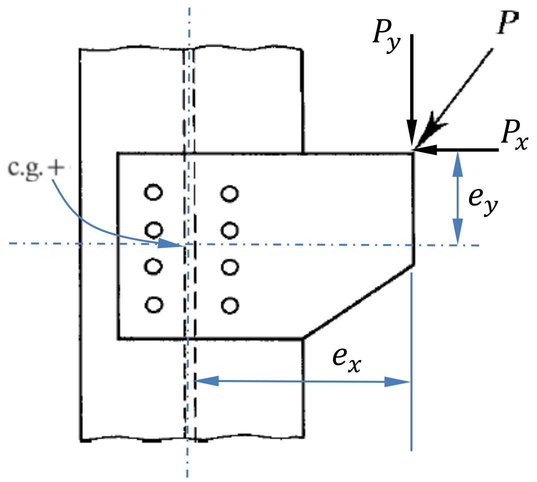
$$\Sigma d^2 = \Sigma h^2 + \Sigma v^2$$

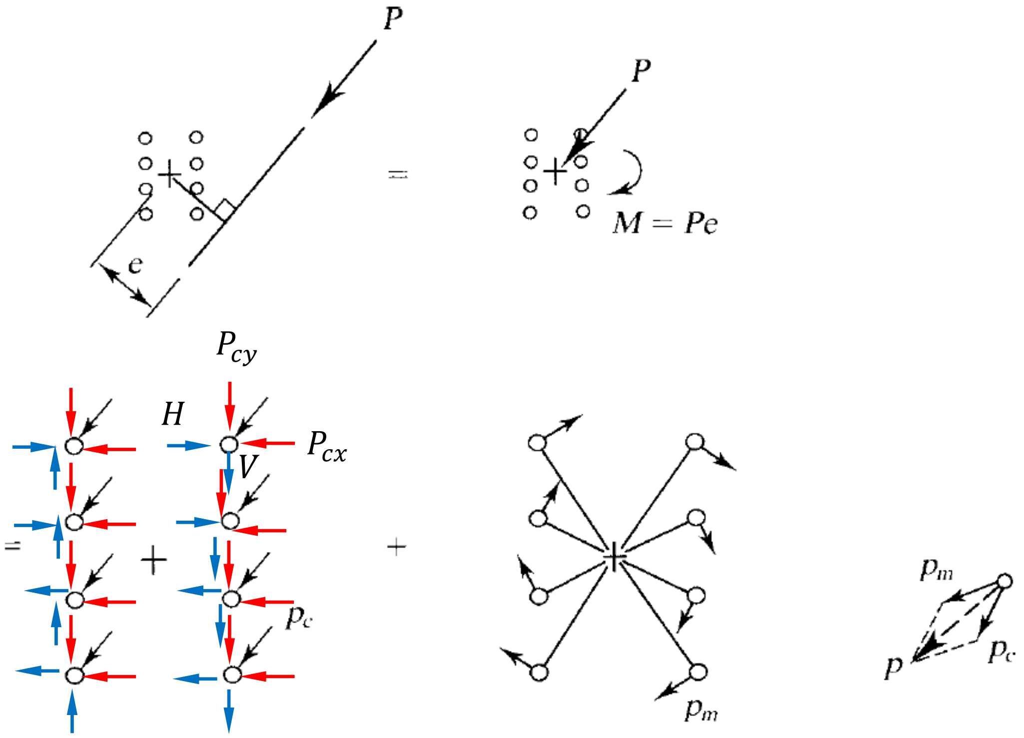
$$H = \frac{Mv}{\Sigma d^2}$$

$$V = \frac{Mh}{\Sigma d^2}$$

$$P_{cx} = \frac{P_x}{n}, \qquad P_{cy} = \frac{P_y}{n}$$

$$R_n = \sqrt{(H + P_x)^2 + \left(V + P_y\right)^2}$$





- **CHECK THE SHEARING STRENGTH OF BOLTS,** AISC Chapter J, Page 108

6. Tension and Shear Strength of Bolts and Threaded Parts

The *design tension* or *shear strength*, ϕR_n, and the *allowable tension* or *shear strength*, R_n/Ω, of a snug-tightened or pretensioned high-strength bolt or threaded part shall be determined according to the *limit states* of *tensile rupture* and *shear rupture* as follows:

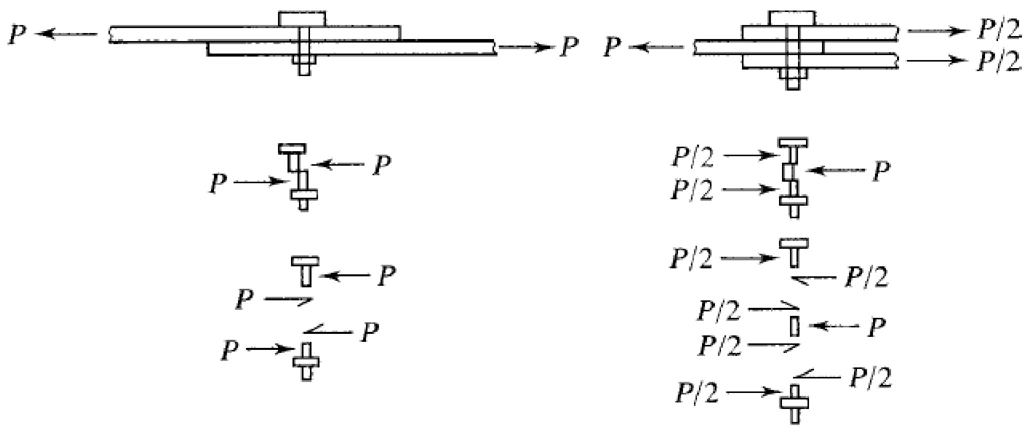
$$R_n = F_n A_b \qquad \text{(J3-1)}$$

$$\phi = 0.75\ \text{(LRFD)} \qquad \Omega = 2.00\ \text{(ASD)}$$

where

F_n = nominal tensile stress F_{nt}, or shear stress, F_{nv} from Table J3.2, ksi (MPa)

A_b = nominal unthreaded body area of bolt or threaded part (for upset rods, see footnote d, Table J3.2), in.^{2} (mm^2)



(a) **Single Shear** (b) **Double Shear**

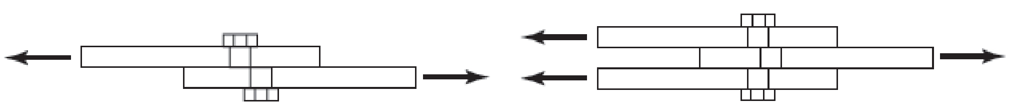


TABLE J3.2
Nominal Stress of Fasteners and Threaded Parts, ksi (MPa)

Description of Fasteners	Nominal Tensile Stress, F_{nt}, ksi (MPa)	Nominal Shear Stress in Bearing-Type Connections, F_{nv}, ksi (MPa)
A307 bolts	45 (310) [a][b]	24 (165) [b][c][f]
A325 or A325M bolts, when threads are not excluded from shear planes	90 (620) [e]	48 (330) [f]
A325 or A325M bolts, when threads are excluded from shear planes	90 (620) [e]	60 (414) [f]
A490 or A490M bolts, when threads are not excluded from shear planes	113 (780) [e]	60 (414) [f]
A490 or A490M bolts, when threads are excluded from shear planes	113 (780) [e]	75 (520) [f]
Threaded parts meeting the requirements of Section A3.4, when threads are not excluded from shear planes	0.75 F_u [a][d]	0.40 F_u
Threaded parts meeting the requirements of Section A3.4, when threads are excluded from shear planes	0.75 F_u [a][d]	0.50 F_u

[a] Subject to the requirements of Appendix 3.

[b] For A307 bolts the tabulated values shall be reduced by 1 percent for each 1/16 in. (2 mm) over 5 diameters of length in the grip.

[c] Threads permitted in shear planes.

[d] The nominal tensile strength of the threaded portion of an upset rod, based upon the cross-sectional area at its major thread diameter, A_D, which shall be larger than the nominal body area of the rod before upsetting times F_y.

[e] For A325 or A325M and A490 or A490M bolts subject to tensile fatigue loading, see Appendix 3.

[f] When bearing-type connections used to splice tension members have a fastener pattern whose length, measured parallel to the line of force, exceeds 50 in. (1270 mm), tabulated values shall be reduced by 20 percent.

- **CHECK THE BEARING STRENGTH OF BOLTS,** AISC Chapter J, Page 111

10. Bearing Strength at Bolt Holes

The *available bearing strength*, ϕR_n and R_n/Ω, at bolt holes shall be determined for the *limit state* of *bearing* as follows:

$$\phi = 0.75 \text{ (LRFD)} \qquad \Omega = 2.00 \text{ (ASD)}$$

(a) For a bolt in a *connection* with standard, oversized, and short-slotted holes, independent of the direction of loading, or a long-slotted hole with the slot parallel to the direction of the bearing *force*:

(i) When deformation at the bolt hole at *service load* is a design consideration

$$R_n = 1.2\, L_c t F_u \leq 2.4\, d t F_u \qquad \text{(J3-6a)}$$

Deformation $\leq 0.25\ in$

(ii) When deformation at the bolt hole at service load is not a design consideration

$$R_n = 1.5\, L_c t F_u \leq 3.0\, d t F_u \qquad \text{(J3-6b)}$$

Deformation $> 0.25\ in$

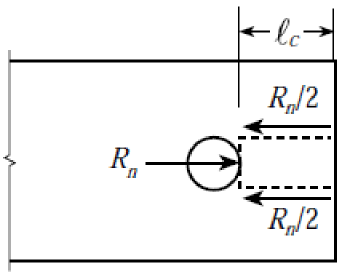
(b) For a bolt in a connection with long-slotted holes with the slot perpendicular to the direction of force:

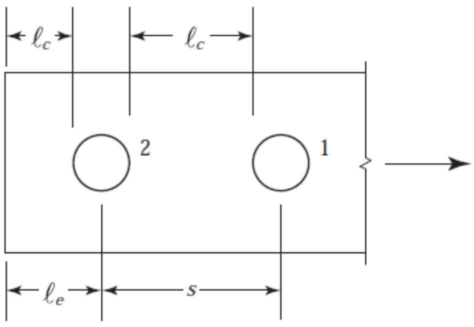
$$R_n = 1.0\, L_c t F_u \leq 2.0\, d t F_u \qquad \text{(J3-6c)}$$

(c) For connections made using bolts that pass completely through an unstiffened box member or *HSS*, see Section J7 and Equation J7-1,

where

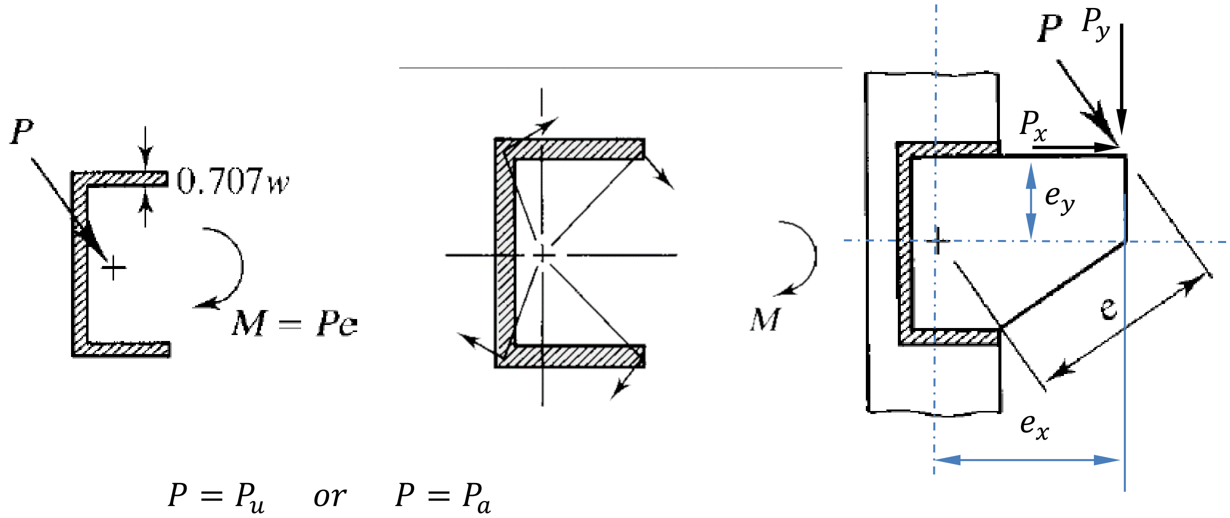
d = nominal bolt diameter, in. (mm)
F_u = *specified minimum tensile strength* of the connected material, ksi (MPa)
L_c = clear distance, in the direction of the force, between the edge of the hole and the edge of the adjacent hole or edge of the material, in. (mm)
t = thickness of connected material, in. (mm)





$$L_c = \min\ [L_{c1}, L_{c2}, L_{c3}, \ldots]$$

ECCENTRICALLY LOADED WELDED CONNECTIONS



$$P = P_u \quad or \quad P = P_a$$

$$M = T = P_y\, e_x + P_x\, e_y$$

$$J = I_x + I_y$$

$$f_h = \frac{Tv}{J} \quad f_v = \frac{Th}{J}$$

$$f_{sh} = \frac{P_x}{L}, \quad f_{sv} = \frac{P_y}{L}$$

$$f_r = \sqrt{(f_h + fs_h)^2 + (f_v + f_{sv})^2}$$

$$w = size\ of\ weld = \frac{f_r}{\phi R_n} \quad (LRFD)$$

$$w = size\ of\ weld = \frac{f_r}{R_n/\Omega} \quad (ASD)$$

$$\phi = 0.75\ (\text{LRFD}) \qquad \Omega = 2.00\ (\text{ASD})$$

where

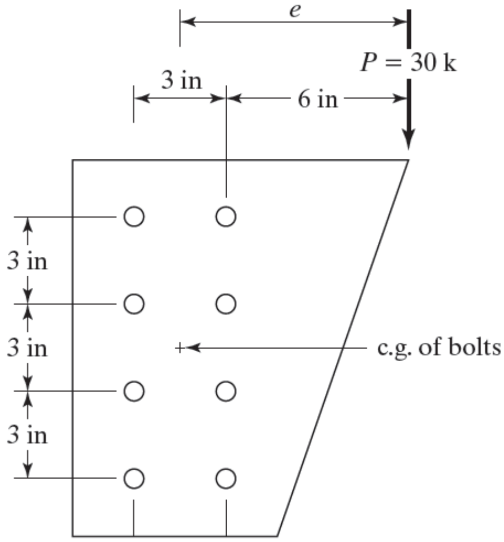
$$R_n = (0.6\, F_{EXX})(0.707\, w)(L)$$

for 1" weld per 1" length

$$R_n = (0.6\, F_{EXX})(0.707 \times 1\,)(1)$$

Example 13.1 — Eccentrically Loaded Bolted and Welded Connections

Determine the force in the most stressed bolt of the group shown in the figure, using the elastic analysis method.



Solution

A sketch of each bolt and the forces applied to it by the direct load and the clockwise moment are shown in the figure shown. From this sketch, the student can see that the upper right-hand bolt and the lower right-hand bolt are the most stressed and that their respective stresses are equal:

$$e = 6 + 1.5 = 7.5 \text{ in}$$

$$M = Pe = (30 \text{ k})(7.5 \text{ in}) = 225 \text{ in-k}$$

$$\Sigma d^2 = \Sigma h^2 + \Sigma v^2$$

$$\Sigma d^2 = (8)(1.5)^2 + (4)(1.5^2 + 4.5^2) = 108 \text{ in}^2$$

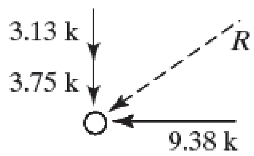
For lower right-hand bolt

$$H = \frac{Mv}{\Sigma d^2} = \frac{(225 \text{ in-k})(4.5 \text{ in})}{108 \text{ in}^2} = 9.38 \text{ k} \quad ;$$

$$V = \frac{Mh}{\Sigma d^2} = \frac{(225 \text{ in-k})(1.5 \text{ in})}{108 \text{ in}^2} = 3.13 \text{ k} \downarrow$$

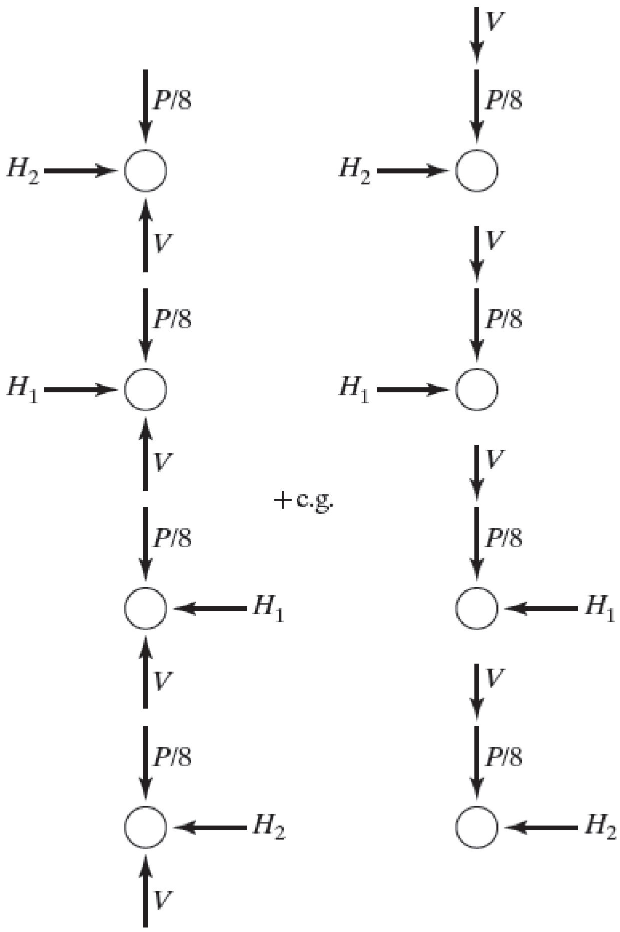
$$\frac{P}{8} = \frac{30 \text{ k}}{8} = 3.75 \text{ k} \downarrow$$

These components for the lower right-hand bolt are sketched as follows:



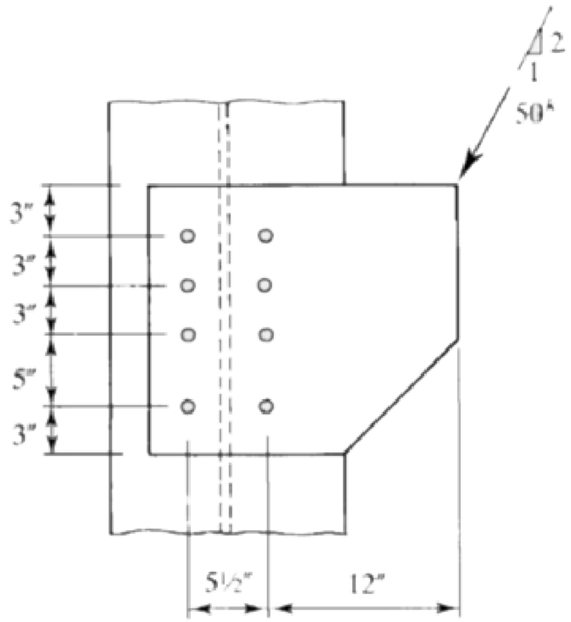
The resultant force applied to this bolt is

$$R = \sqrt{(3.13 + 3.75)^2 + (9.38)^2} = 11.63 \text{ k}$$



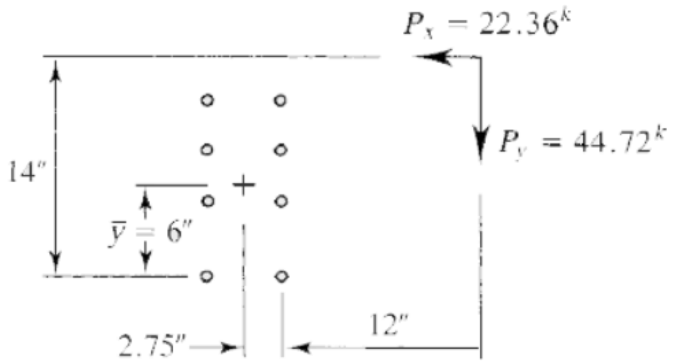
Example 13.2 — Eccentrically Loaded Bolted and Welded Connections

Determine the critical fastener force in the bracket connection shown in the figure.

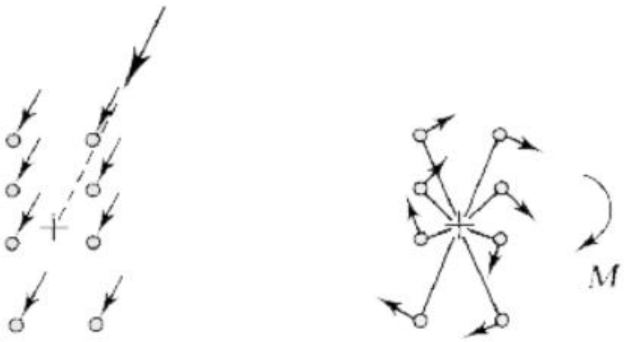


Solution

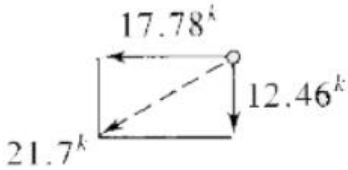
$$\bar{y} = \frac{2(5) + 2(8) + 2(11)}{8} = 6 \text{ in.}$$



(a)



(b)



(c)

The horizontal and vertical components of the load are

$$P_x = \frac{1}{\sqrt{5}}(50) = 22.36 \text{ kips} \leftarrow \quad \text{and} \quad P_y = \frac{2}{\sqrt{5}}(50) = 44.72 \text{ kips} \downarrow$$

Referring to Figure we can compute the moment of the load about the centroid:

$$M = 44.72(12 + 2.75) - 22.36(14 - 6) = 480.7 \text{ in.-kips} \qquad \text{(clockwise)}$$

Figure shows the directions of all component bolt forces and the relative magnitudes of the components caused by the couple. Using these directions and relative magnitudes as a guide and bearing in mind that forces add by the parallelogram law, we can conclude that the lower right-hand fastener will have the largest resultant force.

The horizontal and vertical components of force in each bolt resulting from the concentric load are

$$p_{cx} = \frac{22.36}{8} = 2.795 \text{ kips} \leftarrow \quad \text{and} \quad p_{cy} = \frac{44.72}{8} = 5.590 \text{ kips} \downarrow$$

For the couple,

$$\sum(x^2+y^2) = 8(2.75)^2 + 2[(6)^2 + (1)^2 + (2)^2 + (5)^2] = 192.5 \text{ in.}^2$$

$$p_{mx} = \frac{My}{\sum(x^2+y^2)} = \frac{480.7(6)}{192.5} = 14.98 \text{ kips} \leftarrow$$

$$p_{my} = \frac{Mx}{\sum(x^2+y^2)} = \frac{480.7(2.75)}{192.5} = 6.867 \text{ kips} \downarrow$$

$$\sum p_x = 2.795 + 14.98 = 17.78 \text{ kips} \leftarrow$$

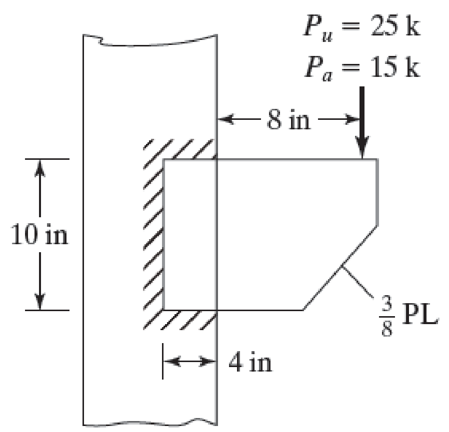
$$\sum p_y = 5.590 + 6.867 = 12.46 \text{ kips} \downarrow$$

$$p = \sqrt{(17.78)^2 + (12.46)^2} = 21.7 \text{ kips}$$

The critical fastener force is 21.7 kips. Inspection of the magnitudes and directions of the horizontal and vertical components of the forces confirms the earlier conclusion that the fastener selected is indeed the critical one.

Example 13.3 Eccentrically Loaded Bolted and Welded Connections

For the A36 bracket shown in the figure, determine the fillet weld size required if E70 electrodes, the AISC Specification, and the SMAW process are used.

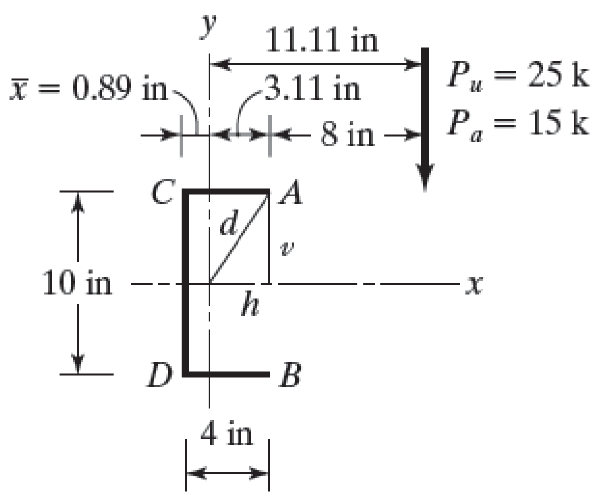


Solution

Assuming a 1-in weld as shown

$$A = 2(4 \text{ in}^2) + 10 \text{ in}^2 = 18 \text{ in}^2$$

$$\bar{x} = \frac{(4 \text{ in}^2)(2 \text{ in})(2)}{18 \text{ in}^2} = 0.89 \text{ in}$$



$$I_x = \left(\frac{1}{12}\right)(1)(10)^3 + (2)(4)(5)^2 = 283.3 \text{ in}^4$$

$$I_y = 2\left(\frac{1}{12}\right)(1)(4)^3 + 2(4)(2 - 0.89)^2 + (10)(0.89)^2 = 28.4 \text{ in}^4$$

$$J = 283.3 + 28.4 = 311.7 \text{ in}^4$$

According to our previous work, the welds perpendicular to the direction of the loads are appreciably stronger than the welds parallel to the loads. However, to simplify the calculations, the author conservatively assumes that all the welds have design strengths or allowable strengths per inch equal to the values for the welds parallel to the loads.

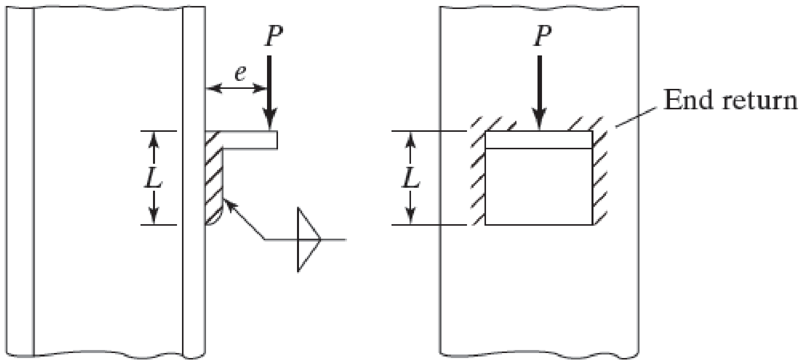
$$R_n \text{ for a 1 in weld} = 0.707 \times 1 \times 0.6 \times 70 = 29.69 \text{ ksi}$$

LRFD $\phi = 0.75$	ASD $\Omega = 2.00$
$\phi R_n = (0.75)(29.69) = 22.27$ ksi	$\frac{R_n}{\Omega} = \frac{29.69}{2.00} = 14.84$ ksi
Forces @ points *C* & *D*	**Forces @ points *C* & *D***
$f_h = \frac{(25 \times 11.11)(5)}{311.7} = 4.46$ k/in	$f_h = \frac{(15)(11.11)(5)}{311.7} = 2.67$ k/in
$f_v = \frac{(25 \times 11.11)(0.89)}{311.7} = 0.79$ k/in	$f_v = \frac{(15)(11.11)(0.89)}{311.7} = 0.48$ k/in
$f_s = \frac{25}{18} = 1.39$ k/in	$f_s = \frac{15}{18} = 0.83$ k/in
$f_r = \sqrt{(0.79 + 1.39)^2 + (4.46)^2} = 4.96$ k/in	$f_r = \sqrt{(0.48 + 0.83)^2 + (2.67)^2} = 2.97$ k/in
Size $= \frac{4.96 \text{ k/in}}{22.27 \text{ k/in}^2} = 0.223$ in, say $\frac{1}{4}$ in	Size $= \frac{2.97 \text{ k/in}}{14.84 \text{ k/in}^2} = 0.200$ in, say $\frac{1}{4}$ in
Forces @ points *A* & *B*	**Forces @ points *A* & *B***
$f_h = \frac{(25 \times 11.11)(5)}{311.7} = 4.46$ k/in	$f_h = \frac{(15 \times 11.11)(5)}{311.7} = 2.67$ k/in

LRFD $\phi = 0.75$	ASD $\Omega = 2.00$
$f_v = \dfrac{(25 \times 11.11)(3.11)}{311.7} = 2.77$ k/in	$f_v = \dfrac{(15 \times 11.11)(3.11)}{311.7} = 1.66$ k/in
$f_s = \dfrac{25}{18} = 1.39$ k/in	$f_s = \dfrac{15}{18} = 0.83$ k/in
$f_r = \sqrt{(2.77 + 1.39)^2 + (4.46)^2}$ $= 6.10$ k/in	$f_r = \sqrt{(1.66 + 0.83)^2 + (2.67)^2}$ $= 3.65$ k/in
Size $= \dfrac{6.10}{22.27} = 0.274$ in, say $\dfrac{5}{16}$ in	Size $= \dfrac{3.65}{14.84} = 0.246$ in, say $\dfrac{1}{4}$ in
Use $\frac{5}{16}$-in fillet welds, E70, SMAW.	**Use $\frac{1}{4}$-in fillet weld, E70, SMAW.**

Example 13.4 — Eccentrically Loaded Bolted and Welded Connections

Using E70 electrodes, the SMAW process, and the LRFD Specification, determine the weld size required for the connection of the figure shown. if $P_D = 10\ k, P_L = 20\ k,\ e = 2\frac{1}{2}$ in, and .$L = 8$ in. Assume that the member thicknesses do not control weld size.



Solution

Initially, assume fillet welds with 1-in leg sizes.

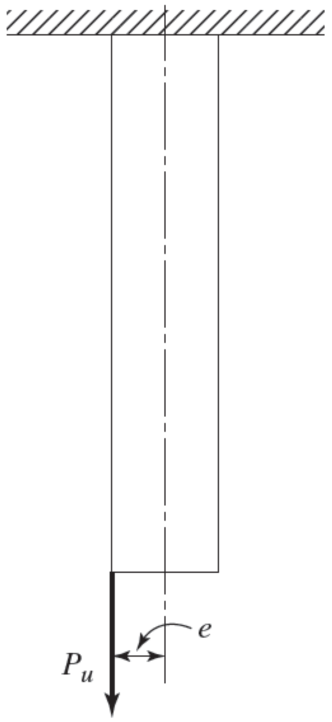
LRFD $\phi = 0.75$	ASD $\Omega_t = 2.00$
$P_u = (1.2)(10) + (1.6)(20) = 44$ k	$P_a = 10 + 20 = 30$ k
$f_v = \dfrac{P_u}{A} = \dfrac{44}{(2)(8)} = 2.75$ k/in	$f_v = \dfrac{P_a}{A} = \dfrac{30}{(2)(8)} = 1.88$ k/in
$f_b = \dfrac{Mc}{I} = \dfrac{(44 \times 2.5)(4)}{2\left(\frac{1}{12}\right)(1)(8)^3} = 5.16$ k/in	$f_b = \dfrac{Mc}{I} = \dfrac{(30 \times 2.5)(4)}{2\left(\frac{1}{12}\right)(1)(8)^3} = 3.52$ k/in

LRFD $\phi = 0.75$	ASD $\Omega_t = 2.00$
$f_r = \sqrt{(2.75)^2 + (5.16)^2} = 5.85$ k/in	$f_r = \sqrt{(1.88)^2 + (3.52)^2} = 3.99$ k/in
weld size reqd $= \dfrac{f_r}{(\phi)(\text{weld size})\ 0.60F_{EXX}}$	weld size reqd $= \dfrac{\Omega f_r}{(\text{weld size})(0.60\ F_{EXX})}$
$= \dfrac{5.85}{(0.75)(0.707 \times 1.0)(0.60 \times 70)}$	$= \dfrac{(2.00)(3.99)}{(0.707 \times 1.0)(0.60 \times 70)}$
$= 0.263$ in, say 5/16 in	$= 0.269$ in, say 5/16 in
Use $\frac{5}{16}$-in weld, E70, SMAW.	**Use $\frac{5}{16}$-in weld, E70, SMAW.**

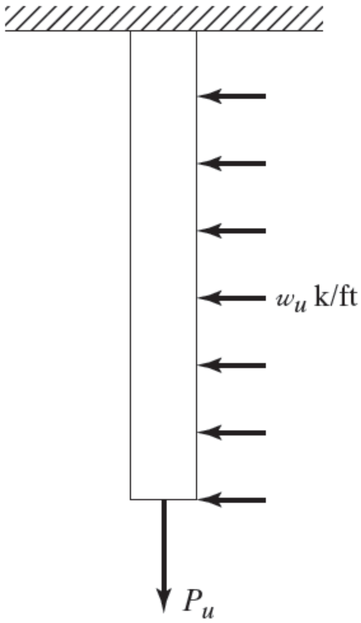
BENDING AND AXIAL FORCE 8

8.1 MEMBERS SUBJECT TO BENDING AND AXIAL TENSION (BEAM-COLUMNS)

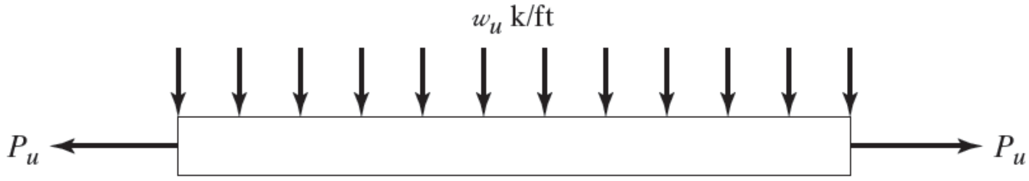
A few types of members subject to both bending and axial tension are shown in the figure



(a) A hanger subject to an off-center tensile load



(b) A hanger subject to an axial tensile load and a lateral load (as wind) or other moment from lateral forces



(c) A beam subject to a uniform gravity load and an axial tensile load

In Section **H1** of the **AISC** Specification (Sect. H1, page 70), the interaction equations that follow are given for symmetric shapes subjected simultaneously to bending and axial tensile forces

(a) For $\frac{P_r}{P_c} \geq 0.2$

$$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0 \qquad \text{(H1-1a)}$$

(b) For $\frac{P_r}{P_c} < 0.2$

$$\frac{P_r}{2P_c} + \left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0 \qquad \text{(H1-1b)}$$

where

For design according to Section B3.3 (LRFD)

P_r = *required tensile strength* using LRFD *load combinations*, kips (N)

$P_c = \phi_t P_n$ = *design tensile strength*, determined in accordance with Section D2, kips (N)

M_r = *required flexural strength* using LRFD load combinations, kip-in. (N-mm)

$M_c = \phi_b M_n$ = *design flexural strength* determined in accordance with Chapter F, kip-in. (N-mm)

ϕ_t = *resistance factor* for tension (see Section D2)

ϕ_b = resistance factor for flexure = 0.90

For doubly symmetric members, C_b in Chapter F may be increased by $\sqrt{1 + \frac{P_u}{P_{ey}}}$ for axial tension that acts concurrently with flexure,

where

$$P_{ey} = \frac{\pi^2 E I_y}{L_b^2}$$

For design according to Section B3.4 (ASD)

P_r = required tensile strength using *ASD load combinations*, kips (N)

$P_c = P_n/\Omega_t$ = *allowable tensile strength*, determined in accordance with Section D2, kips (N)

M_r = required flexural strength using ASD load combinations, kip-in. (N-mm)

$M_c = M_n/\Omega_b$ = *allowable flexural strength* determined in accordance with Chapter F, kip-in. (N-mm)

Ω_t = *safety factor* for tension (see Section D2)

Ω_b = safety factor for flexure = 1.67

For doubly symmetric members, C_b in Chapter F may be increased by $\sqrt{1+\frac{1.5P_a}{P_{ey}}}$ for axial tension that acts concurrently with flexure

where

$$P_{ey} = \frac{\pi^2 EI_y}{L_b^2}$$

For tensile yielding in the gross section:

$$P_n = F_y A_g \qquad \text{(D2-1)}$$

$\phi_t = 0.90$ (LRFD) $\Omega_t = 1.67$ (ASD)

For tensile rupture in the net section:

$$P_n = F_u A_e \qquad \text{(D2-2)}$$

$\phi_t = 0.75$ (LRFD) $\Omega_t = 2.00$ (ASD)

where

A_e = *effective net area*, in.^{2} (mm^2)

A_g = gross area of member, in.^{2} (mm^2)

F_y = *specified minimum yield stress* of the type of steel being used, ksi (MPa)

F_u = *specified minimum tensile strength* of the type of steel being used, ksi (MPa)

In which

C_b = *lateral-torsional buckling* modification factor for nonuniform moment diagrams when both ends of the unsupported segment are braced

$$C_b = \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C} R_m \leq 3.0 \qquad \text{(F1-1)}$$

where

M_{max} = absolute value of maximum moment in the unbraced segment, kip-in. (N-mm)

M_A = absolute value of moment at quarter point of the unbraced segment, kip-in. (N-mm)

M_B = absolute value of moment at centerline of the unbraced segment, kip-in. (N-mm)

M_C = absolute value of moment at three-quarter point of the unbraced segment, kip-in. (N-mm)

R_m = cross-section monosymmetry parameter
= 1.0, doubly symmetric members
= 1.0, singly symmetric members subjected to *single curvature* bending
$= 0.5 + 2\left(\frac{I_{yc}}{I_y}\right)^2$, singly symmetric members subjected to *reverse curvature* bending

I_y = moment of inertia about the principal y-axis, in.^{4} (mm^4)

I_{yc} = moment of inertia about y-axis referred to the compression flange, or if reverse curvature bending, referred to the smaller flange, in.^{4} (mm^4)

Example 8.1 — Bending and Axial Force

A 50 ksi W12 × 40 tension member with no holes is subjected to the axial loads $P_D = 25$ k and $P_L = 30$ k, as well as the bending moments $M_{Dy} = 10$ ft-k and $M_{Ly} = 25$ ft-k. Is the member satisfactory if $L_b < L_p$?

Solution

Using a W12 × 40 ($A = 11.7$ in^2)

LRFD	ASD
$P_r = P_u = (1.2)(25\text{ k}) + (1.6)(30\text{ k}) = 78\text{ k}$	$P_r = P_a = 25\text{ k} + 30\text{ k} = 55\text{ k}$
$M_{ry} = M_{uy} = (1.2)(10\text{ ft-k}) + (1.6)(25\text{ ft-k}) = 52\text{ ft-k}$	$M_{ry} = M_{ay} = 10\text{ ft-k} + 25\text{ ft-k} = 35\text{ ft-k}$
$P_c = \phi P_n = \phi_t F_y A_g = (0.9)(50\text{ ksi})(11.7\text{ in}^2) = 526.5\text{ k}$	$P_c = \dfrac{P_n}{\Omega_c} = \dfrac{F_y A_g}{\Omega_c} = \dfrac{(50\text{ ksi})(11.7\text{ in}^2)}{1.67} = 350.3\text{ k}$
$M_{cy} = \phi_b M_{py} = 63.0$ ft-k (AISC Table 3-4)	$M_{cy} = \dfrac{M_{cy}}{\Omega_b} = 41.9$ ft-k (AISC Table 3-4)
$\dfrac{P_r}{P_c} = \dfrac{78\text{ k}}{526.5\text{ k}} = 0.148 < 0.2$	$\dfrac{P_r}{P_c} = \dfrac{55\text{ k}}{350.3\text{ k}} = 0.157 < 0.2$
∴ **Must use AISC Eq. H1-1b**	∴ **Must use AISC Eq. H1-1b**
$\dfrac{P_r}{2P_c} + \left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) \le 1.0$	$\dfrac{P_r}{2P_c} + \left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) \le 1.0$
$\dfrac{78}{(2)(526.5)} + \left(0 + \dfrac{52}{63}\right)$	$\dfrac{55}{(2)(350.3)} + \left(0 + \dfrac{35}{41.9}\right)$
$= 0.899 < 1.0$ **OK**	$= 0.914 < 1.0$ **OK**

Example 8.2 — Bending and Axial Force

A W10 × 30 tensile member with no holes, consisting of 50 ksi steel and with $L_b = 12.0$ ft, is subjected to the axial service loads $P_D = 30$ k and $P_L = 50$ k and to the service moments $M_{Dx} = 20$ ft-k and $M_{Lx} = 40$ ft-k. If $C_b = 1.0$, is the member satisfactory?

Solution

Using a W10 × 30 ($A = 8.84$ in^2, $L_p = 4.84$ ft and $L_r = 16.1$ ft, $\phi_b M_{px} = 137$ ft-k, BF for LRFD = 4.61, BF for ASD = 3.08 and $M_{px}/\Omega_b = 91.3$ ft-k from AISC Table 3-2)

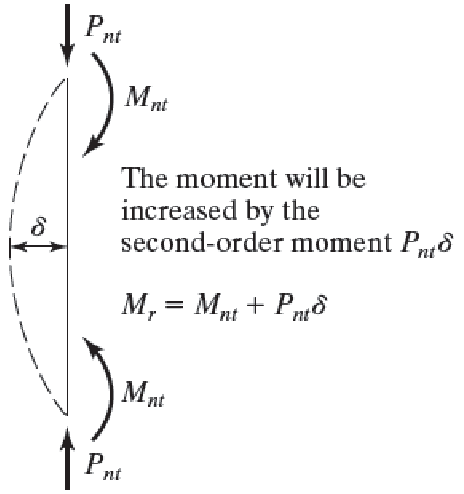
Z_x — Table 3–2 (continued) W Shapes Selection by Z_x — $F_y = 50$ ksi

Shape	Z_x	M_{px}/Ω_b	$\phi_b M_{px}$	M_{rx}/Ω_b	$\phi_b M_{rx}$	BF		L_p	L_r	I_x	V_{nx}/Ω_v	$\phi_v V_{nx}$
		kip-ft	kip-ft	kip-ft	kip-ft	kips	kips				kips	kips
	in.^{3}	ASD	LRFD	ASD	LRFD	ASD	LRFD	ft	ft	in.^{4}	ASD	LRFD
W12×26	**37.2**	**92.8**	**140**	**58.3**	**87.7**	**3.61**	**5.42**	**5.33**	**14.9**	**204**	**56.2**	**84.3**
W10×30	36.6	91.3	137	56.6	85.0	3.08	4.62	4.84	16.1	170	62.8	94.2
W8×35	34.7	86.6	130	54.5	81.9	1.62	2.43	7.17	27.0	127	50.3	75.5

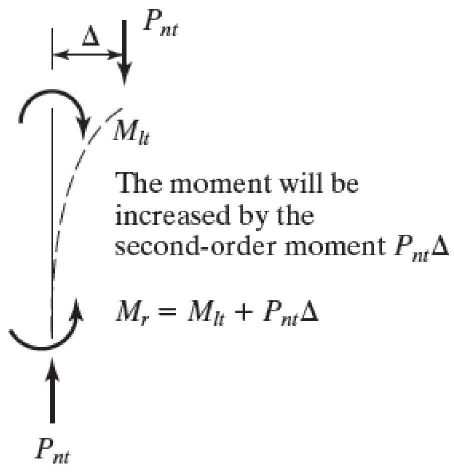
LRFD	ASD
$P_r = P_u = (1.2)(30\text{k}) + (1.6)(50\text{k}) = 116\text{ k}$	$P_r = P_a = 30\text{ k} + 50\text{ k} = 80\text{ k}$
$M_{rx} = M_{ux} = (1.2)(20\text{ ft-k}) + (1.6)(40\text{ ft-k})$	$M_{rx} = M_{ax} = 20\text{ ft-k} + 40\text{ ft-k}$
$= 88\text{ ft-k}$	$= 60\text{ ft-k}$
$P_c = \phi P_n = \phi_t F_y A_g = (0.9)(50\text{ ksi})(8.84\text{ in}^2)$	$P_c = \dfrac{P_n}{\Omega_c} = \dfrac{F_y A_g}{\Omega_c} = \dfrac{(50\text{ ksi})(8.84\text{ in}^2)}{1.67}$
$= 397.8\text{ k}$	$= 264.7\text{ k}$
$M_{cx} = \phi_b M_{nx} = C_b[\phi_b M_{px} - BF(L_b - L_p)]$	$M_{cx} = \dfrac{M_{nx}}{\Omega_b} = C_b\left[\dfrac{M_{px}}{\Omega_b} - BF(L_b - L_p)\right]$
$= 1.0[137 - 4.61(12.0 - 4.84)]$	$= 1.0[91.3 - (3.08)(12 - 4.84)]$
$= 104.0\text{ ft-k}$	$= 69.2\text{ ft-k}$
$\dfrac{P_r}{P_c} = \dfrac{116}{397.8} = 0.292 > 0.2$	$\dfrac{P_r}{P_c} = \dfrac{80}{264.7} = 0.302 > 0.2$
$\therefore$ **Must use AISC Eq. H1-1a**	$\therefore$ **Must use AISC Eq. H1-1a**
$\dfrac{P_r}{P_c} + \dfrac{8}{9}\left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) \le 1.0$	$\dfrac{P_r}{P_c} + \dfrac{8}{9}\left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{rx}}{M_{cy}}\right) \le 1.0$
$\dfrac{116}{397.8} + \dfrac{8}{9}\left(\dfrac{88}{104.0} + 0\right)$	$\dfrac{80}{264.7} + \dfrac{8}{9}\left(\dfrac{60}{69.2} + 0\right)$
$= 1.044 > 1.0$ **N.G.**	$= 1.073 > 1.0$ **N.G.**

8.2 FIRST-ORDER AND SECOND-ORDER MOMENTS FOR MEMBERS SUBJECT TO AXIAL COMPRESSION AND BENDING

8.2.1 APPROXIMATE SECOND-ORDER ANALYSIS



Moment amplification of a column that is braced against sidesway.



Column in an unbraced frame.

The AISC Specification Chapter C.1 states that any rational method of design for stability that considers all of the effects list below is permitted.

1. flexural, shear, and axial member deformation, and all other deformations that contribute to displacement of the structure;
2. second-order effect (both P-Δ and P-δ effects);
3. geometric imperfections;
4. stiffness reductions due to inelasticity;
5. uncertainty in stiffness and strength.

The following is an approximate second-order analysis procedure for calculating the required flexural and axial strengths in members of *lateral load resisting systems.* The required second-order flexural strength, M_r, and axial strength, P_r, shall be determined as follows:

$$M_r = B_1 M_{nt} + B_2 M_{lt} \tag{C2-1a}$$

$$P_r = P_{nt} + B_2 P_{lt} \tag{C2-1b}$$

where

$$B_1 = \frac{C_m}{1 - \alpha P_r / P_{e1}} \geq 1 \tag{C2-2}$$

For members subjected to axial compression, B_1 may be calculated based on the first-order estimate $P_r = P_{nt} + P_{lt}$.

User Note: B_1 is an amplifier to account for second order effects caused by displacements between brace points (P-δ) and B_2 is an amplifier to account for second order effects caused by displacements of braced points (P-Δ).

For members in which $B_1 \leq 1.05$, it is conservative to amplify the sum of the non-sway and sway moments (as obtained, for instance, by a first-order elastic analysis) by the B_2 amplifier, in other words, $M_r = B_2(M_{nt} + M_{lt})$.

$$B_2 = \frac{1}{1 - \dfrac{\alpha \Sigma P_{nt}}{\Sigma P_{e2}}} \geq 1 \tag{C2-3}$$

User Note: Note that the B_2 amplifier (Equation C2-3) can be estimated in preliminary design by using a maximum lateral drift limit corresponding to the story shear ΣH in Equation C2-6b.

and

$$\alpha = 1.00\,(\text{LRFD}) \quad \alpha = 1.60\,(\text{ASD})$$

M_r = required second-order flexural strength using LRFD or ASD load combinations, kip-in. (N-mm)

M_{nt} = first-order moment using LRFD or ASD load combinations, assuming there is no lateral translation of the frame, kip-in. (N-mm)

M_{lt} = first-order moment using LRFD or ASD load combinations caused by lateral translation of the frame only, kip-in. (N-mm)

P_r = required second-order axial strength using LRFD or ASD load combinations, kips (N)

P_{nt} = first-order axial force using LRFD or ASD load combinations, assuming there is no lateral translation of the frame, kips (N)

ΣP_{nt} = total vertical load supported by the story using LRFD or ASD load combinations, including gravity column loads, kips (N)

P_{lt} = first-order axial force using LRFD or ASD load combinations caused by lateral translation of the frame only, kips (N)

C_m = a coefficient assuming no lateral translation of the frame whose value shall be taken as follows:

(i) For beam-columns not subject to transverse loading between supports in the plane of bending,

$$C_m = 0.6 - 0.4(M_1/M_2) \tag{C2-4}$$

where M_1 and M_2, calculated from a first-order analysis, are the smaller and larger moments, respectively, at the ends of that portion of the member unbraced in the plane of bending under consideration. M_1/M_2 is positive when the member is bent in reverse curvature, negative when bent in single curvature.

(ii) For beam-columns subjected to transverse loading between supports, the value of C_m shall be determined either by analysis or conservatively taken as 1.0 for all cases.

P_{e1} = elastic critical buckling resistance of the member in the plane of bending, calculated based on the assumption of zero sidesway, kips (N)

$$P_{e1} = \frac{\pi^2 EI}{(K_1 L)^2} \tag{C2-5}$$

ΣP_{e2} = elastic critical buckling resistance for the story determined by sidesway buckling analysis, kips (N)

For moment frames, where sidesway buckling effective length factors K_2 are determined for the columns, it is permitted to calculate the elastic story sidesway buckling resistance as

$$\Sigma P_{e2} = \Sigma \frac{\pi^2 EI}{(K_2 L)^2} \quad \text{(C2-6a)}$$

For all types of lateral load resisting systems, it is permitted to use

$$\Sigma P_{e2} = R_M \frac{\Sigma HL}{\Delta_H} \quad \text{(C2-6b)}$$

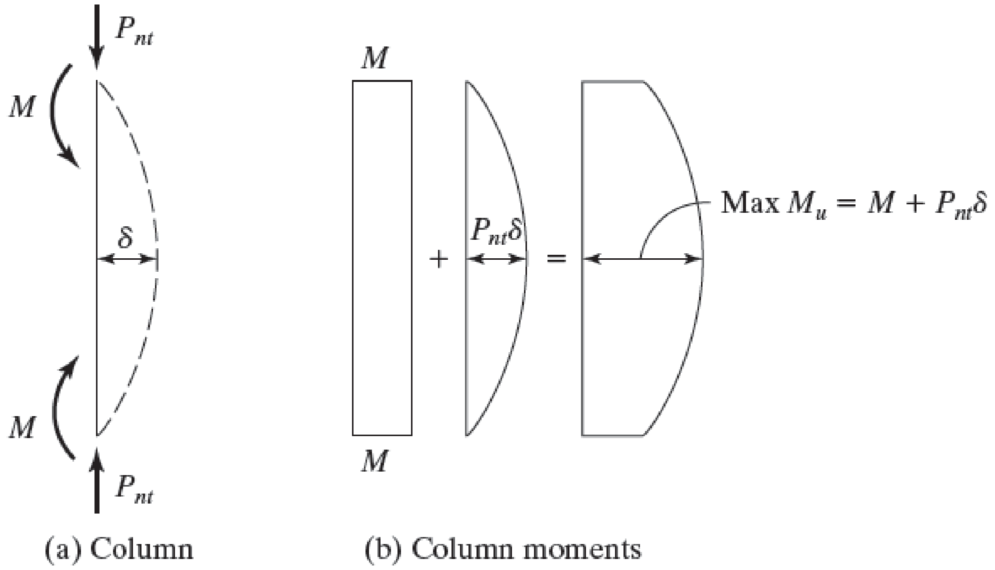
where

E = modulus of elasticity of steel = 29,000 ksi (200 000 MPa)
R_M = 1.0 for braced-frame systems;
= 0.85 for moment-frame and combined systems, unless a larger value is justified by analysis
I = moment of inertia in the plane of bending, in.^{4} (mm^4)
L = story height, in. (mm)
K_1 = effective length factor in the plane of bending, calculated based on the assumption of no lateral translation, set equal to 1.0 unless analysis indicates that a smaller value may be used
K_2 = effective length factor in the plane of bending, calculated based on a sidesway buckling analysis

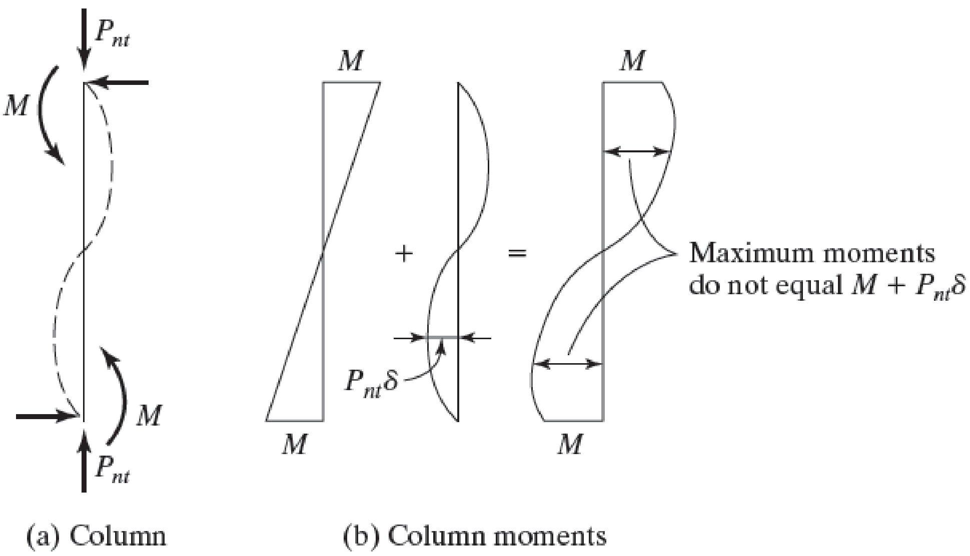
User Note: Methods for calculation of K_2 are discussed in the Commentary.

Δ_H = first-order interstory drift due to lateral forces, in. (mm). Where Δ_H varies over the plan area of the structure, Δ_H shall be the average drift weighted in proportion to vertical load or, alternatively, the maximum drift.
ΣH = story shear produced by the lateral forces used to compute Δ_H, kips (N)

8.2.2 MOMENT MODIFICATION OR C_m FACTORS



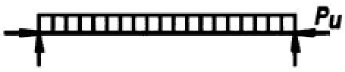
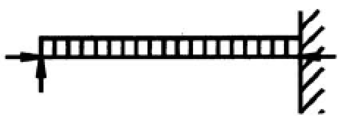
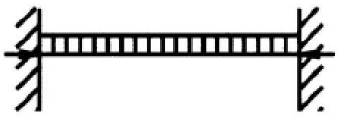
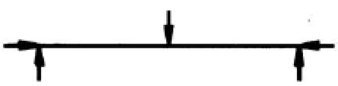
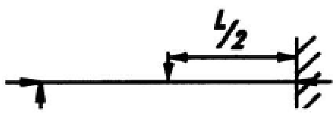
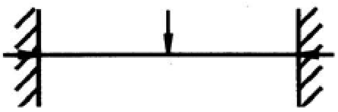
Moment magnification for column bent in single curvature.

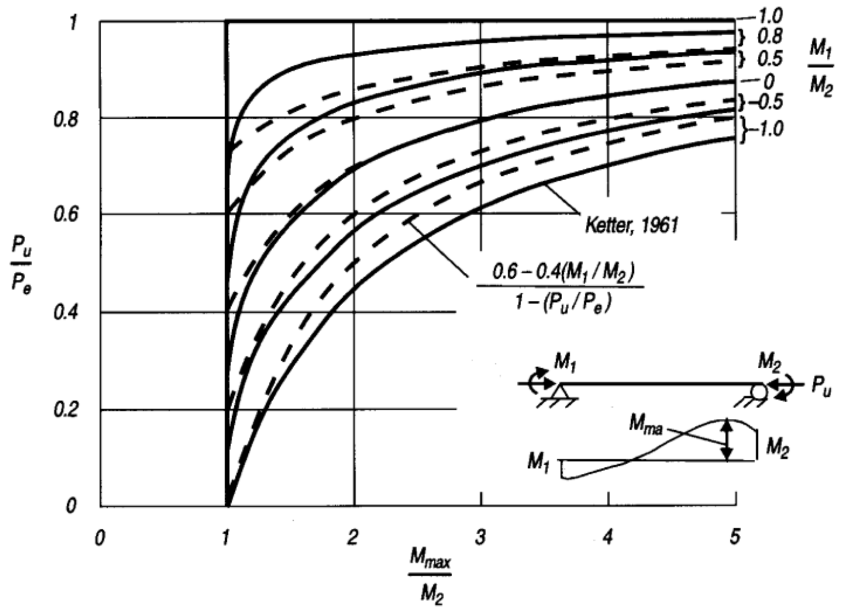


Moment magnification for column bent in double curvature.

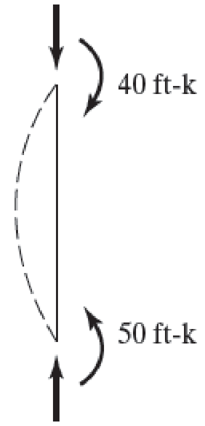
$$C_m = 0.6 - 0.4\frac{M_1}{M_2}$$

TABLE C-C2.1
Amplification Factors ψ and C_m

Case	Ψ	C_m
	0	1.0
	−0.4	$1-0.4\frac{P_u}{P_{e1}}$
	−0.4	$1-0.4\frac{P_u}{P_{e1}}$
	−0.2	$1-0.2\frac{P_u}{P_{e1}}$
	−0.3	$1-0.3\frac{P_u}{P_{e1}}$
	−0.2	$1-0.2\frac{P_u}{P_{e1}}$

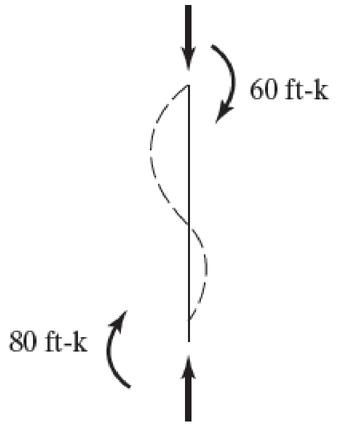


Example 8.3 — Modification or C_m Factors



(a) No sidesway and no transverse loading.
Moments bend member in single curvature.

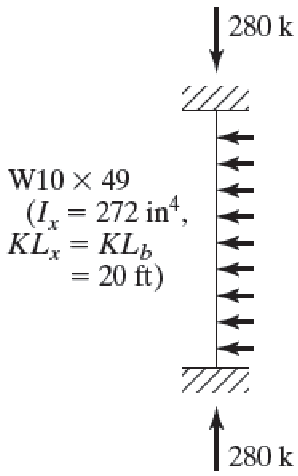
$$C_m = 0.6 - (0.4)\left(-\frac{40}{50}\right) = 0.92$$



(b) No sidesway and no transverse loading.
Moments bend member in reverse curvature.

$$C_m = 0.6 - 0.4\left(+\frac{60}{80}\right) = 0.30$$

Example 8.4 — Modification or C_m Factors



(c) Member has restrained ends and transverse loading and is bent about x axis.

11.1

(AISC Table C-A-8.1) as follows:

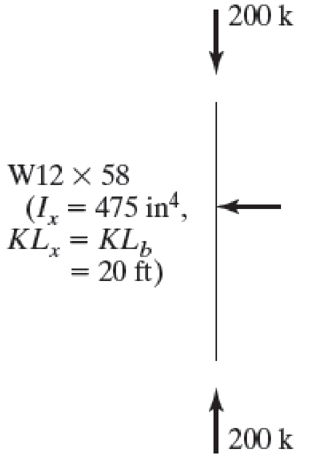
$$\alpha P_r = 280 \text{ k}$$

$$P_{e1} = \frac{\pi^2 EI}{(KL)_x^2} = \frac{(\pi^2)(29 \times 10^3)(272)}{(12 \times 20)^2}$$

$$= 1351 \text{ k}$$

$$C_m = 1 - 0.4\left(+\frac{280}{1351}\right) = 0.92$$

Example 8.5 — Modification or C_m Factors



(d) Member has unrestrained ends and transverse loading and is bent about x axis.

C_m can be determined from Table 11.1 (AISC Table C-A-8.1).

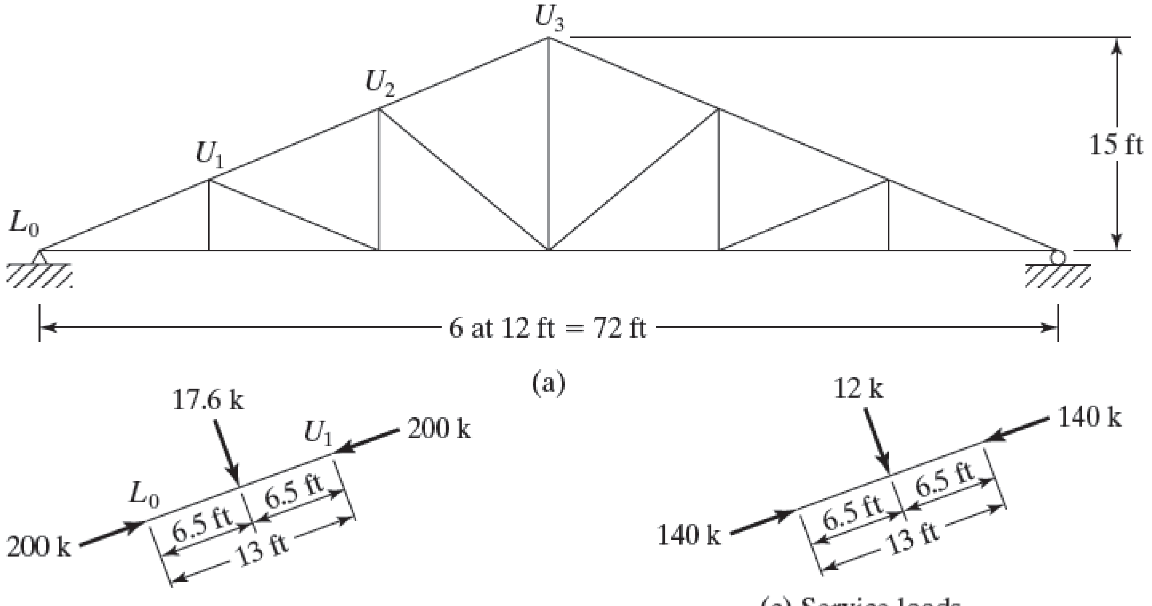
$$\alpha P_r = 200 \text{ k}$$

$$P_{e1} = \frac{(\pi^2)(29 \times 10^3)(475)}{(12 \times 20)^2} = 2360 \text{ k}$$

$$C_m = 1 - 0.2\left(+\frac{200}{2360}\right) = 0.98$$

Example 8.6 — Bending and Axial Force

For the truss shown a W8 × 35 is used as a continuous top chord member from joint L_0 to joint U_3. If the member consists of 50 ksi steel, does it have sufficient strength to resist the loads shown in parts (b) and (c) of the figure? The factored or LRFD loads are shown in part (b), while the service or ASD loads are shown in part (c). The 17.6 k and 12 k loads represent the reaction from a purlin. The compression flange of the W8 is braced only at the ends about the x-x axis, $L_x = 13$ ft, and at the ends and the concentrated load about the y-y axis, $L_y = 6.5$ ft and $L_b = 6.5$ ft.



(b) Factored loads (LRFD)

(c) Service loads (ASD)

Solution

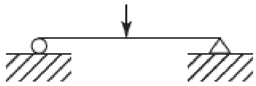
Using a W8 × 35 ($A = 10.3\text{ in}^2$, $I_x = 127\text{ in}^4$, $r_x = 3.51\text{ in}$, $r_y = 2.03\text{ in}$, $L_P = 7.17\text{ ft}$, $\phi_b M_{Px} = 130\text{ ft-k}$, $\dfrac{M_{Px}}{\Omega_b} = 86.6\text{ ft-k}$, $r_x/r_y = 1.73$).

LRFD	ASD
$P_{nt} = P_u$ from figure $= 200\text{ k} = P_r$	$P_{nt} = P_a$ from figure $= 140\text{ k} = P_r$
Conservatively assume $K_x = K_y = 1.0$. In truth, the K-factor is somewhere between $K = 1.0$ (pinned–pinned end condition) and $K = 0.8$ (pinned–fixed end condition) for segment L_oU_i	Conservatively assume $K_x = K_y = 1.0$. In truth, the K-factor is somewhere between $K = 1.0$ (pinned–pinned end condition) and $K = 0.8$ (pinned–fixed end condition) for segment L_oU_i
$\left(\frac{KL}{r}\right)_x = \frac{(1.0)(12 \times 13)}{3.51} = 44.44 \leftarrow$	$\left(\frac{KL}{r}\right)_x = \frac{(1.0)(12 \times 13)}{3.51} = 44.44 \leftarrow$
$\left(\frac{KL}{r}\right)_y = \frac{(1.0)(12 \times 6.5)}{2.03} = 38.42$	$\left(\frac{KL}{r}\right)_y = \frac{(1.0)(12 \times 6.5)}{2.03} = 38.42$
From AISC Table 4-22, $F_y = 50$ ksi	From AISC Table 4-22, $F_y = 50$ ksi
$\phi_c F_{cr} = 38.97$ ksi	$\frac{F_{cr}}{\Omega_c} = 25.91$ ksi
$\phi_c P_n = (38.97)(10.3) = 401.4\text{ k} = P_c$	$\frac{P_n}{\Omega_c} = (25.91)(10.3) = 266.9\text{ k} = P_c$
$\frac{P_r}{P_c} = \frac{200}{401.4} = 0.498 > 0.2$	$\frac{P_r}{P_c} = \frac{140}{266.9} = 0.525 > 0.2$
$\therefore$ Must use AISC Eq. H1-1a	$\therefore$ Must use AISC Eq. H1-1a
Computing P_{e1x} and C_{mx}	Computing P_{e1x} and C_{mx}
$P_{e1x} = \frac{(\pi^2)(29{,}000)(127)}{(1.0 \times 12 \times 13)^2} = 1494\text{ k}$	$P_{e1x} = \frac{(\pi^2)(29{,}000)(127)}{(1.0 \times 12 \times 13)^2} = 1494\text{ k}$
	Computing C_m as in LRFD

Table 4–22 (continued)

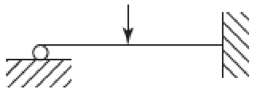
Kl/r	$F_y = 35$ksi F_{cr}/Ω_c ksi ASD	$F_y = 35$ksi $\phi_c F_{cr}$ ksi LRFD	Kl/r	$F_y = 36$ksi F_{cr}/Ω_c ksi ASD	$F_y = 36$ksi $\phi_c F_{cr}$ ksi LRFD	Kl/r	$F_y = 42$ksi F_{cr}/Ω_c ksi ASD	$F_y = 42$ksi $\phi_c F_{cr}$ ksi LRFD	Kl/r	$F_y = 46$ksi F_{cr}/Ω_c ksi ASD	$F_y = 46$ksi $\phi_c F_{cr}$ ksi LRFD	Kl/r	$F_y = 50$ksi F_{cr}/Ω_c ksi ASD	$F_y = 50$ksi $\phi_c F_{cr}$ ksi LRFD
41	19.2	28.9	41	19.7	29.7	41	22.7	34.1	41	24.6	37.0	41	26.5	39.8
42	19.2	28.8	42	19.6	29.5	42	22.6	33.9	42	24.5	36.8	42	26.3	39.5
43	19.1	28.7	43	19.6	29.4	43	22.5	33.7	43	24.3	36.6	43	26.2	39.3
44	19.0	28.5	44	19.5	29.3	44	22.3	33.6	44	24.2	36.3	44	26.0	39.1
45	18.9	28.4	45	19.4	29.1	45	22.2	33.4	45	24.0	36.1	45	25.8	38.8

For



$$C_{mx} = 1 - 0.2\left(\frac{1.0\,(200)}{1494}\right) = 0.973$$

For

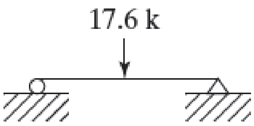


$$C_{mx} = 1 - 0.3\left(\frac{1.0\,(200)}{1494}\right) = 0.960$$

Avg $C_{mx} = 0.967$

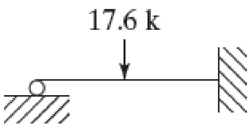
Computing M_{ux}

For



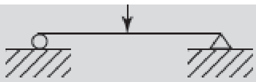
$$M_{ux} = \frac{PL}{4} = \frac{(17.6)(13)}{4} = 57.2 \text{ ft-k}$$

For



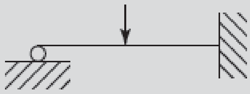
$$M_{ux} = \frac{3\,PL}{16} = \frac{(3)(17.6)(13)}{16} = 42.9 \text{ ft-k}$$

For



$$C_{mx} = 1 - 0.2\left(\frac{1.6\,(140)}{1494}\right) = 0.970$$

For

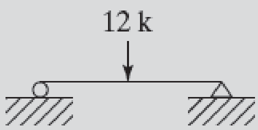


$$C_{mx} = 1 - 0.3\left(\frac{1.6\,(140)}{1494}\right) = 0.955$$

Avg $C_{mx} = 0.963$

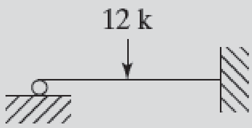
Computing M_{ax}

For

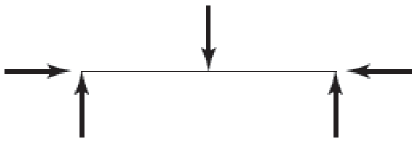
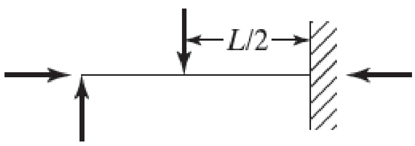


$$M_{ax} = \frac{(12)(13)}{4} = 39 \text{ ft-k}$$

For



$$M_{ax} = \frac{(3)(12)(13)}{16} = 29.25 \text{ ft-k}$$

	-0.2	$1 - 0.2\dfrac{\alpha P_r}{P_{e1}}$
	-0.3	$1 - 0.3\dfrac{\alpha P_r}{P_{e1}}$

Avg $M_{ux} = 50.05 \text{ ft-k} = M_{rx}$

$$B_{1x} = \frac{0.967}{1 - \dfrac{(1)(200)}{1494}} = 1.116$$

$M_r = (1.116)(50.05) = 55.86$ ft-k

Since $L_b = 6.5 \text{ ft} < L_p = 7.17$ ft

$\therefore$ Zone ①

$\phi_b M_{nx} = 130 \text{ ft-k} = M_{cx}$

Using Equation H1-1a

$$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \le 1.0$$

$$\frac{200}{401.4} + \frac{8}{9}\left(\frac{55.86}{130} + 0\right) \le 1.0$$

$0.880 \le 1.0$ Section OK

From AISC Table 6-1

$(KL)_y = 6.5$ ft

$$(KL)_{yEQUIV} = \frac{(KL)_x}{r_x/r_y} = \frac{13}{1.73} = 7.51 \text{ ft} \leftarrow$$

$P = 2.50 \times 10^{-3}$, for $KL = 7.51$ ft

$b_x = 6.83 \times 10^{-3}$, for $L_b = 6.5$ ft

$p\,P_r + b_x\,M_{rx} + b_y\,M_{ry} \le 1.0$

$= (2.50 \times 10^{-3})\,(200) + (6.83 \times 10^{-3})\,(55.86) + 0$

$= 0.882 \le 1.0$ Section OK

Section is Satisfactory.

Avg $M_{ax} = 34.13 \text{ ft-k} = M_{rx}$

$$B_{1x} = \frac{0.967}{1 - \dfrac{(1.6)(140)}{1494}} = 1.138$$

$M_r = (1.138)(34.13) = 38.84$ ft-k

Since $L_b = 6.5 \text{ ft} < L_P = 7.17$ ft

$\therefore$ Zone ①

$$\frac{M_{nx}}{\Omega_b} = 86.6 \text{ ft-k} = M_{cx}$$

Using Equation H1-1a

$$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \le 1.0$$

$$\frac{140}{266.9} + \frac{8}{9}\left(\frac{38.84}{88.6} + 0\right) \le 1.0$$

$0.914 \le 1.0$ Section OK

From AISC Table 6-1

$(KL)_y = 6.5$ ft

$$(KL)_{yEQUIV} = \frac{(KL)_x}{r_x/r_y} = \frac{13}{1.73} = 7.51 \text{ ft} \leftarrow$$

$P = 3.75 \times 10^{-3}$, for $KL = 7.51$ ft

$b_x = 10.3 \times 10^{-3}$, for $L_b = 6.5$ ft

$p\,P_r + b_x\,M_{rx} + b_y\,M_{ry} \le 1.0$

$= (3.75 \times 10^{-3})\,(140) + (10.3 \times 10^{-3})\,(38.84) + 0$

$= 0.925 \le 1.0$ Section OK

Section is Satisfactory.

Table 6–1

Shape		W8×							
		40				35			
Design		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
on r_y	0	2.85	1.89	8.95	5.96	3.25	2.16	10.3	6.83
	6	3.12	2.07	8.95	5.96	3.56	2.37	10.3	6.83
	7	3.22	2.14	8.95	5.96	3.68	2.45	10.3	6.83
	8	3.35	2.23	9.07	6.04	3.82	2.54	10.4	6.94
	9	3.49	2.32	9.23	6.14	3.99	2.66	10.6	7.07

BENDING AND AXIAL FORCE (BEAM-COLUMNS)

AISC Chapter H, Page 70 will be considered

- **Interaction Equations**

1. Doubly and Singly Symmetric Members in Flexure and Compression

(a) For $\frac{P_r}{P_c} \geq 0.2$

$$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0 \qquad \text{(H1-1a)}$$

(b) For $\frac{P_r}{P_c} < 0.2$

$$\frac{P_r}{2P_c} + \left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0 \qquad \text{(H1-1b)}$$

where

P_r = required axial compressive strength, kips (N)
P_c = *available axial compressive strength*, kips (N)
M_r = *required flexural strength*, kip-in. (N-mm)
M_c = *available flexural strength*, kip-in. (N-mm)
x = subscript relating symbol to *strong axis* bending
y = subscript relating symbol to *weak axis* bending

For design according to Section B3.3 (LRFD)

P_r = *required axial compressive strength* using *LRFD load combinations*, kips (N)
$P_c = \phi_c P_n$ = *design axial compressive strength*, determined in accordance with Chapter E, kips (N)
M_r = *required flexural strength* using LRFD load combinations, kip-in. (N-mm)
$M_c = \phi_b M_n$ = *design flexural strength* determined in accordance with Chapter F, kip-in. (N-mm)
ϕ_c = *resistance factor* for compression = 0.90
ϕ_b = resistance factor for flexure = 0.90

For design according to Section B3.4 (ASD)

P_r = required axial compressive strength using *ASD load combinations*, kips (N)

$P_c = P_n/\Omega_c$ = *allowable axial compressive strength*, determined in accordance with Chapter E, kips (N)

M_r = required flexural strength using ASD load combinations, kip-in. (N-mm)

$M_c = M_n/\Omega_b$ = *allowable flexural strength* determined in accordance with Chapter F, kip-in. (N-mm)

Ω_c = *safety factor* for compression = 1.67

Ω_b = safety factor for flexure = 1.67

- **FIRST-ORDER AND SECOND-ORDER MOMENTS FOR MEMBERS SUBJECT TO AXIAL COMPRESSION AND BENDING (M_r and P_r), Zero Sidesway**

AISC Chapter **C**, Section C2, Page 21

1b. Second-Order Analysis by Amplified First-Order Elastic Analysis

$$M_r = B_1 M_{nt} + \cancel{B_2 M_{lt}} \qquad \text{(C2-1a)}$$

$$P_r = P_{nt} + \cancel{B_2 P_{lt}} \qquad \text{(C2-1b)}$$

$\alpha = 1.00$ (LRFD) $\quad \alpha = 1.60$ (ASD)

$$B_1 = \frac{C_m}{1 - \alpha P_r / P_{e1}} \geq 1 \qquad \text{(C2-2)}$$

(i) For beam-columns not subject to transverse loading between supports in the plane of bending,

$$C_m = 0.6 - 0.4(M_1/M_2) \qquad \text{(C2-4)}$$

where M_1 and M_2, calculated from a first-order analysis, are the smaller and larger moments, respectively, at the ends of that portion of the member unbraced in the plane of bending under consideration. M_1/M_2 is positive when the member is bent in reverse curvature, negative when bent in single curvature.

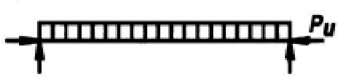
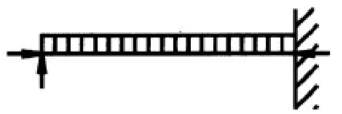
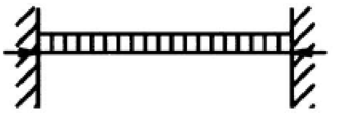
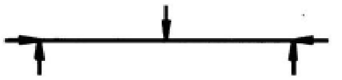
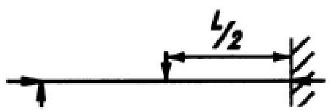
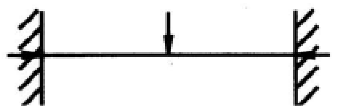
(ii) For beam-columns subjected to transverse loading between supports, the value of C_m shall be determined either by analysis or conservatively taken as 1.0 for all cases.

$$P_{e1} = \frac{\pi^2 EI}{(K_1 L)^2} \qquad \text{(C2-5)}$$

C_m For Lateral Uniformly Distributed Load or Lateral Concentrated Force

AISC Chapter Comm **C2**, Table C-C2.1, Page 237

TABLE C-C2.1
Amplification Factors ψ and C_m

Case	Ψ	C_m
	0	1.0
	−0.4	$1-0.4\frac{P_u}{P_{e1}}$
	−0.4	$1-0.4\frac{P_u}{P_{e1}}$
	−0.2	$1-0.2\frac{P_u}{P_{e1}}$
	−0.3	$1-0.3\frac{P_u}{P_{e1}}$
	−0.2	$1-0.2\frac{P_u}{P_{e1}}$

- **FOR W SHAPES ONLY IN COMBINED AND AXIAL AND BENDING**

AISC Chapter **6**, Table 6-1, Page 6-1 and 6-5

When $P_r/P_c \geq 0.2$, the tabulated values of p, b_x, and b_y can be used as follows to solve the modified form of AISC Specification Equation H1-1a:

$$pP_r + b_xM_{rx} + b_yM_{ry} \leq 1.0$$

When $P_r/P_c < 0.2$, the tabulated values of p, b_x, and b_y can be used as follows to solve the modified form of AISC Specification Equation H1-1b:

$$\tfrac{1}{2}pP_r + \tfrac{9}{8}(b_xM_{rx} + b_yM_{ry}) \leq 1.0$$

$$pP_r + b_xM_{rx} + b_yM_{ry} \leq 1.0 \quad \text{(Modified AISC Equation H1-1a)}$$

$$\frac{1}{2}pP_r + \frac{9}{8}(b_xM_{rx} + b_yM_{ry}) \leq 1.0 \quad \text{(Modified AISC Equation H1-1b)}$$

Table 6–1

Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
0	0.345	0.230	0.220	0.146	0.417	0.278	0.253	0.168	0.476	0.317	0.281	0.187
Other Constants and Properties												
$b_y \times 10^3$ (kip-ft)$^{-1}$	1.51		1.00		1.74		1.16		1.96		1.30	
$t_y \times 10^3$ (kips)$^{-1}$	0.339		0.226		0.390		0.260		0.434		0.289	
$t_r \times 10^3$ (kips)$^{-1}$	0.417		0.278		0.480		0.320		0.534		0.356	
r_x/r_y	5.10				5.10				5.10			

	LRFD	ASD
Axial Compression	$p = \dfrac{1}{\phi_c P_n}$, (kips)$^{-1}$	$p = \dfrac{\Omega_c}{P_n}$, (kips)$^{-1}$
Strong Axis Bending	$b_x = \dfrac{8}{9\phi_b M_{nx}}$, (kip-ft)$^{-1}$	$b_x = \dfrac{8\Omega_b}{9M_{nx}}$, (kip-ft)$^{-1}$
Weak Axis Bending	$b_y = \dfrac{8}{9\phi_b M_{ny}}$, (kip-ft)$^{-1}$	$b_y = \dfrac{8\Omega_b}{9M_{ny}}$, (kip-ft)$^{-1}$

Table 6-1
Combined Axial and Bending
W Shapes

F_y = 50 ksi

W44

Shape	W44× 335^c				W44× 290^c				W44× 262^c			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length *KL* (ft) with respect to least radius of gyration r_y or Unbraced Length L_b (ft) for X-X axis bending												
0	0.345	0.230	0.220	0.146	0.417	0.278	0.253	0.168	0.476	0.317	0.281	0.187
11	0.378	0.251	0.220	0.146	0.454	0.302	0.253	0.168	0.518	0.344	0.281	0.187
12	0.384	0.256	0.220	0.146	0.462	0.307	0.253	0.168	0.526	0.350	0.281	0.187
13	0.393	0.261	0.222	0.148	0.470	0.313	0.255	0.170	0.536	0.356	0.284	0.189
14	0.402	0.267	0.225	0.150	0.480	0.319	0.259	0.173	0.546	0.363	0.289	0.192
15	0.412	0.274	0.229	0.152	0.490	0.326	0.264	0.175	0.557	0.371	0.294	0.196
16	0.423	0.282	0.233	0.155	0.501	0.333	0.268	0.178	0.570	0.379	0.299	0.199
17	0.435	0.290	0.236	0.157	0.514	0.342	0.273	0.181	0.584	0.389	0.304	0.203
18	0.449	0.299	0.240	0.160	0.527	0.351	0.277	0.184	0.599	0.399	0.310	0.206
19	0.463	0.308	0.244	0.162	0.542	0.361	0.282	0.188	0.616	0.410	0.316	0.210
20	0.479	0.319	0.248	0.165	0.559	0.372	0.287	0.191	0.634	0.422	0.322	0.214
22	0.515	0.343	0.257	0.171	0.597	0.397	0.298	0.198	0.676	0.450	0.335	0.223
24	0.558	0.371	0.266	0.177	0.644	0.428	0.309	0.206	0.727	0.484	0.348	0.232
26	0.608	0.405	0.275	0.183	0.702	0.467	0.321	0.214	0.788	0.524	0.363	0.242
28	0.668	0.444	0.286	0.190	0.770	0.513	0.334	0.223	0.862	0.574	0.380	0.253
30	0.738	0.491	0.297	0.198	0.852	0.567	0.349	0.232	0.954	0.635	0.397	0.264
32	0.822	0.547	0.310	0.206	0.948	0.631	0.365	0.243	1.06	0.708	0.417	0.278
34	0.923	0.614	0.323	0.215	1.06	0.708	0.382	0.254	1.20	0.796	0.439	0.292
36	1.04	0.689	0.338	0.225	1.19	0.794	0.401	0.267	1.34	0.892	0.466	0.310
38	1.15	0.767	0.354	0.235	1.33	0.885	0.429	0.285	1.49	0.994	0.508	0.338
40	1.28	0.850	0.377	0.251	1.47	0.981	0.463	0.308	1.66	1.10	0.550	0.366
42	1.41	0.937	0.405	0.269	1.62	1.08	0.498	0.331	1.83	1.21	0.593	0.394
44	1.55	1.03	0.432	0.287	1.78	1.19	0.533	0.355	2.00	1.33	0.636	0.423
46	1.69	1.12	0.459	0.306	1.95	1.30	0.569	0.378	2.19	1.46	0.679	0.452
48	1.84	1.22	0.487	0.324	2.12	1.41	0.604	0.402	2.38	1.59	0.723	0.481
50	2.00	1.33	0.514	0.342	2.30	1.53	0.640	0.426	2.59	1.72	0.767	0.510

Other Constants and Properties

	335^c ASD	335^c LRFD	290^c ASD	290^c LRFD	262^c ASD	262^c LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	1.51	1.00	1.74	1.16	1.96	1.30
$t_y \times 10^3$ (kips)$^{-1}$	0.339	0.226	0.390	0.260	0.434	0.289
$t_r \times 10^3$ (kips)$^{-1}$	0.417	0.278	0.480	0.320	0.534	0.356
r_x/r_y	5.10		5.10		5.10	

[c] Shape is slender for compression with F_y = 50 ksi.

COVER-PLATED BEAMS AND BUILT-UP GIRDERS

9

9.1 COVER-PLATED BEAMS

Should the largest available W section be insufficient to support the loads anticipated for a certain span, several possible alternatives may be taken. Perhaps the most economical solution involves the use of a higher-strength steel W section. If this is not feasible, we may make use of one of the following:

1. Two or more regular W sections side-by-side (**an expensive solution**).
2. A **cover-plated beam**,
3. A **built-up girder**, or
4. A **steel truss**. This section discusses the cover-plated beams, while the remainder of the chapter is concerned with built-up girders.

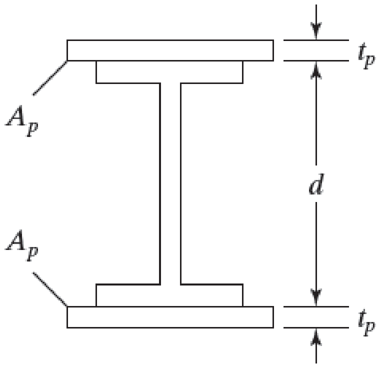
Cover-plated beams frequently will be a very satisfactory solution for situations like these. Furthermore, there may be economical uses for cover-plated beams where the depth is not limited and where there are standard W sections available to support the loads. A smaller W section than required by the maximum moment can be selected and have cover plates attached to its flanges.

For this discussion, reference is made to the figure below. For the derivation to follow, Z is the plastic modulus for the entire built-up section, is the plastic modulus for the W section,

d is the depth of the W section,

t_p is the thickness of one cover plate,and is the area of one cover plate.

A_p is the area of one cover plate.



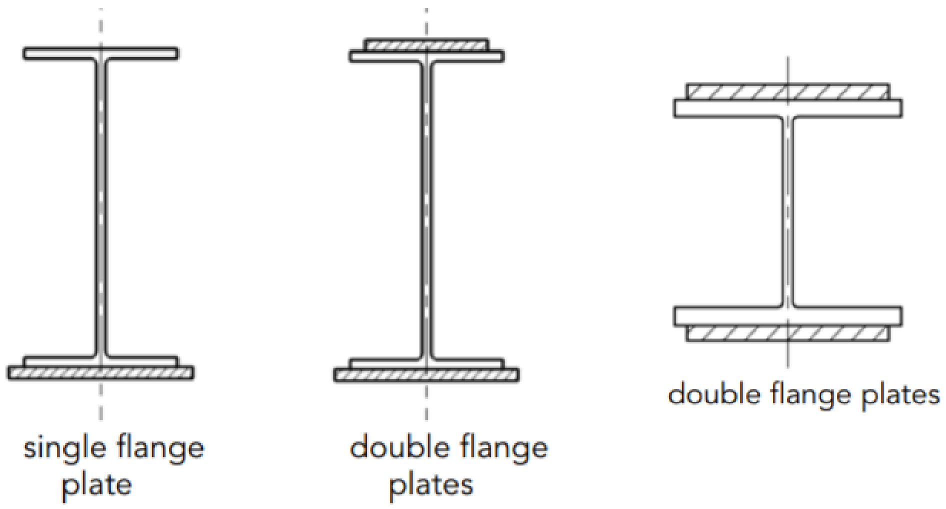
The expressions to follow are written for LRFD design. The designer selects a cover-plated beam and then computes its LRFD design strength and its ASD allowable strength.

An expression for the required area of one flange cover plate can be developed as follows:

$$Z_{\text{reqd}} = \frac{M_u}{\phi_b F_y}$$

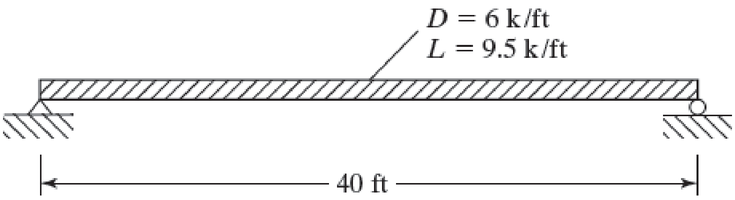
The total Z of the built-up section must at least equal the Z required. It will be furnished by the W shape and the cover plates as follows:

$$\begin{aligned} Z_{\text{reqd}} &= Z_W + Z_{\text{plates}} \\ &= Z_W + 2A_p\left(\frac{d}{2} + \frac{t_p}{2}\right) \\ A_p &= \frac{Z_{\text{reqd}} - Z_W}{d + t_p} \end{aligned}$$



Example 9.1 COVER-PLATED BEAMS

Select a beam limited to a maximum depth of 29.50 in (648 mm) for the loads and span shown. A 50 ksi (345 MPa) steel is used, and the beam is assumed to have full lateral bracing for its compression flange. Span is 40 ft (≈12m), D=6 k/ft (87.6 kN/m), L=9.5 k/ft (138.6 kN/m).



Solution

Assume beam wt = 350 lb/ft

LRFD	ASD
$w_u = (1.2)(6.35) + (1.6)(9.5) = 22.82$ k/ft	$w_a = 6.35 + 9.5 = 15.85$ k/ft
$M_u = \frac{(22.82)(40)^2}{8} = 4564$ ft-k	$M_a = \frac{(15.85)(40)^2}{8} = 3170$ ft-k

$$Z_{\text{reqd}} = \frac{(12)(4564)}{(0.9)(50)} = 1217 \text{ in}^3$$

The only W sections listed in the Manual with depths $\leq$ 29.50 in and Z values $\geq$ 1217 in^3 are the impractical, extremely heavy and expensive, W14 × 605, W14 × 665, and W14 × 730. As a result, the author decided to use a lighter W section with cover plates. He assumes the plates are each 1 in thick.

Try W27 × 146 (d = 27.4 in, Z_x = 464 in^3, b_f = 14.0 in)

Total depth = 27.4 + (2)(1.00) = 29.4 in < 29.5 in **OK**

Area of 1 cover plate for each flange

$$A_p = \frac{Z_{\text{reqd}} - Z_w}{d + t_p} = \frac{1217 - 464}{27.4 + 1.00} = 26.51 \text{ in}^2$$

Try a 1 × 28 in **cover plate each flange**

$$Z_{\text{furnished}} = 464 + (1)(28)(2)\left(\frac{27.4}{2} + 0.5\right)$$

$$= 1259.2 \text{ in}^3 > 1217 \text{ in}^3 \quad \textbf{OK}$$

$$M_n = \frac{F_y Z}{12} = \frac{(50)(1259.2)}{12} = 5246.7 \text{ ft-k}$$

LRFD $\phi_b = 0.9$	ASD $\Omega_b = 1.67$
$\phi_b M_n = (0.9)(5246.7) = 4722 \text{ ft-k} > 4564 \text{ ft-k}$ **OK**	$\frac{M_n}{\Omega_b} = \frac{5246.7}{1.67} = 3142 \text{ ft-k} < 3170 \text{ ft-k}$ **Not quite**

A check of the *b*/*t* ratios for the plates, web, and flanges show them to be satisfactory.

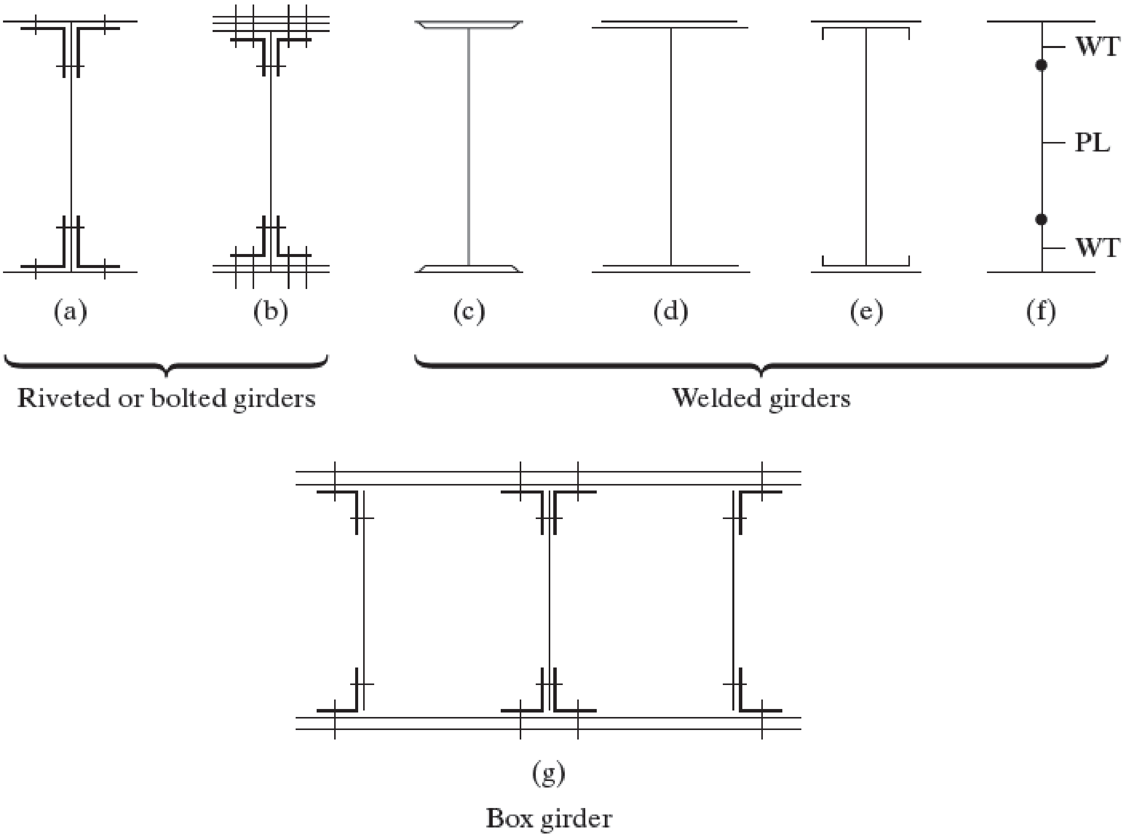
Use W27 × 146 with one 1 × 28 in each flange for LRFD (slightly larger plate needed for ASD).

$$\text{Wt of steel for LRFD design} = 146 + \left(\frac{2 \times 1 \times 28}{144}\right)(490) = 337 \text{ lb/ft} < \text{estimated}$$

$$350 \text{ lb/ft} \quad \textbf{OK}$$

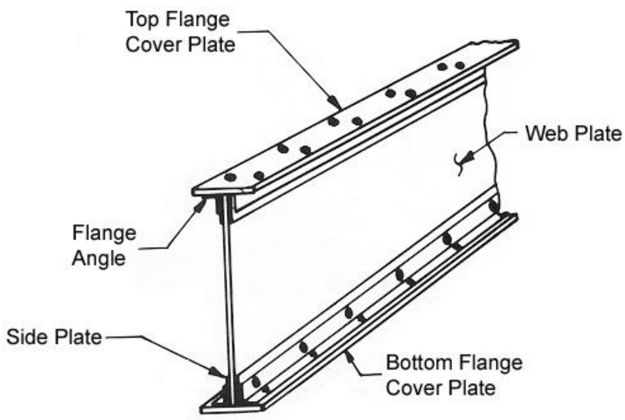
9.2 BUILT-UP GIRDERS (PLATE GIRDERS)

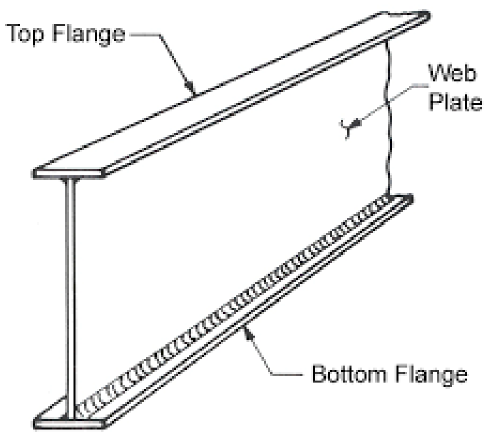
Built-up I-shaped girders, frequently called **plate girders**, are made up with plates and perhaps with rolled sections. They usually have design moment strengths somewhere between those of rolled beams and steel trusses. Several possible arrangements are shown in figure.



- Rather obsolete **bolted girders** are shown in parts (a) and (b) of the figure.
- Several **welded girders** are shown in parts (c) through (f). Since nearly all built-up girders constructed today are welded (although they may make use of bolted field splices), this chapter is devoted almost exclusively to welded girders.
- The welded girder of part (d) shown in the figure is arranged to reduce overhead welding, compared with the girder of part (c), but in so doing may be creating a slightly worse corrosion situation if the girder is exposed to the weather.
- A **box girder**, illustrated in part (g), occasionally is used where moments are large and depths are quite limited. Box girders also have great resistance to torsion and lateral

buckling. In addition, they make very efficient curved members because of their high torsional strengths.





- Most steel highway bridges built today for spans of less than about **80 ft (24 m)** are steel beam bridges. For longer spans, the built-up girder begins to compete very well economically.
- Where loads are extremely large, such as for railroad bridges, built-up girders are competitive for spans as small as **45 or 50 ft (14-15 m)**.
- Generally speaking, built-up girders are very economical for railroad bridges in spans of **50 to 130 ft (15 to 40 m)**, and for highway bridges in spans of **80 to 150 ft (24 to 46 m)**. However, they are often very competitive for much longer spans, particularly when continuous.
- In fact, they are actually common for **200-ft (61-m)** spans and have been used for many spans in excess of **400 ft (122 m)**.
- The main span of the continuous Bonn-Beuel built-up girder bridge over the Rhine River in Germany is **643 ft**.

The usual practical alternative to built-up girders in the spans for which they are economical is the truss. In general, plate girders have the following advantages, particularly compared with trusses:

1. The pound price for fabrication is lower than for trusses, but it is higher than for rolled beam sections.
2. Erection is cheaper and faster than for trusses.
3. Due to the compactness of built-up girders, vibration and impact are not serious problems.
4. Built-up girders require smaller vertical clearances than trusses.
5. The built-up girder has fewer critical points for stresses than do trusses.
6. A bad connection here or there is not as serious as in a truss, where such a situation could spell disaster.
7. There is less danger of injury to built-up girders in an accident, compared with trusses. Should a truck run into a bridge plate girder, it would probably just bend it a little, but a similar accident with a bridge truss member could cause a broken member and, perhaps, failure.
8. A built-up girder is more easily painted than a truss.

9.3 BUILT-UP GIRDER PROPORTIONS

9.3.1 DEPTH

The depths of built-up girders vary from about **1/6** to **1/15** of their spans, with average values of **1/10** to **1/12**, depending on the particular conditions of the job.

$$\frac{d}{span} = \frac{1}{6}\ to\ \frac{1}{15}$$

$$\left(\frac{d}{span}\right)_{\text{Average}} = \frac{1}{10}\ to\ \frac{1}{12}$$

One condition that may limit the proportions of the girder is the largest size that can be fabricated in the shop and shipped to the job. There may be a transportation problem such as clearance requirements that limit maximum depths to **10 or 12 ft** (**3 or 3.66 m**) along the shipping route.

9.3.2 WEB SIZE

After the total girder depth is estimated, the general proportions of the girder can be established from the maximum shear and the maximum moment.

The web depth can be closely estimated by taking the total girder depth and subtracting a reasonable value for the depths of the flanges (roughly **1** to **2 in** each).

$$\textbf{web depth} \approx \textbf{total girder depth} - \textbf{depths of the flanges}\ (\textbf{roughly 1" to 2" each})$$

This problem is handled in **Section G** of the **AISC** Specification.

There, the nominal shearing strength of the webs of stiffened or unstiffened built-up I-shaped girders is presented. In the following equation,

A_w is the depth of the plate girder web times its thickness $A_w = ht_w$

C_v is a web coefficient, values of which are given after the equation:

$$V_n = 0.6F_yA_wC_v \qquad \text{(AISC Equation G2-1)}$$

$$\phi_v = 0.90 \quad \Omega_v = 1.67$$

Values of C_v are as follows:

1. For $h/t_w \le 1.10\sqrt{k_v\, E/F_y}$

$$C_v = 1.0 \qquad \text{(AISC Equation G2-3)}$$

2. For $1.10\sqrt{k_v\, E/F_y} < \dfrac{h}{t_w} \le 1.37\sqrt{k_v\, E/F_y}$

$$C_v = \frac{1.10\sqrt{k_v\, E/F_y}}{h/t_w} \qquad \text{(AISC Equation G2-4)}$$

3. For $h/t_w > 1.37\sqrt{k_v\, E/F_y}$

$$C_v = \frac{1.51Ek_v}{(h/t_w)^2F_y} \qquad \text{(AISC Equation G2-5)}$$

where

A_w is the depth of the plate girder web times its thickness $A_w = ht_w$

The web plate buckling coefficient, k_v, is determined as follows:

(i) For unstiffened webs with $h/t_w < 260$, $k_v = 5$ except for the stem of tee shapes where $k_v = 1.2$.

(ii) For stiffened webs,

$$k_v = 5 + \frac{5}{(a/h)^2}$$

$$= 5 \text{ when } a/h > 3.0 \text{ or } a/h > \left[\frac{260}{(h/t_w)}\right]^2$$

where

a = clear distance between transverse *stiffeners*, in. (mm)

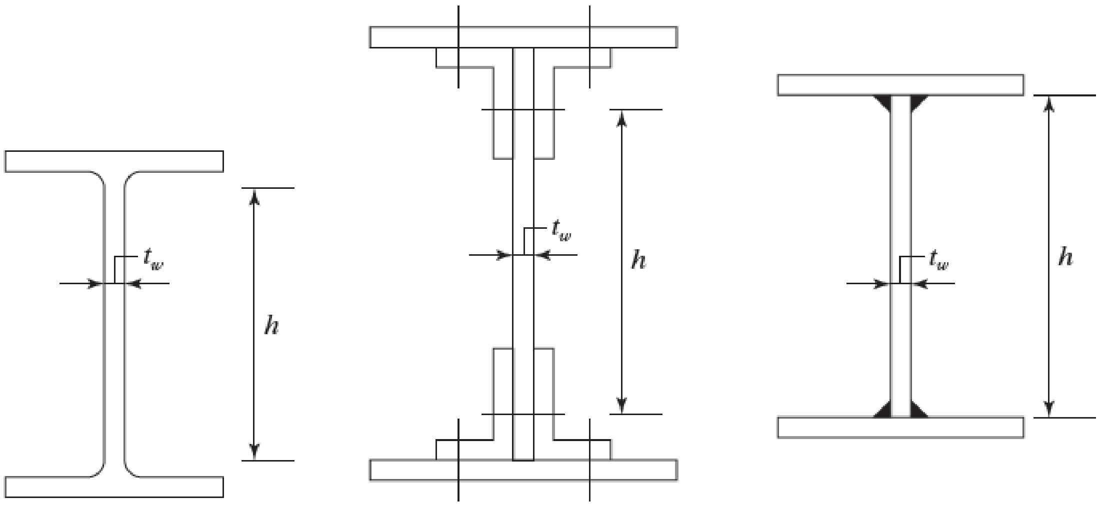
h = for rolled shapes, the clear distance between flanges less the fillet or corner radii, in. (mm)

= for built-up welded sections, the clear distance between flanges, in. (mm)

= for built-up bolted sections, the distance between *fastener* lines, in. (mm)

= for tees, the overall depth, in. (mm)

User Note: For all ASTM A6 W, S, M and HP shapes except M12.5×12.4, M12.5×11.6, M12×11.8, M12×10.8, M12×10, M10×8, and M10×7.5, when $F_y \leq 50$ ksi (345 MPa), $C_v = 1.0$.



(a) Rolled section

(b) Bolted built-up section (seldom used today)

(c) Welded built-up section

9.3.3 FLANGE SIZE

After the web dimensions are selected, the next step is to select an area of the flange so that it will not be overloaded in bending.

The total bending strength of a plate girder equals the bending strength of the flange plus the bending strength of the web.

As almost all of the bending strength is provided by the flange, an approximate expression can be developed to estimate the flange area as follows:

$$Z_{\text{reqd}} = \frac{M_u}{\phi_b F_y}$$

$$Z_{\text{furnished}} = 2A_f\left(\frac{h + t_f}{2}\right) + (2)\left(\frac{h}{2}\right)(t_w)\left(\frac{h}{4}\right)$$

Equating $Z_{\text{reqd}} = Z_{\text{furnished}}$ and solving for A_f

$$\frac{M_u}{\phi_b F_y} = A_f(h + t_f) + (2)\left(\frac{h}{2}\right)(t_w)\left(\frac{h}{4}\right)$$

$$A_f = \frac{M_u}{\phi_b F_y(h + t_f)} - \frac{t_w h^2}{4(h + t_f)}$$

Example 9.2 BUILT-UP GIRDERS (PLATE GIRDERS)

Select trial proportions for a 60-in (1524 mm) deep welded built-up I-shaped section with a 70 ft (21 m) simple span to support a service dead load (not including the beam weight) of 1.1 k/ft (16 kN/m) and a service live load of 3 k/ft (43.8 kN/m). The A36 section will be assumed to have full lateral bracing for its compression flange, and an unstiffened web is to be used.

Solution

Trial proportions

Try 60 in depth $\approx l/14$

$$\text{Estimated beam weight: 2 Flanges} = 2(1.0)(15) = 30 \text{ in}^2$$

$$\text{Web} = (0.75)(58) = 43.5 \text{ in}^2$$

$$A \text{ total} = 73.5 \text{ in}^2$$

$$\text{wt (plf)} = \frac{73.5 \text{ in}^2}{144 \text{ in}^2/\text{ft}^2}\left(490\,\frac{\text{lbs}}{\text{ft}^3}\right) = 250.1 \text{ plf}$$

Assume beam wt = 250 lbs/ft

Maximum moment and shear

LRFD	ASD
$w_u = (1.2)(1.1 + 0.250) + (1.6)(3) = 6.42$ k/ft	$w_a = 1.1 + 0.250 + 3 = 4.35$ k/ft
$R_u = \left(\frac{70}{2}\right)(6.42) = 224.7$ k	$R_a = \left(\frac{70}{2}\right)(4.35) = 152.2$ k
$M_u = \frac{(6.42)(70)^2}{8} = 3932$ ft-k	$M_a = \frac{(4.35)(70)^2}{8} = 2664$ ft-k

Design for a *compact web* and *flange*

$$Z_{\text{reqd}} = \frac{M_u}{\phi F_y} = \frac{(12)(3932)}{(0.9)(36)} = 1456 \text{ in}^3$$

Trial web size

For web to be compact by AISC Table B4.1 $\frac{h}{t_w}$ must be $\leq 3.76\sqrt{\frac{E}{F_y}}$

$$= 3.76\sqrt{\frac{29 \times 10^3}{36}} = 106.7 \qquad \text{(Case 9 AISC Table B4.1b)}$$

Assuming h to be 60 in $-$ 2(1.0 in) $=$ 58 in

$$Min\, t_w = \frac{58}{106.7} = 0.544 \text{ in, Say, } \frac{9}{16} \text{ in } (0.563 \text{ in})$$

Try $\frac{9}{16} \times 58$ web

$$\frac{h}{t_w} = \frac{58}{\frac{9}{16}} = 103.1$$

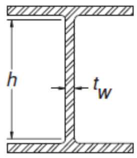
Since 103.1 is $> 2.46\sqrt{\frac{E}{F_y}} = 2.46\sqrt{\frac{29 \times 10^3}{36}} = 69.82$

transverse stiffeners may be required, states AISC Specification G2.2.

But the same specification states that stiffeners are not required if the necessary shear strength for the web is less than or equal to its available shear strength, as stipulated by AISC Specification G2.1, using $k_v = 5.0$.

$$\frac{h}{t_w} = 103.1 > 1.37\sqrt{\frac{k_v E}{F_y}} = 1.37\sqrt{\frac{(5)(29 \times 10^3)}{36}}$$
$$= 86.94$$

TABLE B4.1 (cont.)
Limiting Width-Thickness Ratios for Compression Elements

Case	Description of Element	Width Thickness Ratio	Limiting Width-Thickness Ratios λ_p (compact)	λ_r (noncompact)	Example
9	Flexure in webs of doubly symmetric I-shaped sections and channels	h/t_w	$3.76\sqrt{E/F_y}$	$5.70\sqrt{E/F_y}$	

$$\therefore\ C_v = \frac{1.51Ek_v}{\left(\frac{h}{t_w}\right)^2 F_y} = \frac{(1.51)(29\times10^3)(5.0)}{(103.1)^2(36)} = 0.572$$

$$V_n = 0.6F_yA_wC_v = (0.6)(36)\left(58\times\frac{9}{16}\right)(0.572)$$

$$= 403.1\ \text{k}$$

Available shear strength without stiffeners

LRFD $\phi_v = 0.90$	ASD $\Omega_v = 1.67$
$\phi_v V_n = (0.90)(403.1) = 362.8\ \text{k}$	$\frac{V_n}{\Omega_v} = \frac{403.1}{1.67} = 241.4\ \text{k}$
$> 224.7\ \text{k}$	$> 152.2\ \text{k}$
$\therefore$ **Stiffeners are not required.**	$\therefore$ **Stiffeners are not required.**

Trial flange size

Assume 1-in plates ($t_f = 1.0$ in)

$$A_f = \frac{M_u}{\phi_b F_y(h+t_f)} - \frac{t_w h^2}{4(h+t_f)}$$

$$= \frac{(12)(3932)}{(0.9)(36)(58+1)} - \frac{\left(\frac{9}{16}\right)(58)^2}{4(58+1)} = 16.66\ \text{in}^2$$

Try 1 × 18 plate each flange. Are they compact by AISC Table B4.1 (Case 2)?

$$\frac{b_f}{2t_f} = \frac{18.00}{(2)(1.00)} = 9.00 < 0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29{,}000}{36}} = 10.79 \text{ (Yes, is compact)}$$

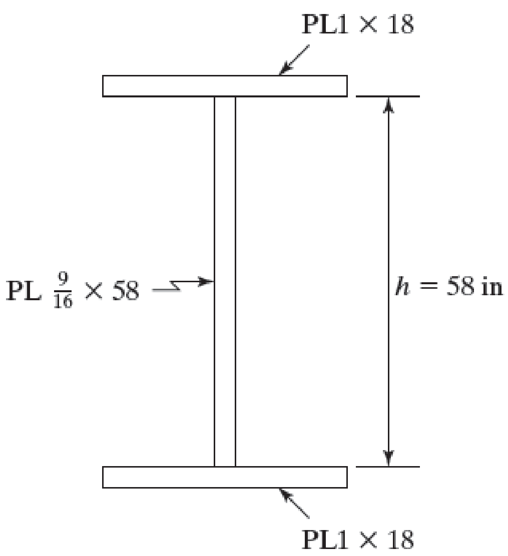
Check Z of section

$$Z = (2)\left(\frac{58}{2}\right)\left(\frac{9}{16}\right)\left(\frac{58}{4}\right) + (2)(1 \times 18)\left(\frac{58}{2} + \frac{1}{2}\right)$$
$$= 1535 \text{ in}^3 > 1456 \text{ in}^3 \quad \textbf{OK}$$

Check girder wt

$$\text{wt} = \frac{\left(\frac{9}{16}\right)(58) + (2)(1 \times 18)}{144}(490) = 233.5 \text{ lb} < 250 \text{ lb estimated} \quad \textbf{OK}$$

Trial Section $\frac{9}{16}$ × 58 web with 1 × 18 PL each flange.

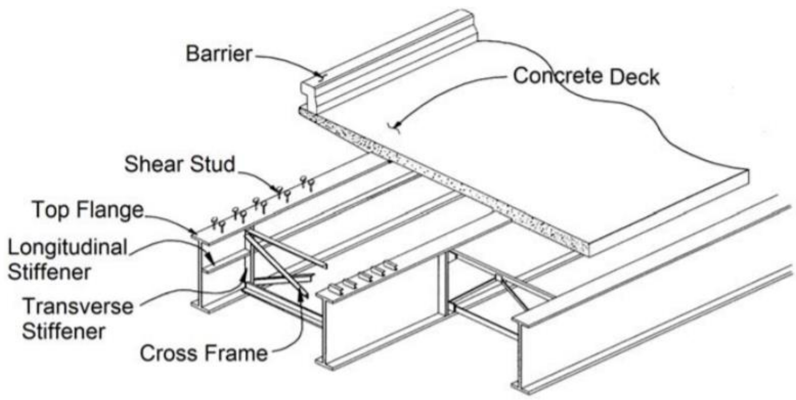


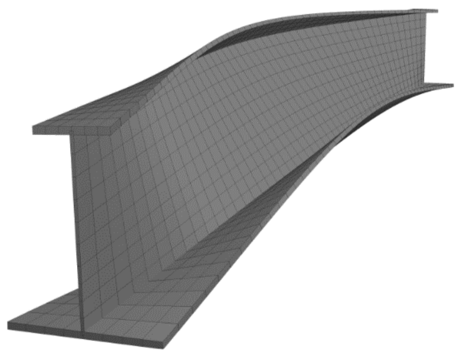
9.3.4 FLEXURAL STRENGTH

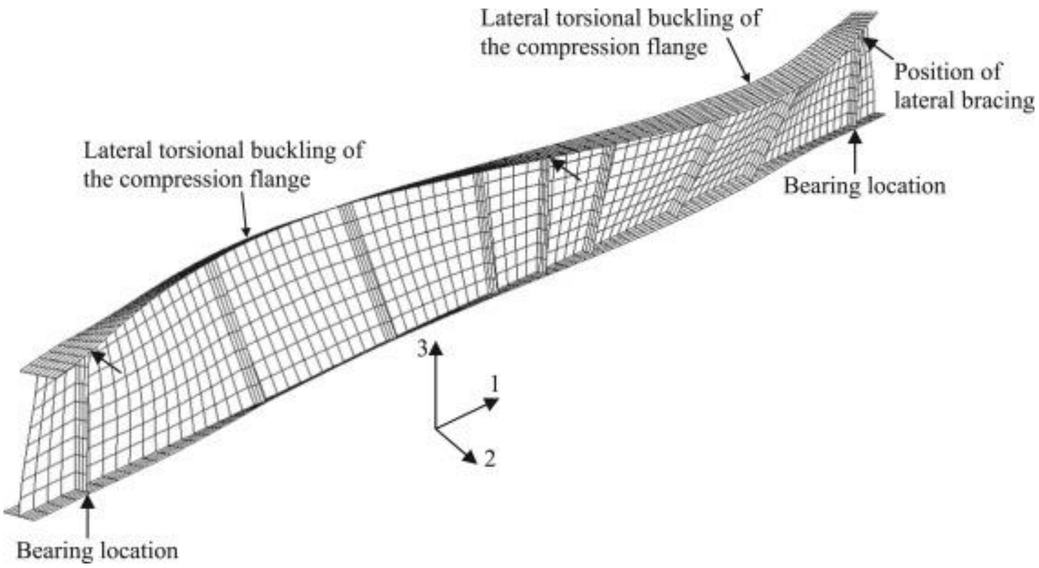
The nominal flexural strength, M_n, of a plate girder bent about its major axis is based on one of the limit states as defined in **Chapter F of the AISC Specification, Section F2 to F5**.

These limit states include:

1. Yielding (**Y**),
2. Lateral-Torsional Buckling (**LTB**),
3. Compression Flange Local Buckling (**FLB**),
4. Compression Flange Yielding (**CFY**),
5. Tension Flange Yielding (**TFY**).



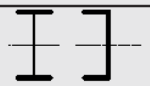
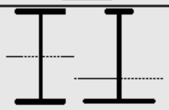
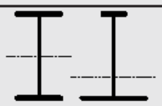




This strength, M_n, is the **lowest value obtained according to these limit states**.

- The application of the limit states defined in **F2 to F5** (Section F1, P. 45)

TABLE User Note F1.1
Selection Table for the Application of Chapter F Sections

Section in Chapter F	Cross Section	Flange Slenderness	Web Slenderness	Limit States
F2		C	C	Y, LTB
F3		NC, S	C	LTB, FLB
F4		C, NC, S	C, NC	Y, LTB, FLB, TFY
F5		C, NC, S	S	Y, LTB, FLB, TFY

C Compact flanges and webs,
NC Non-Compact flanges and webs
S Slender flanges and webs

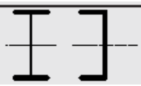
Section F2	was applicable and the limit states of yielding (Y) and lateral-torsional buckling (LTB) would need to be checked to determine Mn.
Section F3	applies to doubly symmetrical I-shaped members having compact webs and non-compact or slender flanges. The nominal flexural strength, Mn, shall be the lower value obtained from the limit states of LTB and FLB.
Section F4	applies to doubly symmetrical I-shaped members with non-compact webs and singly symmetrical I-shaped members with compact or non-compact webs. The nominal flexural strength, Mn, shall be the lowest value obtained from the limit states of CFY, LTB, FLB and TFY.
Section F5	applies to doubly symmetric and singly symmetric I-shaped members with slender webs. The nominal flexural strength, Mn, shall be the lowest value obtained from the limit states of CFY, LTB, FLB and TFY.

F2. DOUBLY SYMMETRIC COMPACT I-SHAPED MEMBERS AND CHANNELS BENT ABOUT THEIR MAJOR AXIS

This section applies to doubly symmetric I-shaped members and channels bent about their major axis, having compact webs and compact flanges as defined in Section B4.

> **User Note:** All current ASTM A6 W, S, M, C and MC shapes except W21×48, W14×99, W14×90, W12×65, W10×12, W8×31, W8×10, W6×15, W6×9, W6×8.5, and M4×6 have compact flanges for $F_y \leq 50$ ksi (345 MPa); all current ASTM A6 W, S, M, HP, C and MC shapes have compact webs at $F_y \leq 65$ ksi (450 MPa).

The *nominal flexural strength*, M_n, shall be the lower value obtained according to the *limit states* of *yielding* (*plastic moment*) and *lateral-torsional buckling*.

F2		C	C	Y, LTB

F3. DOUBLY SYMMETRIC I-SHAPED MEMBERS WITH COMPACT WEBS AND NONCOMPACT OR SLENDER FLANGES BENT ABOUT THEIR MAJOR AXIS

This section applies to doubly symmetric I-shaped members bent about their major axis having compact webs and noncompact or slender flanges as defined in Section B4.

> **User Note:** The following shapes have noncompact flanges for $F_y = 50$ ksi (345 MPa): W21×48, W14×99, W14×90, W12×65, W10×12, W8×31, W8×10, W6×15, W6×9, W6 × 8.5, and M4×6. All other ASTM A6 W, S, M, and HP shapes have compact flanges for $F_y \leq 50$ ksi (345 MPa).

The nominal flexural strength, M_n, shall be the lower value obtained according to the *limit states* of *lateral-torsional buckling* and compression flange *local buckling*.

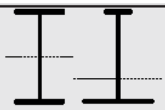
F3		NC, S	C	LTB, FLB

F4. OTHER I-SHAPED MEMBERS WITH COMPACT OR NONCOMPACT WEBS BENT ABOUT THEIR MAJOR AXIS

This section applies to: (a) doubly symmetric I-shaped members bent about their major axis with noncompact webs; and (b) singly symmetric I-shaped members with webs attached to the mid-width of the flanges, bent about their major axis, with compact or noncompact webs, as defined in Section B4.

> **User Note:** I-shaped members for which this section is applicable may be designed conservatively using Section F5.

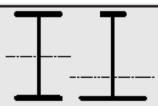
The nominal flexural strength, M_n, shall be the lowest value obtained according to the *limit states* of compression flange *yielding*, *lateral-torsional buckling*, compression flange *local buckling* and tension flange yielding.

F4		C, NC, S	C, NC	Y, LTB, FLB, TFY

F5. DOUBLY SYMMETRIC AND SINGLY SYMMETRIC I-SHAPED MEMBERS WITH SLENDER WEBS BENT ABOUT THEIR MAJOR AXIS

This section applies to doubly symmetric and singly symmetric I-shaped members with slender webs attached to the mid-width of the flanges, bent about their major axis, as defined in Section B4.

The nominal flexural strength, M_n, shall be the lowest value obtained according to the *limit states* of compression flange *yielding*, *lateral-torsional buckling*, compression flange *local buckling* and tension flange yielding.

F5		C, NC, S	S	Y, LTB, FLB, TFY

- Based on whether plate girder has **compact**, **non-compact**, or **slender** flanges and webs as defined in **AISC Specification Section B4.1** (Pages 16 and 17) for flexure and the unbraced length of the compression flange.

TABLE B4.1
Limiting Width-Thickness Ratios for Compression Elements

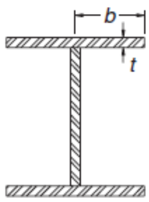
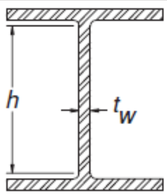
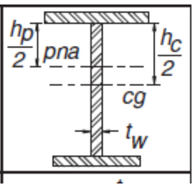
Case	Description of Element	Width Thickness Ratio	Limiting Width-Thickness Ratios: λ_p (compact)	Limiting Width-Thickness Ratios: λ_r (noncompact)	Example
2	Flexure in flanges of doubly and singly symmetric I-shaped built-up sections	b/t	$0.38\sqrt{E/F_y}$	$0.95\sqrt{k_c E/F_L}$ [a],[b]	

TABLE B4.1 (cont.)
Limiting Width-Thickness Ratios for Compression Elements

	Case	Description of Element	Width Thickness Ratio	Limiting Width-Thickness Ratios: λ_p (compact)	Limiting Width-Thickness Ratios: λ_r (noncompact)	Example
Stiffened E	9	Flexure in webs of doubly symmetric I-shaped sections and channels	h/t_w	$3.76\sqrt{E/F_y}$	$5.70\sqrt{E/F_y}$	
	11	Flexure in webs of singly-symmetric I-shaped sections	h_c/t_w	$\dfrac{\dfrac{h_c}{h_p}\sqrt{\dfrac{E}{F_y}}}{\left(0.54\dfrac{M_p}{M_y}-0.09\right)^2} \le \lambda_r$	$5.70\sqrt{E/F_y}$	

Defintion of all parameters used in Section F

F_y = *specified minimum yield stress* of the type of steel being used, ksi (MPa)
Z_x = plastic section modulus about the x-axis, in.^{3} (mm^3)

L_b = length between points that are either braced against lateral displacement of compression flange or braced against twist of the cross section, in. (mm)

E = modulus of elasticity of steel = 29,000 ksi (200 000 MPa)
J = torsional constant, in.^{4} (mm^4)
S_x = elastic section modulus taken about the x-axis, in.^{3} (mm^3)

h_o = distance between the flange centroids, in. (mm)

$$\lambda = \frac{b_f}{2t_f}$$

$\lambda_{pf} = \lambda_p$ is the limiting slenderness for a compact flange, Table B4.1
$\lambda_{rf} = \lambda_r$ is the limiting slenderness for a noncompact flange, Table B4.1
$k_c = \dfrac{4}{\sqrt{h/t_w}}$ and shall not be taken less than 0.35 nor greater than 0.76 for calculation purposes

$M_p = Z_x F_y \leq 1.6 S_{xc} F_y$
S_{xc}, S_{xt} = elastic section modulus referred to tension and compression flanges, respectively, in.^{3} (mm^3)

$$\lambda = \frac{h_c}{t_w}$$

$\lambda_{pw} = \lambda_p$, the limiting slenderness for a compact web, Table B4.1
$\lambda_{rw} = \lambda_r$, the limiting slenderness for a noncompact web, Table B4.1

$$a_w = \frac{h_c t_w}{b_{fc} t_{fc}}$$

b_{fc} = compression flange width, in. (mm)
t_{fc} = compression flange thickness, in. (mm)

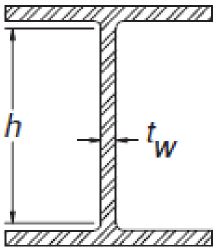
F_L is defined in Equations F4-6a and F4-6b
R_{pc} is the web *plastification* factor, determined by Equations F4-9

$\bar{r}_t$ = radius of gyration of the flange components in flexural compression plus one-third of the web area in compression due to application of major axis bending moment alone, in. (mm)

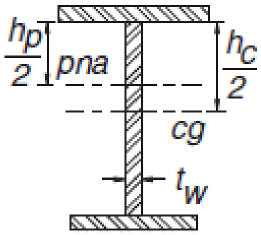
a_w = the ratio of two times the web area in compression due to application of major axis bending moment alone to the area of the compression flange components

a = clear distance between transverse *stiffeners*, in. (mm)

h = for built-up welded sections, the clear distance between flanges, in. (mm)



Double Symmertic Plate Girder Section



Single Symmertic Plate Girder Section

Example 9.3 — BUILT-UP GIRDERS (PLATE GIRDERS)

Determine the *design* flexural strength, $\Phi_b M_n$, and the *allowable* flexural strength, M_n/Ω_b, of the following welded I-shaped plate girder. The flanges are 1 1/4 in × 15 in, the web is 1/4 in × 50 in, and the member is uniformly loaded and simply-supported. Use A36 steel and assume the girder has continuous bracing for its compression flange.

Solution

Determine if flange is compact, non-compact, or slender? *Case* 2 . Table B4.1

$$\frac{b}{t_f} = \frac{b_f/2}{t_f} = \frac{15/2}{1.25} = 6.0 < 0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29{,}000}{36}} = 10.79$$

Compact—Flange

Determine if web is compact, non-compact, or slender? *Case* 9 . Table B4.1

$$\frac{h}{t_w} = \frac{50}{0.25} = 200 > 5.70\sqrt{\frac{E}{F_y}} = 5.70\sqrt{\frac{29{,}000}{36}} = 161.78$$

Slender—Web

∴ (F5) Doubly symmetric section with *slender web* and *compact flange* bent about their major axis.

$\phi_b M_n$ is lowest value of Y, LTB, FLB, TFY

LTB — Since $L_b = 0$, limit state of LTB does not apply.

FLB — Since flange is compact, limit state of FLB does not apply

TFY — Since member is symmetric about x–x axis $S_{xt} = S_{xc}$, limit state of TFY does not apply.

Y — Compression flange yielding.

$$M_n = R_{pg} F_y S_{xc} \qquad \text{(AISC Equation F5-1)}$$

$$a_w = \frac{h_c t_w}{b_{fc} t_{fc}} = \frac{50(1/4)}{15(1.25)} \qquad \text{(AISC Equation F4-12)}$$

$$a_w = 0.667 < 10 \text{ (upper limit)}$$

$$R_{pg} = 1 - \frac{a_w}{1200 + 300\,a_w}\left[\frac{h_c}{t_w} - 5.7\sqrt{\frac{E}{F_y}}\right] \le 1.0 \qquad \text{(AISC Equation F5-6)}$$

$$R_{pg} = 1 - \frac{0.667}{1200 + 300(0.667)}\left[200 - 5.7\sqrt{\frac{29{,}000}{36}}\right] \le 1.0$$

$$R_{pg} = 0.982$$

$$F_y = 36 \text{ ksi}$$

$$I_{xc} = \frac{1}{12}\left(\frac{1}{4}\right)(50)^3 + 2\left(\frac{1}{12}\right)(15)(1.25)^3 + 2(1.25)(15)(25.625)^2$$

$$I_{xc} = 27{,}233 \text{ in}^4$$

$$S_{xc} = \frac{I}{c} = \frac{27{,}233}{26.25} = 1037.4 \text{ in}^3$$

$$M_n = R_{pg} F_y S_{xc} = \frac{(0.982)(36 \text{ ksi})(1037.4 \text{ in}^3)}{12 \text{ in/ft}}$$

$$M_n = 3056 \text{ ft-k}$$

LRFD $\phi = 0.90$	ASD $\Omega = 1.67$
$\phi M_n = 0.9(3056)$ ft-k $\phi M_n = 2750$ ft-k	$M_n/\Omega = 3056$ ft-k/1.67 $M_n/\Omega = 1830$ ft-k

Example 9.4 — BUILT-UP GIRDERS (PLATE GIRDERS)

Determine the *design* flexural strength, $\Phi_b M_n$, and the *allowable* flexural strength, M_n/Ω_b, of the following welded I-shaped plate girder. The flanges are 1 in × 24 in, the web is 5/16 in × 45 in, and the member is uniformly loaded and has a simply-supported 100 ft. span. Use A36 steel and the unbraced length of the compression flange is 20 ft.

Solution

Determine if flange is compact, non-compact, or slender? *Case* 2 . Table B4.1

$$\frac{b}{t_f} = \frac{b_f/2}{t_f} = \frac{24/2}{1} = 12.00 > 0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29{,}000}{36}} = 10.79$$

$$k_c = \frac{4}{\sqrt{h/t_w}} = \frac{4}{\sqrt{45/0.3125}} = 0.333$$

$$\frac{b}{t_f} = 12.00 < 0.95\sqrt{\frac{k_c E}{F_y}} = 0.95\sqrt{\frac{0.333(29{,}000)}{36}} = 15.56$$

Non-Compact—Flange

Determine if web is compact, non-compact, or slender? *Case* 9 . Table B4.1

$$\frac{h}{t_w} = \frac{45}{0.3125} = 144 > 3.76\sqrt{\frac{E}{F_y}} = 3.76\sqrt{\frac{29{,}000}{36}} = 106.72$$

$$< 5.70\sqrt{\frac{E}{F_y}} = 5.70\sqrt{\frac{29{,}000}{36}} = 161.78$$

Non-Compact—Web

∴ (F4) Doubly symmetric I-shaped members with *non-compact webs* bent about their major axis.

$\phi_b M_n$ is lowest value of Y, LTB, FLB, TFY

TFY — Since member is symmetric about *x-x* axis, $S_{xt} = S_{xc}$, limit state of TFY does not apply.

[Y] — Compression flange yielding

$$M_n = R_{pc} F_y S_{xc} \qquad \text{(AISC Equation F4-1}$$

$$\text{Since } \frac{h}{t_w} = 144 \geq \lambda_{pw} = 106.72 = 3.76\sqrt{\frac{E}{F_y}}$$

$$R_{pc} = \left[\frac{M_p}{M_{yc}} - \left(\frac{M_p}{M_{yc}} - 1\right)\left(\frac{\lambda - \lambda_{pw}}{\lambda_{rw} - \lambda_{pw}}\right)\right] \leq \frac{M_p}{M_{yc}} \qquad \text{(AISC Equation F4-9b}$$

$$\frac{M_p}{M_{yc}} = \frac{Z}{S}$$

where: $Z = 2(1)(24)(22.5 + 0.5) + 0.3125(2)(22.5)(11.25)$

$$Z = 1262 \text{ in}^3$$

$$S = \frac{2\left(\frac{1}{12}\right)(24)(1)^3 + \left(\frac{1}{12}\right)(0.3125)(45)^3 + 2(24)(1)(22.5 + 0.5)^2}{23.5}$$

$$S = 1182 \text{ in}^3$$

$$\frac{M_p}{M_{yc}} = \frac{Z}{S} = \frac{1262}{1182} = 1.068$$

$$R_{pc} = \left[1.068 - (1.068-1)\left(\frac{144 - 106.72}{161.78 - 106.72}\right)\right] \leq 1.068$$

$$R_{pc} = 1.022$$

$$M_n = \frac{(1.022)(36 \text{ ksi})(1182 \text{ in}^3)}{12 \text{ in/ft}} = 3624 \text{ ft-k} \qquad \boxed{Y}$$

LTB — Lateral Torsional Buckling check unbraced length of 20 ft,

$$L_b = 20 \text{ ft or } 240 \text{ in}$$

$$L_p = 1.1\, r_t\sqrt{\frac{E}{F_y}} \qquad \text{(AISC Equation F4-7)}$$

$$r_t = \frac{b_{fc}}{\sqrt{12\left(\frac{h_o}{d} + \frac{1}{6}a_w\frac{h^2}{h_o d}\right)}}$$ (AISC Equation F4-11)

$$a_w = \frac{h_c t_w}{b_{fc} t_{fc}} = \frac{45(0.3125)}{24(1.0)} = 0.586$$ (AISC Equation F4-12)

$$b_{fc} = 24 \text{ in}, \quad h_o = 46 \text{ in}, \quad d = 47 \text{ in}, \quad h = 45 \text{ in}$$

$$r_t = \frac{24}{\sqrt{12\left(\frac{46}{47} + \frac{1}{6}(0.586)\left(\frac{45^2}{46(47)}\right)\right)}} = 6.70 \text{ in}$$

$$L_p = 1.1(6.70 \text{ in})\sqrt{\frac{29{,}000}{36}} = 209.1 \text{ in} = 17.42 \text{ ft}$$

$$L_r = 1.95\, r_t \frac{E}{F_L}\sqrt{\frac{J}{S_{xc}h_o} + \sqrt{\left(\frac{J}{S_{xc}h_o}\right)^2 + 6.76\left(\frac{F_L}{E}\right)^2}}$$ (AISC Equation F4-8)

Since $\frac{S_{xt}}{S_{xc}} = 1.0 \geq 0.7$

$\therefore F_L = 0.7\,F_y = 0.7\,(36) = 25.2$ ksi (AISC Equation F4-6a)

$$J = \sum\frac{1}{3}bt^3 = 2\left(\frac{1}{3}\right)(24)(1)^3 + \left(\frac{1}{3}\right)(45)(0.3125)^3 = 16.46 \text{ in}^3$$

$$L_r = 1.95(6.70)\frac{29{,}000}{25.2}\sqrt{\frac{16.46}{1182(46)} + \sqrt{\left(\frac{16.46}{1182(46)}\right)^2 + 6.76\left(\frac{25.2}{29{,}000}\right)^2}}$$

$$L_r = 764.0 \text{ in} = 63.67 \text{ ft}$$

$$M_n = C_b\left[R_{pc}M_{yc} - (R_{pc}M_{yc} - F_L S_{xc})\left[\frac{L_b - L_p}{L_r - L_p}\right]\right] \leq R_{pc}M_{yc}$$ (AISC Equation F4-2)

$$R_{pc}M_{yc} = \frac{1.022(36)(1182)}{12} = 3624 \text{ ft-k}$$

$$M_n = 1.0\left[3624 - \left(3624 - \frac{25.2(1182)}{12}\right)\left[\frac{20 - 17.42}{63.67 - 17.42}\right]\right] \leq 3624$$

$$M_n = 3560 \text{ ft-k} \quad \boxed{\text{LTB}}$$

$\boxed{\text{FLB}}$ – Flange Local Buckling

$$M_n = \left[R_{pc}M_{yc} - (R_{pc}M_{yc} - F_L S_{xc})\left[\frac{\lambda - \lambda_{pf}}{\lambda_{rf} - \lambda_{pf}}\right]\right] \qquad \text{(AISC Equation F4-12)}$$

For sections with non-compact flanges

$$\lambda = \frac{b_f/2}{t_f} = 12.00 \quad \lambda_{pf} = 10.79 = 0.38\sqrt{\frac{E}{F_y}}$$

$$\lambda_{rf} = 15.56 = 0.95\sqrt{\frac{kE}{F_y}}$$

$$M_n = \left[3624 - \left(3624 - \frac{25.2(1182)}{12}\right)\left(\frac{12.00 - 10.79}{15.56 - 10.79}\right)\right]$$

$$M_n = 3334 \text{ ft-k} \quad \boxed{\text{FLB}}$$

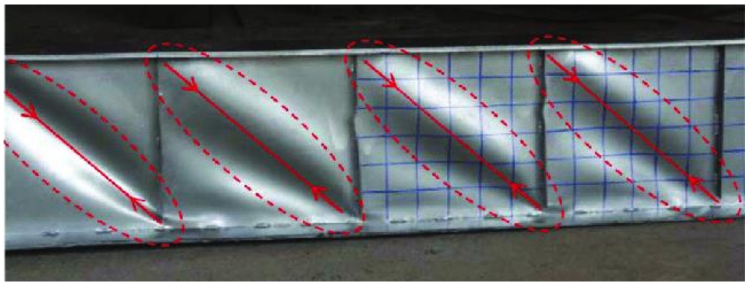
M_n is controlled by least of Y, LTB, FLB

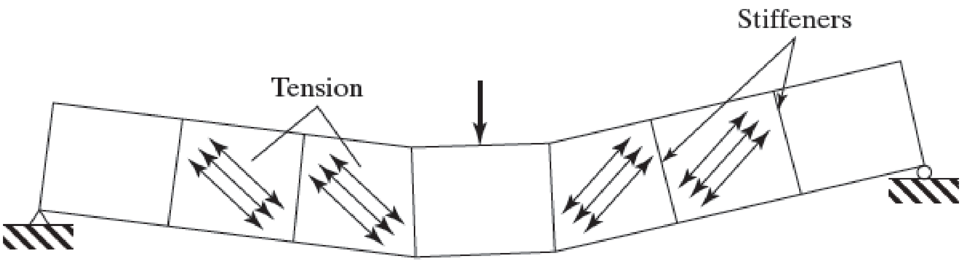
$$\therefore M_n = 3334 \text{ ft-k} \quad \boxed{\text{FLB}}$$

LRFD $\phi = 0.90$	ASD $\Omega = 1.67$
$\phi M_n = 0.9$ (3334 ft-k) $\phi M_n = 3001$ ft-k	$M_n/\Omega = 3334$ ft-k/1.67 $M_n/\Omega = 1996$ ft-k

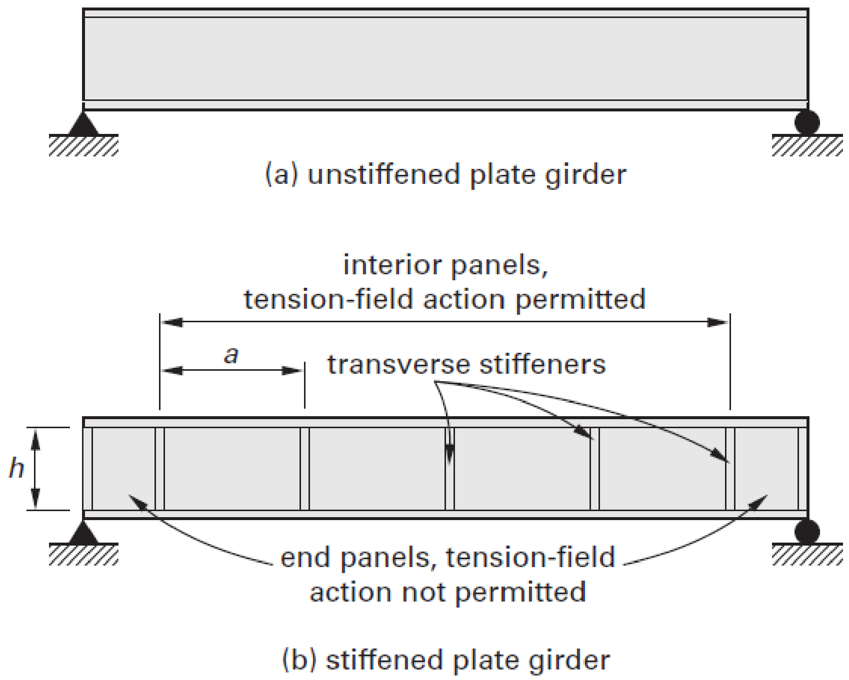
9.4 TENSION FIELD ACTION

The AISC Specification for built-up I-shaped girders permits their design on the basis of **postbuckling strength**. Designs on this basis provide a more realistic idea of the actual strength of a girder. (Such designs, however, do not necessarily result in better economy, because stiffeners are required.) Should a girder be loaded until initial buckling occurs, it will not then collapse, because of a phenomenon known as **tension field action**.

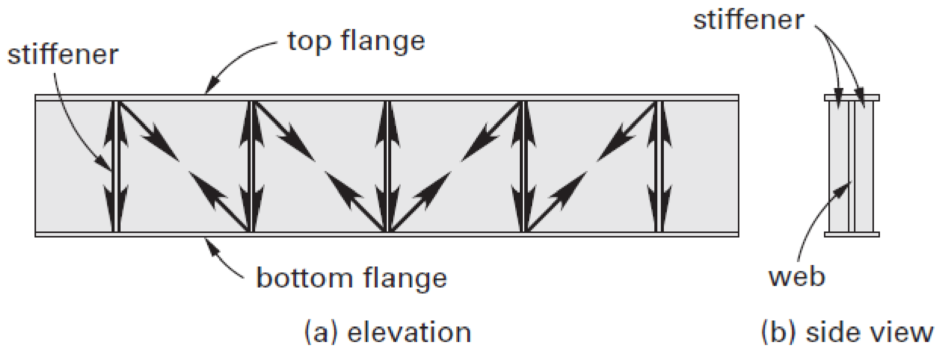




Tension field action in plate-girder web. (Note that end panels cannot develop tension field action.)



Tension-Field Action in a Stiffened Plate Girder



Actually, there the beam behavior, according to this theory, is rather like a Warren truss with the concrete "diagonals" being in compression and the web reinforcing serving as tension verticals.

The stiffeners of the built-up I-girders keep the flanges from coming together, and the flanges keep the stiffeners from coming together. The intermediate stiffeners, which before initial buckling were assumed to resist no load, will after buckling resist compressive loads (or will serve as the compression verticals of a truss) due to diagonal tension. The result is that a plate-girder web probably can resist loads equal to two or three times those present at initial buckling before complete collapse will occur.

The aspect ratio, a, is the ratio of the clear distance between stiffeners in a panel to the height of the panel. According to **AISC Specification G3.1**,

$$a/h \not> 3.0 \text{ or } a/h \not> \left[\frac{260}{(h/t_w)}\right]^2$$

9.4.1 NOMINAL SHEAR STRENGTH

This section applies to webs of singly or doubly symmetric members and channels subject to shear in the plane of the web

The *nominal shear strength*, V_n, of unstiffened or stiffened webs, according to the *limit states* of *shear yielding* and *shear buckling*, is

$$V_n = 0.6F_y A_w C_v \qquad \text{(G2-1)}$$

(a) For webs of rolled I-shaped members with $h/t_w \le 2.24\sqrt{E/F_y}$:

$$\phi_v = 1.00 \text{ (LRFD)} \qquad \Omega_v = 1.50 \text{ (ASD)}$$

and

$$C_v = 1.0 \qquad \text{(G2-2)}$$

User Note: All current ASTM A6 W, S and HP shapes except W44×230, W40×149, W36×135, W33×118, W30×90, W24×55, W16×26 and W12×14 meet the criteria stated in Section G2.1(a) for $F_y \le 50$ ksi (345 MPa).

(i) For unstiffened webs with $h/t_w < 260$, $k_v = 5$ except for the stem of tee shapes where $k_v = 1.2$.

(ii) For stiffened webs,

$$k_v = 5 + \frac{5}{(a/h)^2}$$

$$= 5 \text{ when } a/h > 3.0 \text{ or } a/h > \left[\frac{260}{(h/t_w)}\right]^2$$

9.4.2 LIMITS ON THE USE OF TENSION FIELD ACTION

Consideration of tension field action is permitted for flanged members when the web plate is supported on all four sides by flanges or stiffeners. Consideration of tension field action is not permitted for:

(a) *end panels* in all members with transverse stiffeners;
(b) members when a/h exceeds 3.0 or $[260/(h/t_w)]^2$;
(c) $2A_w/(A_{fc} + A_{ft}) > 2.5$; or
(d) h/b_{fc} or $h/b_{ft} > 6.0$

Tension field action also **may not be considered** if

$$a/h > 3.0 \text{ or } a/h > \left[\frac{260}{(h/t_w)}\right]^2$$

$$\left(\frac{2A_w}{A_{fc} + A_{ft}}\right) > 2.5 \text{ or if } h/b_{fc} \text{ or } h/b_{ft} > 6.0.$$

where

A_{fc} = area of compression flange, in.^{2} (mm^2)
A_{ft} = area of tension flange, in.^{2} (mm^2)
b_{fc} = width of compression flange, in. (mm)
b_{ft} = width of tension flange, in. (mm)

In these cases, the nominal shear strength, V_n, shall be determined according to the provisions of Section G2.

9.4.3 NOMINAL SHEAR STRENGTH WITH TENSION FIELD ACTION

When tension field action is permitted according to Section G3.1, the nominal shear strength, V_n, with tension field action, according to the limit state of tension field yielding, shall be

(a) For $h/t_w \leq 1.10\sqrt{k_v E/F_y}$

$$V_n = 0.6F_y A_w \qquad \text{(G3-1)}$$

(b) For $h/t_w > 1.10\sqrt{k_v E/F_y}$

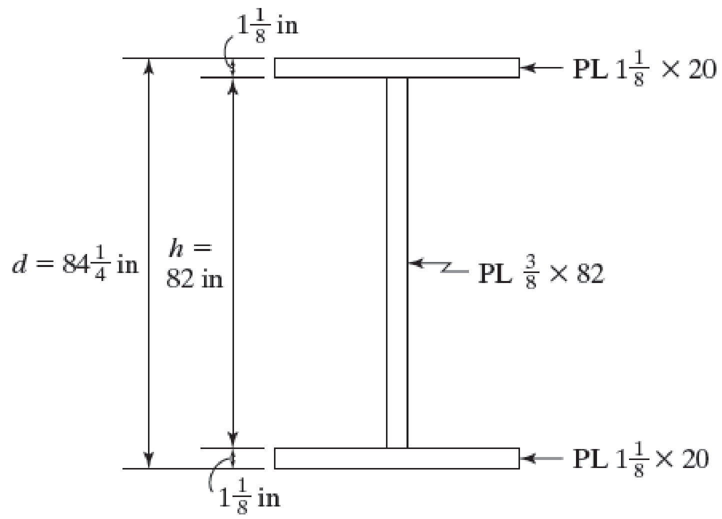
$$V_n = 0.6F_y A_w \left(C_v + \frac{1 - C_v}{1.15\sqrt{1 + (a/h)^2}} \right) \qquad \text{(G3-2)}$$

where

k_v and C_v are as defined in Section G2.1.

Example 9.5 — TENSION FIELD ACTION

The built-up A36 I-shaped girder shown has been selected for a 65-ft (19.8 m) simple span to support the loads $W_D = 1.1$ k/ft (16 kN/m) (not including the beam weight) and $W_L = 2$ k/ft (29 kN/m). Select transverse stiffeners as needed.



d = 2140 mm

h = 2082 mm

PL_{Flange} = PL 29×508 mm

PL_{Web} = PL 9.5×2082 mm

Solution

Computing girder weight

$$A = (2)\left(1\frac{1}{8}\text{ in}\right)(20\text{ in}) + \left(\frac{3}{8}\text{ in}\right)(82\text{ in}) = 75.75\text{ in}^2$$

$$\text{wt per ft} = \left(\frac{75.75\text{ in}^2}{144\text{ in}^2/\text{ft}^2}\right)\left(490\text{ lb/ft}^3\right) = 258\text{ lb/ft}$$

Computing the required shear strength at the support

LRFD	ASD
$w_u = (1.2)(1.1 + 0.258) + (1.6)(2) = 4.83\text{ k/ft}$	$w_a = 1.1 + 0.258 + 2 = 3.358\text{ k/ft}$
$R_u = \left(\frac{65}{2}\right)(4.83) = 156.98\text{ k}$	$R_a = \left(\frac{65}{2}\right)(3.358) = 109.14\text{ k}$

Are stiffeners needed?

$$A_w = dt_w = (84.25\text{ in})\left(\frac{3}{8}\text{ in}\right) = 31.59\text{ in}^2$$

$$\frac{h}{t_w} = \frac{82}{0.375} = 219 < 260 \therefore k_v = 5.0, \text{ says AISC Section G2.1b.}$$

C_v from same AISC section

$$219 > 1.37\sqrt{k_v E/F_y} = 1.37\sqrt{\frac{(5)(29 \times 10^3)}{36}} = 86.95$$

$\therefore$ Must use AISC Equation G2-5.

$$C_v = \frac{1.51Ek_v}{\left(\frac{h}{t_w}\right)^2 F_y} = \frac{(1.51)(29 \times 10^3)(5)}{(219)^2(36)} = 0.1268$$

Computing V_n with AISC Equation G2-1

$$V_n = 0.6F_yA_wC_v = (0.6)(36\text{ k/in}^2)(31.59\text{ in}^2)(0.1268) = 86.52\text{ k}$$

Calculating shear strengths without stiffeners

LRFD $\phi_v = 0.90$	ASD $\Omega_v = 1.67$
$\phi_v V_n = (0.90)(86.52) = 77.87\text{ k}$	$\dfrac{V_n}{\Omega_v} = \dfrac{86.52}{1.67} = 51.81\text{ k}$
$< 156.98\text{ k}$ ∴ **Stiffeners are required.**	$< 109.14\text{ k}$ ∴ **Stiffeners are required.**

Can we use tensile field action? (AISC Specification G3)

a. Not in end panels with transverse stiffeners.

b. Not in members where $\dfrac{a}{h} > 3.0$ or $\left[260\left(\dfrac{h}{t_w}\right)\right]^2$.

c. Not if $\left(\dfrac{2A_w}{A_{fc} + A_{ft}}\right) > 2.5$. Here, A_{fc} = the area of compression flange and A_{ft} = the area of the tension flange.

d. Not if $\dfrac{h}{b_{fc}}$ or $\dfrac{h}{b_{ft}} > 6.0$.

Select stiffener spacing for end panel.

By (a), tension field action may not be used

LRFD	ASD
$\dfrac{\phi V_n}{A_w} = \dfrac{V_u}{A_w} = \dfrac{156.98}{31.59} = 4.97\text{ ksi}$	$\dfrac{V_n}{\Omega_c A_w} = \dfrac{V_a}{A_w} = \dfrac{109.14}{31.59} = 3.45\text{ ksi}$

Referring to AISC Table 3-16a, which provides the available shear stress (tension field action not included). Entering left margin with $h/t_w = 219$ and moving horizontally from that value to the $\phi V_n/A_w$ curve $= 4.97$ ksi (actually interpolating between the curves in table). There, move down vertically to base and read 1.00. This is the a/h value that can be used.

$\therefore$ a = (1.00)(82) = 82 in.

Using the ASD values $h/t_w = 219$ and $V_n/\Omega_v A_w = 3.45$ ksi and entering AISC Table 3-16a, we read at the base the value 0.98. Thus, $a = (0.98)(82) = 80$ in.

Select stiffeners for second panel, noting that tension field action is permitted, since it's not an end panel.

Required shear strength needed for 2nd panel

LRFD (82 in out in span)	ASD (80 in out in span)
$V_u = 156.98 - \left(\frac{82}{12}\right)(4.83)$ $= 123.97$ k	$V_a = 109.14 - \left(\frac{80}{12}\right)(3.358)$ $= 86.75$ k

Computing the available shear strength without stiffeners

LRFD $\phi_v = 0.90$	ASD $\Omega_v = 1.67$
$\phi_v v_n = (0.90)(86.52) = 77.87$ k	$\frac{v_n}{\Omega_v} = \frac{86.52}{1.67} = 51.81$ k
< 123.97 k	< 86.75 k
∴ More stiffeners reqd.	**∴ More stiffeners reqd.**
$\frac{\phi V_n}{A_w} = \frac{123.97}{31.59} = 3.92$ ksi	$\frac{V_n}{\Omega_v A_w} = \frac{V_a}{A_w} = \frac{86.75}{31.59} = 2.75$ ksi

For LRFD with $\phi_v V_n/A_w = 3.92$ ksi and $h/t_w = 219$, we use Table 3-16b, tension field action is included. The stress does not intersect the h/t_w value, so we read the maximum value $a/h = 1.4$. This is obtained by moving horizontally from $h/t_w = 219$ then pivoting on the bold line and moving down vertically to the base and read 1.40.

$$a = (1.4)(82) = 114.8 \text{ in}$$

ASD results are the same with $a = 114.8$ in.

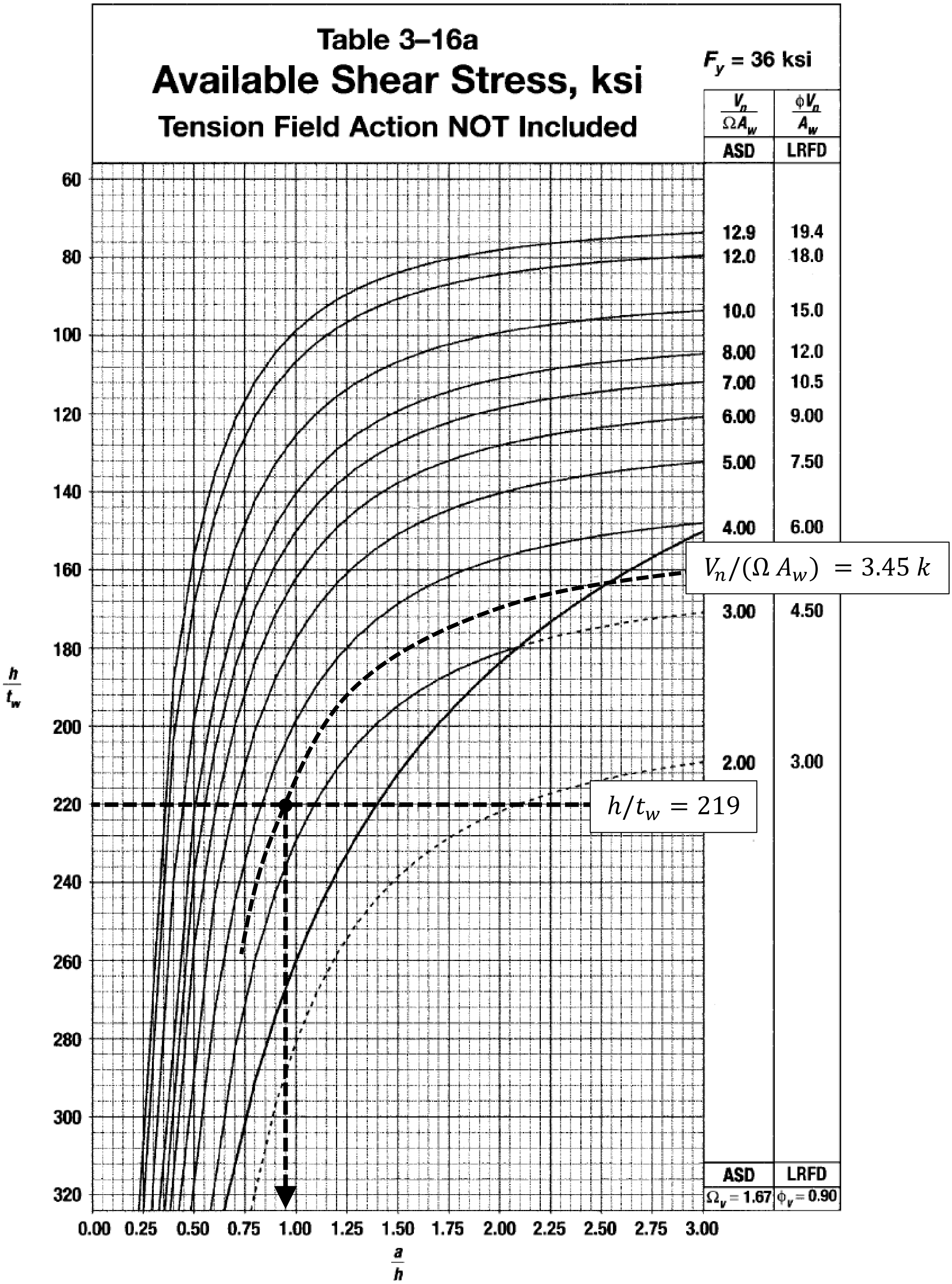


Table 3–16a
Available Shear Stress, ksi
Tension Field Action NOT Included
F_y = 36 ksi
$V_n/\Omega A_w$ ASD
$\phi V_n/A_w$ LRFD
12.9 19.4
12.0 18.0
10.0 15.0
8.00 12.0
7.00 10.5
6.00 9.00
5.00 7.50
4.00 6.00
3.00 4.50
2.00 3.00
$V_n/(\Omega A_w) = 3.45$ k
$h/t_w = 219$
ASD $\Omega_v = 1.67$
LRFD $\phi_v = 0.90$
h/t_w
60 80 100 120 140 160 180 200 220 240 260 280 300 320
0.00 0.25 0.50 0.75 1.00 1.25 1.50 1.75 2.00 2.25 2.50 2.75 3.00
a/h

GUIDE TO THE STEEL CONSTRUCTION MANUAL, 13TH EDITION

G

ANALYSIS OF TENSION MEMBERS

The Steel Construction Manual **AISC Chapter D, Page 26** limit states that will be considered are:

- **SLENDERNESS LIMITATIONS,** AISC Chapter D, Page 26

D1. SLENDERNESS LIMITATIONS

There is no maximum slenderness limit for design of members in tension.

User Note: For members designed on the basis of tension, the slenderness ratio L/r preferably should not exceed 300. This suggestion does not apply to rods or hangers in tension.

- **TENSILE STRENGTH,** AISC Chapter D, Page 26

D2. TENSILE STRENGTH

The *design tensile strength*, $\phi_t P_n$, and the *allowable tensile strength*, P_n/Ω_t, of tension members, shall be the lower value obtained according to the *limit states*

- **TENSILE YIELDING,** AISC Chapter D, Page 26

(a) For tensile yielding in the gross section:

$$P_n = F_y A_g \qquad \text{(D2-1)}$$

$$\phi_t = 0.90 \text{ (LRFD)} \qquad \Omega_t = 1.67 \text{ (ASD)}$$

- **TENSILE RUPTURE,** AISC Chapter D, Page 27

(b) For tensile rupture in the net section:

$$P_n = F_u A_e \qquad \text{(D2-2)}$$

$$\phi_t = 0.75 \text{ (LRFD)} \qquad \Omega_t = 2.00 \text{ (ASD)}$$

- **AREA DETERMINATION,** AISC Chapter D, Page 27

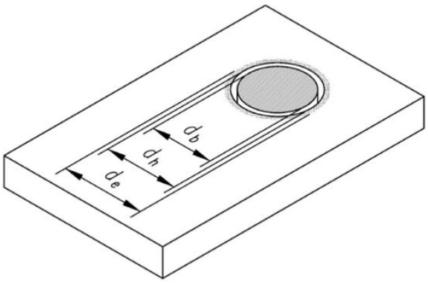
1. **Gross Area**, AISC Chapter D, Page 27

The gross area, A_g, of a member is the total cross-sectional area.

2. **Net Area**, AISC Chapter D, Page 27

$$A_n = A_g - A_{Holes}$$

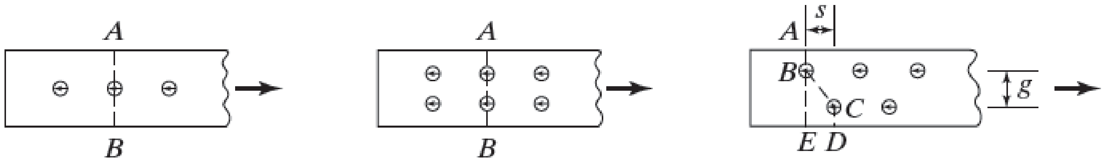
$$d_e = d_b + \frac{1}{8}"$$



Or

For a chain of holes extending across a part in any diagonal or zigzag line, the net width of the part shall be obtained by deducting from the gross width the sum of the diameters or slot dimensions as provided in Section J3.2, of all holes in the chain, and adding, for each *gage* space in the chain, the quantity $s^2/4g$

$$A_n = A_g - A_{Holes} + \sum_{i=1}^{N} \frac{S_i^2}{4g_i} t\,, \quad N\text{: Number of zigzag lines}$$



In determining the net area across plug or *slot welds*, the *weld metal* shall not be considered as adding to the net area.

User Note: Section J4.1(b) limits A_n to a maximum of $0.85A_g$ for splice plates with holes.

$$A_e = A_n \le 0.85A_g$$

3. **Effective Net Area**, AISC Chapter D, Page 28

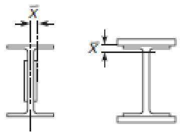
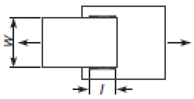
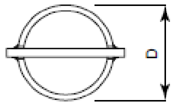
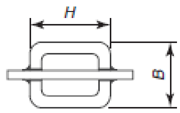
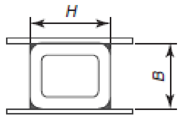
3. Effective Net Area

The effective area of tension members shall be determined as follows:

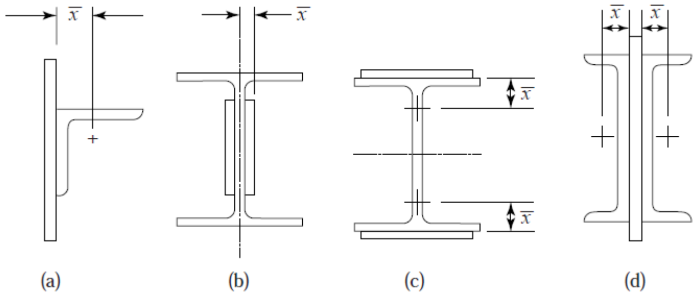
$$A_e = A_n U \qquad \text{(D3-1)}$$

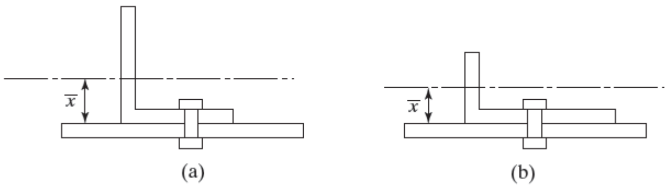
where U, the shear lag factor, is determined as shown in Table D3.1.

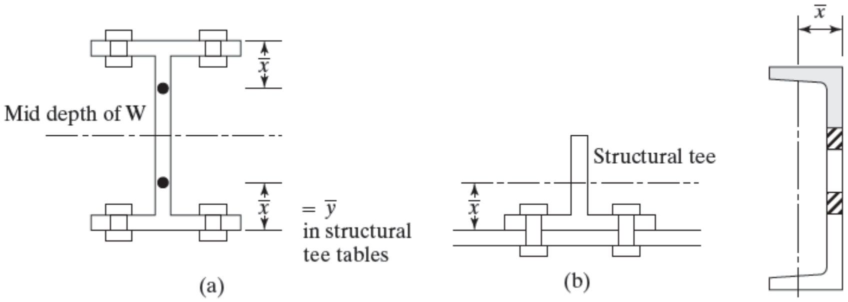
TABLE D3.1
Shear Lag Factors for Connections to Tension Members

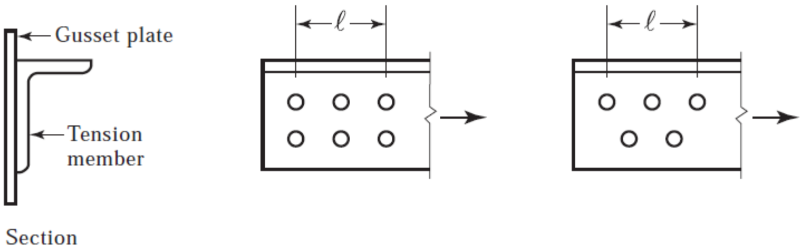
Case	Description of Element		Shear Lag Factor, U	Example
1	All tension members where the tension load is transmitted directly to each of cross-sectional elements by fasteners or welds. (except as in Cases 3, 4, 5 and 6)		$U = 1.0$	—
2	All tension members, except plates and HSS, where the tension load is transmitted to some but not all of the cross-sectional elements by fasteners or longitudinal welds (Alternatively, for W, M, S and HP, Case 7 may be used.)		$U = 1 - \bar{x}/l$	
3	All tension members where the tension load is transmitted by transverse welds to some but not all of the cross-sectional elements.		$U = 1.0$ and A_n = area of the directly connected elements	—
4	Plates where the tension load is transmitted by longitudinal welds only.		$l \geq 2w \ldots U = 1.0$ $2w > l \geq 1.5w \ldots U = 0.87$ $1.5w > l \geq w \ldots U = 0.75$	
5	Round HSS with a single concentric gusset plate		$l \geq 1.3D \ldots U = 1.0$ $D \leq l < 1.3D \ldots U = 1 - \bar{x}/l$ $\bar{x} = D/\pi$	
6	Rectangular HSS	with a single concentric gusset plate	$l \geq H \ldots U = 1 - \bar{x}/l$ $\bar{x} = \dfrac{B^2 + 2BH}{4(B+H)}$	
		with two side gusset plates	$l \geq H \ldots U = 1 - \bar{x}/l$ $\bar{x} = \dfrac{B^2}{4(B+H)}$	
7	W, M, S or HP Shapes or Tees cut from these shapes. (If U is calculated per Case 2, the larger value is permitted to be used)	with flange connected with 3 or more fasteners per line in direction of loading	$b_f \geq 2/3d \ldots U = 0.90$ $b_f < 2/3d \ldots U = 0.85$	—
		with web connected with 4 or more fasteners in the direction of loading	$U = 0.70$	—
8	Single angles (If U is calculated per Case 2, the larger value is permitted to be used)	with 4 or more fasteners per line in direction of loading	$U = 0.80$	—
		with 2 or 3 fasteners per line in the direction of loading	$U = 0.60$	—

l = length of connection, in. (mm); w = plate width, in. (mm); $\bar{x}$ = connection eccentricity, in. (mm); B = overall width of rectangular HSS member, measured 90 degrees to the plane of the connection, in. (mm); H = overall height of rectangular HSS member, measured in the plane of the connection, in. (mm)

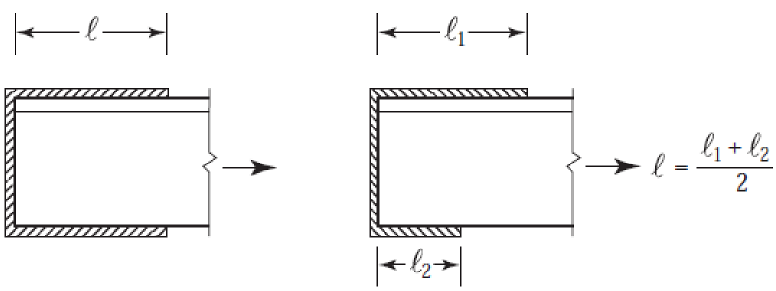








(a) Bolted



(b) Welded

Connection eccentricity $\bar{x}$ for various cases

- **BLOCK SHEAR STRENGTH,** AISC Chapter J, Page 112

3. Block Shear Strength

The *available strength* for the *limit state* of *block shear rupture* along a shear failure path or path(s) and a perpendicular tension failure path shall be taken as

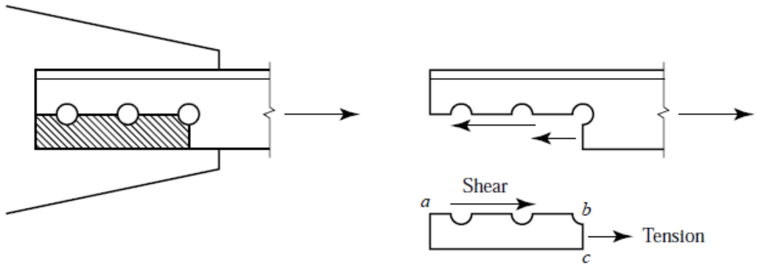
$$R_n = 0.6F_u A_{nv} + U_{bs} F_u A_{nt} \le 0.6F_y A_{gv} + U_{bs} F_u A_{nt} \qquad \text{(J4-5)}$$

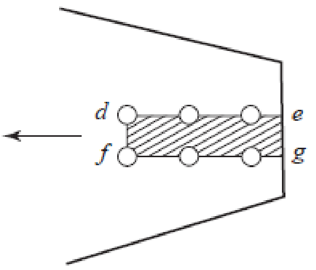
$$\phi = 0.75\,\text{(LRFD)} \qquad \Omega = 2.00\,\text{(ASD)}$$

where

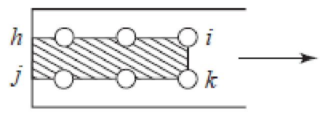
A_{gv} = gross area subject to shear, in.^{2} (mm^2)
A_{nt} = *net area* subject to tension, in.^{2} (mm^2)
A_{nv} = net area subject to shear, in.^{2} (mm^2)

Where the tension *stress* is uniform, $U_{bs} = 1$; where the tension stress is non-uniform, $U_{bs} = 0.5$.





Block shear in gusset plate



Block shear in tension member

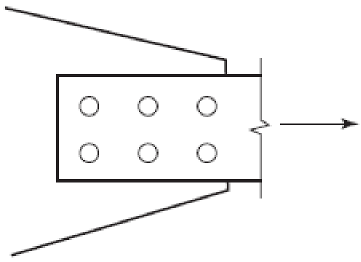
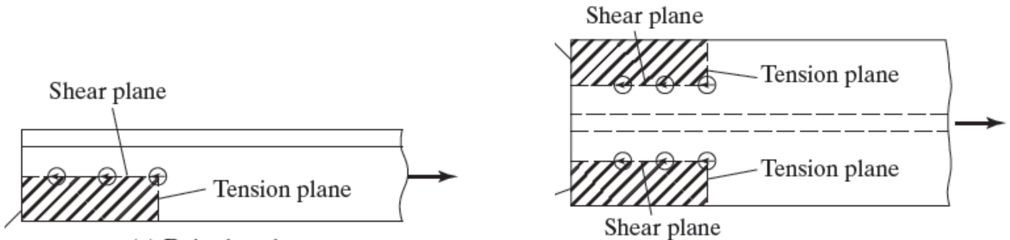


Plate tension member



(a) Bolted angle

(b) Bolted flange of W section

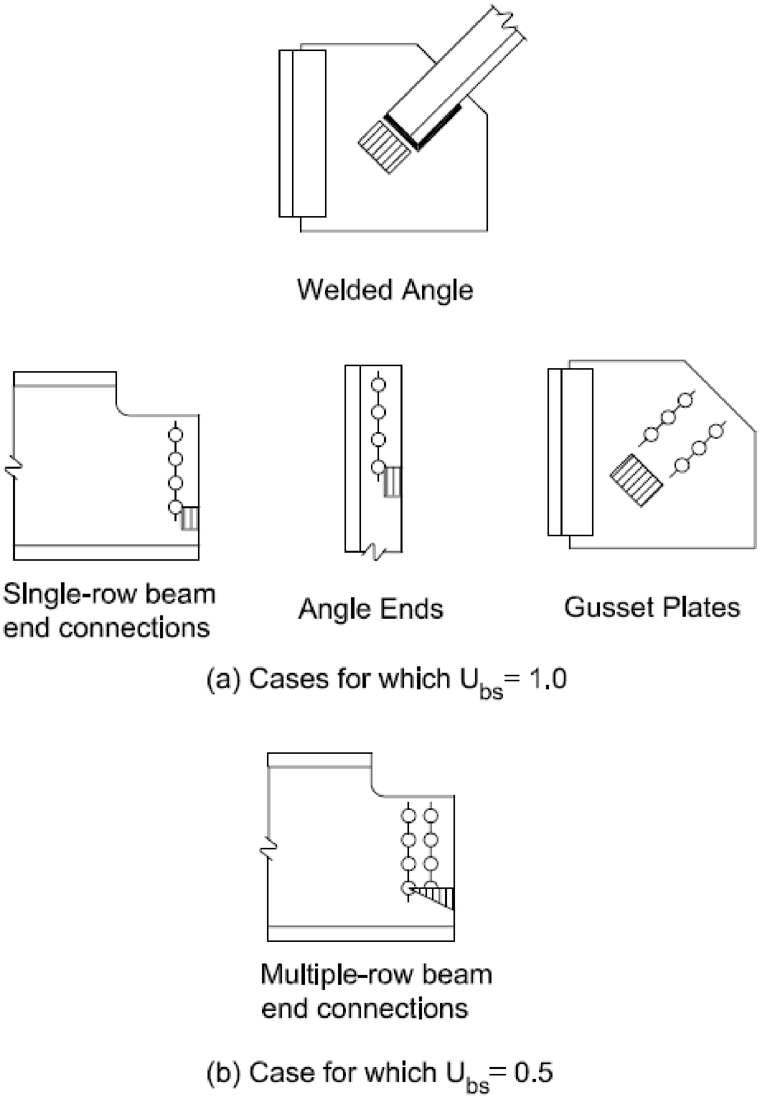
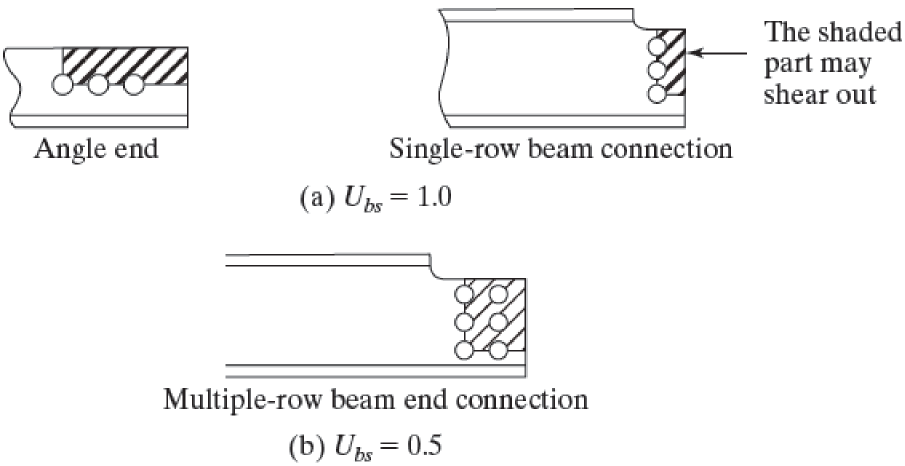


Fig. C-J4.2., **AISC Chapter Comm. J4, Page 352,** Block Shear Tensile Stress Distributions.



BOLTED AND WELDED CONNECTIONS, AISC Chapter J

For bolted and welded connections, the Steel Construction Manual **AISC Chapter J,** limit states that will be considered are:

- **SHEARING STRENGTH OF BOLTS,** AISC Chapter J, Page 108

6. Tension and Shear Strength of Bolts and Threaded Parts

The *design tension* or *shear strength*, ϕR_n, and the *allowable tension* or *shear strength*, R_n/Ω, of a snug-tightened or pretensioned high-strength bolt or threaded part shall be determined according to the *limit states* of *tensile rupture* and *shear rupture* as follows:

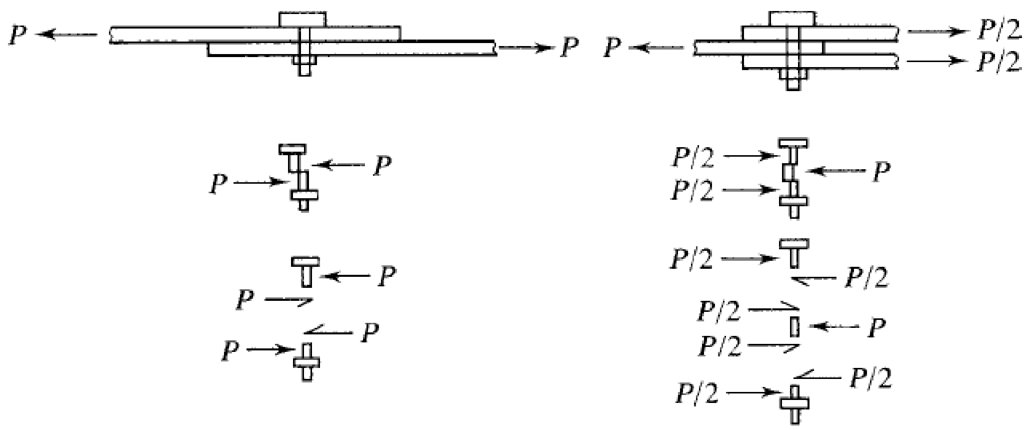
$$R_n = F_n A_b \tag{J3-1}$$

$$\phi = 0.75 \text{ (LRFD)} \qquad \Omega = 2.00 \text{ (ASD)}$$

where

F_n = nominal tensile stress F_{nt}, or shear stress, F_{nv} from Table J3.2, ksi (MPa)

A_b = nominal unthreaded body area of bolt or threaded part (for upset rods, see footnote d, Table J3.2), in.^{2} (mm^2)



(a) Single Shear (b) Double Shear

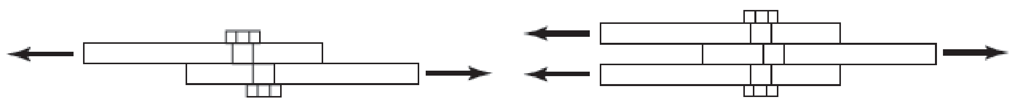


TABLE J3.2
Nominal Stress of Fasteners and Threaded Parts, ksi (MPa)

Description of Fasteners	Nominal Tensile Stress, F_{nt}, ksi (MPa)	Nominal Shear Stress in Bearing-Type Connections, F_{nv}, ksi (MPa)
A307 bolts	45 (310) [a][b]	24 (165) [b][c][f]
A325 or A325M bolts, when threads are not excluded from shear planes	90 (620) [e]	48 (330) [f]
A325 or A325M bolts, when threads are excluded from shear planes	90 (620) [e]	60 (414) [f]
A490 or A490M bolts, when threads are not excluded from shear planes	113 (780) [e]	60 (414) [f]
A490 or A490M bolts, when threads are excluded from shear planes	113 (780) [e]	75 (520) [f]
Threaded parts meeting the requirements of Section A3.4, when threads are not excluded from shear planes	0.75 F_u [a][d]	0.40 F_u
Threaded parts meeting the requirements of Section A3.4, when threads are excluded from shear planes	0.75 F_u [a][d]	0.50 F_u

[a] Subject to the requirements of Appendix 3.
[b] For A307 bolts the tabulated values shall be reduced by 1 percent for each 1/16 in. (2 mm) over 5 diameters of length in the grip.
[c] Threads permitted in shear planes.
[d] The nominal tensile strength of the threaded portion of an upset rod, based upon the cross-sectional area at its major thread diameter, A_D, which shall be larger than the nominal body area of the rod before upsetting times F_y.
[e] For A325 or A325M and A490 or A490M bolts subject to tensile fatigue loading, see Appendix 3.
[f] When bearing-type connections used to splice tension members have a fastener pattern whose length, measured parallel to the line of force, exceeds 50 in. (1270 mm), tabulated values shall be reduced by 20 percent.

- **BEARING STRENGTH OF BOLTS,** AISC Chapter J, Page 111

10. Bearing Strength at Bolt Holes

The *available bearing strength*, ϕR_n and R_n/Ω, at bolt holes shall be determined for the *limit state* of *bearing* as follows:

$$\phi = 0.75 \text{ (LRFD)} \qquad \Omega = 2.00 \text{ (ASD)}$$

(a) For a bolt in a *connection* with standard, oversized, and short-slotted holes, independent of the direction of loading, or a long-slotted hole with the slot parallel to the direction of the bearing *force*:

(i) When deformation at the bolt hole at *service load* is a design consideration

$$R_n = 1.2\, L_c t F_u \leq 2.4\, d t F_u \qquad \text{(J3-6a)}$$

$Deformation \leq 0.25\ in$

(ii) When deformation at the bolt hole at service load is not a design consideration

$$R_n = 1.5\, L_c t F_u \leq 3.0\, d t F_u \qquad \text{(J3-6b)}$$

$Deformation > 0.25\ in$

(b) For a bolt in a connection with long-slotted holes with the slot perpendicular to the direction of force:

$$R_n = 1.0\, L_c t F_u \leq 2.0\, d t F_u \qquad \text{(J3-6c)}$$

(c) For connections made using bolts that pass completely through an unstiffened box member or *HSS*, see Section J7 and Equation J7-1,

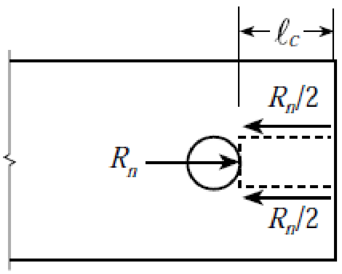
where

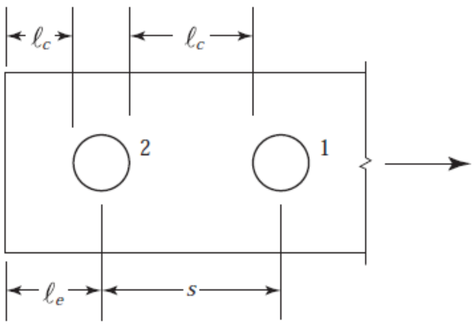
d = nominal bolt diameter, in. (mm)

F_u = *specified minimum tensile strength* of the connected material, ksi (MPa)

L_c = clear distance, in the direction of the force, between the edge of the hole and the edge of the adjacent hole or edge of the material, in. (mm)

t = thickness of connected material, in. (mm)





$$L_c = \min\ [L_{c1}, L_{c2}, L_{c3}, \cdots]$$

- **STRENGTH OF FILLET WELDED CONNECTIONS,** AISC Chapter J2, Page 98

4. Strength

The *design strength*, ϕR_n and the *allowable strength*, R_n/Ω, of welds shall be the lower value of the base material and the *weld metal* strength determined according to the *limit states* of *tensile rupture*, *shear rupture* or *yielding* as follows:

For the base metal

$$R_n = F_{BM} A_{BM} \qquad \text{(J2-2)}$$

For the weld metal

$$R_n = F_w A_w \qquad \text{(J2-3)}$$

where

F_{BM} = *nominal strength* of the base metal per unit area, ksi (MPa)
F_w = nominal strength of the weld metal per unit area, ksi (MPa)
A_{BM} = cross-sectional area of the base metal, in.^{2} (mm^2)
A_w = effective area of the weld, in.^{2} (mm^2)

The values of ϕ, Ω, F_{BM}, and F_w and limitations thereon are given in Table J2.5.

Alternatively, for *fillet welds* loaded in-plane the *design strength*, ϕR_n and the *allowable strength*, R_n/Ω, of welds is permitted to be determined as follows:

$$\phi = 0.75 \text{ (LRFD)} \qquad \Omega = 2.00 \text{ (ASD)}$$

(a) For a linear weld group loaded in-plane through the center of gravity

$$R_n = F_w A_w \qquad \text{(J2-4)}$$

$$R_n = (0.6\, F_{EXX})(0.707\, w)(L)$$

F_{nw} = (nominal strength of base metal $0.60\, F_{EXX}$)
A_{we} = (throat)(weld length) = $(0.707\, w)(L)$
F_{EXX} = electrode classification number, ksi (MPa)
A_w = effective area of the weld, in.^{2} (mm^2)

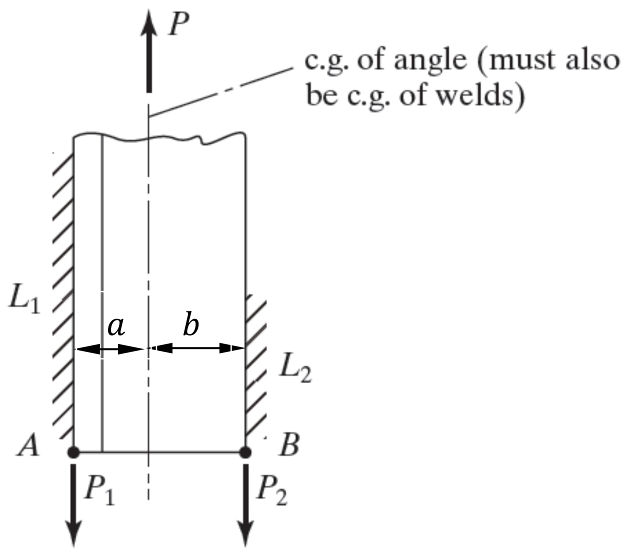
$$\beta = 1.2 - 0.002(L/w) \leq 1.0 \tag{J2-1}$$

where

L = actual length of end-loaded weld, in. (mm)
w = weld leg size, in. (mm)

When the length of the weld exceeds 300 times the leg size, the value of β shall be taken as 0.60.

$$or\ \frac{L}{w} < 100$$



$$L = L_1 + L_2$$

$$L_1 = \frac{b}{a+b}L, \qquad L_1 = \frac{a}{a+b}L, \quad L_1 > L_2, \ \ b > a$$

- **STRENGTH OF WELDED CONNECTIONS WITH BOTH LONGITUDINAL AND TRANSVERSE FILLET WELDS,** AISC Chapter J2, Page 101

(c) For *fillet weld* groups concentrically loaded and consisting of elements that are oriented both longitudinally and transversely to the direction of applied *load*, the combined strength, R_n, of the fillet weld group shall be determined as the greater of

$$R_n = R_{wl} + R_{wt} \tag{J2-9a}$$

or

$$R_n = 0.85R_{wl} + 1.5R_{wt} \tag{J2-9b}$$

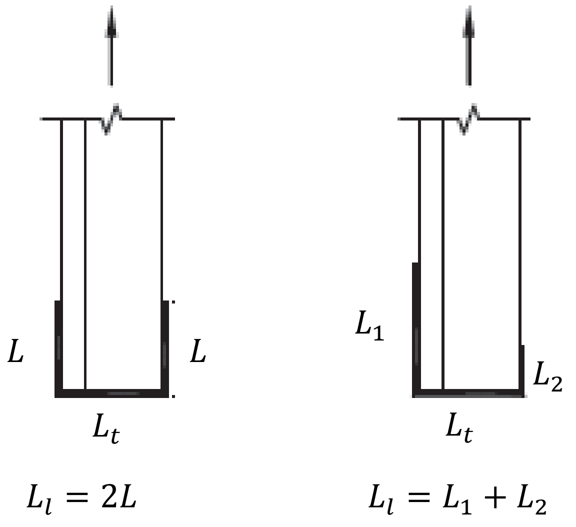
where

R_{wl} = the total nominal strength of longitudinally loaded fillet welds, as determined in accordance with Table J2.5, kips (N)

R_{wt} = the total nominal strength of transversely loaded fillet welds, as determined in accordance with Table J2.5 without the alternate in Section J2.4(a), kips (N)

$$R_{wl} = R_n \text{ for side welds} = (0.6\ F_{EXX})(0.707\ w)(L_l)$$

$$R_{wt} = R_n \text{ for transverse end weld} = (0.6\ F_{EXX})(0.707\ w)(L_t)$$



- **THE MAXIMUM/MINIMUM SIZE OF A FILLET WELD,** AISC Chapter J2, Page 96

TABLE J2.4
Minimum Size of Fillet Welds

Material Thickness of Thinner Part Joined, in. (mm)	Minimum Size of Fillet Weld,[a] in. (mm)
To 1/4 (6) inclusive	1/8 (3)
Over 1/4 (6) to 1/2 (13)	3/16 (5)
Over 1/2 (13) to 3/4 (19)	1/4 (6)
Over 3/4 (19)	5/16 (8)

[a] Leg dimension of fillet welds. Single pass welds must be used.
Note: See Section J2.2b for maximum size of fillet welds.

The maximum size of fillet welds of connected parts shall be:

(a) Along edges of material less than 1/4-in. (6 mm) thick, not greater than the thickness of the material.

(b) Along edges of material 1/4 in. (6 mm) or more in thickness, not greater than the thickness of the material minus 1/16 in. (2 mm), unless the weld is especially designated on the drawings to be built out to obtain full-throat thickness. In the as-welded condition, the distance between the edge of the base metal and the toe of the weld is permitted to be less than 1/16 in. (2 mm) provided the weld size is clearly verifiable.

$$\text{Maximim size of a fillet weld} \le \text{Material thickness} \quad for \quad \text{Material thickness} < \frac{1}{4}"$$

$$\text{Maximim size of a fillet weld} \le \text{Material thickness} - \frac{1}{6}" \quad for \quad \text{Material thickness} \ge \frac{1}{4}"$$

DESIGN OF TENSION MEMBERS

The Steel Construction Manual **AISC Chapter D, Page 26** limit states that will be considered are:

- **LOAD COMBINATIONS,** AISC Chapter 2, Pages 2-8 and 2-9

For LRFD	For ASD
$P_u = 1.4D$ $P_u = 1.2D + 1.6L$	$P_a = D + L$

- **TENSILE YIELDING,** AISC Chapter D, Page 26

To satisfy the first of these expressions, the minimum gross area must be at least equal to

$$\min A_g = \frac{P_u}{\phi_t F_y}.$$

(a) For tensile yielding in the gross section: $\phi_t = 0.90$ (LRFD)

- **TENSILE RUPTURE,** AISC Chapter D, Page 27

To satisfy the second expression, the minimum value of A_e must be at least

$$\min A_e = \frac{P_u}{\phi_t F_u}.$$

And since $A_e = UA_n$ for a bolted member, the minimum value of A_n is

$$\min A_n = \frac{\min A_e}{U} = \frac{P_u}{\phi_t F_u U}.$$

Then the minimum A_g is

$$= \min A_n + \text{estimated area of holes}$$

$$= \frac{P_u}{\phi_t F_u U} + \text{estimated area of holes}$$

(b) For tensile rupture in the net section: $\phi_t = 0.75$ (LRFD)

Assume U, to be checked later

- **CHECK SLENDERNESS LIMITATIONS,** AISC Chapter D, Page 26

$$\min r = \frac{L}{300}$$

- **SELECT A TRIAL SECTION**

Select a Lightest Available Section with a largest Radius of Gyration

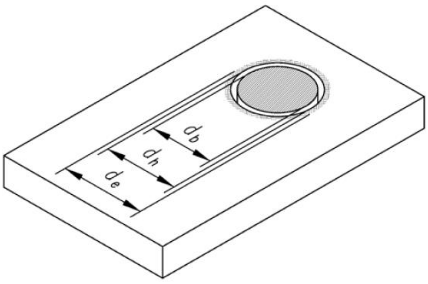
1. **Check the Gross Area**, AISC Chapter D, Page 27

The gross area, A_g, of a member is the total cross-sectional area.

2. **Check the Net Area**, AISC Chapter D, Page 27

$$A_n = A_g - A_{Holes}$$

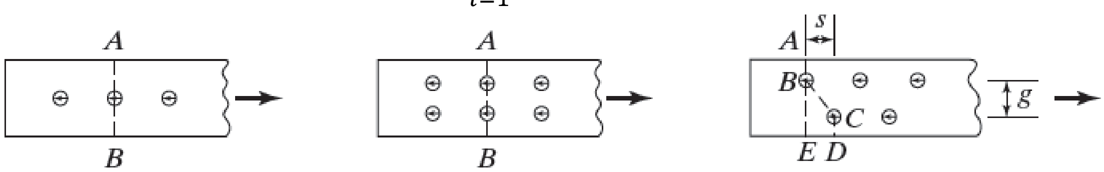
$$d_e = d_b + \frac{1}{8}"$$



Or

For a chain of holes extending across a part in any diagonal or zigzag line, the net width of the part shall be obtained by deducting from the gross width the sum of the diameters or slot dimensions as provided in Section J3.2, of all holes in the chain, and adding, for each *gage* space in the chain, the quantity $s^2/4g$

$$A_n = A_g - A_{Holes} + \sum_{i=1}^{N} \frac{S_i^2}{4g_i} t, \quad N: Number\ of\ zigzag\ lines$$



In determining the net area across plug or *slot welds*, the *weld metal* shall not be considered as adding to the net area.

User Note: Section J4.1(b) limits A_n to a maximum of $0.85A_g$ for splice plates with holes.

$$A_e = A_n \le 0.85A_g$$

3. **Check the Effective Net Area**, AISC Chapter D, Page 28

3. Effective Net Area

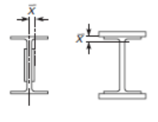
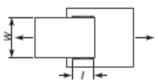
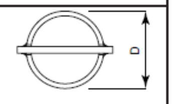
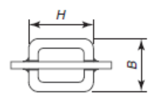
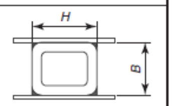
The effective area of tension members shall be determined as follows:

$$A_e = A_n U \qquad \text{(D3-1)}$$

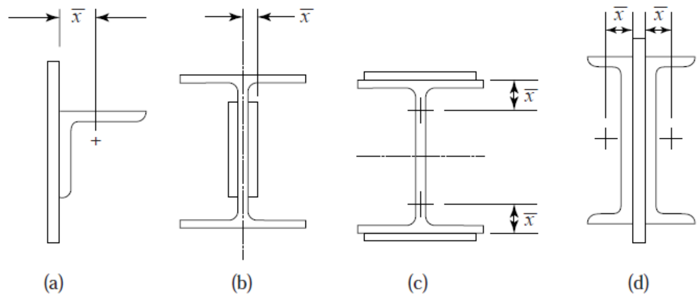
where U, the shear lag factor, is determined as shown in Table D3.1.

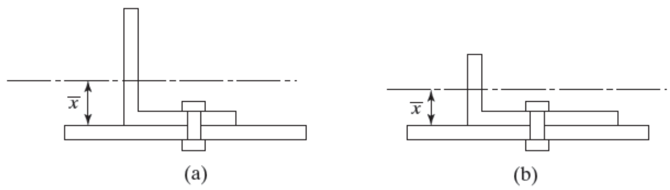
Sect. D5.] PIN-CONNECTED MEMBERS 29

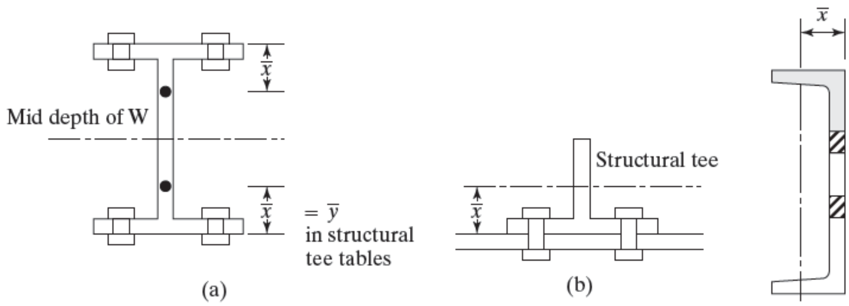
TABLE D3.1
Shear Lag Factors for Connections to Tension Members

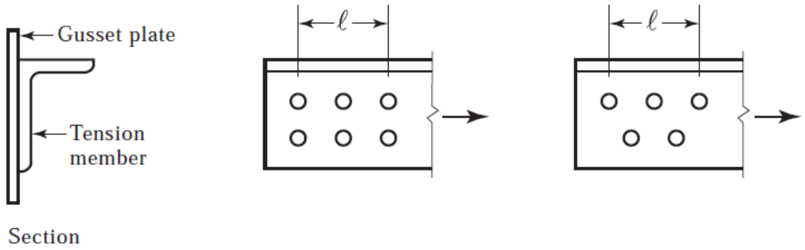
Case	Description of Element		Shear Lag Factor, U	Example
1	All tension members where the tension load is transmitted directly to each of cross-sectional elements by fasteners or welds. (except as in Cases 3, 4, 5 and 6)		$U = 1.0$	—
2	All tension members, except plates and HSS, where the tension load is transmitted to some but not all of the cross-sectional elements by fasteners or longitudinal welds (Alternatively, for W, M, S and HP, Case 7 may be used.)		$U = 1 - \bar{x}/l$	
3	All tension members where the tension load is transmitted by transverse welds to some but not all of the cross-sectional elements.		$U = 1.0$ and A_n = area of the directly connected elements	—
4	Plates where the tension load is transmitted by longitudinal welds only.		$l \geq 2w \ldots U = 1.0$ $2w > l \geq 1.5w \ldots U = 0.87$ $1.5w > l \geq w \ldots U = 0.75$	
5	Round HSS with a single concentric gusset plate		$l \geq 1.3D \ldots U = 1.0$ $D \leq l < 1.3D \ldots U = 1 - \bar{x}/l$ $\bar{x} = D/\pi$	
6	Rectangular HSS	with a single concentric gusset plate	$l \geq H \ldots U = 1 - \bar{x}/l$ $\bar{x} = \dfrac{B^2 + 2BH}{4(B+H)}$	
		with two side gusset plates	$l \geq H \ldots U = 1 - \bar{x}/l$ $\bar{x} = \dfrac{B^2}{4(B+H)}$	
7	W, M, S or HP Shapes or Tees cut from these shapes. (If U is calculated per Case 2, the larger value is permitted to be used)	with flange connected with 3 or more fasteners per line in direction of loading	$b_f \geq 2/3d \ldots U = 0.90$ $b_f < 2/3d \ldots U = 0.85$	—
		with web connected with 4 or more fasteners in the direction of loading	$U = 0.70$	—
8	Single angles (If U is calculated per Case 2, the larger value is permitted to be used)	with 4 or more fasteners per line in direction of loading	$U = 0.80$	—
		with 2 or 3 fasteners per line in the direction of loading	$U = 0.60$	—

l = length of connection, in. (mm); w = plate width, in. (mm); $\bar{x}$ = connection eccentricity, in. (mm); B = overall width of rectangular HSS member, measured 90 degrees to the plane of the connection, in. (mm); H = overall height of rectangular HSS member, measured in the plane of the connection, in. (mm)

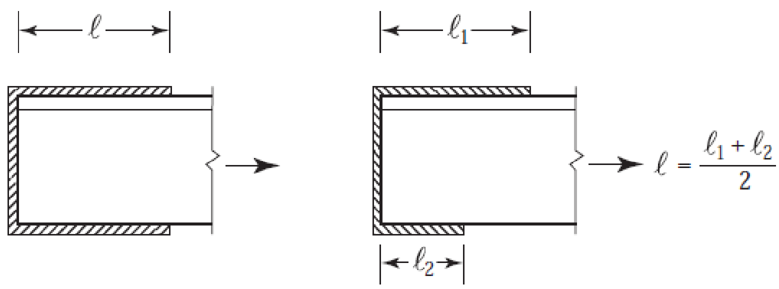








(a) Bolted



(b) Welded

Connection eccentricity $\bar{x}$ for various cases

ANALYSIS OF COMPRESSION MEMBERS

The Steel Construction Manual **AISC Chapter E, Page 32** limit states that will be considered are:

- **SLENDERNESS OF COMPRESSION ELEMENTS,** AISC Chapter **B4** Table B4.1, Page 16

$$\lambda = \frac{b}{t_f} < \lambda_r \ and\ \lambda = \frac{h}{t_w} < \lambda_r, \qquad b = \frac{b_f}{2}, \qquad h = d - 2k$$

TABLE B4.1
Limiting Width-Thickness Ratios for Compression Elements

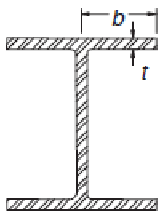
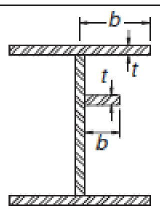
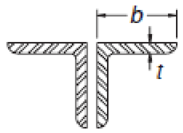
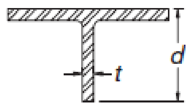
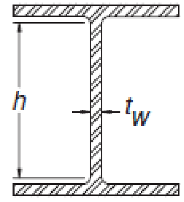
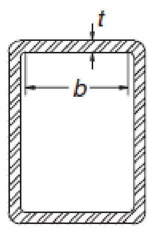
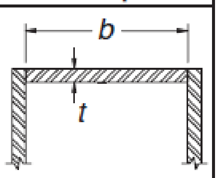
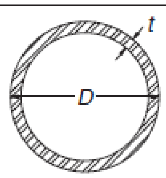
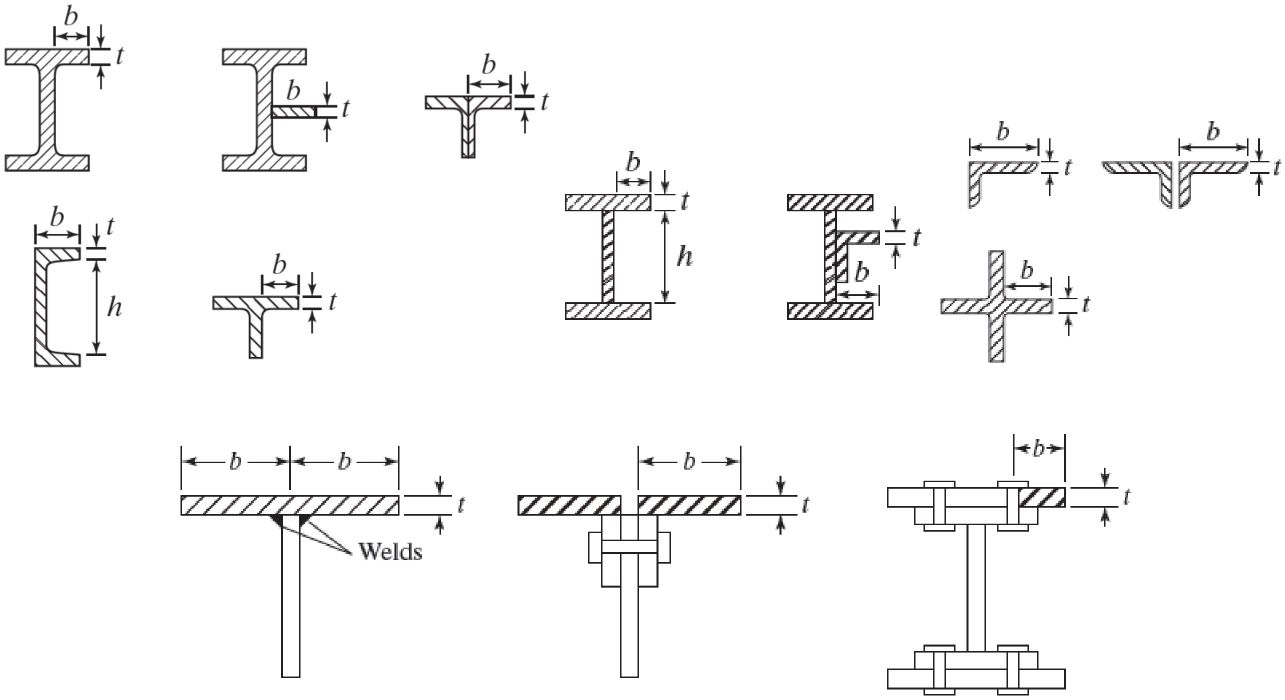
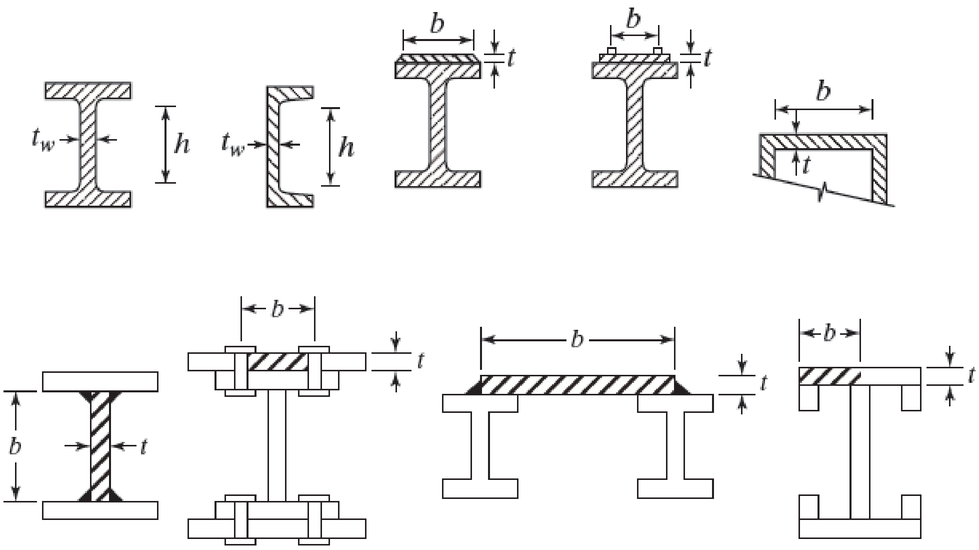
	Case	Description of Element	Width Thickness Ratio	Limiting Width-Thickness Ratios λ_p (compact)	Limiting Width-Thickness Ratios λ_r (noncompact)	Example
Unstiffened Elements	3	Uniform compression in flanges of rolled I-shaped sections, plates projecting from rolled I-shaped sections; outstanding legs of pairs of angles in continuous contact and flanges of channels	b/t	NA	$0.56\sqrt{E/F_y}$	
	4	Uniform compression in flanges of built-up I-shaped sections and plates or angle legs projecting from built-up I-shaped sections	b/t	NA	$0.64\sqrt{k_c E/F_y}$ [a]	
	5	Uniform compression in legs of single angles, legs of double angles with separators, and all other unstiffened elements	b/t	NA	$0.45\sqrt{E/F_y}$	

TABLE B4.1 (cont.)
Limiting Width-Thickness Ratios for Compression Elements

	Case	Description of Element	Width Thick-ness Ratio	Limiting Width-Thickness Ratios λ_p (compact)	Limiting Width-Thickness Ratios λ_r (noncompact)	Example
Elements	8	Uniform compression in stems of tees	d/t	NA	$0.75\sqrt{E/F_y}$	
Elements	10	Uniform compression in webs of doubly symmetric I-shaped sections	h/t_w	NA	$1.49\sqrt{E/F_y}$	
Elements	12	Uniform compression in flanges of rectangular box and hollow structural sections of uniform thickness subject to bending or compression; flange cover plates and diaphragm plates between lines of fasteners or welds	b/t	$1.12\sqrt{E/F_y}$	$1.40\sqrt{E/F_y}$	
Elements	14	Uniform compression in all other stiffened elements	b/t	NA	$1.49\sqrt{E/F_y}$	
Elements	15	Circular hollow sections In uniform compression In flexure	 D/t D/t	 NA $0.07\,E/F_y$	 $0.11\,E/F_y$ $0.31\,E/F_y$	



(a) Unstiffened elements

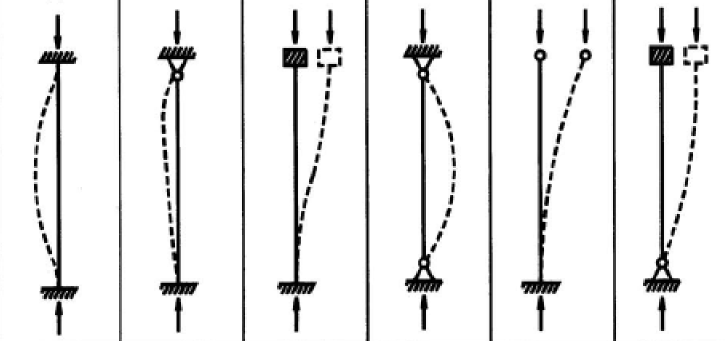


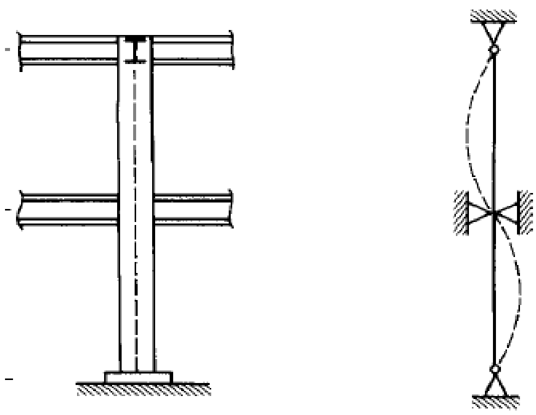
(b) Stiffened elements

- **EFFECTIVE LENGTH FACTOR (K),** AISC Chapter E, Page 26

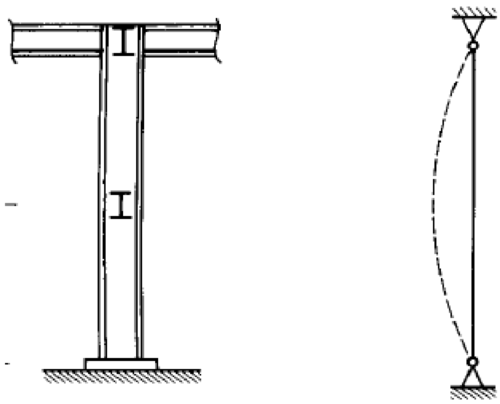
1. **Simple Members**, AISC Chapter Comm. C2, Page 240

TABLE C-C2.2
Approximate Values of Effective Length Factor, *K*

	(a)	(b)	(c)	(d)	(e)	(f)
Buckled shape of column is shown by dashed line.						
Theoretical *K* value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design value when ideal conditions are approximated	0.65	0.80	1.2	1.0	2.10	2.0
End condition code	*Rotation fixed and translation fixed* *Rotation free and translation fixed* *Rotation fixed and translation free* *Rotation free and translation free*					



Minor Axis Buckling



Major Axis Buckling

2. Braced Frames (sidesway inhibited), AISC Chapter Comm. C2, Page 241

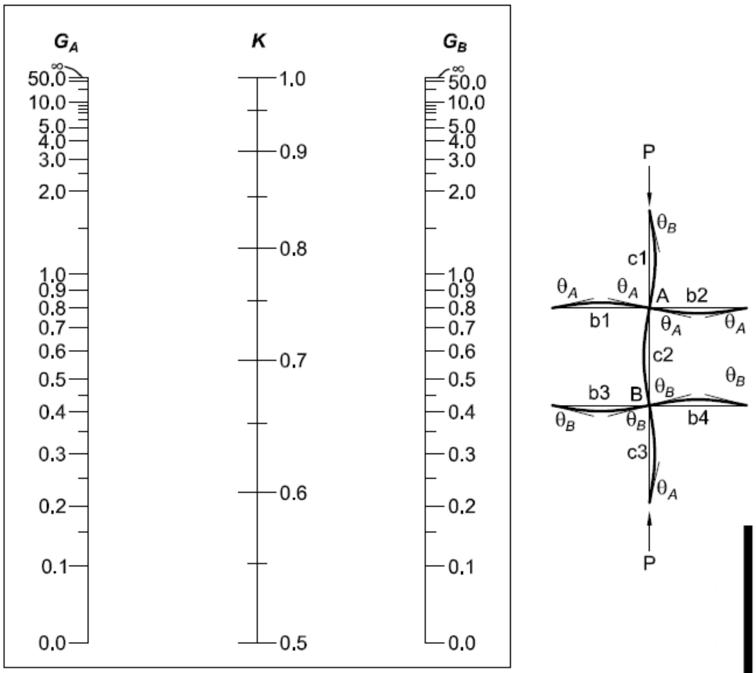


Fig. C-C2.3. Alignment chart—sidesway inhibited (braced frame).


$$G = \frac{\Sigma(E_c I_c / L_c)}{\Sigma(E_g I_g / L_g)} = \frac{\Sigma(EI/L)_c}{\Sigma(EI/L)_g} \qquad G_{Fixed} = 1 \qquad G_{Pinned} = 10$$

$\Sigma E_c I_c / L_c$ = sum of the stiffnesses of all columns at the end of the column under consideration.

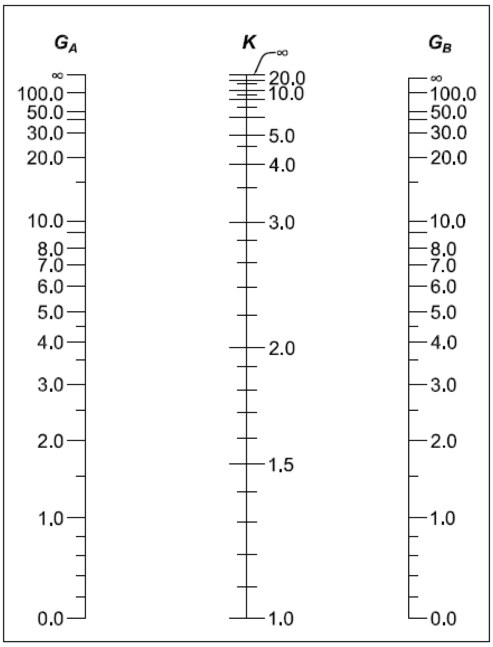
$\Sigma E_g I_g / L_g$ = sum of the stiffnesses of all girders at the end of the column under consideration.

$E_c = E_g = E$, the modulus of elasticity of structural steel.

1. If the far end of a girder is fixed, multiply the $(EI/L)_g$ of the member by 2.0.

2. If the far end of the girder is pinned, multiply the $(EI/L)_g$ of the member by 1.5.

3. **Moment Frames (sidesway uninhibited)**, AISC Chapter Comm. C2, Page 242



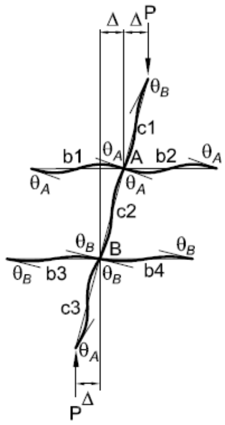
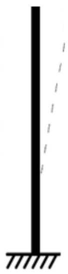


Fig. C-C2.4. Alignment chart—sidesway uninhibited (moment frame).

$$G = \frac{\Sigma(E_c I_c/L_c)}{\Sigma(E_g I_g/L_g)} = \frac{\Sigma(EI/L)_c}{\Sigma(EI/L)_g}$$



$G_{Fixed} = 1$

$G_{Pinned} = 10$

$\Sigma E_c I_c / L_c =$ sum of the stiffnesses of all columns at the end of the column under consideration.

$\Sigma E_g I_g / L_g =$ sum of the stiffnesses of all girders at the end of the column under consideration.

$E_c = E_g = E$, the modulus of elasticity of structural steel.

1. If the far end of a girder is fixed, multiply the $(EI/L)_g$ of the member by 2/3.

2. If the far end of the girder is pinned, multiply the $(EI/L)_g$ of the member by 0.5.

- **SLENDERNESS LIMITATIONS,** AISC Chapter E2, Page 32

E2. SLENDERNESS LIMITATIONS AND EFFECTIVE LENGTH

The effective length factor, K, for calculation of column slenderness, KL/r, shall be determined in accordance with Chapter C,

User Note: For members designed on the basis of compression, the slenderness ratio KL/r preferably should not exceed 200.

- **NOMINAL COMPRESSIVE STRENGTH,** AISC Chapter E3, Page 33

1. By using **AISC Equations E3-1 to E3-4,** Page 33

$$\phi_c P_n = \phi_c F_{cr} A_g = \text{LRFD compression strength } (\phi_c = 0.90)$$

$$\frac{P_n}{\Omega_c} = \frac{F_{cr} A_g}{\Omega_c} = \text{ASD allowable compression strength } (\Omega_c = 1.67)$$

The *nominal compressive strength*, P_n, shall be determined based on the *limit state* of *flexural buckling*.

$$P_n = F_{cr} A_g \qquad \text{(E3-1)}$$

The *flexural buckling stress*, F_{cr}, is determined as follows:

(a) When $\frac{KL}{r} \leq 4.71\sqrt{\frac{E}{F_y}}$ (or $F_e \geq 0.44F_y$)

$$F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right] F_y \qquad \text{(E3-2)}$$

(b) When $\frac{KL}{r} > 4.71\sqrt{\frac{E}{F_y}}$ (or $F_e < 0.44F_y$)

$$F_{cr} = 0.877 F_e \qquad \text{(E3-3)}$$

where

F_e = elastic critical buckling stress determined according to Equation E3-4, Section E4, or the provisions of Section C2, as applicable, ksi (MPa)

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} \qquad \text{(E3-4)}$$

2. By using **AISC Table 4-22,** Page 4-318

Table 4–22
Available Critical Stress for Compression Members

Fy = 35ksi			Fy = 36ksi			Fy = 42ksi			Fy = 46ksi			Fy = 50ksi		
Kl/r	F_{cr}/Ω_c ksi ASD	$\phi_c F_{cr}$ ksi LRFD	Kl/r	F_{cr}/Ω_c ksi ASD	$\phi_c F_{cr}$ ksi LRFD	Kl/r	F_{cr}/Ω_c ksi ASD	$\phi_c F_{cr}$ ksi LRFD	Kl/r	F_{cr}/Ω_c ksi ASD	$\phi_c F_{cr}$ ksi LRFD	Kl/r	F_{cr}/Ω_c ksi ASD	$\phi_c F_{cr}$ ksi LRFD
1	**21.0**	31.5	1	**21.6**	32.4	1	**25.1**	37.8	1	**27.5**	41.4	1	**29.9**	45.0
2	**21.0**	31.5	2	**21.6**	32.4	2	**25.1**	37.8	2	**27.5**	41.4	2	**29.9**	45.0
3	**20.9**	31.5	3	**21.5**	32.4	3	**25.1**	37.8	3	**27.5**	41.4	3	**29.9**	45.0
4	**20.9**	31.5	4	**21.5**	32.4	4	**25.1**	37.8	4	**27.5**	41.4	4	**29.9**	44.9
5	**20.9**	31.5	5	**21.5**	32.4	5	**25.1**	37.7	5	**27.5**	41.3	5	**29.9**	44.9
6	**20.9**	31.4	6	**21.5**	32.3	6	**25.1**	37.7	6	**27.5**	41.3	6	**29.9**	44.9
7	**20.9**	31.4	7	**21.5**	32.3	7	**25.1**	37.7	7	**27.5**	41.3	7	**29.8**	44.8
8	**20.9**	31.4	8	**21.5**	32.3	8	**25.1**	37.7	8	**27.4**	41.2	8	**29.8**	44.8
9	**20.9**	31.4	9	**21.5**	32.3	9	**25.0**	37.6	9	**27.4**	41.2	9	**29.8**	44.7
10	**20.9**	31.3	10	**21.4**	32.2	10	**25.0**	37.6	10	**27.4**	41.1	10	**29.7**	44.7
11	**20.8**	31.3	11	**21.4**	32.2	11	**25.0**	37.5	11	**27.3**	41.1	11	**29.7**	44.6
12	**20.8**	31.3	12	**21.4**	32.2	12	**24.9**	37.5	12	**27.3**	41.0	12	**29.6**	44.5
13	**20.8**	31.2	13	**21.4**	32.1	13	**24.9**	37.4	13	**27.2**	40.9	13	**29.6**	44.4
14	**20.7**	31.2	14	**21.3**	32.1	14	**24.8**	37.3	14	**27.2**	40.9	14	**29.5**	44.4
15	**20.7**	31.1	15	**21.3**	32.0	15	**24.8**	37.3	15	**27.1**	40.8	15	**29.5**	44.3
16	**20.7**	31.1	16	**21.3**	32.0	16	**24.8**	37.2	16	**27.1**	40.7	16	**29.4**	44.2
17	**20.7**	31.0	17	**21.2**	31.9	17	**24.7**	37.1	17	**27.0**	40.6	17	**29.3**	44.1
18	**20.6**	31.0	18	**21.2**	31.9	18	**24.7**	37.1	18	**27.0**	40.5	18	**29.2**	43.9
19	**20.6**	30.9	19	**21.2**	31.8	19	**24.6**	37.0	19	**26.9**	40.4	19	**29.2**	43.8
20	**20.5**	30.9	20	**21.1**	31.7	20	**24.5**	36.9	20	**26.8**	40.3	20	**29.1**	43.7
21	**20.5**	30.8	21	**21.1**	31.7	21	**24.5**	36.8	21	**26.7**	40.2	21	**29.0**	43.6
22	**20.4**	30.7	22	**21.0**	31.6	22	**24.4**	36.7	22	**26.7**	40.1	22	**28.9**	43.4
23	**20.4**	30.7	23	**21.0**	31.5	23	**24.3**	36.6	23	**26.6**	40.0	23	**28.8**	43.3
24	**20.3**	30.6	24	**20.9**	31.4	24	**24.3**	36.5	24	**26.5**	39.8	24	**28.7**	43.1

$$\phi_c P_n = \phi_c F_{cr} A_g = \text{LRFD compression strength } (\phi_c = 0.90)$$

$$\frac{P_n}{\Omega_c} = \frac{F_{cr} A_g}{\Omega_c} = \text{ASD allowable compression strength } (\Omega_c = 1.67)$$

3. By using **AISC Table 4-1 to Table 4-11,** Page 4-10 to Page 4-157

W14

Table 4-1
Available Strength in Axial Compression, kips
W Shapes

F_y = 50 ksi

Shape		W14×											
Wt/ft		730[h]		665[h]		605[h]		550[h]		500[h]		455[h]	
Design		P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
f gyration r_y	0	6440	9670	5870	8820	5330	8010	4850	7290	4400	6610	4010	6030
	11	6070	9130	5530	8310	5010	7530	4550	6840	4120	6200	3750	5640
	12	6010	9030	5470	8220	4950	7440	4500	6760	4070	6120	3710	5570
	13	5940	8920	5400	8110	4890	7350	4440	6670	4020	6040	3660	5500
	14	5860	8810	5330	8010	4820	7250	4380	6580	3960	5950	3600	5420
	15	5780	8690	5250	7890	4750	7140	4310	6480	3900	5860	3550	5330

	730[h]	665[h]	605[h]	550[h]	500[h]	455[h]
L_r (ft)	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
A_g (in.^{2})	215	196	178	162	147	134
I_x (in.^{4})	14300	12400	10800	9430	8210	7190
I_y (in.^{4})	4720	4170	3680	3250	2880	2560
r_y (in.)	4.69	4.62	4.55	4.49	4.43	4.38
Ratio r_x/r_y	1.74	1.73	1.71	1.70	1.69	1.67
$P_{ex}(KL^2)/10^4$ (k-in.^{2})	409000	355000	309000	270000	235000	206000
$P_{ey}(KL^2)/10^4$ (k-in.^{2})	135000	119000	105000	93000	82400	73300

ASD	LRFD	
$\Omega_c = 1.67$	$\phi_c = 0.90$	[h] Flange thickness is greater than 2 in. Special requirements may apply per AISC Specification Section A3.1c.

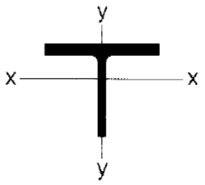
$$(KL)_{Gov.} = \max\ [(KL)_y,\ \ (KL)_{y\,eq}]$$

From **AISC Table 4-1,** Page 4-4

$$(KL)_{y\,eq} = \frac{(KL)_x}{\frac{r_x}{r_y}}$$

Table 4–7
Available Strength in Axial Compression, kips
WT Shapes

F_y = 50 ksi

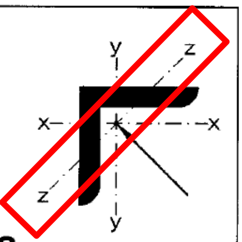


WT18

Shape		WT18×									
Wt/ft		151[c]		141[c]		131[c]		123.5[c]		115.5[c]	
Design		P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD
	0	1210	1820	1050	1580	921	1380	813	1220	708	1060
	10	1170	1760	1020	1530	894	1340	791	1190	690	1040
	12	1150	1730	1010	1510	883	1330	782	1180	682	1030
	14	1130	1700	991	1490	869	1310	771	1160	673	1010
	16	1110	1670	972	1460	854	1280	758	1140	663	997
	18	1090	1630	952	1430	837	1260	744	1120	652	980
X-X Axis	20	1060	1590	930	1400	819	1230	729	1100	639	961
	22	1030	1550	906	1360	799	1200	712	1070	626	940
	24	999	1500	880	1320	778	1170	695	1040	611	919
...d axis	26	967	1450	853	1280	756	1140	676	1020	596	896

Table 4–11
Available Strength in Axial Compression, kips
Concentrically Loaded Single Angles

F_y = 36 ksi



L8

Shape		L8×8×											
		1 1/8		1		7/8		3/4		5/8		9/16[c]	
Wt/ft		57.2		51.3		45.3		39.2		33.0		29.8	
Design		P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD
	0	360	541	323	486	285	428	246	369	207	311	179	270
	1	359	539	322	484	284	426	245	368	206	310	179	269
	2	356	534	319	480	281	422	243	365	204	307	177	267
	3	350	526	314	473	277	416	239	359	201	302	175	263
r_z	4	342	515	308	462	271	407	234	352	197	296	171	258
	5	333	500	299	450	263	396	228	342	192	288	167	251
	6	322	484	289	435	255	383	220	331	185	278	162	243
...of gyration	7	309	464	278	417	245	368	211	318	178	268	156	234

DESIGN OF COMPRESSION MEMBERS

The Steel Construction Manual **AISC Chapter E, Page 32** limit states that will be considered are:

I. By using **AISC Table 4-22,** Trial and Error Procedure**,** Page 4-318

- **LOAD COMBINATIONS,** AISC Chapter 2, Pages 2-8 and 2-9

For LRFD	For ASD
$P_u = 1.4D$ $P_u = 1.2D + 1.6L$	$P_a = D + L$

- **By using** AISC Table 4-22, **Page 4-318**

Assume $\left(\frac{KL}{r}\right) = 50$ to be checked later

Table 4–22 (continued)
Available Critical Stress for Compression Members

	$F_y = 35$ksi			$F_y = 36$ksi			$F_y = 42$ksi			$F_y = 46$ksi			$F_y = 50$ksi	
Kl/r	F_{cr}/Ω_c ksi ASD	$\phi_c F_{cr}$ ksi LRFD	Kl/r	F_{cr}/Ω_c ksi ASD	$\phi_c F_{cr}$ ksi LRFD	Kl/r	F_{cr}/Ω_c ksi ASD	$\phi_c F_{cr}$ ksi LRFD	Kl/r	F_{cr}/Ω_c ksi ASD	$\phi_c F_{cr}$ ksi LRFD	Kl/r	F_{cr}/Ω_c ksi ASD	$\phi_c F_{cr}$ ksi LRFD
48	**18.6**	28.0	48	**19.1**	28.7	48	**21.8**	32.8	48	**23.6**	35.4	48	**25.3**	38.0
49	**18.5**	27.9	49	**19.0**	28.5	49	**21.7**	32.6	49	**23.4**	35.2	49	**25.1**	37.7
50	**18.4**	27.7	50	**18.9**	28.4	50	**21.6**	32.4	50	**23.3**	35.0	50	**24.9**	37.5
51	**18.3**	27.6	51	**18.8**	28.3	51	**21.4**	32.2	51	**23.1**	34.8	51	**24.8**	37.2
52	**18.3**	27.4	52	**18.7**	28.1	52	**21.3**	32.0	52	**23.0**	34.5	52	**24.6**	36.9
53	**18.2**	27.3	53	**18.6**	28.0	53	**21.2**	31.8	53	**22.8**	34.3	53	**24.4**	36.7
54	**18.1**	27.1	54	**18.5**	27.8	54	**21.0**	31.6	54	**22.6**	34.0	54	**24.2**	36.4
55	**18.0**	27.0	55	**18.4**	27.6	55	**20.9**	31.4	55	**22.5**	33.8	55	**24.0**	36.1
56	**17.9**	26.8	56	**18.3**	27.5	56	**20.7**	31.2	56	**22.3**	33.5	56	**23.8**	35.8

- **CALCULATE THE AREA REQUIRED** , AISC Chapter E3, Page 33

$$\text{A Reqd} = \frac{P_u}{\phi_c F_{cr}} \qquad \text{A Reqd} = \frac{P_a}{F_{cr}/\Omega}$$

LRFD compression strength ($\phi_c = 0.90$)

ASD allowable compression strength ($\Omega_c = 1.67$)

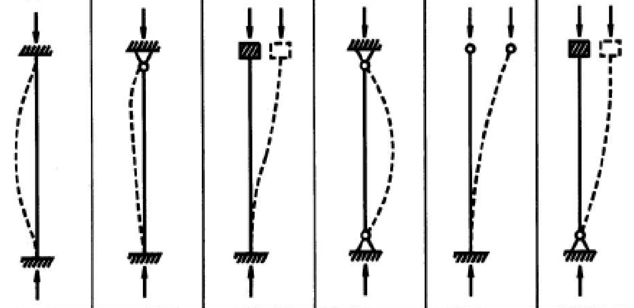
- **SELECT A TRIAL SECTION**

Select a Lightest Available Section with a largest Radius of Gyration

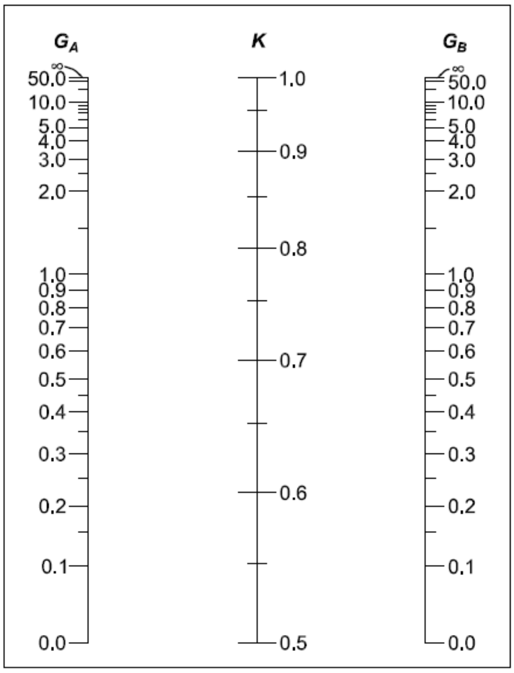
- **CALCULATE THE EFFECTIVE LENGTH FACTOR (K),** AISC Chapter E, Page 26

1. **Simple Members**, AISC Chapter Comm. C2, Page 240

TABLE C-C2.2
Approximate Values of Effective Length Factor, *K*

Buckled shape of column is shown by dashed line.	(a)	(b)	(c)	(d)	(e)	(f)
						
Theoretical *K* value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design value when ideal conditions are approximated	0.65	0.80	1.2	1.0	2.10	2.0
End condition code	 *Rotation fixed and translation fixed* *Rotation free and translation fixed* *Rotation fixed and translation free* *Rotation free and translation free*					

2. **Braced Frames (sidesway inhibited)**, AISC Chapter Comm. C2, Page 241



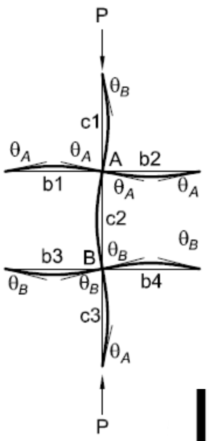
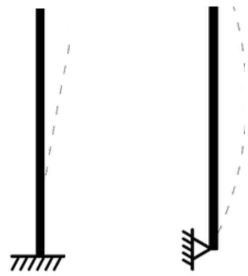


Fig. C-C2.3. Alignment chart—sidesway inhibited (braced frame).



$$G = \frac{\Sigma(E_c I_c / L_c)}{\Sigma(E_g I_g / L_g)} = \frac{\Sigma(EI/L)_c}{\Sigma(EI/L)_g}$$

$G_{Fixed} = 1$ $\quad G_{Pinned} = 10$

$\Sigma E_c I_c / L_c$ = sum of the stiffnesses of all columns at the end of the column under consideration.

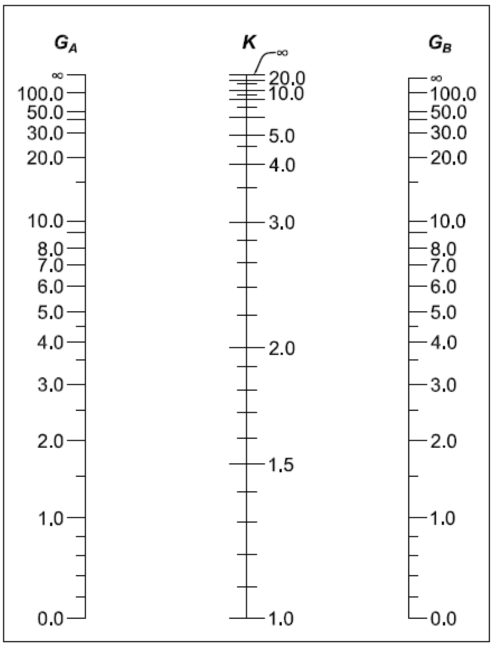
$\Sigma E_g I_g / L_g$ = sum of the stiffnesses of all girders at the end of the column under consideration.

$E_c = E_g = E$, the modulus of elasticity of structural steel.

1. If the far end of a girder is fixed, multiply the $(EI/L)_g$ of the member by 2.0.

2. If the far end of the girder is pinned, multiply the $(EI/L)_g$ of the member by 1.5.

3. **Moment Frames (sidesway uninhibited)**, AISC Chapter Comm. C2, Page 242



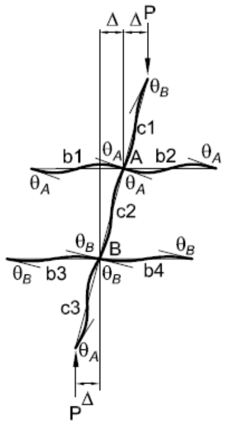
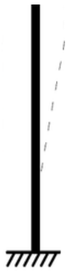


Fig. C-C2.4. Alignment chart—sidesway uninhibited (moment frame).



$$G = \frac{\Sigma(E_c I_c / L_c)}{\Sigma(E_g I_g / L_g)} = \frac{\Sigma(EI/L)_c}{\Sigma(EI/L)_g}$$

$G_{Fixed} = 1$ $G_{Pinned} = 10$

$\Sigma E_c I_c / L_c$ = sum of the stiffnesses of all columns at the end of the column under consideration.

$\Sigma E_g I_g / L_g$ = sum of the stiffnesses of all girders at the end of the column under consideration.

$E_c = E_g = E$, the modulus of elasticity of structural steel.

1. If the far end of a girder is fixed, multiply the $(EI/L)_g$ of the member by 2/3.

2. If the far end of the girder is pinned, multiply the $(EI/L)_g$ of the member by 0.5.

- **CHECK THE SECTION,** by using AISC Table 4-22, **Page 4-318**

Table 4–22
Available Critical Stress for Compression Members

	F_y = 35ksi			F_y = 36ksi			F_y = 42ksi			F_y = 46ksi			F_y = 50ksi	
Kl/r	F_{cr}/Ω_c ksi ASD	$\phi_c F_{cr}$ ksi LRFD	Kl/r	F_{cr}/Ω_c ksi ASD	$\phi_c F_{cr}$ ksi LRFD	Kl/r	F_{cr}/Ω_c ksi ASD	$\phi_c F_{cr}$ ksi LRFD	Kl/r	F_{cr}/Ω_c ksi ASD	$\phi_c F_{cr}$ ksi LRFD	Kl/r	F_{cr}/Ω_c ksi ASD	$\phi_c F_{cr}$ ksi LRFD
1	**21.0**	31.5	1	**21.6**	32.4	1	**25.1**	37.8	1	**27.5**	41.4	1	**29.9**	45.0
2	**21.0**	31.5	2	**21.6**	32.4	2	**25.1**	37.8	2	**27.5**	41.4	2	**29.9**	45.0
3	**20.9**	31.5	3	**21.5**	32.4	3	**25.1**	37.8	3	**27.5**	41.4	3	**29.9**	45.0
4	**20.9**	31.5	4	**21.5**	32.4	4	**25.1**	37.8	4	**27.5**	41.4	4	**29.9**	44.9
5	**20.9**	31.5	5	**21.5**	32.4	5	**25.1**	37.7	5	**27.5**	41.3	5	**29.9**	44.9
6	**20.9**	31.4	6	**21.5**	32.3	6	**25.1**	37.7	6	**27.5**	41.3	6	**29.9**	44.9
7	**20.9**	31.4	7	**21.5**	32.3	7	**25.1**	37.7	7	**27.5**	41.3	7	**29.8**	44.8
8	**20.9**	31.4	8	**21.5**	32.3	8	**25.1**	37.7	8	**27.4**	41.2	8	**29.8**	44.8
9	**20.9**	31.4	9	**21.5**	32.3	9	**25.0**	37.6	9	**27.4**	41.2	9	**29.8**	44.7
10	**20.9**	31.3	10	**21.4**	32.2	10	**25.0**	37.6	10	**27.4**	41.1	10	**29.7**	44.7
11	**20.8**	31.3	11	**21.4**	32.2	11	**25.0**	37.5	11	**27.3**	41.1	11	**29.7**	44.6
12	**20.8**	31.3	12	**21.4**	32.2	12	**24.9**	37.5	12	**27.3**	41.0	12	**29.6**	44.5
13	**20.8**	31.2	13	**21.4**	32.1	13	**24.9**	37.4	13	**27.2**	40.9	13	**29.6**	44.4
14	**20.7**	31.2	14	**21.3**	32.1	14	**24.8**	37.3	14	**27.2**	40.9	14	**29.5**	44.4
15	**20.7**	31.1	15	**21.3**	32.0	15	**24.8**	37.3	15	**27.1**	40.8	15	**29.5**	44.3
16	**20.7**	31.1	16	**21.3**	32.0	16	**24.8**	37.2	16	**27.1**	40.7	16	**29.4**	44.2
17	**20.7**	31.0	17	**21.2**	31.9	17	**24.7**	37.1	17	**27.0**	40.6	17	**29.3**	44.1
18	**20.6**	31.0	18	**21.2**	31.9	18	**24.7**	37.1	18	**27.0**	40.5	18	**29.2**	43.9
19	**20.6**	30.9	19	**21.2**	31.8	19	**24.6**	37.0	19	**26.9**	40.4	19	**29.2**	43.8
20	**20.5**	30.9	20	**21.1**	31.7	20	**24.5**	36.9	20	**26.8**	40.3	20	**29.1**	43.7
21	**20.5**	30.8	21	**21.1**	31.7	21	**24.5**	36.8	21	**26.7**	40.2	21	**29.0**	43.6
22	**20.4**	30.7	22	**21.0**	31.6	22	**24.4**	36.7	22	**26.7**	40.1	22	**28.9**	43.4
23	**20.4**	30.7	23	**21.0**	31.5	23	**24.3**	36.6	23	**26.6**	40.0	23	**28.8**	43.3
24	**20.3**	30.6	24	**20.9**	31.4	24	**24.3**	36.5	24	**26.5**	39.8	24	**28.7**	43.1

$$\phi_c P_n = \phi_c F_{cr} A_g = \text{LRFD compression strength } (\phi_c = 0.90)$$

$$\frac{P_n}{\Omega_c} = \frac{F_{cr} A_g}{\Omega_c} = \text{ASD allowable compression strength } (\Omega_c = 1.67)$$

If $\phi_c P_n < P_u \;\; or \;\; \frac{P_n}{\Omega} < P_a \Rightarrow$ Try the next section, Repeat the Procedure

II. By using AISC Table 4-1 to Table 4-11, Page 4-10 to Page 4-157

- **LOAD COMBINATIONS,** AISC Chapter 2, Pages 2-8 and 2-9

For LRFD	For ASD
$P_u = 1.4D$ $P_u = 1.2D + 1.6L$	$P_a = D + L$

- **By using** AISC Table 4-1 to Table 4-11

Assume $(KL)_y$ to be checked later

- **SELECT A TRIAL SECTION,** by using AISC Table 4-1 to Table 4-11

Select a Lightest Available Section

Table 4-1
Available Strength in Axial Compression, kips
W Shapes

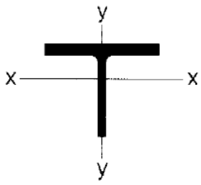
W14

$F_y = 50$ ksi

Shape		W14×											
Wt/ft		730^h		665^h		605^h		550^h		500^h		455^h	
Design		P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
f gyration r_y	0	6440	9670	5870	8820	5330	8010	4850	7290	4400	6610	4010	6030
	11	6070	9130	5530	8310	5010	7530	4550	6840	4120	6200	3750	5640
	12	6010	9030	5470	8220	4950	7440	4500	6760	4070	6120	3710	5570
	13	5940	8920	5400	8110	4890	7350	4440	6670	4020	6040	3660	5500
	14	5860	8810	5330	8010	4820	7250	4380	6580	3960	5950	3600	5420
	15	5780	8690	5250	7890	4750	7140	4310	6480	3900	5860	3550	5330

F_y = 50 ksi

Table 4–7
Available Strength in Axial Compression, kips
WT Shapes

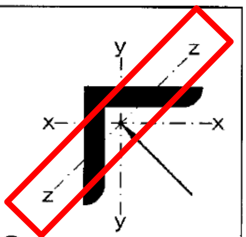


WT18

		Shape	WT18×									
		Wt/ft	151[c]		141[c]		131[c]		123.5[c]		115.5[c]	
		Design	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$
			ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
…d axis	X-X Axis	0	1210	1820	1050	1580	921	1380	813	1220	708	1060
		10	1170	1760	1020	1530	894	1340	791	1190	690	1040
		12	1150	1730	1010	1510	883	1330	782	1180	682	1030
		14	1130	1700	991	1490	869	1310	771	1160	673	1010
		16	1110	1670	972	1460	854	1280	758	1140	663	997
		18	1090	1630	952	1430	837	1260	744	1120	652	980
		20	1060	1590	930	1400	819	1230	729	1100	639	961
		22	1030	1550	906	1360	799	1200	712	1070	626	940
		24	999	1500	880	1320	778	1170	695	1040	611	919
		26	967	1450	853	1280	756	1140	676	1020	596	896

F_y = 36 ksi

Table 4–11
Available Strength in Axial Compression, kips
Concentrically Loaded Single Angles



L8

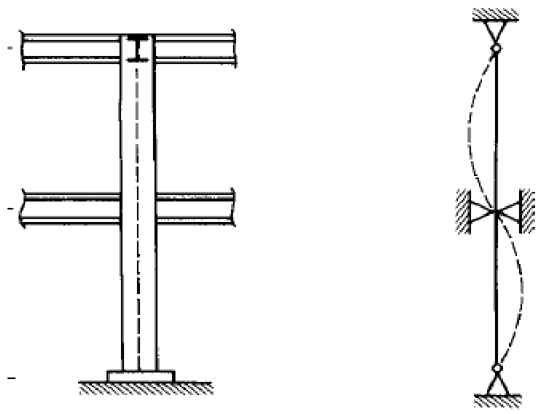
	Shape	L8×8×											
		1 1/8		1		7/8		3/4		5/8		9/16[c]	
	Wt/ft	57.2		51.3		45.3		39.2		33.0		29.8	
	Design	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
…of gyration, r_z	0	360	541	323	486	285	428	246	369	207	311	179	270
	1	359	539	322	484	284	426	245	368	206	310	179	269
	2	356	534	319	480	281	422	243	365	204	307	177	267
	3	350	526	314	473	277	416	239	359	201	302	175	263
	4	342	515	308	462	271	407	234	352	197	296	171	258
	5	333	500	299	450	263	396	228	342	192	288	167	251
	6	322	484	289	435	255	383	220	331	185	278	162	243
	7	309	464	278	417	245	368	211	318	178	268	156	234

- **CALCULATE THE EFFECTIVE LENGTH FACTOR (K),** AISC Chapter E, Page 26

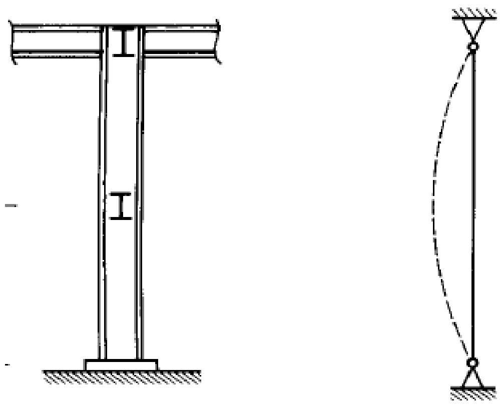
4. Simple Members, AISC Chapter Comm. C2, Page 240

TABLE C-C2.2
Approximate Values of Effective Length Factor, *K*

Buckled shape of column is shown by dashed line.	(a)	(b)	(c)	(d)	(e)	(f)
Theoretical *K* value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design value when ideal conditions are approximated	0.65	0.80	1.2	1.0	2.10	2.0
End condition code	*Rotation fixed and translation fixed* *Rotation free and translation fixed* *Rotation fixed and translation free* *Rotation free and translation free*					



Minor Axis Buckling



Major Axis Buckling

5. **Braced Frames (sidesway inhibited)**, AISC Chapter Comm. C2, Page 241

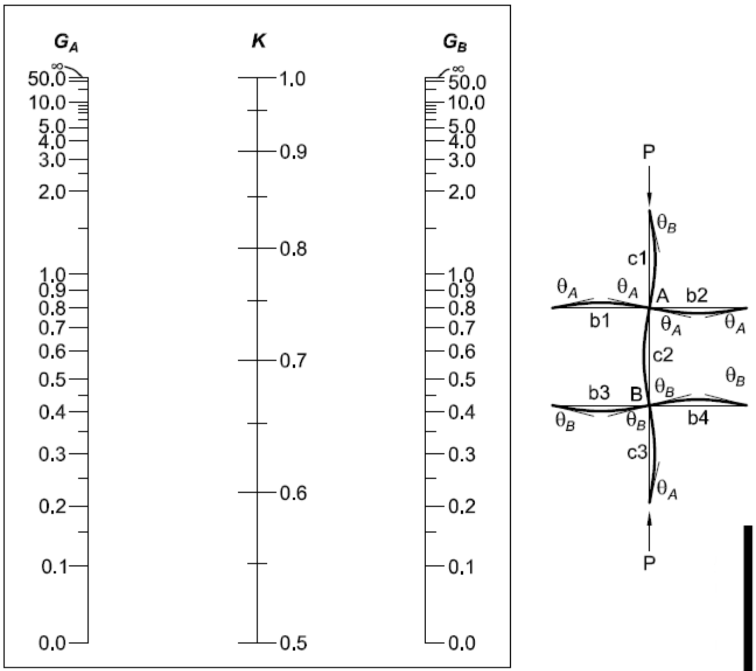
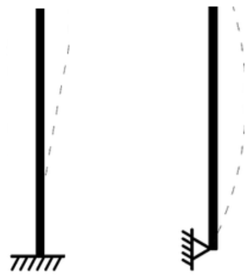


Fig. C-C2.3. Alignment chart—sidesway inhibited (braced frame).



$$G = \frac{\Sigma(E_c I_c / L_c)}{\Sigma(E_g I_g / L_g)} = \frac{\Sigma(EI/L)_c}{\Sigma(EI/L)_g}$$

$G_{Fixed} = 1$ $\quad G_{Pinned} = 10$

$\Sigma E_c I_c / L_c$ = sum of the stiffnesses of all columns at the end of the column under consideration.

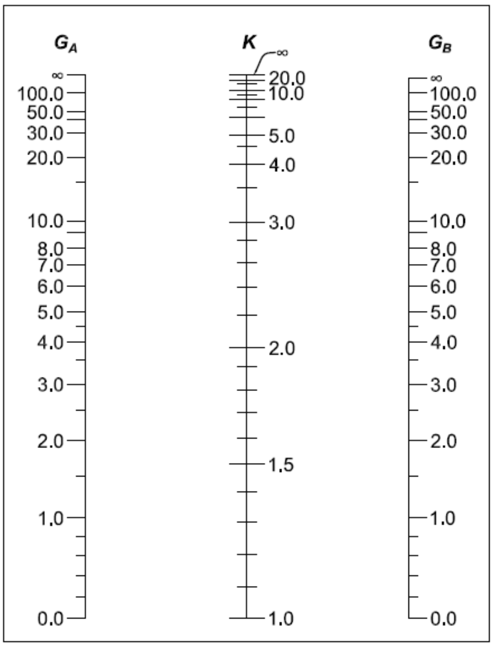
$\Sigma E_g I_g / L_g$ = sum of the stiffnesses of all girders at the end of the column under consideration.

$E_c = E_g = E$, the modulus of elasticity of structural steel.

1. If the far end of a girder is fixed, multiply the $(EI/L)_g$ of the member by 2.0.

2. If the far end of the girder is pinned, multiply the $(EI/L)_g$ of the member by 1.5.

6. **Moment Frames (sidesway uninhibited)**, AISC Chapter Comm. C2, Page 242



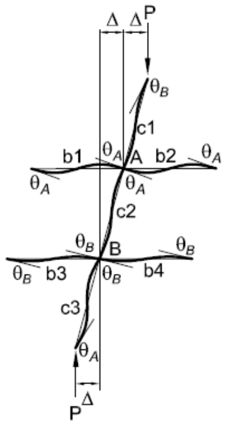


Fig. C-C2.4. Alignment chart—sidesway uninhibited (moment frame).

$$G = \frac{\Sigma(E_c I_c/L_c)}{\Sigma(E_g I_g/L_g)} = \frac{\Sigma(EI/L)_c}{\Sigma(EI/L)_g}$$

$G_{Fixed} = 1$ $\quad G_{Pinned} = 10$

$\Sigma E_c I_c / L_c$ = sum of the stiffnesses of all columns at the end of the column under consideration.

$\Sigma E_g I_g / L_g$ = sum of the stiffnesses of all girders at the end of the column under consideration.

$E_c = E_g = E$, the modulus of elasticity of structural steel.

1. If the far end of a girder is fixed, multiply the $(EI/L)_g$ of the member by 2/3.

2. If the far end of the girder is pinned, multiply the $(EI/L)_g$ of the member by 0.5.

- **CHECK THE EFFECTIVE LENGTH**

$$(KL)_{y\,eq} = \frac{(KL)_x}{\frac{r_x}{r_y}}$$

$$(KL)_y$$

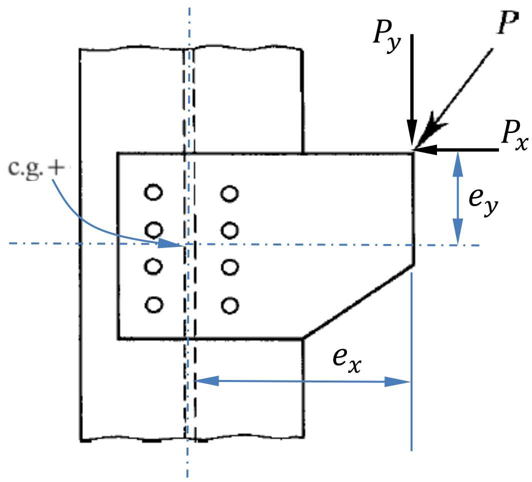
$$(KL)_{Gov.} = \max\ [(KL)_y,\ \ (KL)_{y\,eq}]$$

If $(KL)_{Gov.} > (KL)_{assumed}$ $\Rightarrow$ Try the next section, Repeat the Procedure

Whenever possible, the designer should provide extra support for the weak direction of a column. Otherwise, the member is inefficient: It has an excess of strength in one direction. When K_xL and K_yL are different, K_yL will control unless r_x/r_y is smaller than K_xL/K_yL. When the two ratios are equal, the column has equal strength in both directions. For most of the W-shapes in the column load tables, r_x/r_y ranges between 1.6 and 1.8, but it is as high as 3.1 for some shapes.

ECCENTRICALLY LOADED BOLTED CONNECTIONS

$$M = P_y\, e_x - P_x\, e_y$$



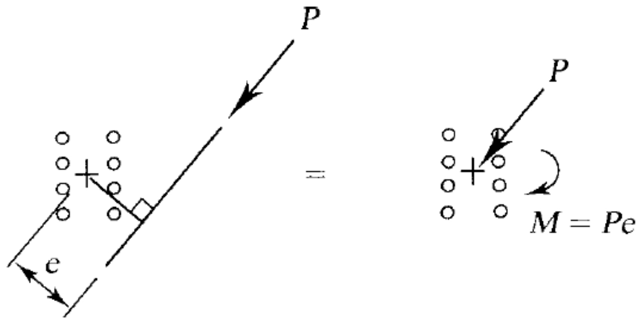
$$\Sigma d^2 = \Sigma h^2 + \Sigma v^2$$

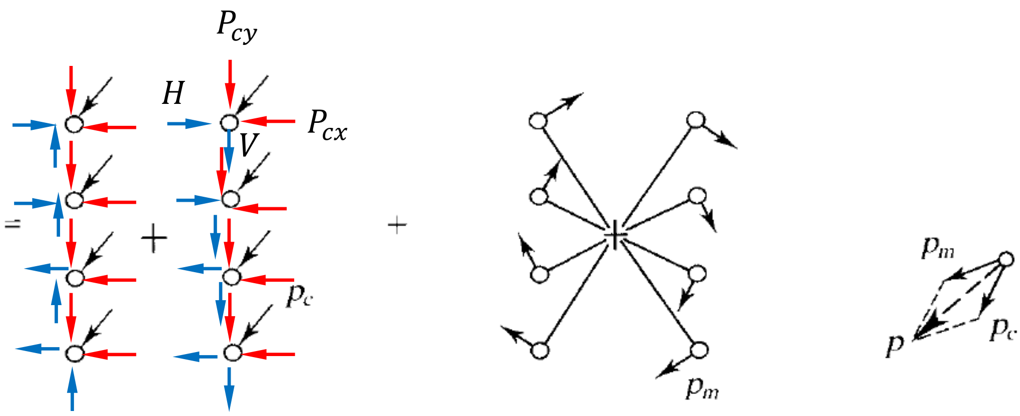
$$H = \frac{Mv}{\Sigma d^2}$$

$$V = \frac{Mh}{\Sigma d^2}$$

$$P_{cx} = \frac{P_x}{n}, \qquad P_{cy} = \frac{P_y}{n}$$

$$R_n = \sqrt{(H + P_x)^2 + \left(V + P_y\right)^2}$$





- **CHECK THE SHEARING STRENGTH OF BOLTS,** AISC Chapter J, Page 108

6. Tension and Shear Strength of Bolts and Threaded Parts

The *design tension* or *shear strength*, ϕR_n, and the *allowable tension* or *shear strength*, R_n/Ω, of a snug-tightened or pretensioned high-strength bolt or threaded part shall be determined according to the *limit states* of *tensile rupture* and *shear rupture* as follows:

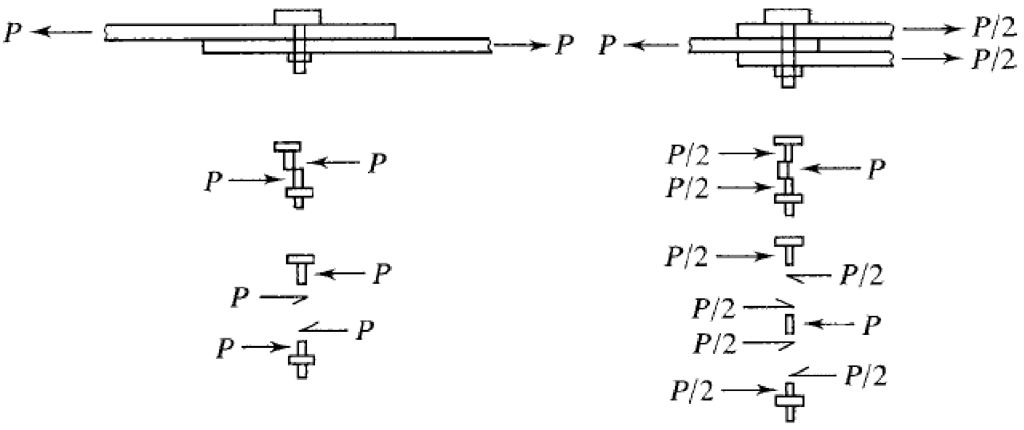
$$R_n = F_n A_b \qquad \text{(J3-1)}$$

$$\phi = 0.75 \text{ (LRFD)} \qquad \Omega = 2.00 \text{ (ASD)}$$

where

F_n = nominal tensile stress F_{nt}, or shear stress, F_{nv} from Table J3.2, ksi (MPa)

A_b = nominal unthreaded body area of bolt or threaded part (for upset rods, see footnote d, Table J3.2), in.^{2} (mm^2)



(a) Single Shear   (b) Double Shear

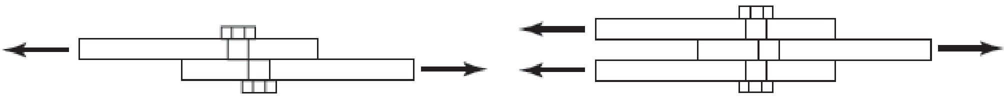


TABLE J3.2
Nominal Stress of Fasteners and Threaded Parts, ksi (MPa)

Description of Fasteners	Nominal Tensile Stress, F_{nt}, ksi (MPa)	Nominal Shear Stress in Bearing-Type Connections, F_{nv}, ksi (MPa)
A307 bolts	45 (310) [a][b]	24 (165) [b][c][f]
A325 or A325M bolts, when threads are not excluded from shear planes	90 (620) [e]	48 (330) [f]
A325 or A325M bolts, when threads are excluded from shear planes	90 (620) [e]	60 (414) [f]
A490 or A490M bolts, when threads are not excluded from shear planes	113 (780) [e]	60 (414) [f]
A490 or A490M bolts, when threads are excluded from shear planes	113 (780) [e]	75 (520) [f]
Threaded parts meeting the requirements of Section A3.4, when threads are not excluded from shear planes	0.75 F_u [a][d]	0.40 F_u
Threaded parts meeting the requirements of Section A3.4, when threads are excluded from shear planes	0.75 F_u [a][d]	0.50 F_u

[a] Subject to the requirements of Appendix 3.

[b] For A307 bolts the tabulated values shall be reduced by 1 percent for each $^1/_{16}$ in. (2 mm) over 5 diameters of length in the grip.

[c] Threads permitted in shear planes.

[d] The nominal tensile strength of the threaded portion of an upset rod, based upon the cross-sectional area at its major thread diameter, A_D, which shall be larger than the nominal body area of the rod before upsetting times F_y.

[e] For A325 or A325M and A490 or A490M bolts subject to tensile fatigue loading, see Appendix 3.

[f] When bearing-type connections used to splice tension members have a fastener pattern whose length, measured parallel to the line of force, exceeds 50 in. (1270 mm), tabulated values shall be reduced by 20 percent.

- **CHECK THE BEARING STRENGTH OF BOLTS,** AISC Chapter J, Page 111

10. Bearing Strength at Bolt Holes

The *available bearing strength*, ϕR_n and R_n/Ω, at bolt holes shall be determined for the *limit state* of *bearing* as follows:

$$\phi = 0.75 \text{ (LRFD)} \qquad \Omega = 2.00 \text{ (ASD)}$$

(a) For a bolt in a *connection* with standard, oversized, and short-slotted holes, independent of the direction of loading, or a long-slotted hole with the slot parallel to the direction of the bearing *force*:

(i) When deformation at the bolt hole at *service load* is a design consideration

$$R_n = 1.2\,L_c t F_u \leq 2.4\,d t F_u \qquad \text{(J3-6a)}$$

Deformation $\leq$ 0.25 *in*

(ii) When deformation at the bolt hole at service load is not a design consideration

$$R_n = 1.5\,L_c t F_u \leq 3.0\,d t F_u \qquad \text{(J3-6b)}$$

Deformation $>$ 0.25 *in*

(b) For a bolt in a connection with long-slotted holes with the slot perpendicular to the direction of force:

$$R_n = 1.0\,L_c t F_u \leq 2.0\,d t F_u \qquad \text{(J3-6c)}$$

(c) For connections made using bolts that pass completely through an unstiffened box member or *HSS*, see Section J7 and Equation J7-1,

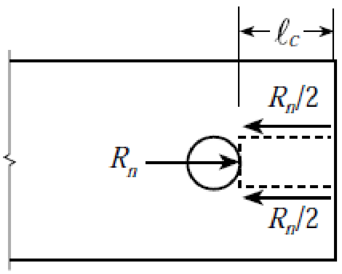
where

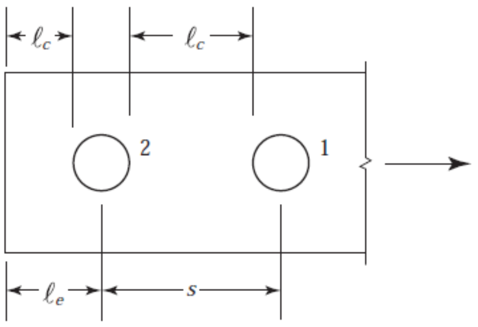
d = nominal bolt diameter, in. (mm)

F_u = *specified minimum tensile strength* of the connected material, ksi (MPa)

L_c = clear distance, in the direction of the force, between the edge of the hole and the edge of the adjacent hole or edge of the material, in. (mm)

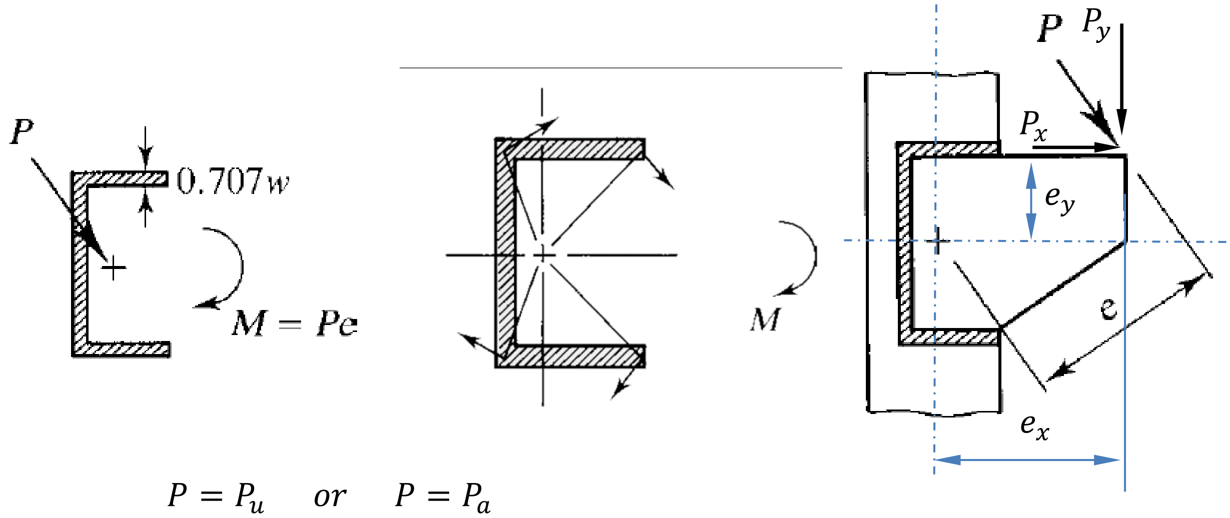
t = thickness of connected material, in. (mm)





$$L_c = \min\ [L_{c1}, L_{c2}, L_{c3}, \ldots]$$

ECCENTRICALLY LOADED WELDED CONNECTIONS



$$P = P_u \quad or \quad P = P_a$$

$$M = T = P_y\, e_x + P_x\, e_y$$

$$J = I_x + I_y$$

$$f_h = \frac{Tv}{J} \quad f_v = \frac{Th}{J}$$

$$f_{sh} = \frac{P_x}{L}, \quad f_{sv} = \frac{P_y}{L}$$

$$f_r = \sqrt{(f_h + fs_h)^2 + (f_v + f_{sv})^2}$$

$$w = size\ of\ weld = \frac{f_r}{\phi R_n} \quad (LRFD)$$

$$w = size\ of\ weld = \frac{f_r}{R_n/\Omega} \quad (ASD)$$

$$\phi = 0.75\,(\text{LRFD}) \qquad \Omega = 2.00\,(\text{ASD})$$

where

$$R_n = (0.6\, F_{EXX})(0.707\, w)(L)$$

for 1" weld per 1" length

$$R_n = (0.6\ F_{EXX})(0.707 \times 1\ )(1)$$

DESIGN OF BEAMS

I. For Flexure, AISC Chapter **3**, Page 3-3 and Chapter **F**, Page 44

II. For shear, AISC Chapter **G**, Page 64

III. Serviceability, AISC Chapter **3**, Page 3-7 and Figure 3-2, Page 3-8

All the above requirements should be satisfied

I. FLEXURAL STRENGTH

- **Classification of Cross-Sections λ,** AISC Chapter **3**, Page 3-5

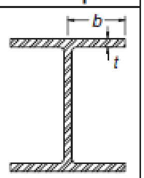
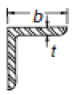
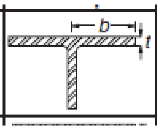
Classification of Cross-Sections

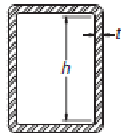
Cross-sections are classified as follows:

- Flexural members are compact (the plastic moment can be reached without local buckling) when λ is equal to or less than λ_p and the flange(s) are continuously connected to the web(s).
- Flexural members are non-compact (local buckling will occur, but only after initial yielding) when λ exceeds λ_p but is equal to or less than λ_r.
- Flexural members are slender-element cross-sections (local buckling will occur prior to yielding) when λ exceeds λ_r.

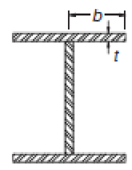
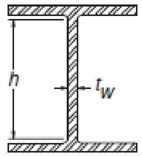
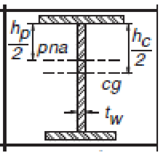
The values of λ_p and λ_r are determined per AISC Specification Section B4.

1. For Roller Sections, AISC Chapter **B**, Table B4.1, Page 16

	Element	Ratio	(compact)	(noncompact)	Example
1	Flexure in flanges of rolled I-shaped sections and channels	b/t	$0.38\sqrt{E/F_y}$	$1.0\sqrt{E/F_y}$	
6	Flexure in legs of single angles	b/t	$0.54\sqrt{E/F_y}$	$0.91\sqrt{E/F_y}$	
7	Flexure in flanges of tees	b/t	$0.38\sqrt{E/F_y}$	$1.0\sqrt{E/F_y}$	

13	Flexure in webs of rectangular HSS	h/t	$2.42\sqrt{E/F_y}$	$5.70\sqrt{E/F_y}$	

2. **For Built-Up Sections,** AISC Chapter **B**, Table B4.1, Page 16

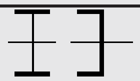
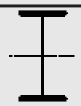
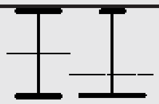
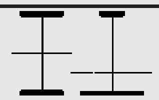
	Case	Description of Element	Width Thickness Ratio	Limiting Width-Thickness Ratios λ_p (compact)	λ_r (noncompact)	Example
	2	Flexure in flanges of doubly and singly symmetric I-shaped built-up sections	b/t	$0.38\sqrt{E/F_y}$	$0.95\sqrt{k_c E/F_L}$ [a],[b]	
	9	Flexure in webs of doubly symmetric I-shaped sections and channels	h/t_w	$3.76\sqrt{E/F_y}$	$5.70\sqrt{E/F_y}$	
Stiffened E	11	Flexure in webs of singly-symmetric I-shaped sections	h_c/t_w	$\dfrac{\frac{h_c}{h_p}\sqrt{\frac{E}{F_y}}}{\left(0.54\frac{M_p}{M_y}-0.09\right)^2} \le \lambda_r$	$5.70\sqrt{E/F_y}$	

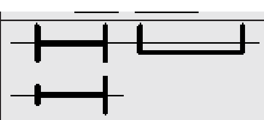
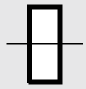
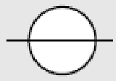
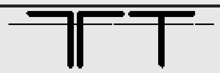
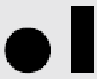
[a] $k_c = \frac{4}{\sqrt{h/t_w}}$, but shall not be taken less than 0.35 nor greater than 0.76 for calculation purposes. (See Cases 2 and 4)
[b] $F_L = 0.7F_y$ for minor-axis bending, major axis bending of slender-web built-up I-shaped members, and major axis bending of compact and noncompact web built-up I-shaped members with $S_{xt}/S_{xc} \ge 0.7$; $F_L = F_y S_{xt}/S_{xc} \ge 0.5F_y$ for major-axis bending of compact and noncompact web built-up I-shaped members with $S_{xt}/S_{xc} < 0.7$. (See Case 2)

- **Nominal Flexural Strength M_n,** AISC Chapter **F**, Page 44

Sect. F1.] GENERAL PROVISIONS 45

TABLE User Note F1.1
Selection Table for the Application of Chapter F Sections

Section in Chapter F	Cross Section	Flange Slenderness	Web Slenderness	Limit States
F2		C	C	Y, LTB
F3		NC, S	C	LTB, FLB
F4		C, NC, S	C, NC	Y, LTB, FLB, TFY
F5		C, NC, S	S	Y, LTB, FLB, TFY

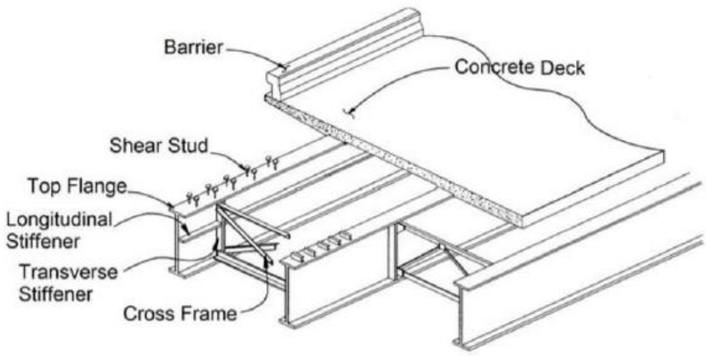
F6		C, NC, S	N/A	Y, FLB
F7		C, NC, S	C, NC	Y, FLB, WLB
F8		N/A	N/A	Y, LB
F9		C, NC, S	N/A	Y, LTB, FLB
F10		N/A	N/A	Y, LTB, LLB
F11		N/A	N/A	Y, LTB
F12	Unsymmetrical shapes	N/A	N/A	All limit states

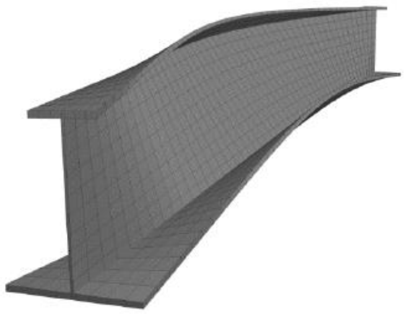
Y = yielding, LTB = lateral-torsional buckling, FLB = flange local buckling, WLB = web local buckling, TFY = tension flange yielding, LLB = leg local buckling, LB = local buckling, C = compact, NC = noncompact, S = slender

The nominal flexural strength, M_n, of a plate girder bent about its major axis is based on one of the limit states as defined in **Chapter F of the AISC Specification, Section F2 to F5**.

These limit states include:

1. Yielding (**Y**),
2. Lateral-Torsional Buckling (**LTB**),
3. Compression Flange Local Buckling (**FLB**),
4. Compression Flange Yielding (**CFY**),
5. Tension Flange Yielding (**TFY**).





- **Classification of Spans for Flexure L_b**, AISC Chapter **3**, Page 3-5

Classification of Spans for Flexure

Flexural members bent about their strong axis are classified on the basis of the length L_b between braced points. Braced points are points at which support resistance against lateral-torsional buckling is provided per AISC Specification Appendix 6.3. Classifications are determined as follows:

- If $L_b \le L_p$, flexural member is not subject to lateral-torsional buckling
- If $L_p < L_b \le L_r$, flexural member is subject to inelastic lateral-torsional buckling
- If $L_b > L_r$, flexural member is subject to elastic lateral-torsional buckling

The values of L_p and L_r are determined per AISC Specification Chapter F. These values are presented in Tables 3–2, 3–6, 3–7, 3–8, and 3–9.

Lateral-torsional buckling does not apply to flexural members bent about their weak axis or HSS bent about either axis, per AISC Specification Sections F6, F7 and F8.

For Section F2: L_p and L_r can be determined by using AISC Chapter **F**, Page 48

The limiting lengths L_p and L_r are determined as follows:

$$L_p = 1.76 r_y \sqrt{\frac{E}{F_y}} \qquad \text{(F2-5)}$$

$$L_r = 1.95 r_{ts} \frac{E}{0.7F_y} \sqrt{\frac{Jc}{S_x h_o}} \sqrt{1 + \sqrt{1 + 6.76 \left(\frac{0.7F_y}{E} \frac{S_x h_o}{Jc}\right)^2}} \qquad \text{(F2-6)}$$

User Note: If the square root term in Equation F2-4 is conservatively taken equal to 1, Equation F2-6 becomes

$$L_r = \pi r_{ts} \sqrt{\frac{E}{0.7F_y}}$$

Note that this approximation can be extremely conservative.

For Section F4: L_p and L_r can be determined by using AISC Chapter **F**, Page 50

The limiting laterally unbraced length for the limit state of *yielding*, L_p, is

$$L_p = 1.1 r_t \sqrt{\frac{E}{F_y}} \qquad \text{(F4-7)}$$

The limiting unbraced length for the limit state of inelastic lateral-torsional buckling, L_r, is

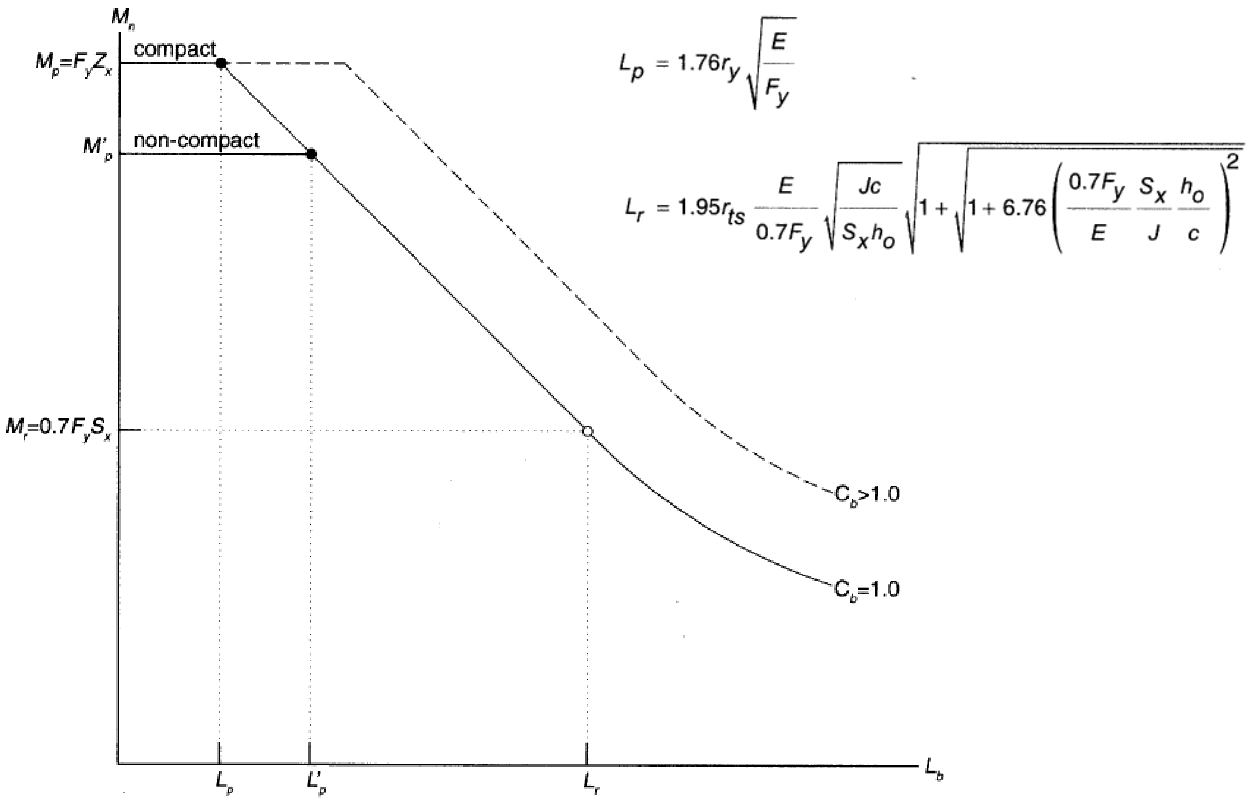
$$L_r = 1.95 r_t \frac{E}{F_L}\sqrt{\frac{J}{S_{xc}h_o}}\sqrt{1+\sqrt{1+6.76\left(\frac{F_L}{E}\frac{S_{xc}h_o}{J}\right)^2}} \quad \text{(F4-8)}$$

For Section F5: L_p and L_r can be determined by using AISC Chapter **F**, Page 53

L_p is defined by Equation F4-7

$$L_r = \pi r_t\sqrt{\frac{E}{0.7F_y}} \quad \text{(F5-5)}$$

For General available flexural strength for beams: L_p and L_r can be determined by using AISC Chapter **3**, Page 3-4



1. **M_n for Roller W Sections $L_b < L_p$**, AISC Chapter **3**, Table 3-2, Page 3-11

Table 3–2
W Shapes
Selection by Z_x

F_y = 50 ksi

Z_x

Shape	Z_x	M_{px}/Ω_b	$\phi_b M_{px}$	M_{rx}/Ω_b	$\phi_b M_{rx}$	BF	BF	L_p	L_r	I_x	V_{nx}/Ω_v	$\phi_v V_{nx}$
	in.³	kip-ft ASD	kip-ft LRFD	kip-ft ASD	kip-ft LRFD	kips ASD	kips LRFD	ft	ft	in.⁴	kips ASD	kips LRFD
W36×800ʰ	3650	9110	13700	5310	7980	47.5	71.4	14.9	94.8	64700	2030	3040
W36×652ʰ	2910	7260	10900	4300	6460	46.8	70.4	14.5	77.8	50600	1620	2430
W40×593ʰ	2760	6890	10400	4090	6140	55.5	83.5	13.4	63.8	50400	1540	2310
W36×529ʰ	2330	5810	8740	3480	5220	46.5	70.0	14.1	64.4	39600	1280	1920
W40×503ʰ	2310	5760	8660	3460	5200	54.7	82.2	13.1	55.3	41600	1290	1940
W36×487ʰ	2130	5310	7990	3200	4800	46.1	69.3	14.0	60.0	36000	1180	1770

2. **M_n for Roller W Sections $L_p < L_b < L_r$**, AISC Chapter **3**, Page 3-8, Table 3-2, Page 3-11

LRFD	ASD
$\phi_b M_n = C_b\,[\phi_b M_{px} - BF(L_b - L_p)] \le \phi_b M_{px}$	$\frac{M_n}{\Omega_b} = C_b \left[\frac{M_{px}}{\Omega_b} - BF(L_b - L_p)\right] \le \frac{M_{px}}{\Omega_b}$

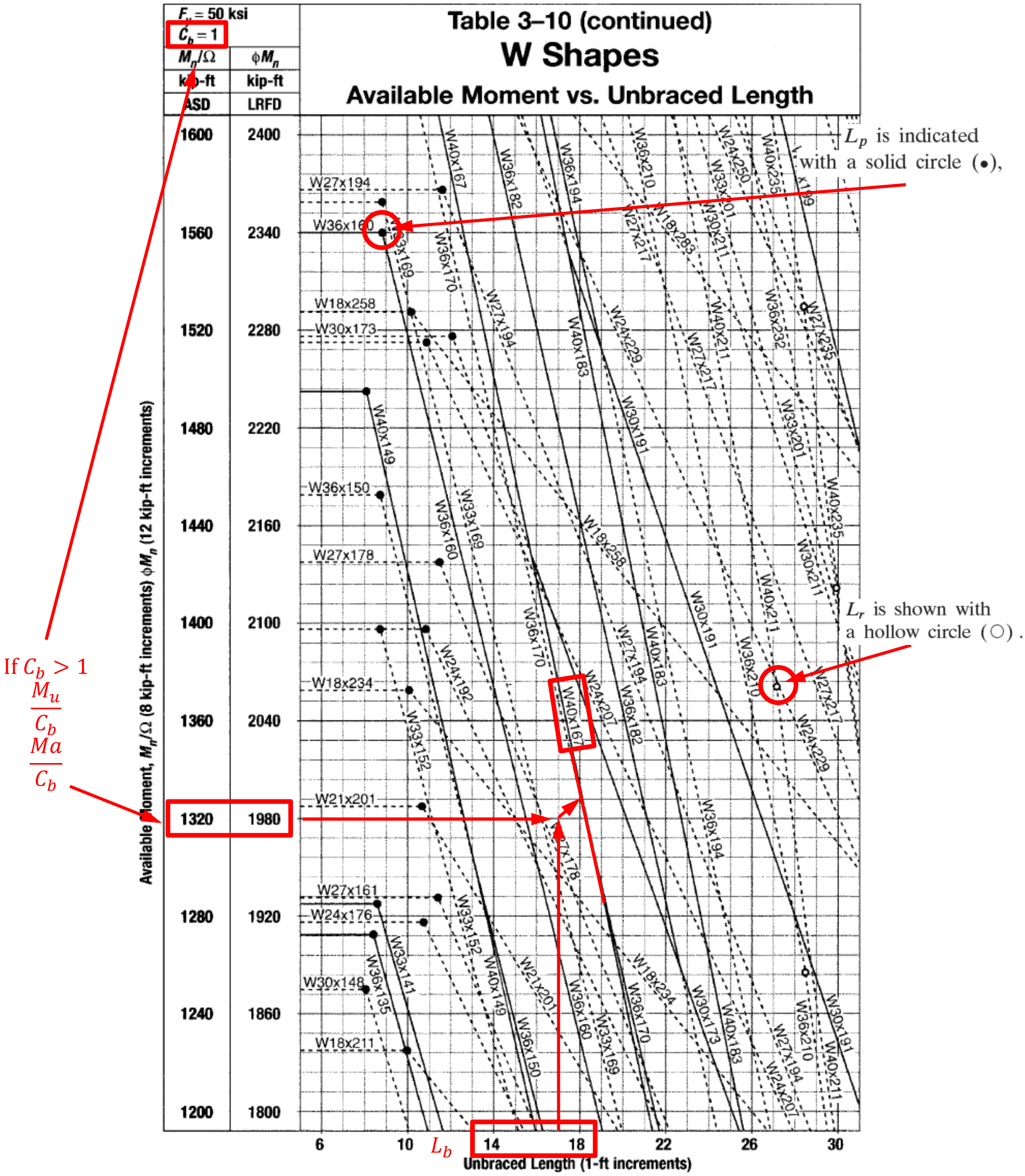
Table 3–2
W Shapes
Selection by Z_x

F_y = 50 ksi

Z_x

Shape	Z_x	M_{px}/Ω_b	$\phi_b M_{px}$	M_{rx}/Ω_b	$\phi_b M_{rx}$	BF	BF	L_p	L_r	I_x	V_{nx}/Ω_v	$\phi_v V_{nx}$
	in.³	kip-ft ASD	kip-ft LRFD	kip-ft ASD	kip-ft LRFD	kips ASD	kips LRFD	ft	ft	in.⁴	kips ASD	kips LRFD
W36×800ʰ	3650	9110	13700	5310	7980	47.5	71.4	14.9	94.8	64700	2030	3040
W36×652ʰ	2910	7260	10900	4300	6460	46.8	70.4	14.5	77.8	50600	1620	2430
W40×593ʰ	2760	6890	10400	4090	6140	55.5	83.5	13.4	63.8	50400	1540	2310

3. M_n for Roller W Sections $L_b > L_r$, AISC Chapter 3, Table 3-10, Page 3-96



- **Consideration of Moment Gradient C_b,** AISC Chapter **3**, Page 3-5 and Page 3-6

Consideration of Moment Gradient

When $L_b > L_p$, the moment gradient between braced points can be considered in the determination of the available strength using the beam bending coefficient C_b. In the case of a

uniform moment between braced points causing single-curvature of the member, $C_b = 1$. This represents the worst case and C_b can be conservatively taken as unity for use with the maximum moment between braced points in all designs per AISC Specification Section F1. However, when desired, a non-uniform moment gradient between braced points can be considered using C_b calculated as given in AISC Specification Equation F1–1. Exceptions are provided as follows:

1. As an alternative, when the moment diagram between braced points is a straight line, C_b can be calculated as given in AISC Commentary Equation C–F1.1.
2. For cantilevered members where the free end is unbraced, C_b must be taken as unity per AISC Specification Section F1.
3. For tees with the stem in compression, C_b should be taken as unity as recommended in AISC Commentary Section F9.

C_b can be determined from AISC Chapter **F**, Section F1, Page 46

C_b = *lateral-torsional buckling* modification factor for nonuniform moment diagrams when both ends of the unsupported segment are braced

$$C_b = \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C} R_m \le 3.0 \quad \text{(F1-1)}$$

where

M_{max} = absolute value of maximum moment in the unbraced segment, kip-in. (N-mm)

M_A = absolute value of moment at quarter point of the unbraced segment, kip-in. (N-mm)

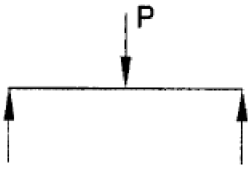
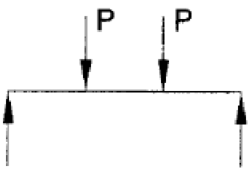
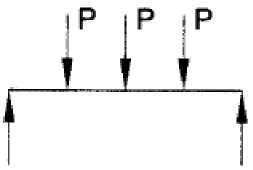
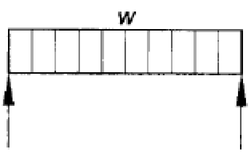
M_B = absolute value of moment at centerline of the unbraced segment, kip-in. (N-mm)

M_C = absolute value of moment at three-quarter point of the unbraced segment, kip-in. (N-mm)

R_m = cross-section monosymmetry parameter
= 1.0, doubly symmetric members
= 1.0, singly symmetric members subjected to *single curvature* bending

C_b for Simply Supported Beams can be found from AISC Chapter **3**, Table 3-1, Page 3-10

Table 3–1
Values for C_b for Simply Supported Beams

Load	Lateral Bracing Along Span	C_b
	None Load at midpoint	1.32
	At load point	1.67, 1.67
	None Loads at third points	1.14
	At load points Loads symmetrically placed	1.67, 1.00, 1.67
	None Loads at quarter points	1.14
	At load points Loads at quarter points	1.67, 1.11, 1.11, 1.67
	None	1.14
	At midpoint	1.30, 1.30
	At third points	1.45, 1.01, 1.45
	At quarter points	1.52, 1.06, 1.06, 1.52
	At fifth points	1.56, 1.12, 1.00, 1.12, 1.56

Note: Lateral bracing must always be provided at points of support per AISC Specification Chapter F.

II. SHEAR STRENGTH

- **Nominal Shear Strength,** AISC Chapter **G**, Page 64

The *design shear strength*, $\phi_v V_n$, and the *allowable shear strength*, V_n/Ω_v, shall be determined as follows.

For all provisions in this chapter except Section G2.1a:

$$\phi_v = 0.90 \text{ (LRFD)} \qquad \Omega_v = 1.67 \text{ (ASD)}$$

The *nominal shear strength*, V_n, of unstiffened or stiffened webs, according to the *limit states* of *shear yielding* and *shear buckling*, is

$$V_n = 0.6 F_y A_w C_v \tag{G2-1}$$

(a) For webs of rolled I-shaped members with $h/t_w \le 2.24\sqrt{E/F_y}$:

$$\phi_v = 1.00 \text{ (LRFD)} \qquad \Omega_v = 1.50 \text{ (ASD)}$$

and

$$C_v = 1.0 \tag{G2-2}$$

(b) For webs of all other doubly symmetric shapes and singly symmetric shapes and channels, except round *HSS*, the web shear coefficient, C_v, is determined as follows:

(i) For $h/t_w \le 1.10\sqrt{k_v E/F_y}$

$$C_v = 1.0 \tag{G2-3}$$

(ii) For $1.10\sqrt{k_v E/F_y} < h/t_w \le 1.37\sqrt{k_v E/F_y}$

$$C_v = \frac{1.10\sqrt{k_v E/F_y}}{h/t_w} \tag{G2-4}$$

(iii) For $h/t_w > 1.37\sqrt{k_v E/F_y}$

$$C_v = \frac{1.51 E k_v}{(h/t_w)^2 F_y} \tag{G2-5}$$

III. SERVICEABILITY

- AISC Chapter **3**, Page 3-7 and Figure 3-2, Page 3-8,

Serviceability

Serviceability requirements, per AISC Specification Chapter L, should be appropriate for the application. This includes an appropriate limit on the deflection of the flexural member and the vibration characteristics of the system of which the flexural member is a part. See also AISC Design Guide No. 3 *Serviceability Design Considerations for Low-Rise Buildings* (Fisher and West, 2004), AISC Design Guide No. 5 *Low- and Medium-Rise Steel Buildings* (Allison, 1991) and AISC Design Guide No. 11 *Floor Vibrations Due to Human Activity* (Murray, Allen and Ungar, 1997).

The maximum vertical deflection, Δ, in., can be calculated using the equations given in Tables 3–22 and 3–23. Alternatively, for common cases of simple-span beams and I-shaped members and channels, the following equation can be used:

$$\Delta = ML^2 / (C_1 I_x)$$

where

M = maximum service-load moment, kip-ft
L = span length, ft
I_x = moment of inertia, in.^{4}
C_1 = loading constant (see Figure 3–2) which includes the numerical constants appropriate for the given loading pattern, E, which has units of ksi, and a ft-to-in. conversion factor of 1,728 in.^{3}/ft^3.

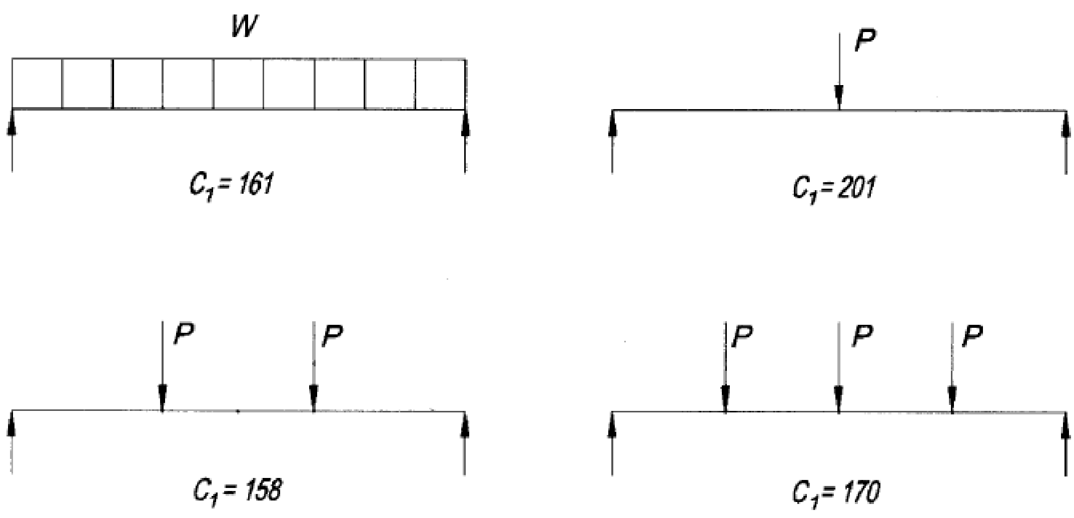
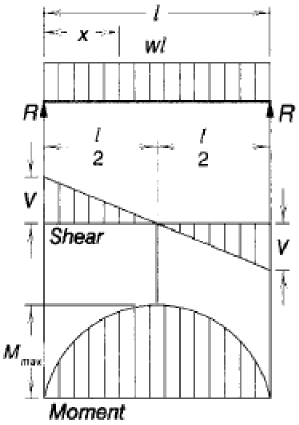


Figure 3–2. Loading constants for use in determining simple beam deflections.

- AISC Chapter **3**, Table 3-23, Page 3-211

Table 3–23
Shears, Moments, and Deflections

1. SIMPLE BEAM — UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load .... $= wl$

$R = V$ .... $= \frac{wl}{2}$

V_x .... $= w\left(\frac{l}{2} - x\right)$

M_{max} (at center) .... $= \frac{wl^2}{8}$

M_x .... $= \frac{wx}{2}(l - x)$

Δ_{max} (at center) .... $= \frac{5wl^4}{384EI}$

Δ_x .... $= \frac{wx}{24EI}\left(l^3 - 2lx^2 + x^3\right)$

2. SIMPLE BEAM — LOAD INCREASING UNIFORMLY TO ONE END

Total Equiv. Uniform Load .... $= \frac{16W}{9\sqrt{3}} = 1.03W$

l

- Deflection Limits

TABLE 10.1 Deflection Limits from IBC 2009

Members	Loading conditions		
	L	D + L	S or W
For floor members	$\frac{L}{360}$	$\frac{L}{240}$	—
For roof members supporting plaster ceiling*	$\frac{L}{360}$	$\frac{L}{240}$	$\frac{L}{360}$
For roof members supporting nonplaster ceilings*	$\frac{L}{240}$	$\frac{L}{180}$	$\frac{L}{240}$
For roof members not supporting ceilings*	$\frac{L}{180}$	$\frac{L}{120}$	$\frac{L}{180}$

*All roof members should be investigated for ponding.

BENDING AND AXIAL FORCE (BEAM-COLUMNS)

AISC Chapter H, Page 70 will be considered

- **Interaction Equations**

1. Doubly and Singly Symmetric Members in Flexure and Compression

(a) For $\frac{P_r}{P_c} \geq 0.2$

$$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0 \qquad \text{(H1-1a)}$$

(b) For $\frac{P_r}{P_c} < 0.2$

$$\frac{P_r}{2P_c} + \left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0 \qquad \text{(H1-1b)}$$

where

P_r = required axial compressive strength, kips (N)
P_c = *available axial compressive strength*, kips (N)
M_r = *required flexural strength*, kip-in. (N-mm)
M_c = *available flexural strength*, kip-in. (N-mm)
x = subscript relating symbol to *strong axis* bending
y = subscript relating symbol to *weak axis* bending

For design according to Section B3.3 (LRFD)

P_r = *required axial compressive strength* using *LRFD load combinations*, kips (N)
$P_c = \phi_c P_n$ = *design axial compressive strength*, determined in accordance with Chapter E, kips (N)
M_r = *required flexural strength* using LRFD load combinations, kip-in. (N-mm)
$M_c = \phi_b M_n$ = *design flexural strength* determined in accordance with Chapter F, kip-in. (N-mm)
ϕ_c = *resistance factor* for compression = 0.90
ϕ_b = resistance factor for flexure = 0.90

For design according to Section B3.4 (ASD)

P_r = required axial compressive strength using *ASD load combinations*, kips (N)

$P_c = P_n/\Omega_c$ = *allowable axial compressive strength*, determined in accordance with Chapter E, kips (N)

M_r = required flexural strength using ASD load combinations, kip-in. (N-mm)

$M_c = M_n/\Omega_b$ = *allowable flexural strength* determined in accordance with Chapter F, kip-in. (N-mm)

Ω_c = *safety factor* for compression = 1.67

Ω_b = safety factor for flexure = 1.67

- **FIRST-ORDER AND SECOND-ORDER MOMENTS FOR MEMBERS SUBJECT TO AXIAL COMPRESSION AND BENDING (M_r and P_r), Zero Sidesway**

AISC Chapter **C**, Section C2, Page 21

1b. Second-Order Analysis by Amplified First-Order Elastic Analysis

$$M_r = B_1 M_{nt} + \cancel{B_2 M_{lt}} \quad \text{(C2-1a)}$$

$$P_r = P_{nt} + \cancel{B_2 P_{lt}} \quad \text{(C2-1b)}$$

$\alpha = 1.00$ (LRFD) $\alpha = 1.60$ (ASD)

$$B_1 = \frac{C_m}{1 - \alpha P_r/P_{e1}} \geq 1 \quad \text{(C2-2)}$$

(i) For beam-columns not subject to transverse loading between supports in the plane of bending,

$$C_m = 0.6 - 0.4(M_1/M_2) \quad \text{(C2-4)}$$

where M_1 and M_2, calculated from a first-order analysis, are the smaller and larger moments, respectively, at the ends of that portion of the member unbraced in the plane of bending under consideration. M_1/M_2 is positive when the member is bent in reverse curvature, negative when bent in single curvature.

(ii) For beam-columns subjected to transverse loading between supports, the value of C_m shall be determined either by analysis or conservatively taken as 1.0 for all cases.

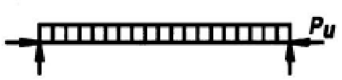
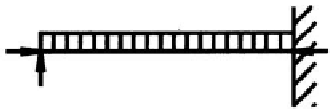
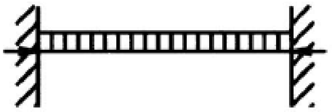
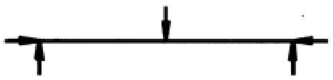
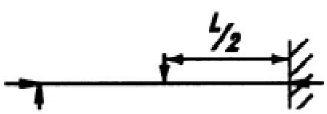
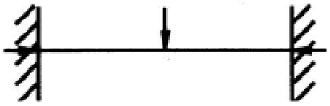
$$P_{e1} = \frac{\pi^2 EI}{(K_1 L)^2} \quad \text{(C2-5)}$$

C_m For Lateral Uniformly Distributed Load or Lateral Concentrated Force

AISC Chapter Comm **C2**, Table C-C2.1, Page 237

Comm. C2.] CALCULATION OF REQUIRED STRENGTHS 237

TABLE C-C2.1
Amplification Factors ψ and C_m

Case	Ψ	C_m
	0	1.0
	−0.4	$1-0.4\frac{P_u}{P_{e1}}$
	−0.4	$1-0.4\frac{P_u}{P_{e1}}$
	−0.2	$1-0.2\frac{P_u}{P_{e1}}$
	−0.3	$1-0.3\frac{P_u}{P_{e1}}$
	−0.2	$1-0.2\frac{P_u}{P_{e1}}$

- **FOR W SHAPES ONLY IN COMBINED AND AXIAL AND BENDING**

AISC Chapter **6**, Table 6-1, Page 6-1 and 6-5

When $P_r/P_c \geq 0.2$, the tabulated values of p, b_x, and b_y can be used as follows to solve the modified form of AISC Specification Equation H1-1a:

$$pP_r + b_xM_{rx} + b_yM_{ry} \leq 1.0$$

When $P_r/P_c < 0.2$, the tabulated values of p, b_x, and b_y can be used as follows to solve the modified form of AISC Specification Equation H1-1b:

$$1/2\,pP_r + 9/8(b_xM_{rx} + b_yM_{ry}) \leq 1.0$$

$$pP_r + b_xM_{rx} + b_yM_{ry} \leq 1.0 \quad \text{(Modified AISC Equation H1-1a)}$$

$$\frac{1}{2}pP_r + \frac{9}{8}(b_xM_{rx} + b_yM_{ry}) \leq 1.0 \quad \text{(Modified AISC Equation H1-1b)}$$

Table 6-1

Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
0	0.345	0.230	0.220	0.146	0.417	0.278	0.253	0.168	0.476	0.317	0.281	0.187
Other Constants and Properties												
$b_y \times 10^3$ (kip-ft)$^{-1}$	1.51		1.00		1.74		1.16		1.96		1.30	
$t_y \times 10^3$ (kips)$^{-1}$	0.339		0.226		0.390		0.260		0.434		0.289	
$t_r \times 10^3$ (kips)$^{-1}$	0.417		0.278		0.480		0.320		0.534		0.356	
r_x/r_y	5.10				5.10				5.10			

	LRFD	ASD
Axial Compression	$p = \frac{1}{\phi_c P_n}$, (kips)$^{-1}$	$p = \frac{\Omega_c}{P_n}$, (kips)$^{-1}$
Strong Axis Bending	$b_x = \frac{8}{9\phi_b M_{nx}}$, (kip-ft)$^{-1}$	$b_x = \frac{8\Omega_b}{9M_{nx}}$, (kip-ft)$^{-1}$
Weak Axis Bending	$b_y = \frac{8}{9\phi_b M_{ny}}$, (kip-ft)$^{-1}$	$b_y = \frac{8\Omega_b}{9M_{ny}}$, (kip-ft)$^{-1}$

F_y = 50 ksi

Table 6–1
Combined Axial and Bending
W Shapes

W44

Shape	W44×											
	335[c]				290[c]				262[c]			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length KL (ft) with respect to least radius of gyration r_y or Unbraced Length L_b (ft) for X-X axis bending												
0	0.345	0.230	0.220	0.146	0.417	0.278	0.253	0.168	0.476	0.317	0.281	0.187
11	0.378	0.251	0.220	0.146	0.454	0.302	0.253	0.168	0.518	0.344	0.281	0.187
12	0.384	0.256	0.220	0.146	0.462	0.307	0.253	0.168	0.526	0.350	0.281	0.187
13	0.393	0.261	0.222	0.148	0.470	0.313	0.255	0.170	0.536	0.356	0.284	0.189
14	0.402	0.267	0.225	0.150	0.480	0.319	0.259	0.173	0.546	0.363	0.289	0.192
15	0.412	0.274	0.229	0.152	0.490	0.326	0.264	0.175	0.557	0.371	0.294	0.196
16	0.423	0.282	0.233	0.155	0.501	0.333	0.268	0.178	0.570	0.379	0.299	0.199
17	0.435	0.290	0.236	0.157	0.514	0.342	0.273	0.181	0.584	0.389	0.304	0.203
18	0.449	0.299	0.240	0.160	0.527	0.351	0.277	0.184	0.599	0.399	0.310	0.206
19	0.463	0.308	0.244	0.162	0.542	0.361	0.282	0.188	0.616	0.410	0.316	0.210
20	0.479	0.319	0.248	0.165	0.559	0.372	0.287	0.191	0.634	0.422	0.322	0.214
22	0.515	0.343	0.257	0.171	0.597	0.397	0.298	0.198	0.676	0.450	0.335	0.223
24	0.558	0.371	0.266	0.177	0.644	0.428	0.309	0.206	0.727	0.484	0.348	0.232
26	0.608	0.405	0.275	0.183	0.702	0.467	0.321	0.214	0.788	0.524	0.363	0.242
28	0.668	0.444	0.286	0.190	0.770	0.513	0.334	0.223	0.862	0.574	0.380	0.253
30	0.738	0.491	0.297	0.198	0.852	0.567	0.349	0.232	0.954	0.635	0.397	0.264
32	0.822	0.547	0.310	0.206	0.948	0.631	0.365	0.243	1.06	0.708	0.417	0.278
34	0.923	0.614	0.323	0.215	1.06	0.708	0.382	0.254	1.20	0.796	0.439	0.292
36	1.04	0.689	0.338	0.225	1.19	0.794	0.401	0.267	1.34	0.892	0.466	0.310
38	1.15	0.767	0.354	0.235	1.33	0.885	0.429	0.285	1.49	0.994	0.508	0.338
40	1.28	0.850	0.377	0.251	1.47	0.981	0.463	0.308	1.66	1.10	0.550	0.366
42	1.41	0.937	0.405	0.269	1.62	1.08	0.498	0.331	1.83	1.21	0.593	0.394
44	1.55	1.03	0.432	0.287	1.78	1.19	0.533	0.355	2.00	1.33	0.636	0.423
46	1.69	1.12	0.459	0.306	1.95	1.30	0.569	0.378	2.19	1.46	0.679	0.452
48	1.84	1.22	0.487	0.324	2.12	1.41	0.604	0.402	2.38	1.59	0.723	0.481
50	2.00	1.33	0.514	0.342	2.30	1.53	0.640	0.426	2.59	1.72	0.767	0.510

Other Constants and Properties

	335[c] ASD	335[c] LRFD	290[c] ASD	290[c] LRFD	262[c] ASD	262[c] LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	1.51	1.00	1.74	1.16	1.96	1.30
$t_y \times 10^3$ (kips)$^{-1}$	0.339	0.226	0.390	0.260	0.434	0.289
$t_r \times 10^3$ (kips)$^{-1}$	0.417	0.278	0.480	0.320	0.534	0.356
r_x/r_y	5.10		5.10		5.10	

[c] Shape is slender for compression with F_y = 50 ksi.

Illustrative Example 1 — Bending and Axial Force, Chapter H and Chapter 6

A 12-ft W12 × 96 (50 ksi steel) is used as a beam-column in a braced frame. It is bent in single curvature with equal and opposite end moments and is not subjected to intermediate transverse loads. Is the section satisfactory if $P_D = 175$ k, $P_L = 300$ k, and first-order $M_{Dx} = 60$ ft-k and $M_{Lx} = 60$ ft-k?

Solution

Using a W12 × 96 ($A = 28.2$ in^2, $I_x = 833$ in^4, $\phi_b M_{px} = 551$ ft-k, $\frac{M_{px}}{\Omega_b} = 367$ ft-k, $L_p = 10.9$ ft, $L_r = 46.7$ ft, $BF = 5.78$ k for LRFD and 3.85 k for ASD).

LRFD	ASD
$P_{nt} = P_u = (1.2)(175) + (1.6)(300) = 690$ k	$P_{nt} = P_a = 175 + 300 = 475$ k
$M_{ntx} = M_{ux} = (1.2)(60) + (1.6)(60) = 168$ ft-k	$M_{ntx} = M_{ax} = 60 + 60 = 120$ ft-k
For a braced frame, let $K = 1.0$	For a braced frame, let $K = 1.0$
$\therefore (KL)_x = (KL)_y = (1.0)(12) = 12$ ft	$\therefore (KL)_x = (KL)_y = (1.0)(12) = 12$ ft
$P_c = \phi_c P_n = 1080$ k (AISC Table 4-1)	$P_c = \frac{P_n}{\Omega_c} = 720$ k (AISC Table 4-1)
$P_r = P_{nt} + \beta_2 P_{lt} = 690 + 0 = 690$ k	$P_r = P_{nt} + \beta_2 P_{lt} = 475 + 0 = 475$ k
$\frac{P_r}{P_c} = \frac{690}{1080} = 0.639 > 0.2$	$\frac{P_r}{P_c} = \frac{475}{720} = 0.660 > 0.2$
$\therefore$ Must use AISC Eq. H1-1a	$\therefore$ Must use AISC Eq. H1-1a
$C_{mx} = 0.6 - 0.4\frac{M_1}{M_2}$	$C_{mx} = 0.6 - 0.4\frac{M_1}{M_2}$
$C_{mx} = 0.6 - 0.4\left(-\frac{168}{168}\right) = 1.0$	$C_{mx} = 0.6 - 0.4\left(-\frac{120}{120}\right) = 1.0$

$P_{e1x} = \frac{\pi^2 EI_x}{(K_1L_x)^2} = \frac{(\pi^2)(29{,}000)(833)}{(1.0 \times 12 \times 12)^2}$ $= 11{,}498$ k	$P_{e1x} = \frac{\pi^2 EI_x}{(K_1L_x)^2} = \frac{(\pi^2)(29{,}000)(833)}{(1.0 \times 12 \times 12)^2}$ $= 11{,}498$ k

$B_{1x} = \frac{C_{mx}}{1 - \frac{\alpha P_r}{P_{e1x}}} = \frac{1.0}{1 - \frac{(1.0)(690)}{11{,}498}} = 1.064$	$B_{1x} = \frac{C_m}{1 - \frac{\alpha P_r}{P_{e1x}}} = \frac{1.0}{1 - \frac{(1.6)(475)}{11{,}498}} = 1.071$
$M_{rx} = B_{1x}M_{ntx} = (1.064)(168) = 178.8$ ft-k	$M_{rx} = (1.071)(120) = 128.5$ ft-k
Since $L_b = 12$ ft $> L_p = 10.9$ ft $< L_r = 46.6$ ft	Since $L_b = 12$ ft $> L_p = 10.9$ ft $< L_r = 46.6$ ft
∴ **Zone 2**	∴ **Zone 2**
$\phi_b M_{px} = 1.0[551 - (5.78)(12 - 10.9)] = 544.6$ ft-k	$\frac{M_{px}}{\Omega_b} = 1.0[367 - 3.85\,(12 - 10.9)] = 362.7$ ft-k
$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) = \frac{690}{1080} + \frac{8}{9}\left(\frac{178.8}{544.6} + 0\right) = 0.931 < 1.0$ OK	$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) = \frac{475}{720} + \frac{8}{9}\left(\frac{128.5}{362.7} + 0\right) = 0.975 < 1.0$ OK
∴ **Section is satisfactory.**	∴ **Section is satisfactory.**

using the AISC simplified method of Part 6 of the Manual

$$pP_r + b_xM_{rx} + b_yM_{ry} \leq 1.0$$

$$\frac{1}{2}pP_r + \frac{9}{8}(b_xM_{rx} + b_yM_{ry}) \leq 1.0$$

LRFD	ASD
From (LRFD)	From (ASD)
$P_r = 690$ k	$P_r = 475$ k
$M_{rx} = 178.8$ ft-k	$M_{rx} = 128.5$ ft-k
From AISC Table 6-1 for a W12 × 96 with $KL = 12$ ft and $L_b = 12$ ft	From AISC Table 6-1 for a W12 × 96 with $KL = 12$ ft and $L_b = 12$ ft
$p = 0.924 \times 10^{-3}$	$p = 1.39 \times 10^{-3}$
$b_x = 1.63 \times 10^{-3}$	$b_x = 2.45 \times 10^{-3}$
$b_y = 3.51 \times 10^{-3}$ (from bottom of table)	$b_y = 5.28 \times 10^{-3}$ (from bottom of table)
Then with the modified equation $(0.924 \times 10^{-3})(690) + (1.63 \times 10^{-3})(178.8) + (3.51 \times 10^{-3})(0) = 0.929 < 1.0$	Then with the modified equation $(1.39 \times 10^{-3})(475) + (2.45 \times 10^{-3})(128.5) + (5.28 \times 10^{-3})(0) = 0.975 < 1.0$
Section is satisfactory.	**Section is satisfactory.**