UNIVERSITY OF BAGHDAD
COLLEGE OF ENGINEERING
CIVIL ENGINEERING DEPARTMENT

DESIGN OF REINFORCED CONCRETE STRUCTURES

THIRD YEAR COURSE (JUNIOR COURSE)

PREPARED BY

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CIVIL ENGINEERING DEPARTMENT
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Academic Year 2018-2019

Syllabus

First Semester

Part I: Introduction to Reinforced Concrete Structures

1. Introduction (1st-14th of October)

1.1 Structural Elements and Structural Forms
1.2 Flooring and Roofing Systems.
1.3 Loads.
1.4 Design Codes and Specifications.
1.5 Design Criteria.
1.6 Design Philosophy.
1.7 Strength Versus Working-Stress Design Methods.
1.8 Fundamental Assumptions For Reinforced Concrete Behavior.
1.9 Syllabus.
1.10 SI Units
1.11 General Problems.
1.12 Additional Examples.

2. Materials (15th-31st of October)

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2.2 Concrete, Chemical Aspects.
2.3 Concrete, Physical Aspects.
2.4 Reinforcing Steels For Concrete.
2.5 General Problems.

3.*[1] Design of Concrete Structures and Fundamental Assumptions

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3.4* Fundamental Assumptions for Reinforced Concrete Behavior.
3.5* Behavior of Members Subject to Axial Loads.
3.6* Bending of Homogeneous Beams.

Part II: Design of Reinforced Concrete Beams

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4.4 Home Work of Article 4.3, Flexure Strength Analysis of Beams with Rectangular Sections.
4.5 Practical Flexure Design of a Rectangular Beam with Tension Reinforcement Only and Pre-specified Dimensions (b and h).
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4.7 Practical Flexure Design of a Rectangular Beam with Tension Reinforcement Only and Non-specified Dimensions.
4.8 Home Work of article 4.7, Practical Flexure Design of a Rectangular Beam with Tension Reinforcement Only and Non-specified Dimensions (b and h).
4.9 Analysis of a Rectangular Beam with Tension and Compression Reinforcements (a Doubly Reinforced Beam).
4.10 Home Work of Article 4.9, Analysis of a Rectangular Section with Tension and Compression Reinforcements Only (a Doubly Reinforced Section).
4.11 Design of a Doubly Reinforced Rectangular Section.
4.12 Home Work of Article 4.11, Design of a Doubly Reinforced Rectangular Section.
4.13 Flexure Analysis of a Section with T Shape.
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4.15 Design of a Beam with T-Shape
4.16 Home Work of Article 4.15, Design of a Section with T Shape.
4.17 Analysis of Beams with Irregular Sections.
4.18 Home Work 4.17, Analysis of Beams with Irregular Sections.

5. Shear and Diagonal Tension in Beams (1st-31st of January)

5.1 Basic Concepts.
5.2 Computing of Applied Factored Shear Force $\boldsymbol{V_u}$.
5.3 Shear Strength Provided by Concrete $\boldsymbol{V_c}$.
5.4 Shear Strength Provided by Shear Reinforcement $\boldsymbol{V_s}$.
5.5 Summary of Practical Procedure for Shear Design.
5.6 Basic Design Examples.
5.7 Problems for Solution on Basic Shear Aspects.

[1] Asterisk, *, indicates more specialized articles that may be terminated without destroying the continuity of the course.

Text Books

1. A. H. Nilson, D. Darwin, and C. W. Dolan, Design of Concrete Structures, 13th Edition, McGraw Hill, 2004.
2. D. Darwin, C. W. Dolan, and A. H. Nilson, Design of Concrete Structures, 15th Edition, McGraw Hill, 2015 (Metric Edition).
3. Building Code Requirements for Structural Concrete (ACI318M-14).

References

1. J. K. Wight and J. G. MacGregor, Reinforced Concrete: Mechanics and Design, 7th Edition, Person/Prentice Hall, 2016.
2. E. G. Nawy, Reinforced Concrete: A Fundamental Approach, 6th Edition, Prentice Hall, 2009.
3. C.K. Wang, C.G. Salmon and J. A. Pincheira, Reinforced Concrete Design, 7th Edition, John Wiley & Sons, 2007.
4. J.C. McCormac and R. H. Brown, Design of Reinforced Concrete, 9th Edition, John Wiley & Sons, 2014.
5. M. N. Hassoun, A. Al-Manaseer, Structural Concrete: Theory and Design, 6th Edition, Wiley, 2015.
6. G.F. Limbrunner and A.O. Aghayere, Reinforced Concrete Design, 7th Edition, Prentice Hall, 2010.
7. M. Setareh, and R. Darvas, Concrete Structure, Prentice Hall, 2007.
8. M. E. Kamara, B. G. Rabbat, Notes on ACI 318-05, 9th Edition, 2005.

CHAPTER 1
INTRODUCTION

1.1 STRUCTURAL DESIGN, STRUCTURAL ELEMENTS, AND STRUCTURAL FORMS

1.1.1 Structural Design

- The main objectives of the structural design are to prepare a structural system that transfers the applied loads from the points of application to the supporting soil safely and with an acceptable cost.
- The first step in the structural design is to select a structural system to be used in transferring the applied loads from the points of application to the supporting soil.

1.1.2 Structural Elements

To deal with an uncountable variety of the structures, they are usually broken into the following structural elements in their analysis and design.

1.1.2.1 Bar Element

- As indicated in ***Figure 1.1-1*** below, the bar element is the structural element that has two dimensions small when compared with the third one.

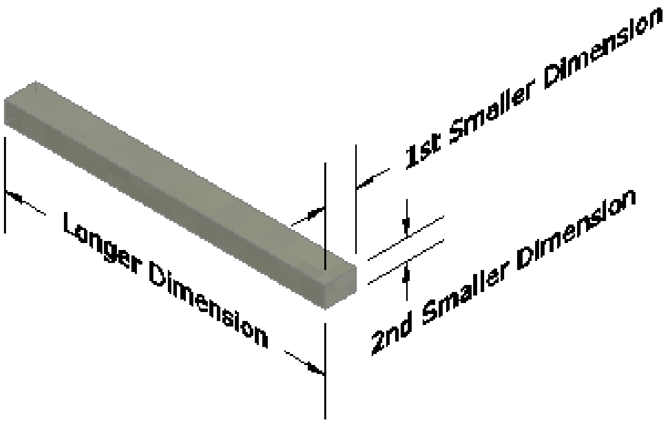


Figure 1.1-1: Bar element.

- A Bar element can be defined as a *beam* when the load is applied transversely to the element axis.

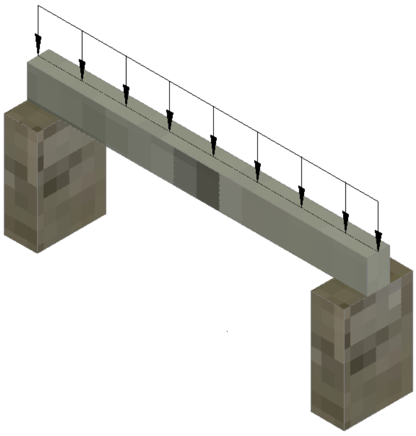


Figure 1.1-2: Beam element.

- A Column can be defined as the bar element that subjected to an axial load with or without bending moment.

Figure 1.1-3: Column element.

1.1.2.2 Plate Element

- The plate element is the structural element that has a one small dimension comparing with other its dimensions.

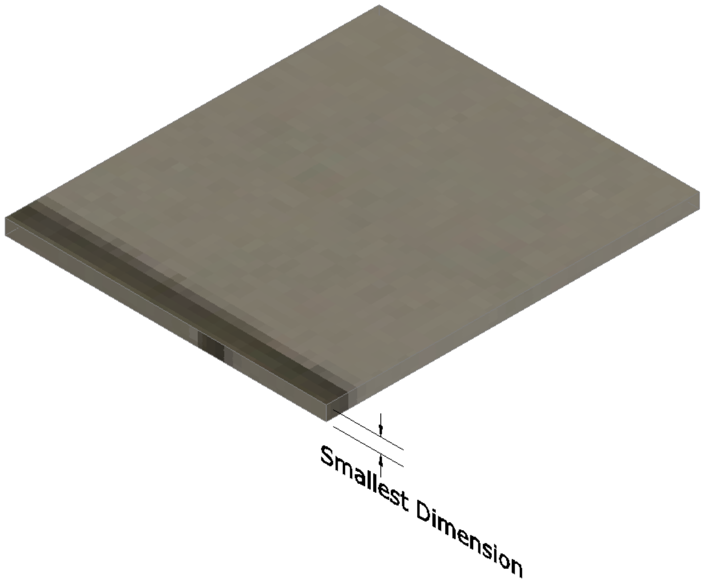


Figure 1.1-4: Plate element.

- The bearing wall is a plate element that subjected to an axial load.

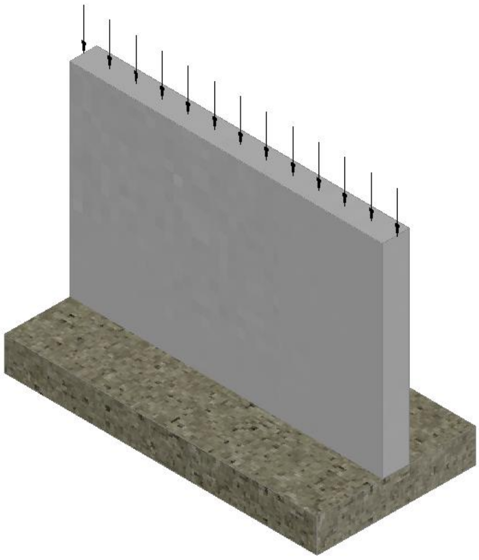


Figure 1.1-5: Bearing wall.

- The slab is a plate element that subjected to transverse loads.

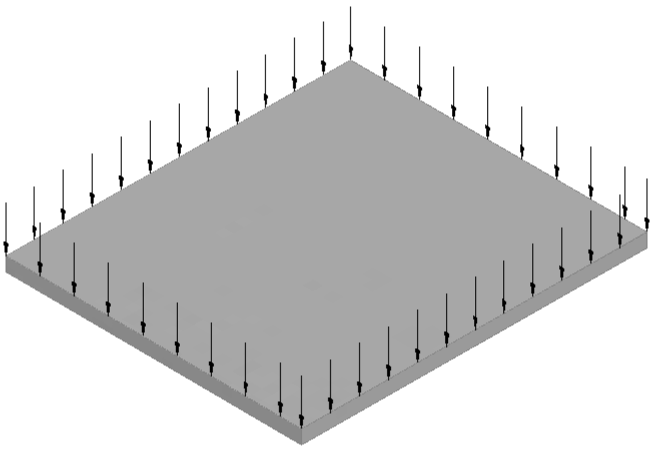


Figure 1.1-6: Structural slab.

1.1.2.3 Shell Element

- The shell element is a curved structural element; one of its dimensions is small when compared with the other two dimensions.
- It may take a form of the dome, or a form cylindrical shell.
- Shell element is out the scope of our course.

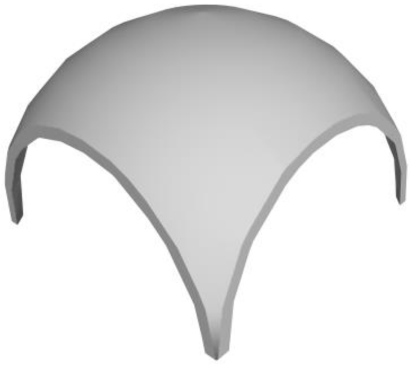


Figure 1.1-7: Dome Shell.

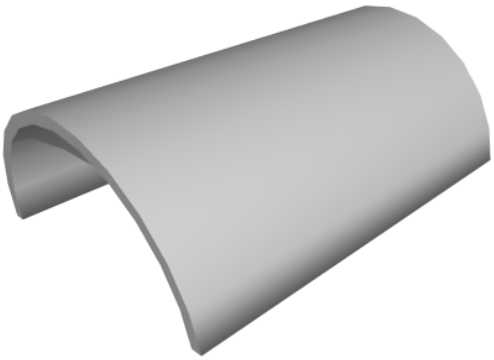


Figure 1.1-8: Cylindrical Shell.

1.2 FLOORING AND ROOFING SYSTEM

Reinforced−concrete floors can be classified into the following systems.

1.2.1 One-way Floor System

- In this system, the applied load acting on the slab is transferred in one direction to the supporting beams, then to the supporting columns.

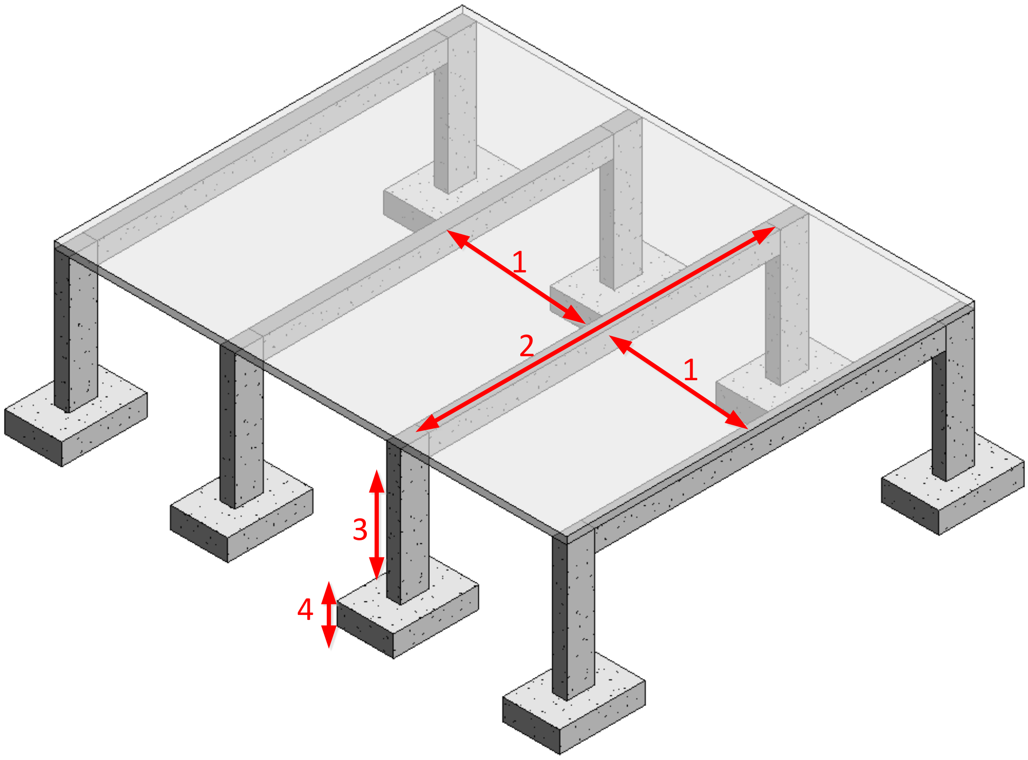


Figure 1.2-1: One-way floor system.

- For a large column spacing, the load may be transferred from the slab to the floor beams, then to larger beams (usually called the girders), and in turn to the supporting columns.

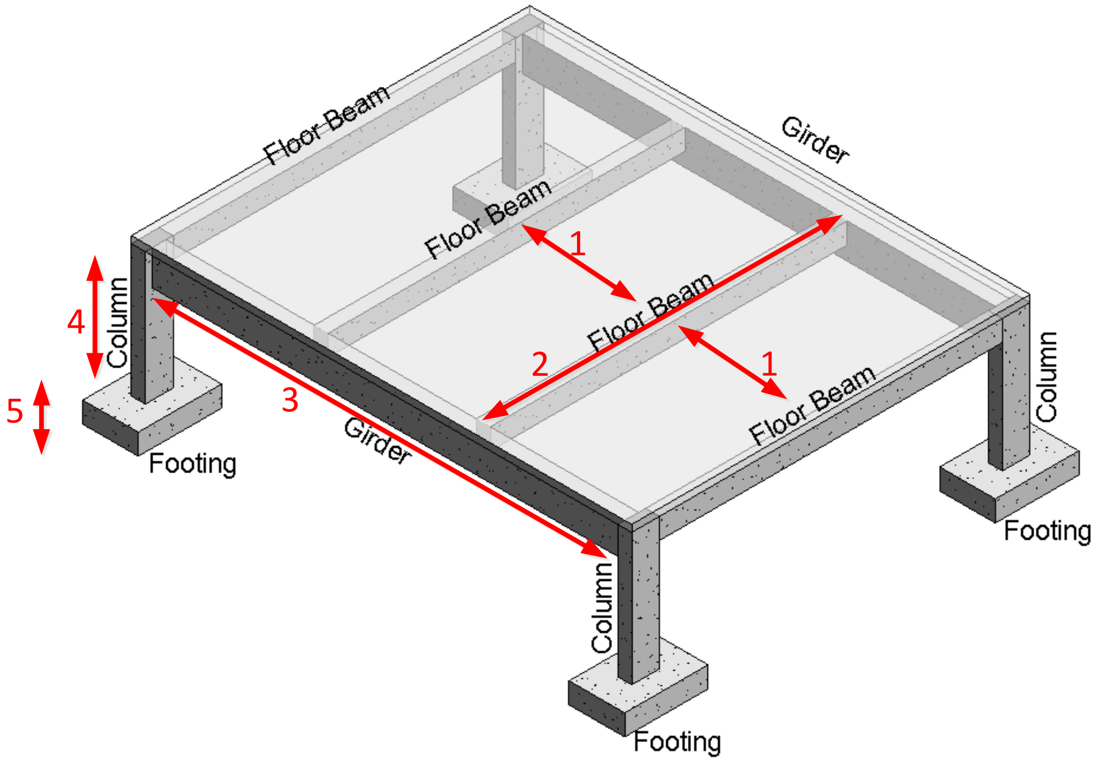


Figure 1.2-2: Slab-beam-girder one-way system.

1.2.2 Two-way Floor System with Beams

In this system, the applied load acting on the slab is transferred in two directions to supporting beams on the slab periphery, and in turn to the supporting columns.

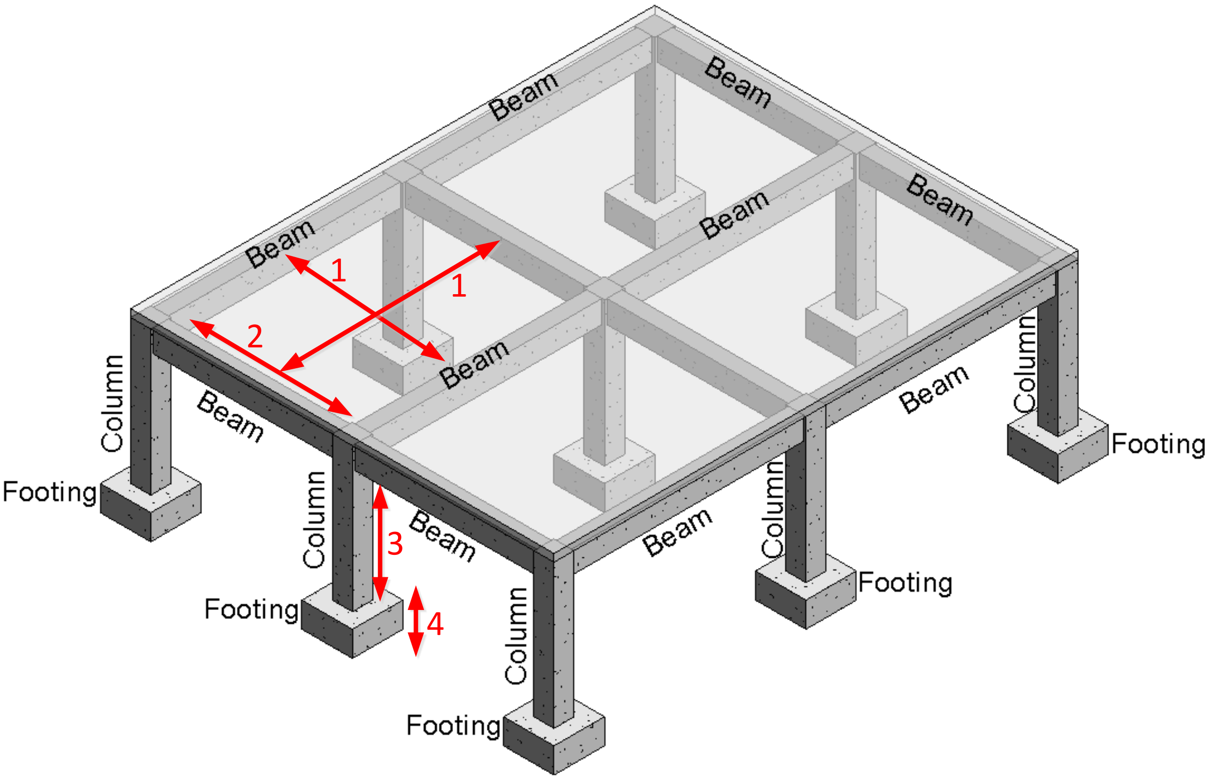


Figure 1.2-3: Two-way floor system with beams.

1.2.3 Two-way Floor System without Beams

- In this system, usually called ***flat plate system***, the slab is supported directly on the columns. Load transferred directly from the slab to the supporting columns.

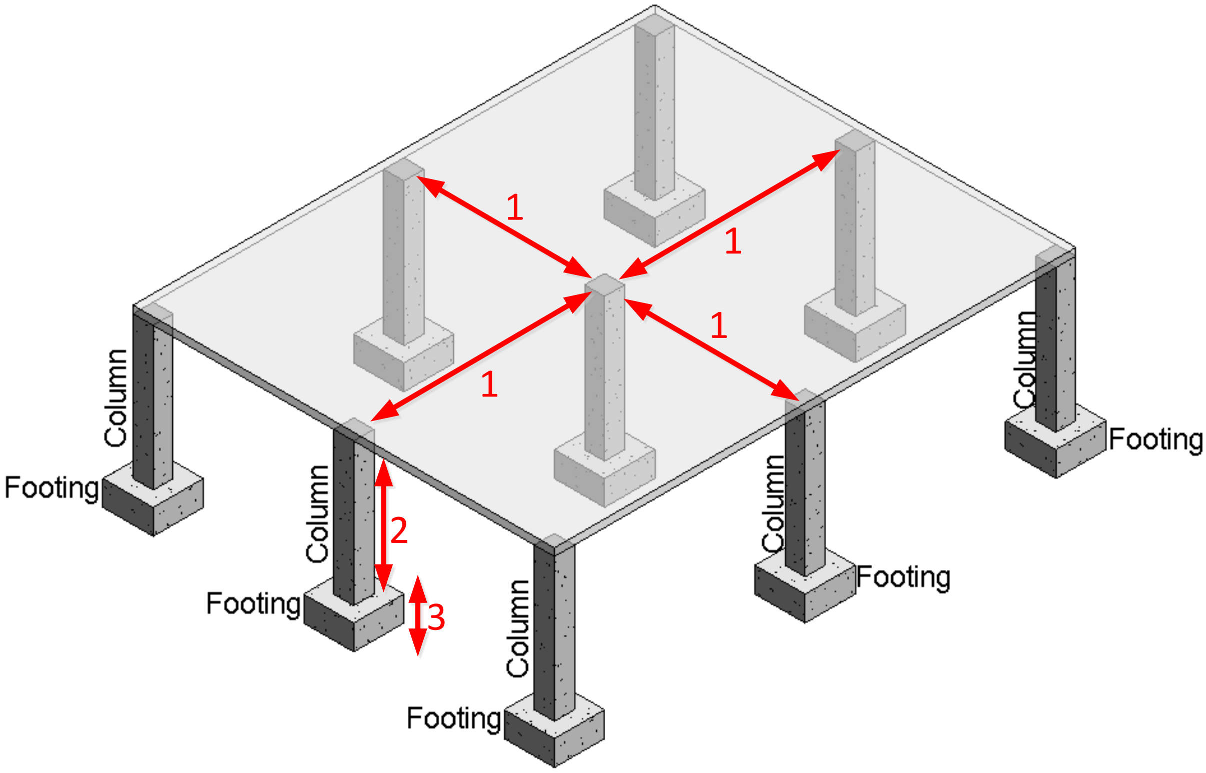


Figure 1.2-4: Flat plate floor system.

- To avoid slab punching due to column concentered forces, aforementioned system may be strengthened with drop panels and/or column capital.
- The resulting system is ***flat slab system***.
- Flat plate and flat slab systems are out the scope of our junior course.

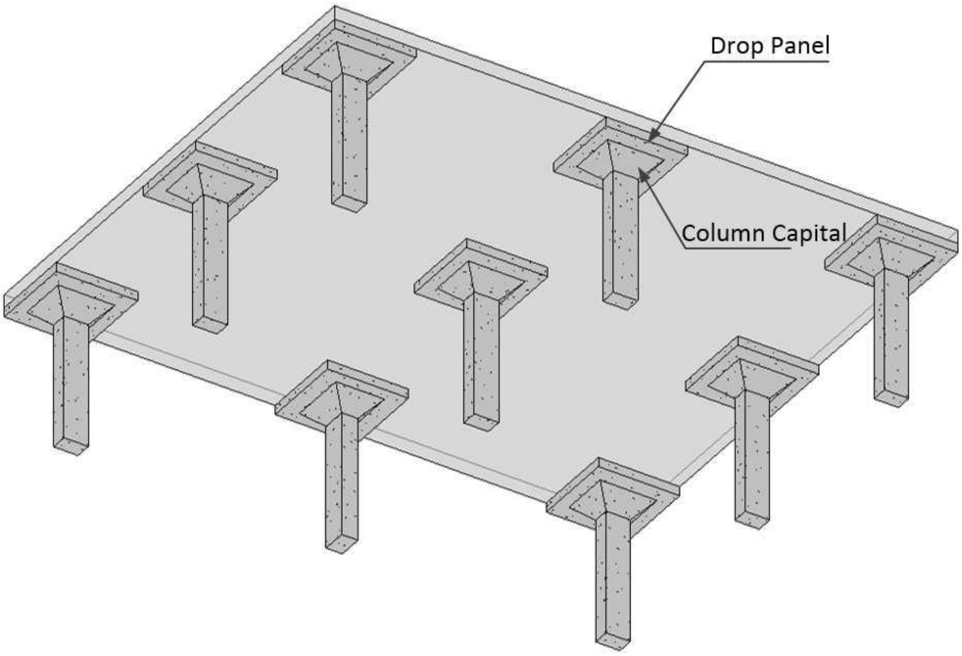


Figure 1.2-5: Flat slab system.

1.3 LOADS

Loads that act on the structures can be classified into three categories: dead loads, live loads, and environmental loads.

1.3.1 Dead Load

1. The major part of it is the weight of the structure itself.
2. It is constant in magnitude and fixed in location throughout the life of the structure.
3. It can be calculated with good accuracy from the dimensions of the structures and density of the materials.
4. Dead loads may be further classified into:
   - Selfweight, which represents own weight of the structural system.
   - Superimposed loads, which represents own weight of surfacing, mechanical, plumbing, and electrical fixtures.

1.3.2 Live Load

1.3.2.1 Floor Live Loads

- It consists of occupancy loads in buildings. According to ***section 5.3.4*** of the code, the live load, L, shall include, see Figure 1.3-1 through Figure 1.3-5.
   - Concentrated live loads,
   - Vehicular loads
   - Crane loads,
   - Loads on hand rails, guardrails, and vehicular barrier systems,
   - Impact effects,
   - Vibration effects.

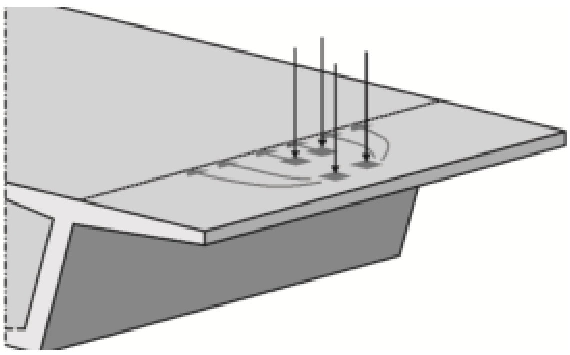


Figure 1.3-1: Concentrated live loads.

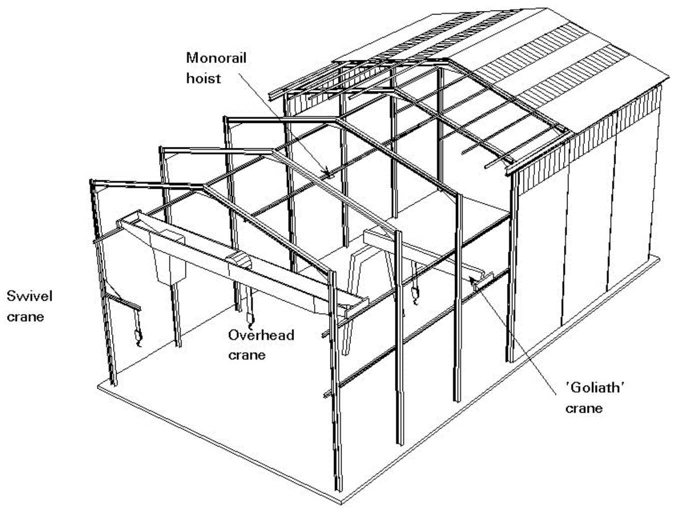


Figure 1.3-2: Crane loads on a building frame.

Figure 1.3-3: Handrail and vehicular guardrail or barrier.

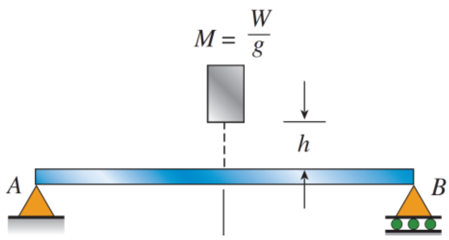


Figure 1.3-4: Impact effects.

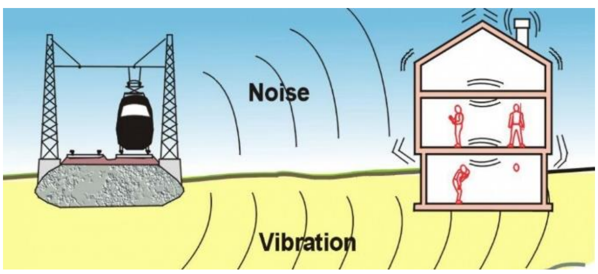


Figure 1.3-5: Vibration effects.

- It may be either fully or partially in place or may not percent at all.
- It may be changed in location.
- Its magnitude and distributions at any given time are uncertain and even their maximum intensities throughout the lifetime of the structures are not known with precision.
- The minimum live loads for which the floors and roof of a building to be designed are usually specified by the building code that governs at the site of constructions.
- Representative values of minimum live loads to be used in many locations including Iraq are presented in Table 1.3-1 below. These values are adopted from (ASCE/SEI 7–10)***, Minimum Design Loads for Buildings and Other Structures.***
- As can be seen from the table, in addition to the uniformly distributed loads, it is recommended that, as an alternative to the uniform loads, floors be designed to support certain concentrated loads if these produce a greater stress.

1.3.2.2 Reduction in Floor Live Load

- As it is improbable that a large floor area be fully loaded with live load at a same time, most of building codes offer relations to relate the value of live load supported by a structural member to the area which supported by this member.
- According to article 4.7.2 of (ASCE/SEI 7–10), reduced live load can be estimated based on following relation:

  $$L = L_o(0.25 + \frac{4.57}{\sqrt{K_{LL}A_T}}) \qquad \textbf{Eq. 1.3-1}$$

  where

  Lo is unreduced design live load per m^2 of area supported by the member (see Table 1.3-1 below),

  L is reduced design live load per m^2 of area supported by the member,

  K_{LL} is live load element factor (see Table 1.3-2 below).

  A_T is tributary area in m^2.

  To be a large area where reduction in live load is permitted, the influence area, $K_{LL}A_T$, should be:

  $K_{LL}A_T \geq 37.16\, m^2$
- L shall not be less than $0.50L_o$ for members supporting one floor and L shall not be less than $0.4L_o$ for members supporting two or more floors.
- Live loads that exceed $4.79\, kN/m^2$ shall not be reduced.

Table 1.3-1: Minimum Uniformly Distributed Live Loads, and Minimum Concentrated Live Loads.

Occupancy or Use	Uniform psf (kN/m^2)	Conc. lb (kN)
Apartments (see Residential)		
Access floor systems		
Office use	50 (2.4)	2,000 (8.9)
Computer use	100 (4.79)	2,000 (8.9)
Armories and drill rooms	150 (7.18)[a]	
Assembly areas and theaters		
Fixed seats (fastened to floor)	60 (2.87)[a]	
Lobbies	100 (4.79)[a]	
Movable seats	100 (4.79)[a]	
Platforms (assembly)	100 (4.79)[a]	
Stage floors	150 (7.18)[a]	
Balconies and decks	1.5 times the live load for the occupancy served. Not required to exceed 100 psf (4.79 kN/m^2)	
Catwalks for maintenance access	40 (1.92)	300 (1.33)
Corridors		
First floor	100 (4.79)	
Other floors, same as occupancy served except as indicated		
Dining rooms and restaurants	100 (4.79)[a]	
Dwellings (see Residential)		
Elevator machine room grating (on area of 2 in. by 2 in. (50 mm by 50 mm))		300 (1.33)
Finish light floor plate construction (on area of 1 in. by 1 in. (25 mm by 25 mm))		200 (0.89)
Fire escapes	100 (4.79)	
On single-family dwellings only	40 (1.92)	
Fixed ladders	See Section 4.5	
Garages		
Passenger vehicles only	40 (1.92)[a,b,c]	
Trucks and buses	[c]	
Handrails, guardrails, and grab bars	See Section 4.5	
Helipads	60 (2.87)[d,e] Nonreducible	[e,f,g]
Hospitals		
Operating rooms, laboratories	60 (2.87)	1,000 (4.45)
Patient rooms	40 (1.92)	1,000 (4.45)
Corridors above first floor	80 (3.83)	1,000 (4.45)
Hotels (see Residential)		
Libraries		
Reading rooms	60 (2.87)	1,000 (4.45)
Stack rooms	150 (7.18)[a,h]	1,000 (4.45)
Corridors above first floor	80 (3.83)	1,000 (4.45)
Manufacturing		
Light	125 (6.00)[a]	2,000 (8.90)
Heavy	250 (11.97)[a]	3,000 (13.40)

Table 1.3-1: Minimum Uniformly Distributed Live Loads, and Minimum Concentrated Live Loads, Continued.

Occupancy or Use	Uniform psf (kN/m^2)	Conc. lb (kN)
Office buildings		
File and computer rooms shall be designed for heavier loads based on anticipated occupancy		
Lobbies and first-floor corridors	100 (4.79)	2,000 (8.90)
Offices	50 (2.40)	2,000 (8.90)
Corridors above first floor	80 (3.83)	2,000 (8.90)
Penal institutions		
Cell blocks	40 (1.92)	
Corridors	100 (4.79)	
Recreational uses		
Bowling alleys, poolrooms, and similar uses	75 (3.59)[a]	
Dance halls and ballrooms	100 (4.79)[a]	
Gymnasiums	100 (4.79)[a]	
Reviewing stands, grandstands, and bleachers	100 (4.79)[a,k]	
Stadiums and arenas with fixed seats (fastened to the floor)	60 (2.87)[a,k]	
Residential		
One- and two-family dwellings		
Uninhabitable attics without storage	10 (0.48)[l]	
Uninhabitable attics with storage	20 (0.96)[m]	
Habitable attics and sleeping areas	30 (1.44)	
All other areas except stairs	40 (1.92)	
All other residential occupancies		
Private rooms and corridors serving them	40 (1.92)	
Public rooms[a] and corridors serving them	100 (4.79)	
Roofs		
Ordinary flat, pitched, and curved roofs	20 (0.96)[n]	
Roofs used for roof gardens	100 (4.79)	
Roofs used for assembly purposes	Same as occupancy served	
Roofs used for other occupancies	[o]	[o]
Awnings and canopies		
Fabric construction supported by a skeleton structure	5 (0.24) nonreducible	300 (1.33) applied to skeleton structure
Screen enclosure support frame	5 (0.24) nonreducible and applied to the roof frame members only, not the screen	200 (0.89) applied to supporting roof frame members only
All other construction	20 (0.96)	
Primary roof members, exposed to a work floor		
Single panel point of lower chord of roof trusses or any point along primary structural members supporting roofs over manufacturing, storage warehouses, and repair garages		2,000 (8.9)
All other primary roof members		300 (1.33)
All roof surfaces subject to maintenance workers		300 (1.33)
Schools		
Classrooms	40 (1.92)	1,000 (4.45)
Corridors above first floor	80 (3.83)	1,000 (4.45)
First-floor corridors	100 (4.79)	1,000 (4.45)
Scuttles, skylight ribs, and accessible ceilings		200 (0.89)
Sidewalks, vehicular driveways, and yards subject to trucking	250 (11.97)[a,p]	8,000 (35.60)[q]
Stairs and exit ways	100 (4.79)	300[r]
One- and two-family dwellings only	40 (1.92)	300[r]

Table 1.3-1: Minimum Uniformly Distributed Live Loads, and Minimum Concentrated Live Loads, Continued.

Occupancy or Use	Uniform psf (kN/m²)	Conc. lb (kN)
Storage areas above ceilings	20 (0.96)	
Storage warehouses (shall be designed for heavier loads if required for anticipated storage)		
Light	125 (6.00)[a]	
Heavy	250 (11.97)[a]	
Stores		
Retail		
First floor	100 (4.79)	1,000 (4.45)
Upper floors	75 (3.59)	1,000 (4.45)
Wholesale, all floors	125 (6.00)[a]	1,000 (4.45)
Vehicle barriers	See Section 4.5	
Walkways and elevated platforms (other than exit ways)	60 (2.87)	
Yards and terraces, pedestrian	100 (4.79)[a]	

[a]Live load reduction for this use is not permitted by Section 4.7 unless specific exceptions apply.
[b]Floors in garages or portions of a building used for the storage of motor vehicles shall be designed for the uniformly distributed live loads of Table 4-1 or the following concentrated load: (1) for garages restricted to passenger vehicles accommodating not more than nine passengers, 3,000 lb (13.35 kN) acting on an area of 4.5 in. by 4.5 in. (114 mm by 114 mm); and (2) for mechanical parking structures without slab or deck that are used for storing passenger vehicles only, 2,250 lb (10 kN) per wheel.
[c]Design for trucks and buses shall be per AASHTO LRFD Bridge Design Specifications; however, provisions for fatigue and dynamic load allowance are not required to be applied.
[d]Uniform load shall be 40 psf (1.92 kN/m²) where the design basis helicopter has a maximum take-off weight of 3,000 lbs (13.35 kN) or less. This load shall not be reduced.
[e]Labeling of helicopter capacity shall be as required by the authority having jurisdiction.
[f]Two single concentrated loads, 8 ft (2.44 m) apart shall be applied on the landing area (representing the helicopter's two main landing gear, whether skid type or wheeled type), each having a magnitude of 0.75 times the maximum take-off weight of the helicopter and located to produce the maximum load effect on the structural elements under consideration. The concentrated loads shall be applied over an area of 8 in. by 8 in. (200 mm by 200 mm) and shall not be concurrent with other uniform or concentrated live loads.
[g]A single concentrated load of 3,000 lbs (13.35 kN) shall be applied over an area 4.5 in. by 4.5 in. (114 mm by 114 mm), located so as to produce the maximum load effects on the structural elements under consideration. The concentrated load need not be assumed to act concurrently with other uniform or concentrated live loads.
[h]The loading applies to stack room floors that support nonmobile, double-faced library book stacks subject to the following limitations: (1) The nominal book stack unit height shall not exceed 90 in. (2,290 mm); (2) the nominal shelf depth shall not exceed 12 in. (305 mm) for each face; and (3) parallel rows of double-faced book stacks shall be separated by aisles not less than 36 in. (914 mm) wide.
[i]In addition to the vertical live loads, the design shall include horizontal swaying forces applied to each row of the seats as follows: 24 lb per linear ft of seat applied in a direction parallel to each row of seats and 10 lb per linear ft of seat applied in a direction perpendicular to each row of seats. The parallel and perpendicular horizontal swaying forces need not be applied simultaneously.
[j]Uninhabitable attic areas without storage are those where the maximum clear height between the joist and rafter is less than 42 in. (1,067 mm), or where there are not two or more adjacent trusses with web configurations capable of accommodating an assumed rectangle 42 in. (1,067 mm) in height by 24 in. (610 mm) in width, or greater, within the plane of the trusses. This live load need not be assumed to act concurrently with any other live load requirement.
[k]Uninhabitable attic areas with storage are those where the maximum clear height between the joist and rafter is 42 in. (1,067 mm) or greater, or where there are two or more adjacent trusses with web configurations capable of accommodating an assumed rectangle 42 in. (1,067 mm) in height by 24 in. (610 mm) in width, or greater, within the plane of the trusses. At the trusses, the live load need only be applied to those portions of the bottom chords where both of the following conditions are met:

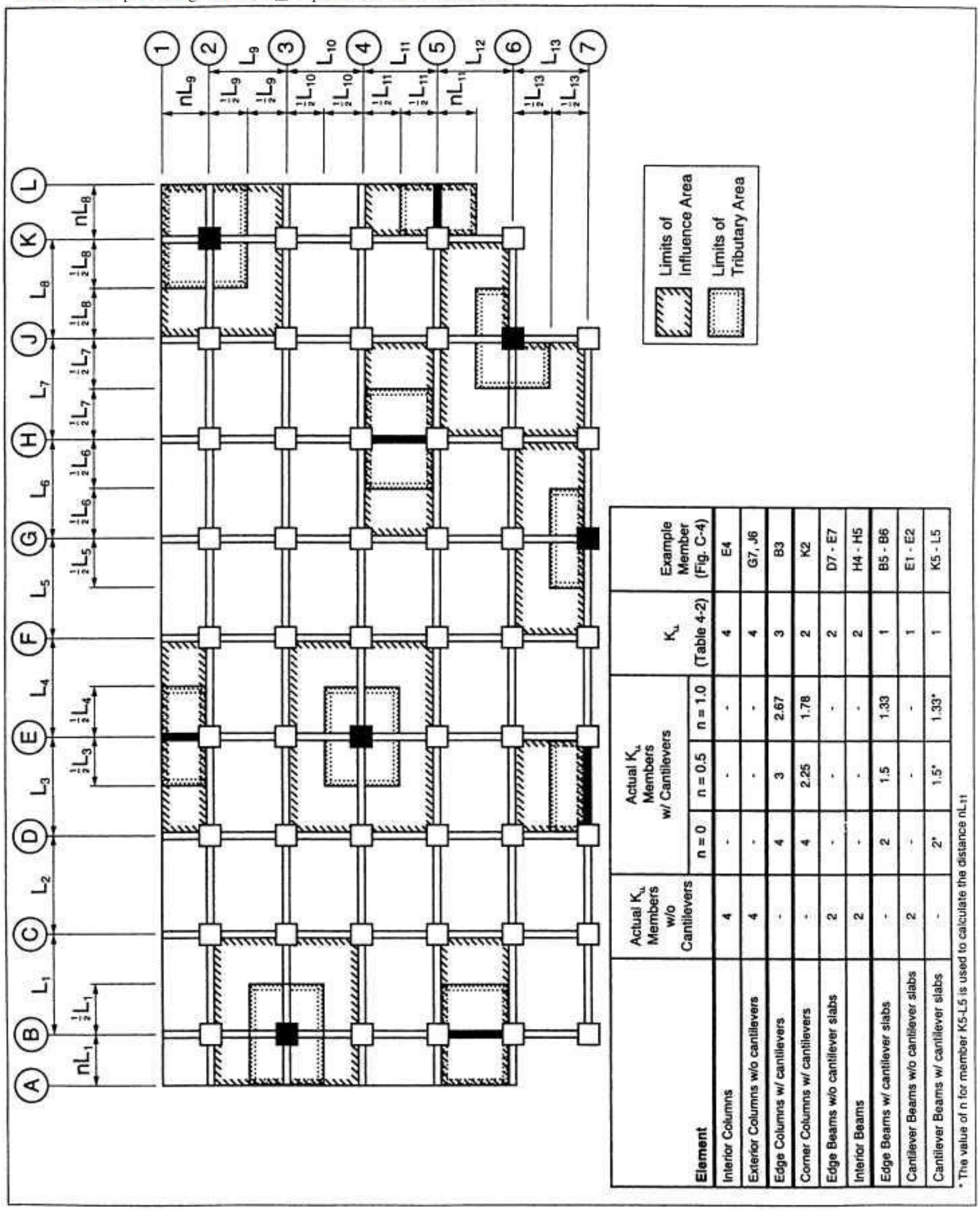
i. The attic area is accessible from an opening not less than 20 in. (508 mm) in width by 30 in. (762 mm) in length that is located where the clear height in the attic is a minimum of 30 in. (762 mm); and
ii. The slope of the truss bottom chord is no greater than 2 units vertical to 12 units horizontal (9.5% slope).

The remaining portions of the bottom chords shall be designed for a uniformly distributed nonconcurrent live load of not less than 10 lb/ft² (0.48 kN/m²).
[l]Where uniform roof live loads are reduced to less than 20 lb/ft² (0.96 kN/m²) in accordance with Section 4.8.1 and are applied to the design of structural members arranged so as to create continuity, the reduced roof live load shall be applied to adjacent spans or to alternate spans, whichever produces the greatest unfavorable load effect.
[m]Roofs used for other occupancies shall be designed for appropriate loads as approved by the authority having jurisdiction.
[n]Other uniform loads in accordance with an approved method, which contains provisions for truck loadings, shall also be considered where appropriate.
[o]The concentrated wheel load shall be applied on an area of 4.5 in. by 4.5 in. (114 mm by 114 mm).
[p]Minimum concentrated load on stair treads (on area of 2 in. by 2 in. [50 mm by 50 mm]) is to be applied nonconcurrent with the uniform load.

Table 1.3-2: Live Load Element Factor, K_{LL}

Element	K_{LL}[a]
Interior columns	4
Exterior columns without cantilever slabs	4
Edge columns with cantilever slabs	3
Corner columns with cantilever slabs	2
Edge beams without cantilever slabs	2
Interior beams	2
All other members not identified, including:	1
Edge beams with cantilever slabs	
Cantilever beams	
One-way slabs	
Two-way slabs	
Members without provisions for continuous shear transfer normal to their span	

[a]In lieu of the preceding values, K_{LL} is permitted to be calculated.



Example 1.3-1

Flat plate system indicated in ***Figure 1.3-6*** below is proposed for a school building. Almost all floors area are classes. For this building,

- According to requirements of ASCE 7-10, select an appropriate value for floor live load.
- Compute live load resultant acting on a typical interior column. Reduce floor live load if possible.

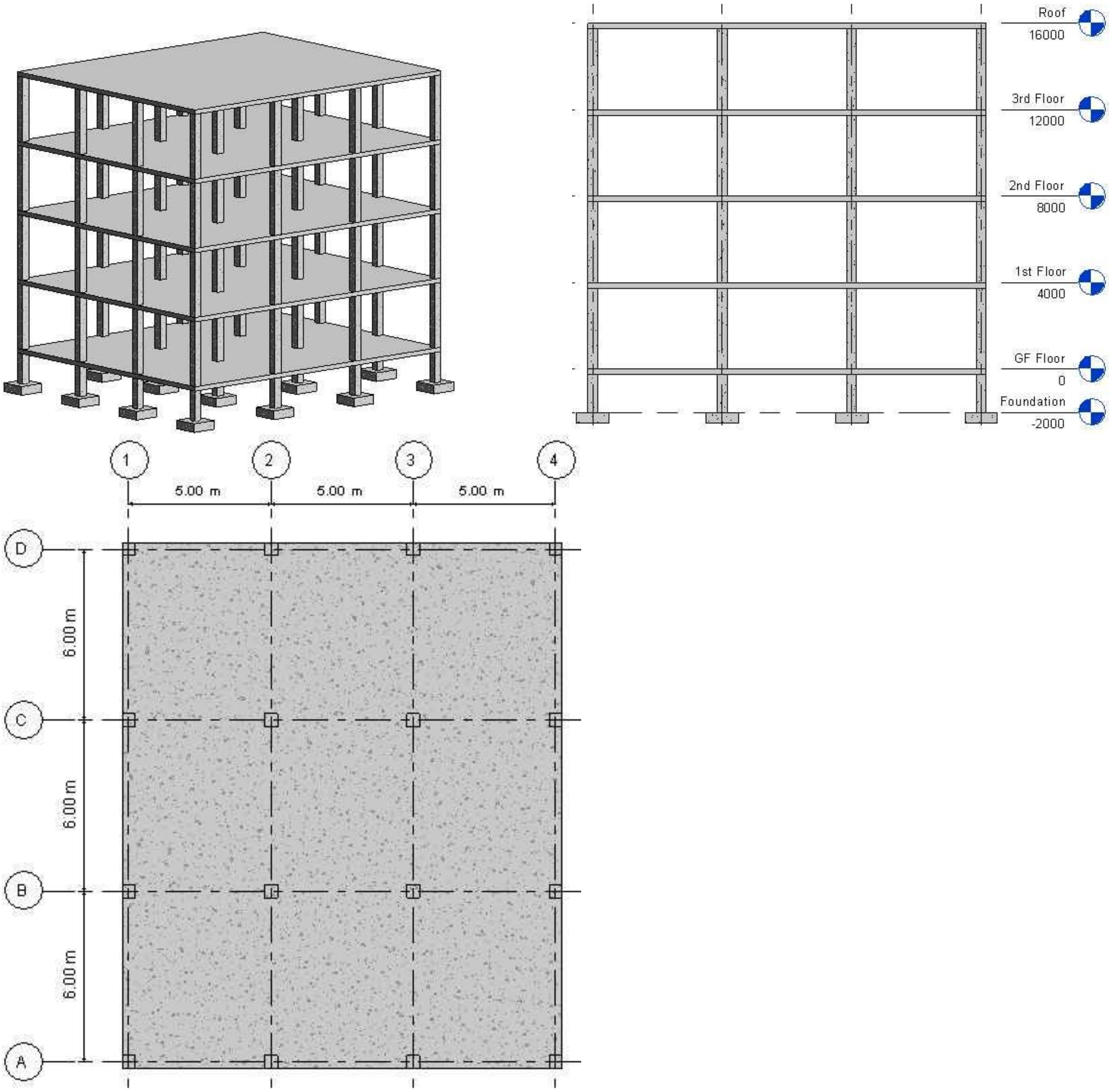


Figure 1.3-6 Flat plate building for Example 1.3-1.

Solution

- Floor Live Load

According to ASCE 7-10, live load for classrooms is:

$$L_L = 1.92\frac{kN}{m^2} = 1.92\ kPa\ \blacksquare$$

- Resultant of an Interior Column

As live load is less than $4.79\ kPa$, therefore it is reducible according to ASCE 7-10. Regarding to live load acting on a typical interior column, its useful to note that in regular system with equal spans, interior column is assumed to support a tributary area bounded by centerlines of adjacent panels, see Figure 1.3-7 below.

$$A_{T\ Supported\ by\ an\ Interior\ Column} = (5 \times 6) \times 4 = 120\ m^2$$

According to Table 1.3-2 above:

$K_{LL} = 4$,

The influence area is:

$K_{LL}A_T = 4 \times 120 = 480\ m^2 > 37.16\ m^2$

Therefore, the reduction in live load is permitted.

$L = L_o\left(0.25 + \frac{4.57}{\sqrt{4 \times 120}}\right) = 0.458\ L_o > 0.4L_O \therefore Ok.$

The resultant of live load acting on a typical interior column is:

$P_L = 0.458 \times 1.92 \times 120 = 106\ kN$ ■

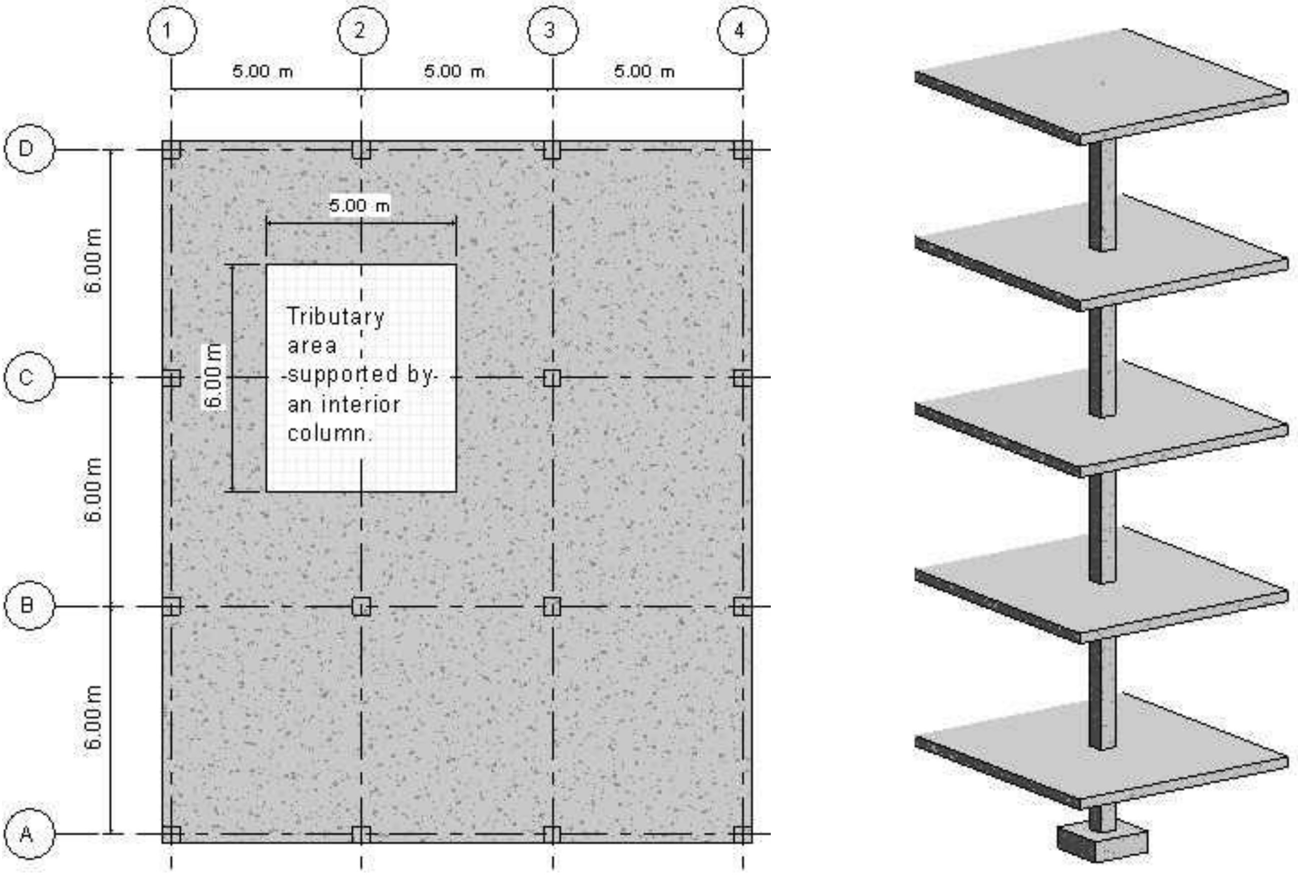


Figure 1.3-7: Tributary area supported by a typical interior column.

Example 1.3-2

Floor system presented in Figure 1.3-8 below is proposed for patient rooms in a hospital building. According to ASCE 7-10:

- Proposed a suitable floor live to be adopted for this floor system,
- Reduce floor live load for a typical interior floor beam,

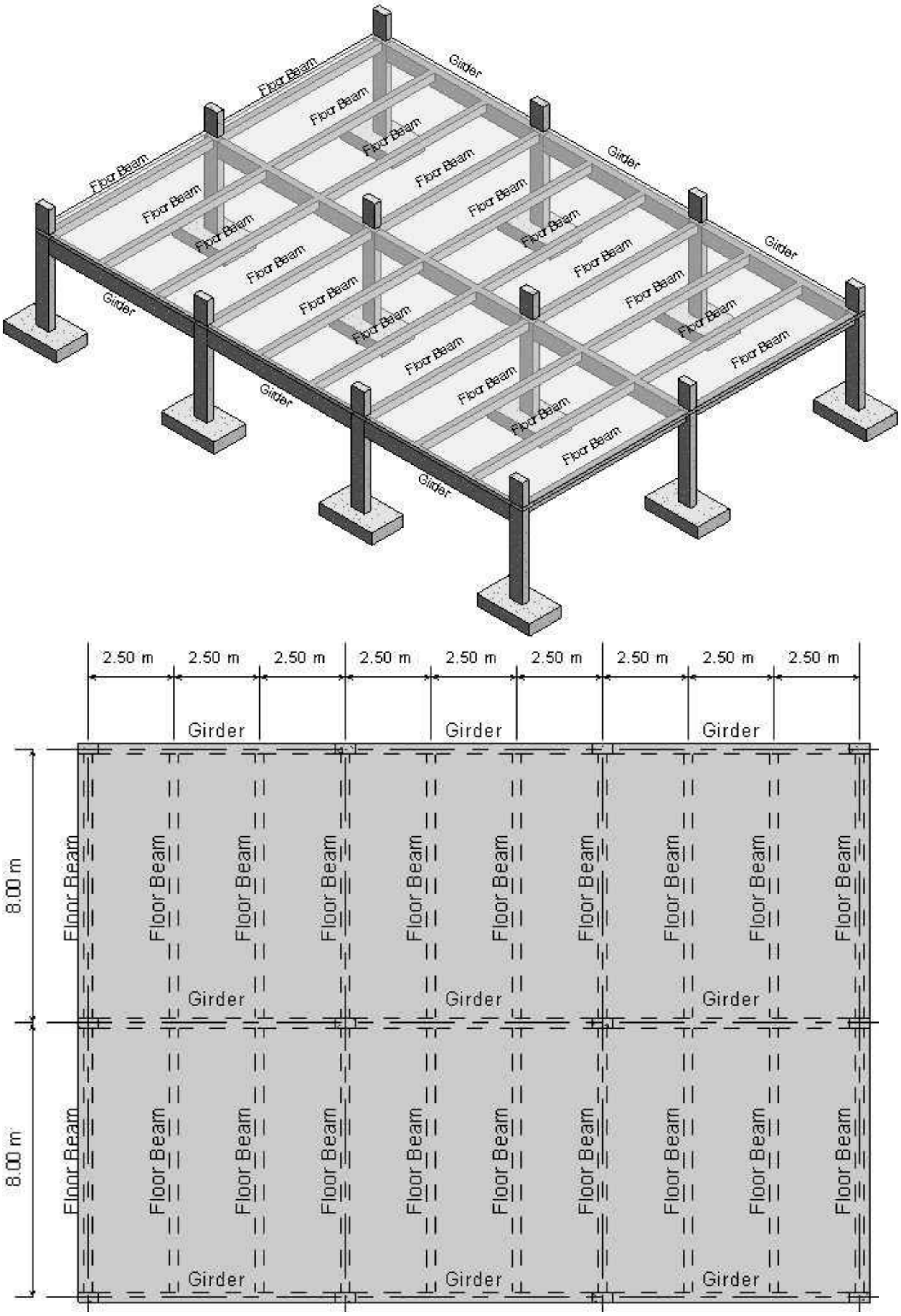


Figure 1.3-8: Floor system for Example 1.3-2.

Solution

According to Table 1.3-1 above, live load for patient rooms in hospital buildings is:

$L_{o\ \text{for patient rooms}} = 1.92\ kPa$ ■

In a one-way floor system, the tributary area supported by a typical interior beam is indicated in Figure 1.3-9 below.

$\therefore A_{T\ for\ typical\ interior\ floor\ beam} = 2.5 \times 8 \times 2 = 40\ m^2$

With K_{LL} factor of 2 according to Table 1.3-2 above, influence area for a typical interior floor beam would be:

$K_{LL}A_T = 2 \times 40 = 80\ m^2 > 37.16\ m^2$

$\because L_o = 1.92\ kPa < 4.79\ kPa$

Therefore, live load of a typical floor beam is reducible and can be estimated from relation below:

$$L = L_o\left(0.25 + \frac{4.57}{\sqrt{80}}\right) = 0.76\ L_O$$

As floor beams contribute in supporting their own story only, therefore reduced live load should be limited by $0.5L_o$.

$L = 0.76L_o = 0.76 \times 1.92 = 1.46\ kPa$ ■

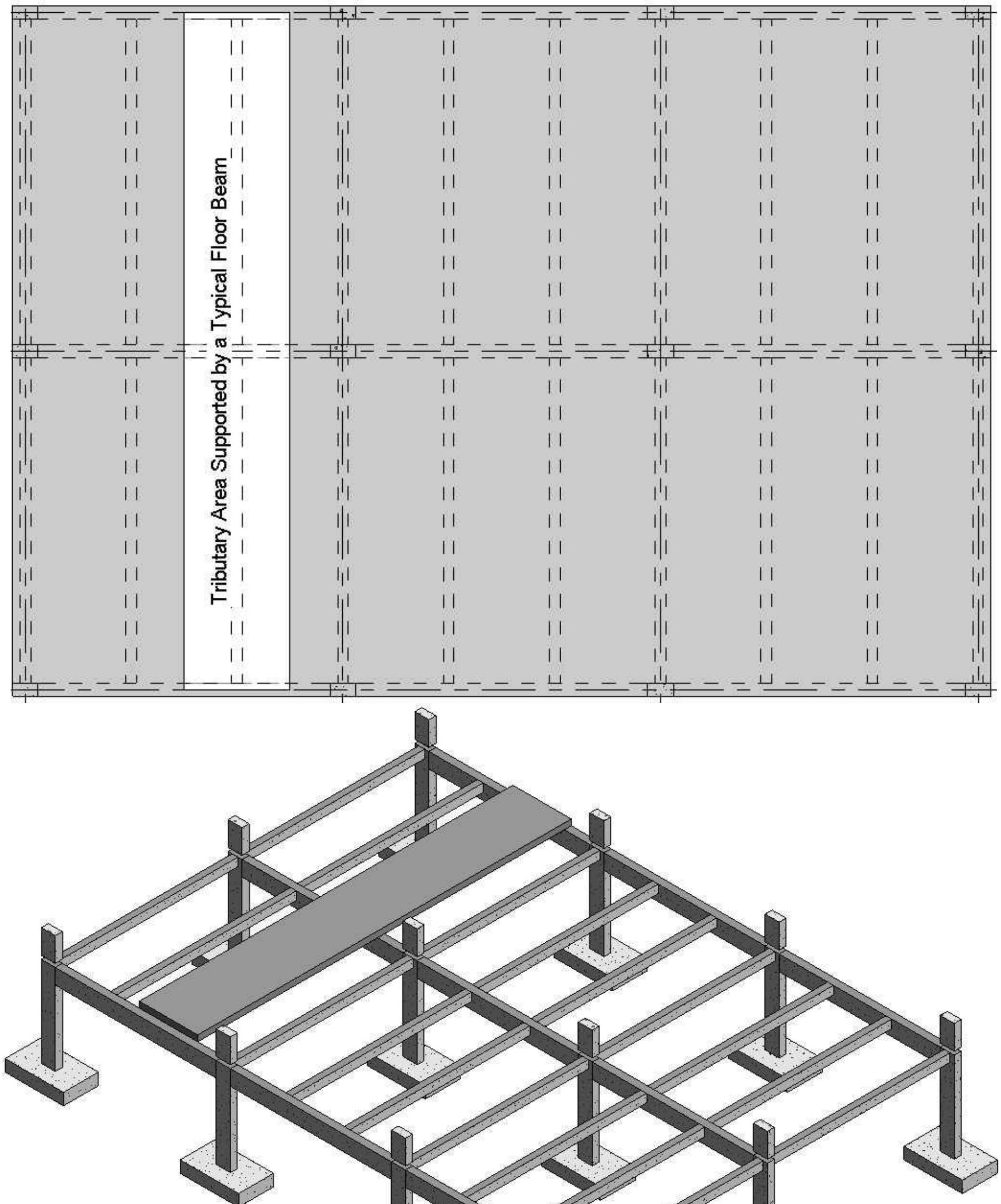


Figure 1.3-9: Tributary area supported by a typical interior floor beam.

– –

1.3.2.3 Roof Live Load

1.3.2.3.1 Basic Value of Roof Live Load

The minimum uniformly distributed roof live loads, L_o, can be estimated from values presnted in Table 1.3-1 above.

1.3.2.3.2 Reduction of Roof Live Load

According to (ASCE/SEI 7–10), roof live load, L_o, can be reduced according to following relation:

$$L_r = L_o R_1 R_2 \qquad 0.58\ kPa \le L_r \le 0.96\ kPa$$ **Eq. 1.3-2**

where

L_r is reduced roof live load per m^2 of horizontal projection supported by the member,
L_o is unreduced design roof live load per m^2 of horizontal projection supported by the member, (see Table 1.3-1 above).

The reduction factor R_1 simulates reduction of roof live load as a function of loaded area and it can be estimated from following relation:

$$R_1 = \begin{cases} 1 & for\ A_T \le 18.58\ m^2 \\ 1.2 - 0.011 A_T & for\ 18.58\ m^2 < A_T < 55.74\ m^2 \\ 0.6 & for\ A_T \ge 55.74\ m^2 \end{cases}$$

where A_T is tributary area in m^2 supported by the member.

While the reduction factor, R_2, simulates reduction in roof live load with increasing in roof slope and it can be estimated from relation below:

$$R_2 = \begin{cases} 1 & for\ F \le 4 \\ 1.2 - 0.05F & for\ 4 < F < 12 \\ 0.6 & for\ F \ge 12 \end{cases}$$

where, for a pitched roof, F = 0.12 × slope, with slope expressed in percentage points.

Example 1.3-3

For ***Example 1.3-1*** above, select an appropriate value for the roof live load and compute the force resultant that supported by an interior column. In your computation, reduce roof live loads if possible.

Solution

As nothing is mentioned in the example statement about the nature of the roof, therefore an ordinary roof has been assumed. The roof live load is:

$L_o = 0.96\ kPa$ ■

Assuming that an interior column supports a tributary area bounded by centerlines of adjacent panels, see Figure 1.3-10,

$A_T = 5 \times 6 = 30\ m^2$

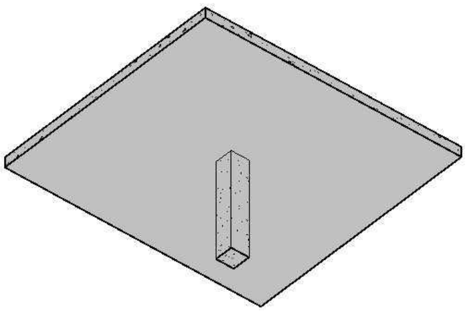


Figure 1.3-10: Roof area supported by a typical interior column for Example 1.3-3.

The reduction factor, R_1, is:

$\because 18.58\ m^2 < A_T < 55.74\ m^2$

$\therefore R_1 = 1.2 - 0.011 A_T = 1.2 - 0.011 \times 30 = 0.87$

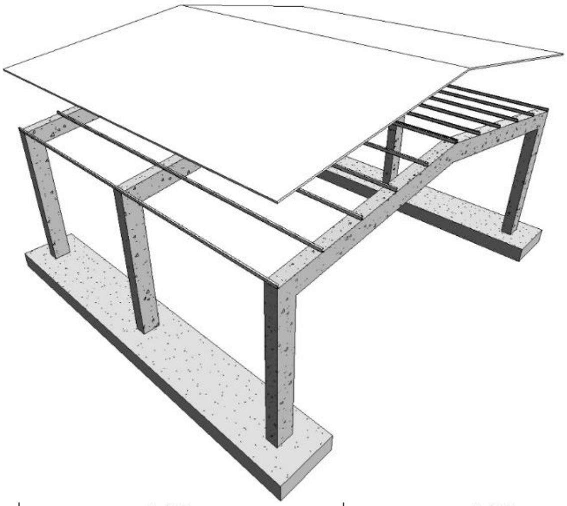
For flat roof,

$R_2 = 1.0$

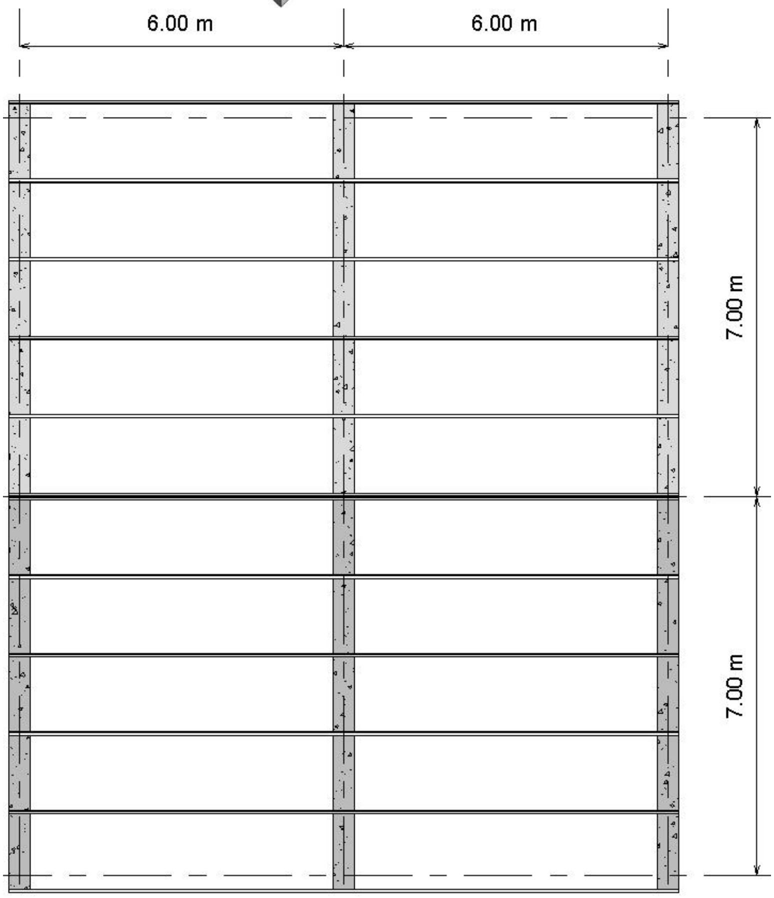
$$P_{Due\ to\ R_o} = \left(0.96 \frac{kN}{m^2} \times 30\ m^2\right) \times 0.87 \times 1.0 = 25.1\ kN$$ ■

Example 1.3-4

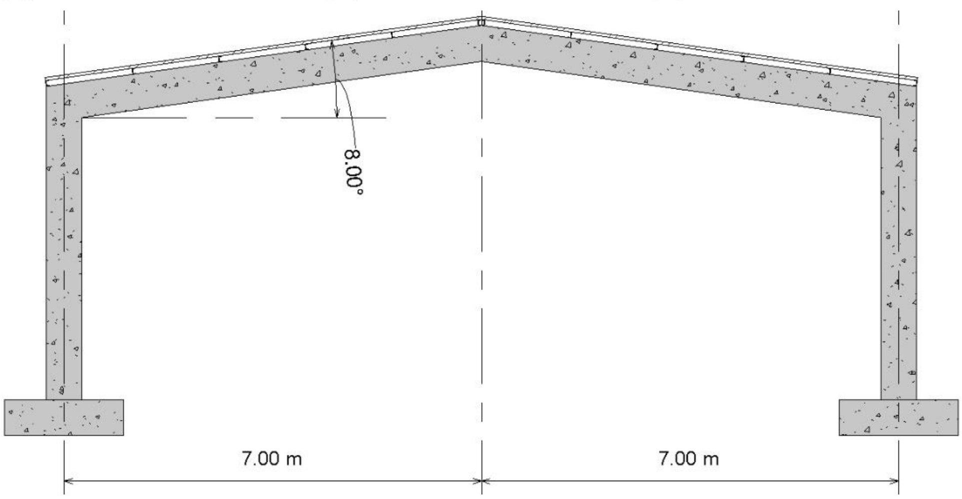
For gable frame presented in ***Figure 1.3-11*** below, select a suitable roof live load then determine the reduced live load that supported by the interior frame.



3D view.



Plan view.



Sectional view.

Figure 1.3-11: Gable frame for Example 1.3-4.

Solution

Assuming an ordinary pitched roof, roof live load according to ***Table 1.3-1*** above would be:

$L_r = 0.96\ kPa$ ■

According to ***Eq. 1.3-2*** above, the reduced roof live load is:

$L_r = L_o R_1 R_2 \qquad 0.58\ kPa\ \leq L_r \leq 0.96\ kPa$

As this live load is acting on the inclined surface, therefore the tributary area would be:

$$A_T = \left(\frac{7}{cos8} \times 2\right) \times \left(\frac{6}{2} \times 2\right) = 84.8\ m^2$$

$\because A_T \geq 55.74\ m^2\ \Rightarrow\ R_1 = 0.6$

In computing R_2, the F factor is determined as follows:

$F = 0.12 \times Slop_{Expressed\ in\ percentage\ units} = 0.12 \times (tan8) \times 100 = 1.69$

$\because F \leq 4 \Rightarrow \therefore R_2 = 1.0$

Hence, the reduced roof live load that supported by the interior frame would be:

$L_r = L_o R_1 R_2 = 0.96 \times 0.6 \times 1.0 \approx 0.58\ kPa = Lower\ bound \therefore Ok.$ ■

Example 1.3-5

For industrial building indicated in ***Figure 1.3-12*** below:

- Select an appropriate value for roof live load. The roof has a slope of 10^o.
- Reduce the selected roof live load, if possible, to determine its resultant on the indicated typical edge column.
- If the building floor is proposed for a light manufacturing process, determined the live load that should be adopted according to ASCE/SEI 7-10.
- Is the selected live load reducible or not? Explain your answer.
- Determine live load resultant on the indicated typical edge column.

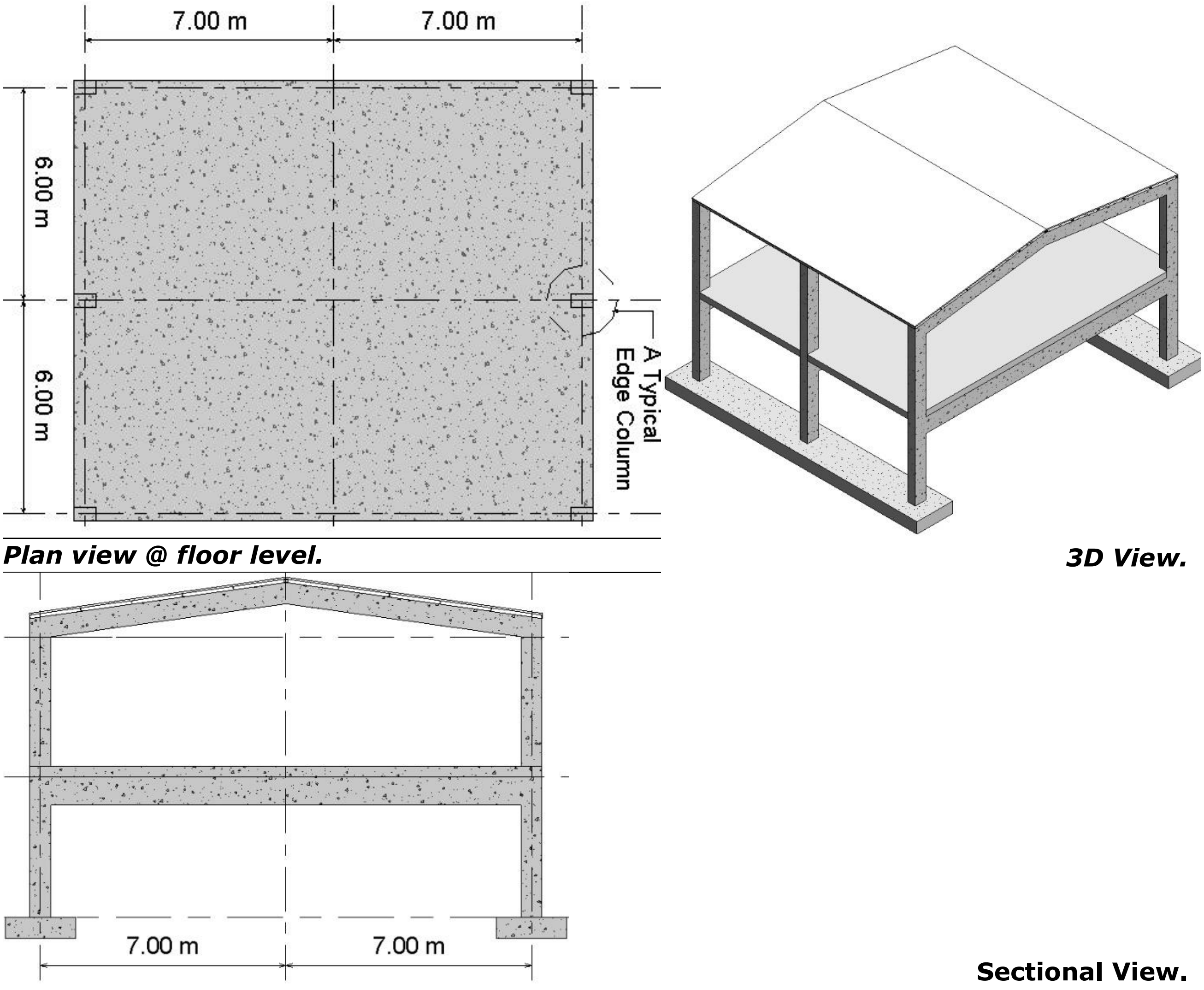


Figure 1.3-12: Structural system for the industrial building Example 1.3-5.

Solutions

- Appropriate value for roof live load:
  According to ASCE 7-10, live load for ordinary flat roof is:
  $L_r = 0.96\ kPa$
- Reduce of roof live load:
  $A_{T\ typical\ edge\ column} = \frac{7}{cos10} \times 6 = 42.6\ m^2 \Rightarrow \because\ 18.58\ m^2 < A_T < 55.74\ m^2$
  $\therefore R_1 = 1.2 - 0.011A_T = 1.2 - 0.011 \times 42.6 = 0.731$
  $F = 0.12 \times Slop_{Expressed\ in\ percentage\ units} = 0.12 \times (tan10) \times 100 = 2.12 \Rightarrow \because F \leq 4 \Rightarrow \therefore R_2 = 1.0$
  $L_r = L_o R_1 R_2 = 0.96 \times 0.731 \times 1.0 \approx 0.702\ kPa > 0.58\ kPa \therefore Ok.$ ■
- Floor live load:
  Assuming a light manufacturing process, the floor live load according to ASCE/SEI 7-10 is:
  $L = 6.0\ kPa$
- Reduction of floor live load:
  Floor live of $6.00\ kPa$ is irreducible according to ASCE/SEI 7-10 as it is greater than $4.79\ kPa$.
- Live load resultant on the indicated edge column:
  $P_L \approx 6\ kPa \times 7m \times 6m = 252\ kN$■

1.3.3 Environmental Loads

Environmental loads can be sub-classified into the following types:

1. Wind Loads.
2. Earthquake Loads.
3. Soil Pressure Loads.
4. Snow Loads.
5. Rain Loads.
6. Force caused by a differential temperature.

Like live loads, environmental loads at any given time are uncertain both in magnitude and in distribution. Therefore, their values and the distribution must be determined based on the codes and specifications like the *International Building Code* or *Minimum Design Loads for Buildings and Other Structures*.

1.3.3.1 Wind Loads

1.4 DESIGN CODES AND SPECIFICATIONS

After selection of a suitable structural system based on the functional and/or architectural requirements, the structural design process can be summarized by following three steps:

Prediction of the applied loads → Make a structural analysis to compute the member internal forces and the structural deformations that related to the applied loads. → Compute the member strength and then assessment the member adequacy based on the comparison between the applied loads and the member strength.

As was shown in the previous article on the loads and as will be shown in the next articles, each one of the above steps contains some kind of uncertainty. To deal with these uncertainties in the design process, the engineers must base their design decision not only on the theoretical aspects but also on the previous experience that usually written in the form of codes or specifications which edited by professional groups and technical institutes.

Following list states most important professional groups and technical institutes:

1.4.1 American Society of Civil Engineers (ASCE)

Produce a document titled "*Minimum Design Loads for Buildings and Other Structures*, *ASCE 7-10*" that is usually used in the definition of loads magnitude, distribution, and load combinations that should be considered in the structural design.

1.4.2 American Concrete Institute (ACI)

Produce documents that including provisions for the concrete design and construction. The "*Buildings Code Requirements for the Structural Concrete (ACI 318M-14)*" that related to the design and construction concrete buildings is an example of these documents.

1.4.3 American Association of State Highway and Transportation Officials (AASHTO)

Produce documents that related to the design and construction of the highway projects and highway bridges.

1.4.4 American Railway Engineering Association (Area)

Produce the documents that related to the design and construction of the railway projects.

1.5 DESIGN CRITERIA

Following criteria are usually adopted in design and assessment of different structural elements:

1.5.1 Criteria for Beams Design

Design and assessment of a reinforced concrete beam are based on the following criteria:

1.5.1.1 Strength criterion

Including checking or design for flexure strength, shear strength, torsion strength, and bond strength of the reinforced concrete beam.

1.5.1.2 Serviceability criterion

Including checking for adequacy of reinforced concrete beams for deflection, crack width, and vibration (vibration is out the scope of this course).

1.5.1.3 Stability criterion

As stated in theory of structure, a plane structure is stable when supported by three reactions or more that neither all parallel nor all concurrent at a single point.

Due to rough nature of surfaces in concrete and masonry structure, most of reinforced concrete beams are stable in nature. Consider for example the reinforced beam indicated in Figure 1.5-1(a) below due to surface roughness, a beam to wall connection can be simulated as a hinge. With two hinge supports indicated in Figure 1.5-1(b), a membrane force develops in the beam in addition to shear force and bending moment. In traditional reinforced concrete design, this membrane force is usually neglected and the beam is simulated as presented Figure 1.5-1(c).

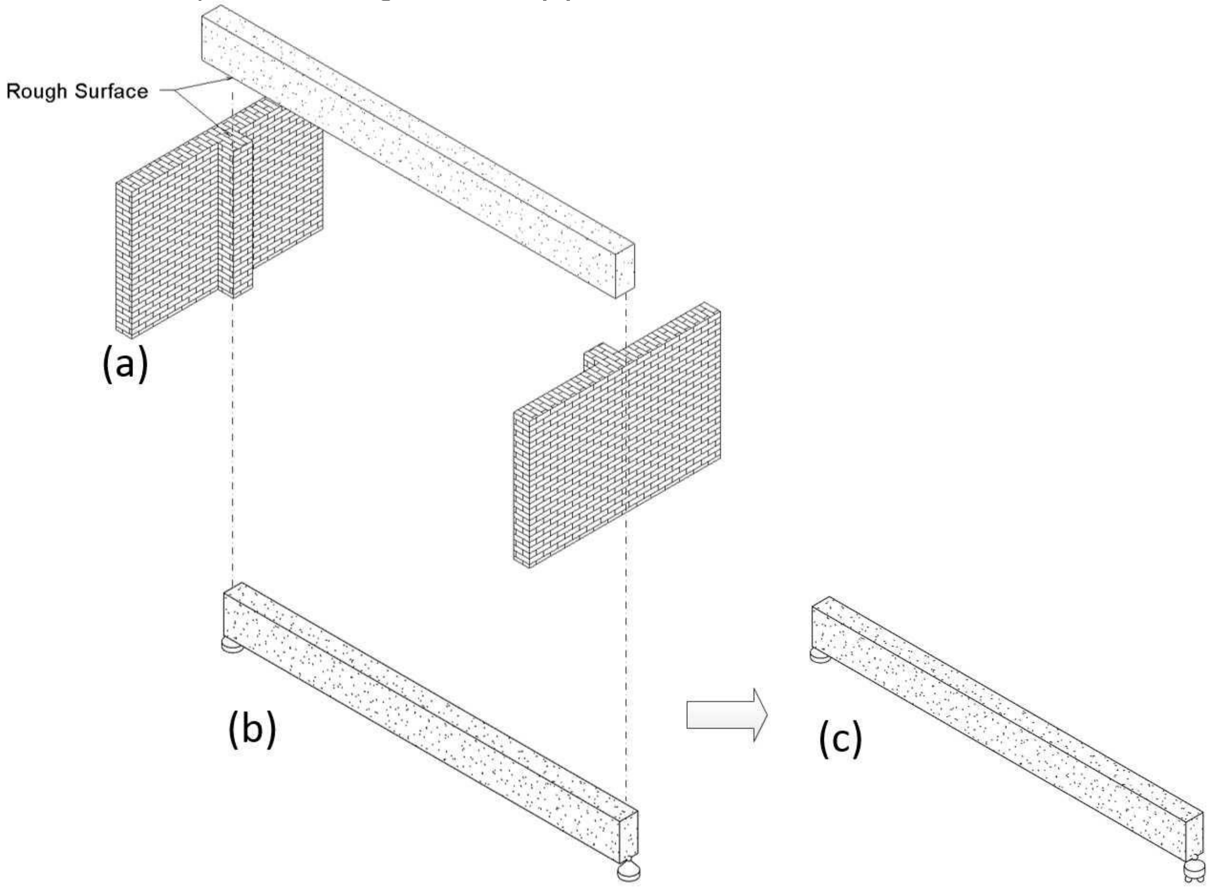


Figure 1.5-1: A simply supported reinforced concrete beam.

1.5.2 Criteria for Slabs Design

Design and assessment of the one-way slabs or two-way slabs are generally based on the following criteria:

1.5.2.1 Strength criterion

Including checking or design for flexure strength, shear strength, and bond strength of the reinforced concrete slab.

1.5.2.2 Serviceability criterion

Including checking for adequacy of reinforced concrete slabs for deflection, crack width, and vibration (vibration out the scope of our course).

1.5.2.3 Stability criterion

As discussed in stability criterion for beams, reinforced concrete slabs are stable in nature due to surface roughness.

1.5.3 Criteria for Columns Design

Design and assessment of the reinforced concrete columns are based on the following criteria.

1.5.3.1 Strength criterion

Including checking for flexure and axial strength of reinforced concrete columns.

1.5.3.2 Stability criterion

In additional to general stability criteria that related to number and arrangement of reactions, stability of some columns, called slender columns, is a function of axial load. For a specific level of axial forces, called column critical load or Euler load, the column is unstable in a sense that it cannot return to its equilibrium position after a small lateral disturbance, see Figure 1.5-2 below.

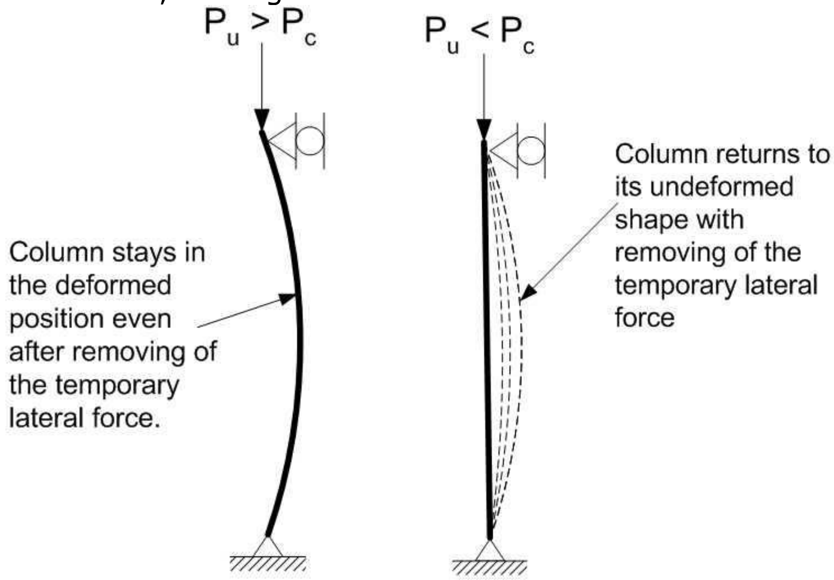


Figure 1.5-2: Physical interpretation of critical load.

In addition to stability aspect, equilibrium equations for a slender column should be formulated in terms of deformed shape instead of undeformed shape to take into account the effects of secondary moments, see Figure 1.5-3 below.

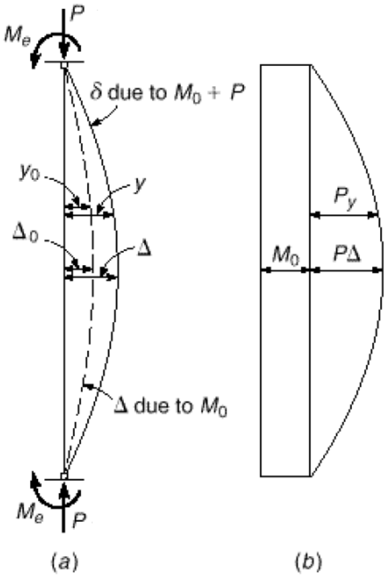


Figure 1.5-3: Secondary moment effects in column analysis.

1.5.3.3 Serviceability

As indicated in Figure 1.5-4 below, generally, axial deformation of columns produce rigid body motion in beams and floor systems and can be disregarded in serviceability checking.

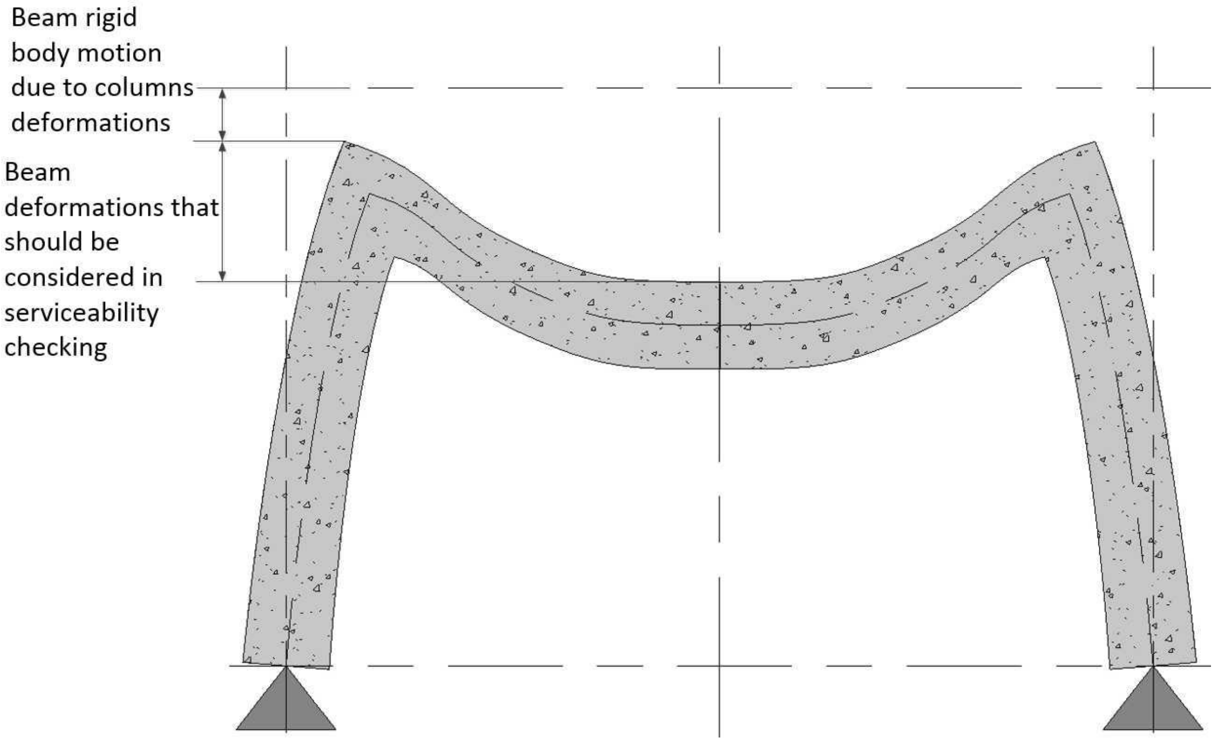


Figure 1.5-4: Rigid body motion and deformation of beams.

1.6 DESIGN PHILOSOPHY

Uncertainties in the analysis, design, and construction of reinforced concrete structures can be summarized in the diagram below:

Uncertainty in the Analysis and Design of Reinforced Concrete Structure can be Classified into:

Uncertainty in Prediction the Applied Load: This is due to

- Usually, there is a difference between the actual and predicted loads.
- Generally, the actual loads have a distribution that differ from the assumed one.

Uncertainty in the Computed Theoretical or Nominal Strength: This is due to

- Actual member dimensions are different from that shown in the drawings and assumed in the calculations.
- Actual materials strength are not exactly the same to that assumed in the design calculations.

1.6.1.1 Load Uncertainty

- Based on statistical data obtained from large-scale survey, load uncertainty can be described in terms of the probability model show below:

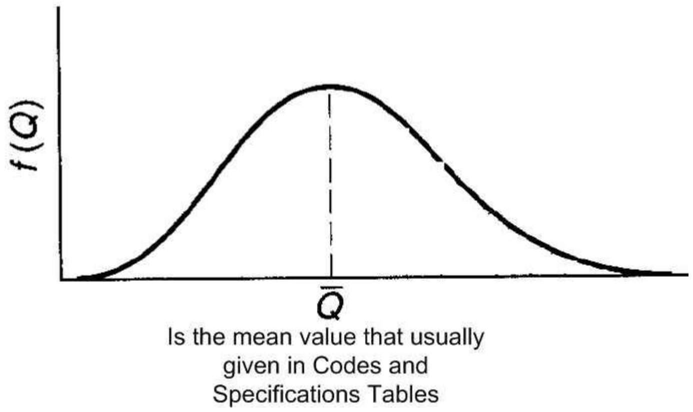


Figure 1.6-1: Frequency curve for load.

- Based on structural type and design code, a designer can select a design load ($\bar{Q}$) from related load Table (e.g. Table 1.3-1).
- If the designer use ($\bar{Q}$) value as a design load, then the designer implicitly accepts a probability of over load in the range of 50% (shaded area in the Figure 1.6-2 below).

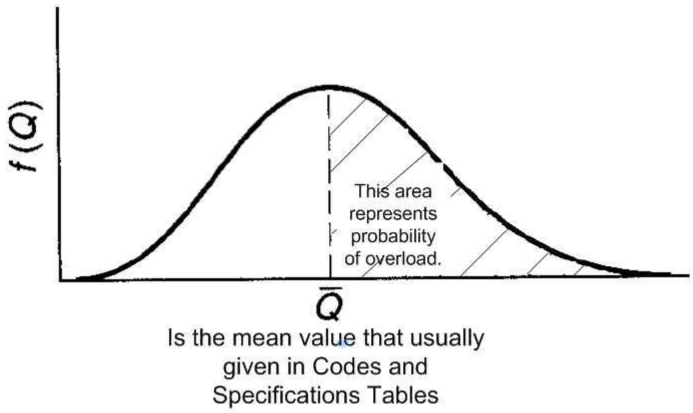


Figure 1.6-2: Adopting of $\bar{Q}$ implicitly equivalent to acceptance a probability of 50% of over load.

- As this probability for over load is so large to be accepted in a design process, the designer should increase the mean value ($\bar{Q}$) to a design value (Q_d) (See Figure below):

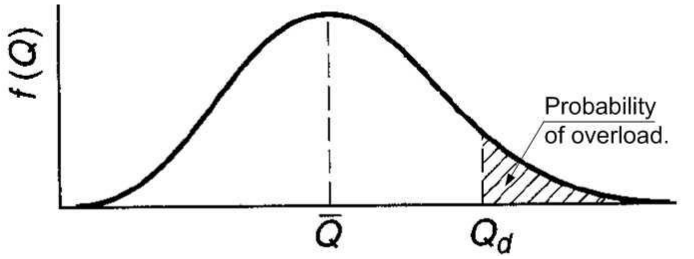


Figure 1.6-3: Factored load with low probability of overload.

- Above increasing or magnification is done based on following relation:
  $Q_d = \gamma \bar{Q}$ **Eq. 1.6-1**
  where
  Q_d is the factored load that will be used in structural design or assessment.
  $\bar{Q}$ is the mean value that usually given in Load Tables or computed theoretically.
  γ Load Factor. It is computed according to ACI 5.3.1 (See Table below):

Table 1.6-1: Load combinations

Load Combination	**Equation Number according to this course**	**Equation Number according to the ACI code**	**Primary load**
$U = 1.4D$	Eq. 1.6-2	5.3.1a	D
$U = 1.2D + 1.6L + 0.5(L_r or S or R)$	Eq. 1.6-3	5.3.1b	L
$U = 1.2D + 1.6(L_r or S or R) + (1.0L \text{ or } 0.5W)$	Eq. 1.6-4	5.3.1c	L_r or S or R
$U = 1.2D + 1.0W + 1.0L + 0.5(L_r or S or R)$	Eq. 1.6-5	5.3.1d	W
$U = 1.2D + 1.0E + 1.0L + 0.2S$	Eq. 1.6-6	5.3.1e	E
$U = 0.9D + 1.0W$	Eq. 1.6-7	5.3.1f	W
$U = 0.9D + 1.0E$	Eq. 1.6-8	5.3.1g	E

- Notes on Wind Load Combinations:
  - In the version of 2010, the ASCE-7 has converted wind loads to ***strength level*** and ***reduced the wind load factor to 1.0***.
  - The Code requires use of the previous ***load factor for wind loads***, ***1.6***, when ***service-level wind loads are used*** as in the case of ***Iraq wind maps***.
  - For serviceability checks, the commentary to Appendix C of ASCE/SEI 7 provides ***service-level wind loads***, W_a.

1.6.1.2 Strength Uncertainty

1. As all of section dimensions and material strength are changed randomly, then if we compute a theoretical or nominal strength of a section (S_n) based on ideal values for design parameters (section dimensions and material strengths), then if a large samples of this sections are test, probability density function of section strength will be as shown in Figure 1.6-4 below:

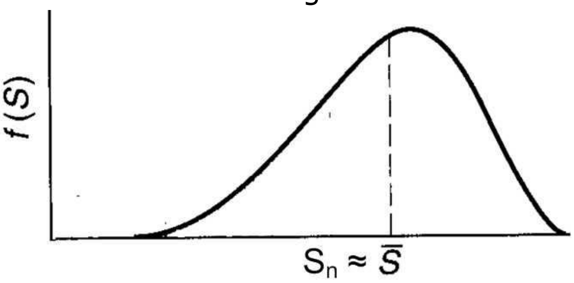


Figure 1.6-4: Frequency curve for strength.

2. If the designer use theoretical or nominal strength of section as a basis for design, he implicitly accept a probability of approximately 50% for overestimation of section strength (see Figure below):

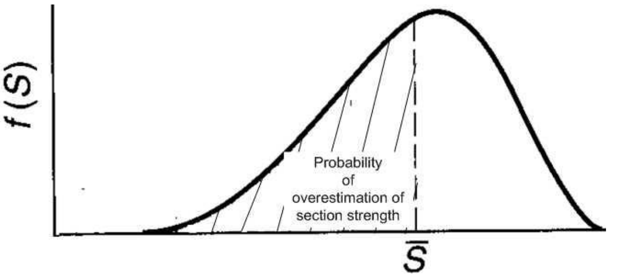


Figure 1.6-5: Adopting of $\bar{S}$ implicitly equivalent to acceptance of 50% of understrength.

3. Therefore, sectional nominal strength (S_n) should be reduced to a design strength (S_d) (see Figure 1.6-6 below).

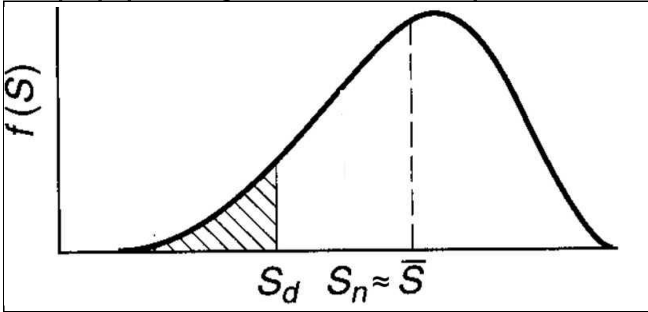


Figure 1.6-6: Factored strength with low probability of understrength.

4. Above reduction will be according to following relation:
   $S_d = \emptyset S_n$ **Eq. 1.6-9**
   where $\emptyset$ is the strength reduction factor that computed based on ACI 21.2.1 (See Table below)

Table 1.6-2: Strength reduction factors φ

Strength Condition	Strength Reduction Factor ϕ
Tension-controlled sections[a]	0.90
Compression-controlled sections[b]	
Members with spiral reinforcement	0.75
Other reinforced members	0.65
Shear and torsion	0.75
Bearing on concrete	0.65
Post-tensioned anchorage zones	0.85
Strut-and-tie models[c]	0.75

[a] Chapter 19 discusses reductions in ϕ for pretensioned members where strand embedment is less than the development length.
[b] Chapter 3 contains a discussion of the linear variation of ϕ between tension and compression-controlled sections. Chapter 8 discusses the conditions that allow an increase in ϕ for spirally reinforced columns.
[c] Strut-and-tie models are described in Chapter 10.

5. Final Design Relation:
   Based on above discussion, a section to be classified as adequate according to strength requirements of the ACI Code, it should satisfied the following relation:
   $\emptyset S_n \geq \gamma \bar{Q}$ **Eq. 1.6-10**

Example 1.6-1

Probability curves for flexural strength of precast beams fabricated by two different manufactures are presented in Figure 1.6-7 below.

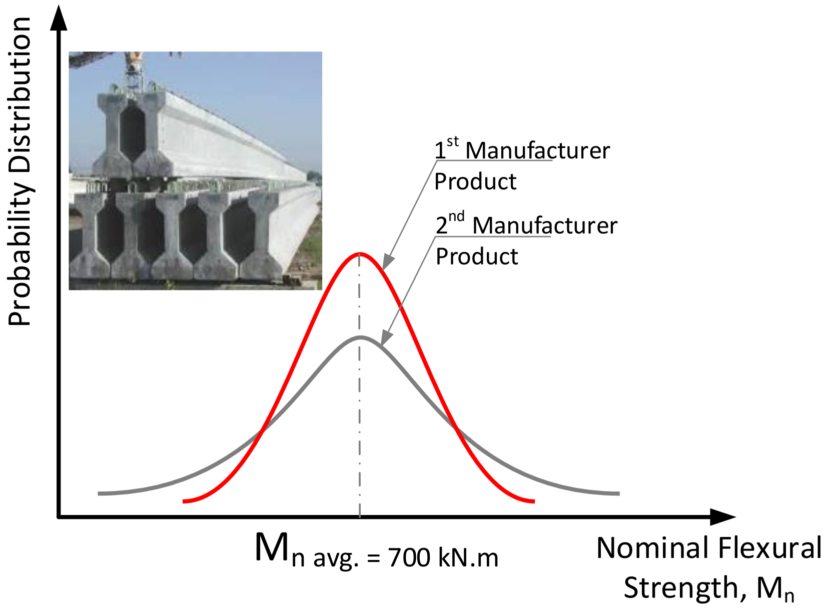


Figure 1.6-7: Probability distribution for precast beams of Example 1.6-1.

1. Which one of two product seems stronger?
2. Which one of two product seems more controlled?
3. Which one of two product needs a larger margin of safety?

Solution

1. In terms of mean strength value, both products have same strength, namely $M_{n\ avg.} = 700\ kN.m$.
2. First product is more controlled than second product as it has lower scatter, deviation, than second product.
3. Second product has a larger margin of safety as it has larger scatter than first product.

Example 1.6-2

Probability curves for two uniformly distributed loads are presented in Figure 1.6-8 below. According to current design philosophy:

- Which one of two loads has larger magnitude?
- Which one of two loads has larger scatter?
- Which curve may represent dead load and which one may represent live load when both loads have same mean value? Explain your answer.

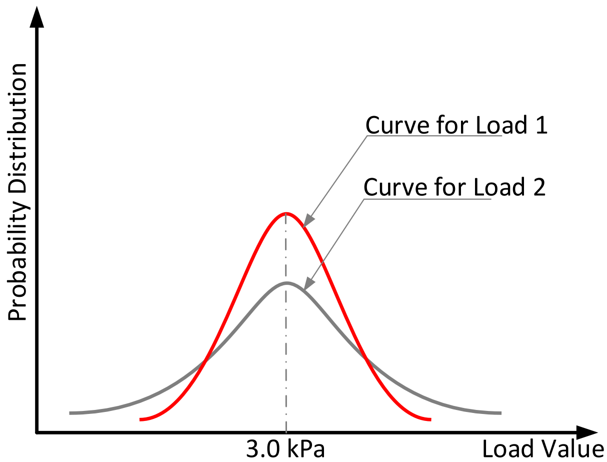


Figure 1.6-8: Probability distribution of uniformly distributed loads for Example 1.6-2.

Solution

- In term of mean value, both loads have same magnitude.
- To be a probability models, area under above curves should equal to one unit. Therefore, width of curve gives an indication on its scatter, its standard deviation, σ, and Load 2 is more scatter than Load 1.

Regarding to their scatters, Load 1 is more suitable to simulate dead load where scatter is smaller than live load.

Example 1.6-3

A factory producing two types of precast beams. According to quality control department, these beams have probability distribution curves shown in Figure 1.6-9 below:

- Which one of two types has larger flexure strength?
- Which one of two types is more controlled?
- Which one of two types need a larger margin of safety?

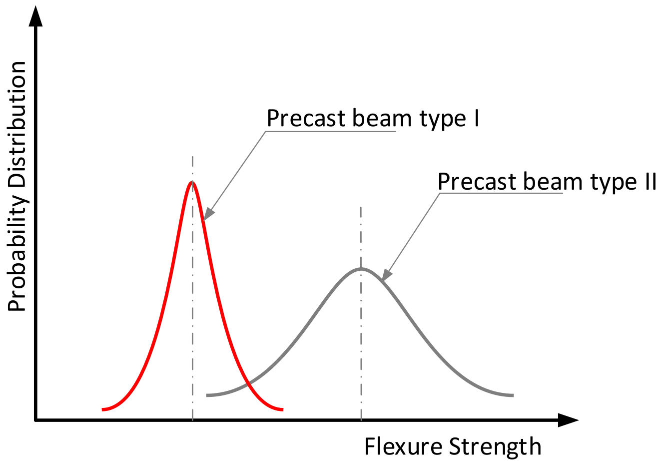


Figure 1.6-9: Probability distributions for flexural strength of precast beams of Example 1.6-3.

Solutions

- In term of mean strength, beam Type II is more strength than Type I.
- Width of curve base gives an indication on standard deviation of the design process. A wider base a less controlled process. Therefore, Type I is more controlled than Type II.

Larger margin of safety should be adopted for beam Type II.

Example 1.6-4

Check adequacy of a simply supported beam presented in Figure 1.6-10 below for flexural strength according to the requirements of ACI318M-14.

Given

$w_D = 20\,\mathrm{kN/m}$ (Not including beam weight)
$w_L = 20\,\mathrm{kN/m}$
$\gamma_{Concrete} = 24\,\mathrm{kN/m^3}$

Nominal (theoretical) flexure strength $M_n = 1000\,\mathrm{kN.m}$ (This will be computed in details in Chapter 3).

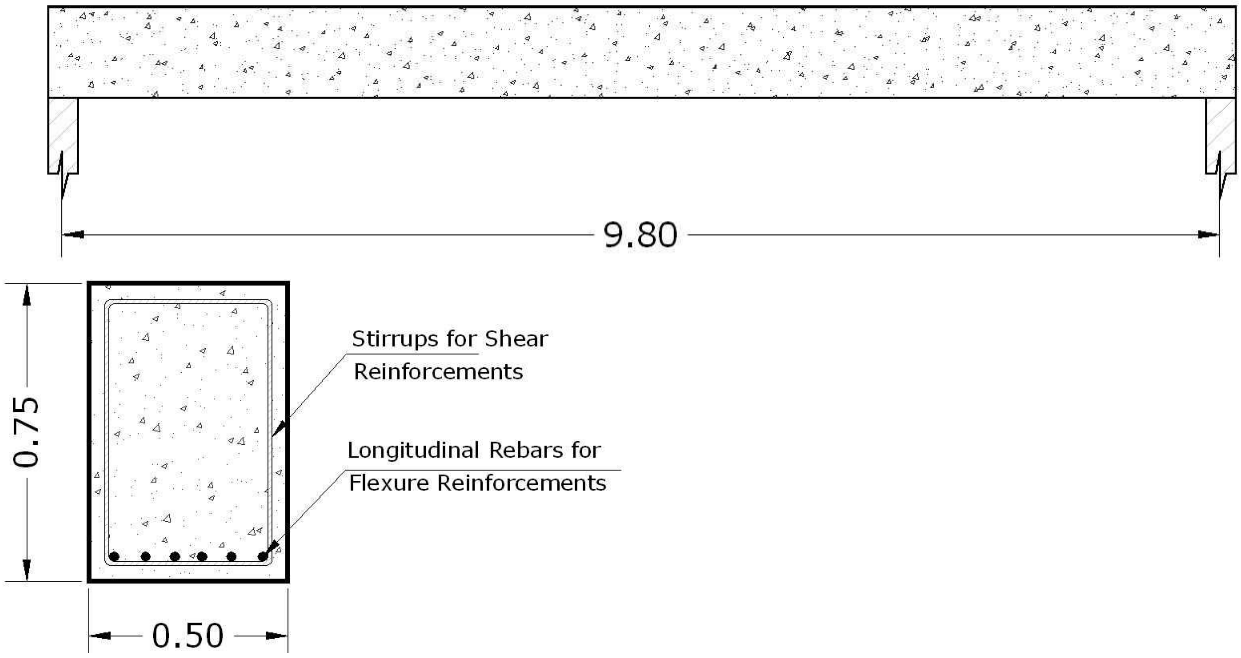


Figure 1.6-10: Simply supported beam for Example 1.6-4.

Solution

1. Compute the Factored Loads:
   Factored load W_u, i.e., the loads that increased to include the load uncertainty can be taken as the maximum of:
   $W_u = 1.4W_{Dead}$
   $W_u = 1.2W_{Dead} + 1.6W_{Live}$
   $w_{Self} = 24\ kN/m^3 \times 0.75m \times 0.5\,m = 9\,kN/m$
   $w_D = 20\,\mathrm{kN/m} + 9\ \mathrm{kN/m} = 29\ \mathrm{kN/m}$
   Then either:
   $$W_u = 1.4 \times 29\frac{kN}{m} = 40.6\frac{kN}{m}$$
   or:
   $$W_u = 1.2W_{Dead} + 1.6W_{Live} = 1.2 \times 29\frac{kN}{m} + 1.6 \times 20\frac{kN}{m} = 66.8\frac{kN}{m}$$
   Therefore, the govern value of the factored load is:
   $$W_{u\,Govern} = 66.8\frac{kN}{m}$$
2. Compute the required flexural strength:
   Bending moment diagram for simply supported beam subjected to uniformly distributed load shows that the maximum bending moment occurs at beam mid span and it has a value of:
   $$M_{u\,maximum} = \frac{W_u l^2}{8} = \frac{66.8 \times 9.8^2}{8} = 802\ kN.m$$
3. Compute the available design strength:
   $\emptyset M_n = 0.9 \times 1000 = 900\,\mathrm{kN.m} > M_u \ \therefore$ ok.

Example 1.6-5

In addition to its own weight, a building supports loads indicated in Figure 1.6-11 below. Check if frame columns with dimensions of 400x400mm and with a nominal strength of, $P_n = 2800\ kN$, at their base, are adequate to support applied loads?

Consider following load combinations in checking,

$U = 1.4D$

$U = 1.2D + 1.6L$

In your solution assume a strength reduction factor, $\phi = 0.65$, and that footings behave as hinges.

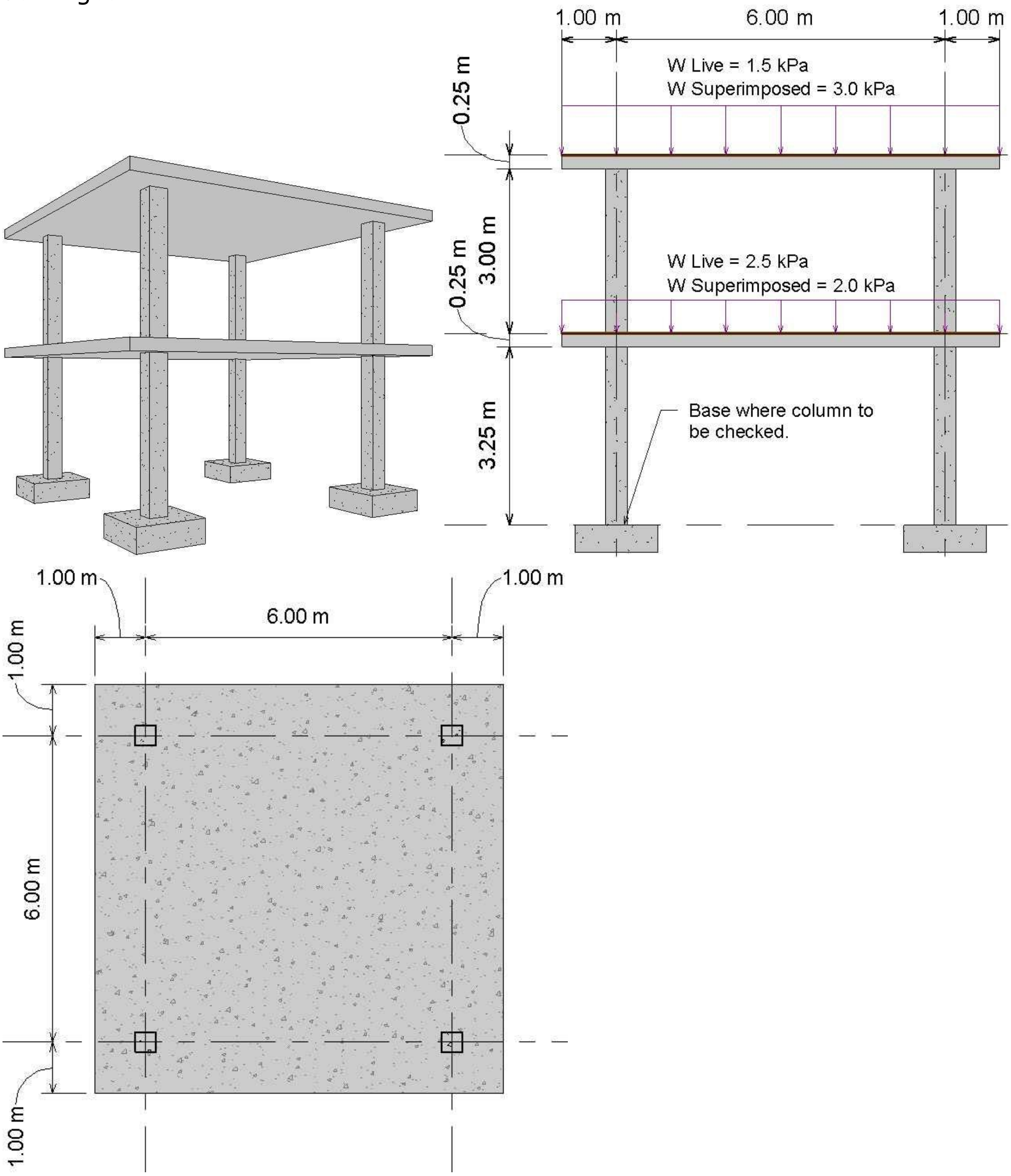
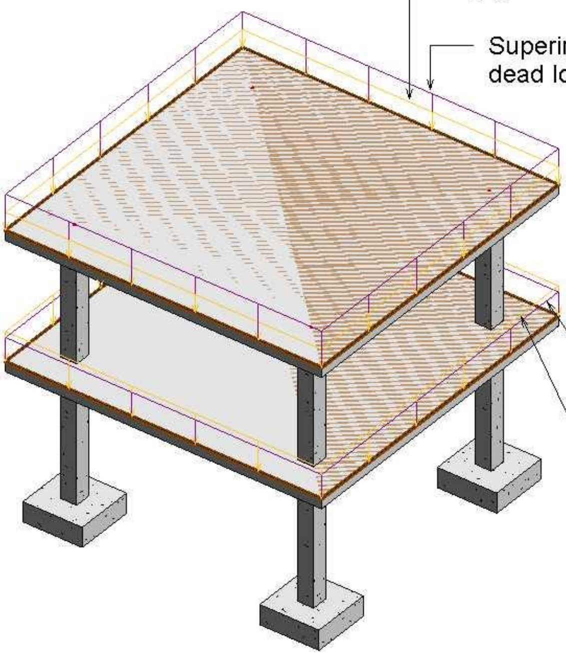


Figure 1.6-11: Frame for Example 1.6-5.

Solution

In 3D, loads acting on slab are as indicated in below:



Resultants for selfweight and superimposed dead loads are:

$R_{Slabs} = 2 \times (8 \times 8 \times 0.25 \times 24) = 768\ kN$

$R_{Columns} = \left((0.4^2 \times (3.25 + 3.00)) \times 24\right) \times 4 = 96\ kN$

$R_{Superimposed} = (3.0 + 2.0) \times 8^2 = 320\ kN$

$R_{Dead} = 768 + 96 + 320 = 1184\ kN$

As the columns are distributed in a symmetrical form, therefore share for each column would be:

$P_{Dead} = \frac{1184}{4} = 296\ kN$

In a similar approach column share due to live load would be:

$P_{Live} = \left((1.5 + 2.5) \times 8^2\right) \times \frac{1}{4} = 64\ kN$

The factored load, P_u, would be:

$P_u = maximum(1.4 \times 296\ or\ 1.2 \times 296 + 1.6 \times 64)$

$P_u = maximum(414.4\ or\ 457.6) \approx 458\ kN$

$P_u = 458\ kN < 0.65 \times 2800 = 1820\ kN \therefore Ok$

Example 1.6-6

Resolve Example 1.6-5 above, with considering the difference between floor live load, L, and roof live load, L_r, in your solution.

Solution

From previous solution:

$P_D = 296\ kN$

The axial force due to floor live is:

$P_L = (2.5 \times 8^2) \times \frac{1}{4} = 40\ kN$

While the axial force due to roof live load is:

$P_{Lr} = (1.5 \times 8^2) \times \frac{1}{4} = 24\ kN$

According Table 1.6-1 above, following load combinations should be considered:

$P_{u1} = 1.4P_D = 1.4 \times 296 = 414\ kN$

$P_{u2} = 1.2P_D + 1.6P_L + 0.5P_{Lr} = 1.2 \times 296 + 1.6 \times 40 + 0.5 \times 24 = 431\ kN$

$P_{u3} = 1.2P_D + 1.0P_L + 1.6P_{Lr} = 1.2 \times 296 + 1.0 \times 40 + 1.6 \times 24 = 434\ kN\ Govern$

$P_u = 434\ kN < 0.65 \times 2800 = 1820\ kN \therefore Ok$

Example 1.6-7

For the one-way structural system of the small apartment building indicated ***Figure 1.3-12***:

- Select an appropriate value for roof live load according to ASCE/SEI 7-10.
- Reduce the selected roof live load, if possible, to determine its resultant on a typical interior beam.
- Determined the floor live load according to ASCE/SEI 7-10. Assume that most of the floor area is for private rooms of more than two families.
- Reduce the selected floor live load, if possible, to determine its resultant on a typical interior beam.
- If a typical interior beam at the floor level is subjected to a dead load (selfweight plus superimposed) of 30kN/m determine the factored negative moment for the cantilever part of the beam.

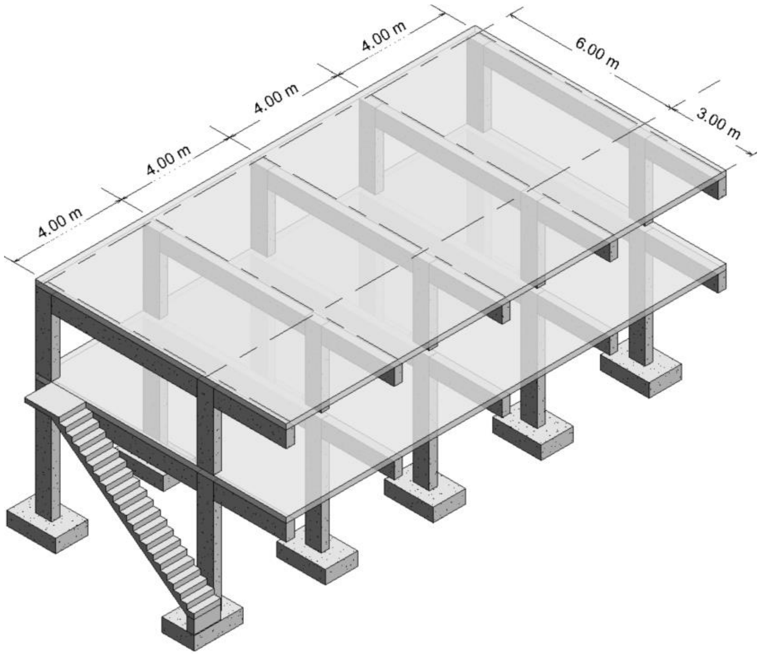


Figure 1.6-12: Structural system for a small apartment building.

Solutions

- Appropriate value for roof live load:
  According to ASCE 7-10, live load for ordinary flat roof is:
  $L_r = 0.96\ kPa$ ■
- Reduce of roof live load:
  The tributary area for a typical interior roof of floor beam is indicated in ***Figure 1.6-13***.
  $A_{T\ typical\ edge\ column} = 4 \times 9 = 36\ m^2 \Rightarrow$
  $\because\ 18.58\ m^2 < A_T < 55.74\ m^2$
  $\therefore R_1 = 1.2 - 0.011 A_T = 1.2 - 0.011 \times 36 = 0.804$

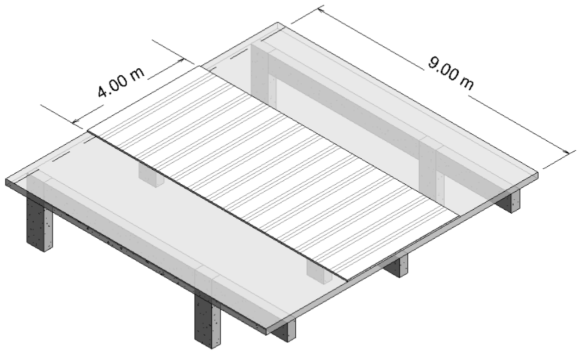


Figure 1.6-13: Tributary area for a typical interior beam.

  As the roof is flat, therefore the reduction factor R_2 is 1.0.
  $L_r = L_o R_1 R_2 = 0.96 \times 0.804 \times 1.0 \approx 0.772\ kPa > 0.58\ kPa\ \therefore Ok.$ ■
  Load share of the roof live load that is supported by a typical interior beam is:
  $W_{Lr} = L_r \times Spacing\ between\ the\ Beams = 0.772 \times 4.0 = 3.01 \frac{kN}{m}$ ■
- Floor live load:
  The building is for residential purpose with floor area mostly for private rooms of more than two families. Therefore, the floor live load according to ASCE/SEI 7-10 is:
  $L = 1.92\ kPa$ ■
- Reduction of floor live load:
  As the live load is smaller than $4.79\ m^2$ and as the influence area is:
  $K_{LL} A_T = 2 \times (4 \times 9) = 72\ m^2 > 37.16\ m^2$
  therefore, the floor live load is reducible according to ASCE/SEI 7-10.
  $L = L_o \left(0.25 + \frac{4.57}{\sqrt{K_{LL} A_T}}\right) = L_o \left(0.25 + \frac{4.57}{\sqrt{72}}\right) = 0.788 L_o > 0.5 L_o\ \therefore Ok.$
  $L = 0.788 L_o = 0.788 \times 1.92 = 1.51\ kPa$ ■
  Load share of the floor live load that is supported by a typical interior beam is:
  $W_L = L \times Spacing\ between\ the\ Beams = 1.51 \times 4.0 \approx 6.00 \frac{kN}{m}$ ■

- Load Combinations

  For a typical beam at the floor level, the following load combinations have to be considered:

  $W_u = \max(1.4W_D, 1.2W_D + 1.6W_L) = \max(1.4 \times 30, 1.2 \times 30 + 1.6 \times 6.00) = 45.6\frac{kN}{m}$

  As nothing is mentioned about column dimensions, therefore the negative moment for cantilever will be conservatively determined at the center of the column:

  $M_u = \frac{W_u \ell^2}{2} = \frac{45.6 \times 3.00^2}{2} = 205\ kN.m$ ■

1.7 STRENGTH (LRFD) VERSUS WORKING-STRESS DESIGN METHODS

As an alternative to the ***Strength Design Method***, members may be proportioned based on the ***Working-Stress Design Method***, where the stresses in the steel and concrete resulting from normal service loads (unfactored loads) should be within a specified limit known as the ***Allowable Stresses***.
Allowable Stresses, in practice, are set at about one-half the concrete compressive strength and one-half the yield stress of steel.
The following Table summarized the main differences between the Strength Design Method and Working-Stress Design Method.

Strength Design Method	**Working-Stress Design Method**
1. Individual load factors may be adjusted to represent different degree of certainty for the various types of loads, and reduction factors likewise may be adjusted to precision with which various types of strength are calculated.	1. All types of loads are treated the same no matter how different in their individual uncertainty.
2. Strength is calculated with explicit regards for inelastic action.	2. Stresses are calculated on the elastic basis.
3. Serviceability with respect to deflection and cracking is considered explicitly.	3. Serviceability with respect to deflection and cracking is considered only implicitly through limits on service loads stress.

Because of these differences, the Strength Design Method has largely displaced the older Allowable Stresses Design Method. *Prior to 2002, Appendix A of the ACI Code allowed design of concrete structures either by Strength Design Method or by Working-Stress Design Method. In 2002, this appendix was deleted.*

1.8 FUNDAMENTAL ASSUMPTIONS FOR REINFORCED CONCRETE BEHAVIOR

As was discussed in the previous article, the uncertainties in the design process have been treated based on the *Strength Reduction Factor* φ and *Load Factors* γ. Therefore, remaining of the design process is based mainly on the *Structural Mechanics* to dealing with the following ***deterministic*** aspects:

1. Compute the theoretical stresses and internal forces (shear, moment, torsion, and axial forces).
2. Compute the theoretical or nominal strength (e.g. M_n, V_n, and P_n).
3. Compute the theoretical deformations and deflections.

The fundamental assumptions on which the *Mechanics of Reinforced Concrete* is based can be summarized as follows:

1.8.1 Assumptions that Related to Equilibrium

All reactions, internal forces, internal stresses and deformations satisfy the equations of equilibrium.

1.8.2 Assumptions that Related to the Compatibility

1. Prefect bonding:
   $\epsilon_{Steel} = \epsilon_{of\ Surrounding\ Concrete}$
2. Cross section, which was plane prior to loading, continues to be plane after loading.

1.8.3 Assumptions that Related to the Constitutive Law

1. Neglecting the tensile strength of concrete.
2. Theory is based on the actual stress-strain relationships and strength properties of the two constituent materials.

1.9 SYLLABUS

1. Based on previous discussions, one can consider any structure like a chain that consists of many rings. Each ring receives loads from previous rings and submits it to the next ones.
2. Syllabus for this course is presented in terms of schematic chain indicated in ***Figure 1.9-1*** below.

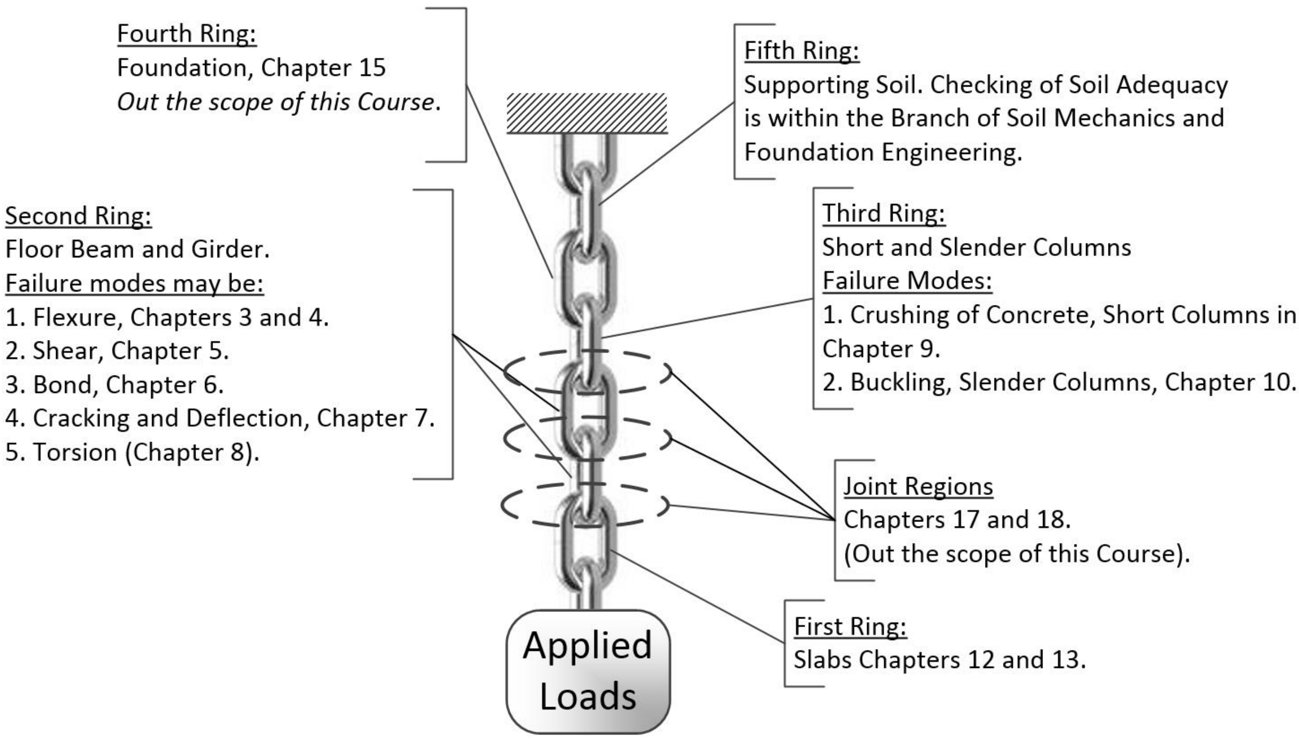


Figure 1.9-1: The chain concept for presenting the syllabus of the course.

1.10 SI UNITS

1.10.1 Metric System in North America

- The American Concrete Institute adopted an SI version of the 1983 ACI Code and continues with the current 2014 SI version.
- In Canada, formal conversion to SI was accomplished with the adoption of the 1977 Canadian Concrete Code.

1.10.2 Different Types of Metric System

- The metric system, although not SI, is used in ***nearly all western hemisphere countries*** (other than the USA and Canada).
- In these countries, the ***MKS*** (meter-kilogram-second) system is used, where instead of the kilogram (kg) as a unit of mass, as in SI, the kilogram (***kgf***) is used as a unit of force.

1.10.3 Soft Metric

- A "***soft conversion***" involves changing a measurement from inch-pound units to equivalent metric units within what the DoD calls "***acceptable measurement tolerances***."
- This is done to ***merely convert the imperial measurements to metric without physically changing the item*** and it is typically used to specify a requirement.
- For example, a ½-in. rebar diameter would be converted to either 12.7 or 13 mm using soft conversion. Although this is not a standard metric rebar size, it expresses the requirement.

1.10.4 Hard Metric

- A "***hard conversion***" involves a change in measurement units that results in a "***physical configuration change***."
- Using the rebar example, this would be ***analogous to changing the diameter of the rebar from ½ in. to an M12 (12-mm) or M14 (14-mm) rebar diameter***.
- Either one of the two new metric rebar diameters would be outside an "***acceptable measurement tolerance***". The new rebar would be considered a "***hard metric***" item.
- This is size substitution, which is one method of using hard conversion; the other method is adaptive conversion, where imperial and metric units are reasonably equivalent, but not exact conversions of each other.

1.11 GENERAL PROBLEMS

Problem 1.11-1

Check adequacy of the beam shown in Figure 1.11-1 below for bending and shear according to the requirements of ACI 318M-14. Assume that $M_n = 400kN.m$ and $V_n = 280kN$. Beam selfweight is not included in the dead load shown. Assume $\phi_{flexure} = 0.9$.

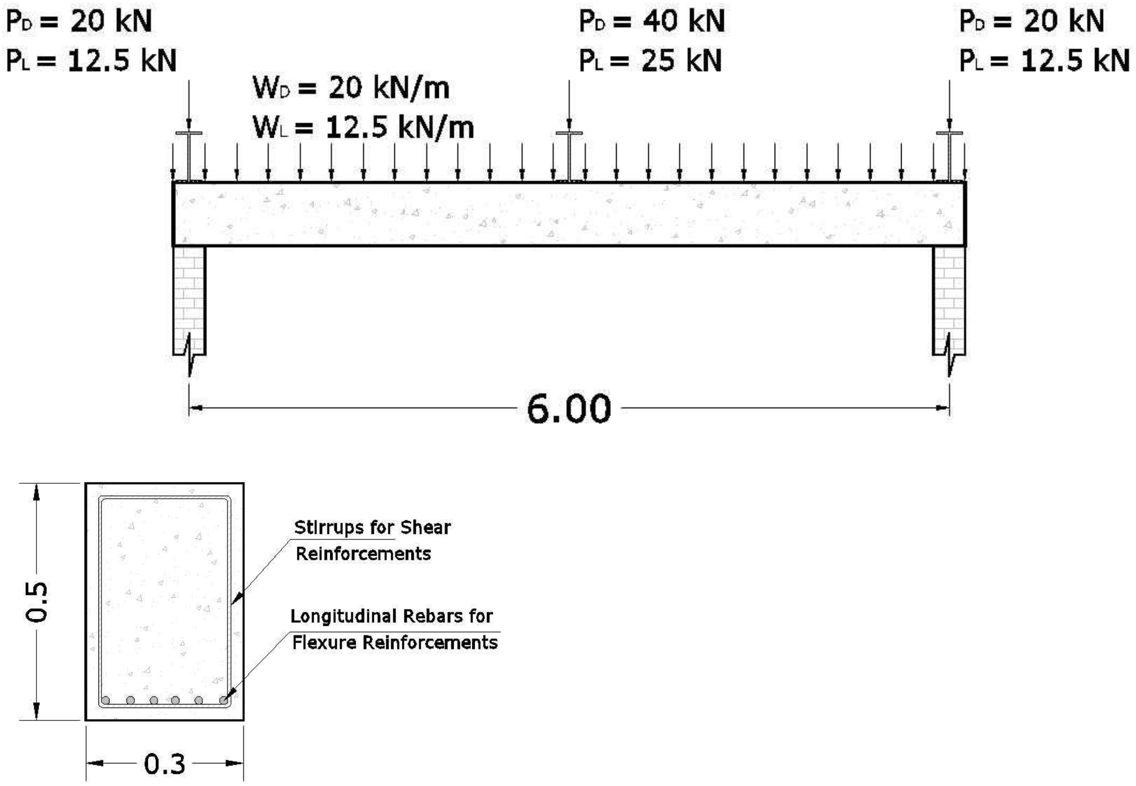


Figure 1.11-1: Beam for Problem 1.11-1.

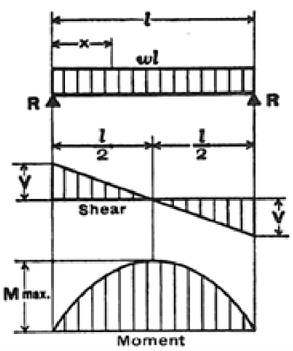
Notes:

1. It is useful to notice that the concentrated loads on girders in civil engineering applications usually resulted from reactions of the floor beams that extend normal to the girder plane, see Figure 1.2-2***.***

2. Designers usually compute the required design internal forces (bending moment, shear force, and axial force) based on the principle of superposition (i.e. compute the effect of each load separately then sum the effects of all loads to obtain the required result), instead of drawing of shear force, bending moment, and axial force diagrams assuming that all loads acting simultaneously.

Design handbooks usually contain shear force, bending moment, and axial force diagrams for beams and simple frames and for typical load conditions. Figures below show the beam diagrams for common load cases in engineering practice:

SIMPLE BEAM—UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load . . . . $= wl$

R = V . . . . $= \frac{wl}{2}$

Vx . . . . $= w\left(\frac{l}{2} - x\right)$

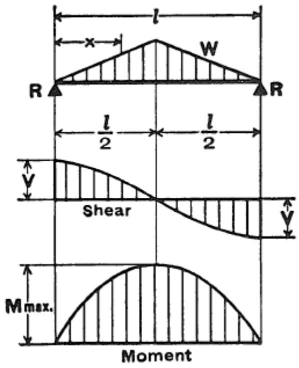
M max. (at center) . . . . $= \frac{wl^2}{8}$

Mx . . . . $= \frac{wx}{2}(l - x)$

Δmax. (at center) . . . . $= \frac{5\,wl^4}{384\,EI}$

Δx . . . . $= \frac{wx}{24EI}(l^3 - 2lx^2 + x^3)$

SIMPLE BEAM—LOAD INCREASING UNIFORMLY TO CENTER



Total Equiv. Uniform Load . . . . $= \frac{4W}{3}$

$R = V$ . . . . $= \frac{W}{2}$

V_x $\left(\text{when } x < \frac{l}{2}\right)$ . . . . $= \frac{W}{2l^2}(l^2 - 4x^2)$

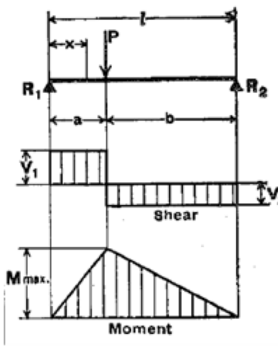
M max. (at center) . . . . $= \frac{Wl}{6}$

M_x $\left(\text{when } x < \frac{l}{2}\right)$ . . . . $= Wx\left(\frac{1}{2} - \frac{2x^2}{3l^2}\right)$

Δmax. (at center) . . . . $= \frac{Wl^3}{60EI}$

Δx $\left(\text{when } x < \frac{l}{2}\right)$ . . . . $= \frac{Wx}{480\,EI\,l^2}(5l^2 - 4x^2)^2$

SIMPLE BEAM—CONCENTRATED LOAD AT ANY POINT



Total Equiv. Uniform Load . . . . . . $= \frac{8\,Pab}{l^2}$

$R_1 = V_1$ (max. when $a < b$) . . . . . $= \frac{Pb}{l}$

$R_2 = V_2$ (max. when $a > b$) . . . . . $= \frac{Pa}{l}$

M max. (at point of load) . . . . . $= \frac{Pab}{l}$

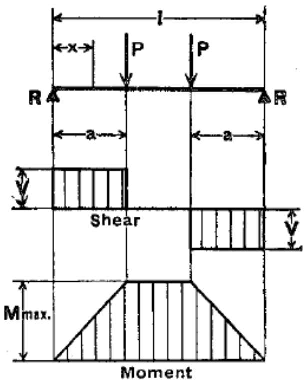
M_x (when $x < a$) . . . . . $= \frac{Pbx}{l}$

Δmax. $\left(\text{at } x = \sqrt{\frac{a(a+2b)}{3}} \text{ when } a > b\right)$ $= \frac{Pab(a+2b)\sqrt{3a(a+2b)}}{27\,EI\,l}$

Δa (at point of load) . . . . . $= \frac{Pa^2b^2}{3EI\,l}$

Δx (when $x < a$) . . . . . $= \frac{Pbx}{6EI\,l}(l^2 - b^2 - x^2)$

SIMPLE BEAM—TWO EQUAL CONCENTRATED LOADS SYMMETRICALLY PLACED



Total Equiv. Uniform Load . . . . . . $= \frac{8\,Pa}{l}$

$R = V$ . . . . $= P$

M max. (between loads) . . . . $= Pa$

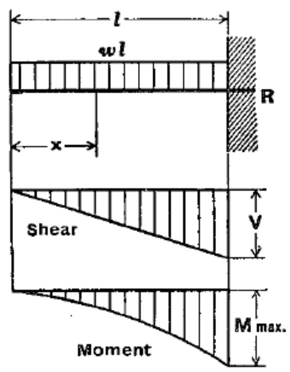
M_x (when $x < a$) . . . . . . $= Px$

Δmax. (at center) . . . . $= \frac{Pa}{24EI}(3l^2 - 4a^2)$

Δx (when $x < a$) . . . . $= \frac{Px}{6EI}(3la - 3a^2 - x^2)$

Δx (when $x > a$ and $< (l - a)$) . . $= \frac{Pa}{6EI}(3lx - 3x^2 - a^2)$

CANTILEVER BEAM—UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load . . . . $= 4wl$

$R = V$ . . . . $= wl$

V_x . . . . $= wx$

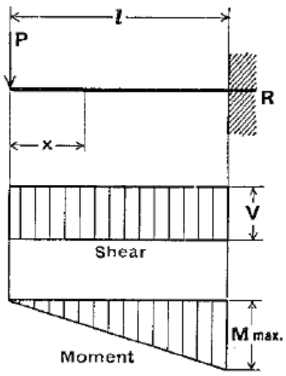
M max. (at fixed end) . . . . . . $= \frac{wl^2}{2}$

M_x . . . . $= \frac{wx^2}{2}$

Δmax. (at free end) . . . . . $= \frac{wl^4}{8EI}$

Δx . . . . $= \frac{w}{24EI}(x^4 - 4l^3x + 3l^4)$

CANTILEVER BEAM—CONCENTRATED LOAD AT FREE END



Total Equiv. Uniform Load . . . . . $= 8P$

$R = V$ . . . . $= P$

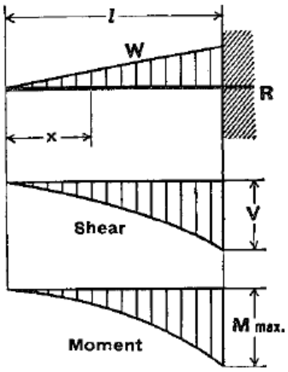
M max. (at fixed end) . . . . . . $= Pl$

M_x . . . . $= Px$

Δmax. (at free end) . . . . $= \frac{Pl^3}{3EI}$

Δx . . . . $= \frac{P}{6EI}(2l^3 - 3l^2x + x^3)$

CANTILEVER BEAM—LOAD INCREASING UNIFORMLY TO FIXED END



Total Equiv. Uniform Load . . . . . $= \frac{8}{3}W$

$R = V$ . . . . $= W$

V_x . . . . $= W\frac{x^2}{l^2}$

M max. (at fixed end) . . . . . . $= \frac{Wl}{3}$

M_x . . . . $= \frac{Wx^3}{3l^2}$

Δmax. (at free end) . . . . . . $= \frac{Wl^3}{15EI}$

Δx . . . . $= \frac{W}{60EIl^2}(x^5 - 5l^4x + 4l^5)$

3. ***Point loads act directly above supports, do not produce shear force nor bending moment in the beam and should be considered only in the design of supports.***

ANSWERS

$M_{u \text{ due to } 1.2DL + 1.6LL} = 349 \text{ kN.m}$, $V_{\text{due to } 1.2DL + 1.6LL} = 189 \text{ kN}$
Section is adequate for flexure and for shear.

Problem 1.11-2

Use bending moment diagrams presented in Problem 1.11-1 above, to compute the bending moment at centerline of the beam presented in Figure 1.11-2 below.

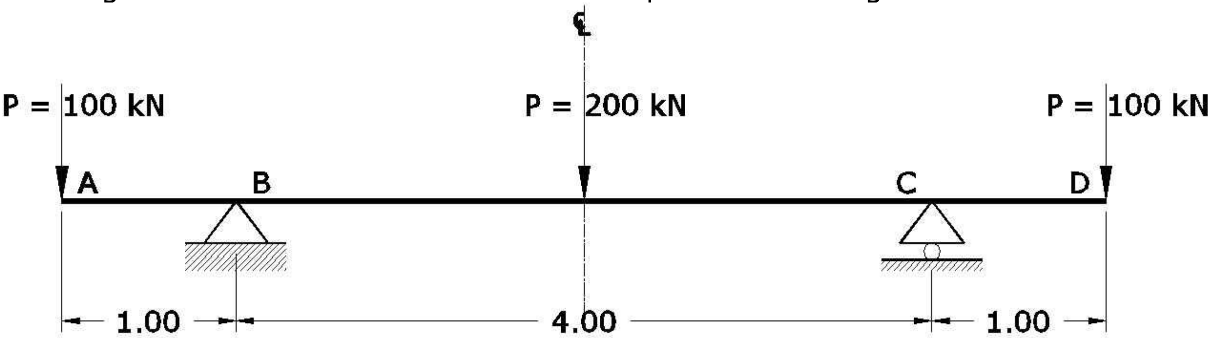
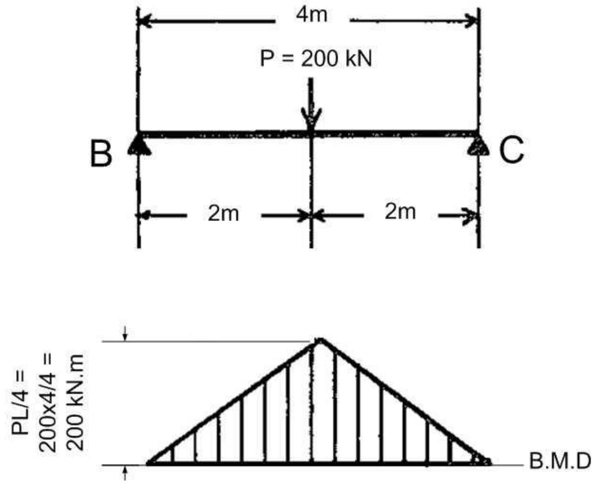


Figure 1.11-2: Overhang beam for Problem 1.11-2.

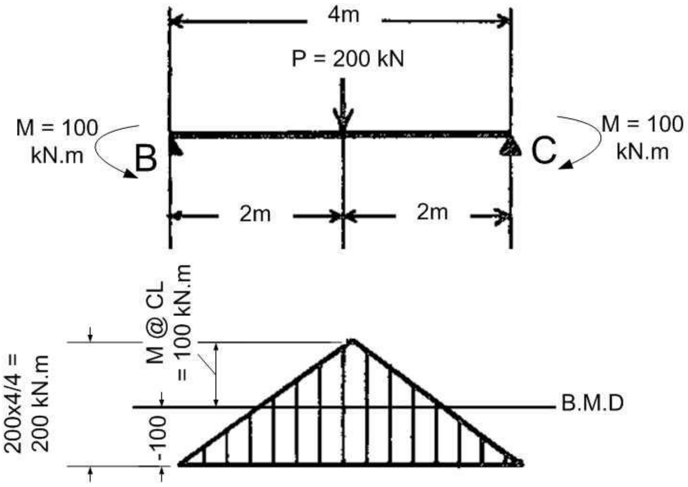
SOLUTION

This problem aims to show that documented diagrams that had been prepared to simple span can be used for problems with overhangs beams and continuous beams. This can be explained as follows:

1. ***Assume that the span BC is simple span, then the bending moment at beam centerline is:***



2. ***It is useful to note that the effects of end moments on moment diagram is that the moment diagram is shifted below for negative end moments and shifted above for positive end moment. For our case with negative end moments of (*** $100\text{kN} \times 1.0\text{m} = 100\ \text{kN.m}$***), the bending moment diagram will be shifted with 100 kN.m in downward direction.***



Problem 1.11-3

Check the adequacy of the beam shown in Figure 1.11-3 below for bending and shear according to the requirements of ACI 318M-14. Assume that $M_n = 650kN.m$ and $V_n = 450kN.m$. Beam selfweight is not included in the dead load shown. Assume $\phi_{flexure} = 0.9$.

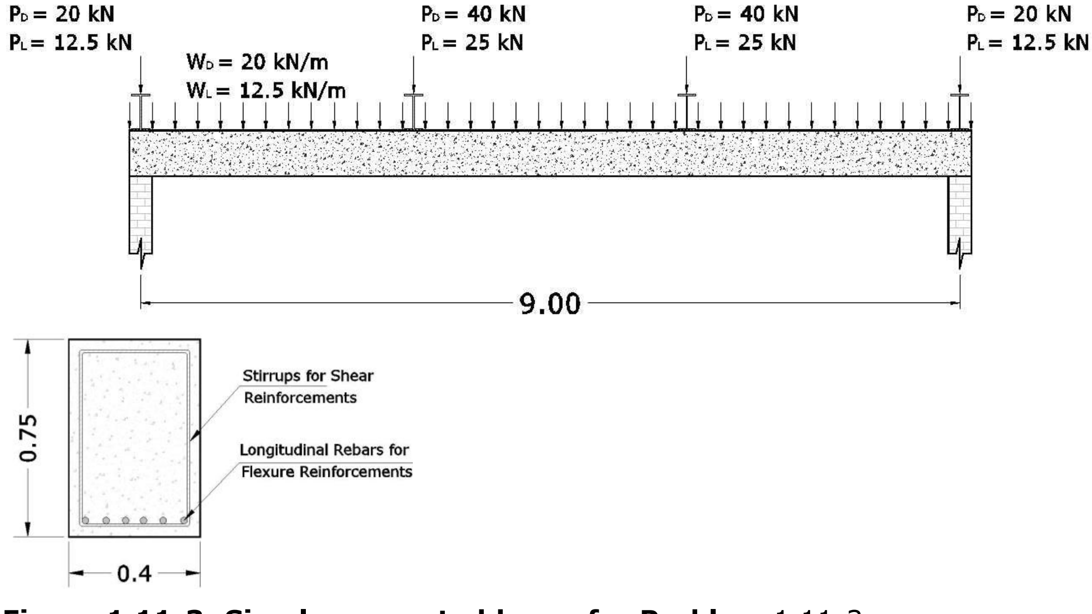


Figure 1.11-3: Simply supported beam for Problem 1.11-3**.**

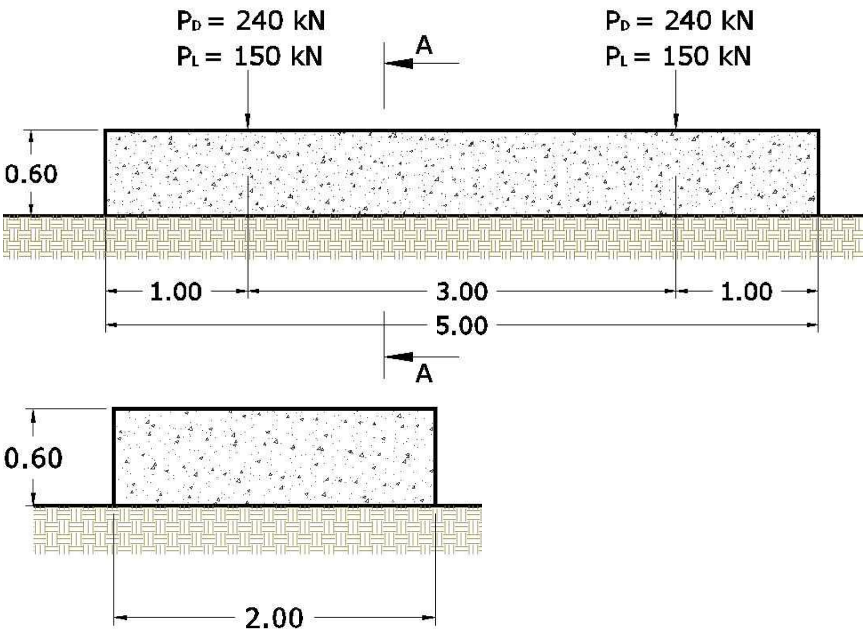
ANSWERS

$M_{u\ due\ to\ 1.2DL+1.6LL} = 797\ kN.m$, $V_{due\ to\ 1.2DL+1.6LL} = 325\ kN$

Section is inadequate for flexure and adequate for shear.

Problem 1.11-4

Check the adequacy of the foundation shown in Figure 1.11-4 below for bending and shear according to the requirements of ACI 318M-14. Assume that $M_n = 200kN.m$ and $V_n = 400kN.m$. Beam selfweight is not included in the dead load shown. Assume $\phi_{flexure} = 0.9$.



Section A-A

Figure 1.11-4: Combined footing for Problem 1.11-4**.**

ANSWERS

$M_{u\ due\ to\ 1.2DL+1.6LL} = 132\ kN.m$, $V_{due\ to\ 1.2DL+1.6LL} = 317\ kN$

Section is adequate for flexure and in adequate for shear.

Problem 1.11-5

For building presented in ***Figure 1.11-5*** below, check adequacy of column (C3) for axial load condition according to the requirements of ACI 318M-14. Assume that:

- Superimposed dead load is $4.0\frac{kN}{m^2}$.
- Live Load is $3\frac{kN}{m^2}$, roof live load is $1\frac{kN}{m^2}$.
- Slab thickness is $200mm$.
- All columns are $400mm \times 400mm$.
- $P_n = 2\ 800kN$.
- Strength reduction factor for column is $\emptyset = 0.65$.
- Your checking must be based on following load conditions:
  - (1.4D), (1.2D + 1.6L+0.5L_r),
  - (1.2D + 1.0L+1.6L_r).

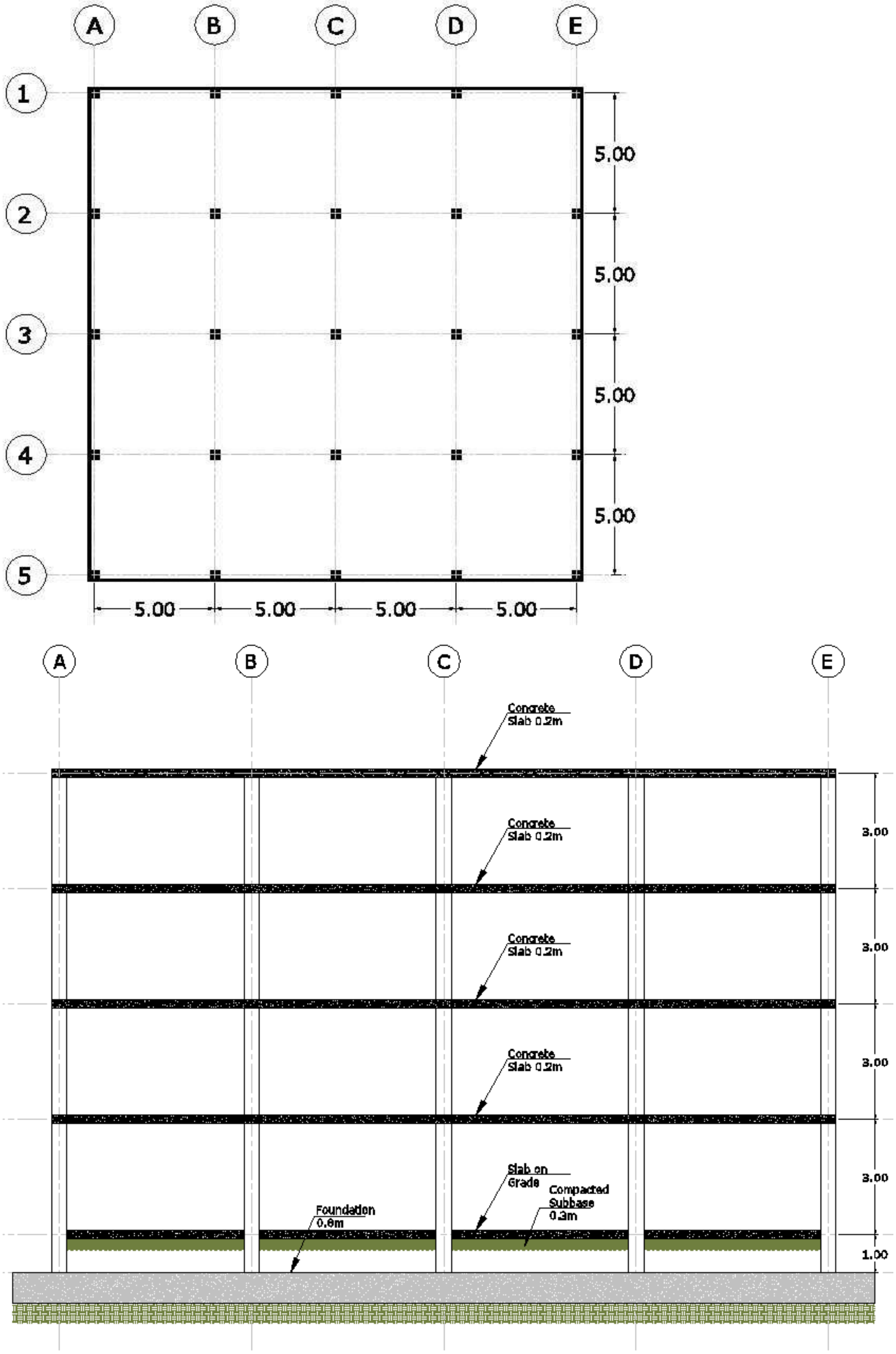


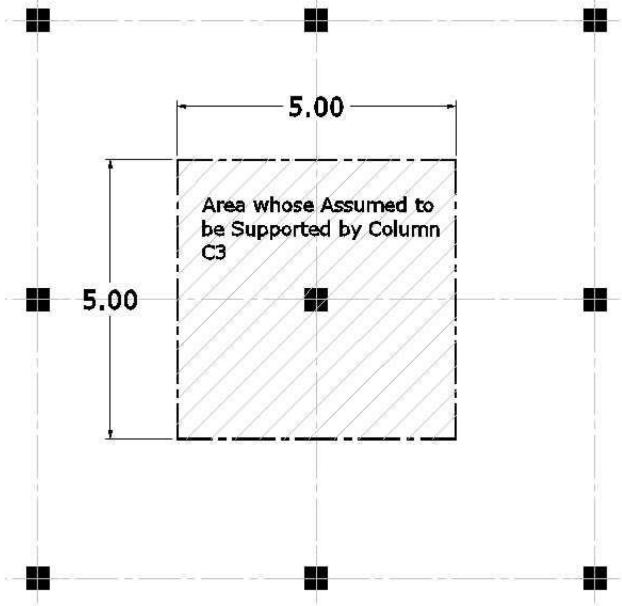
Figure 1.11-5: Building for Problem 1.11-5**.**

Note:

Ground floor is either supported on columns as presented in Example 1.3-1 *or it is supported directly on the underneath soil as assumed in this problem. When supported directly on soil, the slab is usually called as a slab on grade.*

SOLUTION

1. Compute of Basic Load Cases:
   As discussed in Example 1.3-1, it is a common in engineering practice to assume that interior columns are subjected to axial forces only. These forces are computed based on the assumption that an interior column is responsible on supporting an area bounded by centerlines of the four adjacent panels.



$$P_D = \left[\left(0.2m \times 24\frac{kN}{m^3} + 4.0\right)\frac{kN}{m^2} \times 25m^2\right]_{Slab\ Weight} \times 4_{No.of\ Slabs}$$

$$+(0.4^2 m^2 \times 13m)_{Volume\ of\ the\ Column} \times 24\frac{kN}{m^3}$$

$$P_D = 880\ kN + 49.9\ kN = 930\ kN$$

$$P_L = \left(3\frac{kN}{m^2} \times 25\ m^2\right) \times 3_{No.of\ Floors} = 225\ kN$$

$$P_{Lr} = \left(1.0\frac{kN}{m^2} \times 25\ m^2\right) \times 1_{No.of\ Roof} = 25\ kN$$

2. Compute Required Load Combinations:
   $P_{u\ due\ to\ 1.4D} = 1\ 302\ kN$
   $P_{u\ due\ to\ 1.2D\ +\ 1.6L\ +\ 0.5\ Lr} = 1\ 489\ kN$
   $P_{u\ due\ to\ 1.2D\ +\ 1.0L\ +\ 1.6Lr} = 1\ 381\ kN$
   Then the maximum factored load is 1 489 kN due to 1.2D + 1.6L + 0.5L_r.
3. Column Check:
   $\emptyset P_n = 0.65 \times 2\ 800\ kN = 1\ 820\ kN\ \ ?\ P_{u\ due\ to\ 1.2D\ +\ 1.6L\ +\ 0.5\ Lr} = 1\ 489\ kN$
   $\emptyset P_n = 1\ 820\ kN\ > P_{u\ due\ to\ 1.2D\ +\ 1.6L\ +\ 0.5\ Lr} = 1\ 489\ kN\ \ OK.$

Problem 1.11-6

For the elevated reinforced concrete water tank shown Figure 1.11-6 below:

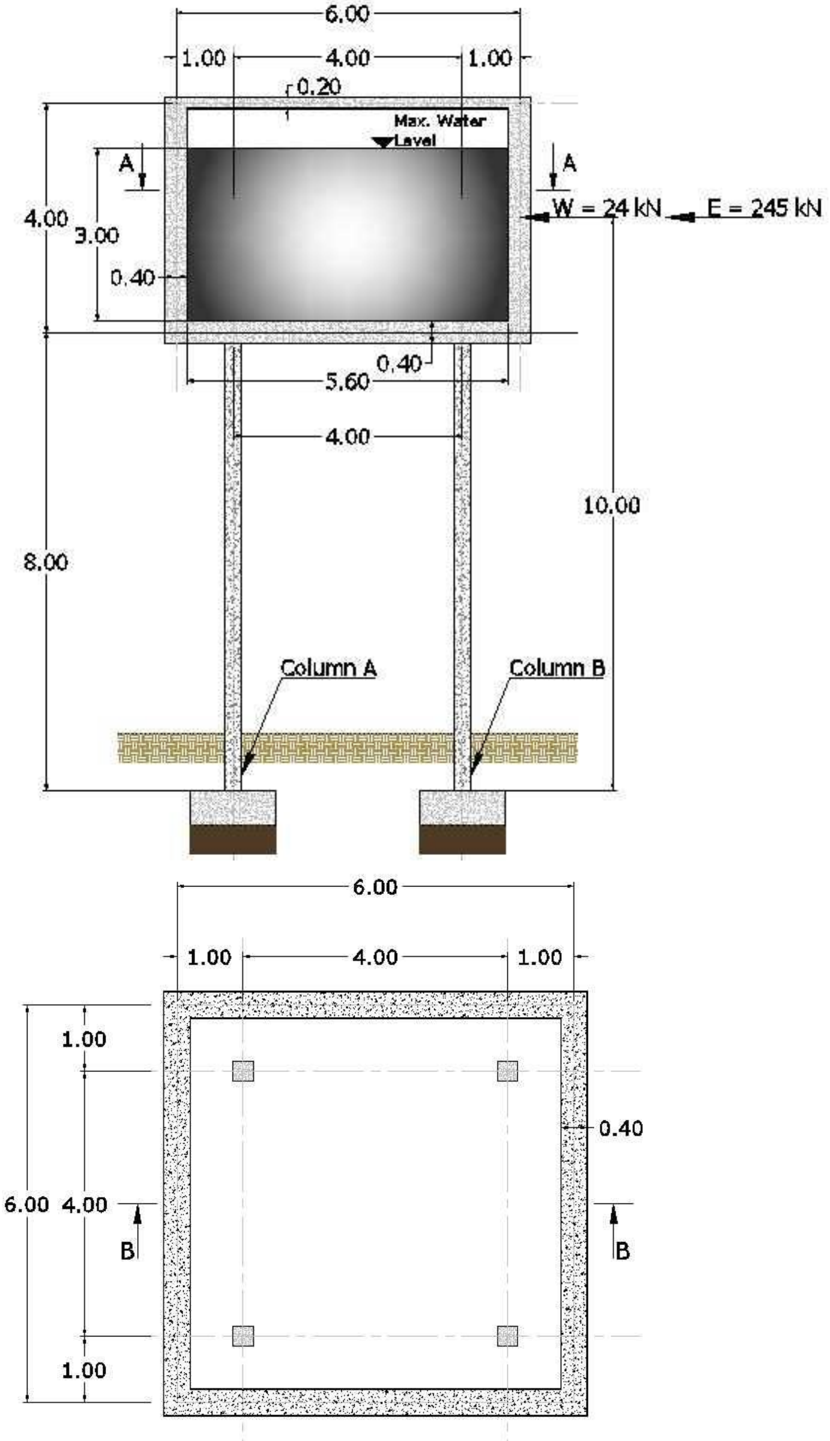
- Check the adequacy of ***Column A*** for axial load condition according to the ***strength requirements*** of ACI 318M-14. Base your strength checking on the following load cases:
  1.4 (D+F), (1.2D + 1.6W), and (1.2D + 1.0E).
- Check the adequacy of column B according to the ***stability requirements***[1] of ACI 318M-14. Base your stability checking on the following load cases:

[1] As most of foundation systems in civil engineering applications are incapable to resist a direct tensile force, then one of the most critical checking is to check that the foundation is not under direct tension. The load combinations with 0.9D are specified for the case where dead load reduce the direct tension effects due to other load conditions (e. g. the direct tension effects of wind and seismic forces on columns B in our Example).

(0.9D + 1.6W), and (0.9D + 1.0E).

In your solution, assume that:

- All columns are $300mm \times 300mm$,
- $P_n = 1\ 700kN$,
- Strength reduction for column is $\emptyset = 0.65$.



Section B-B

Section A-A

Figure 1.11-6: Elevated tank for Problem 1.11-6.

Notes:

One may note that load factor for wind forces adopted in this problem differs from that adopted in Article 1.6.1.1. This difference can be explained as follows, in American codes before (ASCE/SEI 7–10), wind induced forces were having an explicit load factor of 1.6 to simulate uncertainty in wind induced forces. In (ASCE/SEI 7–10), this load factor has been included implicitly through modifying maps for wind speed. Therefore, for wind maps defined by other agencies, the load factor of 1.6 should be included explicitly in definition of wind-induced forces.

ANSWERS

- BASIC LOAD CONDITIONS:
  $\mathrm{P}_{@\text{ Column A \& B due to D}} = 377\ \mathrm{kN}$, $\mathrm{P}_{@\text{ Column A \& B due to F}} = 235\ \mathrm{kN}$,
  $\mathrm{P}_{@\text{ Column A due to W}} = 30\ \mathrm{kN}$, $\mathrm{P}_{@\text{ Column B due to W}} = -30\ \mathrm{kN}$, $\mathrm{P}_{@\text{ Column A due to E}} = 306\ \mathrm{kN}$, and
  $\mathrm{P}_{@\text{ Column B due to E}} = -\ 306\ \mathrm{kN}$.
- Checking Strength for Column A:
  $P_{u\ due\ to\ 1.4\ (D+F)} = 857\ kN$
  $P_{u\ due\ to\ (1.2\mathrm{D}\ +\ 1.6\mathrm{W})} = 500\ kN$
  $P_{u\ due\ to\ (1.2\mathrm{D}\ +\ 1.0\mathrm{E})} = 758\ kN$
  $\therefore P_{u\ Maximum} = 857\ kN < \emptyset P_n = 0.65 \times 1\ 700\ kN = 1\ 105 kN\ \ \therefore Ok.$
- Checking Stability for Column B:
  $P_{u\ due\ to\ (0.9\mathrm{D}\ +\ 1.6\mathrm{W})} = 291\ kN$
  $P_{u\ due\ to\ (0.9\mathrm{D}\ +\ 1.0\mathrm{E})} = 33.3\ kN$
  $\because P_{u\ Minimum} = 33.3\ kN > 0, \therefore Column\ B\ is\ Stable$.

1.12 ADDITIONAL EXAMPLES

Additional Example 1.12-1

Check the adequacy of the reinforced concrete column of the frame shown in Figure 1.12-1 below according to the requirements of ACI Code. In your checking, consider the following load cases:

$U = 1.4D$

$U = 1.2D + 1.6L$

Assume that the column has dimensions of 450mm x 450mm and has a nominal strength of $P_n = 3\,500\ kN$, and a strength reduction factor of 0.65.

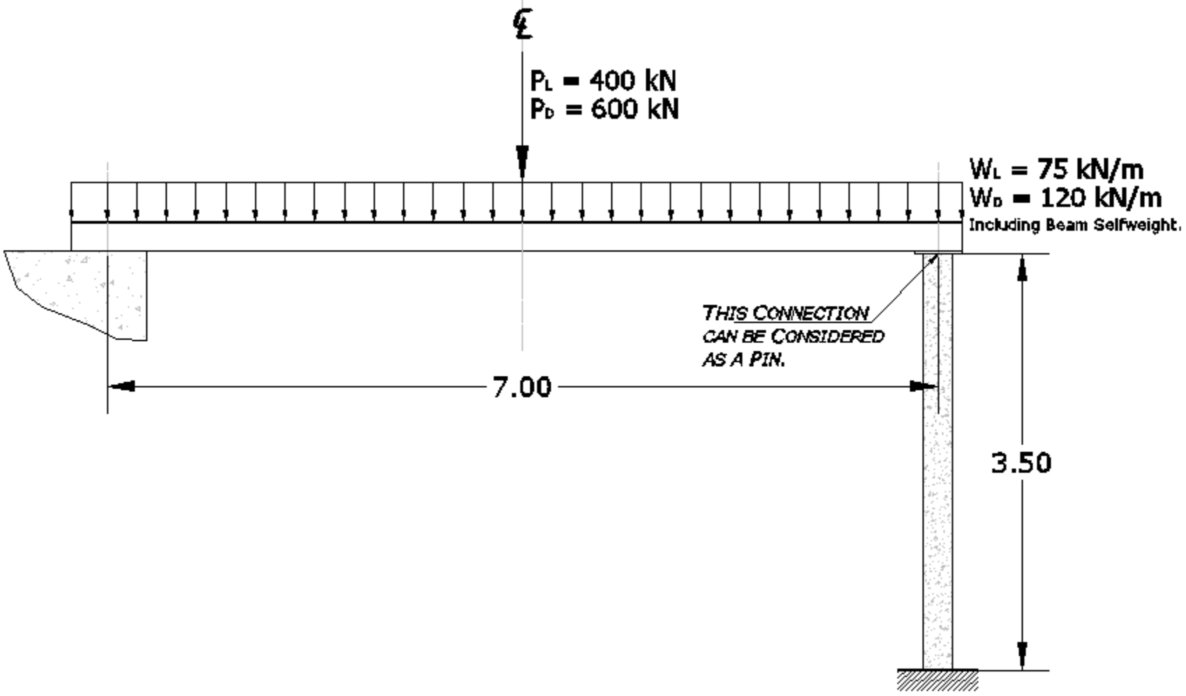


Figure 1.12-1 Frame for Additional Example 1.12-1.

Solution

1. Basic Load Cases:
    a. Compute $P_{Due\ to\ Dead\ Loads}$:

$$P_{Due\ to\ Dead\ Loads} = \frac{\left(120\frac{kN}{m} \times 7.0m\right)}{2} + \frac{600}{2} + (0.45^2 \times 3.5 \times 24) = 737\ kN$$

    b. Compute $P_{Due\ to\ Roof\ Live\ Load}$:

$$P_{Due\ to\ Roof\ Live\ Load} = \frac{75\frac{kN}{m} \times 7.0m}{2} + \frac{400}{2} = 462\ kN$$

2. Load Combinations:

$P_{u\ due\ to\ (1.4D)} = 1.4 \times 737\ kN = 1\,032\ kN$

$P_{u\ due\ to\ [1.2D\ +\ 1.6(L_r or\ S\ or\ R) +\ (1.0L\ or\ 0.8W)]} = 1.2 \times 737 + 1.6 \times 462 = 1\,624\ kN$

3. Check Columns Adequacy:

$P_u\ ?\ \emptyset P_n$

$P_u = 1\,624\ kN < \emptyset P_n = 0.65\ \times 3\,500\ kN = 2\,275\ kN\ \ Ok.$

Therefore, the columns are adequate according to strength requirements of ACI Code.

Additional Example 1.12-2

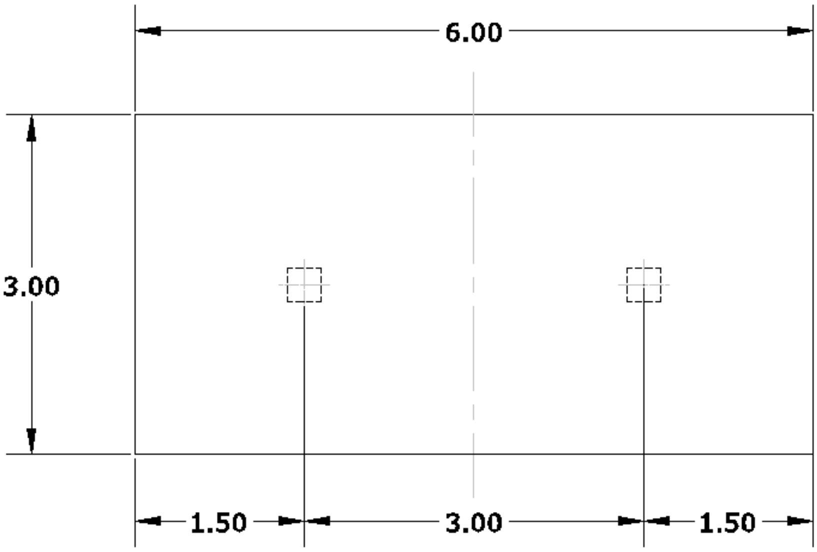
Check columns adequacy for the sunshade with plan and elevation shown in Figure 1.12-2 below according to requirements of ACI Code. Your checking must include following load combination:

$U = 1.4D$

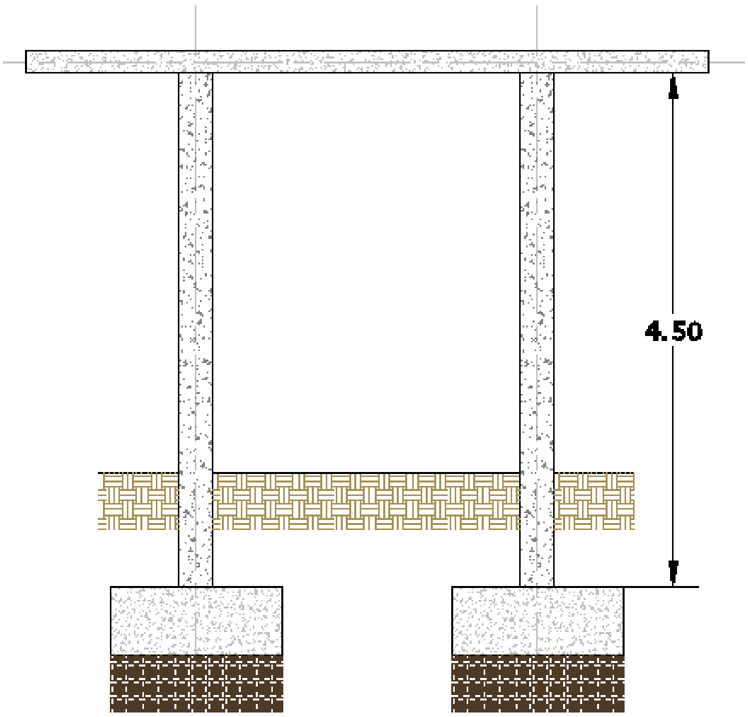
$U = 1.2D + 1.6(L_r or\ S\ or\ R) + (1.0L\ or\ 0.8W)$

Assume that the roof is subjected to a superimposed dead load of 2.0 kPa, roof live load of 1.0 kPa and snow load of 0.75 kPa. Also, assume that wind load effect can be neglected. Concrete slab has a thickness of 200mm.

Assume that all columns have dimensions of 250mm x 250mm and have a nominal strength of $P_n = 1100\ kN$, and a strength reduction factor of 0.65.



A Plan View



An Elevation View

Figure 1.12-2: Shade for Additional Example 1.12-2.

Solution

1. Basic Load Cases:
   a. Compute $P_{Due\ to\ Dead\ Loads}$:

$$P_{Due\ to\ Dead\ Loads} = \frac{\left(0.2m \times 24\frac{kN}{m^3} + 2.0\ kPa\right) \times 18m^2}{2} + (0.25^2 \times 4.5)m^3 \times 24\frac{kN}{m^3}$$

$$= 67.9\ kN$$

   b. Compute $P_{Due\ to\ Roof\ Live\ Load}$:

$$P_{Due\ to\ Roof\ Live\ Load} = \frac{1.0\ kPa\ \times 18\ m^2}{2} = 9.0\ kN$$

As snow load is less than roof live load, then it can be neglected in our checking.

2. Load Combinations:

$P_{u\ due\ to\ (1.4D)} =\ 1.4 \times 67.9\ kN =\ 95.1\ kN$

$P_{u\ due\ to\ [1.2D\ +\ 1.6(L_r or\ S\ or\ R) +\ (1.0L\ or\ 0.8W)]} = 1.2 \times 67.9 + 1.6 \times 9.0 = 95.9\ kN$

3. Check Columns Adequacy:

$P_u\ ?\ \emptyset P_n$

$P_u =\ 95.9\ kN <\ \emptyset P_n = 0.65\ \times 1100\ kN = 715\ kN\ \ Ok.$

Then the columns are adequate according to strength requirements of ACI Code.

Additional Example 1.12-3

Check the adequacy and stability for reinforced concrete columns of the high-elevated cylindrical tank shown in Figure 1.12-3 below according to the requirements of the ACI Code.

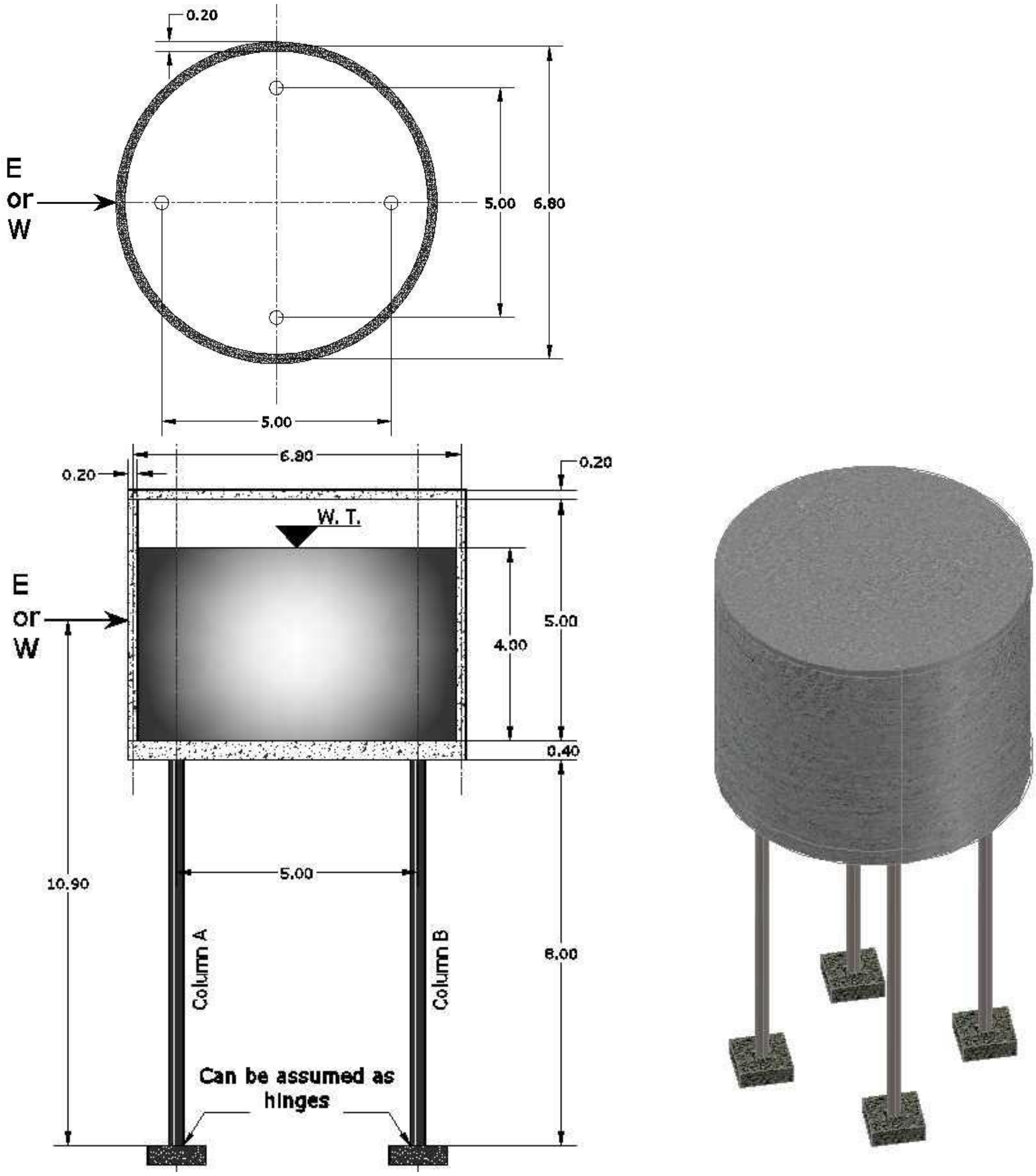


Figure 1.12-3: Elevated concrete tank for Additional Example 1.12-3.

In your solution, assume that:

$W\ =\ 34\ kN$

$E = 0.2D$

Also assume that the column has a diameter of 300mm and has a nominal strength of $P_n = 1\,979\ kN$, and a strength reduction factor of 0.75.

Solution

1. Basic Load Cases:

a. Dead Loads:

$$W_{DRoof} = \frac{(6.8+0.2)^2 \times \pi}{4} \times 0.2 \times 24$$
$$W_{DRoof} = \frac{294\,\pi}{5} = 185\ kN$$
$$W_{DWall} = (6.8 \times \pi \times 0.2 \times 5) \times 24$$
$$W_{DWall} = \frac{816\,\pi}{5} = 512\ kN$$
$$W_{DFloor} = \frac{(6.8+0.2)^2 \times \pi}{4} \times 0.4 \times 24$$
$$W_{DFloor} = \frac{588\,\pi}{5} = 369\ kN$$
$$W_{DColumns} = \left(\frac{0.3^2 \times \pi}{4} \times 8.0\right) \times 24 \times 4$$
$$W_{DColumns} = \frac{432\,\pi}{25} = 54\ kN$$
$$W_D = (185 + 512 + 369 + 54)$$
$$W_D = 1120\ kN$$
$$P_D = 1120 \times \frac{1}{4}$$
$$P_D = 280\ kN$$

b. Fluid Weight:

$$W_{Fluid} = \left(\frac{(6.8-0.2)^2 \times \pi}{4} \times 4\right) \times 10$$
$$W_{Fluid} = \frac{2178\,\pi}{5} = 1368\ kN$$
$$P_{Fluid} = \frac{1368}{4} = 342\ kN$$

c. Wind Loads:

$$P_{Wind} = \pm\left(34 \times \frac{10.9}{5}\right)$$
$$P_{Wind} = \pm\,74\ kN$$

d. Seismic Loads:

$$P_E = (1120 \times 0.2) \times \frac{10.9}{5}$$
$$P_E = \pm\,488\ \text{kN}$$

2. Checking of Column Strength:

$$\text{U} = 1.4(\text{D} + \text{F})$$
$$\text{P}_{u1} = 1.4(280 + 342)$$
$$\text{P}_{u1} = 871\ \text{kN}$$
$$\text{U} = 1.2\text{D} + 1.6\text{W}$$
$$\text{P}_{u2} = 1.2 \times 280 + 1.6 \times 74$$
$$P_{u2} = \frac{2272}{5} = 454\ \text{kN}$$
$$\text{U} = 1.2\text{D} + 1.0\text{E}$$
$$\text{P}_{u3} = 1.2 \times 280 + 1.0 \times 488$$
$$P_{u3} = 824\ kN$$
$$\text{P}_u = \text{Maximum}\ (871\ \text{or}\ 454\ \text{or}\ 824)$$
$$\text{P}_u = 871\ \text{kN} < \phi\text{P}_n = 0.75 \times 1979 = 1484\ \text{kN}\ \ \text{Ok.}\ \blacksquare$$

Therefore, the columns are adequate according to strength requirements of ACI Code.

3. Checking of Columns Stability:

$$U = 0.9D + 1.6W$$
$$P_{u1} = 0.9 \times 280 - 1.6 \times 74$$

$P_{u1} = \frac{668}{5} = 134\ kN > 0 \therefore\ \ Ok.$

$U\ =\ 0.9D\ +\ 1.0E$

$P_{u2} =\ 0.9 \times 280 - 1.0 \times 488$

$P_{u2} = -236\ kN\ < 0\ \therefore\ \ Not\ Ok.$ ■

Then the columns are instable according to stability requirements of ACI Code.

Additional Example 1.12-4

Check the adequacy and stability of reinforced concrete columns for the high-elevated cylindrical tank shown in Figure 1.12-4 below according to the requirements of ACI Code.

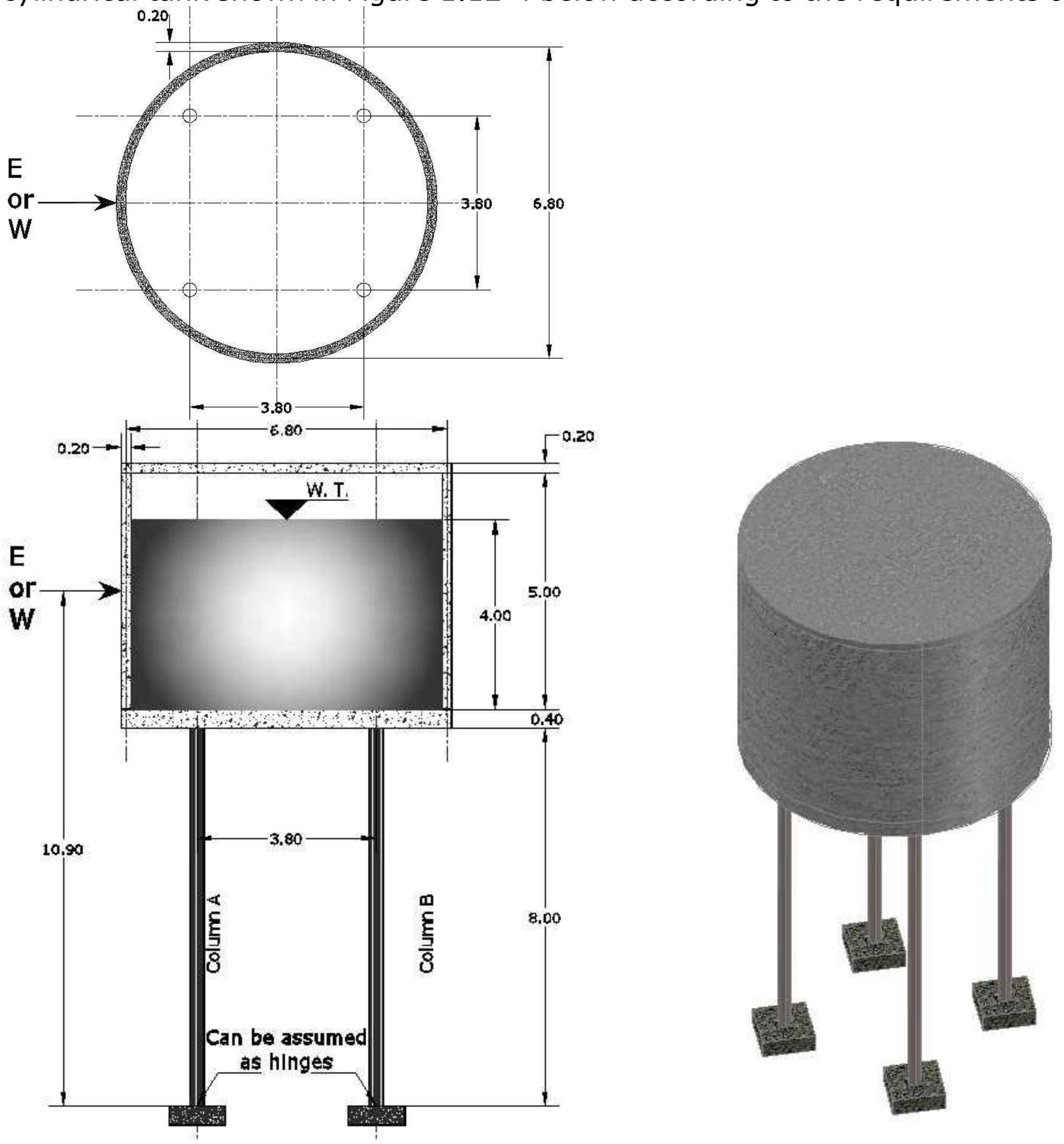


Figure 1.12-4: Elevated tank for Additional Example 1.12-4.

In your solution, assume that:

W = 40 kN

E = 0.25 D

Also assume that the column has a diameter of 300mm and has a nominal strength of P_n = 1979 kN, and a strength reduction factor of 0.75.

Solution

1. Basic Load Cases:

a. Dead Loads:

$$W_{DRoof} = \frac{(6.8+0.2)^2 \times \pi}{4} \times 0.2 \times 24$$
$$W_{DRoof} = \frac{294\,\pi}{5} = 185\ kN$$
$$W_{DWall} = (6.8 \times \pi \times 0.2 \times 5) \times 24$$
$$W_{DWall} = \frac{816\,\pi}{5} = 512\ kN$$
$$W_{DFloor} = \frac{(6.8+0.2)^2 \times \pi}{4} \times 0.4 \times 24$$
$$W_{DFloor} = \frac{588\,\pi}{5} = 369\ kN$$
$$W_{DColumns} = \left(\frac{0.3^2 \times \pi}{4} \times 8.0\right) \times 24 \times 4$$
$$W_{DColumns} = \frac{432\,\pi}{25} = 54\ kN$$
$$W_D = (185 + 512 + 369 + 54)$$
$$W_D = 1120\ kN$$
$$P_D = 1120 \times \frac{1}{4}$$
$$P_D = 280\ kN$$

b. Fluid Weight:

$$W_{Fluid} = \left(\frac{(6.8-0.2)^2 \times \pi}{4} \times 4\right) \times 10$$
$$W_{Fluid} = \frac{2178\,\pi}{5} = 1368\ kN$$
$$P_{Fluid} = \frac{1368}{4} = 342\ kN$$

c. Wind Loads:

$$P_{Wind} = \pm\left(40 \times \frac{10.9}{3.8}\right) \times \frac{1}{2}$$
$$P_{Wind} = \pm 57\ kN$$

d. Seismic Loads:

$$P_E = (1120 \times 0.25) \times \frac{10.9}{3.8} \times \frac{1}{2}$$
$$P_E = \pm 402\ \text{kN}$$

2. Checking of Column Strength:

$$\text{U} = 1.4(\text{D} + \text{F})$$
$$\text{P}_{u1} = 1.4(280 + 342)$$
$$\text{P}_{u1} = 871\ \text{kN}$$
$$\text{U} = 1.2\text{D} + 1.6\text{W}$$
$$\text{P}_{u2} = 1.2 \times 280 + 1.6 \times 57$$
$$P_{u2} = 427\ \text{kN}$$
$$\text{U} = 1.2\text{D} + 1.0\text{E}$$
$$\text{P}_{u3} = 1.2 \times 280 + 1.0 \times 402$$
$$P_{u3} = 738\ kN$$
$$\text{P}_u = \text{Maximum}\ (871\ \text{or}\ 427\ \text{or}\ 738)$$
$$\text{P}_u = 871\ \text{kN} < \phi\text{P}_n = 0.75 \times 1979 = 1484\ \text{kN}\ \ \text{Ok.} \blacksquare$$

Therefore, the columns are adequate according to strength requirements of ACI Code.

3. Checking of Columns Stability:

$$U = 0.9D + 1.6W$$
$$P_{u1} = 0.9 \times 280 - 1.6 \times 57$$
$$P_{u1} = 161\ kN > 0 \therefore\ Ok.$$
$$U = 0.9D + 1.0E$$
$$P_{u2} = 0.9 \times 280 - 1.0 \times 402$$
$$P_{u2} = -150\ kN < 0 \therefore\ Not\ Ok. \blacksquare$$

Therefore, the columns are instable according to stability requirements of ACI Code.

Additional Example 1.12-5

For foundation of pedestrian bridge presented in Figure 1.12-5 below, section A-A has been designed with M_n = 800 kN.m and φ = 0.9. Is this section adequate according to ACI Code requirement?

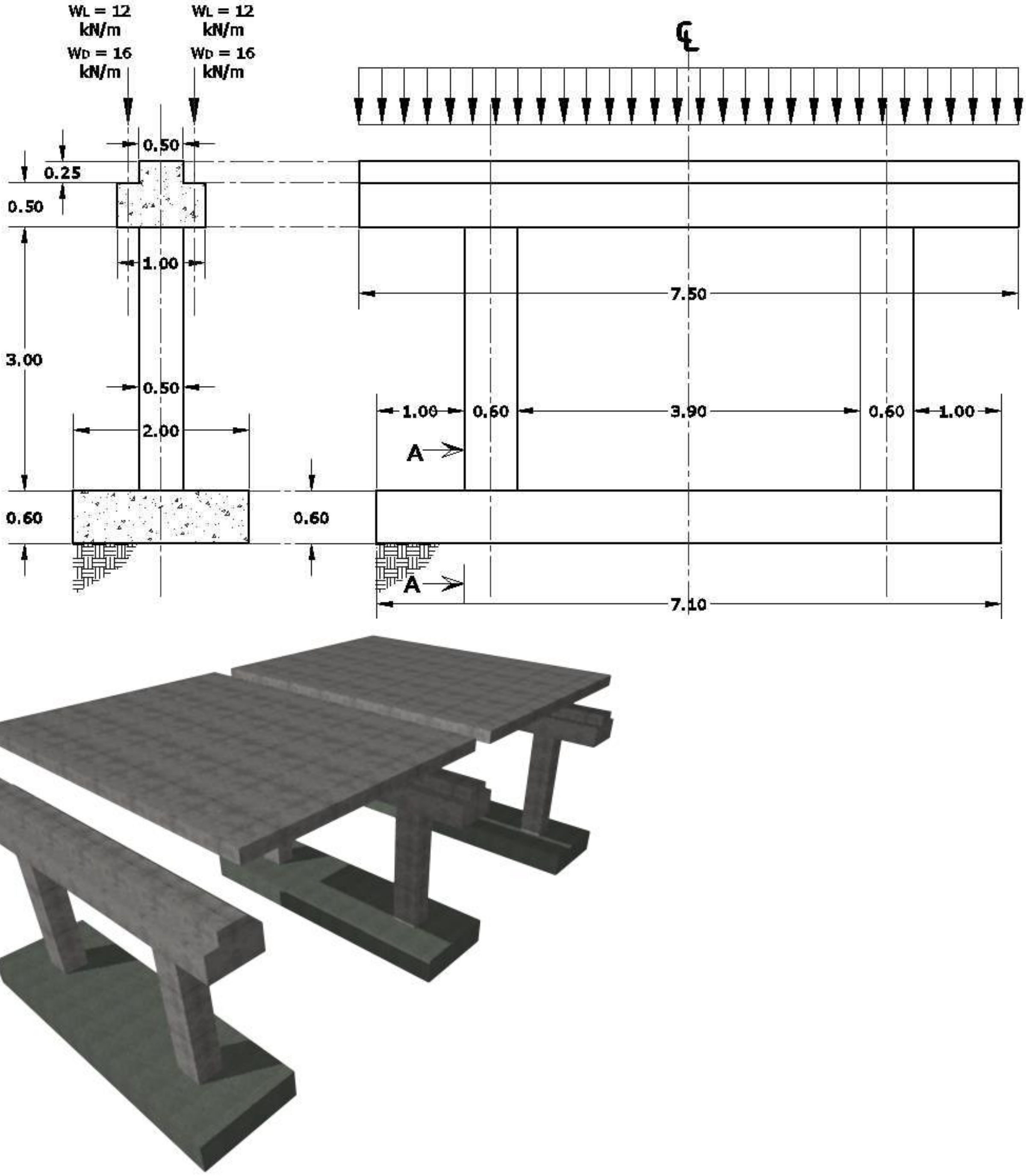


Figure 1.12-5: Pedestrian bridge for Additional Example 1.12-5.

In your Strength Checking, consider following Load Combinations:

- $1.4\,D$,
- $1.2D + 1.6L$.

In your solution, assume that:

- $\gamma_{concrete} = 24\,kN/m^3$,
- Uniform subgrade reaction.

Solution

Dead Load:

$$Volume\ of\ Concrete$$
$$= [(0.5 \times 0.25 + 0.5 \times 1.0)m^2 \times 7.5m]_{Vol\ of\ cross\ beam}$$
$$+ [(0.5 \times 0.6)m^2 \times 3m \times 2]_{Vol.of\ columns}$$
$$Volume\ of\ Concrete\ = [4.69\ m^3]_{Vol\ of\ cross\ beam} + [1.8\ m^3]_{Vol.of\ columns}$$
$$Selfweight\ of\ Concrete\ = \left(4.69m^3 \times 24\frac{kN}{m^3}\right)_{weight\ of\ beam} + \left(1.8\ m^3 \times 24\frac{kN}{m^3}\right)_{weight\ of\ columns}$$
$$Selfweight\ of\ Concrete\ = 112.6 + 43.2 = 156\ kN$$
$$R_D = \left(16\frac{kN}{m} \times 2 \times 7.5m\right) + 156\ kN =$$
$$R_D = 240\ kN + 156\ kN = 396\ kN$$

Live Load:

$$R_{Live} = 12\frac{kN}{m} \times 2 \times 7.5\ m = 180\ kN$$

Subgrade Reactions:

$$W_D = \frac{396\ kN}{7.1\ m} = 55.8\frac{kN}{m}$$
$$W_L = \frac{180\ kN}{7.1\ m} = 25.4\frac{kN}{m}$$

Factored Load:

$$W_u = maximum\ (1.4 \times 55.8\ kN, 1.2 \times 55.8\ kN + 1.6 \times 25.4\ kN)$$
$$W_u = maximum\ \left(78.1\frac{kN}{m}, 108\frac{kN}{m}\right) = 108\frac{kN}{m}$$

Factored Moment:

$$M_u = 108\frac{kN}{m} \times 1.0m \times \frac{1}{2}m = 54\ kN.m$$

Checking of Section Adequacy:

$M_u\ ?\ \phi M_n$

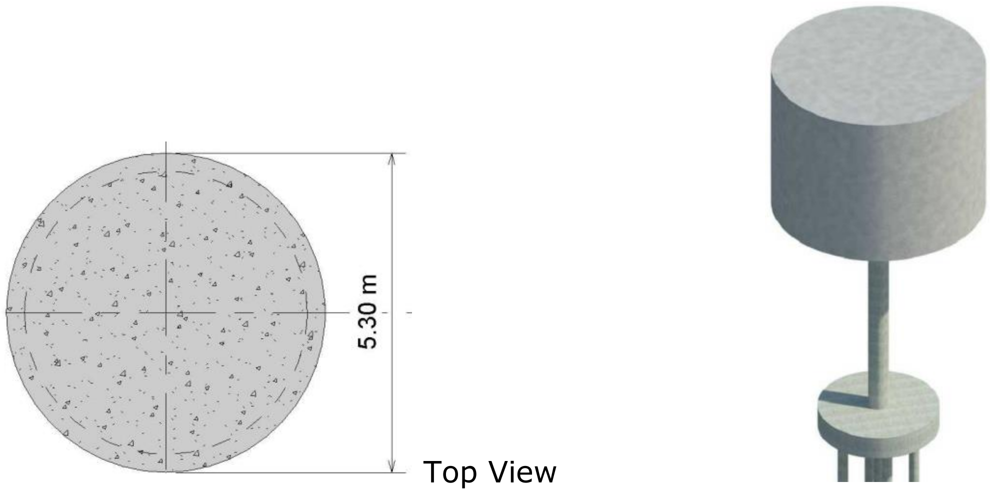
$54\ kN.m\ ?\ 0.9 \times 800\ kN.m$

$54\ kN.m < 720\ kN.m\ Ok.$ ■

Additional Example 1.12-6

Due to weak soil conditions, water tank shown in Figure 1.12-6 below is supported on a piled foundation. With this foundation, tank stability is ensured.

- What are ACI load combinations that should be considered in checking strength adequacy of the supporting column?
- If the column has a nominal axial strength, P_n, of 1200 kN and a design flexural strength, ϕM_n, of 300 kN.m, is it adequate from strength point of view? In your solution, assume reduction strength factor, ϕ, of 0.75.



Top View 3D View

Figure 1.12-6: Tank for Additional Example 1.12-6.

Solution

- Load Combinations for Strength Checking
  As stability is ensured with the piled foundation, then only strength combinations should be considered in the solution.
  $U_{DF} = 1.4(D + F)$
  $U_{DW} = 1.2D + 1.6W$
  $U_{DE} = 1.2D + 1.0E$
- Strength Checking of the Column

Basic Loads

Dead Load

$$P_{D\ Roof} = P_{D\ Floor} = \frac{\pi \times 5.3^2}{4} \times 0.3 \times 24 = 159\ kN$$

$$P_{D\ Wall} = (4.7 + 0.3) \times \pi \times 0.3 \times 3.4 \times 24 = 385\ kN$$

$$P_{D\ Column} = \frac{\pi \times 0.5^2}{4} \times 6.00 \times 24 = 28.3$$

$$P_D = 159 \times 2 + 385 + 28.3 = 731\ kN$$

Fluid

$$P_F = \frac{\pi \times 4.7^2}{4} \times 2.82 \times 10 = 489\ kN$$

Wind

$$M_W = 32 \times 8.0 = 256\ kN.m$$

Seismic

$$M_E = 0.2 \times 731 \times 8.00 = 1170\ kN.m$$

Load Combinations

Load Combination of 1.4(D+F)

$$P_u = 1.4 \times (731 + 489) = 1708\ kN > 0.75 \times 1200 = 900\ kN \therefore Not\ Ok.$$

Load Combination of (1.2D +1.6W)

$$P_u = 1.2 \times 731 = 877\ kN < 0.75 \times 1200 = 900\ kN \therefore Ok.$$

$$M_u = 1.6 \times 256 = 410\ kN.m > 300\ kN.m \therefore Not\ Ok.$$

Load Combination of (1.2D +E)

$$P_u = 1.2 \times 731 = 877\ kN < 0.75 \times 1200 = 900\ kN \therefore Ok.$$

$$M_u = 1.0 \times 1170 = 1170\ kN.m > 300\ kN.m \therefore Not\ Ok.$$

Therefore, the proposed is inadequate.

Important Notes

It will be shown in Chapter 8, Short Columns, that a column has different design strength, ϕP_n and ϕM_n, for different combinations of P_u and M_u. Therefore, using same design strength for different load combinations, as done in this example, is not accurate and has been adopted only to present the probabilistic nature of current ACI design philosophy.

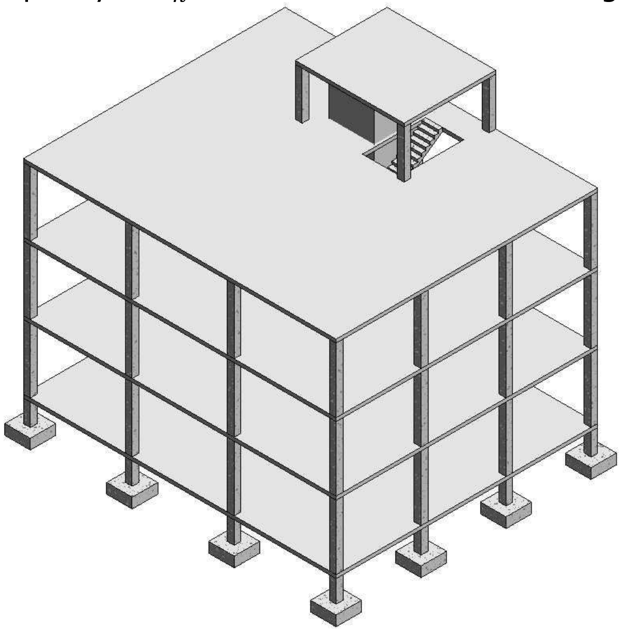
Additional Example 1.12-7

For a hotel building indicated in ***Figure 1.12-7*** below that has flat plate slabs ***200mm*** in thickness, columns of ***400mm by 400mm***, and it is subjected to a floor superimposed dead load of ***2.0 kPa*** and to a roof superimposed dead load of ***3.0 kPa***:

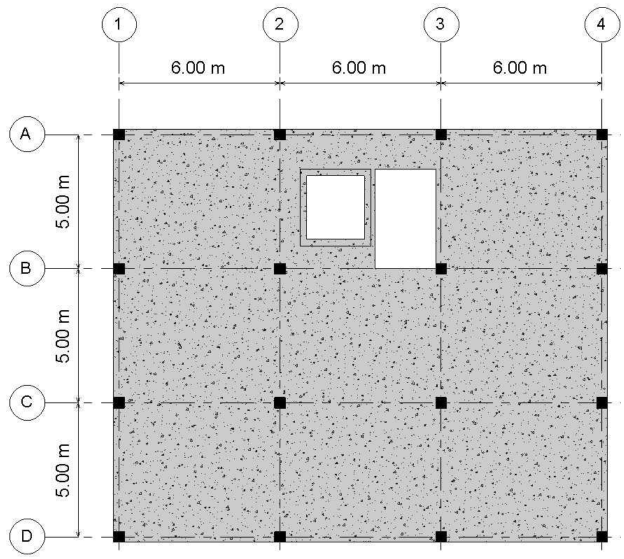
- Select an appropriate value for roof live load,
- Select an appropriate value for floor live load. Most of floor area is proposed for private rooms,
- For a column located at grid lines D-1:
  - What is column axial force at foundation level due to selfweight, i.e. P_{Self}?
  - What is column axial force at foundation level due to superimposed dead load, i.e. $P_{Superimposed\ Dead}$?
  - With adopting reduced live load, if possible, what is column axial force at foundation level due to floor live load, i.e. P_L?
  - With adopting reduced roof live load, if possible, what is column axial force at foundation level due to roof live load, i.e. P_{L_r}?

- What is column maximum ultimate axial load, P_u, due to following load combinations:
  $U = 1.2D + 1.6L + 0.5L_r$
  $U = 1.2D + 1.0L + 1.6L_r$

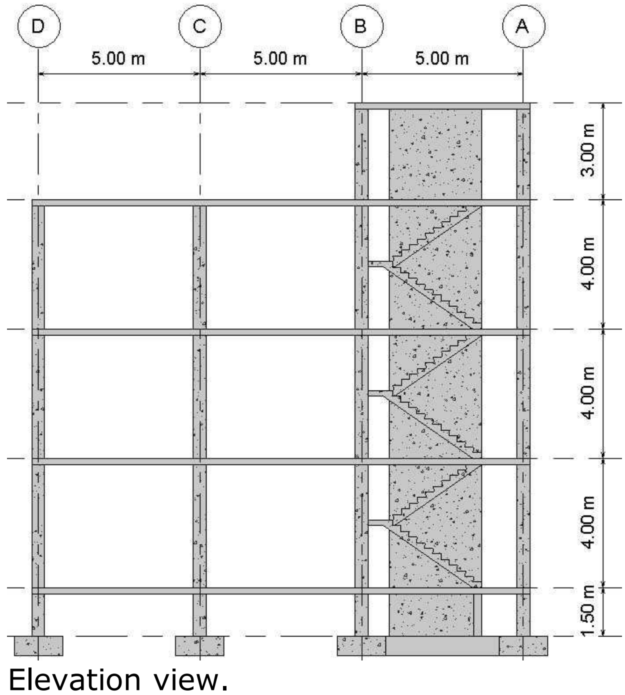
Is the column adequate to support aforementioned axial force, P_u, if it has an axial capacity of P_n of 3550 kN and has a strength reduction factor, ϕ, of 0.65?



3D view.



Plan view.



Elevation view.

Figure 1.12-7: Structural system for a hotel building for Additional Example 1.12-7.

Solution

- Appropriate value for roof live load:
  According to ASCE 7-10, live load for ordinary flat roof is:
  $L_r = 0.96\ kPa$
- Appropriate value for floor live load:
  According to ASCE 7-10, for private rooms in residential or hotel building, a live load of:
  $L = 1.92\ kPa$
  can be adopted.
- Axial force at foundation level due to selfweight:
  Assuming that the corner column located at grid line D-1 supports a tributary of:
  $$A_{Tributary} = \frac{5 \times 6}{4} = 7.5\ m^2$$

$P_{Self} = (7.5 \times 0.2 \times 24) \times 4 + (0.4^2 \times (1.5 - 0.2 + (4.0 - 0.2) \times 3) \times 24 = 193\ kN$ ■

- Axial force at foundation level due to superimposed dead load:
  $P_{Superimposed} = 7.5 \times (3.0 + 3 \times 2.0) = 67.5\ kPa$ ■
- Axial force at foundation level due to floor live load:
  Firstly, check to see if floor live load is reducible:
  According to ***Table 1.3-2*** above, and as there is no cantilever slab,
  $K_{LL} = 4$,
  The influence area is:
  $\because K_{LL}A_T = 4 \times 7.5 \times 3 = 90\ m^2 > 37.16\ m^2 \therefore Ok.$
  $\because L = 1.92\ kPa < 4.79\ kPa \therefore Ok.$
  As two conditions are satisfy, therefore floor live load can be reduced according to following relation:
  $$L = L_o(0.25 + \frac{4.57}{\sqrt{K_{LL}A_T}})$$
  $$L = L_o\left(0.25 + \frac{4.57}{\sqrt{4 \times 7.5 \times 3}}\right) = 0.732\ L_o > 0.4L_O \therefore Ok.$$
  $P_L = 0.732 \times 1.92 \times (7.5 \times 3) = 31.6\ kPa$
- Axial force at foundation level due to roof live load:
  Firstly, check to see if roof live load is reducible:
  $\because A_T = 7.5\ m^2 < 18.58\ m^2$
  Therefore, roof live load cannot be reduced, and the axial force due to roof live would be:
  $P_{L_r} = 7.5 \times 0.96 = 7.2\ kN$ ■
- Maximum ultimate axial load, P_u:
  $P_u = Maximum\ (1.2P_D + 1.6P_L + 0.5P_{L_r}\ or\ 1.2P_D + 1.0P_L + 1.6P_{L_r})$
  $$P_u = Maximum\ (1.2(193 + 67.5) + 1.6 \times 31.6 + 0.5 \times 7.2\ or\ 1.2(193 + 67.5) + 1.0 \times 31.6 + 1.6 \times 7.2)$$
  $P_u = Maximum\ (367\ kN\ or\ 356) = 367\ kN$ ■
- Column Adequacy:
  With a nominal strength, P_n, of 3550 kN, it is easy to show that the proposed column is adequate:
  $P_u = 367\ kN \ll \phi P_n = 0.65 \times 3550 = 2308\ kN \therefore Ok.$

1.13 CONTENTS

CHAPTER 2
MATERIALS

2.1 INTRODUCTION

- The structures and component members treated in this course are composed of:
  - Concrete,
  - Reinforcing steel bars,
- An understanding of the materials characteristics and behavior under load is fundamental to understanding the performance of structural concrete, and to safe, economical, and serviceable design of concrete structures.
- Although ***prior exposure to the fundamentals of material behavior is assumed***, a brief review is presented in this chapter

2.2 CONCRETE, CHEMICAL ASPECTS

2.2.1 Cement

2.2.1.1 Hydraulic Cement

For making structural concrete, hydraulic cements are used exclusively. ***Water is needed for the chemical process*** (***<u>hydration</u>***) in which the cement powder sets and hardens into one solid mass.

2.2.1.2 Portland Cement

- Of the various hydraulic cements that have been developed, ***portland cement***, which was first patented in England in 1824, is by far ***the most common***.
- Portland cement is a finely powdered, grayish material that consists chiefly of ***calcium*** and ***aluminum silicates***.

2.2.1.2.1 Common Raw Materials for Portland Cement

- Limestones, which provide ***CaO***,
- Clays or shales, which furnish SiO_2 and Al_2O_3.

2.2.1.2.2 Manufacturing of Portland Cement

- Raw materials are ground, blended,
- Then fused to clinkers in a kiln, and cooled,
- Gypsum is added to the mixture,
- Mixture is ground to the required fineness.

2.2.1.2.3 Types of Portland Cements

Five standard types of Portland cement have been developed:

- Type I, Normal Portland Cement:
  - It is used for ***over 90 percent of construction***.
  - Concretes made with Type I Portland cement generally need ***one to two weeks to reach sufficient strength so that forms of beams and slabs can be removed and reasonable loads applied***; they reach their ***design strength after 28 days*** and continue to gain strength thereafter at a decreasing rate.
- Type III, High Early Strength Cements:
  - To speed construction when needed, ***Type III*** cement have been developed. They are ***costlier than ordinary Portland cement***, but ***within 7 to 14 days they reach the strength achieved using Type I at 28 days***.
  - Type III Portland cement contains the same basic compounds as Type I, but ***the relative proportions differ*** and it is ***ground more finely***.
- Type V, Sulfate-resisting Cement
  - This cement has a ***low C3A*** content to avoid ***sulfate attack from <u>outside</u> the concrete***; otherwise, the formation of ***calcium sulfoaluminate*** and ***gypsum*** would cause disruption of the concrete due to an increased volume of the resultant compounds.
  - The salts particularly active are ***magnesium*** and ***sodium*** sulfate.
  - Sulfate attack is greatly ***accelerated if accompanied by alternate wetting and drying***, e.g. in marine structures subject to tide or splash.
  - The heat developed by ***sulfate-resisting cement is not much higher than that of low-heat cement***, which is an advantage, but the cost of the former is higher due to the

special composition of the raw materials. Thus, in practice, ***sulfate-resisting cement should be specified only when necessary; it is not a cement for general use***.

2.2.1.2.4 Setting and Hydration

- When cement is mixed with water to form a soft paste, it gradually stiffens until it becomes a solid. This process is known as ***setting and hardening***.
- The cement is said to have ***set*** when it has ***gained sufficient rigidity to support an arbitrarily defined pressure***, after which it continues for a long time to ***harden***, i.e., ***to gain further strength***.
- The water in the paste dissolves material at the surfaces of the cement grains and forms a ***gel*** that gradually increases in volume and stiffness. This leads to a rapid stiffening of the paste ***2 to 4 hours*** after water has been added to the cement.
- Hydration continues to proceed deeper into the cement grains, at decreasing speed, with continued stiffening and hardening of the mass. The ***principal products of hydration*** are ***calcium silicate hydrate***, which is insoluble, and ***calcium hydroxide***, which is soluble.
- Water/Cement Ratio for Hydration
  - For complete hydration of a given amount of cement, an amount of water equal to about ***25 percent*** of that of cement, by weight-i.e., a water-cement ratio of ***0.25***, is needed ***chemically***.
  - An additional amount must be present, however, ***to provide mobility for the water in the cement paste during the hydration process so that it can reach the cement particles*** and to ***provide the necessary workability of the concrete mix***.
  - For ***normal concretes***, the water-cement ratio is generally ***in the range of about 0.40 to 0.60***,
  - For ***high-strength concretes***, ratios ***as low as 0.21*** have been used. In this case, the needed workability is obtained through the use of ***admixtures***.
- Heat of Hydration
  - The chemical process involved in the setting and hardening liberates heat, known as ***heat of hydration***.
  - In ***large concrete masses***, such as dams, ***this heat is dissipated very slowly*** and results in a temperature rise and volume expansion of the concrete during hydration, with subsequent cooling and contraction.

2.2.2 Aggregates

- In ordinary structural concretes, the aggregates occupy ***65 to 75 percent of the volume of the hardened mass***.
- The ***remainder*** consists of ***hardened cement paste***, ***uncombined water*** (i.e., water not involved in the hydration of the cement), and ***air voids***.

2.2.2.1 Gradation of Aggregate

Gradation of Aggregate versus Durability of Concrete

- In general, ***the more densely the aggregate can be packed***, the ***better the durability*** and ***economy*** of the concrete.
- For this reason, the ***gradation of the particle sizes in the aggregate***, to produce ***close packing***, is of considerable importance.

2.2.2.2 Important Properties of Aggregate

It is important that the aggregate

- have good strength,
- have good durability,
- have good weather resistance;
- its surface be free from impurities such as loam, clay, silt, and organic matter that may weaken the bond with cement paste;
- have no unfavorable chemical reaction with the cement.

2.2.2.3 Fine and Coarse Aggregate

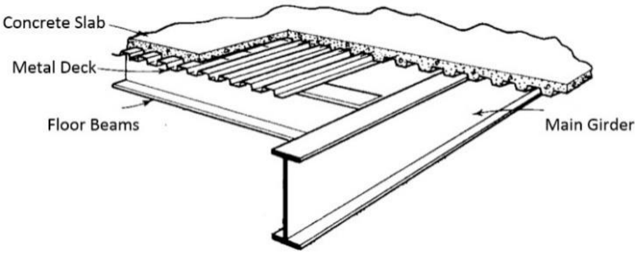
- Natural ***aggregates*** are generally ***classified*** as ***fine*** and ***coarse***.
- Fine aggregate (typically natural sand) is any material that will pass a ***No.4 sieve,*** i.e***., a sieve with four openings per linear inch***.
- Material coarser than this is classified as coarse aggregate.

2.2.2.4 Maximum Size of Coarse Aggregate

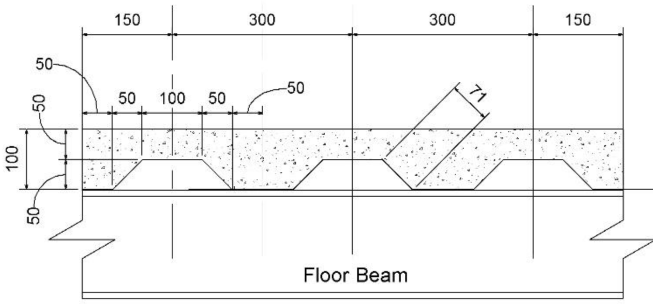
- The ***maximum size of coarse aggregate*** in reinforced concrete is ***governed by*** the requirement that it shall ***easily fit into the forms*** and ***between the reinforcing bars***.
- For this purpose, it should ***not be larger*** than ***one-fifth of the narrowest dimension of the forms*** or ***one-third of the depth of slabs***, nor ***three-quarters of the minimum distance between reinforcing bars***.
- Requirements for satisfactory aggregates are found in ***ASTM C33, "Standard Specification for Concrete Aggregates,"***

Example 2.2-1

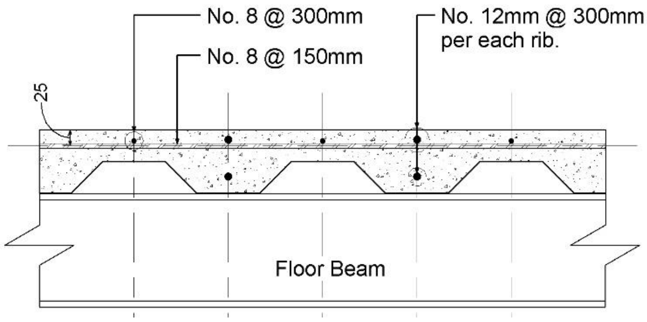
Can a coarse aggregate with a maximum size of 20mm be adopted for the one-way slab indicates in Figure 2.2-1?



Three-dimensional view.



Slab dimensions.



Slab reinforcement.

Figure 2.2-1: One-way slab.

Solution

To fit easily into the forms and between the reinforcing bars, the maximum size of aggregate shall be:

- Not be larger than one-fifth of the narrowest dimension of the forms:

$$\frac{50}{5} > 20 \therefore Not\ ok.$$

- Or one-third of the depth of slabs:

$$\frac{1}{3} \times \left(\frac{50+100}{2}\right) = \frac{75}{3} = 25 > 20 \therefore Ok.$$

- Nor three-quarters of the minimum distance between reinforcing bars:

$$\frac{3}{4} \times 150 = 112.5 > 20 \therefore Ok.$$

- -

Example 2.2-2

Can a coarse aggregate with a maximum size of 20mm be adopted for a precast pile indicates in Figure 2.2-1?

Solution

To fit easily into the forms and between the reinforcing bars, the maximum size of aggregate shall be:

- Not be larger than one-fifth of the narrowest dimension of the forms:
  $$\frac{285}{5} = 57 > 20 \therefore Ok.$$
- Or one-third of the depth of slabs:
  This is inapplicable for pile section.
- Nor three-quarters of the minimum distance between reinforcing bars:
  $$\frac{3}{4} \times \left(\frac{285 - 20 \times 2 - 8 \times 2 - \frac{16}{2} \times 2}{2} \right) \approx 80mm > 20$$
  $$\therefore Ok.$$

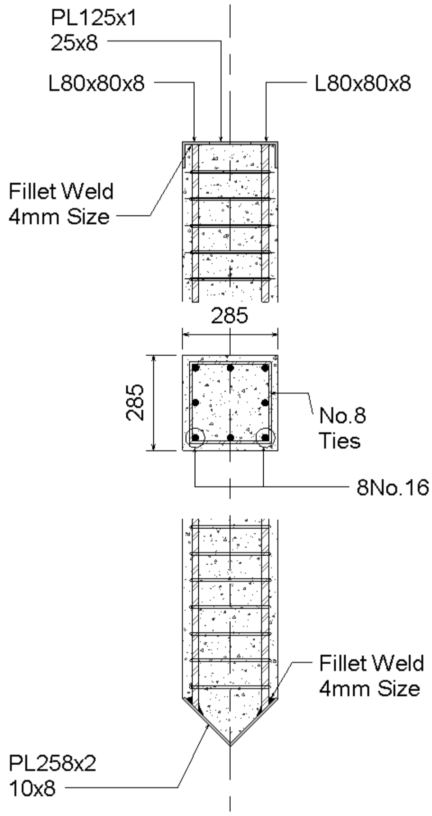


Figure 2.2-2: Precast pile details.

2.2.2.5 Lightweight Aggregate

- A ***variety of lightweight aggregates*** are available. Some ***unprocessed*** aggregates, such as ***pumice*** or ***cinders***, are suitable for ***insulating concretes***.
- For ***structural lightweight concrete***, ***processed aggregates are used because of better control***. These consist of expanded shales, clays, slates, slags, or pelletized fly ash. They are ***light in weight*** because of the ***porous***, ***cellular structure of the individual aggregate particle***, which ***is achieved by gas or steam formation in processing the aggregates in rotary kilns*** at high temperatures (generally in excess of 2000°F).
- Requirements for satisfactory lightweight aggregates are found in ***ASTM C330, "Standard Specification for Lightweight Aggregates for Structural Concrete."***
- Use of lightweight concrete
  - ***low-density concretes***, which are ***chiefly employed for insulation*** and whose unit weight rarely exceeds ***800*** kg/m^3;
  - ***moderate strength concretes***, with unit weights from about ***960 to 1360*** kg/m^3 are chiefly used as ***fill***, e.g., ***over light-gage steel floor panels***;
  - ***structural concretes***, with unit weights from ***1440 to 1920*** kg/m^3.

2.2.2.6 Heavyweight Concrete

- Heavyweight concrete is sometimes required
  - for ***shielding against gamma*** and ***X -radiation*** in ***nuclear reactors*** and similar installations, for protective structures,
  - to ***counterweights*** of ***lift bridges***.
- It consist of
  - heavy iron ores or barite (barium sulfate),
  - rock crushed to suitable sizes,
  - steel in the form of scrap.
- Unit weights of heavyweight concretes with natural heavy rock aggregates range from
  - ***about 3200 to 3680 kg/m^3***;
  - if iron are added to ***high-density ores***, ***weights as high as 4325 kg/m^3 are achieved***.
  - The weight may be as high as ***5290 kg/m^3*** if ores are used for the ***fines only and steel for the coarse aggregate***.

2.2.3 Proportioning and Mixing Concrete

2.2.3.1 Required Properties of Concrete

The various components of a mix are proportioned so that the resulting concrete has

- adequate strength,
- proper workability for placing,
- and low cost,

The third calls for use of the ***minimum amount of cement*** (the most costly of the components) that will achieve adequate properties.

2.2.3.2 Effect of Aggregate Gradation on Concrete Properties

The better the gradation of aggregates, i.e., the smaller the volume of voids, the less cement paste is needed to fill these voids.

2.2.3.3 Water Role in a Concrete Mixture

- Water is required for
    - hydration,
    - wetting the surface of the aggregate.
- As water is added, the plasticity and fluidity of the mix increase (i.e., its workability improves), but the strength decreases because of the larger volume of voids created by the free water.

2.2.3.4 Water-cement Ratio

- To reduce the free water while retaining the workability, cement must be added.
- The water-cement ratio is the chief factor that controls the strength of the concrete. For a given water-cement ratio, one selects the minimum amount of cement that will secure the desired workability.
- ***Figure 2.2-3*** shows the decisive influence of the water-cement ratio on the compressive strength of concrete.
- Its influence on the tensile strength, as measured by the nominal flexural strength or modulus of rupture, is seen to be pronounced but much smaller than its effect on the compressive strength.

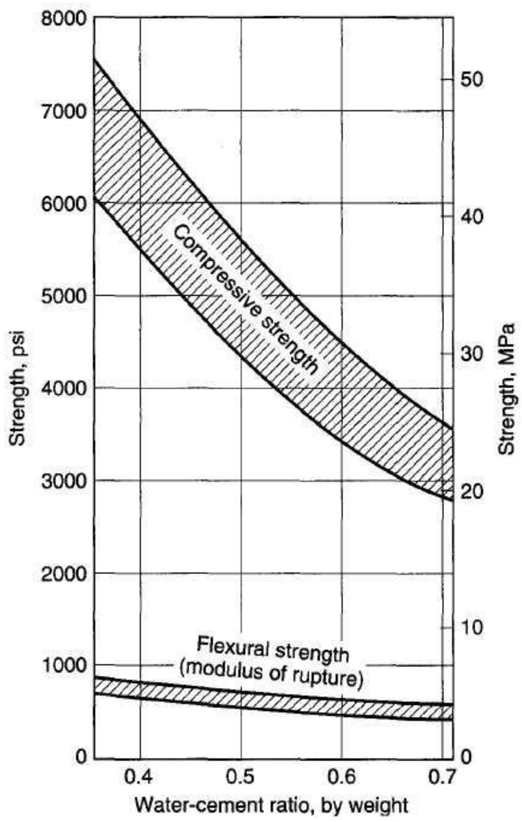


Figure 2.2-3: Effect of water-cement ratio on 28-day compressive and flexural tensile strength.

2.2.4 Conveying, Placing, Compacting, and Curing

2.2.4.1 Conveying

- Conveying of most building concrete from the mixer or truck to the form is done in
    - bottom-dump buckets
    - or by pumping through steel pipelines.
- The chief danger during conveying is that of ***segregation*** which is due to:
    - The individual components of concrete tend to segregate because of their dissimilarity. In overly wet concrete standing in containers or forms, the heavier coarse aggregate particles tend to settle, and the lighter materials, particularly water, tend to rise.
    - Lateral movement, such as flow within the forms, tends to separate the coarse gravel from the finer components of the mix.

2.2.4.2 Placing

- Placing is the process of transferring the fresh concrete from the conveying device to its final place in the forms.
- Causation prior to placing,
    - loose rust must be removed from reinforcement,

  - forms must be cleaned,
  - hardened surfaces of previous concrete lifts must be cleaned and treated appropriately,
- Causations during placing
  Proper placement must avoid
  - segregation,
  - displacement of forms
  - displacement of reinforcement in the forms,
  - poor bond between successive layers of concrete.
- Consolidation with Vibrators
  Consolidation, immediately upon placing, the concrete should be, by ***means of vibrators***, to
  - prevent honeycombing,
  - ensure close contact with forms and reinforcement,
  - serve as a partial remedy to possible prior segregation.

2.2.4.3 Curing

- Fresh concrete gains strength most rapidly during the first few days and weeks.
- Structural design is generally based on the 28-day strength, about ***70 percent of which is reached at the end of the first week after placing***.
- The final concrete strength depends greatly on the conditions of
  - moisture
  - temperature

  during this initial period.
- The maintenance of proper conditions during this time is known as ***curing***.
- ***Thirty percent*** of the strength or ***more*** can be ***lost*** by ***premature drying out*** of the concrete; similar amounts may be lost by permitting the concrete ***temperature to drop to 4°C or lower*** during the first few days unless the concrete is kept continuously moist for a long time.
- ***Curing Period***
  To prevent such damage, concrete should be protected from loss of moisture for
  - at least 7 days and,
  - in more sensitive work, up to 14 days.
  - When high early strength cements are used, curing periods can be cut in half.
- ***Achievement of Curing***
  Curing can be achieved by keeping exposed surfaces continually wet through
  - sprinkling,
  - ponding,
  - covering with plastic film
  - by the use of sealing compounds, which, when properly used, form evaporation-retarding membranes.
- Other Curing Advantage
  In addition to improving strength, proper moist curing provides ***better shrinkage control***.

2.2.5 Quality Control

2.2.5.1 Concrete versus Mill-produced Materials

The quality of mill-produced materials, such as structural or reinforcing steel, is ensured by the producer, who must exercise systematic quality controls, usually specified by pertinent standards. Concrete, in contrast, is produced at or close to the site, and its final qualities are affected by a number of factors. Thus, systematic quality control must be instituted at the construction site.

2.2.5.2 Compressive Strength as the Main Quality Indicator

- The main measure of the structural quality of concrete is its compressive strength.
- Tests for this property are made on cylindrical specimens of height equal to twice the diameter, ***usually 150 × 300 mm***, and subjected to a ***uniaxial monotonic loading***.
- The cylinders are ***moist-cured at about 23°C***, generally for ***28 days***, and then tested in the laboratory at a specified rate of loading.
- The compressive strength obtained from such tests is known as the ***cylinder strength*** f_c'; and is the main property specified for design purposes.

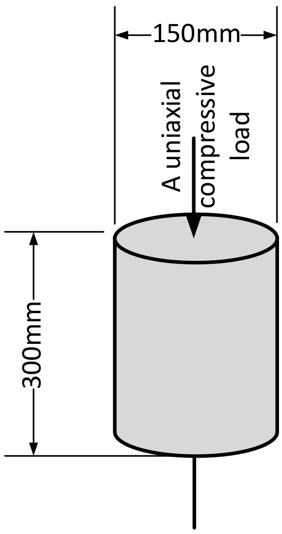


Figure 2.2-4: Standard compressive test.

2.2.5.3 Strength Test Sample

According to article ***26.12.1.1*** of ACI code, ***a strength test shall be average of the strengths of at least two 150 X 300 mm or three 100 X 200 mm cylinders***.

2.2.5.4 Sample Size

According to article ***26.12.2.1*** of ACI code,

- A sample must be tested for each **110m^3** of concrete or for each ***460m^2*** of surface area actually placed, but ***not less than once a day***.
- On a given project, if total volume of concrete is such that frequency of testing would provide fewer than five strength tests for a given concrete mixture, strength test specimens shall be made from at least five randomly selected batches or from each batch if fewer than five batches are used.

If the total quantity of a given concrete mixture is less than 38 m^3, strength tests are not required if evidence of satisfactory strength is submitted to and approved by the building official.

2.2.5.5 Acceptance Criteria

To ensure adequate concrete strength in spite of the scatter, the (ACI318M, 2014), article 26.12.3, stipulates that concrete quality is satisfactory if

- No individual strength test result (the average of two or three cylinder tests depending on cylinder size) falls below the required f_c' by more than 3.5 MPa when f_c' is 35 MPa or less or by more than $0.1f_c'$ when f_c' is more than 35 MPa.
- Every arithmetic average of any three consecutive strength tests equals or exceeds f_c'.

2.2.5.6 Specified versus Mean Compressive Strength

- It is evident that if concrete were proportioned so that its mean strength were just equal to the required strength f_c', it would not pass aforementioned quality requirements, because about one-half of its strength test results would fall below the required f_c'.

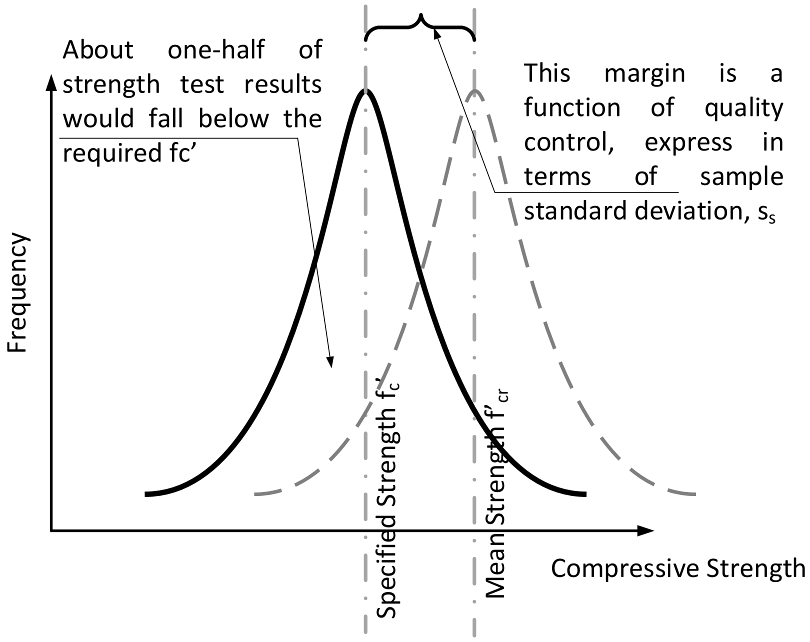


Figure 2.2-5: Strength frequency curve with adopting specified compressive strength in mix design.

- It is therefore necessary to proportion the concrete so that its ***mean strength*** $f_{cr}{}'$, used as the basis for selection of suitable proportions, ***exceeds the required design strength*** $f_c{}'$; ***by an amount sufficient to ensure that the two quoted requirements are met***.
- The minimum amount by which the required mean strength, $f_{cr}{}'$ must exceed $f_c{}'$; ***can be determined only by statistical methods because of the random nature of test scatter***.
- Based on statistical analysis, following recommendation have been adopted by the (ACI318, 2008) to determined required average compressive strength $f_{cr}{}'$ used as the basis for selection of concrete proportions:
  - When data are available to establish a sample standard deviation, f'_{cr} shall be computed based on Table 2.2-1 below.

Table 2.2-1: Required average compressive strength when data are available to establish a sample standard deviation, Table 5.3.2.1 of (ACI318, 2008).

Specified compressive strength, MPa	Required average compressive strength, MPa
$f'_c \leq 35$	Use the larger value computed from Eq. (5-1) and (5-2) $f'_{cr} = f'_c + 1.34 s_s$ (5-1) $f'_{cr} = f'_c + 2.33 s_s - 3.5$ (5-2)
$f'_c > 35$	Use the larger value computed from Eq. (5-1) and (5-3) $f'_{cr} = f'_c + 1.34 s_s$ (5-1) $f'_{cr} = 0.90 f'_c + 2.33 s_s$ (5-3)

  - The sample standard deviation, s_s, calculated as follows:
    - When a concrete production facility has a suitable record of ***30 consecutive tests*** of similar materials and conditions expected, the sample standard deviation, s_s, is calculated from those results in accordance with the following formula:

$$s_s = \left[\frac{\sum(x_i - \bar{x})^2}{(n-1)}\right]^{\frac{1}{2}}$$

where
s_s = sample standard deviation, MPa
x_i = individual strength tests.
$\bar{x}$ = average of n strength test results
n = number of consecutive strength tests

    - If ***less than 30 tests***, but ***at least 15 tests*** are available, the calculated sample standard deviation is increased by the factor given in Table 2.2-2.

Table 2.2-2: Modification factor for sample standard deviation when less than 30 tests are available, Table 5.3.1.2 of (ACI318, 2008).

No. of tests*	Modification factor for sample standard deviation†
Less than 15	Use Table 5.3.2.2
15	1.16
20	1.08
25	1.03
30 or more	1.00

*Interpolate for intermediate numbers of tests.
†Modified sample standard deviation, s_s, to be used to determined required average strength, f'_{cr}, from 5.3.2.1

- When a concrete production facility does not have field strength test records for calculation of s_s, f'_{cr} shall be determined from Table 2.2-3 below.

Table 2.2-3: Required average compressive strength when data are not available to establish a sample standard deviation

Specified compressive strength, MPa	Required average compressive strength, MPa
$f'_c < 21$	$f'_{cr} = f'_c + 7.0$
$21 \le f'_c \le 35$	$f'_{cr} = f'_c + 8.3$
$f'_c > 35$	$f'_{cr} = 1.10f'_c + 5.0$

- The 2014 edition of the Code does not include the statistical requirements for proportioning concrete that were contained in previous editions. This information was removed from the Code because:
  - It is not the responsibility of the licensed design professional to proportion concrete mixtures.
  - This information is available in other ACI documents, such as ACI 301 and ACI 214R.
  - Finally, the quality control procedures of some concrete producers allow meeting the acceptance criteria of the Code without following the process included in previous editions of the Code.

Example 2.2-3

A building design calls for specified concrete strength f'_c of 28 MPa. Calculate the average required strength f'_{cr} if

(a) 30 consecutive tests for concrete with similar strength and materials produce a sample standard deviation s_s of 3.75 MPa,

(b) 15 consecutive tests for concrete with similar strength and materials produce a sample standard deviation s_s of 3.57 MPa,

(c) Less than 15 tests are available.

Solution

(a) 30 consecutive tests are available:

$\because f'_c < 35\ MPa$

$\therefore f'_{cr} = maximum\ (f'_c + 1.34s_s\ or\ f'_c + 2.33s_s - 3.5)$

$f'_{cr} = maximum\ (28 + 1.34 \times 3.75\ or\ 28 + 2.33 \times 3.75 - 3.5)$

$f'_{cr} = maximum\ (33.0\ or\ 33.2) = 33.2\ MPa$ ■

(b) Only 15 consecutive tests are available:

Sample standard deviation, s_s, should be modified according to Table 2.2-2 above,

$s_s = 3.57 \times 1.16 = 4.14\ MPa$

$f'_{cr} = maximum\ (28 + 1.34 \times 4.14\ \ or\ 28 + 2.33 \times 4.14\ - 3.5)$

$f'_{cr} = maximum\ (33.5\ \ or\ 34.1) = \ 34.1\ MPa$ ■

(c) Less than 15 tests are available

According to Table 2.2-3 above,

$\because 21 < f'_c < 35$

$\therefore f'_{cr} = f'_c + 8.3 = 28 + 8.3 = 36.3\ MPa$ ■

This example demonstrates that:

- In cases where test data are available, good quality control, represented by a low sample standard deviation, s_s, can be used to reduce the required average strength $f_{cr}{}'$.

- A lack of certainty in the value of the standard deviation due to the limited availability of data results in higher values for f_{cr}', as shown in parts (b) and (c). As additional test results become available, the higher safety margins can be reduced.

Example 2.2-4

Determine the minimum number of test cylinders that must be cast to satisfy the code minimum sampling frequency for strength tests. Concrete placement is 150 m^3 per day for 7 days, transported by 7.6 m^3 truck mixers.

Solution

- Total concrete placed on project = 150x(7) = 1050 m^3
- Total truck loads (batches) required ≈ 1050/7.6 ≈ 138
- Truck loads required to be sampled per day = 150/110 = 1.36
  Therefore, 2 truck loads must be sampled per day.
- Total truck loads required to be sampled for project = 2 (7) = 14
- Total number of cylinders required to be cast for project = 14 (2 cylinders per test) = 28 (minimum).

It should be noted that the total number of cylinders required to be cast for this project represents a code required minimum number only that is needed for determination of acceptable concrete strength. Addition cylinders should be cast to provide for 7-day breaks, to provide field cured specimens to check early strength development for form removal, and to keep one or two in reserve, should a low cylinder break occur at 28-day.

Example 2.2-5

Determine the minimum number of test cylinders that must be cast to satisfy the code minimum sampling frequency for strength tests. Concrete is to be placed in a 30m × 23m × 200mm slab, and transported by 7.6 m^3 truck mixers.

Solution

Total surface area placed,

$A_{Surface} = 30 \times 23 = 690\ m^2$

Volume to be placed,

$Vol. = 690 \times \frac{200}{1000} = 138\ m^3$

Total truck loads (batches) required,

$Total\ Trucks\ Required = \frac{138}{7.6} \approx 18 > 5$

Therefore,

$Sample\ Size\ based\ on\ Surface\ Area = \frac{690}{460} = 1.5$

$Sample\ Size\ based\ on\ Vol. = \frac{138}{110} = 1.25$

$Sample\ Size\ based\ on\ Trucks\ (Batches) Number\ = 5_{Randomly\ Selected\ from\ 18\ Trucks}$

The govern sample size is that determined based on trucks (batches) number:

$Sample\ Size = 5\ Samples\ or\ 10\ Cylinders\ with\ dimensions\ of\ 150 \times 300\ \text{mm}$ ■

It should again be noted that the total number of cylinders cast represents a code required minimum number only for acceptance of concrete strength. A more prudent total number for a project may include additional cylinders.

Example 2.2-6

The following table lists strength test data from 5 truck loads (batches) of concrete delivered to the job site. For each batch, two cylinders were cast and tested at 28 days. The specified strength of the concrete f_c' is 28 MPa. Determine the acceptability of the concrete based on the strength criteria.

Test No.	Cylinder No. 1, MPa	Cylinder No. 2, MPa
1	28.8	29.8
2	26.9	28.6
3	30.9	31.2
4	25.7	26.7
5	32.3	32.0

Solution

Compute test average and average of three consecutive tests as presented in table below:

Test No.	Cylinder No. 1, MPa	Cylinder No. 2, MPa	Test Average, MPa	Average of 3 Consecutive Tests
1	28.8	29.8	29.3	-
2	26.9	28.6	27.7	-
3	30.9	31.2	31.0	$(29.3 + 27.7 + 31.0)/3 = 29.3$
4	25.7	26.7	26.2	$(27.7 + 31.0 + 26.2)/3 = 28.3$
5	32.3	32.0	32.2	$(31.0 + 26.2 + 32.2)/3 = 29.8$

For concrete to be considered satisfactory,

- No individual test may fall below $f_c' - 3.5$
  $f_c' - 3.5 = 28 - 3.5 = 24.5\ MPa$
  The five tests meet this criterion.
- Every arithmetic average of any three consecutive tests must equal f_c'.
  The five tests meet this criterion.

Thus, based on the code acceptance criteria for concrete strength, the five strength tests results are acceptable, both on the basis of individual test results and the average of three consecutive test results.

Example 2.2-7

The following table lists strength test data from 5 truck loads (batches) of concrete delivered to the job site. For each batch, two cylinders were cast and tested at 28 days. The specified strength of the concrete f_c' is 28 MPa. Determine the acceptability of the concrete based on the strength criteria.

Test No.	Cylinder No. 1, MPa	Cylinder No. 2, MPa
1	25.3	24.9
2	27.8	28.4
3	28.6	28.0
4	34.0	32.9
5	23.7	21.8

Solution

Compute test average and average of three consecutive tests as presented in table below:

Test No.	Cylinder No. 1, MPa	Cylinder No. 2, MPa	Test Average, MPa	Average of 3 Consecutive Tests
1	25.3	24.9	25.1	
2	27.8	28.4	28.1	
3	28.6	28.0	28.3	27.2*
4	34.0	32.9	33.5	29.9
5	23.7	21.8	22.8**	28.2

*Average of 3 consecutive tests low.
**One test more than 3.5 MPa below specified value.

For concrete to be considered satisfactory,

- No individual test may fall below $f_c' - 3.5$
  $f_c' - 3.5 = 28 - 3.5 = 24.5\ MPa$
  Test indicated with ** does not satisfy this requirement.
  Based on experience, the major reasons for low strength test results are:
  - Improper sampling and testing,
  - Reduced concrete quality due to an error in production, or the addition of too much water to the concrete at the job site, caused by delays in placement or requests for wet or high slump concrete. High air content can also be a cause of low strength.

  The test results for the concrete from Truck 5 are below the specified value, especially the value for Cylinder #2, with the average strength being only 22.8 MPa.
  Note that no acceptance decisions are based on the single low cylinder break of 21.8 MPa. Due to the many variables in the production, sampling and testing of concrete, acceptance or rejection is always based on the average of at least 2 cylinder breaks.
- Every arithmetic average of any three consecutive tests must equal f_c':
  Consecutive average indicated with * does not satisfy this criterion.

Thus, based on the code acceptance criteria for concrete strength, the five strength tests results are rejected, both on the basis of individual test results and the average of three consecutive test results.

Example 2.2-8

The first eight compressive strength test results for the building described in Example 2.2-8c are

32.6, 29.5, 27.2, 30.1, 35.7, 33.5, 34.3, and 33.4 MPa.

(a) Are the test results satisfactory?

(b) In what fashion, if any, should the mixture proportions of the concrete be altered?

Solution

(a) For concrete to be considered satisfactory,

1. No individual test may fall below $f_c' - 3.5$

   $f_c' - 3.5 = 28 - 3.5 = 24.5\ MPa$

   The eight tests meet this criterion.

2. Every arithmetic average of any three consecutive tests must equal $f_c{}'$.

$$\frac{32.6 + 29.5 + 27.2}{3} = 29.7\ MPa \qquad \frac{29.5 + 27.2 + 30.1}{3} = 28.9\ MPa$$

$$\frac{27.2 + 30.1 + 35.7}{3} = 31\ MPa \qquad \frac{30.1 + 35.7 + 33.5}{3} = 33.1\ MPa$$

$$\frac{35.7 + 33.5 + 34.3}{3} = 34.5\ MPa \qquad \frac{33.5 + 34.3 + 33.4}{3} = 33.7\ MPa$$

The eight tests meet this criterion.

(b) To determine if the mixture proportions must be altered,

1. we note that the solution to Example 2.2-3c requires that

$$f_{cr}' \geq 36.3\ MPa$$

   The average of the first eight tests is 32.0 MPa, well below the value of $f_{cr}{}'$,. Thus, ***the mixture proportions should be modified by decreasing the water-cement ratio to increase the concrete strength***.

2. Once at least 15 tests are available, the value of $f_{cr}{}'$ can be recalculated with the appropriate factor for s_s from Table 2.2-1.

Example 2.2-9

According to durability requirement, the cylindrical compressive strength, $f_c{}'$, of $30\ MPa$ should be adopted for the spread footings of the indicated building. They have a total volume of $24\ m^3$. For this concrete volume, can an approval by the building official be adopted to accept concrete without sampling?

Solution

According to article 26.12.2.1 of the ACI code, if the total quantity of a given concrete mixture is less than 38 m^3, strength tests are not required if evidence of satisfactory strength is submitted to and approved by the building official. Therefore, for a concrete volume of $24m^3$ an approval by the building official would be adequate.

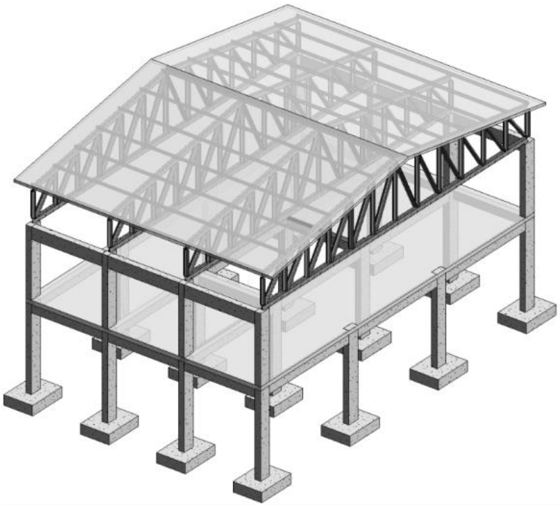


Figure 2.2-6: Building for Example 2.2-9.

2.2.6 Admixtures

2.2.6.1 Beneficial Effects of Admixtures

In addition to the main components of concretes, admixtures are often used to improve concrete performance due:

- To accelerate or retard setting and hardening,
- To improve workability,
- To increase strength,
- To improve durability,
- To decrease permeability,
- To impart other properties.

Chemical admixtures should meet the requirements of ***ASTM C494, "Standard Specification for Chemical Admixtures for Concrete."***

2.2.6.2 Air-entraining Agents

- Air-entraining agents are probably the most commonly used admixtures.
- They cause the entrainment of air in the form of small dispersed bubbles in the concrete.
- Advantages
  - Improve workability and durability (chiefly resistance to freezing and thawing)
  - Reduce segregation during placing.
- Disadvantage

  Decrease concrete density because of the increased void ratio and thereby decrease strength; however, this decrease can be partially offset by a reduction of mixing water without loss of workability.
- The ***chief use of air-entrained concretes is in pavements***, but they are also used for structures, particularly for exposed elements.

2.2.6.3 Accelerating Admixtures

- Usage

  Are used to reduce setting time and accelerate early strength development.
- Composition
  - ***Calcium chloride*** is the most widely used accelerator because of its cost effectiveness, ***but it should not be used in prestressed concrete and should be used with caution in reinforced concrete in a moist environment, because of its tendency to promote corrosion of steel***.
  - Non-chloride, noncorrosive accelerating admixtures are available, the principal one being ***calcium nitrite***.

2.2.6.4 Set-retarding Admixtures

Usage
  - Are used primarily to ***offset the accelerating effect of high ambient temperature*** and ***to keep the concrete workable during the entire placing period***. This helps to ***eliminate cracking due to form deflection***
  - Also keeps concrete workable long enough that succeeding lifts can be placed without the development of ***"cold" joints***.

2.2.6.5 Plasticizers

- Usage

  Are used to ***reduce the water requirement of a concrete mix for a given slump***.
- Advantaged

  Reduction in water demand may result in
  - Either a reduction in the water-cement ratio for a given slump and cement content
  - Or an increase in slump for the same water-cement ratio and cement content.
- Working Principle

  Plasticizers work by ***reducing the inter-particle forces that exist between cement grains in the fresh paste***, thereby ***increasing the paste fluidity***.

2.2.6.6 Superplasticizers

- Usage
    - High-range water-reducing admixtures, or superplasticizers, are used to produce high-strength concrete with a very low water-cement ratio while maintaining the higher slumps needed for proper placement and compaction of the concrete.
    - They are also used to produce flowable concrete at conventional water-cement ratios.
- Difference between Superplasticizers and Conventional Water-reducing Admixture
  Superplasticizers differ from ***conventional water-reducing admixtures in that they do not act as retarders at high dosages; therefore, they can be used at higher dosage rates without severely slowing hydration***.

2.2.6.7 Self-consolidating Concrete, SCC

- Composition
  When superplasticizers are combined with ***viscosity-modifying admixtures***, they can be used to produce self-consolidating concrete (SCC).
- Usage
    - Self-consolidating concrete is highly fluid and does not require vibration to remove entrapped air.
    - The viscosity modifying agents allow the concrete to remain cohesive even with a very high degree of fluidity.
    - As a result, SCC ***can be used for members with congested reinforcement***, such as
        - ***Beam-column joints in earthquake-resistant structures***,
        - Precast concrete, especially precast prestressed concrete, a manufactured product.
- Disadvantage
    - The high fluidity of the mix, however, has been shown to have a ***negative impact on the bond strength*** between the concrete and prestressing steel located in the upper portions of a member, a shortcoming that should be considered in design but is not currently addressed in the ACI Code,
    - The composition of SCC mixtures may result in ***moduli of elasticity***, ***creep***, and ***shrinkage properties*** that ***differ*** from those of more ***traditional mixtures***.

2.2.6.8 Fly Ash and Silica Fume

- Composition and Basic Reaction
  Fly ash and silica fume are pozzolans, highly active silicas, that ***combine with calcium hydroxide***, $Ca(OH)_2$ the soluble product of cement hydration, to form more ***calcium silicate hydrate***, $3CaO.2SiO_2.3H_2O$, the insoluble product of cement hydration.
- Cement versus Cementitious Materials
  Pozzolans qualify as supplementary ***cementitious materials***, also referred to as mineral admixtures, which are used to replace a part of the portland cement in concrete mixes.
- Sustainable Issue
    - In sustainable development, ***the "cost" of concrete lies primarily in the manufacture of portland cement***.
    - The production of ***a ton of portland cement requires roughly the energy needed to operate a typical U.S. household for two weeks*** and ***generates approximately 0.9 ton of $C0_2$*** (a greenhouse gas). The latter ***translates to about 150 kg of $C0_2$ for every cubic yard of concrete that is placed***.
    - The energy and greenhouse gases involved in the production of concrete, however, ***can be viewed as investments because properly designed reinforced concrete structures that take advantage of concrete's thermal mass provide significant reductions in the energy and C02 needed for heating and cooling***, and concrete's inherent durability results in structures with long service lives.
    - Because by-products, such ***as the mineral admixtures fly ash and blast furnace slag, involve minimal energy usage or greenhouse gas production***, they have the potential ***to further improve the sustainability of concrete construction when used as a partial replacement for portland cement***.

2.3 CONCRETE, PHYSICAL ASPECTS

2.3.1 Properties in Compression

2.3.1.1 Short-term Loading

2.3.1.1.1 Important of Stress-Strain Relationship

Performance of a structure under load depends to a large degree on the stress-strain relationship of the material from which it is made, under the type of stress to which the material is subjected in the structure.

2.3.1.1.2 Compressive Stress-strain Curve

Since concrete is used mostly in compression, its compressive stress-strain curve is of primary interest.

2.3.1.1.3 How the Curve is Obtained?

Such a curve is obtained by

- Appropriate strain measurements in cylinder tests.

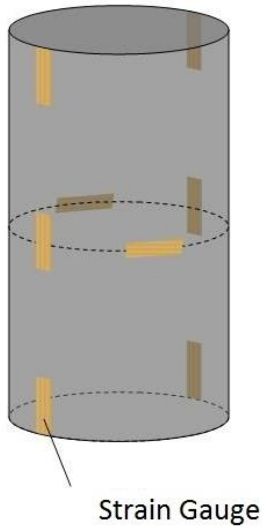


Figure 2.3-1: Stain measurement in cylindrical compression test.

- Appropriate strain measurements on the compression side in beams.

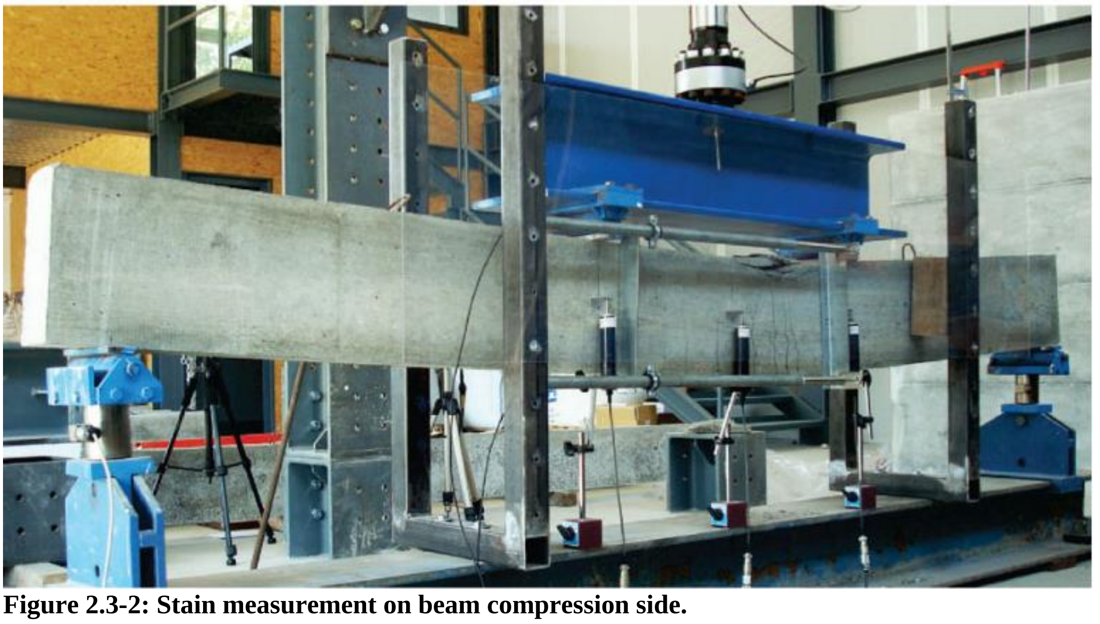


Figure 2.3-2: Stain measurement on beam compression side.

2.3.1.1.4 Typical Compression Stress-strain Diagram

- Figure 2.3-3 shows a typical set of such curves for normal-density concrete obtained from uniaxial compressive tests performed at normal, moderate testing speeds on concretes that are 28 days old.
- All of the curves have following similar character.
    - An initial relatively straight elastic portion in which stress and strain are closely proportional.
    - Curves reaching the maximum stress, i.e., the compressive strength, at a strain that ranges from about 0.002 to 0.003.

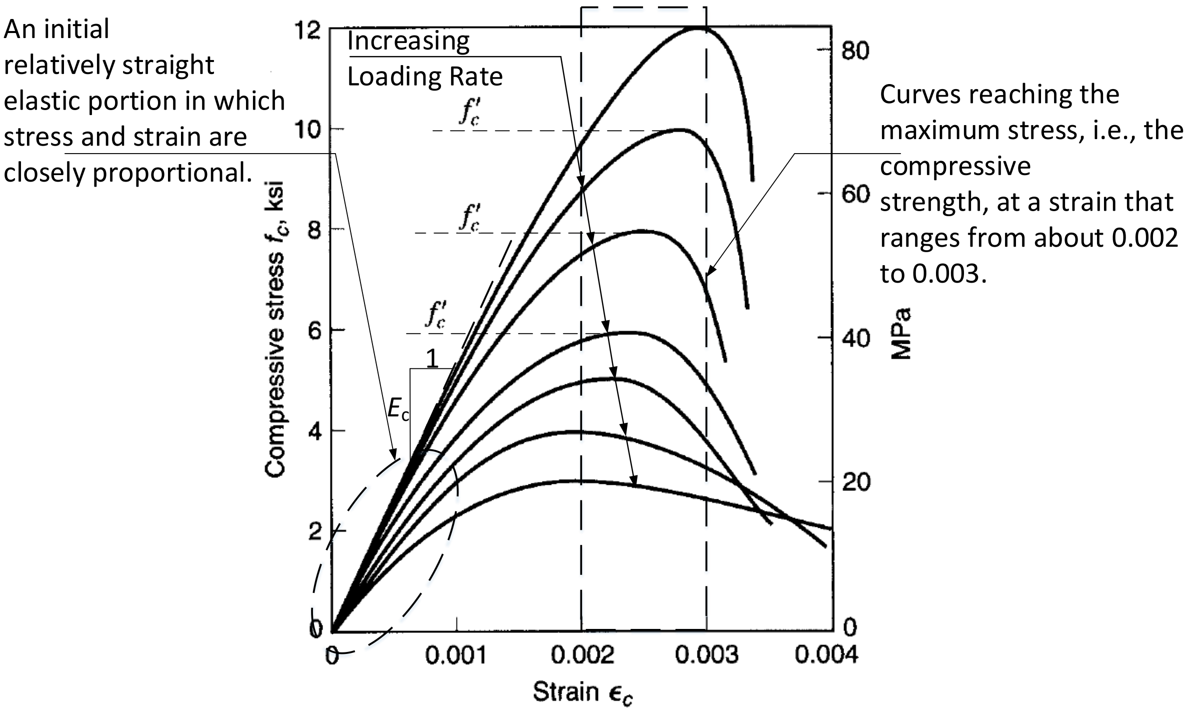


Figure 2.3-3: Typical compressive stress-strain curves for normal-density concrete with $w_c = 2300\ kg/m^3$.

2.3.1.1.5 The Modulus of Elasticity E_c

- The modulus of elasticity E_c (in MPa units), i.e., ***the slope of the initial straight portion of the stress-strain curve***, is seen to be larger as the strength of the concrete increases.
- According to (ACI318M, 2014), article 19.2.2, modulus of elasticity, E_c, for concrete can be estimated based on following correlation:
    - For values of w_c between 1440 and 2560 kg/m^3

      $E_c = w_c^{1.5} 0.043\sqrt{f_c'}\ \ (in\ MPa)$
    - For normalweight concrete

      $E_c = 4700\sqrt{f_c'}\ \ (in\ MPa)$

2.3.1.1.6 Poisson's Ratio

- When compressed in one direction, concrete, like other materials, expands in the direction transverse to that of the applied stress.
- The ratio of the transverse to the longitudinal strain is known as ***Poisson's ratio***.
- It depends somewhat on:
    - strength,
    - composition,
    - other factors.
- At ***stresses lower than about*** $0.7f_c'$, ***Poisson's ratio for concrete falls within the limits of 0.15 to 0.20***.

2.3.1.1.7 *Equations for Compressive Stress–Strain Diagrams of Concrete

2.3.1.1.7.1 Modified Hognestad Stress-strain Curve

- The ***modified Hognestad stress–strain curve,*** shown in ***Figure 2.3-4***, is a common representation of the stress–strain curve for concretes with strengths up to about 42 MPa is
- It consists of a second-degree parabola with apex at a strain of 1.8 f_c''/Ec, where $f_c'' = 0.9f_c'$, followed by a downward-sloping line terminating at a stress of 0.85 f_c'' and a limiting strain of 0.0038.
- The equation given in ***Figure 2.3-4*** describes a second-order parabola with its apex at the strain ϵ_o.
- The reduced strength, $f_c'' = 0.9f_c'$ accounts for the ***differences between cylinder strength and member strength***. These differences result from:
  - o Different curing and placing, which give rise to different water-gain effects due to vertical migration of bleed water.
  - o Differences between the strengths of rapidly loaded cylinders and the strength of the same concrete loaded more slowly.

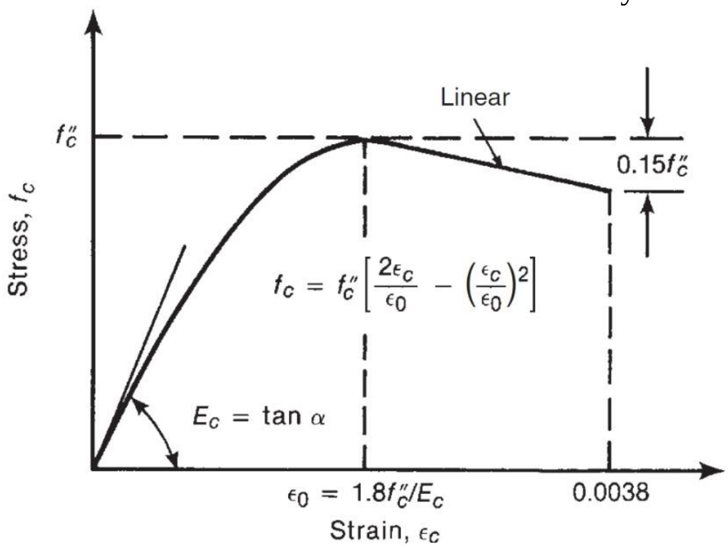


Figure 2.3-4: Modified Hognestad analytical approximations to the compressive stress–strain curve for concrete.

2.3.1.1.7.2 Todeschini Stress–strain Curve

- The Todeschini stress–strain curve presented in ***Figure 2.3-5*** is ***convenient for use in analytical studies*** involving concrete strengths up to about 35 MPa because ***the entire stress–strain curve is given by one continuous function***.
- The highest point in the curve, f_c'' , is taken to equal $0.9f_c'$ to give stress-block properties similar to that of the rectangular stress block of ***Chapter 4*** when $\epsilon_u = 0.003$ for f_c' up to $35\ MPa.$
- The strain ϵ_0, corresponding to maximum stress, is taken as $\epsilon_0 = \frac{1.71f_c'}{E_c}$
- For any given strain ϵ, $x = \epsilon/\epsilon_0$, the stress corresponding to that strain is $f_c = \frac{2f_c''x}{1+x^2}$

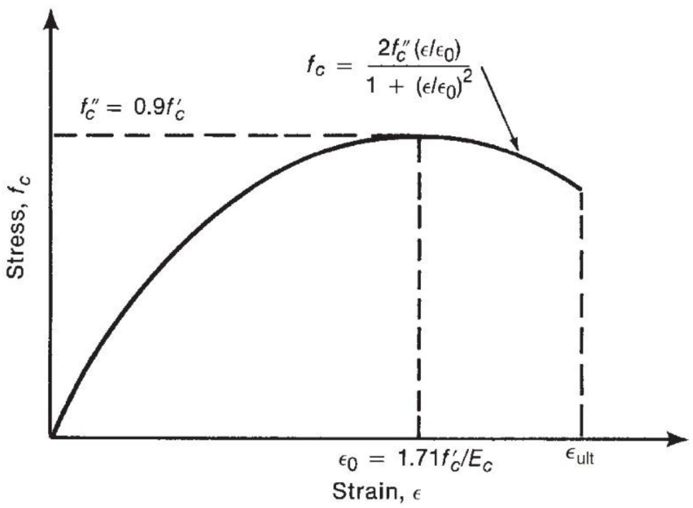


Figure 2.3-5: Todeschini analytical approximations to the compressive stress–strain curve for concrete.

Example 2.3-1

Writ to draw the modified Hognestad approximate stress-strain diagram for a concrete with $f_c' = 28\ MPa$.

Solution

Matlab code is presented in ***Table 2.3-1*** the resulting curve is indicated in ***Figure 2.3-6***.

Table 2.3-1: Matlab code to draw modified Hognestad approximated stress-strain diagram of Example 2.3-1.

```
%
% Matlab code to generate data and darw compressive
% stress-strain of concrete approximated according to
% Modified Hognestad model
%
clc
% Input of Concrete Data
%--------------------------------
fcp =28                    % Concrete cylinderical compressive strength in MPa.
Ec =4700*(fcp)^0.5         % Concrete elastic modulus according to ACI relation.
%
% Model Parameters
%
fcpp = 0.9*fcp             % Member compressive strength to reflect diffrent curing and different loading rate.
ep0 = 1.8*fcpp/Ec          % Concrete strain where stress is reached its maximum value and curved part are ended.
epu = 0.0038               % Ultimate strain where concrete is compeletly crushed.
%
% Stress-strain vectors for curved part
%------------------------------------------------
eps1= 0:0.1*ep0:ep0
fc1 = fcpp.*((2.*eps1./ep0)-(eps1./ep0).^2)
%
% Stress-strain vectors for straight part
%------------------------------------------------
eps2 = [ep0 epu]
fc2 = [fcpp 0.85*fcpp]
%
% Final stress and strain curves
%-------------------------------------
eps = [eps1 eps2]
fc=[fc1 fc2]
%
% Ploting of the curve
%---------------------
plot(eps, fc, '--k*', 'linewidth',2,'markersize',12)
% Formating of x-axis
Xmin = min(eps)
Xmax = max(eps)
xlim([Xmin Xmax])
XR = Xmax-Xmin
xlabel('Concrete strain \epsilon', 'FontSize',14)
% Formating of y-axis
Ymin = min(fc)
Ymax = max(fc)
ylim([Ymin Ymax])
ylabel(' Concrete Stress, f, in MPa', 'FontSize',14)
% Formating Grid
YR = Ymax-Ymin
X_grid=[Xmin:0.1*XR:Xmax]
Y_grid = [Ymin:0.1*YR:Ymax]
set(gca,'XTick',X_grid, 'YTick',Y_grid);
grid
```

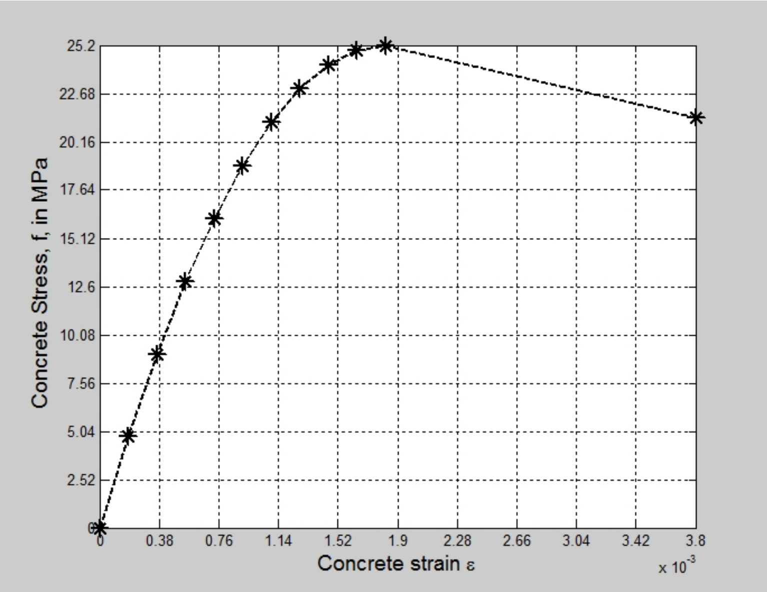


Figure 2.3-6: Modified Hognestad analytical approximations to the compressive stress–strain curve for concrete with f'_c of 28 MPa.

Example 2.3-2

Resolve ***Example 2.3-1*** above but with using of Todeschini stress-strain curve instead of Modified Hognestad curve.

Solution

Matlab code is presented in ***Table 2.3-2*** the resulting curve is indicated in ***Figure 2.3-7***.

Table 2.3-2: Matlab code to draw Todeschini approximated stress-strain diagram of Example 2.3-2.

```
%
% Matlab code to generate data and darw compressive
% stress-strain of concrete approximated according to
% Todeschini model
%
clc
% Input of Concrete Data
%------------------------------
fcp =28                % Concrete cylinderical compressive strength in MPa.
Ec =4700*(fcp)^0.5     % Concrete elastic modulus according to ACI relation.
%
% Model Parameters
%
fcpp = 0.9*fcp         % Member compressive strength to give stress-block properties similar to that of the rectangular stress block.
ep0 = 1.71*fcp/Ec      % Concrete strain where stress is reached its maximum value.
epu = 0.0030           % Ultimate strain where concrete is compeletly crushed.
%
% Stress-strain vectors
%-----------------------------------------------
eps= 0:0.05*epu:epu
fc = (2*fcpp.*(eps./ep0))./(1+(eps/ep0).^2)
%
%
% Ploting of the curve
%-----------------------
plot(eps, fc, '--k*', 'linewidth',2,'markersize',12)
% Formating of x-axis
Xmin = min(eps)
Xmax = max(eps)
xlim([Xmin Xmax])
XR = Xmax-Xmin
xlabel('Concrete strain \epsilon', 'FontSize',14)
% Formating of y-axis
Ymin = min(fc)
Ymax = max(fc)
ylim([Ymin Ymax])
ylabel(' Concrete Stress, f, in MPa', 'FontSize',14)
% Formating Grid
YR = Ymax-Ymin
X_grid=[Xmin:0.1*XR:Xmax]
Y_grid = [Ymin:0.1*YR:Ymax]
set(gca,'XTick',X_grid, 'YTick',Y_grid);
grid
```

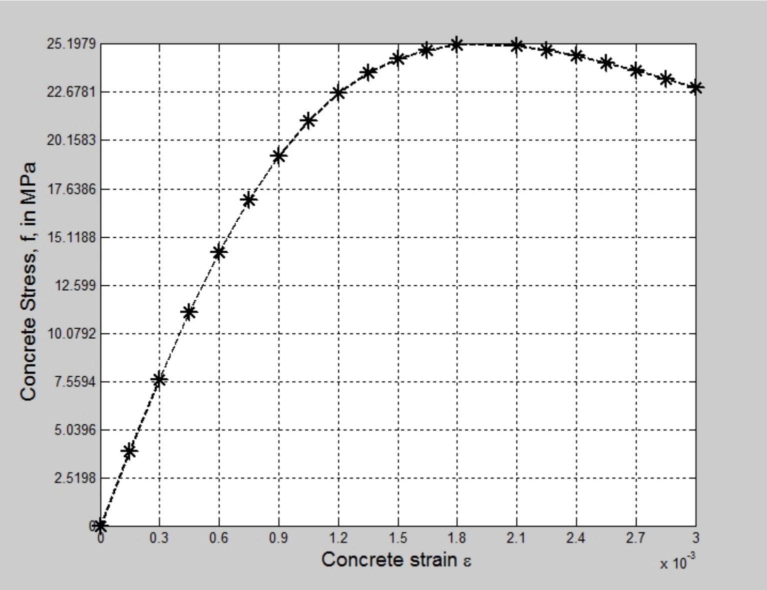


Figure 2.3-7: Todeschini analytical approximations to the compressive stress–strain curve for concrete with f'_c of 28 MPa.

2.3.1.2 Long-Term Loading

2.3.1.2.1 Steel versus Concrete

- In some engineering materials, such as ***steel***, strength and the stress-strain relationships are independent of rate and duration of loading, at least within the usual ranges of rate of stress, temperature, and other variables.
- In contrast, Figure 2.3-3 illustrates the fact that the influence of time, in this case of rate of loading, on the behavior of concrete under load is pronounced.
- The main reason is that concrete ***creeps*** under load, while steel does not exhibit creep under conditions prevailing in buildings, bridges, and similar structures.

2.3.1.2.2 Concrete Creep

- Creep ***is the slow deformation of a material over considerable lengths of time at constant stress or load***.
- The nature of the creep process is shown schematically in Figure 2.3-8.
  - This particular concrete was loaded after 28 days with resulting instantaneous strain ϵ_{inst}.
  - The load was then maintained for 230 days, during which time creep was seen to have increased the total deformation to almost 3 times its instantaneous value.
  - If the load were maintained, the deformation would follow the solid curve.
  - If the load is removed, as shown by the dashed curve, most of the elastic instantaneous strain ϵ_{inst} is recovered, and some creep recovery is seen to occur.
  - If the concrete is reloaded at some later date, instantaneous and creep deformations develop again, as shown.

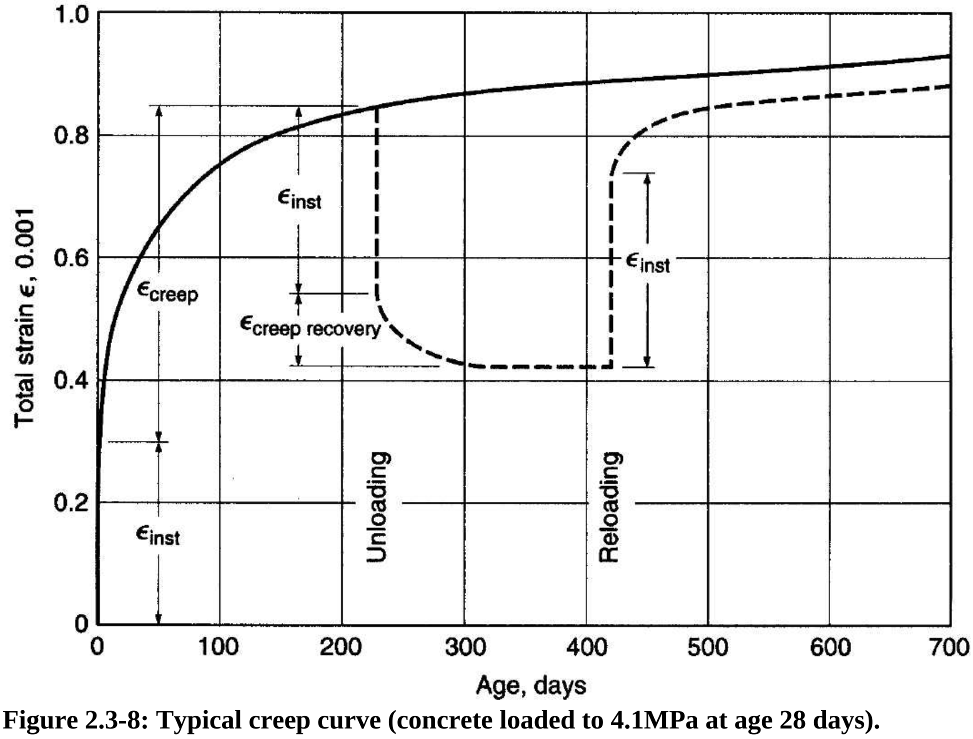


Figure 2.3-8: Typical creep curve (concrete loaded to 4.1MPa at age 28 days).

- Factor Affecting Concrete Creep
  Creep deformations for a given concrete are practically proportional:
  - To the magnitude of the applied stress;
  - To ratio of stress to compressive strength, high-strength concretes show less creep than lower-strength concretes.
  - creep depends on the average ***ambient relative humidity***, being more than ***twice*** as ***large for 50 percent as for 100 percent humidity***. This is so ***because part of the reduction in volume under sustained load is caused by outward migration of free pore water, which evaporates into the surrounding atmosphere***.
  - Other factors of importance include
    - The type of cement and aggregate,
    - age of the concrete when first loaded,
    - concrete strength.

- Creep Progress Rate
  As seen in Figure 2.3-8, with elapsing time, creep proceeds at a decreasing rate and ceases after ***2 to 5 years*** at a final value which, depending on concrete strength and other factors,
- Final Creep versus Instantaneous Strain
  Final creep is about 1.2 to 3 times the magnitude of the instantaneous strain.
- Creep versus Loading Rate
  - If, instead of being applied quickly and thereafter kept constant, the load is increased slowly and gradually, as is the case in many structures during and after construction, then instantaneous and creep deformations proceed simultaneously.
  - The effect is shown in Figure 2.3-3; i.e., the previously discussed difference in the shape of the stress-strain curve for various rates of loading is chiefly the result of the creep deformation of concrete.
- Creep Coefficient
  - For ***stresses not exceeding about one-half the cylinder strength, creep strains are approximately proportional to stress***.
  - Because ***initial elastic strains are also proportional to stress in this range***, this permits definition of the ***creep coefficient***
    $$C_{cu} = \frac{\epsilon_{cu}}{\epsilon_{ci}}$$
    where ϵ_{cu} is the final asymptotic value of the additional creep strain and ϵ_{ci} is the initial, instantaneous strain when the load is first applied.
- Specific Creep
  Creep may also be expressed in terms of the ***specific creep*** δ_{cu} defined as the additional time-dependent strain per MPa stress. It can easily be shown that,
  $C_{cu} = E_c \delta_{cu}$
- Typical Values for Creep Coefficient, C_{cu}, and Specific Creep, δ_{cu}:
  The values of Table 2.3-3, are typical values for ***average humidity conditions***, for ***concretes loaded at the age of 7 days***.

Table 2.3-3: Typical creep parameters

Compressive Strength		Specific Creep δ_{cu}		
psi	MPa	10^{-6} per psi	10^{-6} per MPa	Creep coefficient C_{cu}
3,000	21	1.00	145	3.1
4,000	28	0.80	116	2.9
6,000	41	0.55	80	2.4
8,000	55	0.40	58	2.0
10,000	69	0.28	41	1.6
12,000	83	0.22	33	1.4

Example 2.3-3

What is the final creep deformation for a concrete column with $f_c' = 28\ MPa$ and with length of 6m when subject to a longtime load that causes sustained stress of 8MPa?

Solution

From Table 2.3-3 above,

Compressive Strength		Specific Creep δ_{cu}		
psi	MPa	10^{-6} per psi	10^{-6} per MPa	Creep coefficient C_{cu}
3,000	21	1.00	145	3.1
4,000	28	0.80	116	2.9
6,000	41	0.55	80	2.4
8,000	55	0.40	58	2.0
10,000	69	0.28	41	1.6
12,000	83	0.22	33	1.4

$\delta_{cu} = 116 \times 10^{-6}\ strain\ per\ each\ 1.0\ MPa$

For long-term stress of 8MPa, creep strain would be,

$\epsilon_{cu} = 116 \times 10^{-6} \times 8 = 0.000928$

With length of 6m, final creep deformation would be,

$\Delta_{Creep} = \epsilon_{cu} L = 0.000928 \times 6000 \approx 6\ mm$ ■

- Creep Coefficient

  The creep coefficient at any time C_{ct} can be related to the ultimate creep coefficient C_{cu} can be estimated based on following relation,

  $$C_{ct} = \frac{t^{0.6}}{10 + t^{0.6}} C_{cu}$$

  where t = time in days after loading.

Example 2.3-4

For the column of Example 2.3-4, what is the creep shorting after 1 year of loading?

Solution

$$C_{ct} = \frac{t^{0.6}}{10 + t^{0.6}} C_{cu} = \frac{360^{0.6}}{10 + 360^{0.6}} C_{cu} = 0.77 C_{cu}$$

$\Delta_{creep\ after\ 1\ year} = 0.77 \times 6 = 4.6\ mm$ ■

- Concrete Compressive Strength versus Sustained Loads
  - Sustained loads affect not only the deformation but also the strength of concrete.
  - Based on experimental works, it has been shown that, for concentrically loaded unreinforced concrete prisms and cylinders, the strength under sustained load is ***significantly smaller than*** f_c', ***on the order of 75 percent off; for loads maintained for a year or more***.
  - Thus, ***a member subjected to a sustained overload causing compressive stress of over 75 percent off; may fail after a period of time, even though the load is not increased***.

2.3.1.2.3 Fatigue

- Fatigue is a phenomena noted in all materials:

  When concrete is subject to fluctuating rather than sustained loading, its fatigue strength, ***as for all other materials***, ***is considerably smaller than its static strength***.
- Order of Fatigue Strength for Plain Concrete:

  When plain concrete in compression is ***stressed cyclically from zero to maximum stress***, ***its fatigue limit is from 50 to 60 percent of the static compressive strength, for 2,000,000 cycles***.

2.3.2 Properties in Tension

2.3.2.1 Importance of Concrete Behavior in Tension

While concrete is ***best employed in a manner that uses its favorable compressive strength***, its ***behavior in tension is also important***.

- The ***conditions under which cracks form and propagate on the tension side of reinforced concrete flexural members depend strongly on both the tensile strength and the fracture properties of the concrete***, the latter dealing with the ease with which a crack progresses once it has formed.
- Concrete ***tensile stresses also occur as a result of shear, torsion, and other actions***, and in most cases member behavior changes upon cracking.

2.3.2.2 Predication of Concrete Tensile Strength

- Currently, one of the following three methods can be used to estimate concrete tensile strength.
- The ***results of all types of tensile tests show considerably more scatter than those of compression tests***.

2.3.2.2.1 Direct Tension Tests, f_t'

- Samples of concrete direct tension specimen are presented in Figures below.
- Main Drawbacks of Direct Tension Test:
  In direct tension tests, ***minor misalignments*** and ***stress concentrations*** in the gripping devices are apt to mar the results.

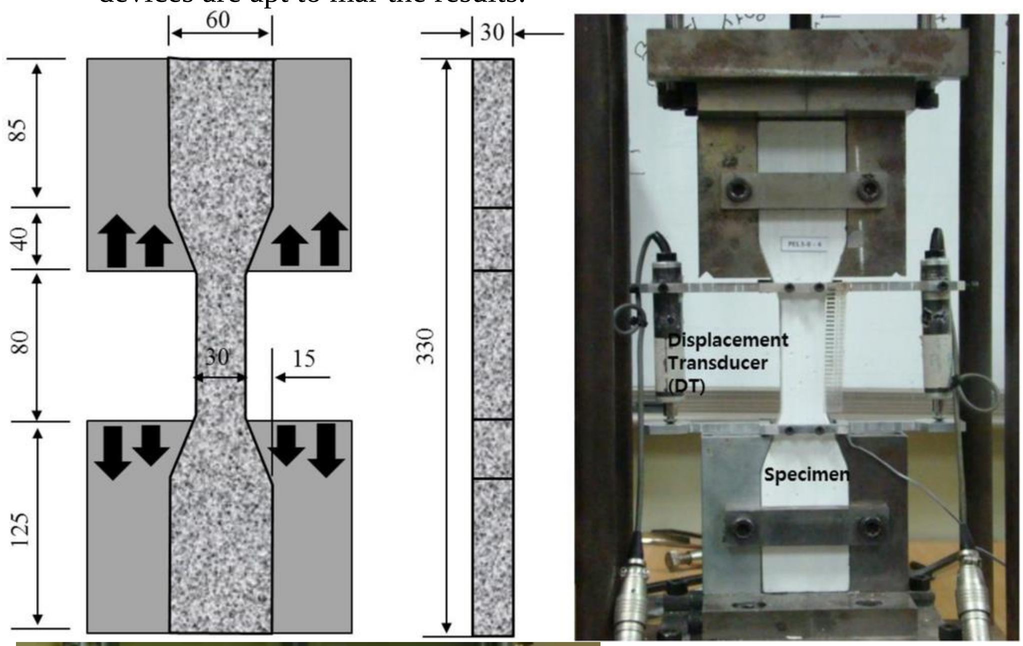


Figure 2.3-9: Samples of concrete direct tension specimen (1).

Figure 2.3-10: Samples of concrete direct tension specimen (2).

Figure 2.3-11: Samples of concrete direct tension specimen (3).

- Direct tension concrete tensile strength, f_t', can be estimated from following correlation with concrete compressive strength, f_c',

  $f_t' = 0.25 \; to \; 0.58 \sqrt{f_c'}$

2.3.2.2.2 Modulus of Rupture, f_r

- For many years, tensile strength has been measured in terms of the ***modulus of rupture***, f_r.
- In ***modulus of rupture test,*** a plain concrete beam, generally $\mathbf{150 \times 150 \times 750}\boldsymbol{mm}$ long, is loaded in flexure at the third points of a 600mm span until it fails due to cracking on the tension face, see Figure 2.3-12 below.
- The flexural tensile strength or modulus of rupture, from a modulus-of-rupture test is calculated from the following equation, assuming a linear distribution of stress and strain:

  $$f_r = \frac{6M}{bh^2}$$

  where

  M is moment

  b is width of specimen

  h is overall depth of specimen

Figure 2.3-12: Modulus of rupture for concrete tensile strength.

- Main Drawback of Modulus of Rupture:
  - Because the f_r is computed on ***the assumption that concrete is an elastic material***, and because ***this bending stress is localized at the outermost surface***, ***it is apt to be larger than the strength of concrete in uniform axial tension***.
  - It is thus a measure of, but not identical with, the real axial tensile strength.
- According to (ACI318M, 2014), article 19.2.3, modulus of rupture, f_r, for concrete can be estimated based on following correlation with concrete compressive strength:

$f_r = 0.62\lambda\sqrt{f_c'}$

The lightweight concrete modification factor λ can be estimated from Table 2.3-4 below:

Table 2.3-4: Modification factor λ, Table 19.2.4.2 of the (ACI318M, 2014).

Concrete	Composition of aggregates	λ
All-lightweight	Fine: ASTM C330M Coarse: ASTM C330M	0.75
Lightweight, fine blend	Fine: Combination of ASTM C330M and C33M Coarse: ASTM C330M	0.75 to 0.85[1]
Sand-lightweight	Fine: ASTM C33M Coarse: ASTM C330M	0.85
Sand-lightweight, coarse blend	Fine: ASTM C33M Coarse: Combination of ASTM C330M and C33M	0.85 to 1[2]
Normalweight	Fine: ASTM C33M Coarse: ASTM C33M	1

[1]Linear interpolation from 0.75 to 0.85 is permitted based on the absolute volume of normalweight fine aggregate as a fraction of the total absolute volume of fine aggregate.

[2]Linear interpolation from 0.85 to 1 is permitted based on the absolute volume of normalweight coarse aggregate as a fraction of the total absolute volume of coarse aggregate.

2.3.2.2.3 Split-cylinder Test, f_{ct}

- More recently the result of the split-cylinder test has established itself as a measure of the tensile strength of concrete.
- A concrete cylinder, the same as is used for compressive tests, is inserted in a compression-testing machine in the horizontal position, so that compression is applied uniformly along two opposite generators. Pads are inserted between the compression platens of the machine and the cylinder to equalize and distribute the pressure.

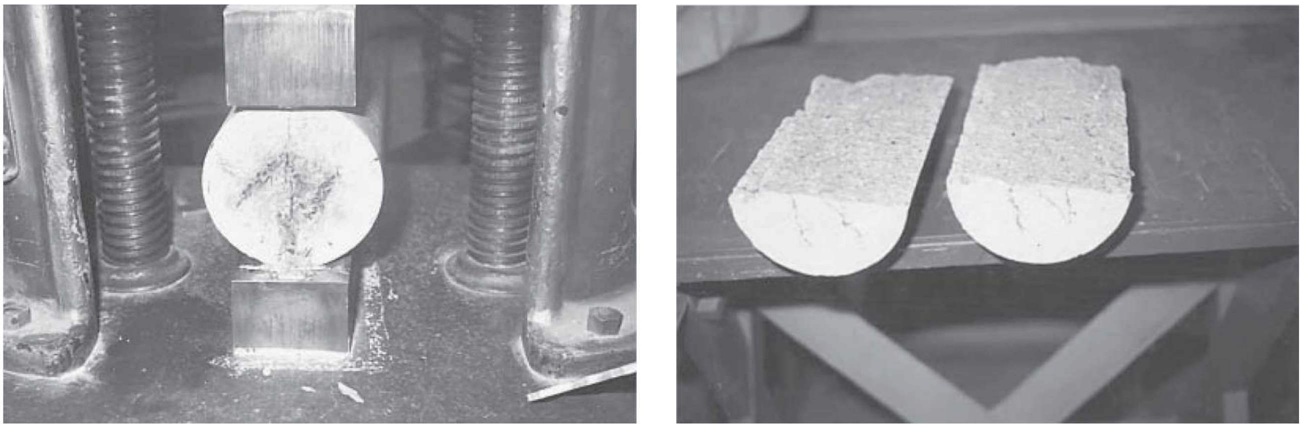


Figure 2.3-13: Concrete cylinder splitting test.

- It can be shown that in an elastic cylinder so loaded, a nearly uniform tensile stress of magnitude, see Figure 2.3-14 below,

  $f_{ct} = \sigma_1 = \frac{2P}{\pi d L}$
- Because of ***local stress conditions at the load lines and the presence of stresses at right angles to the aforementioned tension stresses***, the results of the split-cylinder tests likewise are not identical with (***but are believed to be a good measure of***) the true axial tensile strength.
- Split-cylinder strength, f_{ct}, can be estimated based on following relation with cylindrical compressive strength:

  $f_{ct} = 0.5\ to\ 0.66\sqrt{f_c'}$

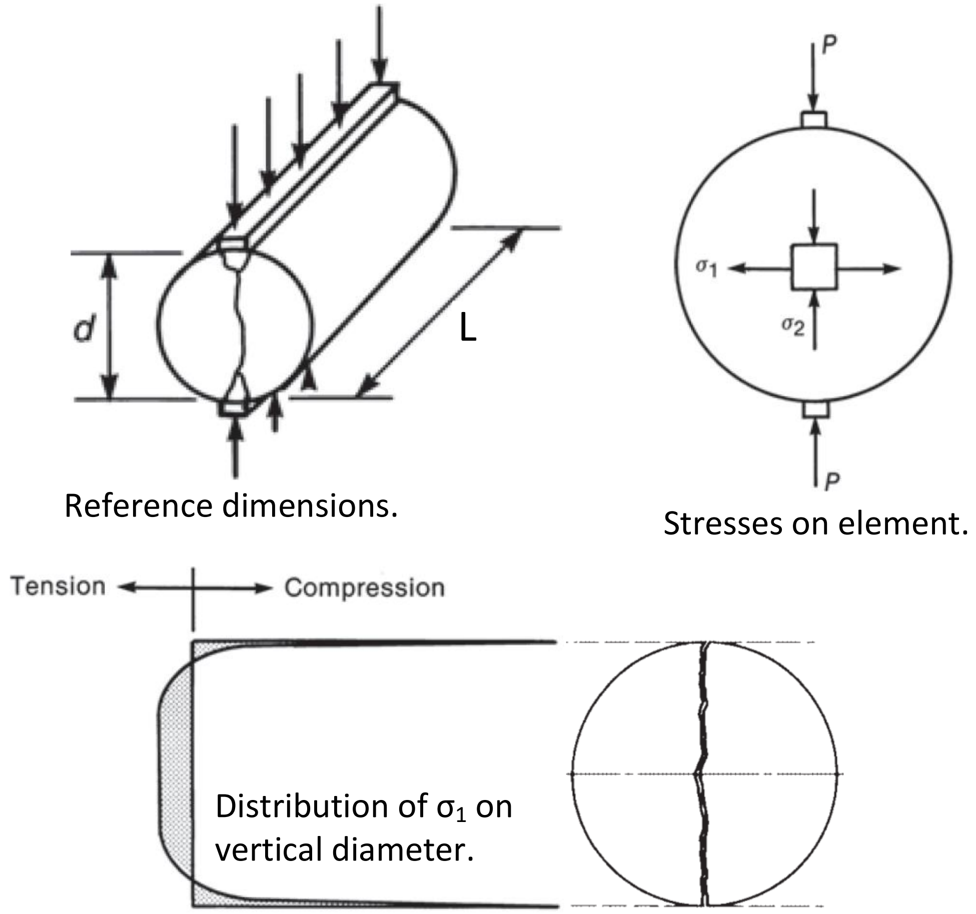
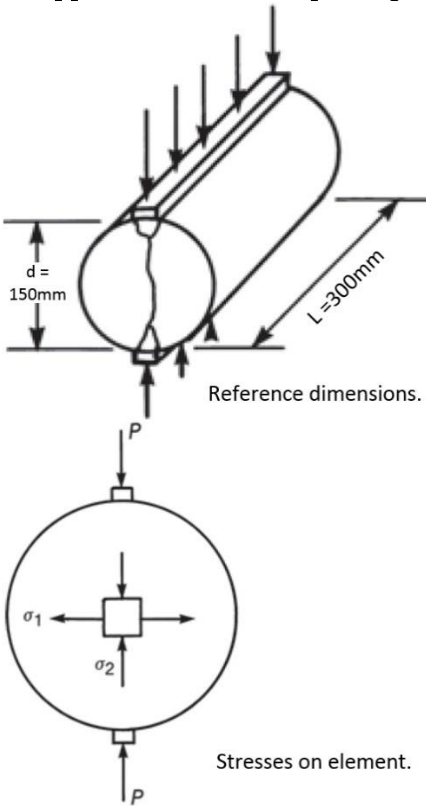


Figure 2.3-14: Stresses in splitting cylinder test.

Example 2.3-5

Use a conservative correlation to estimate the splitting tensile strength, f_{ct}, for a concrete that has a compressive strength, $f_c{}'$, of 30 MPa. Then use the determined f_{ct} to estimate the force P that should be applied in an actual splitting test.

Figure 2.3-15: Concrete cylinder splitting test.



Solution

Split-cylinder strength, f_{ct}, can be estimated based on following relation with cylindrical compressive strength:

$f_{ct} = 0.5 \; to \; 0.66 \sqrt{f_c{}'}$

A conservative value of

$$f_{ct} = 0.5\sqrt{f_c{}'} = 0.5 \times \sqrt{30} = 2.74\ MPa$$

to be adopted according to the problem statement.

It can be shown that in an elastic cylinder so loaded, a nearly uniform tensile stress of magnitude:

$$f_{ct} = \sigma_1 = \frac{2P}{\pi dL} \Rightarrow 2.74 = \frac{2 \times P}{\pi \times 150 \times 300}$$

Solve for P:

$$P = 193679\ N = 194\ kN$$

Example 2.3-6

Use a suitable correlation to estimate the modulus of rapture, f_r, for concrete that has a compressive strength, $f_c{}'$, of 30 MPa. Then use the determined f_r to estimate the force P that should be applied in an actual test for a beam specimen indicated in Figure 2.3-16 that has a cross-section of 100x200mm.

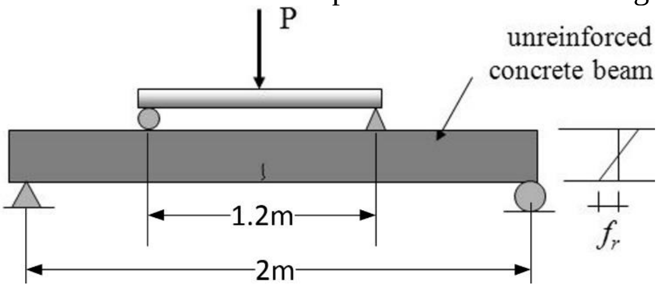


Figure 2.3-16: Concrete specimen for modulus of rapture.

Solution

According to (ACI318M, 2014), Article 19.2.3, modulus of rupture, f_r, for concrete can be estimated based on following correlation with concrete compressive strength:

$$f_r = 0.62\lambda\sqrt{f_c{}'} = 0.62 \times 1.0 \times \sqrt{30} \approx 3.4\ MPa \blacksquare$$

According to traditional flexural formula, the modulus of rapture, f_r, is related to sectional moment, M, as follows:

$$f_r = \frac{6M}{bh^2} \Rightarrow 3.4 = \frac{6M}{100 \times 200^2} \Rightarrow M = 2266667\ N.mm \Rightarrow M = 2.27\ kN.m$$

Finally, based on simple statics, the applied force, P, can be related to the sectional moment, M, as follows:

$$M = \frac{P}{2} \times a \Rightarrow 2.27 = \frac{P}{2} \times (2 - 1.2) \times \frac{1}{2} \Rightarrow P = 11.4\ kN \blacksquare$$

2.4 REINFORCING STEELS FOR CONCRETE

2.4.1 Joint Performance of Steel and Concrete

- The two materials, concrete and reinforcement, are best used in combination if the ***concrete is made to resist the compressive stresses*** and ***the steel the tensile stresses***.
- Flexure
  In reinforced concrete beams, the concrete resists the compressive force, longitudinal steel reinforcing bars are located close to the tension face to resist the tension force.

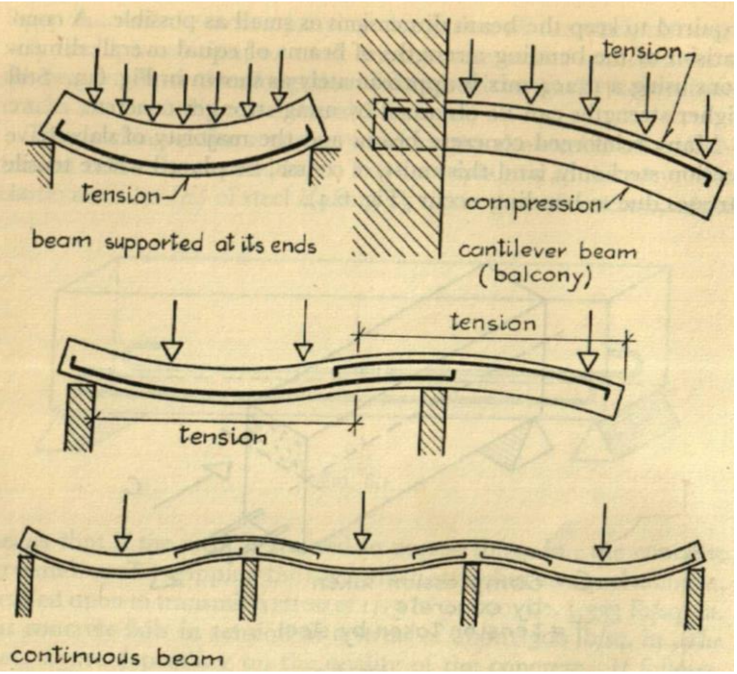


Figure 2.4-1: Reinforcement role in flexure resistance.

- Shear
  Usually additional steel bars are used to resist the inclined tension stresses that are caused by the shear force in the beams.

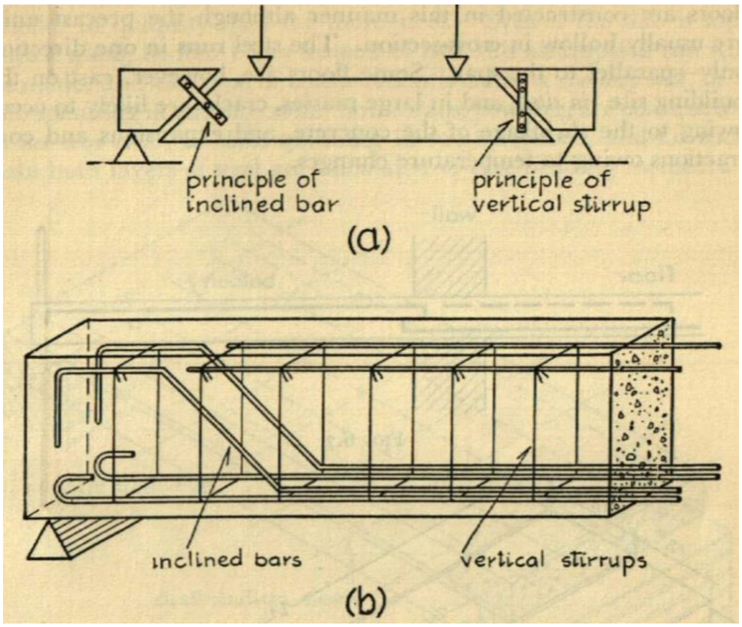


Figure 2.4-2: Reinforcement role in shear resistance.

- Columns
  - However, reinforcement is also used for ***resisting compressive forces primarily where it is desired to reduce the cross-sectional dimensions of compression members***, as in the lower-floor columns of multistory buildings.
  - Even in axially compressed member, ***a minimum amount of reinforcement is placed in all compression members to safeguard them against the effects of small accidental bending moments that might crack and even fail an unreinforced member***.

2.4.2 Additional Notes on Joint Performance of Steel and Concrete

Additional features that make for the satisfactory joint performance of steel and concrete are the following:

- The ***thermal expansion*** coefficients of the two materials, about 11.7×10^{-6} for steel vs. an average of 9.9×10^{-6} for concrete, are sufficiently close to forestall cracking and other undesirable effects of differential thermal deformations.
- While the ***corrosion resistance*** of bare steel is poor, the concrete that surrounds the steel reinforcement provides excellent corrosion protection, minimizing corrosion problems and corresponding maintenance costs.
- The ***fire resistance*** of unprotected steel is impaired by its high thermal conductivity and by the fact that its strength decreases sizably at high temperatures. Conversely, the thermal conductivity of concrete is relatively low. Thus, damage caused by even prolonged fire exposure, if any, is generally limited to the outer layer of concrete, and a moderate amount of concrete cover provides sufficient thermal insulation for the embedded reinforcement.

2.4.3 Reinforcing Bars

- Typical Bars[1]:
  The most common type of reinforcing steel (as distinct from prestressing steel) is in the form of round bars, often called ***rebars***, available in a large range of diameters from about 10 to 36 mm for ordinary applications and in two heavy bar sizes of about 43 and 57mm, see Table 2.4-1.
- Bar Deformed Surface:
  - These bars are furnished with surface deformations for the purpose of ***increasing resistance to slip between steel and concrete***.
  - Minimum requirements for these deformations (spacing, projection, etc.) have been developed in experimental research.
  - Different bar producers use different patterns, all of which satisfy these requirements. Figure 2.4-3 shows a variety of current types of deformations.
- Designation by Number:
  - For many years, bar sizes have been designated by numbers, Nos. 10 to 36 being commonly used and Nos. 43 and 57 representing the two special large-sized bars previously mentioned.
  - Designation by number, instead of by diameter, ***was introduced because the surface deformations make it impossible to define a single easily measured value of the diameter***.

---

[1] A "***soft conversion***" involves changing a measurement from inch-pound units to equivalent metric units within what the DoD calls "***acceptable measurement tolerances***." This is done to merely convert the imperial measurements to metric without physically changing the item, and it is typically used to specify a requirement. For example, a ½-in. rebar diameter would be converted to either 12.7 or 13 mm using soft conversion. Although this is not a standard metric rebar size, it expresses the requirement.
A "***hard conversion***" involves a change in measurement units that results in a "physical configuration change." Using the rebar example, this would be analogous to changing the diameter of the rebar from ½ in. to an M12 (12-mm) or M14 (14-mm) rebar diameter. Either one of the two new metric rebar diameters would be outside an "***acceptable measurement tolerance***". The new rebar would be considered a "***hard metric***" item. This is size substitution, which is one method of using hard conversion; the other method is adaptive conversion, where imperial and metric units are reasonably equivalent, but not exact conversions of each other.

Table 2.4-1: ASTM STANDARD REINFORCING BARS

Bar size, no.*	Nominal diameter, mm	Nominal area, mm^2	Nominal mass, kg/m
10	9.5	71	0.560
13	12.7	129	0.994
16	15.9	199	1.552
19	19.1	284	2.235
22	22.2	387	3.042
25	25.4	510	3.973
29	28.7	645	5.060
32	32.3	819	6.404
36	35.8	1006	7.907
43	43.0	1452	11.38
57	57.3	2581	20.24

*Bar numbers approximate the number of millimeters of the nominal diameter of the bar.

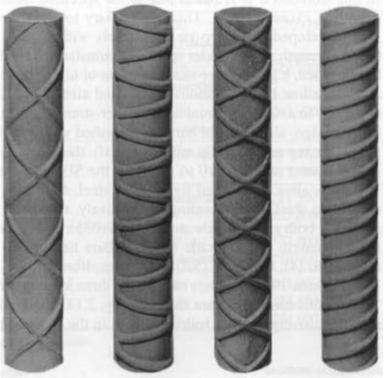


Figure 2.4-3: Types of deformed reinforcing bars.

2.4.4 Grades and Strengths

- Trend toward higher-strength materials:
  In reinforced concrete, a long-term trend is evident toward the use of higher-strength materials, both steel and concrete. Reinforcing bars with 280 MPa yield stress, Grade 40, once standard, have largely been replaced by bars with 420 MPa yield stress, Grade 60, both because:
  - They are more economical,
  - Their use tends to reduce steel congestion in the forms.
- Grads proposed for columns:
  - Bars with a yield stress of 520 MPa are often used in columns,
  - Bars with a yield stress of 690 MPa are allowed to be used as confining reinforcement.
- Table 2.4-2 lists all presently available reinforcing steels, their grade designations, the ASTM specifications that define their properties (including deformations) in detail, and their two main minimum specified strength values.
- Conversion between from US Customary to SI Unit System:

US Customary Unit System	SI Unit System
Grade 40	Grade 280
Grade 60	Grade 420
Grade 75	Grade 520
Grade 100	Grade 690

- Weldability of Rebars:
  - Welding of reinforcing bars;
    - In making splices,
    - Or for convenience in ***fabricating reinforcing cages*** for placement in the forms,

may result in ***metallurgical changes*** that ***reduce both strength and ductility***, and special restrictions must be placed both on the type of steel used and the welding procedures.

- The provisions of ***ASTM A 706*** relate specifically to welding.

Table 2.4-2: Summary of minimum ASTM strength requirements.

Product	ASTM Specification	Designation	Minimum Yield Strength, psi (MPa)	Minimum Tensile Strength, psi (MPa)
Reinforcing bars	A615	Grade 40 Grade 60 Grade 75	40,000 (280) 60,000 (420) 75,000 (520)	60,000 (420) 90,000 (620) 100,000 (690)
	A706	Grade 60	60,000 (420) [78,000 (540) maximum]	80,000 (550)[a]
	A996	Grade 40 Grade 50 Grade 60	40,000 (280) 50,000 (350) 60,000 (420)	60,000 (420) 80,000 (550) 90,000 (620)
	A1035	Grade 100	100,000 (690)	150,000 (1030)
Deformed bar mats	A184	Same as reinforcing bars		
Zinc-coated bars	A767	Same as reinforcing bars		
Epoxy-coated bars	A775, A934	Same as reinforcing bars		
Stainless-steel bars[b]	A955	Same as reinforcing bars		
Wire Plain	A82		70,000 (480)	80,000 (550)
Deformed	A496		75,000 (515)	85,000 (585)
Welded wire reinforcement Plain	A185			
W1.2 and larger			65,000 (450)	75,000 (515)
Smaller than W1.2			56,000 (385)	70,000 (485)
Deformed	A497		70,000 (480)	80,000 (550)
Prestressing tendons Seven-wire strand	A416	Grade 250 (stress-relieved)	212,500 (1465)	250,000 (1725)
		Grade 250 (low-relaxation)	225,000 (1555)	250,000 (1725)
		Grade 270 (stress-relieved)	229,500 (1580)	270,000 (1860)
		Grade 270 (low-relaxation)	243,000 (1675)	270,000 (1860)
Wire	A421	Stress-relieved	199,750 (1375) to 212,500 (1465)[c]	235,000 (1620) to 250,000 (1725)[c]
		Low-relaxation	211,500 (1455) to 225,000 (1550)[c]	235,000 (1620) to 250,000 (1725)[c]
Bars	A722	Type I (plain) Type II (deformed)	127,500 (800) 120,000 (825)	150,000 (1035) 150,000 (1035)
Compacted strand[b]	A779	Type 245 Type 260 Type 270	241,900 (1480) 228,800 (1575) 234,900 (1620)	247,000 (1700) 263,000 (1810) 270,000 (1860)

[a] But not less than 1.25 times the actual yield strength.
[b] Not listed in ACI 318.
[c] Minimum strength depends on wire size.

- Rebar Marking System:
  - To allow bars of various grades and sizes ***to be easily distinguished***, which is necessary ***to avoid accidental use*** of lower-strength or smaller-size bars than called for in the design, all deformed bars are furnished with rolled-in markings.
  - These identify:
    - The producing mill (usually with an initial),
    - The bar size (Nos. 3 to 18 under the inch-pound system and Nos. 10 to 57 under the SI),
    - The type of steel:
      - “S” for carbon steel, (A615)
      - “W” for low-alloy steel, (A706)

- A rail sign for rail steel, (A996)
- "A" for axle steel, (A996)
- "CS" for low-carbon chromium steel, (A1035)

▪ An additional marking to identify higher-strength steels:

- Grade 60 (420) bars have either one longitudinal line or the number 60 (4);
- Grade 75 (520) bars have either two longitudinal lines or the number 75 (5);
- Grade 100 (690) bars have either three longitudinal bars or the number 100 (6).

o The identification marks are shown in Figure 2.4-4.

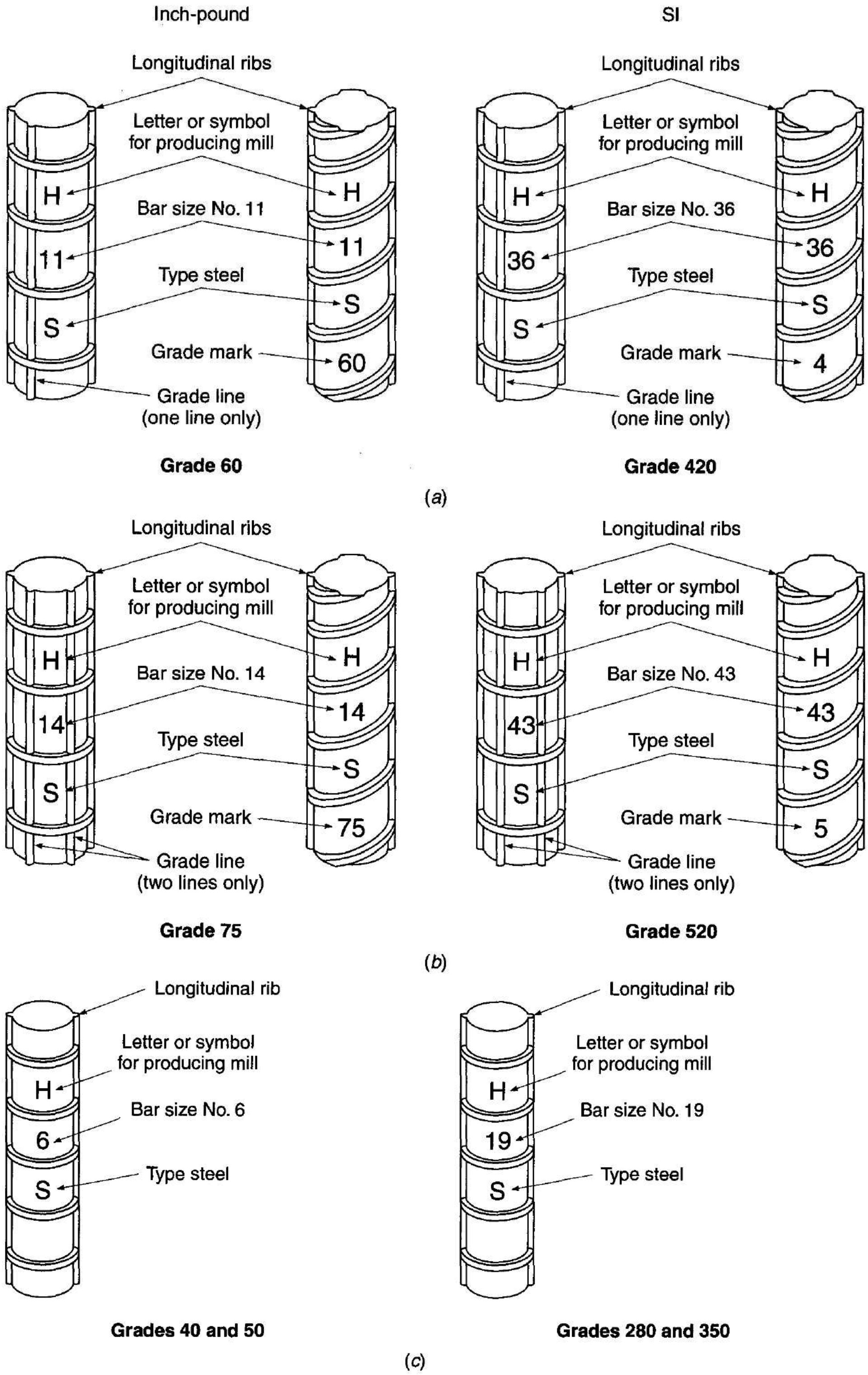


Figure 2.4-4: Marking system for reinforcing bars meeting ASTM Specifications A615, A706, and A996: (a) Grades 60 and 420; (b) Grades 75 and 520; (c) Grades 40, 50, 280, and 350.

2.4.5 Stress-Strain Curves

- Main Characteristics:
  The two chief numerical characteristics that determine the character of bar reinforcement are
  - Its yield point (generally identical in tension and compression)
  - Its modulus of elasticity E_s. It is practically the same for all reinforcing steels (but not for prestressing steels) and is taken as $E_s = 200000\ MPa$, (ACI318M, 2014) ***Article 20.2.2.2***.
- Shape of Stress-strain Curves:
  - Typical stress-strain curves for U.S. reinforcing steels are shown in Figure 2.4-5.
  - The complete stress-strain curves are shown in the left part of the figure; the right part gives the initial portions of the curves magnified 10 times.
  - Low-carbon steels, typified by the Grade 40 curve, show:
    - ***An elastic portion.***
    - ***Yield plateau***, i.e., a horizontal portion of the curve where strain continues to increase at constant stress. For such steels, the ***yield point is that stress at which the yield plateau establishes itself***.
    - ***Strain hardening***, with further strains, the stress begins to increase again, though at a slower rate, a process that is known as ***strain hardening***.
  - Higher-strength carbon steels, e.g., those with 420 MPa (60 ksi) yield stress or higher, have,
    - ***A yield plateau of much shorter length***
    - ***Or enter strain-hardening immediately without any continued yielding at constant stress***.
    - In the latter case, the ACI Code specifies that ***the yield stress*** f_y ***be the stress corresponding to a strain of 0.0035***, as shown in Figure 2.4-5.
  - Low-alloy, high-strength steels rarely show any yield plateau and usually enter strain-hardening immediately upon beginning to yield.

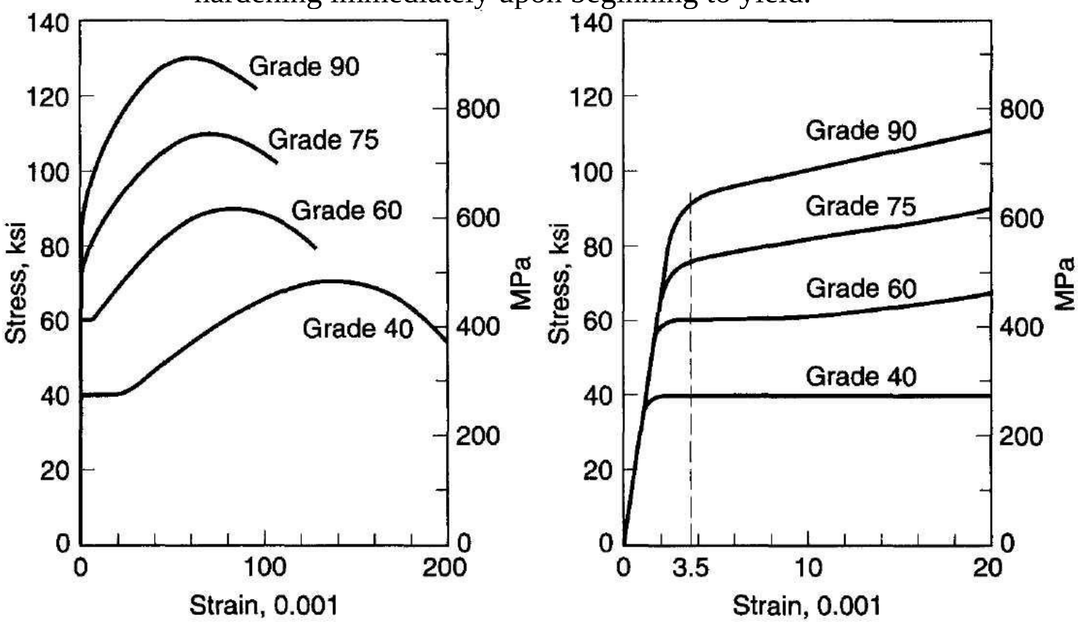


Figure 2.4-5: Typical stress-strain curves for reinforcing bars.

2.4.6 Welded Wire Reinforcement

- Welded wire reinforcement, also described as welded wire fabric, consists of sets of longitudinal and transverse cold-drawn steel wires at right angles to each other and welded together at all points of intersection.

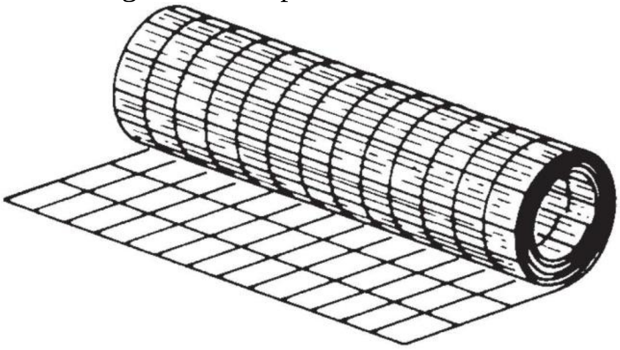


Figure 2.4-6: Welded wire reinforcement.

- Welded wire reinforcement is often used:
    - For reinforcing slabs:

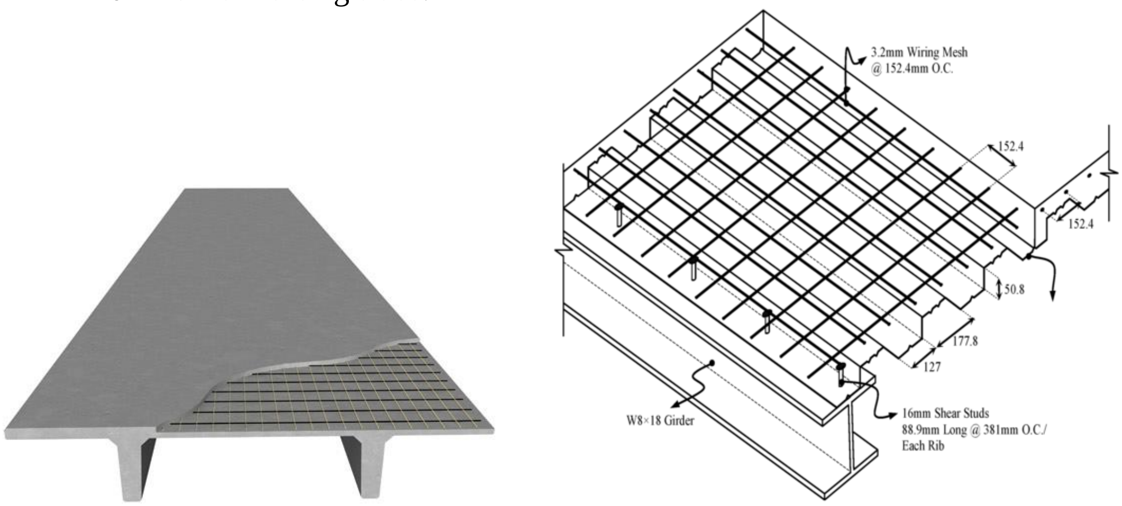


Figure 2.4-7: Slabs reinforced with welded wire reinforcement.

-
    - For reinforcing slabs on grade:

Figure 2.4-8: Slabs on grade reinforced with welded wire reinforcement.

-
    - For shear reinforcement in thin beam webs, particularly in prestressed beams.

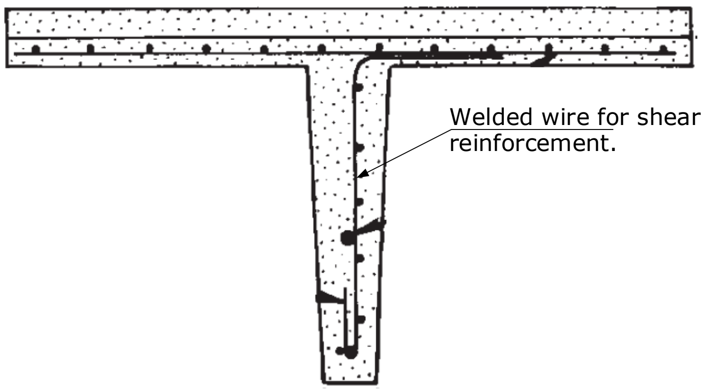


Figure 2.4-9: Welded wire for shear reinforcement.

- Standard wire reinforcement according to ASTM are presented in Table 2.4-3 below. The letter "W" to designate smooth wire while the letter "D" to describe deformed wire.

Table 2.4-3: Standard wire reinforcement

MW & MD size		Nominal diameter, mm	Nominal mass, kg/m	Area, mm^2/m of width for various spacings						
				Center-to-center spacing, mm						
Plain	Deformed			50	75	100	150	200	250	300
MW290	MD290	19.22	2.27	5800	3900	2900	1900	1450	1160	970
MW200	MD200	15.95	1.5700	4000	2700	2000	1300	1000	800	670
MW130	MD130	12.90	1.0204	2600	1700	1300	870	650	520	430
MW120	MD120	12.40	0.9419	2400	1600	1200	800	600	480	400
MW100	MD100	11.30	0.7849	2000	1300	1000	670	500	400	330
MW90	MD90	10.70	0.7064	1800	1200	900	600	450	360	300
MW80	MD80	10.10	0.6279	1600	1100	800	530	400	320	270
MW70	MD70	9.40	0.5494	1400	930	700	470	350	280	230
MW65	MD65	9.10	0.5102	1300	870	650	430	325	260	220
MW60	MD60	8.70	0.4709	1200	800	600	400	300	240	200
MW55	MD55	8.40	0.4317	1100	730	550	370	275	220	180
MW50	MD50	8.00	0.3925	1000	670	500	330	250	200	170
MW45	MD45	7.60	0.3532	900	600	450	300	225	180	150
MW40	MD40	7.10	0.3140	800	530	400	270	200	160	130
MW35	MD35	6.70	0.2747	700	470	350	230	175	140	120
MW30	MD30	6.20	0.2355	600	400	300	200	150	120	100
MW25	MD25	5.60	0.1962	500	330	250	170	125	100	83
MW20		5.00	0.1570	400	270	200	130	100	80	67
MW15		4.40	0.1177	300	200	150	100	75	60	50
MW10		3.60	0.0785	200	130	100	70	50	40	33
MW5		2.50	0.0392	100	67	50	33	25	20	17

2.5 PROBLEMS FOR SOLUTION

Problem 1

Transform the following empirical equations from the SI unit system (where strength and modulus of elasticity are expressed in MPa) to the U.S. customary unit system (where strength and modulus of elasticity are expressed in psi).

$$E_c = 4700\sqrt{f_c'}\ \text{(MPa)} \quad \text{and} \quad f_r = 0.62\sqrt{f_c'}\ \text{(MPa)}$$

Hint for Solution:

It is useful to note that:

1MPa = 145 psi

Notes on Problem 1:

- Relations in engineering and sciences can be classified into ***Rational Relations*** and ***Empirical Relations***.
- ***Rational Relation*** is derived analytically and it represents a cause and effect relation. It must be valid regardless of the system of units used and must be dimensionally homogenous, e.g.:
  $$y'' = \frac{M(x)}{EI}$$
  is a rational equation that shows the relation between the cause (the bending moment $M(x)$) and effect (the beam curvature y''). This relation has unique form regardless it has been written in SI System or in Imperial System.
- On the other hand, ***Empirical Relation*** is the equation that used to express a quantity that is difficult, or costly to be measured directly, in terms of a quantity that can be measured directly with simple procedure and reasonable cost. For example, it is costly to measure (E_c) for concrete directly, then it is usually computed based on an experimental correlation with the concrete compressive strength f_c' that can be measured simply from direct compression test with reasonable cost.
- ***Empirical Relation*** not necessarily represents a cause and effect relation, but usually describe that relation between two quantities that depend on the same agent. For example increasing of f_c' is not a reason for increasing of E_c of concrete, but since both quantities depend on the water cement ratio, then the increasing in f_c' can be taken as an indication on increasing of E_c of concrete and vice versa.
- Above concepts have been used widely in reinforced concrete design. ***ACI code had been used the concrete compressive strength as an index on nearly all concrete properties***. All these relations are empirical in nature.

Answers:

$$E_{\text{Concrete in psi}} = 56\,595\sqrt{f_c'(\text{psi})} \quad \text{and} \quad f_{r\text{ in psi}} = 7.46\sqrt{f_c'(\text{psi})}$$

Problem 2

The specified concrete strength f_c' for a new building is 42 MPa. Calculate the required average strength f_{cr}'; for the concrete:

(a) If there are no prior test results for concrete with a compressive strength within 7 MPa of f_c' made with similar materials,

(b) If 20 test results for concrete with $f_c' = 35\ MPa$ made with similar materials produce a sample standard deviation s_s of 4.0 MPa,

(c) If 30 tests with $f_c' = 38\ MPa$ made with similar materials produce a sample standard deviation s_s of 4.1 MPa.

Problem 3

Ten consecutive strength tests are available for a new concrete mixture with $f_c' = 28\ MPa$: $31.7. 32.8, 36.4, 29.0, 30.8, 28.8, 25.9, 35.2, 32.0,\ and\ 28.8\ MPa$

(a) Do the strength results represent concrete of satisfactory quality? Explain your reasoning.

(b) If f_{cr}' has been selected based on 30 consecutive test results from an earlier project with a sample standard deviation s_s of 3.5 MPa, must the mixture proportions be adjusted? Explain.

Contents

CHAPTER 3

DESIGN OF CONCRETE STRUCTURES AND FUNDAMENTAL ASSUMPTIONS

3.1 INTRODUCTION

- Unknowns which determined in design process:
  Design is the determination of the ***general shape*** and ***specific dimensions***.
- Goals that should be performed in the design of structure:
  - The ***function for which it was created,***
  - S***afety to withstand the influences that will act on it*** throughout its useful life.
- Primary influences act on the structure:
  The influences are primarily:
  - The ***loads*** and ***other forces*** to which it will be subjected,
  - Other ***detrimental agents***, such as ***temperature fluctuations*** and ***foundation settlements***.
- How architect and the engineer work together to select the concept and system:
  - In the case of a ***building***, ***an architect may present an overall concept*** and with ***the engineer develop a structural system***.
  - For ***bridges*** and ***industrial facilities***, the ***engineer is often directly involved in selecting both the concept and the structural system***.
- General Sequence Adopted in Design of Concrete Structures
  Regardless of the application, the design of concrete structures follows the same general sequence.
  - First, an ***initial structural system is defined***, the ***initial member sizes are selected***, and ***a mathematical model of the structure is generated***.
  - Second, ***gravity and lateral loads are determined*** based on the selected system, member sizes, and external loads. Building loads typically are defined in (ASCE/SEI 7–10), as discussed in ***Chapter 1***.
  - Third, the ***loads are applied*** to the ***structural model and the load effects calculated for each member***. This step may be done on a ***preliminary basis*** or by ***using computer-modeling software***.
    This step is ***more complex for buildings*** in ***Seismic Design Categories D though F*** where the ***seismic analysis requires close coordination of the structural framing system and the earthquake load***s (discussed in Chapter 20).
  - Fourth, maximum load effects at ***critical member sections are identified*** and ***each critical section is designed for moment***, ***axial load***, ***shear***, and ***torsion*** as needed.
  - Fourth step may ***become iterative***, For example:
    - If the member initially selected is ***too small***, ***its size must be increased***, ***load effects recalculated for the larger member***, and ***the members redesigned***.
    - If ***the initial member is too large***, a ***smaller section is selected***; however, loads are ***may not be recalculated, as gravity effects are most often conservative***.
  - Fifth, each member is ***checked for serviceability***.
  - Sixth, ***the reinforcement for each member is detailed***, that is, the number and size of reinforcing bars are selected for the critical sections to provide the required strength.
  - Seventh, ***connections are designed*** to ensure that the building performs as intended.
  - Finally, the ***design information is incorporated in the construction documents***.

  This process is illustrated in ***Figure 3.1-1***. In addition to ***the design methodology***, ***Figure 3.1-1*** indicates the chapters in the textbook and in the ACI Code, (ACI318M, 2014), where the topics are covered.

- ACI code versus textbooks:
    - The ACI Code ***is written based on the assumption that the user understands concrete structural behavior and the design process, whereas this text builds that understanding***.
    - Textbooks are organized so that ***the fundamental theory is presented first***, followed by ***the Code interpretation of the theory***. Thus, ***the text remains relevant even as Code provisions are updated***.

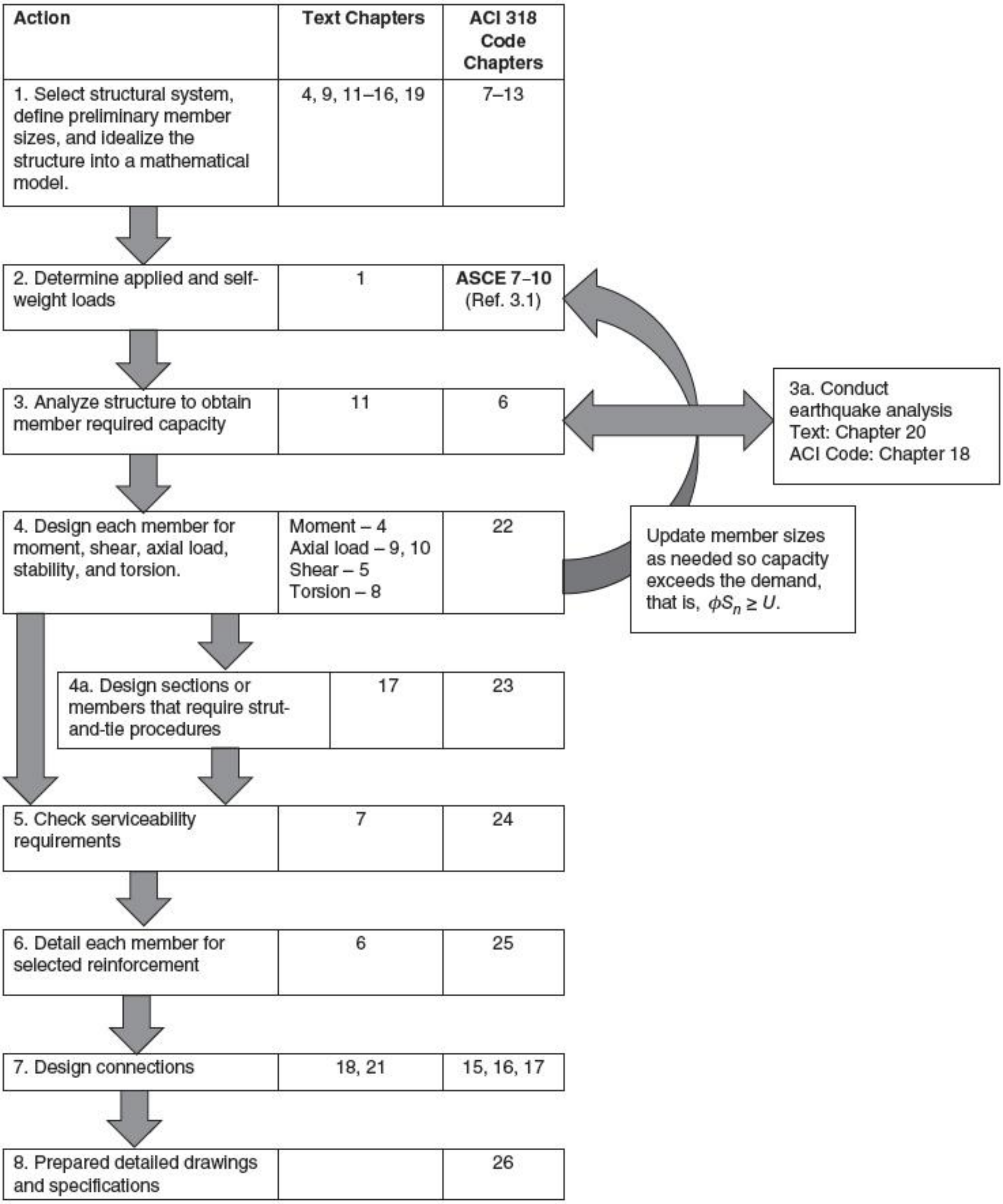
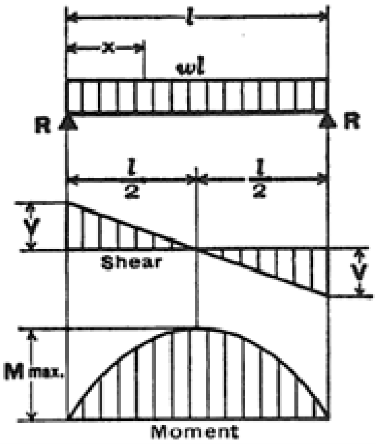


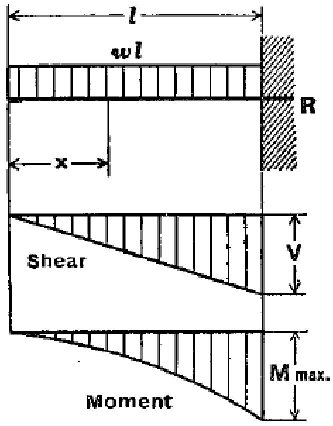
Figure 3.1-1: Design development sequence.

3.2 MEMBERS AND SECTIONS

- The term "***membe***r":
  The term member refers to ***an individual portion of the structure***, such as a ***beam***, ***column***, ***slab***, or ***footing***.
- The term "***section***":
  - Moment, axial load, and shear are distributed along the member, and the member is designed at discrete locations, i.e. discrete sections.
  - The engineer identifies the maximum value of these loads and designs the member at these discrete locations so that the strength at the section exceeds these values. It is not necessary to design every section of a member.
  - For examples, the simply supported and cantilever beams indicated in ***Figure 3.2-1***, have infinite sections but shear forces and bending moments are determined at finite number, discrete number, of sections and then the beams are designed based on critical sections.
    - For the simply supported beam of ***Figure 3.2-1a***, the critical section for flexure is at beam mid-span while the critical section for shear is at support region.
    - For the cantilever beam of ***Figure 3.2-1b***, the critical section for flexure and shear is located at support region.



(a) A simply sumpported beam.



(b) A cantilever beam.

Figure 3.2-1: Critical sections for simply supported and cantilever beams.

- Requirements beyond the critical section:
  - The requirement of:

    $$\phi S_n \geq U \quad \textbf{Eq. 3.2-1}$$

    implies that ***reinforcement for maximum loads can be carried beyond the critical section to ensure that the strength requirements are satisfied for the entire member***.
  - In addition to strength, the reinforcement is designed to provide ***overall structural integrity*** and to ensure that it is anchored to the concrete.

3.3 THEORY, CODES, AND PRACTICE

- The design of concrete structures ***requires an understanding*** the interaction of:
    - Structural theory,
    - The role of building codes,
    - Experience in the practice of structural design itself.
- An example of how practice experience can affect the structural code and theory of structure:
    - A structural failure, a ***practice experience***, may lead to a code revision, i.e. practice experience may alter the design code.
    - The failure may also lead to research, which in turn provides a new theoretical model, i.e. practice experience may alter the structural theory.
    - Changes in practice may also be made to preclude similar failures, even without a code change.
- Insight to the interplay of each of these elements is essential for the engineer to design safe, serviceable, and economical structures.

3.3.1 Theory

- Structural theory includes ***mathematical***, ***physical***, or ***empirical models*** of the behavior of structures.
- For example, in beam theory, equation of ***Eq. 3.3-1*** contains mathematical model of $\kappa \approx y''$, physical models of equilibrium equations and Hook's law, and it may contains an empirical model $E_c \approx 4700\sqrt{f_c'}$.

$$EIy'' = M(x) \qquad \textbf{Eq. 3.3-1}$$

- Mathematical and Physical Models:
    - These models have ***evolved over decades*** of ***research*** and ***practice***.
    - They are used to ***predict the nominal strength of members***.
    - The most robust theories derive from statics, equilibrium, and mechanics of materials.
    - Examples include:
        - Equations for the strength of a concrete section for bending, M_n, (Chapter 4),
        - Bending plus axial load, i.e. M_n and P_n, (Chapters 9 and 10).
- Empirical Models:
  Empirical models consists from the following basic steps:
    - Observations:
      In other cases, an ***empirical understanding of structural behavior***, derived from ***experimental observation***, is combined with theory to develop the prediction of member strength.
    - Fitting:
      In this case, ***equations are then fitted to the experimental data to predict the strength***.
    - Adjusting:
      If the experimental strength of a section is highly variable, then the predicted equations are adjusted for use in design to predict a lower bound of the section capacity.

  This approach is used, for example, to calculate the shear strength of a section, V_n, (***Chapter 5***) and anchorage capacity (***Chapter 21***).

3.3.2 Codes

- Building codes provide minimum requirements for the ***life safety*** and ***serviceability*** for structures.
- Code limits the theory:
    - In their simplest application, codes present the theory needed to ensure that ***sectional and member strengths are provided and define the limits on that theory***.
    - For example, ***a structure could be constructed using a large unreinforced concrete beam that relies solely on the tensile strength of the concrete***. Such ***a structure would be brittle***, and an unanticipated load would lead to sudden collapse. Codes ***prohibit such designs***.

- In a similar manner, ***codes prescribe the maximum and minimum amount of reinforcement allowed in a member***.
- Codes also address ***serviceability considerations***, such as ***deflection*** and ***crack control***.

- Codes impose restrictions out the scope of theory:
  Codes may also ***contain restrictions resulting from failures in practice that were not predicted by the theory*** upon which the code is based.
- Code provisions for structural integrity:
  - Codes require reinforcement to ***limit progressive or disproportional collapse***.
  - Disproportional collapse occurs when ***the failure of a single member leads to the failure of multiple adjacent members***.
  - The failure of a single apartment wall in the ***Ronan Point apartment complex*** in 1968 led to the ***failure of several other units,*** see ***Figure 3.3-1*** and ***Figure 3.3-2***.
  - In response to this collapse, ***codes added requirements for integrity reinforcement based on a rational assessment of the failure***.
  - This integrity reinforcement is a ***prescriptive provision***, that is, ***the requirements are detailed in the code and must be incorporated in the structure <u>without associated detailed calculations</u>***.

Figure 3.3-1: Ronan Point apartment complex.

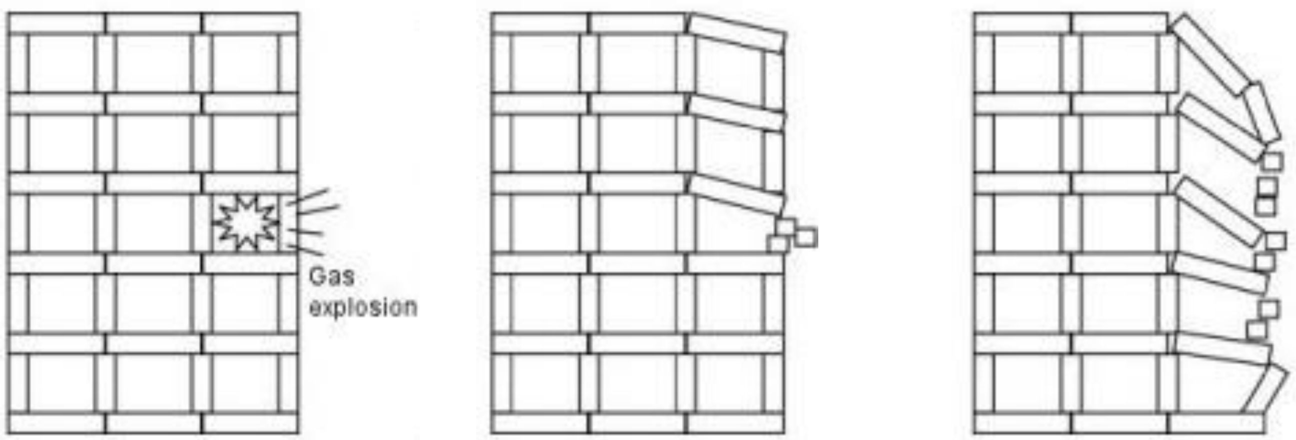


Figure 3.3-2: Chain reaction collapse of a building frame.

- Language of the code:
  - Codes are written in ***terse language***, based on the assumption that the user is a ***competent engineer***.
  - A ***commentar***y accompanies most codes and assists in understanding, ***provides references or background***, and ***offers rationale for the provisions***.
- Codes provide minimum requirements only:
  Because codes provide the minimum requirements for safety and serviceability, ***the engineer is allowed to exceed these requirements***.

3.3.3 Practice

- Structural engineering practice encompasses both the ***art*** and the ***technical practice*** of structural design.
- Throughout history, many extraordinary structures, such as the ***mosques*** and ***cathedrals***, have been ***designed and constructed without the benefit of modern theory and codes***.
- While ***theory and codes provide the mechanics for establishing the strength and serviceability of structures, neither provides the aesthetic, economic, or functional guidance needed for member selection***.
- Questions such as "***Should a beam be slender or stout within the code limits***?" or "***How should the concrete mixture be adjusted for corrosive environments***?" need to be answered by the engineer. To respond, the engineer relies on ***judgment***, ***personal experience***, and the ***broader experience of the profession to adapt the design to meet the overall project requirements***.
- Inclusion of ***long-standing design guidelines for the selection of member sizes is a good example of how that broader experience of the profession is used***.

3.4 BEHAVIOR OF MEMBERS SUBJECT TO AXIAL LOADS

3.4.1 Big Picture for Behavior of Reinforced Concrete Mechanics through Analysis of Axially Loaded Members

- Many of the ***fundamentals of the behavior of reinforced concrete***, through ***the full range of loading from zero to ultimate***, can be ***illustrated*** clearly in ***the context of members subject to simple axial compression or tension***.
- The basic concepts illustrated here will be recognized in later chapters in the analysis and design of beams, slabs, eccentrically loaded columns, and other members subject to more complex loadings.

3.4.2 Axial Compression

- In members that sustain chiefly or exclusively axial compression loads, such as building ***columns***, ***it is economical to make the concrete carry most of the load***.
- Why steel reinforcement are used in an axially loaded member:
  Still, some steel reinforcement is always provided for various reasons.
  - Very few members are ***subjected to truly axial load***; steel is essential for resisting any bending that may exist.
  - If part of the total load is carried by steel with its much greater strength, the ***cross-sectional dimensions of the member can be reduced***—the more so, the larger the amount of reinforcement.
- Two chief column forms:
  - The two chief forms of reinforced concrete columns are shown in Figure 3.4-1.
- The square column:
  - The four longitudinal bars serve as main reinforcement.
  - They are held in place by transverse small-diameter steel ties that:
    - prevent displacement of the main bars ***during construction operations,***
    - counteract any tendency of ***the compression loaded bars to buckle out of the concrete by bursting the thin outer cover***.
- The round column:
  - There are eight main reinforcing bars.
  - These are surrounded by a closely spaced spiral that ***serves the same purpose as the more widely spaced ties*** but also ***acts to confine the concrete within i***t, thereby ***increasing its resistance to axial compression***.
- The discussion that follows applies to ***<u>tied columns</u>***.

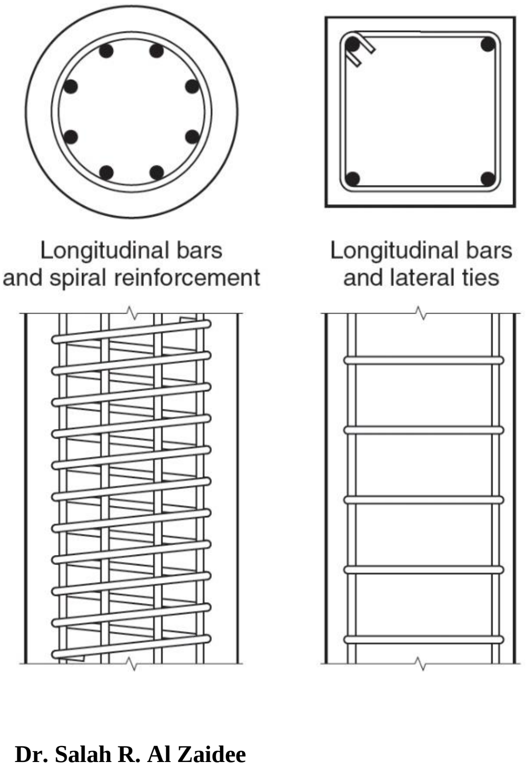


Figure 3.4-1: Reinforced concrete columns.

3.4.2.1 Application of Fundamental Assumption for Analysis of an Axially Loaded Member

- Fundamental assumptions for reinforced concrete behavior of Section 1.8 of Chapter 1 can be adopted to formulate the behavior of axially load member.
- Compatibility and Kinematic Assumption:
  - When axial load is applied, the compression deformation is the same over the entire cross section:
    $\Delta = constant$
  - Hence the strain would be constant for the entire section:
    $\epsilon = \frac{\Delta}{L} = constant$
  - In view of the bonding between concrete and steel, is the same in the two materials:
    $\epsilon_c = \epsilon_{st} = consatnt$
- Stresses-strain Diagrams:
  - Figure 3.4-2 shows two representative stress-strain curves, one for a concrete with compressive strength $f_c' = 28\ MPa$ and the other for a steel with yield stress $f_y = 420\ MPa$.
  - The curves for the two materials are drawn on the same graph using different vertical stress scales.
  - Different Loading Rates for Concrete:
    - ***Curve b*** has the shape that would be obtained in a concrete ***cylinder test***.
    - The rate of loading in most structures is considerably slower than that in a cylinder test, and this affects the shape of the curve.
    - ***Curve c***, therefore, is drawn as being ***characteristic of the performance of concrete under slow loading***.
    - Under these conditions, tests have shown that the maximum reliable compressive strength of reinforced concrete is about $0.85 f_c'$, as shown.

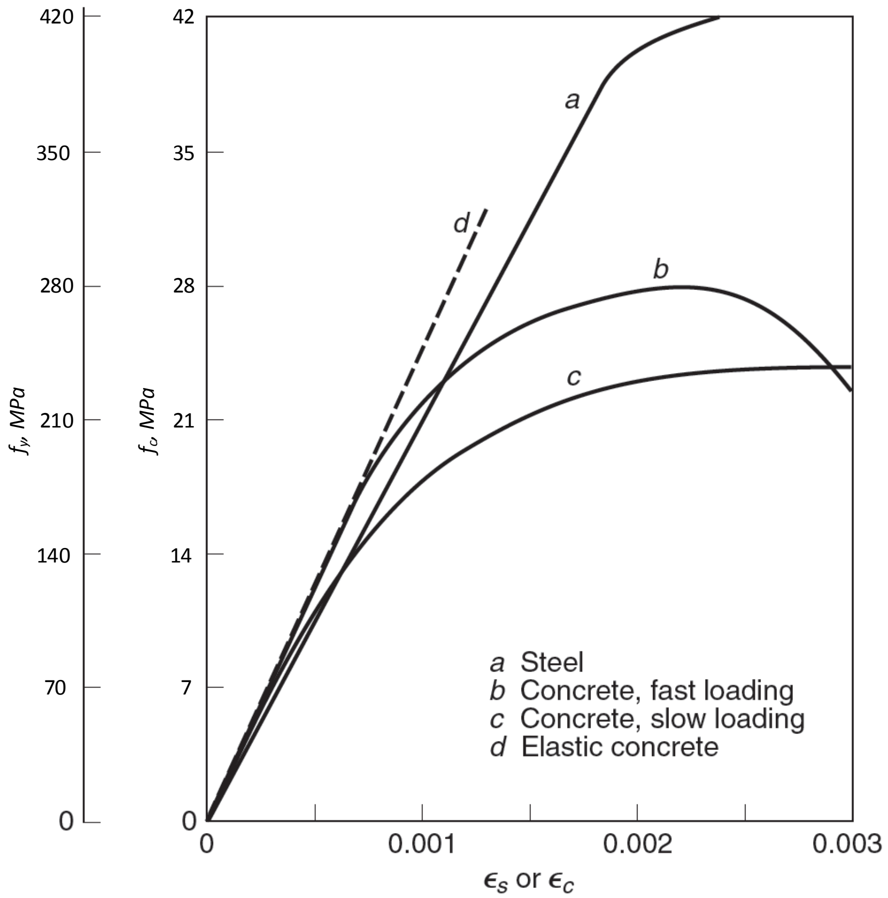


Figure 3.4-2: Concrete and steel stress strain curves.

3.4.2.2 Elastic Behavior

- At low stresses, up to about $f_c'/2$, the concrete is seen to ***behave nearly elastically***, that is, ***stresses and strains are quite closely proportional***; the straight line d represents this range of behavior with little error for both rates of loading.
- For the given concrete, the range extends to a strain of about 0.0005. The steel, on the other hand, is seen to be elastic nearly to its yield point of $420\ MPa$, or to the much greater strain of about 0.002.
- Stress Distribution:
  Because the compression strain in the concrete, at any given load, is equal to the compression strain in the steel,
  $$\epsilon_c = \frac{f_c}{E_c} = \epsilon_s = \frac{f_s}{E_s}$$
  from which the relation between the steel stress f_s and the concrete stress f_c is obtained as:
  $$f_s = \frac{E_s}{E_c} f_c = n f_c \qquad \textbf{Eq. 3.4-1}$$
  where $n = E_s/E_c$ is known as the modular ratio.
- Equilibrium Conditions:
  Let
  A_c = net area of concrete, that is, gross area minus area occupied by reinforcing bars
  A_g = gross area
  A_{st} = total area of reinforcing bars
  P = axial load
  Then from stress distribution and equilibrium condition, namely $\Sigma F_y = 0$, one can obtain:
  $$P = f_c A_c + f_s A_{st} = f_c A_c + n f_c A_{st}$$
  or
  $$P = f_c (A_c + n A_{st}) \qquad \textbf{Eq. 3.4-2}$$
- Concept of the transformed area:
  - The term $A_c + nA_{st}$ can be interpreted as the area of a ***fictitious concrete cross section***, the ***transformed area***, which when subjected to the particular concrete stress f_c results in the same axial load P as the actual section composed of both steel and concrete.
  - This transformed concrete area is seen to consist of the actual concrete area plus n times the area of the reinforcement. It can be visualized as shown in ***Figure 3.4-3***. That is, in ***Figure 3.4-3b*** the three bars along each of the two faces are thought of as being removed and replaced, at the same distance from the axis of the section, with added areas of fictitious concrete of total amount nA_{st}.
  - Alternatively, as shown in ***Figure 3.4-3c*** , one can think of the area of the steel bars as replaced with concrete, in which case one has to add to the gross concrete area A_g so obtained only $(n - 1)A_{st}$ to obtain the same total transformed area.
  - Therefore, alternatively,
    $$P = f_c\big(A_g + (n-1)A_{st}\big) \qquad \textbf{Eq. 3.4-3}$$

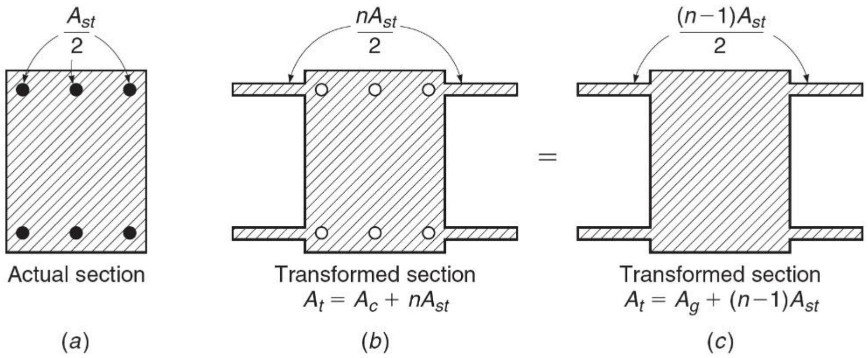


Figure 3.4-3: Transformed section in axial compression.

Example 3.4-1

A column made of the materials defined in ***Figure 3.4-2*** has a cross section of $400 \times 500mm$ and is reinforced by six No. 29 bars, disposed as shown in ***Figure 3.4-3***. Determine the axial load that will stress the concrete to $8.0MPa$. The modular ratio n may be assumed equal to 8, in view of ***the scatter inherent in*** E_c***, it is customary and satisfactory to round off the value of n to the nearest integer and never justified to use more than two significant figures***.

Solution

From Eq. 3.4-3,

$$P = f_c\left(A_g + (n-1)A_{st}\right) \Rightarrow P = 8.0 \times \left((400 \times 500) + (8-1) \times \left(6 \times \frac{\pi \times 29^2}{4}\right)\right)$$

Solve,

$P = 1821935\ N$

Of this total load, the concrete is seen to carry:

$$P_c = f_c A_c = 8.0 \times \left(400 \times 500 - \left(6 \times \frac{\pi \times 29^2}{4}\right)\right) = 1568295\ N$$

and the steel

$$P_s = f_s A_{st} = n f_c A_{st} = 8 \times 8 \times \left(6 \times \frac{\pi \times 29^2}{4}\right) = 253640\ N$$

The percent of load that are supporting by steel would be:

$$Ratio\ of\ P_s = \frac{P_s}{P} \times 100 = \frac{253640}{1821935} \times 100 = 13.9\ \%$$

3.4.2.3 Inelastic Range

- Inspection of ***Figure 3.4-2***, reproduce in below for convenience, shows that the elastic relationships that have been used so far ***cannot be applied beyond a strain of about 0.0005 for the given concrete***.
- To obtain information on the behavior of the member at larger strains and, correspondingly, at larger loads, it is therefore necessary to make direct use of the information in ***Figure 3.4-2***.

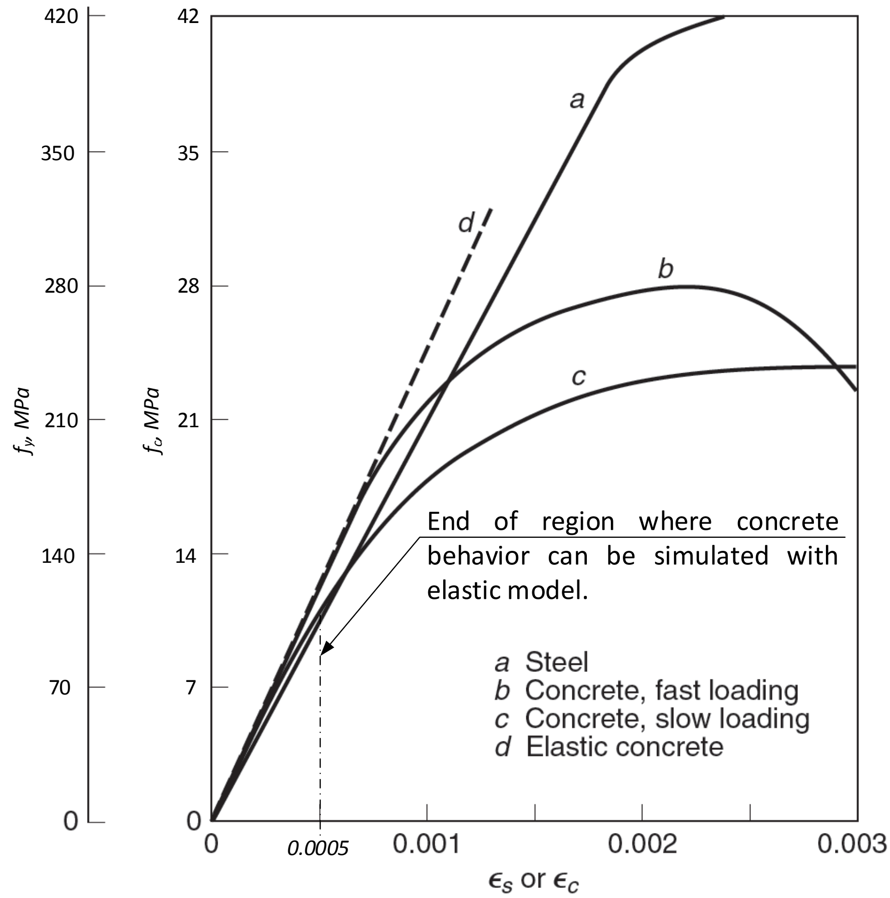


Figure 3.4-2: Concrete and steel stress strain curves. Reproduced for convenience.

Example 3.4-2

Determine the magnitude of the axial load that will produce a strain or unit shortening $\epsilon_c = \epsilon_s = 0.001$ in the column of ***Example 3.4-1***.

Solution

At this strain, the steel is seen to be still elastic, so that the steel stress:

$$f_s = \epsilon_s E_s = 0.001 \times 200000 = 200\ MPa$$

The concrete is in the inelastic range, so that its stress cannot be directly calculated, but it can be read from the stress-strain curve for the given value of strain. Considering load rate, there are two possible solution as indicated in below:

<u>**Fast Loading Rate:**</u>

With referring to ***Figure 3.4-4***, the concrete stress would be:

$$f_c \approx 22\ MPa$$

$$P = f_c A_c + f_s A_s = \frac{\left(22 \times \left(400 \times 500 - \left(6 \times \frac{\pi \times 29^2}{4}\right)\right)\right) + \left(200 \times \left(6 \times \frac{\pi \times 29^2}{4}\right)\right)}{1000}$$

$$= 5105\ kN$$

$$Ratio\ of\ P_s = \frac{P_s}{P} \times 100 = \frac{\left(200 \times \left(6 \times \frac{\pi \times 29^2}{4}\right)\right)}{5105 \times 10^3} \times 100 = 15.5\ \%$$

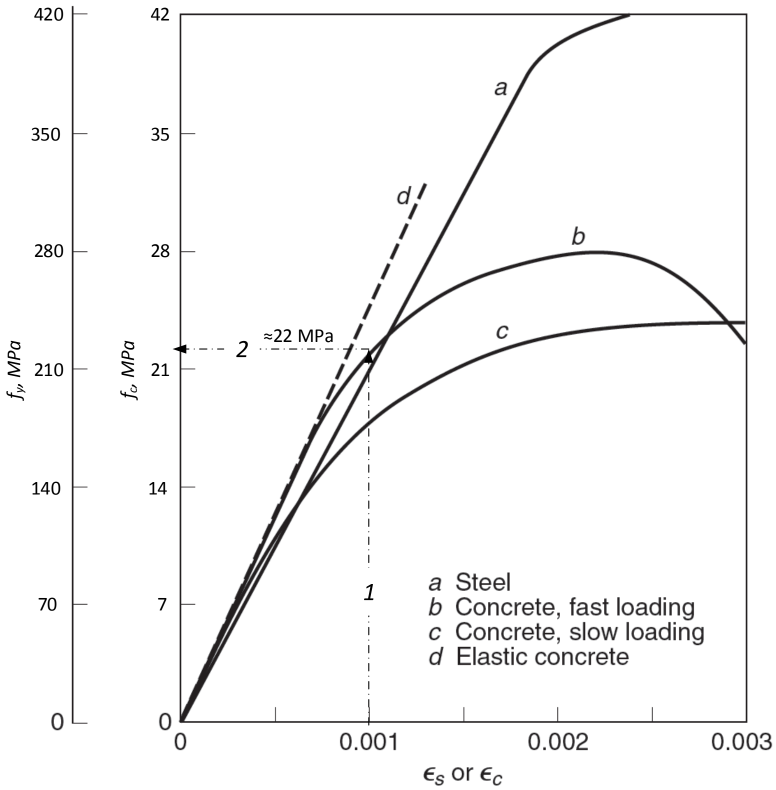


Figure 3.4-4: Concrete stress for the column of Example 3.4-2 when load rate is fast.

Slow Loading Rate:

With referring to ***Figure 3.4-5***, the concrete stress would be:

$f_c \approx 17\ MPa$

$$P = f_c A_c + f_s A_s = \frac{\left(17 \times \left(400 \times 500 - \left(6 \times \frac{\pi \times 29^2}{4}\right)\right)\right) + \left(200 \times \left(6 \times \frac{\pi \times 29^2}{4}\right)\right)}{1000}$$

$$= 4125\ kN$$

$$Ratio\ of\ P_s = \frac{P_s}{P} \times 100 = \frac{\left(200 \times \left(6 \times \frac{\pi \times 29^2}{4}\right)\right)}{4125 \times 10^3} \times 100 = 19.2\ \%$$

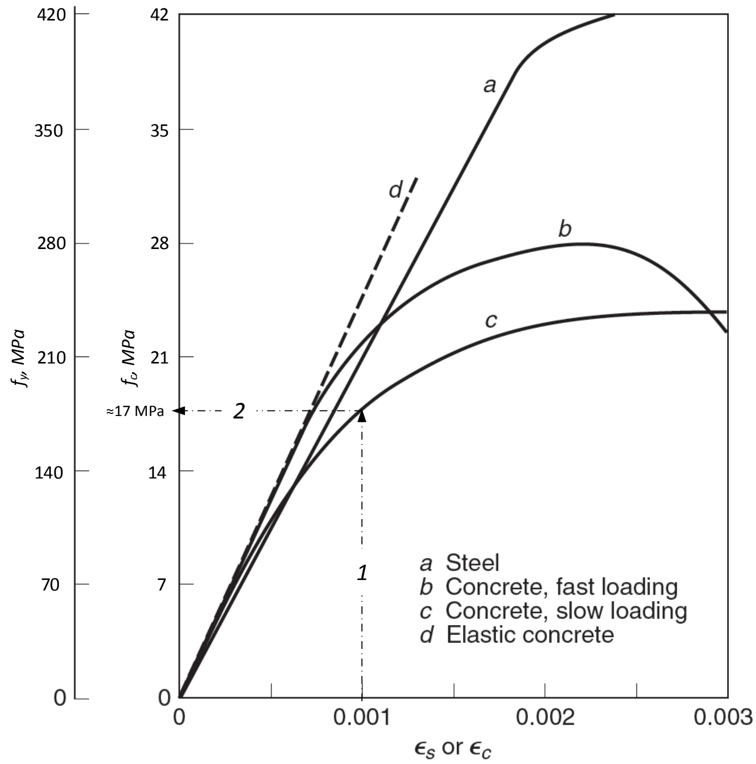


Figure 3.4-5: Concrete stress for the column of Example 3.4-2 when load rate is slow.

Comparison of the results for fast and slow loading shows the following:

- Owing to creep of concrete, ***a given shortening of the column is produced by a smaller load***, $P_{Slow} = 4125$ kN, ***when slowly applied or sustained over some length of time than when quickly applied***.
- More important, the farther the stress is beyond the proportional limit of the concrete, and the more slowly the load is applied or the longer it is sustained, the smaller the share of the total load carried by the concrete and the larger the share carried by the steel.
- In the sample column, the steel was seen to carry ***13.9 percent of the load in the elastic range***, ***15.5 percent for a strain of 0.001 under fast loading***, and ***19.2 percent at the same strain under slow or sustained loading***.

3.4.2.4 Strength

- Importance of Strength:
  The strength is one quantity of chief interest to the structural designer.
- Definition of Strength:
  The strength is the maximum load that the structure or member will carry.
- Parameters to determine the strength:
  Information on ***stresses***, ***strains***, and similar quantities serves chiefly as ***a tool for determining carrying capacity***.
- Performance of the Column:
  The performance of the column discussed so far indicates two things:
  - Large stresses and strains companion to the maximum load:
    - The range of large stresses and strains that precede attainment of the maximum load and subsequent failure.
    - Hence, ***elastic relationships cannot be used***.
  - Different behaviors for different loading rates:
    - The member ***behaves differently under fast and under slow or sustained loading***.
    - It shows ***less resistance to the slow load than to the faster load***.
- Loading rates in usual constructions:
  - Slow rate in general:
    In usual construction, many types of loads, such as the ***weight of the structure*** and any ***permanent equipment housed*** therein, are <u>***sustained***</u>, and ***others are applied at <u>slow rates</u>***.
  - Suitable concrete strength:
    For this reason, to calculate a reliable magnitude of compressive strength, ***curve c*** of ***Figure 3.4-2*** must be used as far as the concrete is concerned.
- Strain for maximum tensile strength of steel:
  The steel reaches its tensile strength (peak of the curve) at strains on the order of 0.08 (see ***Figure 3.4-6***).

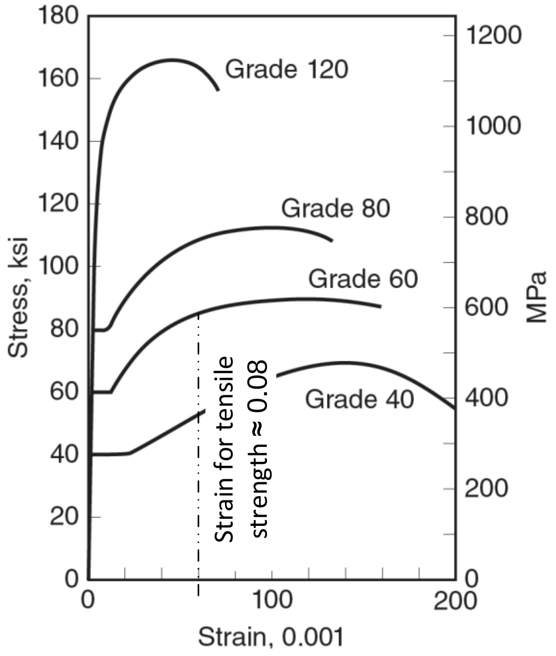


Figure 3.4-6: Strain for tensile strength (peak of the curve).

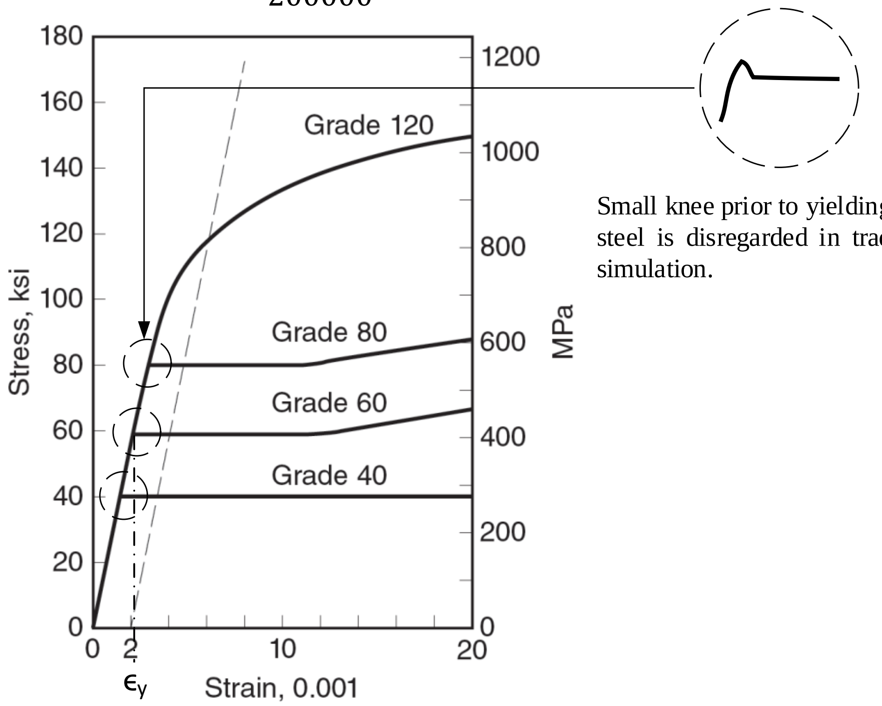
- Crushing strain for concrete:
  Concrete fails by ***crushing at the much smaller strain of about 0.003*** and, as seen from ***Figure 3.4-2*** (***curve c***).
  $\epsilon_u = 0.003$
- Strains for maximum stresses of concrete:
  Concrete reaches its ***maximum stress in the strain range of 0.002 to 0.003***, see ***Figure 3.4-2*** (***curve c***).

- Yielding strains of steel, ϵ_y:
    - If the small knee prior to yielding of the steel is disregarded, that is, if the steel is assumed to be ***sharp-yielding***, ***Figure 3.4-7***, the strain at which it yields is:

$$\epsilon_y = \frac{f_y}{E_s} \quad \textbf{Eq. 3.4-4}$$

    - For Grade 60 steel:

$$\epsilon_y = \frac{420}{200000} \approx 0.002$$



Small knee prior to yielding of the steel is disregarded in traditional simulation.

Figure 3.4-7: Idealized stress-strain diagram of steel and the corresponding yield strain.

- Compatibility conditions and section nature:
  Because the strains in steel and concrete are equal in axial compression, ***the rebars for an axially compressed column are yielded at a strain of*** ϵ_y ***before concrete crushing at strain of*** ϵ_u.
- Nominal compressive capacity, P_n:
  Based on previous discussion, the nominal, theoretical, strength of an axially compressed column is:

$$P_n = 0.85 f_c' A_c + f_y A_{st} \quad \textbf{Eq. 3.4-5}$$

  The factor 0.85 is adopted to calibrate concrete compressive strength at slow loading rate of usual construction to that of fast loading rate for cylindrical test, f_c'.

Example 3.4-3

Determine the nominal compressive strength for the column of ***Example 3.4-1***.

Solution

Based on Eq. 3.4-5, the nominal compressive strength of a column is:

$$P_n = 0.85 f_c' A_c + f_y A_{st} =$$

$$= \frac{\left(0.85 \times 28 \times \left(400 \times 500 - \left(6 \times \frac{\pi \times 29^2}{4}\right)\right)\right) + \left(420 \times \left(6 \times \frac{\pi \times 29^2}{4}\right)\right)}{1000}$$

$$= 6330\ kN \blacksquare$$

$$Ratio\ of\ P_s = \frac{P_s}{P} \times 100 = \frac{\left(420 \times \left(6 \times \frac{\pi \times 29^2}{4}\right)\right)}{6330 \times 10^3} \times 100 = 26.3\ \%$$

3.4.2.5 Summary

- In the ***elastic range***, ***the steel carries a relatively small portion of the total load of an axially compressed member***.
- As ***member strength is approached***, there occurs ***a redistribution of the relative shares of the load resisted by concrete and steel, the latter taking an increasing amount***.
- The nominal capacity, at which the member is on the point of failure, consists of the contribution of the steel when it is stressed to the yield point plus that of the concrete when its stress has attained a value of $0.85 f_c{}'$, as reflected in ***Eq. 3.4-5***.

3.4.3 Axial Tension

- Concrete is not suitable for tension members in general:
  - The tension strength of concrete is only a small fraction of its compressive strength.
  - It follows that reinforced concrete is not well suited for use in tension members because the concrete will contribute little, if anything, to their strength.
- Members where concrete is subjected to direct tension:
  - Still, there are situations in which reinforced concrete is stressed in tension, chiefly in ***tie-rods in structures such as arches and trusses***, see ***Figure 3.4-8***, or in uplift piles, see ***Figure 3.4-9***.


Figure 3.4-8: Concrete trusses with members under tensions.

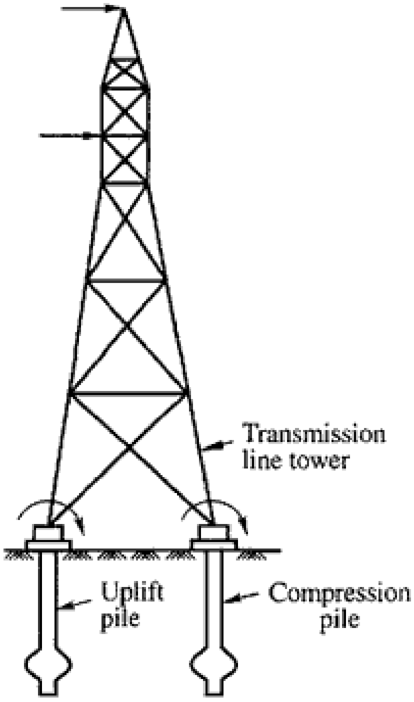


Figure 3.4-9: Tensile piles.

- Reinforcement for concrete tensile members:
  Such members consist of one or more bars embedded in concrete in a symmetric arrangement ***similar to compression members*** (see ***Figure 3.4-1***).
- Elastic behavior under small tensile forces:
  - When the tension force in the member is ***small enough for the stress in the concrete to be considerably below its tensile strength***, ***both steel and concrete behave elastically***.
  - In this situation, ***all the expressions derived for elastic behavior in compressio***n in ***Section 3.4.2.2*** are ***identically valid for tension***. In particular, ***Eq. 3.4-2*** becomes:

$$P = f_{ct}(A_c + nA_{st}) \quad \textbf{Eq. 3.4-6}$$

where f_{ct} is the tensile stress in the concrete.

- Elastic Cracked Section:
  - When the ***load is further increased***, however, ***the concrete reaches its tensile strength at a stress and strain on the order of one-tenth of what it could sustain in compression***. At this stage, ***the concrete cracks across the entire cross section***.
  - When this happens, it ceases to resist any part of the applied tension force, since, evidently, ***no force can be transmitted across the air gap in the crack***.
  - At any load larger than that which caused the concrete to crack, ***the steel is called upon to resist the entire tension force***.
  - Correspondingly, at this stage:

    $P = f_s A_{st}$ **Eq. 3.4-7**
- Tensile Strength (It is determined based on f_y instead of f_u):
  - With further increased load, the tensile stress f_s in the steel reaches the yield point f_y.
  - When this occurs, the tension members cease to exhibit small, elastic deformations but instead ***stretch a sizable and permanent amount at substantially constant load***. ***This does not impair the strength of the member***.
  - Its ***elongation***, however, ***becomes so large*** (***approximately 1 percent or more of its length***) as to render it useless.
  - Therefore, ***the maximum useful strength*** P_{nt} ***of a tension member is the force that will just cause the steel stress <u>to reach the yield point</u>***. That is,

    $P_{nt} = f_y A_{st}$ **Eq. 3.4-8**
- Tensile Strength under Service Conditions:
  - To provide adequate safety, the force permitted in a tension member under normal service loads should be limited to about:

    $P_{Tension\ under\ service\ conditions} = \frac{1}{2} P_{nt}$
  - Because the concrete has cracked at loads considerably smaller than this, ***concrete does not contribute to the carrying capacity of the member in service***.
  - It does serve, however, as ***fire*** and ***corrosion protection*** and ***often improves the appearance of the structure***.
- Tension in Watertight Structures:
  - There are situations, though, in which ***reinforced concrete is used in axial tension under conditions in which the occurrence of tension cracks must be prevented***.
  - A case in point is a circular tank, see ***Figure 3.4-10***, ***to provide watertightness, the hoop tension caused by the fluid pressure must be prevented from causing the concrete to crack***.
  - In this case, Eq. 3.4-2 can be used to determine ***a safe value for the axial tension force P*** by using, for the concrete tension stress f_{ct}, an appropriate fraction of the tensile strength of the concrete, that is, of the stress that would cause the concrete to crack.

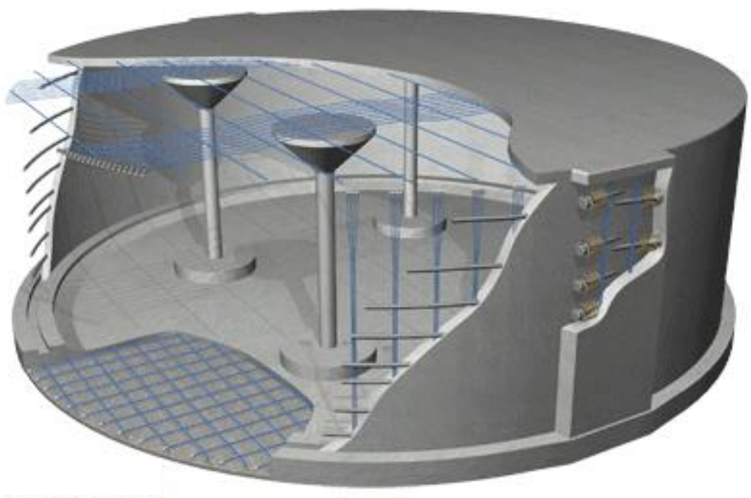


Figure 3.4-10: Circular tank under tensile hoop stresses.

3.5 ADDITIONAL EXAMPLE

Additional Example 3.5-1

A 400 × 500 mm column is made of the same concrete and reinforced with the same six No. 29 bars as the column in ***Example 3.4-1***, except that a steel with yield strength $f_y = 280\ MPa$ is used. The stress-strain diagram of this reinforcing steel is shown in ***Figure 3.5-1*** for $f_y = 280\ MPa$. For this column determine:

- The axial load that will stress the concrete to 8 MPa.
- The load at which the steel starts yielding.
- The maximum load.
- The share of the total load carried by the reinforcement at these three stages of loading.

Compare results with those calculated in the examples for $f_y = 420\ MPa$, keeping in mind, in regard to relative economy, that the price per pound for reinforcing steels with 280 and 420 MPa yield points is about the same.

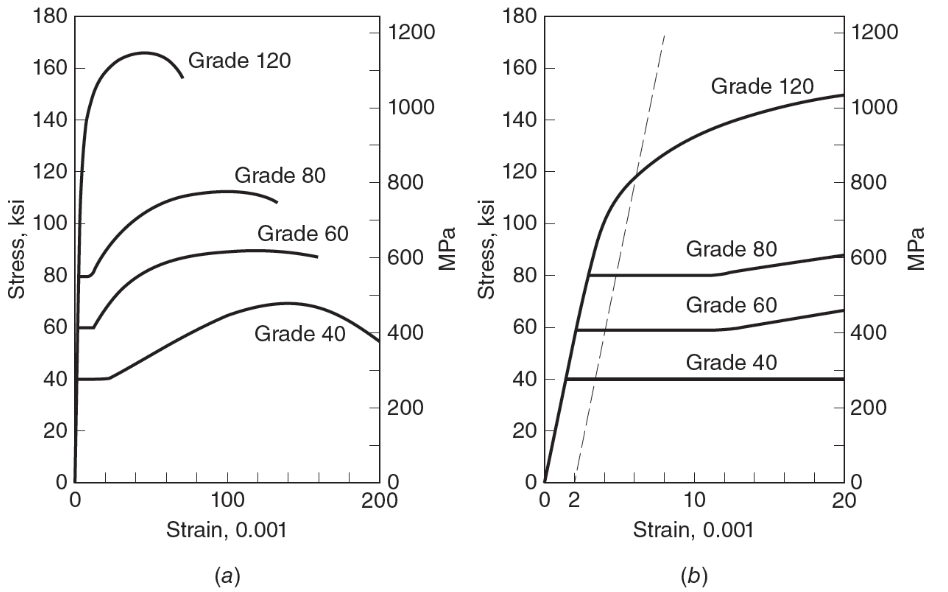


Figure 3.5-1: Typical stress-strain curves for reinforcing bars.

Solution

- The axial load that will stress the concrete to 8 MPa:
  With referring to concrete stress-strain diagram of ***Figure 3.4-2***, concrete behaves elastically when subjected to compressive stress of 8 MPa. Hence the compressive axial force, P, can be determined from ***Eq. 3.4-2***:
  $P = f_c(A_c + nA_{st})$
  As steel elastic modulus, E_s, has a constant value of $200000\ MPa$ irrespective of steel yield stresses, therefore, the modular ratio, n, is equal to 8 as for ***Example 3.4-1***. Substitute into ***Eq. 3.4-2*** to obtain:
  $$P_{@\ stress\ of\ 8\ MPa} = \frac{8.0 \times \left((400 \times 500) + (8-1) \times \left(6 \times \frac{\pi \times 29^2}{4}\right)\right)}{1000} = 1821\ kN\ ■$$
  $$P_{s\ ratio\ @\ elastic\ range} = \frac{8 \times 8 \times \left(6 \times \frac{\pi \times 29^2}{4}\right)}{1821 \times 1000} \times 100 = 13.9\ \%$$
  As steel yield stress has no effect of elastic behavior, these values are same as those of ***Example 3.4-1***.
- The load at which the steel starts yielding:
  The steel yield at strain of:
  $$\epsilon_y = \frac{f_y}{E_s} = \frac{280}{200000} = 0.0014$$
  From concrete stress-strain curve of Figure 3.4-2, reproduce in below for convenience:
  $f_{c\ slow} = 21\ MPa$

$$P_{for\ steel\ yeild} = f_c A_c + f_y A_s$$
$$= \frac{\left(21 \times \left(400 \times 500 - \left(6 \times \frac{\pi \times 29^2}{4}\right)\right)\right) + \left(280 \times \left(6 \times \frac{\pi \times 29^2}{4}\right)\right)}{1000}$$
$$= 5226\ kN\ \blacksquare$$
$$P_{s\ ratio\ @\ yeild} = \frac{\left(280 \times \left(6 \times \frac{\pi \times 29^2}{4}\right)\right)}{5226 \times 1000} \times 100 = 21.2\ \%$$

These values are smaller than those of steel with grade of 420 MPa. Therefore, at yield range, the steel with grade of 420 MPa has more contribution than grade of 280 MPa.

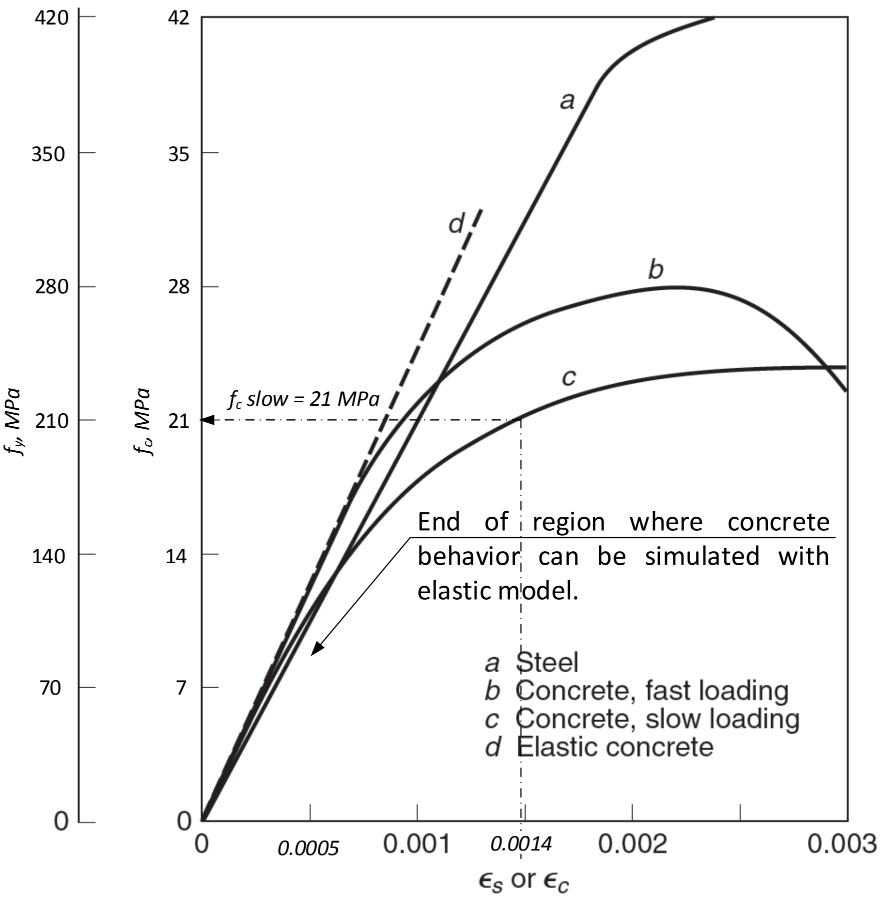


Figure 3.4-2: Concrete and steel stress strain curves. Reproduced for convenience.

- The maximum load:

  The maximum load, nominal strength P_n, can be determined from Eq. 3.4-5:

$$P_n = 0.85 f_c' A_c + f_y A_{st} =$$
$$= \frac{\left(0.85 \times 28 \times \left(400 \times 500 - \left(6 \times \frac{\pi \times 29^2}{4}\right)\right)\right) + \left(280 \times \left(6 \times \frac{\pi \times 29^2}{4}\right)\right)}{1000}$$
$$= 5775\ MPa$$
$$P_{s\ ratio\ @\ ultimate\ range} = \frac{\left(280 \times \left(6 \times \frac{\pi \times 29^2}{4}\right)\right)}{5775 \times 1000} \times 100 = 19.2\ \%$$

These values are smaller than those of steel with grade of 420 MPa. Therefore, at ultimate, the steel with grade of 420 MPa has more contribution than grade of 280 MPa.

Additional Example 3.5-2

The area of steel, expressed as a percentage of gross concrete area, for the column of ***Additional Example 3.5-1*** is lower than would often be used in practice. Recalculate the comparisons of ***Additional Example 3.5-1***, using f_y of $280\,MPa$ as before, but for a 400 × 500 mm column reinforced with eight No. 36 bars. Compare your results with those of ***Additional Example 3.5-1***.

Solution

- The axial load that will stress the concrete to 8 MPa:

$$P_{@\,stress\,of\,8\,MPa} = \frac{8.0 \times \left((400 \times 500) + (8-1) \times \left(8 \times \frac{\pi \times 38^2}{4}\right)\right)}{1000} = 2108\,kN\ ■$$

$$P_{s\,ratio\,@\,elastic\,range} = \frac{8 \times 8 \times \left(8 \times \frac{\pi \times 36^2}{4}\right)}{2108 \times 1000} \times 100 = 24.7\,\%$$

- The load at which the steel starts yielding:

$$P_{for\,steel\,yeild} = f_c A_c + f_y A_s = \frac{\left(21 \times \left(400 \times 500 - \left(6 \times \frac{\pi \times 29^2}{4}\right)\right)\right) + \left(280 \times \left(8 \times \frac{\pi \times 36^2}{4}\right)\right)}{1000}$$

$$= 6397\,kN\ ■$$

$$P_{s\,ratio\,@\,yeild} = \frac{\left(280 \times \left(8 \times \frac{\pi \times 36^2}{4}\right)\right)}{6397 \times 1000} \times 100 = 35.6\,\%$$

- The maximum load:

$$P_n = 0.85 f_c' A_c + f_y A_{st} =$$

$$= \frac{\left(0.85 \times 28 \times \left(400 \times 500 - \left(6 \times \frac{\pi \times 29^2}{4}\right)\right)\right) + \left(280 \times \left(8 \times \frac{\pi \times 36^2}{4}\right)\right)}{1000}$$

$$= 6946\,MPa$$

$$P_{s\,ratio\,@\,ultimate\,range} = \frac{\left(280 \times \left(8 \times \frac{\pi \times 36^2}{4}\right)\right)}{6946 \times 1000} \times 100 = 32.8\,\%$$

- Comments:
  Using larger amount of reinforcement, leads to a larger steel contribution in elastic, yield, and strength ranges.

Additional Example 3.5-3

A square concrete column with dimensions 550 × 550 mm is reinforced with a total of eight No. 32 bars arranged uniformly around the column perimeter. Material strengths are $f_y = 420\,MPa$ and $f_c' = 28\,MPa$, with stress-strain curves as given by curves a and c of ***Figure 3.4-2***. Calculate the percentages of total load carried by the concrete and by the steel as load is gradually increased from 0 to failure, which is assumed to occur when the concrete strain reaches a limit value of 0.0030. Determine the loads at strain increments of 0.0005 up to the failure strain, and graph your results, plotting load percentages vs. strain. The modular ratio may be assumed at n = 8 for these materials.

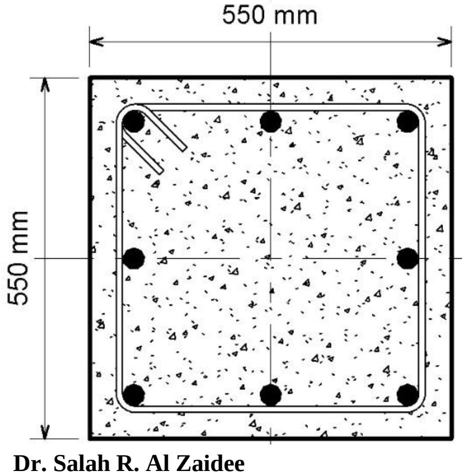


Figure 3.5-2: Cross section for column of Additional Example 3.5-3.

Solution

<u>At $\epsilon = 0$:</u>

$P_c = P_s = 0$

<u>At $\epsilon = 0.0005$:</u>

From ***Figure 3.4-2***, reproduce in below,

$f_c(@\,\epsilon\ of\ 0.0005) = 10.5\ MPa$

From ***Article 3.4.2.3*** and ***Figure 3.4-2***, it is found that concrete behaves almost in linear up to a strain ϵ of 0.0005, then its behavior can be determined from Eq. 3.4-3:

$$P_{@\,\epsilon\ of\ 0.0005} = f_c\left(A_g + (n-1)A_{st}\right) = \frac{10.5 \times \left(550^2 + (8-1) \times 8 \times \frac{(\pi \times 32^2)}{4}\right)}{1000} = 3649\ kN$$

$$Ratio_{of\ P_c @\,\epsilon\ of\ 0.0005} = \frac{f_c A_c}{P} = \frac{10.5 \times \left(550^2 - 8 \times \frac{(\pi \times 32^2)}{4}\right)}{3649 \times 1000} \times 100 \approx 85\%$$

$$Ratio_{of\ P_s @\,\epsilon\ of\ 0.0005} = \frac{n f_c A_s}{P} = \frac{8 \times 10.5 \times 8 \times \frac{(\pi \times 32^2)}{4}}{3649 \times 1000} \times 100 \approx 15\%$$

<u>At $\epsilon = 0.001$:</u>

From ***Figure 3.4-2***, reproduce in below,

$f_c(@\,\epsilon\ of\ 0.001) = 17\ MPa, f_s(@\,\epsilon\ of\ 0.001) = \epsilon E_s = 0.001 \times 200000 = 200\ MPa$

As the strain is within the inelastic range, hence correspond forces can be determined from following relation:

$$P_{@\,\epsilon\ of\ 0.001} = f_c A_c + f_s A_s = \frac{17 \times \left(550^2 - \left(8 \times \frac{\pi \times 32^2}{4}\right)\right) + 200 \times \left(8 \times \frac{\pi \times 32^2}{4}\right)}{1000} = 6320\ kN$$

$$Ratio_{of\ P_c @\,\epsilon\ of\ 0.001} = \frac{f_c A_c}{P} = \frac{17 \times \left(550^2 - \left(8 \times \frac{\pi \times 32^2}{4}\right)\right)}{6320 \times 1000} \times 100 \approx 80\ \%$$

$$Ratio_{of\ P_s @\,\epsilon\ of\ 0.001} = \frac{f_s A_s}{P} = \frac{200 \times 8 \times \frac{(\pi \times 32^2)}{4}}{6320 \times 1000} \times 100 \approx 20\ \%$$

<u>At $\epsilon = 0.0015$:</u>

From ***Figure 3.4-2***, reproduce in below,

$f_c = 21\ MPa, f_s = \epsilon E_s = 0.0015 \times 200000 = 300\ MPa$

$$P_{@\,\epsilon\ of\ 0.0015} = f_c A_c + f_s A_s = \frac{21 \times \left(550^2 - \left(8 \times \frac{\pi \times 32^2}{4}\right)\right) + 300 \times \left(8 \times \frac{\pi \times 32^2}{4}\right)}{1000} = 8148\ kN$$

$$Ratio_{of\ P_c @\,\epsilon\ of\ 0.0015} = \frac{f_c A_c}{P} = \frac{21 \times \left(550^2 - \left(8 \times \frac{\pi \times 32^2}{4}\right)\right)}{8148 \times 1000} \times 100 \approx 76\ \%$$

$$Ratio_{of\ P_s @\,\epsilon\ of\ 0.0015} = \frac{f_s A_s}{P} = \frac{300 \times 8 \times \frac{(\pi \times 32^2)}{4}}{8148 \times 1000} \times 100 \approx 24\ \%$$

<u>At $\epsilon = 0.002$:</u>

From ***Figure 3.4-2,*** reproduce in below,

$f_c = 23\ MPa, f_s = \epsilon E_s = 0.002 \times 200000 = 400\ MPa$

$$P_{@\,\epsilon\ of\ 0.002} = f_c A_c + f_s A_s = \frac{23 \times \left(550^2 - \left(8 \times \frac{\pi \times 32^2}{4}\right)\right) + 400 \times \left(8 \times \frac{\pi \times 32^2}{4}\right)}{1000} = 9383\ kN$$

$$Ratio_{of\ P_c @\,\epsilon\ of\ 0.002} = \frac{f_c A_c}{P} = \frac{23 \times \left(550^2 - \left(8 \times \frac{\pi \times 32^2}{4}\right)\right)}{9383 \times 1000} \times 100 \approx 73\ \%$$

$$Ratio_{of\ P_s\ @\ \epsilon\ of\ 0.002} = \frac{f_s A_s}{P} = \frac{400 \times 8 \times \frac{(\pi \times 32^2)}{4}}{9383 \times 1000} \times 100 \approx 27\ \%$$

At $\epsilon = 0.0025$:

From ***Figure 3.4-2,*** reproduce in below,

$f_c = 23.5\ MPa, f_s = f_y = 420\ MPa$

$$P_{@\ \epsilon\ of\ 0.0025} = f_c A_c + f_s A_s = \frac{23.5 \times \left(550^2 - \left(8 \times \frac{\pi \times 32^2}{4}\right)\right) + 420 \times \left(8 \times \frac{\pi \times 32^2}{4}\right)}{1000}$$
$$= 9660\ kN$$

$$Ratio_{of\ P_c @\ \epsilon\ of\ 0.0025} = \frac{f_c A_c}{P} = \frac{23.5 \times \left(550^2 - \left(8 \times \frac{\pi \times 32^2}{4}\right)\right)}{9660 \times 1000} \times 100 \approx 72\ \%$$

$$Ratio_{of\ P_s\ @\ \epsilon\ of\ 0.0025} = \frac{f_s A_s}{P} = \frac{420 \times 8 \times \frac{(\pi \times 32^2)}{4}}{9660 \times 1000} \times 100 \approx 28\ \%$$

At $\epsilon = 0.003$:

From ***Figure 3.4-2,*** reproduce in below,

$f_c = 24\ MPa, f_s = f_y = 420\ MPa$

$$P_{@\ \epsilon\ of\ 0.003} = f_c A_c + f_s A_s = \frac{24 \times \left(550^2 - \left(8 \times \frac{\pi \times 32^2}{4}\right)\right) + 420 \times \left(8 \times \frac{\pi \times 32^2}{4}\right)}{1000} = 9808\ kN$$

$$Ratio_{of\ P_c @\ \epsilon\ of\ 0.003} = \frac{f_c A_c}{P} = \frac{24 \times \left(550^2 - \left(8 \times \frac{\pi \times 32^2}{4}\right)\right)}{9808 \times 1000} \times 100 \approx 72\ \%$$

$$Ratio_{of\ P_s\ @\ \epsilon\ of\ 0.003} = \frac{f_s A_s}{P} = \frac{420 \times 8 \times \frac{(\pi \times 32^2)}{4}}{9808 \times 1000} \times 100 \approx 28\ \%$$

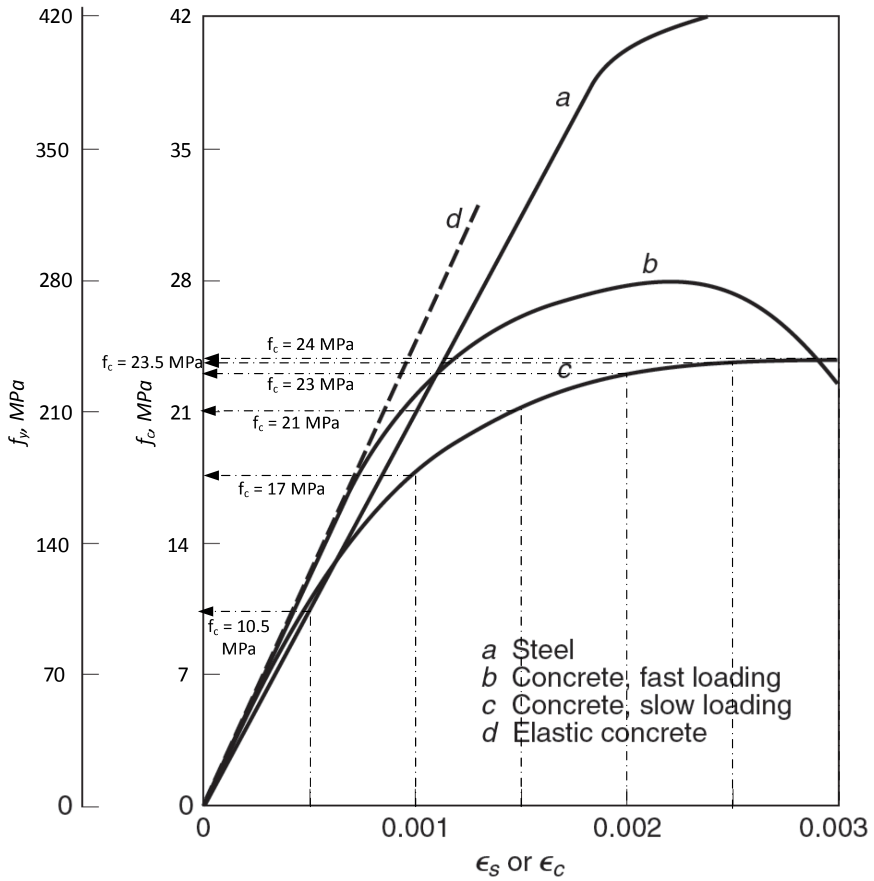


Figure 3.4-2: Concrete and steel stress strain curves. Reproduced for convenience.

Above ratios are summarized in ***Table 3.5-1*** and ***Figure 3.5-3*** below.

Table 3.5-1: Force ratios versus different strains in column of Additional Example 3.5-3.

Strain	Ratio of Pc	Ratio of Ps
0	0	0
0.00005	85	15
0.0001	80	20
0.00015	76	24
0.0002	73	27
0.00025	72	28
0.0003	72	28

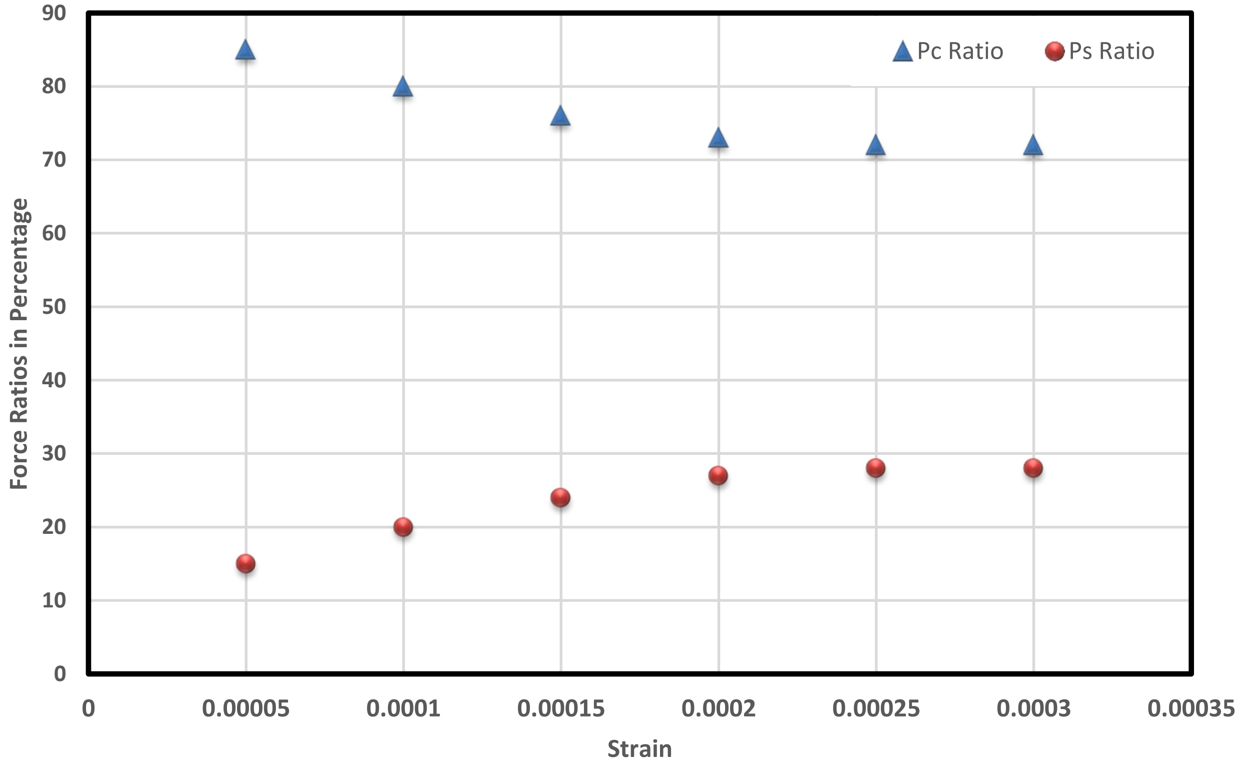


Figure 3.5-3: Force ratios versus different strains in column of Additional Example 3.5-3.

REFERENCES

ACI318M. (2014). Building Code Requirements for Structural Concrete (ACI 318M-14) and Commentary (ACI 318RM-14). Farmington Hills: American Concrete Institute.

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CONTENTS

CHAPTER 4
FLEXURE ANALYSIS AND DESIGN OF BEAMS

4.1 BENDING OF HOMOGENOUS BEAMS

- For the homogenous beams (i.e., the beams made from single homogenous material like steel or wood) and in the elastic range, the bending stresses can be computed based on the following relation:

  $$f = \frac{M.y}{I}$$

  where

  f is the bending stress at distance y from the neutral axis.

  M is the bending moment at the section,

  I is the moment of inertia of cross section about neutral axis.

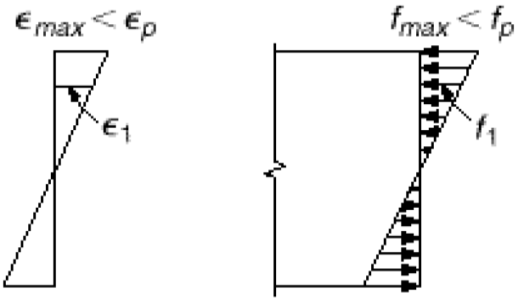


Figure 4.1-1: Stress distribution according to conventional flexural formula.

- For this homogenous elastic beam, the neutral axis passes through the center of gravity of the section.
- In general, the conventional flexure formula, $M.c/I$, is not applicable for RC beams as it has been derived for homogenous materials with linear elastic behavior.
- In Article 4.2 below a more fundamental flexural formula has been derived to take into account the nonlinear and composite nature of RC beams.

4.2 CONCRETE BEAM BEHAVIOR

4.2.1 Behavior of Plain Concrete

Plain concrete beams are inefficient flexure members because the tension strength in bending is a small fraction of the compression strength. Then we will focus on the analysis of reinforced concrete beams only.

4.2.2 Reinforce Concrete Beam Behavior

4.2.2.1 Suitability of Conventional Bending Formula for Analysis of RC Beams

As the reinforced concrete beam

- Is made from two materials
- Cracks in the concrete,
- Behaves no-linear in concrete and steel

above conventional bending relation of $(M.c)/I$ for the homogenous beam cannot be applied to analysis of RC beams.

4.2.2.2 More Rigorous Relations

4.2.2.2.1 Model Beam and Experimental Works

- Experiment works pertained to flexural behavior of RC beams are usually conducted through model beam in below:

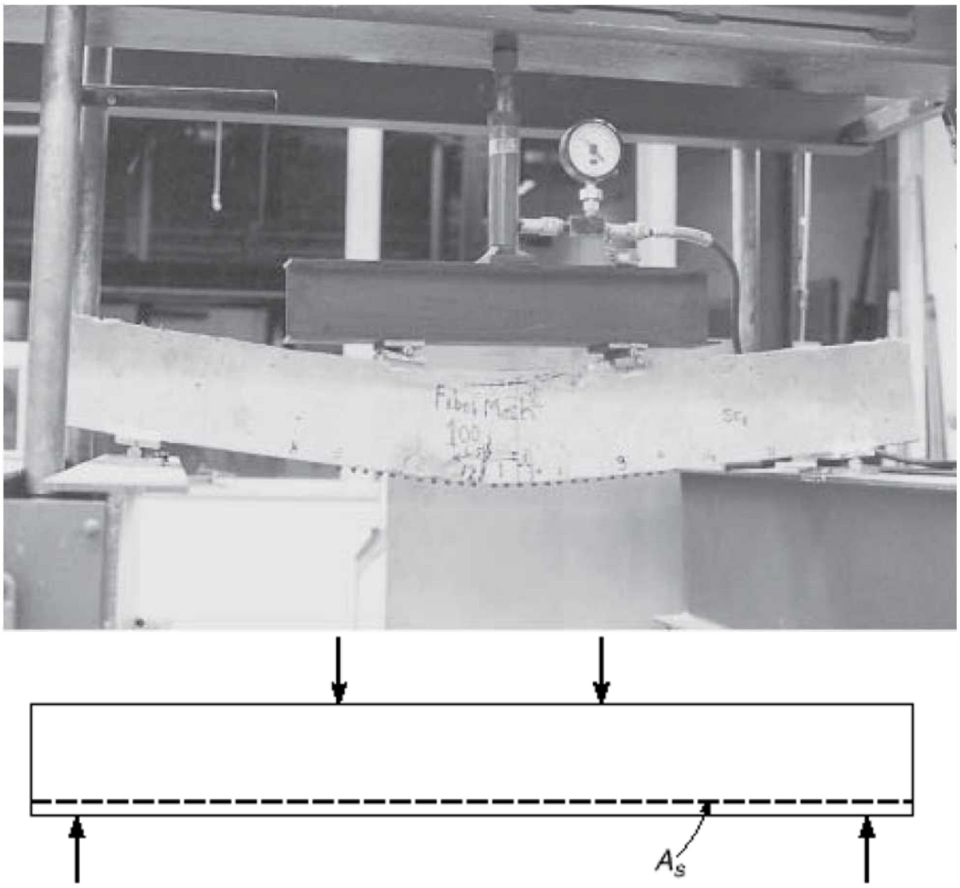


Figure 4.2-1: Model beam for experimental works of RC beams behavior in flexure.

- A beam loaded at third points mainly due to the fact that the mid region is under ***pure bending***, then ***the analysis can exclude the effect of shear stresses and focusing on flexure stresses only***.
- When the load on above beam is gradually increased from zero to the magnitude that will cause the beam to fail ***following three different stages of behavior can be clearly distinguished***.

4.2.2.2.2 Stresses Elastic and Section Uncracked

- At low loads, as long as the maximum tensile stress in the concrete is smaller than the tensile strength of concrete, the entire concrete is effective in resisting stress, in compression on one side and in tension on the other side of the neutral axis.
- At this stage, all stresses in the concrete are of small magnitude and are proportional to strains (i.e. the stresses are varied linearly with the depth). The distribution of strains and stresses in concrete and steel over the depth of the section is as shown in Figure 4.2-2 below.

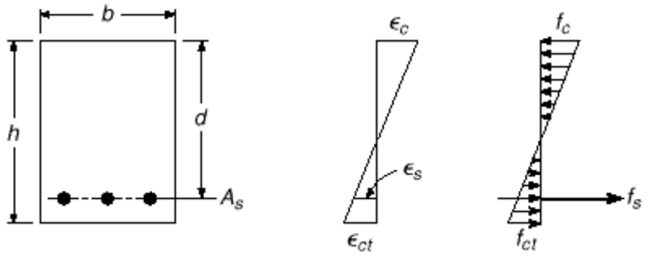


Figure 4.2-2: Strain and stress distribution during elastic uncracked stage.

- Then the only difference from the homogenous beam is in the presence of the steel reinforcement.
- It can be shown (see any text on strength of materials) that one can take account of the presence of the steel reinforcement by replacing the actual steel-and-concrete cross section with a *fictitious section* thought of as consisting of concrete only. In this "***Transformed Section***," the actual area of the reinforcement is replaced with an equivalent concrete area equal to $(n-1)A_s$, located at the level of the steel, as shown in Figure 4.2-3 below:

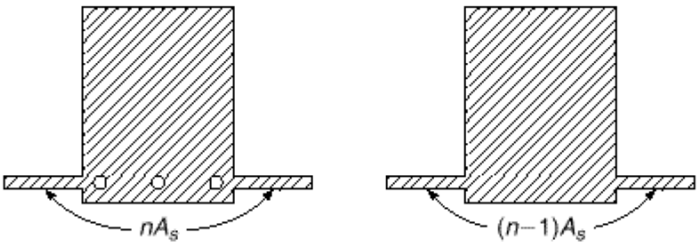


Figure 4.2-3: Transformed section for elastic uncracked RC beam.

where

$$n = \frac{Es}{Ec}$$

is the modular ratio.

- Once the transformed section has been obtained, the usual methods

$$f = \frac{M.c}{I}$$

of analysis of elastic homogeneous beams apply.

- **Computing of E_s and E_c:**

  As discussed in Chapter 2, according to (ACI318M, 2014), article 19.2.2, modulus of elasticity, E_c, for concrete can be estimated based on following correlation:
  - For values of w_c between 1440 and 2560 kg/m^3

    $E_c = w_c^{1.5} 0.043\sqrt{f_c'}\ (in\ MPa)$
  - For normalweight concrete

    $E_c = 4700\sqrt{f_c'}\ \ (in\ MPa)$

  According to the (ACI318M, 2014) (**20.2.2.2**), modulus of elasticity, E_s, for nonprestressed bars and wires shall be permitted to be taken as:

  $E_{Steel} = E_s = \ 200\,000\text{MPa}$

- This stage ends when tensile stress in concrete reaching a limit state. As discussed in Chapter 2, concrete tensile strength can be predicated based on
  - Direct Tensile Strength f_t'.
  - Split-Cylinder Strength f_{ct}.
  - Modulus of Rupture f_r

Example 4.2-1

For the beam shown in Figure 4.2-4 below, find the maximum magnitude of the load "P" such that the section stays in the uncracked elastic state.

Given:

- fc′ = 25 MPa
- fy = 400 MPa
- Neglect the beam selfweight.

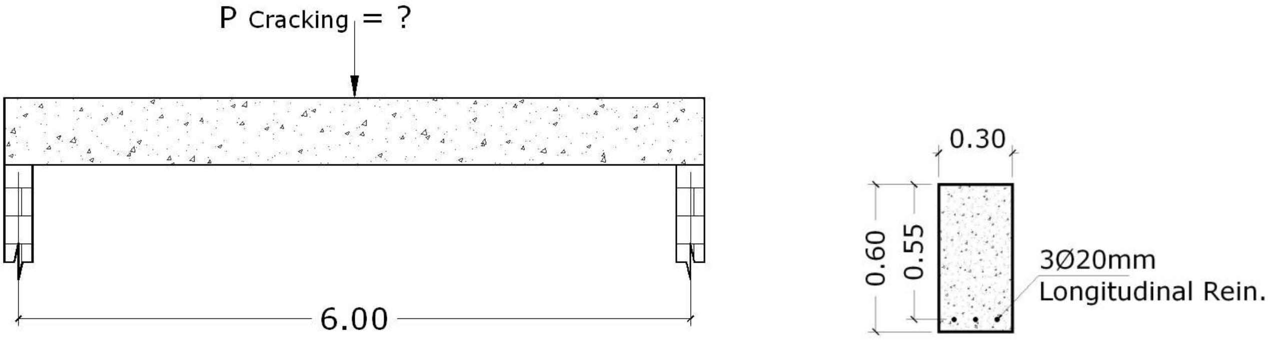
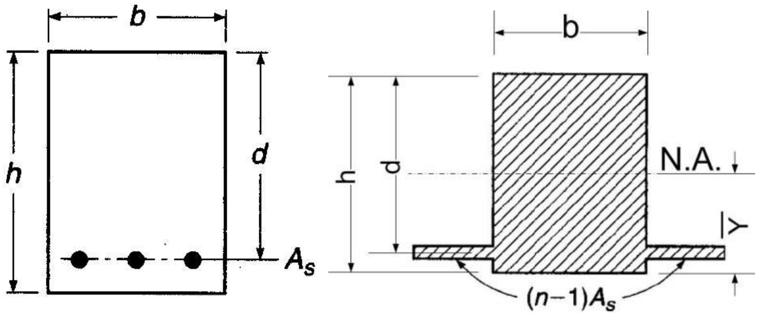


Figure 4.2-4: Beam of elastic uncracked section for Example 4.2-1.

Solution

- As the flexure formula is derived for the ***homogenous section***, then the steel must be transformed for the equivalent concrete to obtain a homogenous section that formed from a single material:



$\because E_S = 200\ 000\ MPa, and\ E_C = 4700\sqrt{f_c'} = 4700\sqrt{25} = 23\ 500\ MPa, \therefore n \approx 8.5$

$A_S = \left(\pi \frac{20^2}{4}\right) \times 3 = 942\ mm^2$

$\therefore (n-1)A_s = 7\ 065\ mm^2$

$\sum M_{of\ Area\ about\ lower\ face} = \bar{y}.A$

$\bar{y}.(300 \times 600 + 7\ 065) = (300 \times 600) \times 300 + (7\ 065) \times 50$

$\Rightarrow\ \bar{y} = 290mm\ < 300\ ok.$

- Compute the moment of inertia for the transformed section:

$I_{N.A.} = \left[\left(300 \times \frac{600^3}{12}\right) + (300 \times 600 \times 10^2)\right] + 7\ 065\ \times (290-50)^2$

$I_{N.A.} = 5.82 \times 10^9\ mm^4$

- Use the flexure formula to compute the cracking moment:

$M_{Crack} = \frac{f_r \times I_{N.A.}}{c}$

$f_r = 0.62\sqrt{f_c'} = 0.62\sqrt{25} = 3.1\ MPa$

$M_{Crack} = \frac{3.1 \frac{N}{mm^2} \times (5.82 \times 10^9 mm^4)}{290mm}\ 62.2 \times 10^6\ N.mm = 62.2\ kN.m$

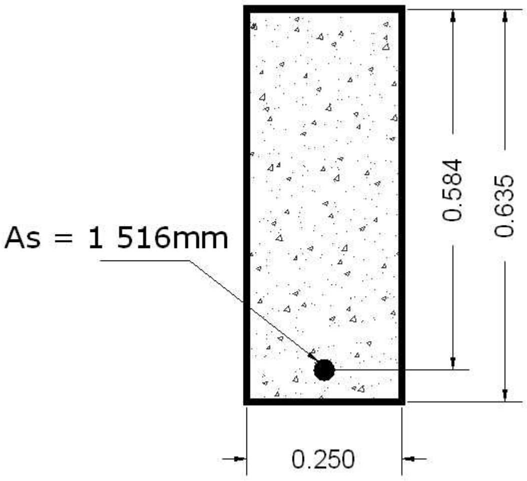
- Compute the P_{Crack}:

$M_{Crack} = \frac{P_{Crack} \times L}{4} \Rightarrow P_{Crack} = \frac{62.2\ kN.m\ \times 4}{6m} = 41.5\ kN$■

4.2.2.2.3 Home Work for Article 4.2.2.2.2: Analysis of Uncracked Elastic Section

Problem 4.2-1

A rectangular beam with dimensions of b= 250mm, h = 635mm, and d = 584mm. The f_c' = 28 MPa, f_y = 400 MPa, and E_{Steel} = 200,00 MPa. Check the state of section and determine the stresses caused by a bending moment of M = 61 kN.m.



Hint:

Start your solution with assumption that the section under a moment of 61 kN.m stills within the 1st Stage. This assumption should be checked later.

Answers:

$f_{c\,Ten.} = 3.04\ MPa < f_r$

Then, the section is uncracked elastic section.

$f_{c\,Comp.} = 3.37\ MPa$

$f_{Steel} = 20.2\ MPa$

4.2.2.2.4 Second Stage: Elastic Cracked Section

- When the load is further increased, the tension strength of the concrete is reached.
- Tension cracks develop and propagate quickly upward to or closed to level of the neutral plane, which in turn shift upward with progressive cracking.
- In ***well-designed beams the width of these cracks is so small*** (***hairline cracks***) that they are not harmful from the view point of the either corrosion protection or appearance ***(i.e. in current design philosophy, the design is based on permitting of hairline cracks***).
- In cracked section, the concrete does not transmit any tension stresses. Hence the steel is called upon to resist the entire tension. If the concrete stress do not exceeded approximately $f_c'/2$ and the steel stress has not reached the yield point, stresses and strains continue to be closely proportional. Then the distribution of strains and stresses are as shown below.

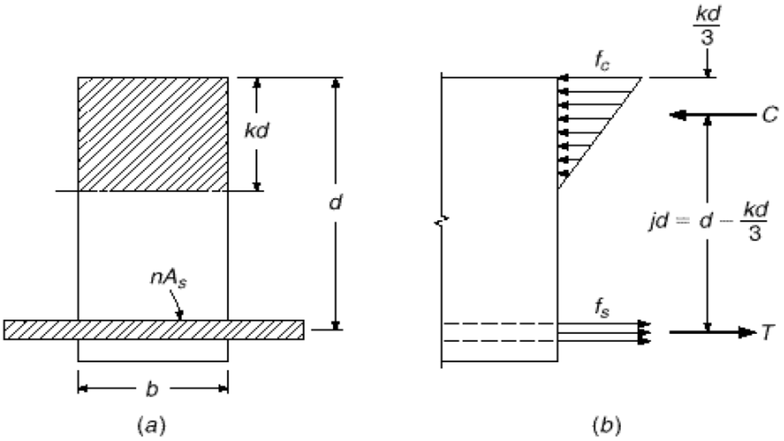


Figure 4.2-5: Strain and stress distribution during elastic cracked stage.

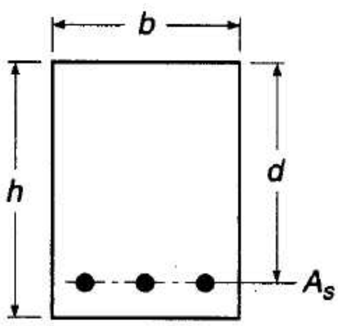
- This situation (elastic cracked section) ***generally occurs in structures under normal service loads (unfactored loads)***.
- The stresses and strains in the elastic cracked section can be computed based on transform the steel to an equivalent concrete, and then use the conventional flexure formula $f = M.y/I$.

Example 4.2-2

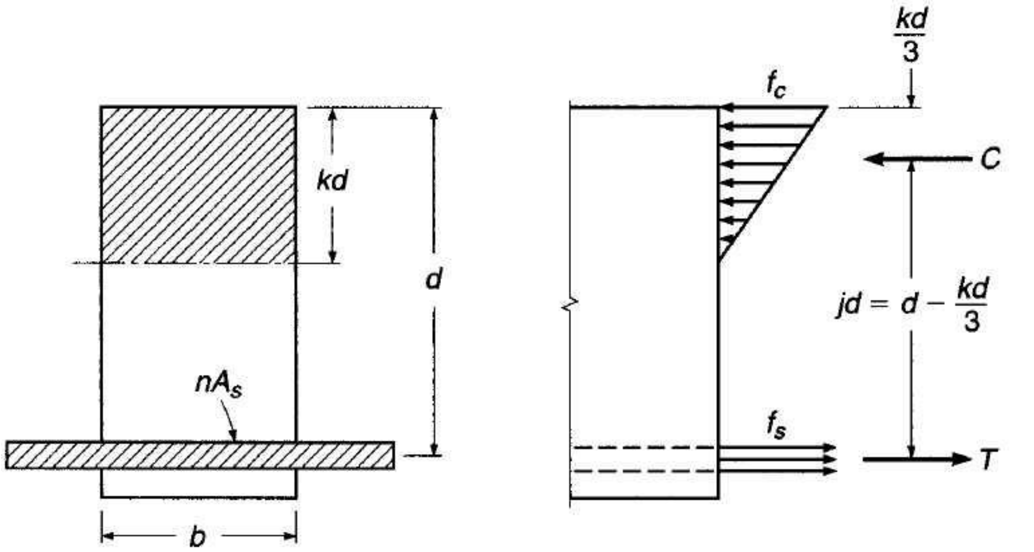
Resolve Example 4.2-1, to compute the maximum magnitude of the load P for the cracked elastic section (assuming that the elastic limit of concrete is equal to $f_c'/2$).

Solution

- As was discussed in the mechanics of materials the application of the conventional flexure formula ($f = M.c/I$) is based on the assumption of homogenous and linear section.
- Linearity of section is assured for concrete and will be assumed for the steel (and should be checked later). While homogeneity of section will be assured through the transformed section concept. That based on transformation of original nonhomogeneous original section shown



- In to the equivalent homogenous section shown below:



Then

$A_S = \left(\pi \frac{20^2}{4}\right) \times 3 = 942 \text{ mm}^2$

$\because E_S = 200\,000 \text{ MPa, and } E_C = 4700\sqrt{f_c'} = 4700\sqrt{25} = 23\,500 \text{ MPa,}$

$\therefore n \approx 8.5$

$\therefore nA_s = 8\,007 \text{ mm}^2$

- Application of flexure formula:
  As the section is transformed to a homogenous one, then the flexure formula can now be applied:
  - Compute kd
    As the N.A. passes through the section centroid:

    $$\sum_{i=1}^{2} \text{M of Area about N.A.} = 0 \Rightarrow (300 \times kd) \times kd/2 = 8007\text{mm}^2 \times d(1 - K)$$

    $$k^2 + 0.0979k - 0.0979 = 0$$

    $$k = \frac{-0.0979 \mp \sqrt{(0.0979^2 + 4 \times 0.0979}}{2 \times 1} = 0.267, \quad kd = 147\text{mm}$$
  - Compute IN.A.

    $$I_{N.A.} = \frac{kd^3 \times b}{3} + nAs \times (d - kd)^2$$

    $$I_{N.A.} = \frac{147^3 \times 300}{3} + 8007 \times (550 - 147)^2 = 1.62 \times 10^9 \text{mm}^4$$
  - Compute M

    $$\because f_c = \frac{(f_c')}{2} = \frac{M.c}{I_{N.A.}}$$

    $$\therefore M = \frac{25}{2} \times \frac{1.62 \times 10^9}{147\text{mm}} = 137.7 \text{ kN.m}$$

    $$\because M = \frac{PL}{4}, \qquad \therefore P = M.\frac{4}{L} = \frac{137.7 \times 4}{6} = 91.8 \text{ kN}$$
- Check the assumption of $f_s \le f_y$ as assumed:

  $$f_s = \frac{M.c}{I} \times n = \frac{137.7 \times 10^6 \times (550 - 147)}{1.62 \times 10^9} \times 8.5 = 291\text{MPa}$$

  $\because f_s = 291 \text{ MPa} < 400\ MPa\ \therefore ok.$

  Then the assumption of $f_s \le f_y$ is correct and the solution that based on it is a final solution.

  $\therefore P = 91.8 \text{ kN}$ ■

Example 4.2-3

Show that the neutral axis of cracked elastic reinforced concrete section with rectangular shape under flexure stress can be located based on the following relation:

$$k = \sqrt{(\rho n)^2 + 2\rho n} \; - \rho n$$

where

ρ is reinforcement ratio that defined as follows:

$$\rho = \frac{A_s}{bd}$$

Solution

As the neutral axis for an elastic beam passes through the centroid of its cross sectional area, then:

$$\sum_{i=1}^{2} \text{M of Area about N.A.} = 0 \;\Rightarrow (b \times kd) \times kd/2 \; = nA_S \times d(1 - K)$$

$$\left[\left(\frac{bd^2}{2}\right)k^2 + (nA_S d)k - nA_S d = 0\right] \div d$$

$$\left[\left(\frac{bd}{2}\right)k^2 + (nA_S)k - nA_S = 0\right] \div bd$$

$$\left(\frac{1}{2}\right)k^2 + (n\rho)k - n\rho = 0$$

Quadratic formula can be used to solve the above quadratic equation[1]:

$$k = \frac{\left(-n\rho \; \pm \; \sqrt{(n\rho)^2 + 4 \times \frac{1}{2} \times n\rho}\right)}{2 \times \frac{1}{2}}$$

As the negative distance has no meaning in our case, then the answer will be in terms of positive root:

$$k = \left(-n\rho \; \pm \; \sqrt{(n\rho)^2 + 2n\rho}\right)$$

As the dimension factor k cannot be a negative value, then the actual root for above equation will be:

$$k = \sqrt{(n\rho)^2 + 2n\rho} \; - n\rho \quad \blacksquare$$

Example 4.2-4

Relocate the neutral axis of Example 4.2-2, based on the general relation that has been derived in Example 3.

Solution

$$n = 8.5$$

$$A_S = 942 \text{ mm}^2$$

$$\rho = \frac{A_s}{bd} = \frac{942 \text{ mm}^2}{550 \times 300 \text{ mm}^2} = 5.71 \times 10^{-3}$$

$$n\rho = 0.0485$$

$$k = \sqrt{(0.0485)^2 + 2 \times 0.0485} \; - 0.0485 \; = 0.267 \quad \blacksquare$$

---

[1] Quadratic equation is a equation that has the following general form:

$$ax^2 + bx + c = \quad where\ a \neq 0$$

This equation can be solved based on the *Quadratic Formula*:

$$x = \frac{\left(-b \; \pm \; \sqrt{b^2 - 4ac}\right)}{2a}$$

Example 4.2-5 Analysis of Working Stresses in a Reinforced Concrete Beam with General Shape

The simply supported beam shown in Figure 4.2-6 below has the following data: $f_{c\,allowable} = 7\,MPa$, $f_{s\,allowable} = 124\,MPa$, and $n = 12$.

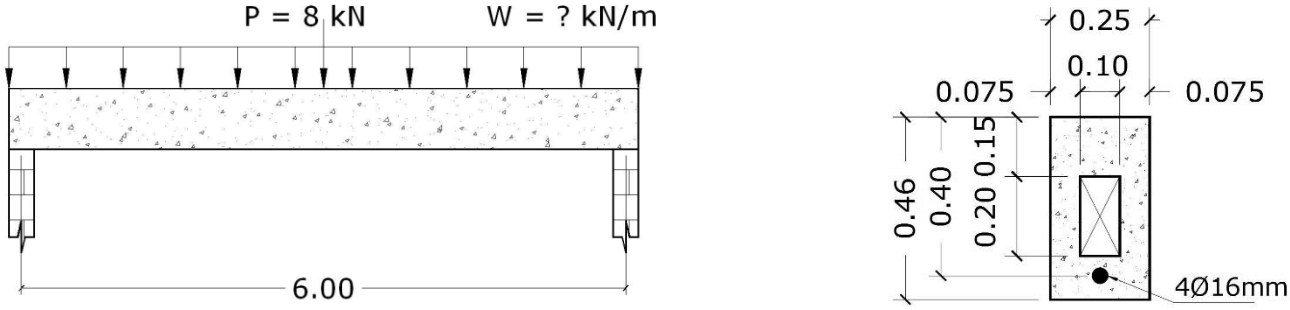


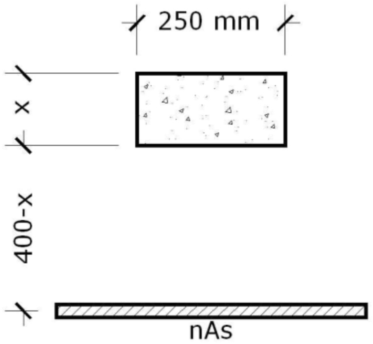
Figure 4.2-6: Simply supported beam for Example 4.2-5.

Compute the following:

- Section allowable bending moment.
- Total uniform load (including beam selfweight) that could the beam carry in addition to a concentrated load of 8 kN at mid-span.

Solution

- Compute the section moment of inertia of transformed section:
  - Assume that the neutral axis to be above the hollow section.



$$A_s = 4 \times \frac{\pi 16^2}{4} = 804\ mm^2 \quad \Rightarrow nA_s = 12 \times 804mm^2 = 9\,648mm^2$$

$$\sum \text{Area Moment about N.A.} = 0$$

$$250mm \times x \times \frac{x}{2} = 9\,648mm^2 \times (400 - x)$$

$$x^2 + 77.2x - 30\,874 = 0$$

$$x = \frac{-77.2 \pm \sqrt{77.2^2 + 4 \times 1 \times 30\,874}}{2 \times 1} = 141mm$$

$$I_{N.A.} = \frac{141^3 \times 250}{3} + 9\,648 \times (400 - 141)^2 = 881 \times 10^6 mm^4$$

  - Compute the allowable bending moment based on concrete allowable stresses:

$$F_{c\,allowable} = \frac{M_{allowable} \cdot c_{Top}}{I_{N.A.}} \Rightarrow M_{allowable} = \frac{7MPa \times 881 \times 10^6 mm^4}{141mm} = 43.7\ kN.m$$

  - Compute the allowable bending moment based on steel allowable stresses:

$$F_{s\,allowable} = n\frac{M_{allowable} \cdot c_{Bottom}}{I_{N.A.}} \Rightarrow M_{allowable} = \frac{124MPa \times 881 \times 10^6 mm^4}{12 \times (400 - 141)mm}$$
$$= 35.1\ kN.m$$

  - Compute the allowable bending moment:

$$M_{allowable} = \text{Minimum of } 43.7\text{ kN.m and } 35.1\text{ kN.m} = 35.1\text{ kN.m} \blacksquare$$

- Compute the allowable total load:

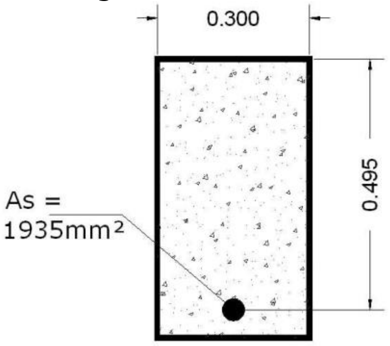
$$M = \frac{WL^2}{8} + \frac{PL}{4} \Rightarrow 35.1\ kN.m = \frac{W\frac{kN}{m} \times 6^2 m^2}{8} + \frac{8kN \times 6m}{4}$$

$$W = 5.13\frac{kN}{m} \blacksquare$$

4.2.2.2.5 Home Work of Article 4.2.2.2.4: Analysis of Working Stresses in Beams with Rectangular Sections

Problem 4.2-2

For the beam shown below if the $E_s = 200\,000\ MPa$, $E_C = 20\,000\ MPa$, $f_c' = 21MPa$, and $f_y = 400\ MPa$, determine the maximum stresses in the steel and concrete if the applied bending moment is 115kN.m.



Answers

$k = 0.396\ kd = 196\ mm\ I_{N.A.} = 2.48 \times 10^9 mm^4$

$f_c = 9.09\ MPa < \frac{f_c'}{2}\ Ok.$ ■ $f_s = 139\ MPa < f_y\ Ok.$ ■

Problem 4.2-3

What is the maximum allowable bending moment for the beam of Problem 1, if the maximum allowable stresses are $f_s = 152MPa$ and $f_C = 8.33\ MPa$.

Answers

$M_{Allowable\ Based\ on\ Steel\ Allowable\ Stresses} = 126\ kN.m$
$M_{Allowable\ Based\ on\ Concrete\ Allowable\ Stresses} = 105\ kN.m$
$M_{Allowable} = Minimum\ (126, 105) = 105\ kN.m$ ■

Problem 4.2-4

What is the maximum allowable bending moment for the beam of Problem 1, if the maximum allowable stresses are $f_s = 132\ MPa$ and $f_C = 9.33\ MPa$.

Answers

$M_{Allowable\ Based\ on\ Steel\ Allowable\ Stresses} = 109\ kN.m$
$M_{Allowable\ Based\ on\ Concrete\ Allowable\ Stresses} = 118\ kN.m$
$M_{Allowable} = Minimum\ (109, 118) = 109\ kN.m$ ■

Problem 4.2-5

A concrete beam shown below has a simple span of 5m. It has $f_c = 9.00MPa$, $f_s = 124MPa$ and n = 10.

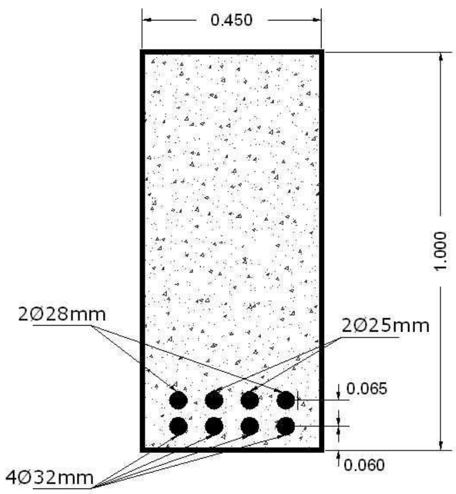
For this beam compute the following:

- Section allowable bending moment.
- Value of concentrated force "P" that the beam could carry at it midspan.

Notes on Problem 4.2-4

Sometimes, beam width is not sufficient to put the required reinforcement in a single layer, and then the reinforcement is put in two or more layers.

For analysis purposes, theses layers are usually replaced with a single layer that has an area equal to area of all layers and located at centroid of steel layers.



Answers

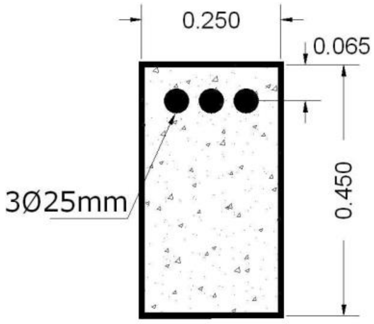
- Section allowable bending moment.
  $A_{of\ Rebar\ 25mm} = 490\ mm^2$
  $A_{of\ Rebar\ 28mm} = 615\ mm^2$
  $A_{of\ Rebar\ 32mm} = 804\ mm^2$
  $\bar{y}_{Measured\ from\ reinforcement\ center\ to\ beam\ lower\ face} = 86.5\ mm < 92.5\ mm\ Ok.$
  $d = 913\ mm$
  $As = 5\,426\ mm^2$
  $\rho = 13.2 \times 10^{-3}$
  $n\rho = 0.132$
  $k = 0.398$
  $kd = 364\ mm$
  $nA_s = 54\,260\ mm^2$
  $I_{N.A.} = 23.6 \times 10^9 mm^4$
  $M_{Allowable\ Based\ on\ Steel\ Allowable\ Stresses} = 533\ kN.m$
  $M_{Allowable\ Based\ on\ Concrete\ Allowable\ Stresses} = 584\ kN.m$
  $M_{Allowable} = 533\ kN.m$ ■
- Value of concentrated force "P" that the beam could carry at it mid-span.
  $W_{Selfweight} = 10.8 \frac{kN}{m}$ $P = 399\ kN$ ■

Problem 4.2-6

The figure shown below is a cross-section of a cantilever beam supporting a uniform load of $2.5 \frac{kN}{m}$ including its own weight and a concentrated load of $30\ kN$ at its free end. It has $f_c = 8.00\ MPa$, $f_s = 124\ MPa$, and $n = 10$.

For this beam compute the following:

- The safe resisting moment of the beam.
- The maximum beam span.



Answers

- The safe resisting moment of the beam:
  $d = 385\ mm$ $As = 1\,470\ mm^2$ $\rho = 15.1 \times 10^{-3}$ $n\rho = 0.151$
  $k = 0.419$ $kd = 163\ mm$ $nA_s = 14\,700\ mm^2$ $I_{N.A.} = 1.12 \times 10^9 mm^4$
  $M_{Allowable\ Based\ on\ Steel\ Allowable\ Stresses} = 61.2\ kN.m$
  $M_{Allowable\ Based\ on\ Concrete\ Allowable\ Stresses} = 55.0\ kN.m$
  $M_{Allowable} = Minimum\ (61.2, 55.0) = 55.0\ kN.m$ ■
- The maximum beam span.
  $L = 1.71\ m$ ■

4.2.2.2.6 Third Stage: Flexure Strength

- When the load is still further increased, flexure strength of the beam is reached. Failure can be caused in one of the following two ways:
  - ***SECONDARY COMPRESSION FAILURE (TENSION-CONTROLLED SECTION):***
    - When relatively ***moderate amount of reinforcement are employed, at some value of the load the steel will reach its yield point***.
    - At that stress the reinforcement yields suddenly and stretched a large amount, and the tension cracks in the concrete widen visibly and propagate upwards, with simultaneous significant deflection of the beam. When this happens, the strains in the remaining compression zone of the concrete increase to such degree that crushing of the concrete. Then the stresses history will be as shown in the figures below:

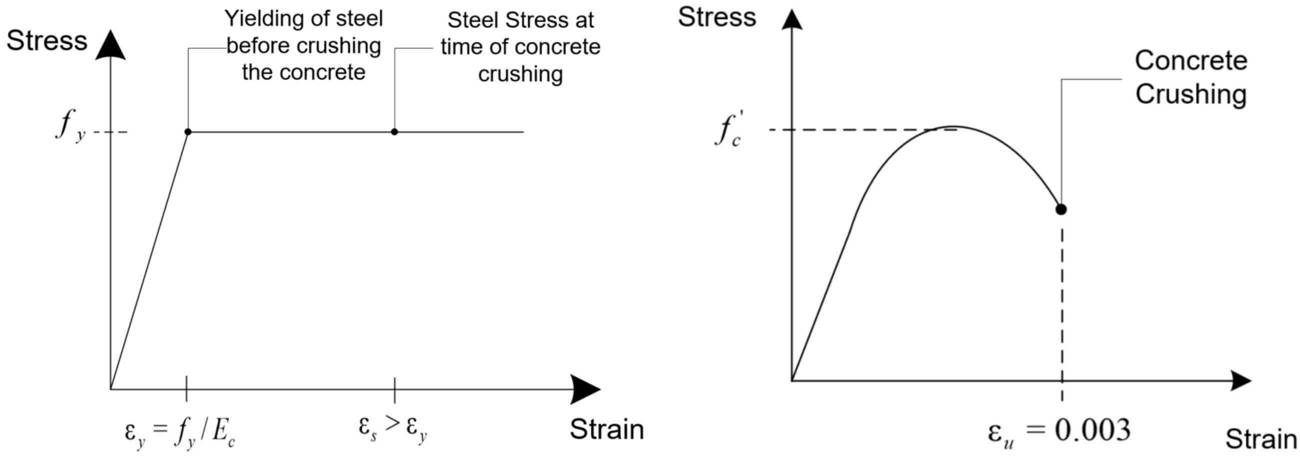


Figure 4.2-7: Stress –strain state for secondary compressive failure.

  - ***COMPRESSION FAILURE (COMPRESSION CONTROLLED SECTION)*:**
    - On the other hand, ***if large amount of reinforcement or normal amount of steel with very high strength are employed, the compression strength of the concrete may be exhausted before the steel start yielding***.
    - Then the stresses history during the compression failure will be as shown below:

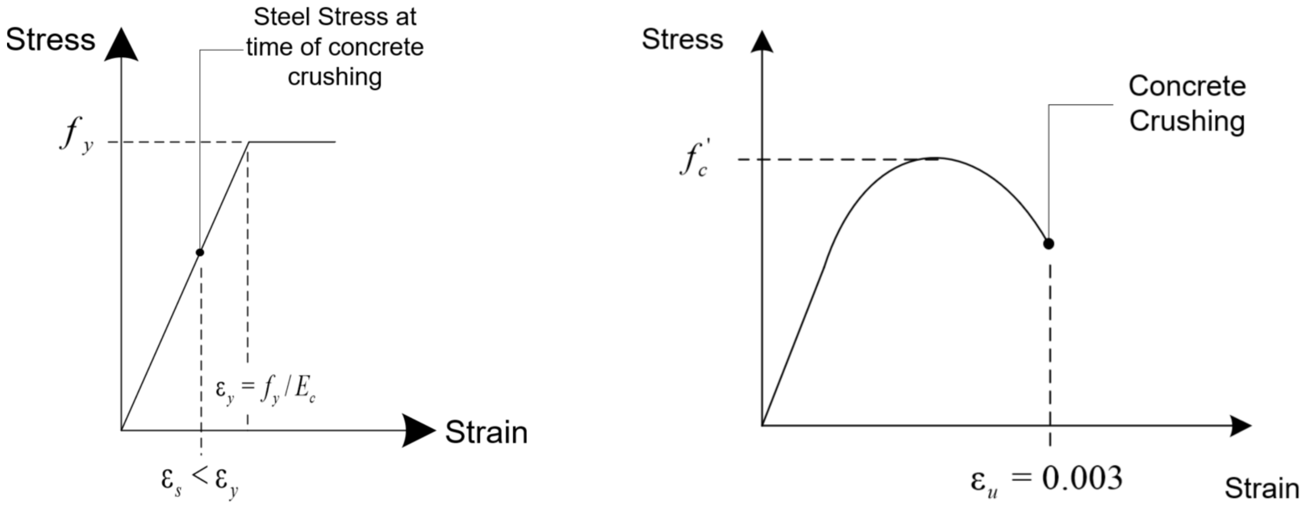


Figure 4.2-8: Stress –strain state for compressive failure.

    - Compression failure is sudden, of an almost explosive nature, and occurs without warning.
    - For this reason, (ACI318M, 2014), ***Article 21.2.1***, required to dimension beams in such a manner that should they be overloaded, failure would be initiated by yielding of the steel rather than by crushing of concrete (*Secondary Compression Failure*).

4.2.2.2.7 Nominal Flexure Strength M_n of a Rectangular Section with Secondary Compression Failure.

- It is clear that at or near ultimate loads, stresses are no longer proportional to the strain, then the conventional flexure formula ($f = \frac{M.c}{I}$) cannot be applied for the analysis and design of the section.
- And the analysis and design of the section must be based on the direct application of the basic principles (compatibility relation, stress-strain relation, and the equilibrium conditions) and as follow:
  - **COMPATIBILITY CONDITIONS**
    Based on
    - The ***kinematic assumption*** of the ***plane section before loading remain plane after loading***. This assumption is adopted by (ACI318M, 2014) in ***article 22.2.1.2***.
    - The ***assumption of the secondary compression failure*** (i.e., the failure starting with the yielding of the steel and the crushing of concrete). According to (ACI318M, 2014), ***article 22.2.2.1***, concrete crushing occurs when maximum strain at the extreme concrete compression fiber reaches a value of $\epsilon_u = 0.003$.

    the strain distribution will be as shown below:

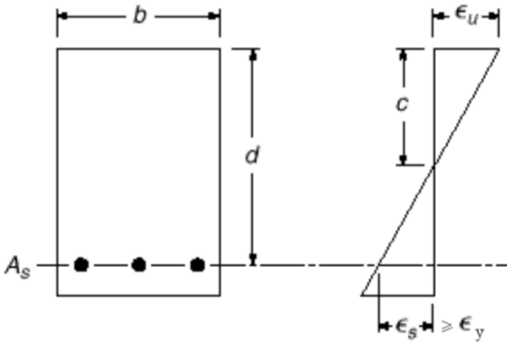


Figure 4.2-9: Strain distribution.

    - The kinematic assumption remains applicable even when materials behave inelastically, (Popov, 1968).
  - **STRESS-STRAIN RELATION:**
    - Based on the actual stress-strain relations of the concrete and reinforcing steel, the stress distribution will be as shown below:

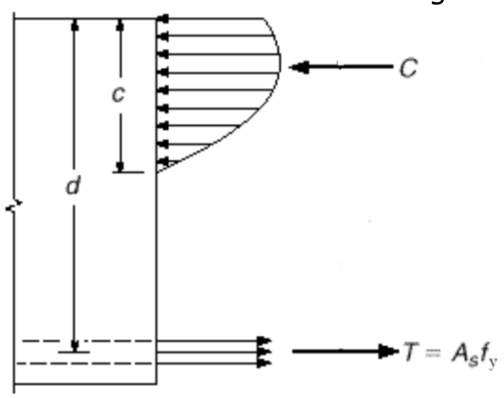


Figure 4.2-10: Stress distribution for a beam with secondary compression failure.

    - It is not really necessary to know the shape of the concrete stress distribution. What is necessary to know is, (ACI318M, 2014) ***article 22.2.2.3***:
      - The total resultant compression force "*C*" in the concrete.
      - Its vertical location.

      Evidently, then, one can think of the actual complex stress distribution as replaced by a fictitious one of some simple geometric shape, provided that this fictitious distribution results in the same total compression force "C" applied at the same location as in the actual member when it is on the point of failure.

- Historically, a number of simplified, fictitious equivalent stress distributions have been proposed by investigators in various countries. The one generally accepted was first proposed by ***C. S. Whitney*** and was subsequently elaborated and checked experimentally by others. The actual stress distribution immediately before failure and the fictitious equivalent distribution are shown in Figure below, (ACI318M, 2014), ***article 22.2.2.4***,

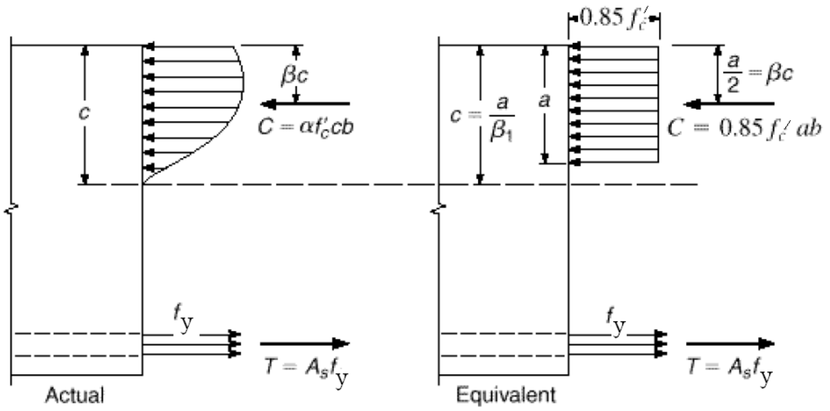


Figure 4.2-11: Whitney simplified equivalent stress distributions.

- According to ACI code (**22.2.2.4.3**) β_1 can be computed based on Table below:

Table 4.2-1: Values of β1 for equivalent rectangular concrete stress distribution, Table 22.2.2.4.3 of (ACI318M, 2014).

f_c', MPa	β_1	
$17 \le f_c' \le 28$	0.85	(a)
$28 < f_c' < 55$	$0.85 - \dfrac{0.05(f_c' - 28)}{7}$	(b)
$f_c' \ge 55$	0.65	(c)

- **EQUILIBRIUM CONDITIONS:**

  According to (ACI318M, 2014), ***article 22.2.1.1***, equilibrium shall be satisfied at each section:

  $$\because \sum F_x = 0 \implies 0.85 f_c' ba = A_s f_y$$

  $$\therefore a = \frac{A_s f_y}{0.85 f_c' b}$$

  $$\because \sum M_{\text{About Centroid of Compressive Force C}} = 0$$

  $$\therefore M_n = A_s f_y \left(d - \frac{a}{2}\right)$$

  Substitute the value of "a" into above equation:

  $$M_n = A_s f_y \left(d - \frac{1}{2}\,\frac{A_s f_y}{0.85 f_c' b}\right) \quad \text{or } M_n = A_s f_y d \left(1 - \frac{1}{2}\,\frac{A_s f_y}{0.85 f_c' bd}\right)$$

  Let $\rho = \frac{A_s}{bd}$

  $$M_n = \rho f_y b d^2 \left(1 - 0.59\,\frac{\rho f_y}{f_c'}\right) \quad \blacksquare$$

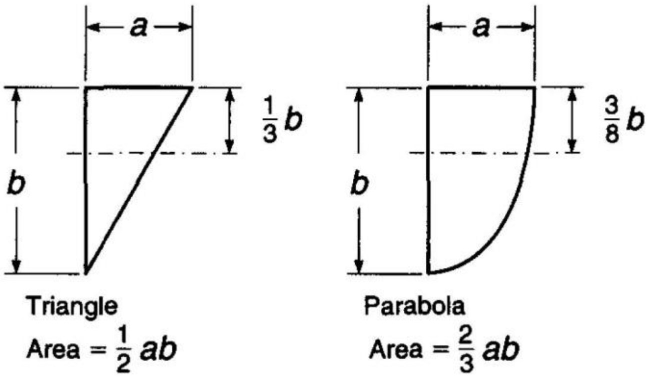
4.2.2.2.8 Home Work of Article 4.2.2.2.7: Behavior of Singly Reinforced Rectangular Concrete Beams

Problem 4.2-7

A rectangular beam made using concrete with $f_c' = 35\,MPa$ and steel with $f_y = 420\,MPa$ has a width b = 450mm, an effective depth d = 540mm, and a total depth h = 600mm. The beam is reinforced with four No. 29 bars. Compute the nominal moment capacity, assuming, see Figure below

a. an equivalent rectangular stress block,
b. a triangular stress block with a peak value of f_c',
c. a parabolic stress block with a peak value of f_c'. (see Fig. P3.13).

Compare and comment on your results, knowing that the rectangular stress block correlates within 4 percent with test results.



Aim of Problem

*This problem aims to highlight code regulations of 22.2.2.3 which states "**The relationship between concrete compressive stress and strain shall be represented by a rectangular, trapezoidal, parabolic, or other shape that results in prediction of strength in substantial agreement with results of comprehensive tests**".*

Answers

a. $a_{Rectangular} = 67.5\,mm \;\; M_{n\,Rectangular} = 549\,kN.m$
b. $a_{Triangular} = 115\,mm \;\; M_{n\,Triangular} = 544\,kN.m$
c. $a_{Parabolic} = 103\,mm \;\; M_{n\,Parabolic} = 543\,kN.m$

$$\frac{M_{n\,Triangular}}{M_{n\,Rectangular}} = \frac{544}{549} = 0.991$$

$$\frac{M_{n\,Parabolic}}{M_{n\,Rectangular}} = \frac{543}{549} = 0.989$$

Comment:

In both cases the results are within a 4% margin or error and the rectangular stress block gives the higher value for the nominal moment.

Problem 4.2-8

A rectangular beam made using concrete with $f_c' = 42\,MPa$ and steel with $f_y = 420\,MPa$ has a width 500mm., an effective depth of d=440mm., and a total depth of h = 500mm. The concrete modulus of rupture $f_r = 3.6\,MPa$. The elastic moduli of the concrete and steel are, respectively, $E_c = 28000\,MPa$ and $E_s = 200000\,MPa$. The tensile steel consists of four No. 36. bars.

a. Find the maximum service load moment that can be resisted without stressing the concrete above $0.45f_c'$ or the steel above $0.4f_y$.
b. Determine whether the beam will crack before reaching the service load.
c. Compute the nominal flexural strength of the beam.
d. Compute the ratio of the nominal flexural strength of the beam to the maximum service load moment, and compare your findings to the ACI load factors and strength reduction factor.

Answers

a. $M_{sc} = 219\ kN.m\ M_{ss} = 179\ kN.m\ M_{ss} = 179\ kN.m$

b. $M_{cr} = \frac{bh^2}{6} f_r = 75\ kN.m$, therefore section cracks.

c. $a = 94.7\ mm\ M_n = 664\ kN.m$

d. $Ratio = \frac{M_n}{M_s} = \frac{664}{179} = 3.7 > \frac{\gamma}{\phi} = \frac{\frac{1.2+1.6}{2}}{0.9} = 1.56$

Comments:

The value of this ratio is greater than the ACI factors for strength, γ, divided by the ϕ factor, thus suggesting that the ***working stress design approach*** is more conservative than the ***strength design, or Load Resisting Factored Design LRFD, approach***.

4.2.2.2.9 Balanced Strain Condition (ACI 10.3.2)

- The secondary compression failure can be assured by keeping the reinforcement ratio ρ below a certain limiting value that called Balanced Steel Ratio ρ_b.
- It represents a limit amount of reinforcement necessary to make the beam fail by crushing of concrete at the same load that causes the steel yield.
- Computing the "***Balanced Steel Ratio***" is also can be written in terms of application of basic principles (Compatibility, Stress-Strain Relation, and Equilibrium):
    - Compatibility Conditions:
      Based on strain conditions shown below:

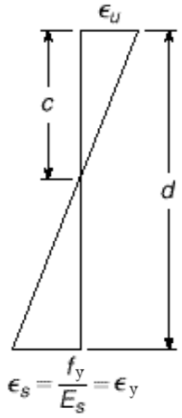


Figure 4.2-12: Strain distribution for balanced condition.

$$\frac{c_b}{\epsilon_u} = \frac{d}{\epsilon_u + \epsilon_y}$$

$$c_b = \frac{\epsilon_u}{\epsilon_u + \epsilon_y} d$$

$$[c_b = \frac{0.003}{0.003 + f_y/E_s} d] \times \frac{E_s}{E_s}$$

$$c_b = \frac{600}{600 + f_y} d$$

Above relation is a general relation and correct not only for rectangular section.

- Stress-Strain Relation:
  Stress distribution for balanced condition can be derived from strain condition and as shown in Figure below:

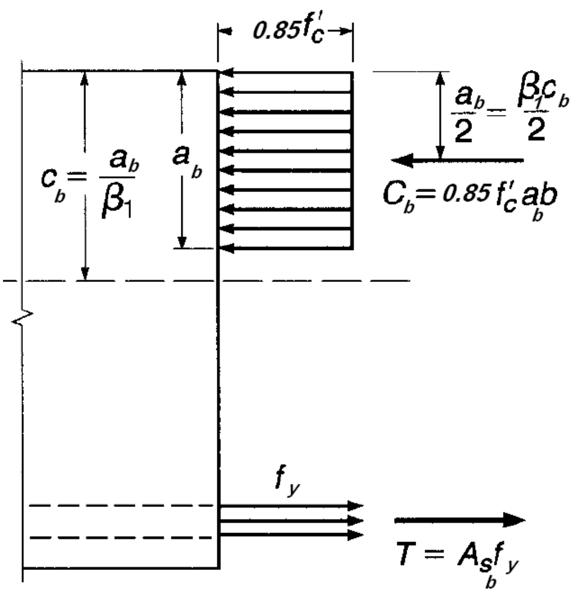


Figure 4.2-13: Stress distribution and forces for balanced condition.

- Equilibrium Conditions:

$$\because \sum F_x = 0 \;\Rightarrow 0.85 f'_c b a_b = A_{sb}\, f_y$$

$$A_{sb}\, f_y = 0.85 f'_c b \beta_1 \left(\frac{600}{600 + f_y} d\right) \quad \div f_y b d$$

$$\rho_b = 0.85\beta_1 \frac{f'_c}{f_y}\left(\frac{600}{600 + f_y}\right) \blacksquare$$

- It useful to notes that the ***Balanced Steel Ratio*** is a function of material strengths (f'_c, and f_y) only and it is independent on beam dimensions.

Example 4.2-6

Compute the Balanced Steel Ratio for concretes that have compressive strength of $f'_c = 21\text{MPa}, 28\text{MPa}, \text{and } 35\text{MPa}$ when reinforced with reinforcing steel have grades of Grade 40, 50, and 60.

Solution

Microsoft Excel is so effective in prepare calculations table has cells that related to each other by algebraic or logical relations.

For our problem, calculation table will take the following form:

f'_c MPa (psi)	β_1	***Steel Grade***		
		40	***50***	***60***
		f_y MPa		
		280	***350***	***420***
21 (3000)	0.850	36.9×10^{-3}	27.4×10^{-3}	21.3×10^{-3}
28 (4000)	0.850	49.3×10^{-3}	36.5×10^{-3}	28.3×10^{-3}
35 (5000)	0.800	51.9×10^{-3}	42.9×10^{-3}	33.3×10^{-3}

4.2.2.2.10 ACI Maximum Steel Ratio ρ_{max}, (ACI318M, 2014), article 9.3.3.1

- In actual practice, the upper limit on ρ should be below ρ_b, for the following reasons:
    - For a beam with ρ exactly equal to ρ_b, the compressive strain limit of the concrete would be reached, theoretically, at precisely the same moment that the steel reaches its yield stress, without significant yielding before failure.
    - Material properties are never known precisely.
    - The actual steel area provided will always be equal to or larger than required, based on selected reinforcement ratio ρ, tending toward overreinforcement.
- Then to ensure under-reinforced behavior (ACI318M, 2014) (**9.3.3.1**) establishes a minimum net tensile strain ϵ_t , at the nominal member strength of **0.004**.
- By way of comparison, ϵ_y, the steel strain at the balanced condition, is 0.002 for Grade 420 (See Figure below).

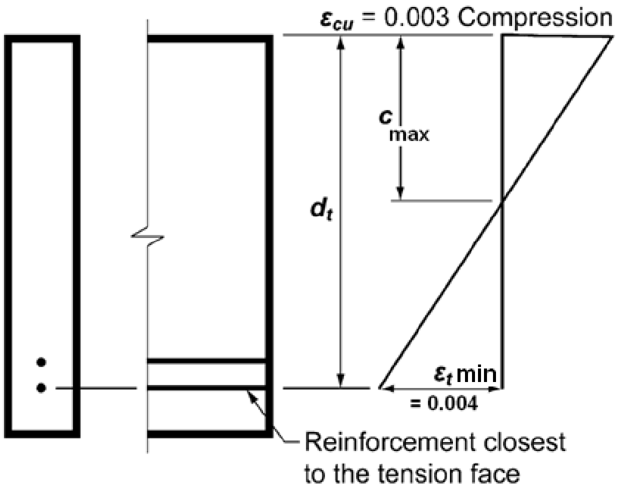


Figure 4.2-14: Strain limits for nonprestressed beams.

where d_t is the distance from extreme compression fiber to centroid of extreme layer of longitudinal tension steel.

- Based on strain distribution (compatibility conditions) c_{max} will be:

$$c_{max} = \frac{\epsilon_u}{\epsilon_u + 0.004} d_t$$

$$c_{max} = \frac{0.003}{0.003 + 0.004} d_t$$

$$c_{max} = 0.429\, d_t \;\blacksquare$$

Above relation is a general relation (i.e., it is applicable for rectangular and non-rectangular section).

- Above relation can be read as follows:
According to ACI, the lowest permitted location of N.A. is located at 42.9% of d_t measured from compressive face.
- Thickness of equivalent rectangular stress distribution will be:

$$a_{max} = \beta_1 \frac{\epsilon_u}{\epsilon_u + 0.004} d_t$$

- Based on stress-strain relation and equilibrium conditions, ***Maximum Steel Area*** $A_{s\,max}$rmitted by the ACI will be:

$$\because \sum F_x = 0 \;\Rightarrow 0.85 f_c' b a_{max} = A_{s\,max}\, f_y$$

$$A_{s\ max}\, f_y = 0.85 f_c' b \left(\beta_1 \frac{\epsilon_u}{\epsilon_u + 0.004} d_t\right)$$

If d_t is conservatively taken equal to d, then:

$$A_{s\ max}\, f_y = 0.85 f_c' b \left(\beta_1 \frac{\epsilon_u}{\epsilon_u + 0.004} d\right) \quad \div f_y b d$$

$$\rho_{max} = 0.85 \beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} \;\blacksquare$$

Example 4.2-7

Compute the ACI ***Maximum Steel Ratio*** ρ_{max} for concretes that have compressive strength of $f_c' = 21MPa, 28MPa, and\ 35MPa$ when reinforced with reinforcing steel have grades of Grade 40, 50, and 60.

Solution

For our problem, calculations table will take the following form:

$f_c'\ MPa\ (psi)$	β_1	***Steel Grade***		
		40	***50***	***60***
		$f_y\ MPa$		
		280	***350***	***420***
21 (3000)	0.850	23.2×10^{-3}	18.6×10^{-3}	15.5×10^{-3}
28 (4000)	0.850	31.0×10^{-3}	24.8×10^{-3}	20.6×10^{-3}
35 (5000)	0.800	36.4×10^{-3}	29.1×10^{-3}	24.3×10^{-3}

4.2.2.2.11 ACI Flexure Strength Reduction Factor ∅ ((ACI318M, 2014) 21.2.2)

- The ACI Code encourages the use of lower reinforcement ratios by allowing higher strength reduction factors (∅) in such beams.
- To do that, ACI Code classified the concrete sections into:
  - Tension Controlled Section:
    - Is the member with a net tensile strain greater than or equal to 0.005. The corresponding strength reduction factor is 0.9.
    - Term member in above definition including beams and columns.
    - The selection of a net tensile strain of (0.005) is included to encompass the yield strain of all reinforcing steel including high-strength rebars.
  - Compression-Controlled Section:
    - Is the member that having a net tensile strain of less than 0.002.
    - Based on comparison with required strain of 0.004 for maximum steel ratio in beam, one can conclude that the term "member" in above definition including columns only, i.e. it must be clear that, it is not permitted by ACI Code to design concrete beams as compression-controlled members.
    - The strength reduction factor for compression-controlled members (columns) is 0.65. A value of 0.75 may be used if the members are spirally reinforced (see Figure below).

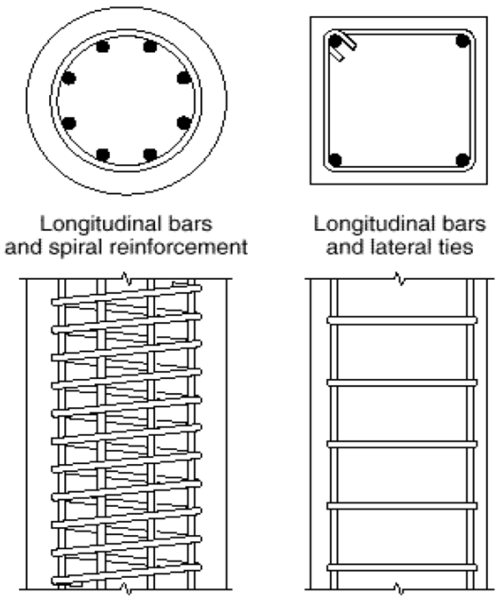


Figure 4.2-15: Spiral and tied columns.

- Difference between Spiral Columns and Tied Columns will be discussed in more detail later.
- A value of 0.002 corresponds approximately to the yield strain for steel with Grade 60.

  - Transition Zone Section:
    - Between net tensile strains of 0.002 and 0.005, the strength reduction factor varies lineally, and The ACI Code allows a linear interpolation of ∅ based on ϵ_t as shown in Figure below.

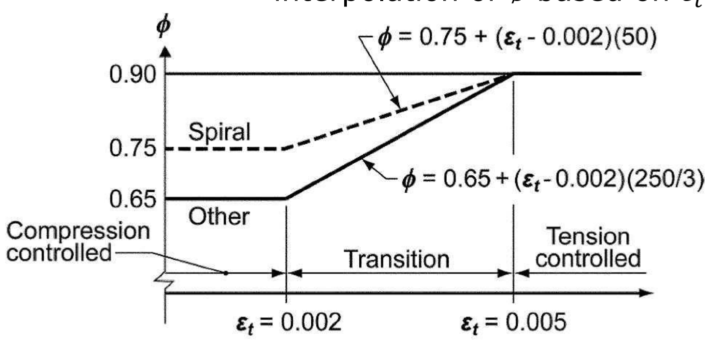


Figure 4.2-16: Strength reduction, ϕ, for transition factor.

- For beams, transition zone reduce to a range of 0.004 to 0.005 instead of range of 0.002 to 0.005 and as shown in Figure below.

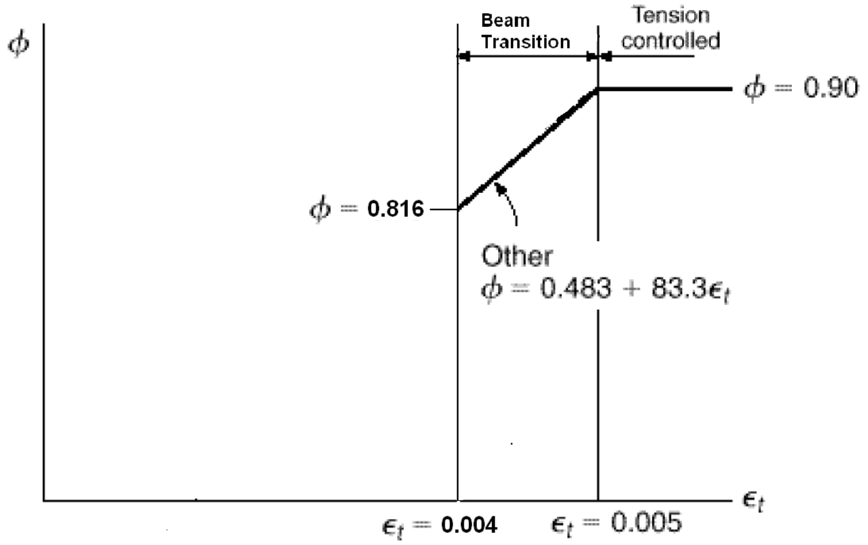


Figure 4.2-17: Transition zone for beams.

- Calculation of the nominal moment capacity frequently involves determination of the depth of the equivalent rectangular stress block "a" that related to "c" by the relation of $a = c/\beta_1$. Then it is some times more convenient to compute c/d ratios in terms the net tensile strain and as shown in Figure below.

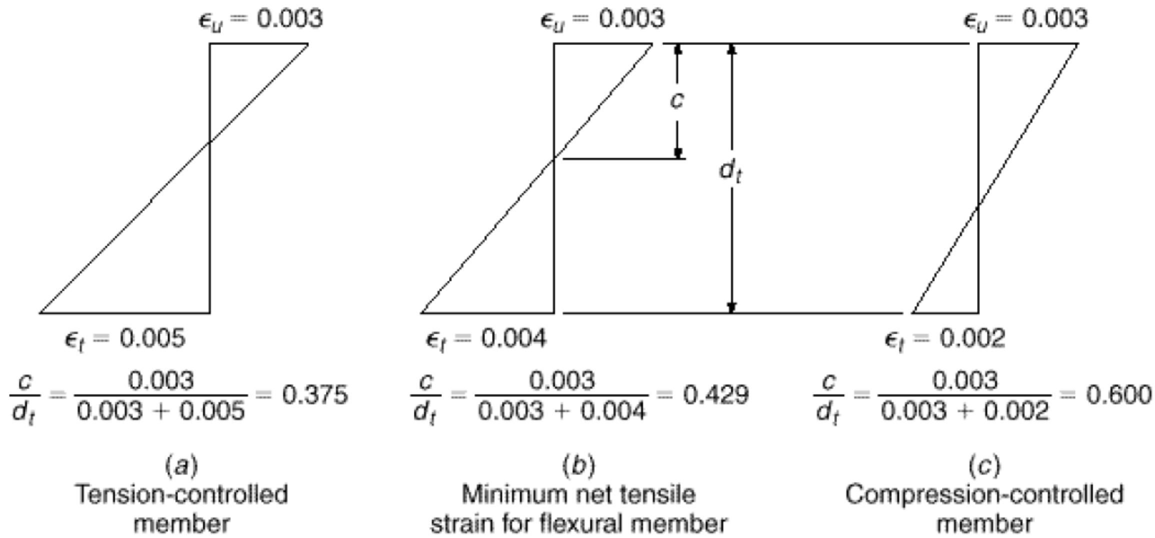


Figure 4.2-18: Section definition in terms of c/d_1 ratio.

Example 4.2-8

According to current design philosophy and for a beam with state of strains shown in Figure 4.2-19 below:

- Is the beam classified as failed or not?
- Is beam ratio, ρ, less than or greater than the maximum steel ratio, $\rho_{maximum}$?
- What is the flexural strength reduction factor, ϕ, for the beam?

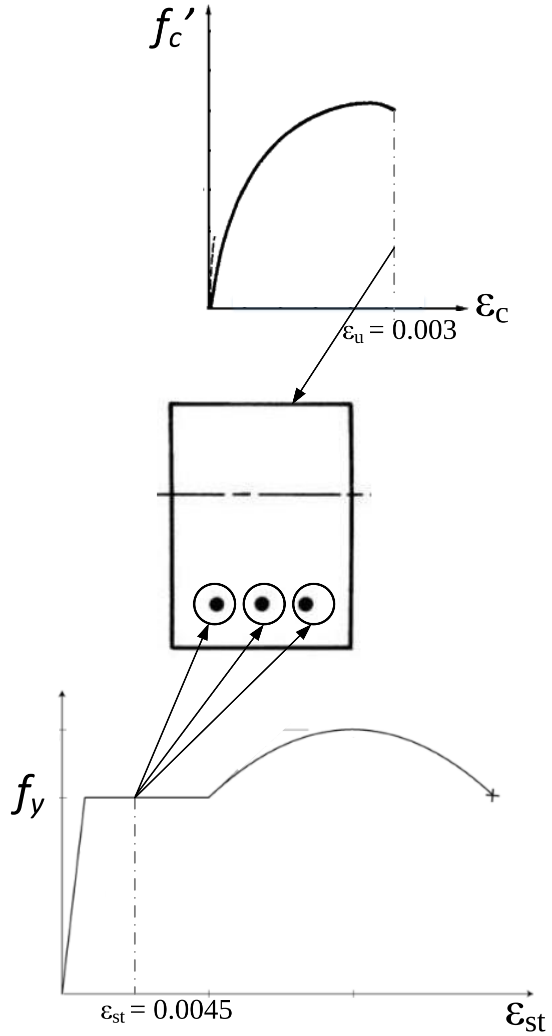


Figure 4.2-19: State of strains for Example 4.2-8.

Solution

- As concrete strain reaches 0.003, then the beam is at failure stage according to current ACI design philosophy.
- As steel strain of 0.0045 is greater than strain of 0.004 for $\rho_{Maximum}$, and as steel strain is inversely proportional to steel ratio, then provided steel ratio is lower than maximum ratio.
- The section is within the transition zone, $0.004 < \epsilon_t < 0.005$, then strength factored should be calculated based on following relation
  $\phi = 0.483 + 83.3\epsilon_t = \ 0.483 + 83.3 \times 0.0045 = 0.858$

4.2.2.2.12 ACI Minimum Reinforcement ((ACI318M, 2014), Article 9.6.1)

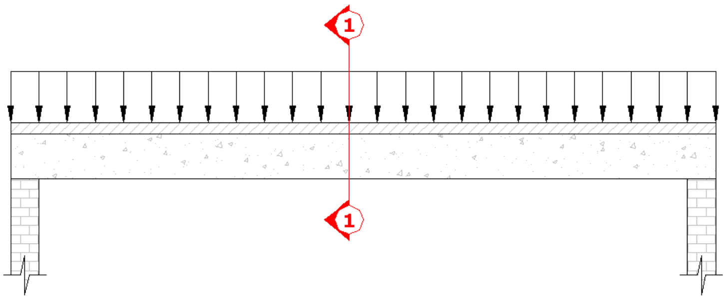
- Another mode of failure may occur in very lightly reinforced beams. If the flexural strength of the cracked section is less than the moment that produced cracking of the previously uncracked section the beam will fail immediately and without warning of distress upon formation of the first flexural crack.
- To ensure against this type of failure, a lower limit has been established for the reinforcement ratio by equating the cracking moment computed from the concrete modulus of rupture to the strength of the cracked section.
  $\because\ M_n = M_{Cracking} \quad \rightrightarrows \quad A_{s\ minimum}$
- Based on above concept, (ACI318M, 2014) (article 9.6.1) gives the following provisions for minimum steel Area:
  - At every section of a flexural member where tensile reinforcement is required by analysis. As provided shall not be less than that given by:
    $$A_{s\ minimum} = \frac{0.25\sqrt{f_c'}}{f_y} b_w d \geq \frac{1.4}{f_y} b_w d$$
    The relation $\frac{1.4}{f_y} b_w d$ had been derived based on substituting $f_c' = 31.4\ MPa$ into more accurate relation $\frac{0.25\sqrt{f_c'}}{f_y} b_w d$.
    For many years the relation $\frac{1.4}{f_y} b_w d$ was used as $A_{s\ min}$. For concrete with high strength, it is not sufficient and $A_{s\ min}$ must be computed based on the more accurate relation $\frac{0.25\sqrt{f_c'}}{f_y} b_w d$.
  - For members that have following properties:
    - Statically determinate.
    - With a flange in tension.

    Asmin shall be computed based on the following equation:
    $$A_{s\ min} = \ minimum \ \left(\frac{0.25\sqrt{f_c'}}{f_y} b_f d, \frac{0.50\sqrt{f_c'}}{f_y} b_w d\right)$$
    It's useful to note that above two conditions usually satisfy in the cantilever spans.
    When the flange of a section is in tension, the amount of tensile reinforcement needed to make the strength of the reinforced section equal that of the unreinforced section is about twice that for a rectangular section or that of a flanged section with the flange in compression. A higher amount of minimum tensile reinforcement is particularly necessary in cantilevers and other statically determinate members where there is no possibility for redistribution of moments.
  - The requirements of $A_{s\ min}$ need not be applied if, at every section, $A_{s\ Provided}$ is at least one-third greater than that required by analysis, i.e.:
    $$A_{S\ Provided} = \ 1\frac{1}{3} A_{S\ Required}$$
    This exception is intended to solve the problem of $A_{s\ min}$ for members that have large cross sectional areas.

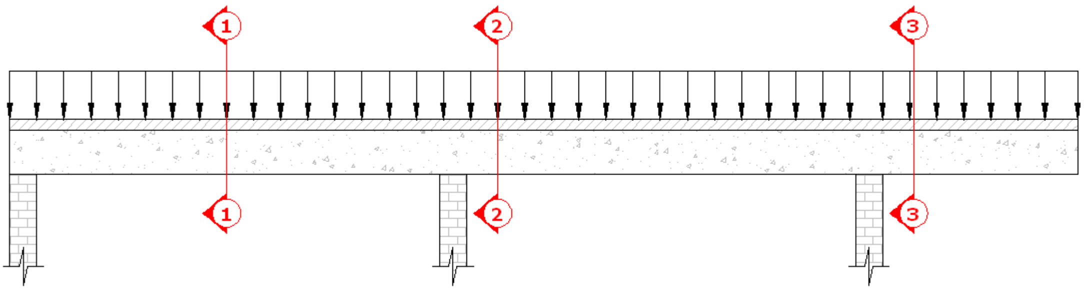
Example 4.2-9

State the relation that must be used for computing $A_{s\,min}$ for beams shown in Figure 4.2-20 below.

- Beam 1:



- Beam 2:



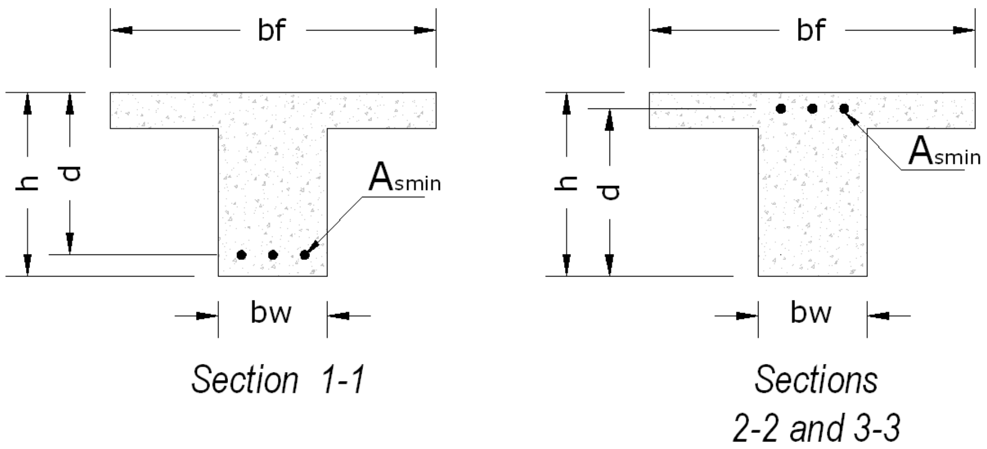


Figure 4.2-20: Beam for Example 4.2-9.

Solution

- For Beam 1:
  Section 1-1:
  As the section flange is under compression stress, then $A_{s\,min}$ is computed based on the following relation:

$$A_{s\,minimum} = \frac{0.25\sqrt{f_c'}}{f_y} b_w d \geq \frac{1.4}{f_y} b_w d$$

- For Beam 2:
  Section 1-1
  As the section flange is under compression stress, then $A_{s\,min}$ is computed based on the following relation:

$$A_{s\,minimum} = \frac{0.25\sqrt{f_c'}}{f_y} b_w d \geq \frac{1.4}{f_y} b_w d$$

  Section 2-2
  In spite of the section flange is under tensile stress, but as the span is a statically indeterminate span then $A_{s\,min}$ is computed based on the following relation:

$$A_{s\,minimum} = \frac{0.25\sqrt{f_c'}}{f_y} b_w d \geq \frac{1.4}{f_y} b_w d$$

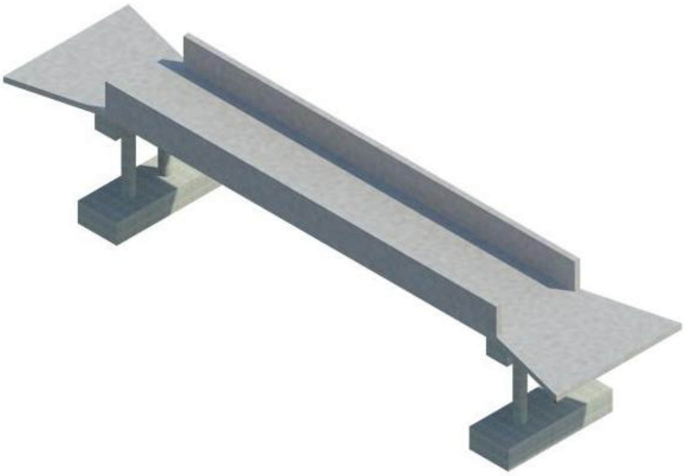
Section 3-3

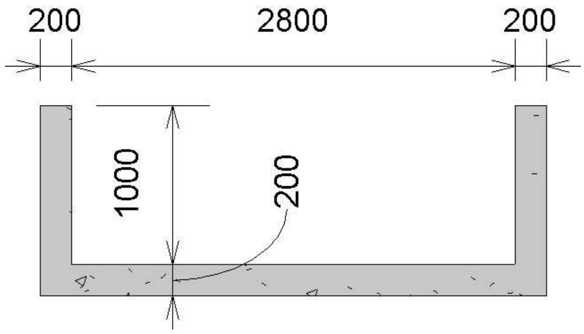
As the section flange is under tensile stress, and the span is a statically determinate span then $A_{s\,min}$ is computed based on the following relation:

$$A_{s\,min} = minimum\left(\frac{0.25\sqrt{f_c'}}{f_y}b_f d, \frac{0.50\sqrt{f_c'}}{f_y}b_w d\right)$$

Example 4.2-10

For a simply supported pedestrian bridge shown in Figure 4.2-21 below, compute the minimum reinforcement area according to ACI requirements.





3D View **Cross Section**

Figure 4.2-21: Pedestrian bridge for Example 4.2-10.

Solution

For this statically determinate pedestrian bridge with a flange in tension, minimum flexure reinforcement should computed based on:

$$A_{s\,min} = minimum\left(\frac{0.25\sqrt{f_c'}}{f_y}b_f d, \frac{0.50\sqrt{f_c'}}{f_y}b_w d\right)$$

Assuming two layers of $\phi 20mm$ longitudinal rebars and $\phi 12mm$ stirrups:

$$d = 1200 - 40 - 12 - 20 - \frac{25}{2} = 1115mm$$

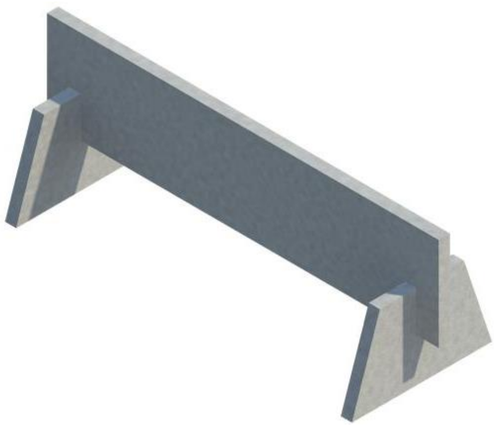
$$A_{s\,\min for\ two\ legs} = minimum\left(\frac{0.25 \times \sqrt{21}}{420} \times 3200 \times 1115, \frac{0.50 \times \sqrt{21}}{420} \times (2 \times 200 \times 1115)\right)$$

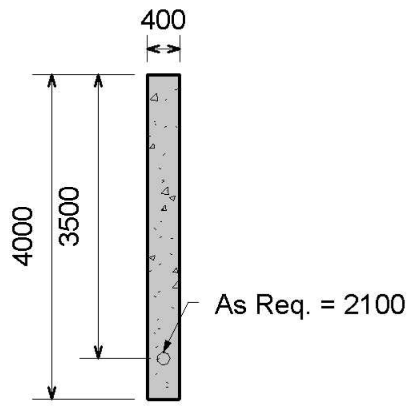
$$A_{s\,\min for\ two\ legs} = minimum\ (9733, \quad 2809)$$

$$A_{s\,\min for\ two\ legs} = 2809\ mm$$

Example 4.2-11

For monument that shown in Figure 4.2-22 below, required steel reinforcement ($A_{S\,Required}$) has been found equal to $2100mm^2$. Compare this area with ACI minimum reinforcement ($A_{s\,Minimum}$). In your solution adopt f_c' of 28 MPa and f_y of 420 MPa.





3D View **Cross Section**

Figure 4.2-22: Monument for **Example 4.2-11**.

Solution

According to ACI 9.6.1.3, the requirements of ACI 9.6.1.2 (traditional $A_{s\,Minimum}$ requirements), and ($A_{s\,Minimum}$ requirements for a statically determinate section with flange in tension) need not be applied if, at every section, $A_{s\,provided}$ is at least one-third greater than that required by analysis.

$$A_{s\,Minimum} = \frac{1.4}{f_y} b_w d = \frac{1.4}{420} \times 400 \times 3500 = 4667\ mm^2$$

$$A_{s\,Minimum} = 4667\ mm^2 > 1\frac{1}{3} \times 2100 = 2799\ mm^2$$

Then, use:

$A_{Provided} = 2799\ mm^2$

4.3 Procedure and Examples for Flexure Analysis of Rectangular Beams with Tension Reinforcement

4.3.1 Procedures

- Generally, in an analysis problem the following information are knowns:
  - Beam dimensions and reinforcement (b, h, d, and A_s).
  - Materials strength (f_y and f_c').

  and the following information are required:
  - To check if the section is adequate to general requirements of ACI code to see if the provided steel reinforcement agrees with ACI limits on $A_{s\,max}$ and $A_{s\,min}$.
  - To compute the design flexural strength of section ($\emptyset M_n$).
  - To compute the maximum live or dead or other loads that can be supported by the considered beam.
- Based on above knowns and requirements, the procedure for analysis of a rectangular beam with tension reinforcement can be summarized as follows:
  - Check if the provided steel reinforcement agrees with ACI limits on $A_{s\,max}$ and $A_{s\,min}$:

    $$\rho \le \rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} \qquad A_s \ge A_{s\,minimum} = \frac{0.25\sqrt{f_c'}}{f_y} b_w d \ge \frac{1.4}{f_y} b_w d$$
  - Compute the nominal strength M_n of the section:

    $$M_n = \rho f_y b d^2 \left(1 - 0.59 \frac{\rho f_y}{f_c'}\right)$$
  - Compute the strength reduction factor $\emptyset$:
    - Compute steel stain based on the following relations:

      $$a = \frac{A_s f_y}{0.85 f_c' b}$$
      $$c = \frac{a}{\beta_1}$$
      $$\epsilon_t = \frac{d - c}{c}\epsilon_u$$
    - If $\epsilon_t \ge 0.005$, then $\emptyset = 0.9$
    - If $\epsilon_t < 0.005$, then $\emptyset = 0.483 + 83.3\epsilon_t$
  - Compute the design strength $\emptyset M_n$ of the section:

    $\emptyset M_n = \emptyset \times M_n$
  - If maximum live, or dead, or other loads are required, then factored moment must be computed based on the following relation

    $M_u = \emptyset M_n$

    and the required loads can be computed based on the bending moment diagram of the problem under consideration.

4.3.2 Examples

Example 4.3-1

Check the adequacy of the beam of ***Example 4.2-1*** according to ACI Code (318M-14) and determine the maximum factored load P_u that can be supported by this beam. In your checking and computation assume that $f_c' = 25\ MPa$, $f_y = 400\ MPa$ and that beam selfweight can be neglected.

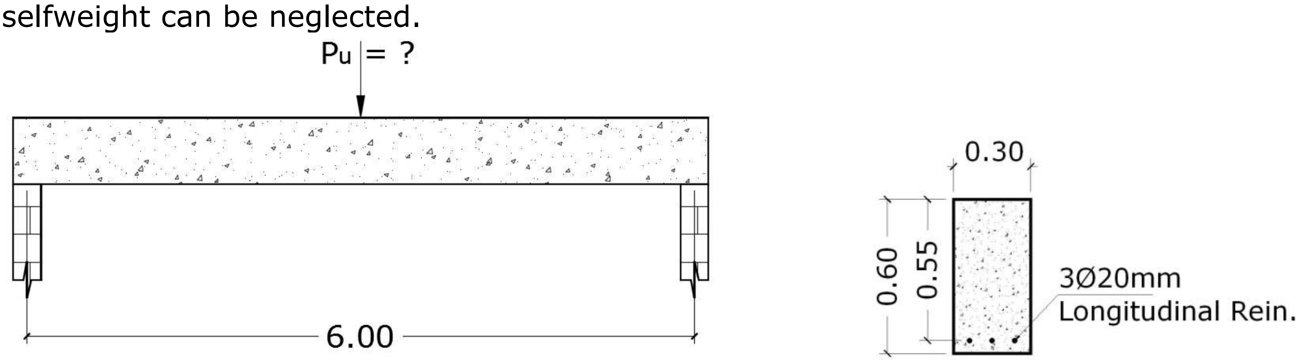


Figure 4.3-1: Simply supported beam for Example 4.3-1.

Solution

- Check if the provided steel reinforcement agrees with ACI limits on $A_{s\,max}$ and $A_{s\,min}$:

$$A_{bar} = \frac{\pi \times 20^2}{4} = 314\ mm^2$$
$$A_s = 3 \times 314 = 942\ mm^2$$
$$\rho = \frac{942}{300 \times 550} = 5.71 \times 10^{-3}$$
$$\rho_{max} = 0.85\beta_1\ \frac{f_c'}{f_y}\ \frac{\epsilon_u}{\epsilon_u + 0.004}$$
$$\because\ f_c' < 28\ MPa \quad \therefore\ \beta_1 = 0.85$$
$$\rho_{max} = 0.85 \times 0.85 \times \frac{25}{400}\ \frac{0.003}{0.003 + 0.004} = 19.4 \times 10^{-3}$$
$$\therefore\ \rho < \rho_{max}\quad Ok. \blacksquare$$
$$A_{s\,minimum} = \frac{0.25\sqrt{f_c'}}{f_y} b_w d \geq \frac{1.4}{f_y} b_w d$$
$$\because\ f_c' < 31\ MPa$$
$$\therefore A_{s\,minimum} = \frac{1.4}{f_y} b_w d = \frac{1.4}{400} \times 300 \times 550 = 525\ mm^2 < A_s\ \ Ok. \blacksquare$$

- Compute the nominal strength M_n of the section:

$$M_n = \rho f_y b d^2 \left(1 - 0.59\ \frac{\rho f_y}{f_c'}\right)$$
$$M_n = 5.71 \times 10^{-3} \times 400 \times 300 \times 550^2 \left(1 - 0.59\ \frac{5.71 \times 10^{-3} \times 400}{25}\right) = 196\ kN.m$$

- Compute strength reduction factor $\emptyset$:
  Compute steel stain based on the following relations:

$$a = \frac{A_s f_y}{0.85 f_c' b} \Rightarrow a = \frac{942\ mm^2 \times 400\ MPa}{0.85 \times 25\ MPa\ \times 300mm} = 59.1\ mm$$
$$c = \frac{a}{\beta_1} \Rightarrow c = \frac{59.1\ mm}{0.85} = 69.5\ mm$$
$$\epsilon_t = \frac{d - c}{c}\epsilon_u \Rightarrow \epsilon_t = \frac{550 - 69.5}{69.5} \times 0.003 = 20.7 \times 10^{-3}$$

As $\epsilon_t > 0.005$, then $\emptyset = 0.9$.

- Compute section design strength $\emptyset M_n$:

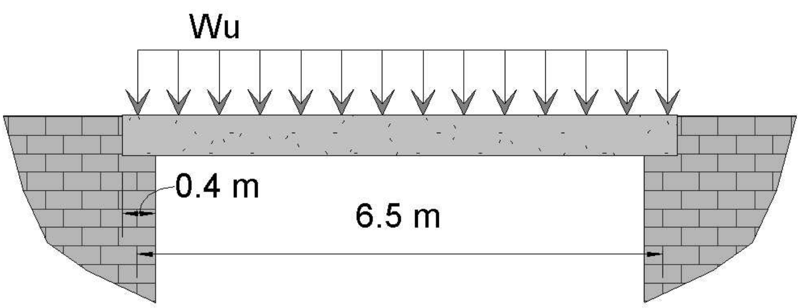
$$\emptyset M_n = \emptyset \times M_n \Rightarrow \emptyset M_n = 0.9 \times 196\ \ kN.m = 176\ kN.m \blacksquare$$

- Compute Maximum Factored Load P_u:
  As the selfweight can be neglected as stated in the example statement, then P_u can be computed based on the following relation:

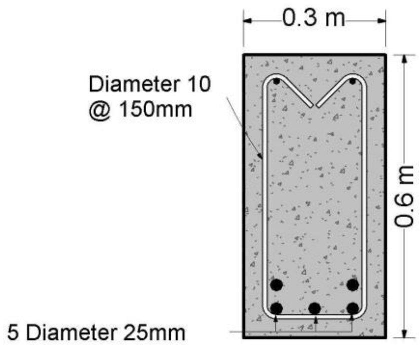
$$\because\ M_u = \frac{P_u L}{4} = \emptyset M_n = 176\ kN.m \Rightarrow P_u = \frac{4 \times 176\ kN.m}{6.0\ m} = 117\ kN\ \ \blacksquare$$

Example 4.3-2

Check flexure adequacy of a simply supported beam shown in Figure 4.3-2 below when subjected to a factored load of $W_u = 70\frac{kN}{m}$ (Including beam selfweight). In your solution assume that $f_c' = 28\ MPa$ and $f_y = 420\ MPa$.



Elevation view.



A mid-span section

Figure 4.3-2: Simply supported beam for Example 4.2-2.

Solution

- Check the proposed beam for general limits of the ACI code:

$d = 600 - 40 - 10 - 25 - \frac{25}{2} = 512\ mm$

$A_s = \frac{\pi \times 25^2}{4} \times 5 = 2454\ mm^2$

$\rho_{Provided} = \frac{2454}{512 \times 300} = 16.0 \times 10^{-3}$

$\rho_{Maximum} = 0.85^2 \times \frac{28}{420} \times \frac{0.003}{0.007} = 20.6 \times 10^{-3} > \rho_{Provided} \therefore Ok.$

$A_{s\ Minimum} = \frac{1.4}{420} \times 300 \times 512 = 512\ mm^2 < A_{s\ Provided} \therefore Ok.$

- Compute its nominal strength, M_n:

Instead of using the relation of

$\mathrm{M_n = \rho f_y bd^2 \left(1 - 0.59\ \frac{\rho f_y}{f_c'}\right)}$

the nominal flexural strength, M_n, can be determined based on simple statics with referring to forces diagram in Figure 4.3-3:

$\Sigma F_x = 0$

$C = T$

$0.85 f_c' ba = A_s f_y$

Solve for a

$a = \frac{(A_s f_y)}{0.85 f_c' b} = \frac{420 \times 2454}{0.85 \times 28 \times 300} = 144\ mm$

$M_n = \Sigma M_{about\ C} = T \times Arm = (A_s f_y) \times \left(d - \frac{a}{2}\right)$

$= (2454 \times 420) \times \left(512 - \frac{144}{2}\right)$

$= 453\ kN.m$

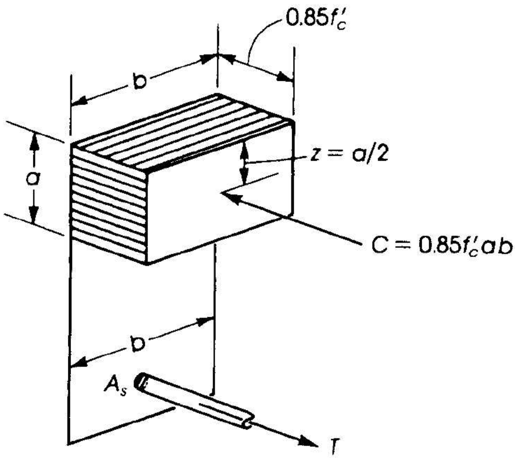


Figure 4.3-3: Forces diagram for a rectangular beam with tension reinforcement only.

This second approach is important as:

- It can be applied to sections other than rectangular sections.
- It focuses on basic principles of the applied mechanics and can be used without the need to remember of a ready relation.

- The strength reduction factor, ϕ, can be determined as follows:

$c = \frac{144}{0.85} = 169\ mm$

$\epsilon_t = \frac{d - c}{c} \epsilon_u \Rightarrow \epsilon_t = \frac{512 - 169}{169} \times 0.003 = 0.00609 > 0.005$

Then:

$\phi = 0.9$

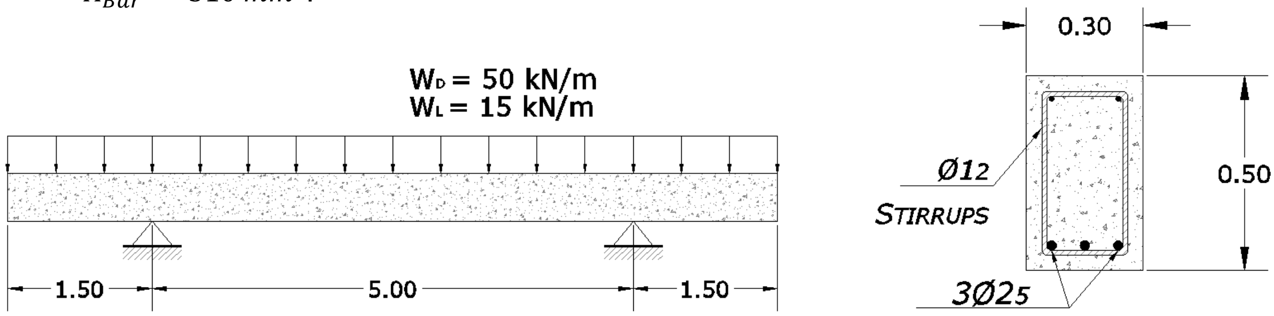
$\phi M_n = 0.9 \times 453 = 408\ kN.m$

$M_u = \frac{W_u l^2}{8} = \frac{70 \times 6.5^2}{8} = 370\ kN.m < \phi M_n \therefore Ok.$

Example 4.3-3

Check the adequacy of the proposed section when used at mid span of the beam shown in Figure 4.3-4. Assume that

- $f_c' = 21 MPa$ and $f_y = 420$MPa.
- Selfweight should be included.
- $A_{Bar} = 510\ mm^2$.



Elevation view. **Cross section**

Figure 4.3-4: Overhang beam for Example 4.2-3.

Solution

- Check if the provided reinforcement is accepted according to ACI requirements:

$A_{bar} = 510\ mm^2$

$A_s = 3 \times 510 = 1530\ mm^2$

$$d = 500 - 40 - 12 - \frac{25}{2} = 435\ mm$$

$$\rho_{Provided} = \frac{1530\ mm^2}{435 \times 300\ mm^2} = 11.7 \times 10^{-3}$$

$$\rho_{Maximum} = 0.85^2 \times \frac{21}{420} \times \frac{0.003}{0.007} = 15.5 \times 10^{-3}$$

$\rho_{Provided} < \rho_{Maximum}\ \therefore Ok.$

$\because f_c' < 31\ MPa$

$$A_{s\ minimum} = \frac{1.4}{420} \times 300 \times 435 = 435 mm^2$$

$A_s > A_{s\ minimum}\ Ok.$

- Compute Nominal Flexure Strength:

$$M_n = 11.7 \times 10^{-3} \times 420 \times 300 \times 435^2 \times (1 - 0.59 \times \frac{11.7 \times 10^{-3} \times 420}{21})$$

$M_n = 240\ kN.m$

- Compute flexure strength reduction factor:

$a = 120\ mm$

$c = 141\ mm$

$$\epsilon_t = \frac{435 - 141}{141} \times 0.003 = 6.26 \times 10^{-3} > 0.005$$

$\therefore \phi = 0.9$

- Design Moment:

$\phi M_n = 0.9 \times 240 = 216\ kN.m$

- Check section adequacy when used at mid-span:

$$W_{self} = 0.3 \times 0.5 \times 24 = 3.6 \frac{kN}{m}$$

$$W_D = 53.6 \frac{kN}{m}$$

$W_u = maximum(1.4 \times 53.6\ or\ 1.2 \times 53.6 + 1.6 \times 15)$

$$W_u = maximum(75.0\ or\ 88.3) = 88.3 \frac{kN}{m}$$

$$M_{u\ support} = \frac{88.3 \times 1.5^2}{2} = 99.8\ kN.m$$

$$M_{u\ @\ mid-span} = \frac{88.3 \times 5^2}{8} - 99.3 = 177\ kN.m < \phi M_n\ Ok$$

4.3.3 Homework Problems

In addition to practice on concepts, *Problem* 4.3-1, *Problem 4.3-2*, and *Problem 4.3-3* aim to show how M_n can be affected by changing in material properties (f_y and f_c').

Problem 4.3-1

A rectangular beam has a width 250 mm, and an effective depth 505 mm. It is reinforced with 3 Φ 25 (assume A_{bar} = 510 mm²). If f_y = 420 MPa and f_c' = 20 MPa. Check the beam adequacy and compute its design flexural strength according to the ACI Code.

Answers

- Check if the provided steel reinforcement agrees with ACI limits on A_{smax} and A_{smin}:
  $A_s = 1\,530\ \text{mm}^2$
  $\rho = 12.1 \times 10^{-3}$
  $\rho_{max} = 14.7 \times 10^{-3}$
  $\therefore\ \rho < \rho_{max}$ Ok. ■
  $A_{s\ minimum} = 421\ \text{mm}^2 < A_s$ Ok. ■
- Compute section nominal strength M_n :
  $M_n = 275\ \text{kN.m}$
- Compute strength reduction factor $\emptyset$:
  a. Compute steel stain:
     $a = 151\ \text{mm}$
     $c = 178\ \text{mm}$
     $\epsilon_t = 0.00551$
  b. $\epsilon_t > 0.005$, then $\emptyset = 0.9$
- Compute section design strength $\emptyset M_n$:
  $\emptyset M_n = 247\ \text{kN.m}$■

Problem 4.3-2

Same as Problem 4.3-1 except that f_c'=40 MPa. Compare the flexure strength for this problem with that of Problem 4.3-1.

Answers

- Check if the provided steel reinforcement agrees with ACI limitss on A_{smax} and A_{smin}:
  $A_s = 1\,530\ \text{mm}^2$
  $\rho = 12.1 \times 10^{-3}$
  $\beta_1 = 0.76 \geq 0.65$ Ok.
  $\rho_{max} = 26.4 \times 10^{-3}$
  $\therefore\ \rho < \rho_{max}$ Ok. ■
  $A_{s\ minimum} = 475\ \text{mm}^2 < A_s$ Ok. ■
- Compute section nominal strength Mn:
  $M_n = 300\ \text{kN.m}$
- Compute strength reduction factor $\emptyset$:
  - Compute steel stain:
    $a = 75.6\ \text{mm}$
    $c = 99.5\ \text{mm}$
    $\epsilon_t = 0.0122$
  - $\epsilon_t > 0.005$, then $\emptyset = 0.9$
- Compute section design strength $\emptyset M_n$:
  $\emptyset M_n = 270\ \text{kN.m}$■
- Comparison with the design strength of Problem 4.3-1:
  Increasing percentage in design strength due to increasing f_c' from 20 MPa to 40 MPa can be computed as follows:
  $$\text{Increasing Percent} = \frac{270 - 247}{247} \times 100\% = 9.31\%$$
  Note that doubling the concrete strength increased $\emptyset M_n$ by only 9.31%■.

Problem 4.3-3

Same as Problem 4.3-1 except that f_y = 300 MPa. Compare the flexure strength for this problem with that of Problem 4.3-1.

Answers

- Check if the provided steel reinforcement agrees with ACI limits on A_{smax} and A_{smin}:
  $A_s = 1\,530\ mm^2$
  $\rho = 12.1 \times 10^{-3}$
  $\rho_{max} = 20.6 \times 10^{-3}$
  $\therefore\ \rho < \rho_{max}$ Ok. ■
  $A_{s\ minimum} = 589\ mm^2 < A_s$ Ok. ■
- Compute section nominal strength M_n :
  $M_n = 207\ kN.m$
- Compute strength reduction factor $\emptyset$:
  - Compute steel stain:
    $a = 108\ mm$
    $c = 127\ mm$
    $\epsilon_t = 0.00893$
  - $\epsilon_t > 0.005$, then $\emptyset = 0.9$
- Compute section design strength $\emptyset M_n$:
  $\emptyset M_n = 186\ kN.m$■
- Comparison with the design strength of Problem 4.3-1:
  $$\text{Increasing Percent} = \frac{|186 - 247|}{247} \times 100\% = 24.7\%$$
  Note that reducing f_y by ($\frac{420-300}{420} = 28.6\%$) reduces $\emptyset M_n$ by 24.7%■.

Based on results of Problem 4.3-1, Problem 4.3-2*, and* Problem 4.3-3 *one concludes that the effect of changing in steel yield stress (f_y) is more significant than the effect of changing in concrete compressive strength (f_c').*

Problem 4.3-4

A rectangular beam has a width of 305 mm, and an effective depth of 444 mm. It is reinforced with $4\emptyset 29mm$ (assume $A_{bar} = 645mm^2$). If $f_y = 414MPa$ and $f_c' = 27.5\ MPa$. Check the beam adequacy and compute its design flexural strength according to the ACI Code.

Aim of the Problem:

This problem aims to show solution procedures for a section in the transition zone.

Answers

- Check if the provided steel reinforcement is in agreement with ACI requirements on A_{smax} and A_{smin}:
  $A_s = 2\,580\ mm^2$
  $\rho = 19.1 \times 10^{-3}$
  $\rho_{max} = 20.6 \times 10^{-3}$
  $\therefore\ \rho < \rho_{max}$ Ok. ■
  $A_{s\ minimum} = 458\ mm^2 < A_s$ Ok. ■
- Compute section nominal strength M_n :
  $M_n = 395\ kN.m$
- Compute strength reduction factor $\emptyset$:
  - Compute steel stain:
    $a = 150\ mm$
    $c = 176\ mm$
    $\epsilon_t = 0.00457$
  - $\epsilon_t < 0.005$, then $\emptyset = 0.864$
- Compute section design strength $\emptyset M_n$:
  $\emptyset M_n = \emptyset \times M_n$
  $\emptyset M_n = 0.864 \times 395\ kN.m = 341\ kN.m$■

Problem 4.3-5

Determine if the beam shown in Figure 4.3-5 is adequate as governed by ACI Code (ACI 318M-14). If $f_y = 414MPa$ and $f_c' = 27.5\ MPa$. Assume that $A_{bar} = 510mm^2$.

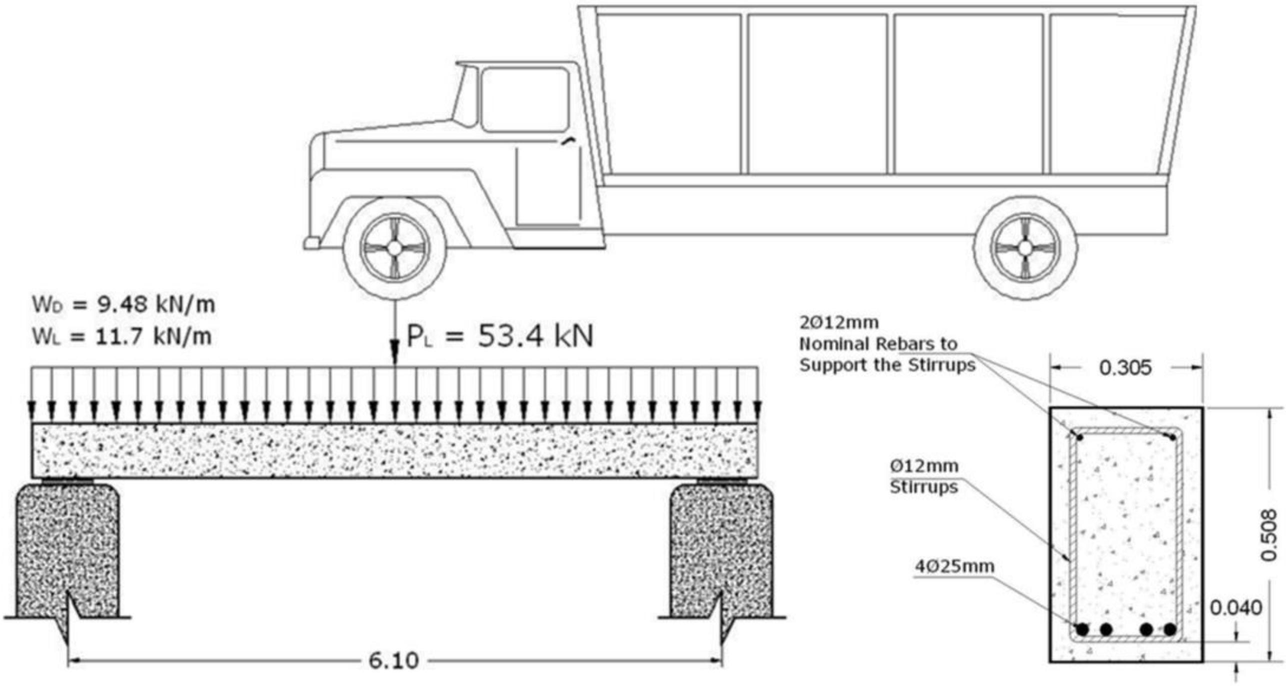


Figure 4.3-5: A simple bridge for the Problem 4.3-5.

Answers

- Check if the provided steel reinforcement agrees with ACI limits on $A_{s\,max}$ and $A_{s\,min}$:
  $A_s = 2\,040\ mm^2$
  $d = 508 - 40_{Cover} - 12_{Stirrups} - \frac{25}{2}_{\text{Half the Bar Diameter}} = 444\ mm$
  $\rho = 15.1 \times 10^{-3}$
  $\rho_{max} = 20.6 \times 10^{-3}\ \Rightarrow \therefore\ \rho < \rho_{max}$ Ok. ■
  $A_{s\,minimum} = 458\ mm^2 < A_s$ Ok. ■
- Compute the nominal strength M_n:
  $M_n = 325\ kN.m$
- Compute the strength reduction factor $\emptyset$:
  - Compute steel stain:
    $a = 118\ mm$
    $c = 139\ mm$
    $\epsilon_t = 0.00658$
  - $\epsilon_t > 0.005$, then $\emptyset = 0.9$
- Compute the design strength $\emptyset M_n$:
  $\emptyset M_n = 293\ kN.m$ ■
- Compute the Factored Moment:
  - Moment due to the Dead Loads:
    $W_{Selfweight} = 3.72\frac{kN}{m} \Rightarrow W_{Dead} = 13.2\frac{kN}{m}$
    $M_{Dead} = 61.4\ kN.m$
  - Moment due to the Live Load:
    $M_{Live} = 136\ kN.m$
  - Factored Moment M_u:
    $M_u = \text{Maximum of}\ (1.4M_D\ \text{or}\ 1.2M_D + 1.6M_L)$
    $M_u = \text{Maximum of}\ [86\ \text{or}\ 291] = 291\ kN.m$ ■
  - Check Section Adequacy:
    $\because\ \emptyset M_n = 293\ kN.m > M_u = 291\ kN.m\ \ \therefore$ Ok. ■

Problem 4.3-6

Check adequacy of the beam shown in Figure 4.3-6 for bending according to the requirements of ACI 318M-14. Assume that $f_c' = 28$ MPa, $f_y = 420$MPa and $A_{Bar} = 510mm^2$.

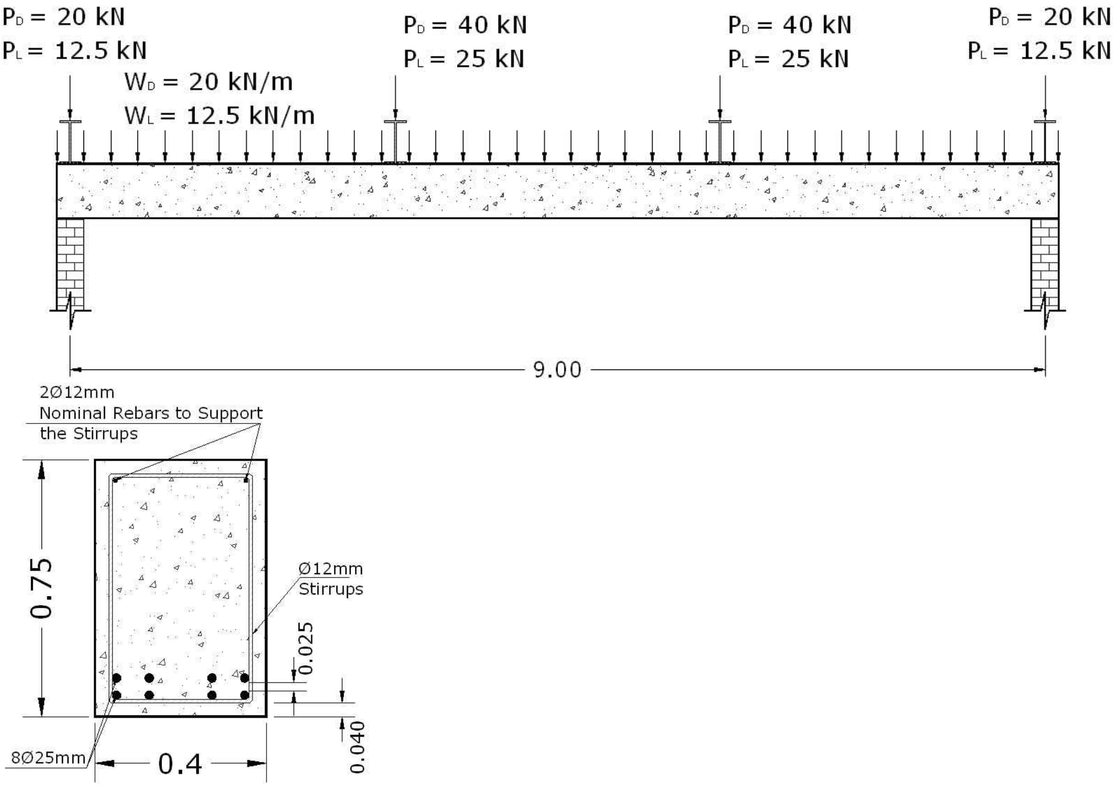


Figure 4.3-6: A simple beam for the Problem 4.3-5.

Answers

- Check if the provided steel reinforcement is in agreement with ACI requirements on $A_{s\,max}$ and $A_{s\,min}$:
  $A_s = 4\,080\ mm^2$
  $d = 750 - 40_{Cover} - 12_{Stirrups} - 25_{\text{ the Bar Diameter}} - \left(\frac{25}{2}\right)_{\text{Half the Spacing between Layers}} = 660\ mm$
  $\rho = 15.5 \times 10^{-3}$
  $\rho_{max} = 20.6 \times 10^{-3} \Rightarrow \therefore\ \rho < \rho_{max}$ Ok. ■
  $A_{s\,minimum} = 880\ mm^2 < A_s$ Ok. ■
- Compute the nominal strength M_n:
  $M_n = 979\ kN.m$
- Compute the strength reduction factor $\emptyset$:
  - Compute steel stain:
    $a = 180\ mm$
    $c = 212\ mm$
    $\epsilon_t = 0.00634$
  - $\epsilon_t > 0.005$, then $\emptyset = 0.9$.
- Compute the design strength $\emptyset M_n$:
  $\emptyset M_n = 881\ kN.m$ ■
- Compute the Factored Moment:
  - Moment due to the dead loads:
    $W_{Selfweight} = 7.2\frac{kN}{m} \Rightarrow W_{Dead} = 27.2\frac{kN}{m} \Rightarrow M_{Dead} = 395\ kN.m$
  - Moment due to the live load:
    $M_{Live} = 202\ kN.m$
  - Factored Moment M_u:
    $M_u = \text{Maximum of } (1.4M_D \text{ or } 1.2M_D + 1.6M_L)$
    $M_u = \text{Maximum of } [553 \text{ or } 797] = 797\ kN.m$ ■
  - Check Section Adequacy:
    $\because\ \emptyset M_n = 881\ kN.m > M_u = 797\ kN.m \quad \therefore$ Ok. ■

4.4 Practical Flexure Design of a Rectangular Beam with Tension Reinforcement Only and Pre-specified Dimensions (b and h)

4.4.1 Essence of Problem

- In the design problem, usually the beam span, beam dimensions (b, and h), dead, live, and other loads are defined based on functional and/or architectural requirements.
- Materials strength (f_c' and f_y) are generally selected based on the available materials in the local market.
- Then, the main unknown in the design process is the reinforcement detail that can be summarized as follows:
  - Number and diameters of rebars.
  - Number of layers that required for these rebars.
  - Required concrete cover to protect the reinforcement against probable corrosion.
  - Points where bars are no longer needed for moments, i.e., points for bending or stopping of reinforcement, out the scope of this chapter and will be discussed thoroughly in Chapter 5.

4.4.2 Design Procedure

Based on above known and unknown quantities, design procedure can be summarized as follows:

1. Computed required factored applied moment (M_u) based on given loads and spans. Beam selfweight can be computed based on given dimensions (b, and h).
2. Computed the required nominal or theoretical flexure strength ($\mathrm{M_n}$) based on the following relation:

   $$\mathrm{M_n} = \frac{\mathrm{M_u}}{\emptyset}$$

   Strength reduction factored can be assumed 0.9, and checked later.
3. Compute the effective beam depth "d":

   Generally, in engineering practice the reinforcements are either put in one or two layers. Depend on number of layers, "d" can be computed based on one of the following relations[2]:

   $$\mathrm{d_{for\ One\ Layer}} = \mathrm{h - Cover - Stirrups} - \frac{\mathrm{Bar\ Diameter}}{2}$$

   $$d_{for\ Two\ Layer} = h - Cover - Stirrups - Bar\ Diameter - \frac{Spacing\ between\ Layers}{2}$$

   Based on above two relations, one can conclude that the designer must assume preliminary values for following items to be able to compute the effective depth "d":

   a. *Number of Layers*

   Heavy loads required a large reinforcement area that cannot be put in one layer and vice versa.

   Diagnosis between heavy loads and light loads is generally depends on designer experience. For examination purposes, number of layers may be mentioned in question statement.

   b. *Concrete Cover*

   To provide the steel with adequate concrete protection against corrosion, the designer must maintain a certain minimum thickness of concrete cover outside of the outermost steel:

   The thickness required will vary, depending upon:

   i. The type of member.

---

[2] It is useful to note that, the equation of effective depth for a beam with two reinforcement layers is based on the assumption that centroid of two layers lies at mid distance between two layers. This assumption is correct for two identical layers. For other conditions, it gives conservative results that accepted in the engineering practice.

ii. Conditions of exposure.
iii. Bar diameter.

According to article 20.6.1.3.1 of the (ACI318M, 2014), concrete cover can be determined based on Table 4.4-1 below.

Table 4.4-1: Specified concrete cover for cast-in-place nonprestressed concrete members, Table 20.6.1.3.1 of the (ACI318M, 2014).

Concrete exposure	Member	Reinforcement	Specified cover, mm
Cast against and permanently in contact with ground	All	All	75
Exposed to weather or in contact with ground	All	No. 19 through No. 57 bars	50
		No. 16 bar, MW200 or MD200 wire, and smaller	40
Not exposed to weather or in contact with ground	Slabs, joists, and walls	No. 43 and No. 57 bars	40
		No. 36 bar and smaller	20
	Beams, columns, pedestals, and tension ties	Primary reinforcement, stirrups, ties, spirals, and hoops	40

As a general case, (ACI318M, 2014) requirements for beams (that not exposed to weather) can be summarized in Figure 4.4-1.

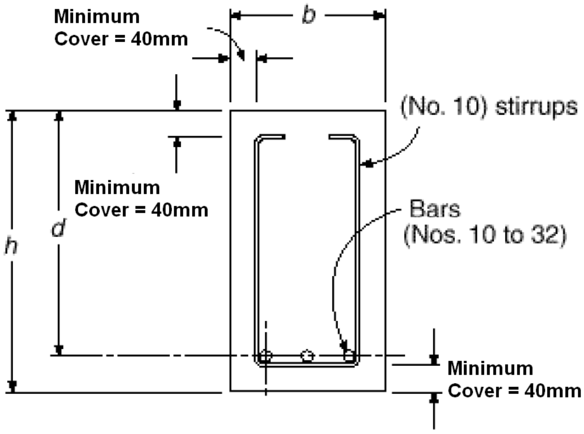


Figure 4.4-1: Cover requirements for beams not exposed to weather.

c. ***Bar Diameter***

As discussed in Chapter 2, data for metric rebars according to ASTM are summarized in Table 4.4-2 below.

No. 16 to No. 25 is usually used in for beam reinforcement. No. 13 rebars may be used in minor works like lintel beam reinforcement.

d. ***Stirrups Diameter***

Stirrups are the hoop reinforcement that used for shear reinforcement.

No. 10 and No. 13 are usually used as stirrups. No. 16 may be used in some large beams with heavy loads.

e. ***Spacing between Layers***

According to article 25.2.2 of (ACI318M, 2014), for parallel nonprestressed reinforcement placed in two or more horizontal layers,

i. reinforcement in the upper layers shall be placed directly above reinforcement in the bottom layer
ii. a clear spacing between layers of at least 25 mm.

Table 4.4-2ASTM standard metric reinforcing bars

Bar size, no.*	Nominal diameter, mm	Nominal area, mm^2	Nominal mass, kg/m
10	9.5	71	0.560
13	12.7	129	0.994
16	15.9	199	1.552
19	19.1	284	2.235
22	22.2	387	3.042
25	25.4	510	3.973
29	28.7	645	5.060
32	32.3	819	6.404
36	35.8	1006	7.907
43	43.0	1452	11.38
57	57.3	2581	20.24

*Bar numbers approximate the number of millimeters of the nominal diameter of the bar.

4. Compute the required steel ratio $\rho_{Required}$:
   The basic relation between variables for rectangular beam with tension reinforcement:
   $$M_n = \rho f_y b d^2 \left(1 - 0.59\ \frac{\rho f_y}{f_c'}\right)$$
   can be solved to compute $\rho_{Required}$ from known f_y, f_c', b, d, and M_n and as follows:
   $$\rho_{Required} = \frac{1 - \sqrt{1 - 2.36 \frac{M_n}{f_c' b d^2}}}{1.18 \times \frac{f_y}{f_c'}} \quad \blacksquare$$
   Why only smaller one of two roots is adopted in the solution of above relation will be discussed thoroughly in Example 4.4-1 below.
   If the quantities under the square root $(1 - 2.36 \frac{M_n}{f_c' b d^2})$ has a negative value, this gives an indication that the failure is a compression failure and that the section is rejected according to ACI Code requirements. Then the designer must increase one or both of beam dimensions (b and h) and resolve the problem from Step 3.
5. Check if the beam failure is secondary compression failure or compression failure:
   If:
   $$\rho_{Required} > \rho_{max} = 0.85\beta_1\ \frac{f_c'}{f_y}\ \frac{\epsilon_u}{\epsilon_u + 0.004}$$
   Then the designer must increase one or both of beam dimensions (b and h) and resolve the problem from Step 3.
6. Compute the required steel area:
   $$A_{S\ Required} = \rho_{Required} \times bd$$
7. Compute required rebars number:
   $$\text{No. of Rebars} = \frac{A_{S\ Required}}{A_{Bar}}$$
   Round the required rebars number to nearest larger integer number.
8. Check if the available width "b" is adequate to put the rebars in a single layer:
   $$b_{required} = 2 \times \text{Side Cover} + 2 \times \text{Stirrups Diamter} + \text{No. of Rebars} \times \text{Bar Diameter} + (\text{No. of Rebars} - 1) \times \text{Spacing between Rebars}$$
   If
   $$b_{required} > b_{available}$$
   Then reinforcement cannot be put in a single layer. If your calculations have be based on assumption of single layer, then you must retain to Step 3 and recalculate "d" based on two reinforcement layers.
   According to article 25.2.1 of (ACI318M, 2014), for parallel nonprestressed reinforcement in a horizontal layer, clear spacing shall be at least the greatest of:

$S_{Clear\ Minimum} = Maximum\left[25mm, d_b, \frac{4}{3}d_{agg}\right]$

As the maximum size of aggregate, d_{agg}, is usually selected to satisfy above relation, then it reduces into:

$S_{Clear\ Minimum} = Maximum\ [25mm, d_b]$

9. Checking cracks width or checking for $S_{Maximum}$:
   a. As was discussed in Chapter 1 (Design Criteria) and in Second Stage of beam behavior (Elastic Cracked Section), current design philosophy doesn't aim to design a concrete beam without cracks but aims to limit these cracks to be fine (called hairline cracks), invisible to a casual observer, permitting little if any corrosion of the reinforcement.
   b. Methods of cracks control:
      i. Previously, ACI 318 requirements were based on computing of actual cracks width (w in Figure below) and compared it with a maximum limits.

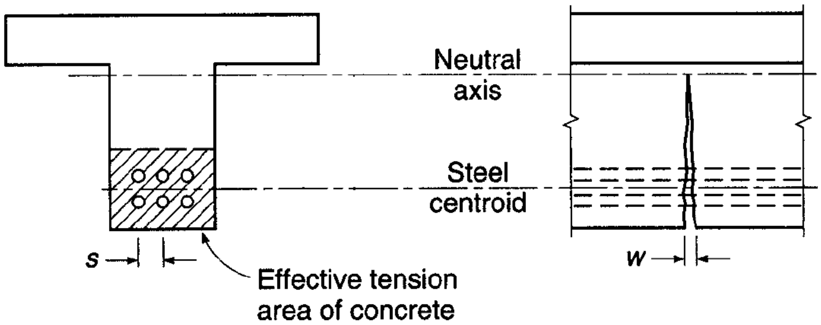


Figure 4.4-2: Crack width computations terminology of previous code editions.

      ii. Currently, ACI Code adopted a simpler approach that can be used for structures that ***not subjected to very aggressive exposure or designed to be watertight***.
      iii. This simplified approach based on following experimental and analytical fact:
      Generally, to control cracking, it is better to use a larger number of smaller-diameter bars to provide the required A_s than to use the minimum number of larger bars.
      iv. Then instead of working with crack width "w", we can work based on <u>center to center</u> spacing between bars "s in Figure above" which gives an indication on bars size as if we use a smaller number of bars with larger diameter instead of larger number of bars with smaller diameter spacing "s" will be larger and vice versa.
      v. According to ACI Code (24.3.2) maximum spacing (center to center) between bars for crack control purposes can be computed as follows:
      $s = 380\frac{280}{f_y} - 2.5c_c \leq 300\frac{280}{f_s}$
      where
         1. f_s is the stress in reinforcement closest to the tension face at service load shall be computed based on the unfactored moment. It shall be permitted to take fs as $2/3\,f_y$.
         2. cc is concrete clear cover.
      vi. For traditional reinforcement of Grade 60 and traditional cover, ACI Code commentary (R24.3.2) shows that s_{max} can be taken as 250 mm.

10. Check with $A_{s\ minimum}$ requirements:
If

$A_{S\ Provided} < A_{s\ minimum} = \frac{0.25\sqrt{f_c'}}{f_y}b_w d \geq \frac{1.4}{f_y}b_w d$

Then used:

$A_{S\ Provided} = A_{s\ minimum}$

And recalculate rebars number based on this area.

11. Check the assumption of $\emptyset = 0.9$:
    a. Compute steel stain based on the following relations:
    $$a = \frac{A_s f_y}{0.85 f_c' b}$$
    $$c = \frac{a}{\beta_1}$$
    $$\epsilon_t = \frac{d - c}{c}\epsilon_u$$
    b. If $\epsilon_t \geq 0.005$, then $\emptyset = 0.9$ Ok.
    c. If $\epsilon_t < 0.005$, then compute more accurate $\emptyset$:
    $$\emptyset = 0.483 + 83.3\epsilon_t$$
    and retain to Step 2.
12. Draw final detailed beam section.

4.4.3 Examples

Example 4.4-1

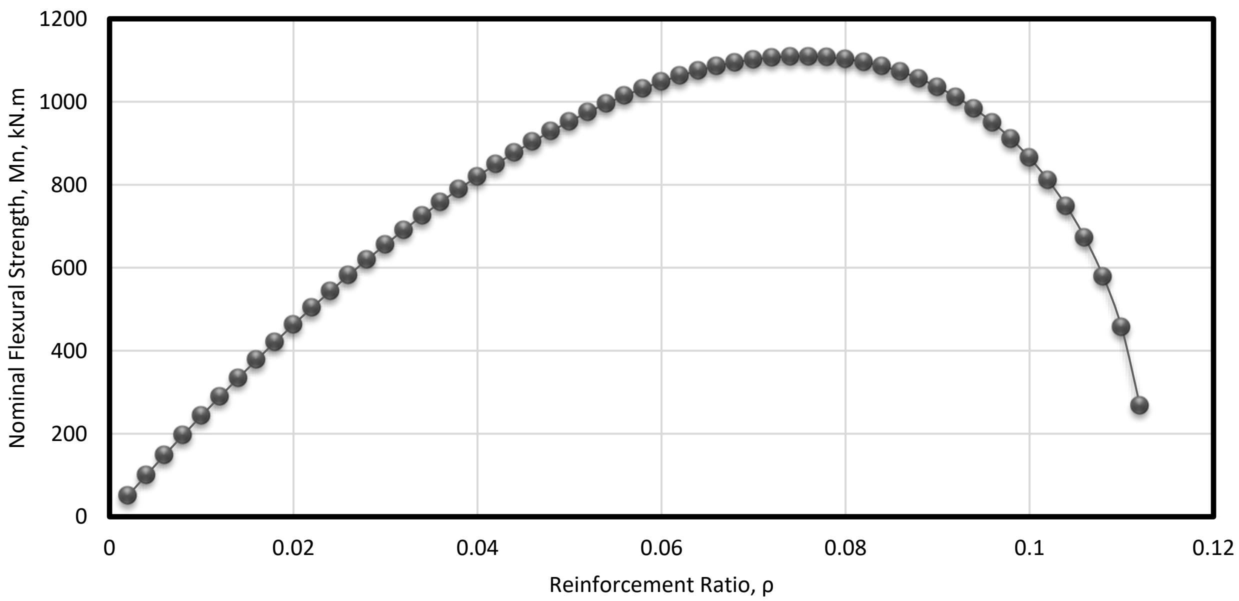
For a beam with an effective depth of 450mm and a width of 300mm, draw a relation between provided reinforcement ratio, $\rho_{Provided}$, and corresponding nominal flexural strength, M_n, and then discuss why only smaller root is adopted in the solution of relation.

$$\rho_{Required} = \frac{1 - \sqrt{1 - 2.36\frac{M_n}{f_c' b d^2}}}{1.18 \times \frac{f_y}{f_c'}}$$

In your solution, assume $f_c' = 28\ MPa$ and $f_y = 420\ MPa$.

Solution

- For each value of ρ, compute corresponding value of M_n based on following relation:
  $$M_n = \rho f_y b d^2\left(1 - 0.59\ \frac{\rho f_y}{f_c'}\right)$$
  and draw resulting values as shown in Figure below.



- For a specific flexural strength, the larger root of above relation either gives reinforcement ratio greater than $\rho_{maximum}$, a rejected design, or gives a larger value compared to first root, uneconomical design. Therefore, only smaller root should be considered in commuting required reinforcement ratio for a specific nominal strength.

Example 4.4-2

Design a simply supported rectangular reinforced concrete beam shown Figure 4.4-3 below. It is known that this beam is not exposed to weather and not in contact with ground.

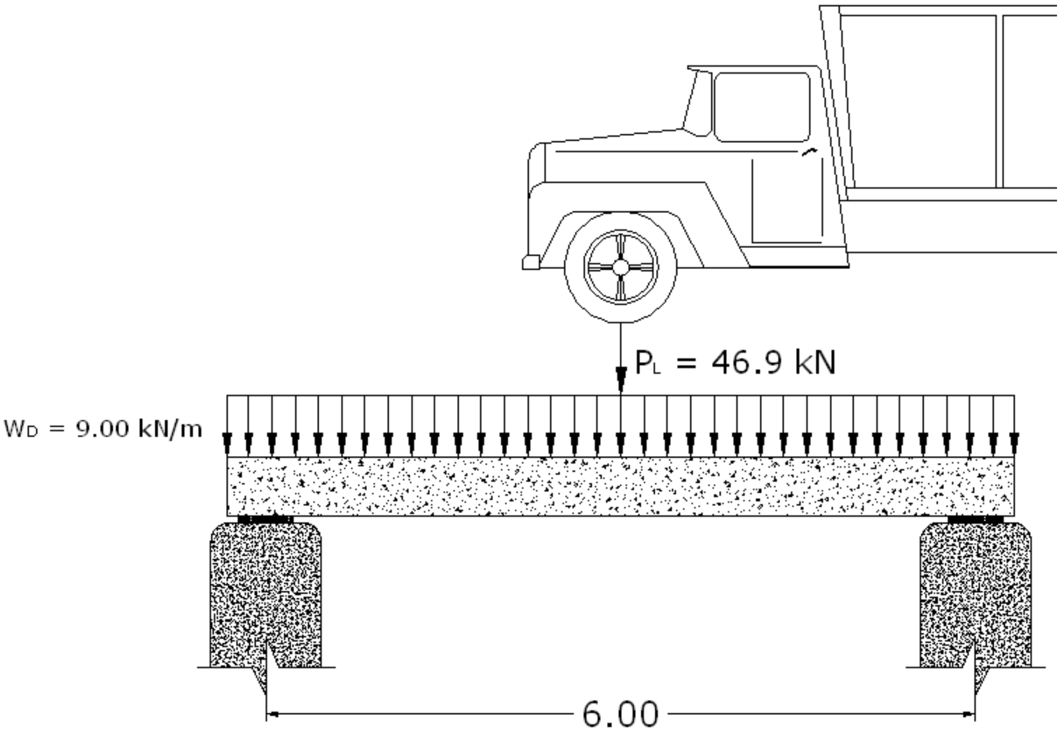


Figure 4.4-3: Simply supported bridge for Example 4.4-2.

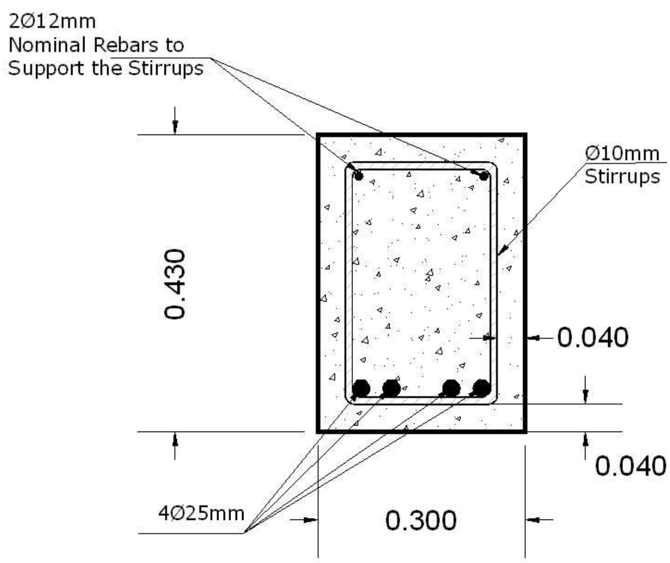
Assume that the designer intends to use:

- Concrete of $f_c{}' = 30\,MPa$.
- Steel of $f_y = 400\text{MPa}$.
- A width of 300mm and a height of 430mm (these dimensions have been determined based on deflection considerations).
- Rebar of No. 25 for longitudinal reinforcement.
- Rebar of No. 10 for stirrups.
- Single layer of reinforcement.

Solution

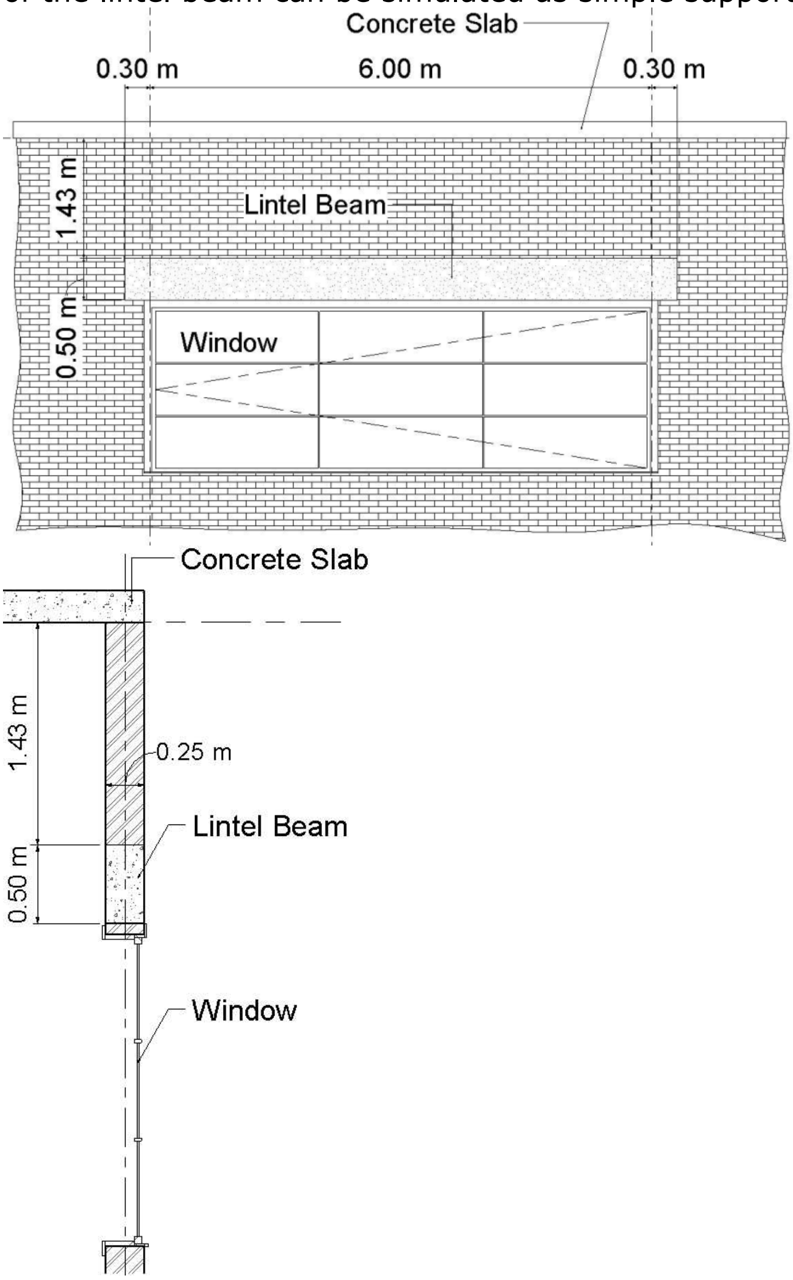
1. Computed required factored applied moment (M_u):
   a. Moment due to Dead Loads:
   $$W_{\text{Selfweight}} = 0.43\text{m} \times 0.3\text{m} \times 24\frac{\text{kN}}{\text{m}^3} = 3.1\frac{\text{kN}}{\text{m}}$$
   $$W_{\text{Dead}} = 9.00\frac{\text{kN}}{\text{m}} + 3.10\frac{\text{kN}}{\text{m}} = 12.1\ \frac{\text{kN}}{\text{m}}$$
   $$M_{\text{Dead}} = \frac{12.1\frac{\text{kN}}{\text{m}} \times 6.0^2\text{m}^2}{8} = 54.5\ \text{kN.m}$$
   b. Moment due to Live Load:
   $$M_{\text{Live}} = \frac{46.9\text{kN} \times 6.0\text{m}}{4} = 70.4\ \text{kN.m}$$
   c. Factored Moment M_u:
   $$M_u = \text{Maximum of } (1.4M_D \ \text{ or } 1.2M_D + 1.6M_L)$$
   $$M_u = \text{Maximum of } [1.4 \times 54.5 \ \text{ or } (1.2 \times 54.5 + 1.6 \times 70.4)] =$$
   $$M_u = \text{Maximum of } [76.3 \ \text{ or } 178] = 178\ \text{kN.m}\ ■$$
2. Computed the required nominal or theoretical flexure strength (M_n) based on the following relation:
   $$M_n = \frac{M_u}{\emptyset}$$
   Strength reduction factored can be assumed 0.9, and checked later.
   $$M_n = \frac{178\ \text{kN.m}}{0.9} = 198\ \text{kN.m}$$

3. Compute the effective beam depth "d":
   Assuming that reinforcement can be put in a single reinforcement, then:
   $$d_{\text{for One Layer}} = 430 - 40 - 10 - \frac{25}{2} = 368 \text{ mm}$$
4. Compute the Required Steel Ratio ρ_{Required}:
   $$\rho_{\text{Required}} = \frac{1 - \sqrt{1 - 2.36\dfrac{M_n}{f'_c bd^2}}}{1.18 \times \dfrac{f_y}{f'_c}}$$
   $$\rho_{\text{Required}} = \frac{1 - \sqrt{1 - 2.36\dfrac{198 \times 10^6 \text{N.mm}}{30 \times 300 \times 368^2}}}{1.18 \times \dfrac{400}{30}} = 13.6 \times 10^{-3}$$
5. Check if the beam failure is secondary compression failure or compression failure:
   $$\rho_{max} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004}$$
   $$\beta_1 = 0.85 - \frac{30 - 28}{7} \times 0.05 = 0.836 > 0.65 \; Ok$$
   $$\rho_{max} = 0.85 \times 0.836 \frac{30}{400} \frac{0.003}{0.003 + 0.004} = 22.8 \times 10^{-3} > \rho_{\text{Required}} \text{ Ok.}$$
6. Compute the required steel area:
   $$A_{S \text{ Required}} = \rho_{\text{Required}} \times bd$$
   $$A_{S \text{ Required}} = 13.6 \times 10^{-3} \times 300 \times 368 = 1501 \text{ mm}^2$$
7. Compute required rebars number:
   $$A_{Bar} = \frac{\pi \times 25^2}{4} = 490 \text{ mm}^2$$
   $$\text{No. of Rebars} = \frac{1\,501 \text{ mm}^2}{490 \text{ mm}^2} = 3.06$$
   Try $4\emptyset25$mm.
   $$A_{S \text{ Provided}} = 4 \times 490 \text{ mm}^2 = 1\,960 \text{ mm}^2$$
8. Check if the available width "b" is adequate to put the rebars in a single layer:
   $$b_{required} = 2 \times \text{Side Cover} + 2 \times \text{Stirrups} + \text{No. of Rebars} \times \text{Bar Diameter} + (\text{No. of Rebars} - 1) \times \text{Spacing between Rebars}$$
   $$b_{required} = 2 \times 40\text{mm} + 2 \times 10 + 4 \times 25\text{mm} + 3 \times 25\text{mm} = 275\text{mm} < 300\text{mm Ok.}$$
9. Checking for $S_{maximum}$ for Crack Control:
   $$s = \frac{300 - 40 \times 2 - 10 \times 2 - 25}{3} = 58.3 \; mm < s_{max} \;\; Ok$$
10. Check with $A_{s \text{ minimum}}$ requirements:
   $$A_{s \text{ minimum}} = \frac{1.4}{f_y} b_w d = \frac{1.4}{400} \times 300 \times 368 = 386 \text{ mm}^2 < A_{S \text{ Provided}} = 1960 \text{ mm}^2 \text{ Ok.}$$
11. Check the assumption of $\emptyset = 0.9$:
   a. Compute steel stain based on the following relations:
   $$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{1\,960 \times 400}{0.85 \times 30 \times 300} = 102 \text{ mm} \Rightarrow c = \frac{102 \text{ mm}}{0.836} = 122 \text{ mm}$$
   $$\epsilon_t = \frac{368 \text{ mm} - 122 \text{ mm}}{122 \text{ mm}} \times 0.003 = 6.05 \times 10^{-3}$$
   b. As $\epsilon_t \geq 0.005$, then $\emptyset = 0.9$ Ok.
12. Final Reinforcement Details:



Example 4.4-3

Design lintel beam shown in Figure 4.4-4 below. In your solution, assume that the beam supports in addition to its own weight all brick works that lie directly on it and supports a dead load of 12 kN/m and live load of 8 kN/m transferred from supported slab. Seats of the lintel beam can be simulated as simple supports in your design.



Elevation View.

Section View.

Figure 4.4-4: Lintel beam for Example 4.4-3.

Solution

$$W_{self} = 0.5 \times 0.25 \times 24 = 3.0 \frac{kN}{m}, W_{Brick} = 1.43 \times 0.25 \times 19 = 6.79 \frac{kN}{m}$$

$$W_D = 3.0 + 6.79 + 12 = 21.8 \frac{kN}{m}$$

$$W_L = 8.0 \frac{kN}{m}$$

$$W_u = maximum\ (1.4 \times 21.8\ or\ 1.2 \times 21.8 + 1.6 \times 8.0) = maximum\ (30.5\ or\ 39) = 39 \frac{kN}{m} \blacksquare$$

$$M_u = \frac{W_u l_n^2}{8} = \frac{39 \times 6.3^2}{8} = 193\ kN.m$$

Try $\phi 20$ for longitudinal reinforcement in two layers and $\phi 12$ for stirrups.

$$d = 500 - 40 - 12 - 20 - \frac{25}{2} = 415\ mm$$

Computed the required nominal or theoretical flexure strength (M_n) based on the following relation:

$M_n = \frac{M_u}{\emptyset}$

Strength reduction factored can be assumed 0.9, and checked later.

$M_n = \frac{193}{0.9} = 214 \text{ kN.m}$

Compute the Required Steel Ratio $\rho_{Required}$:

$$\rho_{Required} = \frac{1 - \sqrt{1 - 2.36 \frac{M_n}{f_c' b d^2}}}{1.18 \times \frac{f_y}{f_c'}} = \frac{1 - \sqrt{1 - 2.36 \times \frac{214 \times 10^6}{28 \times 250 \times 415^2}}}{1.18 \times \frac{420}{28}} = 13.4 \times 10^{-3}$$

Check if the beam failure is secondary compression failure or compression failure:

$$\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} = 0.85 \times 0.85 \frac{28}{420} \frac{0.003}{0.003 + 0.004} = 20.6 \times 10^{-3} > \rho_{Required} \text{ Ok.}$$

Compute the required steel area:

$A_{S\,Required} = \rho_{Required} \times bd = 13.4 \times 10^{-3} \times 250 \times 415 = 1390 \text{ mm}^2$

Compute required rebars number:

$A_{Bar} = \frac{\pi \times 20^2}{4} \approx 314 \text{ mm}^2 \Rightarrow \text{No. of Rebars} = \frac{1390}{314} = 4.43$

Try $5\emptyset 20\text{mm}$.

$A_{S\,Provided} = 5 \times 314 = 1570 \text{ mm}^2$

Check if the available width "b" is adequate to put the rebars in a single layer:

$$b_{required} = 2 \times \text{Side Cover} + 2 \times \text{Stirrups} + \text{No. of Rebars} \times \text{Bar Diameter} + (\text{No. of Rebars} - 1) \times \text{Spacing between Rebars}$$

$b_{required} = 2 \times 40 + 2 \times 12 + 5 \times 20 + 4 \times 25 = 304\text{mm} > 250\text{mm}$

Therefore, reinforcement should be placed in two layers as assumed, $3\phi 20$ for first layer and $2\phi 20$ for the second one.

Checking for $S_{maximum}$ for Crack Control:

$s = \frac{250 - 40 \times 2 - 12 \times 2 - 20}{2} = 63\ mm < s_{max}\ Ok$

Check with $A_{s\,minimum}$ requirements:

$A_{s\,minimum} = \frac{1.4}{f_y} b_w d = \frac{1.4}{420} \times 250 \times 415 = 349 \text{ mm}^2 < A_{S\,Provided} = 1570 \text{ mm}^2 \text{ Ok.}$

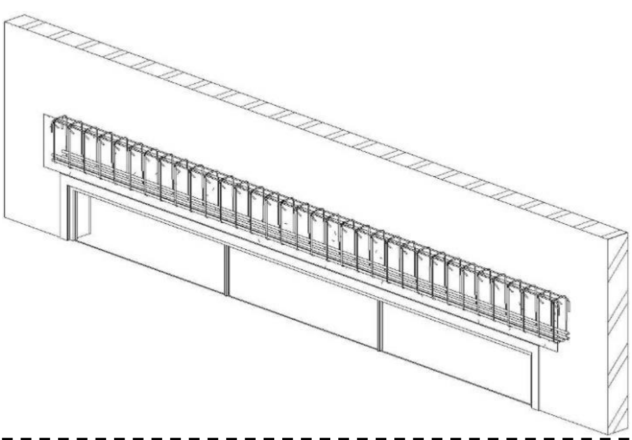
Check the assumption of $\emptyset = 0.9$:

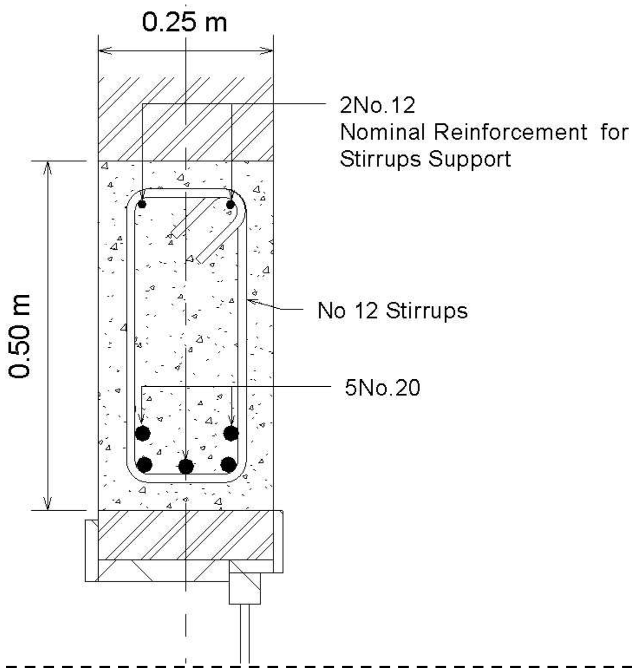
Compute steel stain based on the following relations:

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{1570 \times 420}{0.85 \times 28 \times 250} = 111 \text{ mm} \Rightarrow c = \frac{111}{0.85} = 131 \text{ mm} \Rightarrow \epsilon_t = \frac{415 - 131}{131} \times 0.003 = 6.50 \times 10^{-3}$$

As $\epsilon_t \geq 0.005$, then $\emptyset = 0.9$ Ok.

Finally, beam details are presented in below:





Example 4.4-4

Design a rectangular beam to support a dead load of $35kN/m$ and a live load of $25kN/m$ acting on a simple span of $6m$. Assume that the designer intends to use:

- A width of 300mm and a depth of 700mm. These dimensions have been determined based on architectural considerations.
- $f_c' = 21MPa$ and $f_y = 420MPa$.
- Bar diameter of 25mm for longitudinal reinforcement.
- One layer of reinforcement.
- Bar diameter of 10mm for stirrups.

Solution

1. Computed required factored applied moment (M_u):
    a. Moment due to Dead Loads:

    $$W_{Selfweight} = 0.3m \times 0.7m \times 24\frac{kN}{m^3} = 5.04\frac{kN}{m}$$

    $$W_{Dead} = 35.0\frac{kN}{m} + 5.04\frac{kN}{m} = 40.0\frac{kN}{m}$$

    $$M_{Dead} = \frac{40.0\frac{kN}{m} \times 6.0^2 m^2}{8} = 180 \ kN.m$$

    b. Moment due to Live Load:

    $$M_{Live} = \frac{25 \ kN/m \times 6.0^2 m^2}{8} = 113 \ kN.m$$

    c. Factored Moment M_u:

    $$M_u = Maximum\ of\ (1.4M_D \ \ or\ 1.2M_D + 1.6M_L)$$
    $$M_u = Maximum\ of\ [1.4 \times 180 \ kN.m \ \ or\ (1.2 \times 180 \ kN.m + 1.6 \times 113\ kN.m)]$$
    $$M_u = Maximum\ of\ [252 \ \ or\ 397] = 397\ kN.m \ \blacksquare$$

2. Computed the required nominal or theoretical flexure strength (M_n) based on the following relation:

    $$M_n = \frac{M_u}{\emptyset}$$

    Strength reduction factored can be assumed 0.9, and checked later.

    $$M_n = \frac{397\ kN.m}{0.9} = 441\ kN.m$$

3. Compute the effective beam depth "d":
    Assume that, reinforcement can be put in single reinforcement, then:

    $$d_{for\ One\ Layer} = 700 - 40 - 10 - \frac{25}{2} = 637\ mm$$

4. Compute the Required Steel Ratio $\rho_{Required}$:

    $$\rho_{Required} = \frac{1 - \sqrt{1 - 2.36\frac{M_n}{f_c' b d^2}}}{1.18 \times \frac{f_y}{f_c'}}$$

    $$\rho_{Required} = \frac{1 - \sqrt{1 - 2.36\frac{441 \times 10^6 N.mm}{21 \times 300 \times 637^2}}}{1.18 \times \frac{420}{21}} = 9.75 \times 10^{-3}$$

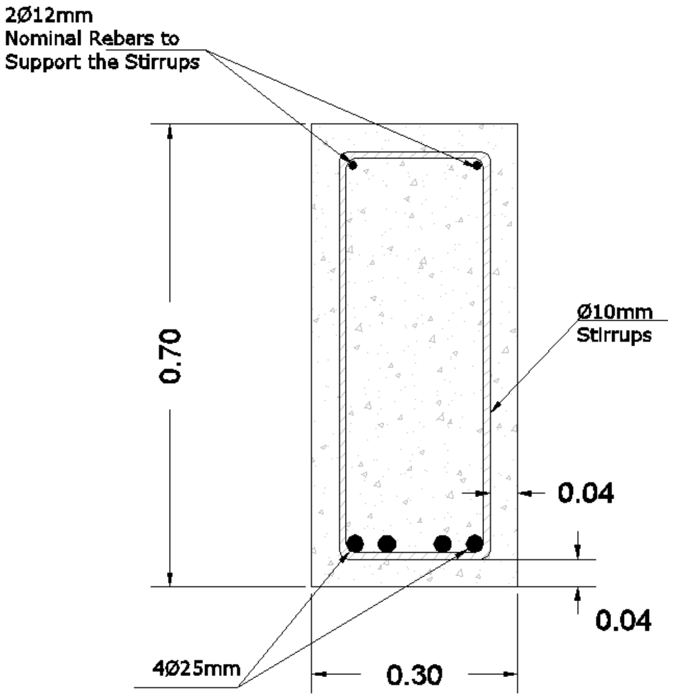
5. Check if the beam failure is secondary compression failure or compression failure:

    $$\rho_{max} = 0.85\beta_1 \ \frac{f_c'}{f_y} \ \frac{\epsilon_u}{\epsilon_u + 0.004}$$

    $$\beta_1 = 0.85$$

    $$\rho_{max} = 0.85 \times 0.85 \ \frac{21}{420} \ \frac{0.003}{0.003 + 0.004} = 15.5 \ \times 10^{-3} > \rho_{Required} \ \ Ok.$$

6. Compute the required steel area:
$A_{S\,Required} = \rho_{Required} \times bd$
$A_{S\,Required} = 9.75 \times 10^{-3} \times 300 \times 637\ mm = 1\,863\ mm^2$
7. Compute required rebars number:
$A_{Bar} = \frac{\pi \times 25^2}{4} = 490\ mm^2$
$No.of\ Rebars = \frac{1\,863\ mm^2}{490\ mm^2} = 3.80$
Try $4\emptyset 25mm$.
$A_{S\,Provided} = 4 \times 490\ mm^2 = 1\,960\ mm^2$
8. Check if the available width "b" is adequate to put the rebars in a single layer:
$b_{required} = 2 \times Side\ Cover + 2 \times Stirrups + No.of\ Rebars \times Bar\ Diameter + (No.of\ Rebars - 1) \times Spacing\ between\ Rebars$
$b_{required} = 2 \times 40mm + 2 \times 10 + 4 \times 25mm + 3 \times 25mm = 275mm < 300mm\ Ok.$
9. Check with $A_{s\,minimum}$ requirements:
$A_{s\,minimum} = \frac{1.4}{f_y} b_w d = \frac{1.4}{420} \times 300 \times 637 = 637\ mm^2 < A_{S\,Provided} = 1\,960\ mm^2\ Ok.$
10. Check $S_{maximum}$:
With four rebars, two stirrup legs, two covers, and a width of 300mm, the requirement of maximum spacing is necessary satisfied.
11. Check the assumption of $\emptyset = 0.9$:
    a. Compute steel stain based on the following relations:
    $a = \frac{A_s f_y}{0.85 f_c' b}$
    $a = \frac{1\,960 \times 420}{0.85 \times 21 \times 300} = 154\ mm$
    $c = \frac{154\ mm}{0.85} = 181\ mm$
    $\epsilon_t = \frac{d - c}{c} \times \epsilon_u$
    $\epsilon_t = \frac{637\ mm - 181\ mm}{181\ mm} \times 0.003 = 7.56 \times 10^{-3}$
    b. As $\epsilon_t \geq 0.005$, then $\emptyset = 0.9$ Ok.
12. Final Reinforcement Details:



Example 4.4-5

What are the required longitdinal reinforcements for Scetion A and Section B of the beam shown in Figure 4.4-5 below?
In your solution assume that:

1. $f_c' = 28\ MPa$ and $f_y = 420\ MPa$.
2. Rebar of 25mm is used for longtudinal reinforcement.
3. Single layer of reinforcement.
4. Rebars of 12mm is used for shear reinforcement.
5. Beam slefweight can be neglcted.
6. Uniform subgarde reaction.

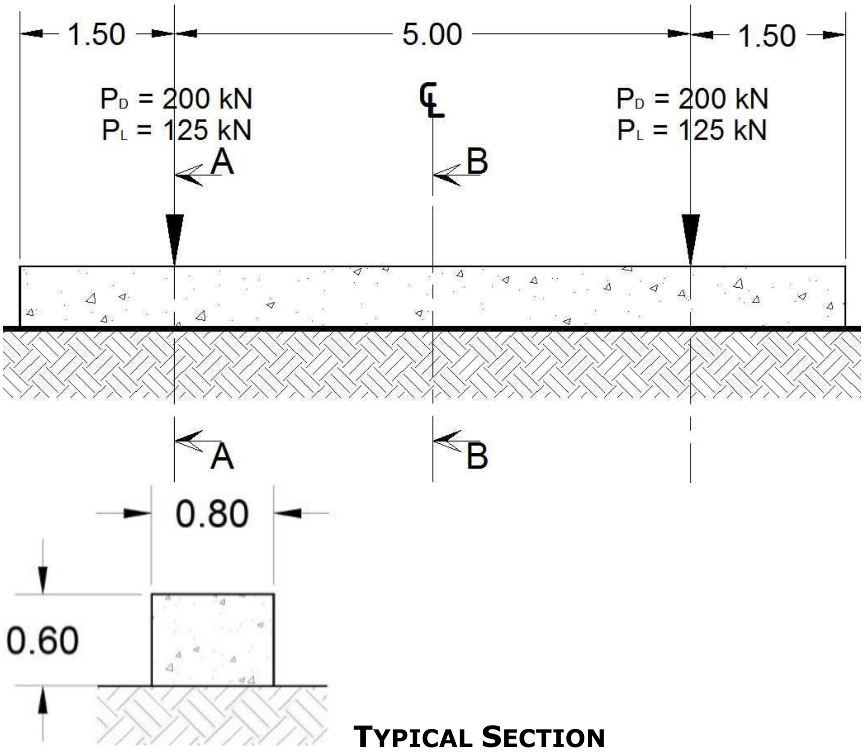


Figure 4.4-5: Foundation beam for Example 4.4-5.

Solution

Design Forces

$$P_u = maximum\ (1.4 P_{Dead}\ Or\ 1.2 P_{Dead} + 1.6 P_{Live})$$
$$P_u = maximum\ (1.4 \times 200\ Or\ 1.2 \times 200 + 1.6 \times 125)$$
$$P_u = maximum\ (280\ Or\ 440)$$
$$P_u = 440\ kN$$
$$W_u = (440\ kN \times 2) \times \frac{1}{8m} = 110 \frac{kN}{m}$$
$$M_{u\ @\ Section\ A-A} = 110 \frac{kN}{m} \times 1.5m \times \frac{1.5m}{2} = 124\ kN.m$$
$$M_{u\ @\ Section\ B-B} = -\left(\frac{110 \frac{kN}{m} \times 5^2 m^2}{8} \right) + 124$$
$$M_{u\ @\ Section\ B-B} = -220\ kN.m$$

Design of Section A-A

1. Computed the required nominal or theoretical flexure strength (M_n) based on the following relation:

$$\mathrm{M_n} = \frac{\mathrm{M_u}}{\emptyset}$$

Strength reduction factored can be assumed 0.9, and checked later.

$$\mathrm{M_n} = \frac{124\ \mathrm{kN.m}}{0.9} = 138\ \mathrm{kN.m}$$

2. Compute the effective beam depth "d":
Assume that, reinforcement can be put in single reinforcement, then:
$$d_{\text{for One Layer}} = 600 - 75_{\text{Casted and exposed to soil}} - 12 - \frac{25}{2} = 505 \text{ mm}$$
3. Compute the Required Steel Ratio ρ_{Required}:
$$\rho_{\text{Required}} = \frac{1 - \sqrt{1 - 2.36 \frac{M_n}{f'_c bd^2}}}{1.18 \times \frac{f_y}{f'_c}}$$
$$\rho_{\text{Required}} = \frac{1 - \sqrt{1 - 2.36 \frac{138 \times 10^6 \text{N.mm}}{28 \times 800 \times 505^2}}}{1.18 \times \frac{420}{28}}$$
$$\rho_{\text{Required}} = 1.63 \times 10^{-3}$$
4. Check if the beam failure is secondary compression failure or compression failure:
$$\rho_{max} = 0.85\beta_1 \ \frac{f'_c}{f_y} \ \frac{\epsilon_u}{\epsilon_u + 0.004}$$
$$\rho_{max} = 0.85^2 \ \frac{28}{420} \ \frac{0.003}{0.003 + 0.004} \ = 20.6 \ \times 10^{-3} > \rho_{\text{Required}} \ \text{Ok.}$$
5. Compute the required steel area:
$$A_{S \text{ Required}} = \rho_{\text{Required}} \times bd$$
$$A_{S \text{ Required}} = 1.63 \times 10^{-3} \times 800 \times 505 = 660 \text{ mm}^2$$
6. Check with $A_{s \text{ minimum}}$ requirements:
$$A_{s \text{ minimum}} = \frac{1.4}{f_y} b_w d = \frac{1.4}{420} \times 800 \times 505 = 1347 \text{ mm}^2$$
$$A_{s \text{ minimum}} = 1347 \text{ mm}^2 > 1\frac{1}{3} A_{S\text{Required}} = 878 \text{ mm}^2$$
Then used
$$A_s = 878 \ mm^2$$
7. Compute required rebars number:
$$A_{Bar} = \frac{\pi \times 25^2}{4} = 490 \ mm^2$$
$$No.of \ Rebars \ = \frac{878 \ mm^2}{490 \ mm^2} = 1.79$$
Try $2\emptyset 25$mm.
$$A_{S \text{ Provided}} = 2 \times 490 \text{ mm}^2 = 980 \text{ mm}^2$$
8. Check if the available width "b" is adequate to put the rebars in a single layer:
$$b_{\text{required}} = 2 \times \text{Side Cover} + 2 \times \text{Stirrups} \ + \text{No. of Rebars} \times \text{Bar Diameter} + (\text{No. of Rebars} - 1) \times \text{Spacing between Rebars}$$
$$b_{\text{required}} = 2 \times 50 + 2 \times 12 + \ 2 \times 25 + 25 = 199 \text{ mm} < 800\text{mm Ok.}$$
9. Checking for Smax for Crack Control:
$$s \ = 800 - 50 \times 2 - 12 \times 2 - 25 = 651 \text{ mm} > s_{max} \ \ Not \ Ok$$
Then use $\boldsymbol{5\phi 16mm}$ instead of $2\phi 25mm$.
$$A_{S \text{ Provided}} = 5 \times 200 \text{ mm}^2 = 1000 \text{ mm}^2$$
10. Check the assumption of $\emptyset = 0.9$:
    a. Compute steel stain based on the following relations:
$$a = \frac{A_s f_y}{0.85 f'_c b}$$
$$a = \frac{10000 \ \times \ 420}{0.85 \times 28 \times 800} = 22.1 \text{ mm}$$
$$c = \frac{22.1 \text{ mm}}{0.85} = 26 \text{ mm}$$
$$\epsilon_t = \frac{505 \text{ mm} - 26 \text{ mm}}{26 \text{ mm}} \times 0.003 = 55.3 \times 10^{-3}$$
    b. As $\epsilon_t \ \geq 0.005$, then $\emptyset = 0.9$ Ok.

Design of Section B-B:

1. Computed the required nominal or theoretical flexure strength (M_n) based on the following relation:
   $$M_n = \frac{M_u}{\emptyset}$$
   Strength reduction factored can be assumed 0.9, and checked later.
   $$M_n = \frac{220\ kN.m}{0.9} = 244\ kN.m$$
2. Compute the effective beam depth "d":
   Assume that, reinforcement can be put in single reinforcement, then:
   $$d_{for\ One\ Layer} = 600 - 50_{Exposed\ to\ Soil} - 12 - \frac{25}{2} = 525\ mm$$
3. Compute the Required Steel Ratio $\rho_{Required}$:
   $$\rho_{Required} = \frac{1 - \sqrt{1 - 2.36 \frac{M_n}{f'_c b d^2}}}{1.18 \times \frac{f_y}{f'_c}}$$
   $$\rho_{Required} = \frac{1 - \sqrt{1 - 2.36 \frac{244 \times 10^6 N.mm}{28 \times 800 \times 525^2}}}{1.18 \times \frac{420}{28}}$$
   $$\rho_{Required} = 2.70 \times 10^{-3}$$
4. Check if the beam failure is secondary compression failure or compression failure:
   $$\rho_{max} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004}$$
   $$\rho_{max} = 0.85^2 \frac{28}{420} \frac{0.003}{0.003 + 0.004} = 20.6 \times 10^{-3} > \rho_{Required}\ Ok.$$
5. Compute the required steel area:
   $$A_{S\ Required} = \rho_{Required} \times bd$$
   $$A_{S\ Required} = 2.70 \times 10^{-3} \times 800 \times 525 = 1134\ mm^2$$
6. Check with $A_{s\ minimum}$ requirements:
   $$A_{s\ minimum} = \frac{1.4}{f_y} b_w d = \frac{1.4}{420} \times 800 \times 525 = 1400\ mm^2$$
   $$A_{s\ minimum} = 1400\ mm^2 < 1\frac{1}{3} A_{SRequired} = 1508\ mm^2\ Ok.$$
   Then used
   $$A_s = A_{s\ minimum} = 1400\ mm^2$$
7. Compute required rebars number:
   $$A_{Bar} = \frac{\pi \times 25^2}{4} = 490\ mm^2$$
   $$No.of\ Rebars = \frac{1400\ mm^2}{490\ mm^2} = 2.85$$
   Try $3\emptyset 25mm$.
   $$A_{S\ Provided} = 3 \times 490\ mm^2 = 1470\ mm^2$$
8. Check if the available width "b" is adequate to put the rebars in a single layer:
   $$b_{required} = 2 \times \text{Side Cover} + 2 \times \text{Stirrups} + \text{No. of Rebars} \times \text{Bar Diameter} + (\text{No. of Rebars} - 1) \times \text{Spacing between Rebars}$$
   $$b_{required} = 2 \times 50 + 2 \times 12 + 3 \times 25 + 2 \times 25 = 249\ mm < 800mm\ Ok.$$
9. Checking for Smax for Crack Control:
   $$s = \frac{800 - 50 \times 2 - 12 \times 2 - 25}{2} = 326\ mm > s_{max}\ Not\ Ok$$
   Then use **$5\phi 20mm$** instead of $3\phi 25mm$.
   $$A_{S\ Provided} = 5 \times 314\ mm^2 = 1570\ mm^2$$

10. Check the assumption of $\emptyset = 0.9$:
    a. Compute steel stain based on the following relations:

$$a = \frac{A_s f_y}{0.85 f_c' b}$$

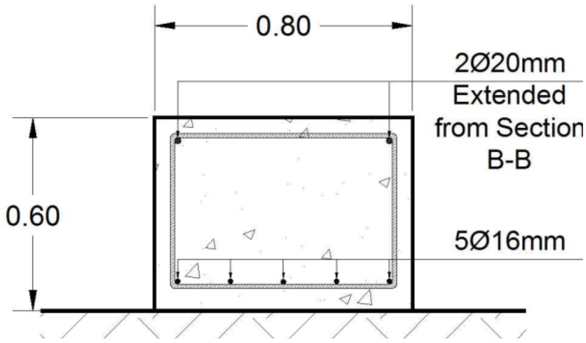
$$a = \frac{1570 \times 420}{0.85 \times 28 \times 800} = 34.6 \text{ mm}$$

$$c = \frac{34.6 \text{ mm}}{0.85} = 40.7 \text{ mm}$$

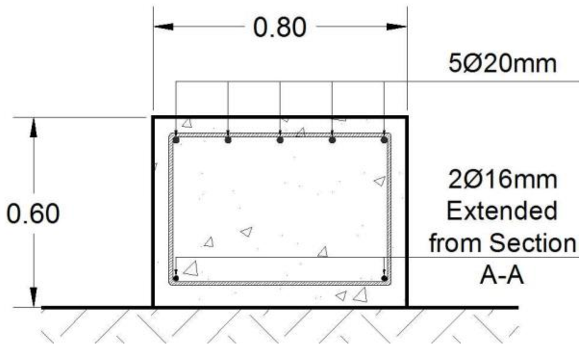
$$\epsilon_t = \frac{525 \text{ mm} - 34.6 \text{ mm}}{34.6 \text{ mm}} \times 0.003 = 42.5 \times 10^{-3}$$

    b. As $\epsilon_t \geq 0.005$, then $\emptyset = 0.9$ Ok.

Sections Details:



SECTION A-A



SECTION B-B

Example 4.4-6

Design Section A-A of beam shown in Figure 4.4-6 below for flexure requirements according to ACI 318M-14.

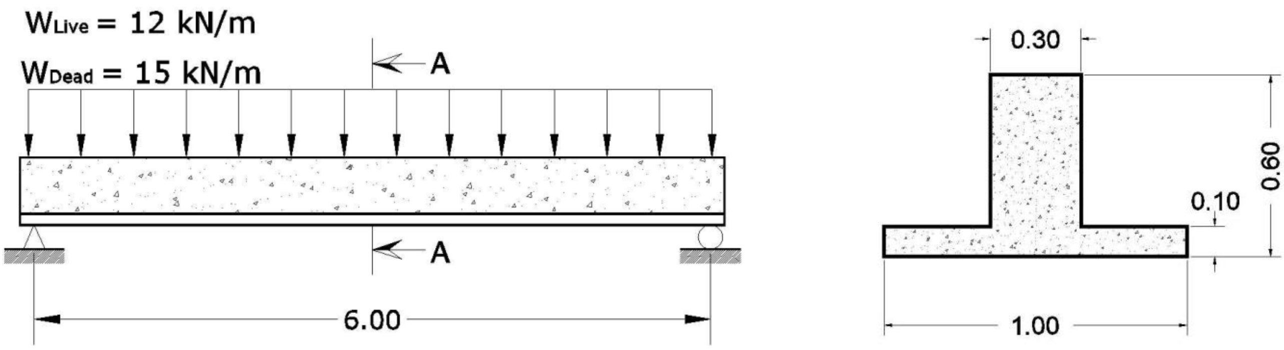


Figure 4.4-6: Inverted beam of Example 4.4-6.

In your solution, assume that:

1. f_c' = 28 MPa.
2. f_y = 420 MPa.
3. Rebar of No. 25 for longitudinal reinforcement.
4. Single layer of reinforcement.
5. Rebar of No. 10 for stirrups.
6. Rebars can be put in a single layer.

Solution

1. Design Moments:

$$W_{selfweight} = (0.1 \times 1.0 + 0.5 \times 0.3)m^2 \times 24\frac{kN}{m^2} = 6\frac{kN}{m}$$

$$W_{Dead} = 15 + 6 = 21\frac{kN}{m}$$

$$W_u = 1.4(21) \text{ or } [1.2 \times 21 + 1.6 \times 12]$$

$$W_u = 29.4\frac{kN}{m} \text{ or } 44.4\frac{kN}{m}$$

$$W_u = 44.4\frac{kN}{m}$$

$$M_u = \frac{W_u l^2}{8} = \frac{44.4\frac{kN}{m} \times 6.0^2 m^2}{8} = 200 \text{ kN.m}$$

2. Section Design:
   As the flange is on the tension side, section should be designed as a rectangular section.
3. Computed the required nominal or theoretical flexure strength (M_n) based on the following relation:

$$M_n = \frac{M_u}{\emptyset}$$

   Strength reduction factored can be assumed 0.9, and checked later.

$$M_n = \frac{200 \text{ kN.m}}{0.9} = 222 \text{ kN.m}$$

4. Compute the effective beam depth "d":
   Assume that, reinforcement can be put in single reinforcement, then:

$$d_{for\ One\ Layer} = 600 - 40 - 10 - \frac{25}{2} = 538 \text{ mm}$$

5. Compute the Required Steel Ratio $\rho_{Required}$:

$$\rho_{Required} = \frac{1 - \sqrt{1 - 2.36\frac{M_n}{f'_c bd^2}}}{1.18 \times \frac{f_y}{f'_c}}$$

$$\rho_{Required} = \frac{1 - \sqrt{1 - 2.36\frac{222 \times 10^6 N.mm}{28 \times 300 \times 538^2}}}{1.18 \times \frac{420}{28}}$$

$$\rho_{Required} = 6.46 \times 10^{-3}$$

6. Check if the beam failure is secondary compression failure or compression failure:

$$\rho_{max} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004}$$

$$\beta_1 = 0.85$$

$$\rho_{max} = 0.85^2 \frac{28}{420} \frac{0.003}{0.003 + 0.004} = 20.6 \times 10^{-3} > \rho_{Required} \text{ Ok.}$$

$$\rho_{max} = 20.6 \times 10^{-3} > \rho_{Required} \text{ Ok.}$$

7. Compute the required steel area:

$$A_{S\ Required} = \rho_{Required} \times bd$$

$$A_{S\ Required} = 6.46 \times 10^{-3} \times 300 \times 538 = 1042 \text{ mm}^2$$

8. Compute required rebars number:

$$A_{Bar} = \frac{\pi \times 25^2}{4} = 490 \text{ mm}^2$$

$$\text{No. of Rebars} = \frac{1042 \text{ mm}^2}{490 \text{ mm}^2} = 2.12$$

   Try $3\emptyset 25$mm.

$$A_{S\ Provided} = 3 \times 490 \text{ mm}^2 = 1\,470 \text{ mm}^2$$

9. Check if the available width "b" is adequate to put the rebars in a single layer:

$$b_{required} = 2 \times \text{Side Cover} + 2 \times \text{Stirrups} + \text{No. of Rebars} \times \text{Bar Diameter} + (\text{No. of Rebars} - 1) \times \text{Spacing between Rebars}$$

$b_{required} = 2 \times 40mm + 2 \times 10 + 3 \times 25mm + 2 \times 25mm = 225mm < 300mm$ Ok.

10. Checking for Smax for Crack Control:

$$S = (300 - 40 \times 2 - 10 \times 2 - 25) \times \frac{1}{2} = 87.5 \text{ mm} < S_{max} \text{ Ok}$$

Remember that S_{max} is c/c distance.

11. Check with $A_{s\ minimum}$ requirements:

As the span is statically determinate, and as flange is in tension side, As min will be computed based on following relation.

$$A_{s\,min} = minimum \left(\frac{0.25\sqrt{f_c'}}{f_y} b_f d, \frac{0.50\sqrt{f_c'}}{f_y} b_w d \right)$$

$$A_{s\,min} = minimum \left(\frac{0.25\sqrt{28}}{420} 1000 \times 538, \frac{0.50\sqrt{28}}{420} 300 \times 538 \right)$$

$$A_{s\,min} = minimum(1695, \quad 1017)$$

$$A_{s\,min} = 1017\ mm^2 < A_{s\,provided} \ \therefore\ ok.$$

12. Check the assumption of $\emptyset = 0.9$:

i. Compute steel stain based on the following relations:

$$a = \frac{A_s f_y}{0.85 f_c' b}$$

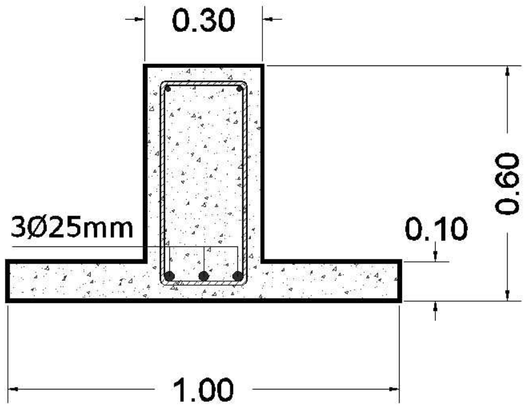
$$a = \frac{1470 \times 420}{0.85 \times 28 \times 300} = 86.5 \text{ mm}$$

$$c = \frac{86.5 \text{ mm}}{0.85} = 102 \text{ mm}$$

$$\epsilon_t = \frac{538 \text{ mm} - 102 \text{ mm}}{102 \text{ mm}} \times 0.003 = 12.8 \times 10^{-3}$$

ii. As $\epsilon_t \geq 0.005$, then $\emptyset = 0.9$ Ok.

13. Final Reinforcement Details:



4.4.4 Homework Problems

Problem 4.4-1

Design a simply supported rectangular reinforced concrete beam to carry a service dead load of $19.7\,\text{kN/m}$ and a service live load of $27.7\,\text{kN/m}$. The span is 5.5m. It is known that this beam is not exposed to weather and not in contact with ground.

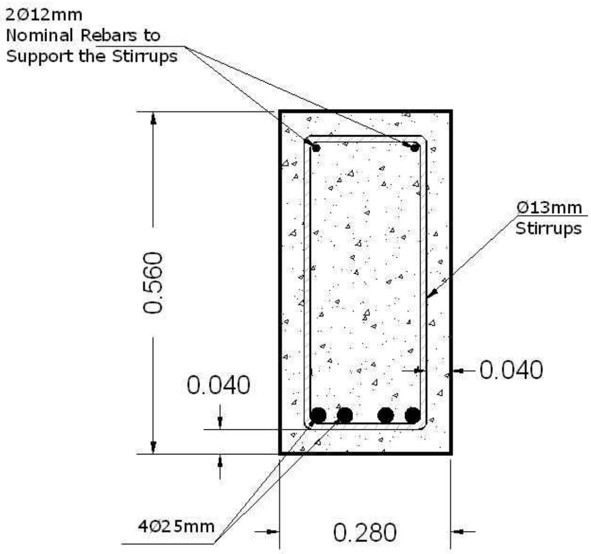
Assume that the designer intend to use:

1. Concrete of $f_c' = 28\,MPa$.
2. Steel of A615 Grade 60.
3. A width of 280mm and a height of 560mm (these dimensions have been determined based on architectural considerations).
4. Rebar of No. 25 for longitudinal reinforcement.
5. Rebar of No. 13 for stirrups.

Answers

- Computed required factored applied moment (M_u):
  a. Moment due to Dead Loads:
  $$\text{W}_{\text{Selfweight}} = 3.76\,\frac{\text{kN}}{\text{m}}$$
  $$\text{W}_{\text{Dead}} = 23.5\,\frac{\text{kN}}{\text{m}}$$
  $$\text{M}_{\text{Dead}} = 88.9\ \text{kN.m}$$
  b. Moment due to Live Load:
  $$\text{M}_{\text{Live}} = 105\ \text{kN.m}$$
  c. Factored Moment M_u:
  $$\text{M}_\text{u} = \text{Maximum of } (1.4\text{M}_\text{D} \text{ or } 1.2\text{M}_\text{D} + 1.6\text{M}_\text{L})$$
  $$\text{M}_\text{u} = \text{Maximum of } [1.4 \times 88.9 \text{ or } (1.2 \times 88.9 + 1.6 \times 105)] =$$
  $$\text{M}_\text{u} = \text{Maximum of } [124 \text{ or } 275] = 275\ \text{kN.m}\ \blacksquare$$
- Computed the required nominal or theoretical flexure strength (M_n) based on the following relation:
  $$\text{M}_\text{n} = \frac{\text{M}_\text{u}}{\emptyset}$$
  Strength reduction factored can be assumed 0.9, and checked later.
  $$\text{M}_\text{n} = 306\ \text{kN.m}$$
- Compute the effective beam depth "d":
  Assume that, reinforcement can be put in a single layer, then:
  $$\text{d}_{\text{for One Layer}} = 495\text{mm}$$
- Compute the Required Steel Ratio ρ_{Required}:
  $$\rho_{\text{Required}} = 11.9 \times 10^{-3}$$
- Check if the beam failure is secondary compression failure or compression failure:
  $$\rho_{\max} = 20.6 \times 10^{-3} > \rho_{\text{Required}}\ \text{Ok.}$$
- Compute the required steel area:
  $$\text{A}_{\text{S Required}} = \rho_{\text{Required}} \times \text{bd}$$
  $$\text{A}_{\text{S Required}} = 1\,649\ \text{mm}^2$$
- Compute required rebars number:
  $$\text{A}_{\text{Bar}} = \frac{\pi \times 25^2}{4} = 490\ \text{mm}^2$$
  $$\text{No. of Rebars} = 3.36$$
  Try $4\emptyset 25\text{mm}$.
  $$\text{A}_{\text{S Provided}} = 1\,960\ \text{mm}^2$$
- Check if the available width "b" is adequate to put the rebars in a single layer:
  $$\text{b}_{\text{required}} = 281\text{mm} \approx 280\text{mm Ok.}$$
- Check for s_{max}:
  As the total width is only 280mm, and as we have used 4 rebars, then the spacing center to center of bars will of course be less than smax.
- Check with $A_{s\,minimum}$ requirements:
  $$\text{A}_{\text{s minimum}} = 462\ \text{mm}^2 < \text{A}_{\text{S Provided}} = 1\,960\ \text{mm}^2\ \text{Ok.}$$

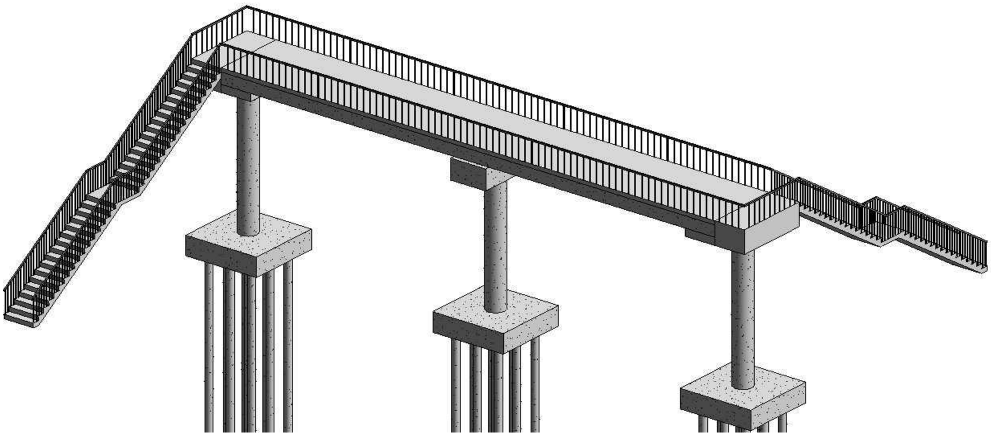
- Check the assumption of $\emptyset = 0.9$:
    a. Compute steel stain based on the following relations:
    $\mathrm{a} = 124\ \mathrm{mm}$
    $\mathrm{c} = 146\ \mathrm{mm}$
    $\epsilon_t = 0.00717$
    b. As $\epsilon_t \geq 0.005$, then $\emptyset = 0.9$ Ok.
- Final Reinforcement Details:



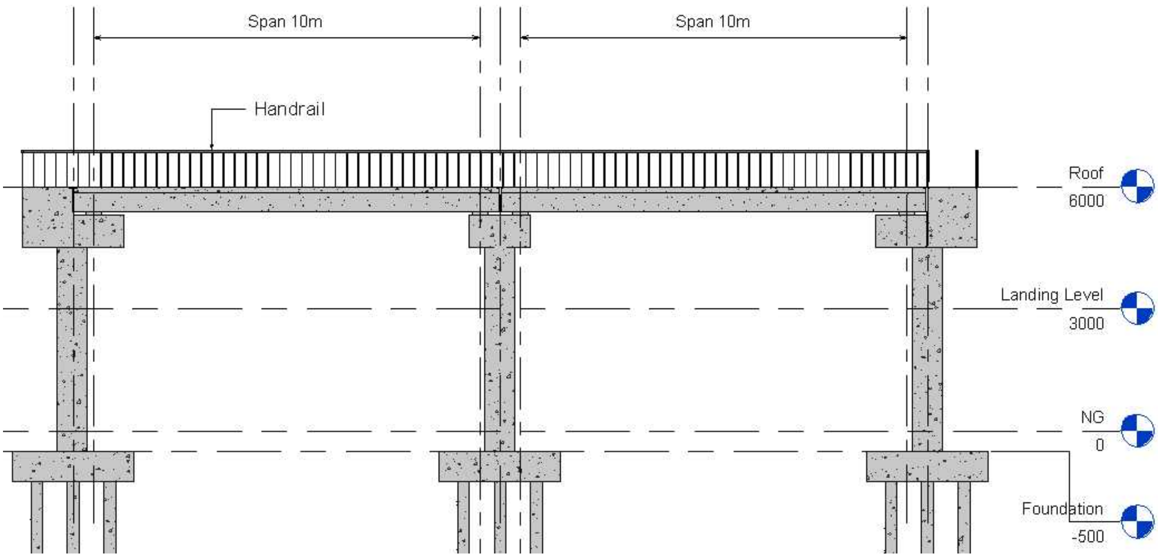
Problem 4.4-2

Design a simply supported pedestrian's bridge to carry the following loads:

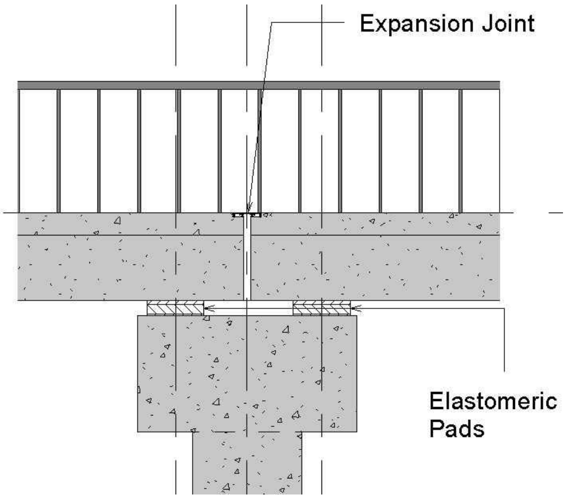
1. Dead load of slab and pavement surfacing is $5.0\ kN/m$.
2. Handrail weight can be taken as $0.5\frac{\mathrm{kN}}{\mathrm{m}}$.
3. Service live load of $6.0\ \mathrm{kN/m}$.



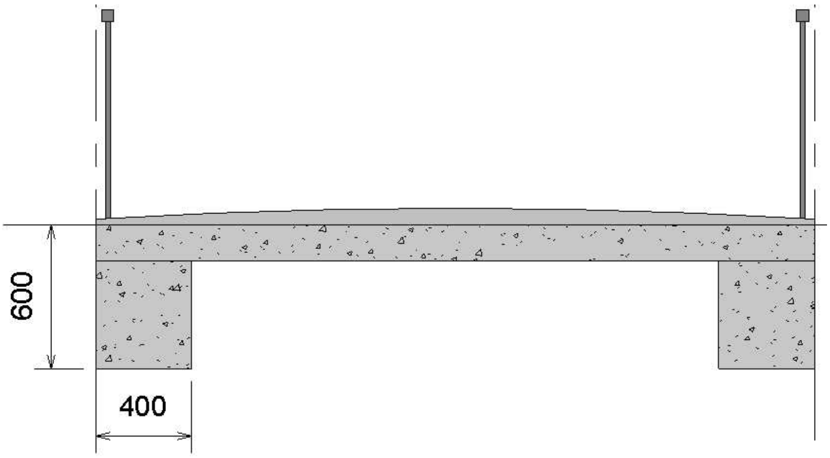
3D View



Elevation View



Callout View



Cross Sectional View

Assume that the designer intends to use:

1. Concrete of $f_c^{'} = 28\text{ MPa}$.
2. Steel of A615 Grade 60.
3. A width of 400mm and a height of 600mm (these dimensions have been determined based on deflection considerations).
4. Rebar of No. 25 for longitudinal reinforcement.
5. Rebar of No. 13 for stirrups.

Notes on the Problem

1. With using of the expansion joint and elastomeric pads shown in the callout view, the beam almost behaves as a simply supported one.
2. Surfacing and live loads should be simulated as load per unit area. Until Chapter 13 where student learns how to transfer loads from slabs to the supporting beams, loads will be given per unit length.
3. Beam is assumed to have a rectangular shape. This conservative assumption equivalent to neglecting slab flanging effect. More detailed modeling will be discussed in analysis and design of beams with "T" shapes.

Answers

- Computed required factored applied moment (M_u):
    a. Moment due to Dead Loads:

    $$W_{Selfweight} = 5.76\frac{kN}{m} \Rightarrow W_{Dead} = 11.3\frac{kN}{m} \Rightarrow$$

    b. Moment due to Live Load:

    $M_{Live} = 75\text{ kN.m}$

    c. Factored Moment M_u:

    $$M_u = \text{Maximum of } (1.4M_D \text{ or } 1.2M_D + 1.6M_L)$$
    $$= \text{Maximum of } [1.4 \times 141 \text{ or } (1.2 \times 141 + 1.6 \times 75)]$$
    $$= \text{Maximum of } [197 \text{ or } 289] = 289\text{ kN.m} \blacksquare$$

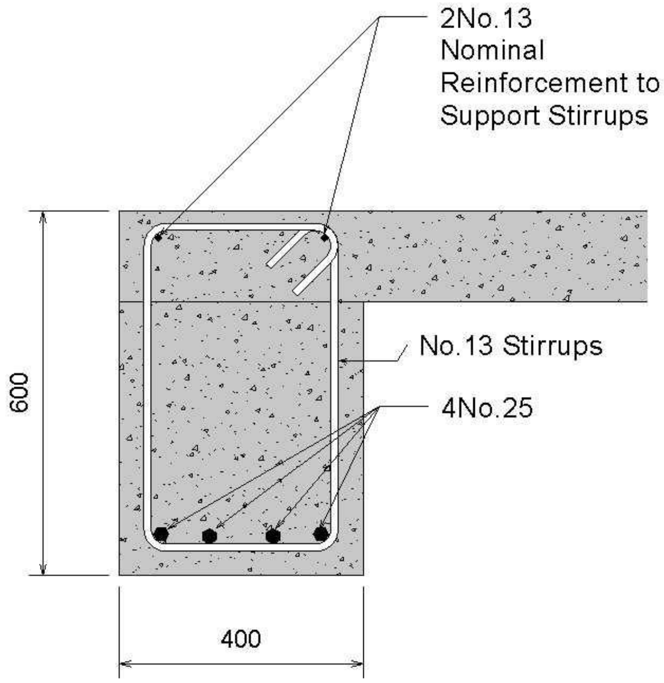
- Computed the required nominal or theoretical flexure strength (M_n) based on the following relation:

$$M_n = \frac{M_u}{\emptyset}$$

Strength reduction factored can be assumed 0.9, and checked later.

$M_n = 321 \text{ kN.m}$

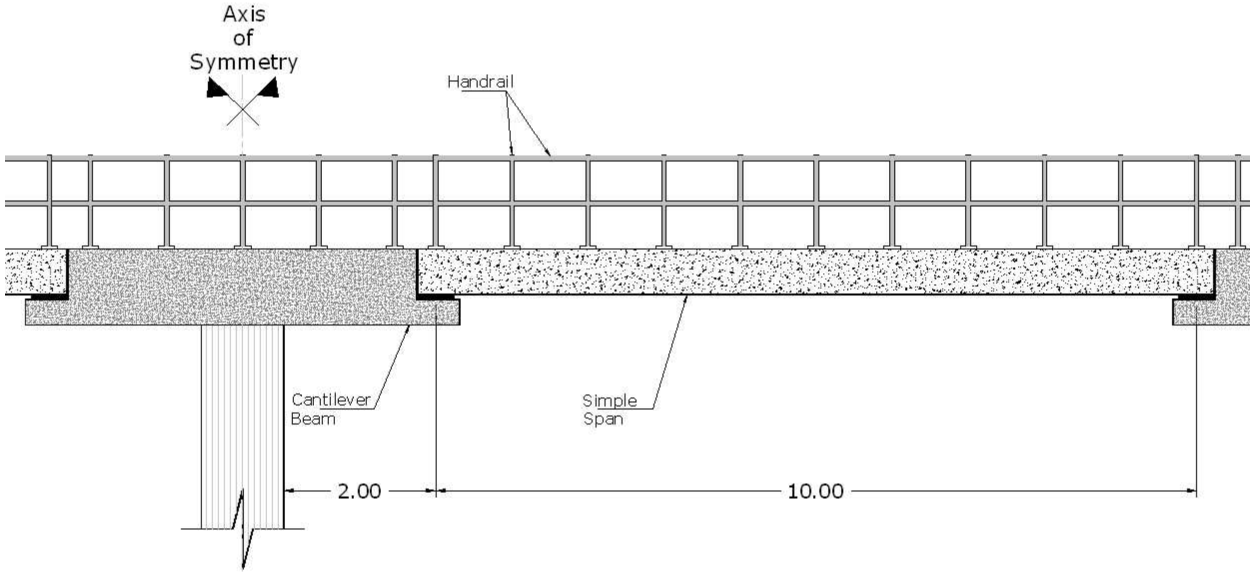
- Compute the effective beam depth "d":
  Assume that, reinforcement can be put in single reinforcement, then:
  $$d_{\text{for One Layer}} = 600 - 40 - 13 - \frac{25}{2} = 534 \text{ mm}$$
- Compute the Required Steel Ratio $\rho_{Required}$:
  $\rho_{Required} = 7.15 \times 10^{-3}$
- Check if the beam failure is secondary compression failure or compression failure:
  $\rho_{max} = 20.6 \times 10^{-3} > \rho_{Required}$ Ok.
- Compute the required steel area:
  $A_{S\ Required} = 1\,527 \text{ mm}^2$
- Compute required rebars number:
  $$A_{Bar} = \frac{\pi \times 25^2}{4} = 490 \text{ mm}^2$$
  No. of Rebars $= 3.12$
  Try $4\emptyset 25\text{mm}$.
  $A_{S\ Provided} = 4 \times 490 \text{ mm}^2 = 1\,960 \text{ mm}^2$
- Check if the available width "b" is adequate to put the rebars in a single layer:
  $b_{required} = 281\text{mm} < 400mm\ Ok.$
- Check for S_{max}:
  $s_{\frac{c}{c}} = 92mm < s_{max}\ Ok.$
- Check with $A_{s\ minimum}$ requirements:
  $A_{s\ minimum} = 712\ mm^2 < A_{S\ Provided} = 1\,960\ mm^2\ Ok.$
- Check the assumption of $\emptyset = 0.9$:
  a. Compute steel stain based on the following relations:
     $a = 86.5$ mm
     $c = 102$ mm
     $\epsilon_t = 12.7 \times 10^{-3}$
  b. As $\epsilon_t \geq 0.005$, then $\emptyset = 0.9$ Ok.
- Final Reinforcement Details:



Problem 4.4-3

Design a cantilever beam of pedestrian's bridge shown below to carry the following loads:

1. Dead load of slab and pavement surfacing is $5.0\ kN/m$.
2. Handrail weight can be taken as $0.5\frac{\mathrm{kN}}{\mathrm{m}}$.
3. Service live load of $6.0\ \mathrm{kN/m}$.

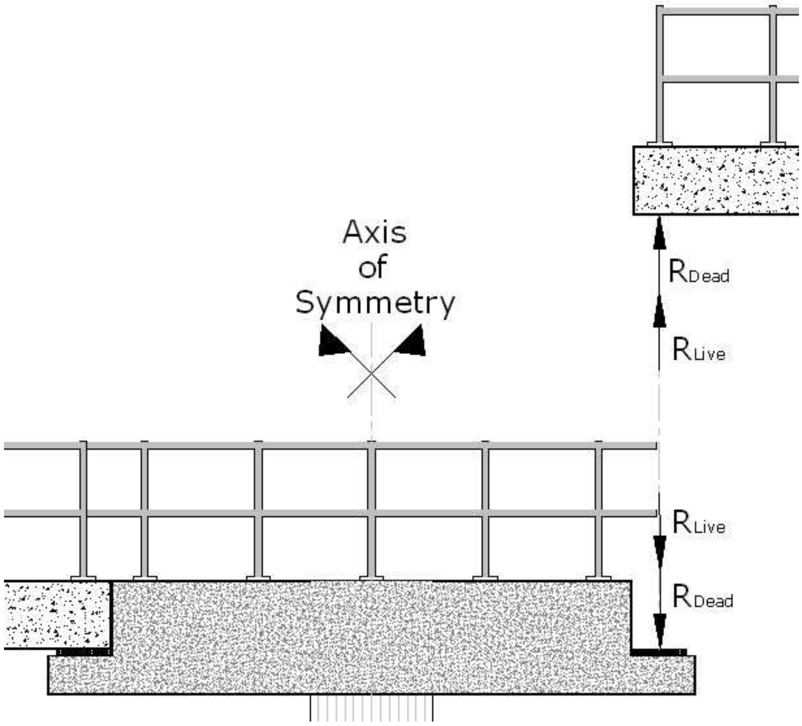


Assume that the designer intends to use:

1. Concrete of $f_c' = 28\ \mathrm{MPa}$.
2. Steel of A615 Grade 60.
3. A width of 400mm and a height of 800mm for cantilever span.
4. A width of 400mm and a height of 600mm for simple span.
5. Rebar of No. 25 for longitudinal reinforcement.
6. Rebar of No. 13 for stirrups.

Answers

- Computed required factored applied moment ($\mathrm{M_u}$):
    a. Compute the reactions of simple span:



b. Reaction due to Dead Loads:

$$W_{Selfweight} = 0.6m \times 0.4m \times 24\frac{kN}{m^3} = 5.76\frac{kN}{m} \Rightarrow W_{Dead} = (5.76 + 5.0 + 0.5)\frac{kN}{m} = 11.3\frac{kN}{m}$$

$$R_{Dead} = \frac{11.3\frac{kN}{m} \times 10.0m}{2} = 56.5\ kN$$

c. Reaction due to Live Load:

$$R_{Live} = \frac{6.0\frac{kN}{m} \times 10.0m}{2} = 30\ kN$$

d. Compute the moments of cantilever span:

Generally, it is useful to note that a negative moment is computed at face of support and not at support centerline. This is due to the fact within support loads transferred in a bearing form instead of shear and bending form.

i. Moment due to Dead Loads:

$$W_{Selfweight} = 0.8m \times 0.4m \times 24\frac{kN}{m^3} = 7.68\frac{kN}{m}$$

$$W_{Dead} = (7.68 + 5.0 + 0.5)\frac{kN}{m} = 13.2\frac{kN}{m}$$

$$M_D = 56.5\ kN \times 2.0m + \frac{13.2\frac{kN}{m} \times 2.0^2 m^2}{2} = 139\ kN.m$$

ii. Moment due to Live Loads:

$$M_L = 30\ kN \times 2.0m + \frac{6.0\frac{kN}{m} \times 2.0^2 m^2}{2} = 72\ kN.m$$

iii. Factored Moment M_u:

$$M_u = \text{Maximum of } (1.4M_D \text{ or } 1.2M_D + 1.6M_L)$$
$$M_u = \text{Maximum of } [1.4 \times 139 \text{ or } (1.2 \times 139 + 1.6 \times 72)]$$
$$M_u = \text{Maximum of } [195 \text{ or } 282] = 282\ kN.m\ \blacksquare$$

- Computed the required nominal or theoretical flexure strength (M_n) based on the following relation:

$$M_n = \frac{M_u}{\emptyset}$$

Strength reduction factored can be assumed 0.9, and checked later.

$$M_n = 313\ kN.m$$

- Compute the effective beam depth "d":

Assume that, reinforcement can be put in single reinforcement, then:

$$d_{for\ One\ Layer} = 734\ mm$$

- Compute the Required Steel Ratio $\rho_{Required}$:

$$\rho_{Required} = 3.57 \times 10^{-3}$$

- Check if the beam failure is secondary compression failure or compression failure:

$$\rho_{max} = 20.6 \times 10^{-3} > \rho_{Required}\ \text{Ok.}$$

- Compute the required steel area:

$$A_{S\ Required} = 1\,048\ mm^2$$

- Compute required rebars number:

$$A_{Bar} = \frac{\pi \times 25^2}{4} = 490\ mm^2$$

$$\text{No. of Rebars} = 2.14$$

Try, 3∅25mm.

$$A_{S\ Provided} = 3 \times 490\ mm^2 = 1\,470\ mm^2$$

- Check if the available width "b" is adequate to put the rebars in a single layer:

$$b_{required} = 231mm < 400mm\ Ok.$$

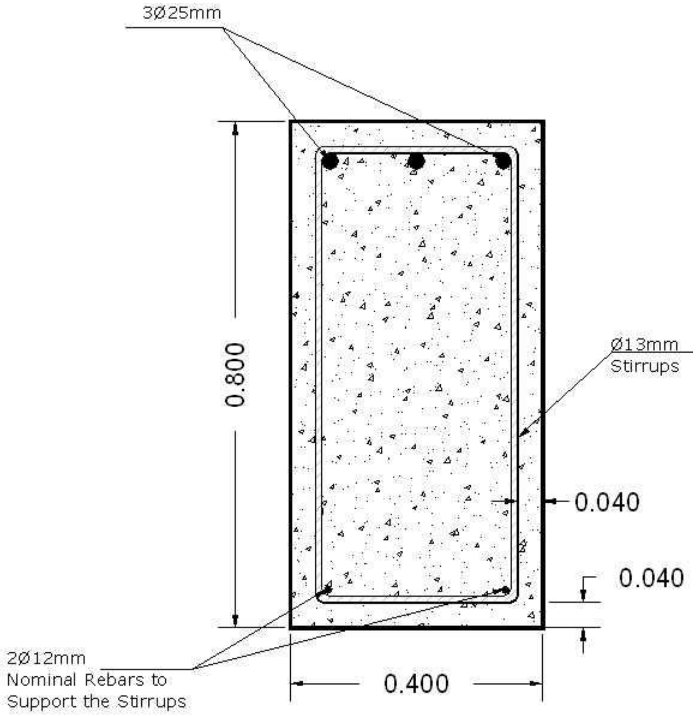
- Check for Smax:

$$s = 137\ mm < s_{max}$$

- Check with $A_{s\ minimum}$ requirements:

$$A_{s\ minimum} = 979\ mm^2 < A_{S\ Provided} = 1\,470\ mm^2\ \text{Ok.}$$

- Check the assumption of $\emptyset = 0.9$:
  a. Compute steel stain based on the following relations:
     $a = 64.9\ \text{mm} \Rightarrow c = 76.3\ \text{mm} \Rightarrow \epsilon_t = 25.9 \times 10^{-3}$
  b. As $\epsilon_t \geq 0.005$, then $\emptyset = 0.9$ Ok.
- Final Reinforcement Details



Problem 4.4-4

Design a simply supported beam with a span of 6.1m to support following loads:

1. A Dead load of $8.22\frac{\text{kN}}{\text{m}}$.
2. Service live load of $24.1\ \text{kN/m}$.
3. Assume that the designer intends to use:
4. Concrete of $f'_c = 35\ \text{MPa}$.
5. Steel of $f_y = 420\text{MPa}$.
6. A width of 325mm and a height of 420mm.
7. Rebar of No. 20 for longitudinal reinforcement.
8. Rebar of No. 13 for stirrups.
9. Two layers of reinforcement.

Aim of the Problem

This problem aims to show how to design a beam within the transition zone, i.e. when the assumption of $\phi = 0.9$ is incorrect.

Answers

- Computed required factored applied moment (M_u):
  a. Moment due to Dead Loads:
     $W_{\text{Selfweight}} = 3.28\frac{\text{kN}}{\text{m}}\ W_{\text{Dead}} = 11.5\frac{\text{kN}}{\text{m}} \Rightarrow M_{\text{Dead}} = 53.5\ \text{kN.m}$
  b. Moment due to Live Load:
     $M_{\text{Live}} = 112\ \text{kN.m}$
  c. Factored Moment M_u:
     $M_u = \text{Maximum of}\ (1.4M_D\ \text{or}\ 1.2M_D + 1.6M_L) =$
     $= \text{Maximum of}\ [1.4 \times 53.5\ \ \text{or}\ (1.2 \times 53.5\ + 1.6 \times 112)]$
     $= \text{Maximum of}\ [74.9\ \text{or}\ 243] = 243\ \text{kN.m}$ ■
- Computed the required nominal or theoretical flexure strength (M_n) based on the following relation:
  $$M_n = \frac{M_u}{\emptyset}$$
  Strength reduction factored can be assumed 0.9, and checked later.
  $M_n = 270\ \text{kN.m}$

- Compute the effective beam depth "d":

  $d_{\text{for Two Layer}} = 420 - 40 - 13 - 20 - \frac{25}{2} = 334 \text{ mm}$
- Compute the Required Steel Ratio ρ_{Required}:

  $\rho_{\text{Required}} = 20.8 \times 10^{-3}$

  Check if the beam failure is secondary compression failure or compression failure:

  $\beta_1 = 0.80$

  $\rho_{\max} = 24.3 \times 10^{-3} > \rho_{\text{Required}}$ Ok. ■
- Compute the required steel area:

  $A_{S\text{ Required}} = 2\,258 \text{ mm}^2$
- Compute required rebars number:

  $A_{\text{Bar}} = 314 \text{ mm}^2$

  $\text{No. of Rebars} = 7.19$

  Try 8Ø20mm.

  $A_{S\text{ Provided}} = 2\,512 \text{ mm}^2$
- Check if the available width "b" is adequate to put the rebars in a single layer:

  $b_{\text{required}} = 2 \times 40\text{mm} + 2 \times 13 + 8 \times 20\text{mm} + 7 \times 25\text{mm} = 441\text{mm} > 325mm$

  Then the reinforcement must be put in two layers as the designer is assumed.
- Check for smax:

  $s = 70mm < s_{max}\ Ok.$
- Check with $A_{s\text{ minimum}}$ requirements:

  $A_{s\text{ minimum}} = 382 \text{ mm}^2 < A_{S\text{ Provided}} = 2\,512 \text{ mm}^2$ Ok.
- Check the assumption of $\emptyset = 0.9$:
  a. Compute steel stain based on the following relations:

     $a = 109 \text{ mm} \Rightarrow c = 136 \text{ mm} \Rightarrow \epsilon_t = 4.37 \times 10^{-3} > 4.0 \times 10^{-3}$ Ok.
  b. As $\epsilon_t < 0.005$, then:

     $\emptyset = 0.483 + 83.3\epsilon_t = 0.847$

     Then the reinforcement will be re-designed based on new Ø.
- Reinforcement Re-design Based on New Ø:
  a. Re-computed the required nominal flexure strength (M_n):

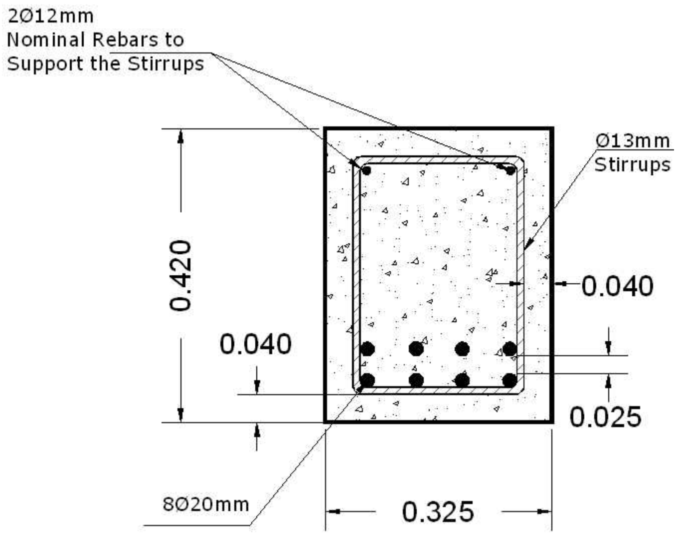
     $M_n = 287 \text{ kN.m}$
  a. Re-compute the Required Steel Ratio ρ_{Required}:

     $\rho_{\text{Required}} = 22.4 \times 10^{-3} < \rho_{\max} = 24.3 \times 10^{-3}$ Ok.
  b. Re-compute the required steel area:

     $A_{S\text{ Required}} = 2\,432 \text{ mm}^2$
  c. Compute required rebars number:

     $\text{No. of Rebars} = 7.74$

  Use 8Ø20mm.
- Final Reinforcement Details:



4.5 PRACTICAL FLEXURE DESIGN OF A RECTANGULAR BEAM WITH TENSION REINFORCEMENT ONLY AND WITH NO PRE-SPECIFIED DIMENSIONS

4.5.1 Essence of the Problem

- A second type of problems may occur when there are no previous functional or architectural limitations on beam dimensions.
- Then, the designer has ***three design parameters***, namely beam width "b", beam depth "h", and beam reinforcement.

4.5.2 Design Procedure

As we have only one relation, namely

$$M_n = \rho f_y b d^2 \left(1 - 0.59\ \frac{\rho f_y}{f_c'}\right)$$

and have three unknowns, then two assumptions to be adopted as summarized in design procedure presented below:

1. Computed the factored moment M_u based on given spans, dead, live, and other loads. Beam selfweight is assumed and checked later.
2. Computed the required nominal or theoretical flexure strength (M_n) based on the following relation:
   $$M_n = \frac{M_u}{\emptyset}$$
   Strength reduction factored can be assumed 0.9 and checked later.
   It will be shown in Step 3 below that we are usually working in the range far from ρ_{max}, then the assumption of $\Phi = 0.9$ seems fair.
3. Select a Reinforcement Ratio (***First Assumption***):
   Theoretically, reinforcement ratio can be selected anywhere between maximum and minimum steel ratios (ρ_{max} or ρ_{min}). However, based on economical and deflection requirements, it is preferred to use a reinforcement ratio in the range of:
   - **a. For Economical Purposes:**
     It can be shown, that an economical design will typically have reinforcement ratios between $0.5\rho_{max}$ to $0.75\rho_{max}$. This recommended range is based on American literature and different recommendations may be adopted for different locations or for the same location but at different periods.
   - **b. For Deflection Control:**
     From mechanics of materials it is known there is an inverse proportionality between the beam deflection, Δ, and its moment of inertia, I:
     $$\Delta \propto \frac{1}{I}$$
     As the concrete dimensions are more effective in increasing the moment of inertia, I, than the reinforcement area, therefore a larger steel ratio is used a smaller moment of inertia resulted with larger potential deflection problem.
     Following criterion can be used to determine the required steel ratio to avoid the potential deflection problem.
     $$\rho_{\text{For Deflection Control}} \le \frac{0.18 f_c'}{f_y}$$
     This criterion is based on previous ACI Code (ACI Code 1963). This stipulation was deleted in more current ACI Code. Nevertheless, it remains a valid guide for selecting a preliminary value for reinforcement ratio.
4. Solve the following relation to compute the required (bd^2):
   $$M_n = \rho f_y (bd^2)_{\text{Required}} \left(1 - 0.59\ \frac{\rho f_y}{f_c'}\right)$$
5. Experience and judgment developed over the years have also established a range of acceptable and economical depth/width ratios for rectangular beams.
   Although there is no code requirement for the d/b ratio to be within a given range, rectangular beams commonly have d/b ratios of (***Second Assumption***):

$$1.0 \le \frac{d}{b} \le 3.0$$

Desirable d/b ratios lie between:

$$1.5 \le \frac{d}{b} \le 2.2$$

6. Compute the required steel area:
   $A_{s\ Required} = \rho \times (bd)$
7. Compute the required rebars number:
   $$\text{No. of Rebars} = \frac{A_s}{A_{Bar}}$$
8. Check if rebars can be put in one or two layers:
   $$b_{required} = 2 \times \text{Side Cover} + 2 \times \text{Stirrups Diamter} + \text{No. of Rebars} \times \text{Bar Diameter} + (\text{No. of Rebars} - 1) \times \text{Spacing between Rebars}$$
   If
   $b_{required} < b_{available}$
   Then reinforcement cannot be put in a single layer.
9. Check Spacing "s" with s_{max} limitations of the ACI Code:
   If
   $s < s_{max}$
   Ok.
   Else, you should used a larger number of smaller bars.
10. Compute the required beam depth "h". Depend on reinforcement layers, one of following two relations can be used:
    $$h_{for\ One\ Layer} = d + \text{Cover} + \text{Stirrups} + \frac{\text{Bar Diameter}}{2}$$
    $$h_{for\ Two\ Layer} = d + \text{Cover} + \text{Stirrups} + \text{Bar Diameter} + \frac{\text{Spacing between Layers}}{2}$$
    Round the computed "h" to a practical number.
11. Check the Assumption of $\emptyset = 0.9$:
    In the previous sections, the strain of tensile reinforcement has been determined directly based similar triangles of the strain distribution.
    In below, another indirect method is proposed to classify the section based on computing the reinforcement ratio required to have a tensile reinforcement strain of 0.005:
    a. Compute the provided effective depth, d.
    b. Compute the provided steel ratio:
       $$\rho_{Provided} = \frac{A_{s\ Provided}}{b \times d_{Provided}}$$
    c. Compute the steel ratio required for steel strain of 0.005:
       $$\rho_{for\ \epsilon_t = 0.005} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.005}$$
       If
       $\rho_{Provided} \le \rho_{for\ \epsilon_t = 0.005}$
       Then the assumption of $\emptyset = 0.9$ is correct.
       Else, compute the more accurate value of $\emptyset$ and retain to step 2.

    This indirect approach may be used by engineers who familiar with older code versions and they do not prefer to use strains in their design calculations.
12. Draw the final reinforcement details.

4.5.3 Skin Reinforcement, (ACI318M, 2014), Article *9.7.2.3*

4.5.3.1 Aim of Skin Reinforcement

- Where h of a beam or joist exceeds 900 mm, longitudinal skin reinforcement shall be uniformly distributed along both side faces of the member.
- These rebars are necessary for crack control. Without such reinforcement, cracks widths in the web wider than those at the level of the main bars have been observed (see ***Figure 4.5-1 below***).

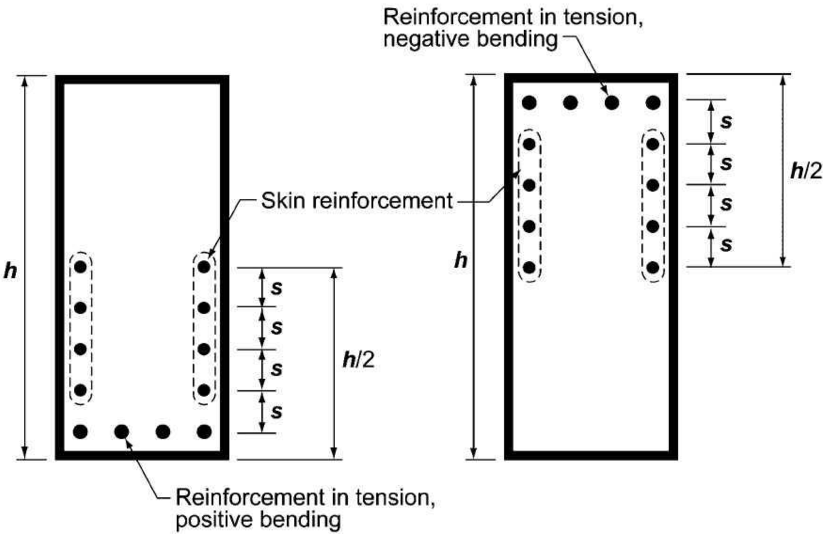


Figure 4.5-1: Skin reinforcement according to Article 9.7.2.3 of ACI code.

4.5.3.2 Extension of Skin Reinforcement

Skin reinforcement shall extend for a distance h/2 from the tension face (see Figure 4.5-1 above).

4.5.3.3 Spacing between Skin Reinforcement

The spacing, s, shall be as provided in Article **24.3.2** of ACI code (i.e. requirements for $s_{Maximum}$), where c_c is the least distance from the surface of the skin reinforcement to the side face.

4.5.3.4 Diameter for Skin Reinforcement

- The size of the skin reinforcement is not specified according to ACI Code. Research has indicated that the spacing rather than bar size is of primary importance.
- According to ACI Commentary (**R9.7.2.3**) bar sizes No. 10 to No. 16 (or welded wire reinforcement with a minimum area of 210 mm^2 per meter of depth) are typically provided.

4.5.3.5 Strength Usefulness of Skin Reinforcement

It shall be permitted to include such reinforcement in strength computations if a strain compatibility analysis is made to determine stress in the individual bars.

4.5.4 Examples

Example 4.5-1

Design a simply supported beam with a span of 7m. Service design loads can be taken as:

$$W_{Dead} = 20\frac{kN}{m}\text{(Including beam selfweight)} \quad W_{Live} = 29.0\frac{kN}{m}$$

Assume that the designer intends to use:

1. Concrete of $f_c' = 35\ \text{MPa}$.
2. Steel of $f_y = 400\text{MPa}$.
3. A reinforcement ratio of $0.5\rho_{max}$.
4. Rebar of No. 25 for longitudinal reinforcement.
5. Rebar of No. 10 for stirrups.

Solution

1. Computed the factored moment M_u:

$$M_{Dead} = \frac{20\frac{kN}{m} \times 7.0^2 m^2}{8} = 123\ kN.m \quad M_{Live} = \frac{29\frac{kN}{m} \times 7.0^2 m^2}{8} = 178\ kN.m$$

$$M_u = \text{Maximum of}\ (1.4M_D\ \text{ or } 1.2M_D + 1.6M_L)$$

$$M_u = \text{Maximum of}\ [1.4 \times 123\ \text{ or } (1.2 \times 123 + 1.6 \times 178)] = \text{Maximum of}\ [172\ \text{ or } 432]$$

$$= 432\ kN.m\ \blacksquare$$

2. Computed the required nominal or theoretical flexure strength (M_n) based on the following relation:

$$M_n = \frac{M_u}{\emptyset}$$

Strength reduction factored can be assumed 0.9 and checked later.

$$M_n = \frac{432\ kN.m}{0.9} = 480\ kN.m$$

3. Select a Reinforcement Ratio:

Assume that:

$$\rho = 0.5\rho_{max}$$

$$\rho_{max} = 0.85\beta_1\ \frac{f_c'}{f_y}\ \frac{\epsilon_u}{\epsilon_u + 0.004}$$

$$\beta_1 = 0.85 - \frac{35-28}{7} \times 0.05 = 0.80$$

$$\rho_{max} = 0.85 \times 0.80\ \frac{35}{400}\ \frac{0.003}{0.003 + 0.004} = 25.5\ \times 10^{-3}$$

$$\rho = 0.5 \times 25.5\ \times 10^{-3} = 12.8 \times 10^{-3}$$

4. Solve the following relation to compute the required (bd^2):

$$M_n = \rho f_y (bd^2)_{Required} \left(1 - 0.59\ \frac{\rho f_y}{f_c'}\right)$$

$$480 \times 10^6 N.mm = 12.8 \times 10^{-3} \times 400 \times (bd^2)_{Required} \left(1 - 0.59\ \frac{12.8 \times 10^{-3} \times 400}{35}\right)$$

$$(bd^2)_{Required} = 103 \times 10^6\ mm^3$$

Use

$$\frac{d}{b} = 2.0$$

and solve for "b":

$$(b \times (2b)^2)_{Required} = 103 \times 10^6\ mm^3$$

$$b = 294\ mm$$

Try

$$b = 300\ mm$$

Then "d" will be:

$$d = \sqrt{\frac{103 \times 10^6\ mm^3}{300\ mm}} = 586\ mm$$

5. Compute the required steel area:

$$A_{s\ Required} = 12.8 \times 10^{-3} \times (300\ mm \times 586\ mm) = 2\ 250\ mm^2$$

6. Compute the required rebars number:

$$No.of\ Rebars\ = \frac{A_s}{A_{Bar}}, A_{Bar} = \frac{\pi \times 25^2}{4} = 490mm^2\ No.of\ Rebars\ = \frac{A_s}{A_{Bar}} = \frac{2\ 250\ mm^2}{490\ mm^2} = 4.59$$

Try $5\emptyset25$.

7. Check if rebars can be put in one or two layers:

$$b_{required} = 2 \times Side\ Cover + \ 2 \times Stirrups\ Diamter + No.of\ Rebars \times Bar\ Diameter + (No.of\ Rebars - 1) \times Spacing\ between\ Rebars$$

$$b_{required} = 2 \times 40 + \ 2 \times 10 + 5 \times 25 + 4 \times 25 = \ 325 > 300$$

Then reinforcement cannot be put in a single layer and should be put in two layers (2Φ25 + 3Φ25 see Figure below).

8. Check Spacing "s" with s_{max} limitations of the ACI Code:

For beams with more than single layer of reinforcement (as in this example), ACI requires that maximum spacing limitations should be checked for the layer most closet to the tension face. Then, for this example, s_{max} will be checked for the layer that has 3Φ25. By inspection, this requirement is satisfied for the beam.

9. Compute the required beam depth "h". depend on reinforcement layers:

$$h_{for\ Two\ Layer} = \ d + Cover + Stirrups + Bar\ Diameter + \frac{Spacing\ between\ Layers}{2}$$

$$h_{for\ Two\ Layer} = \ 586\ mm + 40 + 10 + 25 + \frac{25}{2} = 673.5\ mm$$

Use $300mm \times 675mm$ with $5\emptyset25$.

10. Check the Assumption of $\emptyset = 0.9$:

a. Compute the provided effective depth:

$$d_{Provided} = 675 - 40 - 10 - 25 - \frac{25}{2} = 587 \text{ mm}$$

b. Compute the provided steel ratio:

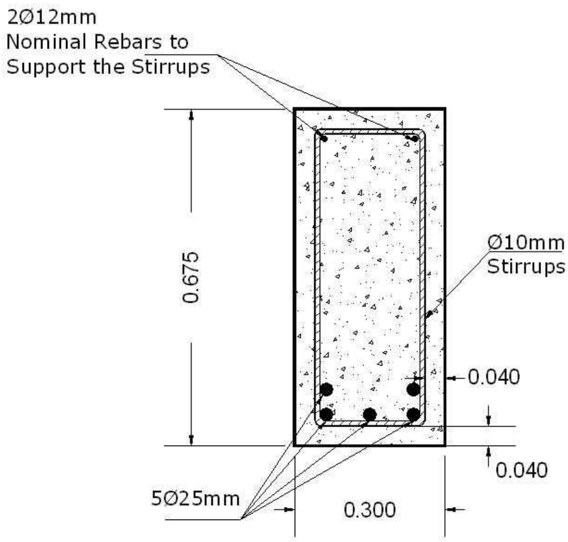
$$\rho_{Provided} = \frac{5 \times 490}{300 \times 587} = 13.9 \times 10^{-3}$$

c. Compute the steel ratio required for steel strain of 0.005:

$$\rho_{for\ \epsilon_t = 0.005} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.005} = 0.85 \times 0.80 \ \frac{35}{400}\ \frac{0.003}{0.008} = 22.3 \times 10^{-3}$$

$$> \rho_{Provided} \implies \emptyset = 0.9$$

11. Draw the final reinforcement details:



Notes:

Smaller section (i.e. a section with $\rho > 0.5\rho_{max}$) can be used if:

- Economic studies show that this section is better. As these studies depended on cost rates of concrete, steel, forms, and labor and as these rates differ from state to state, then a steel ratio that is most economical in a place may not be the best in another place. Usually these studies are out the scope of traditional courses in Reinforced Concrete Design. For more details about these issues, see for example ***Engineering Economy*** by ***Thuesen***.
- Deflection calculations show that this section is adequate based on serviceability requirements.

Example 4.5-2

Design beam shown in Figure 4.5-2 for flexure requirements according to ACI 318M-14.

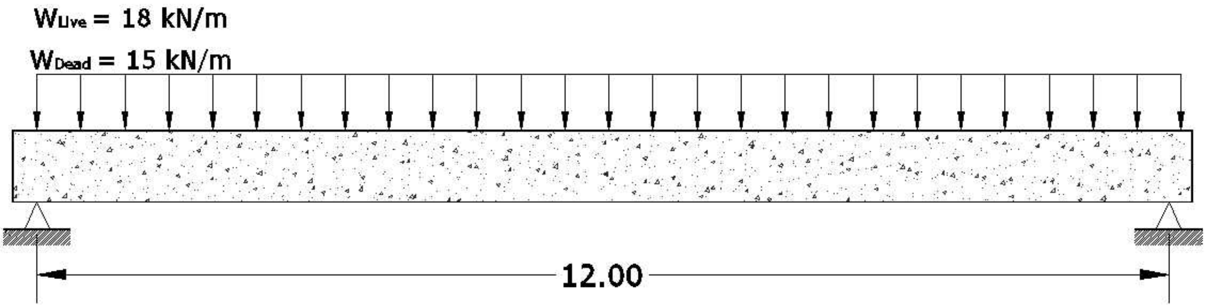


Figure 4.5-2: A Simply supported beam of Example 4.5-2.

In your solution, assume that:

1. $\rho = 0.5\rho_{max}$ (for deflection requirements).
2. Beam selfweight is 10.0 kN/m.
3. h = 1000 mm.
4. Concrete of $f'_c = 28$ MPa.
5. Steel of $f_y = 420$MPa.
6. Rebar of No. 25 for longitudinal reinforcement.
7. Rebar of No. 10 for stirrups.
8. Two layers of reinforcement.

Solution

1. Computed the factored moment M_u:
   Beam selfweight is assumed:
   $$W_{Selfweight} = 10\frac{kN}{m}$$
   Then, total dead load is:

$$W_{Dead} = 25\frac{kN}{m} \Rightarrow M_{Dead} = \frac{W_D l^2}{8} = \frac{25 \times 12^2}{8} = 450\ kN.m$$

$$M_{Live} = \frac{W_L l^2}{8} = \frac{18 \times 12^2}{8} = 324\ kN.m$$

$$M_u = \text{Maximum of } (1.4M_D \text{ or } 1.2M_D + 1.6M_L)$$

$$M_u = \text{Maximum of } [1.4 \times 450 \quad \text{or } (1.2 \times 450 + 1.6 \times 324)] = \text{Maximum of } [630 \text{ or } 1058] = 1058\ kN.m\ \blacksquare$$

2. Computed the required nominal or theoretical flexure strength (M_n) based on the following relation:

$$M_n = \frac{M_u}{\emptyset}$$

Strength reduction factored can be assumed 0.9, and checked later.

$$M_n = \frac{1058}{0.9} = 1176\ kN.m$$

3. Select a Reinforcement Ratio:
For deflection control, the designer will start with reinforcement ratio of:

$$\rho = 0.5\rho_{max}$$

$$\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004}$$

$$\beta_1 = 0.85$$

$$\rho_{max} = 0.85^2 \frac{28}{420} \frac{0.003}{0.003 + 0.004} = 20.6 \times 10^{-3}$$

$$\rho = 0.5 \times 20.6 \times 10^{-3} = 10.3 \times 10^{-3}$$

4. Solve the following relation to compute the required (bd^2):

$$M_n = \rho f_y (bd^2)_{Required}\left(1 - 0.59\ \frac{\rho f_y}{f_c'}\right)$$

$$1176 \times 10^6 = 10.3 \times 10^{-3} \times 420 (bd^2)_{Required}\left(1 - 0.59\ \frac{10.3 \times 10^{-3} \times 420}{28}\right)$$

$$(bd^2)_{Required} = 299 \times 10^6\ mm^3$$

$$d = 1000 - 40 - 10 - 25 - \frac{25}{2} = 913\ mm$$

Solve for b:

$$b = 359\ mm$$

Say

$$b = 375\ mm$$

5. Compute the required steel area:

$$A_{s\ Required} = \rho bd = 10.3 \times 10^{-3} \times 913 \times 375 =$$

$$A_{s\ Required} = 3526\ mm^2$$

6. Compute the required rebars number:

$$No. of\ Rebars = \frac{A_s}{A_{Bar}}$$

$$No. of\ Rebars = \frac{3526}{490} = 7.19$$

Try 8Ø25.

$$A_{S\ Provided} = 490\ mm^2 \times 8 = 3920\ mm^2$$

7. Check if rebars can be put in one or two layers:

$$b_{required} = 40 \times 2 + 10 \times 2 + 4 \times 25 + 3 \times 25$$

$$b_{required} = 275\ mm < 375\ mm\ Ok.$$

8. Check for s_{max}:
By inspection, one can conclude that s_{max} requirement is satisfied.
9. Check the Assumption of $\emptyset = 0.9$:
   a. Compute the provided effective depth:
   $d = 913\ mm$
   b. Compute the provided steel ratio:
   $$\rho_{Provided} = \frac{3920}{375 \times 913} = 11.5 \times 10^{-3}$$
   c. Compute the steel ratio required for steel strain of 0.005:

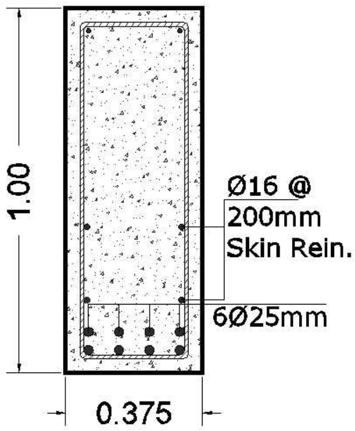
$$\rho_{for\ \epsilon_t=0.005} = 0.85^2 \frac{28}{420} \frac{0.003}{0.003+0.005}$$
$$\rho_{for\ \epsilon_t=0.005} = 18.1 \times 10^{-3}$$
$\because \rho_{Provided} < \rho_{for\ \epsilon_t=0.005}$
$\therefore \emptyset = 0.9$ Ok.

As reinforcement ratio is in the range of $0.5\rho_{maximum}$, then the resulting strain at failure load will be greater than 0.005. From this one can conclude that this checking only has academic value.

10. Check the assumed selfweight:

$$W_{Selfweight} = 0.375 \times 1.0 \times 24 = 9\frac{kN}{m} < 10\frac{kN}{m}\ Ok$$

11. Draw the final reinforcement details. With skin reinforcement, beam section would be as indicated in below.



Example 4.5-3

Design beam shown in Figure 4.5-3 below for flexure requirements according ACI 318M-14.

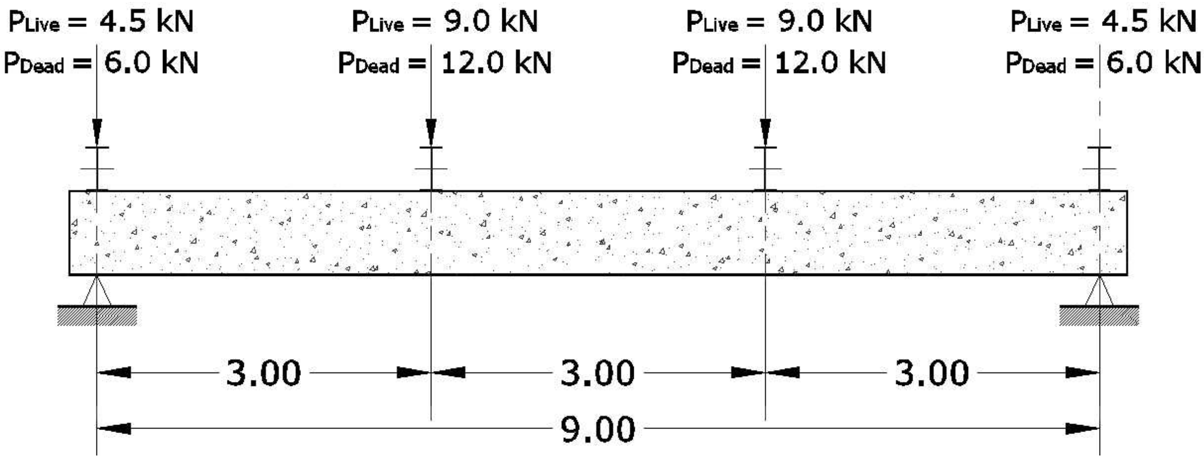


Figure 4.5-3: Simply supported beam for Example 4.5-3.

In your solution, assume that:

1. $\rho = 0.5\rho_{max}$ (for economical and serviceability requirements).
2. Beam selfweight is 3.0 kN/m.
3. b = 250 mm.
4. Concrete of $f_c' = 28$ MPa.
5. Steel of $f_y = 420$MPa.
6. Rebar of No. 25 for longitudinal reinforcement.
7. Rebar of No. 10 for stirrups.

Solution

1. Computed the factored moment M_u:

Beam selfweight is assumed:

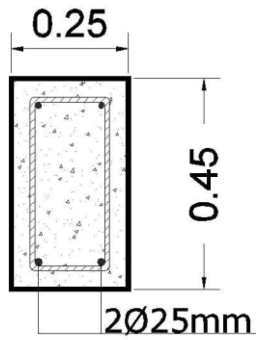
$$W_{Selfweight} = 3\frac{kN}{m}$$

Then, total dead load is:

$$M_{Dead} = \frac{3 \times 9^2}{8} + 12.0 \times 3.0 = 66.4\ kN.m$$
$$M_{Live} = 9 \times 3.0 = 27\ kN.m$$
$$M_u = \text{Maximum of } (1.4M_D \text{ or } 1.2M_D + 1.6M_L)$$
$$M_u = \text{Maximum of } [1.4 \times 66.4 \text{ or } (1.2 \times 66.4 + 1.6 \times 27)]$$
$$M_u = \text{Maximum of } [93.0 \text{ or } 123] = 123\ kN.m\ \blacksquare$$

2. Computed the required nominal or theoretical flexure strength (M_n) based on the following relation:
   $M_n = \frac{M_u}{\emptyset}$
   Strength reduction factored can be assumed 0.9, and checked later.
   $M_n = \frac{123}{0.9} = 137\ kN.m$
3. Select a Reinforcement Ratio:
   For deflection control, the designer starts with reinforcement ratio of:
   $\rho = 0.5\rho_{max}$
   $\rho_{max} = 0.85\beta_1\ \frac{f_c'}{f_y}\ \frac{\epsilon_u}{\epsilon_u + 0.004}$
   $\beta_1 = 0.85$
   $\rho_{max} = 0.85^2\ \frac{28}{420}\ \frac{0.003}{0.003 + 0.004} = 20.6\ \times 10^{-3}$
   $\rho = 0.5 \times 20.6 \times 10^{-3} = 10.3 \times 10^{-3}$
4. Solve the following relation to compute the required (bd^2):
   $M_n = \rho f_y (bd^2)_{Required}\left(1 - 0.59\ \frac{\rho f_y}{f_c'}\right)$
   $137 \times 10^6 = 10.3 \times 10^{-3} \times 420(bd^2)_{Required}$
   $\left(1 - 0.59\ \frac{10.3 \times 10^{-3} \times 420}{28}\right)$
   $(bd^2)_{Required} = 34.8 \times 10^6\ mm^3$
   $\because b = 250mm\ \therefore d\ 373\ mm$
5. Compute the required steel area:
   $A_{s\ Required} = \rho bd = 10.3 \times 10^{-3} \times 373 \times 250 =$
   $A_{s\ Required} = 960\ mm^2$
6. Compute the required rebars number:
   $No. of\ Rebars\ = \frac{A_s}{A_{Bar}}$
   $No. of\ Rebars\ = \frac{960}{490} = 1.96$
   Try 2∅25.
   $A_{S\ Provided} =\ 490\ mm^2 \times 2 = 980\ mm^2$
7. Check if rebars can be put in one or two layers:
   $b_{required} =\ 40 \times 2 + 10 \times 2 + 2 \times 25 + 25$
   $b_{required} =\ 175\ mm < 250\ mm\ Ok.$
8. Check for s_{max}:
   By inspection, one can conclude that S_{max} requirement is satisfied.
9. Compute Required "h":
   $h = 373 + \frac{25}{2} + 10 + 40 = 436\ mm$
   Say h = 450mm
10. Check the assumption of $\emptyset = 0.9$:
    a. Compute the provided effective depth:
       $d =\ 450 - 40 - 10 - \frac{25}{2} = 388\ mm$
    b. Compute the provided steel ratio:
       $\rho_{Provided} = \frac{980}{388 \times 250} = 10.1 \times 10^{-3}$
    c. Compute the steel ratio required for steel strain of 0.005:
       $\rho_{for\ \epsilon_t = 0.005} = 0.85^2\ \frac{28}{420}\ \frac{0.003}{0.003 + 0.005}$
       $\rho_{for\ \epsilon_t = 0.005} = 18.1 \times 10^{-3}$
       $\because \rho_{Provided} < \rho_{for\ \epsilon_t = 0.005}$
       $\therefore\ \emptyset = 0.9\ Ok.$

As reinforcement ratio is in the range of $0.5\rho_{maximum}$, then the resulting strain at failure load will be greater than 0.005. From this one can conclude that this checking only has academic value.



11. Check the assumed selfweight:

$$W_{Selfweight} = 0.45 \times 0.25 \times 24 = 2.7\frac{kN}{m} < 3\frac{kN}{m} \quad Ok$$

12. Draw the final reinforcement details:

4.5.5 Homework Problems

Problem 4.5-1

Design a simply supported rectangular reinforced concrete beam to carry a service dead load of $40\,kN/m$ and a service live load of $17.5\,kN/m$. The span is 12m. It is known that this beam is not exposed to weather and not in contact with ground. Select the beam preliminary steel ratio based on deflection requirements.

Assume that the designer intend to use:

1. Concrete of $f_c' = 21\,MPa$.
2. Steel of A615 Grade 60.
3. A width of 500mm.
4. Rebar of No. 25 for longitudinal reinforcement.
5. Rebar of No. 13 for stirrups.

Answers

- Computed the factored moment M_u:
  Beam selfweight is assumed:
  $$W_{Selfweight} = 8.0\frac{kN}{m}$$
  Then, total dead load is:
  $$W_{Dead} = 48\frac{kN}{m}$$
  $$M_{Dead} = 864\,kN.m$$
  $$M_{Live} = 315\,kN.m$$
  $$M_u = \text{Maximum of } (1.4M_D \text{ or } 1.2M_D + 1.6M_L)$$
  $$M_u = \text{Maximum of } [1.4 \times 864 \quad \text{or } (1.2 \times 864 + 1.6 \times 315)] =$$
  $$M_u = \text{Maximum of } [1\,210 \text{ or } 1\,541] = 1\,541\,kN.m \blacksquare$$
- Computed the required nominal or theoretical flexure strength (M_n) based on the following relation:
  $$M_n = \frac{M_u}{\emptyset}$$
  Strength reduction factored can be assumed 0.9, and checked later.
  $$M_n = 1\,712\,kN.m$$
- Select a Reinforcement Ratio:
  For deflection control, the designer starts with reinforcement ratio of:
  $$\rho = \frac{0.18\,f_c'}{f_y} = \frac{0.18 \times 21\,MPa}{420\,MPa} = 9.0 \times 10^{-3}$$
  $$\rho_{max} = 15.5 \times 10^{-3} > \rho \quad Ok.$$
- Solve the following relation to compute the required (bd^2):
  $$(bd^2)_{Required} = 507 \times 10^6\,mm^3$$
  Use b = 500mm, then "d" will be:
  $$d = 1\,007\,mm$$
- Compute the required steel area:
  $$A_{s\,Required} = 4\,532\,mm^2$$
- Compute the required rebars number:
  $$\text{No. of Rebars} = \frac{A_s}{A_{Bar}}$$

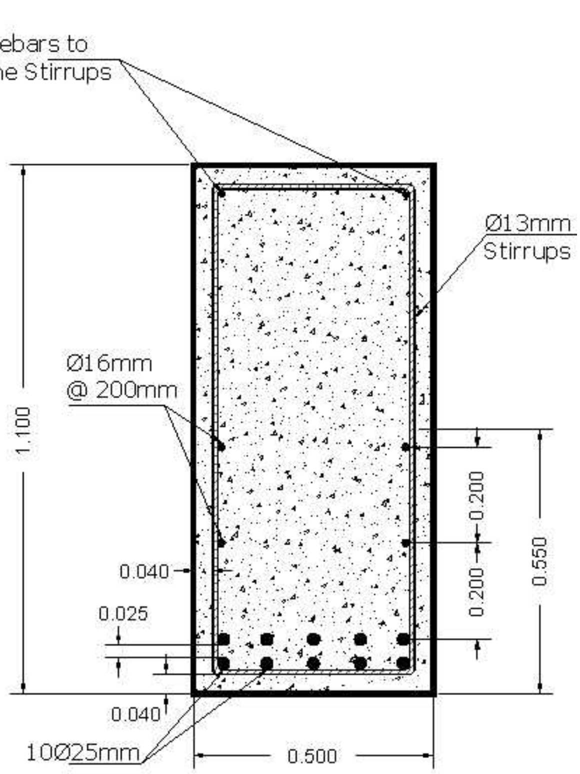
$\text{No. of Rebars} = \frac{A_s}{A_{Bar}} = 9.25$

Try $10\emptyset25$.

- Check if rebars can be put in one or two layers:
  $b_{required} = 581 > 500$
  Then reinforcement cannot be put in a single layer.
- Check for s_{max}:
  By inspection, one can conclude that s_{max} requirement is satisfied (see Figure below).
- Compute the required beam depth "h". depend on reinforcement layers:
  $h_{for\ Two\ Layer} = 1\,097.5\ mm$
  Try $500mm \times 1\,100mm$ with $10\emptyset25$.
- Check the Assumption of $\emptyset = 0.9$:
  a. Compute the provided effective depth:
     $d_{Provided} = 1\,010\ mm$
  b. Compute the provided steel ratio:
     $\rho_{Provided} = 9.7 \times 10^{-3}$
  c. Compute the steel ratio required for steel strain of 0.005:
     $\rho_{for\ \epsilon_t = 0.005} = 13.5 \times 10^{-3}$
     $\because \rho_{Provided} < \rho_{for\ \epsilon_t = 0.005}$
     $\therefore \emptyset = 0.9$ Ok.
- Check the assumed selfweight:
  $W_{Selfweight} = 13.2 \frac{kN}{m} > 8.0 \frac{kN}{m}$
  $W_{Dead} = 53.2 \frac{kN}{m}$
  $M_{Dead} = \frac{53.2 \frac{kN}{m} \times 12^2 m^2}{8} = 958\ kN.m$
  $M_u = \text{Maximum of } (1.4M_D \text{ or } 1.2M_D + 1.6M_L)$
  $M_u = \text{Maximum of } [1.4 \times 958 \text{ or } (1.2 \times 958 + 1.6 \times 315)] =$
  $M_u = \text{Maximum of } [1\,341 \text{ or } 1\,654] = 1\,654\ kN.m$ ■
  $M_n = 1\,840\ kN$
  $\emptyset M_n = 0.9 \times 1\,840\ kN = 1\,656\ kN.m > 1\,654\ kN.m\ Ok.$
- Draw the final reinforcement details:
  With skin reinforcement, beam section would as indicated in below:

ADDITIONAL NOTES:

As was discussed previously, smaller section can be used if deflection calculations show that this section is adequate.

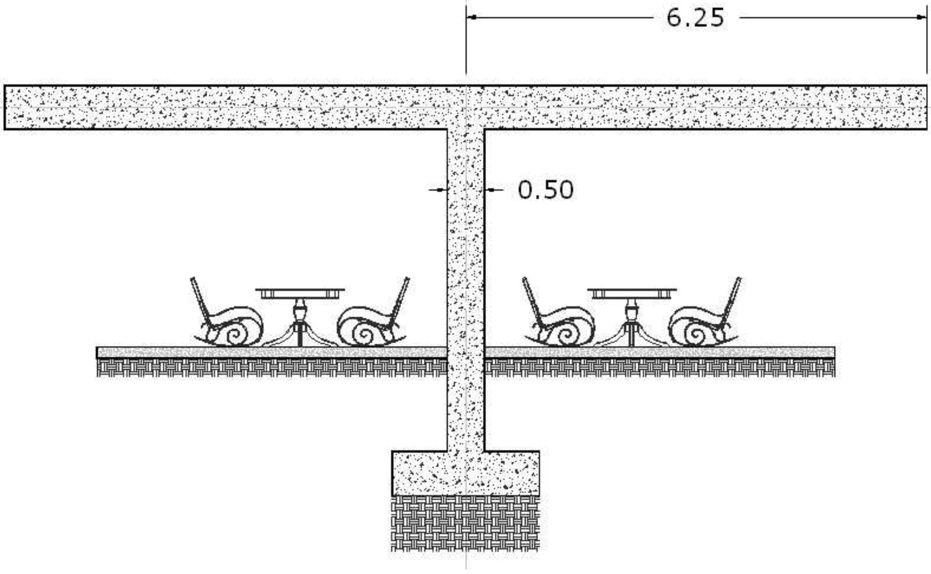


Problem 4.5-2

Design the cantilever beam of canopy structure shown in below to carry a service dead load of $50\,\mathrm{kN/m}$ and a service live load of $7.5\,\mathrm{kN/m}$. Select beam preliminary steel ratio based on deflection requirements.

Assume that the designer intends to use:

1. Concrete of $f_c' = 21\,\mathrm{MPa}$.
2. Steel of A615 Grade 60.
3. A width of 500mm.
4. Rebar of No. 25 for longitudinal reinforcement.
5. Rebar of No. 13 for stirrups.



Answers

- Computed the factored moment M_u:

  Beam selfweight is assumed:

  $W_{Selfweight} = 6.00\,\dfrac{kN}{m}$

  Then, total dead load is:

  $W_{Dead} = 56.0\,\dfrac{kN}{m}$

  $M_{Dead} = 1\,008\,\mathrm{kN.m}$

  $M_{Live} = 135\,\mathrm{kN.m}$

  $M_u = \text{Maximum of }(1.4M_D \text{ or } 1.2M_D + 1.6M_L)$

  $M_u = \text{Maximum of }[1.4 \times 1\,008 \quad \text{or } (1.2 \times 1\,008 + 1.6 \times 135)] =$

  $M_u = \text{Maximum of }[1\,411 \text{ or } 1\,426] = 1\,426\,\mathrm{kN.m}$ ■

- Computed the required nominal or theoretical flexure strength (M_n) based on the following relation:

  $M_n = \dfrac{M_u}{\emptyset}$

  Strength reduction factored can be assumed 0.9, and checked later.

  $M_n = 1\,584\,\mathrm{kN.m}$

- Select a Reinforcement Ratio:

  For deflection control, the designer starts with reinforcement ratio of:

  $\rho = \dfrac{0.18\,f_c'}{f_y} = \dfrac{0.18 \times 21\,\mathrm{MPa}}{420\,\mathrm{MPa}} = 9.0 \times 10^{-3}$

  $\rho_{max} = 15.5 \times 10^{-3} > \rho\ Ok.$

- Solve the following relation to compute the required (bd^2):

  $(bd^2)_{Required} = 469 \times 10^6\,\mathrm{mm^3}$

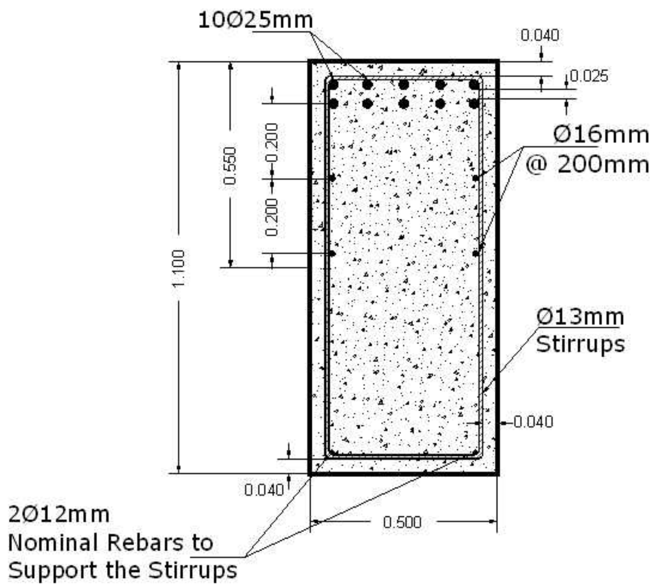
  Use b = 500mm, then "d" will be:

  $d = 969\,\mathrm{mm}$

- Compute the required steel area:

  $A_{s\,Required} = 4\,360\,\mathrm{mm^2}$

- Compute the required rebars number:
  $\text{No. of Rebars} = 9$
  Try $9Ø25$.
- Check if rebars can be put in one or two layers:
  $b_{required} = 531 > 500$
  Then reinforcement cannot be put in a single layer.
- Check for s_{max}:
  By inspection, one can conclude that s_{max} requirement is satisfied (see Figure below).
- Compute the required beam depth "h". depend on reinforcement layers:
  $h_{for\ Two\ Layer} = 1\,059.5\ mm$
  Try $500mm \times 1\,100mm$ with $9Ø25$.
- Check the Assumption of $Ø = 0.9$:
  a. Compute the provided effective depth:
     $d_{Provided} = 1\,010\ mm$
  b. Compute the provided steel ratio:
     $\rho_{Provided} = 8.73 \times 10^{-3}$
  c. Compute the steel ratio required for steel strain of 0.005:
     $\rho_{for\ \epsilon_t = 0.005} = 13.5 \times 10^{-3}$
     $\because \rho_{Provided} < \rho_{for\ \epsilon_t = 0.005}$
     $\therefore Ø = 0.9\ Ok.$
- Check the assumed selfweight:
  $W_{Selfweight} = 13.2 \frac{kN}{m} > 6.0 \frac{kN}{m}$
  $W_{Dead} = 63.2 \frac{kN}{m}$
  $M_{Dead} = 1\,138\ kN.m$
  $M_u = \text{Maximum of } (1.4M_D \text{ or } 1.2M_D + 1.6M_L)$
  $M_u = \text{Maximum of } [1.4 \times 1\,138 \text{ or } (1.2 \times 1\,138 + 1.6 \times 135)] =$
  $M_u = \text{Maximum of } [1\,593 \text{ or } 1\,582] = 1\,593\ kN.m$ ■
  $M_n = 1\,677\ kN$
  $ØM_n = 0.9 \times 1\,677\ kN = 1\,509\ kN.m$
  $< 1\,593\ kN.m\ Not\ Ok.$
  Try $500mm \times 1\,100mm$ with $10Ø25$:
  $\rho_{Provided} = 9.7 \times 10^{-3}$
  $M_n = 1\,840\ kN$
  $ØM_n = 0.9 \times 1\,840\ kN = 1\,656\ kN.m$
  $> 1\,593\ kN.m\ Ok.$
  Use $500mm \times 1\,100mm$ with $10Ø25$ ■
- Draw the final reinforcement details: With skin reinforcement, beam section is presented in below.



4.6 Analysis of a Rectangular Beam with Tension and Compression Reinforcements (a Doubly Reinforced Beam)

4.6.1 Basic Concepts

- Occasionally, beams are built with both tension reinforcement and compression reinforcement. These beams are called as beams with tension and compression reinforcement or doubly reinforced beams.
- Area and ratio of compression reinforcement have notations of A_s' and ρ' respectively (See Figure 4.6-1 below):

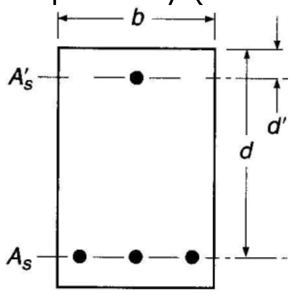


Figure 4.6-1: A doubly reinforced section.

- To be consistence with notations adopted in single reinforced beams, reinforcement ratio for tension reinforcement, ρ, is defined as:

$$\boldsymbol{\rho = \frac{A_s}{bd}}$$

- To simplify algebraic **operation through adopting same denominator, reinforcement notation for compression reinforcement, ρ', is defined as:**

$$\rho' = \frac{A_s'}{bd} \quad \textbf{Eq. 4.6-1}$$

- There are four reasons for using compression reinforcement in beams:
  - **Reduce Sustained-Load Deflection**
    First and most important, the addition of compression reinforcement reduces the long-term deflections of a beam subjected to sustained loads, see Figure 4.6-2 below.

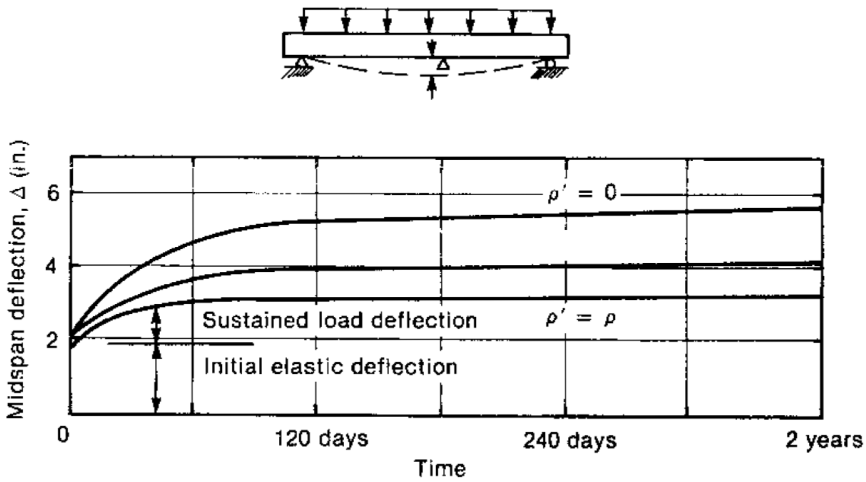


Figure 4.6-2: Compression reinforcement effects on deflection due to sustained loads.

  - **Fabrication Ease**
    When assembling the reinforcing cage for a beam, it is customary to provide bars in the corners of stirrups to hold stirrups in place in the form see Figure 4.6-3 below.

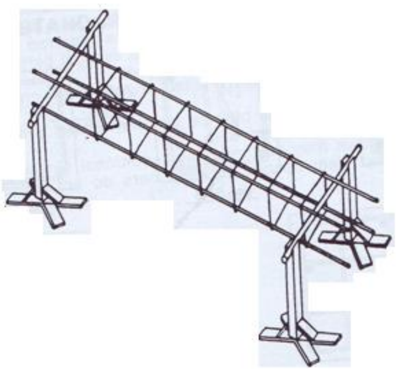


Figure 4.6-3: A reinforcement cage for fabrication ease.

- **Increase Ductility**
  It can be shown that the addition of compression reinforcement causes a reduction in the depth of the compression stress block "a". As "a" decreases the strain in the tension reinforcement at failure increases, resulting in more ductile behavior, see Figure 4.6-4 below.

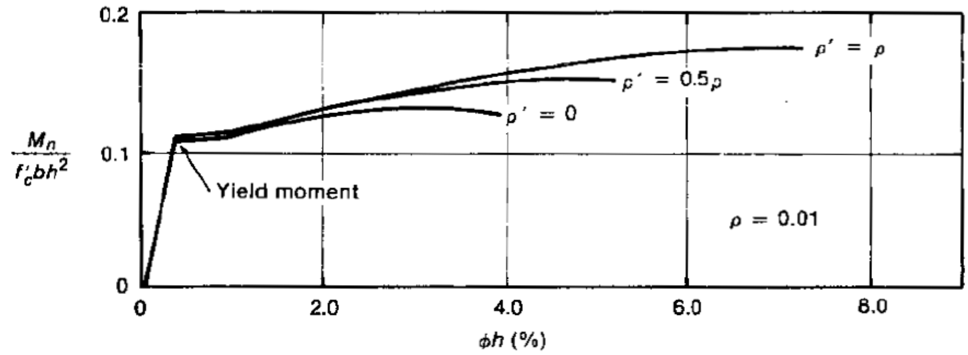


Figure 4.6-4: Ductility increase versus increasing in compression reinforcement.

- **Change the Mode of Failure from a Compression Failure to Secondary Compression Failure:**
  - When $\rho > \rho_b$, a beam fails in brittle manner through crushing of the compressive zone before the steel yields.
  - Adding of compression steel to such beam reduces the depth of the compression stress block "a".
  - As "a" decreases the strain in the tension reinforcement at failure increases, resulting in a more ductile behavior, see Figure 4.6-5 below.
  - The use of compression reinforcement for this reason has decreased markedly with use of strength design method.

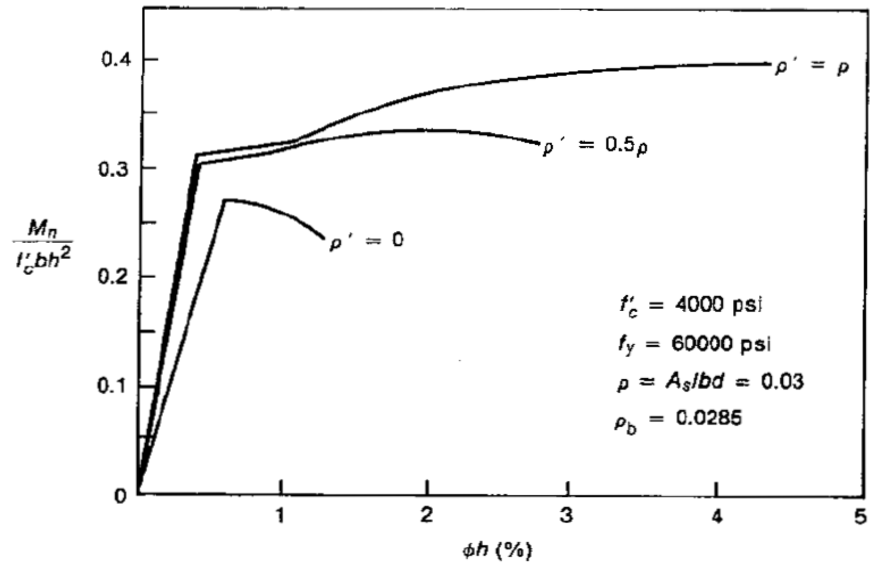


Figure 4.6-5: Changing in failure mode versus compression reinforcement.

- Analysis of a beam with tension and compression reinforcement starts with a checking to diagnose the cause for using the compression reinforcement based on the following argument. if

$$\rho > \rho_{max} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004}$$

then the compression reinforcement has been used to ***Change the Mode of Failure from Compression Failure to Secondary Compression Failure***. Then this reinforcement must be included in the beam analysis. Else, if

$$\rho < \rho_{max} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004}$$

then the compression reinforcement has been ***used either to reduce sustained-load deflection or to fabrication ease or to increase ductility*** and ***its effects can be neglected in the beam analysis***.

- Generally, compression reinforcement increases the value of maximum of steel ratio ρ_{max} and increases the value of nominal strength M_n. These effects will be discussed in paragraphs below.

4.6.2 Maximum Steel Ratio ($\bar{\rho}_{max}$) of a Rectangular Beam with Tension and Compression Reinforcement:

Based on basic tenets of ***Compatibility***, ***Stress-Strain Relation***, and ***Equilibrium***, one can prove that using of compression reinforcement increases the maximum permissible steel ratio from the value of:

$$\rho_{max} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004}$$

to a ratio of:

$$\bar{\rho}_{max} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} + \rho' \frac{f'_s}{f_y}$$

or

$$\bar{\rho}_{max} = \rho_{max} + \rho' \frac{f'_s}{f_y} \blacksquare$$

where f'_s is stress in the compression reinforcement at strains of ρ_{max}. It can be computed from strain distribution and as shown in relation below:

$$f'_s = E_s \left[\epsilon_u - \frac{d'}{d}(\epsilon_u + 0.004)\right] \leq f_y \blacksquare$$

4.6.3 Nominal Flexure Strength of a Rectangular Beam with Tension and Compression Reinforcement:

- The M_n relation of a doubly reinforced beam depends on the yielding of compression reinforcement.
- Then there are two relations for computing of M_n,
  - One for the doubly reinforced beam with compression steel at yield stress,
  - The other for the doubly reinforced beam with compression steel below yield stress.

4.6.3.1 M_n for a Beam with Compression Steel at Yield Stress

- Strains, stresses, and forces diagrams for a beam with tension and compression reinforcement at yield stress can be summarized in Figure 4.6-6 below.

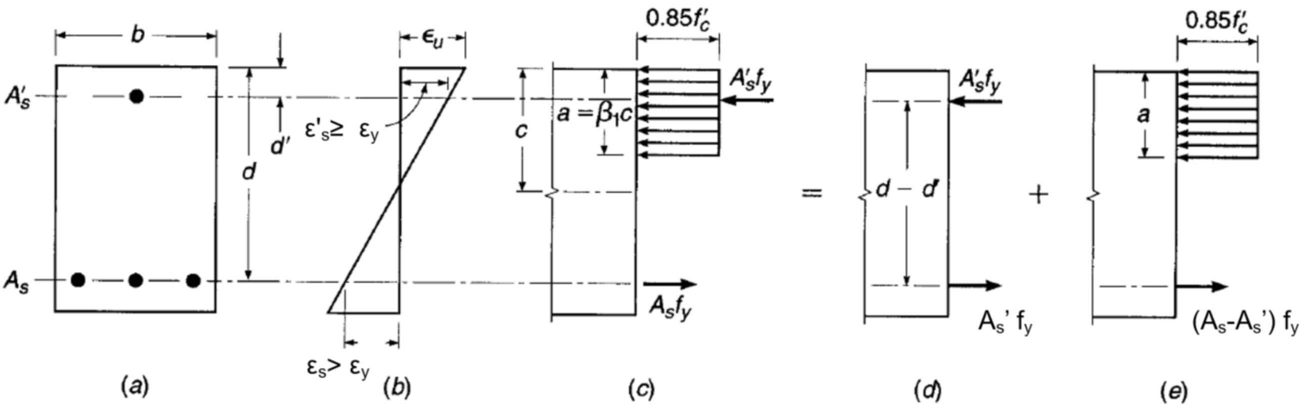


Figure 4.6-6: Strains and stresses for a doubly reinforced rectangular beam with yielded compression reinforcement.

- Then, based on superposition one can conclude that M_n for the section can be computed based on following relation:

$$\sum M_{about\ A'_s} = 0$$

$$M_n = M_{n1} + M_{n2} = A'_s f_y (d - d') + (A_s - A'_s) f_y \left(d - \frac{a}{2}\right) \blacksquare$$

where

$$a = \frac{(A_s - A_s')f_y}{0.85 f'_c b} \blacksquare$$

4.6.3.2 M_n for a Beam with Compression Steel below Yield Stress

- Strains, stresses, and forces diagrams for a beam with compression reinforcement below yield stress can be summarized in Figure 4.6-7 below.

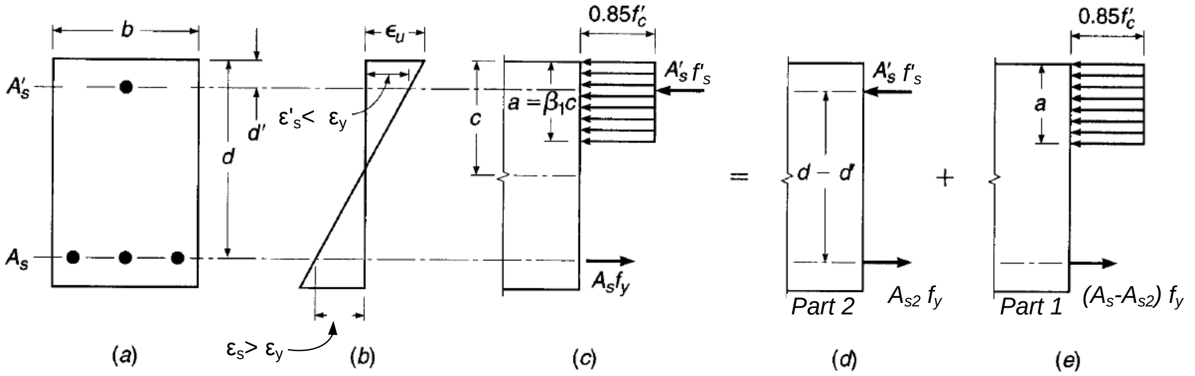


Figure 4.6-7: Strains and stresses for a doubly reinforced rectangular beam with compression reinforcement below the yield.

- Using the superposition, M_n for section can be computed based on following relation:

$$\sum M_{about\,A_s} = 0$$

$$M_n = M_{n1} + M_{n2} = 0.85 f'_c ab(d - \frac{a}{2}) + A'_s f'_s (d - d') \blacksquare$$

where "a" and f_s' can be computed as follows:

- From strain and stress diagram:

$$f'_s = \epsilon_u E_s \frac{(c-d')}{c} \blacksquare \qquad (1)$$

- From equilibrium:

$$\sum F_x = 0.0$$

$$A_s f_y = 0.85 \beta_1 f'_c bc + A'_s f'_s \qquad (2)$$

Substitute of (1) into (2):

$$A_s f_y = 0.85 \beta_1 f'_c bc + A'_s \epsilon_u E_s \frac{(c-d')}{c} \blacksquare \qquad (3)$$

- Solve this quadratic equation for "c" value:

$$c = \sqrt{Q + R^2} - R \blacksquare$$

where:

$$Q = \frac{600 d' A'_s}{0.85 \beta_1 f'_c b} \blacksquare$$

and

$$R = \frac{600 A'_s - f_y A_s}{1.7 \beta_1 f'_c b} \blacksquare$$

- Then substitute "c" into equation (1) to obtain f'_s. Finally "a" value can be computed from:

$$a = \beta_1 c$$

4.6.3.3 Criterion to Check if the Compression Steel is at Yield Stress or Not

- It is clear from above discussion; that the form of relation for computing of M_n is depended on checking of yielding of compression steel.
- Based on basic principles (Compatibility, Stress-Strain Relation, and Equilibrium), following criterion can be derived to check the yielding of compression reinforcement.

  If $\rho \geq \bar{\rho}_{cy}$, then $f'_s = f_y$ and the compression reinforcement is at yield stress.

  Else $f'_s < f_y$ and the compression reinforcement is below the yield stress.
- The minimum tensile reinforcement ratio $\bar{\rho}_{cy}$ that will ensure yielding of the compression reinforcement at failure can be computed as follows:

$$\bar{\rho}_{cy} = 0.85 \beta_1 \frac{f'_c}{f_y} \frac{d'}{d} \frac{\epsilon_u}{\epsilon_u - \epsilon_y} + \rho' \blacksquare$$

4.6.4 Ties for Compression Reinforcement

- If compression bars are used in a flexural member, precautions must be taken to ensure that these bars will not buckle outward under load spelling off the outer concrete, see Figure 4.6-8 below.

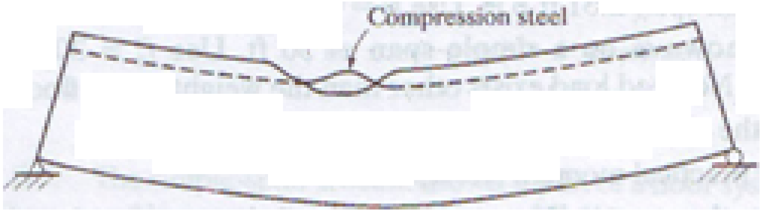


Figure 4.6-8: Buckling of beam compression reinforcement.

- ACI Code (***25.7.2.1***) imposes the requirement that such bars be anchored in the same way that compression bars in columns are anchored by lateral ties. Such ties are designed based on the following procedures:
    - Select bar diameter for ties (**25.7.2.2**):
      All bars of tied columns shall be enclosed by lateral ties at least No 10 in size for longitudinal bars up to No. 32 and at least No. 13 in size for Nos. 36, 43, and 57 and bundled longitudinal bars.
    - The spacing of the ties shall not exceed (**25.7.2.1**):
      $S_{Maximum} = \min[16d_{bar}, 48d_{ties}, \text{Least dimension of column}]$
    - Ties Arrangement (**25.7.2.3**):
      According to ACI (**25.7.2.3**), *rectilinear ties shall be arranged to satisfy (a) and (b):*
      (a) Every corner and alternate longitudinal bar shall have lateral support provided by the corner of a tie with an included angle of not more than 135 degrees.
      (b) No unsupported bar shall be farther than 150 mm clear on each side along the tie from a laterally supported bar.

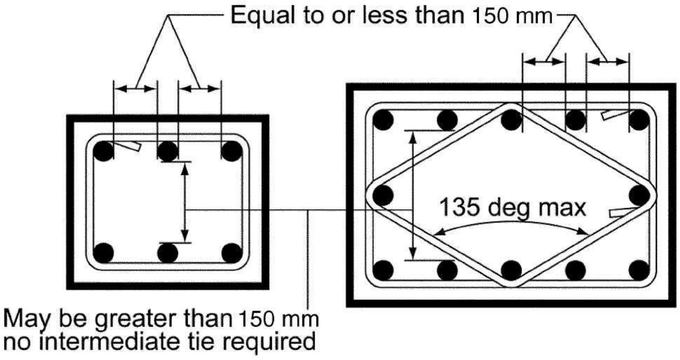


Figure 4.6-9: Ties arrangement according to requirements of ACI Code.

4.6.5 Examples

Example 4.6-1

Check the adequacy of beam shown in Figure 4.6-10 below and compute its design strength according to ACI Code. Assume that: $f_c' = 20\ \text{MPa}$ and $f_y = 300\ \text{MPa}$.

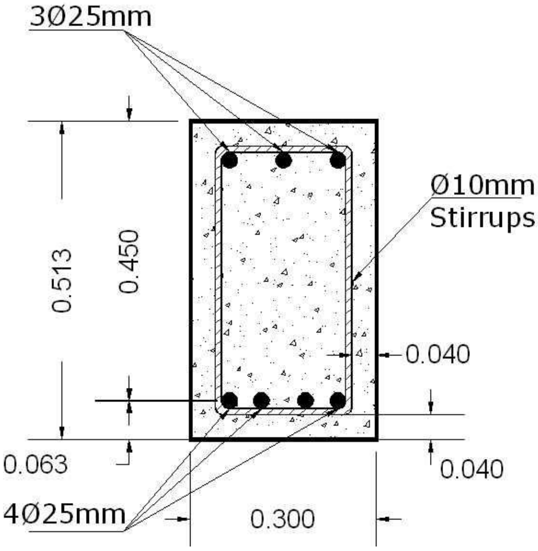


Figure 4.6-10: Cross section for beam of Example 4.6-1.

Solution

- Check the reason for using of compression reinforcement:

$$A_{s\ \text{Provided}} = 4 \times 490 = 1\,960\ \text{mm}^2 \Longrightarrow \rho_{\text{Provided}} = \frac{1\,960\ \text{mm}^2}{300 \times 450} = 14.5 \times 10^{-3}$$

$$\rho_{\max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} = 0.85 \times 0.85 \frac{20}{300} \frac{0.003}{0.003 + 0.004} = 20.6 \times 10^{-3} > \rho_{\text{Provided}}$$

Then, compression reinforcement has been added for a reason other than changing the failure mode from compression failure to secondary compression failure and its effects on section strength can be neglected. Therefore, the section can be analyzed as a singly reinforced section.

$$A_{s\ \text{minimum}} = \frac{0.25\sqrt{f_c'}}{f_y} b_w d \geq \frac{1.4}{f_y} b_w d$$

$$\because\ f_c' < 31\ MPa$$

$$\therefore A_{s\ \text{minimum}} = \frac{1.4}{f_y} b_w d = \frac{1.4}{300} \times 300 \times 450 = 630\ \text{mm}^2 < A_{s\ \text{Provided}}\ \text{Ok.} \blacksquare$$

- Compute section nominal strength M_n :

$$M_n = \rho f_y b d^2 \left(1 - 0.59\ \frac{\rho f_y}{f_c'}\right)$$

$$M_n = 14.5 \times 10^{-3} \times 300 \times 300 \times 450^2 \left(1 - 0.59\ \frac{14.5 \times 10^{-3} \times 300}{20}\right) = 230\ \text{kN.m}$$

- Compute strength reduction factor Ø:
  - Compute steel stain based on the following relations:

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{1\,960\ \text{mm}^2 \times 300\ \text{MPa}}{0.85 \times 20\ \text{MPa}\ \times 300\text{mm}} = 115\ \text{mm} \Longrightarrow c = \frac{a}{\beta_1}\frac{115\ \text{mm}}{0.85} = 135\ \text{mm}$$

$$\epsilon_t = \frac{d - c}{c}\epsilon_u = \frac{450 - 135}{135} \times 0.003 = 7.0 \times 10^{-3}$$

  - $\epsilon_t > 0.005$, then $\emptyset = 0.9$.

- Compute section design strength $\emptyset M_n$:

$$\emptyset M_n = \emptyset \times M_n = 0.9 \times 230\ \text{kN.m} = 207\ \text{kN.m} \blacksquare$$

Example 4.6-2

Check the adequacy of beam shown in Figure 4.6-11 below and compute its design strength according to ACI Code. Assume that: $f_c' = 20\ \mathrm{MPa}$ and $f_y = 300\ \mathrm{MPa}$.

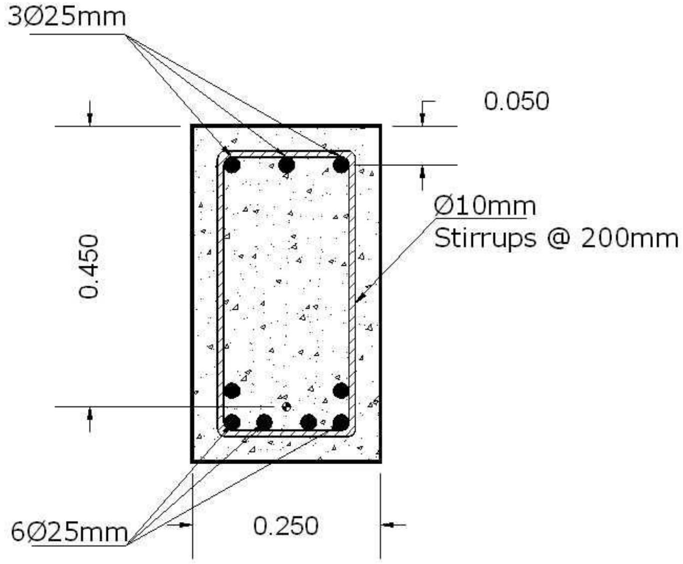


Figure 4.6-11: Cross section for beam of Example 4.6-2.

Solution

- Check the reason for using of compression reinforcement:

$$A_{s\ \text{Provided}} = 6 \times 490 = 2\,940\ \mathrm{mm}^2 \Longrightarrow \rho_{\text{Provided}} = \frac{2\,940\ \mathrm{mm}^2}{250 \times 450} = 26.1 \times 10^{-3}$$

$$\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y}\frac{\epsilon_u}{\epsilon_u + 0.004} \Longrightarrow 0.85 \times 0.85\ \frac{20}{300}\ \frac{0.003}{0.003 + 0.004} = 20.6 \times 10^{-3} < \rho_{\text{Provided}}$$

Then, compression reinforcement has been added for changing the failure mode from compression failure to secondary compression failure and its effects on section strength must be included.

- Checking the Section Type (i.e., check the effect of compression reinforcement on maximum permissible steel ratio):

$$\bar{\rho}_{max} = \rho_{max} + \rho'\frac{f_s'}{f_y}\ \blacksquare$$

where f_s' is stress in the compression reinforcement at strains of ρ_{max}. It can be computed from strain distribution and as shown in relation below:

$$f_s' = E_s\left[\epsilon_u - \frac{d'}{d}(\epsilon_u + 0.004)\right] \le f_y\ \blacksquare$$

$$f_s' = 200\,000\mathrm{MPa}\left[0.003 - \frac{50}{450}(0.003 + 0.004)\right] = 444 > f_y$$

$$f_s' = f_y = 300\mathrm{MPa}$$

$$\therefore\ \bar{\rho}_{max} = \rho_{max} + \rho'$$

$$A_s' = 3 \times 490 = 1\,470\mathrm{mm}^2$$

$$\rho' = \frac{1\,470\mathrm{mm}^2}{250 \times 450} = 13.1 \times 10^{-3}$$

$$\therefore\ \bar{\rho}_{max} = 20.6 \times 10^{-3} + 13.1 \times 10^{-3} = 33.7 \times 10^{-3} > \rho_{\text{Provided}}\ \text{Ok.}$$

- Compute Section Nominal Strength M_n:

First of all, check if the compression reinforcement is yielded on not.

$$\bar{\rho}_{cy} = 0.85\beta_1\frac{f_c'}{f_y}\frac{d'}{d}\frac{\epsilon_u}{\epsilon_u - \epsilon_y} + \rho' = 0.85 \times 0.85\frac{20}{300}\frac{50}{450}\frac{0.003}{0.003 - \dfrac{300}{200\ 000}} + 13.1 \times 10^{-3}$$

$$\bar{\rho}_{cy} = 10.7 \times 10^{-3} + 13.1 \times 10^{-3} = 23.8 \times 10^{-3} < \rho_{\text{Provided}}$$

$$\therefore f_s' = f_y = 300\mathrm{MPa}$$

Then use the relation that derived for yielded compression reinforcement:

$$M_n = M_{n1} + M_{n2} = A_s' f_y(d - d') + (A_s - A_s')f_y\left(d - \frac{a}{2}\right)$$

where,

$$a = \frac{(A_s - A_s')f_y}{0.85 f_c' b} = \frac{(2\,940 - 1\,470) \times 300}{0.85 \times 20 \times 250} = 104\mathrm{mm}$$

$$\mathrm{M_n = M_{n1} + M_{n2} = 1\,470 \times 300 \times (450 - 50) + (2\,940 - 1\,470) \times 300 \times \left(450 - \frac{104}{2}\right)}$$

$$M_n = M_{n1} + M_{n2} = 176.4 \times 10^6 N.mm + 175.5 \times 10^6 N.mm = 352\ kN.m$$

- Compute strength reduction factor $\emptyset$:
  - Compute steel stain based on the following relations:

$$\mathrm{a = 104mm \Rightarrow c = \frac{a}{\beta_1} = \frac{104\ mm}{0.85} = 122\ mm}$$

$$\epsilon_t = \frac{d-c}{c}\epsilon_u = \frac{450-122}{122} \times 0.003 = 8.06 \times 10^{-3}$$

  - $\epsilon_t > 0.005$, then $\emptyset = 0.9$.
- Compute section design strength $\emptyset M_n$:

$$\emptyset M_n = \emptyset \times M_n = 0.9 \times 352\ kN.m = 317\ kN.m \blacksquare$$

- Check Adequacy of Stirrups as Ties:

$$\because \emptyset_{for\ Longitudinal} = 25mm < No.32$$

$$\therefore \emptyset_{for\ Ties} = 10mm\ Ok.$$

$$S_{Maximum} = min[16d_{bar}, 48d_{ties}, Least\ dimension\ of\ column]$$

$$S_{Maximum} = min[16 \times 25mm, 48 \times 10mm, 250mm] = min[400mm, 480mm, 250mm]$$

$$S_{Maximum} = 250mm > S_{Provided} = 200mm\ \ Ok.$$

Checking if alternative rebar is supported or not

$$S_{Clear} = (250 - 40 \times 2 - 10 \times 2 - 3 \times 25) \times \frac{1}{2} = 37.5\ mm < 150mm\ Ok.$$

Example 4.6-3

Recheck the adequacy of the beam of Example 4.6-2 above but with d'=65 mm.

Solution

- Check the reason for using of compression reinforcement:

$$A_{s\ Provided} = 6 \times 490 = 2\,940\ mm^2 \Rightarrow \rho_{Provided} = \frac{2\,940\ mm^2}{250 \times 450} = 26.1 \times 10^{-3}$$

$$\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} = 0.85 \times 0.85\ \frac{20}{300}\ \frac{0.003}{0.003 + 0.004} = 20.6 \times 10^{-3} < \rho_{Provided}$$

Then, compression reinforcement has been added for changing the failure mode from compression failure to secondary compression failure and its effects on section strength must be included.

- Checking the Section Type (i.e., check the effect of compression reinforcement on maximum permissible steel ratio):

$$\bar{\rho}_{max} = \rho_{max} + \rho' \frac{f_s'}{f_y} \blacksquare$$

where f_s' is stress in the compression reinforcement at strains of ρ_{max}. It can be computed from strain distribution and as shown in relation below:

$$f_s' = E_s \left[\epsilon_u - \frac{d'}{d}(\epsilon_u + 0.004)\right] \leq f_y \blacksquare$$

$$f_s' = 200\,000 MPa \left[0.003 - \frac{65}{450}(0.003 + 0.004)\right] = 398 > f_y \Rightarrow f_s' = f_y = 300MPa$$

$$\therefore\ \bar{\rho}_{max} = \rho_{max} + \rho'$$

$$A_s' = 3 \times 490 = 1\,470mm^2 \Rightarrow \rho' = \frac{1\,470mm^2}{250 \times 450} = 13.1 \times 10^{-3}$$

$$\therefore\ \bar{\rho}_{max} = 20.6 \times 10^{-3} + 13.1 \times 10^{-3} = 33.7 \times 10^{-3} > \rho_{Provided}\ \ Ok.$$

- Compute of Section Nominal Strength M_n:

First of all, check if the compression reinforcement is yielded on not.

$$\bar{\rho}_{cy} = 0.85\beta_1 \frac{f_c'}{f_y}\frac{d'}{d}\frac{\epsilon_u}{\epsilon_u - \epsilon_y} + \rho' = 0.85 \times 0.85 \frac{20}{300}\frac{65}{450}\frac{0.003}{0.003 - \frac{300}{200\ 000}} + 13.1 \times 10^{-3}$$

$$\bar{\rho}_{cy} = 13.9 \times 10^{-3} + 13.1 \times 10^{-3} = 27.0 \times 10^{-3} > \rho_{Provided}$$

$$\therefore f_s' < f_y = 300MPa$$

Compute of f_s' can be done based on following relations:

  - Compute "c" based on Quadratic Formula:

$$\mathrm{c = \sqrt{Q + R^2} - R}$$

where:

$$Q = \frac{600d'A'_s}{0.85\beta_1 f'_c b} = \frac{600 \times 65 \text{ mm } \times 1\,470\text{mm}^2}{0.85 \times 0.85 \times 20 \frac{\text{N}}{\text{mm}^2} \times 250\text{mm}} = 15\,870$$

and

$$R = \frac{600A'_s - f_y A_s}{1.7\beta_1 f'_c b} = \frac{600 \times 1\,470\text{mm}^2 - 300 \times 2\,940}{1.7 \times 0.85 \times 20 \times 250} = 0$$

$$c = \sqrt{15\,870 + 0.0^2} - 0.0 = 126\text{mm}$$

- Compute f'_s can be computed based on following relation:

$$f'_s = \epsilon_u E_s \frac{(c - d')}{c} = 0.003 \times 200\,000 \times \frac{126 - 65}{126} = 290 \text{ MPa} < f_y \text{ Ok.}$$

Then use the relation that derived for yielded compression reinforcement:

$$M_n = M_{n1} + M_{n2} = 0.85f'_c ab\left(d - \frac{a}{2}\right) + A'_s f'_s (d - d')\blacksquare$$

where

$$a = \beta_1 c = 0.85 \times 126\text{mm} = 107\text{mm}$$

$$M_n = M_{n1} + M_{n2} = 0.85 \times 20 \times 107 \times 250\left(450 - \frac{107}{2}\right) + 1\,470 \times 290 \times (450 - 65)$$

$$M_n = M_{n1} + M_{n2} = 180.3 \times 10^6 \text{N.mm} + 164.1 \times 10^6 \text{N.mm} = 344 \text{ kN.m}$$

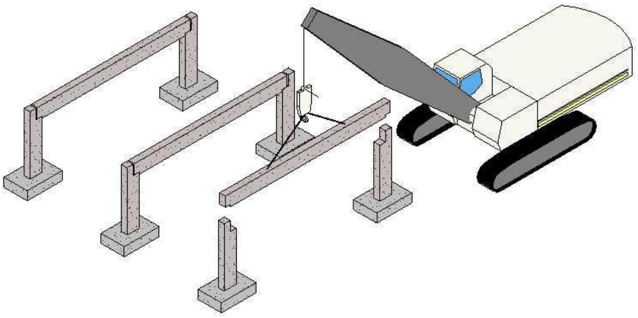
- Compute strength reduction factor $\emptyset$:
  - Compute steel stain based on the following relations:

$$c = 126\text{mm} \Rightarrow \epsilon_t = \frac{d - c}{c}\epsilon_u = \frac{450 - 126}{126} \times 0.003 = 7.71 \times 10^{-3}$$

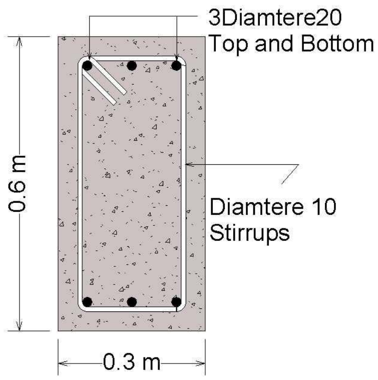
  - $\epsilon_t > 0.005$, then $\emptyset = 0.9$
- Compute section design strength $\emptyset M_n$:
  $\emptyset M_n = \emptyset \times M_n = 0.9 \times 344 \, kN.m = 310 \, kN.m\blacksquare$
- Check Adequacy of Stirrups as Ties:
  See previous example for stirrups checking when used as ties.

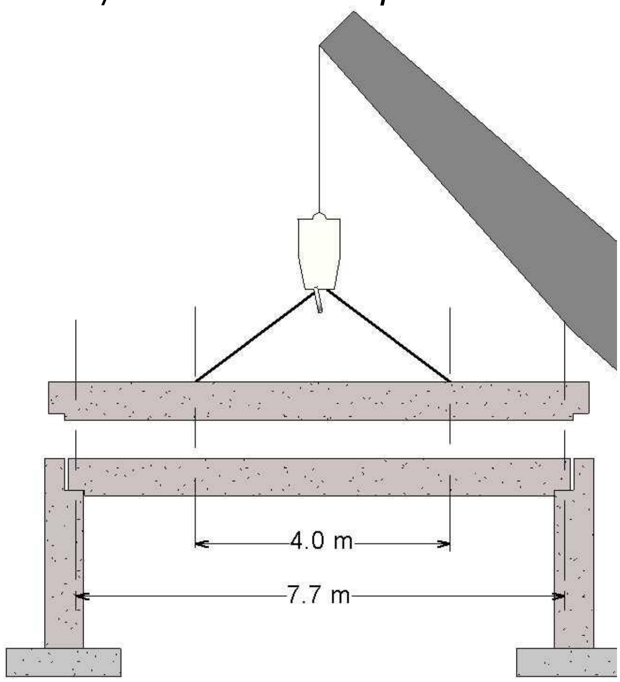
Example 4.6-4

To counteract stresses during lifting process, a simply supported precast concrete beam shown in Figure 4.6-12 below has been symmetrically reinforced with $\mathbf{3\phi 20}$ rebars.



3D View.





Beam Cross Section **Elevation View.**

Figure 4.6-12: Precast beam of Example 4.6-4.

For this precast beam:

- With including effects of compressive reinforcement in your solution, compute section nominal flexural strength ϕM_n.

- What is the maximum uniformly distributed load "Wu" that could be applied on the beam during its work?
- Are the proposed reinforcement adequate during lifting process?

In your solution, assume that, $f_c' = 28\,MPa$ and $f_y = 420\,MPa.$

Solution

- **Section Flexural Strength:**

$$A_s = A_s' = 3 \times \frac{\pi \times 20^2}{4} = 942\,mm^2$$

$$d = 600 - 40 - 10 - \frac{20}{2} = 540\,mm, d' = 40 + 10 + \frac{20}{2} = 60\,mm$$

$$\rho = \rho' = \frac{942}{300 \times 540} = 5.81 \times 10^{-3}$$

$$\rho_{max} = 0.85\beta_1\ \frac{f_c'}{f_y}\ \frac{\epsilon_u}{\epsilon_u + 0.004} = 0.85^2 \times \frac{28}{420} \times \frac{3}{7} = \ 20.6 \times 10^{-3} > \rho$$

In spite of the compression, reinforcement has been used for a reason other than change failure mode; according to problem statement, the compression reinforcement should be included within solution.

$$\because \rho = \rho'$$

$$\therefore \bar{\rho}_{cy} = 0.85\beta_1 \frac{f_c'}{f_y}\frac{d'}{d}\frac{\epsilon_u}{\epsilon_u - \epsilon_y} + \rho' > \rho$$

$$f_s' < f_y$$

$$c = \sqrt{Q + R^2} - R$$

$$Q = \frac{600d'A_s'}{0.85\beta_1 f_c' b} = \frac{600 \times 60\ \times 942}{0.85 \times 0.85 \times 28 \times 300} = 5588$$

$$R = \frac{600A_s' - f_y A_s}{1.7\beta_1 f_c' b} = \frac{600 \times 942 - 420 \times 942}{1.7 \times 0.85 \times 28 \times 300} = 13.9$$

$$c = \sqrt{Q + R^2} - R = \sqrt{5588 + 13.9^2} - 13.9 = 62.1\ \text{mm}$$

$$f_s' = \epsilon_u E_s \frac{(c - d')}{c} = 0.003 \times 200\,000 \times \frac{62.1 - 60}{62.1} = 20\ \text{MPa} < f_y\ \ \text{Ok.}$$

$$M_n = M_{n1} + M_{n2} = 0.85 f_c' ab\left(d - \frac{a}{2}\right) + A_s' f_s'(d - d')$$

$$a = \beta_1 c = 0.85 \times 62.1 = 52.8\,mm$$

$$M_n = 0.85 \times 28 \times 52.8 \times 300\left(540 - \frac{52.8}{2}\right) + 942 \times 20 \times (540 - 60) = 203\ \text{kN.m}$$

$$\epsilon_t = \frac{d - c}{c}\epsilon_u = \frac{540 - 62.1}{62.1} \times 0.003 = 23.1 \times 10^{-3} > 0.005 \Longrightarrow \phi = 0.9$$

$$\phi M_n = 0.9 \times 203 = 183\ kN.m \blacksquare$$

- **Maximum Permissible Factored Load W_u:**

$$M_u = \frac{W_u l^2}{8} = \phi M_n \Longrightarrow M_u = \frac{W_u \times 7.7^2}{8} = 183 \Longrightarrow W_u = 24.7 \frac{kN}{m} \blacksquare$$

- **Section Adequacy during Lifting Process:**

During lifting process, factored load is equal to factored dead load:

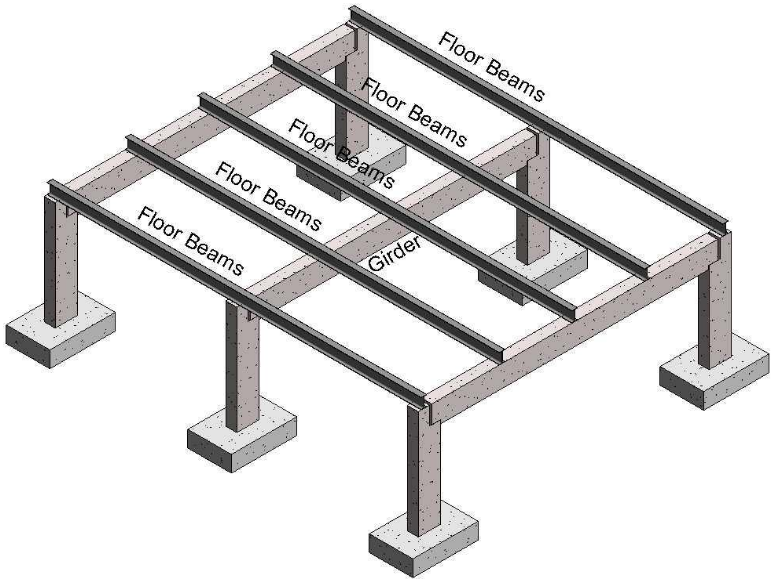
$$W_u = 1.4 W_d = 1.4 \times (24 \times 0.6 \times 0.3) = 6.05 \frac{kN}{m}$$

$$M_{u\,for\,cantilever\,part} = \frac{6.05 \times \left(0.5 \times (7.7 - 4)\right)^2}{2} = 10.4\ kN.m < \phi M_n \therefore Ok.$$

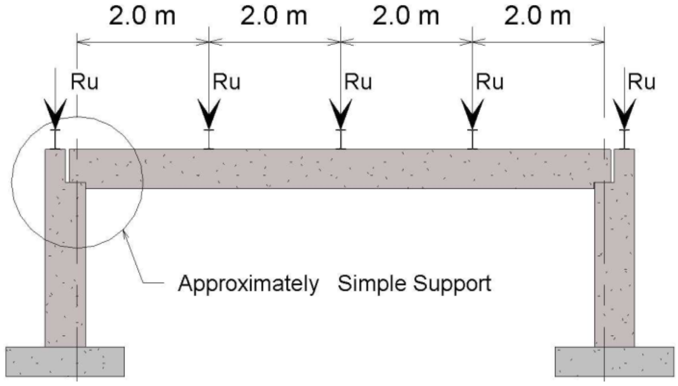
$$M_{u\,mid-span} = \frac{6.05 \times 4^2}{8} - 10.4 = 1.70 < \phi M_n \therefore Ok.$$

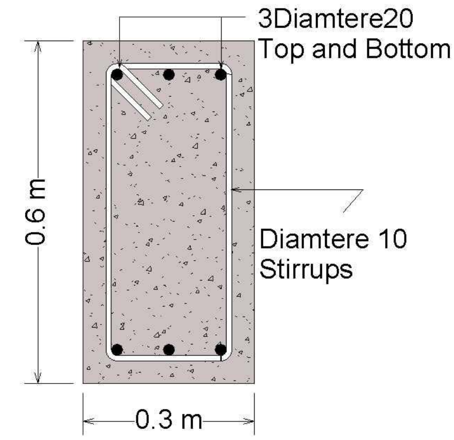
Example 4.6-5

For a frame shown in Figure 4.6-13 below, with neglecting selfweight and with including the effects of compression rebars and based on flexural strength only; what is the maximum factored floor beam reaction "Ru" that could be supported by the girder? In your solution, assume that, $f_c' = 28\,MPa$ and $f_y = 420\,MPa.$



3D View.





Elevation View. **Beam Cross Section.**

Figure 4.6-13: Frame system of Example 4.6-5.

Solution

- **<u>Section Flexural Strength:</u>**

$$A_s = A_s' = 3 \times \frac{\pi \times 20^2}{4} = 942\ mm^2$$

$$d = 600 - 40 - 10 - \frac{20}{2} = 540\ mm, d' = 40 + 10 + \frac{20}{2} = 60\ mm$$

$$\rho = \rho' = \frac{942}{300 \times 540} = 5.81 \times 10^{-3}$$

$$\rho_{max} = 0.85\beta_1\ \frac{f_c'}{f_y}\ \frac{\epsilon_u}{\epsilon_u + 0.004} = 0.85^2 \times \frac{28}{420} \times \frac{3}{7} =\ 20.6 \times 10^{-3} > \rho$$

In spite of the compression reinforcement has been used for a reason other than change failure mode, according to problem statement the compression reinforcement should be included within solution.

$$\because \rho = \rho'$$

$$\therefore \bar{\rho}_{cy} = 0.85\beta_1 \frac{f_c'}{f_y}\frac{d'}{d}\frac{\epsilon_u}{\epsilon_u - \epsilon_y} + \rho' > \rho \Longrightarrow f_s' < f_y$$

$$c = \sqrt{Q + R^2} - R$$

$$Q = \frac{600d'A_s'}{0.85\beta_1 f_c' b} = \frac{600 \times 60\ \times 942}{0.85 \times 0.85 \times 28 \times 300} = 5588, R = \frac{600A_s' - f_y A_s}{1.7\beta_1 f_c' b} = \frac{600 \times 942 - 420 \times 942}{1.7 \times 0.85 \times 28 \times 300} = 13.9$$

$$c = \sqrt{Q + R^2} - R = \sqrt{5588 + 13.9^2} - 13.9 = 62.1\ \text{mm}$$

$$f_s' = \epsilon_u E_s \frac{(c - d')}{c} = 0.003 \times 200\ 000 \times \frac{62.1 - 60}{62.1} = 20\ \text{MPa} < f_y\ \text{Ok.}$$

$$M_n = M_{n1} + M_{n2} = 0.85 f_c' ab\left(d - \frac{a}{2}\right) + A_s' f_s'(d - d')$$

$$a = \beta_1 c = 0.85 \times 62.1 = 52.8\ mm$$

$$M_n = 0.85 \times 28 \times 52.8 \times 300 \left(540 - \frac{52.8}{2}\right) + 942 \times 20 \times (540 - 60) = 203\ \text{kN.m}$$

$$\epsilon_t = \frac{d - c}{c}\epsilon_u = \frac{540 - 62.1}{62.1} \times 0.003 = 23.1 \times 10^{-3} > 0.005 \Longrightarrow \phi = 0.9$$

$$\phi M_n = 0.9 \times 203 = 183\ kN.m \blacksquare$$

- **Maximum Floor Beam Reaction:**

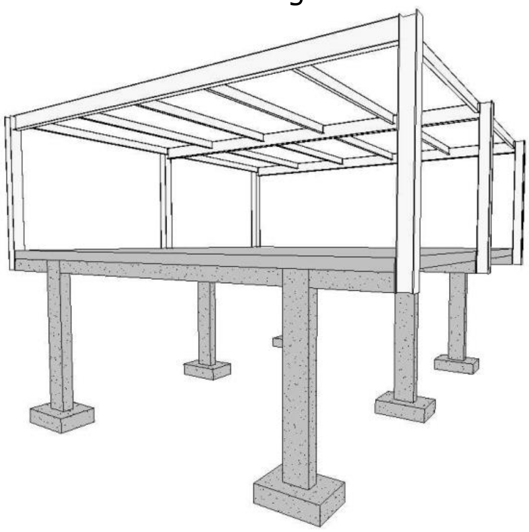
  Let $M_u = \phi M_n$

  With use of superposition principle, factored moment M_u is:

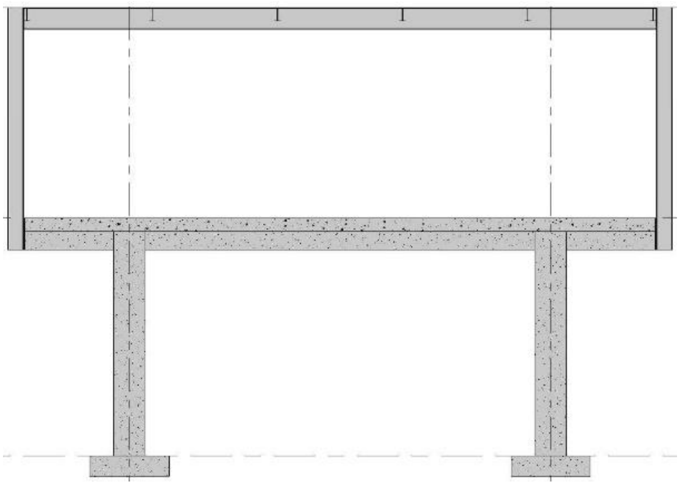
$$M_u = R_u a + \frac{R_u l}{4} = 2R_u + \frac{8}{4}R_u = 4R_u \Longrightarrow M_u = 4R_u = \phi M_n = 183 \Longrightarrow R_u = 45.8\ kN \blacksquare$$

Example 4.6-6

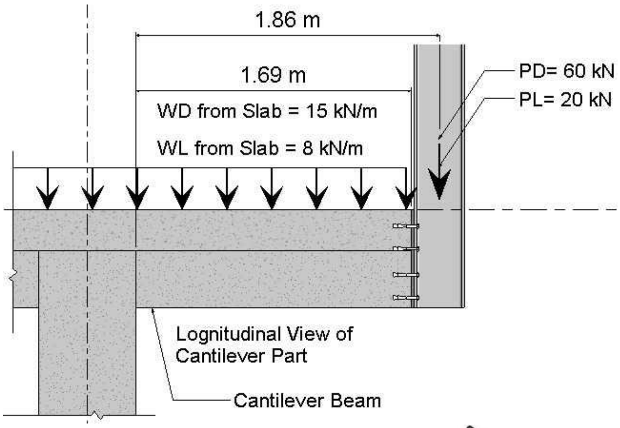
In an attempt to add a new floor for an existing reinforced concrete building, a steel frame shown in Figure 4.6-14 below has been proposed. The steel columns have been supported on cantilever concrete beams of the existing concrete floor. If the cantilever part of the beam is reinforced as shown; can it withstand the applied loads shown based on its flexural strength?

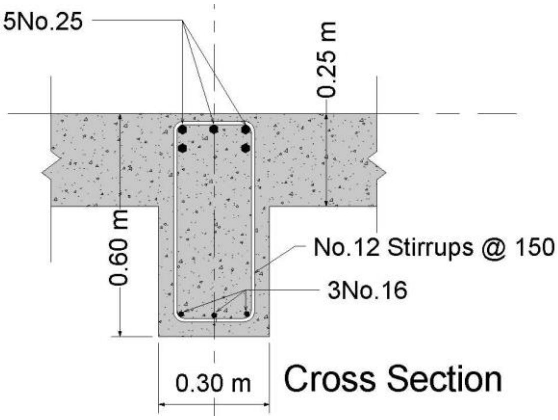


3D View



Elevation





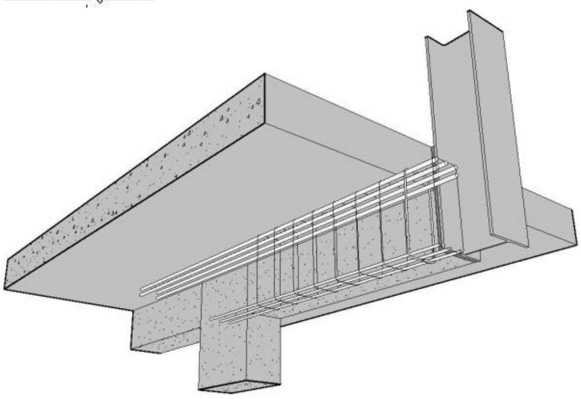


Figure 4.6-14: Building with additional new floor for Example 4.6-6.

Solution

$P_u = maximum(1.4\ P_D\ or\ 1.2P_D + 1.6P_L) = maximum(1.4 \times 60\ or\ 1.2 \times 60 + 1.6 \times 20)$

$P_u = maximum(84\ or\ 104) = 104\ kN$

$W_{self} = 0.3 \times (0.6 - 0.25) \times 24 = 2.52 \frac{kN}{m} \Longrightarrow W_D = 2.52 + 15 = 17.5 \frac{kN}{m}, W_L = 8 \frac{kN}{m}$

$W_u = maximum(1.4 \times 17.5 \; or \; 1.2 \times 17.5 + 1.6 \times 8) = maximum(24.5 \; or \; 33.8) = 33.8 \frac{kN}{m}$

$M_u = \frac{W_u l^2}{2} + P_u l = \frac{33.8 \times 1.69^2}{2} + 104 \times 1.86 = 242 \; kN.m$

Check the reason for using of compression reinforcement:

$A_{Bar} = \frac{\pi \times 25^2}{4} \approx 490 \; mm^2 \Longrightarrow A_{s\,Provided} = 5 \times 490 = 2450 \; mm^2$

$d = 600 - 40 - 12 - 25 - \frac{25}{2} = 510 \; mm \Longrightarrow \rho_{Provided} = \frac{2450}{300 \times 510} = 16.0 \times 10^{-3}$

$\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} = 0.85 \times 0.85 \frac{28}{420} \frac{0.003}{0.003 + 0.004} = 20.6 \times 10^{-3} > \rho_{Provided}$

Then, compression reinforcement has been added for a reason other than to chang the failure mode from compression failure to secondary compression failure and its effects on section strength can be neglected. Therefore, the section can be analyzed as a singly reinforced section.

As the flange is under tension, the span is a statically indeterminate one, and noting is mentioned about flange width, hence second term of second relation for $A_{s\,minimum}$ is adopted:

$A_{s\,minimum} = \frac{0.5\sqrt{f_c'}}{f_y} b_w d = \left(\frac{0.5 \times \sqrt{28}}{420}\right) \times (300 \times 510) = 9640 \; mm^2 < A_{s\,Provided}$ Ok. ■

Compute section nominal strength M_n :

$M_n = \rho f_y b d^2 \left(1 - 0.59 \frac{\rho f_y}{f_c'}\right) = 16.0 \times 10^{-3} \times 420 \times 300 \times 510^2 \times \left(1 - 0.59 \frac{16.0 \times 10^{-3} \times 420}{28}\right)$
$= 450 \; kN.m$

Compute strength reduction factor $\emptyset$:

Compute steel stain based on the following relations:

$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{2450 \times 420}{0.85 \times 28 \times 300} = 144 \; mm \Longrightarrow c = \frac{a}{\beta_1} = \frac{144}{0.85} = 169 \; mm$

$\epsilon_t = \frac{d - c}{c} \epsilon_u = \frac{510 - 169}{169} \times 0.003 = 6.05 \times 10^{-3} \Longrightarrow \emptyset = 0.9$

Compute section design strength $\emptyset M_n$:

$\emptyset M_n = \emptyset \times M_n = 0.9 \times 450 = 405 \; kN.m > M_u \therefore$ Ok.

Therefore, based on its flexural strength, cantilever part is adequate to support intended steel frame.

Example 4.6-7

Based on flexure strength of section A-A, computed the maximum value of P_u that could be supported by the beam presented in Figure 4.6-15 below. In Your solution, assume that:

- $f_c' = 21MPa$ and $f_y = 420MPa$.
- Selfweight could be neglected.
- $A_{Bar} = 500 \; mm^2 \; for \; \phi \; 25mm.$

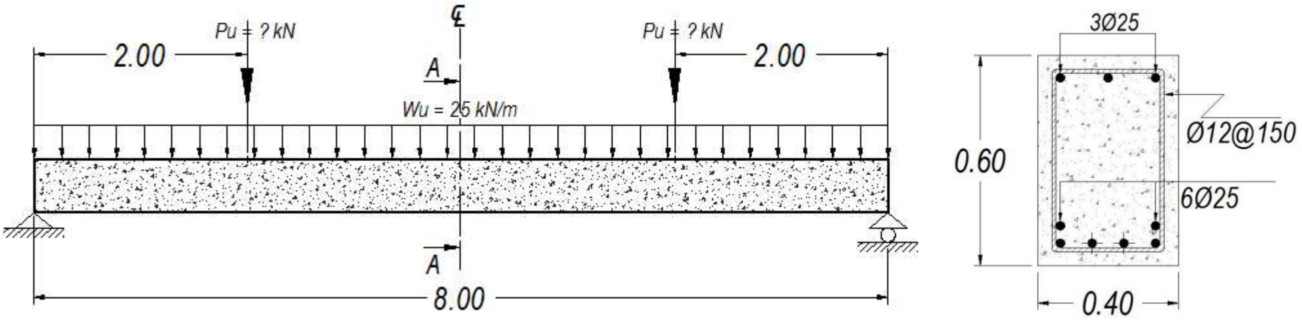


Figure 4.6-15: Simply supported beam for Example 4.6-7. **Section A-A**

Solution

- Check the reason for using of compression reinforcement:

$A_{s\,Provided} = 6 \times 500 = 3000\ mm^2, d = 600 - 40 - 12 - 25 - \frac{25}{2} = 510$

$\rho_{Provided} = \frac{3000}{400 \times 510} = 14.7 \times 10^{-3}$

$\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} = 0.85 \times 0.85 \frac{21}{420} \frac{0.003}{0.003 + 0.004} = 15.5 \times 10^{-3} > \rho_{Provided}$

Then, compression reinforcement has been added for a reason other than changing the failure mode from compression failure to secondary compression failure and its effects on section strength can be neglected.
Then the section can be analyzed as a singly reinforced section.

$A_{s\,minimum} = \frac{0.25\sqrt{f_c'}}{f_y} b_w d \geq \frac{1.4}{f_y} b_w d$

$\because f_c' < 31\ MPa$

$\therefore A_{s\,minimum} = \frac{1.4}{f_y} b_w d = \frac{1.4}{420} \times 400 \times 510 = 680\ mm^2 < A_{s\,Provided}$ Ok. ■

- Compute section nominal strength M_n :

$M_n = \rho f_y b d^2 \left(1 - 0.59 \frac{\rho f_y}{f_c'}\right)$

$M_n = 14.7 \times 10^{-3} \times 420 \times 400 \times 510^2 \left(1 - 0.59 \frac{14.7 \times 10^{-3} \times 420}{21}\right) = 531\ kN.m$

- Compute strength reduction factor $\emptyset$:
Compute steel stain based on the following relations:

$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{3000\ mm^2 \times 420\ MPa}{0.85 \times 21\ MPa \times 400mm} = 176\ mm$

$c = \frac{a}{\beta_1} = \frac{176\ mm}{0.85} = 207\ mm$

$\epsilon_t = \frac{d - c}{c}\epsilon_u = \frac{510 - 207}{207} \times 0.003 = 4.39 \times 10^{-3}$

Then:

$\emptyset = 0.483 + 83.3\epsilon_t = 0.483 + 83.3 \times 4.39 \times 10^{-3} = 0.849$

- Compute section design strength $\emptyset M_n$:

$\emptyset M_n = \emptyset \times M_n = 0.849 \times 531\ kN.m = 451\ kN.m$ ■

- Compute Pu:

$M_u = P_u \times 2.0 + \frac{25 \times 8^2}{8} = 451\ kN.m \implies P_u = 125\ kN$ ■

Example 4.6-8

Compute the maximum factored load P_u that can be supported by a beam shown in Figure 4.6-16 below. In your solution:

- Neglect the selfweight.
- $f_c' = 21\ MPa$
- $f_y = 420\ MPa$.

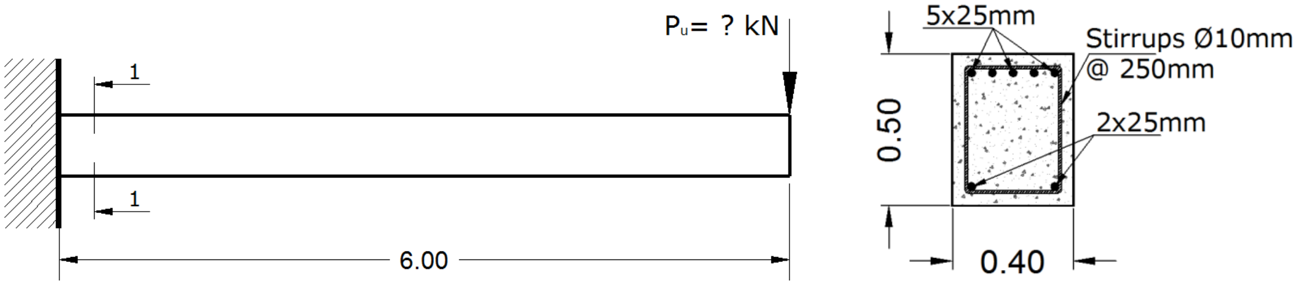


Figure 4.6-16: Cantilever beam for Example 4.6-8. **Section 1-1**

Solution

- Check the cause for using of compression reinforcement:

$A_{Bar} = \frac{\pi \times 25^2}{4} = 490\,mm^2, d = 500 - 40 - 10 - 12.5 = 437.5\,mm$

$\rho_{Provided} = \frac{490 \times 5}{437.5 \times 400} = 14 \times 10^{-3}$

$\rho_{maximum} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{0.003}{0.003 + 0.004} = 0.85^2 \frac{21}{420} \frac{0.003}{0.003 + 0.004} = 15.5 \times 10^{-3}$

As $\rho_{Provided} < \rho_{max}$, then the compression reinforcement has been used to a cause other than the flexure strength. Then the section can be analyzed as singly reinforced section.

- Compute the section flexure nominal strength and design strength:

$\sum F_x = 0$

$0.85 \times 21 \times a \times 400 = (490 \times 5) \times 420 \Rightarrow a = 144\,mm$

$M_n = (490 \times 5) \times 420 \times \left(437.5 - \frac{144}{2}\right) = 376\,kN.m$

- Strength Reduction Factor ϕ:

$c = \frac{144\,mm}{0.85} = 169\,mm \Rightarrow \epsilon_t = \frac{d - c}{c} \times \epsilon_u = \frac{437\,mm - 169\,mm}{169\,mm} \times 0.003 = 4.76 \times 10^{-3}$

$\emptyset = 0.483 + 83.3\epsilon_t = 0.483 + 83.3 \times 4.76 \times 10^{-3} = 0.879$

- Compute ϕM_n:

$\emptyset M_n = 0.879 \times 376\,kN.m = 331\,kN.m$

- Compute the maximum permissible force P_u:

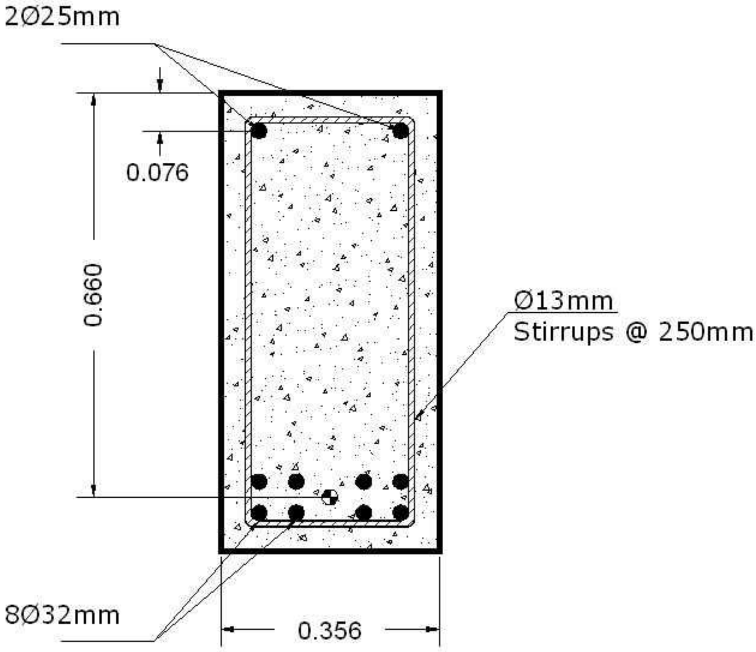
$\emptyset M_n = 331\,kN.m = P_u \times l = P_u \times 6m \Rightarrow P_u = 55.2\,kN.m$ ■

4.6.6 Homework Problems

Problem 4.6-1

Check the adequacy of the beam shown below and compute its design strength according to ACI Code. Assume that:

1. $f_c' = 34.5$ MPa.
2. $f_y = 414$MPa.
3. $A_{of\ Bar\ No.25mm} = 510mm^2$.
4. $A_{of\ Bar\ No.32mm} = 819mm^2$.



Answers

- Check the reason for using of compression reinforcement:

$A_{s\ Provided} = 6\,552\ mm^2 \Rightarrow \rho_{Provided} = 27.9 \times 10^{-3}$

$\beta_1 = 0.804 \Rightarrow \rho_{max} = 24.4 \times 10^{-3} < \rho_{Provided}$

Then, compression reinforcement has been added for changing the failure mode from compression failure to secondary compression failure and its effects on section strength must be included.

- Checking the Section Type (i.e., check the effect of compression reinforcement on maximum permissible steel ratio):

$\bar{\rho}_{max} = \rho_{max} + \rho' \frac{f'_s}{f_y}$ ■

$f'_s = f_y = 414MPa \Longrightarrow \bar{\rho}_{max} = \rho_{max} + \rho'$

$A_s' = 1\,020mm^2 \Longrightarrow \rho' = 4.34 \times 10^{-3} \Longrightarrow \therefore \bar{\rho}_{max} = 28.7 \times 10^{-3} > \rho_{Provided}$ Ok.

- Compute of Section Nominal Strength M_n:

First of all, check if the compression reinforcement is yielded on not.

$\bar{\rho}_{cy} = 25.5 \times 10^{-3} < \rho_{Provided} \Longrightarrow \therefore f'_s = f_y = 414MPa$

Then use the relation that derived for yielded compression reinforcement:

$M_n = M_{n1} + M_{n2} = A'_s f_y (d - d') + (A_s - A'_s) f_y \left(d - \frac{a}{2}\right)$

where

$a = 219mm$

$M_n = M_{n1} + M_{n2} = 247 \times 10^6 N.mm + 1\,261 \times 10^6 N.mm = 1\,508\ kN.m$

- Compute strength reduction factor $\emptyset$:

Compute steel stain based on the following relations:

$a = 219mm \Longrightarrow c = 272\ mm \Longrightarrow \epsilon_t = 4.28 \times 10^{-3}$

$\epsilon_t < 0.005$, then:

$\emptyset = 0.84$

- Compute section design strength $\emptyset M_n$:

$\emptyset M_n = 1\,267\ kN.m$ ■

- Check Adequacy of Stirrups as Ties:

$\because \emptyset_{for\ Longitudinal} = 25mm < No.32$

$\therefore \emptyset_{for\ Ties} = 13mm$ Ok.

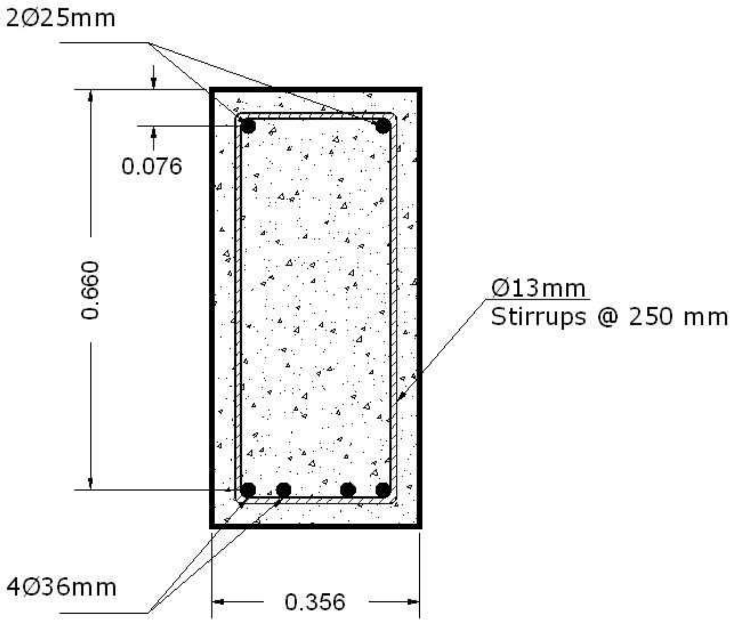
$S_{Maximum} = \min[16 d_{bar}, 48 d_{ties}, \text{Least dimension of column}]$

$S_{Maximum} = 356mm > S_{Provided} = 250mm$ Ok.

Problem 4.6-2

Check the adequacy of the beam shown below and compute its design strength according to ACI Code. Assume that:

1. $f'_c = 34.5$ MPa.
2. $f_y = 414MPa$.
3. $A_{of\ Bar\ No.25mm} = 510mm^2$.
4. $A_{of\ Bar\ No.36mm} = 1\,008mm^2$.



Answers

- Check the reason for using of compression reinforcement:

$A_{s\ Provided} = 4032mm^2$

$\rho_{Provided} = 17.2 \times 10^{-3}$

$\beta_1 = 0.804$

$\rho_{max} = 24.4 \times 10^{-3} > \rho_{Provided}$

Then, compression reinforcement has been added for a reason other than changing the failure mode from compression failure to secondary compression failure and its effects on section strength can be neglected.

Then the section can be analyzed as a singly reinforced section.

$\because f'_c > 31\ MPa \ \therefore A_{s\ minimum} = \frac{0.25\sqrt{f'_c}}{f_y} b_w d = 833\ mm^2 < A_{s\ Provided}$ Ok. ■

- Compute section nominal strength M_n:

$M_n = 970\ kN.m$

- Compute strength reduction factor ∅:
  Compute steel stain based on the following relations:
  $a = 160 \text{ mm} \Rightarrow c = 199 \text{ mm} \Rightarrow \epsilon_t = 6.95 \times 10^{-3}$
  $\epsilon_t > 0.005$, then:
  $\emptyset = 0.9$
- Compute section design strength $\emptyset M_n$:
  $\emptyset M_n = \emptyset \times M_n = 0.9 \times 970 \text{ kN.m} = 873 \text{ kN.m}$■

Problem 4.6-3

Re-compute design strength of beam above according to ACI Code with including the effect of compression reinforcement even it has been used for a purpose other than strength requirement.

Answers

- Compute of Section Nominal Strength M_n:
  First of all, check if the compression reinforcement is yielded on not.
  $\bar{\rho}_{cy} = 21.2 \times 10^{-3} + 4.31 \times 10^{-3} = 25.5 \times 10^{-3} > \rho_{Provided}$
  $\therefore f'_s < f_y$
  Compute of f'_s can be done based on following relations:
  a. Compute "c" based on Quadratic Formula:
     $c = \sqrt{Q + R^2} - R$
     where:
     $$Q = \frac{600 d' A'_s}{0.85 \beta_1 f'_c b} = 5\,541$$
     and
     $$R = \frac{600 A'_s - f_y A_s}{1.7 \beta_1 f'_c b} = -63.0$$
     $c = 160 \text{mm}$
  b. Compute $f_s^{'}$ can be computed based on following relation:
     $$f_s^{'} = \epsilon_u E_s \frac{(c - d')}{c} = 315 \text{ MPa} < f_y \text{ Ok.}$$
     Then use the relation that derived for not yielded compression reinforcement:
     $$M_n = M_{n1} + M_{n2} = 0.85 f'_c a b \left(d - \frac{a}{2}\right) + A'_s f'_s (d - d')$$■
     where
     $a = \beta_1 c = 129 \text{mm}$
     $M_n = M_{n1} + M_{n2} = 802 \times 10^6 \text{N.mm} + 188 \times 10^6 \text{N.mm} = 990 \text{ kN.m}$
- Compute strength reduction factor ∅:
  a. Compute steel stain based on the following relations:
     $c = 160 \text{mm} \Rightarrow \epsilon_t = 9.38 \times 10^{-3}$
     It is useful to note, that using of compression reinforcement has increased strain of tensile reinforcement for $\epsilon_t = 6.95 \times 10^{-3}$ to a strain of $\epsilon_t = 9.38 \times 10^{-3}$. Then using of compression reinforcement has increased section ductility (as was discussed in reasons for using of compression reinforcement).
  b. $\epsilon_t > 0.005$, then $\emptyset = 0.9$
- Compute section design strength $\emptyset M_n$:
  $\emptyset M_n = \emptyset \times M_n = 891 \text{ kN.m}$■

4.7 DESIGN OF A DOUBLY REINFORCED RECTANGULAR SECTION

4.7.1 Essence of the Problem

- This article discusses the design of a doubly reinforced concrete beam to solve a problem related to the fourth one of the four reasons discussed in previous article, i.e. this article discusses the computing of compression reinforcement A_s' when the designer needs a reinforcement ratio greater than ρ_{max} to resist the applied factored moment M_u.
- Therefore, the knowns of the design problem are:
    - Applied factored moment that must be resisted "M_u".
    - Materials strength f_c' and f_y.
    - Pre-specified beam dimensions b and h determined based on architectural or other limitations. These dimensions have been selected relatively small such that the section cannot resist the required moment with tension reinforcement only.
- While, the main unknowns of the design problem are the tension and compression reinforcements and their details. Selection of adequate stirrups that can act as ties for compression reinforcement is a part of the design process.

4.7.2 Design Procedure

This procedure has been written assuming the designer has no previous indication that the proposed dimensions are inadequate and that the section should be designed as a doubly reinforced section.

1. Compute the required factored moment M_u based on the given spans and loads. As the dimensions have been pre-specified, then beam selfweight can be computed and added to applied loads.
2. Compute the required nominal moment based on following relation:
   $$M_n = \frac{M_u}{\emptyset}$$
   where $\emptyset$ will be assumed 0.9 to be checked later.
3. Check if the section can be designed as a singly reinforced section or not based on following reasoning:
    a. If the square roof of following relation has an imaginary value, then the section cannot be designed as singly reinforced section.
    $$\rho_{Required} = \frac{1 - \sqrt{1 - 2.36 \frac{M_n}{f_c' b d^2}}}{1.18 \times \frac{f_y}{f_c'}}$$
    b. If the required steel ratio
    $$\rho_{Required} = \frac{1 - \sqrt{1 - 2.36 \frac{M_n}{f_c' b d^2}}}{1.18 \times \frac{f_y}{f_c'}}$$
    is greater than the maximum steel ratio
    $$\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004}$$
    then the section cannot be designed as singly reinforced section.
4. Re-compute the required nominal moment for the section that must be designed as a doubly reinforced section based on:
   $$M_n = \frac{M_u}{\emptyset}$$
   As the section is at tensile strain range of ρ_{max}, i.e. at tensile strain "ϵ_t" of 0.004, then the strength reduction factor would be as indicated in Figure 4.7-1 below.
   $\emptyset = 0.816$

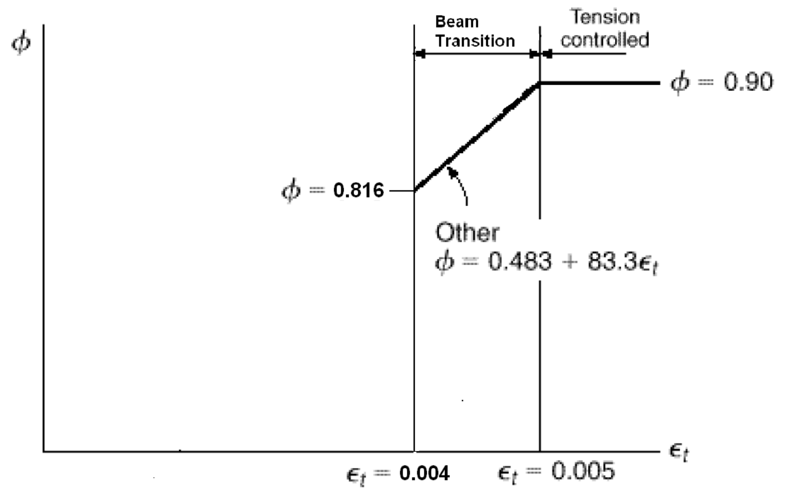


Figure 4.7-1: Strain versus strength reduction factor for beams according to ACI code, reproduced for convenience.

In design process of a doubly reinforced section, it is useful to imagine that the nominal flexure strength M_n is consisting of two parts shown below:

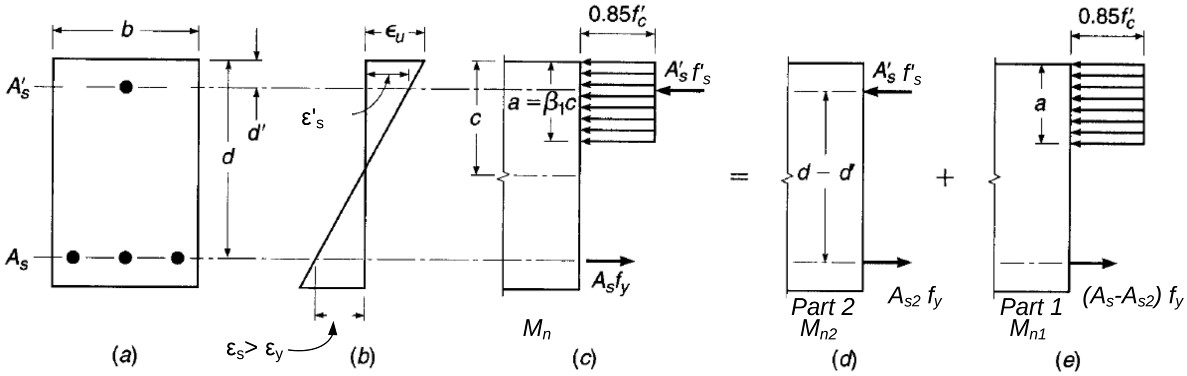


Figure 4.7-2: Strain, stress, and force distribution adopted in design of a doubly reinforced rectangular beam.

5. Compute of Tension Reinforcement A_s:
   a. Compute the nominal moment and tension reinforcement for part 1:
   $$A_{s1} = A_{smax} = \rho_{max} bd$$
   $$M_{n1} = \rho_{max} f_y bd^2 \left(1 - 0.59\,\frac{\rho_{max} f_y}{f'_c}\right)$$
   b. Compute the nominal moment and tension reinforcement for part 2:
   $$M_{n2} = M_n - M_{n1}$$
   $$A_{s2} = \frac{M_{n2}}{f_y(d - d')}$$
   c. Compute the ***Total Tension Reinforcement*** A_s:
   $$A_s = A_{s1} + A_{s2}\ \blacksquare$$
6. Compute of Compression Reinforcement A_s':
   a. Check if compression reinforcement is yielded or not:
   Compute of "a" based on force diagram of Part 1:
   $$a = \frac{A_{s1} f_y}{0.85 f_c' b}$$
   then compute "c":
   $$c = \frac{a}{\beta_1}$$
   and compute of compressive stress in compression reinforcement:
   $$f_s' = \epsilon_u E_s \frac{c - d'}{c}$$
   b. If $f_s' \geq f_y$, then the compression reinforcement has yielded:
   $$f_s' = f_y$$
   $$A_s' = A_{s2}\ \blacksquare$$
   c. Else, the compression reinforcement is not yielded:
   $$A_s' = A_{s2} \frac{f_y}{f_s'}\ \blacksquare$$

7. Compute the Required Rebars Numbers.
8. Ties Design:
   a. Select bar diameter for ties:
      If single compression rebars with diameter of:
      $\emptyset_{Bar} \leq 32mm$
      then
      $\emptyset_{Tie} = 10mm$
      else, use:
      $\emptyset_{Tie} = 13mm$
   b. Compute the required spacing of the ties:
      $S_{Required\ for\ Ties} = \min[16d_{bar}, 48d_{ties}, \text{Least dimension of column}]$
      This spacing must be checked with the shear requirement also. Actual design practice is to select "S" based on shear requirement (As will be discussed in Chapter 5) and then check its adequacy for ties requirements.
   c. Use a suitable ties arrangement as discussed previously.
9. Draw the final section details.

4.7.3 Example

Example 4.7-1

A rectangular beam, that must carry a service live load of 36.0 kN/m and a dead load of 15.3 kN/m (including its selfweight) on a simple span of 5.49 m, is limited in cross section for architectural reasons to 250mm width and 500mm depth. Design this beam for flexure. In your design, assume the following:

- $f_y = 414$ Mpa, $f_c' = 27.5$ Mpa
- No. 29 for longitudinal tension reinforcement.
- No. 19 for compression reinforcement if required.
- No. 10 for stirrups (it's adequacy must be checked when used as a tie).
- Two layers of tension reinforcement.

Solution

- Compute the required factored moment M_u:

$$M_{Dead} = \frac{15.3\frac{kN}{m} \times 5.49^2 m^2}{8} = 57.6\ kN.m \quad M_{Live} = \frac{36.0\frac{kN}{m} \times 5.49^2 m^2}{8} = 136\ kN.m$$

$$M_u = \text{maximum of } [1.4M_{Dead} \text{ or } 1.2M_{Dead} + 1.6M_{Live}]$$

$$M_u = \text{maximum of } [1.4 \times 57.6\ kN.m \text{ or } 1.2 \times 57.6\ kN.m + 1.6 \times 136\ kN.m]$$

$$M_u = \text{maximum of } [80.6\ kN.m \text{ or } 287\ kN.m] = 287\ kN.m$$

- Compute the required nominal moment based on following relation:

$$M_n = \frac{M_u}{\emptyset} = \frac{287}{0.9} = 319\ kN.m$$

where ∅ will be assumed 0.9 to be checked later.

- Check if the section can be design as a singly reinforced section or not based on following reasoning:

$$d = 500 - 40 - 10 - 29 - \frac{25}{2} = 409mm$$

$$\rho_{Required} = \frac{1 - \sqrt{1 - 2.36\frac{M_n}{f_c' bd^2}}}{1.18 \times \frac{f_y}{f_c'}} = \frac{1 - \sqrt{1 - 2.36\frac{319 \times 10^6}{27.5 \times 250 \times 409^2}}}{1.18 \times \frac{414}{27.5}} = 23.2 \times 10^{-3}$$

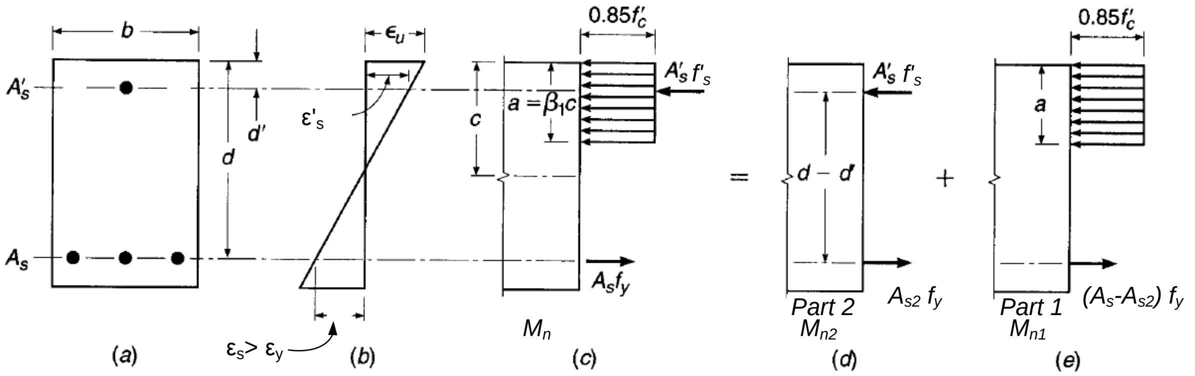
$$\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} = 0.85 \times 0.85 \frac{27.5}{414} \frac{0.003}{0.003 + 0.004} = 20.6 \times 10^{-3} < \rho_{Required}$$

then the section must be design as a doubly reinforced section.

- Re-compute the required nominal for the section based on $\emptyset = 0.816$:

$$M_n = \frac{M_u}{\emptyset} = \frac{287}{0.816} = 352\ kN.m$$

It is useful to imagine that the nominal flexure strength M_n is consisting of two parts shown below:



- Compute of Tension Reinforcement As:
    - Compute the nominal moment and tension reinforcement for part 1:

      $A_{s1} = A_{smax} = \rho_{max} bd == 20.6 \times 10^{-3} \times 250 \times 409 = 2\,106\ \text{mm}^2$

      $M_{n1} = \rho_{max} f_y bd^2 \left(1 - 0.59\,\dfrac{\rho_{max} f_y}{f_c'}\right)$

      $M_{n1} = 20.6 \times 10^{-3} \times 414 \times 250 \times 409^2 \left(1 - 0.59\,\dfrac{20.6 \times 10^{-3} \times 414}{27.5}\right) = 291\ kN.m$

    - Compute the nominal moment and tension reinforcement for part 2:

      $M_{n2} = M_n - M_{n1} = 352\ \text{kN.m} - 291\ \text{kN.m} = 61\ \text{kN.m}$

      $d' = 40 + 10 + \dfrac{19}{2} = 59.2$

      $A_{s2} = \dfrac{M_{n2}}{f_y(d - d')} = \dfrac{61 \times 10^6}{414 \times (409 - 59.2)} = 421\ \text{mm}^2$

    - Compute the Total Tension Reinforcement A_s:

      $A_s = A_{s1} + A_{s2} = 2\,106\ \text{mm}^2 + 421\ \text{mm}^2 = 2\,527\ \text{mm}^2$

- Compute of Compression Reinforcement A_s':
    - Check if compression reinforcement is yielded or not:

      Compute of "a" based on force diagram of Part 1:

      $a = \dfrac{A_{s1} f_y}{0.85 f_c{'} b} = \dfrac{2\,106\ \text{mm}^2 \times 414\ \text{MPa}}{0.85 \times 27.5\text{MPa} \times 250\text{mm}} = 149\ \text{mm}$

      then compute "c":

      $c = \dfrac{a}{\beta_1} = \dfrac{149\ \text{mm}}{0.85} = 175\ \text{mm}$

      and compute of compressive stress in compression reinforcement:

      $f_s{'} = \epsilon_u E_s \dfrac{c - d'}{c} = 0.003 \times 200\,000 \times \dfrac{175\ \text{mm} - 59.5\ \text{mm}}{175\ \text{mm}} = 396\ \text{MPa} < f_y$

    - Then compression reinforcement is not yielded and compression reinforcement will be:

      $A_s{'} = A_{s2} \dfrac{f_y}{f_s'} = 421\ \text{mm}^2 \times \dfrac{414\ \text{MPa}}{396\ \text{MPa}} = 440\ \text{mm}^2$

- Compute the Required Rebars Numbers.

  $\text{Number of Tension Rebars} = \dfrac{(2\,527\ \text{mm}^2)}{\dfrac{\pi \times 29^2}{4}} = \dfrac{(2\,527\ \text{mm}^2)}{660\ \text{mm}^2} = 3.83$

  Then use $4\emptyset 29\text{mm}$ for tension reinforcement.

  Check if these rebars can be put in one layer:

  $b_{Required} = 40 \times 2 + 10 \times 2 + 29 \times 4 + 29 \times 3 = 303\text{mm} > b_{Provided}$

  Then, the rebars must be put in two layers as the designer has assumed.

  $\text{Number of Compression Rebars} = \dfrac{(440\ \text{mm}^2)}{\dfrac{\pi \times 19^2}{4}} = \dfrac{(440\ \text{mm}^2)}{283\ \text{mm}^2} = 1.55$

  Then use $2\emptyset 19\text{mm}$ for compression reinforcement.

- Design of Required Ties:
    - Select bar diameter for ties:

      $\because \emptyset_{Bar} = 19\text{mm} < 32\text{mm}$ and single rebar.

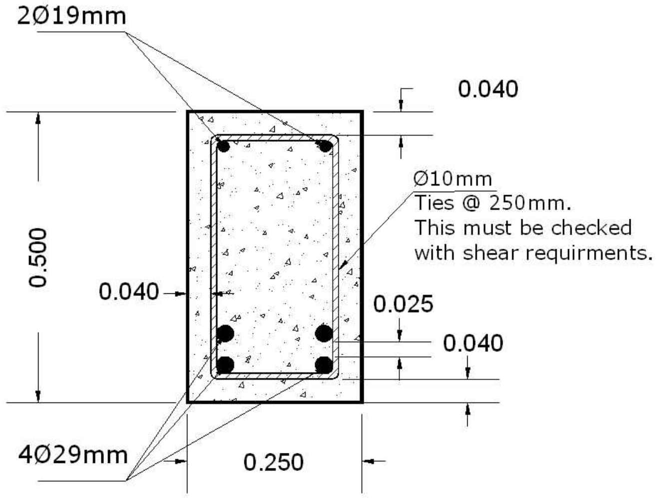
      then $\emptyset_{Tie} = 10\text{mm}\ \text{Ok}.$

Compute the required spacing of the ties:

$$S_{\text{Required for Ties}} = \min[16d_{bar}, 48d_{ties}, \text{Least dimension of column}]$$
$$= \min[16 \times 19, 48 \times 10, 250]$$
$$S_{\text{Required for Ties}} = \min[394, 480, 250] = 250 \text{ mm}$$

Use $\emptyset 10\text{mm} @ 250\text{mm}$ for ties. This spacing must be checked with shear requirement as will be discussed in Chapter 4.

- Draw the final section details:



4.7.4 Homework Problems

Problem 4.7-1

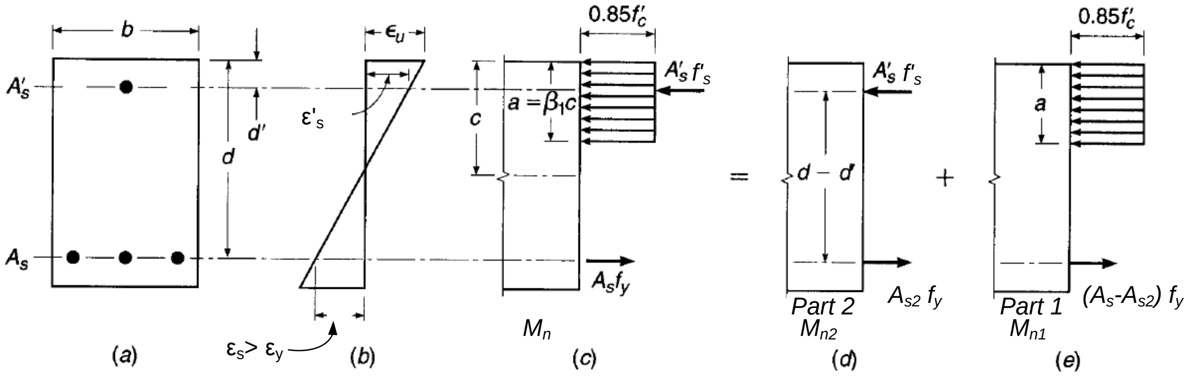
Design a rectangular beam to carry a service live load moment of 561 kN.m and a service dead load of 317 kN.m (including moment due to beam selfweight). In your design assume the following:

1. A width of 350mm and a depth of 750mm (these dimensions have been determined based on architectural limitations).
2. Materials of $f_c' = 34.5 \text{ MPa}$ and $f_y = 414 \text{ MPa}$.
3. Two layers of longitudinal reinforcement.
4. Bar diameter of 25mm for longitudinal reinforcement.
5. Bar diameter of 10 mm for stirrups.

Answers

1. Compute the required factored moment M_u:
$$M_u = \text{maximum of } [1.4M_{Dead} \text{ or } 1.2M_{Dead} + 1.6M_{Live}]$$
$$M_u = maximum\ of\ [1.4 \times 317\ kN.m\ or\ 1.2 \times 317\ kN.m + 1.6 \times 561\ kN.m]$$
$$M_u = \text{maximum of } [444 \text{ kN.m or } 1\,278 \text{kN.m}] = 1\,278 \text{ kN.m}$$
2. Compute the required nominal moment based on following relation:
$$M_n = \frac{M_u}{\emptyset} = 1\,420 \text{ kN.m}$$
where $\emptyset$ will be assumed 0.9 to be checked later.
3. Check if the section can be design as a singly reinforced section or not based on following reasoning:
$$d = 662 \text{ mm}$$
$$\rho_{Required} = \frac{1 - \sqrt{1 - 2.36 \frac{M_n}{f_c' b d^2}}}{1.18 \times \frac{f_y}{f_c'}} = 27.9 \times 10^{-3}$$
$$\beta_1 = 0.8$$
$$\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} = 24.3 \times 10^{-3} < \rho_{Required}$$
then the section must be design as doubly reinforced section
4. Re-compute the required nominal for the section based on $\emptyset = 0.816$:
$$M_n = \frac{M_u}{\emptyset} = 1\,566 \text{ kN.m}$$

The nominal flexure strength M_n is considered to consist of the two parts shown below:

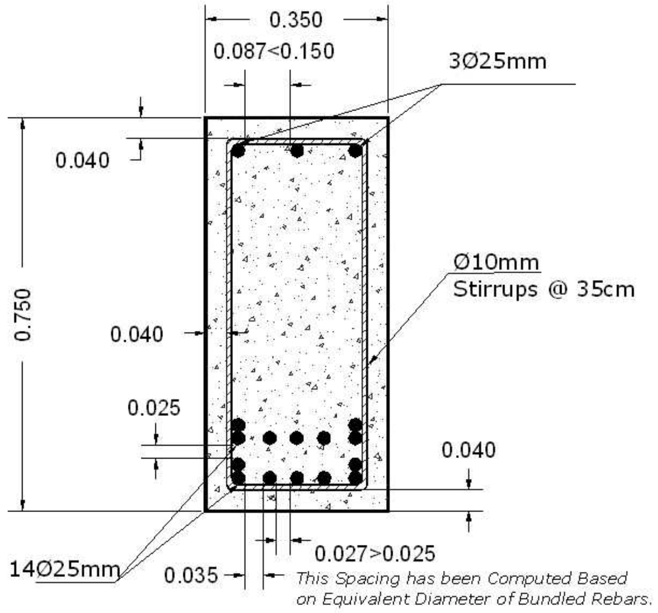


5. Compute of Tension Reinforcement As:
   a. Compute the nominal moment and tension reinforcement for part 1:
   $A_{s1} = A_{smax} = \rho_{max} bd = 5\ 630\ mm^2$
   $M_{n1} = \rho_{max} f_y bd^2 \left(1 - 0.59\ \frac{\rho_{max} f_y}{f'_c}\right) = 1\ 278\ kN.m$
   b. Compute the nominal moment and tension reinforcement for part 2:
   $M_{n2} = M_n - M_{n1} = 288\ kN.m$
   $d' = 62.5$
   $A_{s2} = \frac{M_{n2}}{f_y(d - d')} = 1\ 160\ mm^2$

   c. Compute the Total Tension Reinforcement A_s:
   $A_s = A_{s1} + A_{s2} = 6\ 790\ mm^2$
6. Compute of Compression Reinforcement A_s':
   a. Check if compression reinforcement is yielded or not:
   Compute of "a" based on force diagram of Part 1:
   $a = 227\ mm \Rightarrow c = 284\ mm \Rightarrow f_s' = \epsilon_u E_s \frac{c - d'}{c} = 468\ MPa > f_y$
   b. Then compression reinforcement is yielded and it's area will be:
   $f_s' = f_y = 414\ MPa \Rightarrow A_s' = A_{s2} = 1\ 160\ mm^2$
7. Compute the Required Rebars Numbers.
   $\text{Number of Tension Rebars} = 13.8$
   Then use $14\emptyset 25mm$ for tension reinforcement.
   Check if these rebars can be put in two layers:
   $b_{Required} = 40 \times 2 + 10 \times 2 + 7 \times 25 + 6 \times 25 = 425mm > b_{Provided}$
   Then, the rebars must be put in more than two layers. This problem can be solved through using of Bundled Bars (See Section Details)
   $\text{Number of Compression Rebars} = 2.36$
   Then use $3\emptyset 25mm$ for compression reinforcement.
8. Design of Required Ties:
   a. Select bar diameter for ties:
   $\because \emptyset_{Bar} = 25mm \leq 32mm$
   It is useful to note that ties design is depending on diameter of compression reinforcement and not on tension reinforcement. Therefore, the designer compare with diameter of compression reinforcement instead of comparison with equivalent diameter of Bundled Bars.
   Then
   $\emptyset_{Tie} = 10mm\ Ok.$
   b. Compute the required spacing of the ties:
   $S_{Required\ for\ Ties} = \min[16d_{bar}, 48d_{ties}, \text{Least dimension of column}]$
   $S_{Required\ for\ Ties} = \min[16 \times 25, 48 \times 10\ , 350] = 350\ mm$

Use $\emptyset 10\text{mm} @ 350\text{mm}$ for ties. This spacing must be checked with shear requirement as will be discussed in Chapter 5.

9. Draw the final section details:



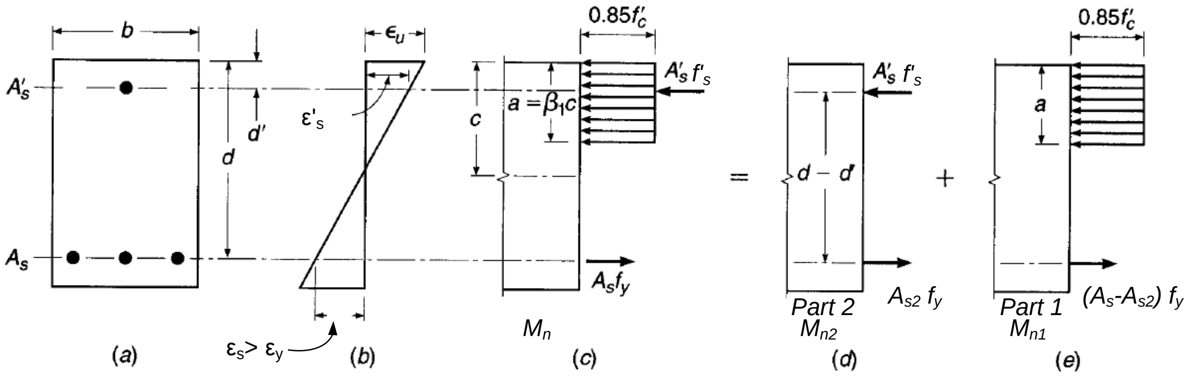
Problem 4.7-2

Resolve previous problem with using of:

1. Materials of $f_c' = 21\ \text{MPa}$ and $f_y = 420\ \text{MPa}$.
2. Two layers of longitudinal reinforcement.
3. Bar diameter of 32 mm for longitudinal reinforcement ($A_{bar} = 819\ \text{mm}^2$).
4. Bar diameter of 12 mm for stirrups.

Answers

1. Compute the required factored moment M_u:
   As for previous problem:
   $M_u = 1\,278\ kN.m$
2. Compute the required nominal moment based on following relation:
   $M_n = \frac{M_u}{\emptyset} = \frac{1\,278}{0.9} = 1\,420\ kN.m$
   where $\emptyset$ will be assumed 0.9 to be checked later.
3. Check if the section can be design as a singly reinforced section or not based on following reasoning:
   $d = 655\ mm$
   $$\rho_{Required} = \frac{1 - \sqrt{1 - 2.36\frac{M_n}{f_c' b d^2}}}{1.18 \times \frac{f_y}{f_c'}} = \frac{1 - 0.257i}{1.18 \times \frac{420}{21}}$$
   As the quantity under square root of above relation has a negative value, then the section cannot be designed as Singly Reinforced Section.
4. Re-compute the required nominal for the section based on $\emptyset = 0.816$:
   $M_n = \frac{M_u}{\emptyset} = 1\,566\ kN.m$
   The nominal flexure strength M_n is considered to consist of the two parts shown below:



5. Compute of Tension Reinforcement As:
   a. Compute the nominal moment and tension reinforcement for part 1:

$$\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} = 15.5 \times 10^{-3}$$

$$A_{s1} = A_{smax} = \rho_{max} bd = 3\,548\, mm^2$$

$$M_{n1} = \rho_{max} f_y bd^2 \left(1 - 0.59\, \frac{\rho_{max} f_y}{f_c'}\right) = 796\, kN.m$$

   b. Compute the nominal moment and tension reinforcement for part 2:

$$M_{n2} = M_n - M_{n1} = 770\, kN.m$$

$$d' = 64.5$$

$$A_{s2} = \frac{M_{n2}}{f_y(d - d')} = 3\,110\, mm^2$$

   c. Compute the Total Tension Reinforcement $\boldsymbol{A_s}$:

$$A_s = A_{s1} + A_{s2} = 3\,548\, mm^2 + 3\,110\, mm^2 = 6\,658\ \text{mm}^2$$

6. Compute of Compression Reinforcement A_s':
   a. Check if compression reinforcement is yielded or not:
   Compute of "a" based on force diagram of Part 1:

$$a = \frac{A_{s1} f_y}{0.85 f_c' b} = 239\, mm \Longrightarrow c = \frac{a}{\beta_1} = 281\, mm$$

   And compute of compressive stress in compression reinforcement:

$$f_s' = \epsilon_u E_s \frac{c - d'}{c} = 462\, MPa > f_y$$

   b. Then compression reinforcement is yielded and it's area will be:

$$A_s' = A_{s2} = 3\,110\, mm^2$$

7. Compute the Required Rebars Numbers.

$$Number\ of\ Tension\ Rebars = \frac{6\,658\ \text{mm}^2}{819} = 8.13$$

Then use $9\emptyset 32mm$ for tension reinforcement.

Check if these rebars can be put in two layers:

$$b_{Required} = 40 \times 2 + 12 \times 2 + 5 \times 32 + 4 \times 32 = 392mm > b_{Provided}\ \text{Not Ok.}$$

To solve a problem that related to distribution Bundled Rebars will be used.

$$Number\ of\ Compression\ Rebars = \frac{3\,110\, mm^2}{819\ \text{mm}^2} = 3.80$$

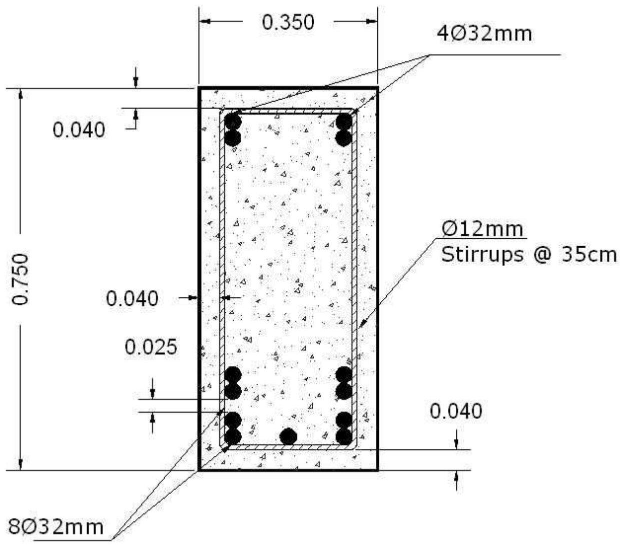


Figure 4.7-3: Detailed section for Problem 4.7-2.

Then use $4\emptyset 32mm$ for compression reinforcement.

8. Design of Required Ties:
    a. Select bar diameter for ties:
       As designer intend to use bundled compression rebars, then
       $\emptyset_{Tie} = 12\ mm\ \ Ok.$
    b. Compute the required spacing of the ties:

$S_{Required\ for\ Ties} = \min[16d_{bar}, 48d_{ties}\ , Least\ dimension\ of\ column]$

$\mathrm{S_{Required\ for\ Ties}} = \min[400, 576, 350]$

$\mathrm{S_{Required\ for\ Ties}} = 350\ \mathrm{mm}$

Use $\emptyset 12mm\ @\ 350mm$ for ties. This spacing must be checked with shear requirement as will be discussed in Chapter 4.

9. Draw the final section details as indicated in Figure 4.7-3.

4.8 FLEXURE ANALYSIS OF A SECTION WITH T SHAPE

4.8.1 Construction Stages

- During construction, the concrete in columns is placed and allowed to harden before the concrete in the floor beam is placed. In next operation, concrete is placed in the slab and beams in a monolithic pour, article 26.5.7.2 of (ACI318M, 2014).
- As a result, the slab serves as the top flange of the beam as indicated by shading area in the Figure 4.8-1 below:

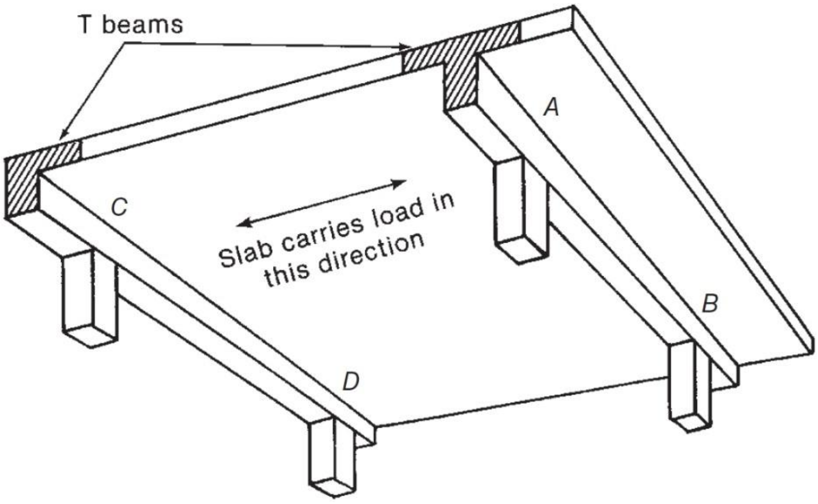


Figure 4.8-1: Slab beam interaction due to monolithic casting.

- Such a beam is referred to as a ***T Beam***. The interior beam, AB, of the Figure 4.8-1 above, has a flange on both sides. The spandrel beam, CD, with flange on one side only, is also referred to as a T Beam.

4.8.2 Behavior of Tee Beams

- An exaggerated deflected view of the interior beam "AB" is shown in Figure 4.8-2 below:

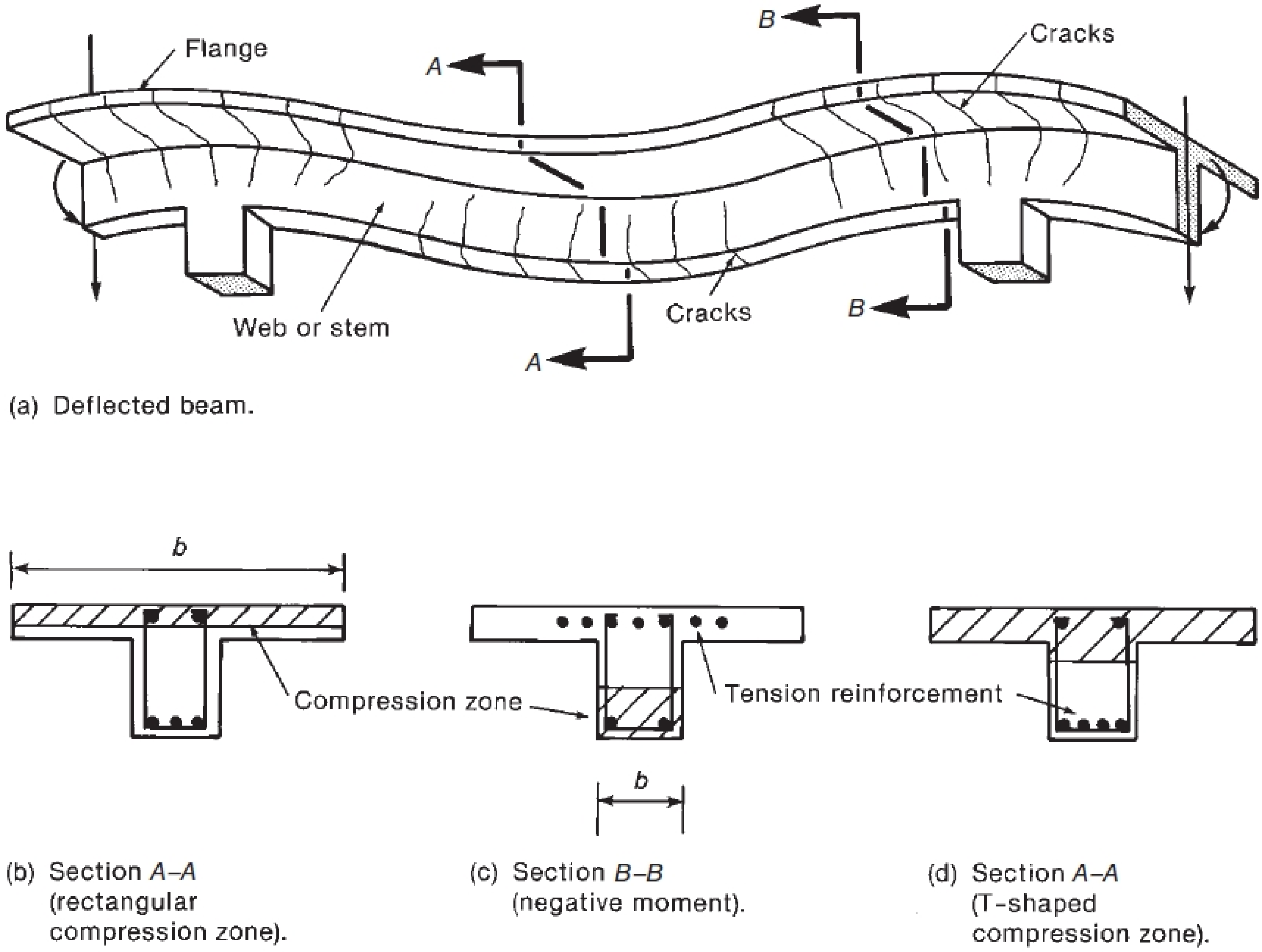


Figure 4.8-2: Exaggerated deflected view for a continuous beam with Tee shape.

- Form above deflected shape, following points can be concluded:
  - At mid-span, the compression zone is in flange as shown in Figure 4.8-2 "b" and "d" above.
  - Generally, it is rectangular as shown in Figure "b", although in a few cases, the neutral axis may shift down into the web, giving a T-shaped compression zone.
  - At the support, the compression zone is at the bottom of the beam and is rectangular, as shown in Figure "c"

4.8.3 Notations Adopted in Design of Tee Beams

Notations indicated in Figure 4.8-3 below are adopted in analysis and design of T Section.

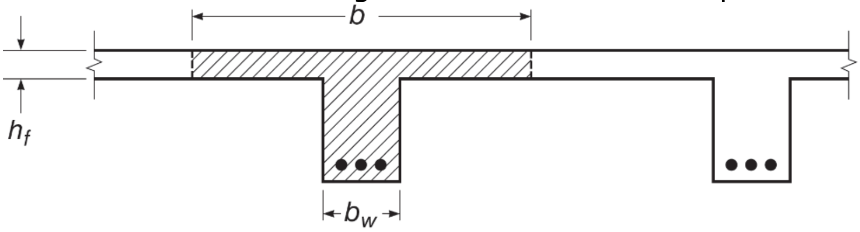


Figure 4.8-3: Notations adopted in analysis and design of Tee beams.

4.8.4 Procedure for Analysis of a Beam with T-Shape

Checking the adequacy of a T-Shape beam according to the requirements of ACI Code can be summarized as follows:

1. Definition of Section Dimensions:
   a. The first question that must be answered in the analysis of T section is "What is the part of the slab that will act as a compression flange for the T beam?"
   Due shearing deformation of the flange, which relieves the more remote elements of some compressive stress, ***shear-lag phenomenon***, actual compression stress in the beam flange varies as indicated in Figure 4.8-4 below.

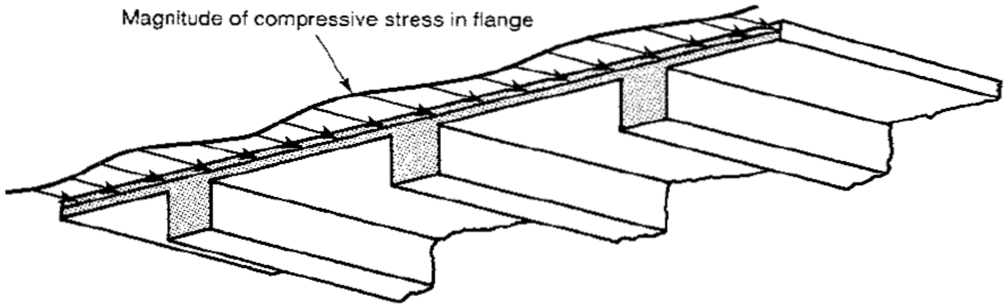


Figure 4.8-4: Actual distribution of compressive stresses in Tee flange.

According to ACI, the variable compressive stresses that acting on the overall width, b_o, in Figure 4.8-5 below can be replaced by an equivalent uniformly distributed compressive force that acting on an effective width, b.

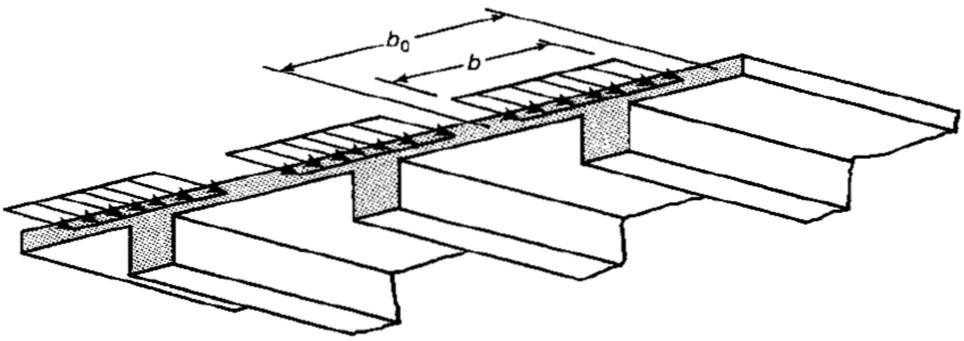


Figure 4.8-5: Equivalent uniform flange stresses adopted by the ACI code, 3D view.

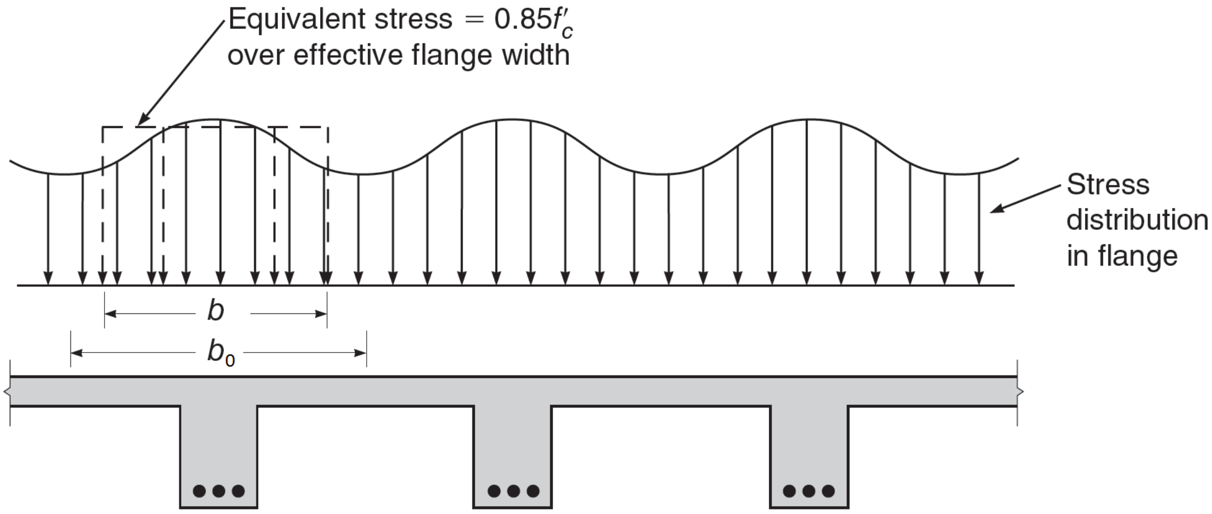


Figure 4.8-5: Equivalent uniform flange stresses adopted by the ACI code, a sectional view.

b. According to ACI Code (6.3.2.1), for nonprestressed T-beams supporting monolithic or composite slabs, the effective flange width, b, shall include the beam web width, b_w, plus an effective overhanging flange width in accordance with Table 4.8-1 below, where h is the slab thickness and s_w is the clear distance to the adjacent web:

Table 4.8-1: Dimensional limits for effective overhanging flange width for T-beams, Table 6.3.2.1 of the (ACI318M, 2014).

Flange location	Effective overhanging flange width, beyond face of web	
Each side of web	Least of:	$8h$
		$s_w/2$
		$\ell_n/8$
One side of web	Least of:	$6h$
		$s_w/2$
		$\ell_n/12$

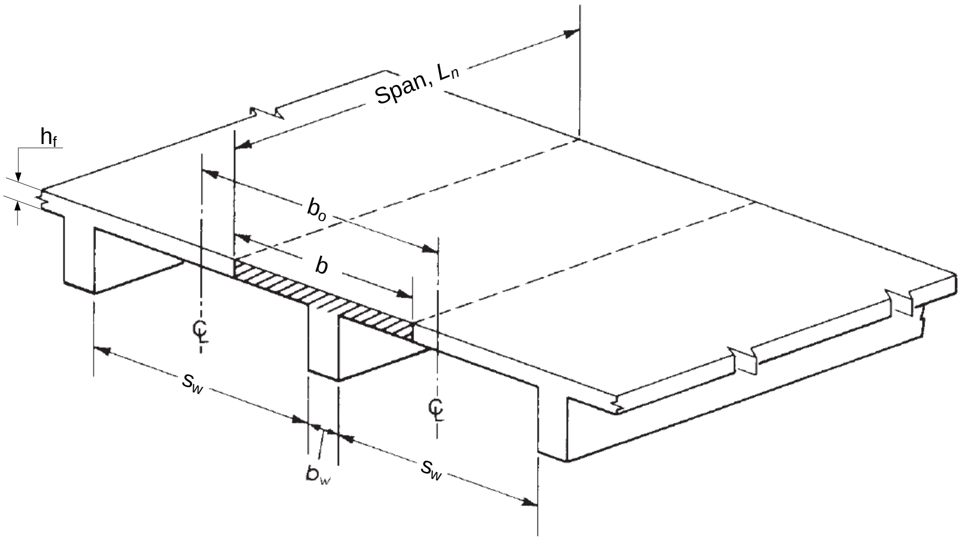


Figure 4.8-6: Notations of Table 4.8-1.

c. According to article 6.3.2.2 of the (ACI318M, 2014), isolated nonprestressed T-beams in which the flange is used to provide additional compression area shall have a flange thickness greater than or equal to $0.5b_w$ and an effective flange width less than or equal to $4b_w$.

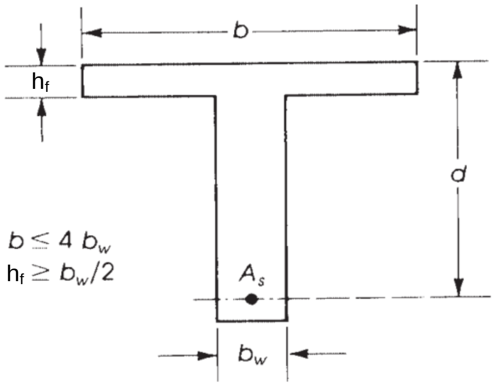


Figure 4.8-7: Isolated T-shaped sections.

2. Checking the Section Type:
Check if the failure is secondary compression failure or compression failure through following comparison:
$\rho_w\ ?\ \rho_{w\,max}$
where
$\rho_w = \frac{A_s}{b_w d}$
To derive a relation for computing of $\rho_{w\,max}$ it is useful to imagine that the T section is consists of following two parts indicated in Figure 4.8-8 below
Then, based on $\sum F_x = 0$, one can show that:
$$\rho_{w\,max} = \frac{A_{s\,max}}{b_w d} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} + \frac{A_{sf}}{b_w d}$$
or $\rho_{w\,max} = \rho_{max} + \rho_f$■
where $\rho_{w\,max} = \frac{A_{s\,max}}{b_w d}$
$$\rho_f = \frac{A_{sf}}{b_w d}, A_{sf} = \frac{0.85 f_c' h_f (b - b_w)}{f_y}$$

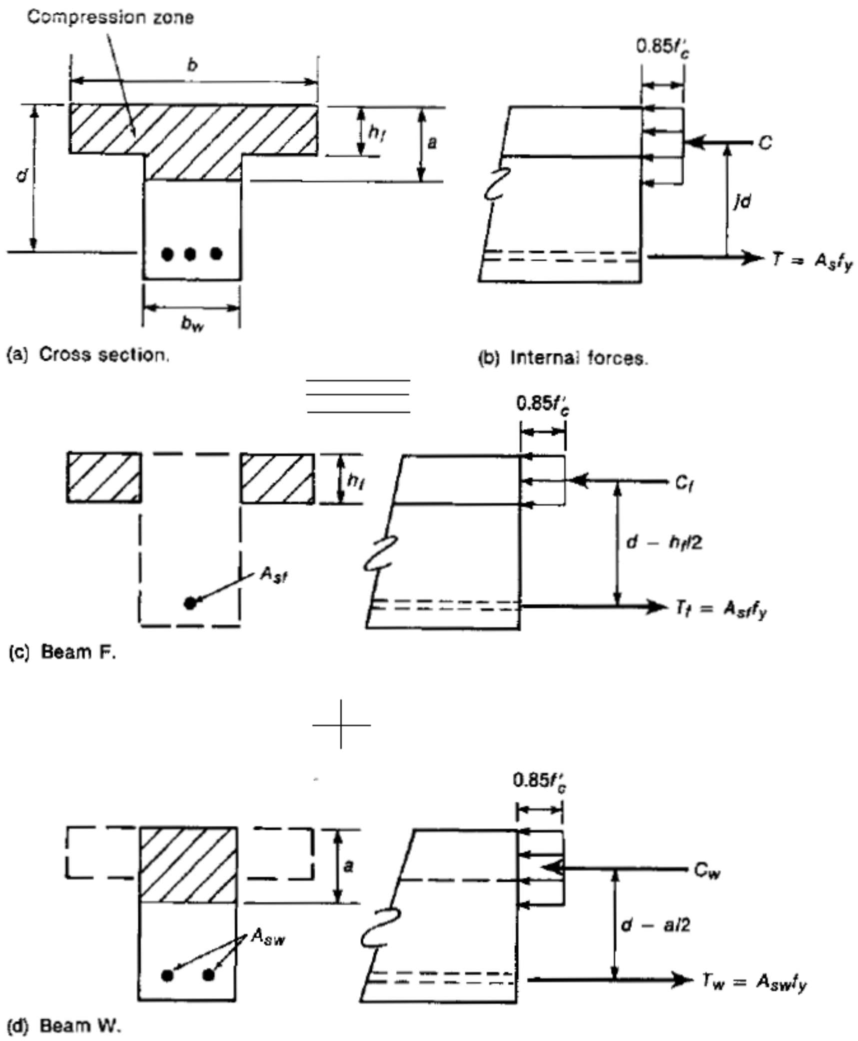


Figure 4.8-8: Imaginary two parts for Tee beam for analysis purpose.

3. Checking of $A_{s\,minimum}$ limitation
   As the flange is under compression stress, then the minimum steel area shall be compute based on ACI (9.6.1.2):
   $$A_{s\,minimum} = \frac{0.25\sqrt{f_c'}}{f_y} b_w d \geq \frac{1.4}{f_y} b_w d$$
4. Computing of Nominal Flexure Strength "M_n":
   As the relation for computing of M_n depends on location of compression block, if it is in the flange or extend to the web. Then the analyzer must first check to see if "a" is less than h_f or not (See Figure 4.8-9 below).

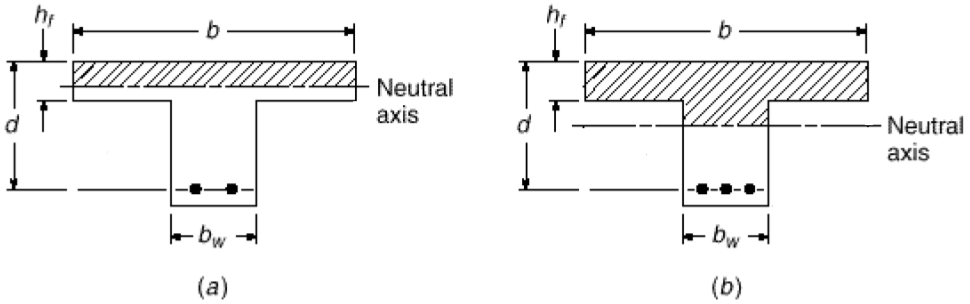


Figure 4.8-9: Possible location of compression block of Tee beams.

   a. Assume that $a \leq h_f$ (based on experience, this can be considered as the general case):

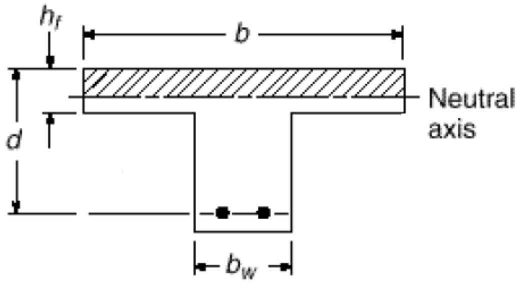


Figure 4.8-10: Compression block within section flange.

$$\because \sum F_x = 0$$
$$\therefore 0.85 f_c' \, b \, a = A_s f_y$$
$$a = \frac{A_s f_y}{0.85 f_c' b}$$

   b. If $a_{Computed} \leq h_f$, then above assumption is correct and nominal flexure strength M_n can be computed based on:
   $$M_n = A_s f_y \left(d - \frac{a}{2}\right) \blacksquare$$
   c. Else (i.e. $a_{Computed} > h_f$) then the nominal flexure strength M_n will be considered to be consist from two parts shown in Figure 4.8-11 below:
   Compute A_{sf} based on:
   $$\sum_{\text{for Part Beam F}} F_x = 0$$
   $$A_{sf} f_y = 0.85 f_c' h_f (b - b_w)$$
   $$A_{sf} = \frac{0.85 f_c' h_f (b - b_w)}{f_y} \blacksquare$$
   Compute the correct value of "a" based on Part "Beam W":
   $$\sum_{\text{for Part Beam W}} F_x = 0$$
   $$(A_s - A_{sf}) f_y = 0.85 f_c' a (b - b_w)$$
   $$a = \frac{(A_s - A_{sf}) f_y}{0.85 f_c' (b_w)}$$
   Compute M_n based on following relation:
   $$M_n = \left[0.85 f_c' h_f (b - b_w)\right] \left(d - \frac{h_f}{2}\right)_{M_n for\ Part\ Beam\ F} + \left[0.85 f_c' a b_w\right] \left(d - \frac{a}{2}\right)_{M_n for\ Part\ Beam\ W} \blacksquare$$

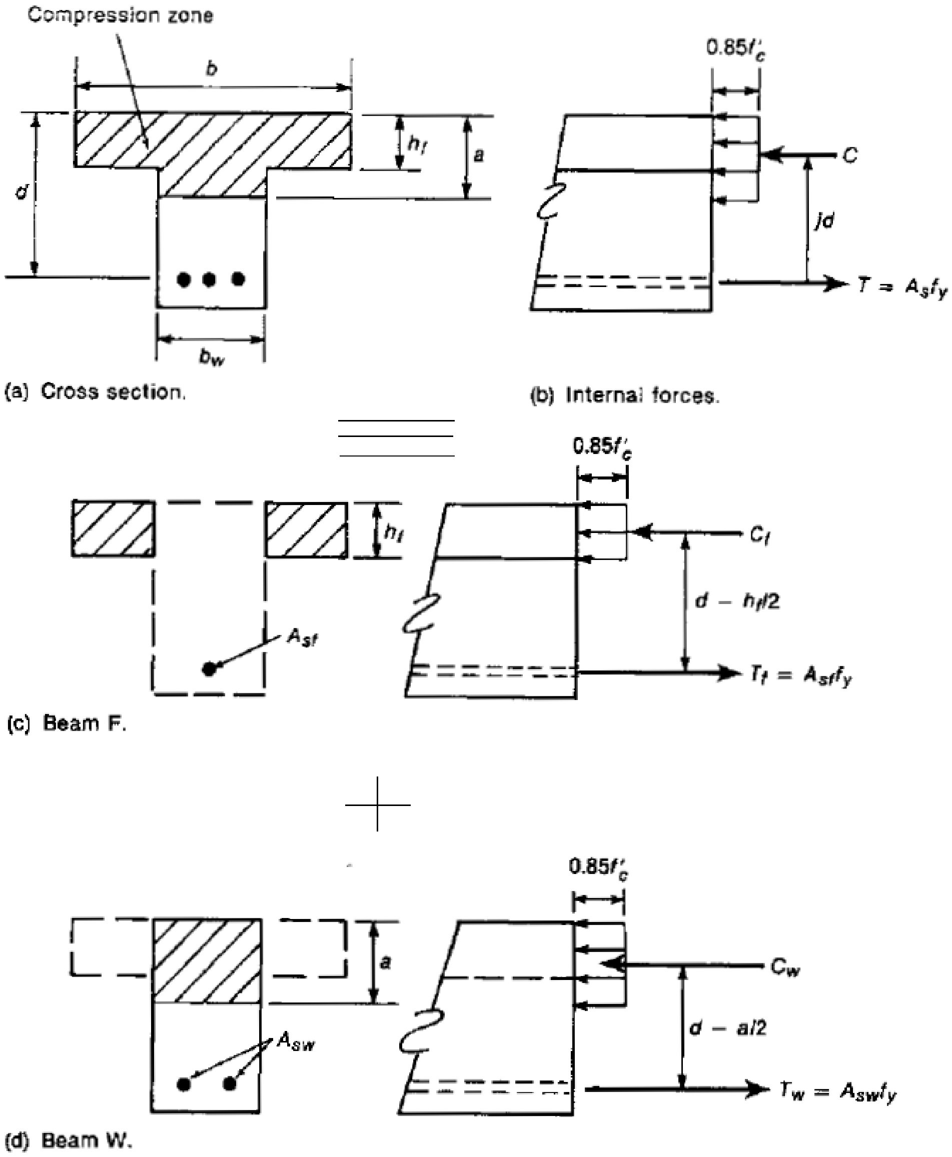


Figure 4.8-11: Analysis parts when compression block within flange.

5. Compute the Strength Reduction Factor ∅ Based on Following Relation:
    a. Compute steel stain based on the following relations:
    $$c = \frac{a}{\beta_1}$$
    $$\epsilon_t = \frac{d - c}{c}\epsilon_u$$
    b. If $\epsilon_t \geq 0.005$, then $\emptyset = 0.9$ Ok.
    c. If $\epsilon_t < 0.005$, then compute more accurate ∅:
    $$\emptyset = 0.483 + 83.3\epsilon_t$$
6. Finally Compute the Design Flexure Strength of Section $\emptyset M_n$:
    $$\emptyset M_n = \emptyset \times M_n$$

4.8.5 Examples

Example 4.8-1

Check the adequacy of the interior T-beam shown below for ACI Code requirements and determine its design strength.

Assume that:

- $f_y = 300$ Mpa
- $f_c' = 20$ Mpa.
- Beam Span 5.5m.
- $A_{Bar\ for\ \emptyset\ 19mm} = 284 mm^2$.

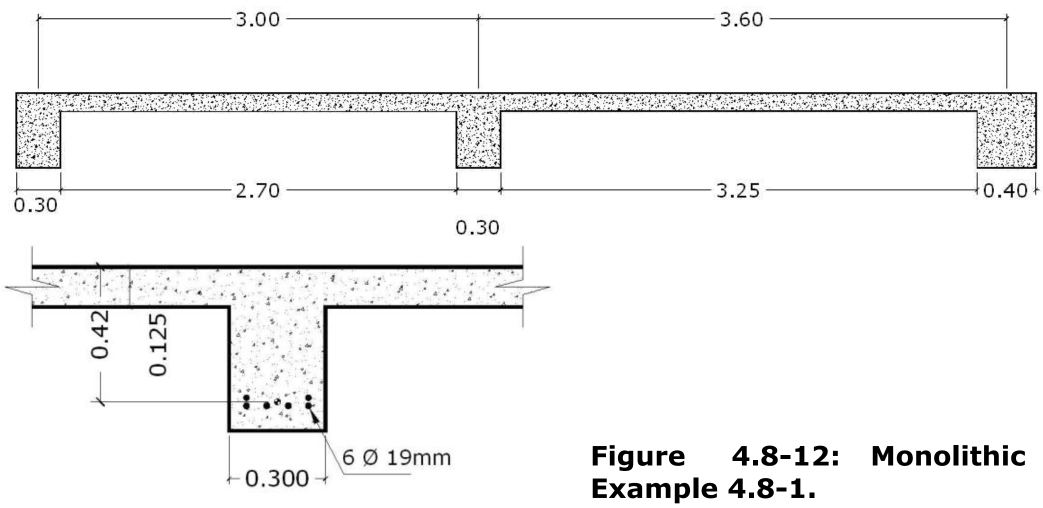
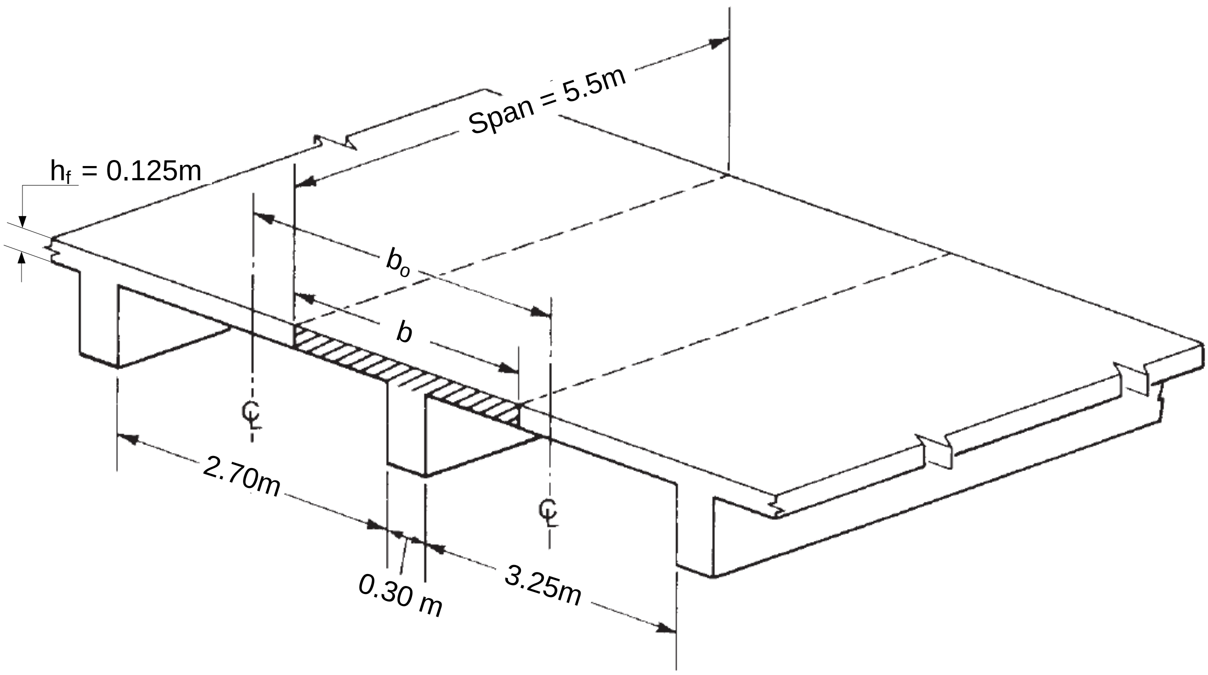


Figure 4.8-12: Monolithic Tee beam for Example 4.8-1.

Solution

1. Definition of Section Dimensions:



$$b = b_w + minimum\left[\frac{S_{w\,left}}{2}\ or\ 8h\ or\ \frac{l_n}{8}\right] + minimum\left[\frac{S_{w\,right}}{2}\ or\ 8h\ or\ \frac{l_n}{8}\right]$$

$$b = 0.3 + minimum\left[\frac{2.7}{2}\ or\ 8\times 0.125\ or\ \frac{5.5}{8}\right] + minimum\left[\frac{3.25}{2}\ or\ 8\times 0.125\ or\ \frac{5.5}{8}\right]$$

$$b = 0.3 + minimum\,[1.35\ or\ 1.0\ or\ 0.688] + minimum[1.63\ or\ 1.0\ or\ 0.688]$$

$$b = 0.3 + 0.688 + 0.688 = 1.68\ m$$

2. Checking the Section Type:
   Check if the failure is secondary compression failure or compression failure through following comparison:

$\rho_w\ ?\ \rho_{w\,max}$

$$\rho_{w\,max} = \frac{A_{s\,max}}{b_w d} = 0.85\beta_1\ \frac{f'_c}{f_y}\ \frac{\epsilon_u}{\epsilon_u + 0.004} + \frac{A_{sf}}{b_w d}$$

$$A_{sf} = \frac{0.85 f'_c h_f (b - b_w)}{f_y} = \frac{0.85\times 20\times 125\times(1680-300)}{300} = 9775\ mm^2$$

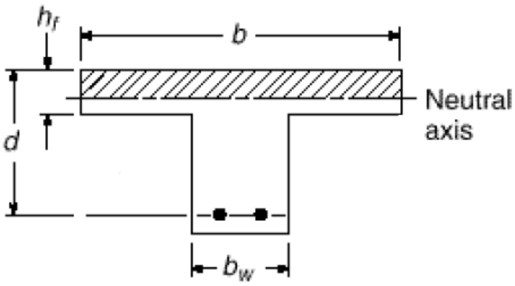
$$\rho_{w\,max} = 0.85^2 \times \frac{20}{300}\times\frac{0.003}{0.003+0.004} + \frac{9775}{300\times 420} = 20.6\times10^{-3} + 77.6\times10^{-3}$$
$$= 98.2\times10^{-3}$$

$$\rho_w = \frac{A_s}{b_w d}$$

$$A_s = 6\times 284mm^2 = 1\,704mm^2$$

$$\rho_w = \frac{1704mm^2}{300\times 420} = 13.5\times10^{-3} << \rho_{w\,max}\ \ Ok.$$

3. Checking of $A_{s\,minimum}$ limitation:
   $\because f'_c < 31\,MPa$
   $$\therefore A_{s\,minimum} = \frac{1.4}{f_y} b_w d = \frac{1.4}{300} \times 300 \times 420 = 588\ mm^2 \ll A_{s\,Provided}\ \text{Ok.}$$
4. Computing of Nominal Flexure Strength "M_n":
   Assume that $a \leq h_f$ (based on experience, this can be considered as the general case):



   $$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{1704 \times 300}{0.85 \times 20 \times 1680} = 17.9\ mm < 125mm\ Ok.$$
   As $a_{Computed} \leq h_f$, then above assumption is correct and nominal flexural strength, M_n, can be computed based on:
   $$M_n = A_s f_y (d - \frac{a}{2}) = 1704 \times 300 \times \left(420 - \frac{17.9}{2}\right) = 210\ kN.m$$
5. Compute the strength reduction factor, ∅, based on following relation:
   Compute steel stain based on the following relations:
   $$c = \frac{a}{\beta_1} = \frac{17.9}{0.85} = 21.1 \Longrightarrow \epsilon_t = \frac{d-c}{c}\epsilon_u = \frac{420 - 21.1}{21.1} \times 0.003 = 56.7 \times 10^{-3}$$
   As $\epsilon_t \gg 0.005$, then $\emptyset = 0.9$
6. Finally Compute the Design Flexure Strength of Section $\emptyset M_n$:
   $\emptyset M_n = \emptyset \times M_n = 0.9 \times 210 = 189\ kN.m$ ■

Example 4.8-2

Check the adequacy of isolated T-beam shown below for ACI Code requirements and determine its design strength.

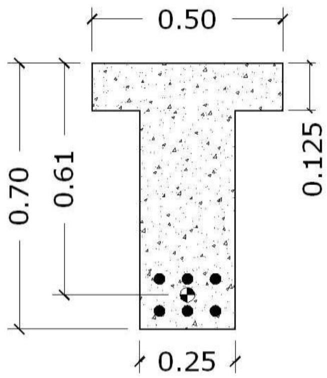


Figure 4.8-13: Isolated Tee beam for Example 4.8-2

Assume that:

- $f_y = 420$ Mpa
- $f_c' = 20$ Mpa.
- Reinforcement is 6∅25mm with $A_{Bar} = 500mm^2$.

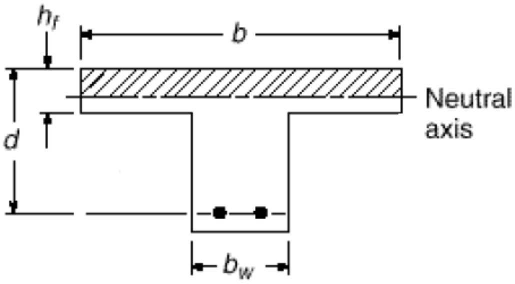
Solution

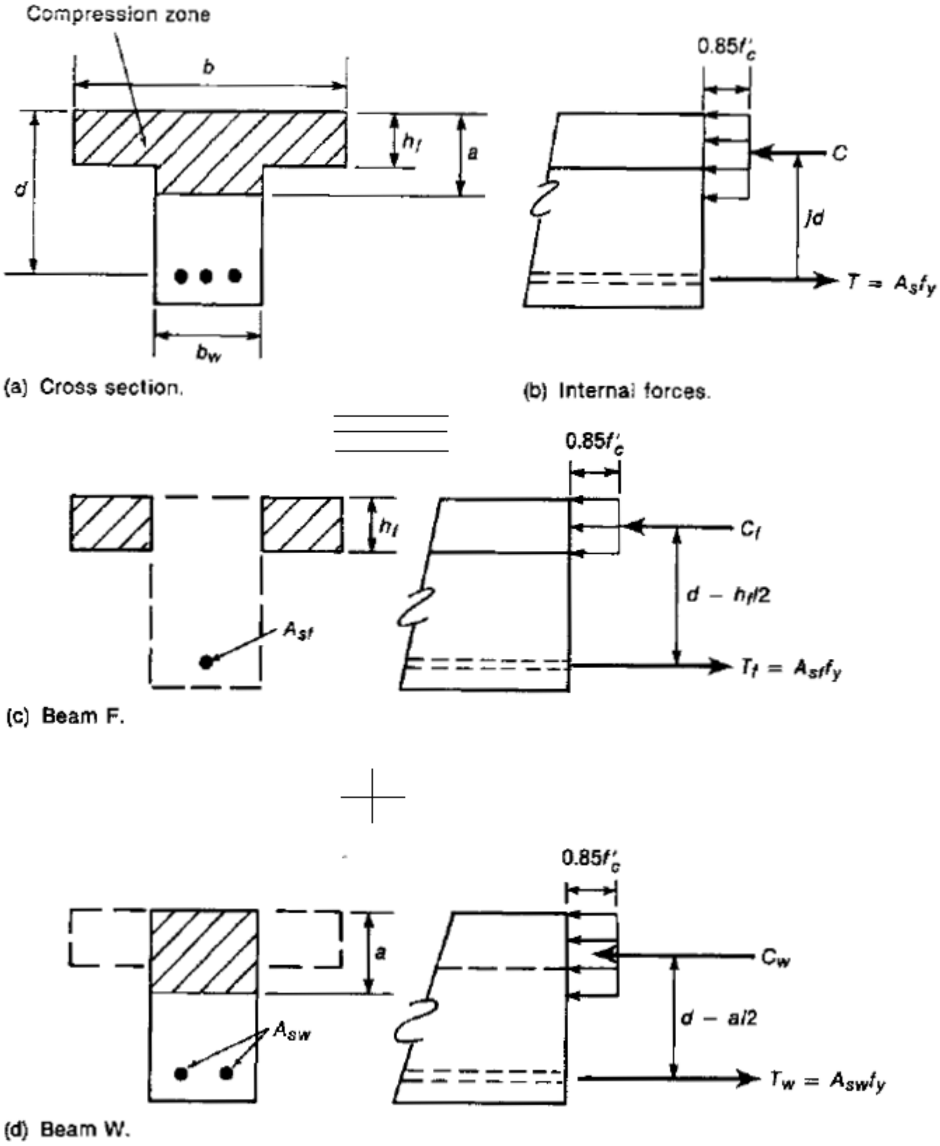
1. Check the Beam Dimensions:
   As the beam is an isolated T-beam, then its flange width and flange thickness must satisfy the following limitations:
   $$h_f \nless \frac{b_w}{2} \Longrightarrow h_f = 125mm = \frac{b_w}{2} = \frac{250}{2}\ \text{Ok.}$$
   $$b \ngtr 4b_w \Longrightarrow b = 500mm < 4 \times 250 mm\ Ok.$$
2. Checking Section Type:
   $\rho_w\ ?\ \rho_{w\,max}$
   $$\rho_{w\,max} = \frac{A_{s\,max}}{b_w d} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} + \frac{A_{sf}}{b_w d}$$
   $$A_{sf} = \frac{0.85 f'_c h_f (b - b_w)}{f_y} = \frac{0.85 \times 20 \times 125 \times (500 - 250)}{420} = 1\,265\ mm^2$$
   $$\rho_{w\,max} = 0.85^2\ \frac{20}{420} \frac{0.003}{0.003 + 0.004} + \frac{1\,265\ mm^2}{250 \times 610} = 14.7 \times 10^{-3} + 8.30 \times 10^{-3} = 23 \times 10^{-3}$$
   $$A_s = 6 \times 500mm^2 = 3\,000mm^2$$
   $$\rho_w = \frac{A_s}{b_w d} = \frac{3\,000mm^2}{250 \times 610} = 19.7 \times 10^{-3} < \rho_{w\,max}\ \text{Ok.}$$
3. Checking of $A_{s\,minimum}$ limitation:
   $\because f'_c < 31\,MPa$
   $$\therefore A_{s\,minimum} = \frac{1.4}{f_y} b_w d = \frac{1.4}{420} \times 250 \times 610 = 508\ mm^2 < A_{s\,Provided}\ \text{Ok.}$$
4. Computing of Nominal Flexure Strength "M_n":

Assume that $a \le h_f$ (based on experience, this can be considered as the general case):

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{3\,000\text{mm}^2 \times 420\text{ MPa}}{0.85 \times 20\text{ MPa} \times 500\text{mm}}$$
$$= 148\text{ mm} > 125mm\ Not\ Ok.$$

As $a_{\text{Computed}} > h_f$ then the nominal flexure strength M_n will be considered to be consist from two parts shown below:





(a) Cross section. (b) Internal forces. (c) Beam F. (d) Beam W.

Compute the correct value of "a" based on Part "Beam W":

$$a = \frac{(A_s - A_{sf}) f_y}{0.85 f_c' (b_w)} = \frac{(3\,000 - 1\,265) \times 420}{0.85 \times 20(250)} = 171\text{ mm}$$

Compute M_n based on following relation:

$$M_n = \left[0.85 f_c' h_f (b - b_w)\right] \left(d - \frac{h_f}{2}\right)_{M_n for\ Part\ Beam\ F} + \left[0.85 f_c' a b_w\right] \left(d - \frac{a}{2}\right)_{M_n for\ Part\ Beam\ W}$$

$$M_n = [0.85 \times 20 \times 125 \times (500 - 250)] \left(610 - \frac{125}{2}\right)_{M_n for\ Part\ Beam\ F}$$
$$+ [0.85 \times 20 \ \times 171 \times 250] \left(610 - \frac{171}{2}\right)_{M_n for\ Part\ Beam\ W}$$

$M_n = 291\text{ kN.m} + 381\text{ kN.m} = 672\text{ kN.m}$■

5. Compute the Strength Reduction Factor ∅ Based on Following Relation:
   Compute steel stain based on the following relations:

$$c=\frac{a}{\beta_1}=\frac{171}{0.85}=201 \Rightarrow \epsilon_t=\frac{d-c}{c}\epsilon_u=\frac{610-201}{201}\times 0.003=0.006$$

As $\epsilon_t \geq 0.005$, then $\emptyset = 0.9$ Ok.

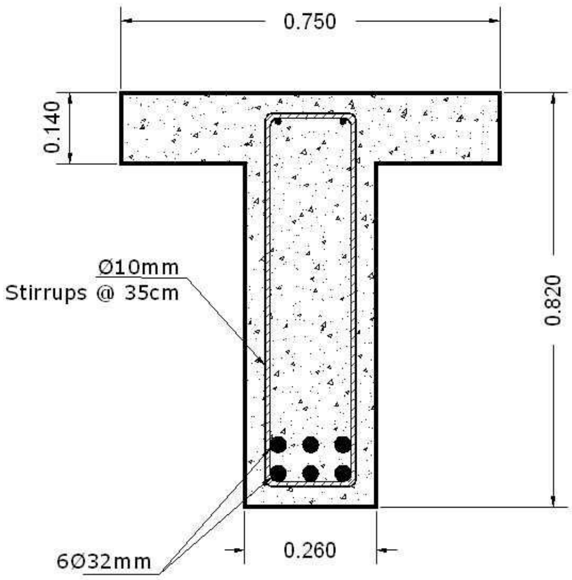
6. Finally Compute the Design Flexure Strength of Section $\emptyset M_n$:

$\emptyset M_n = \emptyset \times M_n = 0.9 \times 672 = 605\ \text{kN.m}$■

4.8.6 Problems for Solution

Problem 4.8-1

Check the adequacy of isolated T-beam shown below for ACI Code requirements and determine it's design strength.

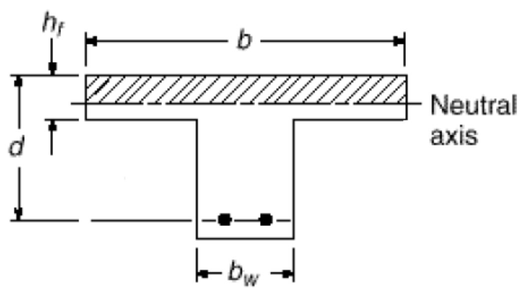


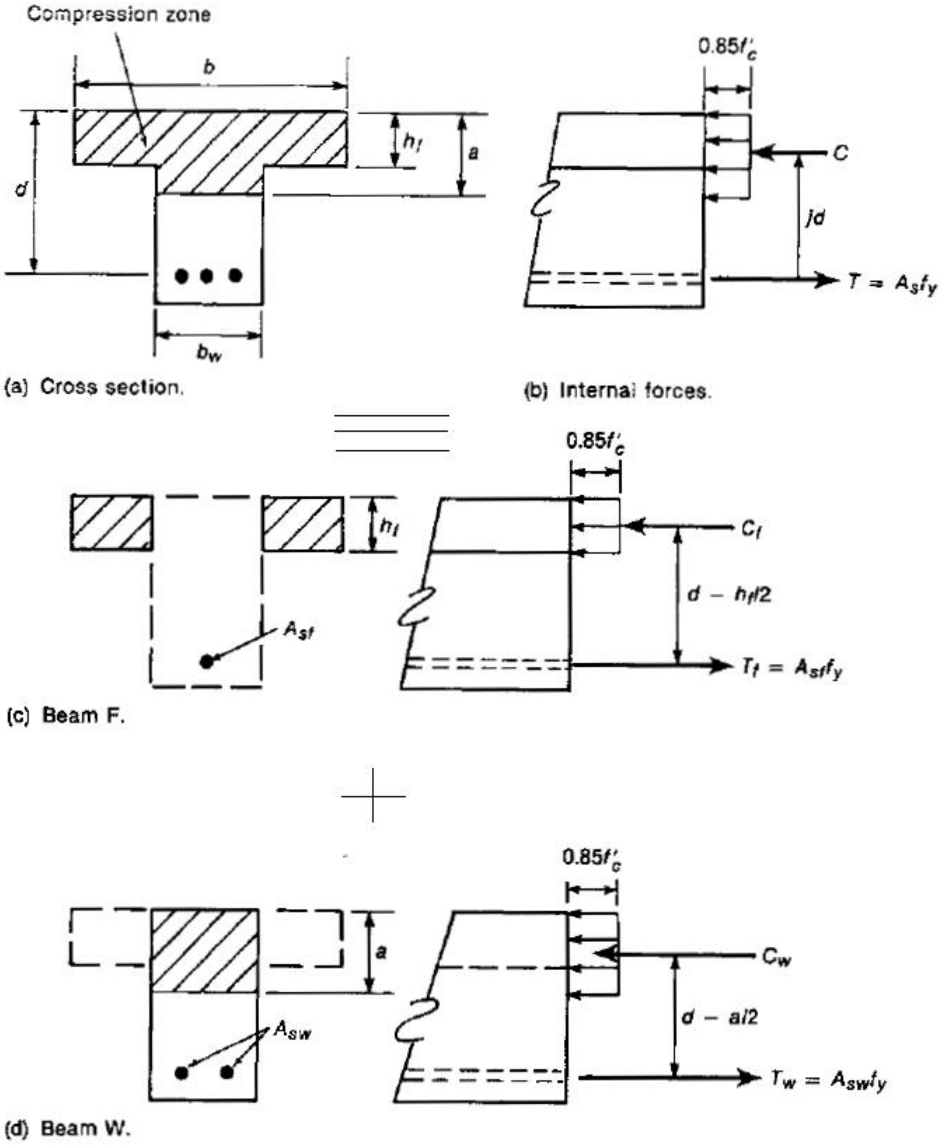
Assume that:

- $f_y = 400$ Mpa
- $f_c' = 20$ Mpa.
- Reinforcement is 6Ø32mm.

Answers

1. Check the Beam Dimensions:
   As the beam is an isolated T-beam, then its flange width and flange thickness must satisfy the following limitations:
   $h_f \nless \frac{b_w}{2}$
   $h_f = 140\text{mm} > \frac{b_w}{2} = \frac{260}{2}$ Ok.
   $b \ngtr 4b_w \Rightarrow b = 750\text{mm} < 4 \times 260 mm\ Ok.$
2. Checking Section Type:
   $\rho_w\ ?\ \rho_{w\,max}$
   $$\rho_{w\,max} = \frac{A_{s\,max}}{b_w d} = 0.85\beta_1 \frac{f_c'}{f_y}\frac{\epsilon_u}{\epsilon_u + 0.004} + \frac{A_{sf}}{b_w d}$$
   $$A_{sf} = \frac{0.85 f_c' h_f (b - b_w)}{f_y} = 2\,916\ \text{mm}^2$$
   $d = 725$ mm
   $\rho_{w\,max} = 15.5 \times 10^{-3} + 15.5 \times 10^{-3} = 31.0 \times 10^{-3}$
   $\rho_w = \frac{A_s}{b_w d}$
   $A_s = 4\,824\text{mm}^2$
   $\rho_w = 25.6 \times 10^{-3} < \rho_{w\,max}$ Ok.
3. Checking of $A_{s\,minimum}$ limitation:
   $\because f_c' < 31\ MPa$
   $\therefore A_{s\,minimum} = \frac{1.4}{f_y} b_w d = 660\ \text{mm}^2 < A_{s\,Provided}$ Ok.
4. Computing of Nominal Flexure Strength "M_n":
   Assume that $a \leq h_f$ (based on experience, this can be considered as the general case):
   $$a = \frac{A_s f_y}{0.85 f_c' b} = 151\ \text{mm} > 140 mm\ Not\ Ok.$$
   As $a_{Computed} > h_f$ then the nominal flexure strength M_n will be considered to be consist from two parts shown below:





Compute the correct value of "a" based on Part "Beam W":

$$a = \frac{(A_s - A_{sf})f_y}{0.85f'_c b_w} = 173 \text{ mm}$$

Compute M_n based on following relation:

$$M_n = \left[0.85f'_c h_f (b - b_w)\right]\left(d - \frac{h_f}{2}\right)_{M_n for\ Part\ Beam\ F} + \left[0.85f'_c a b_w\right]\left(d - \frac{a}{2}\right)_{M_n for\ Part\ Beam\ W}$$

$M_n = 764 \text{ kN.m} + 488 \text{ kN.m} = 1\ 252 \text{ kN.m}$■

5. Compute the Strength Reduction Factor $\emptyset$ Based on Following Relation:
   Compute steel stain based on the following relations:
   $$c = \frac{a}{\beta_1} = 204 \Rightarrow \epsilon_t = \frac{d - c}{c}\epsilon_u = 7.66 \times 10^{-3}$$
   As $\epsilon_t \geq 0.005$, then $\emptyset = 0.9$ Ok.
6. Finally Compute the Design Flexure Strength of Section $\emptyset M_n$:
   $\emptyset M_n = \emptyset \times M_n = 1\ 127 \text{ kN.m}$■

Problem 4.8-2

Resolve previous problem but with $h_f = 180mm$.

Answers

1. Check the Beam Dimensions:
   As the beam is an isolated T-beam, then its flange width and flange thickness must satisfy the following limitations:
   $$h_f \not< \frac{b_w}{2} \Longrightarrow h_f = 180\text{mm} > \frac{b_w}{2} = \frac{260}{2} \text{ Ok.}$$

$b \ngtr 4b_w \Longrightarrow b = 750\text{mm} < 4 \times 260mm\ Ok.$

2. Checking Section Type:

$\rho_w\ ?\ \rho_{w\,max}$

$$\rho_{w\,max} = \frac{A_{s\,max}}{b_w d} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} + \frac{A_{sf}}{b_w d}$$

$$A_{sf} = \frac{0.85 f_c' h_f (b - b_w)}{f_y} = 3\,749\ \text{mm}^2$$

$d = 725\ \text{mm}$

$\rho_{w\,max} = 15.5 \times 10^{-3} + 19.9 \times 10^{-3} = 35.4 \times 10^{-3}$

$$\rho_w = \frac{A_s}{b_w d}$$

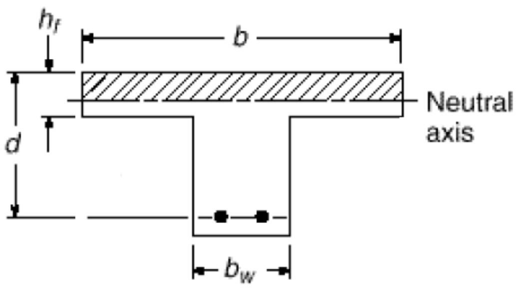
$A_s = 4\,824\text{mm}^2$

$\rho_w = 25.6 \times 10^{-3} < \rho_{w\,max}$ Ok.

3. Checking of $A_{s\,minimum}$ limitation:

$\because f_c' < 31\ MPa \Longrightarrow \therefore A_{s\,minimum} = \frac{1.4}{f_y} b_w d = 660\ \text{mm}^2 < A_{s\,Provided}$ Ok.

4. Computing of Nominal Flexure Strength "M_n":



Assume that $a \le h_f$ (based on experience, this can be considered as the general case):

$$a = \frac{A_s f_y}{0.85 f_c' b} = 151\ \text{mm} < 180mm\ Ok.$$

As $a_{Computed} \le h_f$, then above assumption is correct and nominal flexure strength M_n can be computed based on:

$$M_n = A_s f_y (d - \frac{a}{2}) = 1\,253\ \text{kN.m}$$

5. Compute the Strength Reduction Factor $\emptyset$ Based on Following Relation:

Compute steel stain based on the following relations:

$$c = \frac{a}{\beta_1} = 178\text{mm} \Longrightarrow \epsilon_t = \frac{d - c}{c} \epsilon_u = 9.22 \times 10^{-3}$$

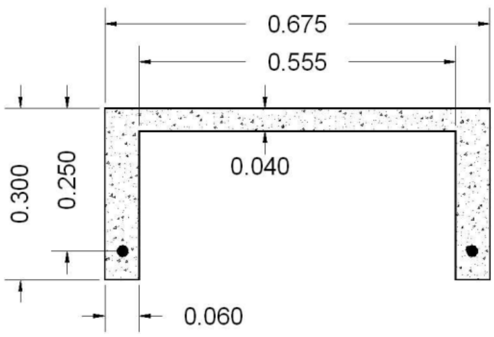
As $\epsilon_t \ge 0.005$, then $\emptyset = 0.9$ Ok.

6. Finally Compute the Design Flexure Strength of Section $\emptyset M_n$:

$\emptyset M_n = \emptyset \times M_n = 1\,128\ \text{kN.m}$■

Problem 4.8-3

Check the adequacy of the precast beam shown below according to ACI Code requirements and compute its flexure design strength. Assume that:

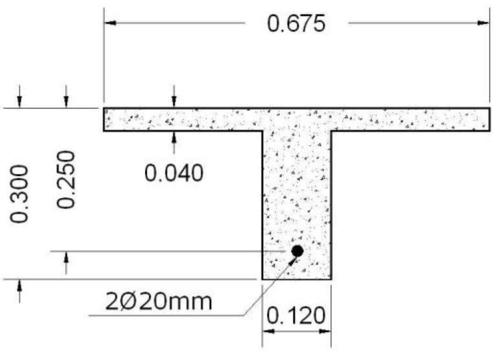


- $f_y = 400$ Mpa
- $f_c' = 20$ Mpa.
- Each leg has been reinforced with one of 20 mm rebar.

Answers

Note: It is easily to show that the horizontal movements of an area in a reinforced concrete beam has no effects on strain or stress distribution if the section remains to have a vertical axis of symmetry.

Then the section will be transformed for the shape below and analyzed as a T shape with web width of 120 mm:



1. Checking Section Type:

$\rho_w \ ? \ \rho_{w\,max}$

$\rho_{w\,max} = \frac{A_{s\,max}}{b_w d} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} + \frac{A_{sf}}{b_w d}$

$A_{sf} = \frac{0.85 f'_c h_f (b - b_w)}{f_y} = 945 \text{ mm}^2$

$d = 250 \text{ mm}$

$\rho_{w\,max} = 15.5 \times 10^{-3} + 31.5 \times 10^{-3} = 47.0 \times 10^{-3}$

$\rho_w = \frac{A_s}{b_w d}$

$A_s = 628 \text{ mm}^2$

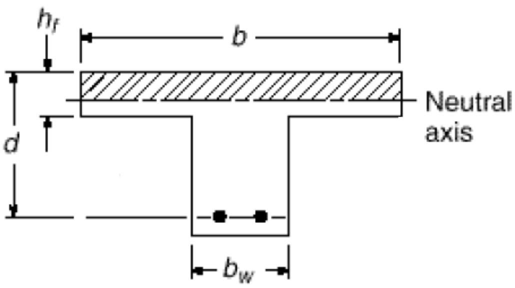
$\rho_w = 20.9 \times 10^{-3} < \rho_{w\,max}$ Ok.

Checking of $A_{s\,minimum}$ limitation:

$\because f'_c < 31\,MPa \Rightarrow \therefore A_{s\,minimum} = \frac{1.4}{f_y} b_w d = \frac{1.4}{400} \times (60 \times 2) \times 250 = 105 \text{ mm}^2$

$< A_{s\,Provided}$ Ok.

2. Computing of Nominal Flexure Strength "M_n":



Assume that $a \le h_f$ (based on experience, this can be considered as the general case):

$a = \frac{A_s f_y}{0.85 f'_c b} = 21.9 \text{ mm} < 40\,mm\ Ok.$

As $a_{Computed} \le h_f$, then above assumption is correct and nominal flexure strength M_n can be computed based on:

$M_n = A_s f_y (d - \frac{a}{2}) = 60.0 \text{ kN.m}$

3. Compute the Strength Reduction Factor $\emptyset$ Based on Following Relations:

Compute steel stain based on the following relations:

$c = \frac{a}{\beta_1} = 25.8 \text{ mm} \Rightarrow \epsilon_t = \frac{d - c}{c} \epsilon_u = 26.1 \times 10^{-3}$

As $\epsilon_t \ge 0.005$, then $\emptyset = 0.9$ Ok.

4. Finally Compute the Design Flexure Strength of Section $\emptyset M_n$:

$\emptyset M_n = \emptyset \times M_n = 54.0 \text{ kN.m}$■

4.9 DESIGN OF A BEAM WITH T-SHAPE

4.9.1 Essence of Problem

- Generally, all design problems for T-section can be classified as a design of a section with pre-specified dimensions (h_f, b, b_w, and h). Usually these dimensions have been determined as follows:
    - h_f, b are both determined from slab design that logically be executed before the design of supporting beams.
    - b_w, and h are determined based on one of following criteria:
        - Based on architectural requirements.
        - Based on strength or deflection requirements in supports region (i.e., region of negative moment), in a continuous T beam.
        - Based on shear requirements (as will be discussed in Chapter 4).
- Therefore, the main unknown of design problem is to determine the required reinforcement and its details.

4.9.2 Design Procedures

Based on above known and unknown, the design procedure can be summarized as follows:

- Computed of M_u:
  Based on given loads and spans the applied factored moment M_u can be computed. As slab weight has been already included in the applied dead load, therefore only selfweight of beam stem should be added.

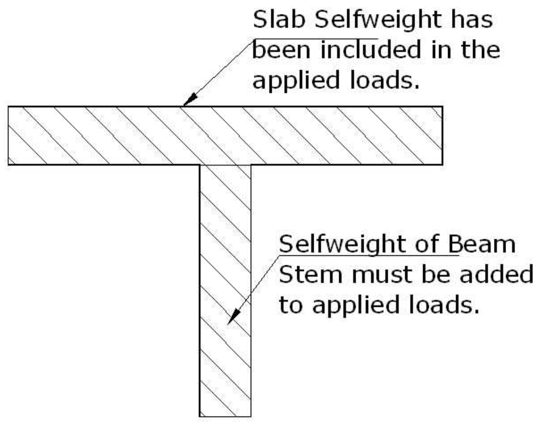


Figure 4.9-1: Selfweight of Tee beams.

- Based on slab and beam data, determine the effective flange width "b" and as was discussed in previous article. For isolated T beam, beam dimensions must be checked based on ACI requirements.
- Compute M_n based on following relation:
  $$M_n = \frac{M_u}{\emptyset}$$
  where $\emptyset$ will be assumed 0.9 to be checked later. This assumption is generally satisfied in the design of T section.
- Check if this section can be design with a compression block in section flange or extend to section web based on following comparison:
  If
  $$M_n \leq 0.85 f_c' h_f b \left(d - \frac{h_f}{2}\right)$$
  then $a \leq h_f$. Else $a > h_f$

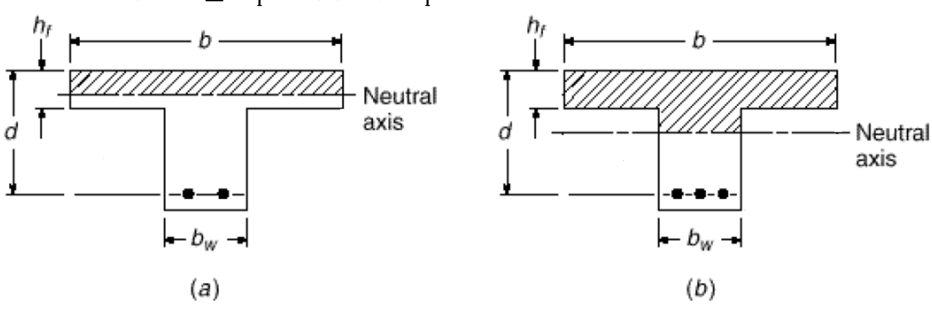


Figure 4.9-2: Possible location of compression block of Tee beams, reproduction of Figure 4.8-9 for quick reference.

- Design of a section with $a \leq h_f$:
  This section can be designed as a rectangular section with dimensions of b and d.

$$\rho_{Required} = \frac{1 - \sqrt{1 - 2.36\frac{M_n}{f'_c bd^2}}}{1.18 \times \frac{f_y}{f'_c}}$$

$$A_{S\ Required} = \rho_{Required} bd \blacksquare$$

- Design of a section with $a > h_f$:
  - Compute the nominal moment that can be supported by flange overhangs:

$$M_{n1} = 0.85 f'_c h_f (b - b_w)(d - \frac{h_f}{2})$$

Steel reinforcement for this part will be:

$$A_{sf} = \frac{0.85 f'_c h_f (b - b_w)}{f_y} \blacksquare$$

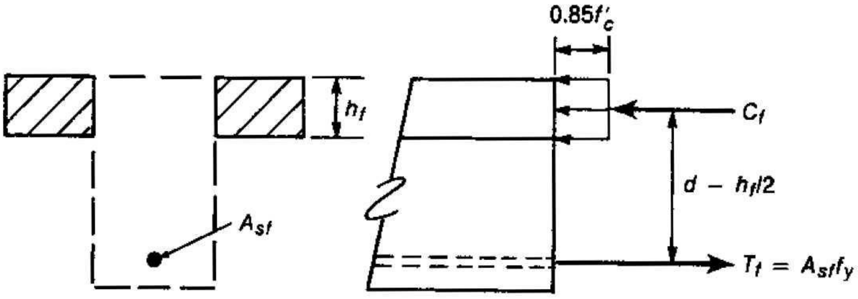


Figure 4.9-3: Forces acting on overhang parts and corresponding streel area.

  - Compute the remaining nominal strength that must be supported by section web:

$$M_{n2} = M_n - M_{n1}$$

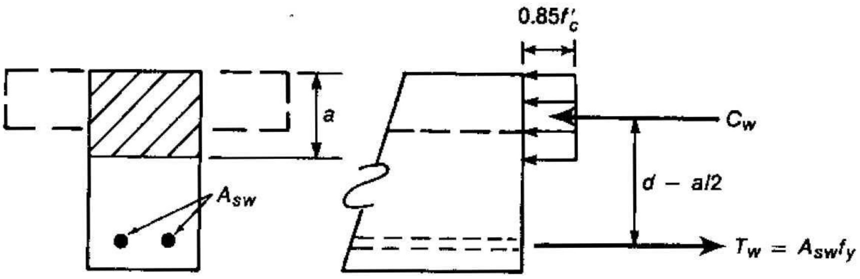


Figure 4.9-4: Forces acting on web part and corresponding steel area.

For this moment "M_{n2}", the section can be designed as a rectangular section with dimensions of b_w and d:

$$\rho_{Required} = \frac{1 - \sqrt{1 - 2.36\frac{M_{n2}}{f'_c b_w d^2}}}{1.18 \times \frac{f_y}{f'_c}}$$

$$A_{s2} = \rho_{Required} b_w d$$

then:

$$A_{s\ Required} = A_{sf} + A_{s2} \blacksquare$$

- Check $A_{s\ Required}$ with minimum steel area permitted by the ACI Code:

$$A_{s\ minimum} = \frac{0.25\sqrt{f'_c}}{f_y} b_w d \geq \frac{1.4}{f_y} b_w d$$

If $A_{s\ Required} > A_{s\ minimum}$ Ok. Else, use:

$$A_{s\ Required} = A_{s\ minimum}$$

- Check the $A_{s\ Required}$ with the maximum steel area permitted by ACI Code:

$$\rho_w = \frac{A_{s\ Required}}{b_w d}\ ?\ \rho_{w\ max} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} + \frac{A_{sf}}{b_w d}$$

If

$\rho_w \leq \rho_{w\ max}$ Ok.

Else, the designer must increase one or more of beam dimensions, i.e., in practice, compression reinforcement is not used in T sections.

- Check the assumption of $\emptyset = 0.9$:
  - Compute "a":
    If $a \leq h_f$
    $$a = \frac{A_s f_y}{0.85 f_c' b}$$
    If $a > h_f$
    $$\mathrm{a} = \frac{(\mathrm{A_s} - \mathrm{A_{sf}})\mathrm{f_y}}{0.85\mathrm{f_c'}(\mathrm{b_w})}$$
  - Compute steel stain based on the following relations:
    $$\mathrm{c} = \frac{\mathrm{a}}{\beta_1}$$
    $$\epsilon_\mathrm{t} = \frac{\mathrm{d} - \mathrm{c}}{\mathrm{c}} \epsilon_\mathrm{u}$$
  - If $\epsilon_\mathrm{t} \geq 0.005$, then $\emptyset = 0.9$ Ok.
  - If $\epsilon_\mathrm{t} < 0.005$, then re-compute a more accurate $\emptyset$:
    $\emptyset = 0.483 + 83.3\epsilon_\mathrm{t}$
    and return to step of computing M_n.
- Finally, compute the required number of rebars and reinforcement layers and draw section details.

4.9.3 Examples

Example 4.9-1

Design the T-beam for the floor system shown in Figure 4.9-5 below. The floor slab supported by 6.71 m simple span beams. Service loads are: $\mathrm{W_{Live}} = 14.6 \frac{\mathrm{kN}}{\mathrm{m}}$ and $\mathrm{W_{Dead}} = 29.2 \frac{\mathrm{kN}}{\mathrm{m}}$.

Assume that the designer intends to use:

- $\mathrm{f_y} = 414\ \mathrm{Mpa}$ $f_c' = 21\ Mpa$.
- $\emptyset 25mm$ for longitudinal reinforcement ($A_{Bar} = 510mm^2$) and $\emptyset 10mm$ for stirrups.
- One layer of reinforcement.

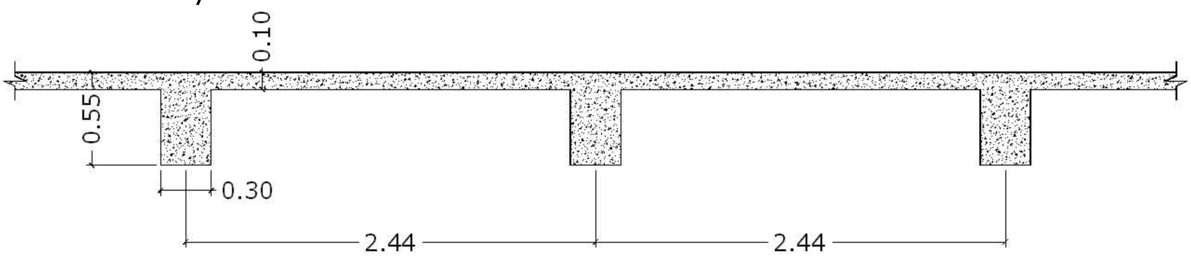


Figure 4.9-5: Floor system for Example 4.9-1.

Solution

- Computed of M_u
  $$\mathrm{W_{Self}} = (0.55 - 0.1)\mathrm{m} \times 0.3\mathrm{m} \times 24 \frac{\mathrm{kN}}{\mathrm{m^3}} = 3.24 \frac{\mathrm{kN}}{\mathrm{m}} \Rightarrow \mathrm{W_{Dead}} = 29.2 \frac{\mathrm{kN}}{\mathrm{m}} + 3.24 \frac{\mathrm{kN}}{\mathrm{m}} = 32.4 \frac{\mathrm{kN}}{\mathrm{m}}$$
  $$\mathrm{M_{Dead}} = \frac{32.4 \frac{\mathrm{kN}}{\mathrm{m}} \times 6.71^2 \mathrm{m^2}}{8} = 182\ \mathrm{kN.m}, \mathrm{M_{Live}} = \frac{14.6 \frac{\mathrm{kN}}{\mathrm{m}} \times 6.71^2 \mathrm{m^2}}{8} = 82.2\ \mathrm{kN.m}$$
  $$\mathrm{M_u} = \text{maximum of } [1.4\mathrm{M_{Dead}} \text{or } 1.2\mathrm{M_{Dead}} + 1.6\mathrm{M_{Live}}]$$
  $$\mathrm{M_u} = \text{maximum of } [1.4 \times 182\ \mathrm{kN.m} \text{ or } 1.2 \times 182\ \mathrm{kN.m} + 1.6 \times 82.2\ \mathrm{kN.m}]$$
  $$\mathrm{M_u} = \text{maximum of } [255\ \mathrm{kN.m} \text{ or } 350\mathrm{kN.m}] = 350\ \mathrm{kN.m}$$
- Compute of Required Nominal Flexure Strength $\mathrm{M_n}$:
  $$\mathrm{M_n} = \frac{\mathrm{M_u}}{\emptyset} = \frac{350\ \mathrm{kN.m}}{0.9} = 389\ \mathrm{kN.m}$$
  where $\emptyset$ will be assumed 0.9 to be checked later.
- Compute the effective flange width "b"
  $$b = \mathrm{b_w} + minimum \left[\frac{s_w}{2} \text{ or } 8h \text{ or } \frac{l_n}{8}\right] \times 2$$
  $$b = 300 + minimum \left[\frac{(2440 - 300)}{2} \text{ or } 8 \times 100 \text{ or } \frac{6710}{8}\right] \times 2$$
  $$b = 300 + minimum\ [1070 \text{ or } 800 \text{ or } 839] \times 2 = 300 + 800 \times 2 = 1900$$

- Check if this section can be design with a compression block in section flange or extend to section web based on following comparison:

$M_n ?\ 0.85f_c' h_f b \left(d - \frac{h_f}{2}\right)$

$d = 550\text{mm} - 40\text{mm} - 10\text{mm} - \frac{25}{2}\text{mm} = 487\text{mm}$

$M_n = 389\ \text{kN.m} ?\ 0.85f_c' h_f b \left(d - \frac{h_f}{2}\right) = 0.85 \times 21 \times 100 \times 1900 \left(487 - \frac{100}{2}\right) = 1482\ \text{kN.m}$

$M_n = 389\ \text{kN.m} < 0.85f_c' h_f b \left(d - \frac{h_f}{2}\right) = 1482\ \text{kN.m}$

Then $a < h_f$

- Design of a section with $a \leq h_f$:
This section can be designed as a rectangular section with dimensions of b and d.

$$\rho_{Required} = \frac{1 - \sqrt{1 - 2.36 \frac{M_n}{f_c' b d^2}}}{1.18 \times \frac{f_y}{f_c'}} = \frac{1 - \sqrt{1 - 2.36 \frac{389 \times 10^6}{21 \times 1900 \times 487^2}}}{1.18 \times \frac{414}{21}} = 2.13 \times 10^{-3}$$

$A_{S\ Required} = \rho_{Required} bd = 2.13 \times 10^{-3} \times 1900 \times 487 = 1971\ \text{mm}^2$

$\text{No of Rebars} = \frac{1971}{510} = 3.86$

Try $4\emptyset 25\text{mm}$

$A_{s\ provided} = 4 \times 510\text{mm}^2 = 2040\ \text{mm}^2$

$b_{Required} = 40 \times 2 + 10 \times 2 + 4 \times 25 + 3 \times 25 = 275\ \text{mm} < 300 mm\ Ok.$

- Check $A_{s\ Provided}$ with minimum steel area permitted by the ACI Code:

$A_{s\ minimum} = \frac{0.25\sqrt{f_c'}}{f_y} b_w d \geq \frac{1.4}{f_y} b_w d \Longrightarrow A_{s\ minimum} = \frac{1.4}{f_y} b_w d = \frac{1.4}{414} \times 300 \times 487 = 494\ \text{mm}^2$

As $A_{s\ Provided} > A_{s\ minimum}$ Ok.

- Check the $A_{s\ Provided}$ with the maximum steel area permitted by ACI Code:

$\rho_w = \frac{A_{s\ Provided}}{b_w d} ?\ \rho_{w\ max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} + \frac{A_{sf}}{b_w d}$

$A_{sf} = \frac{0.85f_c' h_f (b - b_w)}{f_y} = \frac{0.85 \times 21 \times 100 \times (1900 - 300)}{414} = 6899\ \text{mm}^2$

$\rho_w = \frac{2\ 040\ \text{mm}^2}{300 \times 487} ?\ \rho_{w\ max} = 0.85 \times 0.85 \times \frac{21}{414} \frac{0.003}{0.003 + 0.004} + \frac{6899}{300 \times 487}$

$\rho_w = 13.9 \times 10^{-3} \ll \rho_{w\ max} = 15.7 \times 10^{-3} + 47.2 \times 10^{-3} 62.9 \times 10^{-3}$ Ok.

- Check the assumption of $\emptyset = 0.9$:
  - Compute "a":

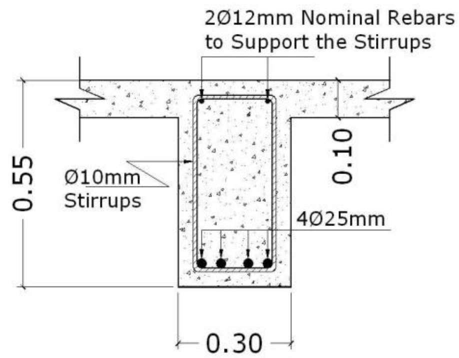
    $\sum F_x = 0 \Longrightarrow 0.85 \times 21 \times a \times 1900$

    $= 2040 \times 414 \Longrightarrow a = 24.9\ \text{mm}$
  - Compute steel stain based on the following relations:

    $c = \frac{a}{\beta_1} = \frac{24.9}{0.85} = 29.3\text{mm} \Longrightarrow \epsilon_t$

    $= \frac{487 - 29.3}{29.3} \times 0.003$

    $= 46.9 \times 10^{-3}$
  - As $\epsilon_t > 0.005$, then $\emptyset = 0.9$ Ok.
- Draw the Section Details:



Example 4.9-2

Design a T-beam having a cross section shown in Figure 4.9-6 below to support a total factored moment M_u of 461 kN.m. Assume that the effective flange width has been computed and as shown in Figure below.

Assume that the designer intends to use:

- $f_y = 414\ \text{Mpa}\ f_c' = 21\ \text{Mpa}$.
- $\emptyset 35\text{mm}$ for longitudinal reinforcement ($A_{Bar} = 1000\text{mm}^2$) and $\emptyset 10\text{mm}$ for stirrups.
- One layer of reinforcement.

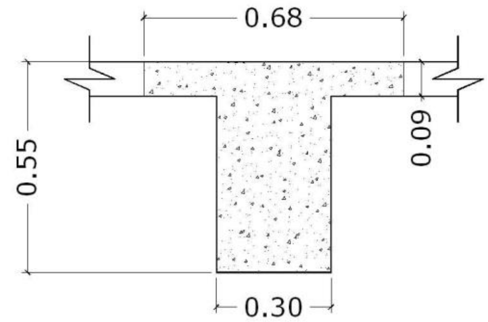


Figure 4.9-6: T section for Example 4.9-2.

Solution

- Compute of Required Nominal Flexure Strength M_n:

$$M_n=\frac{M_u}{\emptyset}=\frac{461\ kN.m}{0.9}=512\ kN.m$$

where ∅ will be assumed 0.9 to be checked later.

- Check if this section can be design with a compression block in section flange or extend to section web based on following comparison:

$$M_n?\ 0.85f_c'\ h_f b\left(d-\frac{h_f}{2}\right)$$

$$d=550mm-40mm-10mm-\frac{35}{2}mm=482mm$$

$$M_n=512\ kN.m>\ 0.85f_c'\ h_f b\left(d-\frac{h_f}{2}\right)=0.85\times21\times90\times680\left(482-\frac{90}{2}\right)=477\ kN.m$$

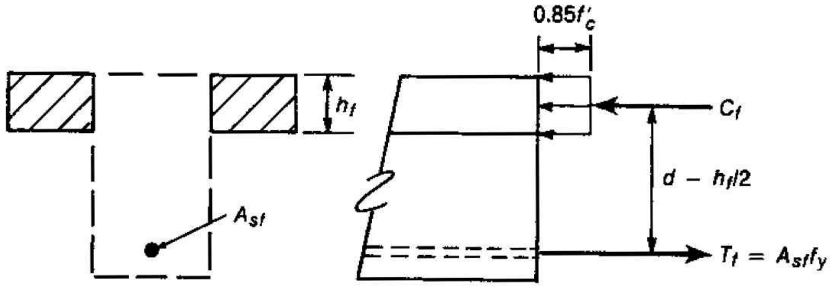
- Design of a section with $a>h_f$:
  - Compute the nominal moment that can be supported by flange overhangs:

$$M_{n1}=0.85f_c'h_f(b-b_w)\left(d-\frac{h_f}{2}\right)=0.85\times21\times90\times(680-300)(482-\frac{90}{2})$$

$$M_{n1}=267\ kN.m$$

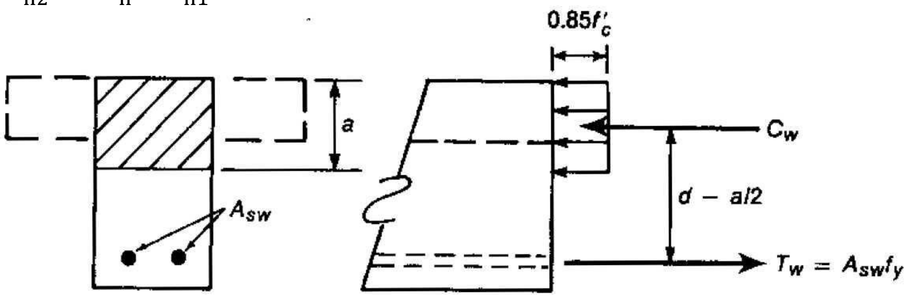
Steel reinforcement for this part will be:

$$A_{sf}=\frac{0.85f_c'h_f(b-b_w)}{f_y}=\frac{0.85\times21\times90\times(680-300)}{414}=1\ 474\ mm^2$$



  - Compute the remaining nominal strength that must be supported by section web:

$$M_{n2}=\ M_n-\ M_{n1}=512-267=245\ kN.m$$



For this moment "M_{n2}", the section can be designed as a rectangular section with dimensions of b_w and d:

$$\rho_{Required}=\frac{1-\sqrt{1-2.36\frac{M_{n2}}{f_c'b_wd^2}}}{1.18\times\frac{f_y}{f_c'}}=\frac{1-\sqrt{1-2.36\frac{245\times10^6}{21\times300\times482^2}}}{1.18\times\frac{414}{21}}=9.55\times10^{-3}$$

$$A_{s2}=\ \rho_{Required}b_wd=\ 9.55\times10^{-3}\times300\times482=1\ 381\ mm^2$$

Then:

$$A_{s\,Required} = A_{sf} + A_{s2} = 1\,474\ mm^2 + 1\,381\ mm^2 = 2\,855\ mm^2$$

$$No.\ of\ Rebars = \frac{2855mm^2}{1000mm^2} = 2.86$$

Try 3∅35mm

$$A_{S\,Provided} = 3\,000\ mm^2$$

$$b_{Required} = 40 \times 2 + 10 \times 2 + 3 \times 35 + 2 \times 35 = 275mm < 300mm\ Ok.$$

- Check $A_{S\,Provided}$ with minimum steel area permitted by the ACI Code:

$$A_{s\,minimum} = \frac{0.25\sqrt{f_c'}}{f_y} b_w d \geq \frac{1.4}{f_y} b_w d$$

$$A_{s\,minimum} = \frac{1.4}{f_y} b_w d = \frac{1.4}{414} \times 300 \times 482 = 489\ mm^2 < A_{S\,Provided}\ Ok.$$

- Check the $A_{S\,Provided}$ with the maximum steel area permitted by ACI Code:

$$\rho_w = \frac{A_{S\,Provided}}{b_w d}\ ?\ \rho_{w\,max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} + \frac{A_{sf}}{b_w d}$$

$$\rho_w = \frac{3000}{300 \times 482} = 20.7 \times 10^{-3}\ ?\ \rho_{w\,max} = 0.85^2 \frac{21}{414} \frac{0.003}{0.003 + 0.004} + \frac{1\,474\ mm^2}{300 \times 482}$$

$$\rho_w = 20.7 \times 10^{-3} < \rho_{w\,max} = 15.7 \times 10^{-3} + 10.2 \times 10^{-3} = 25.9 \times 10^{-3}\ Ok.$$

- Check the assumption of $\emptyset = 0.9$:
  - Compute "a":

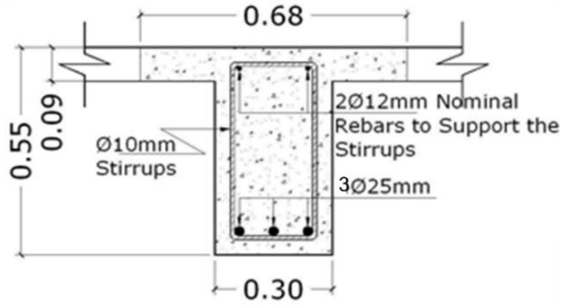
$$a = \frac{(A_s - A_{sf})f_y}{0.85f_c'(b_w)} = \frac{(3\,000 - 1\,474) \times 414}{0.85 \times 21 \times 300} = 118\ mm$$

  - Compute steel stain based on the following relations:

$$c = \frac{a}{\beta_1} = \frac{118\ mm}{0.85} = 139\ mm$$

$$\epsilon_t = \frac{d - c}{c}\epsilon_u = \frac{482 - 139}{139} \times 0.003 = 7.40 \times 10^{-3}$$

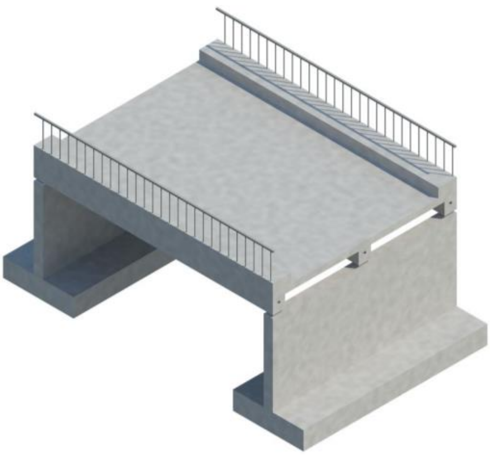
  - As $\epsilon_t > 0.005$, then $\emptyset = 0.9$ Ok.
- Draw the Section Details:



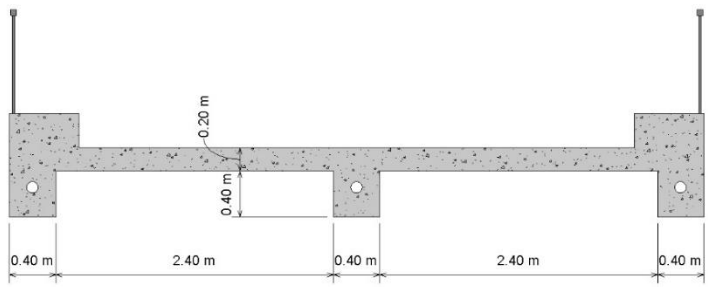
Example 4.9-3

For a pedestrian bridge indicated in ***Figure*** 4.9-7 below, a structural designer includes sleeve with diameter of 100mm for communication and electrical cables. Design for flexural the central supporting beam. In your design, assume that:

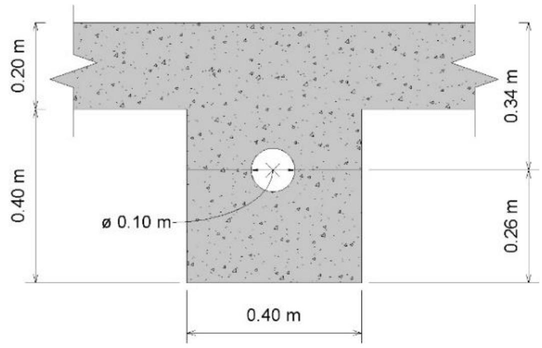
- Materials strength are $f_c' = 28\ MPa$ and $f_y = 420\ MPa$.
- Two layers of reinforcement with a bar diameter of 20mm for longitudinal reinforcement,
- Rebar with a diameter of 10mm for stirrups,
- $W_{Superimposed} = 20\ kN/m$, not including beam own weight, $W_{Live} = 12\ kN/m$

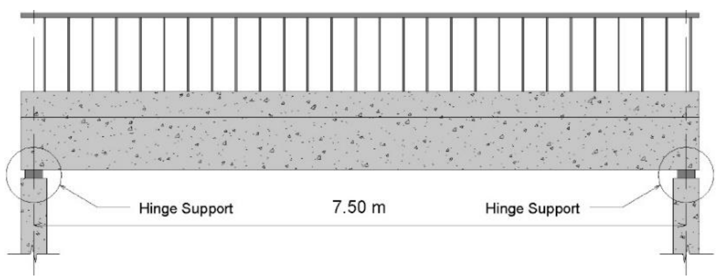


3D view.



Cross sectional view.





Callout view. Elevation view.

Figure 4.9-7: A pedestrian bridge for Example 4.9-3.

Solution

- Compute factored load $\boldsymbol{W_u}$:

Assuming that selfweight of the beam flange has been included in the superimposed dead load of $W_{Superimposed} = 20\,kN/m$, therefore what should be included as a selfweight would include stem selfweight only.

$$W_{Selfweight} = 0.4 \times 0.4 \times 24 = 3.84\frac{kN}{m}$$

Reducing in selfweight due to pipe conduit has been conservatively neglected. The total dead load would be:

$$W_{Dead} = W_{Selfweight} + W_{Superimposed} = 3.84 + 20 = 23.8\frac{kN}{m}$$

The factored uniformly distributed load that acting on the beam would be:

$$W_u = \max(1.4W_D, 1.2W_D + 1.6W_L) = \max(1.4 \times 23.8, 1.2 \times 23.8 + 1.6 \times 12) = 47.8\frac{kN}{m}$$

With the indicated simple supports, the maximum factored moment, M_u, at beam mid-span would be:

$$M_u = \frac{W_u l^2}{8} = \frac{47.8 \times 7.50^2}{8} = 336\,kN.m$$

- Compute $\mathrm{M_n}$:

$$\mathrm{M_n} = \frac{\mathrm{M_u}}{\emptyset}$$

where ∅ will be assumed 0.9 to be checked later. This assumption is generally satisfied in the design of T section.

$$M_n = \frac{336}{0.9} = 373\,kN.m$$

- Beam effective depth, d:

With adopting of two layers of $No.20$ for longitudinal reinforcement and $No.10$ for stirrup reinforcement, the effective depth, d, would be:

$$d = 600 - 40 - 10 - 20 - \frac{25}{2} \approx 517\,mm$$

- Effective flange width:

With referring to Figure 4.9-8:

$$b = \mathrm{b_w} + minimum\left[\frac{S_{w\,left}}{2}\ or\ 8h\ or\frac{l_n}{8}\right] + minimum\left[\frac{S_{w\,right}}{2}\ or\ 8h\ or\frac{l_n}{8}\right]$$

$$b = 0.4 + \min\left(\left(\frac{2.4}{2}\right), (8 \times 0.2), \left(\frac{7.5}{8}\right)\right) + \min\left(\left(\frac{2.4}{2}\right), (8 \times 0.2), \left(\frac{7.5}{8}\right)\right) = 2.275\ \mathrm{m}$$

- Check type of section:

Check to see if the stress block would be in the flange or extends to the web:

$$\mathrm{M_n} = 373\,kN.m \ll 0.85\mathrm{f_c'}\,\mathrm{h_f}\mathrm{b}\left(\mathrm{d} - \frac{\mathrm{h_f}}{2}\right) = \frac{0.85 \times 28 \times 200 \times 2.275 \times 10^3 \times \left(517 - \frac{200}{2}\right)}{10^6}$$
$$= 4516\,kN.m$$

Then $\mathrm{a} < \mathrm{h_f}$ and the section behaves as a rectangular section with dimensions of b by d.

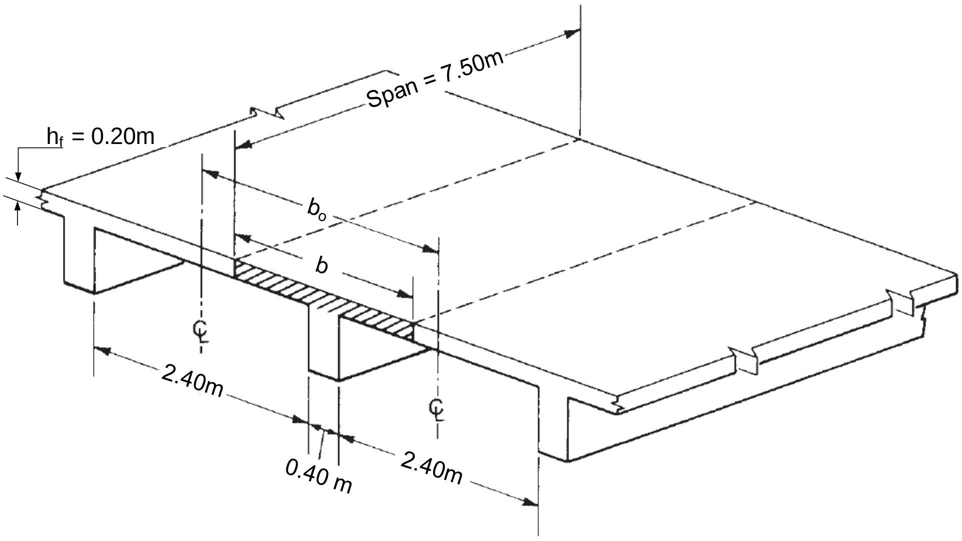


Figure 4.9-8: Flange computation parameters for Example 4.9-3.

- <u>Effect of conduit hole:</u>
  According to aforementioned argument, stress block is located at flange; therefore, the conduit hole has no effect on beam strength, as it is located at the tension zone that completely neglected in traditional concrete theory.
- <u>Required reinforcement ratio, $\rho_{Required}$:</u>

$$\rho_{\text{Required}} = \frac{1-\sqrt{1-2.36\frac{M_n}{f_c' bd^2}}}{1.18\times\frac{f_y}{f_c'}} = \frac{\left(1-\sqrt{1-2.36\times\left(\frac{373\times10^6}{28\times(2.275\times10^3)\times517^2}\right)}\right)}{\left(1.18\times\frac{420}{28}\right)}$$
$$= 1.48\times10^{-3}$$

$A_{s\,Required} = 1.48\times10^{-3}\times(2.275\times10^3)\times517 = 1741\,mm^2$

- <u>Check with $A_{s\,minmum}$:</u>
  With conservative neglecting of the conduit hole, minimum required reinforcement, $A_{s\,minimum}$ would be:
  $\because f_c' < 31\,MPa$

$$A_{s\,\text{minimum}} = \frac{1.4}{f_y}b_w d = \frac{1.4}{420}\times400\times517 = 689\,mm^2 < A_{s\,Required}\;\therefore Ok.$$

- <u>Check with $\rho_{maximum}$:</u>

$$\rho_{w\,\max} = 0.85\beta_1\,\frac{f_c'}{f_y}\,\frac{\epsilon_u}{\epsilon_u+0.004}+\frac{A_{sf}}{b_w d}$$

$$A_{sf} = \frac{0.85f_c'h_f(b-b_w)}{f_y} = \frac{\left(0.85\times28\times200\times\left((2.275\times10^3)-400\right)\right)}{420} = 21250\,mm^2$$

$$\rho_{w\,maximum} = 0.85\times0.85\times\left(\frac{28}{420}\right)\times\left(\frac{0.003}{0.003+0.004}\right)+\left(\frac{21250}{400\times517}\right) = 123\times10^3$$

$$\rho_{w\,Required} = \frac{1741}{400\times517} = 8.42\times10^{-3}$$

$\rho_{w\,maximum} \gg \rho_{w\,Required}\;\therefore Ok.$

As expect, the Tee section has very high ductility level and would fail as a tension control section. This implicitly indicates that ***the assumption of*** $\phi = 0.9$ ***is valid and there is no need to be checked explicitly***.

- <u>Details of the section:</u>

$$No.\,of\ Reabrs = \frac{1741}{\frac{\pi\times20^2}{4}} = 5.54$$

Therefore use $6No.20$ in two layers as indicated in below.

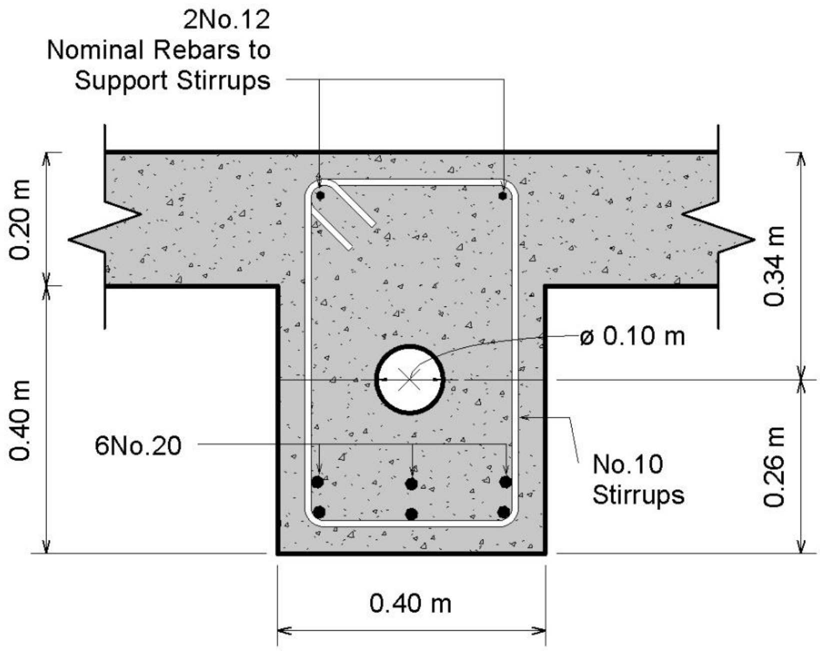


Figure 4.9-9: Final detailed section for Example 4.9-3.

Example 4.9-4

The vertical reaction at end B of an indeterminate propped cantilever beam has been computed as shown in ***Figure 4.9-10*** below:

- Design flexure reinforcement for section at support end (A).
- Check adequacy if same amount of flexure reinforcement calculated at end (A) is used for section of maximum positive bending moment.

Assume that:

- Beam selfweight is neglected.
- Use a rebar of $\phi 25mm$ with A_{bar} of 510 mm^2 and stirrups of $\phi 10mm$.
- $f_c' = 28\ MPa$ and $f_y = 420\ MPa$.

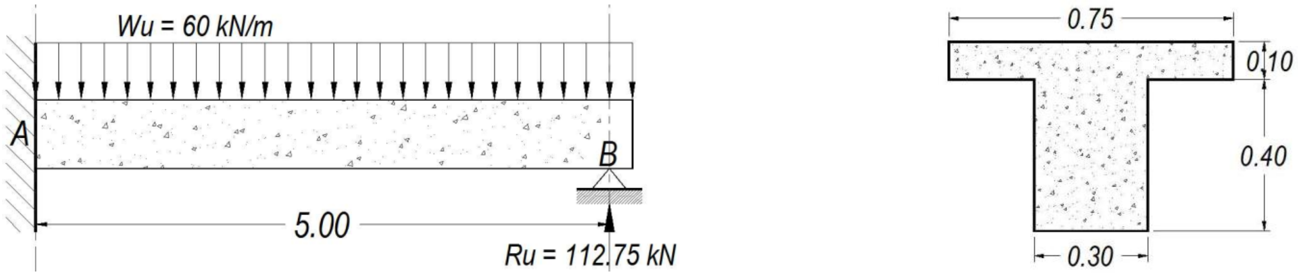


Figure 4.9-10: Propped cantilever beam Example 4.9-4. **Beam Section**

Solution

- **Design flexure reinforcement for section at support end (A):**
  In negative region, section behaves as rectangular section as flange is already cracked.
  - Compute M_n:

    $$M_u = -60 \times 5.00 \times \frac{5.00}{2} + 112.75 \times 5 = -186\ kN.m$$

    Assume ϕ to be 0.9 to be checked later:

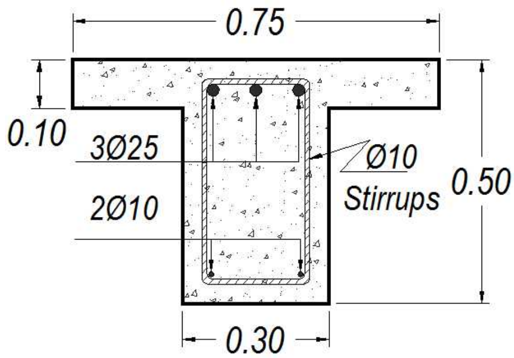
    $$M_n = \frac{186}{0.9} = 207$$

  - Compute ρ:

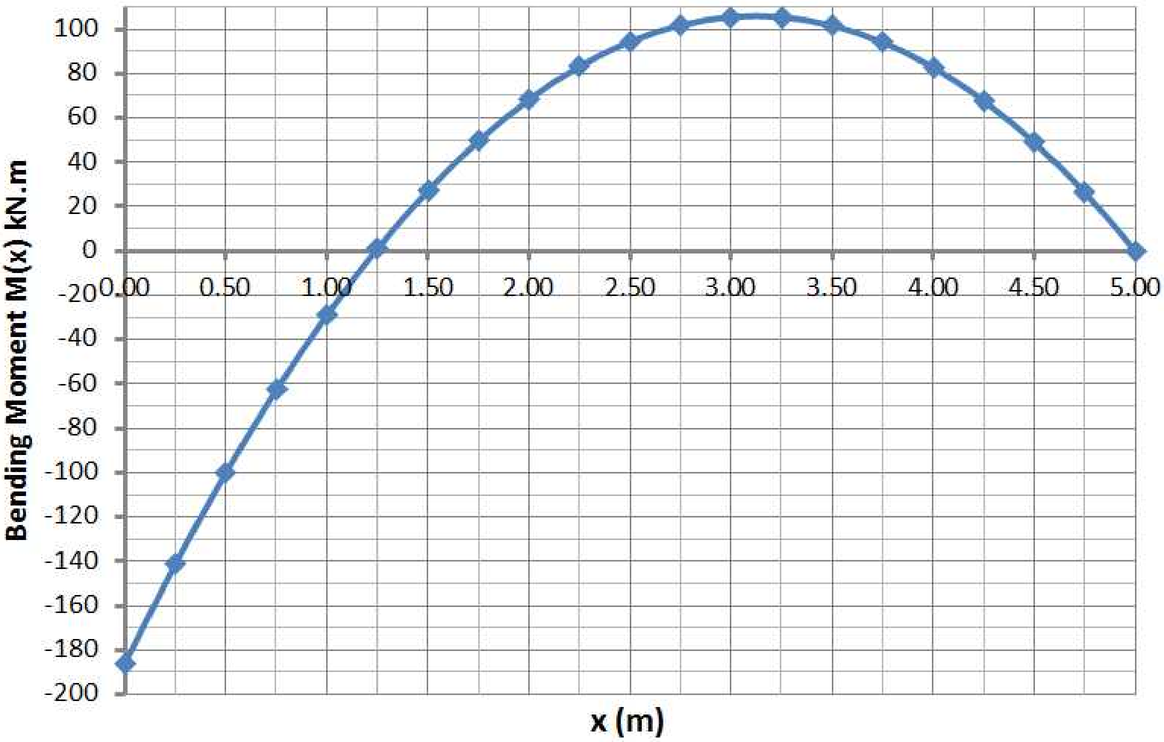
    $$d_{one\ layer} = 500 - 40 - 10 - \frac{25}{2} = 438$$

    $$\rho_{Required} = \frac{1 - \sqrt{1 - 2.36 \frac{M_n}{f_c' b d^2}}}{1.18 \times \frac{f_y}{f_c'}} = \frac{1 - \sqrt{1 - 2.36 \frac{207 \times 10^6}{28 \times 300 \times 438^2}}}{1.18 \times \frac{420}{28}} = 9.33 \times 10^{-3}$$

- Check $\rho_{Maximum}$:
  $\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} = 0.85^2 \times \frac{28}{420} \times \frac{0.003}{0.007} = 20.6 \times 10^{-3} > \rho\ Ok.$
- Compute As:
  $A_s = 9.33 \times 10^{-3} \times 438 \times 300 = 1226\ mm^2$
  $No.of\ rebars = \frac{1226}{510} = 2.4$
  Try $3\phi25$.
- Check b:
  $b_{Required} = 40 \times 2 + 10 \times 2 + 25 \times 3 + 25 \times 2 = 225 < 300\ Ok.$
- Check A_{smin}:
  Since the span is a statically indeterminate span and $f_c' < 31.4\ MPa$, then A_{smin} will be:
  $A_{smin} = \frac{1.4}{fy} b_w d = \frac{1.4}{420} \times 300 \times 438 = 438\ mm^2 < A_{s\ provided}\ Ok.$
- Check ϕ Assumption:
  $a = \frac{3 \times 510 \times 420}{0.85 \times 28 \times 300} = 90\ mm \Rightarrow c = \frac{a}{\beta_1} = \frac{90}{0.85} = 106\ mm$
  $\epsilon_t = \frac{438 - 106}{106} \times 0.003 = 9.40 \times 10^{-3}$
  $> 5 \times 10^{-3} Ok.$
- Draw final section. Drawing shown is a preliminary one as instead of adding two rebars with nominal diameter for lower side, specific amount of positive reinforcement should be extended into support region according to ACI code requirements. This aspect will be discussed in Chapter 5 of the course.



- **Check adequacy if same amount of flexure reinforcement calculated at end (A) is used for section of maximum positive bending moment:**
  - Intuitively one can conclude that reinforcement computed for the negative region would be adequate when used on the bottom side for the positive moment. This is due to the facts that:
    - As will be discussed in ***Chapter 11***, the positive moment is lower than the negative moment for regular spans that subjected to uniformly distributed loads.
    - The flange is effective in the positive region while it is neglected in the negative region.
  - Compute the maximum positive moments:
    The maximum positive moment could either be computed by:
    $R_{u\ left} = 60 \times 5.0 - 112.75 = 187.25\ kN$
    $M(x) = -186.25 + 187.25x - \frac{60x^2}{2}$
    $\frac{dM}{dx} = 187.25 - 60x$
    $x_{maximum} = 3.121$
    $M(3.121) = -186.25 + 187.25 \times 3.121 - \frac{60 \times 3.121^2}{2}$
    $M_{u+ve\ Max} = 106\ kN.m$
    Or by:
    $\Sigma F_y = 0_{for\ right\ side}$
    $60x = 112.75$
    $x = 1.88\ m\ from\ right\ support$
    $M_{u\ +ve\ maximum} = 112.75 \times 1.88 - 60 \times \frac{1.88^2}{2} = 106\ kN.m$



- Check $\rho_{wmax.}$:

$$\rho_{wmax} = \rho_{max} + \frac{A_{sf}}{b_w d}$$

$$A_{sf} = \frac{0.85 \times 28 \times (750 - 300) \times 100}{420} = 2550\ mm^2$$

$$\rho_{wmax.} = 20.6 \times 10^{-3} + \frac{2550}{300 \times 438} = 40.0 \times 10^{-3} \Longrightarrow \rho_w = \frac{3 \times 510}{300 \times 438} = 11.6 \times 10^{-3} < \rho_{w\,max.}\ Ok.$$

- Check A_{smin}:

$$A_{smin} = \frac{1.4}{fy} b_w d = \frac{1.4}{420} \times 300 \times 438 = 438\ mm^2 < A_{s\,provided}\ Ok.$$

- Compute M_n:

Assume $a \le h_f$:

$$a = \frac{3 \times 510 \times 420}{0.85 \times 28 \times 750} = 36mm < 100mm\ Ok.$$

$$M_n = (3 \times 510 \times 420) \times \left(438 - \frac{36}{2}\right) = 270\ kN.m$$

- Compute ϕ:

$$a = 36\ mm \Longrightarrow c = \frac{a}{\beta_1} = \frac{36}{0.85} = 42.3\ mm$$

$$\epsilon_t = \frac{438 - 42.3}{42.3} \times 0.003 = 28.1 \times 10^{-3} > 5 \times 10^{-3} Ok.$$

- ϕM_n:

$$\phi M_n = 0.9 \times 270 = 243\ kN.m > M_u\ Ok.$$

Example 4.9-5

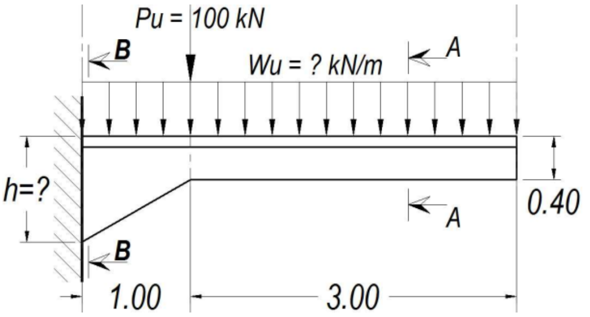
A structural designer has proposed dimensions and reinforcement for the cantilever T-beam shown in Figure 4.9-11 below.

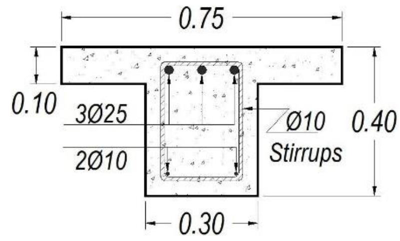
Based on flexure strength for the beam at Section A-A and at Section B-B find:

- Maximum factored uniform load (W_u) that could be supported by the beam.
- Minimum beam depth (h) for Section B-B.

In your solution, assume that:

- Beam selfweight could be neglected.
- As = 510mm^2 for $\phi 25mm$ rebars.
- $f_c' = 28\ MPa$ and $f_y = 420\ MPa$.

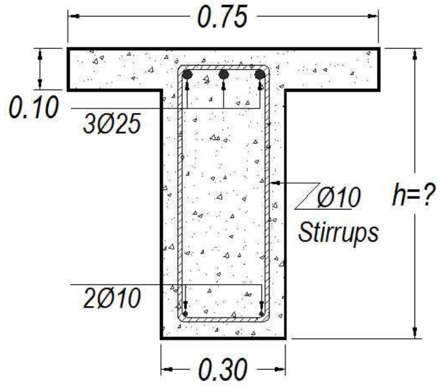




Sec. A-A

Figure 4.9-11: Cantilever T-beam for Example 4.9-5.

Sec. B-B



Solution

As flange is on the tension side, then the beam can be analyzed as a rectangular section except in computing $A_{s\,minimum}$ where flange should be considered.

Find Wu:

$$d = 400 - 40 - 10 - \frac{25}{2} = 338mm, A_s = 3 \times 510 = 1530\ mm^2$$

$$\rho_{provided} = \frac{1530}{300 \times 338} = 15.0 \times 10^{-3}$$

$$\rho_{maximum} = 0.85^2 \times \frac{28}{420} \times \frac{0.003}{0.003 + 0.004} = 20.6 \times 10^{-3} > \rho_{provided}\ Ok$$

For this statically determinate span with a flange in tension, minimum flexure reinforcement should be computed based on:

$$A_{s\,min} = minimum\ \left(\frac{0.25\sqrt{f_c'}}{f_y} bd, \frac{0.50\sqrt{f_c'}}{f_y} b_w d\right)$$

As

$b = 750\ mm > 2b_w = 2 \times 300 = 600\ mm$

then, the second term governs.

$$A_{s\,min} = \frac{0.50\sqrt{28}}{f_y} b_w d = \frac{0.50\sqrt{28}}{420} \times 300 \times 338 = 639\ mm^2 < A_s \therefore Ok.$$

$$M_n = 15.0 \times 10^{-3} \times 420 \times 300 \times 338^2 \times \left(1 - 0.59 \times \frac{15.0 \times 10^{-3} \times 420}{28}\right) = 187\ kN.m$$

Check ϕ:

$$a = \frac{1530 \times 420}{0.85 \times 28 \times 300} = 90\ mm \Rightarrow c = \frac{90}{0.85} = 106\ mm \Rightarrow \epsilon_t = \frac{d-c}{c}\epsilon_u = \frac{338 - 106}{106} \times 0.003 = 0.0065 > 0.005$$

then:

$\phi = 0.9$

$M_u = \phi M_n = 0.9 \times 187 = 168\ kN.m$

$$M_u = \frac{W_u l^2}{2} \Rightarrow W_u = \frac{2M_u}{l^2} = \frac{2 \times 168}{3^2} = 37.3 \frac{kN}{m}\ ∎$$

Find beam depth "d":

$$M_u = \frac{37.3 \times 4^2}{2} + 100 \times 1.0 = 398\ kN.m$$

$$\Sigma F_x = 0 \Rightarrow a = \frac{1530 \times 420}{0.85 \times 28 \times 300} = 90mm$$

Compute M_n (assume that $\phi = 0.9$, to be checked later):

$M_n = \frac{398}{0.9} = 442\ kN.m \Rightarrow 442 \times 10^6 = 1530 \times 420 \times \left(d - \frac{90}{2}\right) \Rightarrow d = 733\ mm$

Check ϕ:

$a = 90\ mm \Rightarrow c = \frac{90}{0.85} = 106\ mm \Rightarrow \epsilon_t = \frac{d-c}{c}\epsilon_u = \frac{733-106}{106} \times 0.003 = 0.0177 > 0.005$

then:

$\phi = 0.9$

Beam depth:

$h = 733 + \frac{25}{2} + 10 + 40 = 796\ mm$

Use

$h = 800\ mm$ ■

4.9.4 Problems for Solution

A T-beam having a span of 6.0 m, a web thickness of 300mm, and an overall depth of 645 mm. The beams spacing is 1.2m center to center and the slab thickness is 100 mm. Design this beam for flexure to carries a total factored moment of 1300 kN.m.

Assume that the designer intends to use:

- $f_y = 400$ Mpa $f_c' = 28$ Mpa
- $\emptyset 32$mm for longitudinal reinforcement ($A_{Bar} = 819 mm^2$) and $\emptyset 10$mm for stirrups.
- Two layers of reinforcement.

Answers

- Compute of Required Nominal Flexure Strength M_n:

  $M_n = \frac{M_u}{\emptyset} = 1\,444$ kN. m

  where Ø will be assumed 0.9 to be checked later.
- Compute the effective flange width "b":

  $b = b_w + minimum\ \left[\frac{s_w}{2}\ or\ 8h\ or \frac{l_n}{8}\right] \times 2$

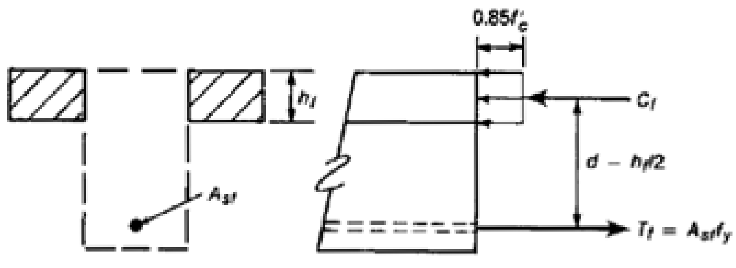
  $b = 300 + minimum\ \left(\frac{1200-300}{2}\ or\ 8 \times 100\ or \frac{6000}{8}\right) \times 2$

  $= 300 + minimum\ (450\ or\ 800\ or 750) \times 2 = 300 + 450 \times 2 = 1200\ mm$
- Check if this section can be design with a compression block in section flange or extend to section web based on following comparison:

  $M_n ?\ 0.85 f_c' h_f b \left(d - \frac{h_f}{2}\right)$

  $d = 550mm$

  $M_n = 1\,444\ kN.m > 0.85 f_c' h_f b \left(d - \frac{h_f}{2}\right) = 1\,428\ kN.m$
- Design of a section with $a > h_f$:
  - Compute the nominal moment that can be supported by flange overhangs:



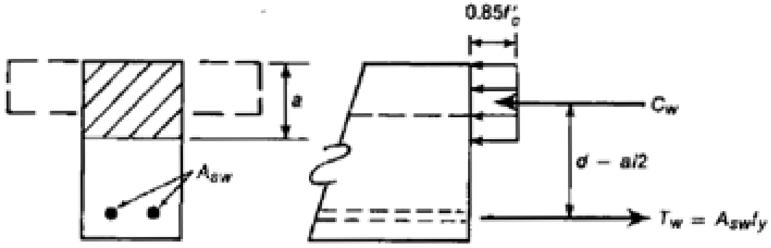
    $M_{n1} = 0.85 f_c' h_f (b - b_w)(d - \frac{h_f}{2}) = 1\,071$ kN. m

    Steel reinforcement for this part will be:

    $A_{sf} = \frac{0.85 f_c' h_f (b - b_w)}{f_y} = 5\,355\ mm^2$
  - Compute the remaining nominal strength that must be supported by section web:

    $M_{n2} = M_n - M_{n1} = 373$ kN. m

For this moment "M_{n2}", the section can be designed as a rectangular section with dimensions of b_w and d:



$$\rho_{Required} = \frac{1 - \sqrt{1 - 2.36\frac{M_{n2}}{f'_c b_w d^2}}}{1.18 \times \frac{f_y}{f'_c}} = 11.4 \times 10^{-3} \Longrightarrow A_{s2} = \rho_{Required} b_w d$$
$$= 1\,881\ mm^2$$

Then:

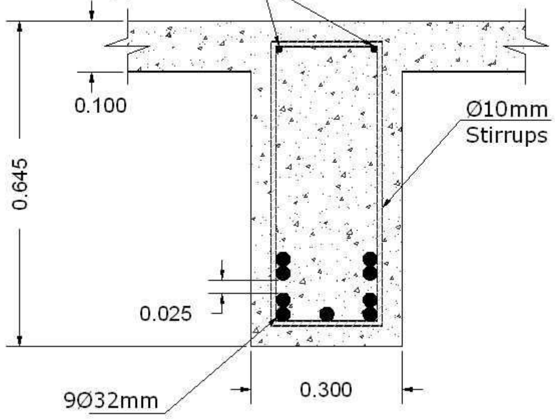
$A_{s\ Required} = A_{sf} + A_{s2} = 7\,236\ mm^2 \Longrightarrow No.\ of\ Rebars = 8.84$

Try $9\emptyset 32mm \Longrightarrow A_{S\ Provided} = 7\,371\ mm^2$

- Check $A_{S\ Provided}$ with minimum steel area permitted by the ACI Code:

$$A_{s\ minimum} = \frac{0.25\sqrt{f'_c}}{f_y} b_w d \geq \frac{1.4}{f_y} b_w d \Longrightarrow A_{s\ minimum} = 578\ mm^2 < A_{S\ Provided}\ Ok.$$

- Check the $A_{S\ Provided}$ with the maximum steel area permitted by ACI Code:



$$\rho_w = \frac{A_{S\ Provided}}{b_w d}\ ?\ \rho_{w\ max}$$
$$= 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} + \frac{A_{sf}}{b_w d}$$
$$\rho_w = 44.7 \times 10^{-3}\ ?\ \rho_{w\ max}$$
$$= 21.7 \times 10^{-3} + 32.5 \times 10^{-3}$$
$$\rho_w = 20.7 \times 10^{-3} < \rho_{w\ max} = 54.2 \times 10^{-3}\ Ok.$$

- Check the assumption of $\emptyset = 0.9$:
  - Compute "a":
  $$a = \frac{(A_s - A_{sf})f_y}{0.85 f'_c (b_w)} = 113\ mm$$
  - Compute steel stain based on the following relations:
  $$c = \frac{a}{\beta_1} = 133\ mm \Rightarrow \epsilon_t = \frac{d-c}{c}\epsilon_u = 9.41 \times 10^{-3}$$
  - As $\epsilon_t > 0.005$, then $\emptyset = 0.9$ Ok.
- Draw the Section Details:

Problem 4.9-1

A reinforced concrete T-beam is to be designed for tension reinforcement. The beam width is 250mm and total depth of 490mm. The flange thickness is 100mm and its effective width has been computed to be 900mm. The applied total factored moment is 300kN.m

Assume that the designer intends to use:

- $f_y = 414\ Mpa, f_c' = 21\ Mpa$
- $\emptyset 28mm$ for longitudinal reinforcement and $\emptyset 10mm$ for stirrups.
- Two layers of reinforcement.

Answers

- Compute of Required Nominal Flexure Strength M_n:
$$M_n = \frac{M_u}{\emptyset} = 333\ kN.m$$
where $\emptyset$ will be assumed 0.9 to be checked later.
- Check if this section can be design with a compression block in section flange or extend to section web based on following comparison:
$$M_n ?\ 0.85 f'_c\ h_f b \left(d - \frac{h_f}{2}\right)$$

$d = 400\ mm$

$M_n = 333\ kN.m > 0.85f'_c\, h_f b\left(d - \frac{h_f}{2}\right) = 562\ kN.m$

- Design of a section with $a \le h_f$:
  This section can be designed as a rectangular section with dimensions of b and d.

$$\rho_{Required} = \frac{1 - \sqrt{1 - 2.36\frac{M_n}{f'_c bd^2}}}{1.18 \times \frac{f_y}{f'_c}} = \frac{1 - \sqrt{1 - 2.36 \times \frac{333 \times 10^6}{21 \times 900 \times 400^2}}}{1.18 \times \frac{414}{21}} = 6.01 \times 10^{-3}$$

$A_{S\ Required} = \rho_{Required} bd = 6.01 \times 10^{-3} \times 900 \times 400 = 2164\ mm^2$

$A_{Bar} = 615\ mm^2$

$\text{No of Rebars} = \frac{2164}{615} = 3.52$

Try $4\emptyset 28mm$

$A_{s\ provided} = 2\,460\ mm^2$

$b_{Required} = 296\ mm > 250mm$

Then the reinforcement must be put in two layers as the designer is assumed.

- Check $A_{s\ Provided}$ with minimum steel area permitted by the ACI Code:

$A_{s\ minimum} = \frac{0.25\sqrt{f'_c}}{f_y} b_w d \ge \frac{1.4}{f_y} b_w d \Rightarrow A_{s\ minimum} = 338\ mm^2$

As $A_{s\ Provided} > A_{s\ minimum}$ Ok.

- Check the $A_{s\ Provided}$ with the maximum steel area permitted by ACI Code:

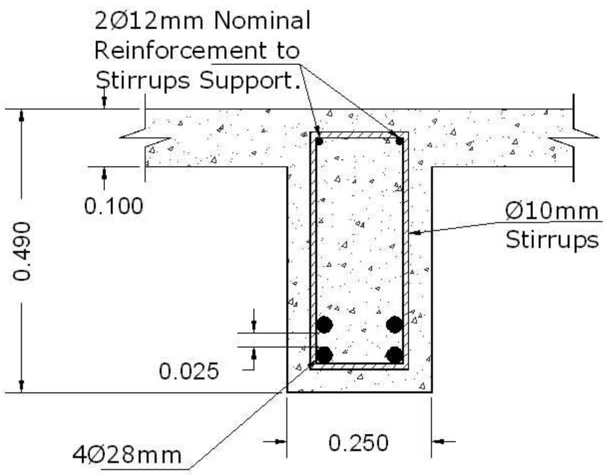
$\rho_w = \frac{A_{s\ Provided}}{b_w d}\ ?\ \rho_{w\ max} = 0.85\beta_1\ \frac{f'_c}{f_y}\ \frac{\epsilon_u}{\epsilon_u + 0.004} + \frac{A_{sf}}{b_w d}$

$A_{sf} = \frac{0.85 f'_c h_f (b - b_w)}{f_y} = 2\,802\ mm^2$

$\rho_w = 24.6 \times 10^{-3}\ ?\ \rho_{w\ max} = 15.7 \times 10^{-3} + 28.0 \times 10^{-3}$

$\rho_w = 24.6 \times 10^{-3} \ll \rho_{w\ max} = 43.7 \times 10^{-3}$ Ok.

- Check the assumption of $\emptyset = 0.9$:
  - Compute "a":
    $\sum F_x = 0 \Rightarrow a = 63.4\ mm$
  - Compute steel stain based on the following relations:
    $c = \frac{a}{\beta_1} = 74.6mm \Rightarrow \epsilon_t = 13.1 \times 10^{-3}$
  - As $\epsilon_t > 0.005$, then $\emptyset = 0.9$ Ok.
- Draw the Section Details:



4.10 Analysis of Beams with Irregular Sections

4.10.1 Basic Concepts

- Beams having shapes other than rectangular and T-shaped cross sections are common, ***particularly in structures using precast elements***.
- The approach for the analysis of such beams is based on applications of basic principles (***compatibility***, ***stress-strain relations***, and ***equilibrium equations***).
- To avoid problems related to unsymmetrical bending:
  - All beams will be assumed to have an axis of symmetry.

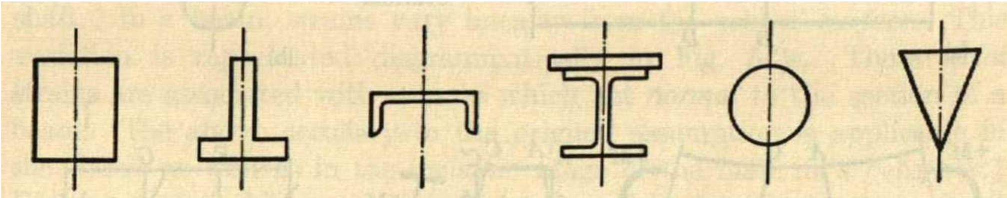


Figure 4.10-1: Different sections with axes of symmetry.

  - All loads will be assumed to act through symmetrical plane.

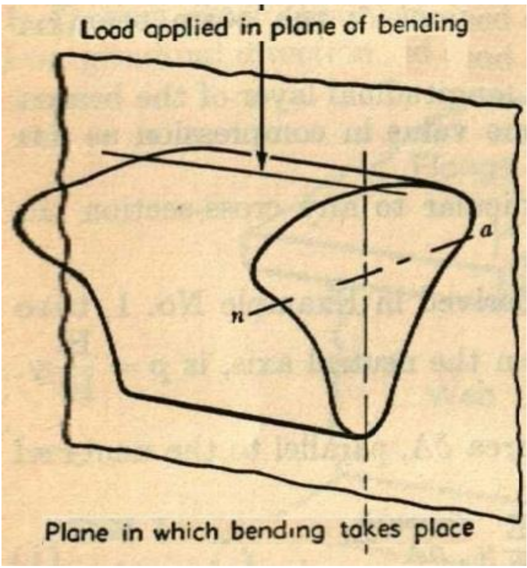


Figure 4.10-2: A beam with loads acting in symmetry plane.

4.10.2 Examples

Example 4.10-1

The cross-section in ***Figure 4.10-3*** below is sometimes referred to as an ***inverted T girder***. Checked if proposed section satisfies ACI requirements and then find its design moment (ϕM_n). In your solution, assume that:

f_c'= 21 MPa and f_y = 420 Mpa

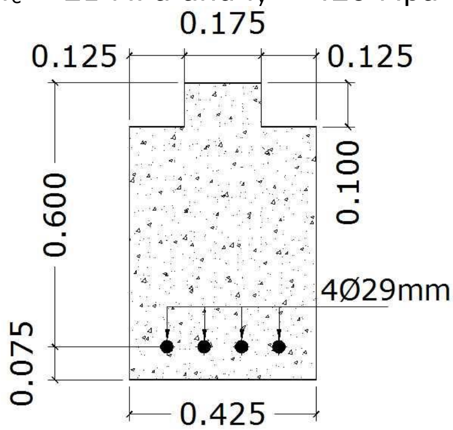
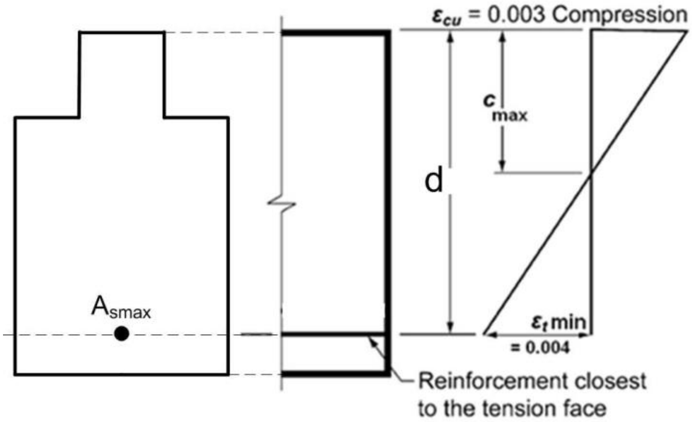


Figure 4.10-3: Inverted T girder for Example 4.10-1.

Solution

- Check type of failure:
  Based on compatibility conditions, and based on the definition of maximum reinforcement area as the area that produces a tensile strain of 0.004 at failure state, the following strain distribution can be concluded:



Based on triangles similarities, following relation for cmax can be concluded:

$$c_{max} = \frac{\epsilon_u}{\epsilon_u + 0.004} d$$
$$= \frac{0.003}{0.003 + 0.004} d$$
$$\Longrightarrow c_{max} = 0.429\, d \;\blacksquare$$

As this relation is derived with considering of strain distribution only, then it is applicable for general shapes.

$$c_{max} = 0.429 \times 600 = 257\ \text{mm}$$

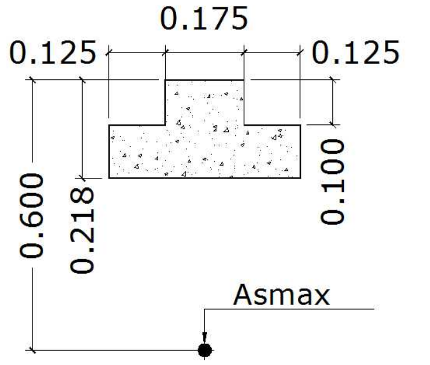
Using Whitney block concept,

$$a_{maximum} = \beta_1 c_{maximum} = 0.85 \times 257 = 218\ mm$$

Based on equilibrium condition,

$$\Sigma F_x = 0 \Rightarrow 0.85 \times 21 \times (175 \times 100 + 118 \times 425)$$
$$= 420 \times A_{s\ Maximum} \Rightarrow A_{s\ Maximum}$$
$$= 2875\ mm^2$$

$$A_{s\ Provided} = 4 \times \frac{\pi \times 29^2}{4} = 2640\ mm^2 < A_{s\ maximum}$$
$$\therefore Ok.$$

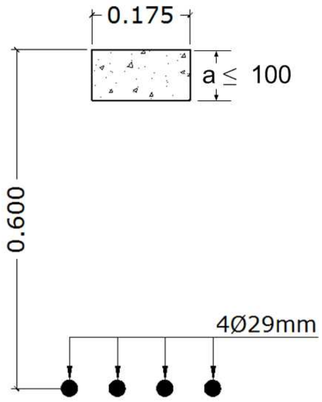


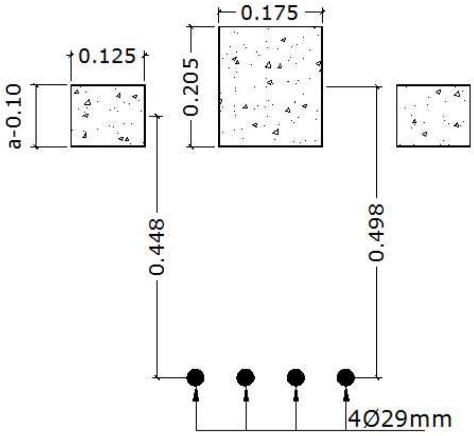
- Check $A_{s\ Minimum}$:
  As divergence from rectangular section is limited to the upper part only, then the minimum reinforcement area can be computed based on traditional relation:

$$A_{s\ Minimum} = \frac{1.4}{420} \times 425 \times 600 = 850\ mm^2 < A_{s\ Provided} \therefore Ok.$$

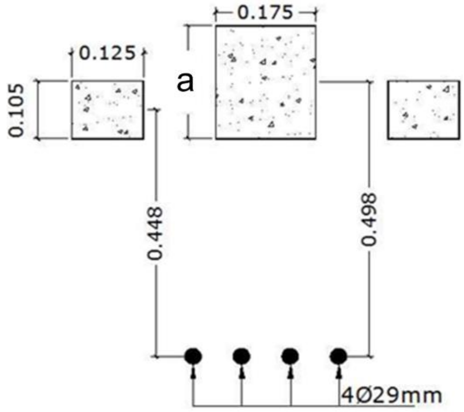
- Compute M_n:
  Let $a \leq 100$:

$$\Sigma F_x = 0 \Rightarrow 0.85 \times 21 \times 175 \times a = 4 \times 660 \times 420 \;\Longrightarrow\; a = 354\ mm$$
$$> 100\ Not\ Ok$$





$$\Sigma Fx = 0 \Rightarrow 0.85 \times 21$$
$$\times \left(175 \times a + 2 \times \left(125 \times (a - 100)\right)\right)$$
$$= 4 \times 660 \times 420$$
$$0.85 \times 21 \times (175 \times a + 250 \times a - 25000)$$
$$= 4 \times 660 \times 420$$
$$(425 \times a - 25000) = 62117.6 \;\Longrightarrow\; a = 205$$
$$\Sigma M_{about\ Reinforcement} = 0$$
$$M_n = 0.85 \times 21$$
$$\times \left(205 \times 175 \times 498\right.$$
$$\left. + 2(125 \times 105 \times 448)\right) = 529\ kN.m$$



- Compute ϕ:
  - Compute "a":
    $a = 205\ \text{mm}$

- Compute steel stain based on the following relations:
  $$c = \frac{a}{\beta_1} = \frac{205}{0.85} = 241\text{ mm} \Rightarrow \epsilon_t = \frac{d-c}{c}\epsilon_u = \frac{600-241}{241} \times 0.003 = 4.47 \times 10^{-3}$$
- Then ϕ should be computed based on following relation:
  $$\phi = 0.483 + 83.3\epsilon_t \Rightarrow = 0.483 + 83.3 \times 4.47 \times 10^{-3} = 0.855$$

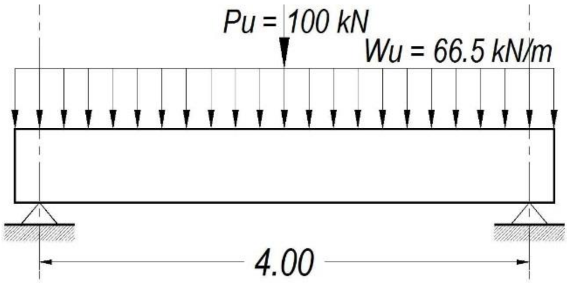
- Compute ϕM_n:
  $$\phi M_n = 0.855 \times 529 = 452\ kN.m \blacksquare$$

Example 4.10-2

For the simply supported beam with a trapezoidal section that shown in Figure 4.10-4 below, and based on flexure strength of the given section find required beam depth (h) and width (b) that are necessary to support the applied loads.

In your solution, assume that:

- Beam selfweight could be neglected.
- As = 510mm^2 for $\phi 25mm$ rebars.
- $f_c' = 28\ MPa$ and $f_y = 420\ MPa$.



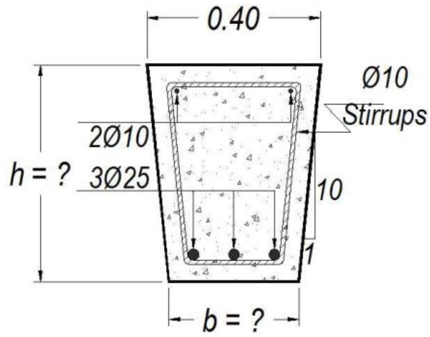
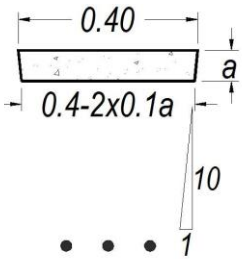


Figure 4.10-4: Trapezoidal beam for Example 4.10-2. **Typical Section**

Solution

Compute depth of compression block "a":



$$\Sigma F_x = 0$$

$$0.85 \times 28\left(\frac{400 + (400 - 0.2a)}{2}\right) \times a = 1530 \times 420$$

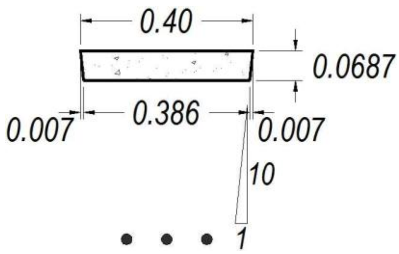
$$(800 - 0.2a) \times a = 54000$$

$$0.2a^2 - 800a + 54000 = 0$$

Solve for a:

$$a = 68.7\ mm$$

Compute effective depth "d":



$$M_u = \frac{W_u l^2}{8} + \frac{P_u l}{4} = \frac{66.5 \times 4.0^2}{8} + \frac{100 \times 4}{4}$$
$$= 233\ kN.m$$

Let $\phi = 0.9$ to be checked later:

$$M_n = \frac{233}{0.9} = 259\ kN.m$$

$$259 \times 10^6 = 0.85 \times 28 \times \left(386 \times 68.7 \times \left(d - \frac{68.7}{2}\right) + \frac{7 \times 68.7}{2} \times 2 \times \left(d - \frac{68.7}{3}\right)\right)$$

$$d = 437\ mm$$

Check ϕ:

$$a = 68.7\ mm \Rightarrow c = \frac{68.7}{0.85} = 80.8\ mm$$

$$\epsilon_t = \frac{d-c}{c}\epsilon_u = \frac{437 - 80.8}{80.8} \times 0.003 = 0.0132 > 0.005$$

then:

$$\phi = 0.9$$

$$\therefore h = 437 + \frac{25}{2} + 10 + 40 = 500\ mm \blacksquare$$

$$b = 400 - \frac{1}{10} \times 500 \times 2 = 300\ mm \blacksquare$$

Example 4.10-3

In a trail to reduce the cost of beam through reducing of concrete on tension side, a structural designer has been proposed section shown in Figure 4.10-5 below to be used through the length of beam show.

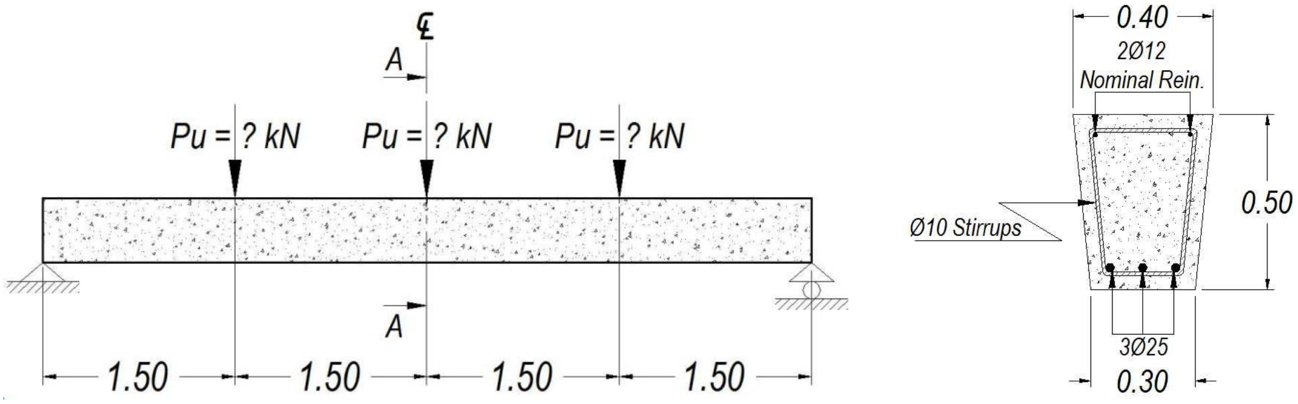


Figure 4.10-5: Trapezoidal beam for Example 4.10-3. **Typical Section**

Check the adequacy of proposed section to ACI flexure requirements and then computed the maximum factored point load (P_u) that can be supported by the beam based on flexural strength.

In your solution, assume that:

- Beam selfweight can be neglected.
- $f_c' = 21\ MPa, f_y = 420\ MPa$
- $A_{Bar} = \ 510\ mm^2\ for\ \phi\ 25mm$

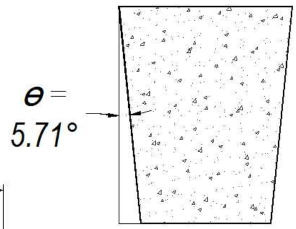
Solution

- Check type of failure:

Based on compatibility conditions, and based on definition of A_{smax} as the reinforcement area that produce a tensile strain of 0.004 at failure state:

$$c_{max} = \frac{\epsilon_u}{\epsilon_u + 0.004} d = \frac{0.003}{0.003 + 0.004} d = 0.429\ d$$

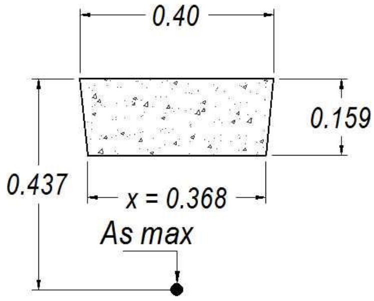
$$d = 500 - 40 - 10 - \frac{25}{2} = 437\ mm \Longrightarrow c_{max} = 0.429 \times 437 = 187\ \text{mm}$$



Using Whitney block concept,

$$a_{maximum} = \beta_1 c_{maximum} = 0.85 \times 187 = 159\ mm$$

$$\theta = \tan^{-1}\frac{50}{500} = 5.71^o$$



$$x = 400 - 2 \times \tan 5.71 \times 159 = 368\ mm$$

$$0.85 \times 21 \times \left(\frac{400 + 368}{2}\right) \times 159 = 420 \times A_{s\ Maximum}$$

$$A_{s\ Maximum} = 2595\ mm^2$$

$$A_{s\ Provided} = 3 \times 510 = 1530\ mm^2 < A_{s\ maximum}\ \therefore Ok.$$

- Compute M_n:

Whitney block depth, a, could be computed based on following relation:

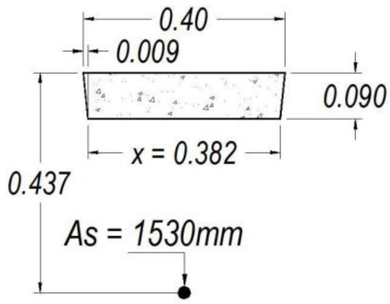
$$\Sigma F_x = 0$$

$$0.85 \times 21 \times \left(400a - 2 \times a \times \frac{a \times tan 5.71}{2}\right) = 1530 \times 420$$

$$0.1a^2 - 400a + 36000 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = \frac{+400 \pm \sqrt{400^2 - 4 \times 0.1 \times 36000}}{2 \times 0.1} = \frac{400 \pm 382}{0.2} = 90mm$$



$$\Sigma M_{about\ steel\ center} = \ 0$$

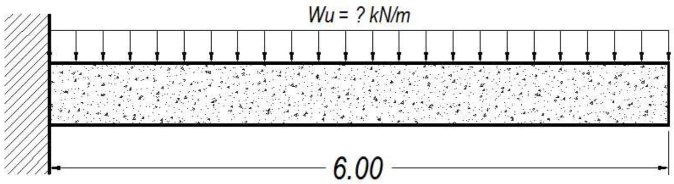
$$M_n = 0.85 \times 21 \times [382 \times 90 \times \left(437 - \frac{90}{2}\right) + 2 \times \frac{1}{2} \times 9 \times 90 \times \left(437 - \frac{90}{3}\right)]$$

$$M_n = 0.85 \times 21 \times [13.5 \times 10^6 + 0.330 \times 10^6] = 247\ kN.m$$

- Compute ϕ:
  - Compute "a":
    $a = 90 \text{ mm}$
  - Compute steel stain based on the following relations:
    $$c = \frac{a}{\beta_1} = \frac{90}{0.85} = 106 \text{ mm} \Longrightarrow \epsilon_t = \frac{d - c}{c}\epsilon_u = \frac{437 - 106}{106} \times 0.003 = 9.37 \times 10^{-3}$$
    Then:
    $\phi = 0.9$
- Compute ϕM_n:
  $\phi M_n = 0.9 \times 247 = 222\ kN.m$
- Compute Pu:
  $$M_u = \frac{P_u \times 6}{4} + P_u \times 1.5 = 222 \Rightarrow P_u = 74\ kN \blacksquare$$

Example 4.10-4

Based on flexure strength for beam shown in Figure 4.10-6 below, what is the maximum factored uniformly distributed load "W_u" that can be supported?



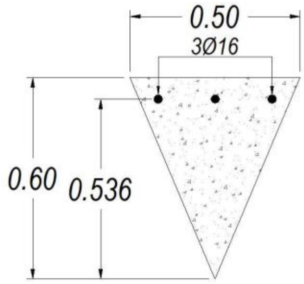


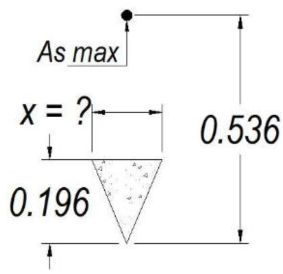
Figure 4.10-6: Beam with triangular section for Example 4.10-4. **Typical Section**

In your solution, assume that:

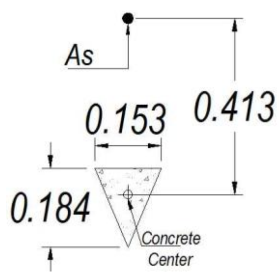
- Beam selfweight can be.
- $f_c' = 21\ MPa, f_y = 420\ MPa$
- $A_{Bar} = 200\ mm^2\ for\ \phi\ 16mm$
- Neglect the checking for As minimum.

Solution

- Check type of failure:
  Based on compatibility conditions, and based on definition of Asmax as the reinforcement area that produce a tensile strain of 0.004 at failure state:
  $$c_{max} = \frac{\epsilon_u}{\epsilon_u + 0.004} d = \frac{0.003}{0.003 + 0.004} d = 0.429\ d \blacksquare$$
  $d = 536\ mm \Longrightarrow c_{max} = 0.429 \times 536 = 230 \text{ mm}$
  Using Whitney block concept,
  $a_{maximum} = \beta_1 c_{maximum} = 0.85 \times 230 = 196\ mm$
  Based on triangles similarities,
  $$\frac{x}{196} = \frac{500}{600} \Rightarrow x = 163\ mm$$
  $\Sigma F_x = 0$
  $$0.85 \times 21 \times \left(\frac{196 \times 163}{2}\right) = 420 \times A_{s\ Maximum} \Rightarrow A_{s\ Maximum} = 679\ mm^2$$
  $A_{s\ Provided} = 3 \times 200 = 600\ mm^2 < A_{s\ maximum} \therefore Ok.$



- Compute M_n:
  Triangle base for a height of "a" could be computed based on following relation:
  $$\frac{x}{a} = \frac{500}{600} \Longrightarrow x = 0.833\ a$$
  $\Sigma F_x = 0$
  $$0.85 \times 21 \times \left(\frac{0.833a^2}{2}\right) = 600 \times 420 \Longrightarrow a = 184 \text{ mm}$$
  $$\Sigma M_{about\ concrete\ center} = 0 \Longrightarrow M_n = 600 \times 420 \times 413 = 104 \text{ kN.m}$$



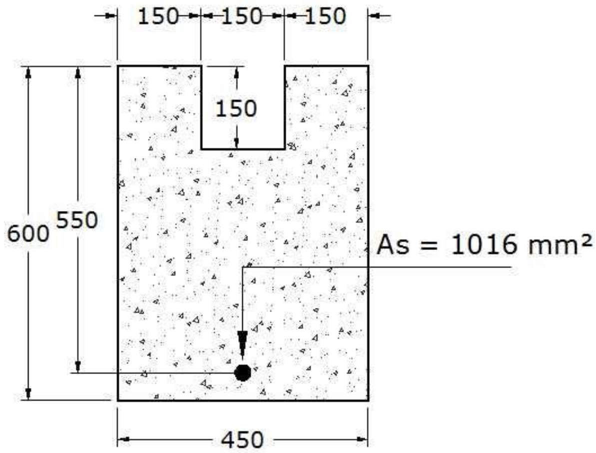
Compute ϕ:
- Compute "a":
  $\mathrm{a} = 184\ \mathrm{mm}$
- Compute steel stain based on the following relations:
  $$\mathrm{c} = \frac{\mathrm{a}}{\beta_1} = \frac{184}{0.85} = 216\ \mathrm{mm} \Longrightarrow \epsilon_t = \frac{\mathrm{d} - \mathrm{c}}{\mathrm{c}} \epsilon_u = \frac{536 - 216}{216} \times 0.003 = 4.44 \times 10^{-3}$$
- Then ϕ should be computed based on following relation:
  $\phi = 0.483 + 83.3\epsilon_t = 0.483 + 83.3 \times 4.44 \times 10^{-3} = 0.853$

- Compute ϕM_n:
  $\phi M_n = 0.853 \times 104 = 88.7\ kN.m$ ■
- Compute Wu:
  $$88.7 = \frac{W_u \times 6^2}{2} \Rightarrow W_u = 4.93 \frac{kN}{m}$$ ■

4.10.3 Problems for Solution

Problem 4.10-1

Check adequacy of the indicated section for ACI requirements of maximum and minimum steel areas and compute its design bending strength if it is satisfied for ACI requirements.
$f_c' = 25\ MPa, f_y = 400\ MPa$

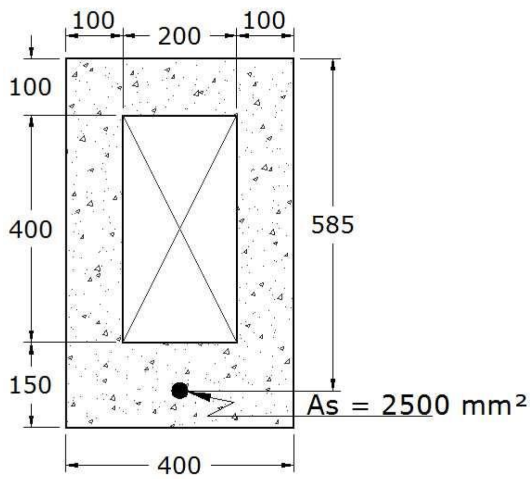


Answers

- Check for A_{smax} and A_{smin}:
  $c_{max} = 0.429\ d = 0.429 \times 550 = 236\ mm$
  $a_{maximum} = 0.85 \times 236 = 201\ mm$
  $\Sigma F_x = 0$
  $0.85 \times 25 \times (2 \times 150 \times 150 + (201 - 150) \times 450) = A_{s\ maximum} \times 400$
  $A_{s\ maximum} = 3610\ mm^2 > A_s\ \ Ok$
  $A_{s\ minimum} = \frac{1.4}{400} \times 450 \times 550 = 866\ mm^2 < A_s\ Ok.$
- Compute M_n:
  Assume $a \leq 150$:
  $0.85 \times 25 \times 2 \times 150 \times a = 400 \times 1016 \Rightarrow a = 63.7\ mm < 150\ Ok$
  $\Sigma M_{about\ T} = 0$
  $M_n = 0.85 \times 25 \times 300 \times 63.7 \left(550 - \frac{63.7}{2}\right) = 210\ kN.m$
- Compute ϕM_n:
  a. Compute "a":
     $\mathrm{a = 63.7\ mm}$
  b. Compute steel stain based on the following relations:
     $\mathrm{c} = \frac{\mathrm{a}}{\beta_1} = \frac{63.7}{0.85} = 74.9\ \mathrm{mm} \Rightarrow \epsilon_t = \frac{\mathrm{d-c}}{\mathrm{c}}\epsilon_u = \frac{550 - 74.9}{74.9} \times 0.003 = 19.0 \times 10^{-3}$
  c. As $\epsilon_t \geq 0.005$ then:
     $\phi = 0.9$
     $\phi M_n = 0.9 \times 210 = 189\ kN.m$

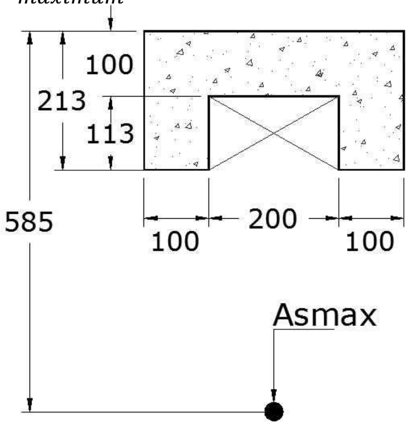
Problem 4.10-2

Check adequacy of the indicated section for ACI requirements of maximum and minimum steel areas and compute its design bending strength if it is satisfied for ACI requirements.
$f_c' = 20\ MPa, f_y = 400\ MPa$



Answers

- Check for As_{max} and As_{min}:
  $c_{max} = 0.429\ d = 0.429 \times 585 = 251\ mm$
  $a_{maximum} = 213$



$\Sigma F_x = 0$
$0.85 \times 20 \times (400 \times 100 + 113 \times 100 \times 2) = A_{s\ Maximum} \times 400$

$A_{s\,maximum} = 2660\ mm^2 > A_s\ \ Ok.$

Asmin could be conservatively computed based on following relation:

$$A_{s\,minimum} = \frac{1.4}{400} \times 400 \times 585 = 819\ mm^2 < As\ Ok.$$

- Compute M_n:

  Assume that $a \leq 100$:

  $\Sigma F_x = 0$

  $0.85 \times 20 \times 400 \times a = 400 \times 2500$

  $a = 147\ mm > 100\ Not\ Ok.$

  $0.85 \times 20 \times (200 \times 100 + 2 \times 100 \times a) = 400 \times 2500$

  $a = 194\ mm$

  $\Sigma M_{about\ T} = 0$

  $$M_n = 0.85 \times 20 \times \left(200 \times 100 \times \left(585 - \frac{100}{2}\right) + 2 \times 100 \times 194 \times \left(585 - \frac{194}{2}\right)\right) = 503\ kN.m$$

- Compute ϕM_n:
  a. Compute "a":

     $a = 194\ mm$
  b. Compute steel stain based on the following relations:

     $$c = \frac{a}{\beta_1} = \frac{194}{0.85} = 228\ mm$$

     $$\epsilon_t = \frac{d - c}{c}\epsilon_u = \frac{585 - 228}{228} \times 0.003 = 4.70 \times 10^{-3}$$
  c. Then ϕ should be computed based on following relation:

     $\phi = 0.483 + 83.3\epsilon_t$

     $\phi = 0.483 + 83.3 \times 4.70 \times 10^{-3} = 0.875$

     $\phi M_n = 0.875 \times 503 = 440\ kN.m$

4.11 USING STAAD PRO SOFTWARE FOR FLEXURAL ANALYSIS AND DESIGN OF RC BEAMS*

In general, most of software, including of STAAD Pro, have been prepared to design problems with pre-specified dimensions where analysis and design processes are simulated as pure iterative. Therefore, only sections analysis and design (with pre-specified dimensions) are presented in this article.

4.11.1 Design of a Singly Reinforced Concrete Beam with a Rectangular Shape

STAAD Pro steps for analysis and design of simply supported beams have been presented in this article with referring for Example 4.4-2 that, for convenient, has been represented in Figure 4.11-1 below. Following data have been adopted for this design:

- Concrete of $f_c' = 30\ MPa$.
- Steel of $f_y = 420\text{MPa}$.
- A width of 300mm and a height of 430mm (these dimensions have been determined based on deflection considerations).
- Rebar of No. 25 for longitudinal reinforcement.
- Rebar of No. 10 for stirrups.
- Single layer of reinforcement.

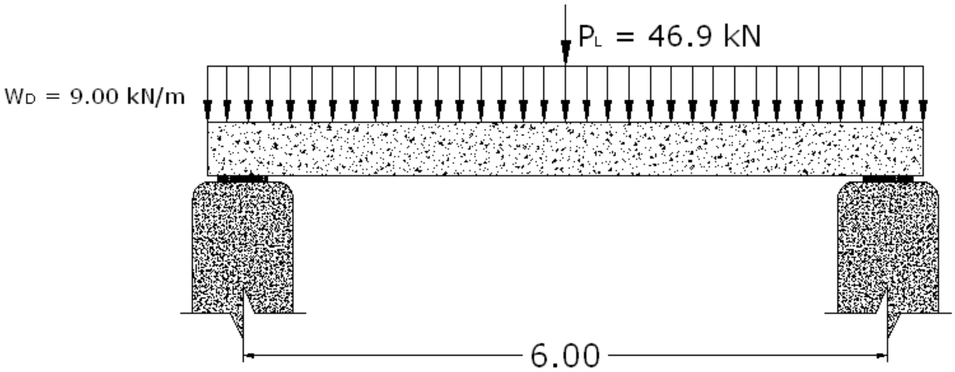


Figure 4.11-1: Simply supported bridge for Example 4.4-2, represented for convenient.

Solution

4.11.1.1 Type of Structure and Units

- Based on interactive box presented in Figure 4.11-2 below, select suitable ***Structure Type*** and suitable ***Units*** to be adopted in simulation of the beam.
- A Structure can be defined as an assemblage of elements. STAAD is capable of analyzing and designing structures consisting of frame, plate/shell and solid elements. Almost any type of structure can be analyzed by STAAD.
  a. A ***SPACE*** structure, which is a three dimensional framed structure with loads applied in any plane, is the most general.
  b. A ***PLANE*** structure is bound by a global X-Y coordinate system with loads in the same plane.
  c. A ***TRUSS*** structure consists of truss members who can have only axial member forces and no bending in the members.
  d. A ***FLOOR*** structure is a two or three-dimensional structure having no horizontal (global X or Z) movement of the structure [FX, FZ & MY are restrained at every joint]. The floor framing (in global X-Z plane) of a building is an ideal example of a FLOOR structure.
- Specification of the correct structure type reduces the number of equations to be solved during the analysis. This results in a faster and more economical solution for the user. The degrees of freedom associated with frame elements of different types of structures is illustrated in Figure 4.11-3.
- The beam of this example is simulated as a plane structure. Using a suitable structure type saving computer resources and avoid stability problems related to some structures, for example plane trusses, when simulated with a three dimensional model.

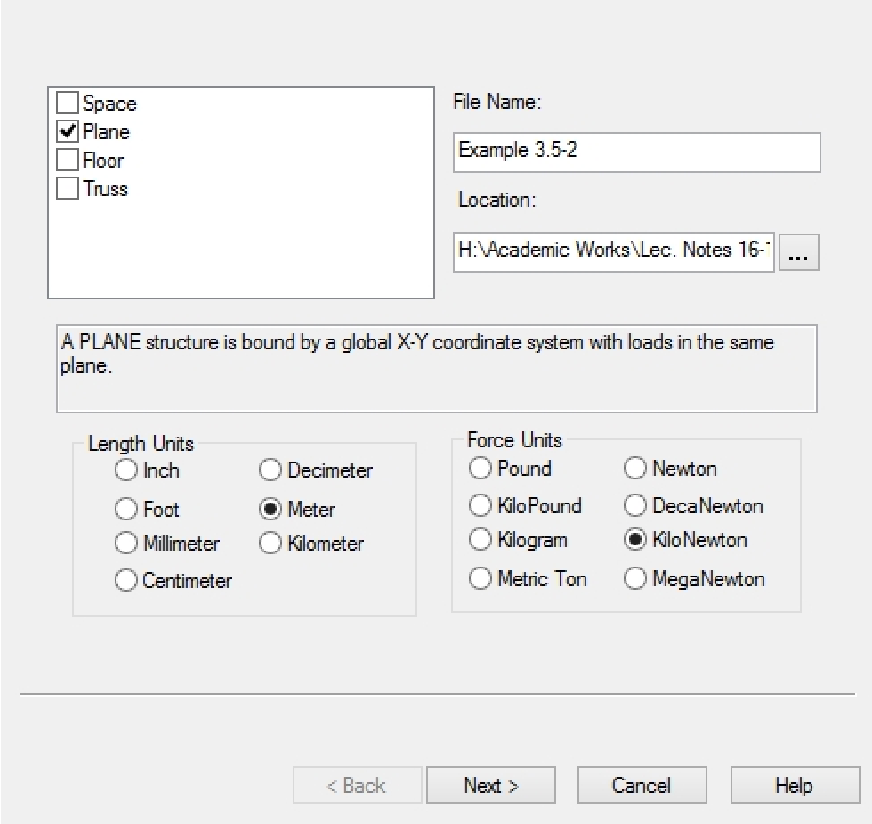


Figure 4.11-2: Different structure types in STAAD environment.

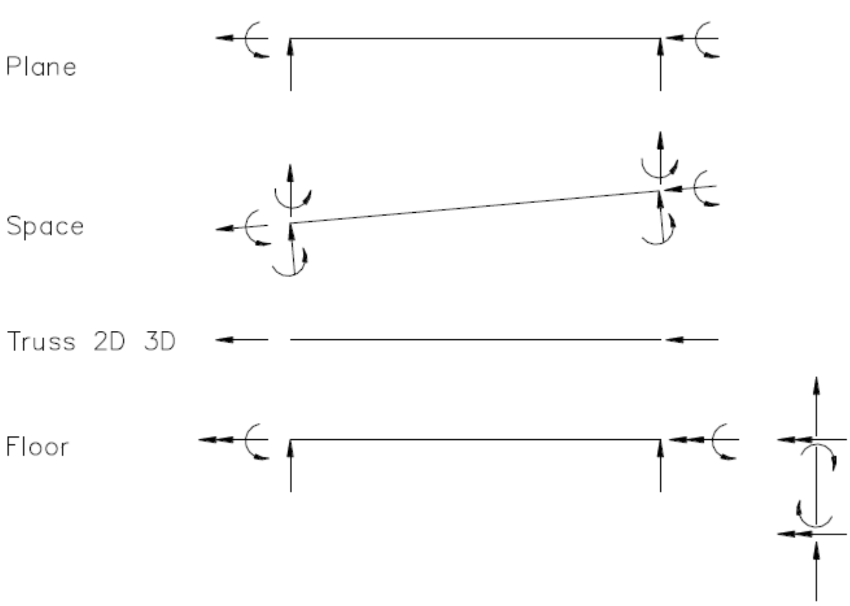
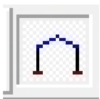


Figure 4.11-3: Degrees of freedom associated with frame elements of different types of structures.

4.11.1.2 STAAD Pages for a Sequential Work

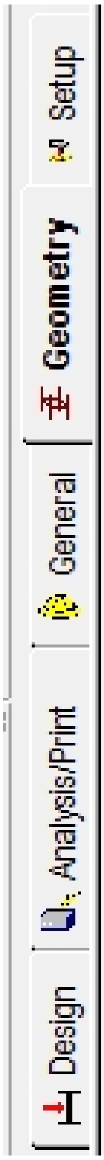
- Workflow in STAAD environment has prepared in form of pages. When these pages are followed, the model would be complete and ready for execution.
- According to STAAD software, the preparation process is called ***Modeling*** and indicated with icon below:

  Main modeling pages in STAAD environment are presented in Figure 4.11-5 Design parameters should be defined in the Modeling stage. Each pages are explained briefly in below.

4.11.1.3 Setup Page

In the Setup page, the user can input all information related to Job based on interactive box indicated Figure 4.11-4.



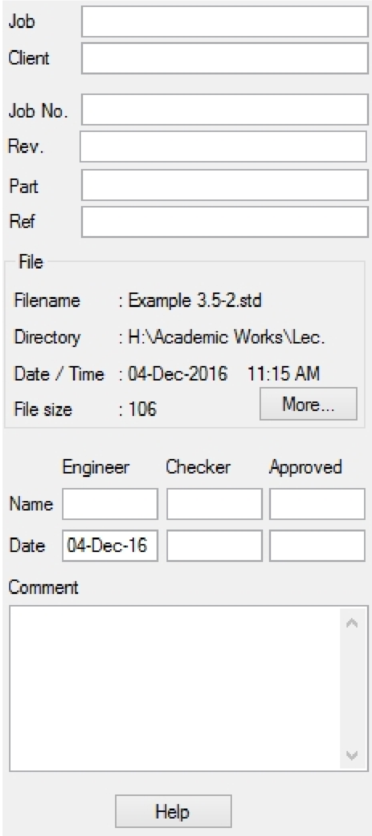


Figure 4.11-4: Setup page in STAAD environment.

Figure 4.11-5: Main modeling pages in STAAD environment.

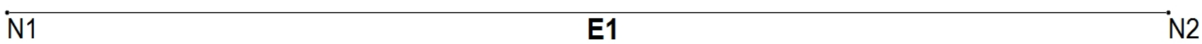
Figure 4.11-6: Geometry Page in STAAD environment.

4.11.1.4 Geometry of the Beam

- Geometry Page has sub-pages indicated in Figure 4.11-6 above.
- STAAD software starts with definition of ***Nodes*** to prepare the geometry of the structure. In skeleton structures, nodes have been physically defined and located at ends of member.
- For the beam of this article, two nodes with coordinates below have been generated.

Node	X m	Y m	Z m
1	0.000	0.000	0.000
2	6.000	0.000	0.000
3			

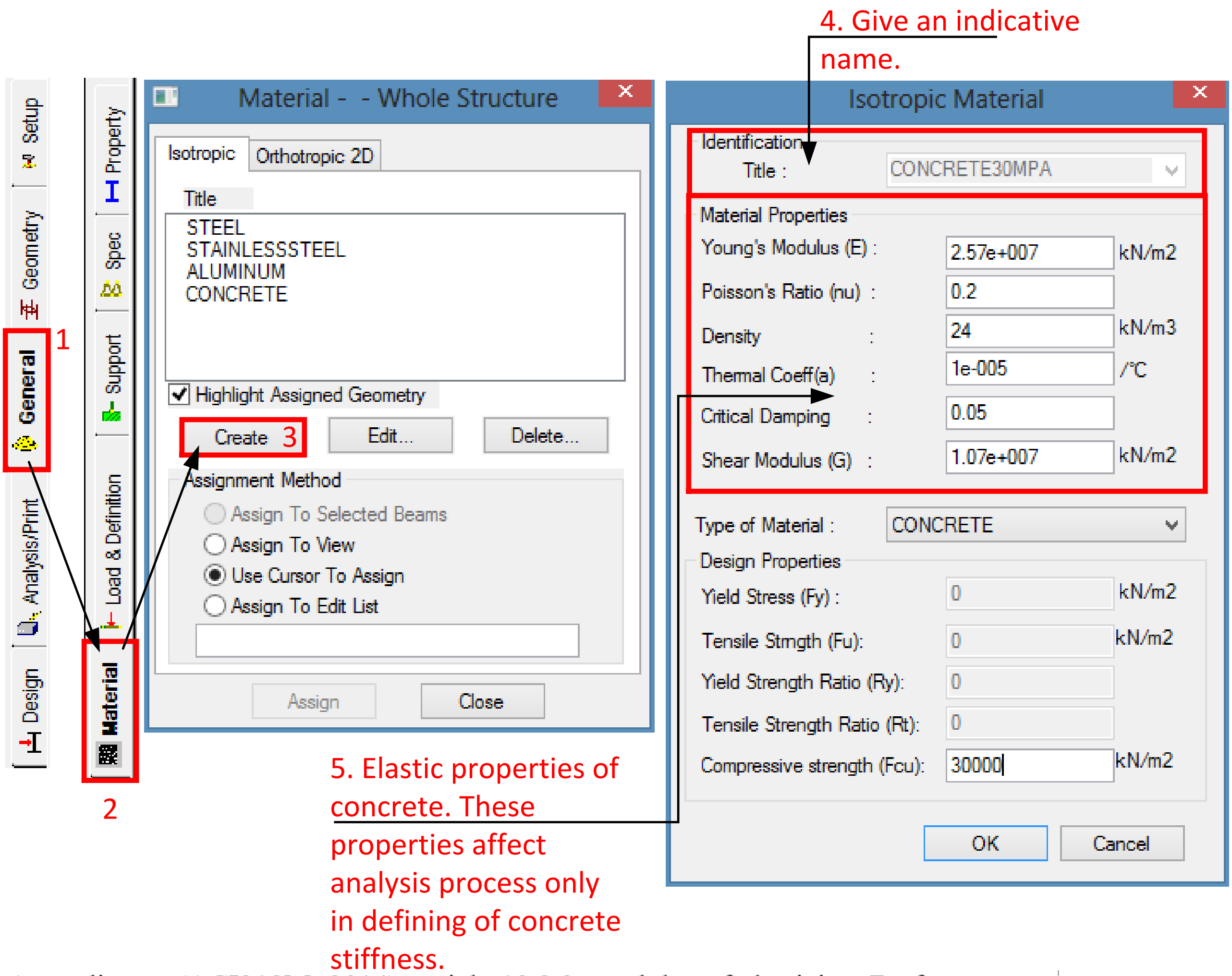
- After definition of nodes, use Add Beam icon  to draw the beam that connecting between nodes.



4.11.1.5 Definition of Material Prosperities

- Definition of new concrete properties has been presented in Figure 4.11-7 below.
- Regarding to shear modulus, G, based on mechanic of materials, one can show that:

$$G = \frac{E}{2(1+\nu)}$$



According to (ACI318M, 2014), article 19.2.2, modulus of elasticity, E_c, for concrete can be estimated based on following correlation:

- For values of w_c between 1440 and 2560 kg/m^3

$$E_c = w_c^{1.5} 0.043\sqrt{f_c'} \ (in\ MPa)$$

- For normalweight concrete

$$E_c = 4700\sqrt{f_c'} \ (in\ MPa)$$

At *stresses lower than about* 0.7f_c', *Poisson's ratio for concrete falls within the limits of 0.15 to 0.20.*

Figure 4.11-7: Definition of material properties in STAAD environment.

4.11.1.6 Properties of the Beam

- Use page General and correspond sub-pages indicated below to defined and assign:
    - Beam Section,
    - Material,
    - Supports.
- Beam Section:

  Use subpage 

 to defined and assign beam section.
- Define section properties as indicated in steps of Figure 4.11-9 below.
- Defined beam section can now be assigned to pertained member based on steps of Figure 4.11-10 below.

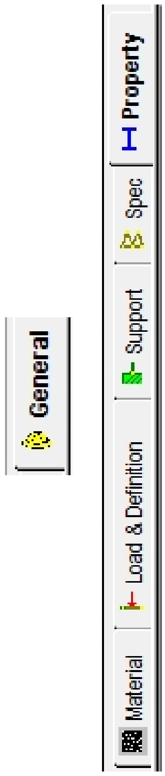


Figure 4.11-8: General page and corresponding subpages in STAAD environment.

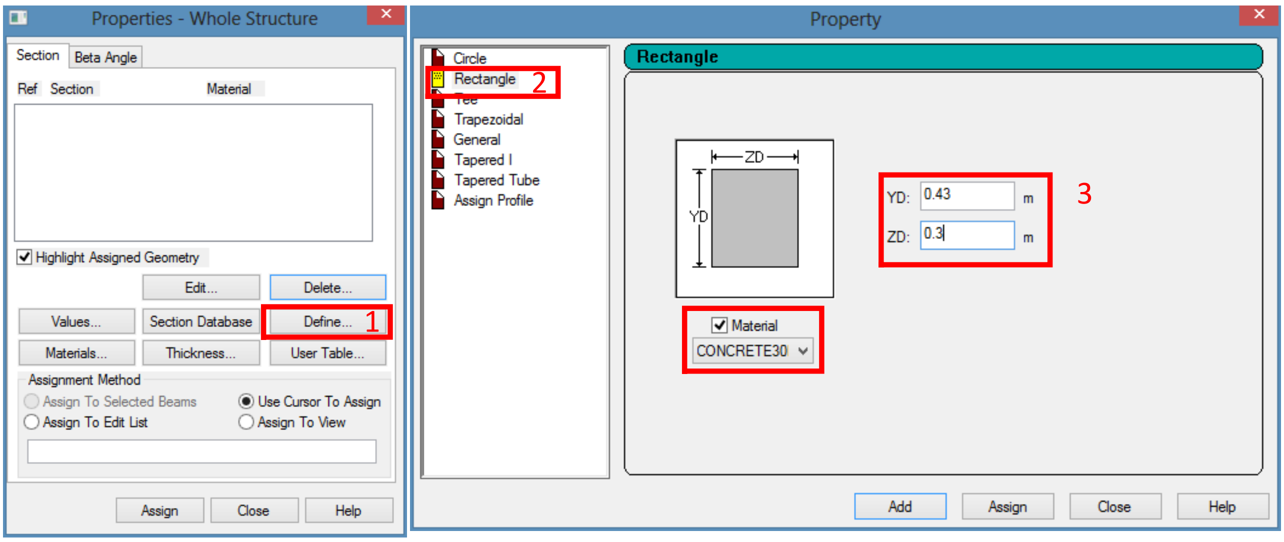


Figure 4.11-9: Steps to define section properties in STAAD environment.

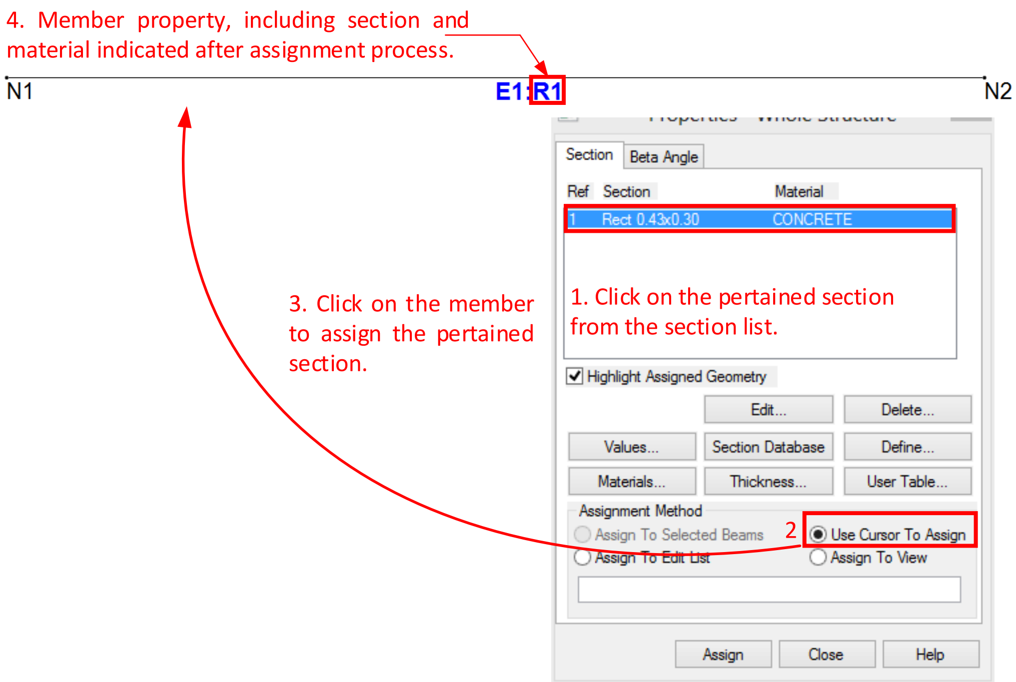


Figure 4.11-10: Steps to section assignment in STAAD environment.

4.11.1.7 Definition and Assignment of the Supports

- Supports can be defined as indicated in steps below.

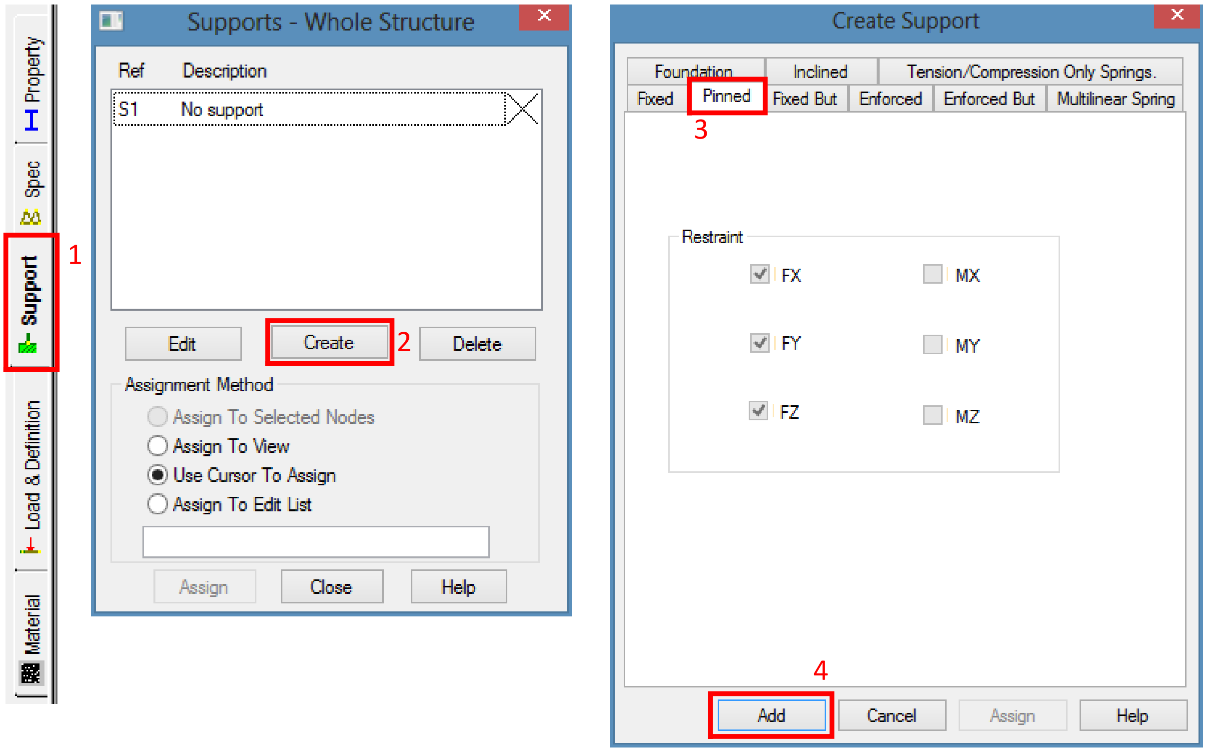


Figure 4.11-11: Steps to define supports in STAAD environment.

- For beam linear analysis, axial forces are already neglected and therefore there is no difference between hinge support and roller support form point of view.
- Defined supports can be assigned to related nodes based on following steps:

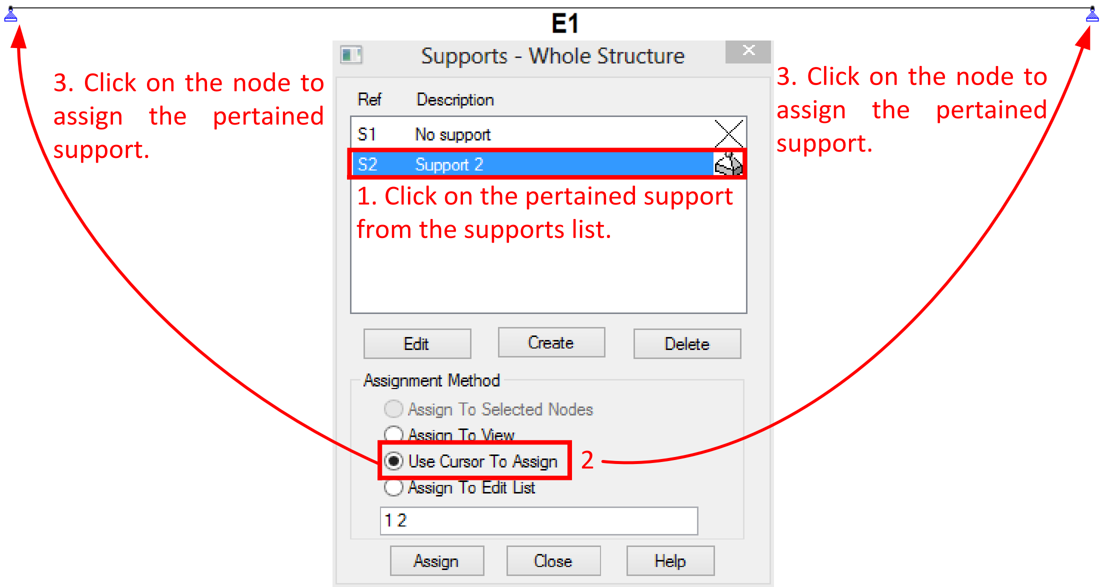


Figure 4.11-12: Steps to assign supports in STAAD environment.

4.11.1.8 Definition of Basic Load Cases, and Load Combinations

4.11.1.8.1 Definition Loads Cases

Basic load cases, namely ***Dead*** and ***Live*** can be defined based on following steps:

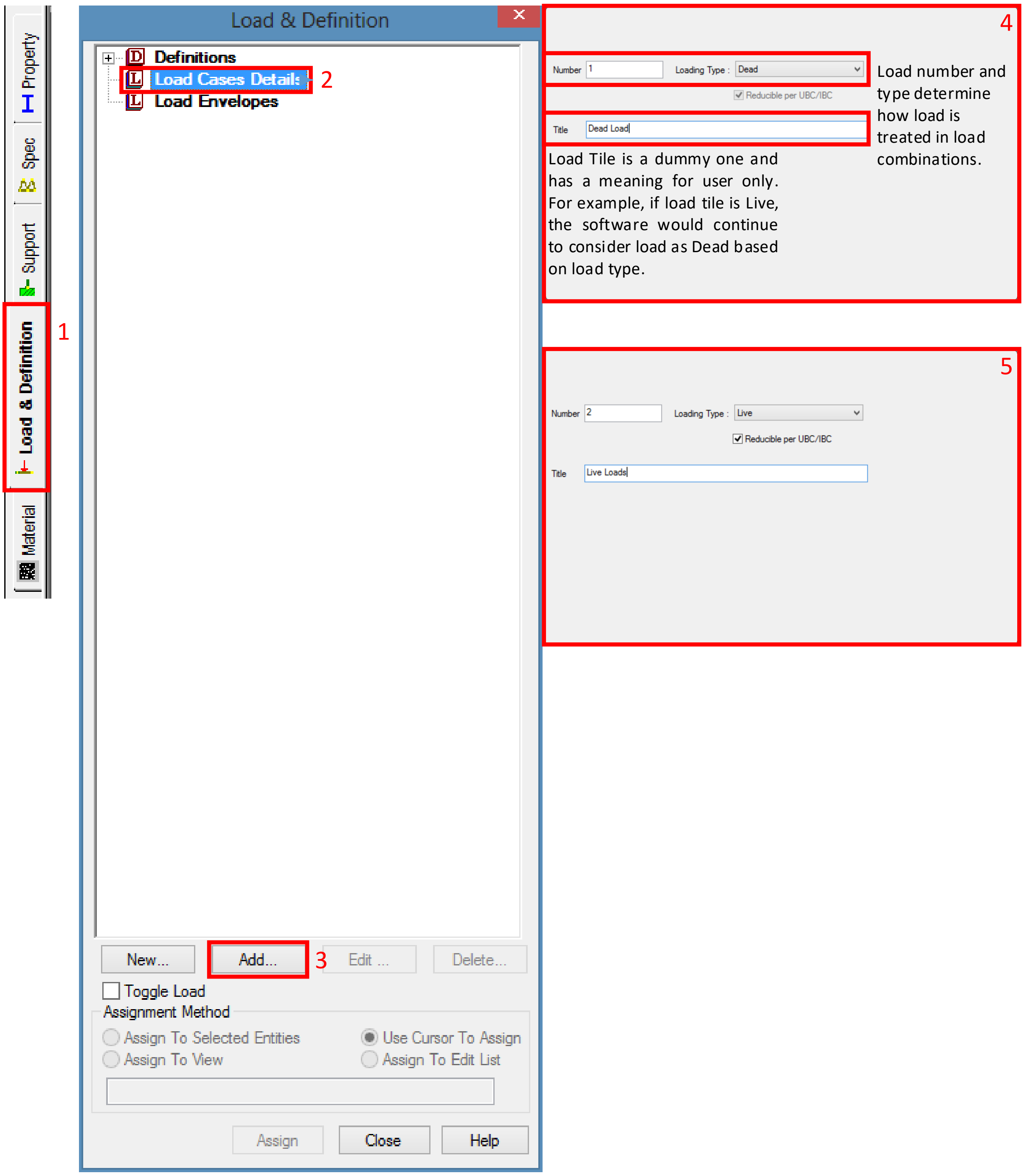


Figure 4.11-13: Definition of basic load cases.

4.11.1.8.2 Definition of Load Values and Assign them to Related Members

- Selfweight can be defined and assigned based on following steps.

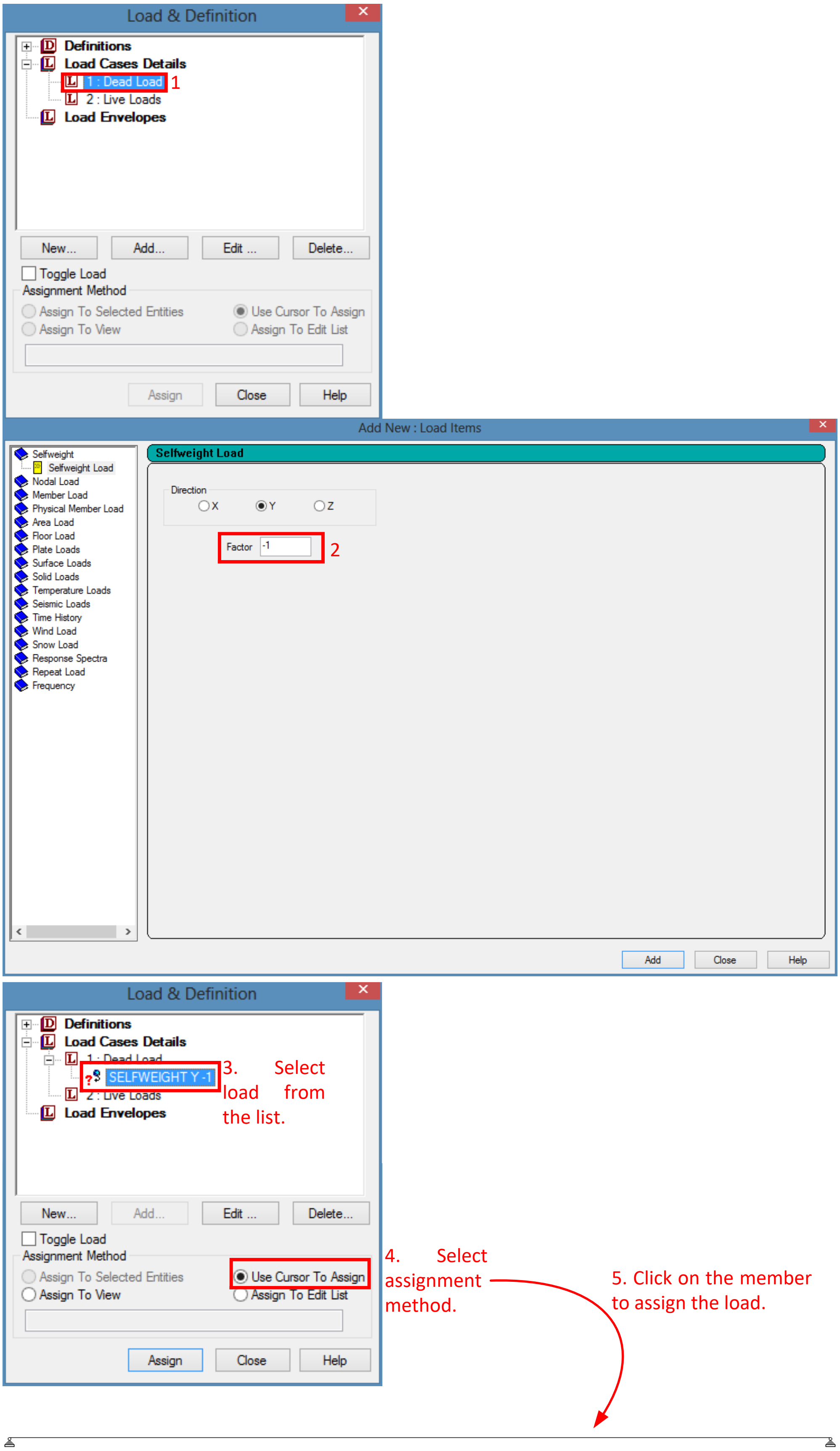


Figure 4.11-14: Definite and assignment of beam selfweight.

- In previous STAAD versions, definition of selfweight implicitly assign it to the whole structure.
- For beams, selfweight is determined based on material density and beam cross sectional dimensions.

- Regarding to the superimposed dead load of $W_{D\ superimposed}$ of $9.00\ kN/m$, and point live load P_L of $46.9\ kN$, except for loads definition that presented in below all other steps are similar to above steps for selfweight definition and assignment.

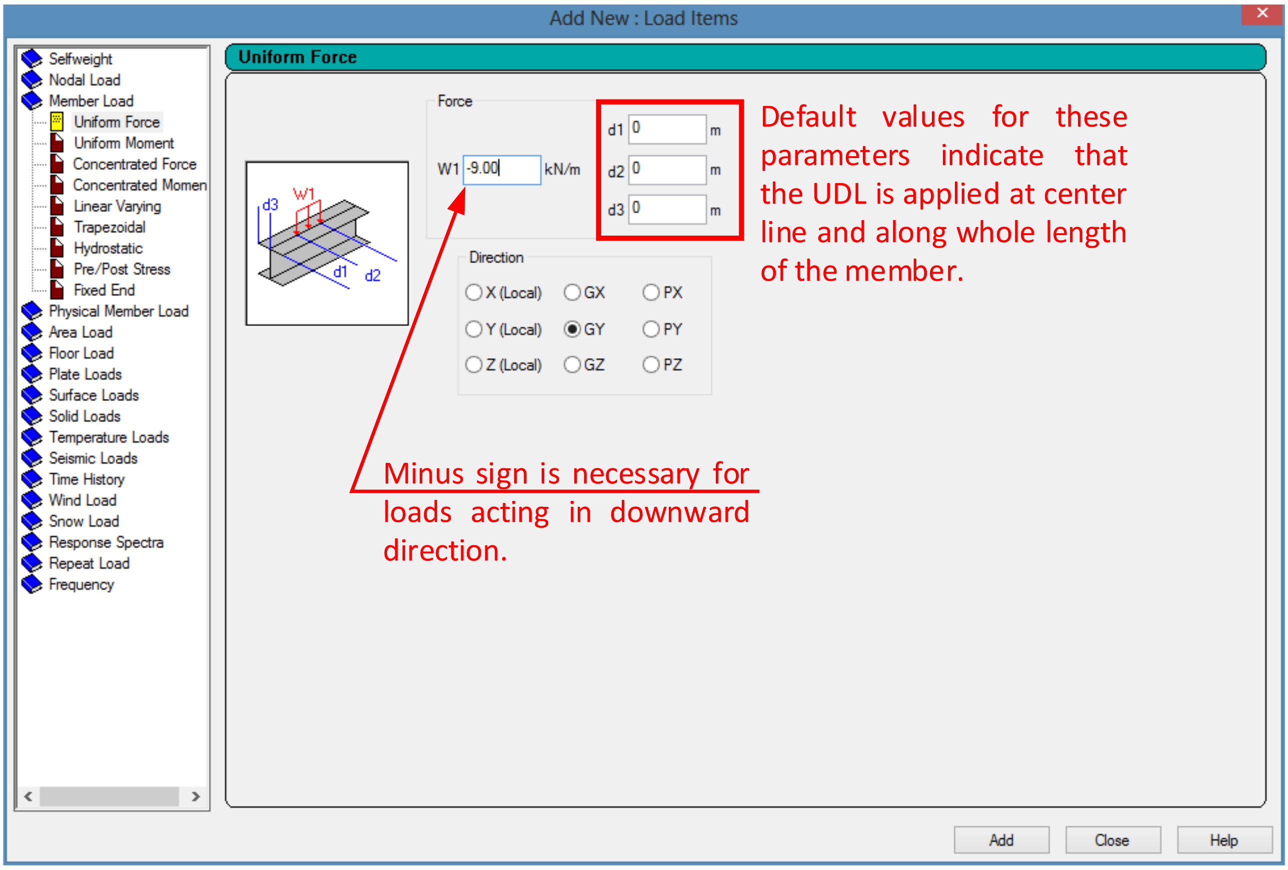


Figure 4.11-15: Definition of uniformly distributed superimposed dead load.

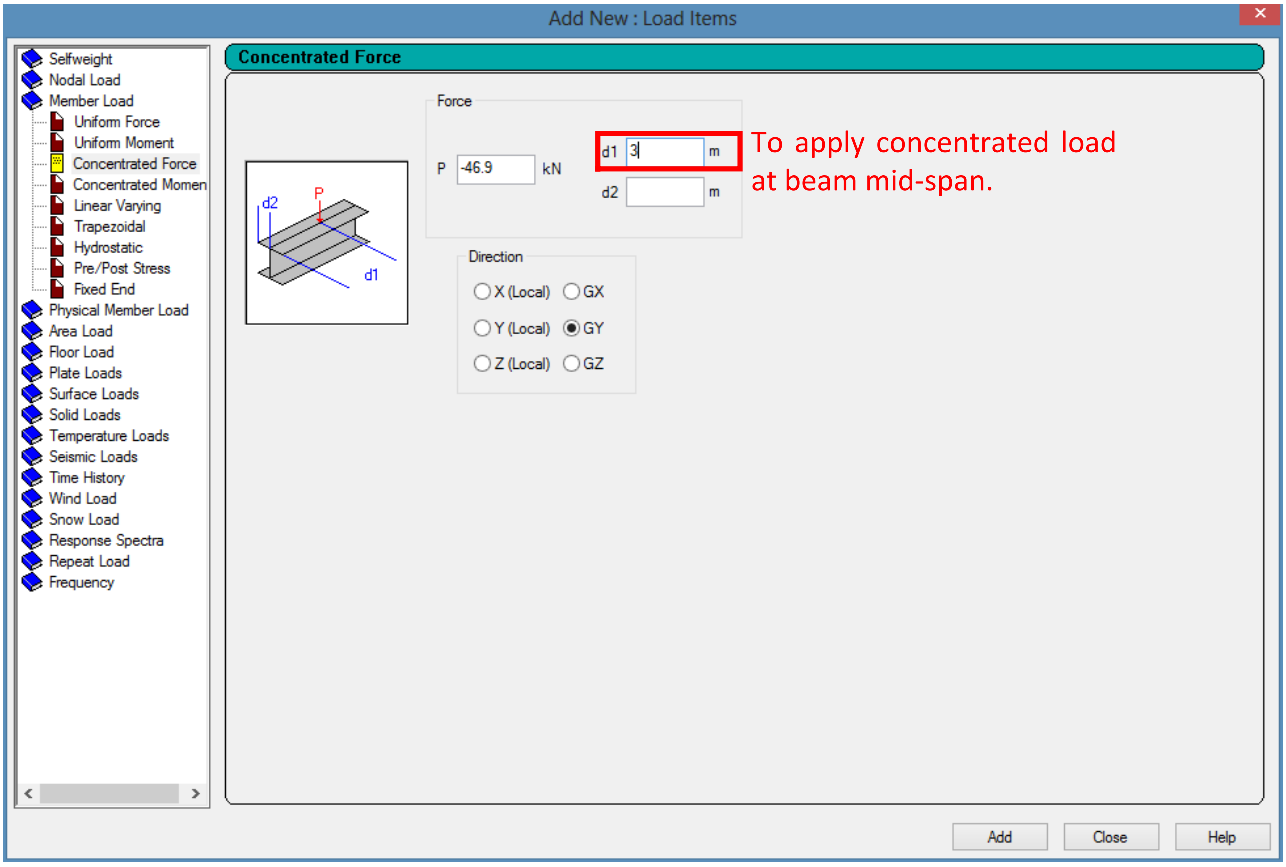


Figure 4.11-16: Definition of concentrated live load at beam mid-span.

4.11.1.8.3 Definition of Load Combinations

As indicated in steps below, load combinations have been generated automatically according to requirements of ACI code.

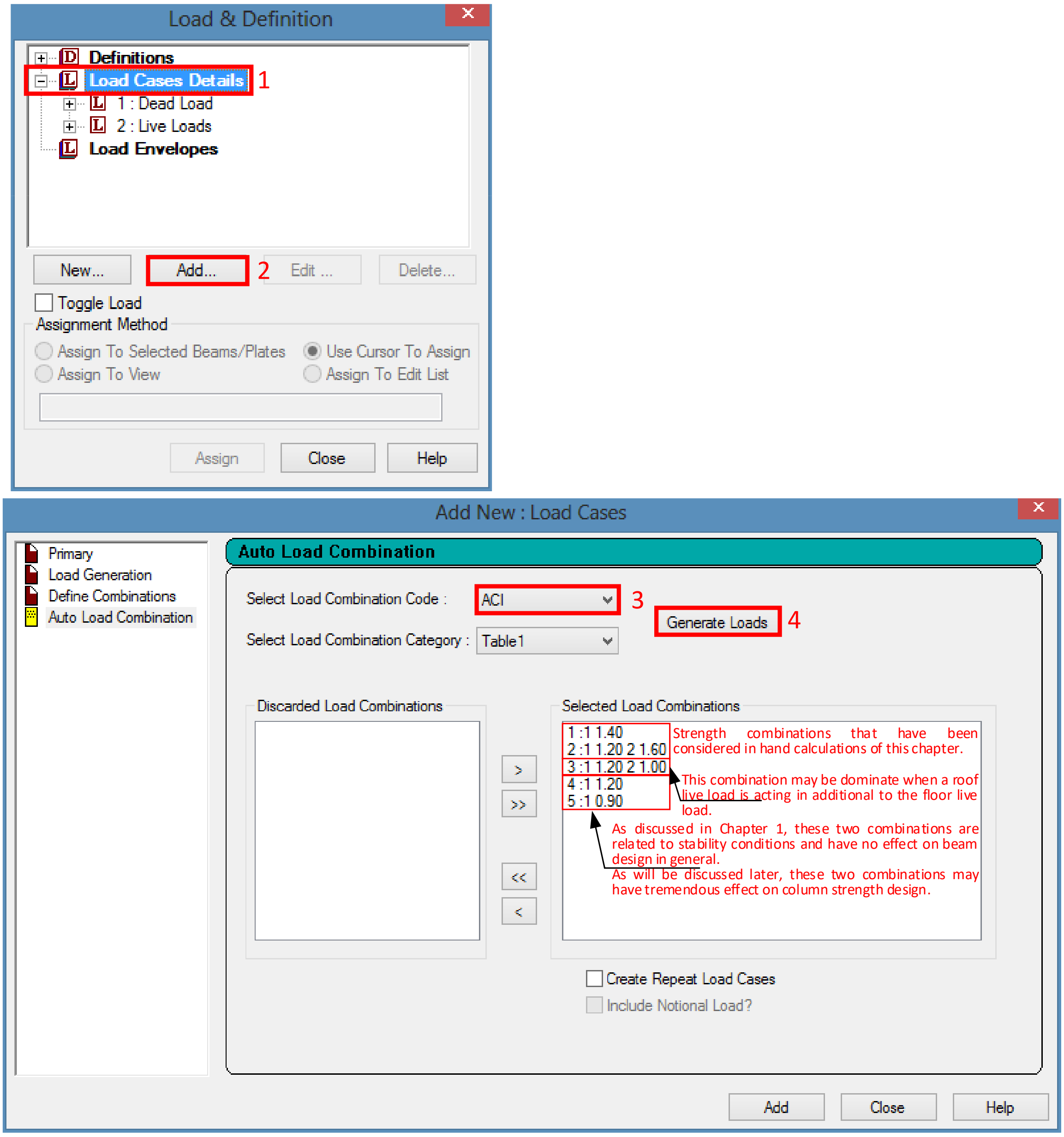


Figure 4.11-17: Steps for automatic generation of load combinations according to ACI code.

4.11.1.9 Definition of Analysis Type

- Traditional linear elastic analysis has been defined based on steps presented in Figure 4.11-18 below.
- When term "***Analysis***" is used in STAAD environment, it refers to a traditional elastic analysis that usually adopted in engineering practice.

4.11.1.10 Review of Input File and Execute the Analysis

- Access of input file is one of the main feature for STAAD software. The input file is a structure program with specific start and end and that executed in a sequencal form.
- The input file is similar to codes of common programming languages like Basic, Fortran, and Matlab.
- STAAD input is so useful to describe problem in a consice form.
- To access to model input file in STADD environment, just click on icon 
- Relation between STAAD command and corresponding GUI is presented in Figure 4.11-19 below.

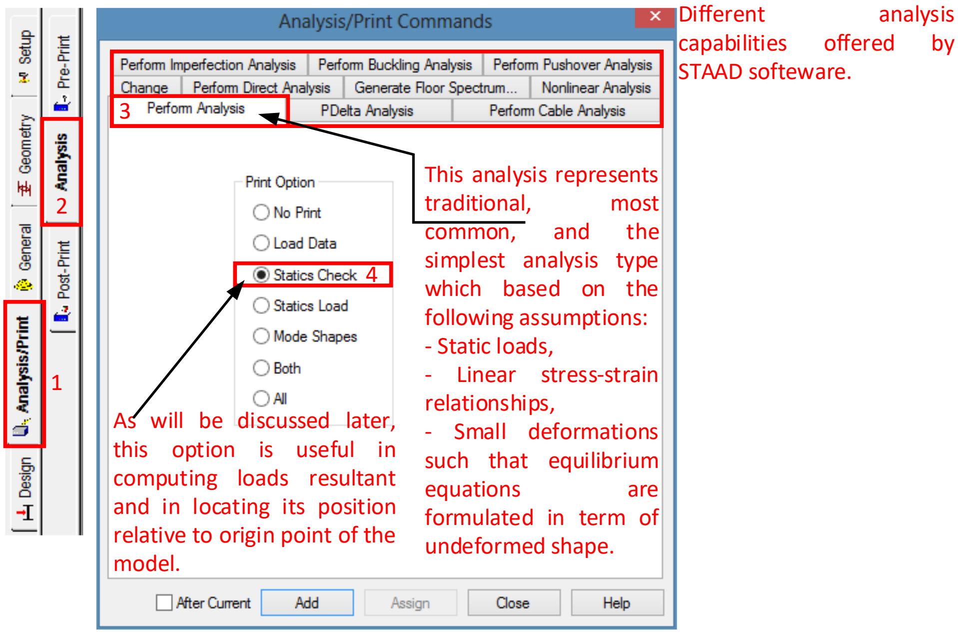


Figure 4.11-18: Steps for definition of analysis type.

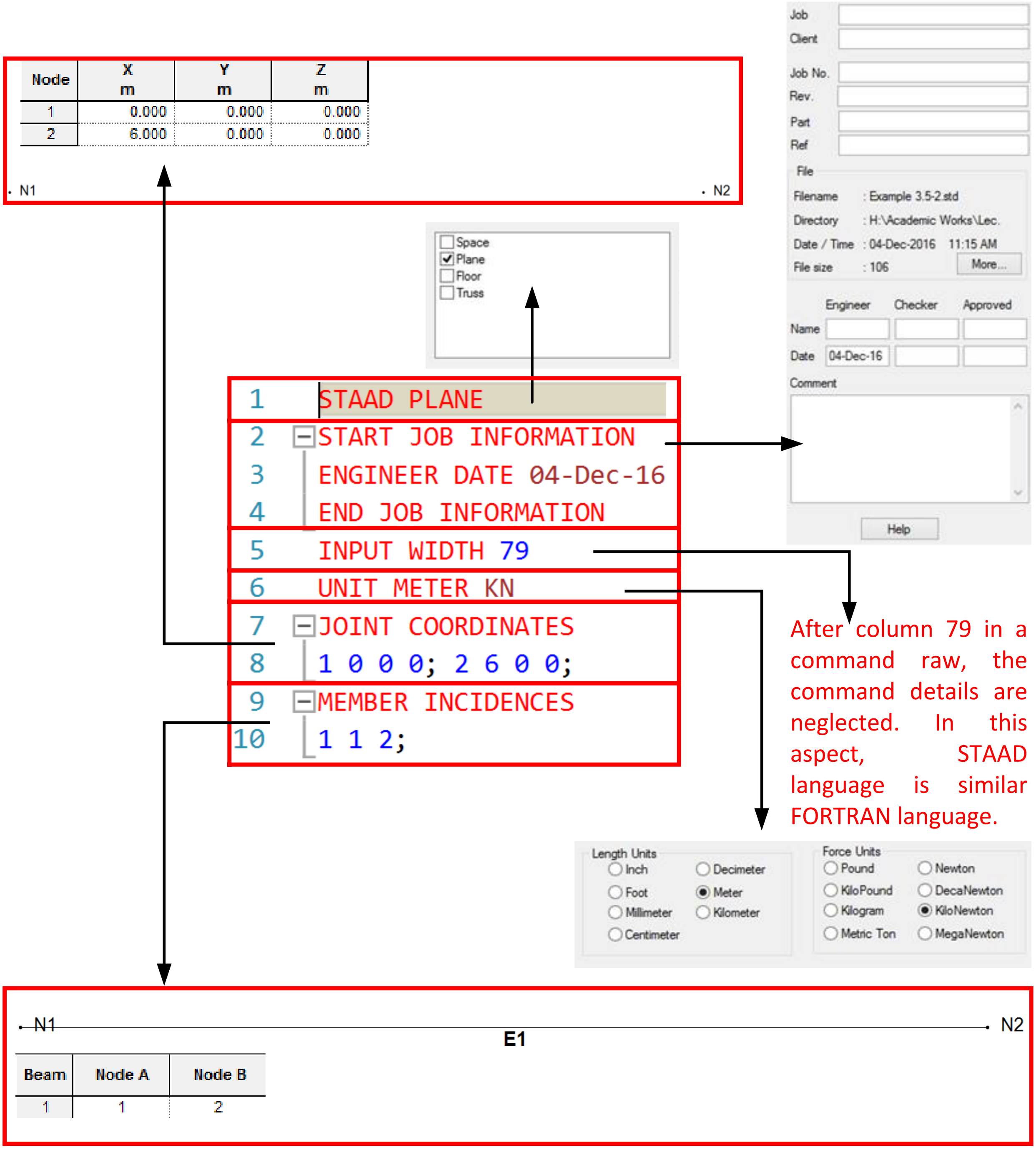


Figure 4.11-19: Relation between STAAD commands and corresponding GUI.

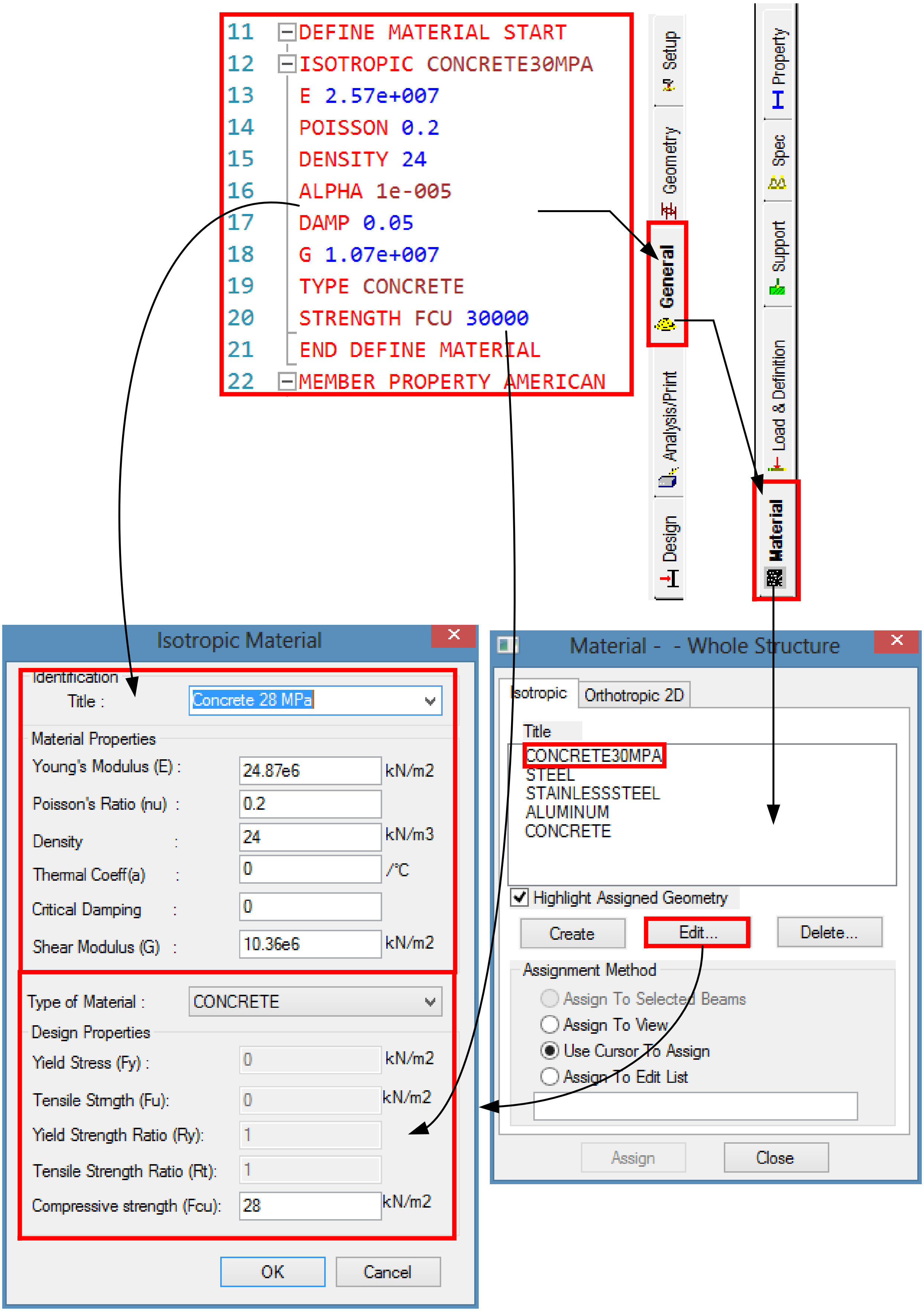


Figure 4.11-19: Relation between STAAD commands and corresponding GUI, continue.

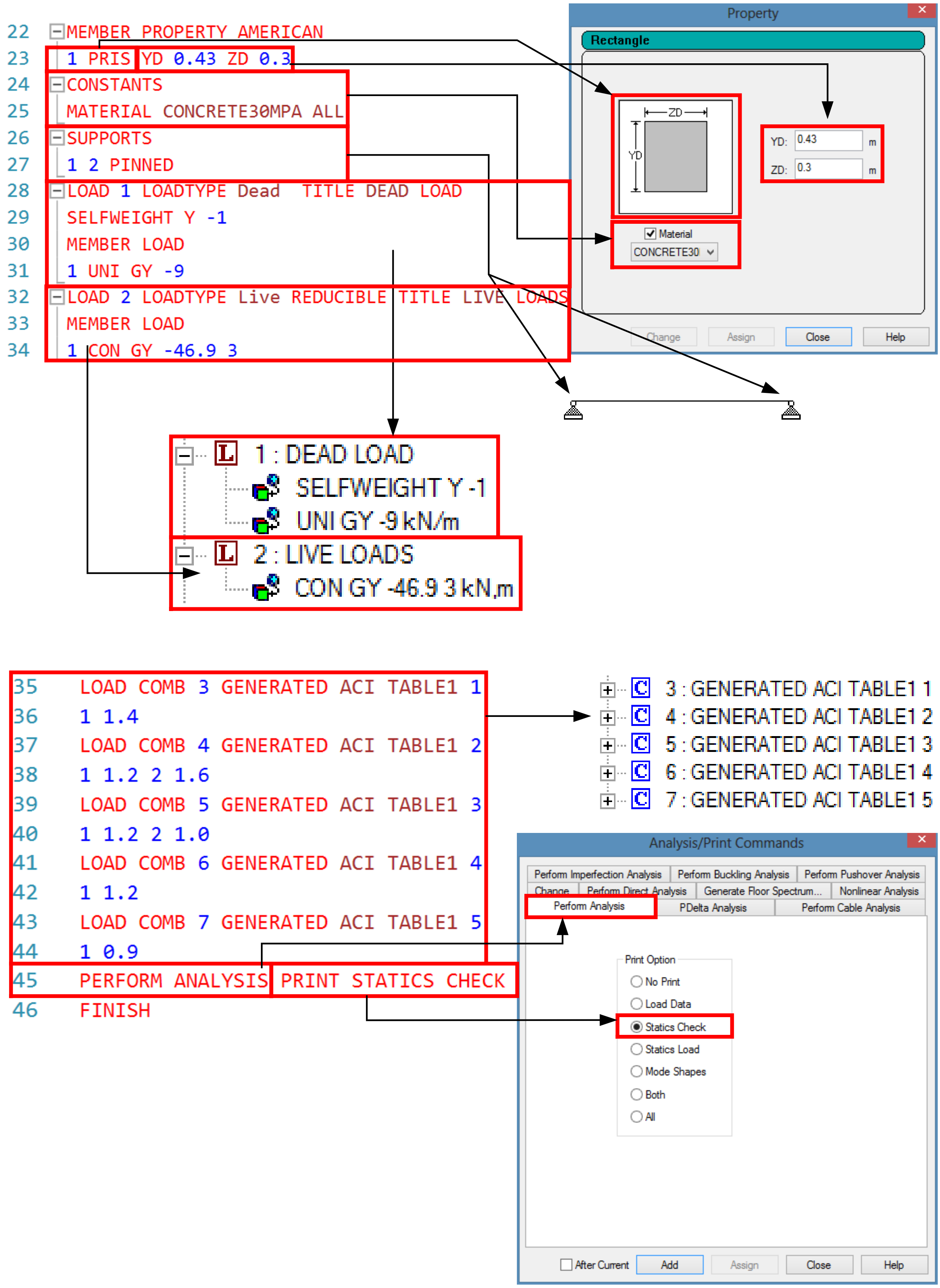


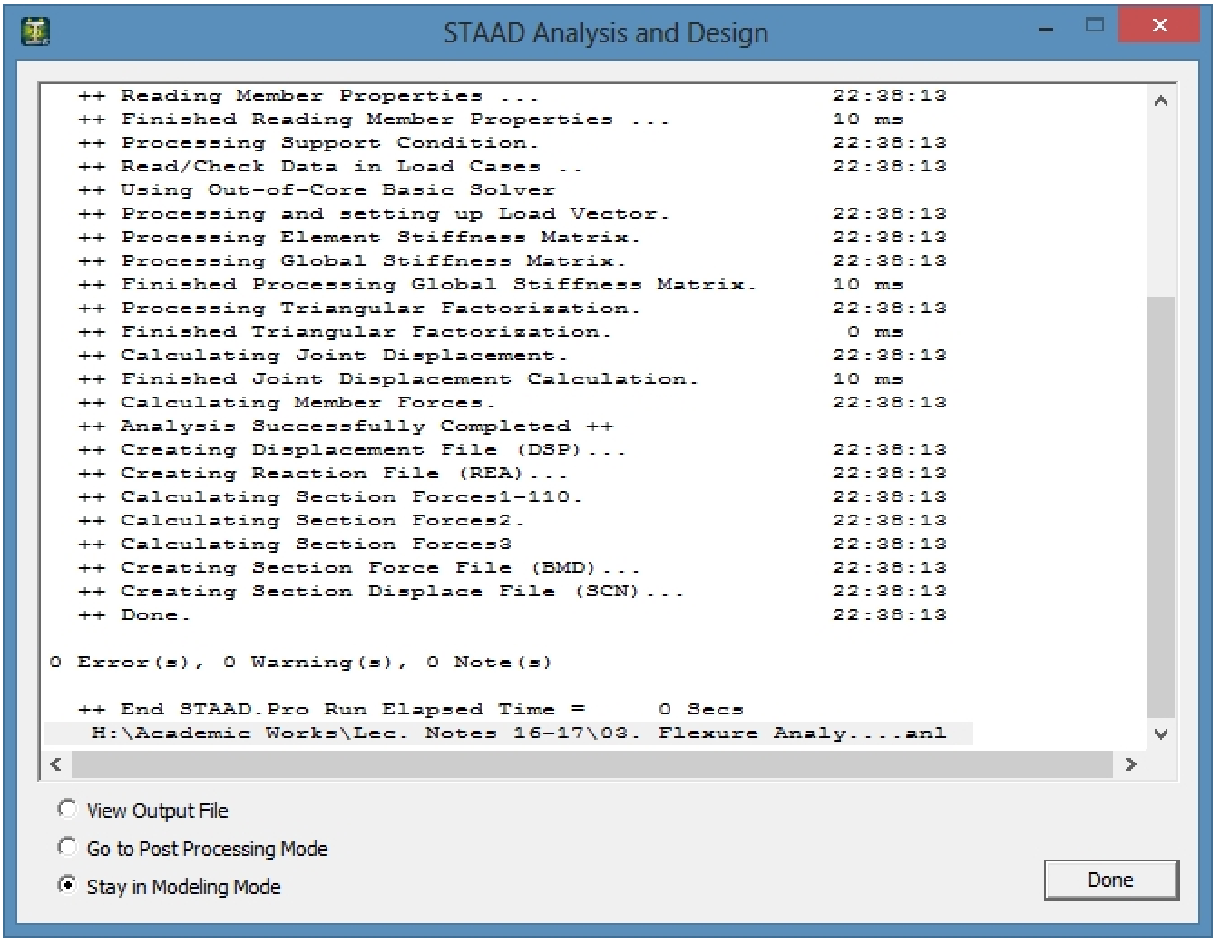
Figure 4.11-19: Relation between STAAD commands and corresponding GUI, continue.

4.11.1.11 Run the Analysis

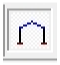
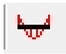
- After completing the preparation part of the input file, it can be executed as indicated in Figure 4.11-20

Figure 4.11-20: Executing of input file.

- As indicated in below, STAAD analysis engine indicate that input file contains no warring and no error.



4.11.1.12 Post Processing

- After structural analysis process, one can review internal moments and shear forces before the design process.
- Reviewing of analysis results starts from change the mode for a Modeling mode with icon of  To post-processing mode with icon of .
- Transformation from ***Modeling*** to ***Post Processing*** can also be done through ***Mode*** page indicated in Figure 4.11-21.

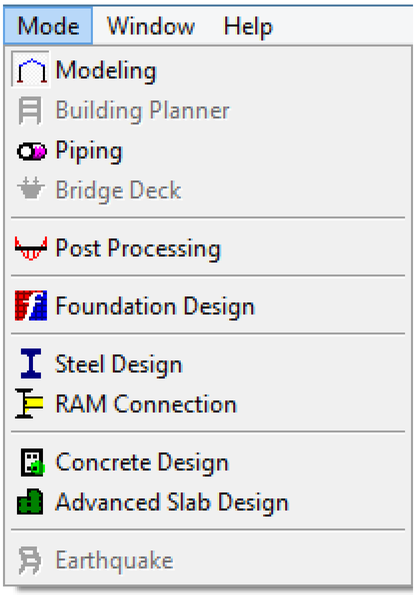


Figure 4.11-21: Mode page to transform from *Modeling* stage to *Post Processing* stage.

- Then, one should select load cases and/or load combinations that he intends to review their results. Usually all load cases and combinations are selected as indicated in Figure 4.11-22.

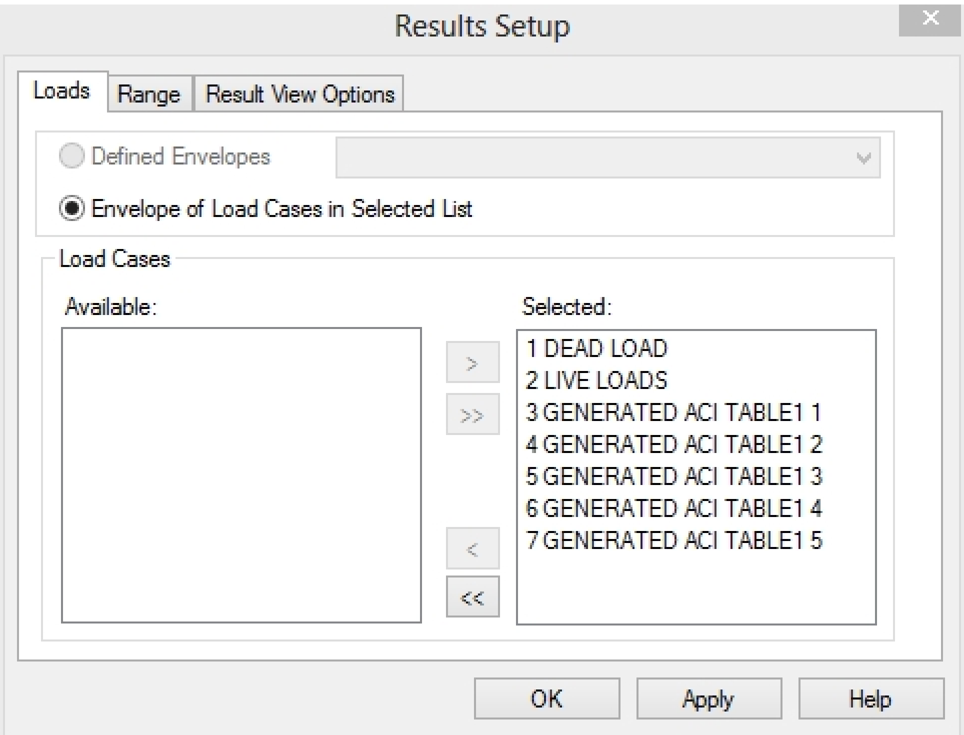
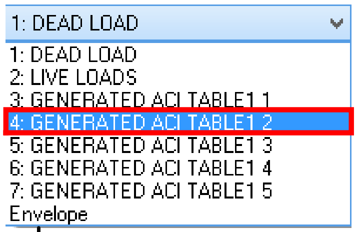
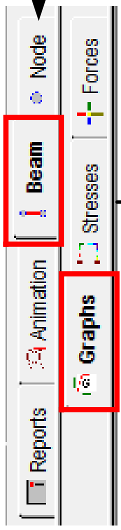


Figure 4.11-22: Selection of load cases and load combinations to review their results.

- The most powerful methods to review of problems with single element have been presented in Figure 4.11-23.



1. From load menu select the most critical load combination. Load combination of GENERATED ACI TABLE1 2 is the combination of U = 1.2D + 1.6L That governs most of beam designs in the course.



2. Select the method of presentation and type of quantities to be reviewed, e.g. stresses of internal forces.

3. The shear force and bending moment diagrams would be:

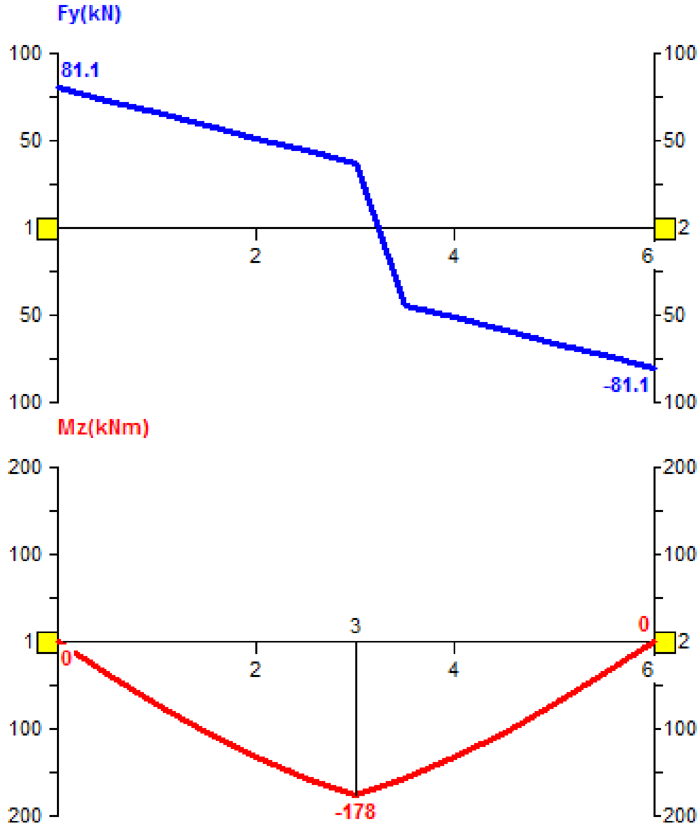


Figure 4.11-23: Selection of load combination, method of presentation, and nature of quantiles to review their analysis results in STAAD environment.

- It is useful to note that the sign convention adopted for bending moments differs from that adopted in analytical solutions. In a more systematic formulation, including STAAD formulation, bending moment is considered positive when produces tensile stresses on side with positive Y, see Figure 4.11-24 below.

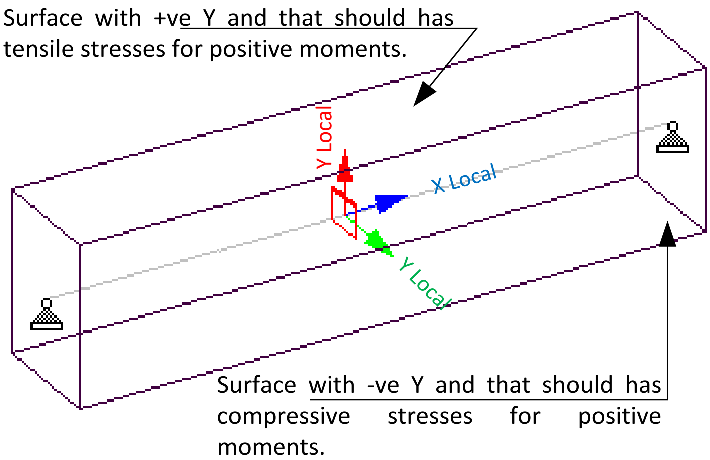


Figure 4.11-24: Definition of positive local moment, M_z, in STAAD environment.

- Based on hand calculations, factored design moment would be:
  - Moment due to Dead Loads:

$$W_{\text{Selfweight}} = 0.43\text{m} \times 0.3\text{m} \times 24\frac{\text{kN}}{\text{m}^3} = 3.1\frac{\text{kN}}{\text{m}}$$

$$W_{Dead} = 9.00\frac{kN}{m} + 3.10\frac{kN}{m} = 12.1\frac{kN}{m}$$

$$M_{Dead} = \frac{12.1\frac{kN}{m} \times 6.0^2 m^2}{8} = 54.5\ kN.m$$

- Moment due to Live Load:

$$M_{Live} = \frac{46.9kN \times 6.0m}{4} = 70.4\ kN.m$$

- Factored Moment M_u:

$$M_u = \text{Maximum of } (1.4M_D \text{ or } 1.2M_D + 1.6M_L)$$
$$M_u = \text{Maximum of } [1.4 \times 54.5 \text{ or } (1.2 \times 54.5 + 1.6 \times 70.4)] =$$
$$M_u = \text{Maximum of } [76.3 \text{ or } 178] = 178\ kN.m = M_{u\ from\ STAAD} \therefore Ok.$$

- While factored shear force would be:

$$V_D = \frac{12.2 \times 6.0}{2} = 36.6\ kN$$

$$V_L = \frac{46.9}{2} = 23.5\ kN$$

- The factored shear force would be:

$$V_u = 1.2 \times V_D + 1.6V_L = 1.2 \times 36.6 + 1.6 \times 23.5 = 81.5\ kN$$
$$\approx V_{u\ @\ center\ of\ support\ computed\ by\ STAAD}$$

4.11.1.13 Design Process

- In STAAD environment, design process is starting by selecting ***Design*** from main pages and then select ***Concrete*** from sub-pages as indicated in ***Figure 4.11-26*** below.
- In general, the design process consists from three basic steps indicated ***Figure 4.11-25*** below. Each step has been discussed in some details in sub-articles below.

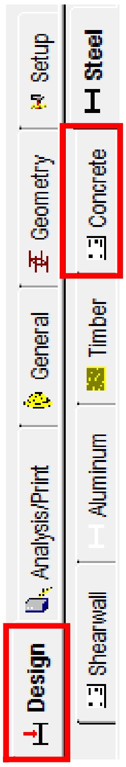


Figure 4.11-26: Starting design process through selecting Design from main pages and selecting Concrete from sub-pages.

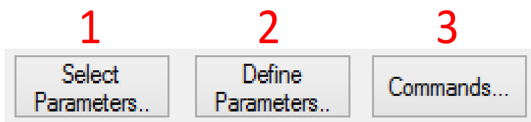


Figure 4.11-25: Basic three steps in design process.

4.11.1.13.1 Step 1: Select Parameters

- In this step, from the list of "***Available Parameters"*** indicated ***Figure 4.11-27*** below, the user can select a list of "***Selected Parameters"*** that pertinent to the design problem.

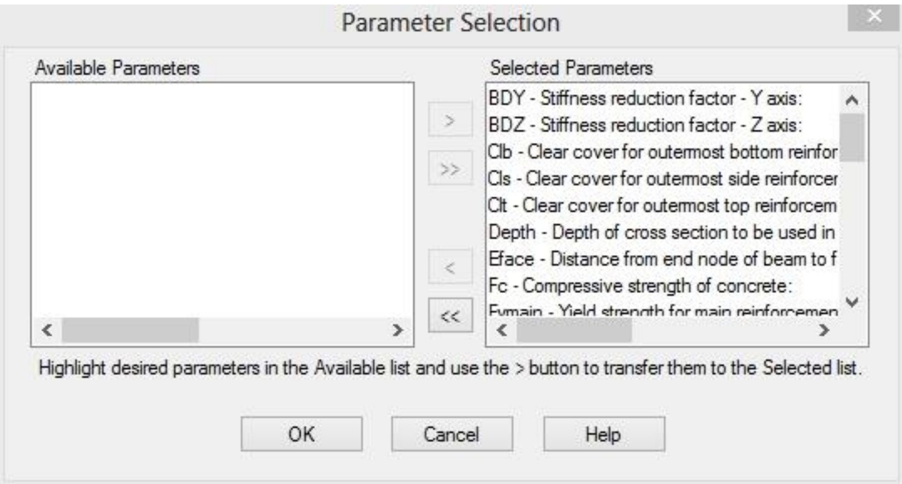


Figure 4.11-27: List of parameters in STAAD Pro software for design of RC members.

- To know which parameters should be selected, one should review definition and default values for the parameters pertinent to beam design.

4.11.1.13.1.1 Beam Dimensions

Beam dimensions can be defined using the two parameters presented in ***Table 4.11-1*** below. With these parameters, the user can adopt in the design process sections other than those that adopted in analysis process.

Table 4.11-1: Dimension Parameters.

Parameter Name	Default Value	Description
DEPTH	YD	Depth of concrete member. This value defaults to YD as provided under MEMBER PROPERTIES.
WIDTH	ZD	Width of concrete member. This value defaults to ZD as provided under MEMBER PROPERTIES.

4.11.1.13.1.2 Reinforcement Covers

Rebar covers have been defined with referring to ***Table 4.11-2*** below. STAAD uses metric in cover conversions; therefore, a cover of 1.5 inch is equivalent to 38mm. To be compatible with metric version of the ACI, the cover should be rounded to 40mm.

Table 4.11-2: Rebar covers.

Parameter Name	Default Value	Description
CLB	1.5 in. for beams	Clear cover for bottom reinforcement.
CLS	1.5 in.	Clear cover for side reinforcement.
CLT	1.5 in. for beams	Clear cover for top reinforcement.

4.11.1.13.1.3 Material Properties

Material properties related to the design process have been defined with referring to ***Table 4.11-3*** below. These material properties, including $f_c{'}$, f_y, f_{yt}, and λ. Default values for dimensional properties have been defined based on the imperial unit system and they would be transformed based on a metric conversion when metric system is adopted.

Table 4.11-3: Material properties in the design process.

Parameter Name	Default Value	Description
FC	4,000 psi	Compressive strength of concrete.
FYMAIN	60,000 psi	Yield stress for main reinforcing steel.
FYSEC	60,000 psi	Yield stress for secondary steel.
LWF	1.0	Modification factor, λ, for lightweight concrete as specified in ACI.

4.11.1.13.1.4 Rebar Size

- STAAD Pro. not only computes required reinforcement areas, $A_{s\,Required}$, but also offers reinforcement distributions including number of rebars, spacing between rebars, and number of reinforcement layers. Therefore, preferable rebar size should be proposed by the user through design parameters indicated in ***Table 4.11-4*** below.

- To enforce STAAD to adopt a single bar diameter, the user should adopt same bar diameter for MAXMAIN and MINMAIN.
- When using metric units for ACI design, provide values for these parameters in actual 'mm' units instead of the bar number. The following metric bar sizes are available: 6 mm, 8 mm, 10 mm, 12 mm, 16 mm, 20 mm, 25 mm, 32 mm, 40 mm, 50 mm and 60 mm.

Table 4.11-4: Parameters for proposing a preferable bar sizes in STAAD environment.

Parameter Name	Default Value	Description
MAXMAIN	#18 bar	Maximum main reinforcement bar size.
MINMAIN	#4 bar	Minimum main reinforcement bar size (Number 4 - 18).
MINSEC	#4 bar	Minimum secondary reinforcement bar size (Number 4 - 18)

4.11.1.13.1.5 Design Sections

- Design parameters related to number and location of design sections have been presented in ***Table 4.11-5*** below.
- Through the parameter of NSECTION, the user can determine the number of sections that should be adopted in the design process. STAAD distributes these sections uniformly along member span.
- After locating of the sections, STAAD computes factored forces, e.g. M_u and V_u, at each section and then computes required reinforcement accordingly.
- STAAD analytical model replaces actual physical members that has definite depth with analytical lines that, by default, are located at centroid of members. Therefore, to determine shear force at face of supports where it has its physical meaning, the user should adopt the parameters of SFACE and EFACE that have been defined and interpreted with referring to ***Table 4.11-5*** and ***Figure 4.11-28*** below.

Table 4.11-5: Number and locations of design sections in STAAD environment.

Parameter Name	Default Value	Description
NSECTION	12	Number of equally spaced sections to be considered in finding critical moments for beam design. NSECTION should have no member list since it applies to all members. ***The minimum value allowed is 12***, ***the maximum is 20***. If more than one NSECTION entered, then highest value is used.
EFACE	0.0	Face of support location at end of beam. If specified, the shear force at end is computed at a distance of EFACE +d from the end joint of the member.
SFACE	0.0*	Face of support location at start of beam. If specified, the shear force at start is computed at a distance of SFACE +d from the start joint of the member.

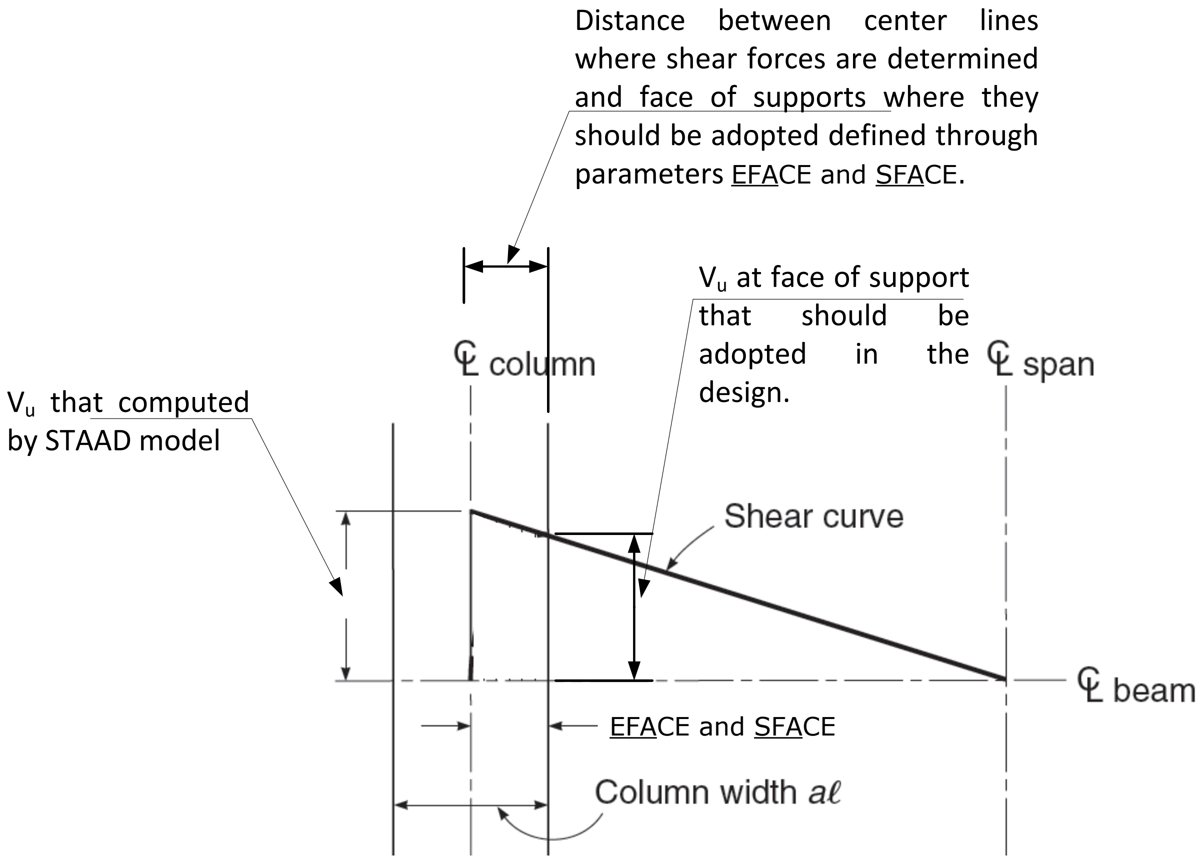


Figure 4.11-28: Interpretation of STAAD parameters EFACE and SFACE.

- Assuming pads that have width of 300mm the EFACE and SFACE would be:

$$\text{EFACE} = \text{SFACE} = \frac{0.300}{2} = 0.150\ m$$

- It will be discussed in ***Chapter 5***, ***Article 5.2.2*** the design shear force can be determined at distance "d" from face of support when three conditions below are satisfied:
    - Support reaction, in direction of applied shear, introduces compression into the end regions of member.
    - Loads are applied at or near the top of the member.
    - No concentrated load occurs between face of support and location of critical.

4.11.1.13.1.6 Detail Level of Outputs

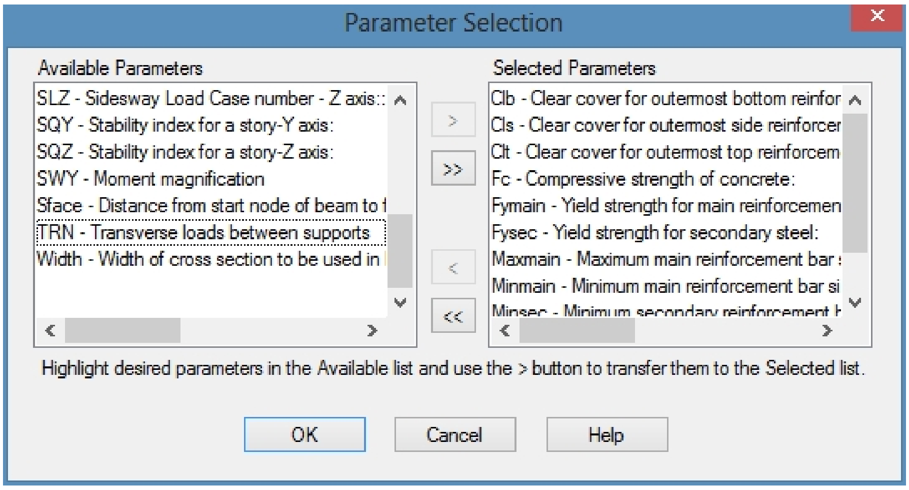
- Finally, based on TRACK parameter, STAAD offers three different levels of output detail that indicated in

Table 4.11-6: Three different levels of output details

Parameter Name	**Default Value**	**Description**
TRACK	0.0	Beam Design: 0.0 = Critical moment will not be printed out with beam design report. 1.0 = Critical moment will be printed out with beam design report 2.0 = Print out required steel areas for all intermediate sections specified by NSECTION.

- Based on above discussion, it is clear that only following parameters should be selected, as values other than their default values should be assigned:
    - Reinforcement covers,
    - Material properties,
    - Rebar size,
    - Design sections for shear,
    - Detail level of outputs.
- These parameters have been selected as indicated in Table 4.11-7 below.

Table 4.11-7: Selected parameters from available list of parameters.



4.11.1.13.2 Step 2: Define Parameters

- Referring to Figure 4.11-25 above, the second step of design process in STAAD environment, is to assign values differ from the default values for the design parameters that have been selected in ***Step 1*** above. As an example, consider ***Figure 4.11-29*** below where a value of ***40mm***, or ***0.04m***, for the ***Clear cover for the outmost bottom reinforcement***.

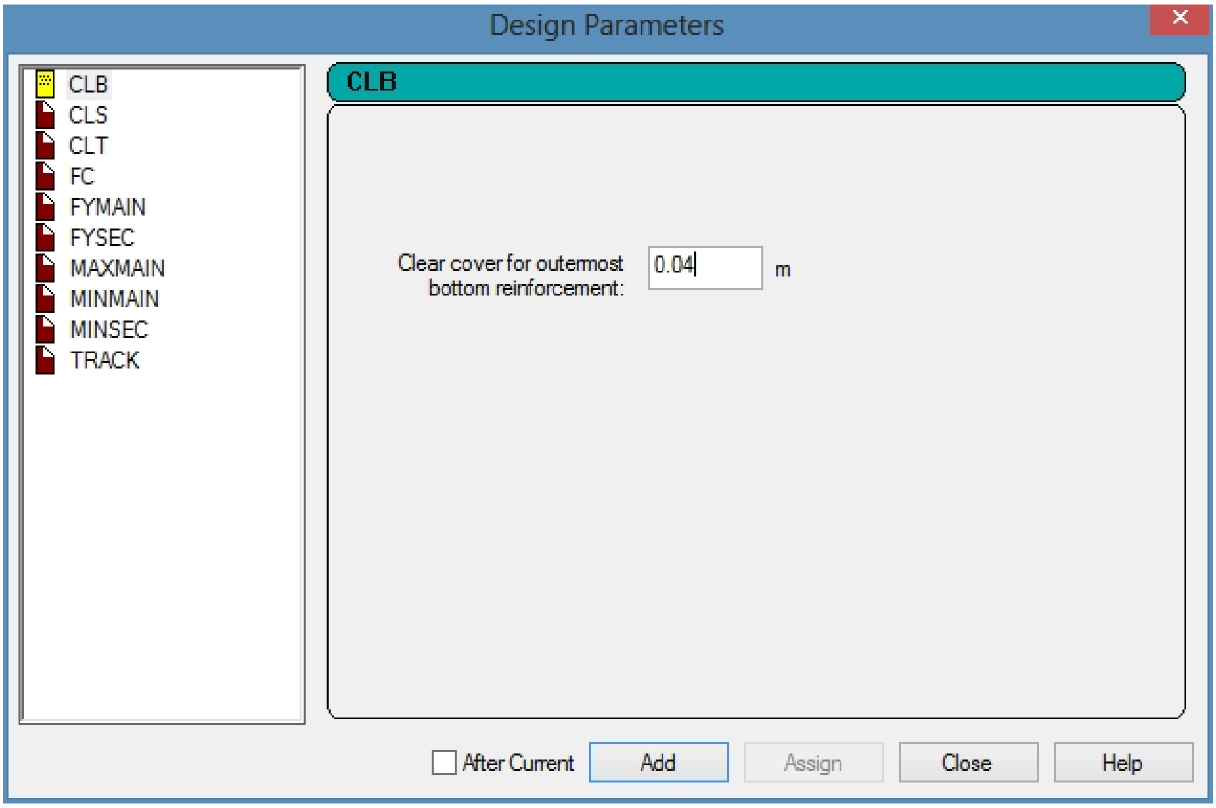


Figure 4.11-29: Assign the Clear cover for the outmost bottom reinforcement.

- In the same approach over values can be assigned to other design parameters as indicated in ***Figure 4.11-30*** below.
- As they not assigned to a specific member yet, all parameters are noted with question mark "?". Steps indicated in ***Figure 4.11-31*** below can be adopted to assign a parameter, e.g. CLB 0.04, to a specific member.
- From pulldown list indicated in ***Figure 4.11-31*** below. Unfortunately, ***ACI 318-14 not included yet in the STAAD environment***.

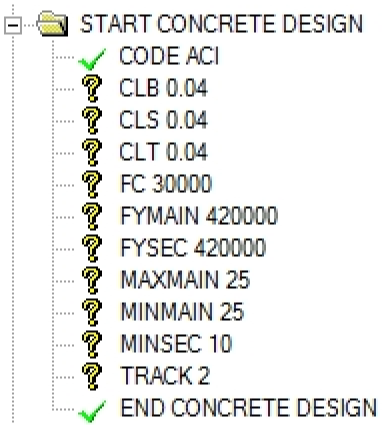


Figure 4.11-30: Assigned values for other pertinent design parameters.

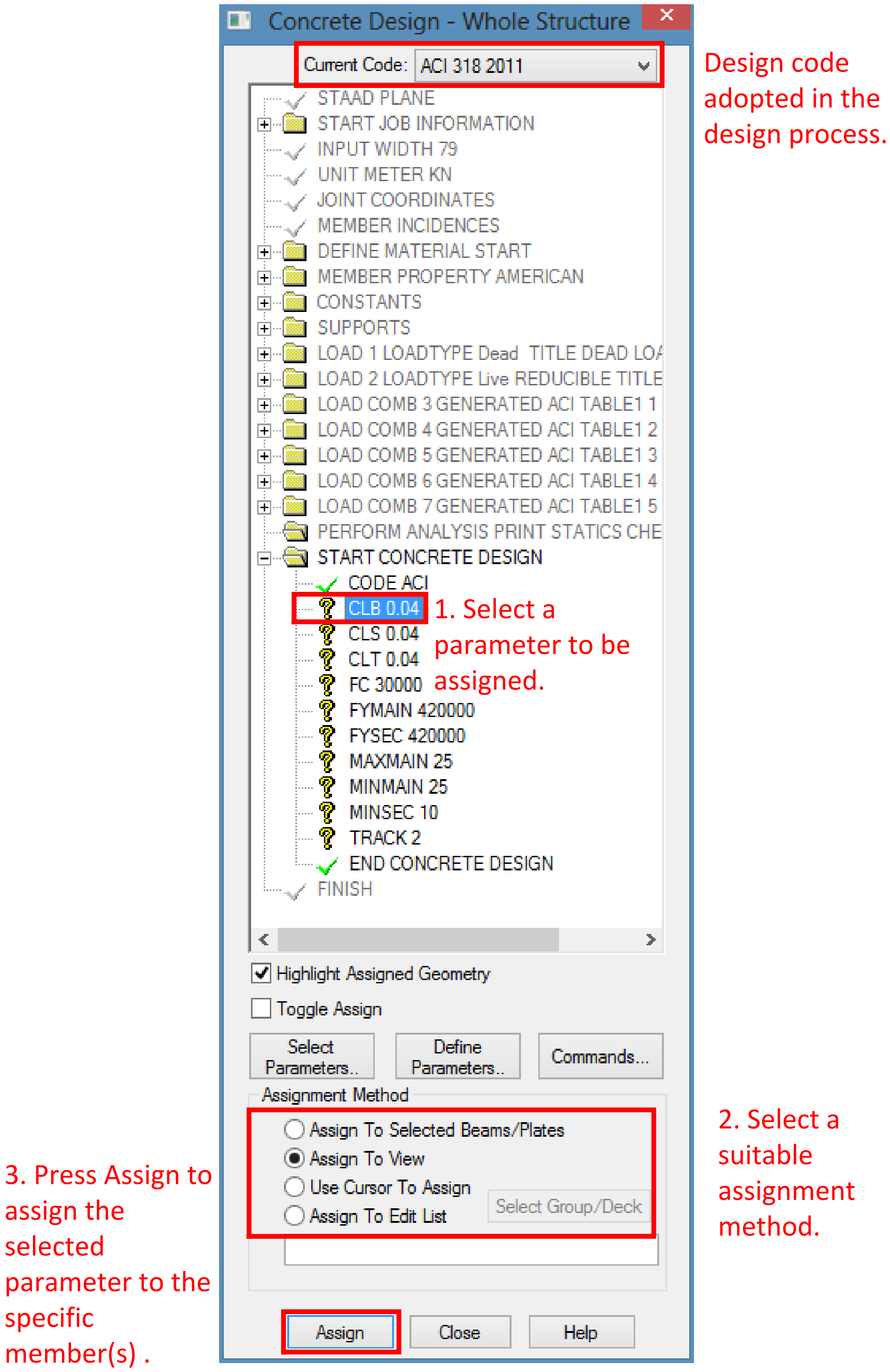


Figure 4.11-31: Assign a design parameter to the pertinent member.

4.11.1.13.3 Step 3: Commands

- In the third and final step, based on interactive box indicated in ***Figure 4.11-32*** below, the user can inform the software to design a specific member as a beam or as a column. As indicated in the interactive box, ***according to STAAD, the beam is the member that should be designed for flexure, shear, and torsion***.
- When including DESIGN BEAM command, the design list would be as indicated in ***Figure 4.11-33*** below. This command can be assigned to the specific member in a method similar to that discussed above.

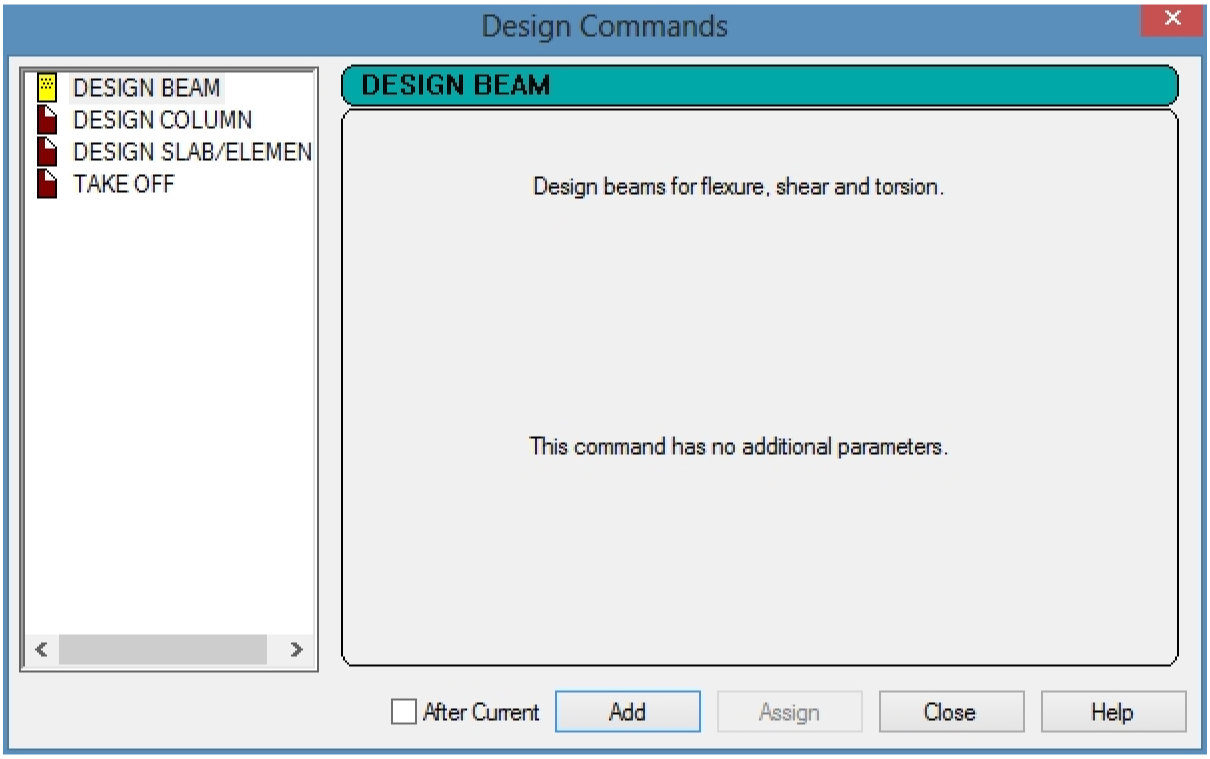


Figure 4.11-32: Commands interactive box in STAAD environment.

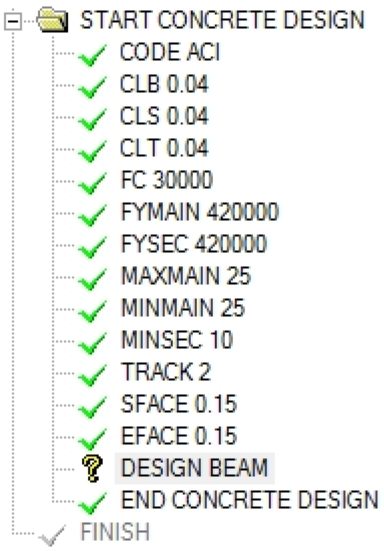


Figure 4.11-33: Updated design list after including DESIGN BEAM command.

4.11.1.13.4 Load Combinations that Adopted in the Design Process

- Design should be done in terms of load combinations only, therefore all basic design cases including DEAD LOAD and LIVE LOAD should be excluded from the list of design loads.
- In STAAD environment, selecting of the loads that should be including in the design process can be executed with ***Load List*** command based on steps indicated in ***Figure 4.11-34*** below.

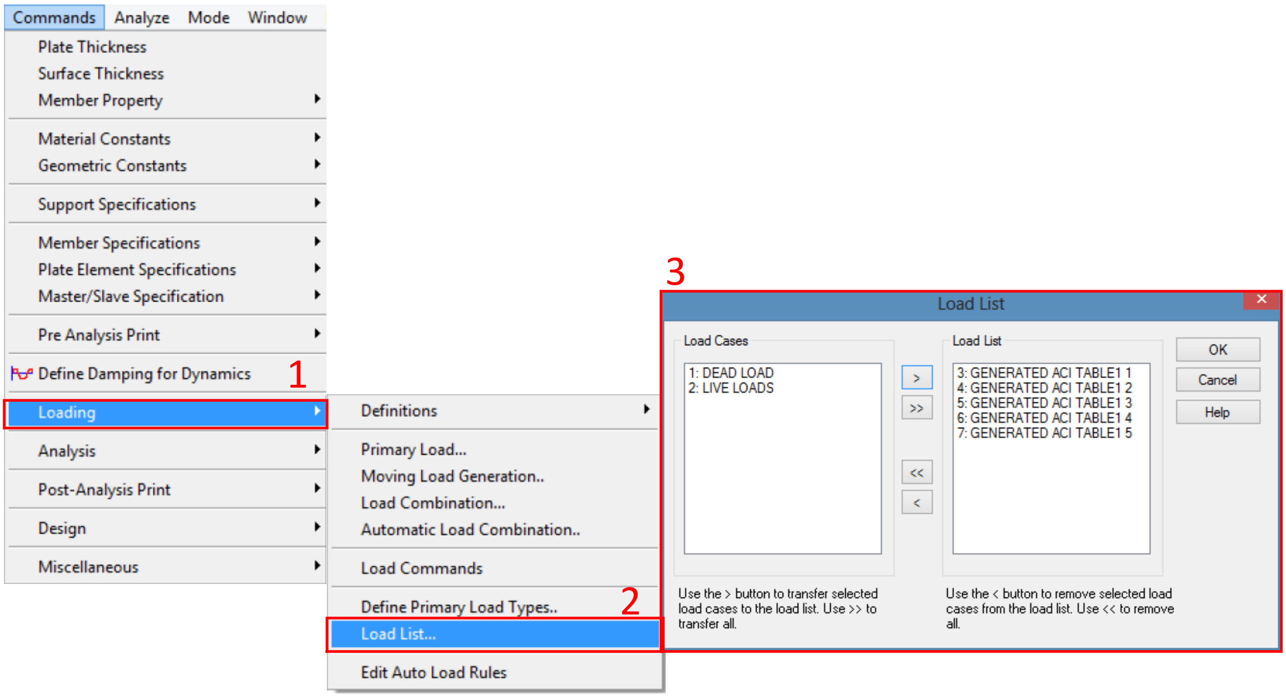


Figure 4.11-34: Steps to select loads that should be adopted in the design process using Load List command.

4.11.1.13.5 Input File with Design Parameters

In input file, the design parameters that have been defined and assigned using GUI above are presented in ***Figure 4.11-35*** below.

```
START CONCRETE DESIGN
CODE ACI
CLB 0.04 ALL
CLS 0.04 ALL
CLT 0.04 ALL
FC 30000 ALL
FYMAIN 420000 ALL
FYSEC 420000 ALL
MAXMAIN 25 ALL
MINMAIN 25 ALL
MINSEC 10 ALL
TRACK 2 ALL
SFACE 0.15 ALL
EFACE 0.15 ALL
END CONCRETE DESIGN
FINISH
```

Figure 4.11-35: Design parameters in the STAAD input file.

4.11.1.13.6 Run Model and Review of Design Results

After completion of definition, and assignment of all design parameters that related to the design process, the STAAD model can be executed or run in the way that discussed in ***Article 4.11.1.11***.

Design results are indicated ***Figure 4.11-36*** through ***Figure 4.11-39*** below. Flexural and shear designs have been discussed in below and compared with those obtained based on hand calculations.

4.11.1.13.6.1 Design for Flexure

Design forces computed by STAAD have been compared with those of hand calculations presented in ***Article 4.4***. Required reinforcement ratio based on hand calculation is:

$$d_{\text{for One Layer}} = 430 - 40 - 10 - \frac{25}{2} = 368 \text{ mm}$$

$$\rho_{Required} = \frac{1 - \sqrt{1 - 2.36\frac{M_n}{f_c' bd^2}}}{1.18 \times \frac{f_y}{f_c'}} = \frac{1 - \sqrt{1 - 2.36\frac{198 \times 10^6 N.mm}{30 \times 300 \times 368^2}}}{1.18 \times \frac{400}{30}} = 13.6 \times 10^{-3}$$

$\approx RHO = 0.0130\ from\ STAAD$

The maximum and minimum reinforcement ratios according to hand calculations would be:

$$\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004}$$

$$\beta_1 = 0.85 - \frac{30 - 28}{7} \times 0.05 = 0.836 > 0.65\ Ok$$

$$\rho_{max} = 0.85 \times 0.836 \frac{30}{400} \frac{0.003}{0.003 + 0.004} = 22.8 \times 10^{-3} \approx \text{RHOMX} = 0.0223 \text{ from STAAD}$$

$$A_{s\ minimum} = \frac{1.4}{f_y} b_w d \Rightarrow \rho_{Minimum} = \frac{A_{s\ Minimum}}{b_w d} = \frac{1.4}{f_y} = \frac{1.4}{420} = 0.00333 = \text{RHOMN from STAAD}$$

```
          ACI 318-11    BEAM  NO.      1   DESIGN RESULTS
          ==============================================

 LEN -  6000. MM  FY -  420.  FC -  30.  MPA, SIZE -  300. X  430. MMS

 LEVEL    HEIGHT     BAR INFO       FROM         TO          ANCHOR
           (MM)                     (MM)        (MM)        STA  END
 ______________________________________________________________________

   1        62.     3 - 25MM         0.        6000.        YES  YES
 |-----------------------------------------------------------------|
 |   CRITICAL POS MOMENT=    177.88 KN-MET  AT  3000.MM, LOAD    4|
 |   REQD STEEL=  1434.MM2, RHO=0.0130, RHOMX=0.0223 RHOMN=0.0033 |
 |   MAX/MIN/ACTUAL BAR SPACING=   250./   50./   88. MMS         |
 |   REQD. DEVELOPMENT LENGTH =   989. MMS                        |
 |-----------------------------------------------------------------|

   Cracked Moment of Inertia Iz at above location =    85985.2 cm^4
```

Figure 4.11-36: Details of flexure design at the most critical section.

```
 REQUIRED REINF. STEEL SUMMARY :
 -------------------------------
 SECTION      REINF STEEL(+VE/-VE)      MOMENTS(+VE/-VE)     LOAD(+VE/-VE)
 ( MM )          (SQ.  MM )                (KNS-MET )

    0.            0./        0.           0./        0.          4/    7
  500.          286./        0.          39./        0.          4/    0
 1000.          557./        0.          74./        0.          4/    0
 1500.          811./        0.         105./        0.          4/    0
 2000.         1045./        0.         133./        0.          4/    0
 2500.         1256./        0.         157./        0.          4/    0
 3000.         1443./        0.         178./        0.          4/    0
 3500.         1256./        0.         157./        0.          4/    0
 4000.         1045./        0.         133./        0.          4/    0
 4500.          811./        0.         105./        0.          4/    0
 5000.          557./        0.          74./        0.          4/    0
 5500.          286./        0.          39./        0.          4/    0
 6000.            0./        0.           0./        0.          3/    4
```

Figure 4.11-37: Design Summary for different design sections.

4.11.1.13.6.2 Design for Shear

As will be discussed in ***Chapter 5***, ***Article 5.2.2***, design shear force can be determined at distance "d" from face of support when three conditions below are satisfied:

- Support reaction, in direction of applied shear, introduces compression into the end regions of member.
- Loads are applied at or near the top of the member.
- No concentrated load occurs between face of support and location of critical.

As all these conditions are satisfied, therefore the design shear force, V_u, can be determined at distance d from face of support.

$$V_{u\,@\,distance\ d} = \frac{1}{2}\left(1.2 \times 12.1 \times \left(6 - 2 \times \left(\frac{0.300}{2} + 0.368\right)\right) + 1.6 \times 46.9\right) = 73.6\ kN \approx V_u\ in\ STAAD$$

According to ACI code, concrete shear strength, V_c, is:

$$\mathrm{V_c} = 0.17\lambda\sqrt{f_c'}\,b_w d = \left(0.17 \times 1.0 \times \sqrt{30} \times 300 \times 368\right) \times \frac{1}{1000} = 102.7\ kN \approx V_{c\ from\ STAAD}$$

Based on V_u and V_c, required shear force that should be supported by shear reinforcement can be determined accordingly.

$$\because V_u = \phi(V_c + V_s) \Longrightarrow \therefore V_s = \frac{V_u - \emptyset V_c}{\emptyset} = \frac{\left(73.6 - \frac{102.7}{0.75}\right)}{0.75} = 0.0$$

Therefore, no theoretical reinforcement are required and only nominal reinforcement of ***Article 11.5.5.1*** should be adopted:

$$\because V_s \le 0.33\sqrt{f_c'}b_w d$$

$$\therefore s_{maximum} = \text{Minimum}\left[\frac{d}{2}\text{ or } 600\text{mm}\right] = \min\left(\frac{368}{2}, 600\right) = 184\ mm$$

```
      B E A M  N O.    1 D E S I G N  R E S U L T S - SHEAR


AT START SUPPORT - Vu=   68.18 KNS  Vc=  104.66 KNS  Vs=    0.00 KNS
Tu=    0.00 KN-MET  Tc=    3.9 KN-MET  Ts=    0.0 KN-MET  LOAD     4
NO STIRRUPS ARE REQUIRED FOR TORSION.
REINFORCEMENT FOR SHEAR IS PER CL.11.5.5.1.
PROVIDE 10 MM 2-LEGGED STIRRUPS AT 184. MM  C/C FOR 2112. MM
AT END   SUPPORT - Vu=   68.18 KNS  Vc=  104.66 KNS  Vs=    0.00 KNS
Tu=    0.00 KN-MET  Tc=    3.9 KN-MET  Ts=    0.0 KN-MET  LOAD     4
NO STIRRUPS ARE REQUIRED FOR TORSION.
REINFORCEMENT FOR SHEAR IS PER CL.11.5.5.1.
PROVIDE 10 MM 2-LEGGED STIRRUPS AT 184. MM  C/C FOR 2112. MM
```

Figure 4.11-38: Detailed design for shear reinforcement.

```
 ___    1J____________________   6000X 300X  430_____________________    2J____
|                                                                              |
|                                                                              |
|                                                                              |
| 13*10c/c184                                               13*10c/c184        |
| 3No25 H  62.   0.TO 6000                                                     |
||============================================================================||
|                                                                              |
|______________________________________________________________________________|

 _________   _________   _________   _________   _________    _________
|         | |         | |         | |         | |         |  |         |
|         | |         | |         | |         | |         |  |         |
|         | |         | |         | |         | |         |  |         |
|         | |         | |         | |         | |         |  |         |
| 3#25    | | 3#25    | | 3#25    | | 3#25    | | 3#25    |  | 3#25    |
|   ooo   | |   ooo   | |   ooo   | |   ooo   | |   ooo   |  |   ooo   |
|         | |         | |         | |         | |         |  |         |
|_________| |_________| |_________| |_________| |_________|  |_________|
```

Figure 4.11-39: Longitudinal and cross sections to present required reinforcements and their distribution.

4.11.2 Design of a Doubly Reinforced Concrete Beam

This article aims to show how STAAD Pro software can be adopted for analysis and design of doubly reinforced beam. Analysis and design process are presented with referring to Example 4.7-1 on page 92. Data for this example has been represented in below for convenient:

- The beam has a simple span of 5.49m and subjected to dead load of $15.3\ \mathrm{kN/m}$ (including its selfweight) and to service live load of $36.0\ \mathrm{kN/m}$.
- Beam dimensions are $250\mathrm{mm}$ width and $500\mathrm{mm}$ depth.
- $f_y = 414\ \mathrm{Mpa}$
- $f_c' = 27.5\ \mathrm{Mpa}$
- No. 29 for longitudinal tension reinforcement.
- No. 19 for compression reinforcement if required.
- No. 10 for stirrups (it's adequacy must be checked when used as a tie).
- Two layers of tension reinforcement.

STAAD Pro input file has been prepared in same steps of Section 4.11.1 and summarized in Table 4.11-8.

Table 4.11-8: STAAD input file for Example 4.7-1.

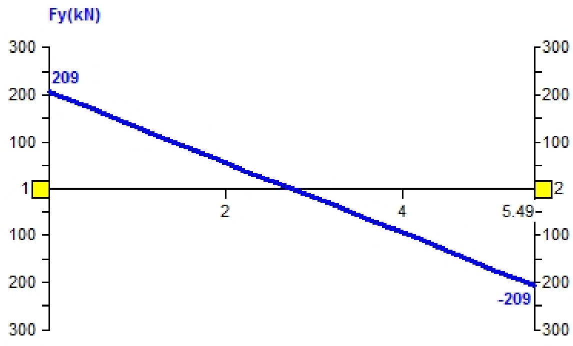
```
STAAD PLANE
START JOB INFORMATION
ENGINEER DATE 11-Feb-18
END JOB INFORMATION
INPUT WIDTH 79
UNIT METER KN
JOINT COORDINATES
1 0 0 0; 2 5.49 0 0;
MEMBER INCIDENCES
1 1 2;
DEFINE MATERIAL START
ISOTROPIC CONCRETE
E 2.17185e+007
POISSON 0.17
DENSITY 23.5616
ALPHA 1e-005
DAMP 0.05
TYPE CONCRETE
STRENGTH FCU 27579
END DEFINE MATERIAL
MEMBER PROPERTY
1 PRIS YD 0.5 ZD 0.25
CONSTANTS
MATERIAL CONCRETE ALL
SUPPORTS
1 2 PINNED
LOAD 1 LOADTYPE Dead  TITLE DEAD
MEMBER LOAD
1 UNI GY -15.3
LOAD 2 LOADTYPE Live  TITLE LIVE
MEMBER LOAD
1 UNI GY -36
LOAD COMB 3 Generated ACI Table1 1
1 1.4
LOAD COMB 4 Generated ACI Table1 2
1 1.2 2 1.6
LOAD COMB 5 Generated ACI Table1 3
1 1.2 2 1.0
LOAD COMB 6 Generated ACI Table1 4
1 1.2
LOAD COMB 7 Generated ACI Table1 5
1 0.9
PERFORM ANALYSIS
START CONCRETE DESIGN
CODE ACI
CLB 0.04 ALL
CLS 0.04 ALL
CLT 0.04 ALL
FC 27579.2 ALL
FYMAIN 414000 ALL
FYSEC 414000 ALL
MAXMAIN 25 ALL
MINMAIN 20 ALL
MINSEC 10 ALL
TRACK 2 ALL
DESIGN BEAM 1
END CONCRETE DESIGN
FINISH
```

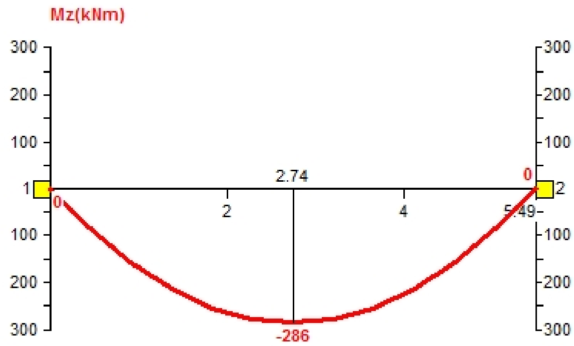
Based on STAAD analysis factored shear force and bending moment diagrams have been determined and presented in Figure 4.11-40. Based on hand calculations, factored forces would be:

$$W_u = \max(1.4 \times 15.3, 1.2 \times 15.3 + 1.6 \times 36.0) \approx 76 \frac{\mathrm{kN}}{\mathrm{m}} \Rightarrow M_u = \frac{76 \times 5.49^2}{8} = 286\ \mathrm{kN.m}$$

$$\approx \mathrm{M_{u\ from\ STAAD\ analysis}}$$

$$V_u = \frac{76 \times 5.49}{2} = 209\ kN = V_{u\ from\ STAAD\ analysis}$$





FSFD FBMD

Figure 4.11-40: STAAD analysis factored shear force and bending moment diagrams for beam of Example 4.7-1.

Flexural design results are presented in Figure 4.11-41. Comparing required reinforcement from STAAD design with those of hand calculation:

$A_{s\,required\;from\;hand\;calculations} = 2\,527\ \mathrm{mm}^2 > A_{s\,from\;STAAD} = 2131\ mm^2$

It useful to note that based on 0.04m value for CLB parameter, STAAD implicitly assume single layer of reinforcement and adopt a n overestimate for effective depth d and hence a lower estimate for required reinforcement as indicated in aforementioned comparison. To have a more accurate analysis, CLB parameter should be modified to reflect the two layer of reinforcement: $CLB = 40 + \frac{25}{2} + \frac{25}{2} = 65mm$

When this value is adopted, STAAD design results will be updated to those indicated in Figure 4.11-42 to indicate that the section could not be designed as a single reinforced section and it is should be designed as a doubly reinforced one. To design a doubly reinforced section, STAAD Pro RC Design Module should be adopted. This module is out of the scope of this course.

```
          ACI 318-11    BEAM  NO.     1   DESIGN RESULTS
          ===============================================

LEN -  5490. MM  FY -  414.  FC -  28.  MPA, SIZE -  250. X  500. MMS

LEVEL     HEIGHT     BAR INFO       FROM          TO           ANCHOR
           (MM)                     (MM)         (MM)         STA   END
________________________________________________________________________


*** A SUITABLE BAR ARRANGEMENT COULD NOT BE DETERMINED.
REQD. STEEL =   2131. MM2,  MAX. STEEL PERMISSIBLE =   2326. MM2
MAX POS MOMENT =     286.18 KN-MET,  LOADING     4
```

Figure 4.11-41: Details of flexure design at the most critical section Example 4.7-1.

```
          ACI 318-11    BEAM  NO.     1   DESIGN RESULTS
          ===============================================

LEN -  5490. MM  FY -  414.  FC -  28.  MPA, SIZE -  250. X  500. MMS

LEVEL     HEIGHT     BAR INFO       FROM          TO           ANCHOR
           (MM)                     (MM)         (MM)         STA   END
________________________________________________________________________


***MEMBER FAILS IN MAX REINFORCEMENT.
   INCREASE MEMBER SIZE.
MAX POS MOMENT =     286.18 KN-MET,  LOADING     4
```

Figure 4.11-42: Details of flexure design at the most critical section Example 4.7-1 with updated CLB parameter.

4.11.3 Design of Tee Section

- This article presents analysis and design of T beams using STAAD Pro software.
- It has been presented with referring to beam of ***Example 4.9-1*** that has been represented in below. This beam is a simply supported one with span of 6.71m, and it is subjected to superimposed load of 29.2 kN, and to a live load of 14.6kN/m. Material properties are $f_y = 414\ \text{Mpa}$ and $f_c' = 21\ Mpa$. Adopted rebars are one-layer of $\emptyset 25mm$ for longitudinal reinforcement ($A_{Bar} = 510mm^2$) and $\emptyset 10mm$ for stirrups.
- As discussed previously, STAAD can deal only with sections that have predefined dimensions. Hence, flange width, b, should be determined manually based on ACI provisions, see Table 4.8-1, and feedback to the software. Based on calculations of ***Example 4.9-1,*** $b = 1900\ mm.$

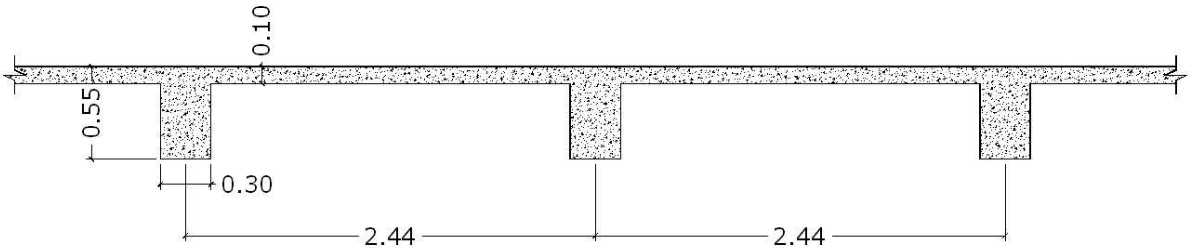


Figure 4.9-5: Floor system for Example 4.9-1. Reproduced for convenient.

- As indicated in Figure 4.11-43, T section can be defined from ***General*** page and from ***Property*** subpage.

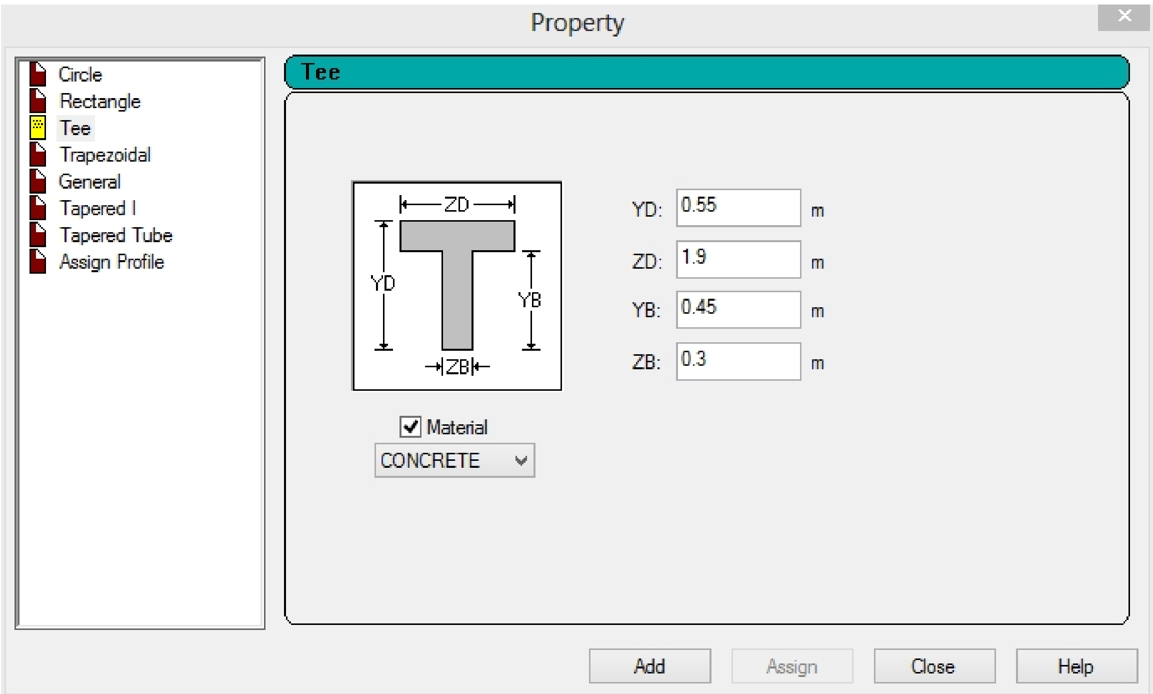


Figure 4.11-43: Definition of T section for Example 4.9-1 in STAAD environment.

- Beam rendered view indicated in Figure 4.11-44 can be reviewed from 3D Rendered View icon, .
- As discussed in Section 4.11.1.8.1, STAAD computes beam selfweight based on proposed section and material densities. In this example, it duplicates flange selfweight, which is already included in the superimposed load, see Section 4.9.2. To avoid this duplication, dead load of

  $$W_{\text{Dead}} = 29.2\frac{\text{kN}}{\text{m}} + 3.24\frac{\text{kN}}{\text{m}} = 32.4\frac{\text{kN}}{\text{m}}$$

  that includes selfweight and superimposed dead is determined and assigned to the beam.
- Other steps and parameters can be executed and defined in same approach discussed in Section 4.11.1 and Section 4.11.2 above. STAAD input file is presented in Table 4.11-9.

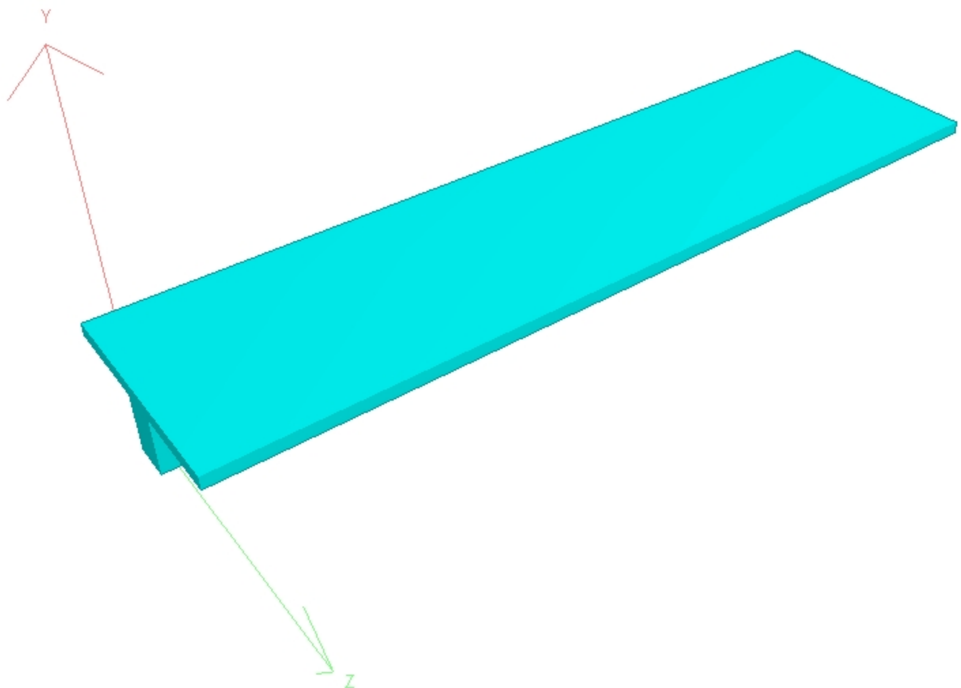
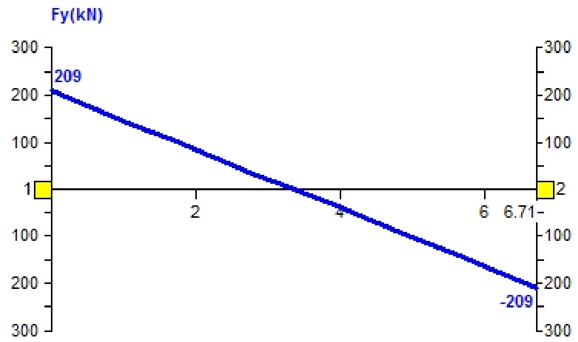


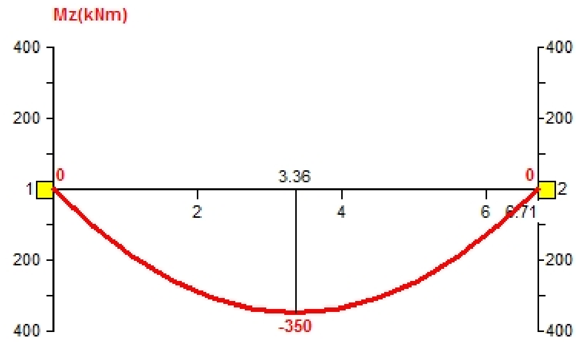
Figure 4.11-44: Render view for beam of Example 4.9-1 in STAAD environment.

Table 4.11-9: STAAD input file for Example 4.9-1.

```
STAAD PLANE
START JOB INFORMATION
ENGINEER DATE 21-Feb-18
END JOB INFORMATION
INPUT WIDTH 79
UNIT METER KN
JOINT COORDINATES
1 0 0 0; 2 6.71 0 0;
MEMBER INCIDENCES
1 1 2;
DEFINE MATERIAL START
ISOTROPIC CONCRETE
E 2.17185e+007
POISSON 0.17
DENSITY 23.5616
ALPHA 1e-005
DAMP 0.05
TYPE CONCRETE
STRENGTH FCU 27579
END DEFINE MATERIAL
MEMBER PROPERTY
1 PRIS YD 0.55 ZD 1.9 YB 0.45 ZB 0.3
CONSTANTS
MATERIAL CONCRETE ALL
SUPPORTS
1 2 PINNED
LOAD 1 LOADTYPE Dead  TITLE Dead Load
MEMBER LOAD
1 UNI GY -32.4
LOAD 2 LOADTYPE Live REDUCIBLE TITLE Live
MEMBER LOAD
1 UNI GY -14.6
LOAD COMB 3 Generated ACI Table1 1
1 1.4
LOAD COMB 4 Generated ACI Table1 2
1 1.2 2 1.6
LOAD COMB 5 Generated ACI Table1 3
1 1.2 2 1.0
LOAD COMB 6 Generated ACI Table1 4
1 1.2
LOAD COMB 7 Generated ACI Table1 5
1 0.9
PERFORM ANALYSIS
START CONCRETE DESIGN
CODE ACI
CLB 0.04 ALL
CLS 0.04 ALL
CLT 0.04 ALL
FC 21000 ALL
FYMAIN 414000 ALL
FYSEC 414000 ALL
MAXMAIN 25 ALL
MINMAIN 25 ALL
MINSEC 10 ALL
TRACK 2 ALL
DESIGN BEAM 1
END CONCRETE DESIGN
FINISH
```

- STAAD factored shear force and bending moment diagrams are presented in Figure 4.11-45. The maximum bending moment of 350 kN.m is equal to that determined based on simple statics in ***Example 4.9-1***.





FSFD FBMD

Figure 4.11-45: STAAD analysis factored shear force and bending moment diagrams for beam of Example 4.9-1.

- STAAD flexural design output is presented in Figure 4.11-46. The output indicates that required reinforcement from STAAD analysis of $2021\ mm^2$ is close to $1971\ mm^2$ that has been determined manually in ***Example 4.9-1***. It also indicates that, the required number of rebars, four according to hand calculation, cannot be distributed within the available width of 300mm. This seems natural as hand calculations indicates a width of 275mm is essential to accommodate the required reinforcement. When the number of rebars increases to about five to satisfy required reinforcement according to STAAS, available width would be insufficient.

```
     ACI 318-11    BEAM  NO.     1   DESIGN RESULTS
     ============================================

LEN -   6710. MM  FY -  414.  FC -  21.  MPA, SIZE - 1900. X  550. MMS
                                     TEE BEAM ZB/YB 300.00 /450.00

LEVEL     HEIGHT      BAR INFO        FROM         TO          ANCHOR
           (MM)                       (MM)        (MM)        STA  END
_____________________________________________________________________


*** A SUITABLE BAR ARRANGEMENT COULD NOT BE DETERMINED.
REQD. STEEL =    2021. MM2,   MAX. STEEL PERMISSIBLE =    2330. MM2
MAX POS MOMENT =     350.29 KN-MET,   LOADING    4
```

Figure 4.11-46: STAAD details of flexure design at the most critical section Example 4.9-1.

CONTENTS

CHAPTER 5
SHEAR AND DIAGONAL TENSION IN BEAMS

5.1 BASIC CONCEPTS

5.1.1 Shear versus Flexural Failures

Due to the following points, shear, or diagonal tension, failure may be more dangerous than flexural failure:

- It has greater uncertainty in predicting,
- It is not yet fully understood, in spite of many decades of experimental research and the use of highly sophisticated analytical tools,
- If a beam without properly designed shear reinforcement is overloaded to failure, shear collapse is likely to occur suddenly, with no advance warning of distress, see ***Figure 5.1-1*** below.

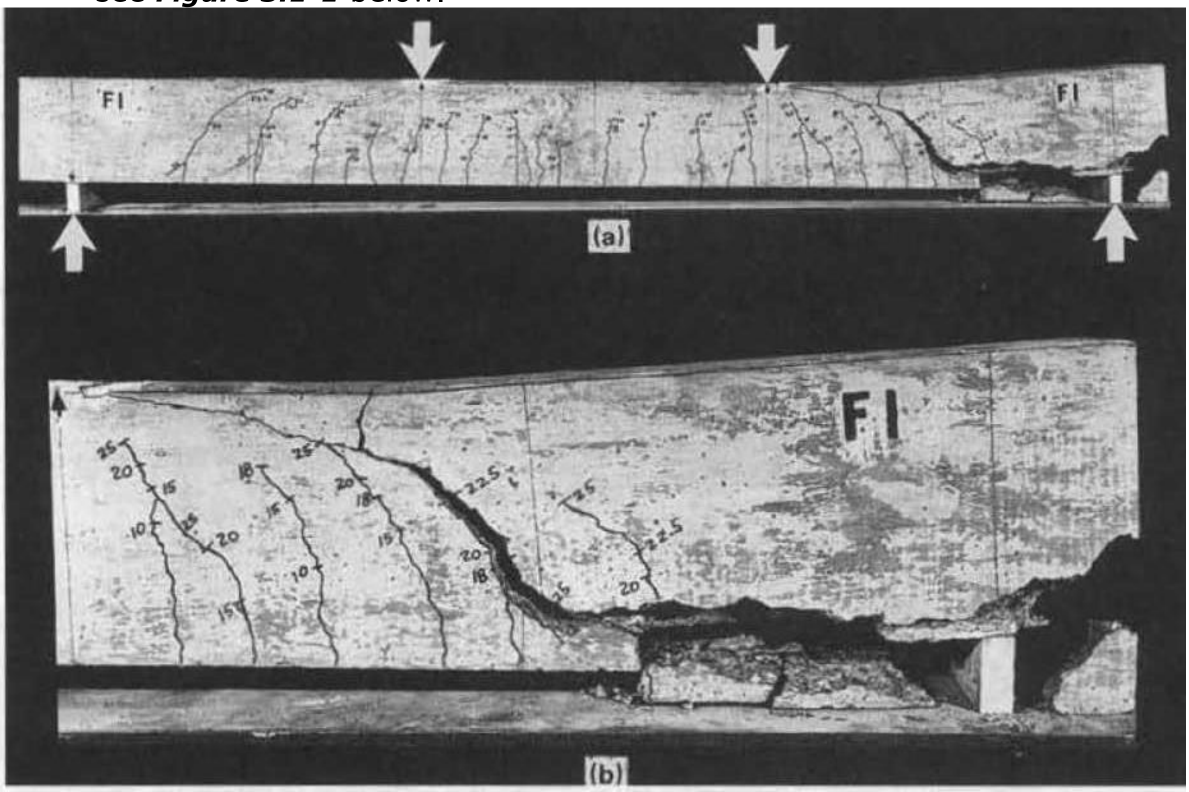


Figure 5.1-1: Shear failure of reinforced concrete beam: (a) overall view, (b) detail near right support.

5.1.2 Direct Shear versus Diagonal Tension

- It is important to realize that shear analysis and design in reinforced concrete structure are not really concerned with shear as such.
- The shear stresses in most beams are far below the direct shear strength of the concrete.
- The real concern is with diagonal tension stress, resulting from the combination of shear stress and longitudinal flexural stress.
- Difference between direct shear and diagonal tension is presented in sub article below.

5.1.2.1 Vertical and Horizontal Shears

- The simplest form of shear is the ***Vertical Shear Stress*** indicated in ***Figure 5.1-2*** below.

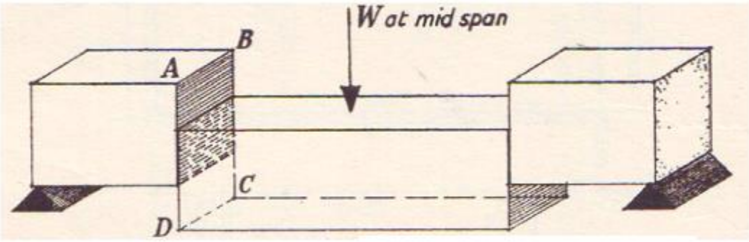


Figure 5.1-2: Vertical shear stresses.

- For homogenous beams and plain concrete beams before cracking, vertical shear stresses can be estimated from the following relation:

$$v = \frac{V.Q}{Ib} \quad \textbf{Eq. 5.1-1}$$

where:
V is total shear at section,
Q is statical moment about the neutral axis of that portion of cross section lying between a line through the point in question parallel to the neutral axis and nearest face (upper or lower) of the beam,
I is the moment of inertia of cross section about neutral axis,
b is width of beam at a given point.

- Distribution of vertical shear stress along beam depth is presented in ***Figure 5.1-3*** below:

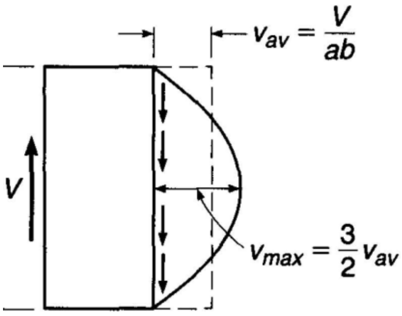


Figure 5.1-3: Shear stress distribution in homogeneous rectangular beams.

5.1.2.2 Horizontal Shear Stresses

- Referring to ***Figure 5.1-4*** below, imagine that a ball is placed between the two cut sections at X, because of the vertical shear action, the ball will turn in a clockwise direction.

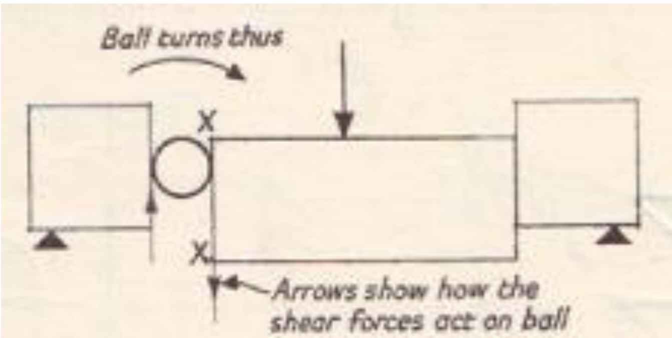


Figure 5.1-4: Conceptual view to imagine role of horizontal shear in resisting possible elemental rotation.

- Then in order to prevent turning, the cube shown below must be acted upon by horizontal forces shown in ***Figure 5.1-5*** below (Morgan, 1958):

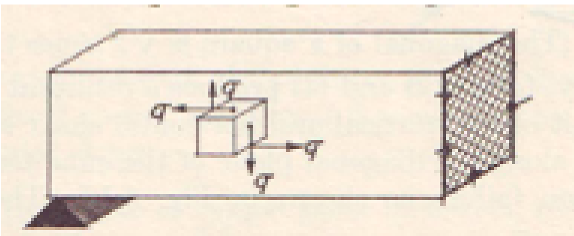


Figure 5.1-5: Horizontal shear stresses.

- These horizontal forces produce another type of shear stress called as ***Horizontal Shear Stress***.
- Thus, one can conclude that the ***vertical shear stress is accompanied by horizontal shear stress of equal intensity*** (Morgan, 1958).

5.1.2.3 Diagonal Tension and Compression

- Force (1) in ***Figure 5.1-6*** below can be combined with force (3) to produce a resultant force of $q\sqrt{2}$. Similarly force (2) and (4) produce a resultant force of $q\sqrt{2}$.

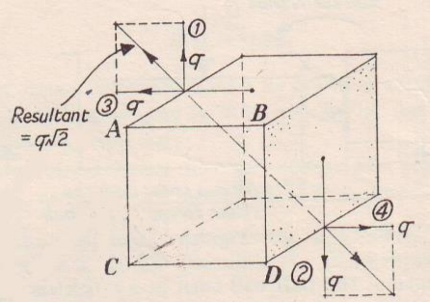


Figure 5.1-6: Diagonal tensile resultant of horizontal and vertical shear stresses.

- Thus, resultant of the vertical and horizontal shear stresses is a pull that exerted along the diagonal plane of the cube tending to cause the diagonal tension failure indicated in ***Figure 5.1-7*** below.

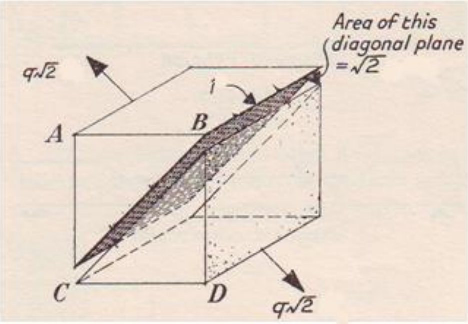


Figure 5.1-7: Diagonal tensile resultant of horizontal and vertical shear stresses, 2.

- Similarly, the vertical and horizontal shear stresses produce a compression force by combining force (2) with force (3) and force (1) with force (4) (see ***Figure 5.1-8*** below).

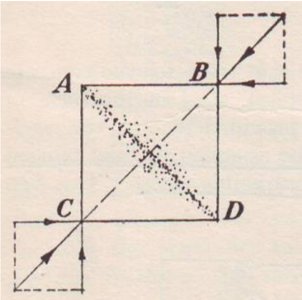


Figure 5.1-8: Diagonal compression resultant of horizontal and vertical shear stresses.

- Therefore***, whenever pure shear stress is acting on an element, it may be thought of as causing tension along one of the diagonals and compression along the other*** (Popov, 1976).

5.1.2.4 Stress Trajectories

Based on the above discussion for the relation between shear stresses and corresponding diagonal stresses, ***stress trajectories*** in a homogeneous simply supported beam with a rectangular section are presented in ***Figure 5.1-9*** below.

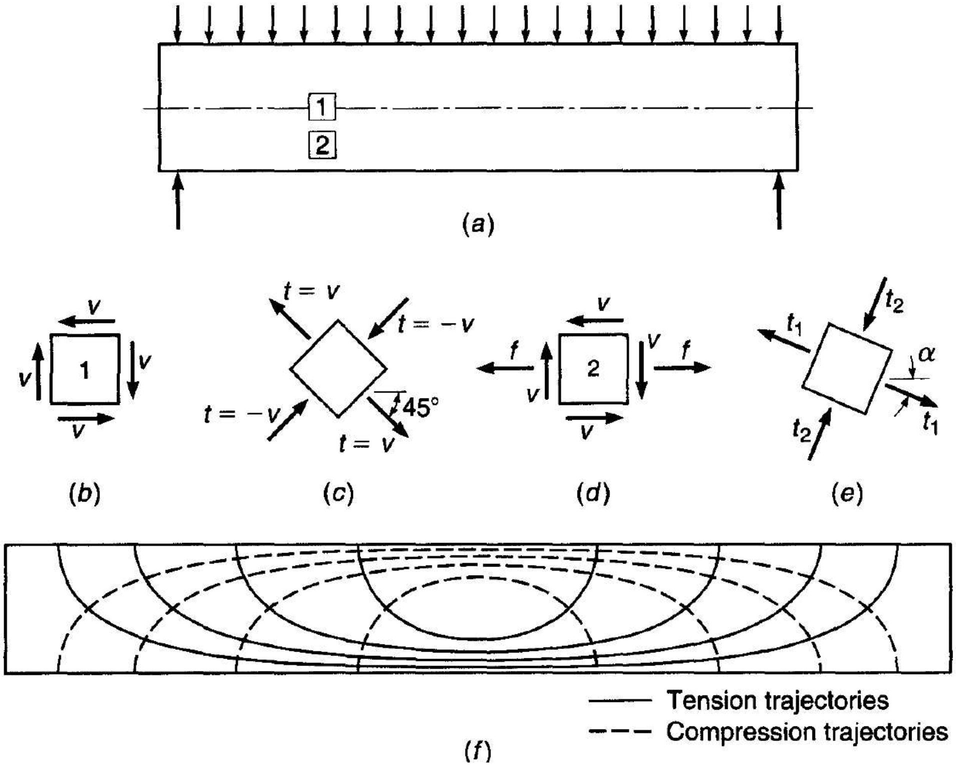


Figure 5.1-9: Stress trajectories in homogeneous rectangular beam.

5.1.2.5 Modes of Failure due to Shear or Diagonal Stresses

- Failure due to vertical shear stress is as shown in ***Figure 5.1-10*** below.

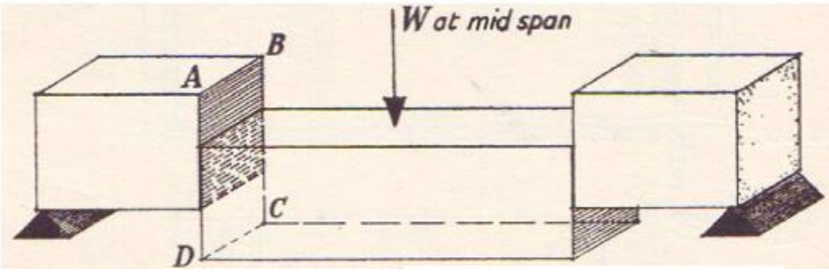


Figure 5.1-10: Mode of failure due to vertical shear stresses.

- Failure due to horizontal shear stress is as shown in ***Figure 5.1-11*** below.

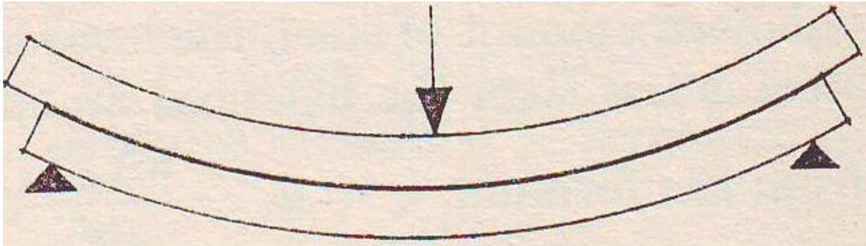


Figure 5.1-11: Mode of failure due to horizontal shear stresses.

- Failure due to diagonal compression stress is as shown in ***Figure 5.1-12*** below.

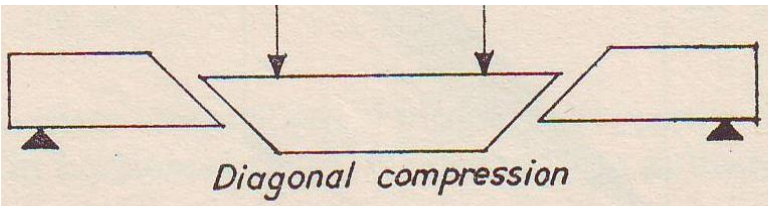


Figure 5.1-12: Mode of failure due to diagonal compression stresses.

- Failure due to diagonal tension stress is as shown in ***Figure 5.1-13*** below.

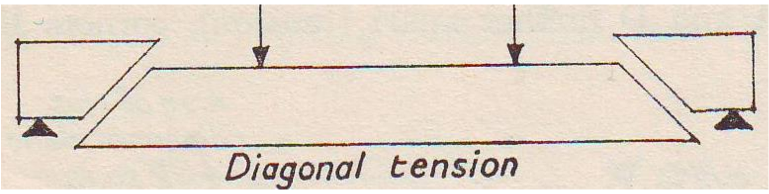


Figure 5.1-13: Mode of failure due to diagonal tensile stresses.

- It is known that concrete is at least ten times as strong in compression as it is in tension, so the typical shear failure in reinforced concrete beams is actually a diagonal tension failure.

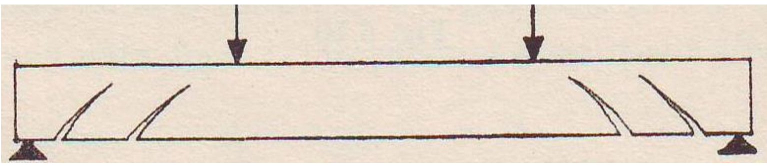


Figure 5.1-14: Cracks in concrete beams due to diagonal tension.

- When the shear stress is higher than the safe value of the concrete, steel in the form of vertical stirrups or inclined bars must be provided to take the exceed shear force.

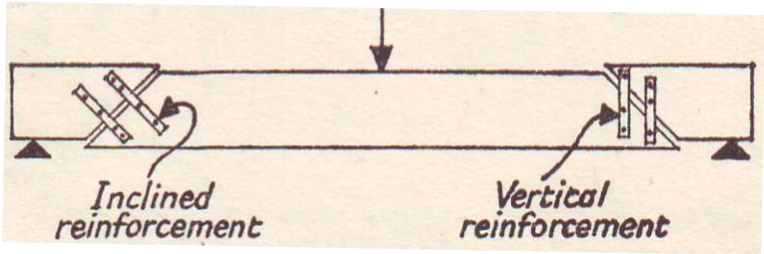


Figure 5.1-15: Conceptual view of vertical and inclined shear reinforcement.

5.1.3 Direct Shear

There are some circumstances in which consideration of direct shear is appropriate:

- The design of composite members combining precast beams with a cast-in-place top slab.

Figure 5.1-16: Composite members combining precast beams with a cast-in-place top slab.

- The horizontal shear stresses on the interface between components are important. The ***shear-friction theory*** is useful in this and other cases. This theory is out of our scope.

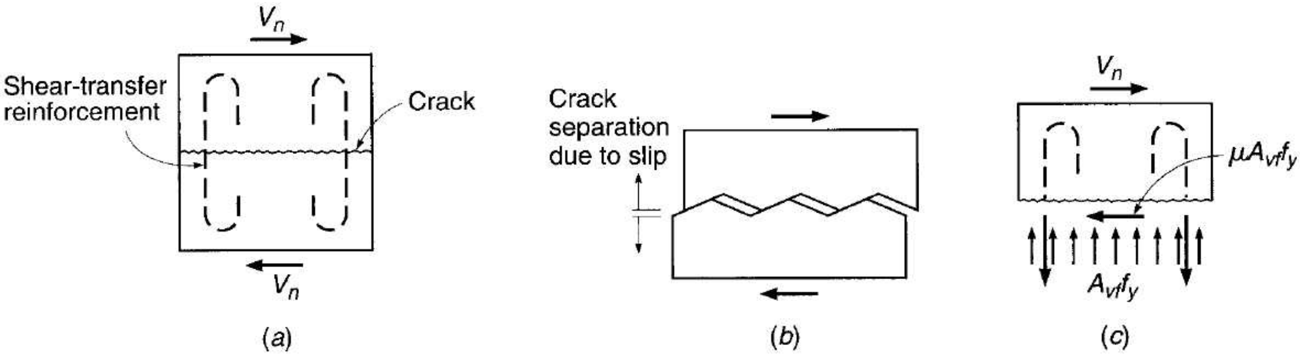


Figure 5.1-17: Basis of shear-friction design method: (a) applied shear; (b) enlarged representation of crack surface; (c) free-body sketch of concrete above crack.

5.1.4 ACI Code Provisions for Shear Design

- It is clear from the previous discussion; the problem under consideration is a problem of diagonal tension stresses.
- As the ACI Code uses the shear forces as an indication of the diagonal tension, then all design equations according to ACI Code are presented regarding shear forces.
- According to ACI Code (***9.5.1.1***), the design of beams for shear is to be based on the relation:

$$V_u \leq \emptyset V_n \quad \textbf{Eq. 5.1-2}$$

- According to the ACI Code (21.2.1), the strength reduction factor, ϕ, for shear is 0.75.
- According to article ***22.5.1.1***, nominal shear strength, V_n, can be computed based on the following relation:

$$V_n = V_c + V_s$$ **Eq. 5.1-3**

or,

$$V_u \leq V_n = \emptyset(V_c + V_s)$$ **Eq. 5.1-4**

where:

V_u is the total shear force applied at a given section of the beam due to factored loads,

V_n is the nominal shear strength, equal to the sum of the contributions of the concrete (V_c) and the steel (V_s) if present.

- Thus, according to ACI Code, the design problem for shear can be reduced to provisions for computing of V_u, V_c, and V_s if present. Each one of these quantities is discussed in some details in the articles below.

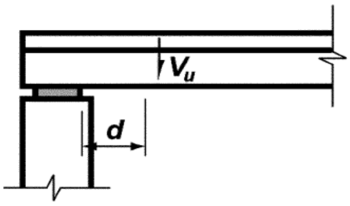
5.2 COMPUTING OF APPLIED FACTORED SHEAR FORCE V_u

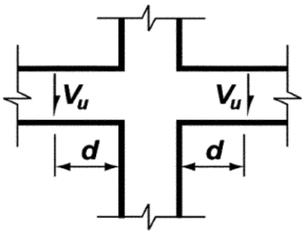
5.2.1 Basic Concepts

- The applied shear force can be computed based on given loads and spans.
- Generally, the applied factored shear force V_u is computed at the face of supports.
- According to ACI Code (**9.4.3.2**), ***sections between the face of support and a critical a section located "d" from the face of support for nonprestressed shall be permitted to be designed for*** V_u ***at that critical section*** if following conditions are satisfied: Discussion similar to that of classroom is preferable to add here to explain physical aspects of the three conditions below.
  - Support reaction, in the direction of applied shear, introduces compression into the end regions of the member.
  - Loads are applied at or near the top of the member.
  - No concentrated load occurs between the face of support and location of critical.

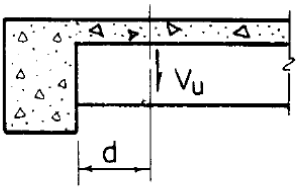
5.2.2 Examples on Computing of V_u

- For the figures below, critical section for computing of V_u will be taken at a distance "d" from the face of support as all above conditions are satisfied (Nilson, Design of Concrete Structures, 14th Edition, 2010). It is preferable to put these cases in groups, for example, floor beam supported on a deeper girder and a girder with same depth can be put in the same group. Besides, it is preferable that each group and corresponding figures have subtitle and caption.

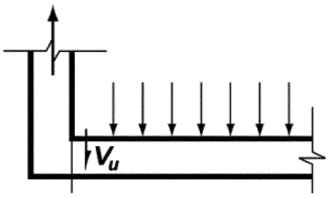




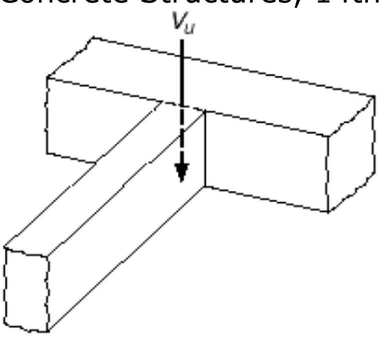
- For the figure below, the critical section for computing of V_u is at distance "d" from the face of support for a floor beam supported by a deeper main girder as all above conditions are satisfied (Kamara, 2005) (Page 12-3).



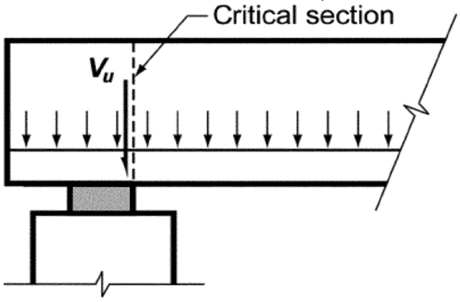
- For the figure below, the critical section for computing of V_u is at the face of support as member framing into a supporting member in tension (Nilson, Design of Concrete Structures, 14th Edition, 2010) (Page 131).



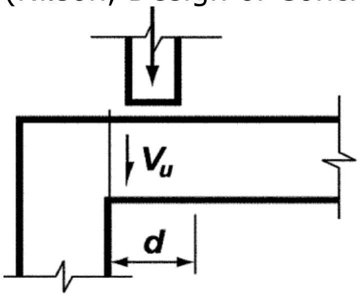
- For the figure below, the critical section for computing of V_u is at the face of support if the beam is supported by a girder of similar depth (Nilson, Design of Concrete Structures, 14th Edition, 2010).



- For the figure below, the critical section for computing of V_u is at the face of support as loads are not applied at or near the top of the member (Nilson, Design of Concrete Structures, 14th Edition, 2010).



- For the figure below, the critical section for computing of V_u is at the face of support as concentrated load occurs within a distance "d" from the face of support (Nilson, Design of Concrete Structures, 14th Edition, 2010).



5.3 SHEAR STRENGTH PROVIDED BY CONCRETE V_c

5.3.1 Upper Bound of Concrete Compressive Strength, f_c', in Estimating V_c

- According to ***article 22.5.3.1*** of ACI code, except for ***article 22.5.3.2***, related to prestressed beams and joist construction, ***the value of*** $\sqrt{f_c'}$ ***used to calculate*** V_c ***shall not exceed 8.3 MPa***.
- The above statement is because of a lack of test data and practical experience with concretes having compressive strengths greater than 70 MPa.

5.3.2 Plain Concrete Beams

- As the load increases in such a beam, a tension crack will form where the tensile stresses are largest, and it will immediately cause the beam to fail.
- Except for beams of very unusual proportions, the largest tensile stresses are those caused at the outer fiber by bending alone, at the section of maximum bending moment. In this case, shear has little, if any, influence on the strength of a beam.

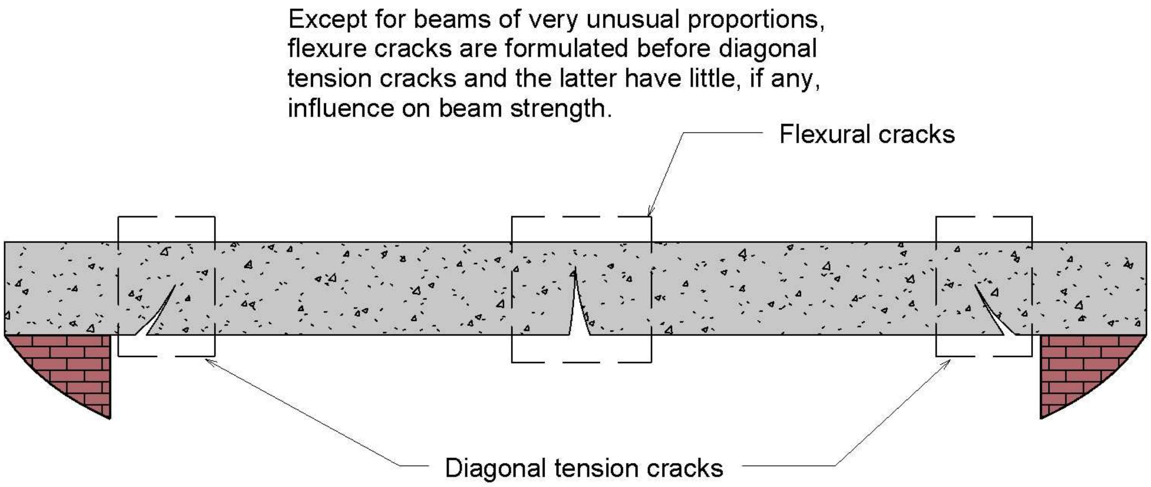


Figure 5.3-1: Behavior of plain concrete beams.

5.3.3 Reinforced Concrete Beams without Shear Reinforcement

- For beams designed properly for flexure, diagonal cracks may propagate faster than flexural cracks, and shear aspects may govern the beam failure.

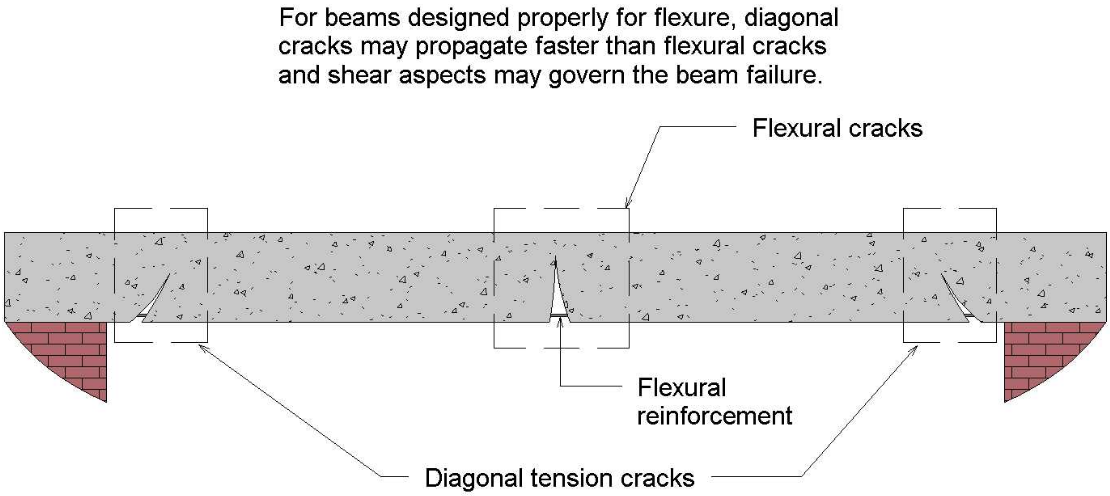


Figure 5.3-2: Behavior of a beam reinforced for flexure only.

- For concrete beams ***reinforced for flexure only***, shear force required ***to initiates diagonal cracks in web-shear cracks region***, or ***to propagate cracks in a flexure-shear region*** can be estimated from relation below, ***Article 22.5.5.1*** of (ACI318M, 2014):

$$V_c = 0.17\lambda\sqrt{f_c'}\, b_w d \qquad \textbf{Eq. 5.3-1}$$

where:

λ is the lightweight modification factor that taken from ***Table 5.3-1*** below, Table 19.2.4.2 of (ACI318M, 2014).

Table 5.3-1: Modification factor λ, Table 19.2.4.2 of (ACI318M, 2014).

Concrete	Composition of aggregates	λ
All-lightweight	Fine: ASTM C330M Coarse: ASTM C330M	0.75
Lightweight, fine blend	Fine: Combination of ASTM C330M and C33M Coarse: ASTM C330M	0.75 to 0.85[1]
Sand-lightweight	Fine: ASTM C33M Coarse: ASTM C330M	0.85
Sand-lightweight, coarse blend	Fine: ASTM C33M Coarse: Combination of ASTM C330M and C33M	0.85 to 1[2]
Normalweight	Fine: ASTM C33M Coarse: ASTM C33M	1

[1]Linear interpolation from 0.75 to 0.85 is permitted based on the absolute volume of normalweight fine aggregate as a fraction of the total absolute volume of fine aggregate.

[2]Linear interpolation from 0.85 to 1 is permitted based on the absolute volume of normalweight coarse aggregate as a fraction of the total absolute volume of coarse aggregate.

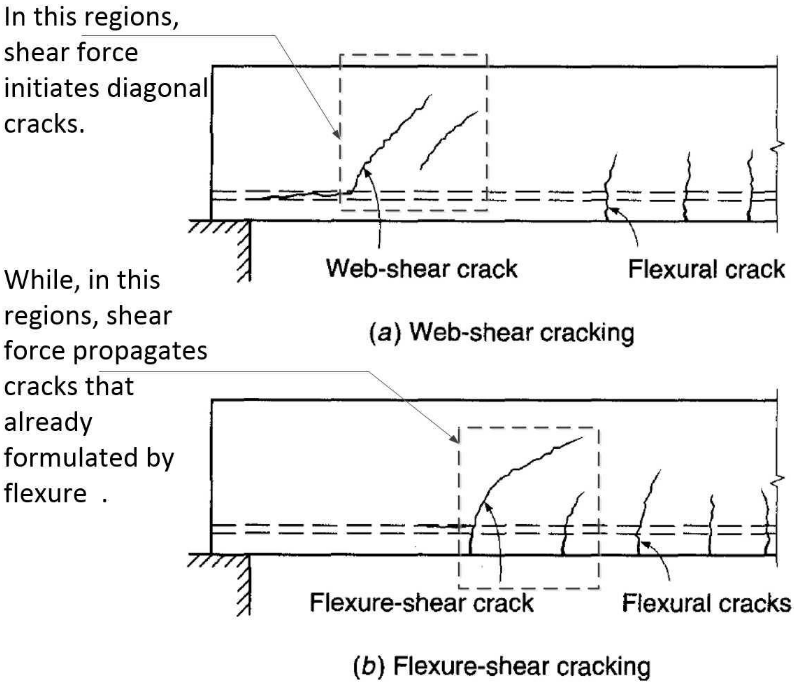


Figure 5.3-3: Diagonal tension cracking in reinforced concrete beams.

- With referring to ***Figure 5.3-3*** above, it is useful to note that the Eq. 5.3-1 is ***more suitable flexure-shear crack*** and ***relatively conservative for web-shear cracks***. A more accurate relation has been presented in ***Article 5.8*** of this chapter.
- In spite of its conservative nature in the web-shear crack region, in practice, most of the beams are usually designed based on Eq. 5.3-1.
- For solid circular members, the area used to compute V_c shall be taken as shown in ***Figure 5.3-4*** (***Article 22.5.2.2*** of ACI Code).

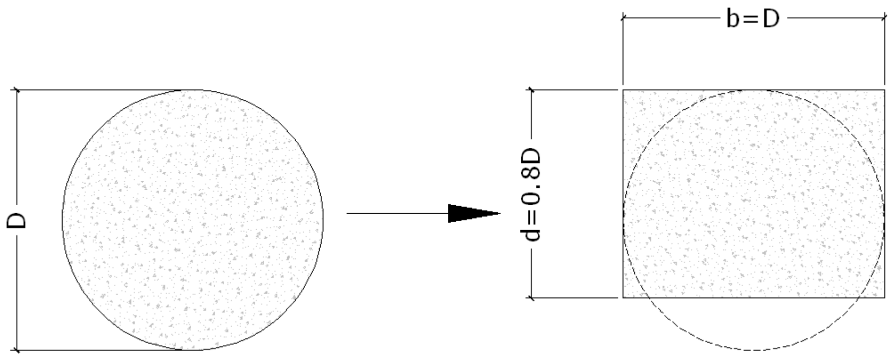


Figure 5.3-4: Effective area for shear in solid circular sections.

5.3.4 Beams Reinforced for Shear

- As for flexural behavior, current ACI code permits formation of web-shear cracks and flexure-shear cracks when beams are reinforced for shear and diagonal tension.
- With shear reinforcements, that resist propagation of web-shear cracks, the free body diagram for one side of crack at failure stage would be as shown ***Figure 5.3-5*** below.

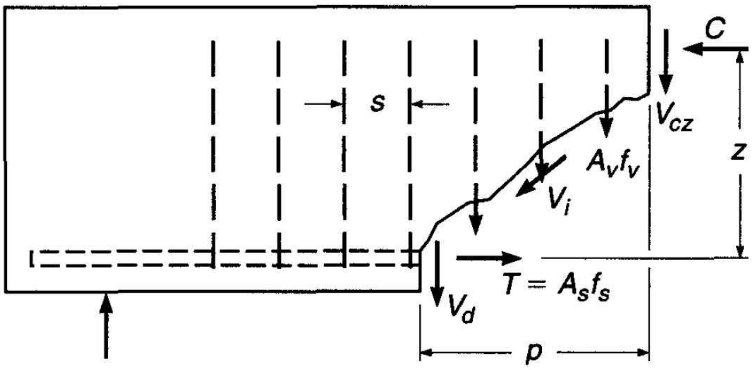


Figure 5.3-5: Forces at a diagonal crack in a beam with vertical stirrups.

where

$A_v f_v$ is shear force resisted by each stirrup, will be discussed in detail in ***Article 5.4.2*** of this chapter,

V_{cz} shear force resisted by uncracked concrete portion,

V_i shear force resisted by the interlocking of concrete on two sides of the crack,

V_d shear force resisted by longitudinal rebars, dowel action,

- From equilibrium in vertical direction,

$$V_{ext} = V_{cz} + V_d + V_{iy} + V_s \quad \textbf{Eq. 5.3-2}$$

- ***Empirically*** and ***conservatively*** current ACI code assumes that:

$$V_{cz} + V_d + V_{iy} \approx V_c = 0.17\lambda\sqrt{f_c'}\, b_w d \quad \textbf{Eq. 5.3-3}$$

- Therefore, in the current ACI code, the relation:

$$V_c = 0.17\lambda\sqrt{f_c'}\, b_w d \quad \textbf{Eq. 5.3-4}$$

has two roles:
  - It is used ***rationally*** to estimate the shear force that either initiates web-shear cracks or propagate flexure-shear cracks.
  - It is used ***empirically*** to estimate the order for summation of V_{cz}, V_d, and V_{iy}.

5.4 SHEAR STRENGTH PROVIDED BY SHEAR REINFORCEMENT V_s

5.4.1 Type of Shear Reinforcement

- Several types and arrangements of shear reinforcement permitted by ACI are illustrated in ***Figure 5.4-1*** (Kamara, 2005) (Page 12-6).

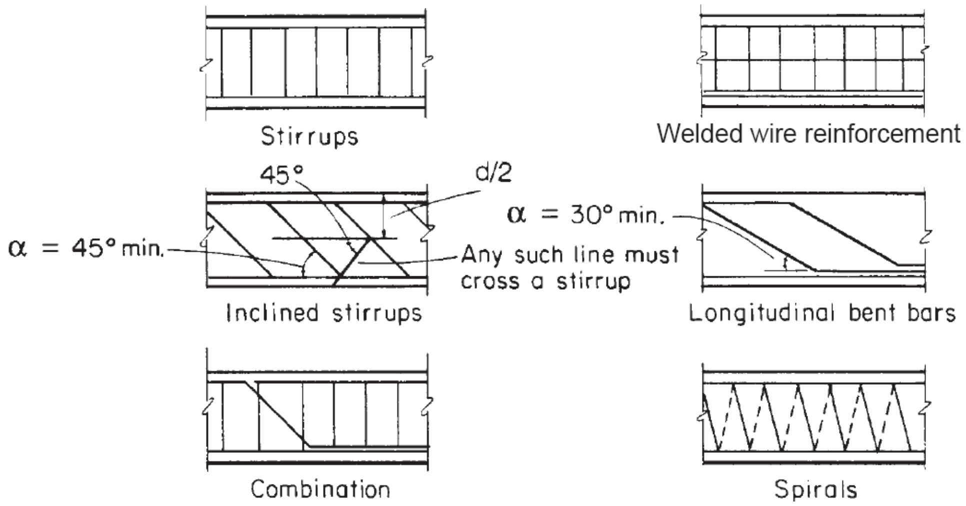


Figure 5.4-1: Types of shear reinforcement.

- Spirals, circular ties, or hoops are explicitly recognized as types of shear reinforcement starting with the 1999 code (Kamara, 2005) (Page 12-6).
- Vertical stirrups are the most common type of shear reinforcement.
- Inclined stirrups and longitudinal bent bars are rarely used as they require a special care during placement in the field.
- U-shaped bars similar to those presented in ***Figure 5.4-2*** below are the most common, although multiple-leg stirrups such as shown are sometimes necessary.

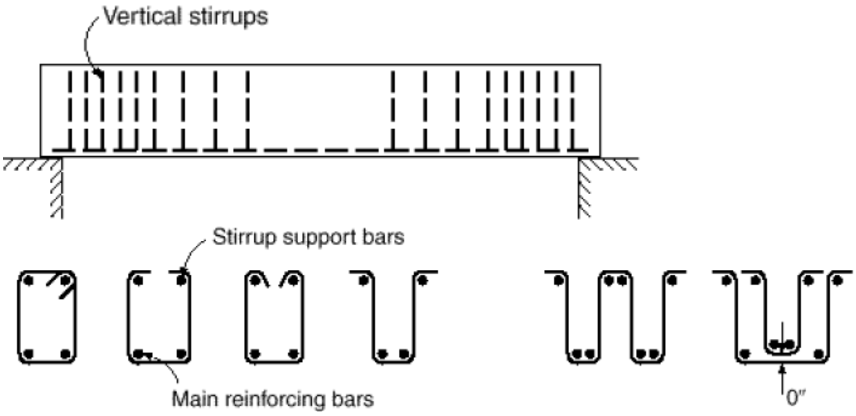


Figure 5.4-2: U stirrups shear reinforcement.

5.4.2 Theoretical Spacing between Vertical Stirrups

- Theoretical spacing for vertical stirrups can be related to other design parameters based on following relations:

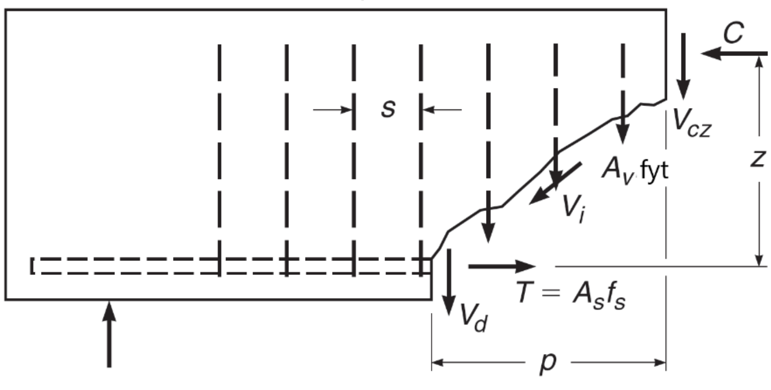


Figure 5.4-3: Forces at a diagonal crack in a beam with vertical stirrups, reproduced for convenience.

$$V_s = \text{Fore per each stirrup} \times \text{No. of stirrups through the inclined crack}$$

$$V_s = \left(A_v \times f_{yt}\right)_{\text{Fore per each stirrup}} \times \left(\frac{p}{s}\right)_{\text{No.of stirrups through the inclined crack}}$$

where:

$A_v = \text{ area of shear reinforcement } = \frac{\pi \emptyset_{Stirrups}^2}{4} \times \text{No. of Legs}$

- If the crack is assumed to have an angle of 45 degree with the horizon, then p can be computed approximately based on following relation:
  $p \approx d$
  Then:
  $V_s = \frac{A_v f_{yt} d}{s}$ **Eq. 5.4-1**
  Above relation that suitable for analysis purpose, can be solved for s to be more suitable for design purpose:
  $s = \frac{A_v f_{yt} d}{V_s}$ ■ **Eq. 5.4-2**
- In addition to this theoretical spacing for shear reinforcement, ACI Code also includes many other nominal requirements that related to shear reinforcement. ACI practical procedure for shear design has been summarized in article below.

5.5 SUMMARY OF PRACTICAL PROCEDURE FOR SHEAR DESIGN

5.5.1 Essence of the Problem

- Generally, beam dimensions (b and h) are determined based on considerations other than shear and diagonal tension requirements.
- Then, in a shear problem, the designer deals with a beam that has pre-specified dimensions and main unknowns in the design problem are the shear reinforcement (if needed) and its details that can be summarized as follows:
  - o The diameter of shear reinforcement.
  - o Spacing (for economic aspect, a beam may be divided to sub-regions with different shear reinforcements) for shear reinforcements.
  - o Anchorage requirements for shear reinforcements.
- The detailed procedure for each one of the above three unknowns will be discussed below.

5.5.2 Bar Diameter for Stirrups and Stirrups Support Bars

- As was previously discussed in Chapter 4, bar diameters that used for shear reinforcements usually include 10mm, or 13mm.
- A Bar diameter of 16mm rarely used as shear reinforcement.
- Where no top bars are required for flexure, stirrups support bars must be used. These are usually about the same diameter as the stirrups themselves (Nilson, Design of Concrete Structures, 14th Edition, 2010).

5.5.3 Spacing for Shear Reinforcements

Computing of required spacing can be summarized as follows:

- Draw the shear force diagram based on factored load and span length, and divide the diagram into the three distinct regions shown in ***Figure 5.5-1*** (Kamara, 2005) (Page 12-9):

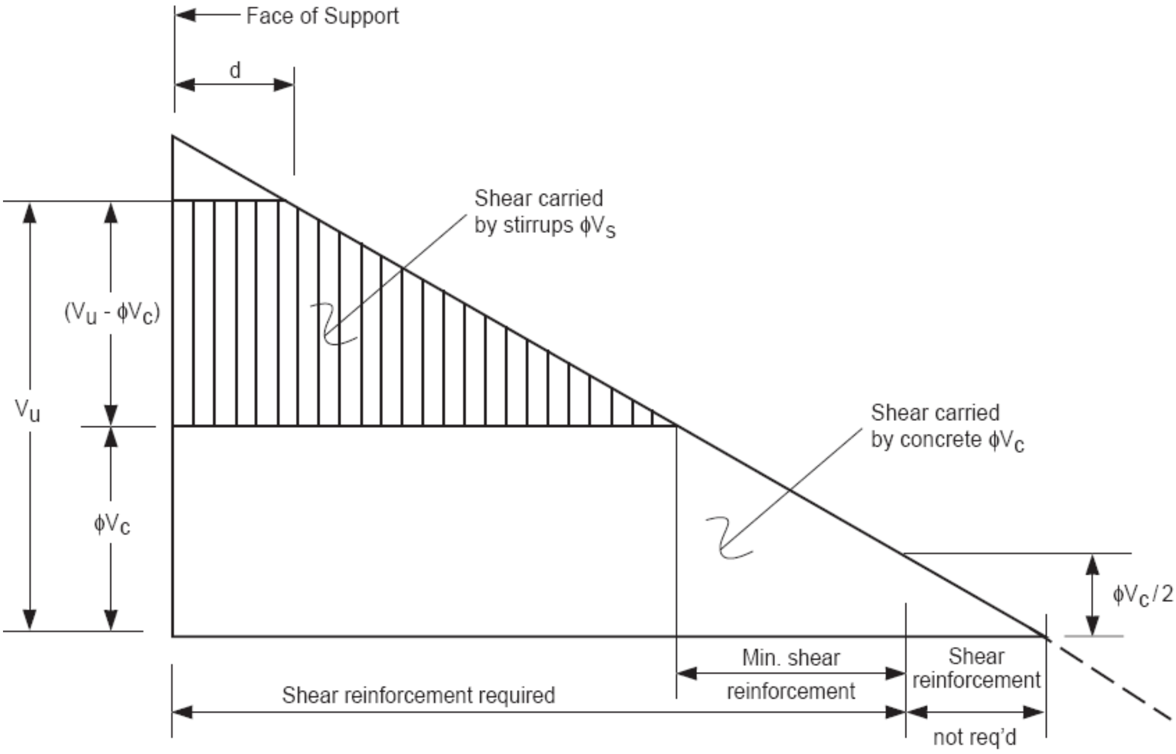


Figure 5.5-1: Three distinguish regions of shear force diagram.

- Based on ***Table 5.5-1***, compute the required spacing for each one of the regions shown above (if shear reinforcement is required for this region) (Kamara, 2005) (Page 12-8):

Table 5.5-1: ***ACI provisions for shear design.***

Region	$V_u \le \emptyset \frac{V_c}{2}$	$\emptyset \frac{V_c}{2} < V_u \le \emptyset V_c$	$\emptyset V_c \le V_u$
V_s	None	None	$= \frac{V_u - \emptyset V_c}{\emptyset} \le 0.66\sqrt{f_c'} b_w d$ Else, change beam dimensions.
$S_{Theoretical}$	None	None	$= \frac{A_v f_{yt} d}{V_s}$
$S_{for\ Av\ minimum}$ (9.6.3.3)	None	minimum $(\frac{A_v f_{yt}}{0.062\sqrt{f_c'} b_w}$ or $\frac{A_v f_{yt}}{0.35 b_w})$	minimum $(\frac{A_v f_{yt}}{0.062\sqrt{f_c'} b_w}$ or $\frac{A_v f_{yt}}{0.35 b_w})$
$S_{maximum}$ (9.7.6.2.2)	None	Minimum $\left[\frac{d}{2} \text{ or } 600\text{mm}\right]$	$V_s \le 0.33\sqrt{f_c'} b_w d$ Minimum $\left[\frac{d}{2} \text{ or } 600\text{mm}\right]$ $V_s > 0.33\sqrt{f_c'} b_w d$ Minimum $\left[\frac{d}{4} \text{ or } 300\text{mm}\right]$
$S_{Required}$	None	Minimum $\left[S_{for\ Av\ minimum}, S_{maximum}\right]$	Minimum $\left[S_{Theoretical}, S_{for\ Av\ minimum}, S_{maximum}\right]$

- Notes on A_{vmin}:
  According to ***Article 9.6.3.1***, for cases presented in Table below, $A_{v\,minimum}$ is not required even with $\emptyset \frac{V_c}{2} < V_u \le \emptyset V_c$:

Table 5.5-2: Cases where $A_{v\,\min}$ is not required if $0.5\phi Vc < Vu \le \phi Vc$, Table 9.6.3.1 of (ACI318M, 2014).

Beam type	**Conditions**
Shallow depth	$h \le 250$ mm
Integral with slab	$h \le$ greater of $2.5t_f$ or $0.5b_w$ and $h \le 600$ mm
Constructed with steel fiber-reinforced normalweight concrete conforming to 26.4.1.5.1(a), 26.4.2.2(d), and 26.12.5.1(a) and with $f_c' \le 40$ MPa	$h \le 600$ mm and $V_u \le \phi 0.17\sqrt{f_c'} b_w d$
One-way joist system	In accordance with 9.8

5.5.4 Anchorage Requirement for Shear Reinforcements

5.5.4.1 Design Assumptions Regarding to Anchorage

Above design is based on assumption that the stirrups will yield at ultimate load. This will be true only if the stirrups are well anchored.

5.5.4.2 General Anchor Requirements

- Generally, the upper end of the inclined crack approach very closed to the compression face of the beam. Thus, the portion of the stirrups shown shaded in ***Figure 5.5-2*** must be able to anchor the stirrups.

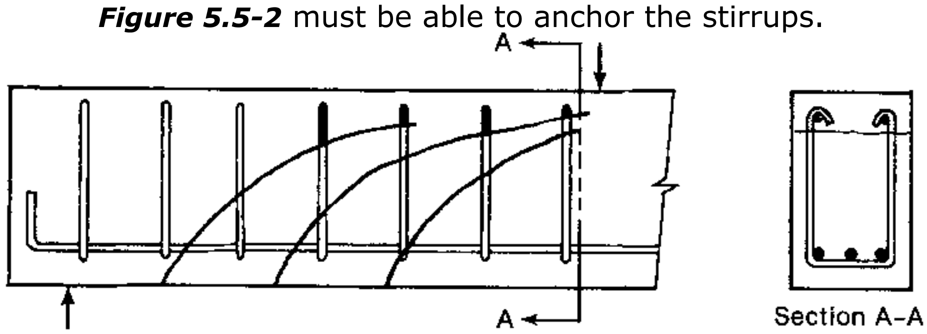


Figure 5.5-2: General requirements for anchorage of stirrups.

- ACI general anchor requirement can be summarized in ***Figure 5.5-3***.

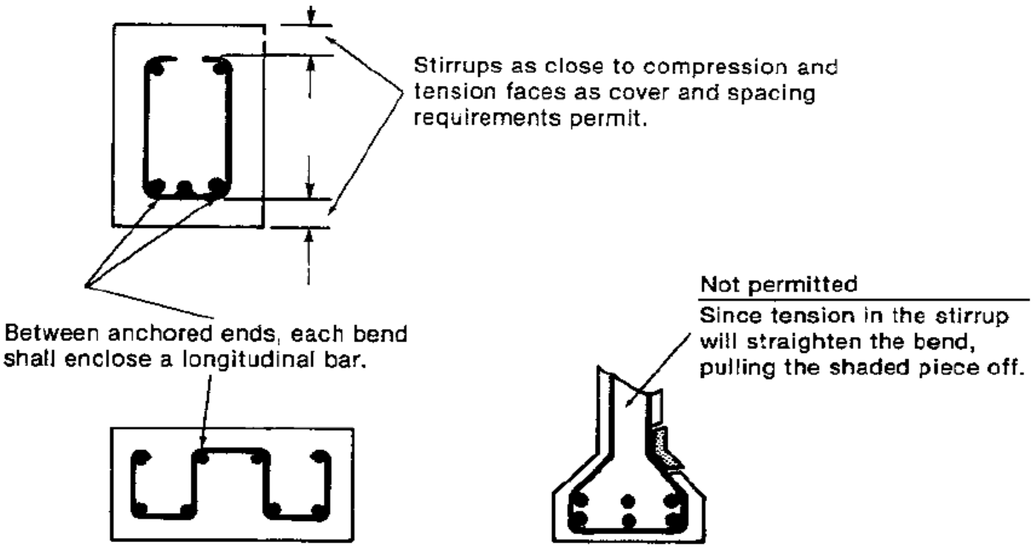


Figure 5.5-3: General requirements for anchorage of stirrups, continued.

- According to anchorage requirements, stirrups may be classified into the following two types.

5.5.4.3 Open Stirrups

- They may take any one of the shapes indicated in ***Figure 5.5-4***.
- As shown in ***Figure 5.5-5***, anchorage of an open stirrup depends on using standard hooks at the corners of the stirrups supporting rebars.

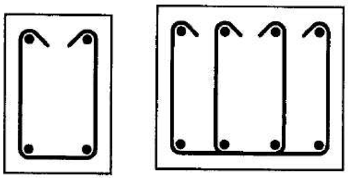


Figure 5.5-4: Open stirrups.

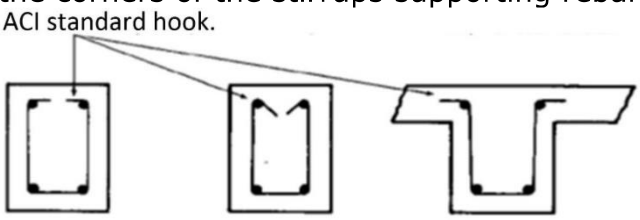
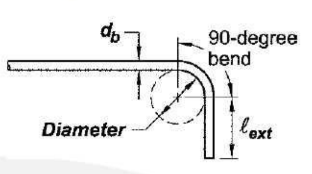
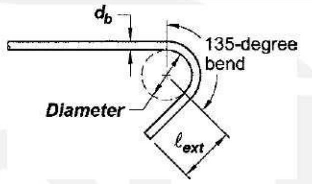
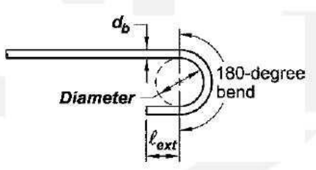


Figure 5.5-5: Standard hook anchorage for open stirrups.

- Minimum inside bend diameters and standard hook geometry ***for stirrups, ties, and hoops*** are presented in ***Table 5.5-3***.

Table 5.5-3: Minimum inside bend diameters and standard hook geometry for stirrups, ties, and hoops, Table 25.3.2 of (ACI318M, 2014).

Type of standard hook	Bar size	Minimum inside bend diameter, mm	Straight extension[1] ℓ_{ext}, mm	Type of standard hook
90-degree hook	No. 10 through No. 16	$4d_b$	Greater of $6d_b$ and 75 mm	
	No. 19 through No. 25	$6d_b$	$12d_b$	
135-degree hook	No. 10 through No. 16	$4d_b$	Greater of $6d_b$ and 75 mm	
	No. 19 through No. 25	$6d_b$		
180-degree hook	No. 10 through No. 16	$4d_b$	Greater of $4d_b$ and 65 mm	
	No. 19 through No. 25	$6d_b$		

[1]A standard hook for stirrups, ties, and hoops includes the specific inside bend diameter and straight extension length. It shall be permitted to use a longer straight extension at the end of a hook. A longer extension shall not be considered to increase the anchorage capacity of the hook.

- According to ACI (25.7.1.3b), for No. 19, through No. 25 stirrups with f_{yt} greater than 280 MPa, a standard stirrup hook around a longitudinal bar plus an embedment between mid-height of the member and the outside end of the hook equal to or greater than $0.17 d_b\, f_{yt}\,/(\lambda\,\sqrt{f_c'})$.

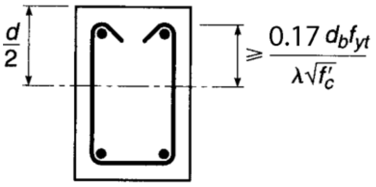


Figure 5.5-6: Embedment length for open stirrups with for No. 19, through No. 25 stirrups with f_{yt} greater than 280 MPa.

- This requirement has been included as it is not possible to bend a No. 19, No. 22, or No. 25 stirrup tightly around a longitudinal.

5.5.4.4 Closed Stirrups

- Its typical shapes are shown ***Figure 5.5-7***.

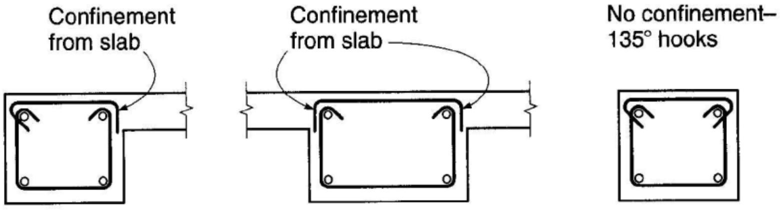


Figure 5.5-7: Typical closed stirrups.

- It may be taking the form of closed tie shown in ***Figure 5.5-8***.
- Closed stirrups or closed ties should be used for:
  - For beams with compression reinforcements.
  - For members subjected to torsion.

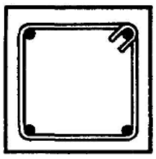


Figure 5.5-8: Tie reinforcement.

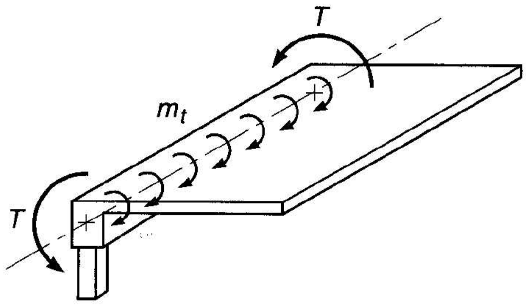


Figure 5.5-9: Beam subjected to torsion where closed stirrups should be adopted.

  - For reversals stresses.

5.5.4.5 Spliced Stirrup

- According to article ***25.7.1.7*** of (ACI318M, 2014), ***except where used for torsion*** or ***integrity reinforcement***, closed stirrups are permitted to be made using pairs of U-stirrups spliced to form a closed unit where lap lengths are at least $1.3 l_d$.
- In members with a total depth of at least 450 mm, such splices with $A_b f_{yt} \leq\ 40\ kN$ per leg shall be considered adequate if stirrup legs extend the full available depth of member.

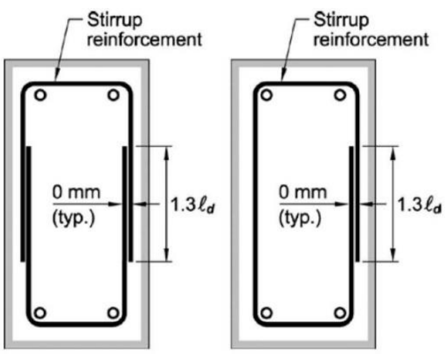


Figure 5.5-10: Closed stirrup configurations.

- The development length may be defined as ***the length of embedment necessary to develop the full tensile strength of the bar***. It will be discussed in details in ***Chapter 6***.
- Its approximate value can be computed from Table below:

Table 5.5-4: Simplified tension development length in bar diameters l_d/d_b for uncoated bars and normalweight concrete

		No. 6 (No. 19) and Smaller[a]			No. 7 (No. 22) and Larger		
		f'_c, psi			f'_c, psi		
	f_y, ksi	4000	5000	6000	4000	5000	6000
(1) Bottom bars							
Spacing, cover and ties as per Case *a* or *b*	40	25	23	21	32	28	26
	50	32	28	26	40	35	32
	60	38	34	31	47	42	39
Other cases	40	38	34	31	47	42	39
	50	47	42	39	59	53	48
	60	57	51	46	71	64	58
(2) Top bars							
Spacing, cover and ties as per Case *a* or *b*	40	33	29	27	41	37	34
	50	41	37	34	51	46	42
	60	49	44	40	62	55	50
Other cases	40	49	44	40	62	55	50
	50	62	55	50	77	69	63
	60	74	66	60	92	83	76

Case *a*: Clear spacing of bars being developed or spliced $\geq d_b$, clear cover $\geq d_b$, and stirrups or ties throughout l_d not less than the Code minimum.

Case *b*: Clear spacing of bars being developed or spliced $\geq 2d_b$, and clear cover not less than d_b.

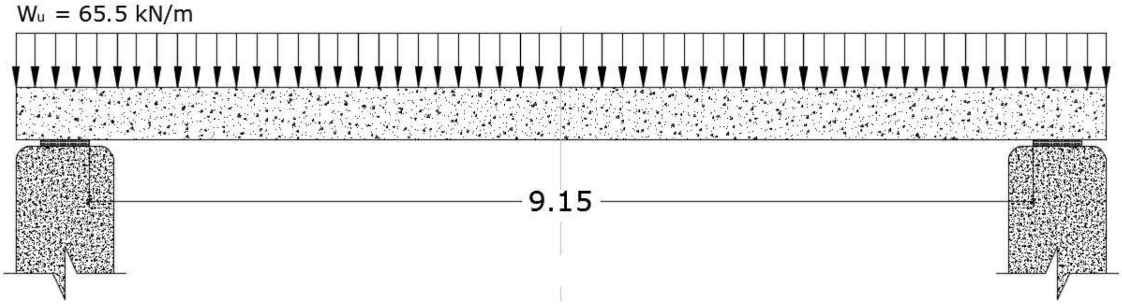
[a]ACI Committee 408 recommends that the values indicated for bar sizes No. 7 (No. 22) and larger be used for all bar sizes.

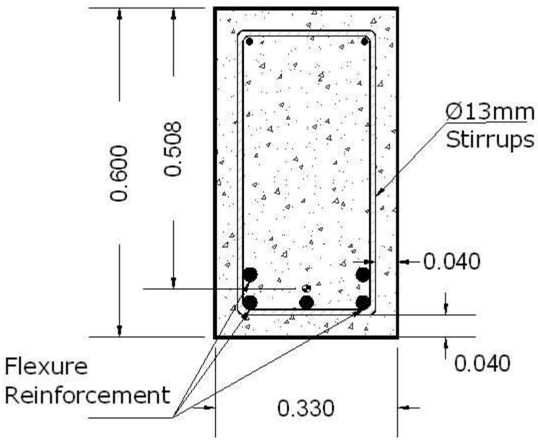
5.6 BASIC DESIGN EXAMPLES

Example 5.6-1

Check adequacy of proposed size and determine required spacing of vertical stirrups for a 9.15m span simply supported beam with following data:

$b_w = 330\text{mm},\ d = 508\text{mm},\ f_c' = 21\text{ MPa},\ f_{yt} = 275\text{ MPa},\ W_u = 65.5\,\frac{\text{kN}}{\text{m}}$



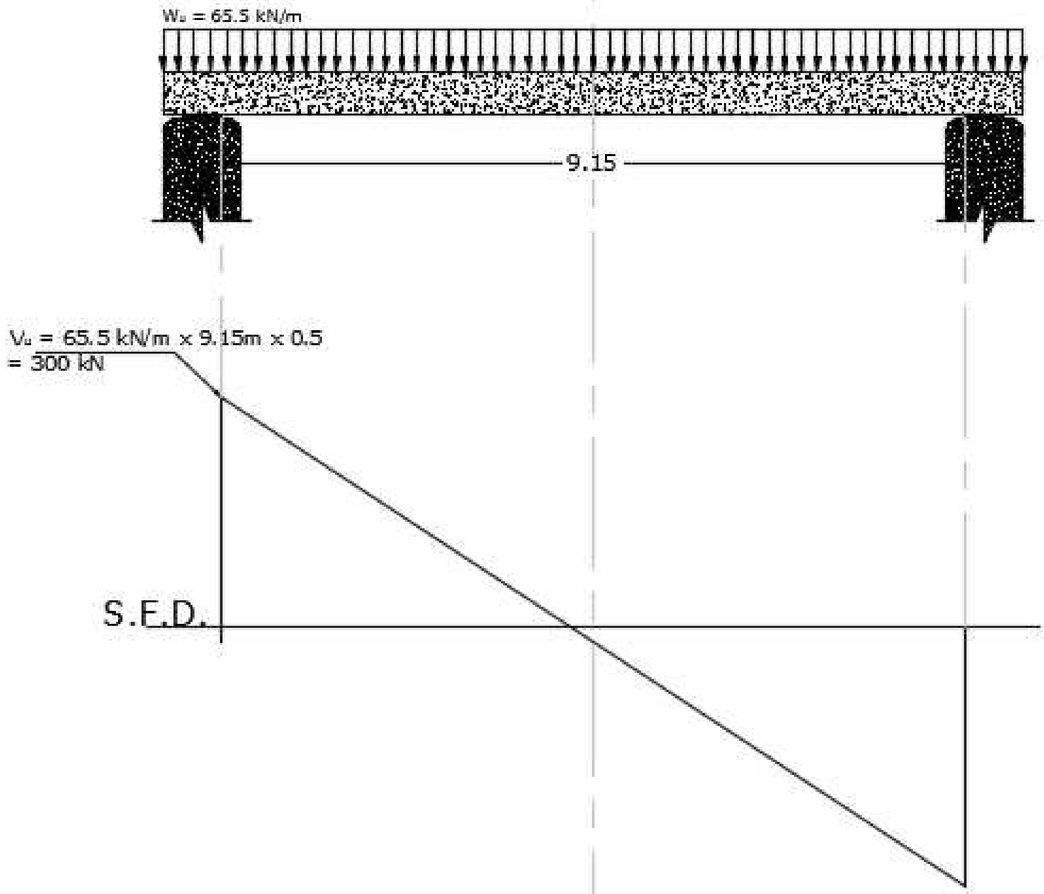


Proposed beam section.

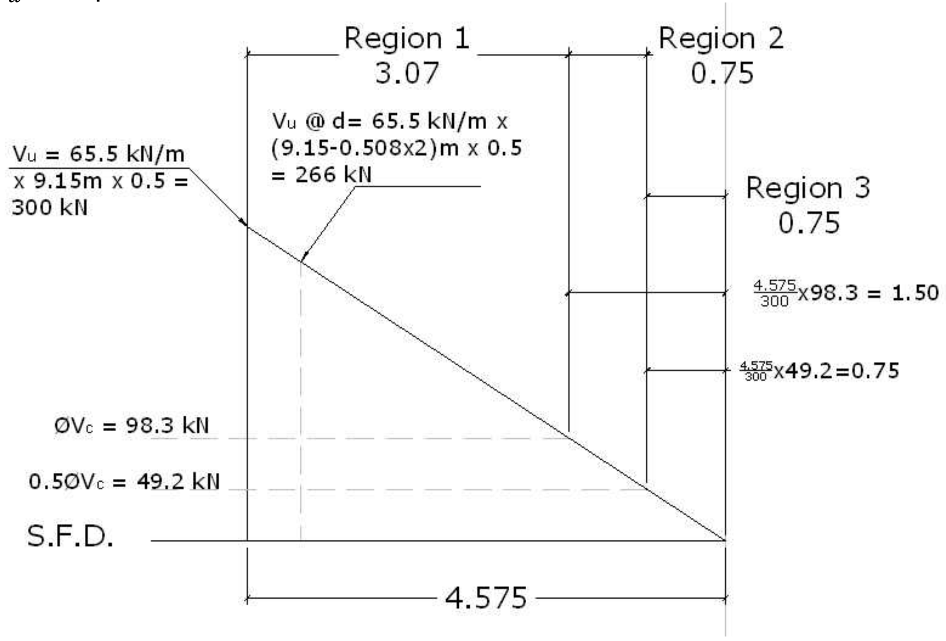
Figure 5.6-1: Simply supported beam for Example 5.6-1.

Solution

- Regarding to bar diameter for stirrups, the proposed diameter of 13mm is common and accepted one.
- Draw the shear force diagram for the beam:



- Compute of Shear Strength Provided by Concrete V_c:
  $V_c = 0.17\lambda\sqrt{f'_c}\, b_w d$
  As $\lambda = 1.0$ for normal weight concrete, then:
  $V_c = 0.17\sqrt{f'_c}\, b_w d = 0.17 \times \sqrt{21}\frac{N}{mm^2} \times 330mm \times 508mm = 131\,000\text{ N} = 131\text{ kN}$
  $\emptyset V_c = 0.75 \times 131\text{ kN} = 98.3\text{ kN}$
- Based on value of $\emptyset V_c$ divide the shear force diagram into three regions indicated below. As all limitations of article (**9.4.3.2**) are satisfied, then sections located less than a distance "d" from face of support shall be permitted to be designed for V_u computed at a distance "d".



To compute shear force at distance "d" from face of supported any one of the following two approaches can be adopted:

- Based on differential equations of equilibrium:

From mechanics of materials, to satisfy equilibrium of an infinitesimal element, following differential equations should be satisfied:

$$w = \frac{dV}{dx}$$

$$V = \frac{dM}{dx}$$

The first equation indicates that the load value, w, represents the slope for shear diagram while the second equation indicates that the value of shear force represents the slope of the bending moment diagram. It is useful to note that both equations are consistent in units.

From the first equation:

$$dV = wdx$$

Integrate to obtain

$$V_2 - V_1 = \int_1^2 w\, dx$$

Or

$$V_{u\,@\,distance\,d} = \int_{From\ face\ of\ support}^{To\ a\ distance\ d\ from\ face\ of\ support} w\, dx \; + V_{u\,@\,face\ of\ support}$$

It is worthwhile to note that the above ***finite integral is equal to area under load diagram from face of support to a distance "d" from face of support***.

$$V_{u\,@\,distance\,d} = (-65.5 \times 0.508 \; + 300) \approx 266\, kN$$

- Based on Symmetry

From problems that have symmetry, shear force at distance "d" can be determined based on following relation:

$$V_{u\ at\ distance\ d\ from\ face\ of\ support} = \frac{1}{2} \times W_u(l_n - 2d)$$

where l_n is the clear span measured from face to face of supports.

$$V_{u\ at\ distance\ d\ from\ face\ of\ support} = \frac{1}{2} \times 65.5 \times (9.15 - 2 \times 0.508) = 266\ kN$$

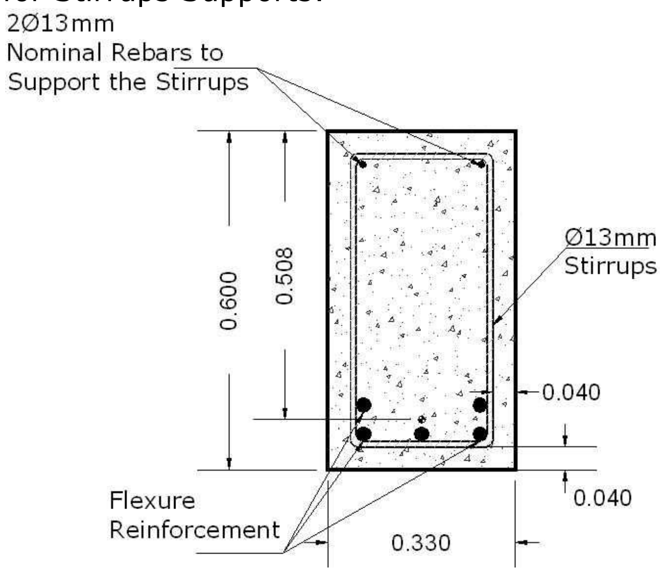
- Compute stirrups spacing for each region based on the table presented below: Try U Shape stirrups of 13mm diameter, then A_v will be:

$$A_v = \frac{\pi \times 13^2}{4} \times 2 = 265\ mm^2$$

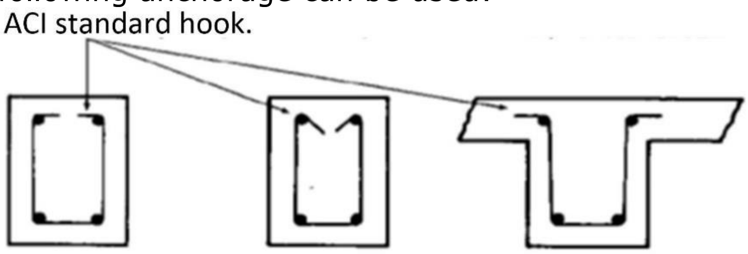
Stirrups Spacing for Example 5.6-1

Region	$V_u \le \emptyset \frac{V_c}{2}$	$\emptyset \frac{V_c}{2} < V_u \le \emptyset V_c$	$\emptyset V_c \le V_u$
V_s	None	None	$= \frac{V_u - \emptyset V_c}{\emptyset} = 0.66\sqrt{f_c'} b_w d$ $\frac{266 - 98.3}{0.75}$? $0.66 \times \sqrt{21} \times 330 \times 508$ $224kN < 507\ kN\ Ok$ Beam dimensions are adequate.
$S_{Theoretical}$	None	None	$= \frac{A_v f_{yt} d}{V_s} = \frac{265 \times 275 \times 508}{224\ 000} = 165\ mm$
$S_{for\ Av\ minimum}$	None	$minimum\ (\frac{A_v f_{yt}}{0.062\sqrt{f_c'} b_w}\ or\ \frac{A_v f_{yt}}{0.35 b_w})$ $= 630\ mm$	$minimum\ (\frac{A_v f_{yt}}{0.062\sqrt{f_c'} b_w}\ or\ \frac{A_v f_{yt}}{0.35 b_w})$ $minimum\ (\frac{265 \times 275}{0.062\sqrt{21} \times 330}\ or\ \frac{265 \times 275}{0.35 \times 330})$ $minimum\ (\ 777\ or\ 630)$ $= 630\ mm$
$S_{maximum}$	None	$Minimum\ \left[\frac{d}{2}\ or\ 600mm\right]$ $= 254\ mm$	$V_s \le 0.33\sqrt{f_c'} b_w d$ $224kN \le 0.33\sqrt{21} \times 330 \times 508$ $224\ kN \le 254\ kN$ $Minimum\ \left[\frac{d}{2}\ or\ 600mm\right]$ $Minimum\ \left[\frac{508}{2}\ or\ 600mm\right] = 254\ mm$ --- $V_s > 0.33\sqrt{f_c'} b_w d$ $Minimum\ \left[\frac{d}{4}\ or\ 300mm\right]$
$S_{Required}$	None	$Minimum\ [S_{for\ Av\ minimum}, S_{maximum}]$ $Minimum\ [630\ \ mm, 254\ mm]$ = 254 mm Use $\emptyset 13mm$ @ $250mm$	$Minimum\ \left[\begin{matrix} S_{Theoretical}, \\ S_{for\ Av\ minimum}, S_{maximum} \end{matrix}\right]$ $Minimum\ \left[\begin{matrix} 165\ mm, \\ 630\ \ mm, 254\ mm \end{matrix}\right]$ $= 165\ mm$ Use $\emptyset 13mm$ @ $150mm$

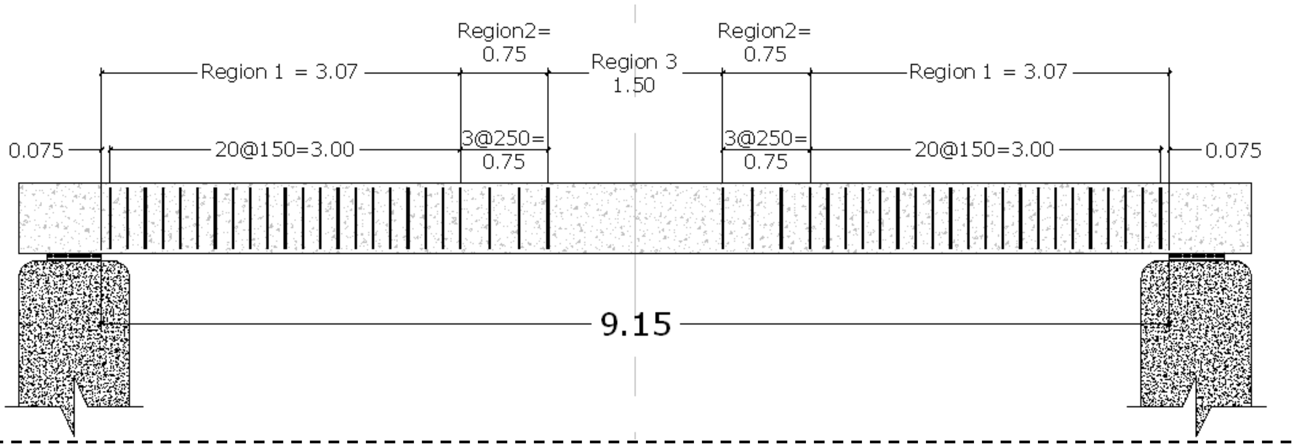
- Selecting of Nominal Reinforcement for Stirrups Supports: As no top bars are required for flexure, stirrups support bars must be used. These are usually about the same diameter as the stirrups themselves (Nilson, Design of Concrete Structures, 3th Edition , 2003) (Page 180).



- Anchorage Requirement for Shear Reinforcements: If one assumes that no compression reinforcement is required for this beam, any one of following anchorage can be used:



- Final stirrup spacing would be as indicated in below:



Example 5.6-2

Re-design ***Example 5.6-1*** but with using same spacing along beam span. Then compare the two designs.

Solution

It practices, structural designers may use the same spacing along beam span. This spacing should be computed based on maximum shear force and can be used in other regions where shear forces are less than the force that used in design.

- Compute V_u:

  As all limitations of article (**9.4.3.2**) are satisfied, then sections located less than a distance "d" from face of support shall be designed for V_u computed at a distance "d".

  $$V_u = \frac{\left[65.5\frac{kN}{m} \times (9.15 - 2 \times 0.508)m\right]}{2} = 266\ kN$$

- Compute Concrete Shear Strength V_c:

  $V_c = 0.17\lambda\sqrt{f_c'}\, b_w d$

  With $\lambda = 1.0$ for normal weight concrete:

  $$V_c = 0.17\sqrt{f_c'}\, b_w d = 0.17 \times \sqrt{21}\frac{N}{mm^2} \times 330mm \times 508mm = 131\,000\ N = 131\ kN$$

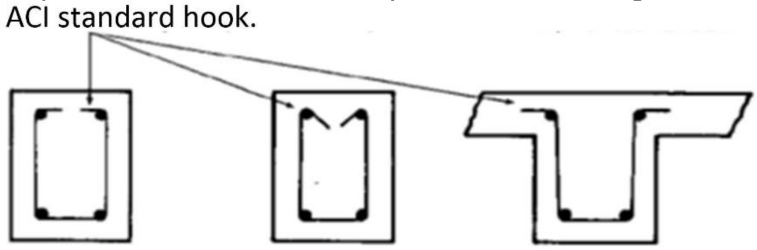
  $\emptyset V_c = 0.75 \times 131\ kN = 98.3\ kN$

  $\because V_u = 266\ kN > \emptyset V_c = 98.3\ kN$

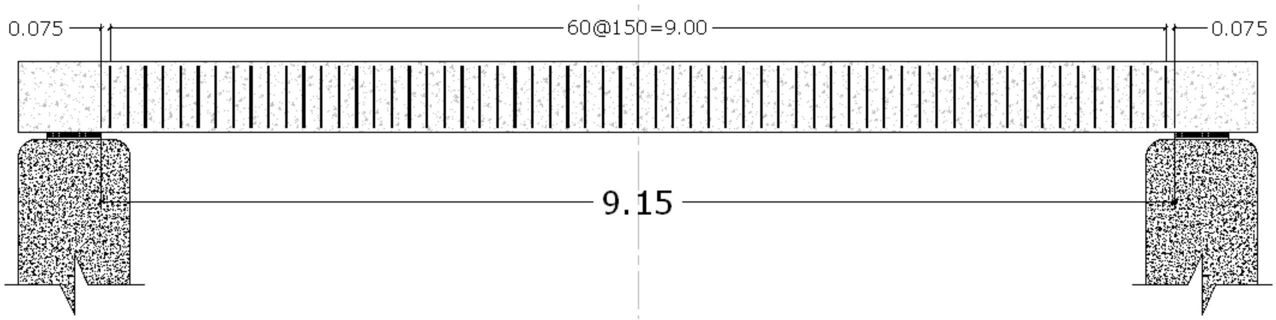
  Then the beam will be designed based of provisions of $V_u > \phi V_c$.

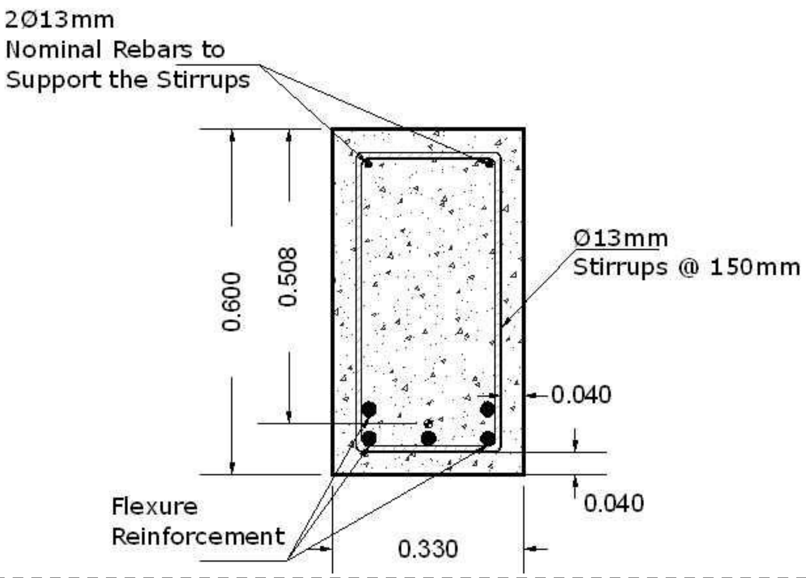
Shear Spacing Design of Example 5.6-2	
Region	$\emptyset V_c \leq V_u$
V_s	$= \frac{V_u - \emptyset V_c}{\emptyset} = 0.66\sqrt{f_c'}b_w d$ $\frac{266 - 98.3}{0.75}\ ?\ 0.66 \times \sqrt{21} \times 330 \times 508 \Longrightarrow 224kN < 507\ kN\ Ok$ Beam dimensions are adequate.
$S_{Theoretical}$	$= \frac{A_v f_{yt} d}{V_s} = \frac{265 \times 275 \times 508}{224\,000} = 165\ mm$
S for Av minimum	$minimum\ (\frac{A_v f_{yt}}{0.062\sqrt{f_c'}b_w}\ or\ \frac{A_v f_{yt}}{0.35 b_w})$ $minimum\ \left(\frac{265 \times 275}{0.062\sqrt{21} \times 330}\ or\ \frac{265 \times 275}{0.35 \times 330}\right) = minimum\ (777\ or\ 630)$ $= 630\ mm$
$S_{maximum}$	$V_s \leq 0.33\sqrt{f_c'}b_w d$ $224kN \leq 0.33\sqrt{21} \times 330 \times 508 \Longrightarrow 224\ kN \leq 254\ kN$ $Minimum\ \left[\frac{d}{2}\ or\ 600mm\right] \Longrightarrow Minimum\ \left[\frac{508}{2}\ or\ 600mm\right] = 254\ mm$
	$V_s > 0.33\sqrt{f_c'}b_w d \Longrightarrow Minimum\ \left[\frac{d}{4}\ or\ 300mm\right]$
$S_{Required}$	$Minimum\ [S_{Theoretical}, S_{for\ Av\ minimum}, S_{maximum}]$ $Minimum\ [165\ mm, 630\ mm, 254\ mm] = 165\ mm$ **Use $\emptyset 13mm\ @\ 150mm$**

- Anchorage Requirement for Shear Reinforcements:
  As for previous example, if one assumes that no compression reinforcement is required for this beam, any one of following anchorage can be used:



- Comparison between two designs:
  Required Number of Stirrups for the more accurate design of Example 5.6-1 is:
  $$\text{No. of Stirrups} = \left[\left(\frac{3.0}{0.150} + 1\right) + \frac{0.75}{0.250}\right] \times 2 = 48 \text{ U Stirrups}$$
  Required Number of Stirrups for the simplified design of Example 5.6-2 is:
  $$\text{No. of Stirrups} = \left(\frac{9.0}{0.150} + 1\right) = 61 \text{ U Stirrups}$$
  Then dividing the beam into three regions and design of each region for its shear force can save 13 stirrups.





Example 5.6-3

Design Region 1 and Region 2 of floor beam indicated in ***Figure 5.6-2*** for shear. The beam has a width of 375mm and an effective depth of 775mm. Assume that the designer intends to use:

- $f_c' = 27.5\ \text{MPa}$.
- $f_{yt} = 414\ \text{MPa}$.
- Stirrups of 10mm diameter ($A_{Bar} = 71\text{mm}^2$).

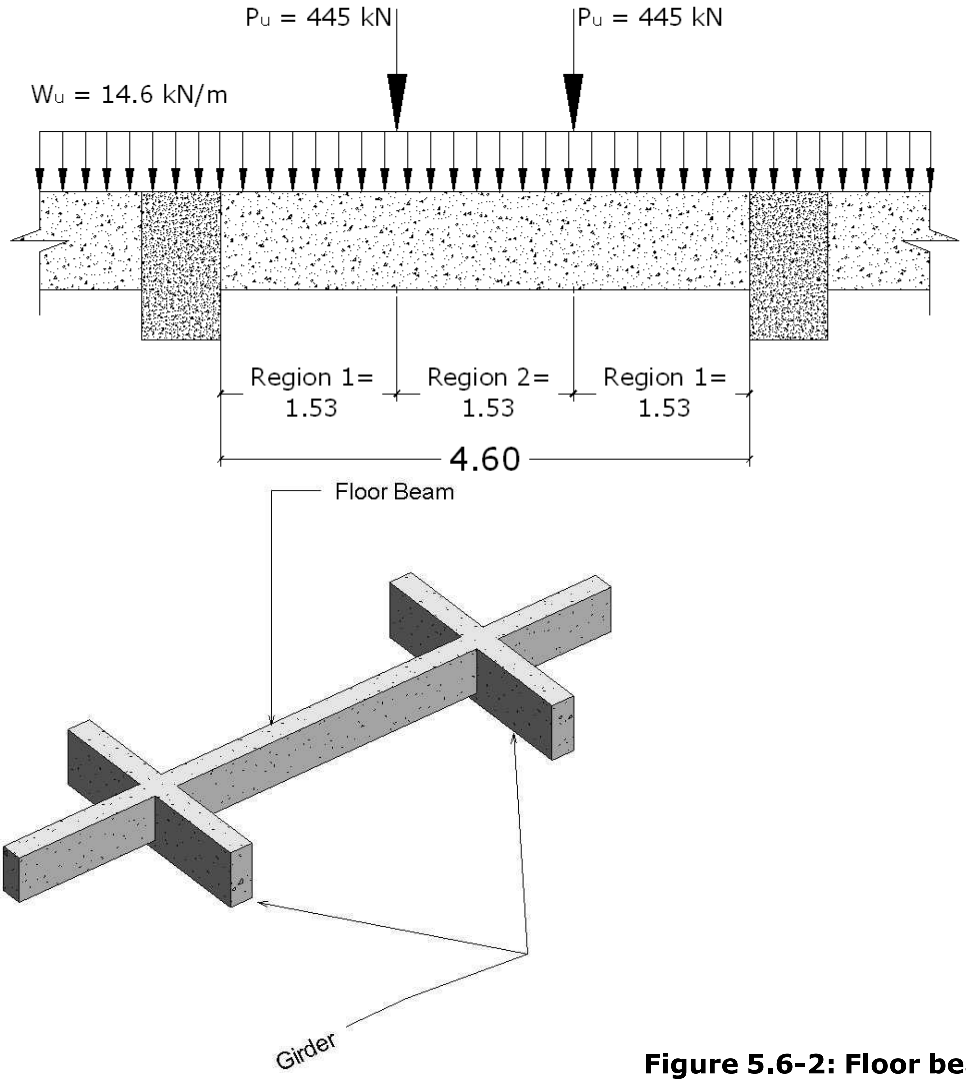


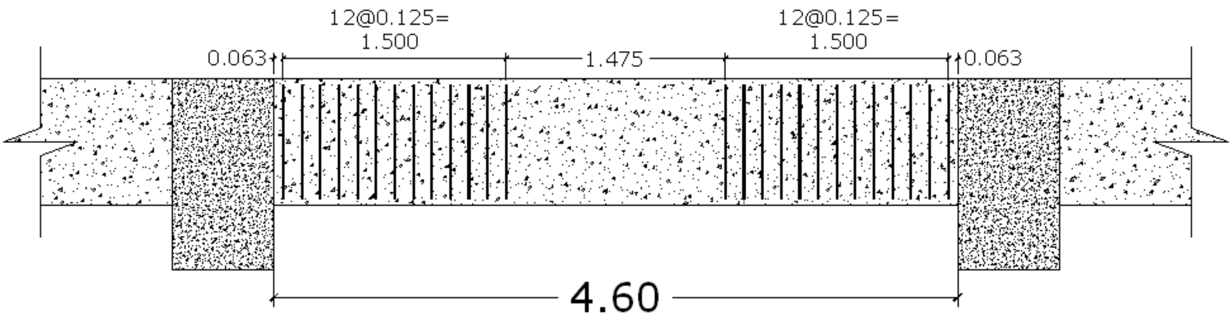
Figure 5.6-2: Floor beam for Example 5.6-3.

Solution

- Shear Reinforcement for Region 1:
  - Compute factored shear force V_u:
    As girder is deeper than floor beam, then all ACI limitations are satisfied and the shear force for Region 1 can be determined at distance "d" from face of support.
    $$V_u = 14.6\frac{\text{kN}}{\text{m}} \times (4.6\text{m} - 2 \times 0.775\text{m}) \times \frac{1}{2} + 445\ \text{kN} = 467\ \text{kN}$$
  - Shear strength of concrete V_c:
    $$V_c = 0.17\lambda\sqrt{f_c'}\,b_w d$$
    with $\lambda = 1.0$ for normal weight concrete:
    $$V_c = 0.17\sqrt{f_c'}\,b_w d = 0.17 \times \sqrt{27.5}\frac{\text{N}}{\text{mm}^2} \times 375\text{mm} \times 775\text{mm} = 259\ \text{kN}$$
  - Stirrups spacing:
    $\emptyset V_c = 0.75 \times 259\ \text{kN} = 194\ \text{kN}$
    $\because V_u = 467\ \text{kN} > \emptyset V_c = 194\ \text{kN}$
    Then, shear reinforcement must be used and its spacing can be computed from Table below:
    $A_v = 71 \times 2 = 142\ \text{mm}^2$

Shear Spacing Design of Example 5.6-3 for Region 1	
Region	$\emptyset V_c \leq V_u$
V_s	$= \frac{V_u - \emptyset V_c}{\emptyset} = 0.66\sqrt{f_c'}b_w d$ $\frac{467-194}{0.75} \ ?\ 0.66 \times \sqrt{27.5} \times 375 \times 775$ $364 < 1\ 006\ kN\ Ok$ Beam dimensions are adequate.
$S_{Theoretical}$	$= \frac{A_v f_{yt} d}{V_s} = \frac{142 \times 414 \times 775}{364\ 000} = 125\ mm$
$S_{for\ Av\ minimum}$	$minimum\ (\frac{A_v f_{yt}}{0.062\sqrt{f_c'}b_w}\ or\ \frac{A_v f_{yt}}{0.35 b_w})$ $minimum\ (\frac{142 \times 414}{0.062\sqrt{27.5} \times 375}\ or\ \frac{142 \times 414}{0.35 \times 375})$ $minimum\ (482\ or\ 448)$ $= 448\ mm$
$S_{maximum}$	$V_s \leq 0.33\sqrt{f_c'}b_w d$ $364kN \leq 0.33\sqrt{27.5} \times 375 \times 775$ $364kN\ kN \leq 503\ kN$ $Minimum\ \left[\frac{d}{2}\ or\ 600mm\right]$ $Minimum\ \left[\frac{775}{2}\ or\ 600mm\right] = 387\ mm$
	$V_s > 0.33\sqrt{f_c'}b_w d$ $Minimum\ \left[\frac{d}{4}\ or\ 300mm\right]$
$S_{Required}$	$Minimum\ [S_{Theoretical}, S_{for\ Av\ minimum}, S_{maximum}]$ $Minimum\ [125\ mm, 448\ mm, 387\ mm]$ $= 125\ mm$ ***Use*** $\boldsymbol{\emptyset 10mm\ @\ 125mm}$

- Shear Reinforcement for Region 2:
  - Factored shear force V_u:
    Due to symmetry
    $V_u = \left(14.6\frac{kN}{m} \times 1.53m\right) \times \frac{1}{2} = 11.1\ kN$
  - Shear strength of concrete V_c:
    According to simplified equation of the code, concrete shear force is constant along span of prismatic beam. Therefore, concrete shear strength of Region 2 would be equal to that of Region 1.
    $V_c = 259\ kN$
    $\emptyset V_c = 0.75 \times 259\ kN = 194\ kN$
    $\because \frac{\emptyset V_c}{2} = \frac{194\ kN}{2} = 97\ kN > V_u$
    Then, no shear reinforcement is required for Region 2.
- Anchorage
  As nothing is mentioned about longitudinal reinforcement, then one cannot select between closed or open stirrups.

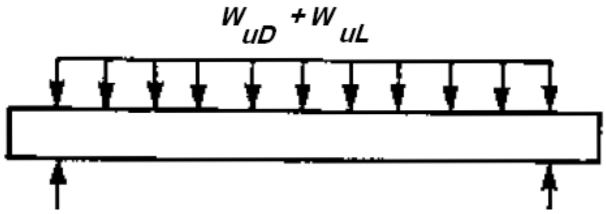


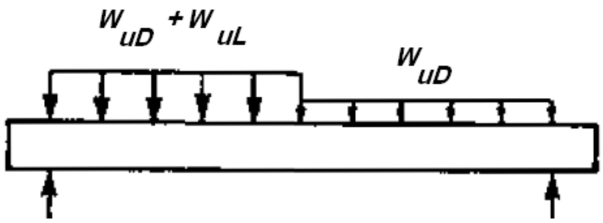
Example 5.6-4

For a simply supported beam, that has a clear span of 6m, design 10mm U stirrup at a mid-span section. In your design, assume that load pattern must be included and assume:

- $f_c' = 21\ MPa, f_{yt} = 420\ MPa$
- $h = 500, d = 450\ mm, b_w = 300mm$
- $W_{ud} = 60\frac{kN}{m}$ (Including Beam Selfweight) and $W_{ul} = 200\frac{kN}{m}$

Solution





- Compute V_u

  Although the dead load is always present over the full span, the live load may act over the full span as shown or over a part of span as shown in below.

  Based on influence line for shear at mid-span of simply supported beam, the maximum effect of live load occurs when this load acting on one half of beam span as indicated in above. Therefore, for design case when load pattern is important, shear force must be computed based on partial loading of one half of beam span:

$$V_{u\ @\ mid\ span} = 0.0_{\text{Shear due to } W_D} + \frac{W_{uL}L}{8}_{\text{Shear Due to LL on half of Beam Span}} = \frac{W_{uL}L}{8}$$

$$= \frac{200\frac{kN}{m} \times 6m}{8} = 150\ kN$$

- Compute V_c

$$V_c = 0.17\sqrt{f_c'}\,b_w d = 0.17 \times \sqrt{21}\frac{N}{mm^2} \times 300mm \times 450mm = 105\ kN \Rightarrow \emptyset V_c = 0.75 \times 105\ kN = 78.8\ kN$$

- Stirrups Design

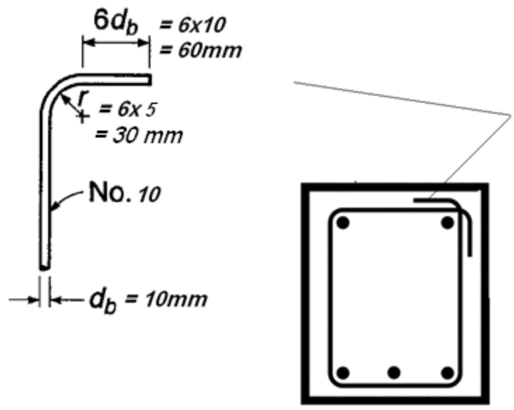
  As

  $V_u > \emptyset V_c$

  then shear stirrups is designed as presented in Table below.

Shear Spacing Design of Example 5.6-4	
Region	$\emptyset V_c \leq V_u$
V_s	$= \frac{V_u - \emptyset V_c}{\emptyset} = 0.66\sqrt{f_c'}b_w d$ $\frac{150 - 78.8}{0.75}$? $0.66 \times \sqrt{21} \times 300 \times 450 \Rightarrow 94.9\ kN < 408\ kN\ Ok$ Beam dimensions are adequate.
$S_{Theoretical}$	$= \frac{A_v f_{yt} d}{V_s} = \frac{157 \times 420 \times 450}{94.9 \times 10^3} = 313\ mm$
$S_{for\ Av\ minimum}$	$minimum\ (\frac{A_v f_{yt}}{0.062\sqrt{f_c'}b_w}\ or\ \frac{A_v f_{yt}}{0.35 b_w})$ $minimum\ (\frac{157 \times 420}{0.062\sqrt{21} \times 300}\ or\ \frac{157 \times 420}{0.35 \times 300}) \Rightarrow minimum\ (774\ or\ 628) = 628\ mm$
$S_{maximum}$	$V_s \leq 0.33\sqrt{f_c'}b_w d$ $94.9\ kN \leq 0.33\sqrt{21} \times 300 \times 450$ $94.9\ kN < 204\ kN$ $Minimum\ \left[\frac{d}{2}\ or\ 600mm\right] \Rightarrow Minimum\ \left[\frac{450}{2}\ or\ 600mm\right] = 225\ mm$
	$V_s > 0.33\sqrt{f_c'}b_w d \Rightarrow Minimum\ \left[\frac{d}{4}\ or\ 300mm\right]$
$S_{Required}$	$Minimum\ [S_{Theoretical}, S_{for\ Av\ minimum}, S_{maximum}]$ $Minimum\ [313\ mm, 628\ mm, 225\ mm] = 225\ mm$ **Use** $\emptyset 10mm\ @\ 225mm$

- Stirrups Details
  As movable live load is a reversal load, then closed stirrup must be used here as shown in the figure below.



Example 5.6-5

For the roof system shown in ***Figure 5.6-3*** below, design shear reinforcement for a typical floor beam and a typical girder.

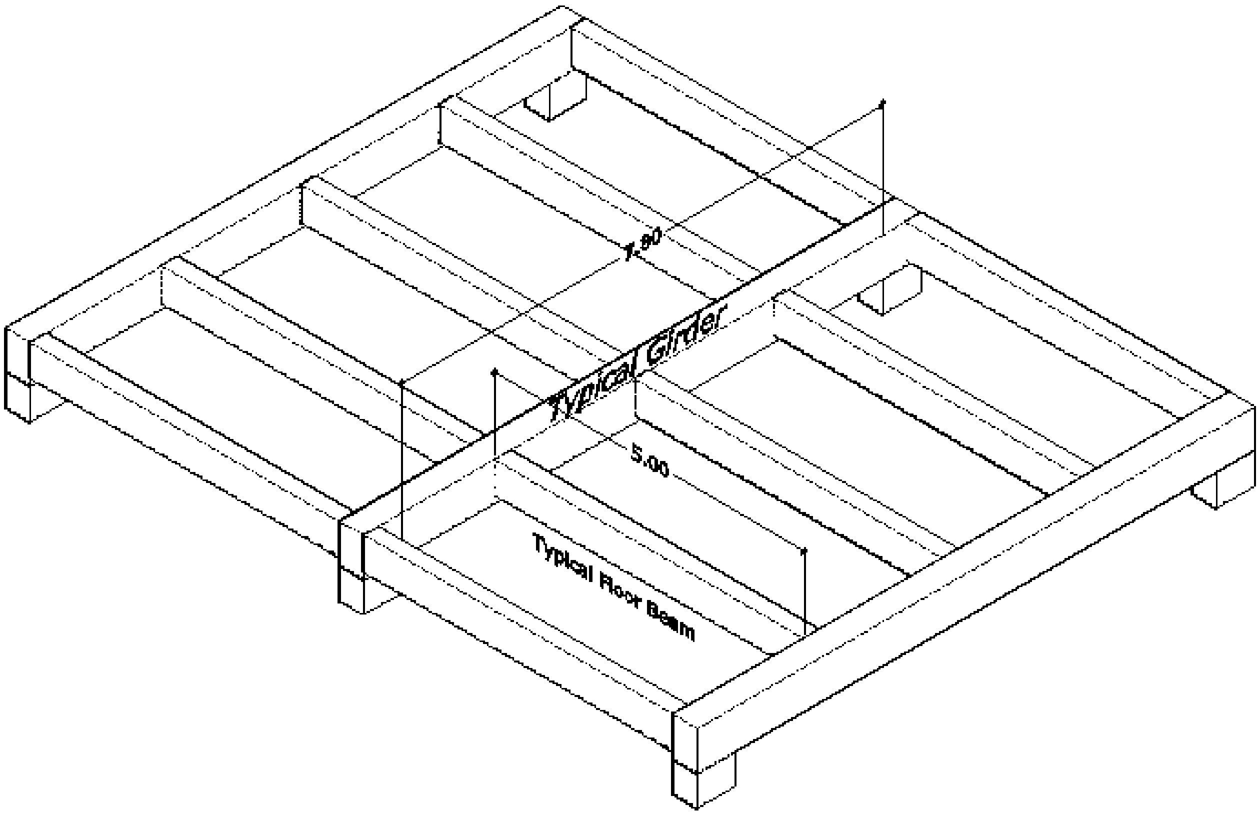
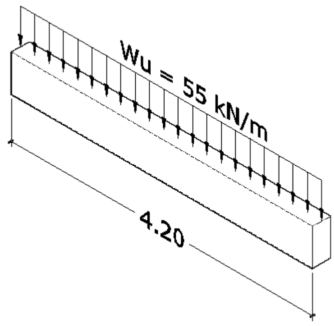


Figure 5.6-3: Roof system for Example 5.6-5.

In your design, assume that:

- $f_c' = 21\ MPa$ and $f_{yt} = 420\ MPa$.
- Floor beams have b = 250mm, h = 450mm, and d = 400mm and subjected to a uniformly distributed factored load of W_u = 55 kN/m transferred from the supported slab.
- Girders have b = 400mm, h = 600mm, and d = 520mm.
- Selfweight of floor beams and girders should be included in your design.
- Try 10mm U stirrups for the floor beam and 12mm U stirrups for the girder.

Solution



- Design Shear Reinforcement for Floor Beam:
  - Computing of Vu:
    As the girder is deeper than the floor beam, then critical section for the floor beam can be taken at distance "d" from face of support (girder in this case).

$$W_u = 55\frac{kN}{m} + \left((0.45 \times 0.25\ m^2) \times 24\frac{kN}{m^3}\right) \times 1.2 = 58\frac{kN}{m}$$

$$V_{u\ @\ d\ from\ face\ of\ support} = \left(58\frac{kN}{m} \times (5.0 - 0.4 \times 2)m\right) \times \frac{1}{2} = 122\ kN$$

- Compute V_c:

$$V_c = 0.17\sqrt{f_c'}\, b_w d = 0.17 \times \sqrt{21}\frac{N}{mm^2} \times 250mm \times 400mm = 77.9\ kN$$

$$\emptyset V_c = 0.75 \times 77.9\ kN = 58.4\ kN$$

- Design of Shear Reinforcement:

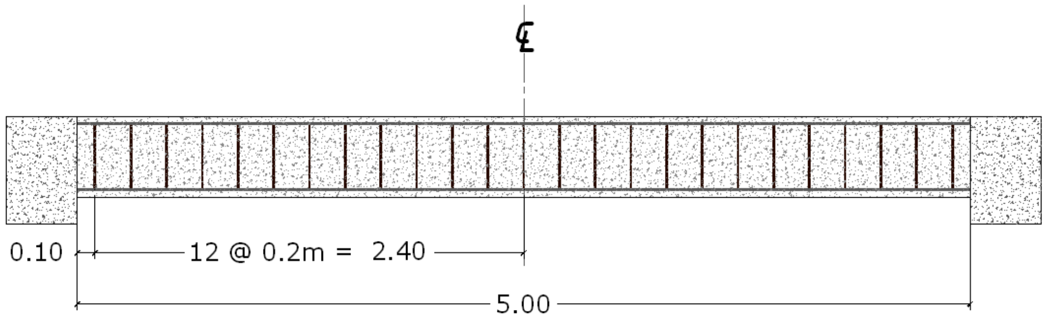
As

$V_u > \emptyset V_c$

Then shear reinforcement must be designed based on zone 1 (see the table below).

Stirrups Design of Example 5.6-5 (Floor Beam)	
Region	$\emptyset V_c \le V_u$
V_s	$= \frac{V_u - \emptyset V_c}{\emptyset} = 0.66\sqrt{f_c'}b_w d$ $\frac{122 - 58.4}{0.75}$? $0.66 \times \sqrt{21} \times 250 \times 400 \Rightarrow 84.8\ kN < 302\ kN\ Ok$ Beam dimensions are adequate.
$S_{Theoretical}$	$= \frac{A_v f_{yt} d}{V_s} = \frac{157 \times 420 \times 400}{84.8 \times 10^3} = 311\ mm$
$S_{for\ Av\ minimum}$	$minimum\ (\frac{A_v f_{yt}}{0.062\sqrt{f_c'}b_w}\ or\ \frac{A_v f_{yt}}{0.35 b_w})$ $minimum\ (\frac{157 \times 420}{0.062\sqrt{21} \times 250}\ or\ \frac{157 \times 420}{0.35 \times 250}) \Rightarrow minimum\ (928\ or\ 754)$ $= 754\ mm$
$S_{maximum}$	$V_s \le 0.33\sqrt{f_c'}b_w d$ $84.8\ kN \le 0.33\sqrt{21} \times 250 \times 400 \Rightarrow 84.8\ kN \le 151\ kN$ $Minimum\ \left[\frac{d}{2}\ or\ 600mm\right] \Rightarrow Minimum\ \left[\frac{400}{2}\ or\ 600mm\right] = 200\ mm$
	$V_s > 0.33\sqrt{f_c'}b_w d$ $Minimum\ \left[\frac{d}{4}\ or\ 300mm\right]$
$S_{Required}$	$Minimum\ [S_{Theoretical}, S_{for\ Av\ minimum}, S_{maximum}]$ $Minimum\ [311\ mm,\ 754\ mm,\ 200\ mm] = 200\ mm$ **Use $\emptyset 10mm\ @\ 200mm$**

- Draw of Stirrups:



- Design of Shear Reinforcement for Girder:
  - Compute of Vu:

Forces acting on the girder are summarized in the figure below. Shear force, R_u, transfers from floor beams to the supporting girder can be computed as follows:

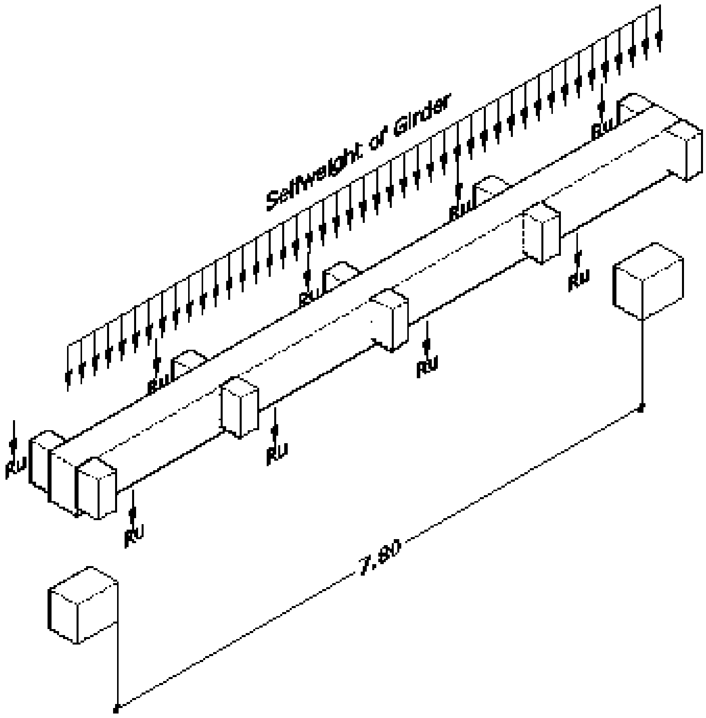
$$R_u = 58\frac{kN}{m} \times \frac{5m}{2} = 145\ kN$$

Shear force due to girder selfweight is

$$V_{u\ Due\ to\ Girder\ Selfweight} = \left((0.6 \times 0.4)m^2 \times 24\frac{kN}{m^3} \times (7.8 - 0.52 \times 2)m \times \frac{1}{2}\right)1.2 = 23.4\ kN$$

Therefore, the total factored shear force would be:

$$V_u = \left((3 \times 145\ kN_{Reactions\ from\ 3\ floor\ beam}) \times 2_{Two\ faces}\right)\frac{1}{2} + 23.4\ kN = 458\ kN$$



- Compute V_c:

  $V_c = 0.17\sqrt{f'_c}\, b_w d = 0.17 \times \sqrt{21}\,\frac{N}{mm^2} \times 400mm \times 520mm = 162\ kN$

  $\varnothing V_c = 0.75 \times 162\ kN = 121\ kN$
- Design of Shear Reinforcement:

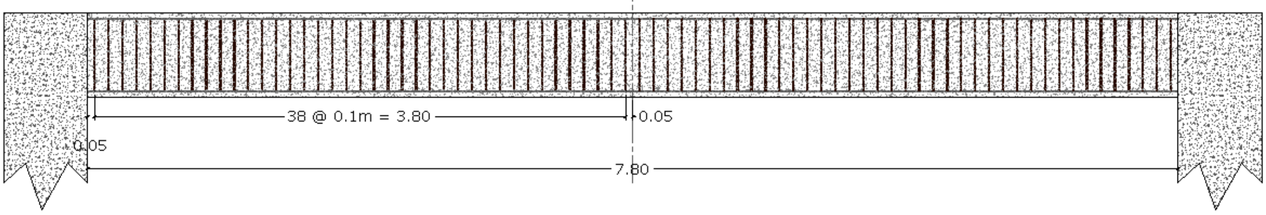
  As

  $V_u > \varnothing V_c$

  then, shear reinforcement is designed as indicated in the table below.

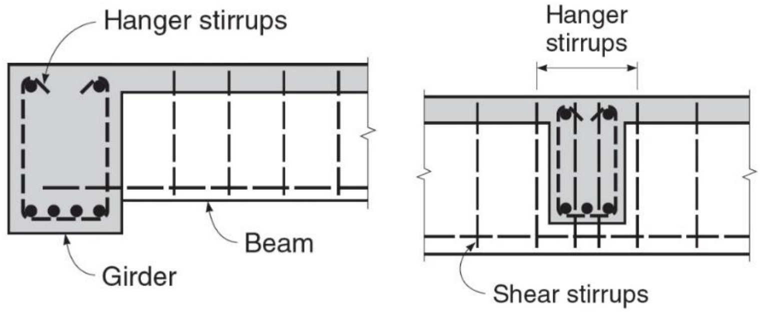
Stirrups Design of Example 5.6-5 (Girder Design)	
Region	$\varnothing V_c \le V_u$
V_s	$= \frac{V_u - \varnothing V_c}{\varnothing} = 0.66\sqrt{f_c'}b_w d \Rightarrow \frac{458-121}{0.75}\ ?\ 0.66 \times \sqrt{21} \times 400 \times 520$ $\Rightarrow 449\ kN < 629\ kN\ Ok$ Beam dimensions are adequate.
$S_{Theoretical}$	$= \frac{A_v f_{yt} d}{V_s} = \frac{226 \times 420 \times 520}{449 \times 10^3} = 110\ mm$
$S_{for\ Av\ minimum}$	$minimum\ (\frac{A_v f_{yt}}{0.062\sqrt{f_c'}b_w}\ or\ \frac{A_v f_{yt}}{0.35 b_w})$ $minimum\ (\frac{226 \times 420}{0.062\sqrt{21} \times 400}\ or\ \frac{226 \times 420}{0.35 \times 400}) \Rightarrow minimum\ (835\ or\ 678)$ $= 678\ mm$
$S_{maximum}$	$V_s \le 0.33\sqrt{f_c'}b_w d$
	$V_s > 0.33\sqrt{f_c'}b_w d$ $449\ kN > 0.33\sqrt{21} \times 400 \times 520 \Rightarrow 449\ kN > 314\ kN$ $Minimum\ \left[\frac{d}{4}\ or\ 300mm\right] \Rightarrow Minimum\ \left[\frac{520}{4}\ or\ 300mm\right] = 130\ mm$
$S_{Required}$	$Minimum\ [S_{Theoretical}, S_{for\ Av\ minimum}, S_{maximum}]$ $\Rightarrow Minimum\ [110\ mm,\ 678mm,\ 130\ mm]$ $= 110\ mm$ ***Use*** $\boldsymbol{\varnothing 12mm\ @\ 100mm}$

- Draw stirrups for the girder.

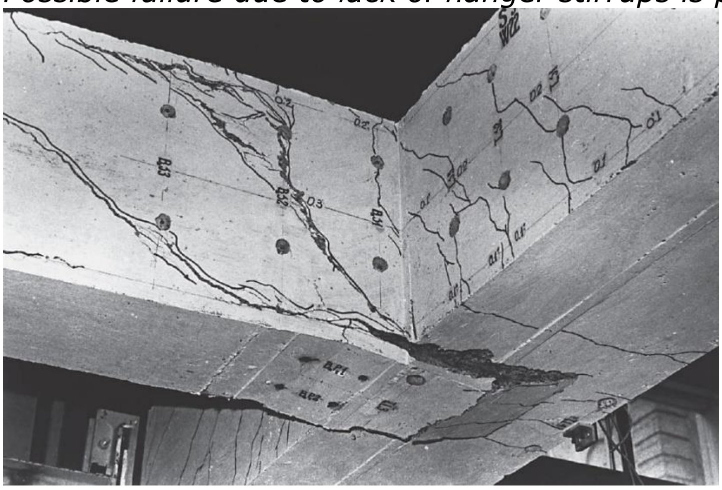


Important Notes

- *It is useful to note that shear forces in this example have been determined based on assumption of equal shear at beam-ends. More accurate assumption will be discussed later when we study the analysis and design of slabs and continuous beams.*
- *Hanger Stirrups:*
  - *Proper detailing of steel in the region of beam-to-girder connection such a joint requires the use of well-anchored "hanger" stirrups in the girder, as shown in below:*



  - *The hanger stirrups are required in addition to the normal girder stirrups.*
  - *Possible failure due to lack of hanger stirrups is presented in below:*

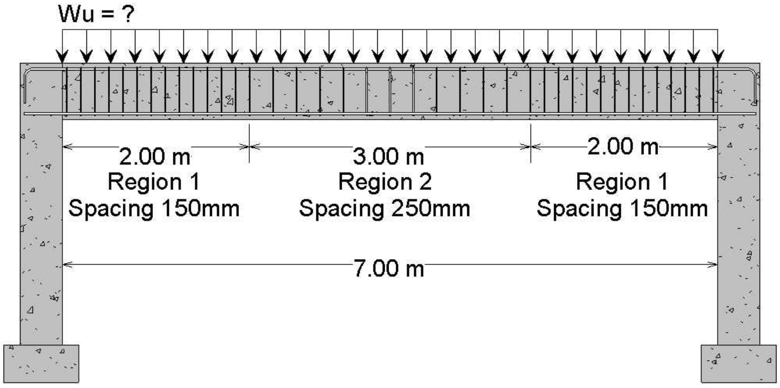


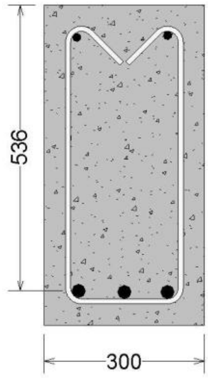
  - *Design of hanger stirrups is out of our scope, for more information about their design see* (Darwin, Dolan, & Nilson, 2016)*, page 557.*

Example 5.6-6

For the singly reinforced beam of the portal frame shown in ***Figure 5.6-4*** below, a designer has proposed to use open U stirrups with diameter of 10mm and with indicated spacing for shear reinforcement of the beam.

- Is using of open U stirrups justified according to ACI requirements? Explain your answer.
- Based on proposed spacing and beam shear strength, what is the maximum uniformly factored load W_u that could be applied? In your solution assume $f_c' = 28\ MPa$ and $f_y = 420\ MPa$.





Longitudinal Sectional View. **Beam Cross Section**

Figure 5.6-4: Frame for Example 5.6-6.

Solution

- Using of Open U Stirrups:
  As the beam is singly reinforced and with assuming that it is not subjected to torsion nor to reversal loads, then using of open U stirrups is justified according to ACI code.
- Maximum Uniformly Distributed Load $\boldsymbol{W_u}$:
  Based on Shear Strength of Region 1:

$$V_c = 0.17 \times 1.0 \times \sqrt{28} \times 300 \times 536 = 145\ kN$$

$$A_v = \frac{\pi \times 10^2}{4} \times 2 = 157\ mm^2$$

$$V_s = \frac{A_v f_y d}{s} = \frac{157 \times 420 \times 536}{150} = 236\ kN < 0.33 \times 1.0 \times \sqrt{28} \times 300 \times 536 = 281\ kN$$

$$S_{maximum} = minimum\left(\frac{536}{2}, 600\right) = 268\ mm > 150mm \therefore Ok.$$

$$\phi V_n = 0.75 \times (145 + 236) = 286\ kN$$

$$V_u = \frac{W_u \times (7.0 - 2 \times 0.536)}{2} = 286 \Rightarrow W_u = 96.5 \frac{kN}{m}$$

Based on Shear Strength of Region 2:

$$V_s = \frac{A_v f_y d}{s} = \frac{157 \times 420 \times 536}{250} = 141\ kN$$

$$S_{maximum} = 268\ mm > 250mm \therefore Ok.$$

$$\phi V_n = 0.75 \times (145 + 141) = 214\ kN$$

$$V_u = \frac{W_u \times 3}{2} = 214$$

$$W_u = 143 \frac{kN}{m}$$

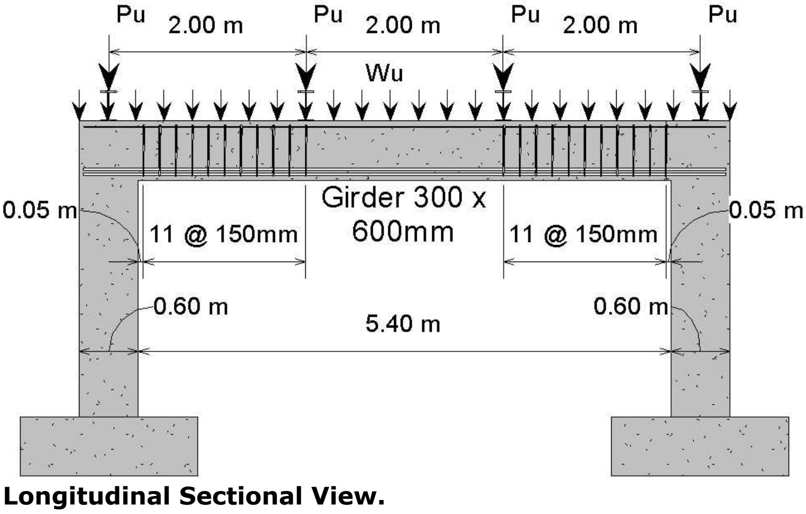
Finally,

$$W_u = minimum\ (96.5, 143) = 96.5 \frac{kN}{m}\ \blacksquare$$

Example 5.6-7

For a frame shown in ***Figure 5.6-5*** below, based on shear capacity of Girder 300x600, what are maximum values for point load "P_u", and distributed load "W_u" that can be supported by the beam?

In your solution, assume that selfweight could be neglected, $f_c' = 28\ MPa$ and $f_y = 420\ MPa$.



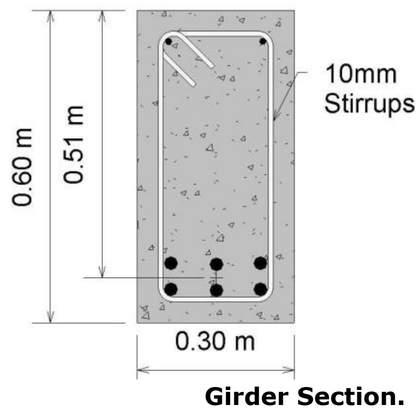


Figure 5.6-5: Frame for Example 5.6-7.

Solution

Distributed load "W_u" could be computed from middle region where no shear reinforcement are used:

$$V_u = \frac{\phi V_c}{2} = \frac{1}{2} \times \left(0.75 \times 0.17 \times \sqrt{28} \times 300 \times 510\right) \Rightarrow V_u = \frac{\phi V_c}{2} = 51.6\ kN$$

$$V_u = \frac{W_u \times 2.00}{2} = 51.6\ kN \Rightarrow W_u = 51.6\ kN\ \blacksquare$$

Point load "Pu" could be computed from support regions where stirrups of $\phi 10 @ 150mm$ are used.

$V_s = 2 \times \frac{\pi \times 10^2}{4} \times 420 \times \frac{510}{150} = 224\ kN \Rightarrow \mathrm{V_s} = 224 < 0.66\sqrt{28} \times 300 \times 510 = 534\ kN \therefore Ok.$

$\because \mathrm{V_s} = 224 < 0.33\sqrt{28} \times 300 \times 510 = 267\ kN \Rightarrow \mathrm{S} = 150\mathrm{mm} < \mathrm{Minimum}\ \left[\frac{510}{2} \mathrm{or}\ 600\right]$

$\mathrm{S} = 150\mathrm{mm} < 255\ \mathrm{mm} \therefore Ok.$

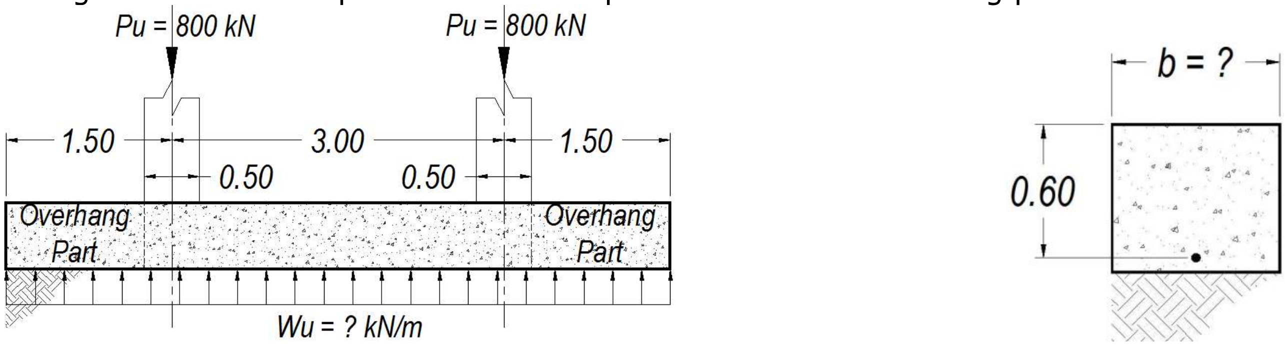
$V_c = 0.17 \times \sqrt{28} \times 300 \times 510 = 138\ kN \Rightarrow V_u = 0.75 \times (138 + 224) = 272\ kN$

$(W_u \times (5.4 - 0.51 \times 2) + 2P_u) = 2 \times 272 \Rightarrow (51.6 \times (5.4 - 0.51 \times 2) + 2P_u) = 2 \times 272$

$\Rightarrow P_u = 159\ kN$ ■

Example 5.6-8

For beam shown in ***Figure 5.6-6*** below, select beam width such that concrete shear strength would be adequate for shear requirements in the overhang parts.



Logitudinal veiw **A Section in Overhang Region**

Figure 5.6-6: Foundation for Example 5.6-8.

In your solution, assume that:

- Beam selfweight can be neglected.
- $f_c' = 21\ MPa$

Solution

$W_u = \frac{800 \times 2}{6} = 267 \frac{kN}{m} \Rightarrow V_{u\ @\ d\ fram\ face\ of\ support} = 267 \frac{kN}{m}(1.5 - 0.25 - 0.6)m = 174\ kN$

$V_u = \frac{\phi V_c}{2} \Rightarrow 174000\ N = \frac{1}{2}\left(0.75\left(0.17\sqrt{21} \times b \times 600\right)\right) \Rightarrow b = 993\ mm$

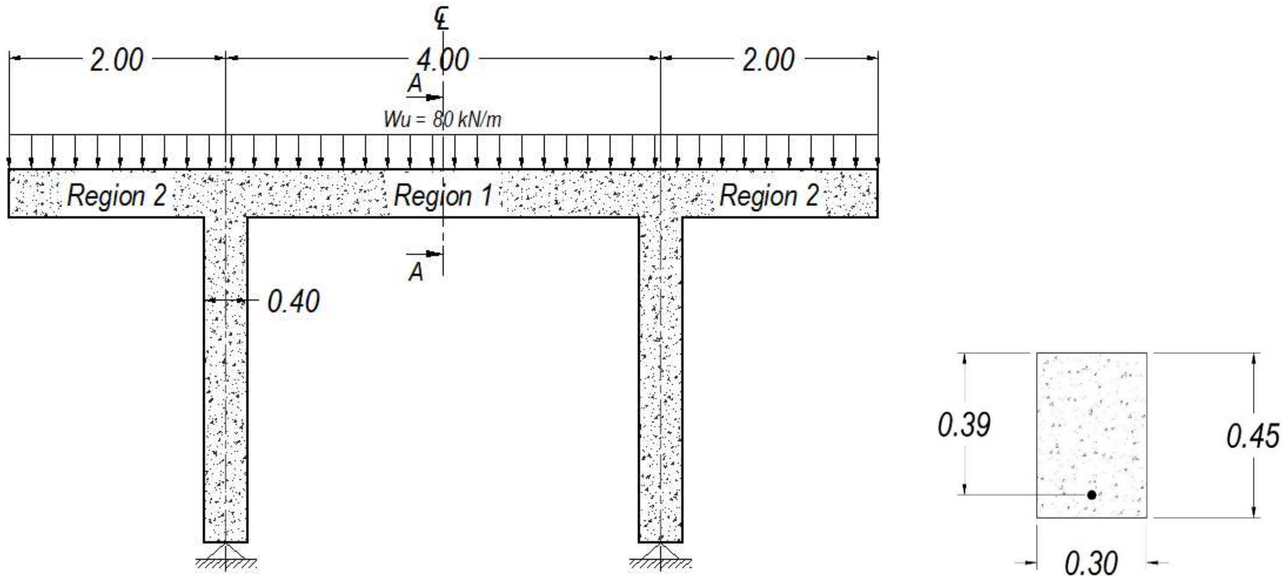
Say

$b = 1000\ mm$ ■

Example 5.6-9

For the frame shown in Figure 5.6-7 below,

- Design Region 1 for shear according ACI requirements.
- Is shear reinforcement for Region 1 adequate for Region 2?



Elevation view. **Section A-A**

Figure 5.6-7: Frame for Example 5.6-9.

In your solution, assume that:

- $f_c' = 21\ MPa$, $f_y = 420\ MPa$
- No.10 for stirrups.

Solution

Region 1:

$$V_{u\ for\ Region\ 1} = 80\frac{kN}{m}(4 - 0.4 - 2 \times 0.39)m \times \frac{1}{2} = 113\ kN$$

$$\phi V_c = 0.75 \times 0.17 \times \sqrt{21} \times 300 \times 390 = 68.4\ kN < V_u$$

Shear Spacing Design of Region 1	
Region	$\emptyset V_c \le V_u$
V_s	$= \frac{V_u - \emptyset V_c}{\emptyset} = 0.66\sqrt{f_c'}b_w d \Rightarrow \frac{113 - 68.4}{0.75}\ ?\ 0.66 \times \sqrt{21} \times 300 \times 390$ $59.5kN < 354\ kN\ Ok$ Beam dimensions are adequate.
$S_{Theoretical}$	$= \frac{A_v f_{yt} d}{V_s} = \frac{157 \times 420 \times 390}{59.5 \times 10^3} = 432\ mm$
S_{for Av minimum}	$minimum\ (\frac{A_v f_{yt}}{0.062\sqrt{f_c'}b_w}\ or\ \frac{A_v f_{yt}}{0.35 b_w}) \Rightarrow minimum\ (\frac{157 \times 420}{0.062\sqrt{21} \times 300}\ or\ \frac{157 \times 420}{0.35 \times 300})$ $minimum\ (773\ or\ 628) = 628\ mm$
$S_{maximum}$	$V_s \le 0.33\sqrt{f_c'}b_w d \Rightarrow 59.5kN \le 0.33\sqrt{21} \times 300 \times 390 \Rightarrow 59.5kN \le 177\ kN$ $Minimum\ \left[\frac{d}{2}\ or\ 600mm\right] \Rightarrow Minimum\ \left[\frac{390}{2}\ or\ 600mm\right] = 195\ mm$
	$V_s > 0.33\sqrt{f_c'}b_w d \Rightarrow Minimum\ \left[\frac{d}{4}\ or\ 300mm\right]$
$S_{Required}$	$Minimum\ [S_{Theoretical}, S_{for\ Av\ minimum}, S_{maximum}]$ $Minimum\ [432\ mm, 628\ mm, 195\ mm] = 195\ mm$ **Use $\emptyset 10mm\ @\ 175mm$**

Region 2:

$$V_{u\ for\ Region\ 2} = 80\frac{kN}{m}\left(2.0 - \frac{0.4}{2} - 0.39\right)m = 113\ kN$$

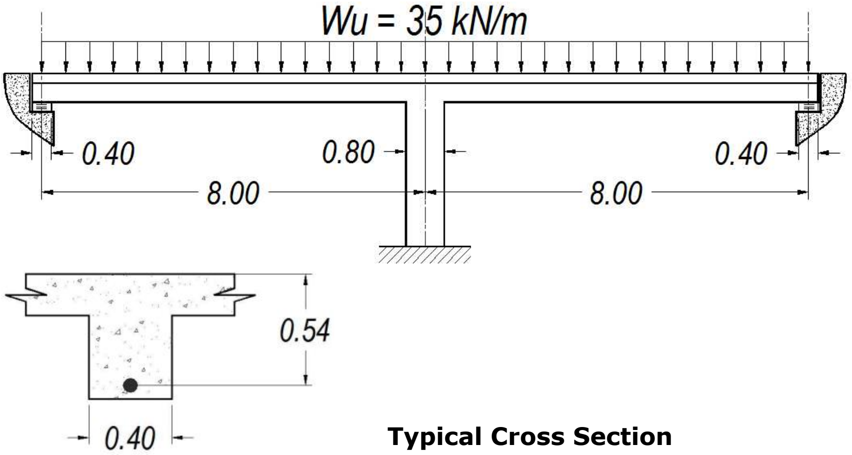
$$\because\ V_{u\ for\ Region\ 2} = V_{u\ for\ Region\ 1}$$

Therefore, the shear reinforcement for Region 1 is adequate for Region 2.

Example 5.6-10

Design for shear the most critical region of pedestrian bridge shown in Figure 5.6-8 below. In your solution, assume that:

- Shear force at interior support to be increased by 15%.
- Beam selfweight could be neglected.
- U stirrups with 10mm diameter.
- $f_c' = 28\ MPa\ f_{yt} = 420\ MPa$



Longitudinal Section

Typical Cross Section

Figure 5.6-8: Pedestrian bridge for Example 5.6-10.

Solution

- Design Shear Force:

  As will be discussed in design of ***one-way slabs*** and ***continuous beams***, according to ACI code, the most critical shear for continuous beams occurs at the exterior face of first interior support with a shear force of 15% greater than average shear force for simple beams.

$$V_{u\,@\,face\,of\,support} = 1.15\frac{W_u l_n}{2}, l_n = 8.0 - \frac{0.8}{2} - \frac{0.4}{2} = 7.4\,m$$

$$V_{u\,@\,face\,of\,support} = 1.15\frac{\left(35\frac{kN}{m} \times 7.4m\right)}{2} = 149\,kN$$

As all related conditions are satisfied, then shear at distance "d" could be used in beam design.

$$V_{u\,@\,distance\,d\,from\,face\,of\,support} = 149\,kN - 35\frac{kN}{m} \times 0.54m = 130\,kN$$

- Concrete Shear Strength:

$$\phi V_c = \phi(0.17\lambda\sqrt{f_c'}\,b_w d) = \phi(0.17 \times 1 \times \sqrt{28} \times 400 \times 540) = 146\text{ kN}$$

$$\because \emptyset\frac{V_c}{2} < V_u \le \emptyset V_c$$

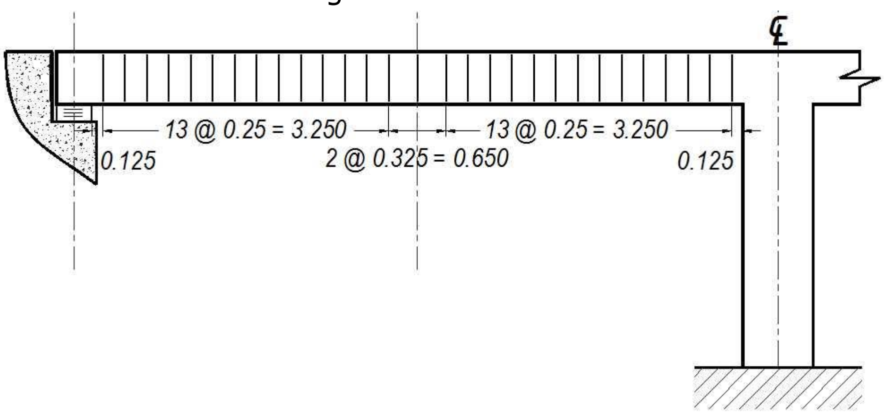
then only nominal shear reinforcement is required.

- Required Shear Reinforcement:

$$A_v = \frac{\pi \times 10^2}{4} \times 2 = 157\,mm^2$$

Shear reinforcement for Example 5.6-10	
Region	$\emptyset\frac{V_c}{2} < V_u \le \emptyset V_c$
V_s	None
$S_{Theoretical}$	None
$S_{for\ Av\ minimum}$	$\text{minimum}\left(\frac{A_v f_{yt}}{0.062\sqrt{f_c'}b_w}\text{ or }\frac{A_v f_{yt}}{0.35 b_w}\right)$ $\text{minimum}\left(\frac{157 \times 420}{0.062 \times \sqrt{28} \times 400}, \frac{157 \times 420}{0.35 \times 400}\right) \Rightarrow \text{minimum}\ (502, 471) = 471\,mm$
$S_{maximum}$	$\text{Minimum}\left[\frac{d}{2}\text{ or }600\text{mm}\right] = 270$
$S_{Required}$	$\text{Minimum}\begin{bmatrix}471,\\270\end{bmatrix} = 270\,mm$ ***Use U Stirrups ϕ10mm @ 250mm***

- Reinforcement Drawings:



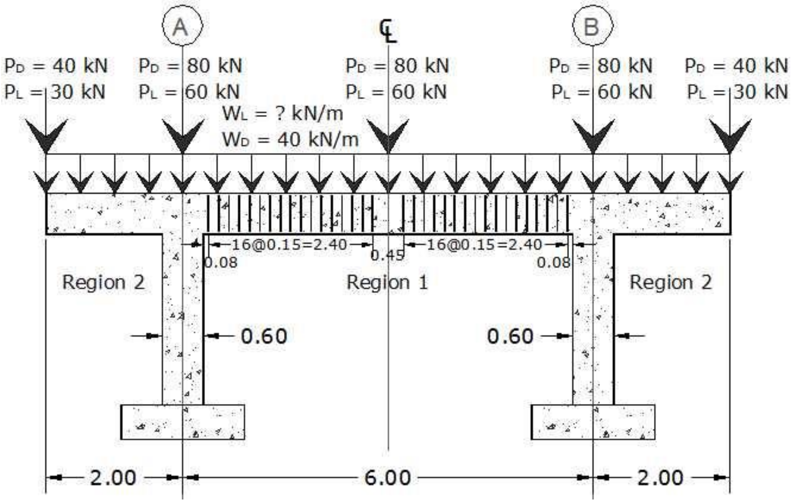
Example 5.6-11

For the frame that shown in Figure 5.6-9 below.

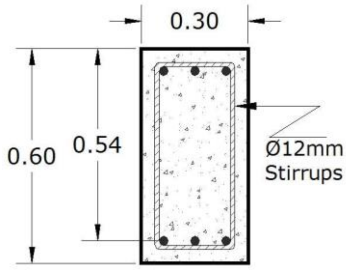
- Based on shear reinforcement that proposed for Region 1, what is maximum uniform distributed live load "W_L" that could be supported?
- Is shear reinforcement that proposed for Region 1 adequate when used in Region 2?

In your solution, assume that:

- U stirrups with 12mm diameter.
- $f_c' = 28\,MPa\; f_{yt} = 420\,MPa$
- $W_u = 1.2D + 1.6L$



Longitudinal Section



Typical Cross Section.

Figure 5.6-9: Frame for Example 5.6-11.

Solution

- Based on shear reinforcement that proposed for Region 1, the maximum uniform distributed live load "W_L" that could be supported would be:

$$A_v = \frac{\pi \times 12^2}{4} \times 2 = 226\,mm^2 \Longrightarrow V_s = \frac{A_v f_{yt} d}{s} = \frac{226 \times 420 \times 540}{150} = 342\,kN$$

$$V_c = \left(0.17\lambda\sqrt{f_c'}\,b_w d\right) = 0.17 \times \sqrt{28} \times 300 \times 540 = 146\,kN \Longrightarrow \phi V_n = \phi(V_c + V_s)$$

$$= 0.75 \times (146 + 342) = 366\,kN$$

$$V_{u\,@\,face\,of\,support} = \phi V_n = 366\,kN$$

$$P_u = 1.2 \times 80 + 1.6 \times 60 = 192\,kN$$

$$(W_u \times (6.0 - 0.6 - 0.54 \times 2) + 192) \times \frac{1}{2} = 366 \Longrightarrow W_u = 125\,\frac{kN}{m}$$

$$W_D = 40 + (0.6 \times 0.3 \times 24) = 44.3\,kN$$

$$W_u = 125 = 1.2 \times 44.3 + 1.6 \times W_L \Longrightarrow W_l = 44.9\,kN \;\blacksquare$$

- Check if the shear reinforcement that proposed for Region 1 is adequate when used in Region 2?

$$P_u = 1.2 \times 40 + 1.6 \times 30 = 96.0\,kN$$

$$V_{u\,at\,d} = 125 \times \left(2.0 - \frac{0.6}{2} - 0.54\right) + 96.0 \Longrightarrow V_{u\,at\,d} = 241\,kN < \phi V_n \;\therefore Ok. \blacksquare$$

5.7 PROBLEMS FOR SOLUTION ON BASIC SHEAR ASPECTS

Problem 5.7-1

A reinforced concrete beam with a rectangular cross section is reinforced for moment only and subjected to a shear V_u of 40.0 kN. Beam width b=300mm and effective depth d=184mm, $f_c' = 21\text{MPa}$ and $f_y = 414\text{MPa}$. Is beam satisfactory for shear?

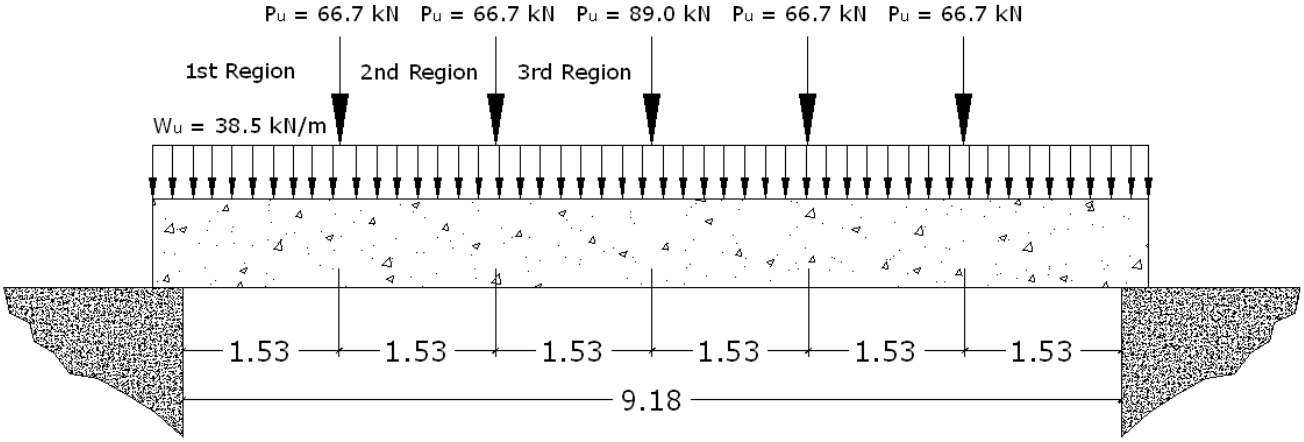
Answers

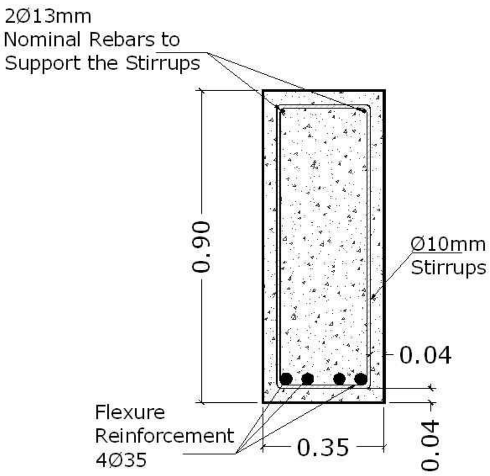
$V_c = 43.0, \frac{1}{2}\emptyset V_c \; ? \; V_u, \quad \because \frac{1}{2}\emptyset V_c < V_u$

Then shear reinforcement is required for this beam. As no shear reinforcement is provided, then the beam is inadequate for shear.

Problem 5.7-2

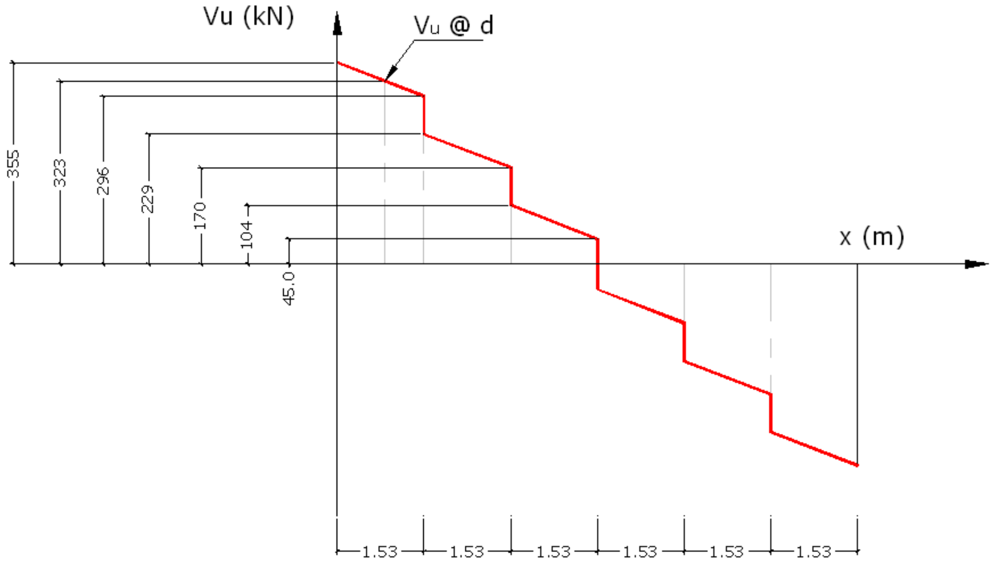
For beam shown below, design single-loop stirrups. The loads shown are factored loads. Use $f_c' = 21\text{MPa}$ and $f_y = 414\text{MPa}$. The uniformly load includes the beam selfweight.





Answers

Draw the shear force diagram:



$\text{d} = 833 \text{ mm}$

Shear Design for 1st Region:

$\text{V}_{\text{u @ d from Face of Support}} = 323 \text{ kN}$

$\text{ØV}_\text{c} = 170 \text{ kN}$

$\text{A}_\text{v} = 157 \text{ mm}^2$

Stirrups Design of Problem 5.7-2 (Region 1)	
Region	$ØV_c \leq V_u$
V_s	$\frac{V_u - ØV_c}{Ø} = 0.66\sqrt{f_c'}b_w d \Longrightarrow 204kN < 882\ kN\ Ok$ Beam dimensions are adequate.
$S_{Theoretical}$	$= \frac{A_v f_{yt} d}{V_s} = 266\ mm$
$S_{for\ Av\ minimum}$	$minimum\ (\frac{A_v f_{yt}}{0.062\sqrt{f_c'}b_w}\ or\ \frac{A_v f_{yt}}{0.35 b_w}) \Longrightarrow minimum\ (654\ or\ 531) = 531\ mm$
$S_{maximum}$	$V_s \leq 0.33\sqrt{f_c'}b_w d \Longrightarrow 204\ kN \leq 441\ kN$ $Minimum\ \left[\frac{833}{2}\ or\ 600mm\right] = 416\ mm$
	$V_s > 0.33\sqrt{f_c'}b_w d \Longrightarrow Minimum\ \left[\frac{d}{4}\ or\ 300mm\right]$
$S_{Required}$	$Minimum\ [S_{Theoretical}, S_{for\ Av\ minimum}, S_{maximum}] \Longrightarrow Minimum\ [266mm, 531\ mm, 416\ mm]$ $= 266\ mm$ Use $Ø10mm\ @\ 250mm$

Shear Design for 2nd Region:

Stirrups Design of Problem 5.7-2 (Region 2)	
Region	$ØV_c \leq V_u$
V_s	$= \frac{V_u - ØV_c}{Ø} = 0.66\sqrt{f_c'}b_w d \Longrightarrow 78.7kN < 882\ kN\ Ok$ Beam dimensions are adequate.
$S_{Theoretical}$	$= \frac{A_v f_{yt} d}{V_s} = 688\ mm$
$S_{for\ Av\ minimum}$	$minimum\ (\frac{A_v f_{yt}}{0.062\sqrt{f_c'}b_w}\ or\ \frac{A_v f_{yt}}{0.35 b_w}) \Longrightarrow minimum\ (654\ or\ 531) = 531\ mm$
$S_{maximum}$	$V_s \leq 0.33\sqrt{f_c'}b_w d \Longrightarrow 204\ kN \leq 441\ kN \Longrightarrow Minimum\ \left[\frac{833}{2}\ or\ 600mm\right] = 416\ mm$
	$V_s > 0.33\sqrt{f_c'}b_w d \Longrightarrow Minimum\ \left[\frac{d}{4}\ or\ 300mm\right]$
$S_{Required}$	$Minimum\ [S_{Theoretical}, S_{for\ Av\ minimum}, S_{maximum}]$ $Minimum\ [688\ mm, 531\ mm, 416\ mm] = 416mm$ Use $Ø10mm\ @\ 400mm$

Shear Design for 3rd Region:

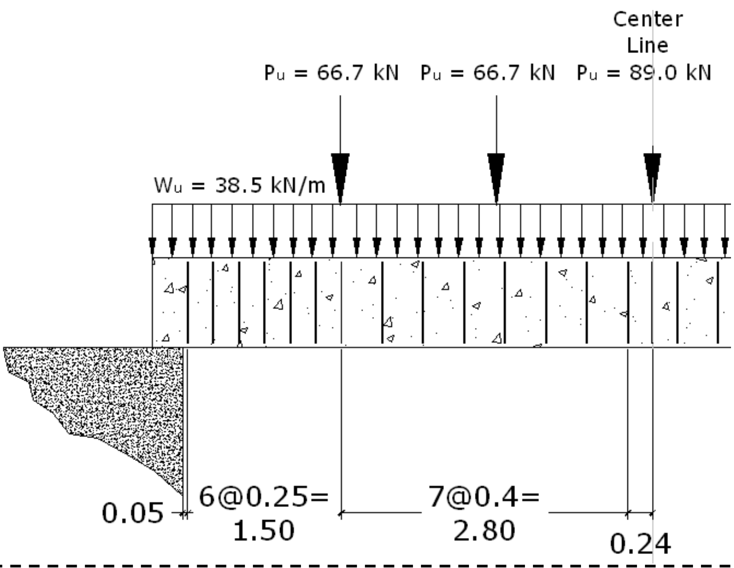
$\because \text{V}_\text{u} = 104 \text{ kN} < \text{ØV}_\text{c} = 170 \text{ kN}$

Then, only nominal requirement is required for 2nd Region:

$\text{S}_{\text{Required}} = \text{Minimum [531 mm, 416 mm]}$

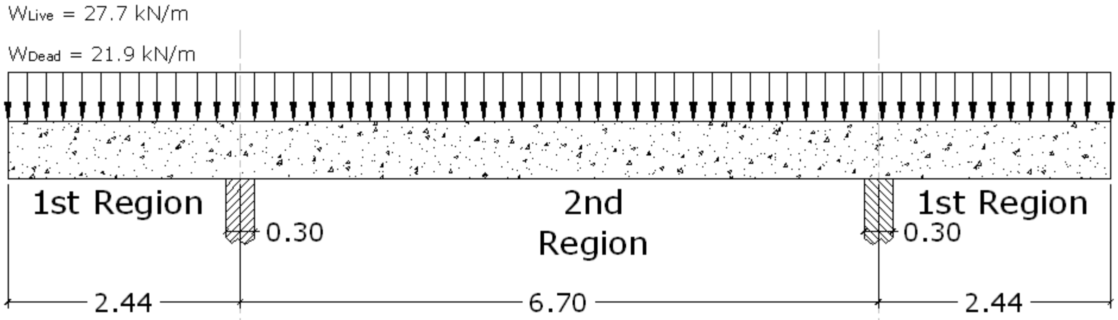
$\text{S}_{\text{Required}} = 416\text{mm}$

$\Longrightarrow \text{Use Ø10mm @ 400mm}$



Problem 5.7-3

Design stirrups for the beam shown. Service loads are 21.9 kN/m dead load (including beam selfweight) and 27.7 kN/m live load. Beam width "b" is 325mm and effective depth "d" is 600mm for both top and bottom reinforcement. Use $f_c' = 21\text{MPa}$ and $f_y = 414\text{MPa}$. Use 10mm U Stirrups.



Answers

Computed the factored load:

$W_u = \text{maximum of } [1.4\text{ Dead or } 1.2\text{ Dead } + 1.6\text{ Live}]$

$W_u = \text{maximum of } \left[31.0\frac{\text{kN}}{\text{m}} \text{ or } 70.6\frac{\text{kN}}{\text{m}}\right] = 70.6\frac{\text{kN}}{\text{m}}$

Shear Design for Region 1:

$V_{u\,@\,d} = 70.6(2.44 - 0.15 - 0.6) = 119\ kN$

$\emptyset V_c = 0.75 \times 152\text{ kN} = 114\text{ kN}$

$A_v = 157\text{ mm}^2$

Summary of stirrups design for this region is given in Table below.

Shear Design for Region 2:

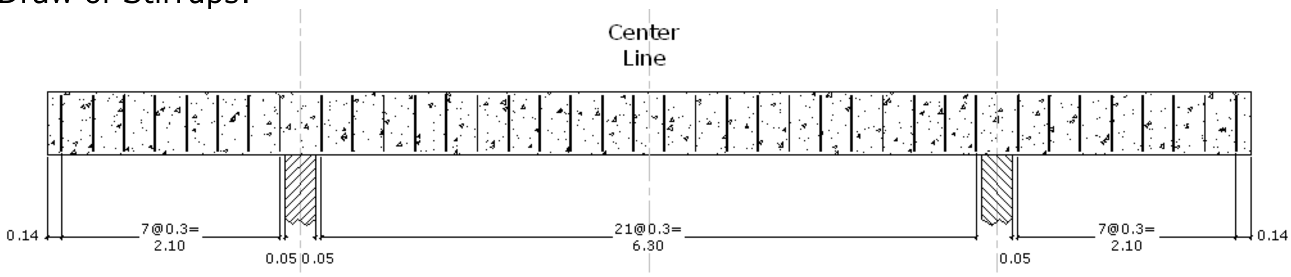
$V_{u\,@\,d} = 70.6\,(6.7 - 0.15 \times 2 - 0.6 \times 2) \times \frac{1}{2} = 184\ kN$

$\emptyset V_c = 114\text{ kN}$

$A_v = 157\text{ mm}^2$

Summary of stirrups design for this region is given in the table below.

Draw of Stirrups:



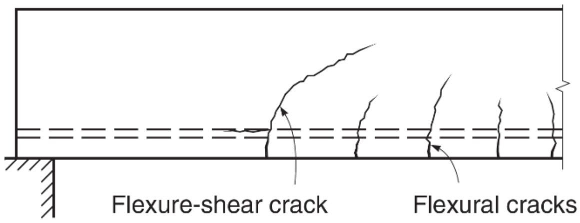
Stirrups Design of (Region 1)	
Region	$\emptyset V_c \leq V_u$
V_s	$= \frac{V_u - \emptyset V_c}{\emptyset} = 0.66\sqrt{f_c'}b_w d$ $6.67kN < 590\ kN\ Ok$ Beam dimensions are adequate.
$S_{Theoretical}$	$= \frac{A_v f_{yt} d}{V_s} = 5847\ mm$
$S_{for\ Av\ minimum}$	$minimum\ (\frac{A_v f_{yt}}{0.062\sqrt{f_c'}b_w}\ or\ \frac{A_v f_{yt}}{0.35 b_w}) \Rightarrow minimum\ (704\ or\ 571) = 571\ mm$
$S_{maximum}$	$V_s \leq 0.33\sqrt{f_c'}b_w d \Rightarrow 6.67kN \leq 295\ kN \Rightarrow Minimum\ \left[\frac{d}{2}\ or\ 600mm\right]$ $Minimum\ \left[\frac{600}{2}\ or\ 600mm\right] = 300\ mm$
	$V_s > 0.33\sqrt{f_c'}b_w d \Rightarrow Minimum\ \left[\frac{d}{4}\ or\ 300mm\right]$
$S_{Required}$	$Minimum\ [S_{Theoretical}, S_{for\ Av\ minimum}, S_{maximum}] \Rightarrow Minimum\ [5847\ mm, 571\ mm, 300\ mm]$ $= 300\ mm$ **Use** $\emptyset 10mm\ @\ 300\ mm$

Stirrups Design of (Region 2)	
Region	$\emptyset V_c \leq V_u$
V_s	$\frac{V_u - \emptyset V_c}{\emptyset} = 0.66\sqrt{f_c'} b_w d \Rightarrow 93.3kN < 590\ kN\ Ok$ Beam dimensions are adequate.
$S_{Theoretical}$	$= \frac{A_v f_{yt} d}{V_s} = 418\ mm$
$S_{for\ Av\ minimum}$	$minimum\ (\frac{A_v f_{yt}}{0.062\sqrt{f_c'} b_w}\ or\ \frac{A_v f_{yt}}{0.35 b_w}) \Rightarrow minimum\ (704\ or\ 571) = 571\ mm$
$S_{maximum}$	$V_s \leq 0.33\sqrt{f_c'} b_w d \Rightarrow 93.3kN \leq 295\ kN$ $Minimum\ \left[\frac{d}{2}\ or\ 600mm\right] \Rightarrow Minimum\ \left[\frac{600}{2}\ or\ 600mm\right] = 300\ mm$
	$V_s > 0.33\sqrt{f_c'} b_w d \Rightarrow Minimum\ \left[\frac{d}{4}\ or\ 300mm\right]$
$S_{Required}$	$Minimum\ [S_{Theoretical}, S_{for\ Av\ minimum}, S_{maximum}]$ $Minimum\ [418\ mm, 571\ mm, 300\ mm] = 300mm$ Use $\emptyset 10mm\ @\ 300\ mm$

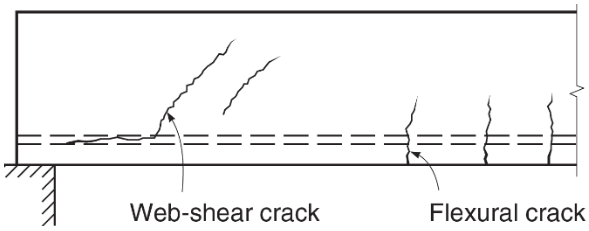
5.8 *SHEAR DESIGN BASED ON THE MORE DETAILED RELATION FOR V_c

5.8.1 Basic Concepts

- As discussed in Article 5.3 above, there are two types of shear or diagonal tension cracks:
    - First Type (Flexure-shear Crack): Shear cracks of this type occur after formation of flexural cracks and growth form the end of flexural cracks.



    - Second Type (Web-shear crack): Shear cracks of this type occur in region with small bending moments and form mainly due to applied shear force.



- For flexure-shear cracks, value of V_c represents shear force that is required to expand preexisting flexural cracks. While for web-shear crack, V_c represents shear force required to initiate web cracks and has a value greater than that required for expands preexisting flexural cracks.
- Factors Affecting V_c:
    - Based on above definition, it is evident that the shear value at which diagonal cracks developed or/and propagate depends on the ratio of shear force to bending moment. This ratio can be expressed in terms of $V_u d/Mu$.
    - It can also be shown that increasing values of reinforcing ratio ρ_w have a beneficial effect in that they increase the shear at which diagonal cracks develop. This is so because larger amount of longitudinal steel results in smaller and narrower flexural tension cracks prior to the formation of diagonal cracks, leaving a larger area of uncracked concrete available to resist shear.
- Based on above reasoning, ACI offers Table 5.8-1 below to simulate the effects of $V_u d/Mu$ and ρ_w on concrete cracking shear strength.
- Expression (b) in Table 5.8-1 limits V_c near points of inflection.

Table 5.8-1: Detailed method for calculating V_c, Table 22.5.5.1 of the code.

V_c		
Least of (a), (b), and (c):	$\left(0.16\lambda\sqrt{f_c'}+17\rho_w\frac{V_u d}{M_u}\right)b_w d$ [1]	(a)
	$(0.16\lambda\sqrt{f_c'}+17\rho_w)b_w d$	(b)
	$0.29\lambda\sqrt{f_c'}b_w d$	(c)

[1]M_u occurs simultaneously with V_u at the section considered.

5.8.2 Detailed versus Simplified Relations for V_c

- In simplified equation of Article 5.3, the second term in expressions (a) and (b) of ***Table 5.8-1*** have been assumed equals $0.01\lambda\sqrt{f_c}$ and use V_c equal to $V_c = 0.17\lambda\sqrt{f_c'}\, b_w d$

  This simplified relation has been used in solutions of previous examples and problems.
- It is useful to note that the simplified equation has been derived based assumption of low $\frac{(V_u d)}{M_u}$ and low ρ_w that lead to a second term that has a small value of $(0.01\lambda\sqrt{f_c'})$. Therefore, it gives an accurate estimation of V_c in regions with large moment but gives an underestimation (conservative value) in regions with small moment.

5.8.3 Which Relation Should be Adopted

Use of more detailed or simplified ACI relations can be summarized with refers to Figure 5.8-1, Figure 5.8-2, and Figure 5.8-3 below.

- Simple Span with Uniformly Distributed Load:

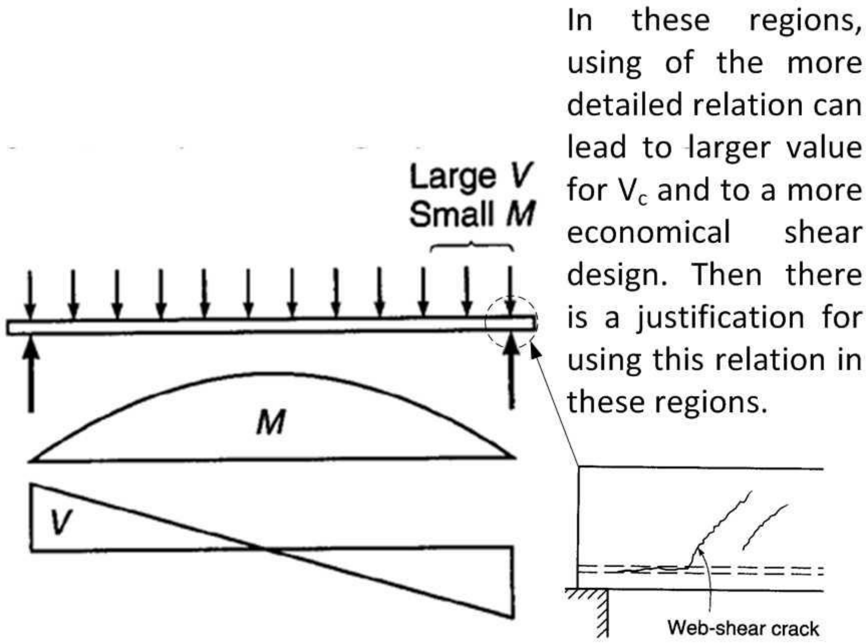


Figure 5.8-1: Simple span with uniformly distributed loads.

- Simple Span with a Concentrated Load at Mid-span:

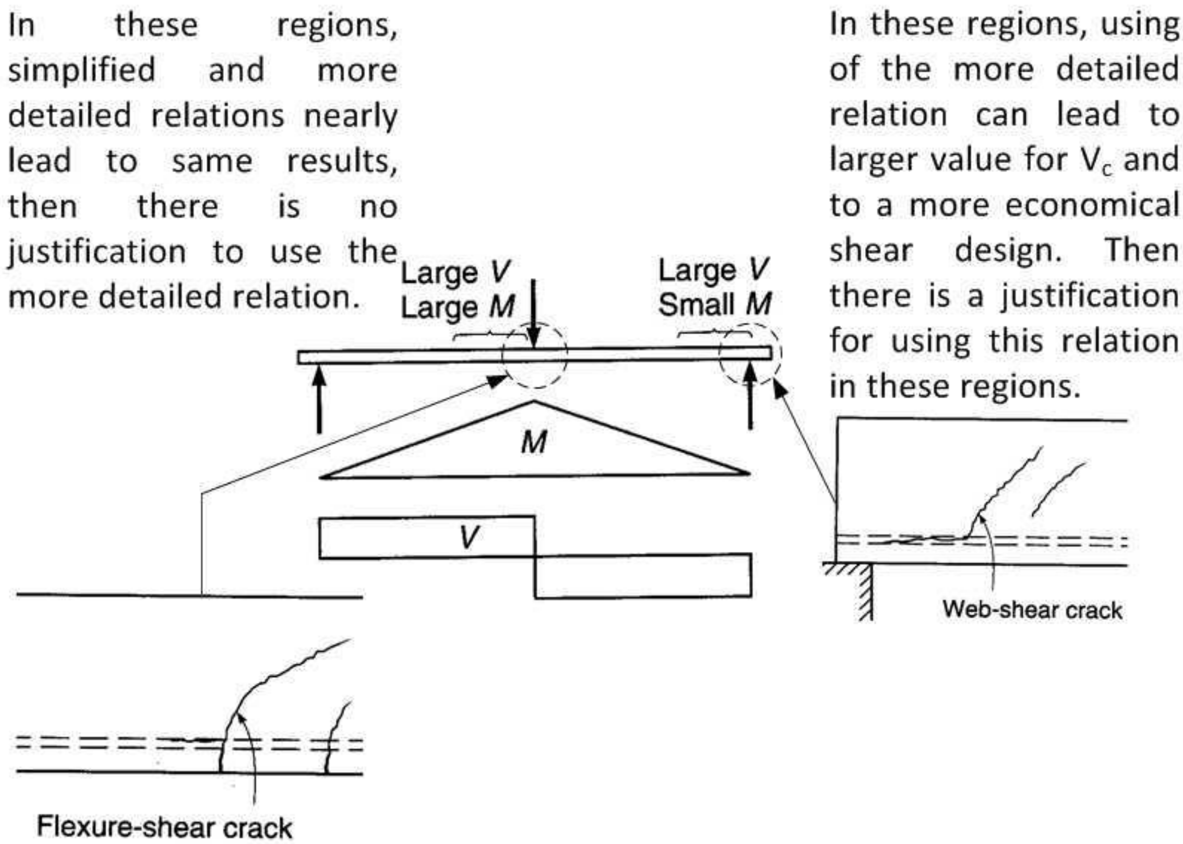


Figure 5.8-2: Simple span with a point load.

- Continuous Span with Uniformly Distributed:

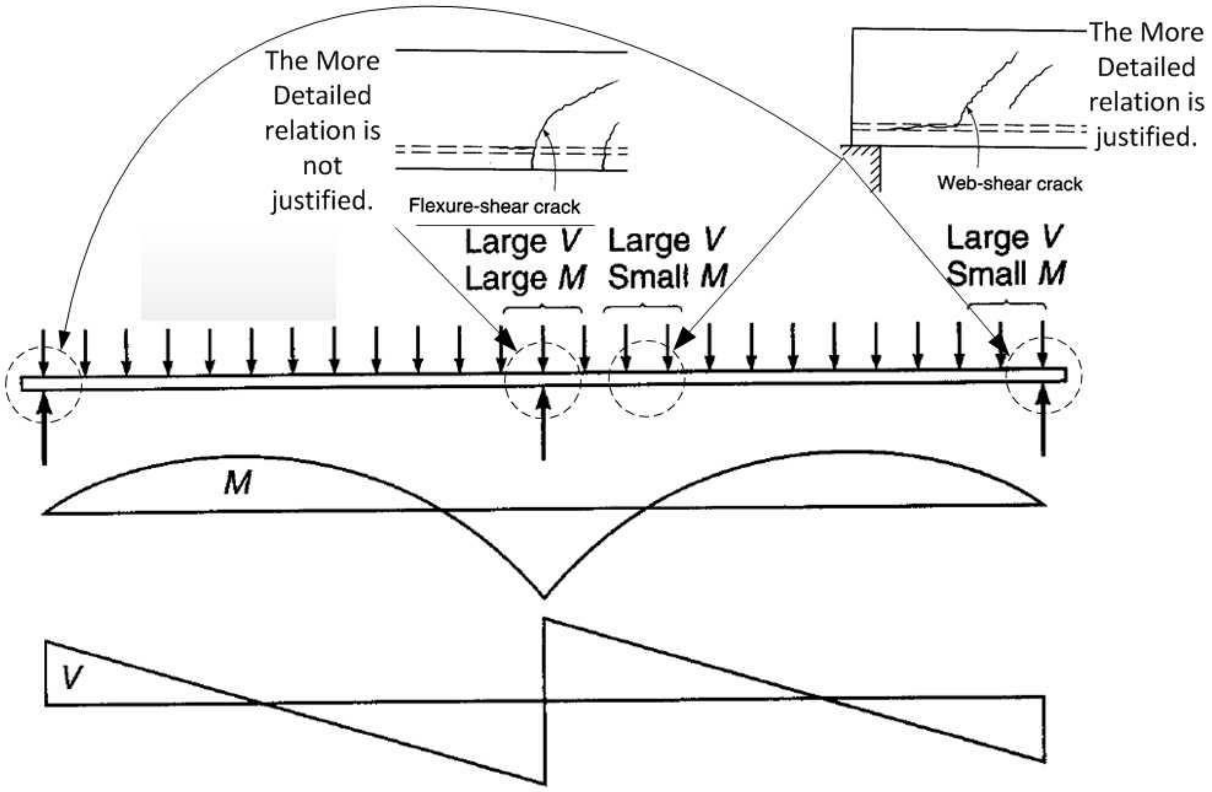


Figure 5.8-3: Continuous span with uniformly distributed loads.

5.8.4 Examples

Example 5.8-1

Based on a statically indeterminate analysis, shear force and bending moment have been computed and drawn for the continuous beam shown in ***Figure 5.8-4*** below. For this beam, compute V_c based on simplified relation and more detailed relation at exterior and interior supports. Assume that $f_c' = 21\ MPa$ and $f_y = f_{yt}\ =\ 420\ MPa$.

Based on flexural design following values have been determined:

b = 300mm, d= 535mm, h = 600mm.

$\rho_{-ve} = 19.4\ \times 10^{-3}, \rho_{+ve} = 10.6\ \times 10^{-3}$

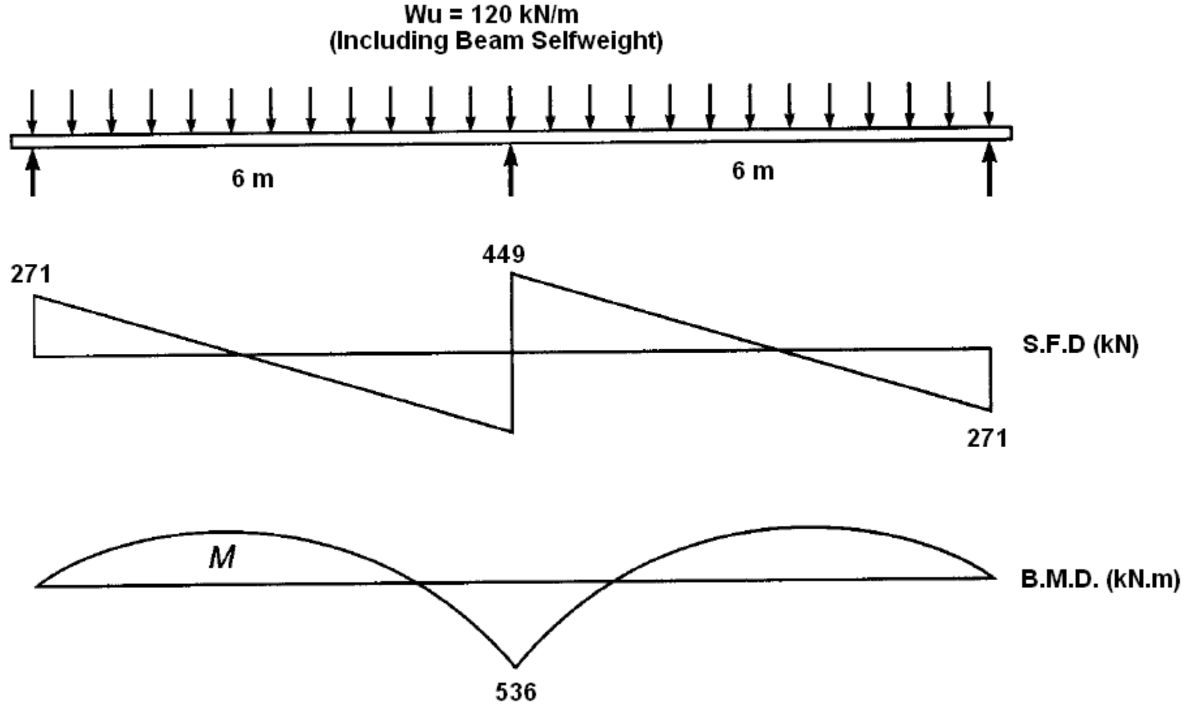


Figure 5.8-4: Continuous beam for Example 5.8-1 with its shear force and bending moment diagrams.

Solution

At Exterior Support

Based on Simplified Relation:

$V_c = 0.17\lambda\sqrt{f_c'}\, b_w d \Rightarrow V_c = 0.17\sqrt{21} \times 300 \times 535 = 0.779\, MPa \times 300 \times 535 = 125\, kN$

Based on the more detailed relation:

$$V_c = \left(0.16\lambda\sqrt{f_c'} + 17\rho_w \frac{(V_u d)}{M_u}\right) b_w d \leq 0.29\lambda\sqrt{f_c'} b_w d$$

For inflection points that have zero moment, $\frac{(V_u d)}{M_u}$ taken equal to 1.0, then:

$V_c = \left(0.16\sqrt{21} + 17 \times (10.6 \times 10^{-3}) \times 1.0\right)300 \times 535 \ ?\ 0.29\lambda\sqrt{f_c'} b_w d$

$V_c = (0.733\, MPa + 0.180\, MPa)300 \times 535 \ ?\ 0.29\lambda\sqrt{f_c'} b_w d$

$V_c = (0.913\, MPa)300 \times 535 \ ?\ 0.29\lambda\sqrt{f_c'} b_w d \Rightarrow V_c = 147\, kN < 213\, kN$ Ok. $\Rightarrow V_c = 147\, kN$

Increase percentage due to use of the more detailed relation:

$$Increase\ Percentage = \frac{147 - 125}{125} \times 100\% = 17.6\ \%$$

At Interior Support

Based on Simplified Relation:

$V_c = 0.17\lambda\sqrt{f_c'}\, b_w d \Rightarrow V_c = 0.17\sqrt{21} \times 300 \times 535 = 0.779\, MPa \times 300 \times 535 = 125\, kN$

Based on the more detailed relation:

$$V_c = \left(0.16\lambda\sqrt{f_c'} + 17\rho_w \frac{(V_u d)}{M_u}\right) b_w d \leq 0.29\lambda\sqrt{f_c'} b_w d$$

$$\frac{(V_u d)}{M_u} = \left(\frac{449000\, N \times 535\, mm}{536 \times 10^6\, N.mm}\right) = 0.448 < 1.0\ Ok.$$

$V_c = \left(0.16\sqrt{21} + 17 \times (19.4 \times 10^{-3}) \times 0.448\right)300 \times 535 \ ?\ 0.29\lambda\sqrt{f_c'} b_w d$

$V_c = (0.733\, MPa + 0.147\, MPa)300 \times 535 \ ?\ 0.29\lambda\sqrt{f_c'} b_w d \Rightarrow V_c = 141\, kN < 213\, kN$ Ok.

$$Increase\ Percentage = \frac{141 - 125}{125} \times 100\% = 12.8\ \%$$

As it is expected, using of the more detailed ACI equation is more useful in regions of large shear and small moment (regions of inflection points).

Example 5.8-2

For the simply supported beam shown in ***Figure 5.8-5*** below, make a complete shear design use same spacing of 10mm U stirrups along beam span. In your design assume that:

- Beam selfweight can be neglected,
- $f_c' = 28\, MPa$ and $f_y = f_{yt} = 420\, MPa$,
- Beam has dimensions of d = 550 mm and b = 300mm,
- V_c must be computed from the more detailed ACI relation. Use the same V_c value along beam span.
- Steel reinforcement area for positive moment has been computed to be $2835\, mm^2$.

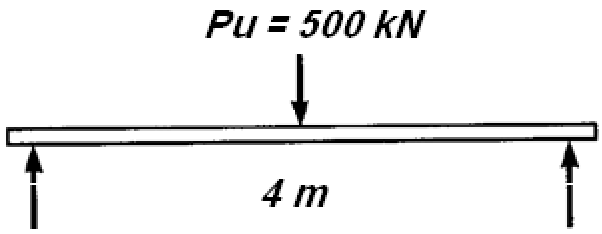
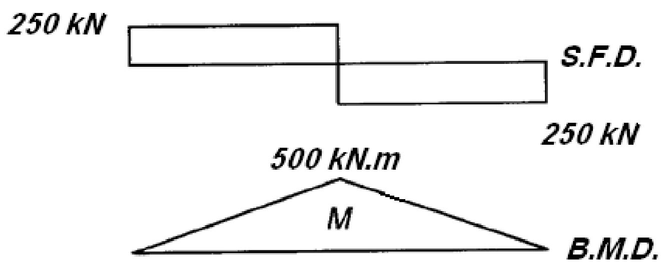


Figure 5.8-5: Simply supported beam for Example 5.8-2.

Solution

- Compute of V_c

  As same V_c must be used along beam span, therefore V_c must be computed based on a region of large shear and large moment (under concentrated load in this example) to obtain a value that is conservative along beam span.



$$V_c = \left(0.16\lambda\sqrt{f_c'} + 17\rho_w \frac{(V_u d)}{M_u}\right) b_w d \leq 0.29\lambda\sqrt{f_c'} b_w d$$

$$\rho_w = \frac{2835\ mm^2}{550 \times 300\ mm^2} = 17.2 \times 10^{-3}$$

$$\frac{V_u d}{M_u} = \frac{(250\ kN \times 0.55m)}{500\ kN.m} = 0.275 < 1.0\ \therefore Ok.$$

$$V_c = \left(0.16\sqrt{28} + 17 \times 17.2 \times 10^{-3} \times \frac{(250\ kN \times 0.55m)}{500\ kN.m}\right) 300 \times 550\ ?\ 0.29\sqrt{28} \times 300 \times 550$$

$$V_c = (0.847\ MPa + 0.080\ MPa\ )300 \times 550\ ?\ 0.29\sqrt{28} \times 300 \times 550 \Longrightarrow V_c = 153\ kN\ <\ 253\ kN$$

As $V_c < V_u$, therefore shear must be designed based on region of theoretical and nominal reinforcement.

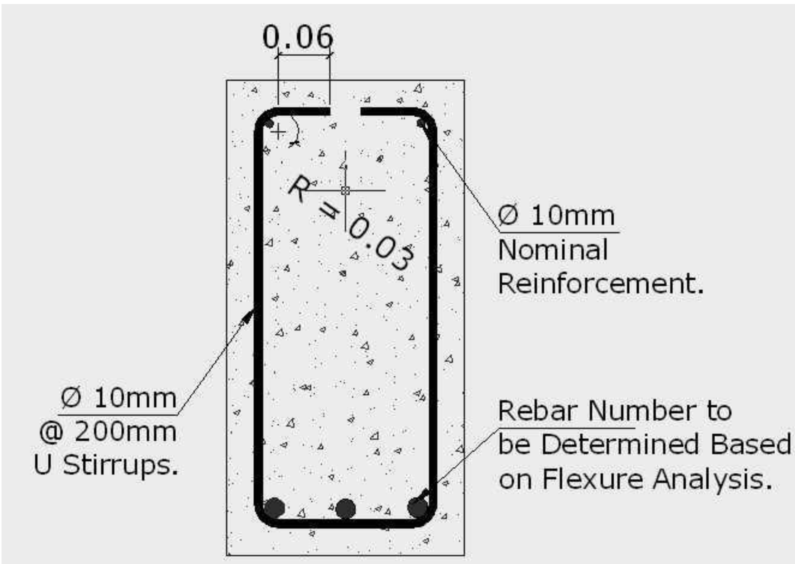
- Shear Design:

Shear design is summarized in Table below.

Stirrups Design of Example 5.8-2	
Region	$\emptyset V_c \leq V_u$
V_s	$= \frac{V_u - \emptyset V_c}{\emptyset}\ ?\ 0.66\sqrt{f_c'} b_w d \Longrightarrow \frac{250 - 0.75 \times 153}{0.75}\ ?\ 0.66\sqrt{28} \times 300 \times 550$ $180kN <\ 576\ kN\ Ok$ Beam dimensions are adequate.
$S_{Theoretical}$	$= \frac{A_v f_{yt} d}{V_s} = \frac{157\ mm^2 \times 420 \frac{N}{mm^2} \times 550mm}{180\ 000\ N} = 201\ mm$
$S_{for\ Av\ minimum}$	$minimum\ (\frac{A_v f_{yt}}{0.062\sqrt{f_c'} b_w}\ or\ \frac{A_v f_{yt}}{0.35 b_w})$ $minimum\ (\frac{157\ mm^2 \times 420 \frac{N}{mm^2}}{0.062\sqrt{28} \times 300}\ or\ \frac{157\ mm^2 \times 420 \frac{N}{mm^2}}{0.35 \times 300})$ $minimum\ (\ 700\ or\ 628) = 628\ mm$
$S_{maximum}$	$V_s \leq 0.33\sqrt{f_c'} b_w d \Longrightarrow 180kN \leq 0.33\sqrt{28} \times 300 \times 550$ $180\ \ kN < 288\ \ kN$ $Minimum\ \left[\frac{d}{2}\ or\ 600mm\right] \Longrightarrow Minimum\ \left[\frac{550}{2}\ or\ 600mm\right] = 225\ mm$
	$V_s > 0.33\sqrt{f_c'} b_w d \Longrightarrow Minimum\ \left[\frac{d}{4}\ or\ 300mm\right]$
$S_{Required}$	$Minimum\ [S_{Theoretical}\ , S_{for\ Av\ minimum}\ , S_{maximum}]$ $Minimum\ [201\ mm\ , 628\ mm, 225\ mm] = 201\ mm$ ***Use*** $\boldsymbol{\emptyset 10mm\ @\ 200\ mm}$

- Drawing and Details of Stirrups:

The longitudinal section through beam cannot be drawn in this example, as nothing has been mentioned about supports widths. Cross section for this beam is shown below:



5.9 *Shear Design with Effects of Axial Loads

5.9.1 Scope

- The beams considered in the preceding sections were subjected to shear and flexure only.
- Reinforced concrete beams may also be subjected to axial forces, acting simultaneously with shear and flexure, due to a variety of causes. These include:
  - External axial loads,
  - Longitudinal prestressing,
  - Restraint forces introduced as a result of shrinkage of the concrete or temperature changes.
- Axial forces due to prestressing are out of our scope where this article deals only with non-prestressed members.
- As for members without axial forces, the ACI codes offers simplified and detailed relations to simulate the effect of axial forces on shear strength of concrete. Only simplified equations are considered in this article.

5.9.2 Effects of Axial Forces on Shear Strength of Concrete

- The main effect of axial load is to modify the diagonal cracking load of the member.
- It was shown in Article 5.1 that diagonal tension cracking will occur when the principal tensile stress in the web of a beam, resulting from combined action of shear and bending, reaches the tensile strength of the concrete.
- It is clear that the introduction of longitudinal force, which modifies the magnitude and direction of the principal tensile stresses, may significantly alter the diagonal cracking load. ***Axial compression will increase the cracking load, while axial tension will decrease it***.

5.9.3 Members with Compressive Axial Forces

- According to ***Article 22.5.6.1*** of the ACI code, for nonprestressed members with axial compression, V_c shall be calculated by:

$$V_c = 0.17\left(1 + \frac{N_u}{14A_g}\right)\lambda\sqrt{f_c'}b_w d \qquad \textbf{Eq. 5.9-1}$$

  where
  - N_u is the factored axial force normal to cross section occurring simultaneously with V_u; ***to be taken as positive for compression*** , it is expressed in unit of "N",
  - A_g is gross area of concrete section, mm^2. For a hollow section, A_g is the area of the concrete only and does not include the area of the void(s).
- From equation above, it is clear that the term of $N_u/14A_g$ represents the increasing in concrete shear strength, V_c, due to existing of the compressive axial force N_u.

5.9.4 Member with Significant Axial Tensile Forces

- According code commentary, R22.5.7.1, the term "significant" is adopted to recognize that judgment is required in deciding whether axial tension needs to be considered. Axial tension often occurs due to volume changes, but the levels may not be detrimental to the performance of a structure with adequate expansion joints and minimum reinforcement.
- According to ***Article 22.5.7.1*** of the code, for nonprestressed members with ***significant axial tension***, V_c shall be calculated by:

$$V_c = 0.17\left(1 + \frac{N_u}{3.5A_g}\right)\lambda\sqrt{f_c'}b_w d \geq 0 \qquad \textbf{Eq. 5.9-2}$$

  where
  - N_u is the factored axial force normal to cross section occurring simultaneously with V_u; ***to be taken as negative for tension***, it is expressed in unit of "N",
  - A_g is gross area of concrete section, mm^2. For a hollow section, A_g is the area of the concrete only and does not include the area of the void(s).

- From equation above, it is clear that the term of $N_u / 3.5A_g$ represents the decreasing in concrete shear strength, V_c, due to existing of the tensile axial force N_u.

- According commentary ***Article R22.5.7.1***, ***it may be desirable to design shear reinforcement to resist the total shear if there is uncertainty about the magnitude of axial tension***.

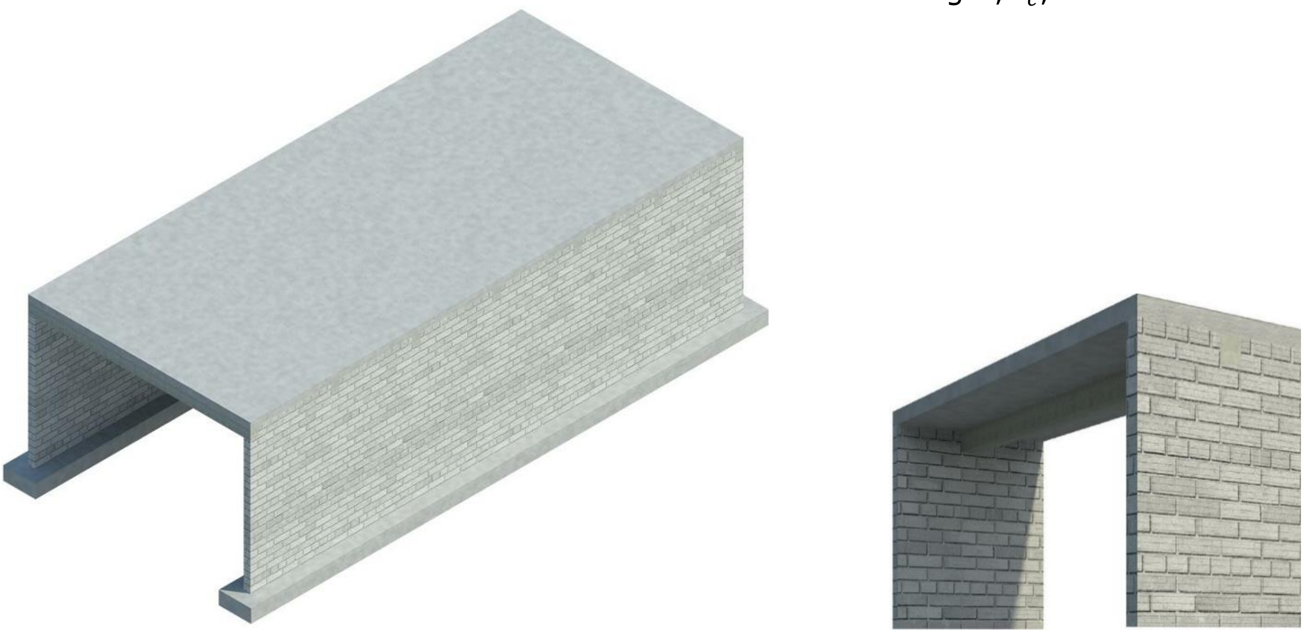
5.9.5 Comparing between Effect of Axial Compression and Axial Tension

Comparing between effect of axial compression, $N_u / 14A_g$, and the effect of axial tension, $N_u / 3.5A_g$, one concludes that the code is more conservative in estimating decreasing in shear strength, V_c, due to tensile stresses than its increasing due to compressive stresses.

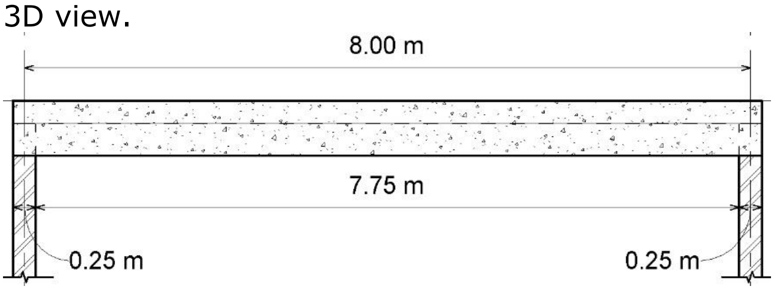
5.9.6 Examples

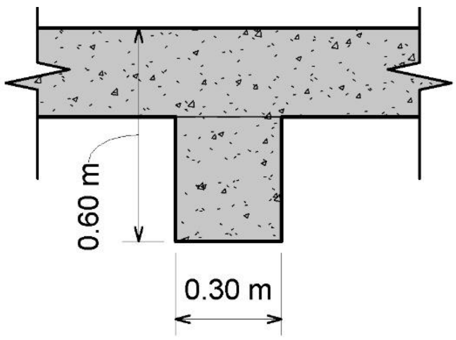
Example 5.9-1

Concrete roof slab and its supporting beams indicated in ***Figure 5.9-1*** below have been casted against and supported on brick bearing walls. The slab and beams have been concreted monolithically at a temperature of 20°C. Determine axial forces that are developed in a typical beam when the temperature decreases into 0°C or increases into 40°C then show how these forces can alter concrete shear strength, V_c, for the beam.



3D view.





Longitudinal section. Cross section.

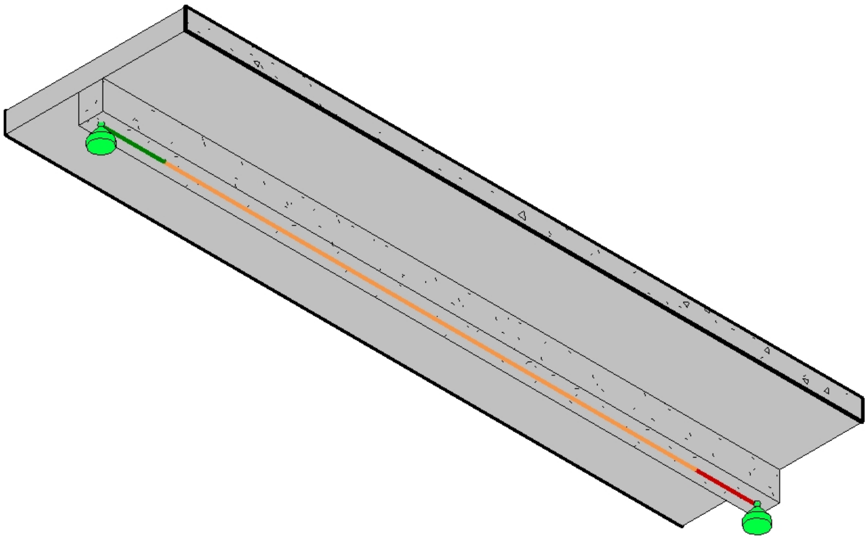
Figure 5.9-1: Roof slab and its supporting beams for Example 5.9-1.

In your analysis,

- Assume that the contact surface between beams and walls is rough enough to restrained beam movement,
- Assume that f_c' is 28 MPa and that the coefficient for thermal expansion of concrete is $\alpha_{Concrete} = 11 \times 10^{-6}\ 1/^{o}C$.
- Assume a load factor of 1.6 for forces due to temperature change.

Solution

With assumption of rough surface, the analytical model for a typical supporting beam would be as indicated in below:



From strength of materials, with equating of strain due to temperature to that due to restrained forces, the relation would be:

$\because \epsilon_{Forces} = \epsilon_{Temp}.$

$\therefore \frac{\sigma}{E_C} = \alpha \Delta T$

$\sigma = E_C \alpha \Delta T$

$\frac{N}{A_g} = E_C \alpha \Delta T$ ■

With a compressive strength of f_c' of 28MPa, the modulus of elasticity of concrete, E_c in MPa would be:

$E_c = 4700\sqrt{f_c'} = 4700 \times \sqrt{28} = 24870\ MPa$

The normal force per gross area would be:

$\frac{N}{A_g} = \pm(24870 \times 11 \times 10^{-6} \times 20) = \pm 5.47\ \text{MPa}$

These stresses would be tensile when temperature decreases while it would be compressive when temperature increases. With a load factor of 1.6, the ultimate stresses due to temperature change would be:

$\frac{N_u}{A_g} = \pm 1.6 \times 5.47 = 8.75\ \text{MPa}$

When these stresses are compressive, the shear strength of concrete, V_c, would increase by 62.5% as indicated in below:

$V_c = 0.17\left(1 + \frac{N_u}{14A_g}\right)\lambda\sqrt{f_c'}b_w d = V_c = 0.17\left(1 + \frac{8.75}{14}\right)\lambda\sqrt{f_c'}b_w d = 0.17(1 + 0.625)\lambda\sqrt{f_c'}b_w d$

While, when these stresses are tensile, the shear strength of concrete, V_c, would decrease to zero as indicated in below.

$V_c = 0.17\left(1 + \frac{N_u}{3.5A_g}\right)\lambda\sqrt{f_c'}b_w d = 0.17\left(1 - \frac{8.75}{3.5}\right)\lambda\sqrt{f_c'}b_w d = 0.17(1 - 2.5)\lambda\sqrt{f_c'}b_w d < 0.0$

$\therefore Not\ Ok.$

$V_c = 0.0$

Example 5.9-2: A Portal Frame Subjected to Gravity Loads Only

Based on a statically indeterminate analysis, reactions for the portal frame shown in ***Figure 5.9-2*** above have been computed and presented as shown. Design 12mm U Stirrups for the beam BC. In your design, assume that:

- $f_c' = 21\ MPa$ and $f_y = f_{yt} = 420\ MPa$,

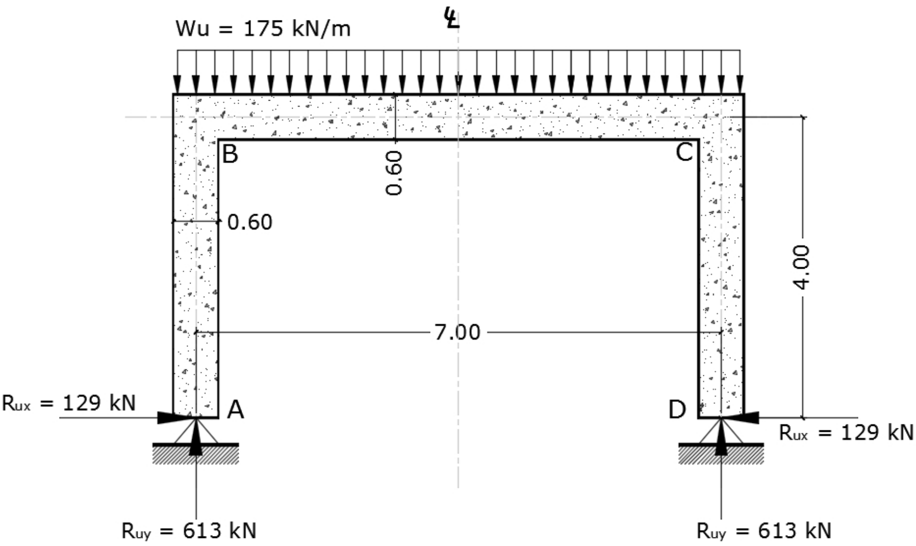
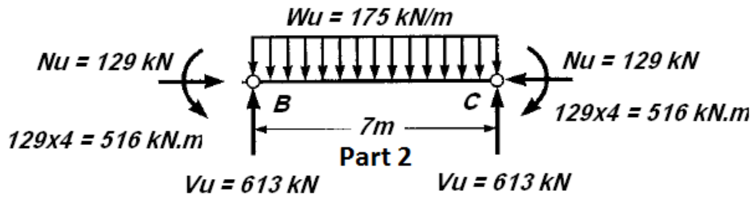


Figure 5.9-2: A portal frame subjected for gravity loads.

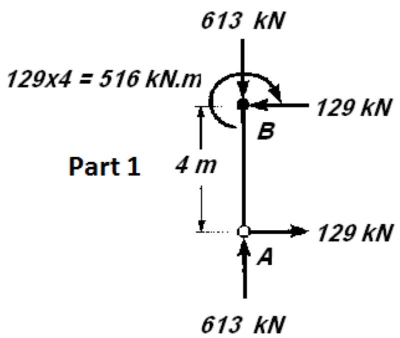
- $b_w = 300mm$, $h = 600mm$ and $d = 550mm$,
- Selfweight of the frame can be neglected,
- Effects of axial forces on concrete shear strength V_c of beam BC should be included.

Solution

- Computing of V_u:
  Applied factored shear force V_u can be computed based on any one of the following two approaches:
  - First Approach (Based on Forces Diagrams):
    - Based on simple static of column AB and beam BC, forces acting on beam BC can be determined based on indicated figure.



    - From above Figure, ***it is clear that column shear force transfer to beam axial force and column axial force transfer to beam shear***.

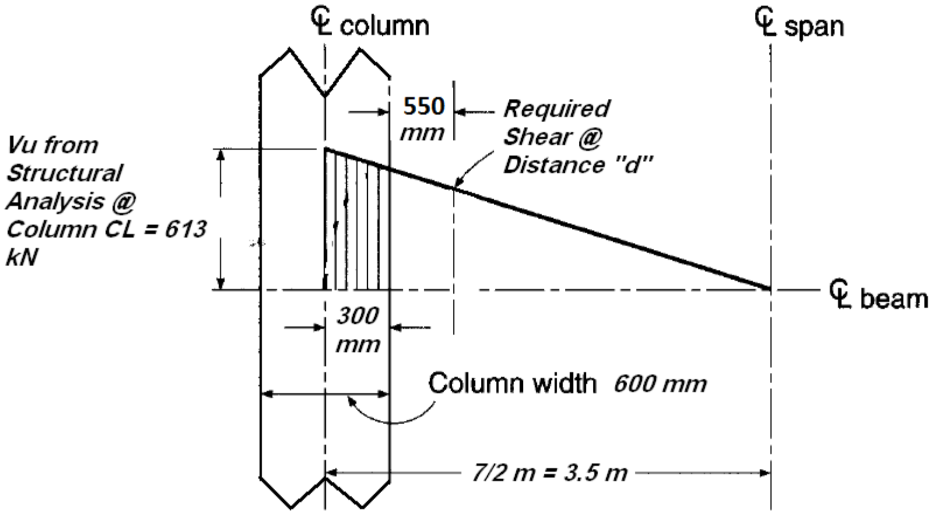


    - As the forces that computed based on structural analysis represent forces at center lines, then two transformation of beam shear force ($V_u = 613\ kN$) seems necessary to obtain required $V_{u\,@d}$. The first one transforms shear force for column center line to the face of column and the second one transforms shear force from face of column to a distance (d) from face of column as all ACI conditions are satisfied (See Figure below). These transformation can be done based on following relation:

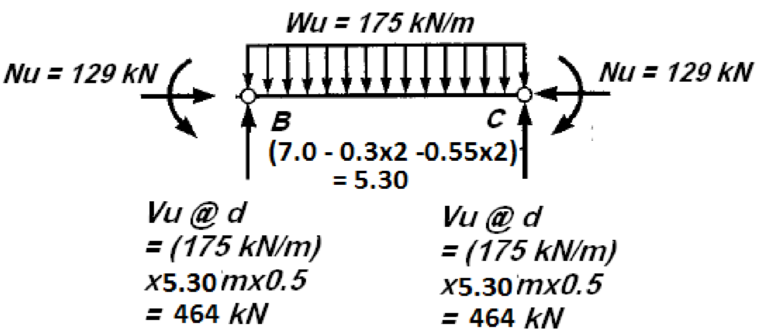
$$\because W = \frac{dV}{dx} \Rightarrow dV = Wdx \Rightarrow V_{u\,@\,d} - V_{@\,CL} = \int_{from\ CL}^{To\ Distance\ d\ from\ Face\ of\ Support} Wdx$$

$$V_{u\,@\,d} = \int_{from\ CL}^{To\ Distance\ d\ from\ Face\ of\ Support} Wdx \ + \ V_{@\,CL}$$

$$V_{u\,@\,d} = \left(-175\frac{kN}{m}\right) \times \left(\frac{0.6m}{2} + 0.55m\right) \ + \ 613 = 464\ kN$$



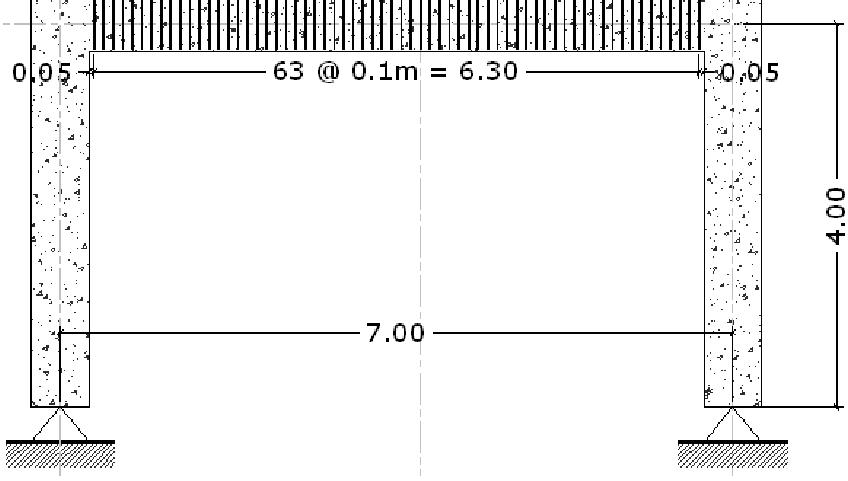
- Second Approach (Based on Symmetry):
  - For our symmetrical problem, principle of symmetry can be used to computed required shear force at distance (d) from face of support as indicated in the figure.



- Compute of V_c:
  Based on ACI code, for member with a compression axial force, concrete shear strength will be:
  $$V_c = 0.17\left(1+\frac{N_u}{14A_g}\right)\lambda\sqrt{f_c'}\,b_w d \Longrightarrow V_c = 0.17\left(1+\frac{129\,000}{14(300\times 600)}\right)\times 1.0\times\sqrt{21}\times 300\times 550$$
  $$V_c = 0.17(1+0.051)\times 1.0\times\sqrt{21}\times 300\times 550 = 135\text{ kN}$$
  Relation above indicates that concrete shear strength, V_c, increases by about 5% due to existing of the axial compressive force, N_u, with magnitude of 129 kN. Including the strength reduction factor, ϕ, for shear, design shear strength of the concrete would be:
  $$\emptyset V_c = 0.75\times 135\,kN = 101\,kN$$
- Shear Design:
  As applied shear force V_u is greater that $\emptyset V_c$, then beam should be designed on region with theoretical and nominal shear reinforcement (See Table below):

SHEAR DESIGN OF Example 5.9-2	
Region	$\emptyset V_c \le V_u$
V_s	$=\frac{V_u-\emptyset V_c}{\emptyset} = 0.66\sqrt{f_c'}b_w d \Longrightarrow \frac{464-101}{0.75}\ ?\ 0.66\times\sqrt{21}\times 300\times 550$ $484\,kN < 499\,kN\ Ok$ Beam dimensions are adequate.
$S_{Theoretical}$	$=\frac{A_v f_{yt} d}{V_s} = \frac{226\times 420\times 550}{484\,000} = 108\,mm$
$S_{for\ Av\ minimum}$	$minimum\ (\frac{A_v f_{yt}}{0.062\sqrt{f_c'}b_w}\ or\ \frac{A_v f_{yt}}{0.35 b_w}) \Longrightarrow minimum\ (\frac{226\times 420}{0.062\sqrt{21}\times 300}\ or\ \frac{226\times 420}{0.35\times 300})$ $minimum\ (1\,114\ or\ 904) = 904\,mm$
$S_{maximum}$	$V_s \le 0.33\sqrt{f_c'}b_w d$
	$V_s\ ?\ 0.33\sqrt{f_c'}b_w d \Longrightarrow V_s = 484 > 0.33\sqrt{21}\times 300\times 550 = 249\,kN$ $Minimum\ \left[\frac{d}{4}\ or\ 300mm\right] \Longrightarrow Minimum\ \left[\frac{550}{4}\ or\ 300mm\right] = 137\,mm$
$S_{Required}$	$Minimum\ [S_{Theoretical}, S_{for\ Av\ minimum}, S_{maximum}]$ $\Longrightarrow Minimum\ [108\,mm,\ 904\,mm,\ 137\,mm] = 108\,mm$ **Use $\emptyset 12mm\ @\ 100mm$**

- Stirrups Details:



Example 5.9-3: A Portal Frame Subjected to Gravity and Lateral Loads.

Based on a statically indeterminate analysis, reactions for the portal frame shown ***Figure 5.9-3*** below have been computed and presented as shown. Design 12mm U Stirrups for the beam BC. In your design, assume that:

- $f_c' = 21\,MPa$ and $f_{yt} = 420\,MPa$,
- $b_w = 300mm$, $h = 600mm$ and $d = 550mm$,
- Selfweight of the frame can be neglected,
- Effects of axial forces on concrete shear strength, V_c, of beam BC should be included.
- Ultimate forces to be determined based on following load combination:
  U = 1.2D + 1.6L$_r$ + 0.8W.

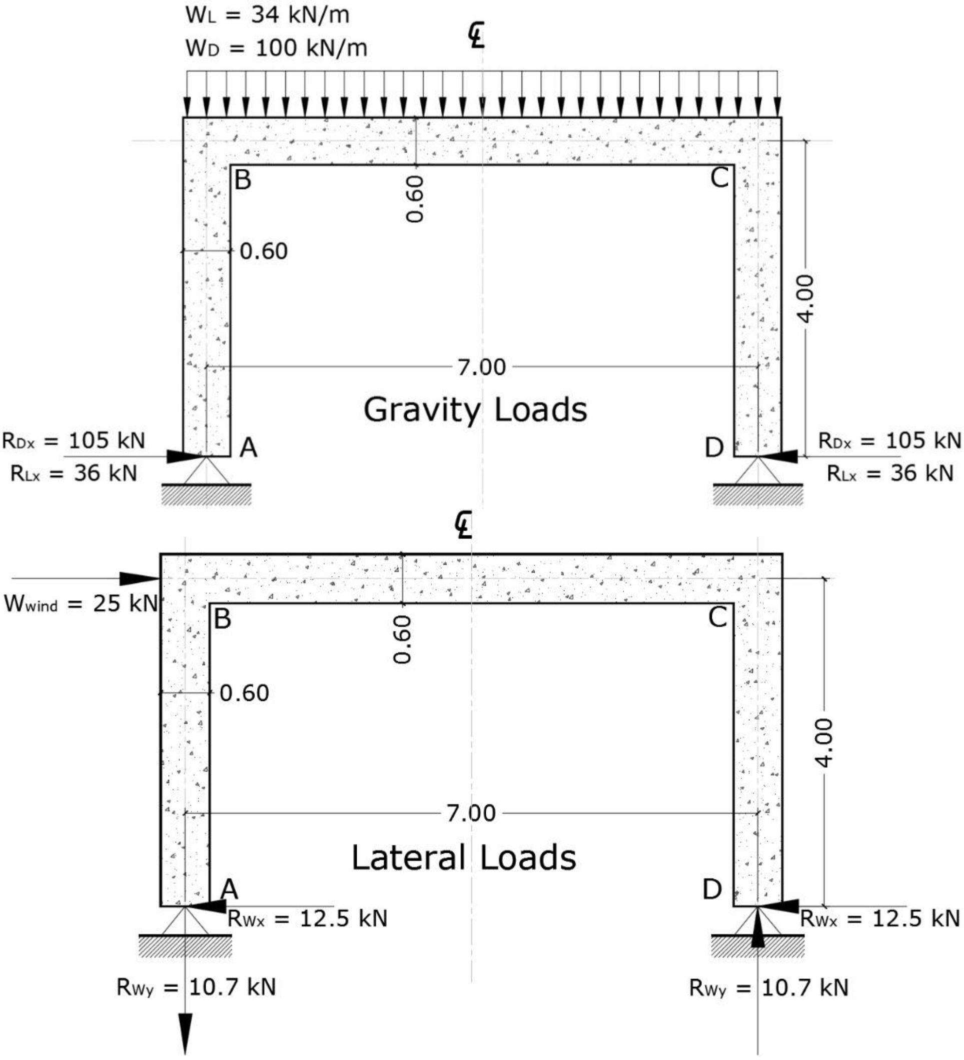


Figure 5.9-3: A portal frame subjected to gravity and lateral forces.

Solution

Compute V$_u$:

<u>V$_u$ @ Left Support (Support B):</u>

To determine if reaction at support is compression or tension, the resultant for R_D, R_L, and R_W should be determined:

$$R_D = (100 \times 7) \times \frac{1}{2} = 350\ kN, R_{Lr} = (34 \times 7) \times \frac{1}{2} = 119\ kN, R_w = -10.7\ kN$$

The ultimate reaction due to indicated load combination would be:

$$R_u = 1.2R_D + 1.6R_{Lr} + 0.8R_W = 1.2 \times 350 + 1.6 \times 119 - 0.8 \times 10.7 = 602\ kN$$

As the ultimate reaction is compressive, therefore shear force can be determined at distance "d" from face of support "B":

$$V_{D\ @\ d\ from\ Support\ B} = 100\frac{kN}{m}\ (7.0 - \frac{0.6}{2} \times 2 - 0.55 \times 2)m \times \frac{1}{2} = 265\ kN$$

$$V_{L\ @\ d\ from\ Support\ B} = 34\frac{kN}{m}\left(7.0 - \frac{0.6}{2} \times 2 - 0.55 \times 2\right)m \times \frac{1}{2} = 90.1\ kN$$

$V_{W\ @\ d\ from\ Support\ B} = R_{Wy} = -10.7\ kN$
$V_u = 1.2V_D + 1.6V_L + 0.8V_W$
$V_{u\ @\ d\ from\ face\ of\ Support\ B} = 1.2 \times 265 + 1.6 \times 90.1 - 0.8 \times 10.7 = 454\ kN$

V_u @ Right Support (Support C):

As all reactions (due to dead, live, and wind) are compression reactions, therefore shear force could be computed at distance "d" from face of right support "C".

$V_{D\ @\ d\ from\ Support\ C} = 100\frac{kN}{m}\ (7.0 - \frac{0.6}{2} \times 2 - 0.55 \times 2)m \times \frac{1}{2} = 265\ kN$

$V_{L\ @\ d\ from\ Support\ C} = 34\frac{kN}{m}\left(7.0 - \frac{0.6}{2} \times 2 - 0.55 \times 2\right)m \times \frac{1}{2} = 90.1\ kN$

$V_{W\ @\ d\ from\ Support\ C} = R_{Wy} = +10.7\ kN$
$V_u = 1.2V_D + 1.6V_L + 0.8V_W$
$V_{u\ @\ d\ from\ face\ of\ Support\ C} = 1.2 \times 265 + 1.6 \times 90.1 + 0.8 \times 10.7 = 471\ kN$

Critical V_u:

$V_u = maximum\ (V_{u\ @\ face\ of\ Support\ B}, V_{u\ @\ d\ from\ face\ of\ Support\ C})$
$V_u = maximum\ (454\ kN, 471\ kN) = 471\ kN$

Compute V_c:

$$V_c = 0.17\left(1 + \frac{N_u}{14A_g}\right)\lambda\sqrt{f_c'}\ b_w d$$

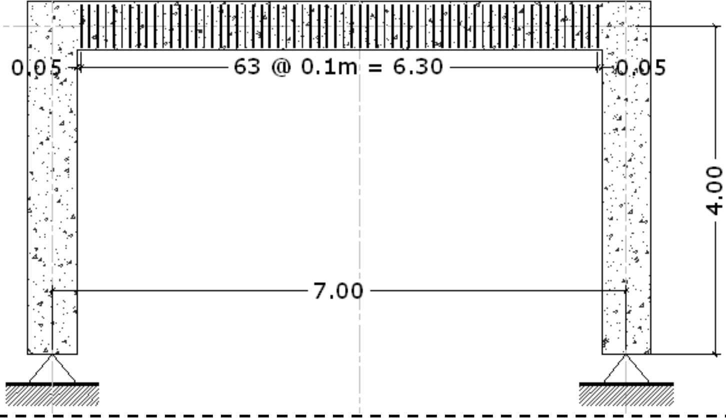
$N_D = R_{Dx} = 105\ kN, N_L = R_{Lx} = 36\ kN$
$N_W = W - R_{Wx} = 25 - 12.5 = 12.5\ kN, N_u = 1.2 \times 105 + 1.6 \times 36 + 0.8 \times 12.5 = 194\ kN$

$$V_c = 0.17\left(1 + \frac{194\ 000}{14(300 \times 600)}\right) \times 1.0 \times \sqrt{21} \times 300 \times 550$$
$$= 0.17(1 + 0.077) \times 1.0 \times \sqrt{21} \times 300 \times 550 = 138\ \text{kN}$$

$\phi V_c = 0.75 \times 138\ kN = 104\ kN$

Shear design for Example 5.9-3	
Region	$\emptyset V_c \leq V_u$
V_s	$= \frac{V_u - \emptyset V_c}{\emptyset} = 0.66\sqrt{f_c'}b_w d \Rightarrow \frac{471 - 104}{0.75}\ ?\ 0.66 \times \sqrt{21} \times 300 \times 550 \Rightarrow 489\ kN < 499\ kN\ Ok$ Beam dimensions are adequate.
$s_{Theoretical}$	$= \frac{A_v f_{yt} d}{V_s} = \frac{226 \times 420 \times 550}{489000} = 107\ mm$
$s_{for\ Av\ minim}$	$minimum\ (\frac{A_v f_{yt}}{0.062\sqrt{f_c'}b_w}\ or\ \frac{A_v f_{yt}}{0.35b_w}) \Rightarrow minimum\ (\frac{226 \times 420}{0.062\sqrt{21} \times 300}\ or\ \frac{226 \times 420}{0.35 \times 300})$ $minimum\ (1\ 114\ or\ 904) = = 904\ mm$
$s_{maximum}$	$V_s \leq 0.33\sqrt{f_c'}b_w d$ $V_s\ ?\ 0.33\sqrt{f_c'}b_w d \Rightarrow V_s = 484 > 0.33\sqrt{21} \times 300 \times 550 = 249\ kN$ $Minimum\ \left[\frac{d}{4}\ or\ 300mm\right] \Rightarrow Minimum\ \left[\frac{550}{4}\ or\ 300mm\right] = 137\ mm$
$s_{Required}$	$Minimum\ [s_{Theoretical}, s_{for\ Av\ minimum}, s_{maximum}]$ $Minimum\ [107\ mm, \quad 904\ mm, \quad 137\ mm] = 107\ mm$ ***Use*** $\boldsymbol{\emptyset 12mm\ @\ 100mm}$

- Stirrups Details:



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Contents

CHAPTER 6
BOND, ANCHORAGE, AND DEVELOPMENT LENGTH

6.1 FUNDAMENTALS OF FLEXURAL BOND

6.1.1 BOND ROLE

- An experiment to show importance of the bond:
  - If the reinforced concrete beam of ***Figure 6.1-1a*** below were constructed using plain round reinforcing bars, and, furthermore, if those bars were to be greased or otherwise lubricated before the concrete were cast, the beam would be very little stronger than if it were built of plain concrete, without reinforcement.
  - If a load were applied, as shown in ***Figure 6.1-1b***, the bars would tend to maintain their original length as the beam deflected. The bars would slip longitudinally with respect to the adjacent concrete, which would experience tensile strain due to flexure.
  - Then, ***the assumption that the strain in an embedded reinforcing bar is the same as that in the surrounding concrete***, would ***not be valid***.
- For reinforced concrete to behave as intended, it is essential that bond forces be developed on the interface between concrete and steel, such as to prevent significant slip from occurring at that interface. ***Figure 6.1-1c*** shows the bond forces that act on the concrete at the interface as a result of bending, while ***Figure 6.1-1d*** shows the equal and opposite bond forces acting on the reinforcement. It is through the action of these interface bond forces that the slip indicated in ***Figure 6.1-1b*** is prevented.

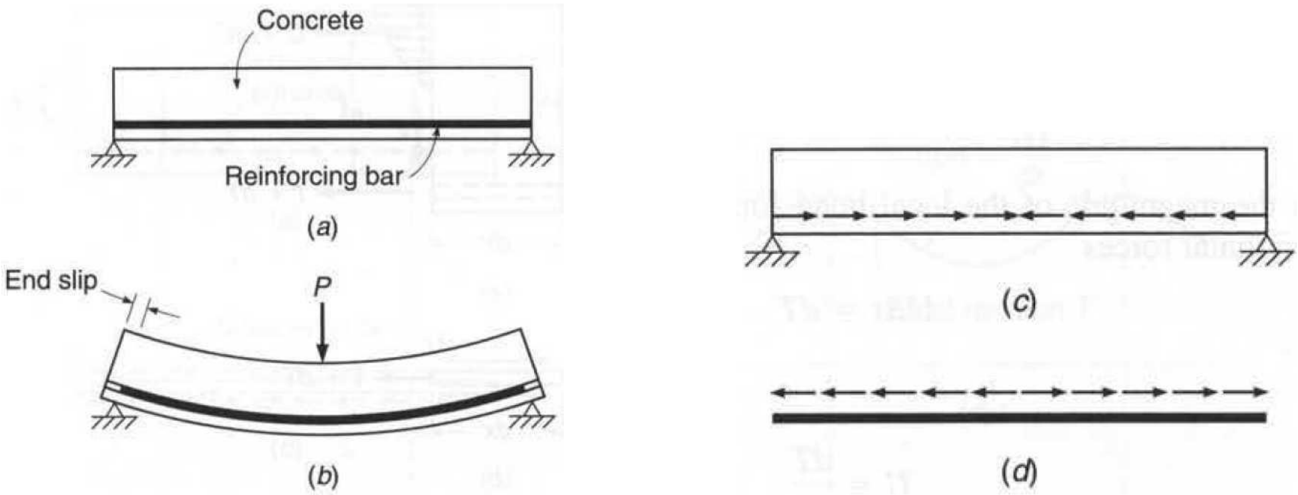


Figure 6.1-1: Bond forces due to flexure: (a) beam before loading; (b) unrestrained slip between concrete and steel; (c) bond forces acting on concrete; (d) bond forces acting on steel.

6.1.2 PLAIN AND DEFORMED REBARS

6.1.2.1 Plain Rebars

- Some years ago, when ***plain bars*** without surface deformations were used, ***initial bond strength*** was provided only by the relatively weak ***chemical adhesion*** and ***mechanical friction*** between steel and concrete. Once adhesion and static friction were overcome at larger loads, small amounts of slip led to interlocking of the natural roughness of the bar with the concrete.
- However, this natural bond strength ***is so low that in beams reinforced with plain bars***, the bond between steel and concrete was frequently broken.
- Such a beam will collapse as the bar is pulled through the concrete. To prevent this, ***end anchorage was provided***, chiefly in the form of hooks, as in ***Figure 6.1-2***.

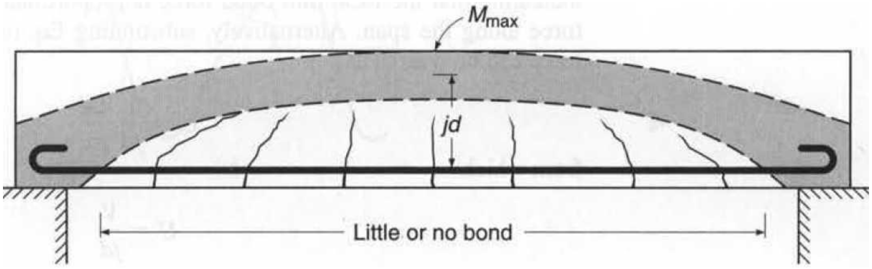


Figure 6.1-2: Tied-arch action in a beam with little or no bond.

- If the anchorage is adequate, such a beam will not collapse, even if the bond is broken over the entire length between anchorages. This is so because the member acts as a ***tied arch***, as shown in ***Figure 6.1-2***, with the uncracked concrete shown shaded representing the arch and the anchored bars the tie-rod.
- Main Disadvantage of Plain Rebars:

  In this case, over the length in which the bond is broken, bond forces are zero. This means that over the entire unbonded length the force in the steel is constant and equal to:

  $$T = \frac{M_{maximum}}{jd}$$

  As a consequence, the total steel elongation in such beams is larger than in beams in which bond is preserved, ***resulting in larger deflections*** and ***greater crack widths***.

6.1.2.2 Deformed Bars

- To improve this situation, deformed bars are now universally used in the United States and many other countries.

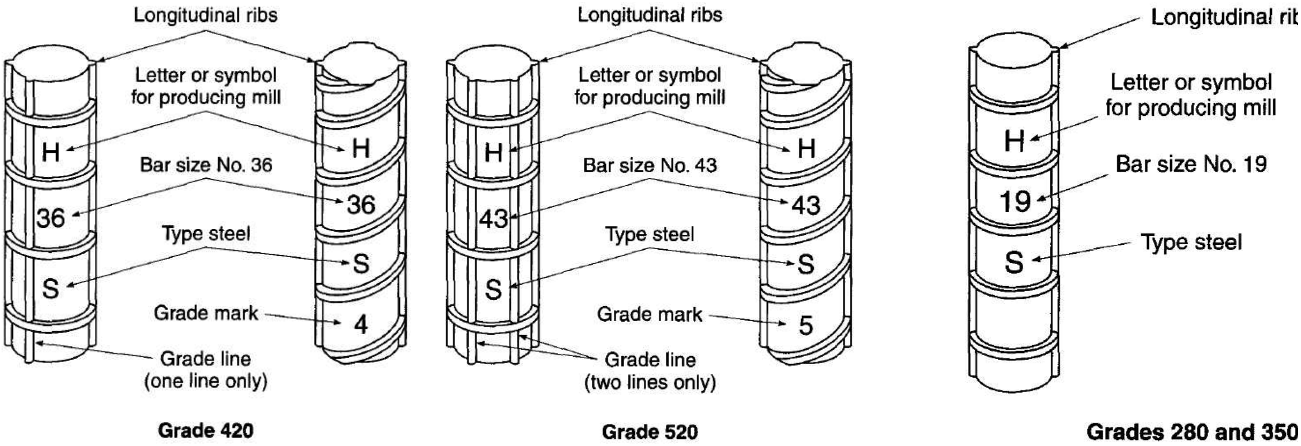


Figure 6.1-3: Marking system for reinforcing bars meeting ASTM Specifications.

6.1.3 Bond Force Based on Simple Cracked Section Analysis

- In a short piece of a beam of length dx, such as shown in ***Figure 6.1-4a***, the moment at one end will generally differ from that at the other end by a small amount dM. If this piece is isolated, and if one assumes that, after cracking, the concrete does not resist any tension stresses, the internal forces are those shown in ***Figure 6.1-4a***.

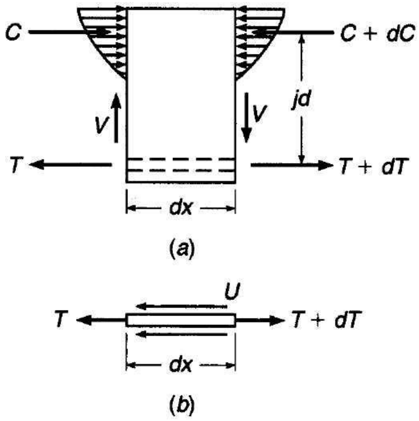


Figure 6.1-4: Forces acting on elemental length of beam: (a) free-body sketch of reinforced concrete element; (b) free-body sketch of steel element.

- Theoretical Relation for Bond Stresses:
  - The change in bending moment, dM, produces a change in the bar force:
    $$dT = \frac{dM}{jd}$$
  - If U is the magnitude of the local bond force per unit length of bar, then, by summing horizontal forces
    $$Udx = dT \Rightarrow U = \frac{dT}{dx}$$
    $$U = \frac{1}{jd}\frac{dM}{dx} \Rightarrow U = \frac{1}{jd}V$$
- Main Conclusions for the Relation:
  - Equation above is the "***elastic cracked section equation***" for flexural bond force, and it indicates that ***the bond force per unit length is proportional to the shear*** at a particular section, i.e., to the rate of change of bending moment.
  - Basic Assumption that Used in Relation:

    Equation assumes that ***concrete zone to be fully cracked***, with the concrete ***resisting no tension***.
- Applicability of Relation:

  The relation applies, therefore, to
  - The tensile bars in simple spans,
  - The continuous spans, either to the bottom bars in the positive bending region between inflection points or to the top bars in the negative bending region between the inflection points and the supports,
  - It does not apply to compression reinforcement.

6.1.4 ACTUAL DISTRIBUTION OF FLEXURAL BOND FORCE:

- The actual distribution of bond force along deformed reinforcing bars is ***much more complex than that represented by*** $U = \frac{1}{jd}V$.
- Beam with Pure Bending:

  According to $U = \frac{1}{jd}V$, beam with ***pure bending has no bond stresses***, <u>***but***</u> ***as the concrete fails*** to resist tensile stresses only where the actual crack is located and as between cracks, the concrete does resist moderate amounts of tension, therefore ***bond stresses distribution will be as shown*** in ***Figure 6.1-5*** below:

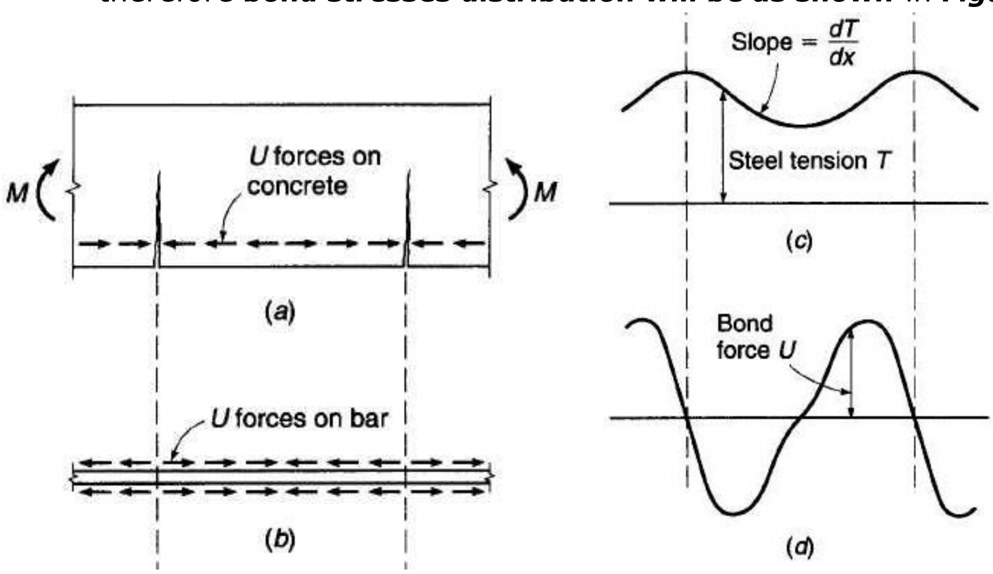


Figure 6.1-5: Variation of steel and bond forces in a reinforced concrete member subject to pure bending: (a) cracked concrete segment; (b) bond forces acting on reinforcing bar; (c) variation of tensile force in steel; (d) variation of bond force along steel.

- Beams with Bending and Shear Forces:
  - Beams are seldom subject to pure bending moment; they generally carry transverse loads producing shear and moment that vary along the span. ***Figure***

6.1-6a shows a beam carrying a distributed load. The cracking indicated is ***typical***.

- The steel force T predicted by simple cracked section analysis is proportional to the moment diagram and is as shown by the dashed line in ***Figure 6.1-6b***.
- However, the actual value of T is ***less than*** that predicted by the simple analysis everywhere except at the actual crack locations.
- In ***Figure 6.1-6c***, the bond forces predicted by the simplified theory are shown by the dashed line, and the actual variation is shown by the solid line.

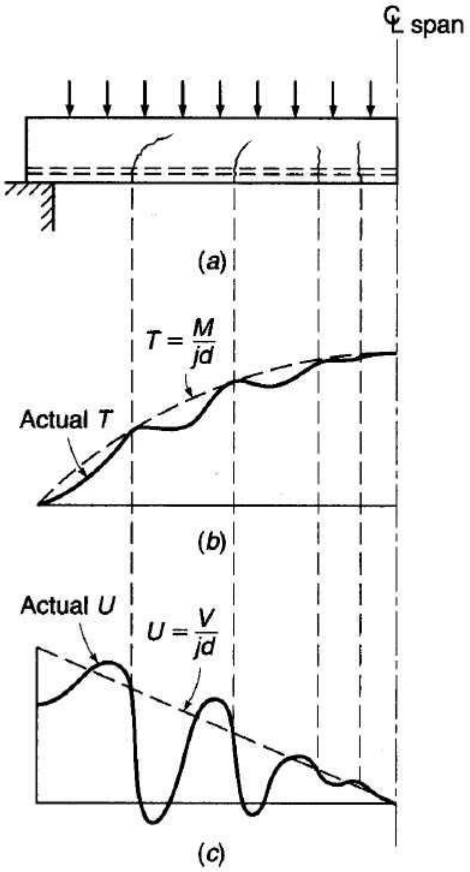


Figure 6.1-6: Effect of flexural cracks on bond forces in beam: (a) beam with flexural cracks; (b) variation of tensile force Tin steel along span; (c) variation of bond force per unit length U along span.

6.1.5 Main Conclusion About Bond Stresses

It is evident that actual bond forces in beams bear very little relation to those predicted by $U = \frac{1}{jd} V$, except in the general sense that they are highest in the regions of high shear.

6.1.6 Bond Strength

For reinforcing bars in tension, two types of bond failure have been observed:

6.1.6.1 Pullout Mode

- Occurs when ***ample confinement*** is provided by the surrounding concrete.
- It could be expected when ***relatively small-diameter bars*** are used with sufficiently large concrete cover distances and bar spacing.

6.1.6.2 Splitting Mode

- Occurs along the bar when cover, confinement, or bar spacing is insufficient to resist the lateral concrete tension resulting from the ***wedging effect*** of the bar deformations.
- It is ***more common*** in beams than direct pullout.
- It may occur either in a vertical plane as in ***Figure 6.1-7a*** or horizontally in the plane of the bars as in ***Figure 6.1-7b***.

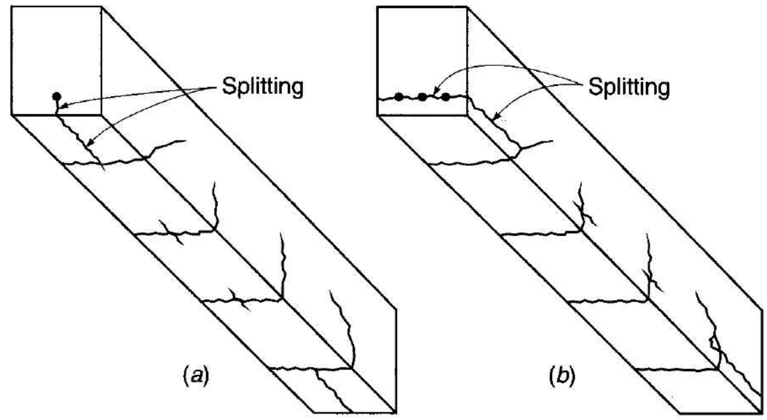


Figure 6.1-7: **Splitting of concrete along reinforcement.**

6.1.7 CONCEPT OF DEVELOPMENT LENGTH

- Based on above discussion, ***local failures result in small local slips*** and some widening of cracks and increase of deflections but ***will be harmless as long as failure does not propagate all along the bar, with resultant total slip***.
- This fact suggests ***the concept of development length of a reinforcing bar*** which ***could be defined as that length of embedment necessary to develop the full tensile strength of the bar, controlled by either pullout or splitting***.

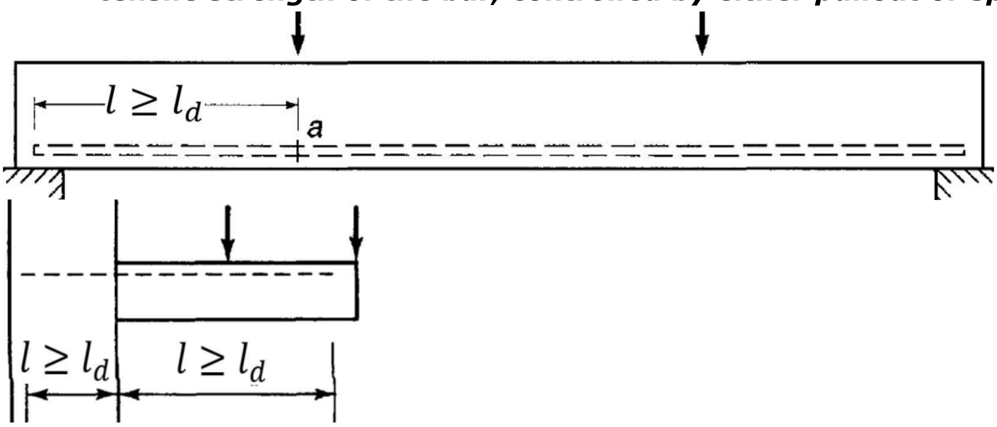


Figure 6.1-8: Concepts of development length.

6.2 ACI Provisions for Development of Reinforcement

6.2.1 Strength Reduction Factor, ϕ

- According to (ACI318M, 2014), article ***25.4.1.3***, ***the strength reduction factor*** ϕ ***is not used in the development length and lap splice length equations***.
- An allowance for strength reduction ***is already included*** in the expressions for determining development and splice lengths.

6.2.2 Maximum Value for f_c'

- According to (ACI318M, 2014), article ***25.4.1.4***, the values of $\sqrt{f_c'}$ used to calculate development length shall not ***exceed 8.3 MPa***. This equivalent to a f_c' of 68.9 MPa.
- Why this limitation:
  - Tests show that the force developed in a bar in development and lap splice tests increases at a lesser rate than $\sqrt{f_c'}$ with increasing compressive strength.
  - Using $\sqrt{f_c'}$, however, is sufficiently accurate for values of $\sqrt{f_c'}$ up to 8.3 MPa.
  - ACI Committee 318 has chosen not to change the exponent applied to the compressive strength used to calculate development and lap splice lengths, but rather to set an upper limit of 8.3 MPa on $\sqrt{f_c'}$.

6.3 ACI Code Provisions for Development of Tension Reinforcement

6.3.1 Basic Equation for Development of Tension Bars

- According to ACI Code **25.4.2.3**, for deformed bars or deformed wires,

However, the product $\psi_t\psi_e$ need not be greater than 1.7.

(a) Where horizontal reinforcement is placed such that more than 300 mm of fresh concrete is cast below the development length or splice, $\psi_t = 1.3$. For other situations, $\psi_t = 1.0$.

(b) For epoxy-coated bars or wires with cover less than $3d_b$, or clear spacing less than $6d_b$, $\psi_e = 1.5$. For all other epoxy-coated bars or wires, $\psi_e = 1.2$. For uncoated and zinc-coated (galvanized) reinforcement, $\psi_e = 1.0$.

$$\ell_d = \left(\frac{f_y}{1.1\lambda\sqrt{f_c'}}\frac{\psi_t\psi_e\psi_s}{\left(\frac{c_b+K_{tr}}{d_b}\right)}\right)d_b$$

λ is modification factor to reflect the reduced mechanical properties of lightweight concrete relative to normalweight concrete of the same compressive Strength. Estimated either from Table 19.2.4.2 or from relation below:

$$\lambda = f_{ct}/(0.56\sqrt{f_c'}) \le 1.0$$

where f_{ct} is the average splitting tensile strength of lightweight concrete.

in which the confinement term $(c_b + K_{tr})/d_b$ shall not be taken greater than 2.5, and

(c) For No. 19 and smaller bars and deformed wires, $\psi_s = 0.8$. For No. 22 and larger bars, $\psi_s = 1.0$.

- Confinement term has been explained in more detailed below:

c_b = smaller of: (a) the distance from center of a bar or wire to nearest concrete surface, and (b) one-half the center-to-center spacing of bars or wires being developed, mm,

$$\left(\frac{c_b+K_{tr}}{d_b}\right) \qquad K_{tr} = \frac{40A_{tr}}{sn}$$

A_{tr} = total cross-sectional area of all transverse reinforcement within spacing s that crosses the potential plane of splitting through the reinforcement being developed, mm^2,

where n is the number of bars or wires being spliced or developed along the plane of splitting. It shall be permitted to use $K_{tr} = 0$ as a design simplification even if transverse reinforcement is present.

s = maximum spacing of transverse reinforcement within l_d center to center,

- Excess reinforcement:

  According to (ACI318M, 2014), article ***25.4.10.1***, reduction of development lengths shall be permitted by use of the ratio (As,required)/(As,provided).

- Important Notes on ACI Basic Equation:
  - Above single basic equation includes all the influences ***thus appears highly complex because of its inclusiveness***.
  - However,

  1. It does permit the designer to see the effects of all the controlling variables
  2. It allows more rigorous calculation of the required development length when it is critical.

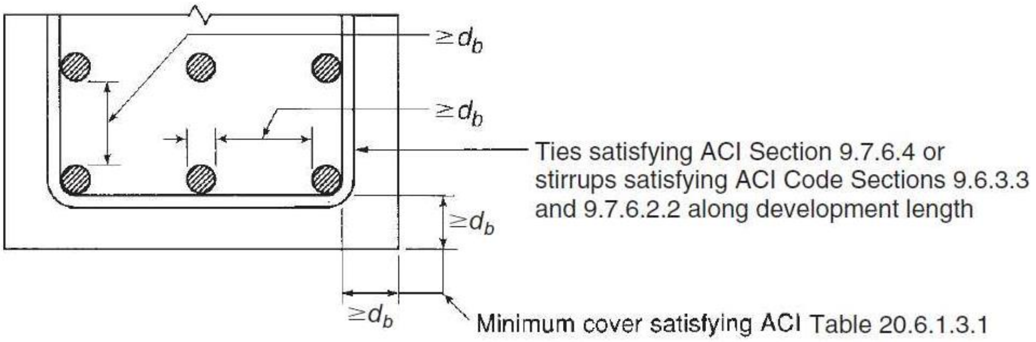
6.3.2 SIMPLIFIED EQUATIONS FOR DEVELOPMENT LENGTH

- Calculation of required development length (in terms of bar diameter) by above basic equation requires that the term $(c_b + K_{tr})/d_b$ be calculated for each particular combination of cover, spacing, and transverse reinforcement.
- Alternatively, according to the Code, article ***25.4.2.2***, a simplified form of basic equation may be used in which $\mathbf{(c_b + K_{tr})/d_b}$ is set equal to ***1.5***, provided that certain restrictions are placed on cover, spacing, and transverse reinforcement. These requirements have been presented in term of confinement cases 1 and 2 that indicated in below.
- If confinement cases 1 and 2 are not satisfied, a confinement factor $\mathbf{(c_b + K_{tr})/d_b}$ of 1.0 is adopted.

Table 6.3-1: Simplified ACI Relations for Development Length (Table 25.4.2.2 of (ACI318M, 2014)).

	Spacing and cover	No. 19 and smaller bars and deformed wires ($\Psi s = 0.8$)	No. 22 and larger bars ($\Psi s = 1.0$)	
Case 1 Case 2	Clear spacing of bars or wires being developed or spliced not less than d_b, clear cover not less than d_b, and stirrups or ties throughout ℓ_d not less than the Code minimum or Clear spacing of bars or wires being developed or spliced not less than $2d_b$ and clear cover not less than d_b	$\left(\dfrac{f_y \psi_t \psi_e}{2.1\lambda\sqrt{f'_c}}\right) d_b$	$\left(\dfrac{f_y \psi_t \psi_e}{1.7\lambda\sqrt{f'_c}}\right) d_b$	← $(c_b + k_{tr})/d_b = 1.5$
	Other cases	$\left(\dfrac{f_y \psi_t \psi_e}{1.4\lambda\sqrt{f'_c}}\right) d_b$	$\left(\dfrac{f_y \psi_t \psi_e}{1.1\lambda\sqrt{f'_c}}\right) d_b$	← $(c_b + k_{tr})/d_b = 1.0$

- Case 1 and Case 2 have been presented graphically in ***Figure 6.3-1*** below:



(a) Case 1.

Figure 6.3-1: Explanation of Cases 1 and 2.

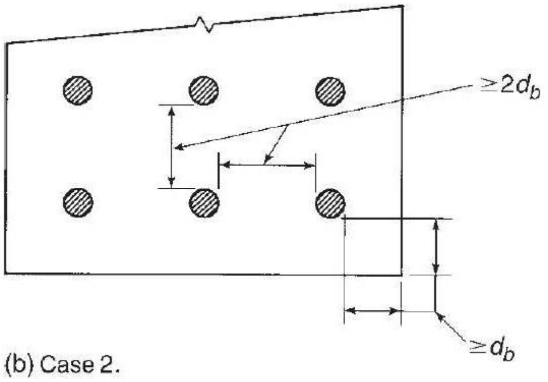


Figure 6.3-1: Explanation of Cases 1 and 2. Continue.

6.3.3 Summary of ACI Modification Factors of Deformed Bars in Tension

ACI modification factors adopted in basic equation, ***Article 6.3.1***, and simplified equations, ***Article 6.3.2***, have been summarized in ***Table 6.3-2***.

Table 6.3-2: Modification factors for development of deformed bars and deformed wires in tension, Table 25.4.2.4 of (ACI318M, 2014).

Modification factor	Condition	Value of factor
Lightweight λ	Lightweight concrete	0.75
	Lightweight concrete, where f_{ct} is specified	In accordance with 19.2.4.3
	Normalweight concrete	1.0
Epoxy[1] ψ_e	Epoxy-coated or zinc and epoxy dual-coated reinforcement with clear cover less than $3d_b$ or clear spacing less than $6d_b$	1.5
	Epoxy-coated or zinc and epoxy dual-coated reinforcement for all other conditions	1.2
	Uncoated or zinc-coated (galvanized) reinforcement	1.0
Size ψ_s	No. 22 and larger bars	1.0
	No. 19 and smaller bars and deformed wires	0.8
Casting position[1] ψ_t	More than 300 mm of fresh concrete placed below horizontal reinforcement	1.3
	Other	1.0

[1]The product $\psi_t\psi_e$ need not exceed 1.7.

6.3.4 Further Simplified Tabular Values for Development Length

- Further simplifications are possible for the most common condition of
  - Normal density concrete ($\lambda = 1.0$),
  - Uncoated reinforcement ($\psi_e = 1.0$).
- With these simplifications, the development lengths, in terms of bar diameters, would be a function of f_c', f_y and the bar location factor ψ_t.
- Thus, development lengths are easily tabulated for the
  - Usual combinations of material strengths,
  - Bottom or top bars,
  - And for the restrictions on bar spacing, cover, and transverse steel defined.
- Results are given in ***Table 6.3-3*** below.

Table 6.3-3: Further Simplified tension development length in bar diameters l_d/d_b for uncoated bars and normal weight concrete, adopted from (Nilson, Design of Concrete Structures, 14th Edition, 2010):

		No. 6 (No. 19) and Smaller[a]			No. 7 (No. 22) and Larger		
		f'_c, MPa			f'_c, MPa		
	f_y, MPa	28	35	42	28	35	42
(1) Bottom bars							
Spacing, cover and ties as per Case *a* or *b*	280	25	23	21	32	28	26
	350	32	28	26	40	35	32
	420	38	34	31	47	42	39
Other cases	280	38	34	31	47	42	39
	350	47	42	39	59	53	48
	420	57	51	46	71	64	58
(2) Top bars							
Spacing, cover and ties as per Case *a* or *b*	280	33	29	27	41	37	34
	350	41	37	34	51	46	42
	420	49	44	40	62	55	50
Other cases	280	49	44	40	62	55	50
	350	62	55	50	77	69	63
	420	74	66	60	92	83	76

Case *a*: Clear spacing of bars being developed or spliced $\geq d_b$, clear cover $\geq d_b$, and stirrups or ties throughout l_d not less than the Code minimum.

Case *b*: Clear spacing of bars being developed or spliced $\geq 2d_b$, and clear cover not less than d_b.

[a]ACI Committee 408 recommends that the values indicated for bar sizes No. 7 (No. 22) and larger be used for all bar sizes.

6.3.5 Notes on the Three Different Methods

- When the confinement term, $(c_b + K_{tr})/d_b$, differs from the assumed values of 1.5 or 1.0, there would be a significant difference between values of the basic method and those of the other two methods.
- Slight differences between second and third methods are due to the unit systems. Where values for the third method have been originally prepared in US customary system.

6.3.6 ACI Lower Bound Limitation on l_d for Tension Rebars

According to ACI 25.4.2.1 l_d shall not be less than 300 mm.

6.3.7 Design Examples for Rebars in Tension

Example 6.3-1

Figure 6.3-2 below shows a beam-column joint in a continuous building frame with the following data:

- Based on frame analysis, the negative steel required at the end of the beam is 1870mm^2; two No. 36 bars are used providing A_s of 2012mm^2.
- The design will include No. 10 stirrups spaced four at 75mm, followed by a constant 125mm spacing in the region of the support, with 40mm clear cover.
- Normal weight concrete is to be used, with $f'_c = 28\ MPa$, and reinforcing bars have $f_y = 420\ MPa$.

Find the minimum distance l_d at which the negative bars can be cut off, based on development of the required steel area at the face of the column, using:

- The simplified equations,
- Further simplified tabulated values,
- The basic equation.

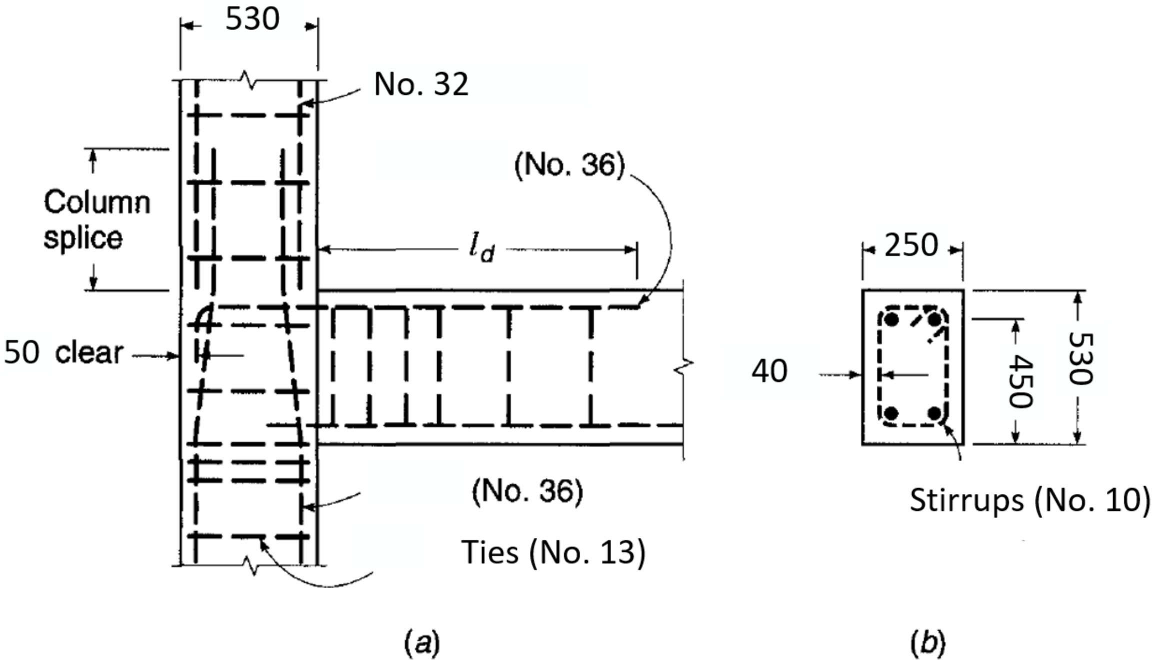


Figure 6.3-2: Bar details at beam-column joint for bar development examples.

Solution

- The simplified equations:

Checking for lateral spacing in the No. 36 bars determines that the clear distance between the bars is:

$$\frac{Clear\ distance}{d_b}=\frac{250-40\times 2-2\times 10-2\times 36}{36}=\ 2.17>2d_b$$

$$\frac{Clear\ side\ cover}{d_b}=\frac{40+10}{36}=1.39>d_b$$

$$\frac{Clear\ top\ cover}{d_b}=\frac{530-450-\frac{36}{2}}{36}=1.72$$

These dimensions meet the restrictions stated in the second row of Table 6.3-1, and as $d_b>No.22$ then:

Spacing and cover	No. 19 and smaller bars and deformed wires	No. 22 and larger bars
Clear spacing of bars or wires being developed or spliced not less than d_b, clear cover not less than d_b, and stirrups or ties throughout ℓ_d not less than the Code minimum or Clear spacing of bars or wires being developed or spliced not less than $2d_b$ and clear cover not less than d_b	$\left(\frac{f_y\Psi_t\Psi_e}{2.1\lambda\sqrt{f_c'}}\right)d_b$	$\left(\frac{f_y\Psi_t\Psi_e}{1.7\lambda\sqrt{f_c'}}\right)d_b$
Other cases	$\left(\frac{f_y\Psi_t\Psi_e}{1.4\lambda\sqrt{f_c'}}\right)d_b$	$\left(\frac{f_y\Psi_t\Psi_e}{1.1\lambda\sqrt{f_c'}}\right)d_b$

$$l_d=\left(\frac{f_y\psi_t\psi_e}{1.7\lambda\sqrt{f_c{}'}}\right)d_d$$

For uncoated top bars with normal-density concrete:

$\psi_t=1.3, \psi_e=1.0, \lambda=1.0$

$$l_d=\left(\frac{420\times 1.3\times 1.0}{1.7\times 1.0\times\sqrt{28}}\right)d_d=60.7d_b=2185\ mm$$

This can be reduced by the ratio of steel required to that provided, so that the final development length is

$$l_d = 2185 \times \frac{1870}{2012} = 2031\, mm > 300\text{mm} \therefore \text{Ok.} \blacksquare$$

- Further simplified tabulated values:

	f_y, MPa	No. 6 (No. 19) and Smaller[a] f_c', MPa 28	35	42	No. 7 (No. 22) and Larger f_c', MPa 28	35	42
(1) Bottom bars							
Spacing, cover and ties as per Case *a* or *b*	280	25	23	21	32	28	26
	350	32	28	26	40	35	32
	420	38	34	31	47	42	39
Other cases	280	38	34	31	47	42	39
	350	47	42	39	59	53	48
	420	57	51	46	71	64	58
(2) Top bars							
Spacing, cover and ties as per Case *a* or *b*	280	33	29	27	41	37	34
	350	41	37	34	51	46	42
	420	49	44	40	62	55	50
Other cases	280	49	44	40	62	55	50
	350	62	55	50	77	69	63
	420	74	66	60	92	83	76

Case *a*: Clear spacing of bars being developed or spliced $\geq d_b$, clear cover $\geq d_b$, and stirrups or ties throughout l_d not less than the Code minimum.

Case *b*: Clear spacing of bars being developed or spliced $\geq 2d_b$, and clear cover not less than d_b.

[a] ACI Committee 408 recommends that the values indicated for bar sizes No. 7 (No. 22) and larger be used for all bar sizes.

$$\frac{l_d}{d_b} = 62 \Longrightarrow l_d = 62 \times 36 \times \frac{1870}{2012} = 2074mm > 300mm \therefore Ok. \blacksquare$$

- The basic equation:

$$l_d = \left(\frac{f_y}{1.1\lambda\sqrt{f_c'}} \frac{\psi_t \psi_e \psi_s}{\frac{c_b + K_{tr}}{d_b}} \right) d_b$$

$$c_b = minimum\left(side\ cover\ to\ center\ of\ bar, top\ cover\ to\ center\ of\ bar, \frac{1}{2} S_{\frac{c}{c}}\right)$$

$$c_b = minimum\left(40 + 10 + \frac{36}{2}, 530 - 450, \frac{1}{2} \times (250 - 40 \times 2 - 10 \times 2 - \frac{36}{2} \times 2)\right)$$

$$c_b = minimum(68, 80, 57) = 57\ mm$$

The smallest of these three distances controls and $c_b = 57mm$. Potential splitting would be in the horizontal plane of the bars.

$$K_{tr} = \frac{40A_{tr}}{sn} = \frac{40 \times 2 \times \frac{(\pi \times 10^2)}{4}}{125 \times 2} = 25$$

$$Confinement\ Term = \frac{c_b + K_{tr}}{d_b} = \frac{57 + 25}{36} = 2.27 < 2.5 \therefore Ok.$$

$$\because d_b > 19 \therefore \psi_s = 1.0$$

$$l_d = \left(\frac{420}{1.1 \times 1.0 \times \sqrt{28}} \times \frac{1.3 \times 1.0 \times 1.0}{2.27}\right) d_b = 41.3 d_d = 41.3 \times 36 = 1487\ mm$$

This can be reduced by the ratio of steel required to that provided, so that the final development length is

$$l_d = 1487 \times \frac{1870}{2012} = 1382\ mm > 300mm \therefore Ok \blacksquare$$

- Main conclusions about different approaches to compute l_d:
  - Clearly, the use of the more accurate equation permits a considerable reduction in development length.
  - Even though its use requires much more time and effort, it is justified if the design is to be repeated many times in a structure.

Example 6.3-2

Figure 6.3-3 below shows cross-section of a simply supported beam reinforced with four No. 25 bars that are confined with No. 10 stirrups spaced at 150mm. Determine the development length of the bars if the beam is made of normal-weight concrete, bars are not coated, $f_c' = 21\ MPa$, and $f_y = f_{yt} = 420\ MPa$.

In your solution, use:

- The simplified equations,
- Simplified tabulated values,
- The basic equation.

In your solution, assume that $S_{maximum}$ could be computed based on:

$$S_{maximum} = minimum\left(\frac{d}{2}, 600\right)$$

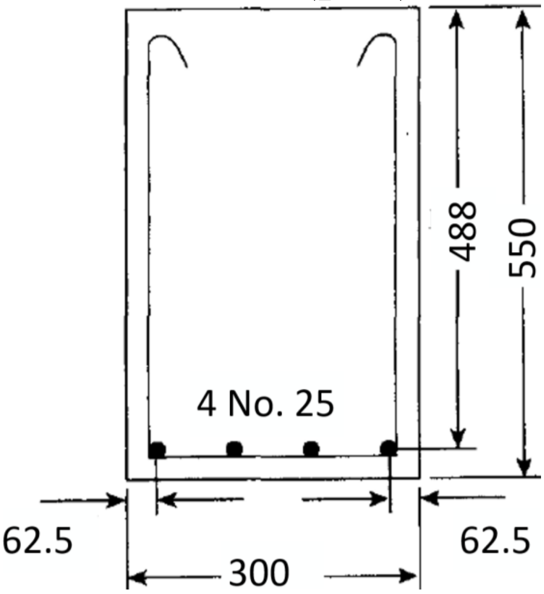
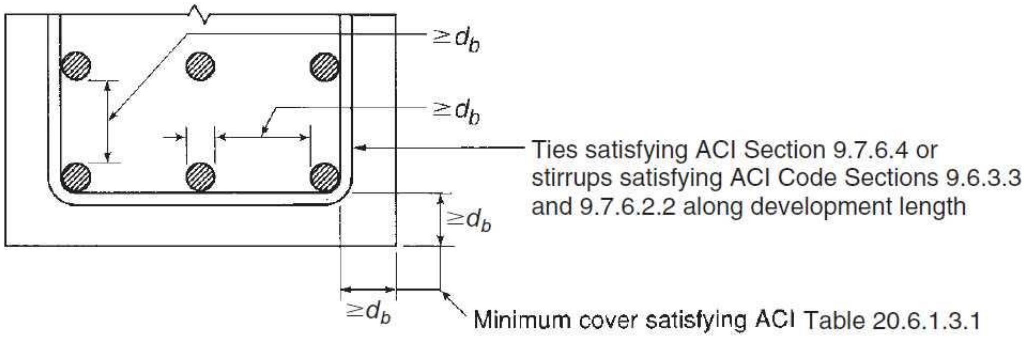


Figure 6.3-3: Beam Cross Section for Example 6.3-2.

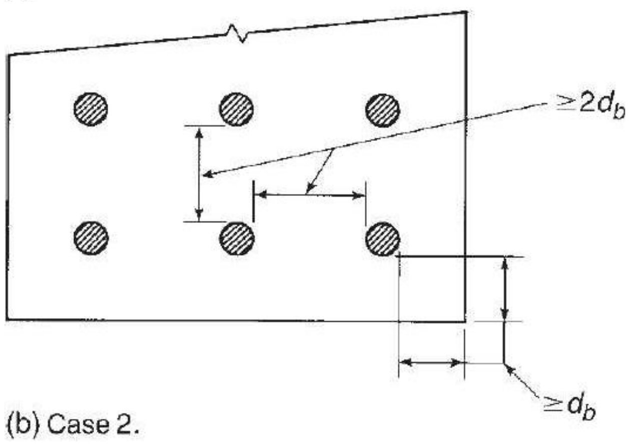
Solution

- The Simplified Relation:
  - Case for Spacing and Concrete Cover:

    Check conditions to see if spacing and concrete cover are to be classified as case 1, case 2, or other cases.



(a) Case 1.



(b) Case 2.

- For No. 25 bars, $d_b = 25mm$.

  $$Clear\ cover = 62.5 - \frac{25}{2} = 50mm > d_b$$

  $$Clear\ spacing\ between\ bars = \frac{300 - 2 \times 62.5 - 3 \times 25}{3} = 33.3 > d_b$$

  $$\because Clear\ Spacing = 33.3 < 2d_b$$

- Therefore, the provided stirrups should be compared with minimum limitations required by ACI Code:

  $$S_{maximum} = minimum\left(\frac{488}{2}, 600\right) = 244 > 150mm \therefore Ok.$$

  $$S_{\text{for Av minimum}} = \text{minimum}\ \left(\frac{A_v f_{yt}}{0.062\sqrt{f_c{'}} b_w} \text{ or } \frac{A_v f_{yt}}{0.35 b_w}\right)$$

  $$S_{\text{for Av minimum}} = \text{minimum}\ \left(\frac{\frac{\pi \times 10^2}{4} \times 2 \times 420}{0.062\sqrt{21} \times 300} \text{ or } \frac{\frac{\pi \times 10^2}{4} \times 2 \times 420}{0.35 \times 300}\right)$$

  $$S_{\text{for Av minimum}} = \text{minimum}\ (774 \text{ or } 628) = 628\ mm > 150mm \therefore Ok.$$

  Then, rebars confinement could be classified as case 1, and as bar diameter is greater 22mm, then development could be computed based on following relation:

  $$l_d = \left(\frac{f_y \psi_t \psi_e}{1.7\lambda\sqrt{f_c{'}}}\right) d_b$$

  For bottom rebars:

  $\psi_t = 1.0$

  For uncoated rebars:

  $\psi_e = 1.0$

  For normal weight concrete:

  $\lambda = 1.0$

  $$l_d = \left(\frac{1.0 \times 1.0 \times 420}{1.7 \times 1.0 \times \sqrt{21}}\right) d_d = 53.9 d_b = 53.9 \times 25 \Rightarrow l_d = 1348\ mm > 300mm\ ■$$

- Simplified Tabulated Values:

  As $f_c' = 21\ MPa$ has not been tabulated within the Table, then tabulated values cannot be used in this example.

- The basic equation:

$$l_d = \left(\frac{f_y}{1.1\lambda\sqrt{f_c{'}}} \frac{\psi_t \psi_e \psi_s}{\frac{c_b + K_{tr}}{d_b}}\right) d_b$$

$$K_{tr} = \frac{40 A_{tr}}{sn} = \frac{40 \times 2 \times \frac{(\pi \times 10^2)}{4}}{150 \times 4} = 10.5$$

$$c_b = minimum\left(side\ cover\ to\ center\ of\ bar, bottom\ cover\ to\ center\ of\ bar, \frac{1}{2} S_{\frac{c}{c}}\right)$$

$$c_b = minimum\left(62.5, 550 - 488, \left((300 - 2 \times 62.5) \times \frac{1}{3}\right) \times \frac{1}{2}\right)$$

$$c_b = minimum(62.5, 62, 29.2) = 29.2$$

$$\frac{c_b + K_{tr}}{d_b} = \frac{29.2 + 10.5}{25} = 1.59 < 2.5 \therefore Ok.$$

$$l_d = \left(\frac{420}{1.1 \times 1.0 \times \sqrt{21}} \frac{1.0 \times 1.0 \times 1.0}{1.59}\right) d_b = 52.4\ d_b$$

$$l_d = 52.4 \times 25 = 1310\ mm > 300mm \therefore Ok.\ ■$$

Example 6.3-3

Repeat ***Example 6.3-2*** if the beam is made of lightweight aggregate concrete, the bars are epoxy coated, and A_s required from analysis is 1800 mm^2. In your solution, use the simplified equations.

Solution

As confinement is same as Example 6.3-2, then confinement case could be classified as case 1, and development length could be computed based on following relation:

$$l_d = \left(\frac{f_y \psi_t \psi_e}{1.7\lambda\sqrt{f_c'}}\right) d_d$$

For bottom rebars:

$\psi_t = 1.0$

For coated rebars, and with cover less than 3d, then:

$\psi_e = 1.5$

According to ACI:

$\psi_t \psi_e = 1.0 \times 1.5 = 1.5 < 1.7 \therefore Ok.$

For a lightweight aggregate concrete:

$\lambda = 0.75$

$$l_d = \left(\frac{1.0 \times 1.5 \times 420}{1.7 \times 0.75 \times \sqrt{21}}\right) d_b \Rightarrow l_d = 108 d_d$$

Development could be reduced by ratio of steel required to that provided,

$$l_d = 108 \times \frac{1800}{4 \times \left(\pi \times \frac{25^2}{4}\right)} d_b$$

$$l_d = 108 \times \frac{1800}{4 \times \left(\pi \times \frac{25^2}{4}\right)} d_b = 99 \times 25 = 2475\, mm > 300mm \therefore Ok. \blacksquare$$

Example 6.3-4

Check adequacy of the embedded length of $1000mm$ indicated in ***Figure 6.3-4*** for development requirement of tension member. In your checking assume the following:

- $f_c' = 28\, MPa$ and $f_y = 420\, MPa$,
- Use ACI basic equation,
- Uncoated rebar,
- $A_{s\, required}/A_{s\, provided} \approx 1.0$.

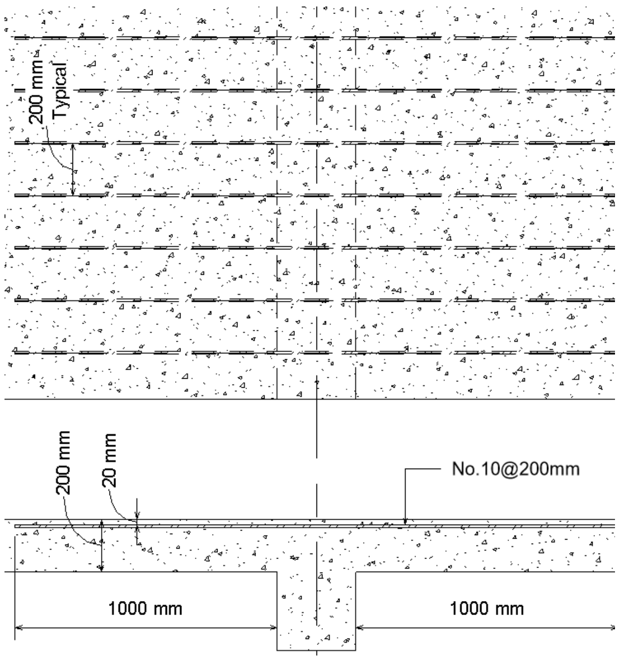


Figure 6.3-4: Slab rebars for Example 6.3-4.

Solution

According to the ACI basic relation, the development length for tension is:

$$l_d = \left(\frac{f_y}{1.1\lambda\sqrt{f_c'}} \frac{\psi_t \psi_e \psi_s}{\frac{c_b + K_{tr}}{d_b}}\right) d_b$$

As no more 300mm of fresh concrete is cast below the rebar, therefore $\psi_t = 1.0$. For the uncoated rebars, $\psi_e = 1.0$. As the rebars have a size smaller than No.22, $\psi_s = 0.8$. Regarding the confinement term, as there is no shear reinforcement, therefore $K_{tr} = 0$. The c_b is:

$$c_b = minimum\left(\left(\frac{10}{2} + 20\right) or \frac{200}{2}\right) = 25mm \Rightarrow \frac{c_b + K_{tr}}{d_b} = \frac{(25 + 0)}{10} = 2.5 \not> Code\ limit\ of\ 2.5 \therefore Ok.$$

The development length would be:

$$l_d = \left(\frac{f_y}{1.1\lambda\sqrt{f_c'}} \frac{\psi_t \psi_e \psi_s}{\frac{c_b + K_{tr}}{d_b}}\right) d_b = \left(\left(\frac{420}{1.1 \times 1.0 \times \sqrt{28}}\right) \times \left(\frac{1.0 \times 1.0 \times 0.8}{2.5}\right)\right) d_b = 23 d_b = 23 \times 10$$

$$= 230\, mm < 300mm \therefore No\ Ok.$$

$\therefore l_d = minimum\ value\ according\ to\ ACI\ code = 300mm < l_{\text{embedded}} = 1000\, mm \therefore Ok$

Therefore, the proposed embedment is adequate from bond point of view but for the final decision, it should be checked for the requirements of the cutoff points.

Example 6.3-5

Use ACI basic relation to determine the development length of the bottom tensile rebars indicated in ***Figure 6.3-5***. In your solution assumes that $A_{s\,required} \approx 750\,mm^2$

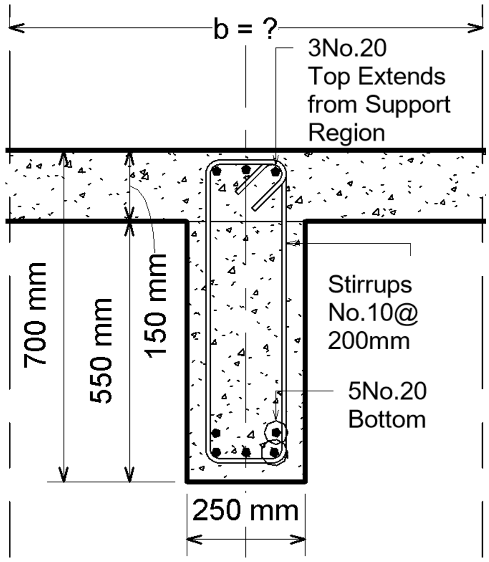


Figure 6.3-5: Beam section for Example 6.3-5.

Solution

The basic equation:

$$l_d = \left(\frac{f_y}{1.1\lambda\sqrt{f_c'}}\frac{\psi_t\psi_e\psi_s}{\frac{c_b+K_{tr}}{d_b}}\right)d_b$$

$$c_b = minimum\left(\begin{array}{c} side\ cover\ to\ center\ of\ bar, \\ top\ cover\ to\ center\ of\ bar, \frac{1}{2}S_{\frac{c}{c}} \end{array}\right)$$

$$c_b = minimum\left(40+10+\frac{20}{2}, 40+10+\frac{20}{2}, \frac{1}{2}\times(250 - 40\times 2 - 10\times 2 - \frac{20}{2}\times 2)\right)$$

$$c_b = minimum(60, 60, 65) = 60\ mm$$

The smallest of these three distances controls and $c_b = 60mm$.

$$K_{tr} = \frac{40A_{tr}}{sn} = \frac{40\times 2\times\frac{(\pi\times 10^2)}{4}}{150\times 5} = 8.38$$

$$Confinement\ Term = \frac{c_b+K_{tr}}{d_b} = \frac{60+8.38}{20} = 3.42 > 2.5 \therefore \text{Not } Ok.$$

Therefore, use confinement term of 2.5.

For uncoated bottom bars with normal-density concrete:

$\psi_t = 1.0, \psi_e = 1.0, \lambda = 1.0$

$\because d_b > 19 \therefore \psi_s = 1.0$

$$l_d = \left(\frac{420}{1.1\times 1.0\times\sqrt{28}}\times\frac{1.0\times 1.0\times 1.0}{2.5}\right)d_b = 28.9d_d = 28.9\times 20 = 578\ mm$$

This can be reduced by the ratio of steel required to that provided, so that the final development length is

$$l_d = 578\times\frac{750}{5\times 314} = 276\ mm < 300mm \therefore \text{Not } Ok$$

$\therefore \ell_{\rm d} = 300\text{mm}$ ■

Example 6.3-6

Based on a structural analysis and design, a designer has adopted reinforcement of $No.12@200mm$ for the cantilever slab indicated in ***Figure 6.3-6*** above. Using ACI basic equation, determine the development length, l_d, for the adopted negative reinforcements and then check to see if the available overhang part is adequate for their anchorage.

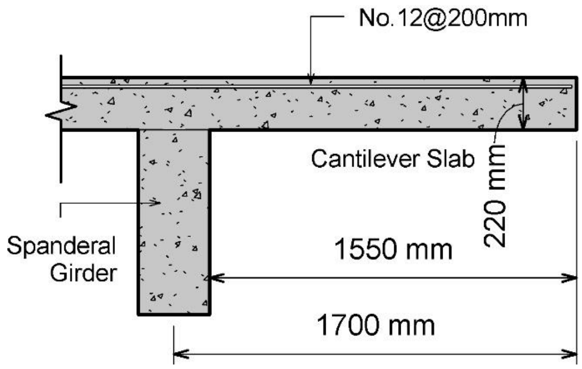


Figure 6.3-6: Reinforcement for the cantilever slab of Example 6.3-6.

Solution

According to basic equation of the ACI code, the development length for tension rebars, l_d, would be:

$$l_d = \left(\frac{f_y}{1.1\lambda\sqrt{f_c'}}\frac{\psi_t\psi_e\psi_s}{\frac{c_b+K_{tr}}{d_b}}\right)d_b$$

$$c_b = minimum\left(\begin{array}{c} side\ cover\ to\ center\ of\ bar, \\ top\ cover\ to\ center\ of\ bar, \frac{1}{2}S_{\frac{c}{c}} \end{array}\right)$$

$$c_b = \min\begin{pmatrix} \left(20+\frac{12}{2}\right), \\ \left(20+\frac{12}{2}\right), \left(\frac{1}{2}\times 200\right) \end{pmatrix} \Rightarrow c_b = \min(26,26,100) = 26\,mm$$

As there is no shear reinforcement in the cantilever slab, therefore:

$K_{tr} = 0$

As the concrete is normal weight concrete, $\lambda = 1.0$. For uncoated rebars, $\psi_e = 1.0$. As slab has thickness less than $300mm$, therefore, the rebars are considered bottom rebars from bond point of view. Finally, for rebars with size less than 19mm, $\psi_s = 0.8$.

$$l_d = \left(\left(\frac{420}{1.1\times 1.0\times \sqrt{28}}\right)\times\left(\frac{1.0\times 1.0\times 0.8}{\frac{26+0}{12}}\right)\right)d_b = 26.6d_b$$

As nothing is mentioned about $A_{s\,required}/A_{s\,provided}$, therefore it can be conservatively assumed 1.0.

$l_d = 26.6d_b = 26.6\times 12 = 319\,mm > 300\,mm \therefore Ok.$

Check adequacy of the available overhang part for anchorage of negative slab reinforcement:

$\because l_d = 319mm \ll 1550mm \therefore Ok.$

Example 6.3-7

Referring to beam section of ***Figure 6.3-7*** and based on ACI basic equation, what is the development length, l_d, for bottom rebars with 25mm diameter? In your solution, assume that the coefficient of $A_{s\,required}/A_{s\,provided}$ can conservatively be neglected. Based on your calculations, is a standard hook should adopt for the bottom rebars?

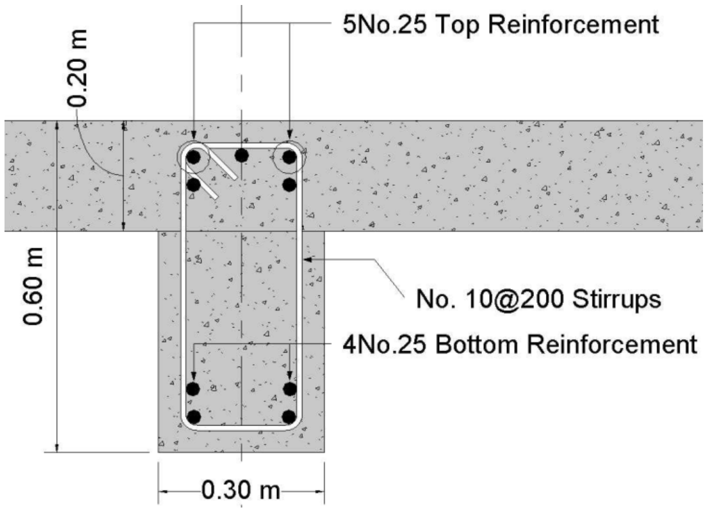


Figure 6.3-7: Beam for Example 6.3-7.

Solution

According to basic relation, development length for rebars in tension is:

$$l_d = \left(\frac{f_y}{1.1\lambda\sqrt{f_c'}}\frac{\psi_t\psi_e\psi_s}{\frac{c_b+K_{tr}}{d_b}}\right)d_b$$

$$c_b = minimum\begin{pmatrix} side\ cover\ to\ center\ of\ bar, \\ \text{bottom}\ cover\ to\ center\ of\ bar, \frac{1}{2}S_{\frac{c}{c}} \end{pmatrix}$$

$$c_b = minimum\left(40+10+\frac{25}{2}, 40+10+\frac{25}{2}, \frac{1}{2}\times(300-40\times 2-10\times 2-\frac{25}{2}\times 2)\right)$$

$$c_b = minimum(62.5, 62.5, 87.5) = 62.5\,mm \Rightarrow K_{tr} = \frac{40A_{tr}}{sn} = \frac{40\times 2\times\frac{(\pi\times 10^2)}{4}}{200\times 4} = 7.9$$

$$Confinement\ Term = \frac{c_b+K_{tr}}{d_b} = \frac{62.5+7.9}{25} = 2.82 > 2.5 \therefore Not\ Ok.$$

To avoid overemphasis of confinement role, use:

$$\frac{c_b+K_{tr}}{d_b} = 2.5$$

For bottom bars, uncoated, and with normal-density concrete, we have the values of:

$\psi_t = 1.0, \psi_e = 1.0, \lambda = 1.0$

For 25mm rebars, greater than 19mm, we have:

$\psi_s = 1.0$

$$l_d = \left(\frac{420}{1.1\times 1.0\times\sqrt{28}}\frac{1.0\times 1.0\times 1.0}{2.5}\right)d_b = 28.7\,d_b = 28.7\times 25 = 717.5\,mm < \frac{6000}{2}\,mm \therefore Ok$$

Therefore, bottom rebars can be developed with available room and no hook should be adopted.

Example 6.3-8

Referring to beam section of ***Figure 6.3-7*** and based on ACI basic equation, what is the development length, l_d, for bottom rebars with 20mm diameter? In your solution, assume that the coefficient of $A_{s\,required}/A_{s\,provided}$ can conservatively be neglected. Based on your calculations, is a standard hook should adopt for the bottom rebars?

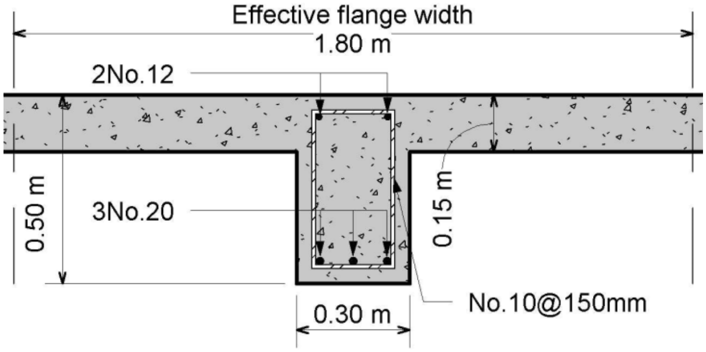


Figure 6.3-8: Beam for Example 6.3-8

Solution

According to basic relation, development length for rebars in tension is:

$$l_d = \left(\frac{f_y}{1.1\lambda\sqrt{f_c'}}\,\frac{\psi_t\psi_e\psi_s}{\frac{c_b + K_{tr}}{d_b}}\right)d_b$$

$$c_b = minimum\left(\begin{matrix}side\ cover\ to\ center\ of\ bar,\\ \text{bottom}\ cover\ to\ center\ of\ bar, \frac{1}{2}S_{\frac{c}{c}}\end{matrix}\right)$$

$$c_b = minimum\left(40 + 10 + \frac{20}{2}, 40 + 10 + \frac{20}{2}, \frac{1}{2} \times (300 - 40 \times 2 - 10 \times 2 - \frac{20}{2} \times 2)\right)$$

$$c_b = minimum(60, 60, 90) = 60\ mm \implies K_{tr} = \frac{40A_{tr}}{sn} = \frac{40 \times 2 \times \frac{(\pi \times 10^2)}{4}}{150 \times 3} \approx 14$$

$$Confinement\ Term = \frac{c_b + K_{tr}}{d_b} = \frac{60\ + 14}{20} = 3.7 > 2.5 \therefore Not\ Ok.$$

To avoid overemphasis of confinement role, use:

$$\frac{c_b + K_{tr}}{d_b} = 2.5$$

For bottom uncoated bars and with normal-density concrete, values of ψs would be:

$\psi_t = 1.0, \psi_e = 1.0, \lambda = 1.0$

For 20mm rebars, i.e. greater than 19mm, ψ_s is:

$\psi_s = 1.0$

$$l_d = \left(\frac{420}{1.1 \times 1.0 \times \sqrt{28}}\,\frac{1.0 \times 1.0 \times 1.0}{2.5}\right)d_b = 28.7\ d_b = 28.7 \times 20 = 574\ mm < \frac{6300 + 300 \times 2}{2} = 3450mm \therefore Ok$$

Therefore, bottom rebars can be developed with available room and no hook should be adopted.

6.4 ANCHORAGE OF TENSION BARS BY HOOKS

In the event that the desired tensile stress in a bar cannot be developed by bond alone, it is necessary to provide special anchorage at the ends of the bar, usually by means of:

- a 90° hook,
- a 180° hook,
- a headed bar (out the scope of this course).

Figure 6.4-1: 90° Hook, 180° Hook, and Headed Bar.

6.4.1 BASIC CONCEPTS

- Forces Acting on Hooks:
  - A 90° hook loaded in tension develops forces in the manner shown in ***Figure 6.4-2*** below.
  - The stress in the bar is resisted by the bond on the surface of the bar and by the bearing on the concrete inside the hook.

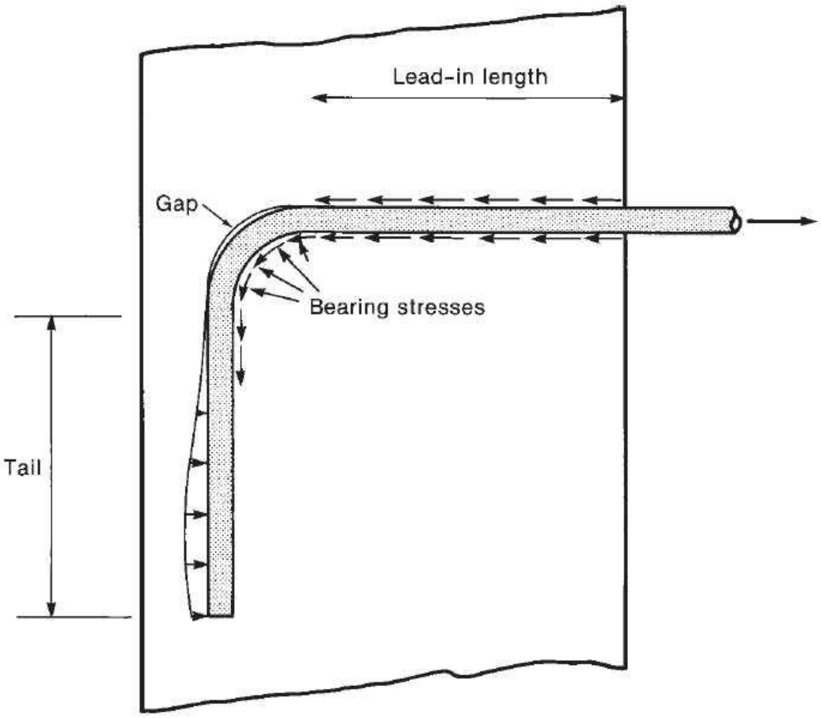


Figure 6.4-2: Forces Acting on a 90-degree Hooked Bars.

- Mode of Failure:
  - Hooked bars resist pullout by the combined actions of bond along the straight length of bar leading to the hook and anchorage provided by the hook. Then ***Pullout strength of hook is okay.***
  - Tests indicate that the ***main cause of failure of hooked bars in tension is splitting of the concrete in the plane of the hook***.
- Splitting Stresses:
  - This splitting is due to the very high stresses in the concrete inside of the hook; these stresses are influenced mainly by:
    - the bar diameter d_b for a given tensile force,
    - the radius of bar bend.
  - Resistance to splitting:

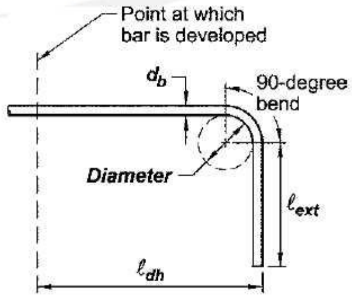
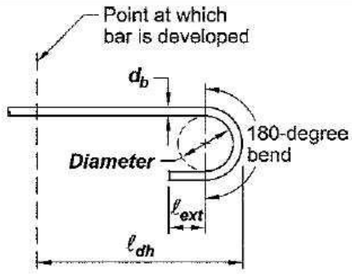
    Resistance to splitting has been found to depend on ***the concrete cover for the hooked bar***, measured laterally from the edge of the member to the bar perpendicular to the plane of the hook, and measured to the top (or bottom) of the member from the point where the hook starts, parallel to the plane of the hook.

- Increase resistance to splitting:
  The strength of the anchorage can be substantially increased by providing confinement steel in the form of closed stirrups or ties.

6.4.2 STANDARD HOOK DIMENSIONS

According to (ACI318M, 2014), ***25.3.1***, Standard hooks for the development of deformed bars in tension shall conform to ***Table 6.4-1*** below, ***Table 25.3.1*** of the (ACI318M, 2014).

Table 6.4-1: Standard hook geometry for development of deformed bars in tension, Table 25.3.1 of the (ACI318M, 2014).

Type of standard hook	Bar size	Minimum inside bend diameter, mm	Straight extension[1] ℓ_{ext}, mm	Type of standard hook
90-degree hook	No. 10 through No. 25	$6d_b$	$12d_b$	
	No. 29 through No. 36	$8d_b$		
	No. 43 and No. 57	$10d_b$		
180-degree hook	No. 10 through No. 25	$6d_b$	Greater of $4d_b$ and 65 mm	
	No. 29 through No. 36	$8d_b$		
	No. 43 and No. 57	$10d_b$		

[1]A standard hook for deformed bars in tension includes the specific inside bend diameter and straight extension length. It shall be permitted to use a longer straight extension at the end of a hook. A longer extension shall not be considered to increase the anchorage capacity of the hook.

6.4.3 DESIGN OF HOOKED ANCHORAGES

- The design process described in ACI Code does not distinguish between 90° and 180° hooks or between top and bottom bar hooks.
- ACI design procedure for hooked anchorage can be summarized as follows:
  - Compute l_{dh} based on a basic relation of ***ACI 25.4.3.1***,
  - Reduce l_{dh} by multiplier of ***ACI 25.4.3.2*** when applicable,
  - Check ACI provisions related to discontinuous end (***ACI 25.4.3.3***),
  - Check l_{dh} with minimum code limitations (***ACI 25.4.3.1***).

6.4.3.1 Basic Relation for l_{dh}

According to ***ACI 25.4.3.1***, a total development length, l_{dh}, defined as shown in ***Table 6.4-1*** above, for deformed bars in tension terminating in a standard hook shall be:

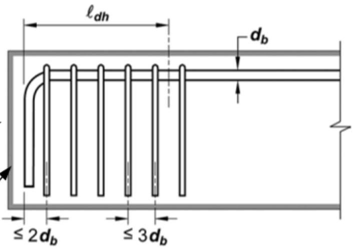
$$l_{dh} = \left(\frac{0.24\, f_y \psi_e \psi_c \psi_r}{\lambda \sqrt{f_c'}}\right) d_b \qquad \text{Eq. 6.4-1}$$

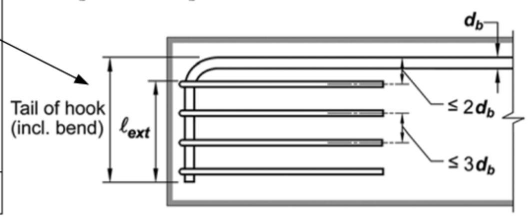
6.4.3.2 Multiplier Factors of ACI 12.5.3

- According to ***ACI 25.4.3.2***, for the calculation of l_{dh}, modification factors shall be in accordance with ***Table 6.4-2*** below, ***Table 25.4.3.2*** of (ACI318M, 2014).
- Factors ψ_c and ψ_r shall be permitted to be taken as 1.0.
- According to ***25.4.10.1*** of (ACI318M, 2014), reduction of development lengths defined by provision above shall be permitted by use of the ratio: $(As_{required})/(As_{provided})$

Table 6.4-2: Modification factors for development of hooked bars in tension, Table 25.4.3.2 of (ACI318M, 2014).

Modification factor	Condition	Value of factor
Lightweight λ	Lightweight concrete	0.75
	Normalweight concrete	1.0
Epoxy ψ_e	Epoxy-coated or zinc and epoxy dual-coated reinforcement	1.2
	Uncoated or zinc-coated (galvanized) reinforcement	1.0
Cover ψ_c	For No. 36 bar and smaller hooks with side cover (normal to plane of hook) $\geq$ 65 mm and for 90-degree hook with cover on bar extension beyond hook $\geq$ 50 mm	0.7
	Other	1.0
Confining reinforcement ψ_r[2]	For 90-degree hooks of No. 36 and smaller bars (1) enclosed along ℓ_{dh} within ties or stirrups[1] perpendicular to ℓ_{dh} at $s \leq 3d_b$, or (2) enclosed along the bar extension beyond hook including the bend within ties or stirrups[1] perpendicular to ℓ_{ext} at $s \leq 3d_b$	0.8
	For 180-degree hooks of No. 36 and smaller bars enclosed along ℓ_{dh} within ties or stirrups[1] perpendicular to ℓ_{dh} at $s \leq 3d_b$	0.8
	Other	1.0





[1]The first tie or stirrup shall enclose the bent portion of the hook within $2d_b$ of the outside of the bend.

[2]d_b is the nominal diameter of the hooked bar.

6.4.3.3 Transverse Confinement Steel at Discontinuous Ends

6.4.3.3.1 Provisions for Discontinues Ends

According to (ACI318M, 2014), ***25.4.3.3***, for bars being developed by a standard hook:

- At discontinuous ends of members,
- With **both** side cover and top (or bottom) cover to hook less than 65 mm

provisions (a) through (c) shall be satisfied, see ***Figure 6.4-3*** below:

(a) The hook shall be enclosed along l_{dh} within ties or stirrups perpendicular to ℓ_{dh} at $s \leq 3d_b$,

(b) The first tie or stirrup shall enclose the bent portion of the hook within $2d_b$ of the outside of the bend,

(c) ψ_r shall be taken as 1.0 in calculating ℓdh in accordance with Table 6.4-2 above.

where d_b is the nominal diameter of the hooked bar.

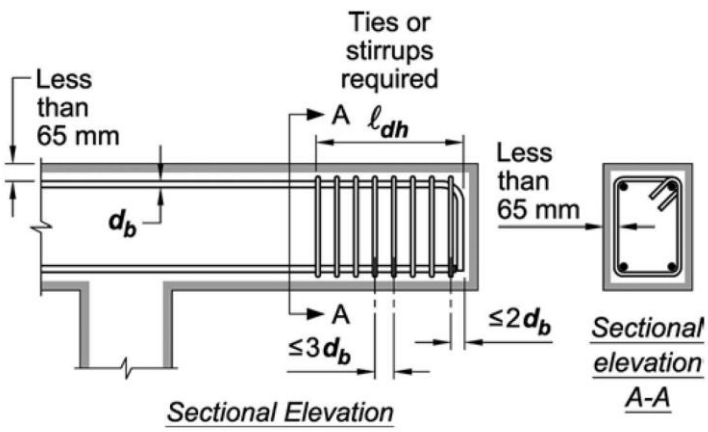


Figure 6.4-3: Transverse confinement steel at discontinuous ends.

6.4.3.3.2 Discontinuous Ends

Cases where hooks may require ties or stirrups for confinement are, adopted from ***R25.4.3.3*** of (ACI318M, 2014)

- At ends of simply-supported beams,
- At the free end of cantilevers,
- At ends of members framing into a joint where members do not extend beyond the joint.

6.4.3.3.3 Discontinuous Ends of Slabs

Above provisions do not apply for hooked bars at discontinuous ends of slabs where confinement is provided by the slab on both sides and perpendicular to the plane of the hook.

6.4.3.4 ACI Minimum Limitations on Hook Development Length

According to ***ACI 25.4.3.1***,

$l_{dh} \geq maximum\ (8d_b, 150mm)$

6.4.3.5 Hooks Effectiveness in Compression Rebars

According to ***ACI 25.4.1.2***, hooks shall not be considered effective in developing bars in compression.

6.4.4 DESIGN EXAMPLES FOR TENSION ANCHORAGE WITH HOOK

Example 6.4-1

Referring to the beam-column joint of ***Example 6.3-1*** that is represented below for convenience, the No. 36 negative bars are to be extended into the column and terminated in a standard 90° hook, keeping 50mm clear to the outside face of the column. The column width in the direction of beam width is 400mm. Find the minimum length of embedment of the hook past the column face, and specify the hook details. As in ***Example 6.3-1***, assume that normal weight concrete is to be used, with $f_c' = 28\ MPa$, and reinforcing bars have $f_y = 420\ MPa$.

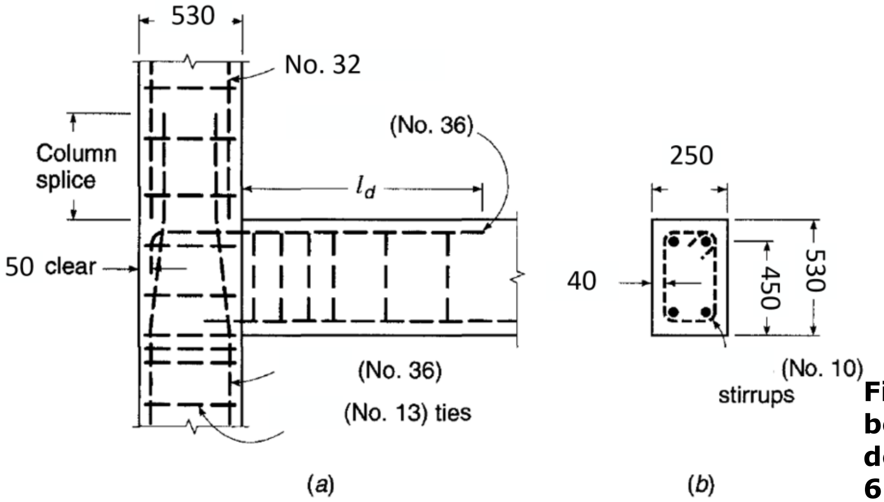


Figure 6.4-4: Bar details at beam-column joint for bar development of Example 6.3-1 (Re-presenting).

Figure 6.4-5: Bar details at beam-column joint for bar development examples (3D Views).

Solution

Basic Relation

The development length for hooked bars, measured from the critical section along the bar to the far side of the vertical hook, is given by:

$$l_{dh} = \left(\frac{0.24 f_y \psi_e \psi_c \psi_r}{\lambda \sqrt{f_c'}}\right) d_b$$

For uncoated or zinc-coated (galvanized):

$\psi_e = 1.0$

Confinement Provided by Increased Cove:

$$Side\ cover = 40 + \frac{400 - 250}{2} = 115\ mm > 65mm$$

In this case, side cover for the (No. 36) bars exceeds 65mm and cover beyond the bent bar is adequate, so:

$\psi_c = 0.7$

As there is no confinement reinforcement, therefore:

$\psi_r = 1.0$

$$l_{dh} = \left(\frac{0.24 \times 420 \times 1.0 \times 0.7 \times 1.0}{1.0 \times \sqrt{28}}\right) d_b = 13.3\ d_b$$

Multiplication Factor of $A_{S\ Required}$/ $A_{S\ Provided}$

As

$$\frac{A_{S\ Required}}{A_{S\ Provided}} = \frac{1870}{2012} = 0.929$$

Then:

$l_{dh} = 0.929 \times 13.3\ d_b = 12.4\ d_b = 12.4 \times 36 = 446\ mm$

Code Minimum Limitations

Check with code minimum limitations:

$l_{dh} =\ 446mm\ ?\ maximum\ (8 \times 36, 150mm) \Rightarrow l_{dh} =\ 446mm\ ?\ maximum\ (288, 150mm)$

$l_{dh} =\ 446mm >\ 288\ \therefore Ok.$

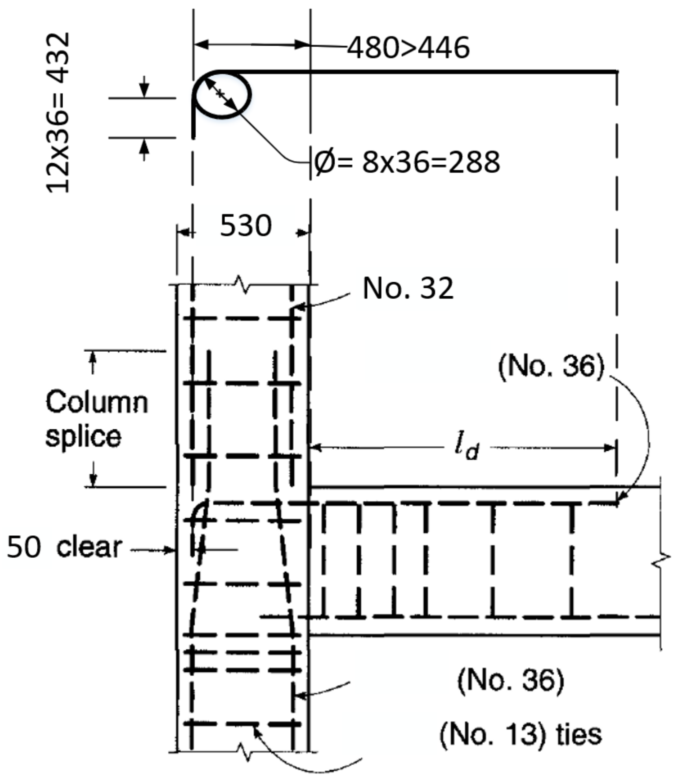
$l_{dh} =\ 446\ mm < 530 - 50 = 480\ \therefore Ok.$

$l_{dh} =\ 445\ mm$ ■

Transverse Confinement Steel at Discontinuous Ends:

As side, bottom, and top coves are greater than 65mm, then no need for transverse confinement reinforcement.

Final Details for Hooked Bar:



Example 6.4-2

Design standard hook details to anchor exterior negative slab reinforcement to the supporting beam, see ***Figure 6.4-6*** below. In your solution assume $f_c' = 28\,MPa$, $f_y = 420\,MPa$, and $A_{s\,required}/A_{s\,provided}$ is $\frac{216}{524}$.

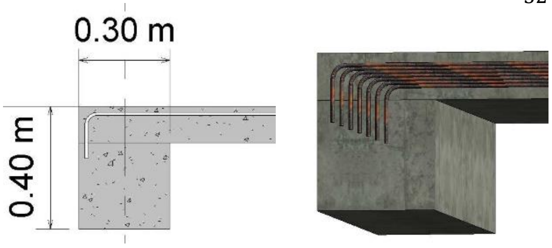


Figure 6.4-6: Details for exterior negative reinforcement of a slab to the supporting beam.

Solution

Basic Relation

The development length for hooked bars, measured from the critical section along the bar to the far side of the vertical hook, is given by:

$$l_{dh} = \left(\frac{0.24 f_y \psi_e \psi_c \psi_r}{\lambda\sqrt{f_c'}}\right) d_b$$

For normal weight concrete:

$\lambda = 1.0$

For uncoated or zinc-coated (galvanized):

$\psi_e = 1.0$

As cover extends beyond hook is less than 50mm, then:

$\psi_c = 1.0$

As there are no transverse confinement reinforcement, then

$\psi_r = 1.0$

$$l_{dh} = \left(\frac{0.24 \times 420 \times 1.0 \times 1.0 \times 1.0}{1.0 \times \sqrt{28}}\right) d_b = 19 d_b = 19 \times 10 = 190mm$$

Multiplication Factor of As Required/ As Provided

Finally, reduction factor of $A_{s\,Required}/A_{s\,Provided}$ is applicable.

$$l_{dh} = 190 \times \frac{216}{524} = 78.3\,mm$$

Transverse Confinement Steel at Discontinuous Ends:

Provisions of discontinuous edges do not apply for hooked bars at discontinuous ends of slabs where confinement is provided by the slab on both sides and perpendicular to the plane of the hook.

ACI Minimum Limitations on Hook Development Length:

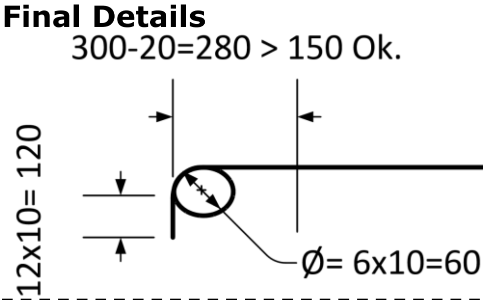
$l_{dh} > maximum\ (8d_b, 150mm)$

$l_{dh} = 78.3\,mm\ > maximum\ (8 \times 10, 150mm) = 150mm \therefore Not\ Ok.$

Then, use

$l_{dh} = 150mm$

Final Details



Example 6.4-3

The short beam shown in ***Figure 6.4-7*** cantilevers from a supporting column at the left. It must carry a calculated dead load of $29\,kN/m$ including its own weight and a service live load of $38\,kN/m$. Based on these loads, required reinforcement of $A_{s\,Required} = 1593\ mm^2$ have been determined. Tensile flexural reinforcement consists of ***two No. 36 bars*** have been provided. ***Transverse No. 10 U stirrups with 40mm*** cover are provided at the following spacings from the face of the column: $\mathbf{100mm}$, $\mathbf{3\ at\ 200mm}$, and $\mathbf{5\ at\ 260mm}$.

a) If the flexural and shear steel use $f_y = 420\,MPa$ and if the beam uses ***lightweight concrete*** having $f_c' = 28\,MPa$, check to see if proper development length can be provided for the No. 36 bars. Use the ***simplified development length equations***.

b) If the column material strengths are $f_y = 420\,MPa$ and $f_c' = 35\,MPa$ (***normalweight concrete***), check to see if adequate embedment can be provided within the column for the No. 36 bars. In your checking, use ***the basic equation***.

c) If hooks are required, specify detailed dimensions.

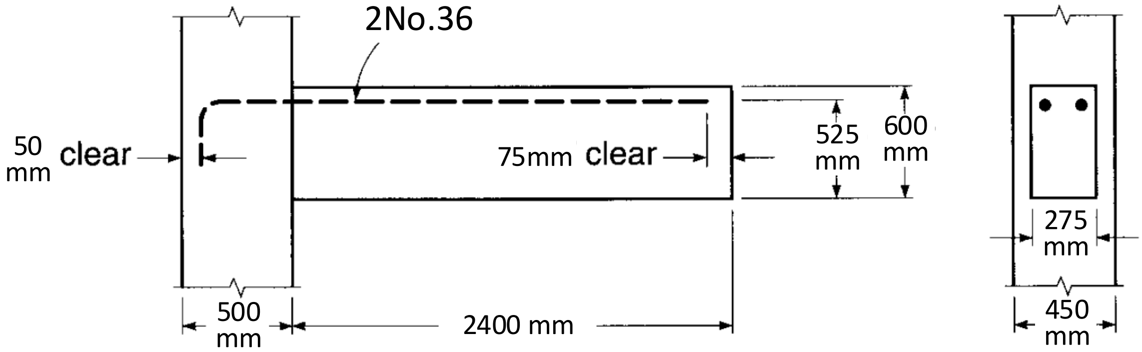
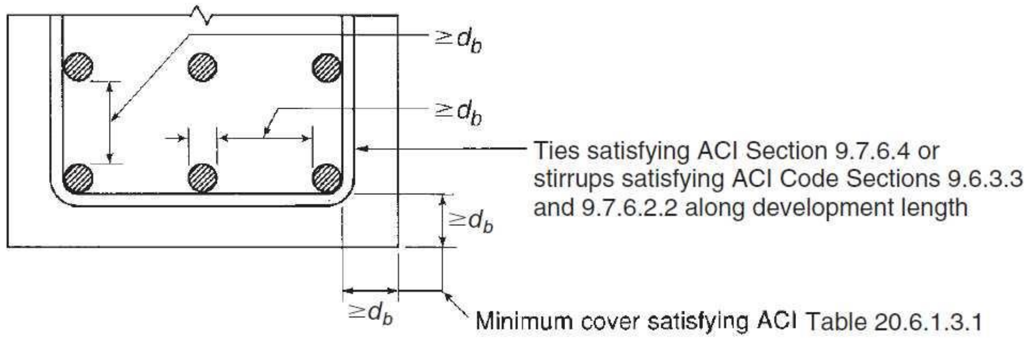


Figure 6.4-7: Cantilever beam for Example 6.4-3.

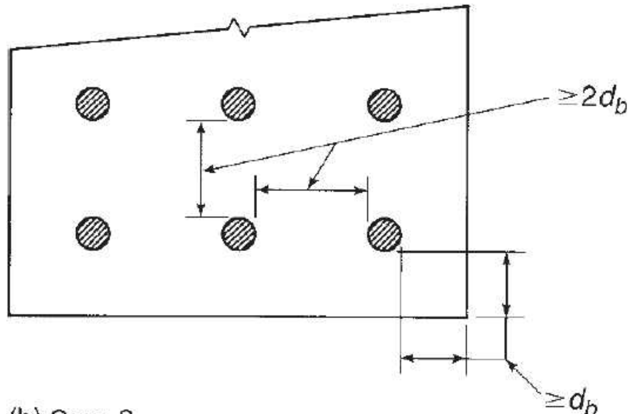
Solution

a) Anchorage to beam based on the simplified equations:

Checking for lateral spacing to determine if rebar is confined according to Case 1 or Case 2 below:



(a) Case 1.



(b) Case 2.

Start with checking for Case 2 where confinement depends on concrete mass only:

$$\frac{Clear\ distance}{d_b} = \frac{275 - 40 \times 2 - 2 \times 10 - 2 \times 36}{36} = 2.86 > 2d_b \therefore \text{Ok.}$$

$$\frac{Clear\ side\ cover}{d_b} = \frac{40 + 10}{36} = 1.39 > d_b \therefore Ok.$$

$$\frac{Clear\ top\ cover}{d_b} = \frac{600 - 525 - \frac{36}{2}}{36} = 1.58 > \mathrm{d_b} \therefore Ok.$$

Therefore, the rebars can be considered confined depends on concrete mass, Case 2. With bar diameter of $36mm$, i.e. greater than No. 22, the required development length would be:

Spacing and cover	No. 19 and smaller bars and deformed wires	No. 22 and larger bars
Clear spacing of bars or wires being developed or spliced not less than d_b, clear cover not less than d_b, and stirrups or ties throughout ℓ_d not less than the Code minimum or Clear spacing of bars or wires being developed or spliced not less than $2d_b$ and clear cover not less than d_b	$\left(\frac{f_y\psi_t\psi_e}{2.1\lambda\sqrt{f_c'}}\right)d_b$	$\left(\frac{f_y\psi_t\psi_e}{1.7\lambda\sqrt{f_c'}}\right)d_b$
Other cases	$\left(\frac{f_y\psi_t\psi_e}{1.4\lambda\sqrt{f_c'}}\right)d_b$	$\left(\frac{f_y\psi_t\psi_e}{1.1\lambda\sqrt{f_c'}}\right)d_b$

$$l_d=\left(\frac{f_y\psi_t\psi_e}{1.7\lambda\sqrt{f_c{}'}}\right)d_d$$

Then for top bars, uncoated, and with lightweight concrete, we have the values of:

$\psi_t=1.3, \psi_e=1.0, \lambda=0.75$

$$l_d=\left(\frac{420\times1.3\times1.0}{1.7\times0.75\times\sqrt{28}}\right)d_d=80.9d_b=80.9\times36=2912\ mm$$

This can be reduced by the ratio of steel required to that provided, so that the final development length is

$$l_d=2912\times\frac{\mathrm{A_{s\,Required}}}{A_{s\,Provided}}=2912\times\frac{1593}{2\times\frac{\pi\times36^2}{4}}=2279>300\mathrm{mm}\therefore\mathrm{Ok.}$$

$l_d=2279mm$ ■

$\because l_d=2279mm<2400mm\therefore Ok.$

b) Anchorage to the column using straight rebar with l_d determined based on the basic relation:

$$l_d=\left(\frac{f_y}{1.1\lambda\sqrt{f_c{}'}}\frac{\psi_t\psi_e\psi_s}{\frac{c_b+K_{tr}}{d_b}}\right)d_b$$

$$c_b=minimum\left(side\ cover\ to\ center\ of\ bar, top\ cover\ to\ center\ of\ bar, \frac{1}{2}S_{\frac{c}{c}}\right)$$

$$c_b=\min\left(\left(\frac{450-275}{2}+40+10+\frac{36}{2}\right),\infty,\left(\frac{1}{2}\times(275-40\times2-10\times2-\frac{36}{2}\times2)\right)\right)$$

$$=\min(156,\infty,69.5)=69.5mm$$

As no stirrups have been adopted in column joint region, therefore the parameter of k_{tr} which simulate stirrup confinement would be:

$$K_{tr}=\lim_{s\to\infty}\frac{40A_{tr}}{sn}=0$$

Finally, the confinement term would be

$$Confinement\ Term\ =\frac{c_b+K_{tr}}{d_b}=\frac{69.5+0}{36}=1.93<2.5\therefore Ok.$$

$\because d_b>19\therefore\psi_s=1.0$

$$l_d=\left(\frac{420}{1.1\times1.0\times\sqrt{35}}\times\frac{1.3\times1.0\times1.0}{1.93}\right)d_b=43.5d_d=\ 43.5\times36=1566\ mm$$

This can be reduced by the ratio of steel required to that provided, so that the final development length is:

$$l_d=1566\times\frac{\mathrm{A_{s\,Required}}}{A_{s\,Provided}}=1566\times\frac{1593}{2\times\frac{\pi\times36^2}{4}}=1225mm>300\mathrm{mm}\therefore\mathrm{Ok.}$$

$l_d=1225\ mm$ ■ $\Longrightarrow\because l_d=1225mm>(500-50)=450mm\therefore Not\ Ok.$

Therefore, hook should be adopted to anchor the rebar to the column.

c) Anchorage to the column using standard hook:

The development length for hooked bars, measured from the critical section along the bar to the far side of the vertical hook, is given by:

$$l_{dh}=\left(\frac{0.24 f_y \psi_e \psi_c \psi_r}{\lambda\sqrt{f_c'}}\right) d_b$$

For uncoated or zinc-coated (galvanized):

$\psi_e = 1.0$

Confinement provided by increased cove:

$$Side\ cover = 40+\frac{450-275}{2}=127.5\ mm > 65mm$$

In this case, side cover for the (No. 36) bars exceeds 65mm and cover beyond the bent bar is adequate, so:

$\psi_c = 0.7$

As there is no confinement reinforcement, therefore:

$\psi_r = 1.0$

With above parameters, the relation for l_{dh} would be:

$$l_{dh}=\left(\frac{0.24\times 420\times 1.0\times 0.7\times 1.0}{1.0\times\sqrt{35}}\right) d_b = 11.9\ d_b$$

Multiplication factor of $A_{s\ Required}/A_{s\ Provided}$:

As

$$\frac{A_{S\ Required}}{A_{S\ Provided}}=\frac{1593}{2\times\frac{\pi\times 36^2}{4}}$$

Then:

$$l_{dh}=\frac{1593}{2\times\frac{\pi\times 36^2}{4}}\times 11.9\ d_b = 9.31\ d_b = 9.31\times 36 = 335\ mm$$

Code Minimum Limitations

Check with code minimum limitations:

$l_{dh} = \ 335mm\ ?\ maximum\ (8\times 36, 150mm) \Rightarrow l_{dh} = \ 335mm\ ?\ maximum\ (288, 150mm)$

$l_{dh} = \ 335mm > \ 288 \therefore Ok.$

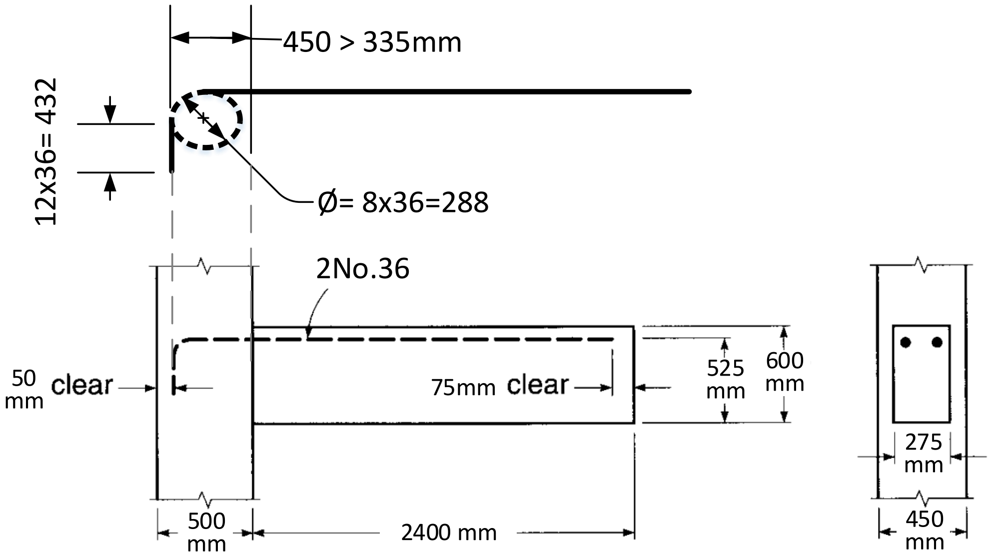
Check with available room:

$l_{dh} = \ 335\ mm < 500-50 = 450 \therefore Ok.$

Transverse Confinement Steel at Discontinuous Ends:

As side, or bottom or top coves are greater than 65mm, then no need for transverse confinement reinforcement.

Final details for hooked bar:



6.5 Anchorage Requirements for Web Reinforcement

This issue has been discussed thoroughly in ***Chapter 5***.

6.6 Development of Bars in Compression

6.6.1 Basic Concepts

- Reinforcement may be required to develop its compressive strength by embedment under various circumstances, e.g., where bars transfer their share of column loads to a supporting footing or basement walls or where lap splices are made of compression bars in column.
- In the case of bars in compression,
  - A part of the total force is transferred by bond along the embedded length,
  - And a part is transferred by end bearing of the bars on the concrete.
- Main difference between development length in tension and in compression:
  - Because the surrounding concrete is relatively free of cracks
  - And because of the beneficial effect of end bearing,

  shorter basic development lengths are permissible for compression bars than for tension bars.
- Transverse confinement steel:

  If transverse confinement steel is present, such as spiral column reinforcement or special spiral steel around an individual bar, the ***required development length is further reduced***.

6.6.2 ACI Relations

- Basic Relation

  According to ACI ***25.4.9.2***, development length for rebars in compression (l_{dc}) shall be computed based on following relation:

$$l_{dc} = maximum\left(\frac{0.24 f_y \psi_r}{\lambda\sqrt{f_c{}'}} d_b \; or \; 0.043 f_y \psi_r d_b\right) \qquad \text{Eq. 6.6-1}$$

- Modifications Factors
  - According to ***25.4.9.3***, for the calculation of ℓ_{dc}, modification factors shall be in accordance with ***Table 6.6-1*** above, except ψ_r shall be permitted to be taken as 1.0.
  - According to ACI ***25.4.10.1***, length l_{dc} shall be permitted to reduce by ratio of $A_{s\,required}/A_{s\,provided}$.
- $\boldsymbol{l_{dc}}$ Lower Bound

  According to ACI **25.4.9.1**, $l_{dc} \geq 200\,mm$

Table 6.6-1: Modification factors for deformed bars and wires in compression, Table 25.4.9.3 of (ACI318M, 2014).

Modification factor	Condition	Value of factor
Lightweight λ	Lightweight concrete	0.75
	Lightweight concrete, if f_{ct} is specified	In accordance with 19.2.4.3
	Normalweight concrete	1.0
Confining reinforcement ψ_r	Reinforcement enclosed within (1), (2), (3), or (4): (1) a spiral (2) a circular continuously wound tie with $d_b \geq 6$ mm and pitch 100 mm (3) No. 13 bar or MD130 wire ties in accordance with 25.7.2 spaced ≤ 100 mm on center (4) hoops in accordance with 25.7.4 spaced ≤ 100 mm on center	0.75
	Other	1.0

6.6.3 EXAMPLES

Example 6.6-1

The forces in the column bars of ***Figure 6.6-1*** below are to be transferred into the footing with No. 29 dowels.

Determine the development lengths needed for the dowels:

- Down into the footing.
- Up into the column.

In your solution assume that, $f_y = 420\ MPa$ and that column is under a compressive force.

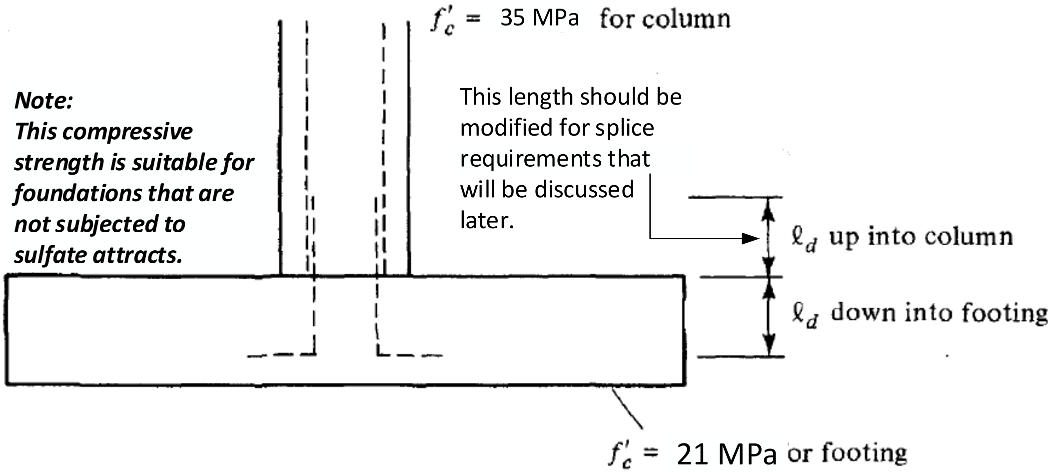


Figure 6.6-1: Foundation and column dowels for Example 6.6-1.

Solution

<u>Down into the Footing</u>

$$l_{dc} = maximum\left(\frac{0.24 f_y \psi_r}{\lambda\sqrt{f_c'}} d_b \ or\ 0.043 f_y \psi_r d_b\right)$$

As no confining reinforcement are included,

$\psi_r = 1.0$

For normal weight concrete,

$\lambda = 1.0$

$$l_{dc} = maximum\left(\frac{0.24 \times 420 \times 1.0}{1.0 \times \sqrt{21}} d_b \ or\ 0.043 \times 420 \times 1.0 d_b\right)$$

$$l_{dc} = maximum(22d_b \ or\ 18d_b) = 22\ d_b = 22 \times 29 = 638mm > 200\ mm \therefore Ok.$$

<u>Up into the Column</u>

$$l_{dc} = maximum\left(\frac{0.24 f_y \psi_r}{\lambda\sqrt{f_c'}} d_b \ or\ 0.043 f_y \psi_r d_b\right)$$

As no confining reinforcement are included,

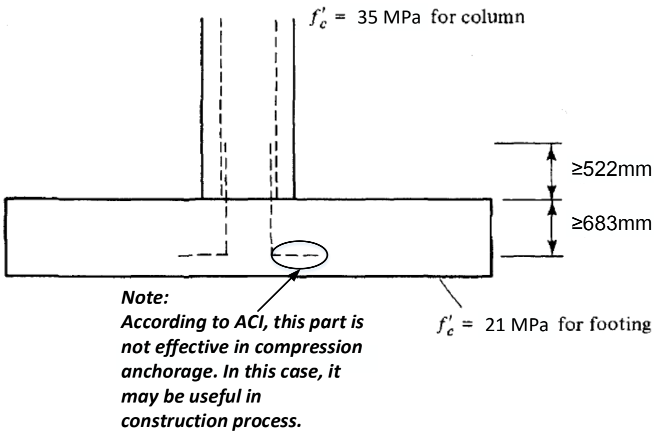
$\psi_r = 1.0$

For normal weight concrete,

$\lambda = 1.0$

$$l_{dc} = maximum\left(\frac{0.24 \times 420 \times 1.0}{1.0 \times \sqrt{35}} d_b \ or\ 0.043 \times 420 \times 1.0 d_b\right)$$

$$l_{dc} = maximum(17d_b \ or\ 18d_b) = 18\ d_b = 18 \times 29 = 522mm > 200mm \therefore Ok.$$



Example 6.6-2

To anchor an axially compressed column, that reinforced with $4\phi25$, to its foundation, a designer has proposed the detail shown in below.

Assuming that $A_{s\ provided} \approx A_{s\ Required}$, $f_c' = 28\ MPa$, and $f_y = 420\ MPa$.

- Is the proposed down in to foundation anchorage adequate according to ACI code?
- If proposed anchorage is inadequate, propose two different alternatives to solve the problem.

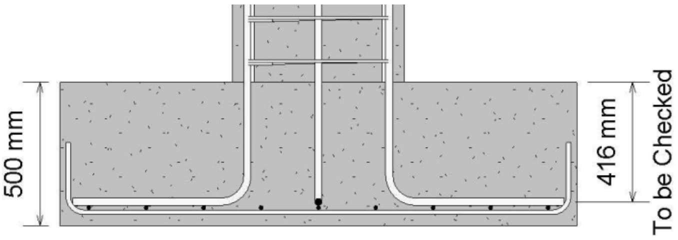


Figure 6.6-2: Column to foundation anchor of Example 6.6-2.

Solution

Adequacy Checking

Down into the footing:

$$l_{dc} = maximum\left(\frac{0.24 f_y \psi_r}{\lambda\sqrt{f_c'}} d_b \; or \; 0.043 f_y \psi_r d_b\right)$$

As no confining reinforcement are included,

$\psi_r = 1.0$

For normal weight concrete,

$\lambda = 1.0$

$$l_{dc} = maximum\left(\frac{0.24 \times 420 \times 1.0}{1.0 \times \sqrt{28}} d_b \; or \; 0.043 \times 420 \times 1.0 d_b\right)$$

$l_{dc} = maximum(19 d_b \; or \; 18 d_b) = 19\, d_b = 19 \times 25 = 475 mm > 416 \text{ mm} \therefore \text{Not ok.}$

Proposed Alternatives

1st Alternative:

First alternative is to use $\phi 16mm$ instead of $\phi 25mm$ for column longitudinal reinforcement and recalculate required number accordingly:

$l_{dc\,for\,\phi 16} = 19 \times 16 = 304\, mm < 416\, mm \therefore Ok.$

$$No.of\ \phi 16 = \frac{\left(4 \times \frac{\pi \times 25^2}{4}\right)}{\pi \times \frac{16^2}{4}} = 9.76$$

Used $10\phi 16mm$.

2nd Alternative:

This alternative is based on using an area for longitudinal reinforcement greater than the required one to activate the reduction factor of $A_{s\,Required}/A_{s\,Provided}$.

$$\frac{A_{s\,Required}}{A_{s\,Provided}} \times 475 = 416 \Rightarrow \frac{A_{s\,Required}}{A_{s\,Provided}} = 0.876$$

$$A_{s\,Provided} = \frac{\left(\frac{\pi \times 25^2}{4} \times 4\right)}{0.876} = 2241\, mm^2$$

$$Modified\ No.of\ \phi 25 = \frac{2241}{\pi \times \frac{25^2}{4}} = 4.56$$

Then use $6\phi 25$ instead of $4\phi 25$.

6.7 Development of Bundled Bars

6.7.1 General Requirements

- According to (ACI318M, 2014), ***25.6.1.1***, groups of parallel reinforcing bars bundled in contact to act as a unit ***shall be limited to <u>four</u> in any one bundle***.
- According to (ACI318M, 2014), ***25.6.1.2***, bundled bars in compression members shall be enclosed by ***transverse reinforcement at least No. 13 in size***. This aspect has been discussed thoroughly ***in analysis and design of doubly reinforced beams in Chapter 3***.
- According to (ACI318M, 2014), ***25.6.1.3***, ***bars larger than a No. 36 shall not be bundled in beams***.

6.7.2 Development Length for Bundled Bars

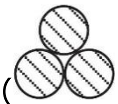
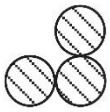
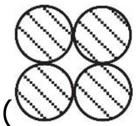
- According to (ACI318M, 2014), ***25.6.1.5***, development length of individual bars within a bundle, ***in tension or compression***, shall be that for the individual bar,
  - Increased 20 percent for three-bar bundle, ( or )
  - Increased 33 percent for four-bar bundle ().
- The extra extension is needed because the grouping makes it more difficult to mobilize bond resistance from the core between the bars.
- According to (ACI318M, 2014), ***25.6.1.6***, a unit of bundled bars shall be treated as a single bar of a diameter derived from the equivalent total area and having a centroid that coincides with that of the bundled bars for determining the following:
  - Spacing limitations based on d_b,
  - Cover requirements based on d_b,
  - Spacing and cover values in ***Article 25.4.2.2***, i.e. cover and spacing related to Table 6.3-1. ***For bundled bars, bar diameter*** d_b ***outside the brackets is that of a single bar***.

Table 6.3-1: Simplified ACI Relations for Development Length (Table 25.4.2.2 of (ACI318M, 2014)). Represented for convenience.

Spacing and cover	No. 19 and smaller bars and deformed wires	No. 22 and larger bars
Clear spacing of bars or wires being developed or lap spliced not less than d_b, clear cover at least d_b, and stirrups or ties throughout ℓ_d not less than the Code minimum or Clear spacing of bars or wires being developed or lap spliced at least $2d_b$ and clear cover at least d_b	$\left(\frac{f_y\Psi_t\Psi_e}{2.1\lambda\sqrt{f_c'}}\right)d_b$	$\left(\frac{f_y\Psi_t\Psi_e}{1.7\lambda\sqrt{f_c'}}\right)d_b$
Other cases	$\left(\frac{f_y\Psi_t\Psi_e}{1.4\lambda\sqrt{f_c'}}\right)d_b$	$\left(\frac{f_y\Psi_t\Psi_e}{1.1\lambda\sqrt{f_c'}}\right)d_b$

  - Confinement term in 25.4.2.3, i.e., the term $(c_b + K_{tr})/d_b$ in the basic equation below. ***For bundled bars, bar diameter*** d_b ***outside the brackets is that of a single bar***.

$$\ell_d = \left(\frac{f_y}{1.1\lambda\sqrt{f_c'}} \frac{\psi_t \psi_e \psi_s}{\left(\frac{c_b + K_{tr}}{d_b}\right)} \right) d_b$$

- ψ_e factor in ***Article 25.4.2.4***, i.e., ψ_e in the Table 6.3-2, represented in below for convenience in below.

Table 6.3-2: Modification factors for development of deformed bars and deformed wires in tension, Table 25.4.2.4 of (ACI318M, 2014). Represented for convenience.

Modification factor	Condition	Value of factor
Lightweight λ	Lightweight concrete	0.75
	Lightweight concrete, where f_{ct} is specified	In accordance with 19.2.4.3
	Normalweight concrete	1.0
Epoxy[1] ψ_e	Epoxy-coated or zinc and epoxy dual-coated reinforcement with clear cover less than $3d_b$ or clear spacing less than $6d_b$	1.5
	Epoxy-coated or zinc and epoxy dual-coated reinforcement for all other conditions	1.2
	Uncoated or zinc-coated (galvanized) reinforcement	1.0
Size ψ_s	No. 22 and larger bars	1.0
	No. 19 and smaller bars and deformed wires	0.8
Casting position[1] ψ_t	More than 300 mm of fresh concrete placed below horizontal reinforcement	1.3
	Other	1.0

[1]The product $\psi_t\psi_e$ need not exceed 1.7.

6.7.3 EXAMPLES

Example 6.7-1

Compute the development length required for the uncoated bundled bars shown in ***Figure 6.7-1*** below if $f_y = 420\,MPa$ and $f_c' = 28\,MPa$ with normal weight concrete. Use ACI basic relation and assume that $K_{rt} = 0$.

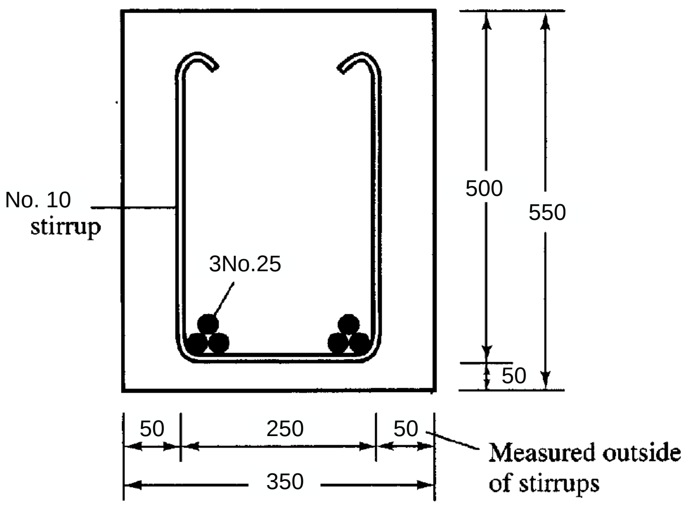


Figure 6.7-1: Cross sectional area for a beam reinforced with bundled bars.

Solution

$$l_d = \left(\frac{f_y}{1.1\lambda\sqrt{f_c'}} \frac{\psi_t \psi_e \psi_s}{\frac{c_b + K_{tr}}{d_b}} \right) d_b$$

$\psi_t = \psi_e = \psi_s = \lambda = 1.0$

In confinement term, c_b and d_b will be computed based on an equivalent single rebar:

$$\frac{\pi d_{b\ equivalent}^2}{4} = 3 \times \frac{\pi \times 25^2}{4}$$

$$d_{b\ equivalent} = \sqrt{3 \times 25^2} = 43.3\ mm$$

$$c_b = minimum\left(side\ cover\ to\ center\ of\ bar, bottom\ cover\ to\ center\ of\ bar, \frac{1}{2} S_{\frac{c}{c}}\right)$$

$$c_b = minimum\left(50 + 10 + \frac{43.3}{2}, \quad 50 + 10 + \frac{43.3}{2}, \quad \frac{1}{2}\left(250 - 2 \times 10 - \frac{43.3}{2} \times 2\right)\right)$$

$$c_b = minimum(81.6, \quad 81.6, \quad 93.4) = 81.6\ mm$$

$$\frac{c_b + K_{tr}}{d_{b\ equivalent}} = \frac{81.6 + 0.0}{43.3} = 1.88 < 2.5 \therefore Ok$$

$$l_d = \left(\frac{420}{1.1 \times 1.0 \times \sqrt{28}} \frac{1.0 \times 1.0 \times 1.0}{1.88}\right) d_b$$

$$l_d = 38.4 d_b$$

This value should be increased 20% for a 3-bar bundle according to ACI Section ***25.6.1.5***.

$$l_d = 1.2 \times 38.4 \times 25 = 1152\ mm$$ ■

6.8 LAP SPLICES

6.8.1 BASIC CONCEPTS

- Need for Splices:

  In general, reinforcing bars are stocked by suppliers in lengths of 12m. For this reason, and because it is often more convenient to work with shorter bar lengths, it is frequently necessary to splice bars in the field.
- Splice Types:

  Rebars are spliced to each other by:
  - Lap Splices:

    In this type, rebars are usually made simply by lapping the bars a sufficient distance to transfer stress by bond from one bar to the other. The lapped bars are usually placed in contact and lightly wired so that they stay in position as the concrete is placed.
  - Mechanical Splices:

    Sample of mechanical splice is presented in ***Figure 6.8-2***.

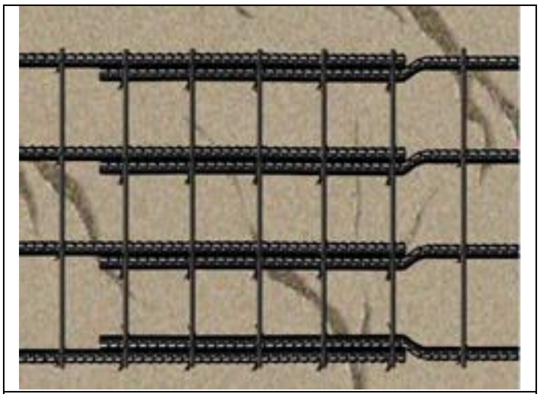


Figure 6.8-1: Lap Splices.

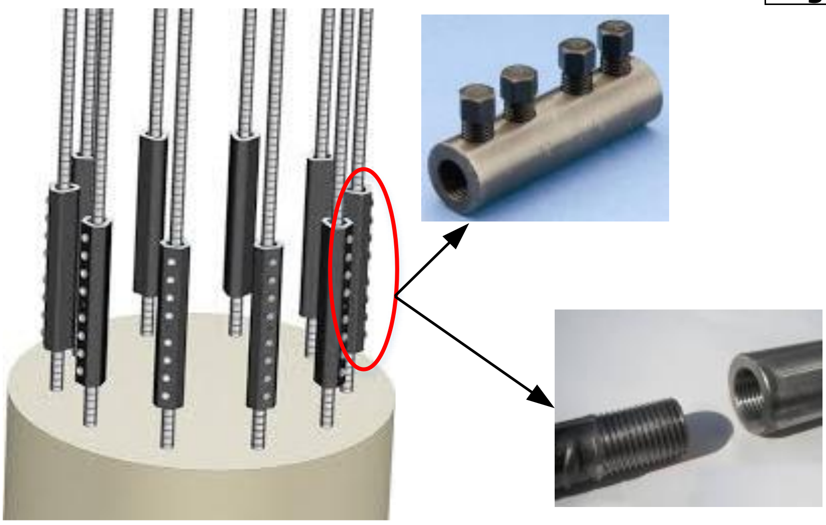


Figure 6.8-2: Mechanical Splice for Rebars.

  - Welding Splice:

    Splice with welding splice, with fillet weld, is presented in ***Figure 6.8-3***.

Figure 6.8-3: Welding Splice for Rebars.

- Only lap splice is considered in this article.

6.8.2 GENERAL NOTES ON LAP SPLICES

- According to (ACI318M, 2014), ***Article 25.5.1.1***, ***Lap splices shall not be used for bars larger than No. 36 except as provided in 25.5.5.3 (***compression lap splices of No. 43 and No. 57 bars with smaller bars***)***. This because of lack of adequate experimental data on lap splices for larger diameters.
- According to (ACI318M, 2014), ***Article 25.5.1.4***, ***Lap splices of bars in a bundle shall be based on the lap splice length required for individual bars within the bundle, increased in accordance with Article 25.6.1.7 (***increased by 20 percent and 33 percent for 3- and 4-bar bundles, respectively***).***
- According to (ACI318M, 2014), ***Article 25.5.1.4***, reduction of development length in accordance with ${A_{s\,required}}/{A_{s\,provided}}$ is not permitted in calculating lap splice lengths because the splice classifications already reflect any excess reinforcement at the splice location.

6.8.3 Lap Splices in Tension

- According to (ACI318M, 2014), ***Article 25.5.2.1***, tension lap splice length l_{st} for deformed bars and deformed wires in tension shall be in accordance with ***Table 6.8-1*** below, Table 25.5.2.1 of (ACI318M, 2014):

Table 6.8-1: Lap splice lengths of deformed bars and deformed wires in tension, Table 25.5.2.1 of (ACI318M, 2014).

$A_{s,provided}/A_{s,required}$[1] over length of splice	Maximum percent of A_s spliced within required lap length	Splice type	ℓ_{st}	
≥ 2.0	50	Class A	Greater of:	$1.0\ell_d$ and 300 mm
	100	Class B	Greater of:	$1.3\ell_d$ and 300 mm
< 2.0	All cases	Class B		

[1]Ratio of area of reinforcement provided to area of reinforcement required by analysis at splice location.

- The two-level lap splice requirements encourage splicing bars at points of minimum stress and staggering splices to improve behavior of critical details.
- For calculating ℓ_d for staggered splices, the clear spacing is taken as the minimum distance between adjacent splices, as illustrated in ***Figure 6.8-4*** below.
- According to (ACI318M, 2014), ***Article 25.5.2.2***, if bars of different size are lap spliced in tension, ℓ_{st} shall be the greater of ℓ_d of the larger bar and ℓ_{st} of the smaller bar.

6.8.4 Tension Lap Splice for Columns

- For tension lap splice in columns, see (ACI318M, 2014) Article ***10.7.5.2***.
- Tension lap splice in columns is out of our scope in this article.

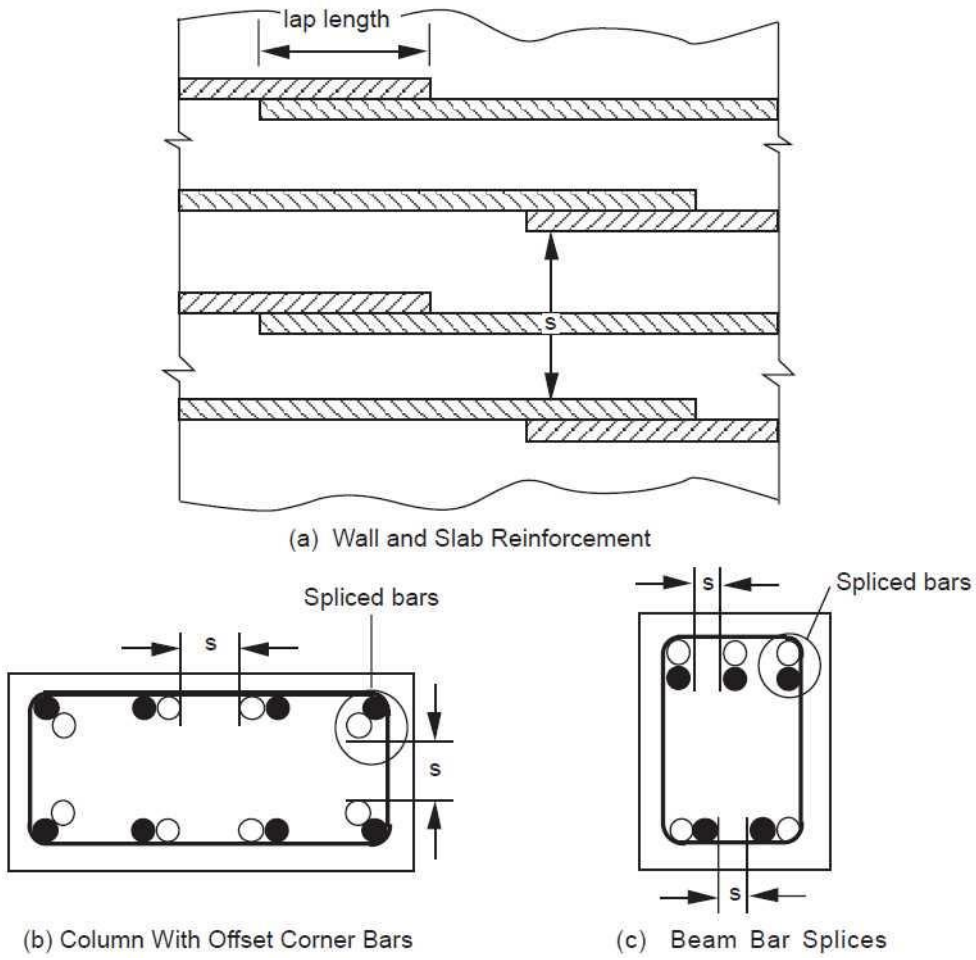


Figure 6.8-4: Effective Clear Spacing of Spliced Bars for determination of ℓ_d for staggered splices, adopted from (Kamara, 2005).

Example 6.8-1

Calculate the lap-splice length for six No. 25 tension bottom bars (in two rows) with clear spacing of 63.5 mm, clear cover of 40 mm and stirrups of 10mm for the following cases:

- When three bars are spliced and (As provided)/(As required) > 2.
- When four bars are spliced and (As provided)/(As required) < 2.
- When all bars are spliced at the same location.

In your solution assume $f_c' = 35\ MPa$ and $f_y = 420\ MPa$ and use basic equation to compute l_d

Solution

Compute the development length, l_d:

$$l_d = \left(\frac{f_y}{1.1\lambda\sqrt{f_c'}} \frac{\psi_t\psi_e\psi_s}{\frac{c_b + K_{tr}}{d_b}} \right) d_b$$

$$c_b = minimum\left(side\ cover\ to\ center\ of\ bar, top\ cover\ to\ center\ of\ bar, \frac{1}{2} S_{\frac{c}{c}}\right)$$

$$c_b = minimum\left(40 + 10 + \frac{25}{2}, 40 + 10 + \frac{25}{2}, \frac{1}{2} \times (63.5 + 25)\right)$$

$$c_b = minimum(62.5, 62.5, 44.3) = 44.3\ mm$$

For bottom rebars,

$\psi_t = 1.0$

For epoxy uncoated rebars,

$\psi_e = 1.0$

For bar with diameter of 25mm > 19mm,

$\psi_s = 1.0$

For normal weight concrete,

$\lambda = 1.0$

As nothing has been mentioned about stirrups spacing,

$K_{tr} = 0$

and the confinement factor would be:

$$\frac{c_b + K_{tr}}{d_b} = \frac{44.3 + 0}{25} = 1.77 < 2.5 \therefore Ok.$$

$$l_d = \left(\frac{420}{1.1 \times 1.0 \times \sqrt{35}} \frac{1.0 \times 1.0 \times 1.0}{\frac{44.3 + 0}{25}} \right) d_b = 36.4\ d_b = 910\ mm$$

Splice

When three bars are spliced and (As provided)/(As required) > 2:

As

$$\frac{A_{s\ provided}}{A_{s\ required}} > 2.0$$

and only 50% of reinforcement to be spliced, therefore splice can be classified as Class A.

$$l_{splice} = 1.0\ l_d = 910\ mm > 300mm \therefore Ok.$$

When four bars are spliced and (As provided)/(As required) < 2:

Class B splice should be adopted.

$$l_{splice} = 1.3 l_d = 1.3 \times 910 = 1183\ mm > 300\ mm \therefore Ok.$$

When all bars are spliced at the same location:

Class B splice should be adopted.

$$l_{splice} = 1.3 l_d = 1.3 \times 910 = 1183\ mm > 300\ mm \therefore Ok.$$

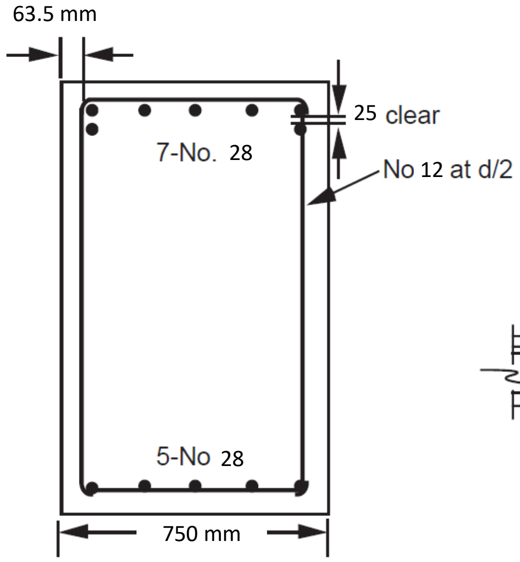
Example 6.8-2

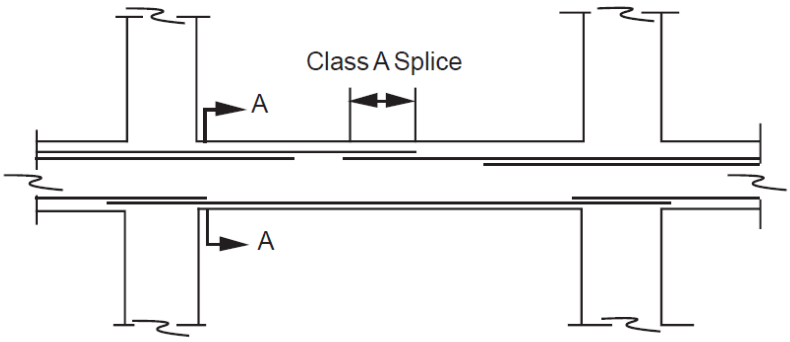
A beam at the perimeter of the structure has 7-No. 28 top bars over the support. Structural integrity provisions require that at least one-sixth of the tension reinforcement be made continuous, but not less than 2 bars (9.7.7.1).

Bars are to be spliced with a Class A splice at mid-span. Determine required length of Class A lap splice for the following two cases:

Case I - Development length computed based on simplified equation.
Case II - Development length computed based basic equation.
In your solution assume:

- Lightweight concrete
- Epoxy-coated bars
- $f_c' = 28\ MPa$
- $f_y = 420\ MPa$





Solution

<u>Case I - Development computed from simplified equation:</u>

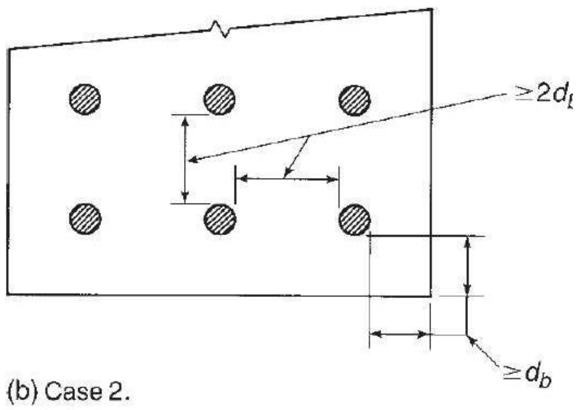
Check confinement: try Case 2:

$$Clear\ Spacing = \frac{(750 - 2 \times 63.5 - 2 \times 12 - 2 \times 28)}{28} = 19.4 \gg 2$$

$$Side\ Cover = \frac{63.5 + 12}{28} = 2.7 > 2$$

Then rebar is confined according to requirement of Case 2. As rebar diameter is greater than No. 19, therefore development length would be:

$$l_d = \left(\frac{f_y \psi_t \psi_e}{1.7\lambda\sqrt{f_c'}}\right) d_b$$



(b) Case 2.

	Spacing and cover	No. 19 and smaller bars and deformed wires	No. 22 and larger bars
Case 1 Case 2	Clear spacing of bars or wires being developed or spliced not less than d_b, clear cover not less than d_b, and stirrups or ties throughout ℓ_d not less than the Code minimum or Clear spacing of bars or wires being developed or spliced not less than $2d_b$ and clear cover not less than d_b	$\left(\frac{f_y \Psi_t \Psi_e}{2.1\lambda\sqrt{f_c'}}\right) d_b$	$\left(\frac{f_y \Psi_t \Psi_e}{1.7\lambda\sqrt{f_c'}}\right) d_b$
	Other cases	$\left(\frac{f_y \Psi_t \Psi_e}{1.4\lambda\sqrt{f_c'}}\right) d_b$	$\left(\frac{f_y \Psi_t \Psi_e}{1.1\lambda\sqrt{f_c'}}\right) d_b$

For top rebars, assuming concrete below rebar is greater than 300 as it is clear from Figure above assuming it has been drawn to scale.

$\psi_t = 1.3$

For epoxy coated with clear cover less than 3d,

$\psi_e = 1.5$

$\psi_t \psi e = 1.3 \times 1.5 = 1.95 > 1.7 \therefore Not\ ok$

Let

$\psi_t \psi_e = 1.7$

For lightweight concrete:

$\lambda = 0.75$

$$l_d = \left(\frac{f_y \psi_t \psi_e}{1.7\lambda\sqrt{f_c'}}\right) d_b = \left(\frac{420 \times 1.7}{1.7 \times 0.75 \times \sqrt{28}}\right) d_b = 106\ d_b = 106 \times 28 = 2968\ mm$$

For Case A splice:

$l_{splice} = 1.0\ l_d = 2968\ mm > 300\ mm$ ■

<u>Case II - Development computed from basic equation.</u>

According to basic equation below:

$$l_d = \left(\frac{f_y}{1.1\lambda\sqrt{f_c'}} \frac{\psi_t \psi_e \psi_s}{\frac{c_b + K_{tr}}{d_b}}\right) d_b$$

As discussed above,

$\psi_t \psi_e = 1.7$

$\lambda = 0.75$

$\because d_b = 28mm > 19mm \therefore \psi_s = 1.0$

For rebars in the second layer, only side cover and center to center rebar spacing to be considered:

$$c_b = minimum\left(63.5 + 12 + \frac{28}{2}, \frac{1}{2} \times (750 - 2 \times 63.5 - 2 \times 12 - 28)\right) = minimum\ (89.5\ or\ 286)$$
$$= 89.5\ mm$$

Without computing K_{tr}, one can conclude that:

$$\frac{c_b + K_{tr}}{d_b} = \frac{89.5}{28} = 3.2 > 2.5$$

Therefore,

$$\frac{c_b + K_{tr}}{d_b} = 2.5$$

$$l_d = \left(\frac{f_y}{1.1\lambda\sqrt{f_c'}} \frac{\psi_t \psi_e \psi_s}{\frac{c_b + K_{tr}}{d_b}}\right) = \left(\frac{420}{1.1 \times 0.75 \times \sqrt{28}} \frac{1.7 \times 1.0}{2.5}\right) d_b = 65.4\ d_b$$

$l_d = 65.4 \times 28 = 1831\ mm$

For Case A splice:

$l_{splice} = 1.0\ l_d = 1831\ mm > 300mm$ ■

Example 6.8-3

Based on structural analysis and design, a designer has adopted bottom reinforcement of No. 12@200mm for the slab indicated in ***Figure 6.8-5***. Using ACI basic equation, determine the development length, l_d, for the adopted positive slab reinforcement and then compute the corresponding lap splice length.

Solution

Computing of the development length, $\boldsymbol{l_d}$:

According to basic equation of the ACI code, the development length for tension rebars, l_d, would be:

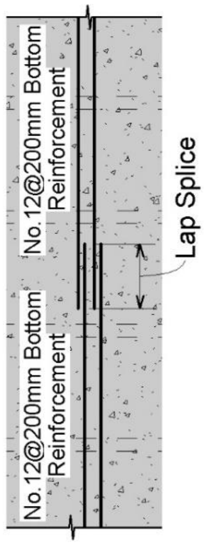


Figure 6.8-5: Slab reinforcement for Example 6.8-3

$$l_d = \left(\frac{f_y}{1.1\lambda\sqrt{f_c'}}\frac{\psi_t\psi_e\psi_s}{\frac{c_b+K_{tr}}{d_b}}\right)d_b$$

$$c_b = \text{minimum}\begin{pmatrix} side\ cover\ to\ center\ of\ bar, \\ top\ cover\ to\ center\ of\ bar, \frac{1}{2}S_{\frac{c}{c}} \end{pmatrix} \Rightarrow c_b = \min\begin{pmatrix} \left(20+\frac{12}{2}\right), \\ \left(20+\frac{12}{2}\right), \left(\frac{1}{2}\times 200\right) \end{pmatrix}$$

$c_b = \min(26,26,100) = 26\ \text{mm}$

As there is no shear reinforcement in the cantilever slab, therefore:

$K_{tr} = 0$

As the concrete is normal weight concrete, $\lambda = 1.0$. For uncoated rebars, $\psi_e = 1.0$.

As the rebars are bottom rebars from bond point of view.

$\psi_t = 1.0$

Finally, for rebars with size less than 19mm, $\psi_s = 0.8$.

$$l_d = \left(\left(\frac{420}{1.1\times 1.0\times\sqrt{28}}\right)\times\left(\frac{1.0\times 1.0\times 0.8}{\frac{26+0}{12}}\right)\right)d_b = 26.6\text{d}_\text{b}$$

As nothing is mentioned about $A_{s\,required}/A_{s\,provided}$, therefore it can be conservatively assumed 1.0.

$l_d = 26.6\text{d}_\text{b} = 26.6\times 12 = 319\ \text{mm} > 300\ \text{mm} \therefore \text{Ok.} \Rightarrow l_d = 319\ mm$ ■

Splice Length:

As nothing has been mentioned about $A_{s\,provided}/A_{s\,required}$, therefore the splice would conservatively be classified as ***Class B***.

$l_{st} = maximum(1.3l_d, 300) = 1.3\times 319 = 415\ mm$ ■

6.8.5 Lap Splice Lengths of Deformed Bars in Compression

6.8.5.1 General Requirements

According to (ACI318M, 2014), ***25.5.5.2***, compression lap splices shall not be used for bars larger than No. 36, except to No. 36 or smaller bars.

6.8.5.2 Compression Lap Splice Length ℓ_{sc}

- According to (ACI318M, 2014), ***25.5.5.1***, compression lap splice length ℓ_{sc} of No. 36 or smaller deformed bars in compression shall be calculated in accordance with (a) or (b):

  (a) For $f_y \leq 420\ MPa$:

  $\ell_{sc} = maximum\ (0.071 f_y d_b and\ 300\ mm)$

  (b) For $f_y > 420\ MPa$:

  $\ell_{sc} = maximum\ ((0.13 f_y - 24) d_b and\ 300\ mm)$

- For $f_c' < 21\ MPa$, the length of lap shall be increased by one-third.

6.8.5.3 Reducing in Compression Lap Splice Length ℓ_{sc}

According to (ACI318M, 2014), ***10.7.5.2.1***, it shall be permitted to decrease the compression lap splice length in accordance with (a) or (b), but the lap splice length shall be at least 300 mm.

(a) For tied columns, where ties throughout the lap splice length have an effective area not less than $\mathbf{0.0015hs}$ in ***both directions***, lap splice length shall be permitted to be multiplied by ***0.83***. Tie legs perpendicular to dimension h shall be considered in calculating effective area.

(b) For spiral columns, where spirals throughout the lap splice length satisfy ***25.7.3***, ***this will be discussed thoroughly in column design***, lap splice length shall be permitted to be multiplied by ***0.75***.

6.8.5.4 Lap Splice for Bars with Different Size

- According to (ACI318M, 2014), ***25.5.5.3***, compression lap splices of No. 43 or No. 57 bars to No. 36 or smaller bars shall be permitted and shall be in accordance with (ACI318M, 2014), ***25.5.5.4***, presented in below.
- According to (ACI318M, 2014), ***25.5.5.4***, where bars of different size are lap spliced in compression, ℓ_{sc} shall be:

  $l_{sc} = maximum\ (l_{dc}\ for\ larger\ bar\ or\ \boldsymbol{l_{sc}}\ \text{of smaller bar})$

6.8.5.5 Common Details for Columns Splices

- The ***most common method of splicing column steel*** is ***the simple lapped bar splice***, with the bars in contact throughout the lapped length.
- It is standard practice to offset the lower bars, as shown in ***Figure 6.8-6*** below

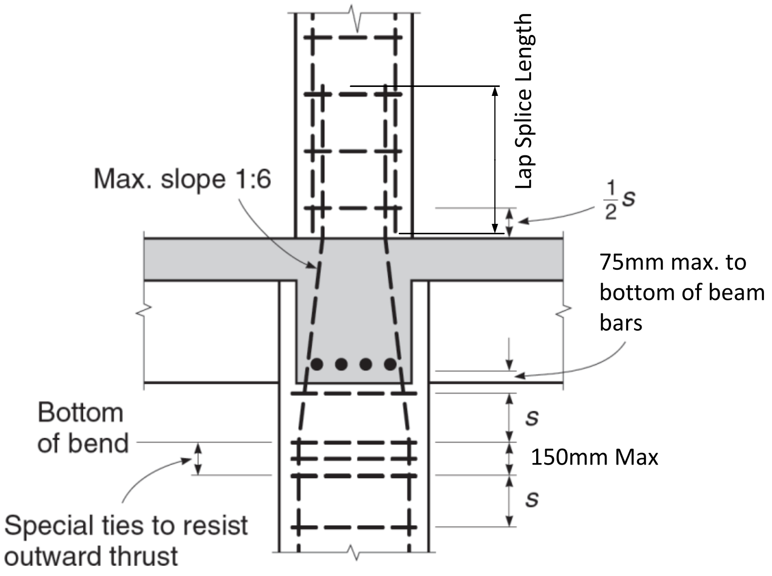
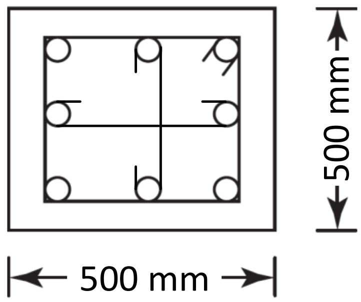


Figure 6.8-6: Splice details at typical interior column. Beams frame into joint from four directions.

6.8.5.6 Examples

Example 6.8-4

Calculate the lap-splice length for a tied column. The column has eight No. 32 longitudinal bars and No. 10 ties with spacing of 450mm. Given $f_c' = 35\ MPa$, $f_y = 420\ MPa$, $f_y = 560\ MPa$ and all rebars under compression.



Solution

For $f_y \leq\ 420\ MPa$:

$\ell_{sc} =\ maximum\ \left(0.071 f_y d_b and\ 300\ mm\right)$

$\ell_{sc} =\ maximum\ (0.071 \times 420 \times 32\ and\ 300\ mm)$

$\ell_{sc} =\ maximum\ (954\ \ and\ 300\ mm) = 954\ mm$

Reduction in l_{sc}:

Determine column tie requirements to allow 0.83 reduce lap-splice length according to ACI Code, Section 10.7.5.2.1.

$Effective\ area\ of\ ties\ \geq 0.0015\ hs$

$3 \times \frac{\pi \times 10^2}{4} = 236\ mm^2 < 0.0015 \times 500 \times 450 = 338\ mm^2$

Modifier 0.83 will not apply. Lap-splice length is $954\ mm$.

For $f_y >\ 420\ MPa$:

$\ell_{sc} =\ maximum\ \left((0.13 f_y - \ 24) d_b and\ 300\ mm\right)$

$\ell_{sc} =\ maximum\ \left((0.13 \times 560 -\ 24) \times 32\ and\ 300\ mm\right)$

$\ell_{sc} =\ maximum\ (1562\ and\ 300\ mm) = 1562\ mm$

Modifier 0.83 will not apply as previously calculated.

6.11 Contents

CHAPTER 7
SERVICEABILITY

7.1 INTRODUCTION

- What have been achieved in Chapters 4, 5, and 6?
  - Chapters 4, 5, and 6 have dealt mainly with the ***strength design*** of reinforced concrete beams.
  - Methods have been developed to ensure that beams will have ***a proper safety margin against failure*** in ***flexure*** or ***shear***, or due to ***inadequate bond and anchorage*** of the reinforcement.
  - The member has been ***assumed*** to be at a ***<u>hypothetical</u> overload state*** for this purpose.
- Performance in normal service:
  - It is also important that ***member performance*** in ***normal service*** be ***satisfactory***.
  - ***Normal service conditions*** are ***when*** loads are ***those actually expected to act***, that is, when ***load factors are 1.0***.
- Member adequacy in strength is not necessarily adequate in service conditions.
  - ***Normal service conditions are not guaranteed*** simply by ***providing adequate strength***.
- Aspects that such be checked under normal conditions:
  - ***Deflection:***
    It may be:
    - Excessively large under full-service,
    - Long-term due to sustained loads,

    such that may ***cause damage***.
  - ***Tension cracks***:
    They in beams may be wide enough to:
    - Be visually disturbing,
    - reduce the durability of the structure.
  - ***Vibration or Fatigue***:
    Vibration or Fatigue are other questions that require consideration under service conditions. These aspects are out the scope of this course.
- The theory adopted to study the elastic conditions:
  - ***Serviceability studies*** are ***carried out based on elastic theory***.
  - Assumptions of the elastic theory:
    The elastic theory for analysis assumes that:
    - ***Stresses*** in ***both concrete*** and ***steel*** are ***proportional to strain***.
    - The ***concrete on the tension side*** of the neutral axis may be ***uncracked***, ***partially cracked***, or ***fully cracked***, depending on the loads and material strengths.
- Past versus current design philosophies:
  - ***In early*** reinforced concrete designs, ***questions of serviceability*** were dealt with ***indirectly***, by ***limiting the stresses in concrete and steel at service loads*** to the rather ***conservative values*** that had resulted in satisfactory performance.
  - The current design methods:
    - It ***permits more slender members*** through:
      1. More accurate assessment of capacity,
      2. Higher-strength materials.
    - It contributs to the trend toward ***smaller member sizes***, such that the old indirect methods no longer work.
    - The ***current approach*** is to ***investigate service load cracking*** and ***deflections*** specifically, ***<u>after</u> proportioning members based on strength requirements***.
- Scope of this Chapter:
  According to the text book:
  - ***Tension cracks***
    - This chapter develops methods to ensure that the cracks associated with flexure of reinforced concrete beams are narrow and well distributed.
    - For structures other than ***liquid retaining structures***, the concept of $s_{maximum}$ that has been discussed in Chapter 4 is adequate to ensure narrow and well distributed cracks.

    - Therefore, due to limited time, the explicit checking of cracks may be skipped in buildings-oriented design courses.
- Deflection control:
  After reviewing the deflection determinations from the mechanics of material and theory of structures, this chapter aims to:
    - Modify deflections determined based on assumptions of the uncracked section and short-term effect to be more accurate and representative for actual structures where the sections are fully or partially cracked, and the loads are sustained in nature.
    - Give permissible limits for the deflections.

7.2 Control of Deflections

7.2.1 Basic Concepts

- Main Concerns of Excessive Deflection:
  Excessive deflections can lead to:
  - Cracking of supported walls and partitions,
  - Ill-fitting doors and windows,
  - Poor roof drainage,
  - Misalignment of sensitive machinery and equipment,
  - Visually offensive sag.

  It is important, therefore, to maintain control of deflections, in one way or another. ***So that members designed mainly for strength at prescribed overloads will also checked in normal service.***
- Approaches for Deflection Control:
  There are presently two approaches to deflection control:
  - Indirect Approach.
  - Direct Approach.

  These approaches are discussed in some details in following articles, and different illustrated examples are presented.

7.2.2 Indirect Approach

- The approach consists of setting proper upper limits on the span-depth ratio. These limits are as follows:
  - For one-way slabs:
    According to the code ***7.3.1.1***, for ***solid nonprestressed slabs not supporting or attached to partitions or other construction likely to be damaged by large deflections***, overall slab thickness h shall not be less than the limits in ***Table 7.2-1***, ***unless the calculated deflection limits are satisfied***.
  - For beams:
    According to the code ***9.3.1.1***, ***for nonprestressed beams not supporting or attached to partitions or other construction likely to be damaged by large deflections***, overall beam depth h shall satisfy the limits in ***Table 7.2-2***, ***unless the calculated deflection limits are satisfied***.

Table 7.2-1: Minimum thickness of solid nonprestressed one-way slabs, Table 7.3.1.1 of the code.

Support condition	Minimum h[1]
Simply supported	$\ell/20$
One end continuous	$\ell/24$
Both ends continuous	$\ell/28$
Cantilever	$\ell/10$

[1]Expression applicable for normalweight concrete and f_y = 420 MPa. For other cases, minimum h shall be modified in accordance with 7.3.1.1.1 through 7.3.1.1.3, as appropriate.

Table 7.2-2: Minimum depth of nonprestressed beams, Table 9.3.1.1 of the code.

Support condition	Minimum h[1]
Simply supported	$\ell/16$
One end continuous	$\ell/18.5$
Both ends continuous	$\ell/21$
Cantilever	$\ell/8$

[1]Expressions applicable for normalweight concrete and Grade 420 reinforcement. For other cases, minimum h shall be modified in accordance with 9.3.1.1.1 through 9.3.1.1.3, as appropriate.

- This approach is simple, and it is ***satisfactory*** in many cases ***where spans***, ***loads*** and ***load distributions***, and ***member sizes*** and ***proportions fall in the usual ranges***.
- The condition of "***Members not supporting or attached to partitions or other construction likely to be damaged by large deflections***" has been left for designer judgment.

7.2.3 Direct Approach

- In this approach, deflection has to be calculated and to be compared with specific limitations that may be imposed by codes or by special requirements.
- Methods for predicating deflection in RC beams and ACI limitations on deflection are discussed in articles below.

7.2.4 Deflection Types in RC Beams

Two types of deflections are usually noted in RC beams:

- Immediate Deflection:
  As its name implies, this type of deflection occurs immediately when the load is applied.
- Long-term Deflection (Time Dependent Deflections):
  - These time-dependent deformations take place gradually over an extended time.
  - They are chiefly due to concrete creep and shrinkage.
  - Because of these influences, reinforced concrete members continue to deflect with the passage of time. Long-term deflections continue over a period of several years.
  - They may eventually be 2 or more times the initial elastic deflections.

7.3 IMMEDIATE DEFLECTION

- Safety provisions of the ACI Code and similar design specifications ensure that, under loads up to the full-service load, ***stresses in both steel and concrete remain within the elastic ranges***.
- Consequently, ***deflections that occur at once upon application of load, can be calculated based on the properties of the uncracked elastic member, the cracked elastic member, or some combination of these***.
- From the mechanics of materials, it is well known that ***elastic deflections*** can be expressed in the general form:

$$\Delta_{Immediate\ or\ Elastic} = \frac{f(Loads, Spans, Supports)}{EI} \qquad \text{Eq. 7.3-1}$$

- Deflection relations can easily be computed and tabulated for many loadings and spans arrangements as shown in ***Table 7.3-1***, ***Table 7.3-2***, and ***Table 7.3-3***.
- With the tabulated relations, the deflection computing in reinforced concrete structures are reduced into:
  - What should load values be in deflection computing?
  - What is the appropriate flexural rigidity EI for the member?

  These two issues are discussed below.

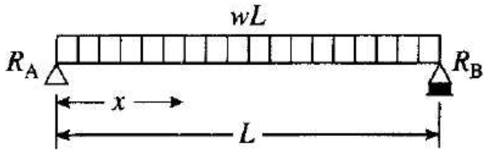
7.3.1 LOADS USED IN DEFLECTION CALCULATIONS

- The deflections of concern are generally those that occur during the ***normal service life of the member***.
- In service, a member sustains ***the full dead load***, plus ***some fraction*** or ***all*** of the specified ***service live load***.

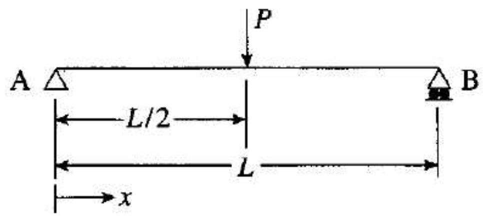
Table 7.3-1: Deflection of Simply Supported Beams.

$$\Delta_{max} = \frac{5}{384} \times \frac{WL^3}{EI} \quad \text{(at center)}$$

$$W = \text{total load} = wL$$



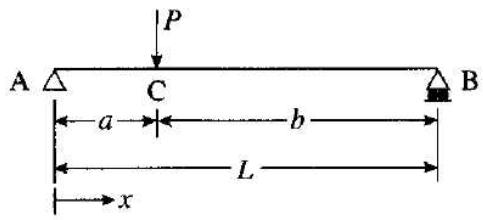
$$\Delta_{max} = \frac{PL^3}{48EI} \quad \text{(at midspan)}$$



$$\Delta_C = \frac{Pa^2b^2}{3EIL} \quad \text{(at point load)}$$

$$\Delta_{max} = \frac{PL^3}{48EI}\left[\frac{3a}{L} - 4\left(\frac{a}{L}\right)^3\right] \quad \text{(when } a \geq b\text{)}$$

$$\text{at } x = \sqrt{a(b+L)/3}$$



$$\Delta_{max} = \frac{PL^3}{6EI}\left[\frac{3a}{4L} - \left(\frac{a}{L}\right)^3\right] \quad \text{(at midspan)}$$

$$\Delta_{max} = \frac{23PL^3}{648EI} \quad \text{(at midspan) when } a = L/3$$

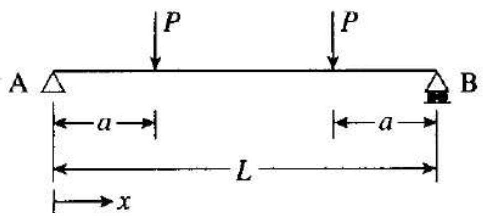
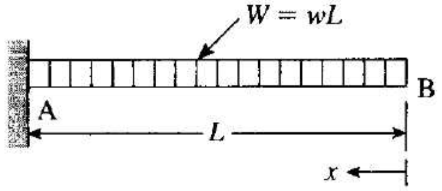


Table 7.3-2: Deflection for Cantilever Beams.

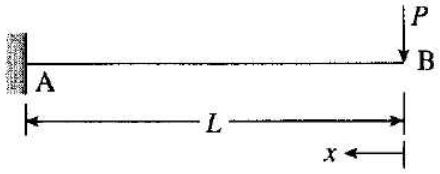
$$\Delta_B \max = \frac{WL^3}{8EI} \quad (W = wL)$$

$$\Delta_x = \frac{w}{24EIL}(x^4 - 4L^3x + 3L^4)$$



$$\Delta_B \max = \frac{PL^3}{3EI}$$

$$\Delta_x = \frac{P}{6EI}(2L^3 - 3L^2x + x^3)$$



$$\Delta_C = Pa^3/3EI$$

$$\Delta_B \max = \frac{Pa^3}{3EI}\left(1 + \frac{3b}{2a}\right) \quad \text{(at free end)}$$

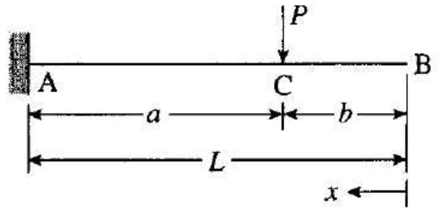
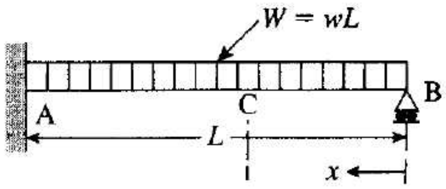


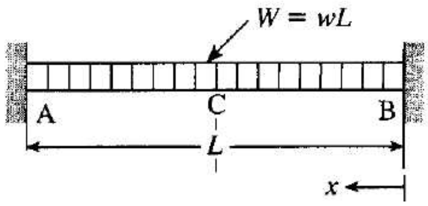
Table 7.3-3: Deflection of Statically Indeterminate Single Span Beams.

$$\Delta_{max} = \frac{WL^3}{185EI}$$

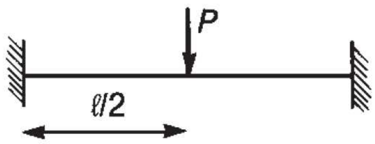
at a distance $x = 0.4215L$ (from support B)



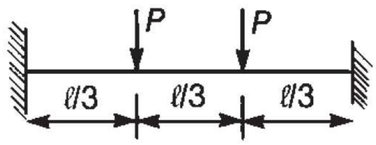
$$\Delta_{max} = \frac{WL^3}{384EI} \quad \text{(at midspan)}$$



$$\Delta_{mid} = \frac{1}{192} \cdot \frac{P\ell^3}{EI}$$



$$\Delta_{mid} = \frac{5}{684} \cdot \frac{P\ell^3}{EI}$$



7.3.2 MEMBER FLEXURAL RIGIDITY EI

7.3.2.1 MODULUS OF ELASTICITY E

- According to ACI 24.2.3.4, ***immediate deflection shall be computed with the modulus of elasticity for concrete,*** E_c.

  $E_{for\ immddiate\ deflection\ calculations} = E_c$ Eq. 7.3-2

- According to ACI 19.2.2, ***modulus of elasticity,*** E_c ***for normal weight concrete*** could be computed based on following relation.

  $E_c = 4700\sqrt{f_c'}$ Eq. 7.3-3

7.3.2.2 EFFECTIVE MOMENT OF INERTIA I_e

7.3.2.2.1 Uncracked Elastic Range

- If the maximum moment in a flexural member is so small that the tensile stress in the concrete does not exceed the modulus of rupture f_r, no flexural tension cracks will occur. The full, uncracked section is then available for resisting stress and providing rigidity, see ***Figure 7.3-1***.

- Then for applied moment M_a less than cracking moment M_{cr}, effective moment of inertia I_e that could be used for composite RC beams is:

  $\because M_a \leq M_{cr} \Rightarrow \therefore I_e \approx I_g$ Eq. 7.3-4

  where:

  I_e is effective moment of inertia for computation of deflection, mm4.

  I_g is moment of inertia of gross concrete section about centroidal axis, neglecting reinforcement, mm4.

  M_{cr} is cracking moment, according to ACI ***24.2.3.5***, it could be computed as follows:

  $$M_{cr} = \frac{f_r I_g}{y_t}$$ Eq. 7.3-5

  and

  f_r is modulus of rupture of concrete, MPa:

  $$f_r = 0.62\lambda\sqrt{f_c'}$$ Eq. 7.3-6

  y_t is distance from centroidal axis of gross section, neglecting reinforcement, to tension face, mm. Above terms are more clarified with referring to Figure 7.3-2.

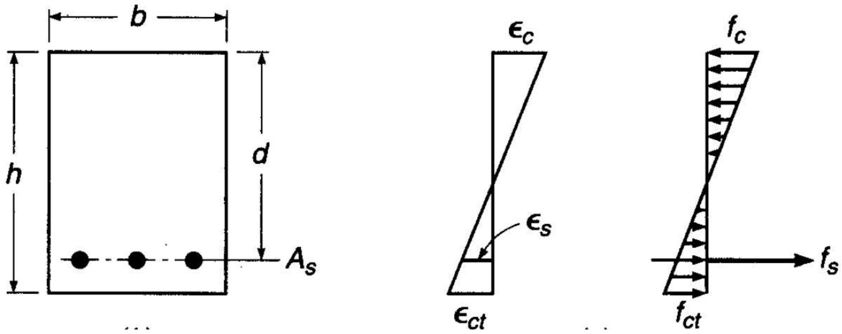


Figure 7.3-1: Uncracked elastic section.

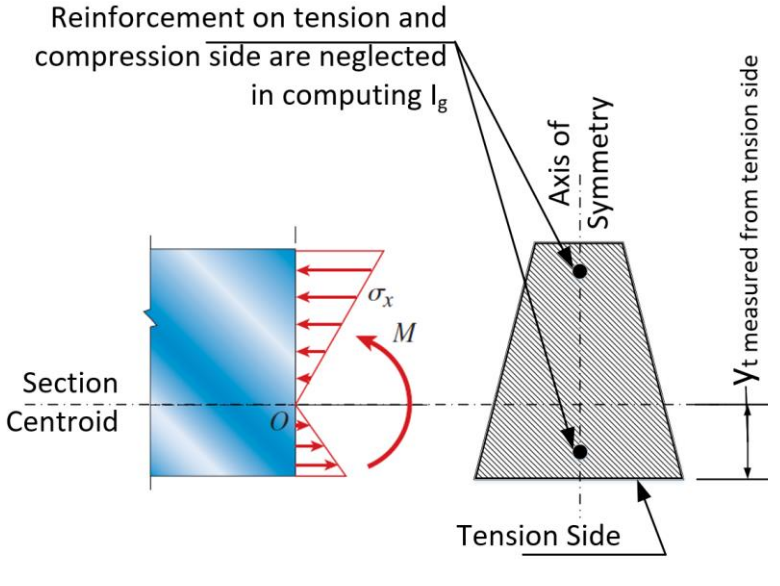


Figure 7.3-2: Gross moment of inertia for a RC beam with general symmetrical shape.

Example 7.3-1

For the rectangular concrete section that shown in ***Figure 7.3-3***, calculate

- Modulus of rupture, f_r,
- Gross moment of inertia, I_g,
- Cracking moment, M_{cr}.

Use $f_c' = 28\ MPa$ and $f_y = 420\ MPa$.

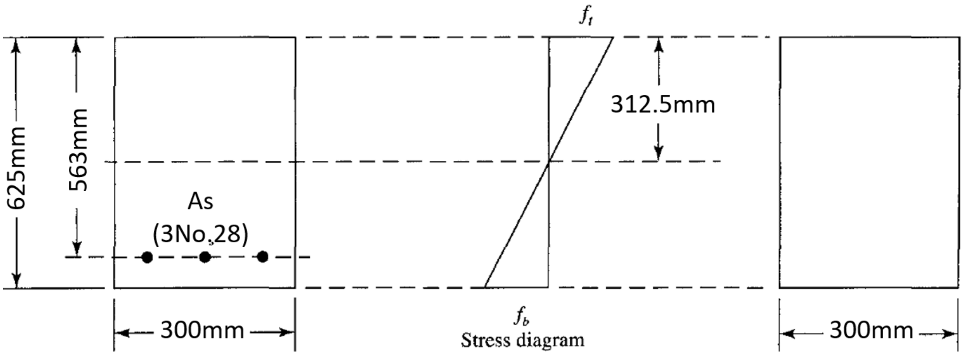


Figure 7.3-3: Beam for Example 7.3-1.

Solution

Modulus of rupture:

$f_r = 0.62\lambda\sqrt{f_c'}$

With normal weight concrete,

$\lambda = 1.0$

$f_r = 0.62 \times 1.0 \times \sqrt{28} = 3.28\ MPa$ ■

Gross Moment of Inertia:

With neglecting of reinforcement, gross section is a rectangular one:

$$I_g = \frac{bh^3}{12} = \frac{300 \times 625^3}{12} = 6104 \times 10^6\ mm^4 ■$$

$$y_t = \frac{625}{2} = 313\ mm$$

$$M_{cr} = \frac{f_r I_g}{y_t} = \frac{3.28 \times 6104 \times 10^6}{313} = 64.0\ kN.m ■$$

7.3.2.2.2 Partially to Fully Cracked Range

- According to ACI ***24.2.3.5***, when applied bending moment, M_a, greater than section cracking moment, M_{cr}, section would be in ***partially*** to ***fully cracked stage*** and its effective moment of inertia could be estimated based on following relation (***Eq. 24.2.3.5a of ACI Code***).

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left(1 - \left(\frac{M_{cr}}{M_a}\right)^3\right) I_{cr} \qquad \text{Eq. 7.3-7}$$

- Trends of ***Eq. 7.3-7*** are presented in ***Figure 7.3-4***. Graphically it is presented in ***Figure 7.3-5***.

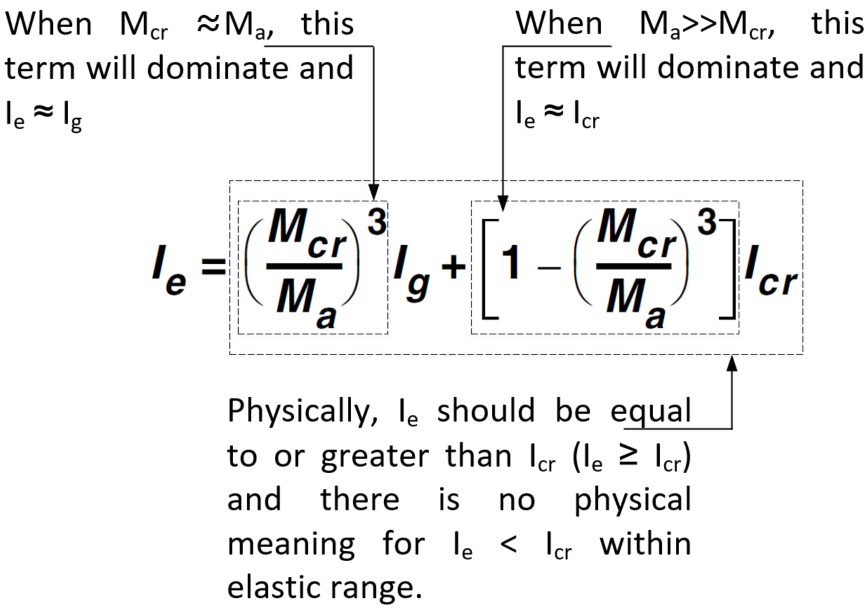


Figure 7.3-4: Trends of Eq. 7.3-7.

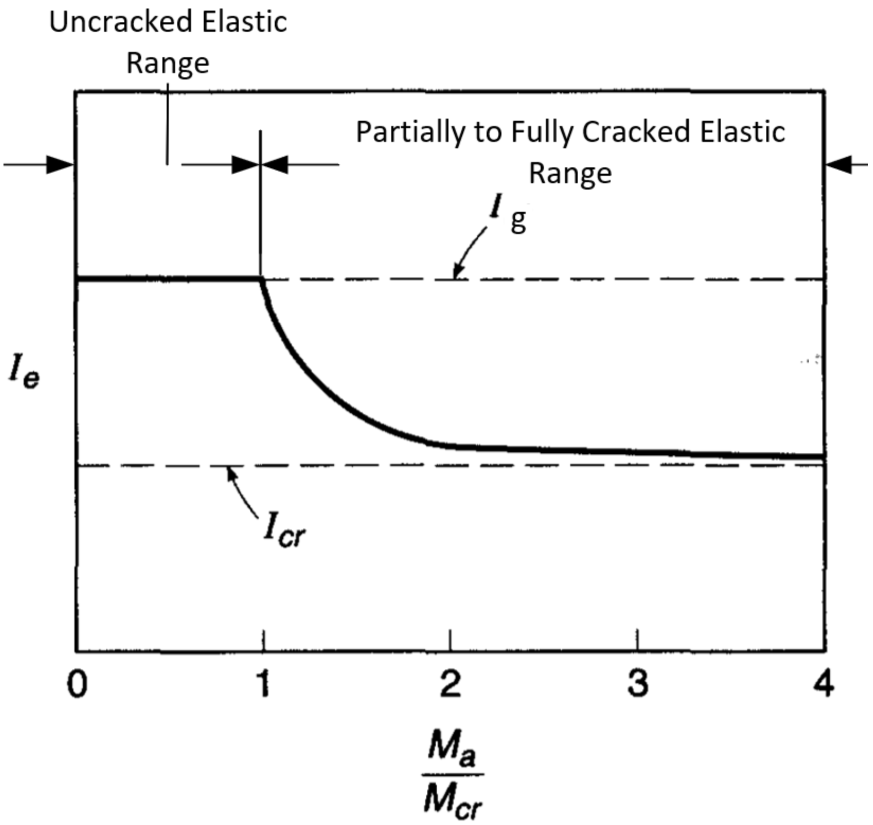


Figure 7.3-5: Variation of I_e with Moment Ratio.

- Variation of I_e along Beam Span:

  As I_e depends on $\frac{M_{cr}}{M_a}$, then it inversely varies with M_a along beam span as indicated in ***Figure 7.3-6***.

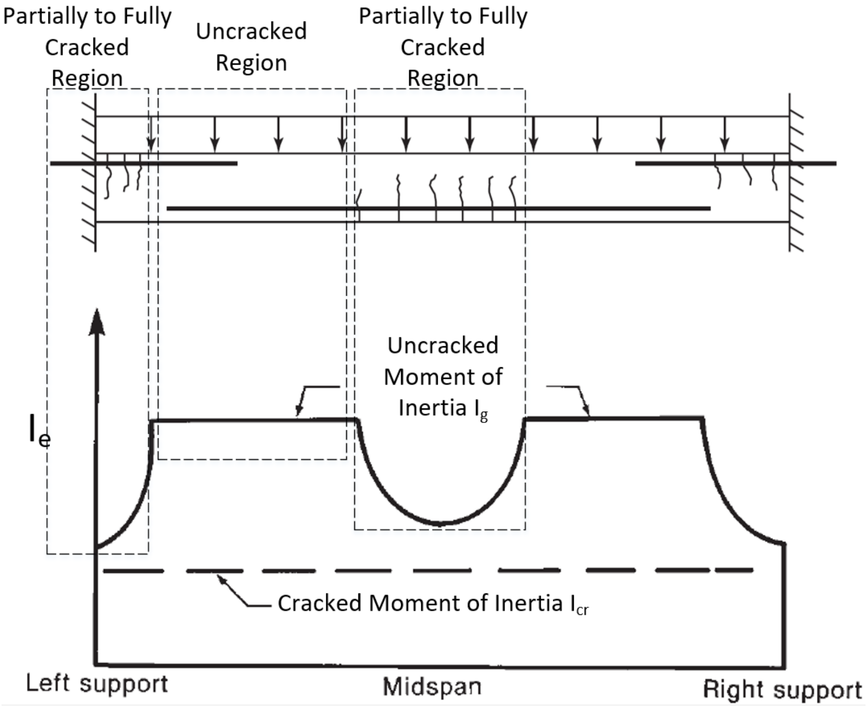


Figure 7.3-6: Variation of I_e along the Length of a Continuous Beam.

- According to ***ACI 24.2.3***, above variation of I_e along beam span could be approximated as follows for different support conditions:
  - Simply Supported Beam:

$$I_{e\ for\ simply\ supported\ beam} = I_{e\ @\ mid-span}$$

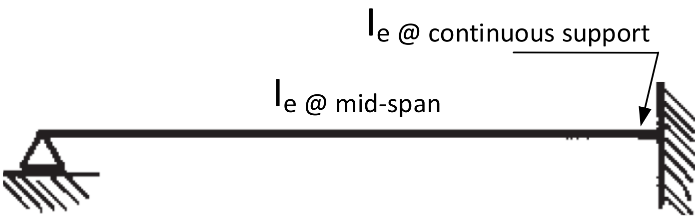
  - Both-end Continuous Beam:

$$I_{e\ for\ Both\ end\ continuous\ beam} = 0.5 I_{e\ @\ mid-span} + 0.25\left(I_{e\ @\ left\ support} + I_{e\ @\ right\ support}\right)$$

or:

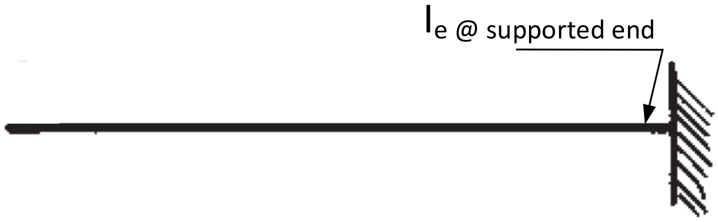
$$I_{e\ for\ Both\ end\ continuous\ beam} \approx I_{e\ @\ mid-span}$$

  - One-end Continuous Beam (Nilson, Design of Concrete Structures, 14th Edition, 2010):



$$I_{e\ for\ one\ end\ continuous\ beam} = 0.85 I_{e\ @\ mid-span} + 0.15 I_{e\ @\ continuous\ support}$$

  - Cantilever Beam:



$$I_{e\ for\ cantilever\ beam} = I_{e\ @\ supported\ end}$$

7.4 Deflections Due to Long-term Loads

- ***Initial deflections*** are ***increased significantly*** if ***loads are sustained over a long period of time***, due to the ***effects of shrinkage and creep***.
- Creep or shrinkage, which one is more dominant?
  - These two effects are usually combined in deflection calculations.
  - ***Creep generally dominates***,
  - But for ***some types of members***, ***shrinkage deflections are large*** and ***should be considered separately***.
- On the basis of ***empirical studies***, ACI Code ***24.2.4.1*** specifies that ***additional long-term deflections*** Δ_t due to the ***combined effects of creep and shrinkage*** be calculated by multiplying the immediate deflection Δ_i by a factor λ_Δ:

$$\lambda_\Delta = \frac{\xi}{1 + 50\rho'} \qquad \text{Eq. 7.4-1}$$

where

$$\rho' = \frac{A_s'}{bd}$$

and ξ is a time-dependent coefficient. It is a ***material property*** depending on ***creep*** and ***shrinkage*** characteristics and it can be estimated from ***Figure 7.4-1*** or from ***Table 7.4-1***.

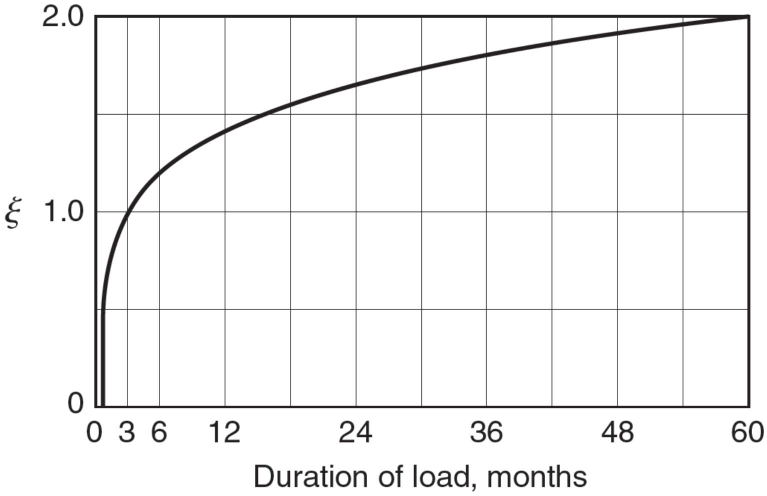


Figure 7.4-1: Time variation of ξ for long-term deflections.

Table 7.4-1: Time-dependent factor, ξ, for sustained loads, Table 24.2.4.1.3 of the code.

Sustained load duration, months	Time-dependent factor ξ
3	1.0
6	1.2
12	1.4
60 or more	2.0

- In Eq. 7.4-1, the quantity

$$\frac{1}{1 + 50\rho'}$$

  is a ***reduction factor*** that is essentially a ***section property***, reflecting the ***beneficial effect of compression reinforcement*** A_s' in ***reducing long-term deflections***.
- When should ρ' be determined along the beam span:
  According to the ACI Code the value of ρ' used in Eq. 7.4-1 should be:
  - For simple and continuous spans that at the midspan section,
  - For cantilevers that at the support.

7.5 Permissible Deflections

- To ensure satisfactory performance in service, ACI Code ***24.2.2*** imposes ***certain limits*** on ***deflections calculated according to the procedures just described***.
- These limits are given in Table 7.5-1.
- Limits depend on:
  - Whether or not the member supports or is attached to other nonstructural elements,
  - Whether or not those nonstructural elements are likely to be damaged by large deflections.
- Span length ℓ:
  - According notations and terminology in Chapter 2 of the code, the length ℓ has been defined as ***span length of beam or one-way slab; clear projection of cantilever***.
  - According the ***textbook***, this statement has been understood as that ***center to center span should be used for ℓ for spans other than cantilever where clear span should be used***.

Table 7.5-1: Maximum permissible calculated deflections, Table 24.2.2 of the code.

Member	Condition		Deflection to be considered	Deflection limitation
Flat roofs	Not supporting or attached to nonstructural elements likely to be damaged by large deflections		Immediate deflection due to maximum of L_r, S, and R	$\ell/180$[1]
Floors			Immediate deflection due to L	$\ell/360$
Roof or floors	Supporting or attached to non-structural elements	Likely to be damaged by large deflections	That part of the total deflection occurring after attachment of nonstructural elements, which is the sum of the time-dependent deflection due to all sustained loads and the immediate deflection due to any additional live load[2]	$\ell/480$[3]
		Not likely to be damaged by large deflections		$\ell/240$[4]

[1]Limit not intended to safeguard against ponding. Ponding shall be checked by calculations of deflection, including added deflections due to ponded water, and considering time-dependent effects of sustained loads, camber, construction tolerances, and reliability of provisions for drainage.

[2]Time-dependent deflection shall be calculated in accordance with 24.2.4, but shall be permitted to be reduced by amount of deflection calculated to occur before attachment of nonstructural elements. This amount shall be calculated on basis of accepted engineering data relating to time-deflection characteristics of members similar to those being considered.

[3]Limit shall be permitted to be exceeded if measures are taken to prevent damage to supported or attached elements.

[4]Limit shall not exceed tolerance provided for nonstructural elements.

- When ***long-term deflections are computed***, ***that part of the deflection that occurs before attachment of the nonstructural elements may be deducted***; information from ***Figure 7.4-1*** or from ***Table 7.4-1*** may be useful for this purpose.
- As indicated in footnotes (3) and (4), the last two limits of ***Table 7.5-1 may be exceeded under certain conditions***, according to the ACI Code.

7.6 A Step by Step Procedure to Check the Deflection

The aforementioned discussions of Sections 7.2 through 7.5 have been put in a step by step procedure for to be used in the practical checking of deflection problem.

1. Determination of the deflections due to dead and live loads:
   - Use the mechanics of materials relations presented in ***Table 7.3-1***, ***Table 7.3-2***, and ***Table 7.3-3*** to determine the deflections due to the dead load, Δ_d, and the live load, Δ_ℓ.
   - Other analytical methods such as ***moment-area method*** can be used to determine these deflections.
   - ***Almost in all of current practical problems***, these deflections are ***determined by the software***.
   - If the computations give the total defection due to dead and live load together, $\Delta_{d+\ell}$, the deflection due to each part can be determined based on the following linear interpolations:

     $$\Delta_d = \frac{W_d}{W_d + W_\ell} \times \Delta_{d+\ell} \quad \text{Eq. 7.6-1}$$

     $$\Delta_\ell = \frac{W_\ell}{W_d + W_\ell} \times \Delta_{d+\ell} \quad \text{Eq. 7.6-2}$$

     As traditional structural analysis in civil engineering applications are based on ***linear behavior assumptions***[1], the above ***linear proportionalities seem justifiable***.
   - Irrespective of the computation approach, these deflections are usually ***instantaneous in nature*** and ***determined based on gross moment of inertia***, I_g. Therefore, they ***should be modified for the cracking effect and the long-term effect***.
2. Modification for the crack effect if necessary:
   - Determine the service moment, M_a, due to the dead and live loads.
   - Determine the cracking moment, M_{cr}, based on ***Eq. 7.3-5***.
   - If $M_a < M_{cr}$ then ***the section is uncracked***, and the deflections determined based on I_g are correct and ***no modification is required***.
   - If $M_a > M_{cr}$, the section is ***partially*** to ***fully cracked stage***, and the deflections should be modified as follows:
     1. Determine the effective moment of inertia, I_e, based on ***Eq. 7.3-7***.
     2. Modified the deflection based on the following relation:

     $$\Delta_{with\ crack\ effects} = \Delta_{without\ crack\ effect} \times \frac{I_g}{I_e} \quad \text{Eq. 7.6-3}$$
3. Modification for the log-term effect:
   The deflection due to sustained loads including ***selfweight***, ***superimposed dead load***, and a ***permanent*** part of the live load should be modified with the factor λ_Δ of ***Eq. 7.4-1***.
4. Determine the final deflections and compare with code permissible values:
   As indicated in ***Table 7.5-1***, the code offers two deflection checking, one for the immediate live loads and the second for the total loads.
   - Checking for immediate live load deflection:
     1. Classify the structural system into a flat roof system or into a floor system.
     2. Determine the immediate live load deflection with modification for the crack effect if necessary and compare with the permissible value of the code:

     $$\Delta_{\text{immediate}\,\ell} = \Delta_\ell \times \left(\frac{I_g}{I_e}\right)_{modification\ for\ crack} \leqslant \begin{Bmatrix} if\ flat\ roof & \dfrac{\ell}{180} \\ if\ floor & \dfrac{\ell}{360} \end{Bmatrix}$$

---

[1] Analytically, this assumption is valid only when the ***materials are linearly elastic***, and the ***deformations are small***.

- Checking for the total deflection:
  i. Classify the structural system into a ***floor system*** supports ***nonstructural elements likely to be damaged by large deflections elements*** or ***not***.
  ii. Compute the ***total deflection*** occurring ***after attachment of nonstructural elements***, which is the sum of the ***time-dependent deflection due to all sustained loads*** and ***the immediate deflection due to any additional live load***.

$$\Delta_{\text{total}} = \left(\left(\Delta_d + \Delta_\ell\left(\frac{Live\ sustained}{Live\ total}\right)\right) \times \left(\frac{I_g}{I_e}\right)_{modification\ for\ crack} \times (\lambda_\Delta)_{modification\ for\ long-term} + (\Delta_\ell) \times \left(\frac{I_g}{I_e}\right)_{modification\ for\ crack}\right)$$

$$\leqslant \begin{cases} if\ \text{nonstructural elements likely to be damaged} & \dfrac{\ell}{480} \\ if\ \text{nonstructural elements not likely to be damaged} & \dfrac{\ell}{240} \end{cases}$$

7.7 Examples for Deflection Control

Example 7.7-1

Check adequacy for the simply supported beam indicated in ***Figure 7.7-1*** for the deflection control requirements of the code. In your checking assume that:

- The selfweight of the beam is already included in the indicated dead loads,
- Sixty percent of the live load is sustained,
- The beam is part from a flooring system, and it supports non-structural element likely to be damaged by the deflection,
- Material strengths are $f_c' = 28\ MPa$ and $f_y = 420\ MPa$.

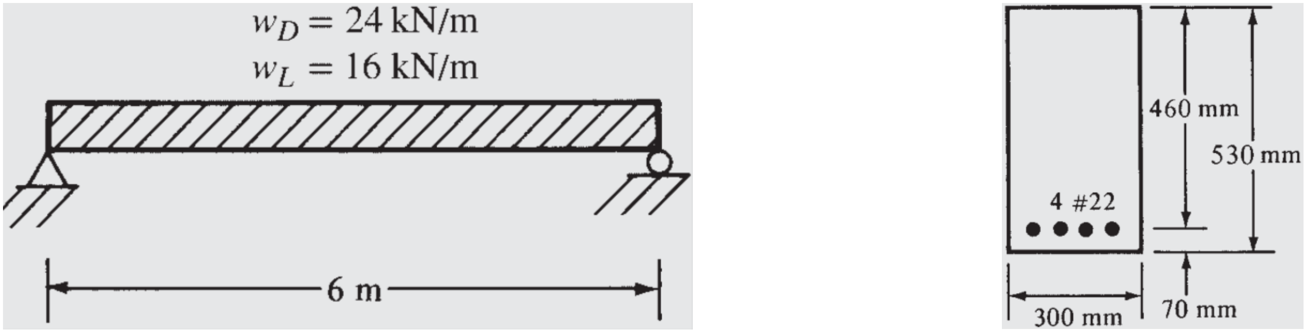


Figure 7.7-1: Simply supported beam for Example 7.7-1.

Solution

1. Determination of the deflections due to dead and live loads:
   Based on the mechanics of materials, see ***Table 7.3-1***, the immediate deflection in terms of I_g would be:

$$\Delta = \frac{5}{384}\left(\frac{w\ell^4}{E_c I_g}\right)$$

$$E_c = 4700\sqrt{f_c'} = 4700 \times \sqrt{28} = 24870\ MPa, E_s = 200000\ MPa \Rightarrow n = \frac{E_s}{E_c} = \frac{200000}{24870} \approx 8$$

$$I_g = \frac{bh^3}{12} = \frac{300 \times 530^3}{12} = 3.72 \times 10^9\ mm^4$$

Before substitution in the above relation, it is useful to note that:

$$\frac{kN}{m} = \frac{N}{mm}$$

Therefore, no unit transformation is required for the distributed loads.

$$\Delta_{\mathrm{d}} = \frac{5}{384} \times \left(\frac{24 \times 6000^4}{24870 \times 3.72 \times 10^9}\right) = 4.38\ mm$$

$$\Delta_\ell = \frac{5}{384} \times \left(\frac{16 \times 6000^4}{24870 \times 3.72 \times 10^9}\right) = 2.92\ mm$$

2. Modification for the crack effect if necessary:

$$M_a = \frac{w_{d+\ell}\ell^2}{8} = \frac{(24+16) \times 6^2}{8} = 180\ kN.m$$

$$M_{cr} = \frac{f_r I_g}{y_t} = \left(\frac{\left((0.62 \times 1.0 \times \sqrt{28}) \times 3.72 \times 10^9\right)}{\left(\frac{530}{2}\right)}\right) \times \left(\frac{1}{10^6}\right) = 46.1\ kN.m < M_a$$

Therefore, the section is a ***partially*** or ***full cracked*** one.
The centroid for the cracked section measured from the top face is:

$$(\bar{y} \times b) \times \frac{\bar{y}}{2} = nA_s \times (d - \bar{y})$$

$$nA_s = 8 \times \left(4 \times \frac{\pi \times 22^2}{4}\right) = 12164\ mm^4$$

$$(\bar{y} \times 300) \times \frac{\bar{y}}{2} = (12164) \times (460 - \bar{y}) \Rightarrow \bar{y} = 157\ mm$$

$$I_{cr} = \frac{300 \times 157^3}{3} + 12164 \times (460 - 157)^2 = 1.5 \times 10^9\ mm^4$$

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left(1 - \left(\frac{M_{cr}}{M_a}\right)^3\right) I_{cr}$$

$$I_e = \left(\left(\frac{46.1}{180}\right)^3 \times 3.72 + \left(1 - \left(\frac{46.1}{180}\right)^3\right) \times 1.50\right) \times 10^9 \Rightarrow I_e = 1.54 \times 10^9 mm^4$$

Therefore, the modification factor for crack would be:

$$\frac{I_g}{I_e} = \frac{3.72}{1.54} = 2.42 \blacksquare$$

3. Modification for the log-term effect:
   It is next necessary to find the sustained-load deflection multiplier, λ_Δ given by ***Eq. 7.4-1***:

$$\lambda_\Delta = \frac{\xi}{1 + 50\rho'}$$

   The time-dependent coefficient, ξ, can be taken as 2.0 based on ***Figure 7.4-1*** or ***Table 7.4-1***. As the beam is singly reinforced, therefore $\rho' = 0$, then λ_Δ would be:

$$\lambda_\Delta = \frac{2}{1 + 50\rho'} = 2 \blacksquare$$

4. Determine the final deflections and compare with the code permissible values:

- Checking for immediate live load deflection:
  i. Classify the structural system into a flat roof system or into a floor system:
     In the examples statement, the structural system is a floor system.
  ii. Determine the immediate live load deflection with modification for the crack effect if necessary and compare with the permissible value of the code:

$$\Delta_{\text{immediate}\,\ell} = \Delta_\ell \times \left(\frac{I_g}{I_e}\right)_{modification\ for\ crack} \leqslant \frac{\ell}{360}$$

$$\Delta_{\text{immediate}\,\ell} = 2.92 \times 2.42 = 7.05\ mm < \frac{\ell}{360} = \frac{6000}{360} = 16.7\ mm \therefore Ok.$$

- Checking for the total deflection:
  i. Classify the structural system into a ***floor system*** supports ***nonstructural elements likely to be damaged by large deflections elements*** or ***not***.
     Example statements mentions that ***the beam supports nonstructural partitions that would be damaged if large deflections were to occur***.
  ii. Compute the ***total deflection*** occurring ***after attachment of nonstructural elements***, which is the sum of the ***time-dependent deflection due to all sustained loads*** and ***the immediate deflection due to any additional live load***.

$$\Delta_{\text{total}} = \left(\left(\Delta_d + \Delta_\ell\left(\frac{Live\ sustained}{Live\ total}\right)\right) \times \left(\frac{I_g}{I_e}\right)_{modification\ for\ crack} \times (\lambda_\Delta)_{modification\ for\ long-term} + (\Delta_\ell) \times \left(\frac{I_g}{I_e}\right)_{modification\ for\ crack}\right) \leqslant \frac{\ell}{480}$$

$$\Delta_{\text{total}} = \left(\left(4.38 + 2.92 \times \left(\frac{60}{100}\right)\right) \times (2.42) \times (2) + (2.92) \times (2.42)\right) = 36.7\ mm < \frac{\ell}{480} = \frac{6000}{480} = 12.5\ mm \therefore Not\ Ok.$$

Aforementioned computations and comparisons indicating that ***the stiffness of the proposed member is insufficient***.

Example 7.7-2

The beam shown in ***Figure 7.7-2*** is a part of the floor system of an apartment house and is designed to carry calculated ***dead load*** w_d ***of 24 kN/m*** and a ***service live load*** w_ℓ ***of 48 kN/m***. Of the total live load, ***20 percent is sustained in nature***, while ***80 percent will be applied only intermittently over the life of the structure***. Under ***full dead and live load***, the moment diagram is as shown in ***Figure 7.7-2c*** and the ***total deflection is*** $\Delta_{d+\ell} = 2.82$ mm.

The ***beam will support nonstructural partitions that would be damaged if large deflections were to occur***. They will be installed shortly after construction shoring is removed and dead loads take effect, but before significant creep occurs.

Check beam adequacy for deflection requirements of the ACI code. Material strengths are $f_c' = 28\ MPa$ and $f_y = 420\ MPa$.

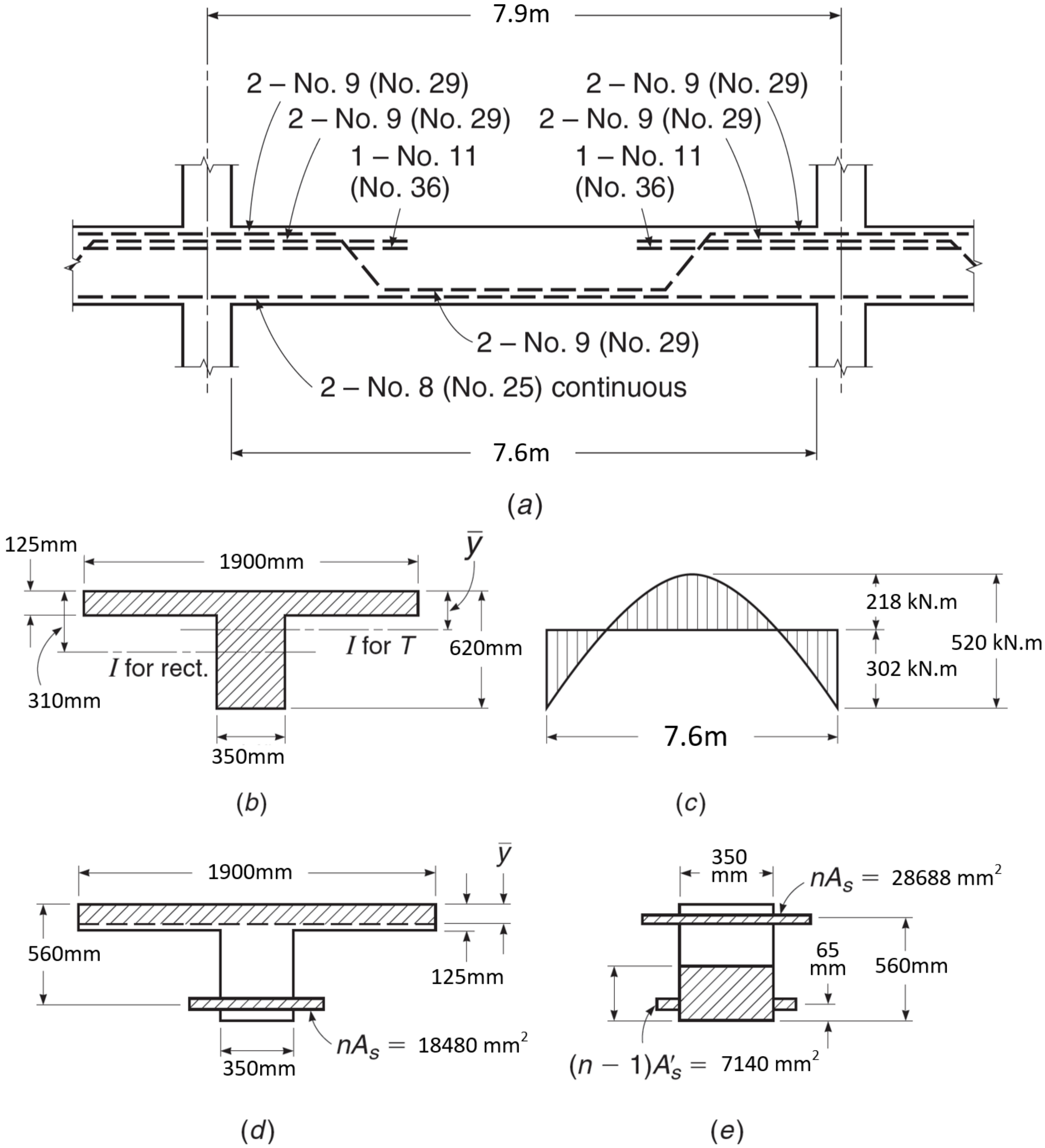


Figure 7.7-2: Continuous T beam for deflection calculations in Example 7.7-2. The uncracked section is shown in (b), the cracked transformed section in the positive moment region is shown in (d), and the cracked transformed section in the negative moment region is shown in (e).

Solution

1. Determination of the deflections due to dead and live loads:
   As the deflection due to dead and live loads, $\Delta_{d+\ell}$, is already given in the example statement, therefore what is necessary at this step is to determine the deflection due to dead load alone, Δ_d , and the live load alone, Δ_ℓ based on linear proportionalities of ***Eq. 7.6-1*** and ***Eq. 7.6-2***:
   $$\Delta_d = \frac{W_d}{W_d + W_\ell} \times \Delta_{d+\ell} = \frac{24}{24+48} \times 2.82 = 0.94\,mm$$
   $$\Delta_\ell = \frac{W_\ell}{W_d + W_\ell} \times \Delta_{d+\ell} = \frac{48}{24+48} \times 2.82 = 1.88\,mm$$
2. Modification for the crack effect if necessary:
   For the specified materials:
   $$E_c = 4700\sqrt{f_c'} = 4700 \times \sqrt{28} = 24870\,MPa, E_s = 200000\,MPa \Rightarrow n = \frac{E_s}{E_c} = \frac{200000}{24870} \approx 8$$
   The modulus of rupture, f_r, is:
   $$f_r = 0.62\lambda\sqrt{f_c'} = 0.62 \times 1 \times \sqrt{28} = 3.28\,MPa$$
   The effective moment of inertia will be calculated for the moment diagram shown in ***Figure 7.7-2c*** corresponding to ***the full-service load***, on the basis that ***the extent of cracking will be governed by the full-service load***, ***even though that load is intermittent***.
   Determine the instantaneous deflection due to dead and live loads:
   As the structure is assumed linear in traditional structural analysis, therefore the instantaneous deflection due to deal load, Δ_d, and due to live load, Δ_ℓ, can be determined from $\Delta_{d+\ell}$ based on the following linear proportionalty
   <u>The positive region:</u>
   In the positive-moment region, the centroidal axis of the uncracked T section of ***Figure 7.7-2b*** is found by taking moments about the top surface, to be;
   $$\bar{y}_{for\ the\ gross\ positve\ section} = \frac{\Sigma A_i y_i}{\Sigma A_i}$$
   $$= \frac{\left(1900 \times 125 \times \frac{125}{2} + 350 \times (620-125) \times \left(\frac{(620-125)}{2} + 125\right)\right)}{(1900 \times 125 + 350 \times (620-125))}$$
   $$= 193\,mm < 310 mm \therefore Ok.$$
   The moment of inertia, I_g, for the gross section is:
   $$I_g = \left(\frac{350 \times 620^3}{12} + 350 \times 620 \times (310-193)^2\right) + \frac{(1900-350) \times 125^3}{12}$$
   $$+ (1900-350) \times 125 \times \left(193 - \frac{125}{2}\right)^2 = 1.347 \times 10^{10} mm^4$$
   The cracking moment, M_{cr}, is then found by means of Eq. 7.3-5:
   $$M_{cr} = \frac{f_r I_g}{y_t} = \left(\frac{3.28 \times 1.347 \times 10^{10}}{620-193}\right) \times \frac{1}{1000000} = 104\,kN.m$$
   With
   $$\frac{M_{cr}}{M_a} = \frac{104}{218} = 0.477 < 1.0$$
   Therefore, the section is cracked and I_{cr} and I_e should be determined and the deflection should be modified accordingly.
   The centroidal axis of the cracked transformed T section shown in ***Figure 7.7-2d*** is determined as follows, assume that $\bar{y} \le 125mm$ to be checked later:
   $$(\bar{y} \times b) \times \frac{\bar{y}}{2} = nA_s \times (d - \bar{y}) \Rightarrow (\bar{y} \times 1900) \times \frac{\bar{y}}{2} = 18480 \times (560 - \bar{y})$$
   $$\Rightarrow \bar{y} = 95.7\,mm < 125mm \therefore Ok.$$
   below the top of the slab and I_{cr} would be:
   $$I_{cr} = \frac{1900 \times 95.7^3}{3} + 18480 \times (560-95.7)^2 = 0.4539 \times 10^{10}\,mm^4$$
   The effective moment of inertia in the positive bending region is found from Eq. 7.3-7 to be:

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left(1 - \left(\frac{M_{cr}}{M_a}\right)^3\right) I_{cr} \Rightarrow I_e$$
$$= \left(\frac{104}{218}\right)^3 \times 1.347 \times 10^{10} + \left(1 - \left(\frac{104}{218}\right)^3\right) \times 0.4539 \times 10^{10}$$
$$I_e = 0.551 \times 10^{10}\ mm^4$$

<u>The negative region:</u>

In the negative bending region, the gross moment of inertia will be based on the rectangular section shown in ***Figure 7.7-2b***. For this area, the centroid is:

$$\bar{y} = \frac{620}{2} = 310mm$$

from the top surface and I_g would be:

$$I_g = \frac{bh^3}{12} = \frac{350 \times 620^3}{12} = 0.695 \times 10^{10} mm^4$$

Therefore, the crack moment, M_{cr}, would be:

$$M_{cr} = \frac{f_r I_g}{y_t} = \left(\frac{3.28 \times 0.695 \times 10^{10}}{310}\right) \times \frac{1}{1000000} = 73.5\ kN.m$$
$$\frac{M_{cr}}{M_a} = \frac{73.5}{302} = 0.243 < 1.0$$

Therefore, the section is cracked and I_{cr} and I_e should be determined and the deflection should be modified accordingly.

For the cracked transformed section shown in ***Figure 7.7-2e***, the centroidal axis is found, taking moments about the bottom surface, to be:

$$(\bar{y} \times 350) \times \frac{\bar{y}}{2} + 7140 \times (\bar{y} - 65) = 28688 \times (560 - \bar{y}) \Rightarrow \bar{y} = 222mm$$

from that level, and I_{cr} would be:

$$I_{cr} = \frac{350 \times 222^3}{3} + 7140 \times (222 - 65)^2 + 28688 \times (560 - 222)^2 = 0.473 \times 10^{10} mm^4$$

Thus, for the negative-moment regions,

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left(1 - \left(\frac{M_{cr}}{M_a}\right)^3\right) I_{cr} \Rightarrow I_e$$
$$= \left(\frac{73.5}{302}\right)^3 \times 0.695 \times 10^{10} + \left(1 - \left(\frac{73.5}{302}\right)^3\right) \times 0.473 \times 10^{10}$$
$$I_e = 0.476 \times 10^{10} mm^4$$

The average effective moment of inertia:

The average value of I_e to be used in calculation of deflection is:

$$I_{e\ avg.} = \frac{1}{2}(0.551 + 0.476) \times 10^{10} = 0.514 \times 10^{10}\ mm^4$$

<u>The modification factor for the crack:</u>

Based on Eq. 7.6-2, the deflection should be modified for the crack based on the following relation:

$$\Delta_{with\ crack\ effects} = \Delta_{without\ crack\ effect} \times \frac{I_g}{I_e} \Rightarrow$$
$$\Delta_{with\ crack\ effects} = \frac{1.347 \times 10^{10}}{0.514 \times 10^{10}} \Delta_{without\ crack\ effect} = 2.62 \Delta_{without\ crack\ effect} \blacksquare$$

3. Modification for the log-term effect:

   It is next necessary to find the sustained-load deflection multiplier, λ_Δ given by Eq. 7.4-1:

   $$\lambda_\Delta = \frac{\xi}{1 + 50\rho'}$$

   The time-dependent coefficient, ξ, can be taken as 2.0 based on ***Figure 7.4-1*** or ***Table 7.4-1***. For the positive bending zone, with no compression reinforcement, $\rho' = 0$, then λ_Δ would be:

   $$\lambda_\Delta = \frac{2}{1 + 50\rho'} = 2 \blacksquare$$

4. Determine the final deflections and compare with the code permissible values:
- Checking for immediate live load deflection:
  i. Classify the structural system into a flat roof system or into a floor system: In the examples statement, the structural system is a floor system.
  ii. Determine the immediate live load deflection with modification for the crack effect if necessary and compare with the permissible value of the code:

$$\Delta_{\text{immediate }\ell} = \Delta_\ell \times \left(\frac{I_g}{I_e}\right)_{modification\ for\ crack} \leqslant \frac{\ell}{360}$$

$$\Delta_{\text{immediate }\ell} = 1.88 \times 2.62 = 4.92\ mm < \frac{\ell}{360} = \frac{7900}{360} = 21.9\ mm \therefore Ok.$$

- Checking for the total deflection:
  i. Classify the structural system into a ***floor system*** supports ***nonstructural elements likely to be damaged by large deflections elements*** or ***not***. Example statements mentions that ***the beam will support nonstructural partitions that would be damaged if large deflections were to occur***.
  ii. Compute the ***total deflection*** occurring ***after attachment of nonstructural elements***, which is the sum of the ***time-dependent deflection due to all sustained loads*** and ***the immediate deflection due to any additional live load***.

$$\Delta_{\text{total}} = \left(\left(\Delta_d + \Delta_\ell\left(\frac{Live\ sustained}{Live\ total}\right)\right) \times \left(\frac{I_g}{I_e}\right)_{modification\ for\ crack} \times (\lambda_\Delta)_{modification\ for\ long-term} + (\Delta_\ell) \times \left(\frac{I_g}{I_e}\right)_{modification\ for\ crack}\right) \leqslant \frac{\ell}{480}$$

$$\Delta_{\text{total}} = \left(\left(0.94 + 1.88 \times \left(\frac{20}{100}\right)\right) \times 2.62 \times (2) + (1.88) \times 2.62\right) = 11.8\ mm < \frac{\ell}{480} = \frac{7900}{480} = 16.5mm$$

$\therefore Ok.$

Aforementioned computations and comparisons indicating that ***the stiffness of the proposed member is sufficient***.

Contents

CHAPTER 8
ANALYSIS AND DESIGN FOR TORSION

8.1 INTRODUCTION

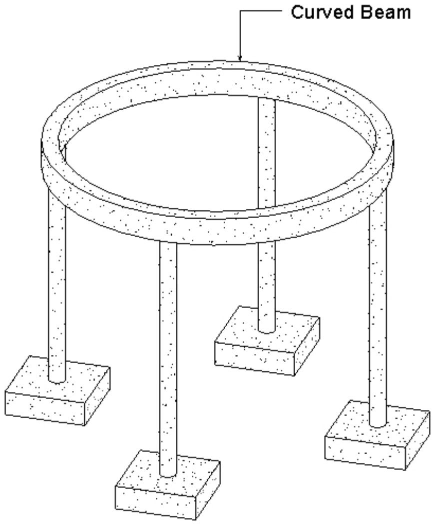
- Torsional forces may act, tending to twist a member about its longitudinal axis.
- Torsional forces seldom act alone and are ***usually concurrent*** with ***bending moment*** and ***transverse shear***, and sometimes with axial force.

8.1.1 Torsion in Old Design Philosophy

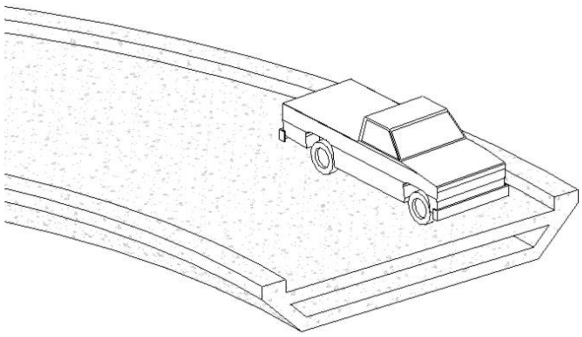
For many years, ***torsion was regarded as a secondary effect*** and ***was not considered explicitly in design***, its influence being absorbed in the overall factor of safety of rather conservatively designed structures.

8.1.2 Torsion in Current Design Philosophy

- Current methods of analysis and design have ***resulted in less conservatism***, ***leading to somewhat smaller members that***, in many cases, ***must be reinforced to increase torsional strength***.
- Torsion should be included explicitly especially with the increasing use of structural members for which ***torsion is a central feature of behavior***; examples include ***curved beam***, ***curved bridge girders***, and ***helical stairway slabs***.



(a)



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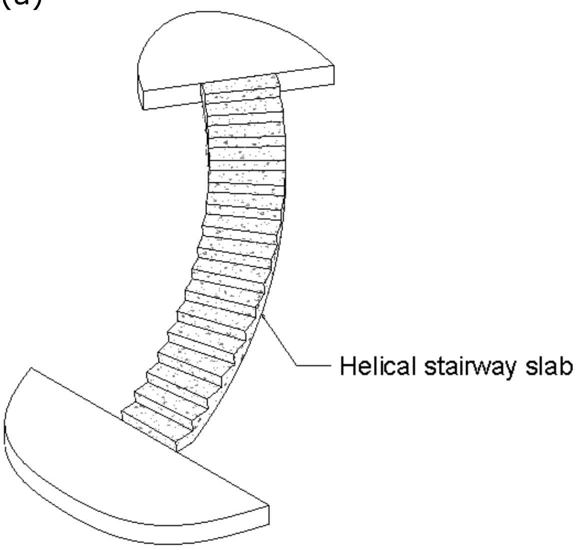


(c)

Figure 8.1-1: Members subjected to significant torsion: (a) curved beams; (b) bridge girders; (c) helical stairway slabs.

8.1.3 Primary versus Secondary Torsions

It is useful in considering torsion to distinguish between ***primary*** and ***secondary*** torsion in reinforced concrete structures.

8.1.3.1 Primary Torsion

- Sometimes called ***equilibrium torsion*** or ***statically determinate torsion***, exists when the external load has no alternative load path but must be supported by torsion.
- For such cases, the torsion required to maintain static equilibrium could be uniquely determined.
- An example is the cantilevered slab of ***Figure 8.1-2*** below. Loads applied to the slab surface cause twisting moments m_t to act along the length of the supporting beam. These are equilibrated by the resisting torque T provided at the columns. Without the torsional moments, the structure will collapse.

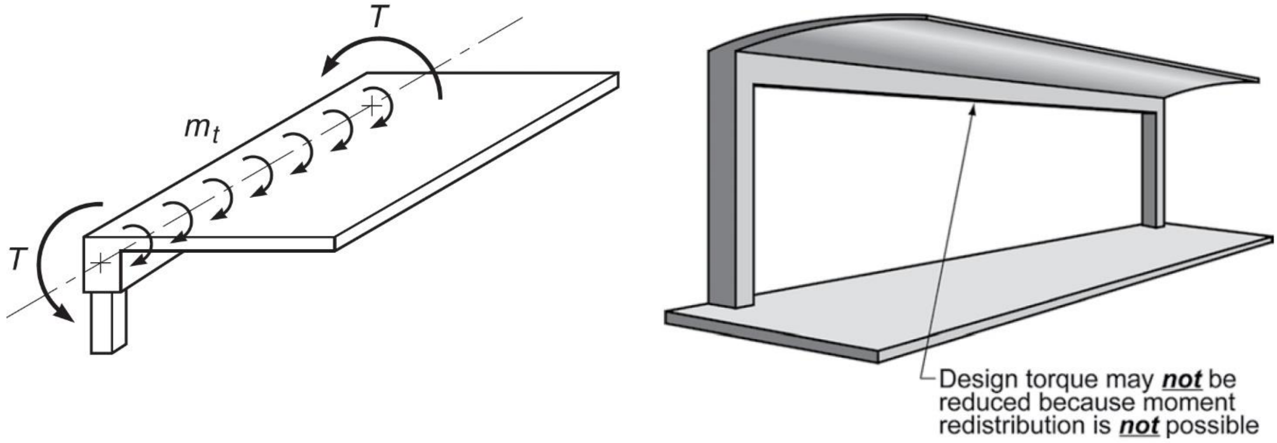
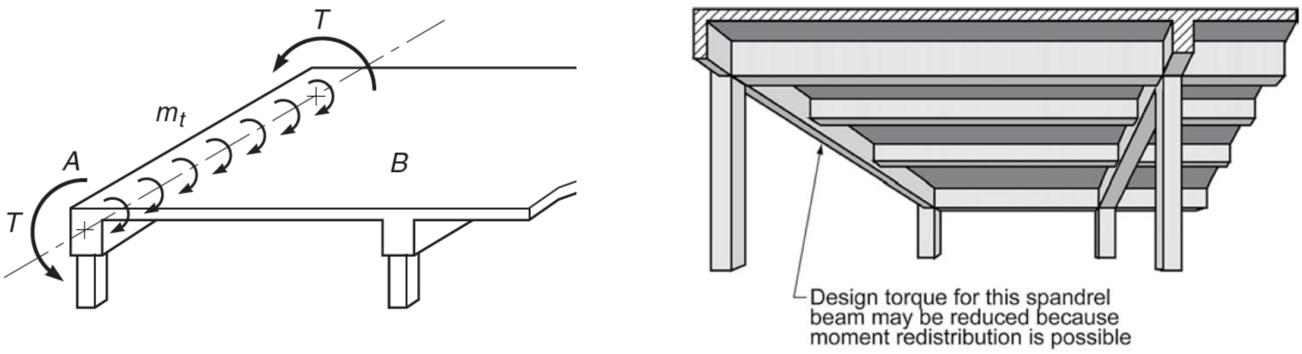


Figure 8.1-2: Primary or equilibrium torsion at a cantilevered slab.

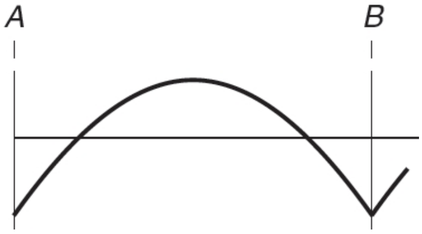
8.1.3.2 Secondary Torsion,

- Also called ***compatibility torsion*** or ***statically indeterminate torsion***, arises from the requirements of continuity, that is, compatibility of deformation between adjacent parts of a structure.
- For this case, the torsional moments cannot be found based on static equilibrium alone. ***Disregard of continuity in the design will often lead to extensive cracking, but generally will not cause collapse***. ***An internal readjustment of forces is usually possible and an alternative equilibrium of forces found***.
- An example of secondary torsion is found in the spandrel or edge beam supporting a monolithic concrete slab, shown in ***Figure 8.1-3a***.
  - **First Load Path:**
    If the spandrel beam is torsionally stiff and suitably reinforced, and if the columns can provide the necessary resisting torque T, then the slab moments will approximate those for a rigid exterior support as shown in ***Figure 8.1-3b***.
  - **Second Load Path:**
    However, if the beam has little torsional stiffness and inadequate torsional reinforcement, cracking will occur to further reduce its torsional stiffness, and the slab moments will approximate those for a hinged edge, as shown in ***Figure 8.1-3c***.

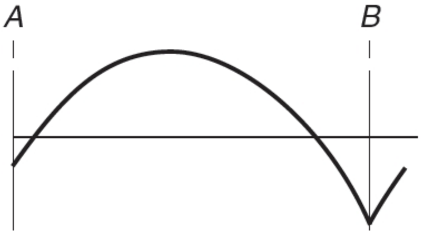
  If the slab is designed to resist the altered moment diagram, collapse will not occur.



(a) Secondary or compatibility torsion at an edge beam.



(b) slab moments if edge beam is stiff torsionally



(c) Slab moments if edge beam is flexible torsionally.

Figure 8.1-3: Secondary or compatibility torsion.

8.1.4 Torsion in Uncracked Plain Concrete Members

If the material is elastic, St. Venant's torsion theory indicates that torsional shear stresses are distributed over the cross section, as shown in ***Figure 8.1-4*** below.

- Stress Distribution in Elastic Martial:
  The largest shear stresses occur at the middle of the wide faces.
- Stress Distribution in Inelastic Martial:
  If the material deforms inelastically, as expected for concrete, the stress distribution is closer to that shown by the dashed line.
- Diagonal Stresses Associated with Torsional Shear Stresses:
  - Shear stresses in pairs act on an element at or near the wide surface, as shown in ***Figure 8.1-4a***.
  - As explained in strength of materials texts, this state of stress corresponds to equal tension and compression stresses on the faces of an element at 45° to the direction of shear.
  - These inclined tension stresses are of the same kind as those caused by transverse shear, discussed in ***Chapter 5***.
  - However, in the case of torsion, since the torsional shear stresses are of opposite sign on opposing sides of the member (***Figure 8.1-4b***), the corresponding diagonal tension stresses are at right angles to each other (***Figure 8.1-4a***).

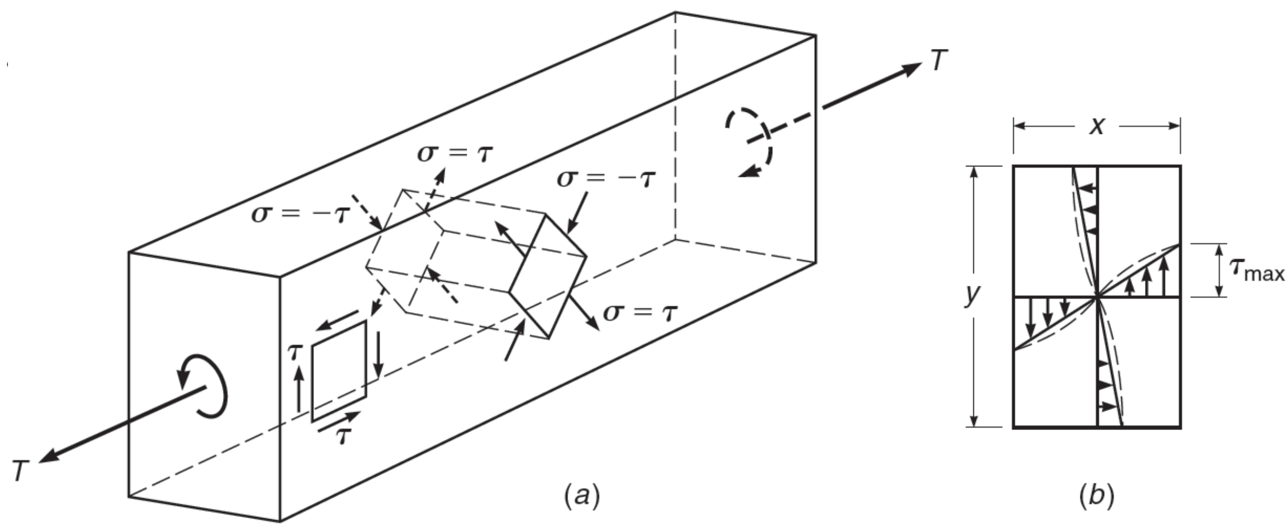


Figure 8.1-4: Stresses caused by torsion.

8.1.5 Cracking Torque T_{cr}

- Definition of Cracking Torque T_{cr}
  - When the diagonal tension stresses exceed the tensile resistance of the concrete, a crack forms at some accidentally weaker location and spreads immediately across the beam.
  - The value of torque corresponding to the formation of this diagonal crack is known as the cracking torque T_{cr}.
- Using ***Thin-walled Tube***, ***Space Truss Analogy*** to Compute T_{cr}:
  - The nonlinear stress distribution shown by the dotted lines in ***Figure 8.1-4b*** lends itself to the use of the ***thin-walled tube***, ***space truss analogy***.
  - Using this analogy, ***the shear stresses are treated as constant over a finite thickness t around the periphery of the member***, allowing the beam to be represented by an equivalent tube, as shown in ***Figure 8.1-5*** below.
- Shear Flow According to Thin-walled Tube Model:
  In the analogy, shear flow q is treated as a constant around the perimeter of the tube and related to applied torque, T, as follows:
  $T = q(x_0 t)_{Area} \times y_{0\ Arm} + q(y_0 t)_{Area} \times x_{0\ Arm} \Rightarrow T = 2q\ x_0 y_0 t$
  The product $x_0 y_0$ represents the area enclosed by the shear flow path A_0 , giving
  $\because x_0 y_0 = A_0 \Rightarrow \therefore T = 2qA_0 \Rightarrow q = \dfrac{T}{2A_0}$ ■
- Shear Stress τ According to Thin-walled Tube Model:
  $\because \tau = \dfrac{q}{t} \Rightarrow \therefore \tau = \dfrac{T}{2A_0 t}$
- Corresponding Diagonal Tension:
  From ***Figure 8.1-4a*** above
  $\sigma = \tau$
  Let tensile strength of concrete approximated with
  $\sigma = 0.33\lambda\sqrt{f_c'}$
  Therefore, the cracking torque would be:
  $T_{cr} = 0.33\lambda\sqrt{f_c'}(2A_0 t)$
  Let
  $A_0 \approx \dfrac{2}{3}A_{cp}, t = \dfrac{3}{4}\dfrac{A_{cp}}{p_{cp}}$
  The cracking moment would be:
  $T_{cr} = 0.33\lambda\sqrt{f_c'}\left(2 \times \dfrac{2}{3}A_{cp} \times \dfrac{3}{4}\dfrac{A_{cp}}{p_{cp}}\right)$
  $T_{cr} = 0.33\lambda\sqrt{f_c'}\left(\dfrac{A_{cp}^2}{p_{cp}}\right)$ ■
  where
  A_{cp} is area enclosed by outside perimeter of concrete cross section, in mm^2,
  p_{cp} is outside perimeter of concrete cross section, in mm.

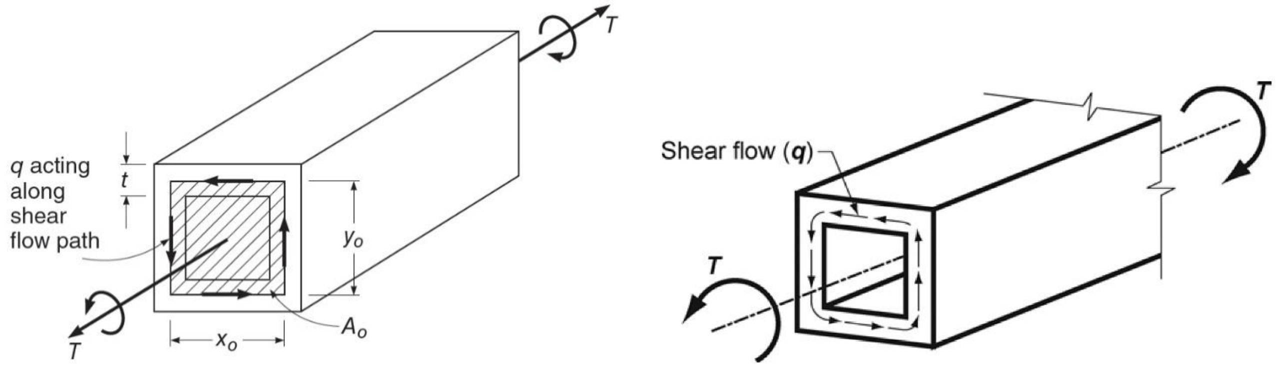


Figure 8.1-5: Thin-walled tube under torsion.

8.1.6 Torsion in Reinforced Concrete Members

8.1.6.1 Reinforcement for Torsion

- To resist torsion for values of T above T_{cr} , reinforcement must consist of ***closely spaced stirrups*** and ***longitudinal bars***,
- Reinforcement for torsion with cracking pattern are presented in ***Figure 8.1-6*** below.

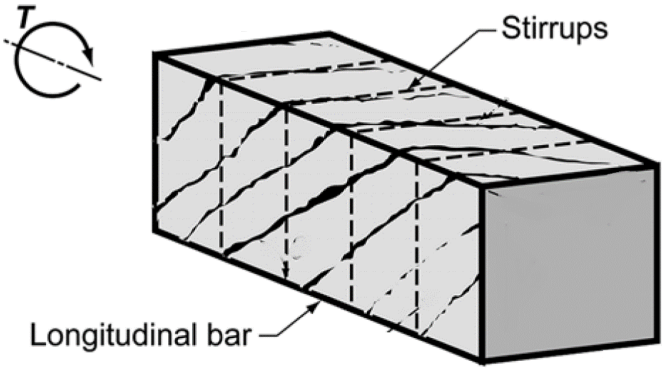


Figure 8.1-6: Reinforcement for torsion.

8.1.6.2 Shear Path after Cracking

- Tests show that, after cracking, the area enclosed by the shear path is defined by the dimensions x_0 and y_0 ***measured to the centerline of the outermost closed transverse reinforcement***.
- These dimensions define the gross area
  $A_{oh} = x_o\, y_o$
  and the shear perimeter
  $p_h = 2(x_o + y_o)$
  measured at the steel centerline, see ***Figure 8.1-7*** above

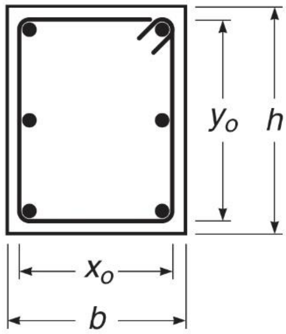


Figure 8.1-7: Notations for shear flow path after cracking of a reinforced concrete beam.

8.1.6.3 Basic Relation for Stirrups Torsional Reinforcement

- With referring to ***Figure 8.1-8*** below, the relation for stirrups torsional reinforcement can be formulated based on basic principles of equilibrium as presented in below:
  $T_4 = \frac{V_4 x_0}{2}$
- With referring to ***Figure 8.1-9*** below, the vertical shear force, V_4, can be related to the provided stirrups as follows:
  $V_4 = A_t f_{yt} n$
  where
  A_t is area of one leg of a closed stirrup,
  f_{yt} is yield strength of transverse reinforcement,
  n is number of stirrups intercepted by torsional crack.
  $\because n = y_0 \frac{\cot\theta}{s} \Rightarrow \therefore V_4 = \frac{A_t f_{yt} y_0}{s}\cot\theta$
  and the pertained torsion, T_4, would be:
  $T_4 = \frac{A_t f_{yt} y_0 x_0}{2s}\cot\theta$
- The contributions of the horizontal walls T_1, T_2, and T 3 can be determined in the same way. Summing over all four sides, the nominal capacity of the section is:
  $$T_{Resisted\ by\ stirrups} = T_n = \sum_{i=1}^{4} \mathrm{T_i} = \frac{2A_t f_{yt} y_0 x_0}{s}\cot\theta \quad \blacksquare$$
- Noting that $y_o x_o = A_{oh}$ and rearranging slightly give
  $T_n = \frac{2A_{0h} A_t f_{yt}}{s}\cot\theta$

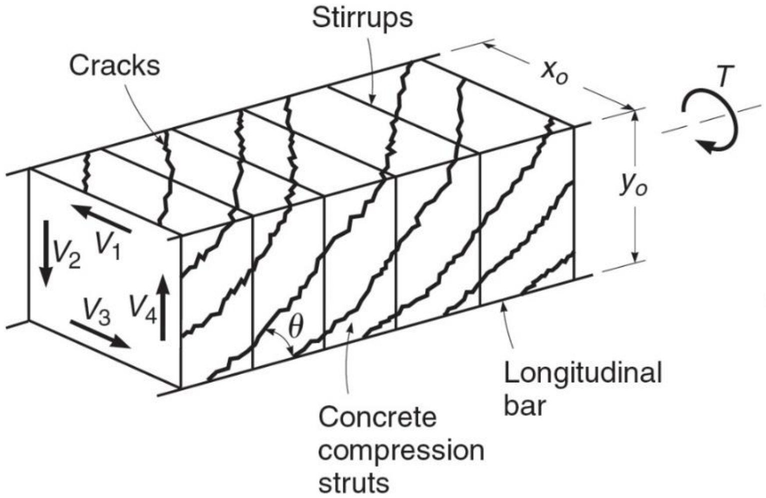


Figure 8.1-8: Space truss analogy.

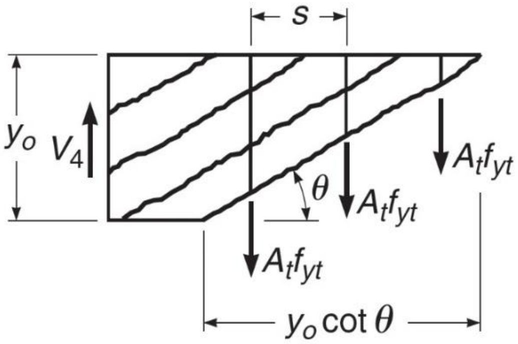
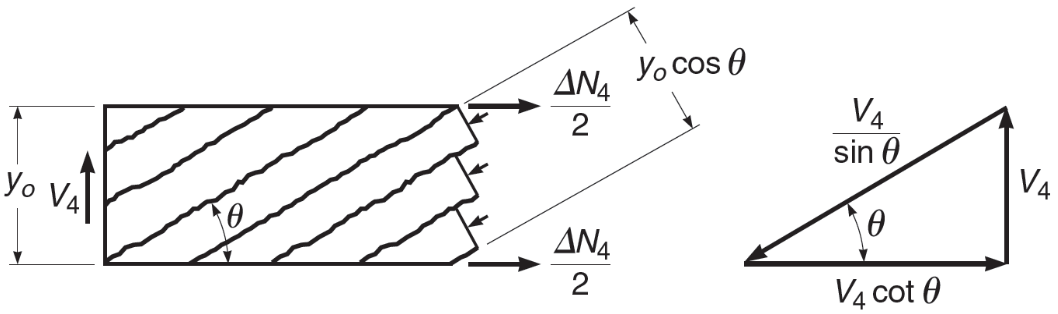


Figure 8.1-9: Vertical tension in stirrups.

8.1.6.4 Basic Relation for Longitudinal Reinforcement

- As shown in ***Figure 8.1-10 a*** and ***b***, the horizontal component of compression in the struts in the vertical wall must be equilibrated by an axial tensile force ΔN_4.



(a) (b)

Figure 8.1-10: Basis for contribution of longitudinal reinforcement for torsional strength: (a) diagonal compression in vertical wall of beam; and (b) equilibrium diagram of forces due to shear in vertical wall.

- Based on the assumed ***uniform distribution of shear flow around the perimeter of the member***, the ***diagonal stresses in the struts must be uniformly distributed***, resulting in a line of action of the resultant axial force that ***coincides with the mid-height of the wall***.
- Referring to ***Figure 8.1-10***b , the total contribution of the right-hand vertical wall to the change in axial force of the member due to the presence of torsion is:

  $$\Delta N_4 = V_4 \cot\theta = \frac{A_t f_{yt} y_0}{s} \cot^2\theta$$

- Summing over all four sides, the total increase in axial force for the member is:

  $$\Delta N = \sum_{i=1}^{4} \Delta \mathrm{N_i} = \frac{A_t f_{yt}}{s} 2(x_0 + y_0) \cot^2\theta \Rightarrow \Delta N = \frac{A_t f_{yt} p_h}{s} \cot^2\theta$$

  where p_h is the perimeter of the centerline of the closed stirrups.
- Longitudinal reinforcement must be provided to carry the added axial force Δ N. If that steel is designed to yield, then:

  $$A_l f_y = \frac{A_t f_{yt} p_h}{s} \cot^2\theta$$

  Solve for A_l to obtain:

  $$A_l = \frac{A_t}{s} p_h \frac{f_{yt}}{f_y} \cot^2\theta$$

  where A_ℓ is total area of longitudinal reinforcement to resist torsion,
- Finally, as

  $$T_n = \frac{2A_{0h} A_t f_{yt}}{s} \cot\theta$$

  therefore,

$$A_t f_{yt} = \frac{T_n s}{2A_{0h} A_t f_{yt} \cot\theta}$$

Substitute $A_t f_{yt}$ into equation above for A_l

$$A_l = \left(\frac{1}{s} p_h \frac{1}{f_y} \cot^2\theta\right)\left(\frac{T_n s}{2A_{0h} A_t f_{yt} \cot\theta}\right)$$

and for T_n to obtain:

$$T_n = \frac{2A_{oh} A_l f_y}{p_h} \tan\theta$$

8.1.7 Torsion plus Shear

- Members are rarely subjected to torsion alone. The prevalent situation is that of a beam subject to the usual flexural moments and shear forces, which, in addition, must resist torsional moments.
- Basic Shear and Torsion Stresses in Reinforced Concrete Members:
  Using the usual representation for reinforced concrete, the nominal shear stress caused by an applied shear force V is:

$$\tau_v = \frac{V}{b_w d}$$

While using the concept of thin-walled tube, the shear stress caused by torsion would be:

$$\tau_t = \frac{T}{2A_0 t}$$

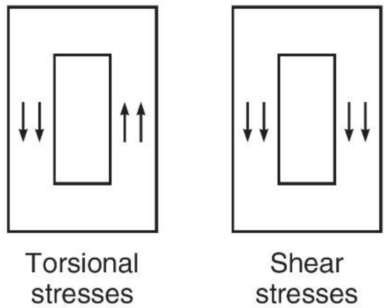


Figure 8.1-11: Addition of torsional and shear stresses in a hollow section.

- Superposition of Shear and Torsion Stresses in a Hollow Section:
- As shown in ***Figure 8.1-11*** for hollow sections, these stresses are directly additive on one side of the member. Thus, for a cracked concrete cross section with
  $A_o = 0.85\,A_{oh}$ and $t = \frac{A_{oh}}{p_h}$
  the maximum shear stress can be expressed as:

$$\tau = \tau_v + \tau_t = \frac{V}{b_w d} + \frac{T p_h}{1.7A_{oh}^2}$$

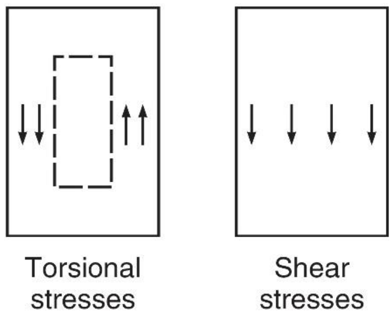


Figure 8.1-12: Addition of torsional and shear stresses in a solid section.

- For a member with a solid section, ***Figure 8.1-12***, τ_t is predominately distributed around the perimeter, as represented by the hollow tube analogy, but the full cross section contributes to carrying τ_v.
- ***Comparisons with experimental results show that equation above for a hollow section is somewhat overconservative for solid sections*** and that a better representation for maximum shear stress is provided by the square root of the sum of the squares, SRSS[1], of the nominal shear stresses:

$$\tau = \sqrt{\left(\frac{V}{b_w d}\right)^2 + \left(\frac{T p_h}{1.7A_{oh}^2}\right)^2}$$

[1] SRSS summation is usually adopted to superimpose two quantities that their maximum values occurs at different positions or at different times.

8.2 ACI CODE PROVISIONS FOR TORSION DESIGN

8.2.1 Basic Design Principle

The basic principles upon which ACI Code design provisions are based have been presented in the preceding chapters for flexure and shear. ACI Code ***9.5.1.1*** safety provisions require that:

$T_u \leq \phi T_n$

where

T_n = nominal torsional strength of member,

T_u = required torsional strength at factored loads. The strength reduction factor $\phi = 0.75$ applies for torsion.

8.2.2 Computing of T_u

- In accordance with ***ACI Code 9.4.4.3***, sections located less than a distance d from the face of a support may be designed for the same torsional moment T_u as that computed at a distance d, recognizing the beneficial effects of support compression.
- However, if a concentrated torque is applied within this distance, the critical section must be taken at the face of the support.
- ***These provisions parallel those used in shear design***.

8.2.3 Effective Section

8.2.3.1 Before Cracking

- For T beams, a portion of the overhanging flange contributes to the cracking torsional capacity and, ***if reinforced with closed stirrups***, to the torsional strength.
- According to ***ACI Code 9.2.4.4***, the contributing width of the overhanging flange on either side of the web would be as indicated in ***Figure 8.2-1*** below.

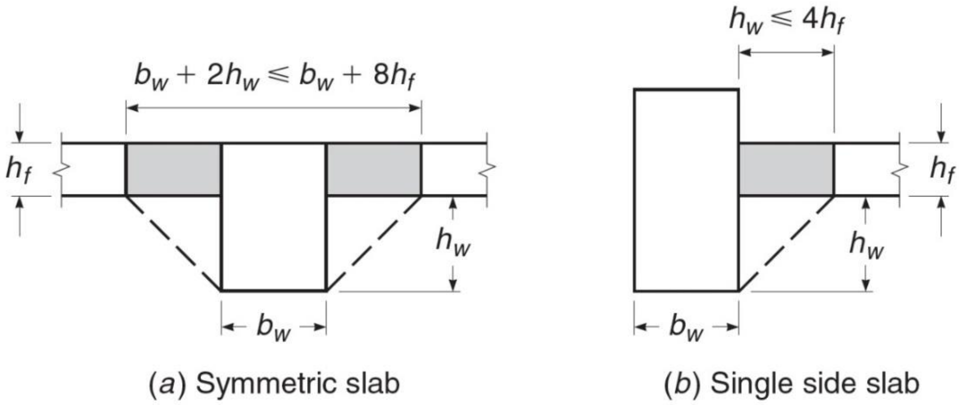


Figure 8.2-1: Portion of slab to be included with beam for torsional design.

- The overhanging flanges ***shall be neglected in cases where the parameter*** A_{cp}^2/p_{cp} ***for solid sections or*** A_g^2/p_{cp} ***for hollow sections calculated for a beam with flanges is less than that calculated for the same beam ignoring the flanges***.

8.2.3.2 After Cracking

After torsional cracking, the applied torque is resisted by the portion of the section represented by A_{oh}, the area enclosed by the centerline of the outermost closed transverse torsional reinforcement. For ***rectangular***, ***box***, and ***T sections***, A_{oh} is illustrated in ***Figure 8.2-2*** below.

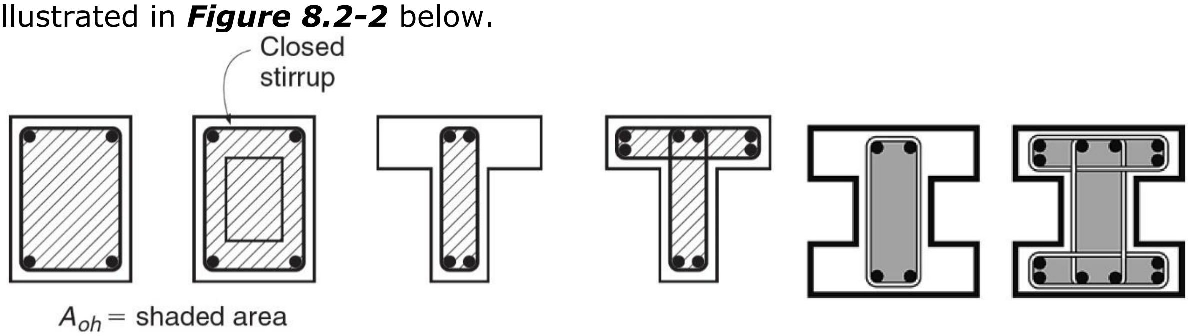


Figure 8.2-2: Definition of A_{oh}.

8.2.3.3 Sections before and after Cracking

For sections with flanges, the Code does not require that the section used to establish A_{cp} coincide with that used to establish A_{oh}.

8.2.4 Threshold Torsion

- If the value of factored torsional moment T_u is low enough, the effects of torsion may be neglected, according to ***ACI Code 22.7.1.1***.
- This lower limit is ϕ times the threshold torsion T_{th}, which equals 25 percent of the ***cracking torque***, given by:

  $$T_{th} = \frac{1}{4} T_{cr}$$

  $$\because T_{cr} = 0.33\lambda\sqrt{f_c{}'}\left(\frac{A_{cp}^2}{p_{cp}}\right)$$

  $$\therefore T_{th\,for\,solid\,section} = 0.083\lambda\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)$$

- For hollow cross sections, the threshold torsion is:

  $$T_{th\,for\,hollow\,section} = 0.083\lambda\sqrt{f_c'}\left(\frac{A_g^2}{p_{cp}}\right)$$

- The value of λ is as specified in ***ACI Code 19.2.4.2*** and previously described with λ = 0.85, 0.75, and 1.0 for ***sand-lightweight***, ***all-lightweight***, and ***normalweight*** concrete, respectively.

8.2.5 Equilibrium vs. Compatibility Torsion

- As discussed in Article 8.1.3, a distinction is made in the ACI Code between ***equilibrium*** (***primary***) torsion and ***compatibility*** (***secondary***) torsion.
- For the ***equilibrium*** (***primary***) torsion***, the supporting member must be designed to provide the torsional resistance required by static equilibrium***.
- For ***secondary torsion*** resulting from compatibility requirements, it is assumed that cracking will result in a redistribution of internal forces; and according to ***ACI Code 22.7.3.2***, the maximum torsional moment T_u may be reduced to:

  $$T_u = \phi T_{cr}$$

  or

  $$T_u = \phi\left(0.33\lambda\sqrt{f_c{}'}\left(\frac{A_{cp}^2}{p_{cp}}\right)\right)$$

8.2.6 Limitations on Shear Stress

- Based largely on ***empirical observations***, the ***width of diagonal cracks*** caused by combined shear and torsion under service loads can be limited by limiting the calculated shear stress under factored shear and torsion.
- In accordance with ACI Code 22.7.7.1, shear stresses should be limited to the following values:
  - For hollow sections:

    $$\left(\frac{V_u}{b_w d}\right) + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right) \le \phi\left(\frac{V_c}{b_w d} + 0.66\sqrt{f_c{}'}\right)$$

  - For solid sections:

    $$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right)^2} \le \phi\left(\frac{V_c}{b_w d} + 0.66\sqrt{f_c{}'}\right)$$

8.2.7 Reinforcement for Torsion

8.2.7.1 Stirrups for Torsion

- As discussed in Article 8.1.6 above, stirrups for torsion can be determined from following relation:

  $$A_t = \frac{T_u s}{2\phi A_o f_{yt} \cot\theta}$$

- According to ACI Code ***22.7.6.1***, the angle θ may assume any value between ***30***

and 60°, with a value of ***θ = 45°*** suggested.

- The Code limits f_{yt} ***to a maximum of 420 MPa*** for reasons of crack control.
- The reinforcement provided for torsion must be combined with that required for shear. Based on the typical two-leg stirrup, this may be expressed as
$$\frac{A_{v+t}}{s} = \frac{A_v}{s} + 2\frac{A_t}{s}$$
- Anchorage of torsional stirrups is presented in ***Figure 8.2-3*** below.

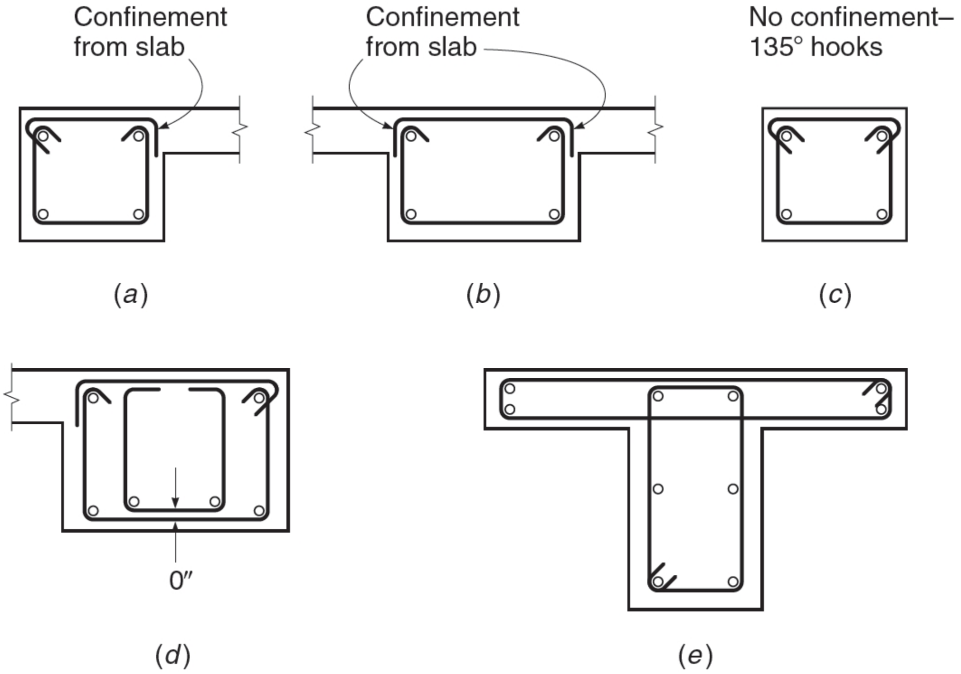


Figure 8.2-3: Stirrup-ties and longitudinal reinforcement for torsion: (a) spandrel beam with flanges on one side; (b) interior beam; (c) isolated rectangular beam; (d) wide spandrel beam; and (e) T beam with torsional reinforcement in flanges.

- Maximum Spacing for Torsional Stirrups
According to ACI Code ***9.6.4.2***, to ***control spiral cracking***, the maximum spacing of torsional stirrups should be:
$$s_{Maximum} = Minimum\left(\frac{p_h}{8} \text{ or } 300mm\right)$$
- Minimum Area of Closed Stirrups
In addition, for members requiring both shear and torsion reinforcement, the minimum area of closed stirrups is equal to:
$$A_v + 2A_t = 0.062\sqrt{f_c'}\,\frac{b_w s}{f_{yt}} \geq 0.35\,\frac{b_w s}{f_{yt}}$$

8.2.7.2 Longitudinal Reinforcement

- Based on discussion of Article ***8.1.6*** above, the area of longitudinal bar reinforcement A_ℓ required to resist T_n is given by:
$$A_\ell = \left(\frac{A_t}{s}\right) p_h \left(\frac{f_{yt}}{f_y}\right)\cot^2\theta$$
where θ must have the same value used to calculate A_t.
- The term A_t/s should be taken as the value calculated, not modified based on minimum transverse steel requirements.
- Based on an evaluation of the performance of reinforced concrete beam torsional test specimens, ACI Code ***9.6.4.3*** requires a minimum value of A_ℓ equal to the lesser:
$$a.\;\; 0.42\sqrt{f_c'}\left(\frac{A_{cp}}{f_{yt}}\right) - \left(\frac{A_t}{s}\right) p_h \left(\frac{f_{yt}}{f_y}\right)$$
$$b.\;\; 0.42\sqrt{f_c'}\left(\frac{A_{cp}}{f_{yt}}\right) - \left(\frac{0.175 b_w}{f_{yt}}\right) p_h \left(\frac{f_{yt}}{f_y}\right)$$

8.3 DESIGN PROCEDURES AND EXAMPLES

Designing a reinforced concrete flexural member for torsion involves a series of steps. The following sequence ensures that each is covered:

- Compute V_u and T_u. When pertinent conditions are satisfied, V_u and T_u can be determined at distance d from face of support.
- Determine if the factored torque is less than:

  $$\phi T_{Th} = \phi 0.083\lambda\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)$$

  If so, torsion may be neglected. If not, proceed with the design. Note that in this step, portions of over-hanging flanges, as defined in ***Figure 8.2-1*** above, must be included in the calculation of A_{cp} and p_{cp}.

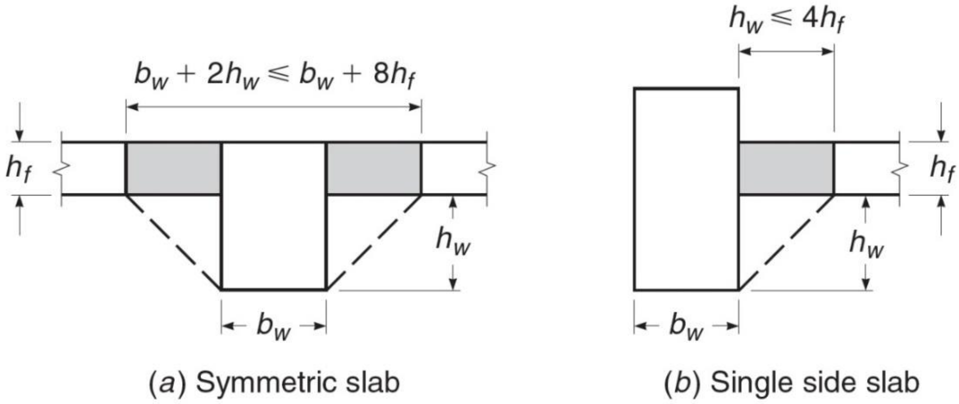


Figure 8.2-1: Portion of slab to be included with beam for torsional design. Reproduce for convenience.

- If the ***torsion is compatibility torsion***, rather than equilibrium torsion, as described in ***Section 8.1.3*** above, the maximum factored torque may be reduced to:

  $$\phi 0.33\lambda\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)$$

  Equilibrium torsion cannot be adjusted.
- Check the shear stresses in the section under combined torsion and shear, using the following criteria:
  - For hollow sections:

    $$\left(\frac{V_u}{b_w d}\right) + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right) \leq \phi\left(\frac{V_c}{b_w d} + 0.66\sqrt{f_c'}\right)$$
  - For solid sections:

    $$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right)^2} \leq \phi\left(\frac{V_c}{b_w d} + 0.66\sqrt{f_c'}\right)$$
- Calculate the required transverse reinforcement for torsion using following relation:

  $$A_t = \frac{T_u s}{2\phi A_o f_{yt}\cot\theta}$$

  Combine A_t and A_v using following relation:

  $$\frac{A_{v+t}}{s} = \frac{A_v}{s} + 2\frac{A_t}{S}$$
- Check that the minimum transverse reinforcement requirements are met for both torsion and shear. These include:
  - The maximum spacing:

    $$s_{Maximum} = Minimum\left(\frac{p_h}{8}\ or\ 300mm\right)$$
  - The minimum area:

    $$A_v + 2A_t = Maximum\left(0.062\sqrt{f_c'}\ \frac{b_w s}{f_{yt}}, 0.35\ \frac{b_w s}{f_{yt}}\right)$$

    As in ***Chapter 5***, solve for $s_{\text{For minimum value of } A_v + 2A_t}$

$$s_{\text{For minimum value of } A_v+2A_t} = \text{minimum}\left(\frac{(A_v + 2A_t)f_{yt}}{0.062\sqrt{f_c'}b_w}, \frac{(A_v + 2A_t)f_{yt}}{0.35b_w}\right)$$

- Calculate the required longitudinal torsional reinforcement A_ℓ, using the following relation:
$$A_l = \frac{A_t}{s} p_h \frac{f_{yt}}{f_y} \cot^2\theta$$
then comparing with $A_{l\,minimum}$ given by:
$$A_{l\,minimum} = minimum\left(0.42\sqrt{f_c'}\frac{A_{cp}}{f_{yt}} - \left(\frac{A_t}{s}\right)p_h\frac{f_{yt}}{f_y}, 0.42\sqrt{f_c'}\frac{A_{cp}}{f_{yt}} - \left(\frac{0.175b_w}{f_{yt}}\right)p_h\frac{f_{yt}}{f_y}\right)$$
- Details for Torsional Longitudinal Bars:
According to ***ACI Code 9.7.5***,
  - The spacing of the longitudinal bars should not exceed 300mm,
  - They should be distributed around the perimeter of the cross section to control cracking and to ensure that the centroid of the additional longitudinal reinforcement for torsion should approximately coincide with the centroid of the section.
  - The bars shall have a diameter at least 0.042 times the transverse reinforcement spacing, but not less than 10 mm.
  - At least one longitudinal bar must be placed at each corner of the stirrups.
  - Careful attention must be paid to the anchorage of longitudinal torsional reinforcement so that it is able to develop its yield strength at the face of the supporting columns, where torsional moments are often maximum.

Example 8.3-1

The ***8.5m*** span beam shown in ***Figure 8.3-1*** below carries a monolithic slab cantilevering ***1.8m*** past the beam centerline. The resulting L beam supports a live load of ***11.5 kN/m*** along the beam centerline plus ***2.4 kPa*** uniformly distributed over the upper slab surface. The effective depth to the flexural steel centroid is ***546mm***, and the distance from the beam surfaces to the centroid of stirrup steel is ***45mm***. Material strengths are $f_c' = 35\ MPa$ and $f_y = f_{yt} = 420\ MPa$. Using same stirrup spacing along beam span, design the torsional and shear reinforcement for the beam.

It is useful to note that based on flexure requirement a longitudinal reinforcement of $1191\ mm^2$ should be provided for negative region and about $900mm^2$ should be provided for positive region.

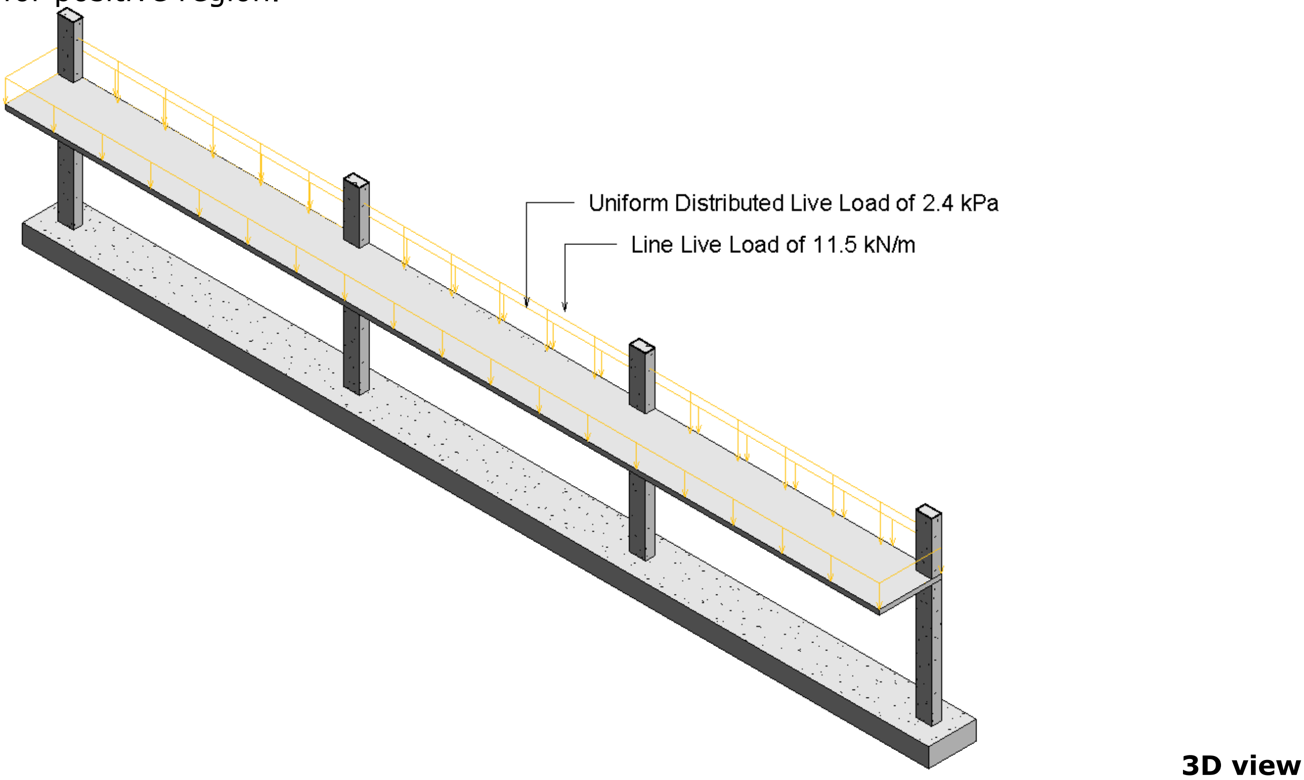
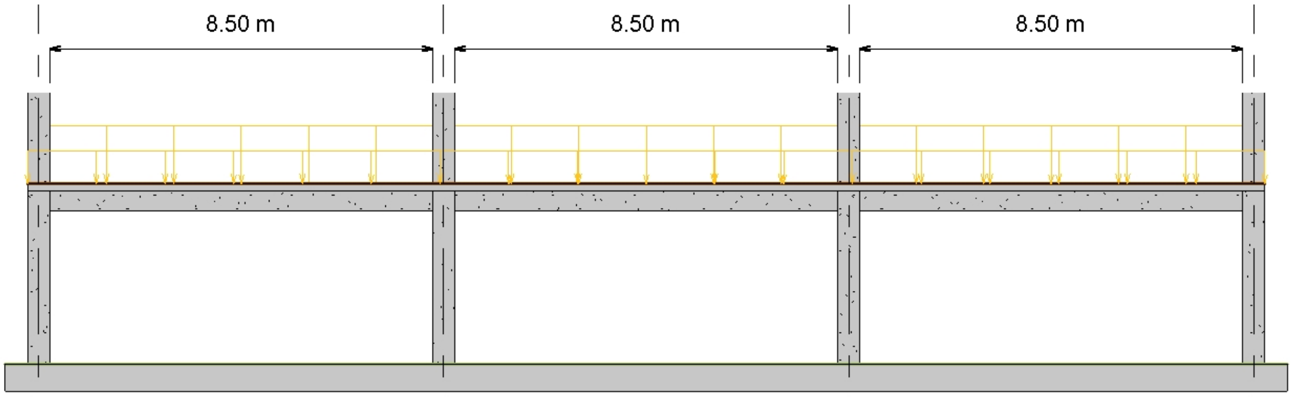
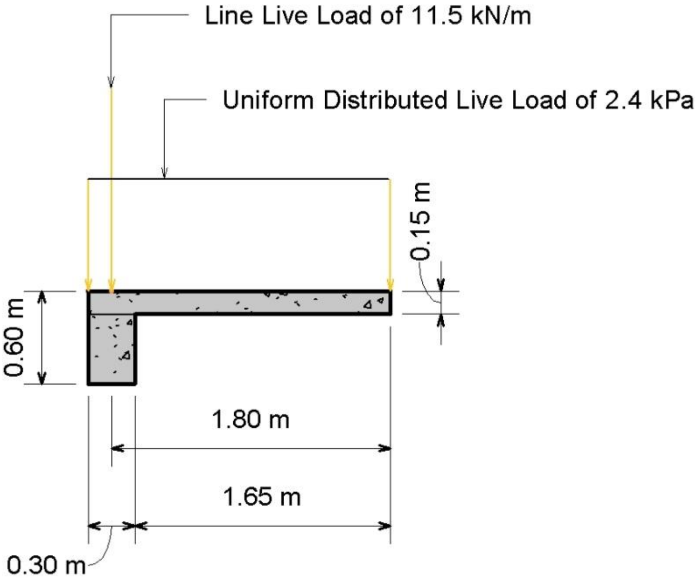


Figure 8.3-1: Structure for Example 8.3-1.



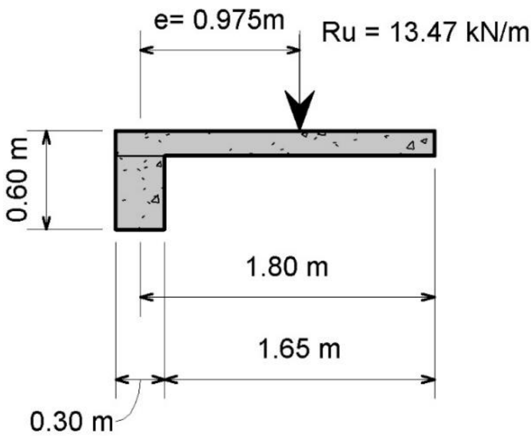
Elevation View.



Sectional View.

Figure 8.3-1: Structure for Example 8.3-1. Continue.

Solution



Factored Loads

Factored uniformly distributed load:

$W_u = 1.2 \times (0.15 \times 24) + 1.6 \times 2.4 = 8.16\ kPa$

The resultant for this UDL would be:

$R_{u\ of\ UDL} = 8.16 \times 1.65 = 13.47 \frac{kN}{m}$

Located at eccentricity of:

$e = \frac{1.65}{2} + \frac{0.30}{2} = 0.975\ m$

Factored live load:

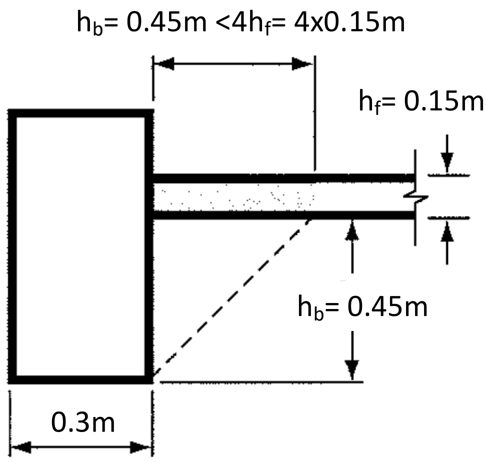
$q_u = 1.2 \times (0.3 \times 0.6 \times 24) + 1.6 \times 11.5 \approx 24\ kN/m$

Factored Shear Force and Torsion

As all related conditions are satisfied, therefore shear force and torsion can be determined at distance

$$V_{u\ @\ distance\ d\ from\ face\ of\ support} = \frac{1}{2}\left((24 + 13.47) \times (8.50 - 0.546 \times 2)\right) \approx 139\ kN$$

$$T_{u\ @\ distance\ d\ from\ face\ of\ support} = \frac{1}{2}\left((13.47 \times 0.975) \times (8.50 - 0.546 \times 2)\right) = 48.6\ kN.m$$



Comparing with ϕT_{th}

$$\phi T_{Th} = \phi 0.083 \lambda \sqrt{f_c'} \left(\frac{A_{cp}^2}{p_{cp}}\right)$$

The effective section would be as indicated in below:

$A_{cp} = (300 \times 600 + 150 \times 450) = 247500\ mm^2$

$p_{ch} = (300 + 600) \times 2 + 450 \times 2 = 2700\ mm$

$$\phi T_{Th} = \phi 0.083\lambda\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right) = \frac{0.75 \times 0.083 \times 1.0 \times \sqrt{35}\left(\frac{247500^2}{2700}\right)}{10^6} = 8.36\ kN.m < T_u$$

Clearly, torsion must be considered in the present case.

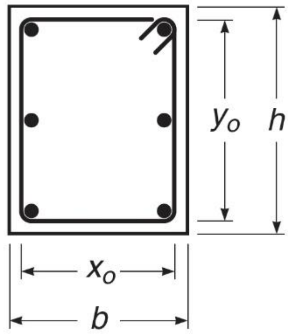
<u>Primary versus compatibility torsion:</u>

Since the torsional resistance of the beam is required for equilibrium, no reduction in T_u may be made.

<u>Checking for shear stresses:</u>

Check the shear stresses in the section under combined torsion and shear. As the section is a solid one, therefore shear stresses would be checked using the following criteria:

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right)^2} \le \phi\left(\frac{V_c}{b_w d} + 0.66\sqrt{f_c'}\right)$$



Proposed the stirrups indicated in below, with $45mm$ cover to the center of the stirrup bars from all faces,

$x_0 = 300 - 90 = 210\ mm, y_0 = 600 - 90 = 510\ mm$

$A_{oh} = 210 \times 510 = 107100\ mm^2, p_h = 2 \times (210 + 510) = 1440\ mm$

Substitute in the criterion to obtain

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right)^2} = \sqrt{\left(\frac{139 \times 10^3}{300 \times 546}\right)^2 + \left(\frac{48.6 \times 10^6 \times 1440}{1.7 \times 107100^2}\right)^2}$$

$$\le 0.75 \times \left(0.17\sqrt{35} + 0.66\sqrt{35}\right)$$

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right)^2} = \sqrt{\left(\frac{139 \times 10^3}{300 \times 546}\right)^2 + \left(\frac{48.6 \times 10^6 \times 1440}{1.7 \times 107100^2}\right)^2} = 0.75 \times \left(0.17\sqrt{35} + 0.66\sqrt{35}\right)$$

$$= 3.68MPa = 3.68\ MPa \therefore Ok.$$

Therefore, the cross section is of adequate size for the given concrete strength.

<u>Design of Transverse Reinforcement</u>

The values of A_t and A_v will now be calculated at the distance d from column face. With choosing $\theta = 45°$,

$$A_t = \frac{T_u s}{2\phi A_o f_{yt} \cot\theta}$$

$A_o = 0.85A_{oh} = 0.85 \times 210 \times 510 = 91035\ mm^2$

$$A_t = \frac{48.6 \times 10^6}{2 \times 0.75 \times 91035 \times 420 \times 1.0}s = 0.847s$$

$$\phi V_c = \frac{0.75 \times 0.17 \times \sqrt{35} \times 546 \times 300}{1000} = 124\ kN$$

$$V_s = \frac{V_u - \phi V_c}{\phi} = \frac{139 - 124}{0.75} = 20\ kN$$

From Chapter 5,

$$V_s = \left(A_v f_{yt}\right) \times \frac{d}{s} \Rightarrow A_v = \frac{V_s}{f_{yt} d}s = \frac{20000}{420 \times 546}s = 0.0872s$$

Combine A_t and A_v using following relation:

$2A_t + A_v = 2 \times 0.847s + 0.0872s = 1.78s$

Try stirrups of $No. 13$

$$2A_t + A_v = 2 \times \frac{\pi \times 13^2}{4} = 1.78s$$

Solve for spacing s:

$s = 149\ mm$

Try $No. 13$ @ $125\ mm$

Check with maximum spacing for shear and torsion:

$\because V_s < 0.33\lambda\sqrt{f_c'}b_w d$

Therefore,

$$s_{Maximum\ for\ shear} = Minimum\left(\frac{d}{2}, 600\right)$$

While the maximum spacing for torsion is:

$s_{Maximum\ for\ torsion} = Minimum\left(\frac{p_h}{8}\ or\ 300mm\right)$

Therefore, $s_{Maximum}$ for both aspects would be:

$$s_{Maximum} = Minimum\left(\frac{d}{2}, \frac{p_h}{8}, 300\right) = Minimum\left(\frac{546}{2}, \frac{1440}{8}, 300\right) = Minimum(273, 180, 300)$$
$$= 180mm > s_{Provided} \therefore Ok.$$

Finally, checking the limitation on minimum area of transverse reinforcement:

$$s_{\text{For minimum value of } A_v+2A_t} = \text{minimum}\left(\frac{(A_v + 2A_t)f_{yt}}{0.062\sqrt{f_c'}b_w}, \frac{(A_v + 2A_t)f_{yt}}{0.35b_w}\right)$$

$$s_{\text{For minimum value of } A_v+2A_t} = \text{minimum}\left(\frac{2 \times \frac{\pi \times 13^2}{4} \times 420}{0.062 \times \sqrt{35} \times 300}, \frac{2 \times \frac{\pi \times 13^2}{4} \times 420}{0.35 \times 300}\right)$$

$$= \text{minimum}\ (1013, 1061) = 1013\ mm \gg s_{Provided} \therefore Ok.$$

Therefore use $No.13$ @ $125mm$ along whole span of the beam.

<u>Design for Longitudinal Reinforcement for Torsion:</u>

Calculate the required longitudinal torsional reinforcement A_l, using the following relation:

$$A_l = \frac{A_t}{s} p_h \frac{f_{yt}}{f_y} \cot^2\theta$$

The longitudinal steel required for torsion at a distance d from the column face is:

$$\because A_t = 0.847s \Rightarrow \frac{A_t}{s} = 0.847$$

Then

$$A_l = \frac{A_t}{s} p_h \frac{f_{yt}}{f_y} \cot^2\theta = 0.847 \times 1440 \times 1.0 \times 1.0^2 = 1219\ mm^2$$

Comparing with $A_{l\ minimum}$ given by:

$$A_{l\ minimum} = minimum\left(0.42\sqrt{f_c'}\frac{A_{cp}}{f_{yt}} - \left(\frac{A_t}{s}\right)p_h\frac{f_{yt}}{f_y}, 0.42\sqrt{f_c'}\frac{A_{cp}}{f_{yt}} - \left(\frac{0.175b_w}{f_{yt}}\right)p_h\frac{f_{yt}}{f_y}\right)$$

$$A_{l\ minimum} = minimum\Big(0.42 \times \sqrt{35} \times \frac{247500}{420} - (0.847) \times 1440 \times 1.0,$$
$$0.42 \times \sqrt{35} \times \frac{247500}{420} - \left(\frac{0.175 \times 300}{420}\right) \times 1440 \times 1.0\Big) = minimum(225, 1284)$$
$$= 225\ mm^2 < A_l \therefore Ok.$$

Reinforcement will be placed at the top, mid-depth, and bottom of the member each level to provide not less than $1219/3 = 406$. Try rebar with No.20:

$$No.of\ rebars\ at\ mid\ depth = \frac{406}{\frac{\pi \times 20^2}{4}} = 1.29$$

Use $2No.20$ @ $mid\ depth$

$$No.of\ top\ rebars = \frac{1191 + 406}{\frac{\pi \times 20^2}{4}} \approx 5.0$$

Use $5No.20$ @ top

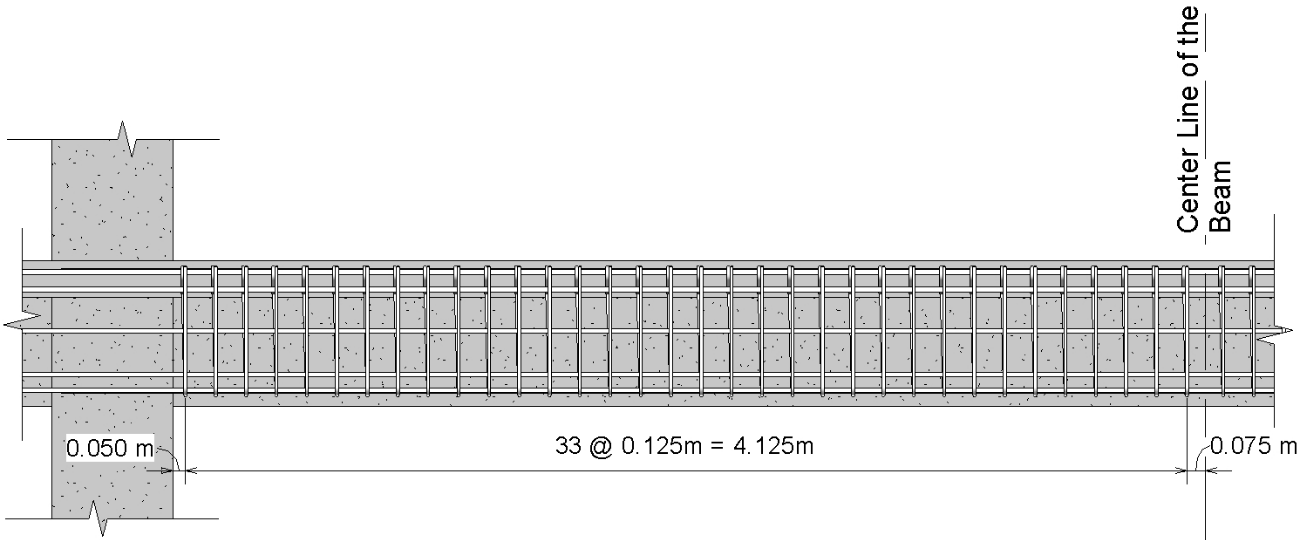
$$No.of\ bottom\ rebars = \frac{900 + 406}{\frac{\pi \times 20^2}{4}} = 4.15$$

Use $5No.20$ @ $Bottom$.

Proposed beam reinforcement are presented in ***Figure 8.3-2*** below. To be a final decision, proposed reinforcement should be checked for ACI requirements for details of torsional longitudinal bars, ACI Code 9.7.5,

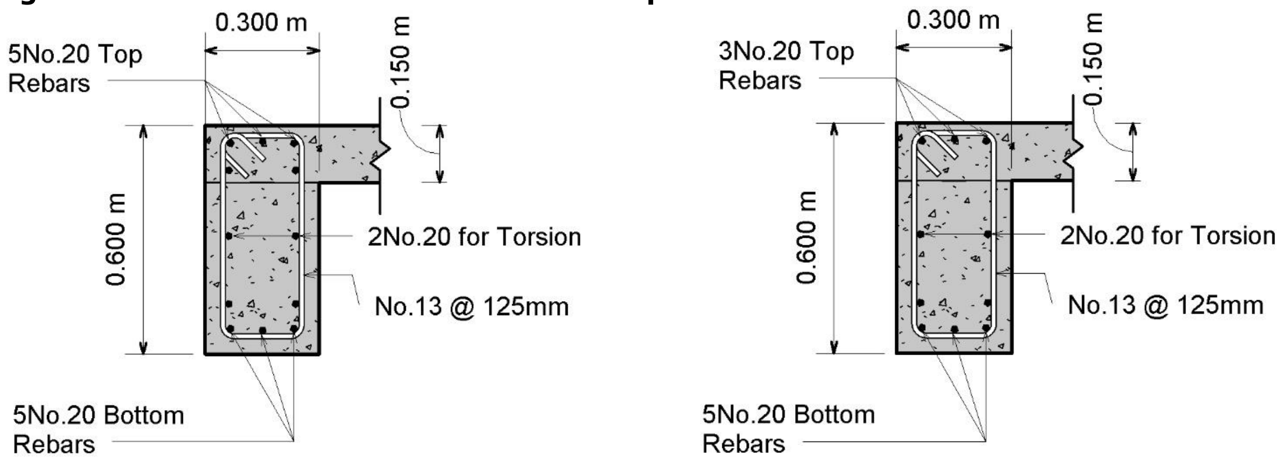
- The spacing of the longitudinal bars should not exceed 300mm, Ok.
- They should be distributed around the perimeter of the cross section to control cracking, Ok.
- The bars shall have a diameter at least 0.042 times the transverse reinforcement spacing, but not less than 10 mm., Ok.
- At least one longitudinal bar must be placed at each corner of the stirrups, Ok.

- Careful attention must be paid to the anchorage of longitudinal torsional reinforcement so that it is able to develop its yield strength at the face of the supporting columns, where torsional moments are often maximum. ***This should be as discussed in Chapter 7***.



Longitudinal section view.

Figure 8.3-2: Beam reinforcement for Example 8.3-1.



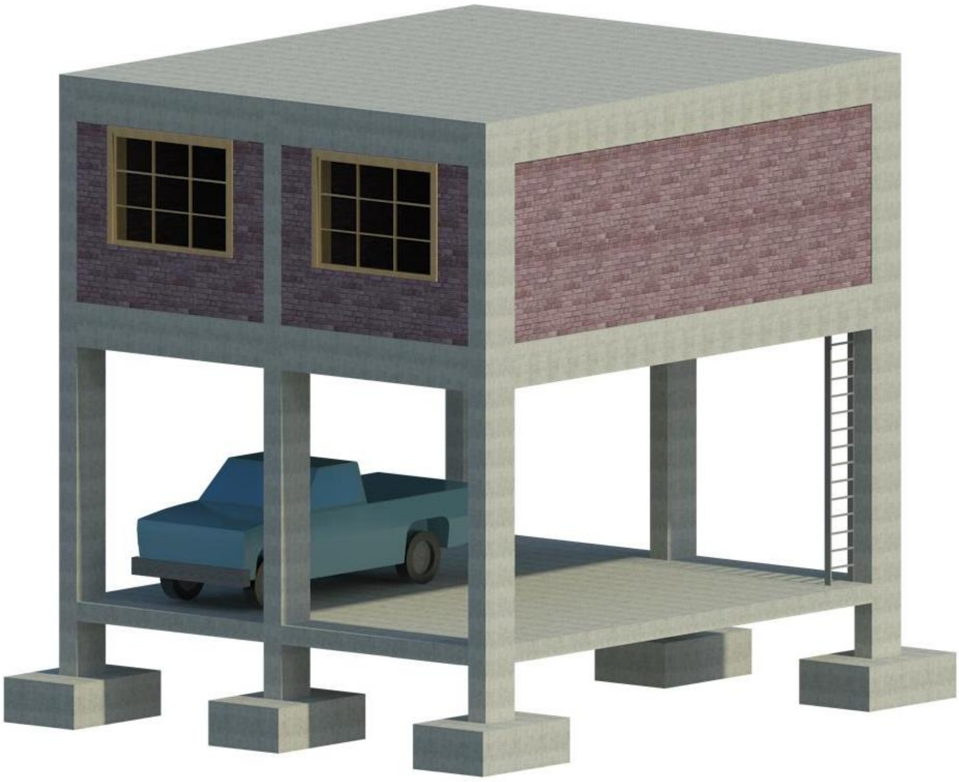
Section @ supports **Section @ mid-span**

Figure 8.3-2: Beam reinforcement for Example 8.3-1. Continue.

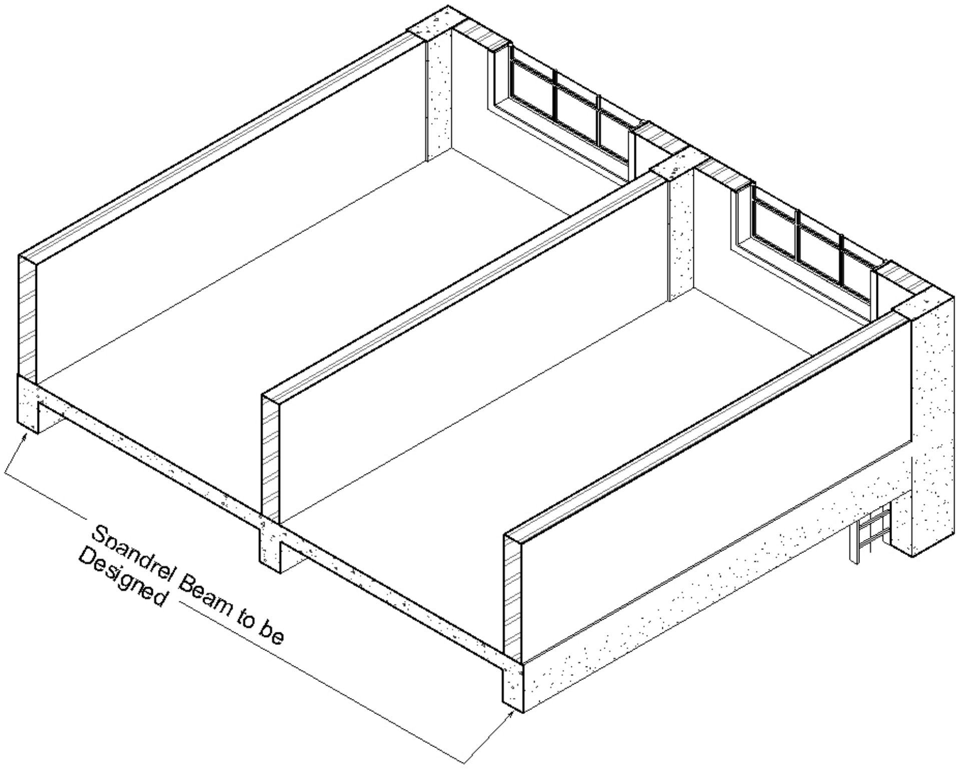
Example 8.3-2

For a maintenance shop indicated in ***Figure 8.3-3*** below, design a floor supporting spandrel beam for torsion and shear. The floor slab is subjected to a ***live load of 2.5 kPa*** and a ***superimposed dead load of 2.0 kPa*** in addition of its own weight. In your design, assume that:

- $f_c' = 28\ MPa$, and $f_y = f_{yt} = 420\ MPa$,
- Based on flexural design, $A_{top\ required} \approx 700\ mm^2$ and $A_{bottom\ required} \approx 610\ mm^2$,
- Try two layers with $No. 20$ for longitudinal reinforcement and $No. 10$ for stirrups.

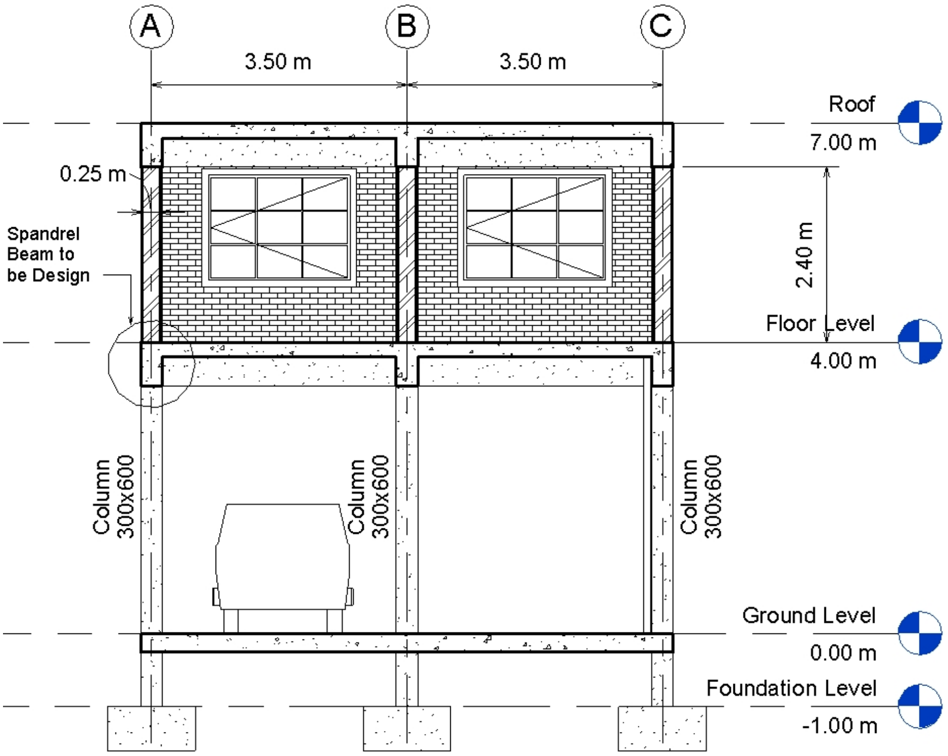


3D View.

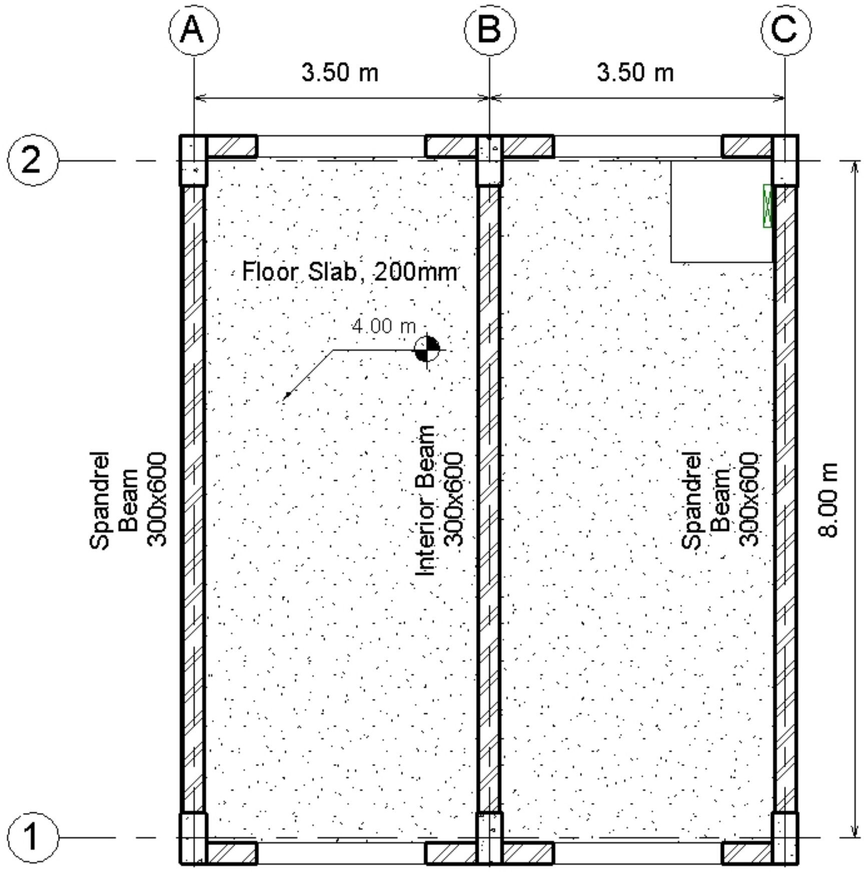


3D Sectional View.

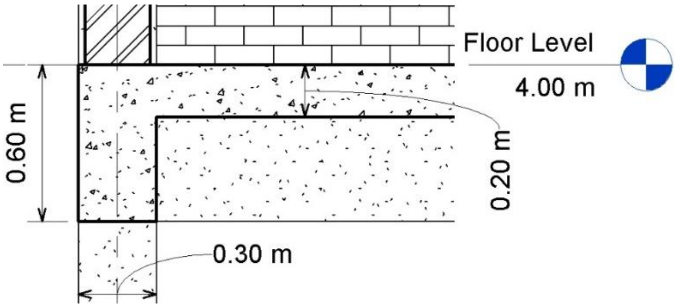
Figure 8.3-3: Maintenance shop for Example 8.3-2.



Sectional View.



Plan View.



Callout View for the Spandrel Beam.

Figure 8.3-3: Maintenance shop for Example 8.3-2. Continue.

Solution

Factored Loads

Factored uniformly distributed, W_u, that acting on the slab would be:

$W_D = 0.2 \times 24 + 2.0 = 6.8\ kPa$

$W_u = \max(1.4 \times 6.8, 1.2 \times 6.8 + 1.6 \times 2.5) = 12.2\ kPa$

With considering brick cladding as a dead load and with assuming $\gamma_{Brick} = 19\ kN/m^3$, the factored line load, q_u, would be:

$$q_u = 1.2 \times \left((0.25 \times 2.40 \times 19)_{Weight\ of\ brick\ wall} + (0.3 \times 0.6 \times 24)_{selfweight\ of\ the\ beam}\right) = 18.9 \frac{kN}{m}$$

Factored Shear Force, V_u, and Torsion, T_u, Acting on Beam

As would be discussed in ***Chapter 12***, ***Analysis and Design of One-way Slabs***, an edge supported slab is classified as one-way slab when its length to width ratio is more than 2.

$$\frac{l}{s} = \frac{8.00}{3.50} = 2.28 > 2$$

Therefore, floor system is classified as one-way slab system.

In ***Chapter 12***, it is shown that shear force and torsion transferred from the slab to the supporting beam can be estimated from following relations, see Figure 8.3-4 below:

$$M_{u\ exterior-ve\ of\ slab} = T_{u\ torsional\ moment\ acting\ on\ beam} = \frac{W_u l_n^2}{24}$$

$$V_{shear\ force\ acting\ on\ slab} = Load\ acting\ on\ beam = \frac{W_u l_n}{2}$$

where:

$W_u = Factored\ UDL\ acting\ on\ slab = 12.2\ kPa$

$$l_n = clear\ span\ of\ slab = clear\ spacing\ between\ supporting\ beam = 3.5 - \frac{0.3}{2} \times 2 = 3.2\ m$$

$$M_{u\ exterior-ve\ of\ slab} = T_{u\ torsional\ moment\ acting\ on\ beam} = \frac{12.2 \times 3.2^2}{24} = 5.21\ kN.m\ per\ m\ ■$$

$$V_{shear\ force\ acting\ on\ slab} = Load\ acting\ on\ beam = \frac{12.2 \times 3.2}{2} = 19.5 \frac{kN}{m}$$

Including the factored loads that acting directly on beam, the total factored line load acting on the beam would be:

$$q_{u\ total\ line\ load\ acting\ on\ the\ beam} = 19.5 + 18.9 = 38.4 \frac{kN}{m}\ ■$$

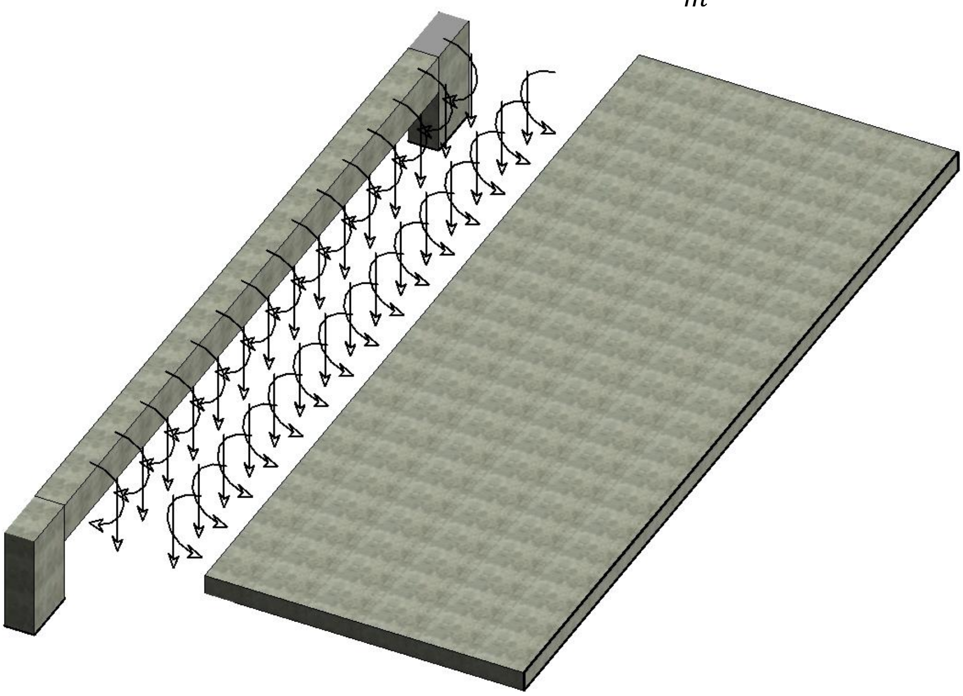


Figure 8.3-4: Forces transformed from supported slab to the supporting beam.

As all pertinent conditions are satisfied, therefore, design force can be determined at distance d from face of support. With two layers of reinforcement and with adopting of $No.20$ for longitudinal reinforcement and $No.10$ for stirrups, the effective depth would be:

$$d = 600 - 40 - 10 - 20 - \frac{25}{2} = 517\ mm$$

$$V_{u\,@\,distance\,d\,from\,face\,of\,support} = \frac{38.4 \times \left(8.0 - \frac{0.6}{2} \times 2 - 0.517 \times 2\right)}{2} = 122\,kN$$

$$T_{u\,@\,distance\,d\,from\,face\,of\,support} = \frac{5.21 \times \left(8.0 - \frac{0.6}{2} \times 2 - 0.517 \times 2\right)}{2} = 16.6\,kN.m\;per\;m$$

A more accurate torque can be determined with considering of offset between transferred shear force, V_u, and the center line of the spandrel beam:

$$T_{u\,@\,distance\,d\,from\,face\,of\,support} = \frac{\left(5.21 + 19.5 \times \frac{0.3}{2}\right) \times \left(8.0 - \frac{0.6}{2} \times 2 - 0.517 \times 2\right)}{2}$$
$$= 25.9\,kN.m\;per\;m$$

Comparing with ϕT_{th}

$$\phi T_{Th} = \phi 0.083\lambda\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)$$

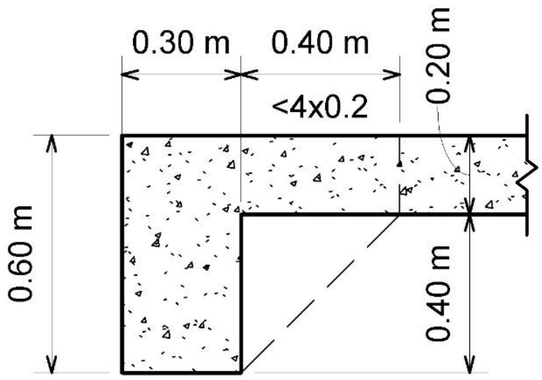
The effective section would be as indicated in below:

$A_{cp} = (300 \times 600 + 200 \times 400) = 260000\,mm^2$

$p_{ch} = (300 + 600) \times 2 + 400 \times 2 = 2600\,mm$

$$\phi T_{Th} = \phi 0.083\lambda\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right) = \frac{\left(0.75 \times 0.083 \times 1.0 \times \sqrt{28} \times \left(\frac{260000^2}{2600}\right)\right)}{10^6} = 8.56\,kN.m < T_u$$

Clearly, torsion must be considered in the present case.



Primary versus compatibility torsion:

Since the torsional resistance of the beam is required for computability, therefore T_u .can be reduced to the value indicated in below:

$$T_{u\,reduced} = \phi 0.33\lambda\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)$$
$$= \frac{0.75 \times 0.33 \times 1.0 \times \sqrt{28} \times \left(\frac{260000^2}{2600}\right)}{10^6}$$
$$= 34.1\,kN.m > T_{u\,applied}$$

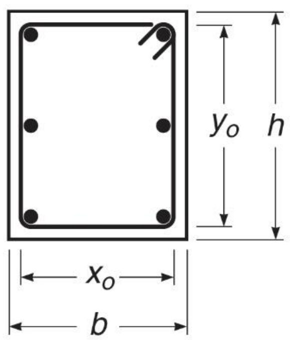
Therefore, no benefit can be obtained for torque reduction and the design should be based on T_u of $16.6\;kN.m$.

Checking for shear stresses:

Check the shear stresses in the section under combined torsion and shear. As the section is a solid one, therefore shear stresses would be checked using the following criteria:

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right)^2} \leq \phi\left(\frac{V_c}{b_w d} + 0.66\sqrt{f_c'}\right)$$

Proposed the stirrups indicated in below, with $45mm$ cover to the center of the stirrup bars from all faces,



$$x_0 = 300 - 40 \times 2 - \frac{10}{2} \times 2 = 210\,mm$$

$$y_0 = 600 - 40 \times 2 - \frac{10}{2} \times 2 = 510\,mm$$

$A_{oh} = 210 \times 510 = 107100\,mm^2$

$p_h = 2 \times (210 + 510) = 1440\,mm$

Substitute in the criterion to obtain

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right)^2} = \sqrt{\left(\frac{122 \times 10^3}{300 \times 517}\right)^2 + \left(\frac{25.9 \times 10^6 \times 1440}{1.7 \times 107100^2}\right)^2}$$
$$\leq 0.75 \times \left(0.17\sqrt{28} + 0.66\sqrt{28}\right)$$

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right)^2} = 2.06\,MPa \leq 3.29\,MPa \therefore Ok.$$

Therefore, the cross section is of adequate size for the given concrete strength.

<u>Design of Transverse Reinforcement</u>

The values of A_t and A_v will now be calculated at the distance d from column face. With choosing $\theta = 45°$,

$$A_t = \frac{T_u s}{2\phi A_o f_{yt} \cot\theta}$$

$$A_o = 0.85 A_{oh} = 0.85 \times 210 \times 510 = 91035\ mm^2$$

$$A_t = \frac{25.9 \times 10^6}{2 \times 0.75 \times 91035 \times 420 \times 1.0} s = 0.452s$$

$$\phi V_c = \frac{0.75 \times 0.17 \times \sqrt{28} \times 517 \times 300}{1000} = 105\ kN$$

$$V_s = \frac{V_u - \phi V_c}{\phi} = \frac{122 - 105}{0.75} = 22.6\ kN$$

From Chapter 5,

$$V_s = (A_v f_{yt}) \times \frac{d}{s} \Rightarrow A_v = \frac{V_s}{f_{yt} d} s = \frac{22.6 \times 10^3}{420 \times 517} s = 0.104s$$

Combine A_t and A_v using following relation:

$$2A_t + A_v = 2 \times 0.452s + 0.104s = 1.00s$$

Try stirrups of $No. 10$

$$2A_t + A_v = 2 \times \frac{\pi \times 10^2}{4} = 1.00s$$

Solve for spacing s:

$$s = 157\ mm$$

Try $No. 10$ @ $150\ mm$

Check with maximum spacing for shear and torsion:

$$\because V_s < 0.33\lambda\sqrt{f_c'} b_w d$$

Therefore,

$$s_{Maximum\ for\ shear} = Minimum\left(\frac{d}{2}, 600\right)$$

While the maximum spacing for torsion is:

$$s_{Maximum\ for\ torsion} = Minimum\left(\frac{p_h}{8}\ or\ 300mm\right)$$

Therefore, $s_{Maximum}$ for both aspects would be:

$$s_{Maximum} = \min\left(\frac{d}{2}, \frac{p_h}{8}, 300\right) = \min\left(\frac{517}{2}, \frac{1440}{8}, 300\right) = 180mm > s_{Provided} \therefore Ok.$$

Try $No. 10$ @ $150\ mm$

Finally, checking the limitation on minimum area of transverse reinforcement:

$$s_{\text{For minimum value of } A_v + 2A_t} = \min\left(\frac{(A_v + 2A_t) f_{yt}}{0.062\sqrt{f_c'} b_w}, \frac{(A_v + 2A_t) f_{yt}}{0.35 b_w}\right)$$

$$s_{\text{For minimum value of } A_v + 2A_t} = \min\left(\frac{2 \times \frac{\pi \times 10^2}{4} \times 420}{0.062 \times \sqrt{28} \times 300}, \frac{2 \times \frac{\pi \times 10^2}{4} \times 420}{0.35 \times 300}\right) = 628\ \text{mm} > s_{Provided}$$

$$\therefore Ok.$$

Therefore ***use*** No. 10 @ 150mm ***along whole span of the beam***.

<u>Design for Longitudinal Reinforcement for Torsion:</u>

Calculate the required longitudinal torsional reinforcement A_l, using the following relation:

$$A_l = \frac{A_t}{s} p_h \frac{f_{yt}}{f_y} \cot^2\theta$$

The longitudinal steel required for torsion at a distance d from the column face is:

$$\because A_t = 0.452s \Rightarrow \frac{A_t}{s} = 0.452$$

Then

$$A_l = \frac{A_t}{s} p_h \frac{f_{yt}}{f_y} \cot^2\theta = 0.452 \times 1440 \times 1.0 \times 1.0^2 = 650\ mm^2$$

Comparing with $A_{l\ minimum}$ given by:

$$A_{l\,minimum} = \min\left(0.42\sqrt{f_c'}\frac{A_{cp}}{f_{yt}} - \left(\frac{A_t}{s}\right)p_h\frac{f_{yt}}{f_y}, 0.42\sqrt{f_c'}\frac{A_{cp}}{f_{yt}} - \left(\frac{0.175b_w}{f_{yt}}\right)p_h\frac{f_{yt}}{f_y}\right)$$

$$A_{l\,minimum} = \min\left(0.42 \times \sqrt{28} \times \frac{260000}{420} - (0.452) \times 1440 \times 1.0,\right.$$

$$\left. 0.42 \times \sqrt{28} \times \frac{260000}{420} - \left(\frac{0.175 \times 300}{420}\right) \times 1440 \times 1.0\right) = 724\,mm^2 > A_l \therefore Not\ Ok.$$

$\therefore A_l = 724\,mm^2$

Reinforcement will be placed at the top, mid-depth, and bottom of the member. Each level to provide not less than $724/3 = 241$. Try No.20 rebar:

$$No.of\ rebars\ at\ mid\ depth\ = \frac{241}{\frac{\pi \times 20^2}{4}} \approx 0.767$$

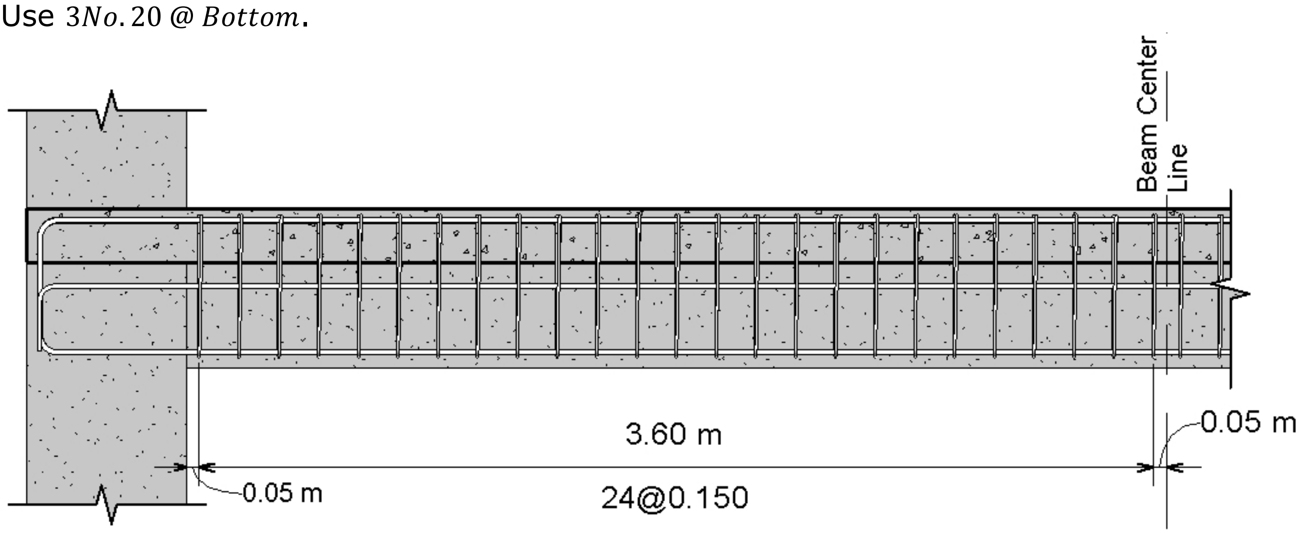
Therefore, using $2No.16\ @\ mid - depth$ seems more suitable and economical.

$$No.of\ top\ rebars = \frac{700 + 241}{\frac{\pi \times 20^2}{4}} = 2.99$$

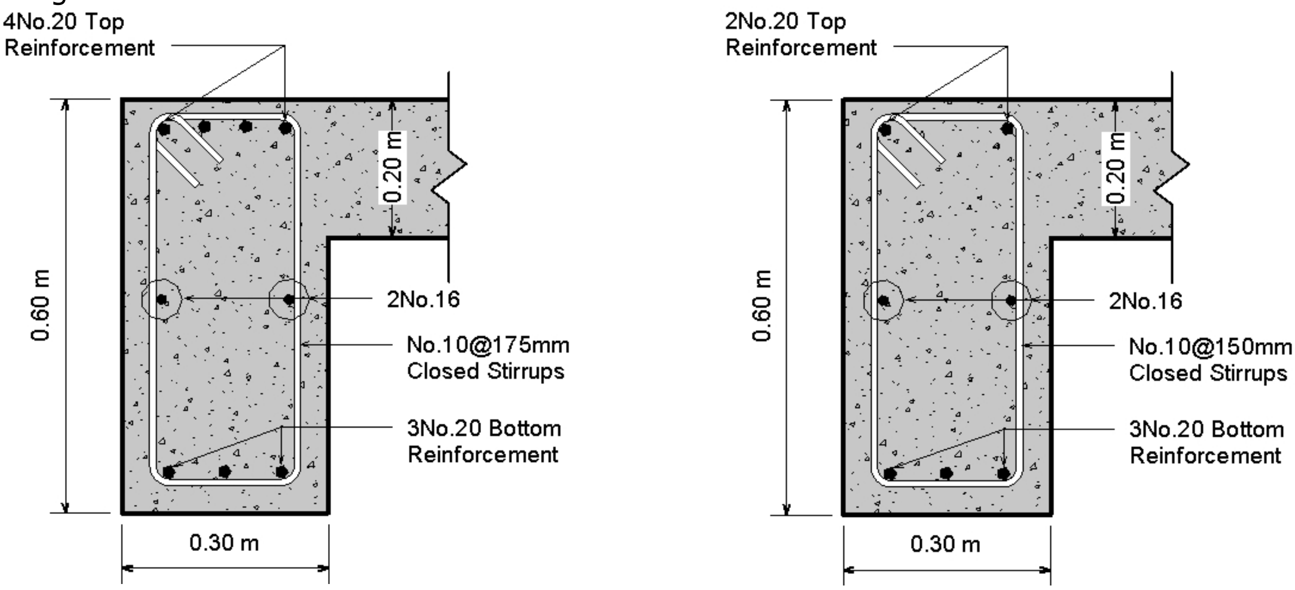
Use $3No.20\ @\ top$

$$No.of\ bottom\ rebars = \frac{610 + 241}{\frac{\pi \times 20^2}{4}} = 2.7$$

Use $3No.20\ @\ Bottom$.



Longitudinal section.



Beam Section at Supports **Section at Mid-span**

Figure 8.3-5: Reinforcement for Example 8.3-2.

Proposed beam reinforcement are presented in ***Figure 8.3-5*** above. To be a final decision, proposed reinforcement should be checked for ACI requirements for details of torsional longitudinal bars, ACI Code 9.7.5,

- The spacing of the longitudinal bars should not exceed 300mm, Ok.

- They should be distributed around the perimeter of the cross section to control cracking, Ok.
- The bars shall have a diameter at least 0.042 times the transverse reinforcement spacing, but not less than 10 mm, Ok.
- At least one longitudinal bar must be placed at each corner of the stirrups, Ok.
- Careful attention must be paid to the anchorage of longitudinal torsional reinforcement so that it is able to develop its yield strength at the face of the supporting columns, where torsional moments are often maximum. ***This should be as discussed in Chapter 6***.

Contents

CHAPTER 9
SHORT COLUMNS

9.1 INTRODUCTION

9.1.1 Definition of Column

- Columns are defined as ***members that carry loads chiefly in compression with a ratio of height to least lateral dimension exceeding 3***
- According to ***Article 14.3.3*** of the code, vertical member with ratio of ***unsupported*** height to average least lateral dimension ***not exceed 3***, is classified as ***pedestal*** and can be designed as a plain concrete member.
- Pedestals are usually used in steel structures to protect steel against corrosion due to soil contact, see ***Figure 9.1-1*** and ***Figure 9.1-2*** shown below.
- Usually columns ***carry bending moments as well, about one or both axes of the cross section***, and ***the bending action may produce tensile forces over a part of the cross section***. Even in such cases, columns are generally referred to as compression members, because the compression forces dominate their behavior.

9.1.2 Other Compression Members

- In addition to the most common type of compression member, i.e., vertical elements in structures, other member can be classified as compression members and design on same basis adopted for column.
- These compression members include ***arches***, ***inclined members in gable frame***, and ***compression elements in trusses***. It is interested to know that truss can be constructed with reinforced concrete, for more information ***Advanced Reinforced Concrete Design by N. K. Raju Page 281***.

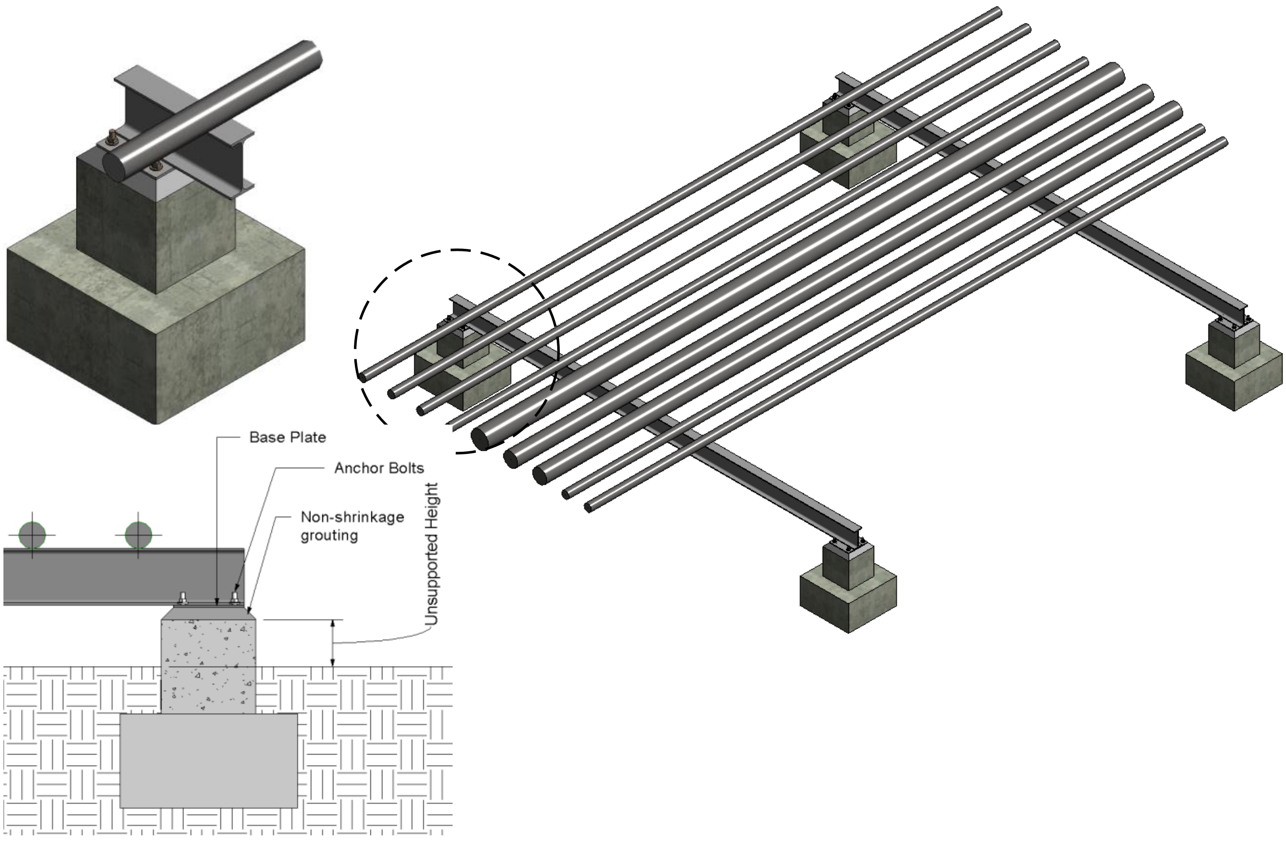


Figure 9.1-1: Pedestal used in a pipe supporting structure.

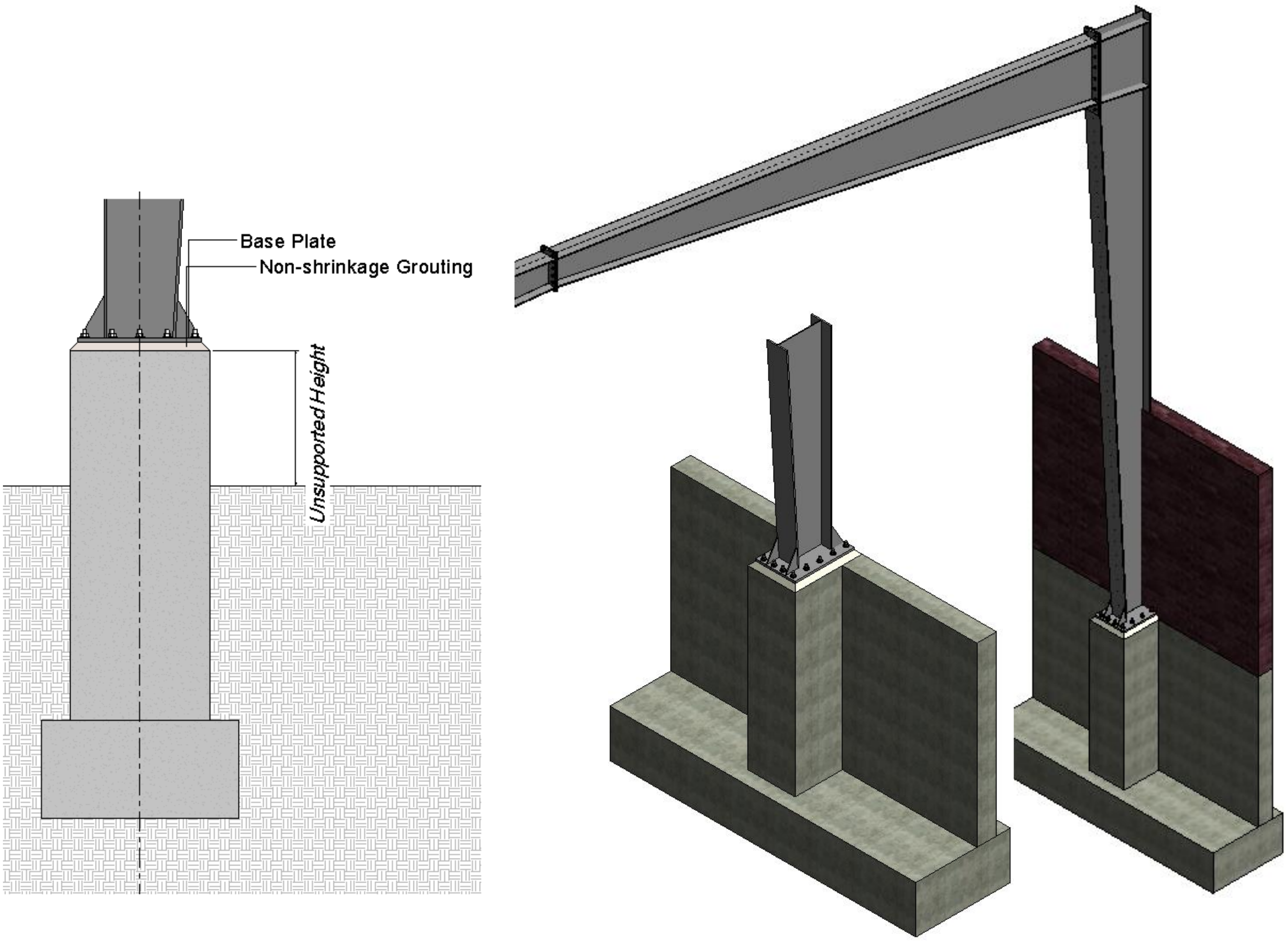


Figure 9.1-2: Pedestal used in a gable steel structure.

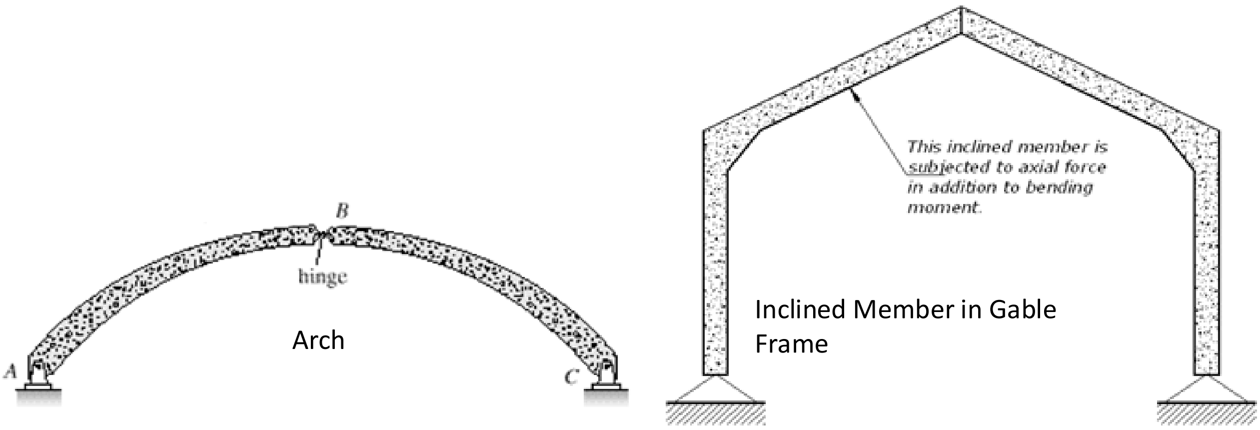


Figure 9.1-3: Other compression members.

9.1.3 Columns Classification According to Their Reinforcement

According to details of their reinforcement, reinforced concrete columns can be classified into:

- Tied Columns:
  Members reinforced with longitudinal bars and lateral ties, see ***Figure 9.1-4*** below.
- Spiral Columns
  Members reinforced with longitudinal bars and continuous spirals, ***Figure 9.1-5*** below.

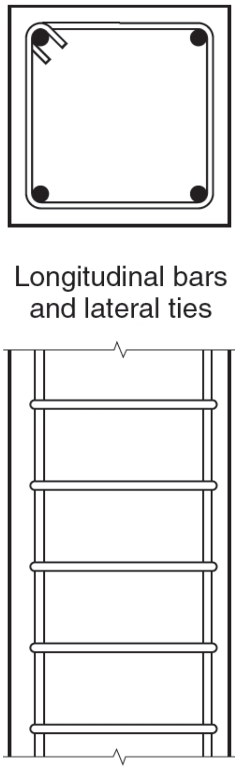


Figure 9.1-4: Tied columns.

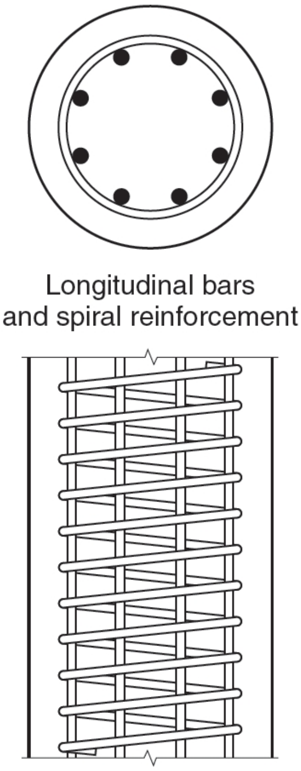


Figure 9.1-5: Spiral columns.

- Composite columns:
  Composite compression members reinforced longitudinally with structural steel shapes, pipe, or tubing, with or without additional longitudinal bars, and various types of lateral reinforcement, ***Figure 9.1-6*** above.
- Types 1 and 2 are by far the most common, and the discussion of this chapter will refer to them.

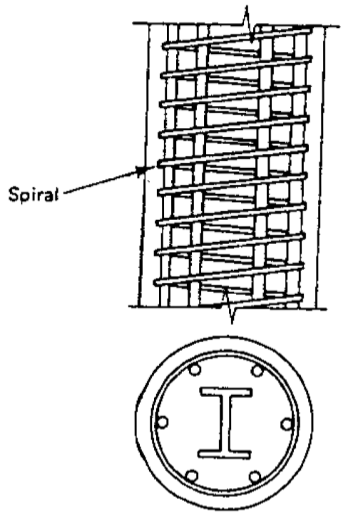


Figure 9.1-6: Composite columns.

9.1.4 Columns Classification According to Their Slenderness

- According to their length or slenderness, columns may be divided into two broad categories:
  - Short columns, for which the strength is governed by the strength of the materials and the geometry of the cross section.
  - Slender columns, for which the strength may be significantly reduced by lateral deflections.
- Only short columns will be discussed in this Chapter; the effects of the slenderness in reducing column strength will be covered in ***Chapter 10***.

9.1.5 Columns Classification According to Nature of Applied Forces

According to the nature of applied loads, columns can be calcified into following types.

9.1.5.1 Axially Loads Columns

- Sometimes columns are almost subjected to concentric forces with negligible moments, ***Figure 9.1-7*** above.
- Interior columns in building with equal spans are with in this category when subjected to gravity loads, see columns B2, C2 of ***Figure 9.1-10*** below.
- Analysis of columns under axial loads, i.e., checking the adequacy of proposed longitudinal and lateral reinforcements for given axial loads has been presented in ***Article 9.2***. While design of columns under axial loads, i.e., select the required longitudinal and lateral reinforcements for the axial loads has been presented in ***Article 9.3*** below.

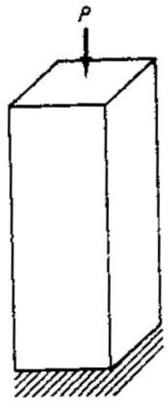


Figure 9.1-7: Axially loaded columns.

9.1.5.2 Columns Subjected to Axial Force and Uniaxial Moment

- In buildings with equal spans, edge columns are mainly subjected to axial force and uniaxial moment, see ***Figure 9.1-8*** and see ***columns B1, C1, A2, D2, B2, and D2*** of ***Figure 9.1-10*** below.
- Analysis and design of columns that subjected to axial force and uniaxial moment are presented in ***Article 0*** and ***Article 9.6*** respectively.

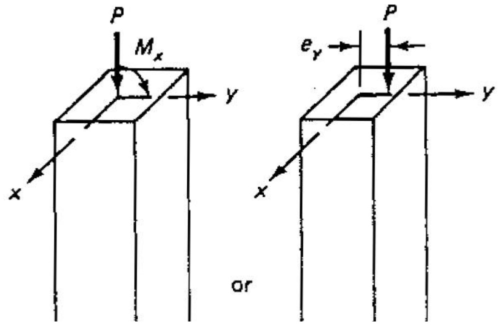


Figure 9.1-8: Columns subjected to axial force and uniaxial moment.

9.1.5.3 Columns Subjected to Axial Force and Biaxial Moments

- Columns at corner of buildings, columns A1, A2, D1, and D2 of ***Figure 9.1-10***, are usually subjected to axial force and biaxial moments as indicated in ***Figure 9.1-9*** below.
- As the design of these columns is iterative in nature, only their analysis is presented in ***Article 9.8***.

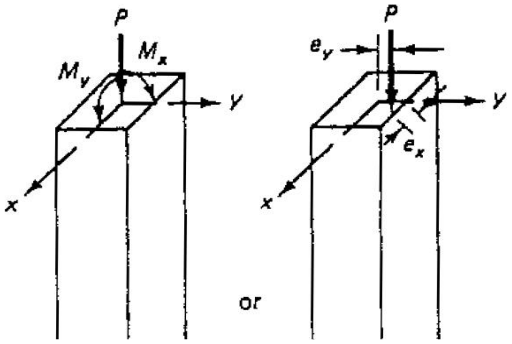


Figure 9.1-9: Columns subjected to axial force and biaxial moments.

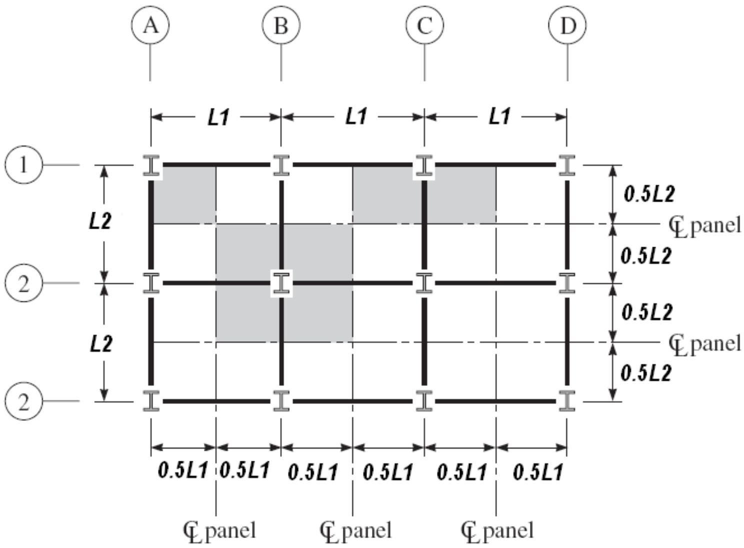


Figure 9.1-10: Columns layout for a building with equal spans.

9.2 ACI ANALYSIS PROCEDURE FOR A SHORT COLUMN UNDER AN AXIAL LOAD (SMALL ECCENTRICITY)

- Earlier ACI versions have defined small eccentricity as follows:
  - For spirally reinforced columns: $e/h \leq 0.05$.
  - For tied reinforced columns: $e/h \leq 0.10$.
- For short columns, definition of minimum eccentricity is implicitly included as will be discussed in ***Articles 9.5*** and ***9.6***.
- ACI procedures for the analysis of short columns under axial loads can be summarized as follows:

9.2.1 Checking of Longitudinal Reinforcement for Nominal Requirements

Reinforcement Limits

- Check ρ_g within acceptable limits.

$$0.01 \leq \rho_g = \frac{A_{St}}{A_g} \leq 0.08$$

- According to AC1 Code (10.6.1.1), the ratio of longitudinal steel area A_{st}, to gross concrete cross section A_g should be in the range from 0.01 to 0.08.
- The lower limit is necessary:
  - To ensure resistance to bending moments not accounted for in the analysis.
  - To reduce the effects of creep and shrinkage of the concrete under sustained compression.
- Ratios higher than 0.08 not only are uneconomical, but also would cause difficulty owing to congestion of the reinforcement, particularly where the steel must be spliced.
- Most columns are designed with ratios below 0.04. Larger-diameter bars are used to reduce placement costs and to avoid unnecessary congestion.
- The special large-diameter, No. 43 and No. 37 bars are produced mainly for use in columns.

Number of Rebars

- Check the rebar number with the minimum number of longitudinal bars:
  - Four bars for tied columns.
  - Six bars for spiral columns.
- According to ACI Code 10.7.3.1, a minimum of four longitudinal bars is required when the bars are enclosed by spaced rectangular or circular ties, and a minimum of six bars must be used when the longitudinal bars are enclosed by a continuous spiral.

Minimum Spacing between Longitudinal Bras

- According to (ACI318M, 2014), article ***25.2.3***, for longitudinal reinforcement in columns, pedestals, struts, and boundary elements in walls, clear spacing between bars shall be

$$\mathrm{S_{Minimum}} = \mathrm{Maximum}\left(1.5\mathrm{d_{Bar}}, 40^{\mathrm{mm}}, \frac{4}{3} \times \text{maximum size of aggregate}\right)$$

- As the student in his course on concrete technology how to select the maximum size of aggregate as a function of rebar spacing, the third condition related to maximum size of aggregate is assumed satisfied in this course.

9.2.2 Design Strength of Axially Loaded Columns

- According to (ACI318M, 2014), article 22.4, design strength of axially loaded column, can be determined as follows:
  - For spiral column the design strength is:
    $\emptyset P_{nMaximum} = 0.85\emptyset[0.85 f_c' (A_g - A_{st}) + A_{st} f_y]$
    with $\emptyset = 0.75$.
  - For tied columns:
    $\emptyset P_{nMaximum} = 0.80\emptyset[0.85 f_c' (A_g - A_{st}) + A_{st} f_y]$
    with $\emptyset = 0.65$.

- In articles 0 and 9.6, it will be shown that a column has its maximum strength when it is subjected to concentrically loaded with a compression force.
- Nominal strength of axially loaded column can be derived as follows:
  Nominal Strength of an Axially Loaded Column can be found recognizing the nonlinear response of both materials (steel and concrete) by:
  $P_n = 0.85f_c' A_c + A_{st} f_y$
  or
  $P_n = 0.85f_c'(A_g - A_{st}) + A_{st} f_y$
  i.e., by summing the strength contributions of the two components of the column.
- Strength Reduction Factor for Columns:
  The ACI strength reduction factors, ϕ, are lower for columns than for beams, see article 21.2.1. of the (ACI318M, 2014),
  - ***Reflecting their greater importance in a structure***,
  - A beam failure would normally affect only a local region whereas a column failure could result in the collapse of the entire structure,

  In addition, these factors reflect differences in the behavior of tied columns and spirally reinforced columns that shown in Figure 9.2-1 below.
  $\emptyset_{Tied\ Column} = 0.65$
  $\emptyset_{Spiral\ Column} = 0.75$

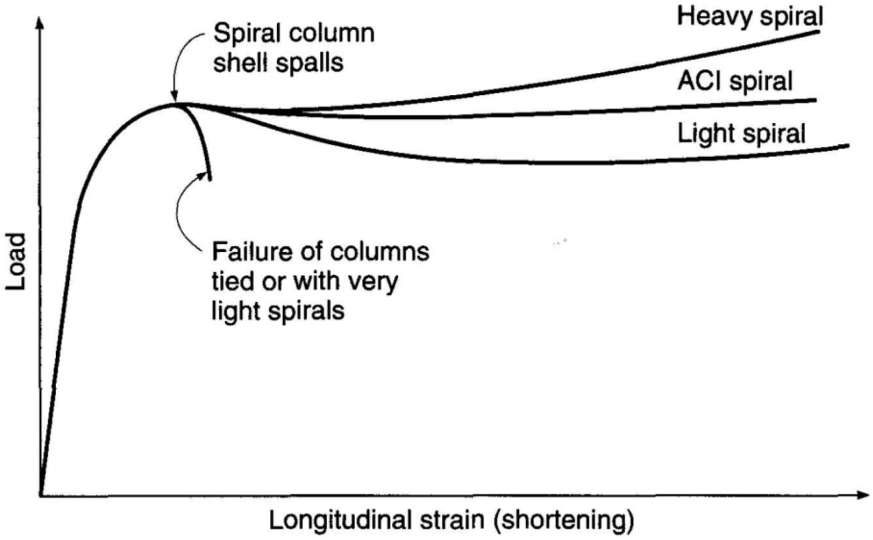


Figure 9.2-1: Behavior of spirally reinforced and tied columns.

- Provisions for Small Eccentricity:
  - A farther limitation on column strength is imposed by ACI Code 22.4 to allow for accidental eccentricities of loading not considered in the analysis.
  - This is done by imposing an upper limit on the axial load that is less than the calculated design strength:
    $Reduction\ Factor\ for\ Accidental\ Eccentricities_{Tied\ Column} = 0.8$
    $Reduction\ Factor\ for\ Accidental\ Eccentricities_{Spiral\ Column} = 0.85$
- Based on above discussion, design strength of axially loaded columns would be:
  - For spiral column the design strength is:
    $\emptyset P_{nMaximum} = 0.85\emptyset\left[0.85f_c'\left(A_g - A_{st}\right) + A_{st} f_y\right]$
    with $\emptyset = 0.75$.
  - For tied columns:
    $\emptyset P_{nMaximum} = 0.80\emptyset\left[0.85f_c'\left(A_g - A_{st}\right) + A_{st} f_y\right]$
    with $\emptyset = 0.65$.

9.2.3 Checking of Lateral Reinforcement (Ties), (ACI318M, 2014), Article 25.7.2

- All bars of tied columns shall be enclosed by lateral ties at least No 10 in size for longitudinal bars up to No. 32 and at least No. 13 in size for Nos. 36, 43, and 57 and bundled longitudinal bars.
- The spacing of the ties shall not exceed:
  $S_{Maximum} = \min[16d_{bar}, 48d_{ties}, Least\ dimension\ of\ column]$

- Arrangement of Rectilinear Ties
  - The ties shall be so arranged that every corner and alternate longitudinal bar shall have lateral support provided by the corner of a tie having an included angle of not more than 135, and no bar shall be farther than 150mm clear on either side from such a laterally supported bar.
  - Rectilinear ties arrangement according to ACI Code requirements can be summarized as follows, ***Figure 9.2-2*** below.

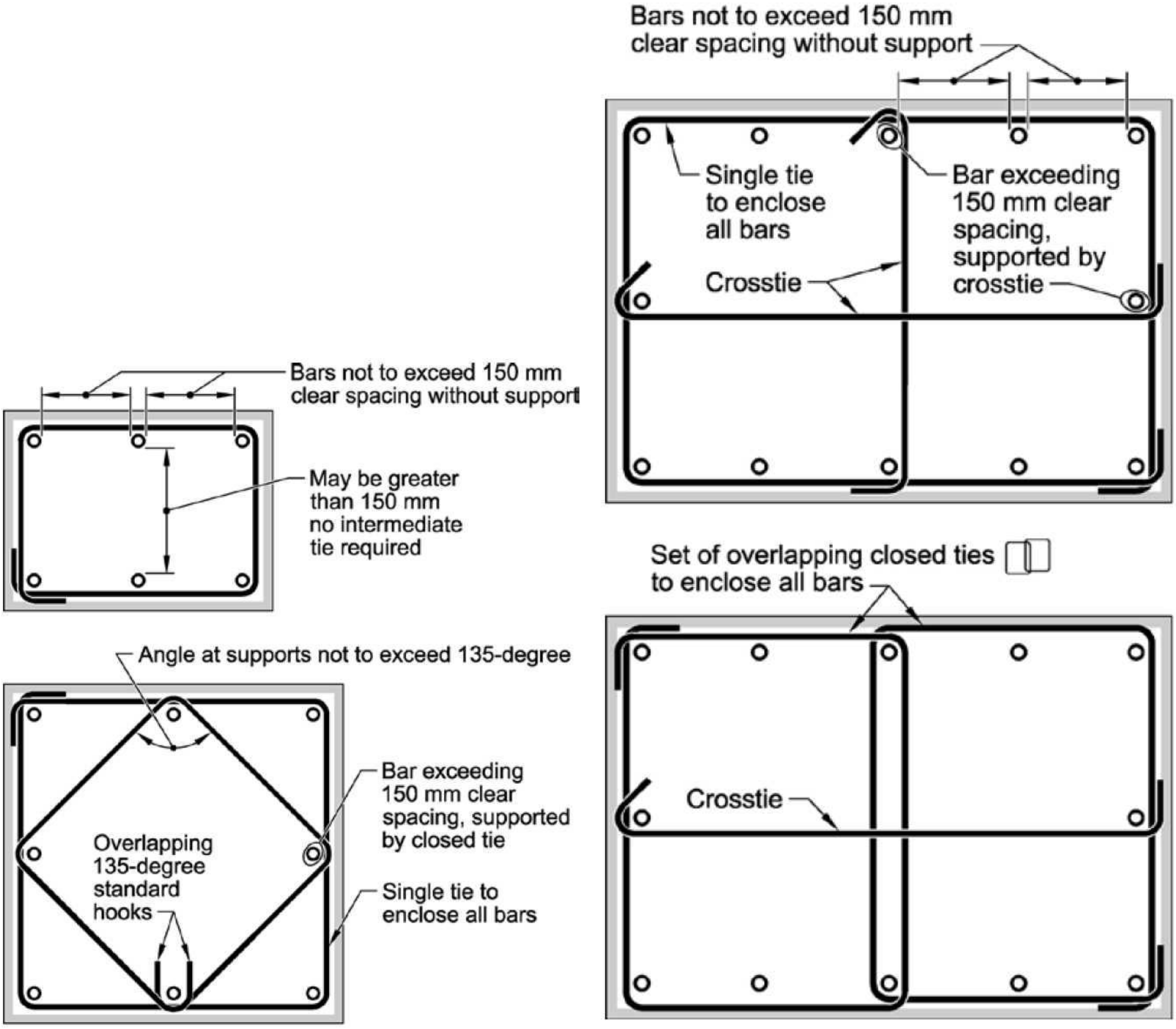


Figure 9.2-2: Tie arrangements for square and rectangular columns.

- Anchorage of Circular Ties
  - Circular ties shall be permitted where longitudinal bars are located around the perimeter of a circle.
  - Anchorage of individual circular ties shall be in accordance with:
    i. Ends shall overlap by at least 150 mm
    ii. Ends shall terminate with standard hooks,
    iii. Overlaps at ends of adjacent circular ties shall be staggered around the perimeter enclosing the longitudinal bars.
  - Above anchorage requirements have been summarized in ***Figure 9.2-3*** above.

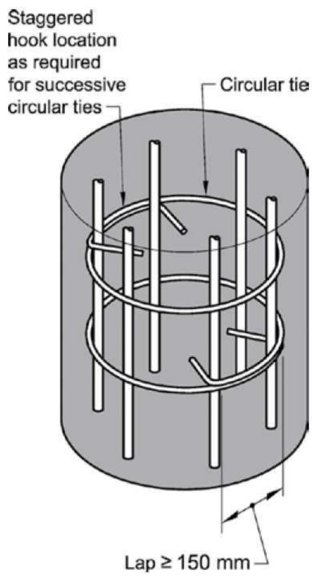


Figure 9.2-3: Circular tie anchorage.

9.2.4 Checking of Lateral Reinforcement (Spiral)

- For spirally reinforced columns, ACI Code requirements (25.7.3) for lateral reinforcement may be summarized as follows:
- Spirals shall consist of a continuous bar or wire not less than 10mm. in diameter.
- Compare the spiral ratio provided by the designer ($\rho_{s\ Provided}$) with the minimum recommended spiral ratio by the ACI Code ($\rho_{s\ Minimum}$):

$$\rho_{s\ Provided} = \frac{Volume\ of\ the\ spiral\ steel\ in\ one\ revolution}{volume\ of\ concrete\ core\ contained\ in\ one\ revoltion}$$

$$= \frac{4A_{sp}}{D_c\ S}$$

$$\rho_{sMinimum} = 0.45\left(\frac{A_g}{A_c} - 1\right)\frac{f_c'}{f_{sy}}$$

- The provided clear spacing ($S_{Provided\ CLear}$) between turns of the spiral must be:

$S_{Provided\ CLear} \leq 80^{mm}$ and $S_{Provided\ Clear} \geq 25^{mm}$

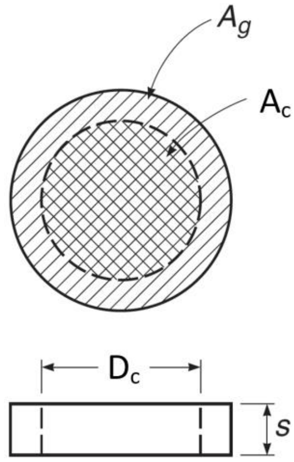


Figure 9.2-4: Notations adopted in analysis and design of spiral reinforcements.

Example 9.2-1

For a column that has the cross section area shown in ***Figure 9.2-5***, check the column adequacy with ACI Code requirements and compute the design axial load. Use $f_c' = 27.5\ \text{MPa}$, and $f_y = 420\text{MPa}$.

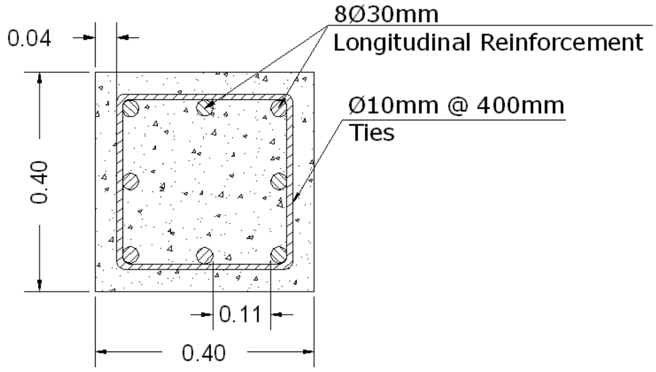


Figure 9.2-5: Proposed tied column for Example 9.2-1.

Solution

Longitudinal reinforcement

Check ρ_g within acceptable limits:

$A_g = 400^2 = 160\ \ 000\ mm^2$

$$A_{st} = \frac{\pi \times 30^2}{4} \times 8 = 5\ \ 652\ mm^2$$

$$0.01 < \rho_g = \frac{5\ 652}{160\ 000} = 3.53\% < 0.08$$

Check minimum number of longitudinal bars:

$8 > 4\quad \therefore Ok.$

Check minimum distance between longitudinal bars:

$S_{Minimum} = Maximum[1.5 \times 30^{mm}, 40^{mm}] = 45^{mm} < 110^{mm}\ \ \therefore Ok.$

Design Axial Strength, ϕP_n

Calculate the maximum design axial load strength $\emptyset P_{n(max)}$:

$\emptyset P_{nMaximum} = 0.80 \times 0.65[0.85 \times 27.5(160\ 000 - 5\ 652) + 5\ 652 \times 420] =$

$\emptyset P_{nMaximum} = 3\ \ 110\ kN$

Lateral reinforcement (Ties)

Checking of Lateral Reinforcement (Ties):

Ties diameter:

$\because\ \emptyset = 30^{\text{mm}} < 32^{\text{mm}}, \therefore$ we can use $\emptyset = 10^{\text{mm}}$ for ties

Ties spacing:

$S_{\text{Maximum}} = \min[16 \times 30^{\text{mm}}, 48 \times 10^{\text{mm}}, 400^{\text{mm}}] = 400^{\text{mm}} = S_{\text{Provided}}\ \ \therefore \text{Ok}.$

Ties arrangement:

$\because S_{\text{Spacing between longitudinal bars}} < 150^{\text{mm}}$

Then, alternate longitudinal bars will be supported by corner bars.

Example 9.2-2

Check the column shown in ***Figure 9.2-6*** with general requirements of the ACI Code, then determine whether this column is adequate to carry a factored load of P_u= 2250 kN or not.

In your analysis:

- Assume small eccentricity.
- Use $f_c' = 27.5\ \text{MPa}$, and $f_y = 420\text{MPa}$.

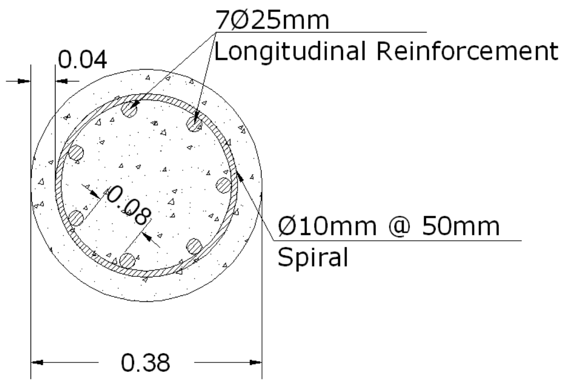


Figure 9.2-6: Spiral column of Example 9.2-2.

Solution

Longitudinal reinforcement

Check ρ_g within acceptable limits:

$$A_g = \frac{\pi \times 380^2}{4} = 113\ \ 354\ mm^2$$

$$A_{st} = \frac{\pi \times 25^2}{4} \times 7 = 3\ 434\ mm^2 \Rightarrow \rho_g = \frac{3\ \ 434}{113\ \ 354} = 3.0\% \Rightarrow 0.01 < \rho_g < 0.08\ \therefore Ok.$$

Check minimum number of longitudinal bars

$7 > 6\quad \therefore Ok.$

Check minimum distance between longitudinal bars

$S_{Minimum} = Maximum[1.5 \times 25^{mm}, 40^{mm}] = 40.0^{mm} < 80^{mm}\ \ \therefore Ok.$

Design Axial Strength

Calculate the maximum design axial load strength $\emptyset P_{n(max)}$:

$\emptyset P_{nMaximum} = 0.85 \times 0.75[0.85 \times 27.5(113\ 354 - 3\ 434) + 3\ 434 \times 420] = 2\ \ 557\ kN > P_u\ \ \therefore Ok.$

<u>Lateral reinforcement (Ties)</u>

Check the lateral reinforcement (Spiral):

Check Spiral Diameter:

$\emptyset_{Spiral} = 10mm\ Ok.$

Check Spiral Steel Ratio:

$$A_{sp} = \frac{\pi \times 10^2}{4} = 78.5^{mm^2} \Longrightarrow \rho_{sProvided} = \frac{4 \times 78.5^{mm^2}}{(380 - 2 \times 40)^{mm} \times 50^{mm}} = 0.0209$$

$$\rho_{sMinimum} = 0.45 \times \left(\frac{113\ 354}{\frac{\pi \times 300^2}{4}} - 1 \right) \times \frac{27.5}{420} = 0.0178 < 0.0209\ \therefore Ok.$$

Check the Clear Spacing:

$25^{mm} < [S_{Clear\ Provided} = 50^{mm} - 10^{mm} = 40^{mm}] < 80^{mm}\ \therefore Ok.$

9.3 ACI DESIGN PROCEDURE FOR A SHORT COLUMN UNDER AN AXIAL LOAD (SMALL ECCENTRICITY)

ACI Code procedure for design of a short column under an axial compression force can be summarized as follows:

- Determine the applied factored axial load P_u:
- Establish a desired ρ_g.
- Determine the required gross column area A_g:

  For tied column:

  $$A_{gRequired} = \frac{P_U}{0.80 \times \emptyset[0.85f_c'(1-\rho_g) + f_y\rho_g]}$$

  For spiral column:

  $$A_{gRequired} = \frac{P_U}{0.85 \times \emptyset[0.85f_c'(1-\rho_g) + f_y\rho_g]}$$

- Select the column dimensions. Round the answer to the nearest 25^{mm}.
- Find the load that carried by the concrete:

  For tied column:

  $$\emptyset P_{n\text{ Carried by Concrete}} = 0.80 \times \emptyset[0.85f_c'A_g(1-\rho_g)]$$

  For spiral column:

  $$\emptyset P_{n\text{ Carried by Concrete}} = 0.85 \times \emptyset[0.85f_c'A_g(1-\rho_g)]$$

- Determine the load required to be carried by the longitudinal steel:

  $$\emptyset P_{n\text{Carried by Steel}} = P_u - \emptyset P_{n\text{ Carried by Concrete}}$$

- Determine the required steel area of longitudinal bars:

  For tied column:

  $$\emptyset P_{n\text{Carried by Steel}} = 0.80\emptyset[A_{stRequired}f_y]$$

  For spiral column:

  $$\emptyset P_{n\text{Carried by Steel}} = 0.85\emptyset[A_{stRequired}f_y]$$

- Determine the required number of bars:

  $$No.of\ Bar_{Required} = \frac{A_{stRequired}}{A_{Bar}}$$

  Round required number to the nearest integer and check with requirement of the ACI for the minimum number of longitudinal bars:

  $$\text{No. of Bars}_{\text{Provided}} \geq 4_{\text{for tied columns}}$$

  $$\text{No. of Bars}_{\text{Provided}} \geq 6_{\text{for spiral columns}}$$

- Check the spacing between the longitudinal bars:

  $$S_{Provided} \geq Maximum\ of\ [1.5d_{Bar}, 40^{mm}]$$

- Design the lateral reinforcement:

  Ties:

  Select ties diameter:

  If $\varphi_{Longitudinal} \leq 32^{mm}$ then:

  $$\varphi_{Ties} = 10^{mm}$$

  Else

  $$\varphi_{Ties} = 13^{mm}$$

  Select ties spacing:

  $$S_{Required} \leq Minimum[16\varphi_{Bar}, 48\varphi_{ties}, Least\ Column\ Dimensions]$$

  Arrange the ties according to requirements of the ACI for maximum spacing between longitudinal bars (use the standard arrangements of Figure 9.2-2 above).

  Spiral:

  $$\varphi_{Spiral} \geq 10^{mm}$$

  Compute $\rho_{sMinimum}$

  $$\rho_{sMinimum} = 0.45\left(\frac{A_g}{A_c} - 1\right)\frac{f_c'}{f_{sy}}$$

  Let $\rho_s = \rho_{sMinimum}$ to compute the required $S_{Required}$:

$$S_{Required} = \frac{4A_{sp}}{D_c\ \rho_{sMinimum}}$$

The clear spacing $S_{Required\ Clear}$ between turns of the spiral must be:

$25 \leq S_{Clear} \leq 80^{mm}$

Example 9.3-1

Design a tied column to carry a factored axial load of P_u=3 184 kN.

- Assume that there is no identified applied moment.
- Assume that the column is short.
- Assume $\rho_{Preferable} = 0.03$.
- Assume $f_c' = 27.5\text{MPa}$, $f_y = 420\text{MPa}$.
- Try square section.
- Try $\varphi_{Longitudinal\ Bar} = 29^{mm}$, $A_{Bar} = 645\text{mm}^2$
- Try $\varphi_{Lateral\ Reiforcement} = 10^{mm}$.

Solution

Compute $A_{gRequired}$:

$$A_{gRequired} = \frac{3184 \times 10^3 \text{N}}{0.80 \times 0.65[0.85 \times 27.5(1 - 0.03) + 420 \times 0.03]} = 173\ 587\text{mm}^2$$

Try square section:

$B = \sqrt{173\ 587\text{mm}^2} = 416.6^{mm}$

Try $B = 425^{mm}, \therefore A_g = 180\ \ 625\text{mm}^2$.

Compute $\emptyset P_{n\ Carried\ by\ Concrete}$:

$\emptyset P_{n\ Carried\ by\ Concrete} = 0.8 \times 0.65[0.85 \times 27.5 \times 180\ 625(1 - 0.03)] = 2\ \ 130\text{kN}$

Compute $\emptyset P_{n\ Carried\ by\ Steel}$:

$\emptyset P_{n\ Carried\ by\ Steel} = 3\ 184 - 2\ \ 130 = 1\ 054\ \text{kN}$

Compute $A_{stReqired}$:

$1\ 054 \times 10^3 = 0.8 \times 0.65 \times [420 \times A_{stReqired}] \Longrightarrow A_{stReqired} = 4\ 826\ \text{mm}^2$

Compute Number of longitudinal bars:

Try $\varphi_{Longitudinal} = 29^{mm}$:

$$\text{No.} = \frac{4\ 826}{645} = 7.48$$

Try $8\varphi 29^{mm}$:

$\because 8 \geq 4\ \therefore \text{Ok}.$

Check spacing between longitudinal bars:

$$S_{Provided} = [425^{mm} - 2 \times 40^{mm} - 2 \times 10^{mm} - 3 \times 29^{mm}]\frac{1}{2} = 119^{mm}$$

$$S_{Minimum} = \text{Maximum}[1.5d_{bar}, 40^{mm}] = 43.5^{mm} < 119^{mm}$$

$\therefore$ Ok.

Design of Ties:

Ties diameter:

$\because \varphi_{Longitudinal\ Bars} < 32^{mm},$

$\therefore Use\ \varphi_{Ties} = 10^{mm}$

Tie spacing:

$$S_{Required} = Minimum[16 \times 29^{mm}, 48 \times 10^{mm}, 425^{mm}] = 425^{mm}$$

Try $\varphi 10^{mm}@425^{mm}$

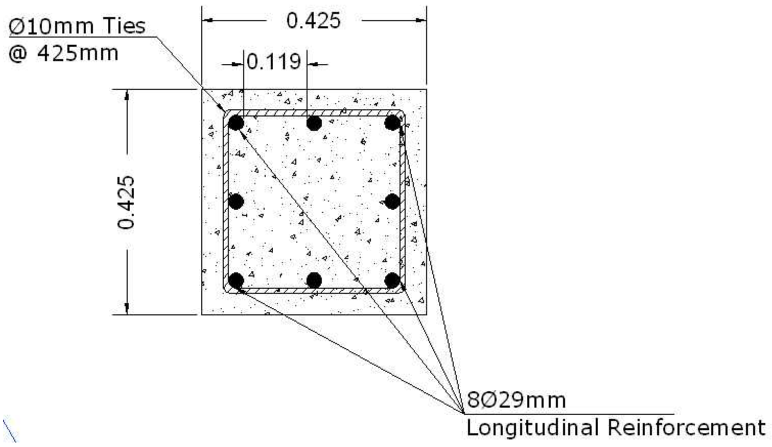


Figure 9.3-1: Final design section for the column of Example 9.3-1.

Ties arrangement:

As we intend to use eight rebars and spacing between rebars is less than 150mm, then the ties reinforcement is presented in ***Figure 9.3-1***.

Example 9.3-2

Redesign the column of Example 9.3-1 as a circular spirally reinforced column with P_u = 3 429 kN.

Solution

Compute $A_{gRequired}$:

$$A_{gRequired} = \frac{3429 \times 10^3 N}{0.85 \times 0.75[0.85 \times 27.5(1-0.03) + 420 \times 0.03]} = 152\,488\ mm^2$$

$$\frac{\pi D^2}{4} = 152\,488\ mm^2, \therefore D_{Required} = 441^{mm}, Try\ D_{Provided} = 450^{mm}$$

Compute $\emptyset P_{n\ \text{Carried by Concrete}}$:

$$\emptyset P_{n\ \text{Carried by Concrete}} = 0.85 \times 0.75 \times \left[0.85 \times 27.5 \times \frac{\pi \times 450^2}{4}(1-0.03)\right] = 2\,298 kN$$

Compute $\emptyset P_{n\ \text{Carried by Steel}}$

$$\emptyset P_{n\ \text{Carried by Steel}} = 3\,429 kN - 2\,298 kN = 1\,131\ kN$$

Compute $A_{stRequired}$

$$0.85 \times 0.75 \times [A_{stRequired} \times 420] = 1\,131\,000\ N$$

$$A_{stRequired} = 4\,224 mm^2$$

Compute number of longitudinal bars:

Try $\varphi_{Longitudinal} = 29^{mm}$, $A_{Bar} = 645 mm^2$.

$$No. = \frac{4\,224}{645} = 6.55$$

Try $7\varphi 29^{mm}$.

$\because 7 \geq 6\ \therefore Ok.$

Check spacing between longitudinal bars:

$$D_{Center\ of\ Longitudinal\ Bars} = 450^{mm} - 2 \times 40^{mm} - 2 \times 10^{mm} - 29^{mm} = 321^{mm}$$

$$S_{Provided} = \frac{[\pi \times 321^{mm} - 7 \times 29^{mm}]}{7} = 115^{mm}$$

$$S_{Minimum} = Maximum\ [1.5 d_{Bar}, 40^{mm}] = 43.5^{mm} < 115^{mm}\ \therefore Ok.$$

Spiral Design:

Spiral diameter:

$\because \varphi_{Spiral} = 10^{mm}\ \therefore Ok.$

Compute $\rho_{sMinimum}$:

$$D_c = 450^{mm} - 2 \times 40^{mm} = 370^{mm}$$

$$A_c = \frac{\pi \times 370^2}{4} = 107\,467\ mm^2$$

$$A_g = \frac{\pi \times 450^2}{4} = 158\,962\ mm^2$$

$$\rho_{sMinimum} = 0.45\left(\frac{158\,962}{107\,467} - 1\right) \times \frac{27.5}{420} = 14.2 \times 10^{-3}$$

$$A_{sp} = \frac{\pi \times 10^2}{4} = 78.5^{mm^2}$$

$$\therefore S_{Required} = \frac{4 \times 78.5^{mm^2}}{370^{mm} \times 14.2 \times 10^{-3}} = 59.8^{mm}$$

Try $\varphi 10^{mm}$ @ 60^{mm}

$\because S_{Clear} = 50^{mm} < 80^{mm}\ \therefore Ok.$

$\because S_{Clear} = 50^{mm} > 25^{mm}\ \therefore Ok.$

Use $\varphi 10^{mm}$ @ 60^{mm}

The final section of the column is shown in Figure 9.3-2.

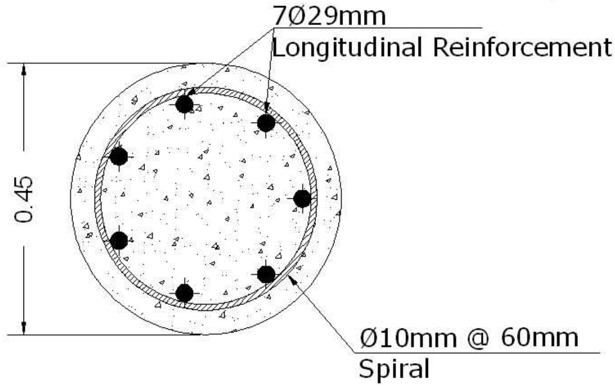


Figure 9.3-2: Final design section for the column of Example 9.3-2.

9.4 HOMEWORK: ANALYSIS AND DESIGN OF AXIALLY LOADED COLUMNS

Problem 9.4-1

Check the adequacy of the column that shown below according to the requirement of the ACI Code and compute its design strength.

Assume:

- Short column
- $f_c' = 27.5\ \mathrm{MPa}$
- $f_y = 420\ \mathrm{MPa}$
- $A_{Bar} = 637.5\ mm^2$

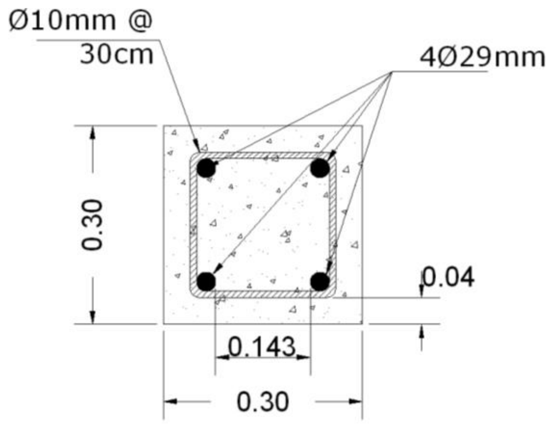


Figure 9.4-1: Proposed tied column for Problem 9.4-1.

Answers

Longitudinal reinforcement:

Check ρ_g within acceptable limits:

$A_g = 90\ 000\ mm^2, A_{st} = 2\ 550\ mm^2$

$0.01 < \rho_g = 2.83\% < 0.08$

Check minimum number of longitudinal bars:

$No. of\ Bars = 4\quad \therefore Ok.$

Check minimum distance between longitudinal bars:

$S_{Minimum} = 43^{mm} < 143^{mm}\ \therefore Ok.$

Calculate the maximum design axial load strength $\emptyset P_{n(max)}$:

$\emptyset P_{nMaximum} = 1\ 620\ kN$

Lateral reinforcement (Ties):

Ties diameter:

$\because\ \emptyset = 29^{mm} < 32^{mm}, \therefore\ we\ can\ use\ \emptyset = 10^{mm}\ for\ ties$

Ties spacing:

$S_{Maximum} = 300^{mm} = S_{Provided}\ \therefore Ok.$

Ties arrangement:

For a column with four rebars only, no interior ties are required.

Problem 9.4-2

Design a square tied column to support an axial load of $P_u = 4\ 078\ kN$. Design the necessary ties also.

Assume:

- Short column
- $f_c' = 34.5\ \mathrm{MPa}$
- $f_y = 420\ \mathrm{MPa}$
- $\rho_g = 0.05$
- $\varphi_{\text{Longitudinal Bars}} = 32^{\text{mm}}$
- $\varphi_{\text{Ties}} = 10^{\text{mm}}$

Answers

Compute $A_{gRequired}$:

$A_{gRequired} = 160\ 510 mm^2$

Try square section:

$B \approx 400^{mm}$

Try $B = 400^{mm}, \therefore A_g = 160\ \ 000 mm^2$.

Compute $\emptyset P_{n\ Carried\ by\ Concrete}$:

$\emptyset P_{n\ Carried\ by\ Concrete} = 2\ \ 318 kN$

Compute $\emptyset P_{n\ Carried\ by\ Steel}$:

$\emptyset P_{n\ Carried\ by\ Steel} = 1\ 760 kN$

Compute $A_{stReqired}$:

$A_{stReqired} = 8\ 059\ mm^2$

Compute Number of longitudinal bars:

$No. \approx 10$

$\because 10 > 4\ \therefore Ok.$

Check spacing between longitudinal bars:

$S_{Provided} = 57.3^{mm}, S_{Minimum} = Maximum[1.5d_{bar}, 40^{mm}] = 48^{mm} < 57.3^{mm} \therefore Ok.$
Design of Ties:
Ties diameter:
$\because \varphi_{Longitudinal\ Bars} = 32^{mm}, \therefore Use\ \varphi_{Ties} = 10^{mm}$
Tie spacing
$S_{Required} = 400^{mm}$
Try $\varphi 10^{mm}@400^{mm}$
Ties arrangement:
Sketch for details of longitudinal and lateral reinforcements are shown in ***Figure 9.4-2***.

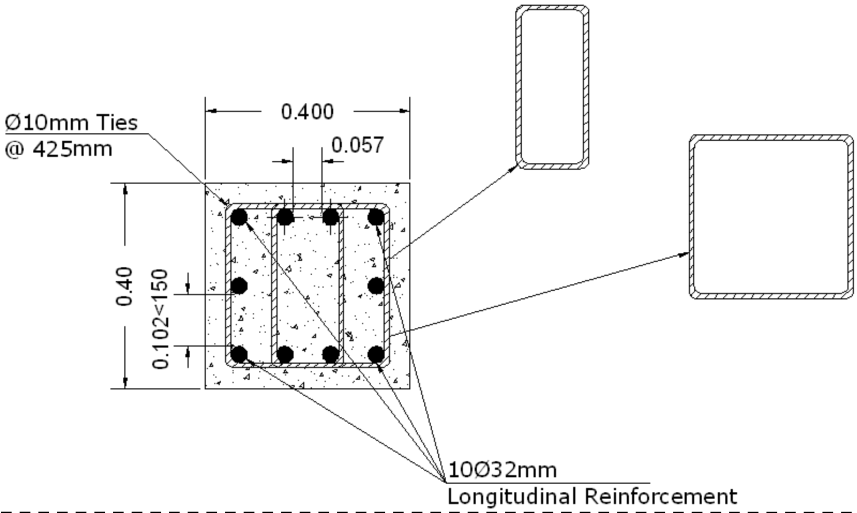


Figure 9.4-2: Final design section Problem 9.4-2.

Problem 9.4-3

Repeat Problem 9.4-2, using a rectangular section that has width $b = 350^{mm}$.

Answers

Compute $A_{gRequired}$:
$A_{gRequired} = 160\ 510mm^2$
Try rectangular section with $b = 350^{mm}$, therefore $h = 459\ mm$
Try $b = 350^{mm}, h = 460^{mm} \therefore A_g = 161\ 000\ mm^2$.
Compute $\emptyset P_{n\ Carried\ by\ Concrete}$:
$\emptyset P_{n\ Carried\ by\ Concrete} = 2\ \ 332kN$
Compute $\emptyset P_{n\ Carried\ by\ Steel}$:
$\emptyset P_{n\ Carried\ by\ Steel} = 1\ 746kN$
Compute $A_{stReqired}$:
$A_{stReqired} = 7\ 994\ mm^2$
Compute Number of longitudinal bars:
$No. = \frac{7\ 994}{804} = 9.94$, Try $10\varphi 32^{mm}$. $\because 10 > 4 \therefore Ok.$
Check spacing between longitudinal bars:
$S_{Provided} = 77.3^{mm}, S_{Minimum} = Maximum[1.5d_{bar}, 40^{mm}] = 48^{mm} < 74^{mm} \therefore Ok.$
Design of Ties:
Ties diameter:
$\because \varphi_{Longitudinal\ Bars} = 32^{mm}, \therefore Use\ \varphi_{Ties} = 10^{mm}$
Tie spacing
$S_{Required} = 350^{mm}$
Try $\varphi 10^{mm}@350^{mm}$
Ties arrangement:
Sketch for details of longitudinal and lateral reinforcements are shown in Fig. below.
$S_{Provided} = 77^{mm} < 150^{mm}$
No additional interior ties are required.

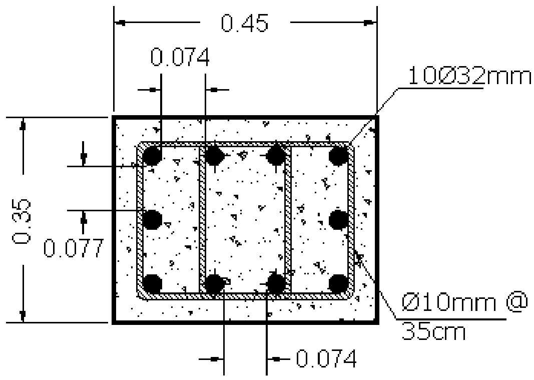


Figure 9.4-3: Final design section for Problem 9.4-3.

Problem 9.4-4

Design the spiral column that supports four girders of bridge shown in Figure 9.4-4 below. In your design, assume that.

- Each girder has a dead load reaction of 150 kN and has a live load reaction of 100 kN.
- f_c' = 28 MPa and f_y = 420 MPa.
- Rebar No. 25 for longitudinal reinforcement and No. 10 for spiral reinforcement.
- Column has a height of 4m, and it is assumed short.
- Column and cap selfweight should be included in your solution.

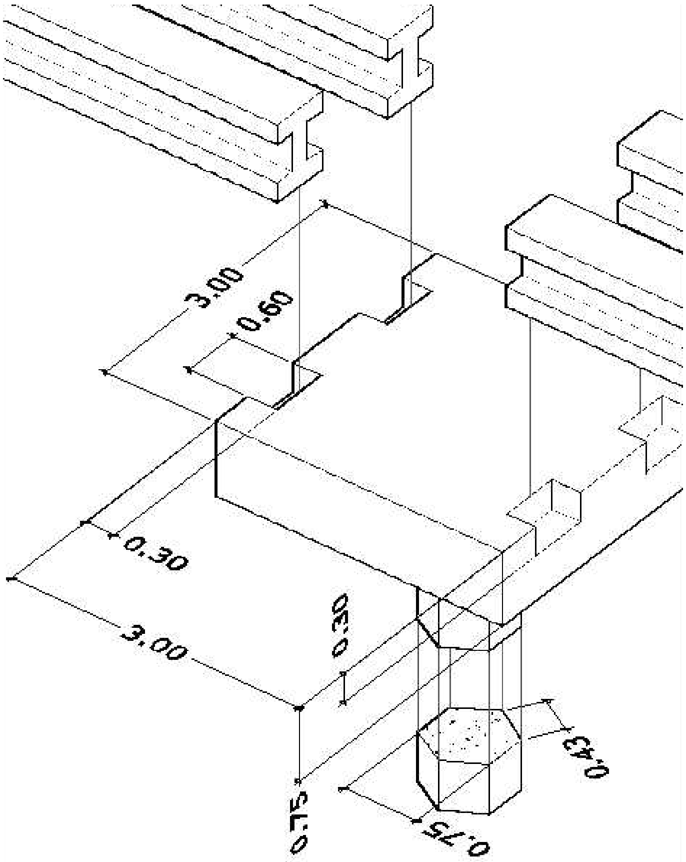


Figure 9.4-4: Four girders that supported on the column of Problem 9.4-4.

Hint for Solution: According to ACI Code, ***version 2011***, ***Article (10.8.3)*** "As an alternative to using the full gross area for design of a compression member with a square, octagonal, or other shaped cross section, it shall be permitted to use a circular section with a diameter equal to the least lateral dimension of the actual shape. Gross area considered, required percentage of reinforcement, and design strength shall be based on that circular section". Then this column can be transformed from hexagonal shape to the following circular shape.

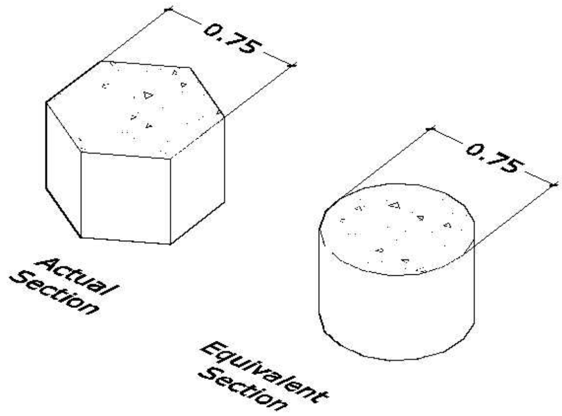


Figure 9.4-5: Transformation of an octagonal column into the equivalent circular section.

9.5 ANALYSIS OF A COLUMN WITH COMPRESSION LOAD PLUS UNIAXIAL MOMENT

9.5.1 Introduction

- Members that are axially loaded, i.e., concentrically compressed, occur rarely, if ever in buildings and other structures. Components such as columns chiefly carry loads in compression but simultaneous bending is usually present.
- Bending moments are caused by:
  - Continuity, i.e., by the fact that building columns are parts of monolithic frames in which the support moments of the girders are partly resisted by the abutting columns.

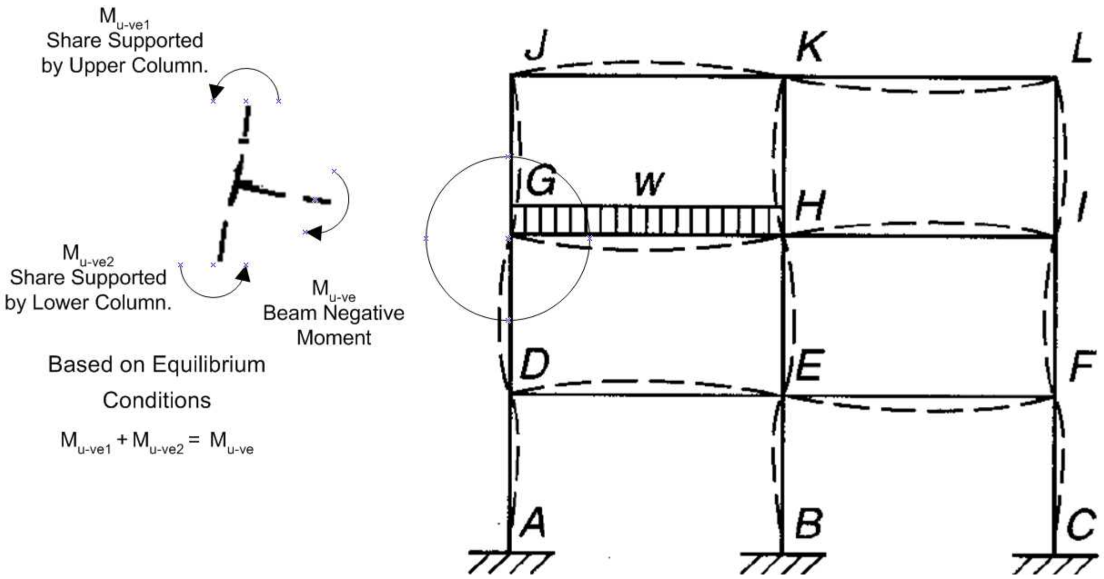


Figure 9.5-1: Moments in columns due to frame continuity.

  - Transverse loads such as wind forces.

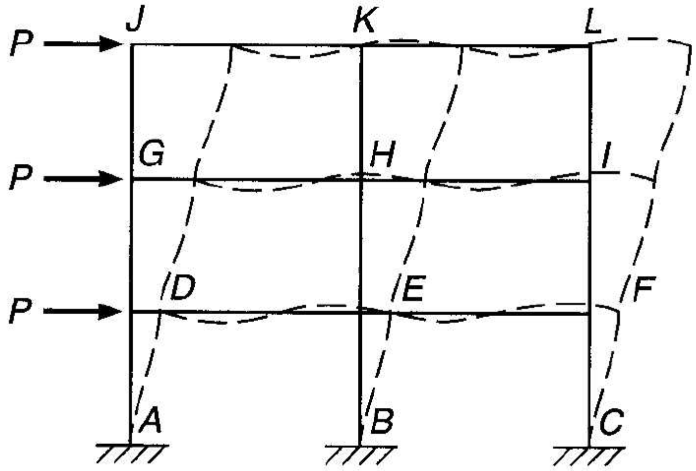


Figure 9.5-2: Moments in columns due to lateral forces.

  - Loads carried eccentrically on column brackets when the column axis does not coincide with the pressure line.
  - Imperfections of construction.

    For these reasons, members that should be designed for simultaneous compression and bending are very frequent in almost all types of concrete structures.
- When a member is subjected to combined axial compression P and moment M, it is usually convenient to replace the axial load and moment with an equal load P applied at eccentricity $e = M/P$. The two loadings are statically equivalent.

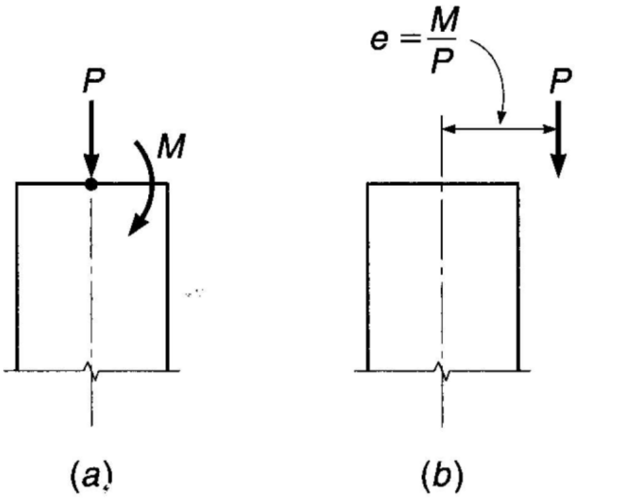


Figure 9.5-3: Equivalent eccentricity of column load.

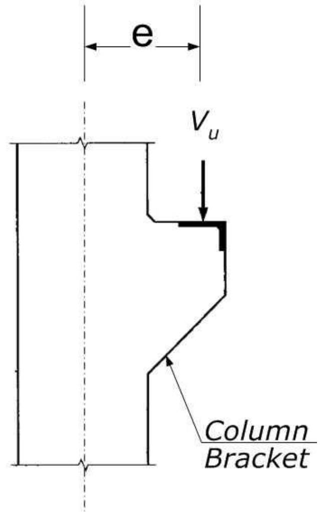
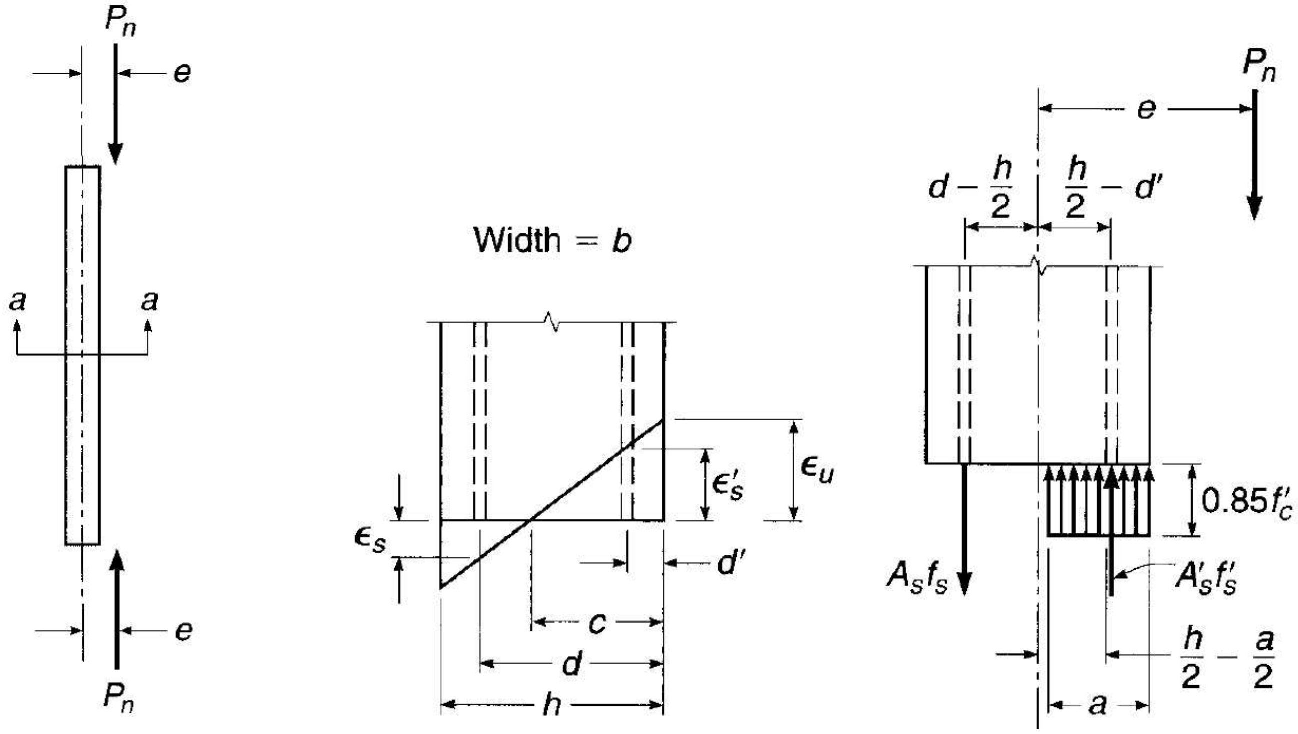


Figure 9.5-4: Moments in columns in precast frames.

- Two approaches for analysis of a column with axial force and uniaxial moment will be discussed in ***Articles 9.5.2*** and ***9.5.3*** below.

9.5.2 Column Analysis by Direct Application of Basic Principles

- Figure 9.5-5 "a" shows a member loaded parallel to its axis by a compressive force P, at an eccentricity e measured from the centerline.



(a) (b) (c)

Figure 9.5-5: Column subject to eccentric compression: (a) loaded column; (b) strain distribution at section a-a; (c) stresses and forces at nominal strength.

- Above column can be analyzed based on direct application of basic principles of applied mechanics and as follows:

9.5.2.1 Compatibility

- With plane sections assumed to remain plane, concrete strains vary linearly with distance from the neutral axis which is located a distance "c" from the more heavily loaded side of the member.
- With full compatibility of deformations, the steel strains at any location are the same as the strains in the adjacent concrete; thus if the ultimate concrete strain is ϵ_u, the strain in the bars nearest the load is ϵ'_s while that in the tension bars at the far side is ϵ_s.

- Compression steel having area A_s' and tension steel with area A_s, are located at distances d and d', respectively, from the compression face (See Figure 9.5-5 "b" above).

9.5.2.2 Constitutive Relationships

The corresponding stresses and forces are shown in Figure 9.5-5 "c", just as for simple bending, the actual concrete compressive stress distribution is replaced by an equivalent rectangular distribution having depth $a = \beta_1 c$.

9.5.2.3 Equilibrium Equations

- Equilibrium between external and internal axial forces shown in Figure 9.5-5 "c"; requires that:

  $$\sum \boldsymbol{F_y} = \mathbf{0.0}$$

  $$P_n = 0.85 f_c' ab + A_s' f_s' - A_s f_s$$

- Also, the moment about the centerline of the section of the internal stresses and forces must be equal and opposite to the moment of the external force. P_n, so that:

  $$\sum M = 0.0$$

  $$M_n = P_n e = 0.85 f_c' ab \left(\frac{h}{2} - \frac{a}{2}\right) + A_s' f_s' \left(\frac{h}{2} - d'\right) + A_s f_s (d - \frac{h}{2})$$

- These are the two basic equilibrium relations for rectangular eccentrically compressed members.
- For a given eccentricity determined from the frame analysis (i.e., $e = \frac{M}{P}$) it is possible to solve above equations for the load and moment M_n that would result in failure as follows:
  - In both equations, f_s', f_s, and a can be expressed in terms of a single unknown c, the distance to the neutral axis. This is easily done based on the geometry of the strain diagram, with ϵ_u taken equal to 0.003 as usual, and using the stress-strain curve of the reinforcement.
  - The result is that the two equations contain only two unknowns, P_n and c, and can be solved for those values simultaneously. However, to do so in practice would be complicated algebraically particularly because of the need to incorporate the limit f_y on both f_s', and f_s.

9.5.3 Concept of Interaction Diagram

9.5.3.1 Basic Concepts

- A better approach, providing the basis for practical design, is to construct a *Strength Interaction Diagram* defining the failure load and failure moment for a given column for the full range of eccentricities from zero *to* infinity (see Fig. below):

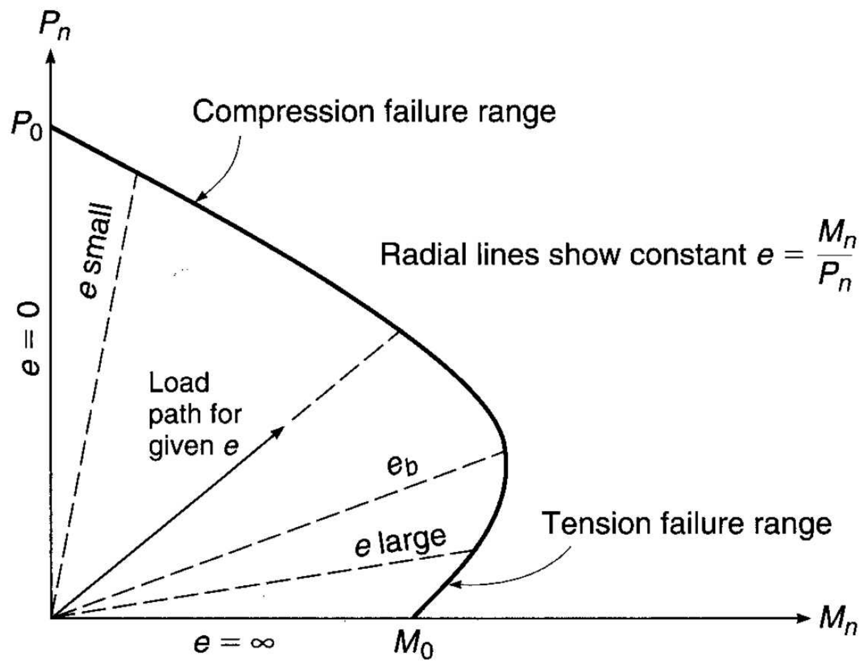


Figure 9.5-6: Interaction diagram for nominal column strength in combined bending and axial load.

- On such a diagram, any radial line represents a particular eccentricity $e = \frac{M}{P}$. For that eccentricity, gradually increasing the load will define a load path as shown, and when that load path reaches the limit curve, failure will result.
- The vertical axis corresponds to $e = 0$, and P_0 is the capacity of the column if concentrically loaded, as given by equations of articles 9.2 and 9.3.
- The horizontal axis corresponds to an infinite value of e, i.e., pure bending at moment capacity M_0.
- Failure Regions on Interaction Diagram:
  - Small eccentricities will produce failure governed by concrete compression.
  - Large eccentricities give a failure triggered by yielding of the tension steel.

9.5.3.2 Construction of A nominal Interaction Diagram

For a given column, the interaction diagram is most easily constructed by following procedure:

- Selecting successive choices of neutral axis distance "c", from infinity (axial load with eccentricity 0) to a very small value found by trial to give $P_n = 0$ pure bending).
- For each selected value of "c", the steel strains and stresses and the concrete force are easily calculated as follows:
  - For the tension steel:

$$\epsilon_s = \epsilon_u \frac{d-c}{c}$$

$$f_s = E_s \epsilon_u \frac{d-c}{c} \le f_y$$

  - While for the compression steel:

$$\epsilon_s' = \epsilon_u \frac{c-d'}{c}$$

$$f_s' = E_s \epsilon_u \frac{c-d'}{c} \le f_y$$

  - The concrete stress block has depth:

$$a = \beta_1 c \ \le h$$

- Substitute the values of f_s, f_s', and a into the following relations to compute the values of P_n and M_n that corresponding to assume "c" value.

$$\sum F_y = 0.0 \Rightarrow P_n = 0.85 f_c' ab + A_s' f_s' - A_s f_s$$

$$\sum M = 0.0 \Rightarrow M_n = P_n e = 0.85 f_c' ab \left(\frac{h}{2} - \frac{a}{2}\right) + A_s' f_s' \left(\frac{h}{2} - d'\right) + A_s f_s (d - \frac{h}{2})$$

- These steps (starting from assuming of "c" to obtain the corresponding P_n and M_n) represent a point on the interaction diagram. Then these will be repeated until enough number of points on interaction is obtained to draw the required diagram.
- Construct interaction diagram through connecting between points drawn.

9.5.3.3 Design Interaction Diagram

- As was discussed in Chapter 3, the strength reduction factor "∅" is a function of steel strain and as shown in ***Figure 9.5-7*** below.

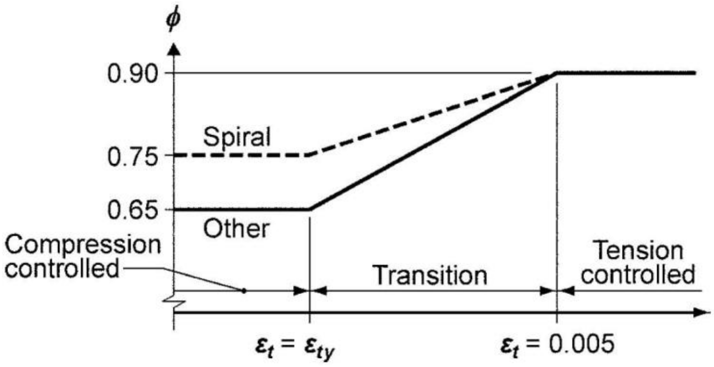


Figure 9.5-7: Variation of ϕ with net tensile strain in extreme tension reinforcement, ε_t.

- Each point on the interaction diagram has its strain, see ***Figure 9.5-8*** below, and in turn has own factor of safety, see ***Figure 9.5-9*** below.

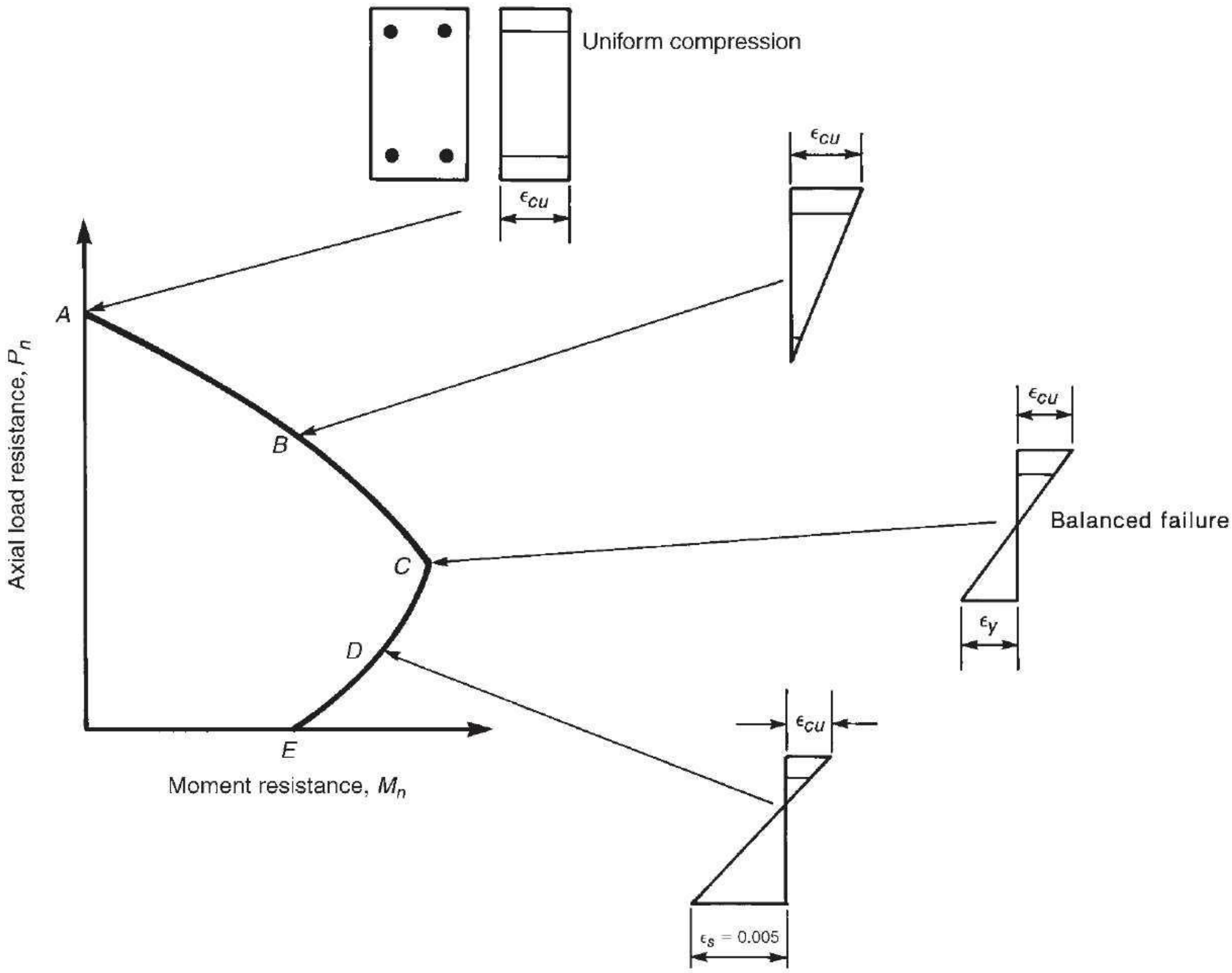


Figure 9.5-8: Strain distributions corresponding to points on the interaction diagram.

- Column design strengths ($\emptyset P_n$, $\emptyset M_n$) can be obtained by multiplied the nominal strengths (P_n, M_n) by the corresponding factor of safety "∅" to obtained the *Design Interaction Diagram* and as shown ***Figure 9.5-9*** below.

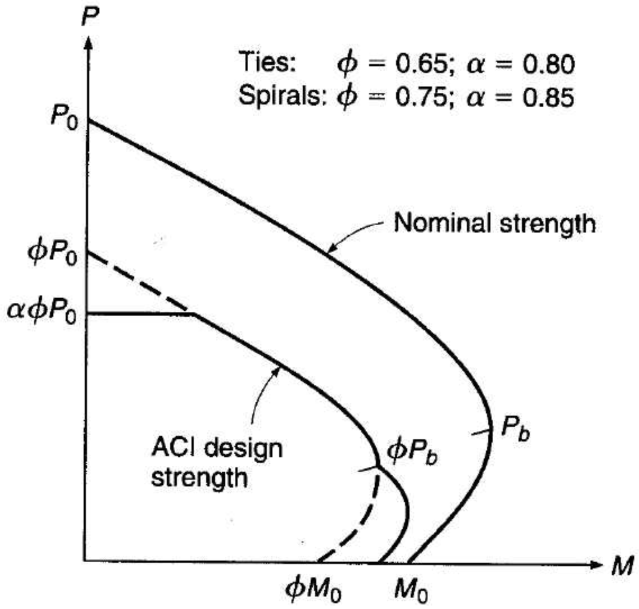


Figure 9.5-9: ACI safety provisions superimposed on column interaction diagram.

9.5.3.4 Notes on Design Interaction Diagram

- For high eccentricities, as the eccentricity increases to infinity (pure: bending), the ACI Code recognizes that the member behaves progressively more like a flexural member and less like a column. This is acknowledged in ACI Code by providing a linear transition in ∅ from values of 0.65 (for tied column) and 0.75 (for spiral column) to 0.90 (for beam) as the net tensile strain in the extreme tensile steel ϵ_s increases from 0.002 for Grade 60 reinforcement to 0.005.
- At the other extreme, for columns with very small or zero calculated eccentricities, the. ACI Code recognizes that accidental construction misalignments and other unforeseen factors may produce actual eccentricities in excess of these small design values. Therefore, regardless of the magnitude of the calculated eccentricity, ACI Code limits the maximum design strength to $0.80\,\emptyset P_{nmax}$, for tied columns and to $0.85\emptyset\,P_{nmax}$ for spirally reinforced.

9.5.3.5 A Set of Design Interaction Curves

Our textbook (Design of Concrete Structures, 15th Edition, by David Darwin, Charles W. Dolan, and A. H. Nilson) includes the group of useful ***Design Interaction Diagrams***.

Example 9.5-1

Check the adequacy of column shown below for general ACI requirement then use an appropriate interaction diagram to find its design axial load and design bending moment.

Use $f_c' = 28\ MPa$ and $f_y = 420\ MPa$. $A_{Bar\ of\ 29mm} = 645mm$.

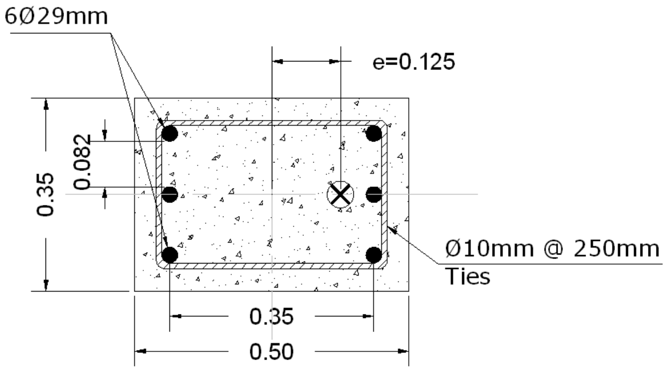


Figure 9.5-10: Proposed column section for Example 9.5-1.

Solution

- The procedure for analysis of an eccentrically loaded column is exactly similar to the procedure of a concentrically loaded column in all steps except in the computing of design axial force and bending moment $(\emptyset P_n, \emptyset M_n)$.
- Longitudinal reinforcement:

  Check ρ_g within acceptable limits:

  $A_g = 500 \times 350 = 175\ 000\ mm^2$

  $A_{st} = 645 \times 6 = 3\ 870\ mm^2$

  $0.01 < \rho_g = \dfrac{3\ 870}{175\ 000} = 2.2\% < 0.08$

  Check minimum number of longitudinal bars:

  $6 > 4 \quad \therefore Ok.$

  Check minimum distance between longitudinal bars:

  $S_{Minimum} = Maximum[1.5 \times 29^{mm}, 40^{mm}]$

  $S_{Minimum} = 43.5^{mm} < 82^{mm} \ \therefore Ok.$
- Calculate the design axial load strength and bending moment for given eccentricity *($\emptyset P_n, \emptyset M_n$)*:

  $\gamma = \dfrac{350}{500} = 0.7$

  Based on γ value and as the reinforcements are distributed on two faces of the rectangular column, then the interaction diagram that will be used is as shown in Figure below.

  For

  $\dfrac{e}{h} = \dfrac{125}{500} = 0.25$

  the R_n for the interaction diagram will be:

  $R_n = \dfrac{P_n.e}{f_c' A_g h} = 0.17$

  $M_n = P_n.e = 0.17 \times 28 \times (500 \times 350) \times 500 = 417\ kN.m$

  As we working with compression controlled section (i.e. with a section has $\epsilon_t < 0.002$) then the strength reduction factor is $\emptyset = 0.65$

  $\emptyset M_n = 0.65 \times 417\ kN.m = 271\ kN.m$■

  and the K_n for the interaction diagram will be:

  $K_n = 0.69 = \dfrac{P_n}{f_c' A_g}$

  $P_n = 0.69 \times 28 \times 500 \times 350 = 3\ 381\ kN$

  $\emptyset P_n = 0.65 \times 3\ 381\ kN = 2\ 198\ kN$ ■

- Lateral reinforcement (Ties):
  Ties diameter:
  $\because\ \emptyset = 29^{mm} < 32^{mm}, \therefore$ we can use $\emptyset = 10^{mm}$ for ties
  Ties spacing:
  $S_{Maximum} = \min[16 \times 29^{mm}, 48 \times 10^{mm}, 350^{mm}] = 350^{mm} > S_{Provided}\ \therefore$ Ok.
  Ties arrangement:
  $\because\ S_{Spacing\ between\ longitudinal\ bars} < 150^{mm}$
  Then, alternate longitudinal bars will be supported by corner bars.

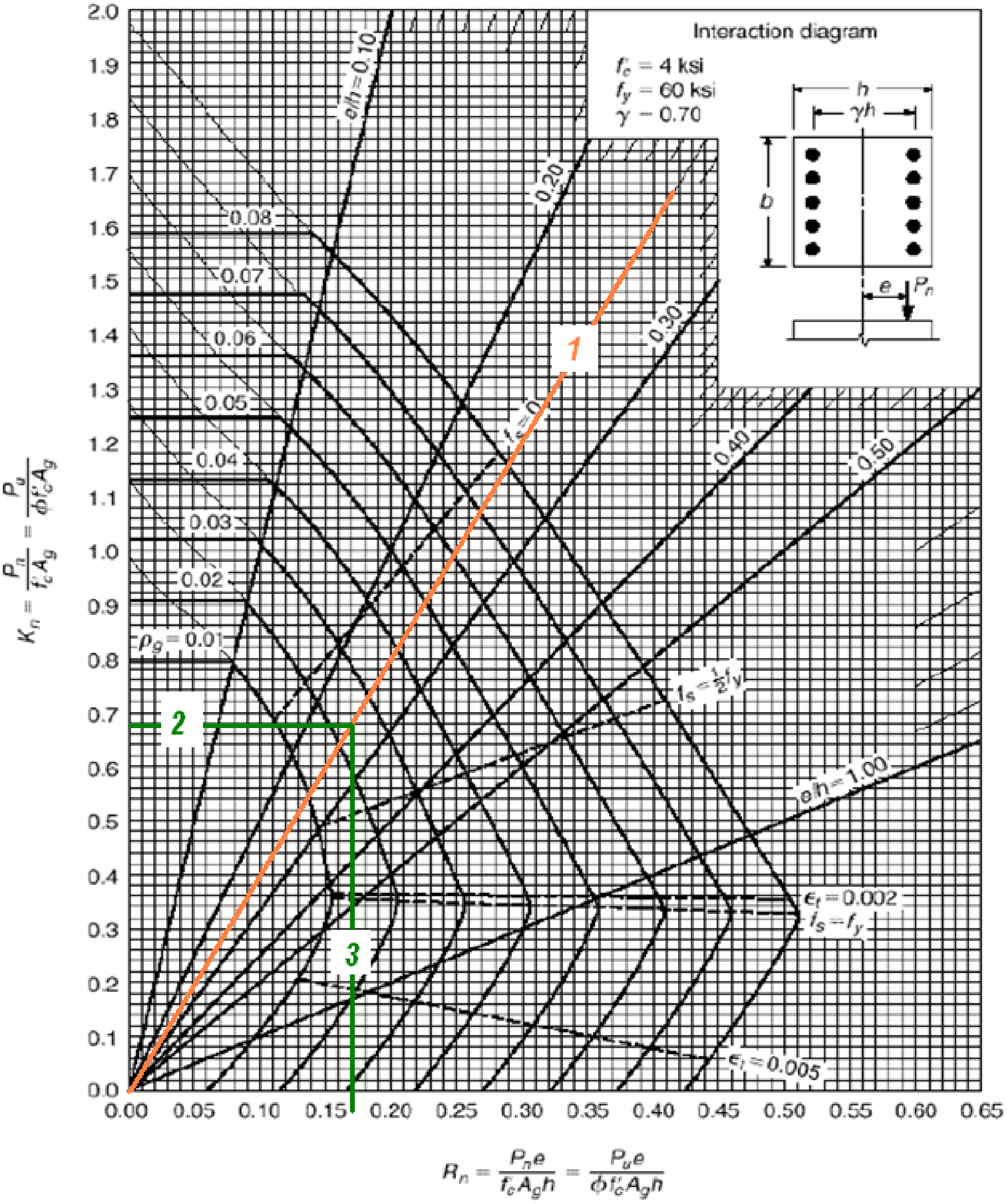


Figure 9.5-11: Adopting interaction diagram to computed design strength for the column of Example 9.5-1.

Example 9.5-2

In ***Example 9.2-1*** above, it was required to check the column shown in Figure below to general requirements of ACI code and to compute its design strength. Material properties where $f_c' = 27.5\,\text{MPa}$, and $f_y = 420\text{MPa}$. Resolve this example based on interaction diagram instead of equations for axially loaded columns.

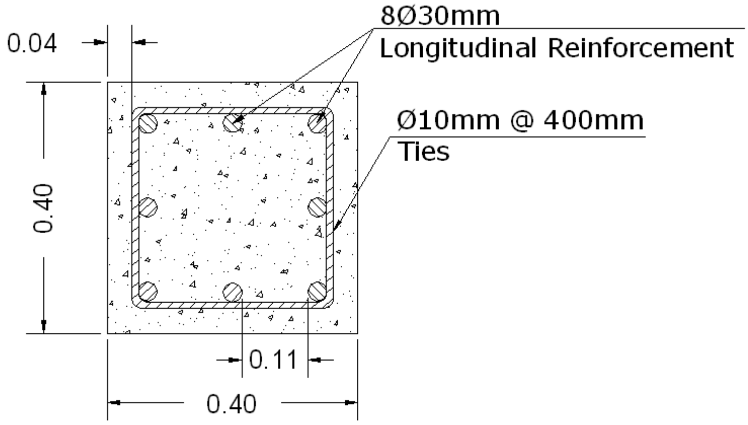


Figure 9.2-5: Proposed tied column for Example 9.2-1. Reproduce for convenience.

Solution

<u>Checking for General Requirements</u>

General requirements of ACI code are nominal in nature and do not related to use of interaction diagram or use equation in computing of column design strength:

<u>Longitudinal reinforcement</u>

Check ρ_g within acceptable limits:

$A_g = 400^2 = 160\ 000\,mm^2$

$A_{st} = \dfrac{\pi \times 30^2}{4} \times 8 = 5\ 652\,mm^2$

$0.01 < \rho_g = \dfrac{5\ 652}{160\ 000} = 3.53\% < 0.08$

Check minimum number of longitudinal bars:

$8 > 4 \quad \therefore Ok.$

Check minimum distance between longitudinal bars:

$S_{Minimum} = Maximum[1.5 \times 30^{mm}, 40^{mm}]$

$S_{Minimum} = 45^{mm} < 110^{mm} \ \therefore Ok.$

<u>Lateral reinforcement (Ties)</u>

Checking of Lateral Reinforcement (Ties):

Ties diameter:

$\because\ \emptyset = 30^{\text{mm}} < 32^{\text{mm}}, \therefore$ we can use $\emptyset = 10^{\text{mm}}$ for ties

Ties spacing:

$S_{\text{Maximum}} = \min[16 \times 30^{\text{mm}}, 48 \times 10^{\text{mm}}, 400^{\text{mm}}] = 400^{\text{mm}} = S_{\text{Provided}} \ \therefore \text{Ok.}$

Ties arrangement:

$\because S_{\text{Spacing between longitudinal bars}} < 150^{\text{mm}}$

Then, alternate longitudinal bars are supported.

<u>Column Design Strength</u>

Strength of axially loaded columns are not related to whether reinforcement are distributed on two faces or on four faces nor related to γ value. To emphasize this fact, two extremes interaction diagrams, the first one for reinforcement distributed on four faces and with γ value of 0.6 while the other with reinforcement on two faces and with γ value of 0.9, have been compared in below.

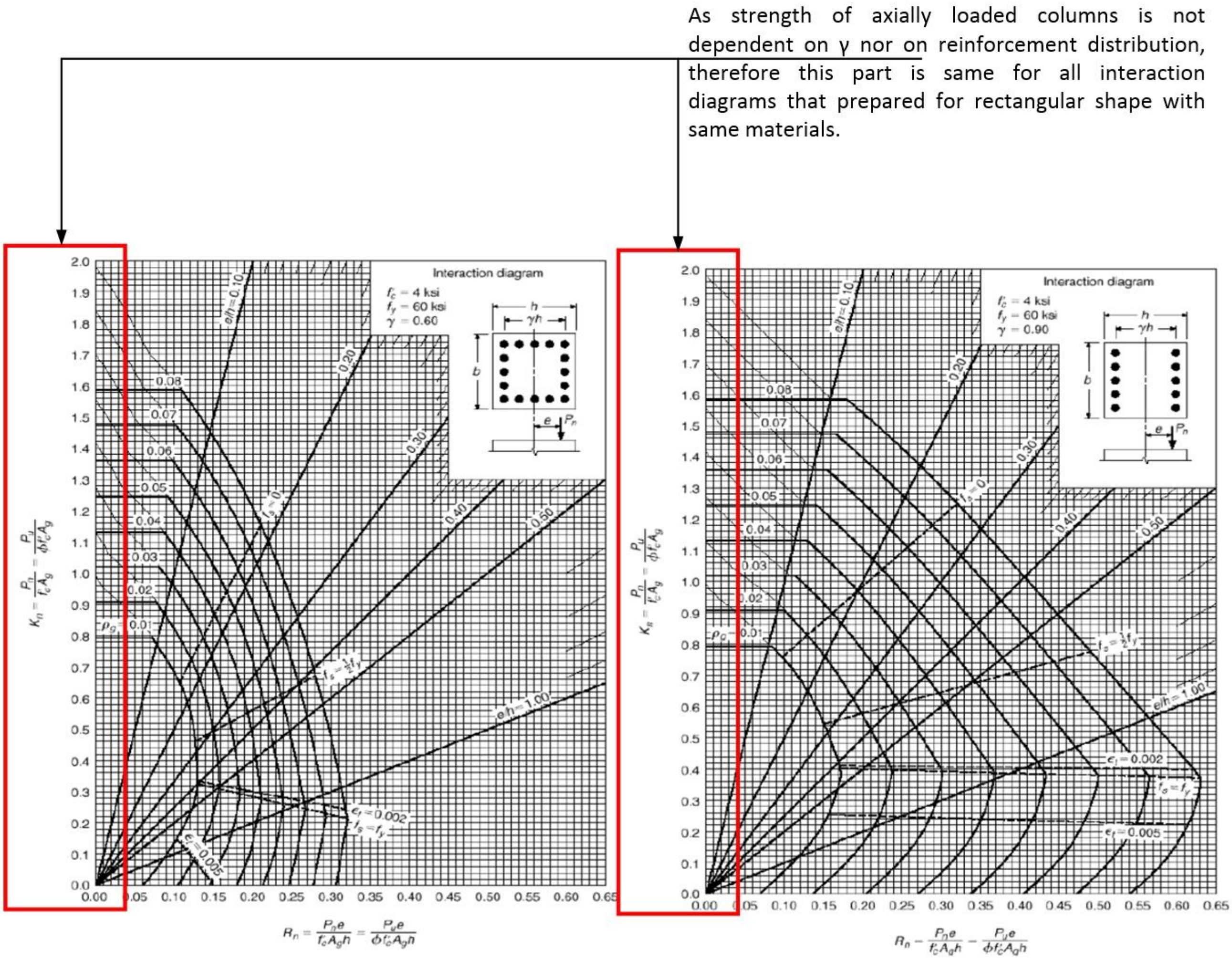


Figure 9.5-12: Two interaction diagrams that are equally applicable to solve the axially loaded column of Example 9.5-2.

Adopting any one of interaction diagrams for rectangular columns with $f_c' = 4\,ksi$ and $f_y = 28\,MPa$ will leads to:

For

$$\rho_g = \frac{5\,652}{160\,000} = 3.53\%$$

$$K_n = \frac{P_n}{f_c' A_g} \approx 1.06$$

$$P_n = \frac{(1.06 \times 27.5 \times 400 \times 400)}{1000} = 4664\,kN$$

For compression control region and with tied columns:

$$\phi = 0.65$$

$$\phi P_n = 0.65 \times 4664 = 3032\,kN$$

This design strength is close to that computed based on equations in Example 9.2-1, $\emptyset P_{nMaximum} = 3110\,kN$

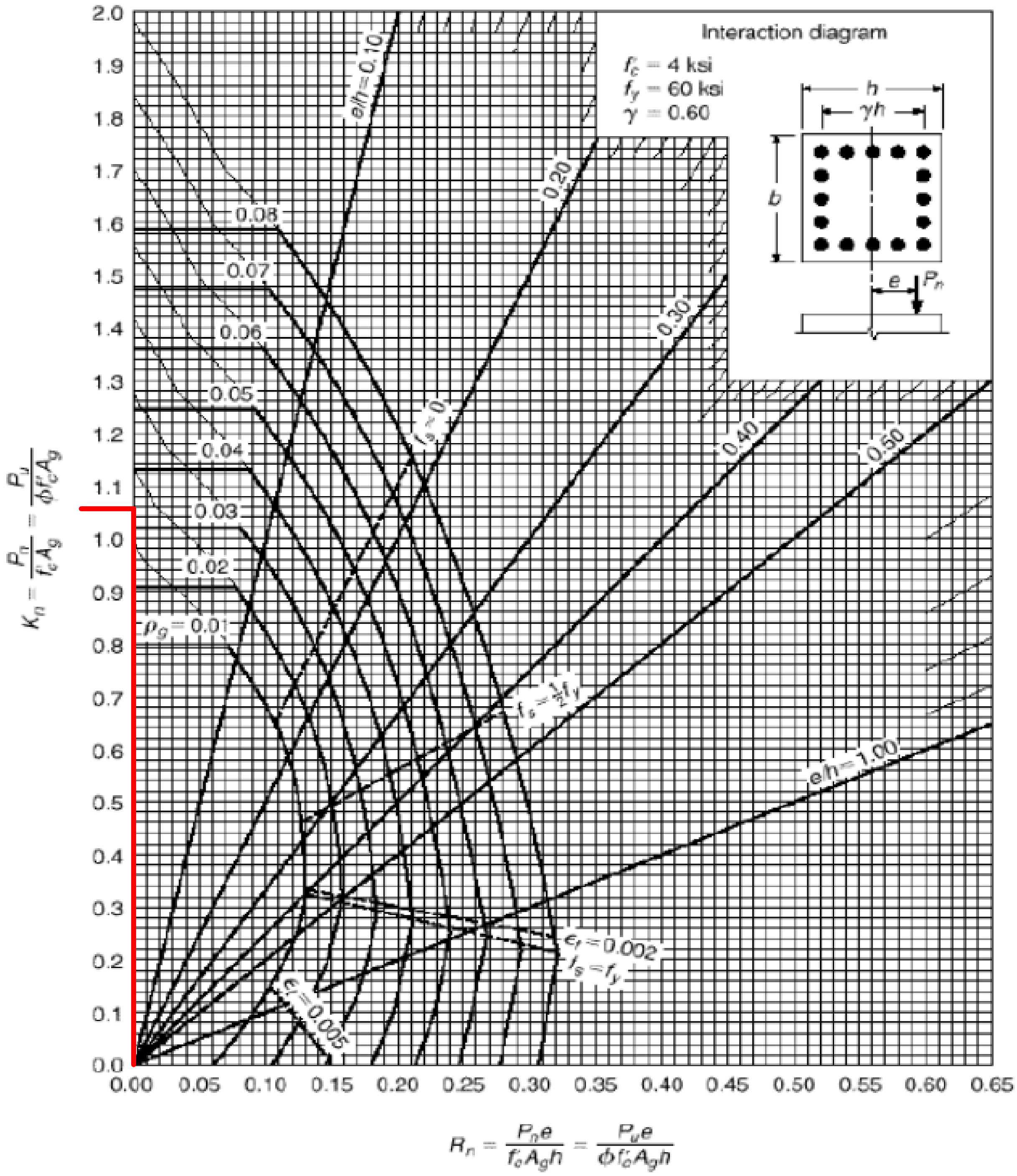


Figure 9.5-13: Sample interaction diagram adopted to solve the axially loaded column of Example 9.5-2.

Example 9.5-3

In ***Example 9.2-2*** above, it was required to check the column shown in ***Figure 9.2-6*** to general requirements of ACI code and then determine whether this column is adequate to carry a factored load of P_u= 2250 kN. Material properties where $f'_c = 27.5\,\text{MPa}$, and $f_y = 420\text{MPa}$. Resolve this example based on interaction diagram instead of equations for axially loaded columns.

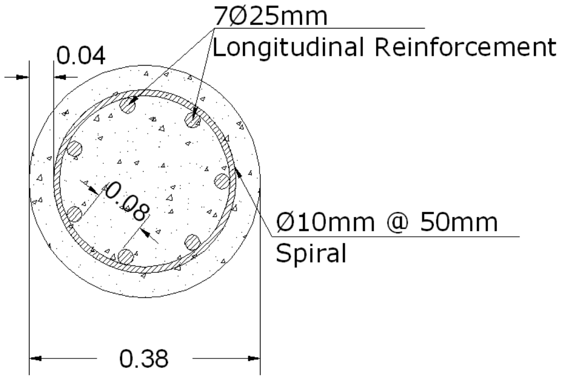


Figure 9.2-6: Spiral column of Example 9.2-2. Reproduce for convenience.

Solution

<u>Checking for General Requirements of ACI Code</u>

<u>Longitudinal reinforcement</u>

Check ρ_g within acceptable limits:

$$A_g = \frac{\pi \times 380^2}{4} = 113\ \ 354\, mm^2$$

$$A_{st} = \frac{\pi \times 25^2}{4} \times 7 = 3\ 434\, mm^2$$

$$\rho_g = \frac{3\ \ 434}{113\ \ 354} = 3.0\%$$

$0.01 < \rho_g < 0.08\ \therefore Ok.$

Check minimum number of longitudinal bars

$7 > 6\ \ \therefore Ok.$

Check minimum distance between longitudinal bars

$S_{Minimum} = Maximum[1.5 \times 25^{mm}, 40^{mm}]$

$S_{Minimum} = 40.0^{mm} < 80^{mm}\ \therefore Ok.$

<u>Check the lateral reinforcement (Spiral):</u>

Check Spiral Diameter:

$\emptyset_{Spiral} = 10mm\ Ok.$

Check Spiral Steel Ratio:

$$A_{sp} = \frac{\pi \times 10^2}{4} = 78.5^{mm^2}$$

$$\rho_{sProvided} = \frac{4 \times 78.5^{mm^2}}{(380 - 2 \times 40)^{mm} \times 50^{mm}} = 0.0209$$

$$\rho_{sMinimum} = 0.45 \times \left(\frac{113\ \ 354}{\frac{\pi \times 300^2}{4}} - 1\right) \times \frac{27.5}{420} = 0.0178 < 0.0209\ \therefore Ok.$$

Check the Clear Spacing:

$25^{mm} < [S_{Clear\ Provided} = 50^{mm} - 10^{mm} = 40^{mm}] < 80^{mm}\ \therefore Ok.$

<u>Axial Design Strength of the Column</u>

With any of interaction diagrams for circular columns,

With

$$\rho_g = \frac{3\ \ 434}{113\ \ 354} = 3.0\%$$

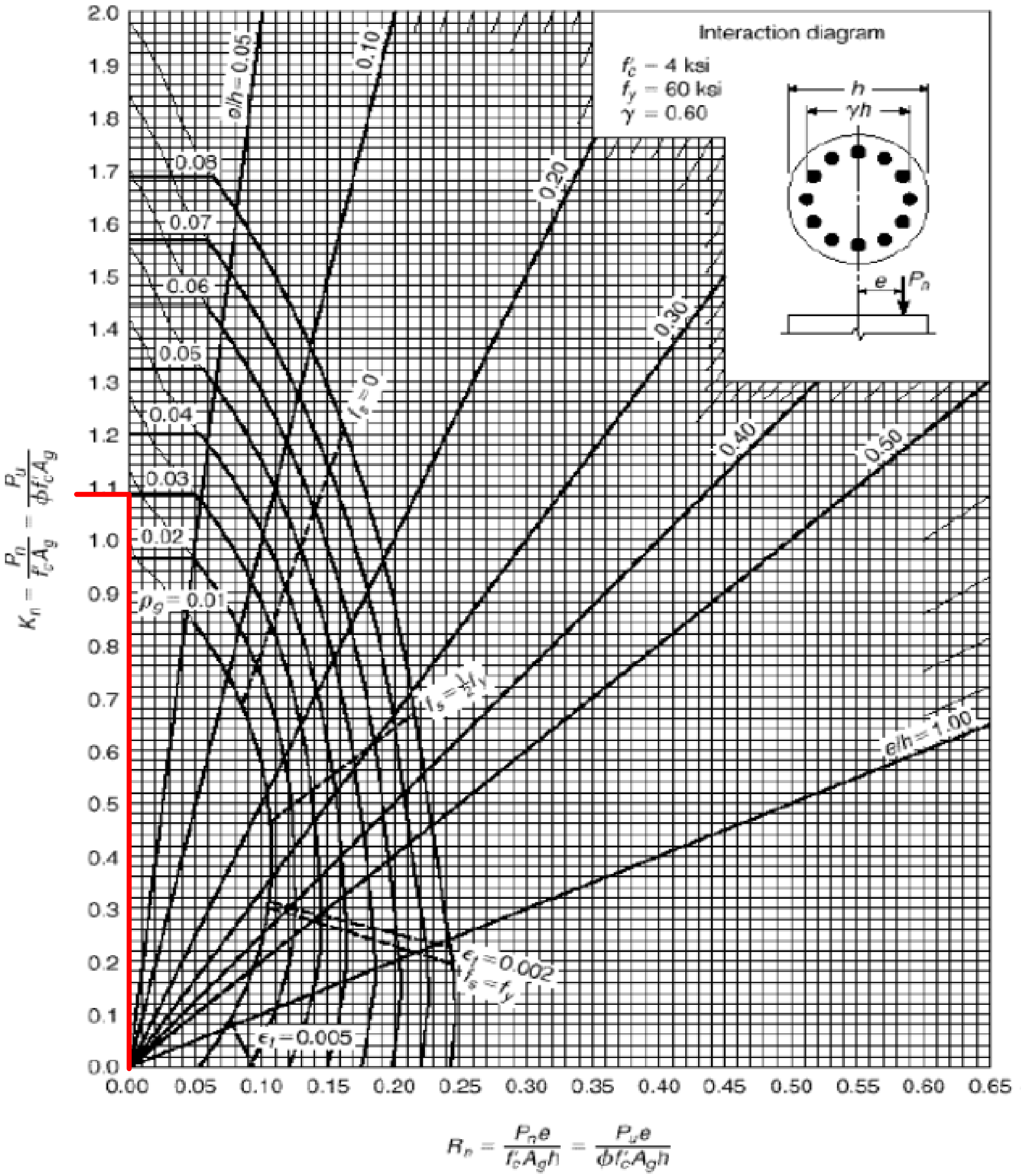


Figure 9.5-14: A sample interaction diagram adopted to solve the Example 9.5-3.

$$K_n = \frac{P_n}{f'_c A_g} \approx 1.1$$

$$P_n = \frac{\left(1.1 \times 27.5 \times \left(\frac{\pi \times 380^2}{4}\right)\right)}{1000} = 3431\ kN$$

As axially loaded columns are located in compression-controlled regions, therefore ϕ of 0.75 is adopted for this spiral column:

$$\phi P_n = 0.75 \times 3431 = 2573\ kN$$

This value is close to that of 2557 kN which computed based on relations for axially loaded columns in ***Example 9.2-2***. The proposed column is adequate as:

$$\phi P_n = 2573\ kN > P_u = 2250\ \text{kN}$$

9.6 DESIGN OF A COLUMN WITH COMPRESSION LOAD PLUS UNIAXIAL MOMENT

9.6.1 General Guides for Columns Design

- The following guides that related to columns design have been proposed by ***J. G. MacGregor*** in his book "***Reinforced Concrete: Mechanics and Design, 4th Edition***):
- Type of Column:
  - For eccentricity, e/h, greater than 0.2, a tied column with bars in the faces farthest from axis of bending is most efficient. Even more efficiency can be obtained by using of a rectangular column.
  - Tied columns with bars in four faces are used for e/h ratios of less than about 0.2 and also when moments exist about both axes. ***Many designers prefer this arrangement because there is less possibility of construction error in the field if there are equal numbers of rebars in each face of the column***.
  - Spiral columns are relatively infrequent in non-seismic areas. In seismic areas or in other situations where ductility is important, spiral columns are used frequently.
- Estimating the Column Size:
  - The initial stage in column design involves estimating the required size of column. There is no simple rule for doing this, since the axial-load capacity of a given cross section varies with the moment acting on section. For very small moments following relations can be used (these relations similar to that derived in ***Article 9.3***):
  - For Tied Columns:
    $$A_{gTrail} \geq \frac{P_U}{0.4[f_c' + f_y \rho_g]}$$
  - For spiral column:
    $$A_{gTrail} \geq \frac{P_U}{0.5[f_c' + f_y \rho_g]}$$
  - Both of these relations will tend to underestimate the column size if there are moments present.
- Column Thickness "b":
  - The Fire Codes usually specified minimum column size as follows:

Table 9.6-1: Minimum column thickness for fire rating requirements, adopted from

Fire Rating (hours)	*Minimum Column Thickness (mm)*
1 hour	$b \geq 225\ mm$
2-3 hours	$b \geq 300\ mm$

  - Although the ACI Code does not specify a minimum column size, ***the minimum dimension of cast-in-place tie column should not be less than 200mm and preferably not less than 250mm***.
  - ***The diameter of a spiral column should not be less than about 300mm***.

9.6.2 Using Interaction Charts in Design Process

Conventional design charts permit the direct design of eccentrically loaded columns throughout the common range of strength and geometric variables. They may be used in one of two ways as presented in ***Article 9.6.2.1*** and ***Article 0*** below.

9.6.2.1 Selection of Reinforcement for Column of Given Size

For a given factored load P_u and equivalent eccentricity $e = \frac{M_u}{P_u}$ and given cross section this direct procedure can be summarized as follows:

Design of Longitudinal Reinforcement:

- Calculate the ratio γ based on required cover distances to the bar centroid, and select the corresponding column design chart.
- Calculate $K_n = \frac{P_u}{\emptyset f_c' A_g}$ and $R_n = \frac{P_u e}{\emptyset f_c' A_g h}$ where A_g is section gross area.
- Strength reduction value is selected based on type of section (i.e. is the member a compression controlled member or a tension controlled member or in the transition region).
- From the graph, for the values found in above, read the required reinforcement ratio ρ_g.
- Calculate the total steel area A_{st}.
- Compute the required number of longitudinal bars:
  $$\text{No. of Longitudinal Bars} = \frac{A_{st}}{A_{Bar}}$$
- The limitations on the number and arrangement of longitudinal bars are as discussed in the design of columns for axial loads.

Design of Lateral Reinforcement

Design of lateral reinforcement is exactly as discussed in the design of columns for axial loads, ***Article 9.3***. For convenience, these procedures have been represented in below:

Ties:

- Select ties diameter:
- If $\varphi_{Longitudinal} \leq 32^{mm}$ then:
  $\phi_{Ties} = 10^{mm}$
  Else
  $\phi_{Ties} = 13^{mm}$
- Select ties spacing:
  $S_{Required} \leq Minimum[16\varphi_{Bar}, 48\varphi_{ties}, Least\ Column\ Dimensions]$
- Arrange the ties according to requirements of the ACI for maximum spacing between longitudinal bars (use the standard arrangements of ***Figure 9.2-2*** above).

Spiral:

- Spiral Diameter
  $\phi_{Spiral} \geq 10^{mm}$
- Compute $\rho_{sMinimum}$
  $$\rho_{sMinimum} = 0.45\left(\frac{A_g}{A_c} - 1\right)\frac{f_c'}{f_{sy}}$$
  Let $\rho_s = \rho_{sMinimum}$ to compute the required $S_{Required}$:
  $$S_{Required} = \frac{4A_{sp}}{D_c\ \rho_{sMinimum}}$$
- Check with Limitation for Clear Spacing
  The clear spacing $S_{Required\ Clear}$ between turns of the spiral must be:
  $25 \leq S_{Clear} \leq 80^{mm}$

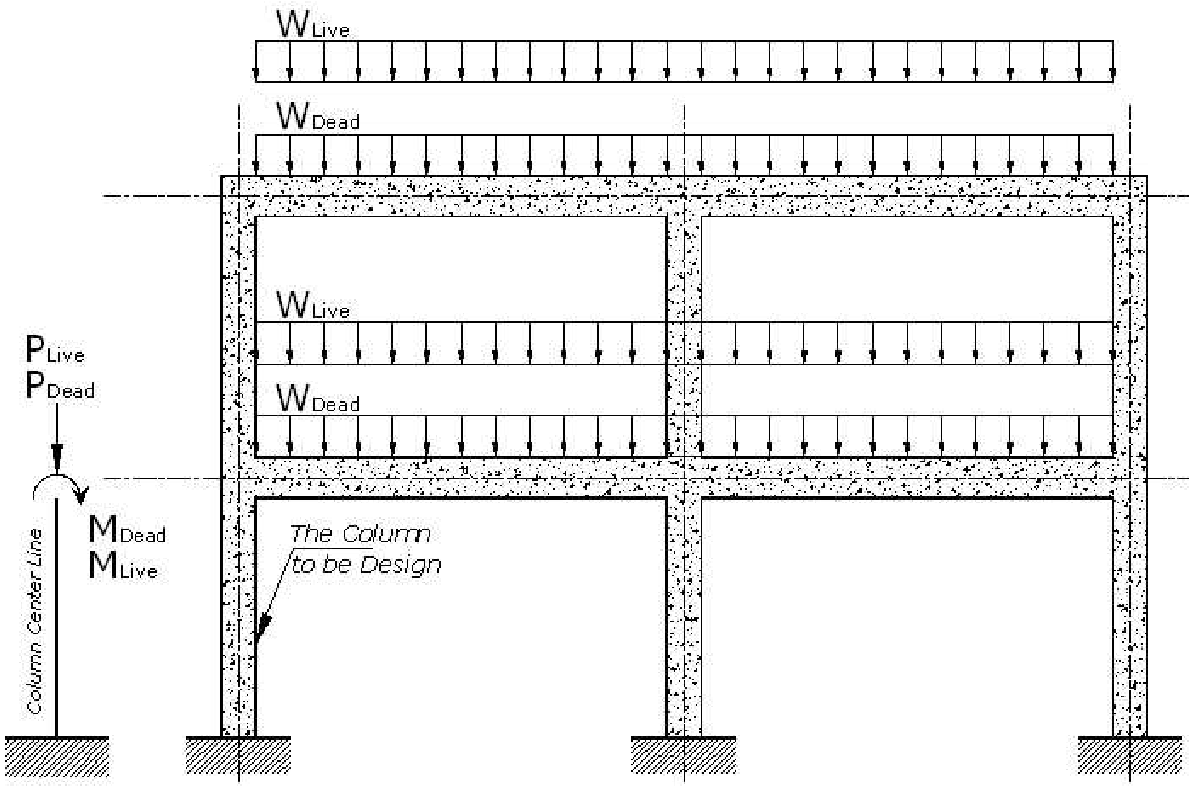
Example 9.6-1

In a two-story building that shown in ***Figure 9.6-1*** below an exterior column is to be designed for the following loading:

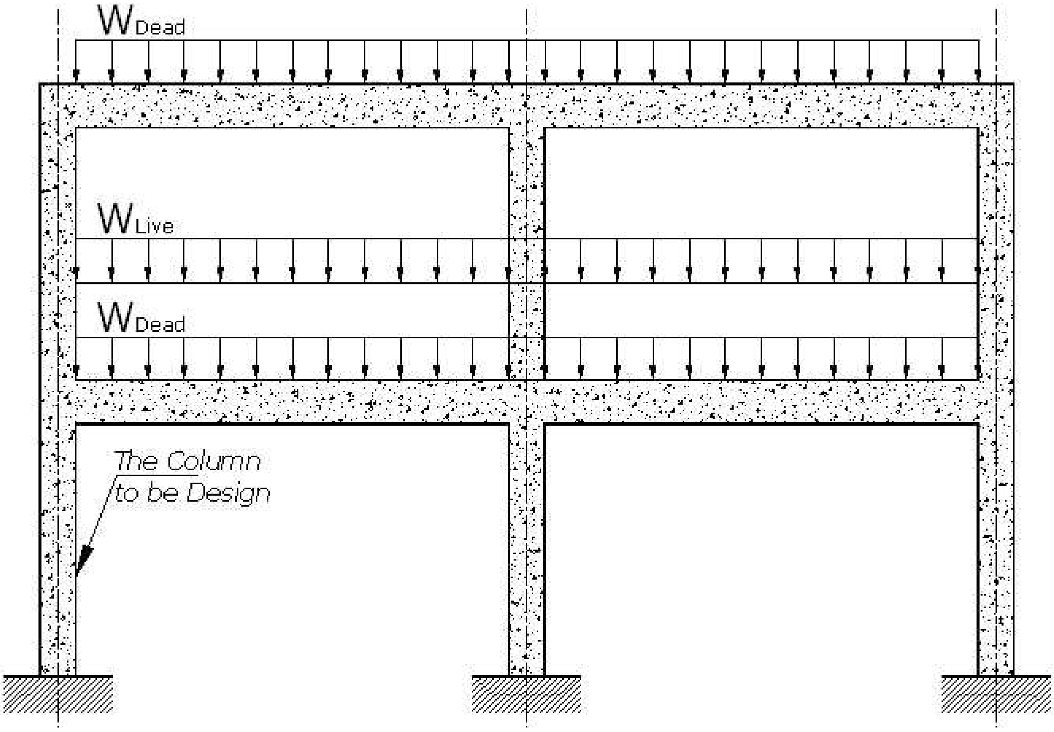
- First Load Pattern:
  $P_{Dead} = 987\ kN$
  $P_{Live} = 1\ 481\ kN$
  $M_{Dead} = 220\ kN.m$
  $M_{Live} = 315\ kN.m$
- Second Load Pattern:
  $P_{Dead} = 987\ kN$
  $P_{Live} = 738\ kN$
  $M_{Dead} = 220\ kN.m$
  $M_{Live} = 315\ kN.m$

Architectural considerations required that a rectangular column to be used, with dimensions:
$b = 500^{mm}$ and $h = 625^{mm}$
Materials:
$f_c' = 28\ MPa$
$f_y = 420\ MPa$
Reinforcement:
Try $\varphi = 32^{mm}$ for longitudinal reinforcement ($A_{Bar} = 819\ mm^2$).
Try $\varphi = 10^{mm}$ for lateral reinforcement.
Based on above data

- Design the column for first load pattern.
- Check to ensure that the column is adequate for the second load pattern.



First Load Pattern
Figure 9.6-1: Building and the edge column for the Example 9.6-1.



Second Load Pattern
Figure 9.6-1: Building and the edge column for the Example 9.6-1. Continued.

Solution

Design of Column for First Load Pattern

The column will be designed initially for full load, then it would be checked for adequacy when live load is partially removed.

According to the ACI safety provisions, the column must be designed for a factored load:

$P_{u\ Maximum} = 1.2 \times 987 + 1.6 \times 1\,481 = 3\,554\ \text{kN}$

$M_u = 1.2 \times 220 + 1.6 \times 315 = 768\ kN.m$

Design of Longitudinal Reinforcement:

- Calculate the ratio γ based on required cover distances to the bar centroid, and select the corresponding column design chart.

  $\gamma h = 625 - 2 \times 40 - 2 \times 10 - 32 = 493\ mm$

  $$\gamma = \frac{\gamma h}{h} = \frac{493}{625} = 0.79$$

  Say $\gamma = 0.80$ and assume that the reinforcement will be distributed on four faces. Then the interaction diagram that used in the design is that shown in ***Figure 9.6-2*** below.

- Calculate $K_n = \frac{P_u}{\emptyset f'_c A_g}$ and $R_n = \frac{P_u e}{\emptyset f'_c A_g h}$:

$$e = \frac{M_u}{P_u} = \frac{768\ kN.m}{3\,554\ \text{kN}} = 0.216\ m$$

$$\frac{e}{h} = \frac{0.216m}{0.625m} = 0.35$$

- Based on $\frac{e}{h}$ ratio, one can see that the tensile strain for this column under proposed loads is less than 0.002. Therefore the section is compression controlled section and strength reduction factor is $\emptyset = 0.65$.

$$K_n = \frac{P_u}{\emptyset f'_c A_g} = \frac{3\,554\,000\ \text{N}}{0.65 \times 28 \frac{\text{N}}{\text{mm}^2} \times (625 \times 500)\text{mm}^2} = 0.625$$

$$R_n = \frac{P_u e}{\emptyset f'_c A_g h} = \frac{768\ \times 10^6\ N.mm}{0.65 \times 28 \frac{N}{mm^2} \times (625 \times 500)\text{mm}^2 \times 625mm} = 0.216$$

- From the graph, the required reinforcement ratio, ρ_g, would be:
  $\rho_g = 0.04$
- Calculate the total steel area A_{st}:
  $A_{st} = 0.04 \times 625 \times 500 = 12\,500\ mm^2$
- Compute the required number of longitudinal bars:
  $$\text{No. of Longitudinal Bars} = \frac{A_{st}}{A_{Bar}} = \frac{12\,500\ mm^2}{819\ mm^2} = 15.3$$
  Try 16 Ø 32.

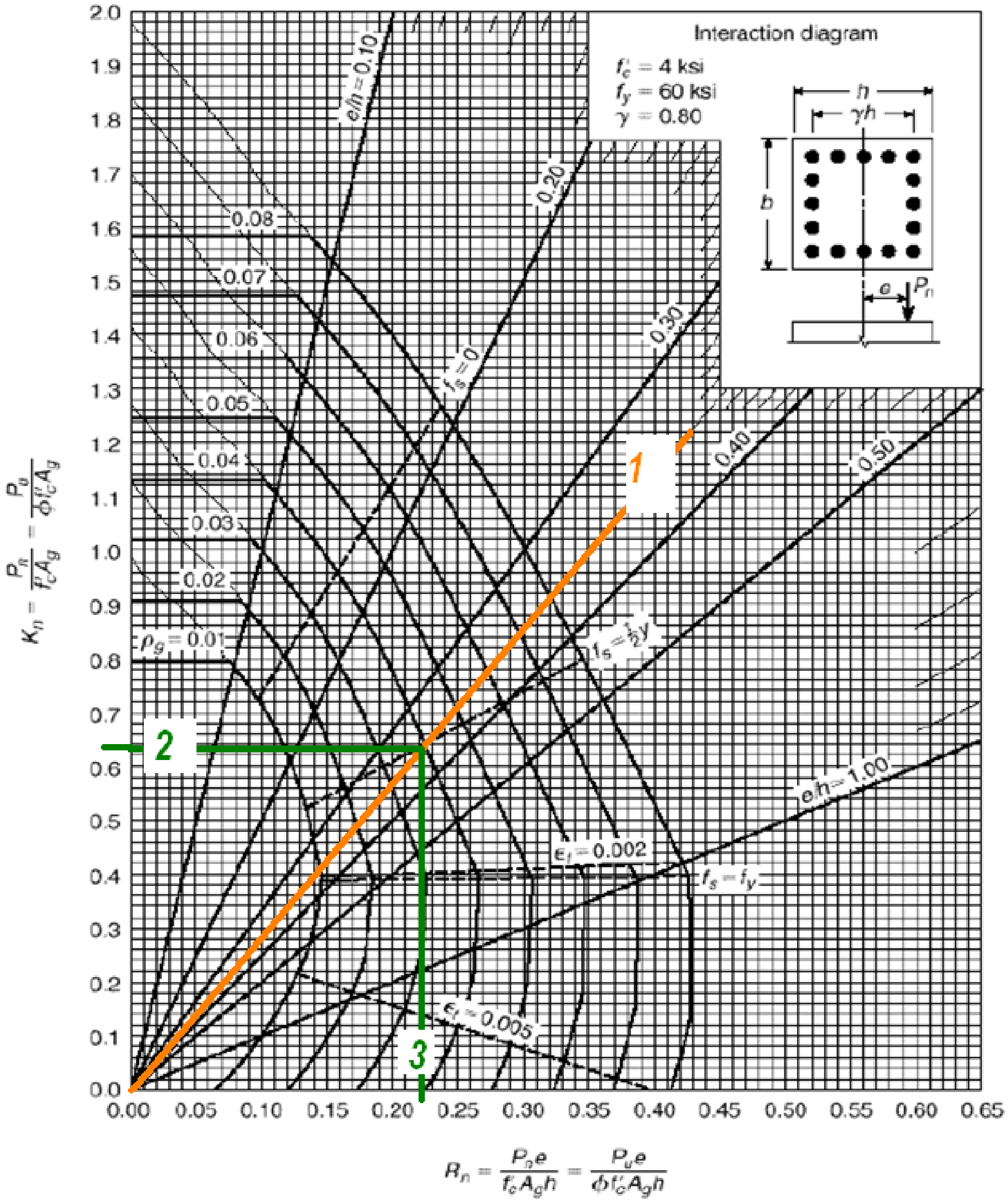


Figure 9.6-2: Interaction diagram for Example 9.6-1, first load pattern.

<u>Design of Lateral Reinforcement:</u>

- Ties diameter:
  $\because\ \emptyset = 32^{mm}, \therefore$ we can use $\emptyset = 10^{mm}$ for ties
- Ties spacing:
  $S_{Required} = \min[16 \times 32^{mm}, 48 \times 10^{mm}, 500^{mm}] = 480^{mm}$
  Use $\emptyset 10mm$ @ $475\ mm$
- Ties arrangement:
  The following arrangement can be used for our column:

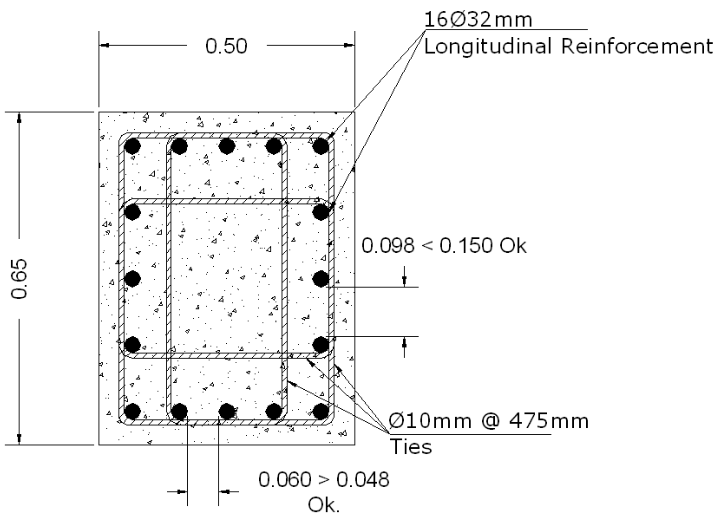


Figure 9.6-3: Design section for the column of Example 9.6-1 based on first load pattern.

Column Checking for Second Load Pattern

Aim of Checking for the Second Load Pattern:

- Before starting the checking, it is useful to discuss the aim of this checking.
- At first sight this checking seems unnecessary as the column that designed with live load acting on all floors and roofs of course will be adequate when live loads acting on the floor under consideration only.
- Unfortunately, the problem is not so simple as appear, i.e. some columns that are adequate for full live loads may be not adequate for partially live load, this strange fact can be explained as follows:
  - Assume that required reinforcement has been selected based on full live loads as was done in previous article and assume that load path for dead and full live loads will be as shown in Figure below.
  - As the axial force in a column resulting from accumulation of loads acting on the floor under consideration and on above floors and roof, then removing live loads from above floors and roof will decrease the axial force in that column.
  - For gravity loads, bending moments in a column are mainly resulting from negative moments of beams that connected directly to the column, the removing of live loads from above floors and roof does not change the bending moments in the column. Based on this reasoning, bending moments have been assumed the same in first and second load patterns.
- Then with second load pattern, load path will move vertically in downward direction (as we have negative ΔP and zero ΔM). With this movement, load case that was inside the interaction diagram may move to be outside it. Therefore, the section that was pass under full live load may fail under partial live load!

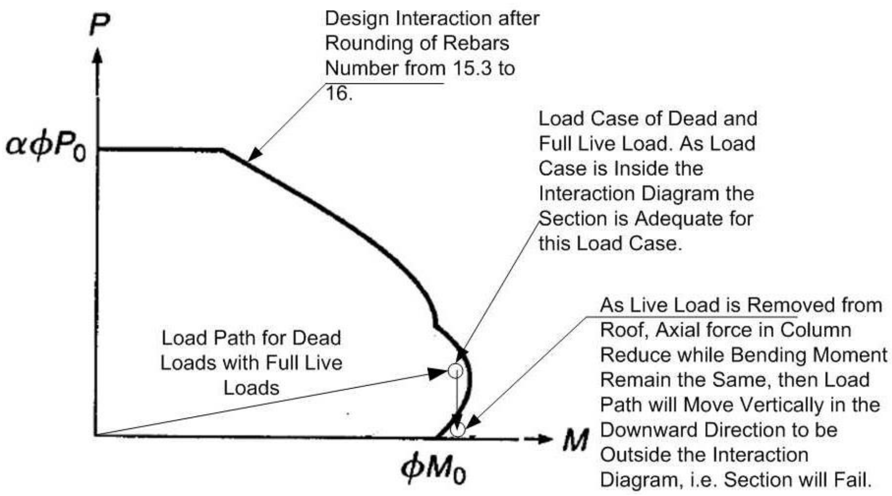


Figure 9.6-4: Schematic integration diagram to show the aim of checking for the second pattern.

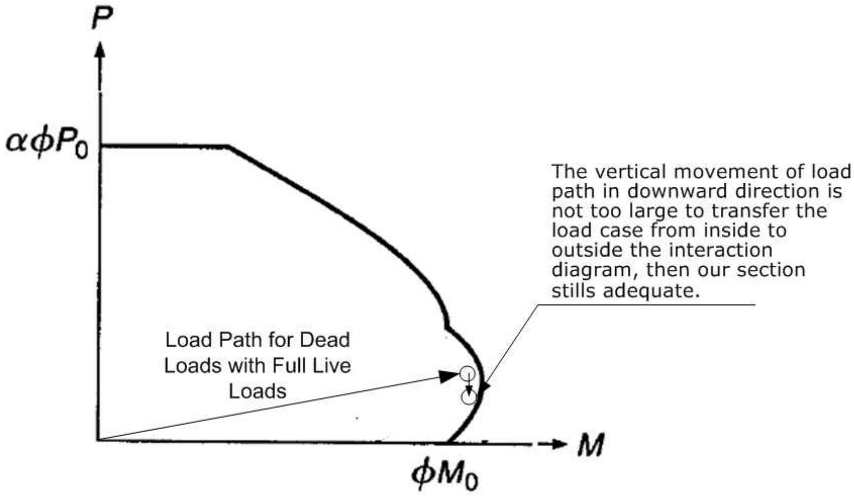


Figure 9.6-4: Schematic integration diagram to show the aim of checking for the second pattern. Continued.

Checking Details:

- Check to ensure that the column is adequate for the second load pattern:

  $P_{u\,Minimum} = 1.2 \times 987 + 1.6 \times 738 = 2\,365\; kN \qquad M_u = 768\; kN.m$

  Say $\gamma = 0.80$

  $e = \frac{M_u}{P_u} = \frac{768\; kN.m}{2\,365\;\text{kN}} = 0.325\; m \quad \frac{e}{h} = \frac{0.325\; m}{0.625 m} = 0.52$

  $K_n = \frac{P_u}{\emptyset f'_c A_g} = \frac{2\,365\;\;000\;\text{N}}{0.65 \times 28 \frac{\text{N}}{\text{mm}^2} \times (625 \times 500)\text{mm}^2} = 0.416$

  $R_n = \frac{P_u e}{\emptyset f'_c A_g h} = \frac{768\;\times 10^6\; N.mm}{0.65 \times 28 \frac{N}{mm^2} \times (625 \times 500)\text{mm}^2 \times 625 mm} = 0.216$

- From ***Figure 9.6-5*** with $\rho_{g\,\text{Required}} = 0.028 < \rho_{\text{Provided}}$, one concludes that vertical movement of load path in downward direction is not too large to transfer the load case from inside to outside the interaction diagram, then the section stills adequate.

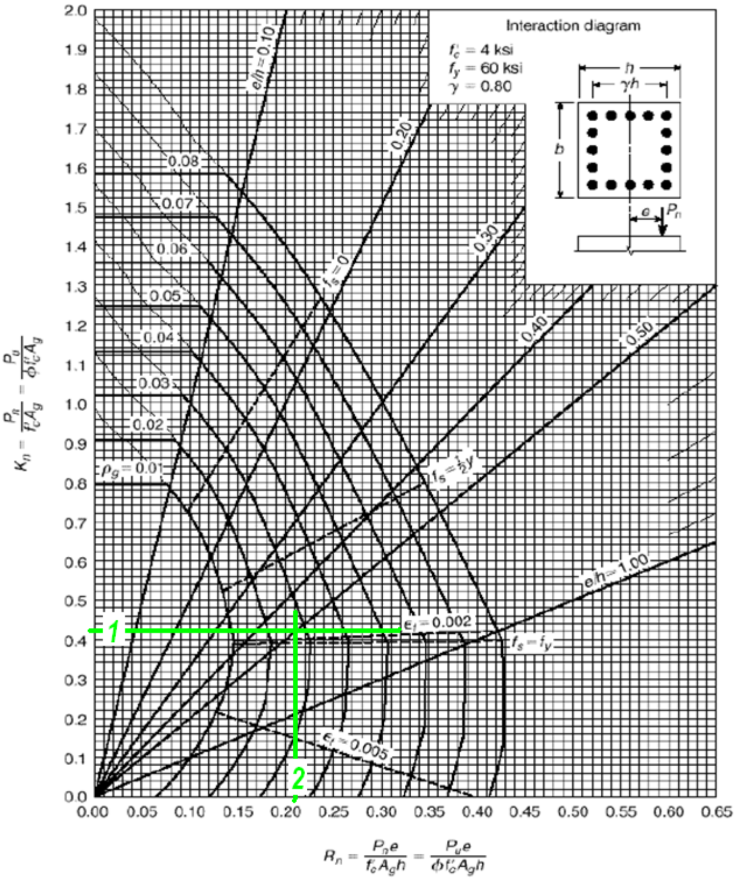


Figure 9.6-5: Interaction diagram to check the second load pattern for the column of Example 9.6-1.

9.6.2.2 Selecting of Column Size for a Given Reinforcement Ratio

Example 9.6-2

A column is to be designed to carry factored loads of:

$P_u = 2\,139\ kN$

$M_u = 667\ kN.$

Assume that:

- Bending moment about major or strong axis.
- Material strengths $f_y = 420MPa$ and $f_c' = 28\ MPa$ are specified.
- Cost studies for the particular location indicate that a reinforcement ratio of about 0.03 is optimum.
- Column depth: h=625mm.
- $\emptyset = 36mm$ for longitudinal reinforcements ($A_{Bar} = 1\,006\ mm^2$).
- Steel with bars concentrated in two layers, adjacent to the outer faces of the column and parallel to the axis of beading, will be used.

Find the required column width "b" and design the longitudinal lateral reinforcements.

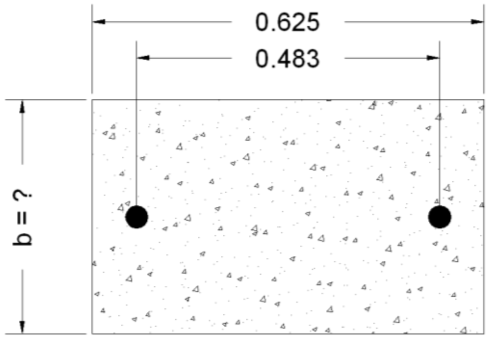


Figure 9.6-6: Proposed section for Example 9.6-2.

Solution

Column Width "b" and Design of Longitudinal Reinforcement

- Calculate the ratio γ based on required cover distances to the bar centroid, and select the corresponding column design chart.

  $\gamma h = 483\ mm$

  $$\gamma = \frac{\gamma h}{h} = \frac{483}{625} = 0.78$$

- Say $\gamma = 0.80$ and as steel is assumed to be concentrated in two layers, then the design interaction diagram will be as ***Figure 9.6-7*** below.

  As

  $$e = \frac{M_u}{P_u} = \frac{667}{2\,139} = 0.31$$

  and

  $$\frac{e}{h} = \frac{0.31}{0.625} = 0.496 \approx 0.5$$

  then (from Figure above)

  $$K_n = \frac{P_u}{\emptyset f_c' bh} = 0.51$$

- As we working in the compression controlled region, then the strength reduction factor ∅ is 0.65.

  $$b = \frac{2\,139\,000\ \text{N}}{0.65 \times 28 \frac{\text{N}}{\text{mm}^2} \times 0.51 \times 625\text{mm}} = 369\ \text{mm}$$

  Use 375mm by 625mm section.

  $$A_{st\ Required} = 0.03 \times 625\text{mm} \times 375\text{mm} = 7\,031\ \text{mm}^2$$

  $$\text{No. of Rebars} = \frac{7\,031\ \text{mm}^2}{1\,006\ \text{mm}^2} = 6.99$$

  Use $8\emptyset 36mm$ rebars.

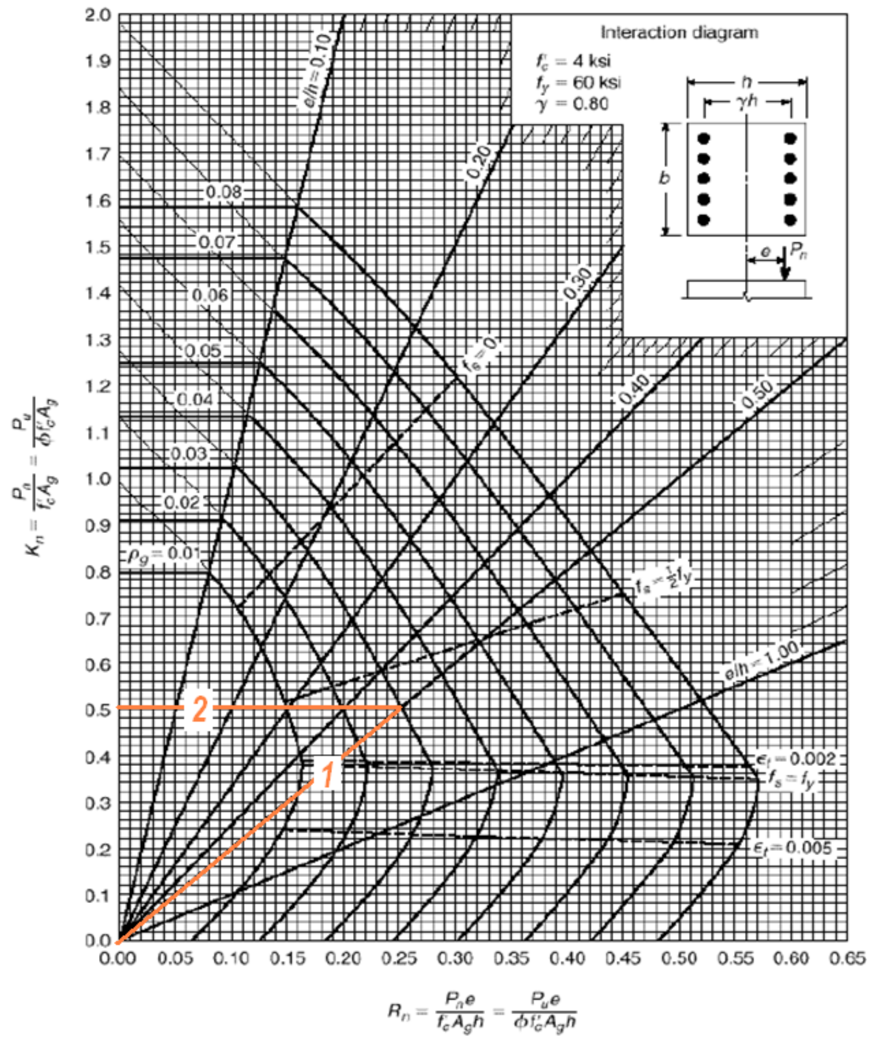


Figure 9.6-7: Interaction diagram to design the column of Example 9.6-2.

Design of Lateral Reinforcement

- Ties diameter:
  $\because\ \emptyset = 36^{\text{mm}} > 32^{\text{mm}}, \therefore$ we must use $\emptyset = 13^{\text{mm}}$ for ties
- Ties spacing:
  $S_{\text{Required}} = \min[16 \times 36^{\text{mm}}, 48 \times 13^{\text{mm}}, 375^{\text{mm}}] = 375^{\text{mm}}$, Use $\emptyset 13\text{mm} @ 375$ mm
- Ties arrangement:
  As can be shown from ***Figure 9.6-8*** below, the proposed distribution does not satisfy the ACI Code requirements related to minimum spacing between longitudinal rebars. Then bundled rebars must be used in our design.

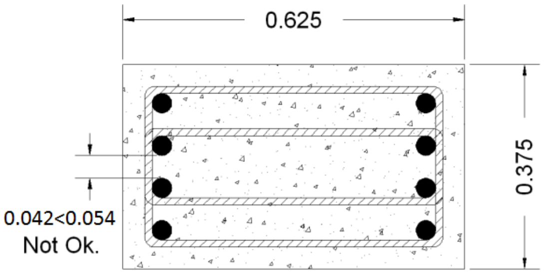


Figure 9.6-8: Final design section for the column of Example 9.6-2.

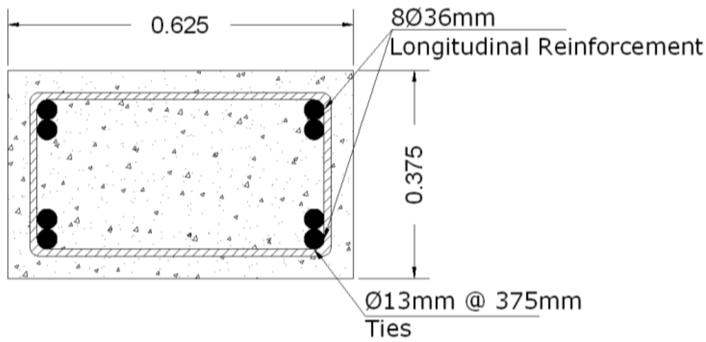


Figure 9.6-8: Final design section for the column of Example 9.6-2. Continued.

9.7 Homework Problems: Analysis and Design of a Column under Axial Load and Uniaxial Moment

Problem 9.7-1

Using an appropriate interaction curve, determine the value of P_n for the short tied column shown in ***Figure 9.7-1*** below. Assume that $f_c' = 28\,MPa$ and $f_y = 420\,MPa$.

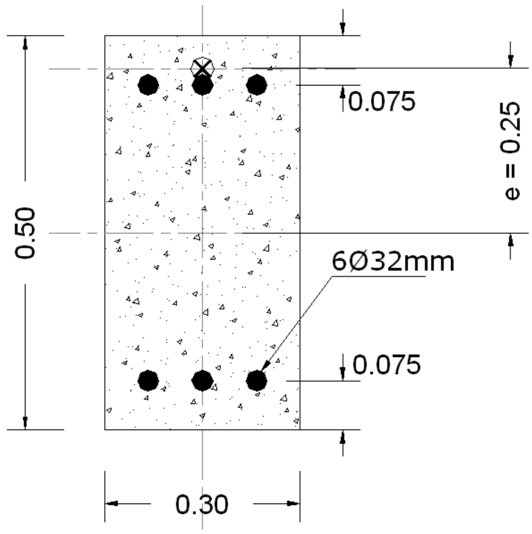


Figure 9.7-1: Column for Problem 9.7-1.

Answers

- Compute γ:
  $\gamma = 0.70$
- As the reinforcement is distributed along two faces only and as $\gamma = 0.07$, then use corresponding interaction diagram:
  $\because \dfrac{e}{h} = 0.50$
  $A_g = 150\,000\,mm^2, A_{Bar} = 804\,mm^2, A_{st} = 4\,824\,mm^2, \therefore\ \rho_g = 0.032$
  $K_n = \dfrac{P_n}{f_c{'}A_g} = \ 0.51 \Longrightarrow P_n = 2\,142\,kN$ ■

Problem 9.7-2

For the column shown in ***Figure 9.7-2*** below, based on structural calculations a designer has proposed the attached section. Check the adequacy of this section to ACI Code requirements and to the applied load. Assume that selfweight can be neglected.

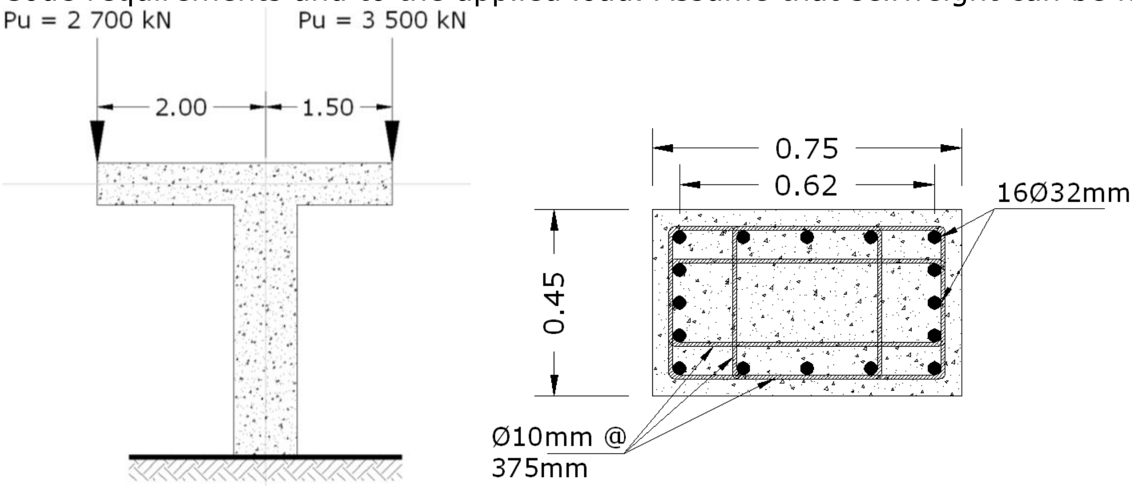


Figure 9.7-2: Frame and proposed column section for Problem 9.7-2.

Answers

Checking of Longitudinal Reinforcement:

- Check if ρ_g within acceptable limits:
  $A_g = 337\,500\,mm^2, A_{st} = 12\,864\,mm^2 \Longrightarrow 0.01 < \rho_g = 3.81\,\% < 0.08$
- Check minimum number of longitudinal bars:
  $16 > 4 \quad \therefore Ok.$
- Check minimum distance between longitudinal bars:

$S_{Minimum} = 48^{mm}, S_{Provided} = 47.5\ mm \approx 48mm\ Ok.$

Section Strength

- Calculate the design axial load strength and bending moment for given eccentricity $(\emptyset P_n, \emptyset M_n)$:

  $P_u = 6\,200\ kN, M_u = 150\ kN.m \Longrightarrow e = 0.024\ m, \because \frac{e}{h} = 0.032 < 0.10$
- Then this column can be analyzed as an axially loaded column, i.e. the applied moment can be neglected.

  $\emptyset P_{nMaximum} = 6\,827\ kN > P_u\ Ok.$

Checking of Ties:

- Ties diameter:

  $\because \varphi_{Longitudinal\ Bars} = 32^{mm}$

  Then using of $\varphi_{Ties} = 10^{mm}$ is okay.
- Tie spacing

  $S_{Required} = 450^{mm} > S_{Provided}\ \ Ok.$
- Ties arrangement:

  The proposed distribution is adequate according to ACI requirements.

Problem 9.7-3

The short tied column shown in ***Figure 9.7-3*** below is to be used to support the following loads and moments:

$P_D = 556\ kN,\ \ P_L = 623\ kN, M_D = 102\ kN.m\ , and\ \ M_L = 122\ kN.m$

Select longitudinal bars to be placed in its end faces only using appropriate ACI column interaction diagram, and design the ties.

Assume: Short column, $\emptyset\,32\,mm$ for longitudinal reinforcement, $f_c' = 28\ \text{MPa}$, and $f_y = 420\ \text{MPa}$.

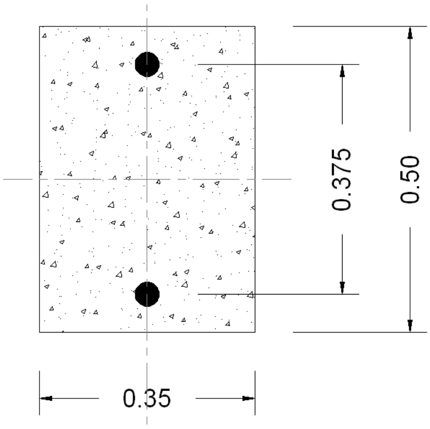


Figure 9.7-3: Column section for Problem 9.7-3.

Answers

Applied Factored Loads:

$P_u = 1\,664\ kN, M_u = 318\ kN.m \Longrightarrow e = 0.191\ m\ \Longrightarrow \frac{e}{h} = 0.38$

Longitudinal reinforcement:

- Compute γ:

  $\gamma = 0.75$
- Based on $\frac{e}{h}$, the strength reduction factor "$\emptyset$" can assumed to be 0.65:

  $K_n = \frac{P_u}{\emptyset f_c' A_g} = 0.52$
- Steel ratio ρ_g can be computed from interpolation from curves of $\gamma = 0.70$ and $\gamma = 0.80$.

γ	0.70	0.75	0.80
ρ_g	2.2%	2.1%	2.0%

$A_{st} = 3\,675\ mm^2 \Longrightarrow No.of\ Rebars = 4.57$, Try $6\ \emptyset\ 32\text{mm}$.

Design of Lateral Reinforcement (Ties):

- Ties diameter:

  $\because\ \emptyset = 32^{\text{mm}}, \therefore$ we can use $\emptyset = 10^{\text{mm}}$ for ties
- Ties spacing:

  $S_{Maximum} = 350^{mm}$

- Ties arrangement:
  $\because S_{\text{Spacing between longitudinal bars}} < 150^{\text{mm}}$
  Then, alternate longitudinal bars will be supported by corner bars.

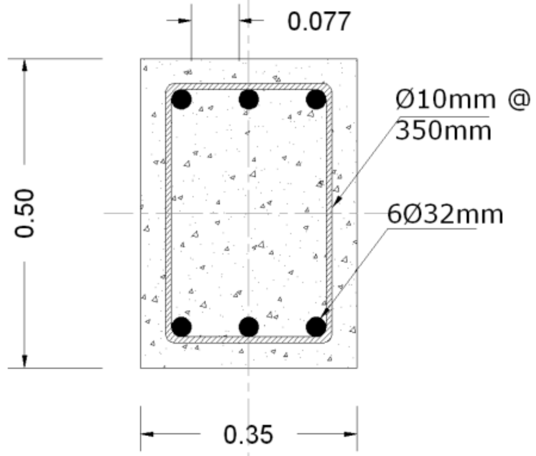


Figure 9.7-4: Final design section for Problem 9.7-4.

Problem 9.7-4

Design the spiral column that supports four girders of bridge shown in ***Figure 9.7-5*** below. In your design assume that.

- Each girder has a dead load reaction of 150 kN and has a live load reaction of 100 kN.
- Assume that live load acting on right span only.
- f_c' = 28 MPa and f_y = 420 MPa.
- Rebar No. 25 for longitudinal reinforcement (A_{bar} = 510mm^2) and No. 10 for spiral reinforcement.
- Column has a height of 4m, and it is assumed short.
- Column and cap selfweight should be included in your solution.

As was discussed previously, your solution can be based on the equivalent circular section instead of actual hexagonal section, see ***Figure 9.4-5***.

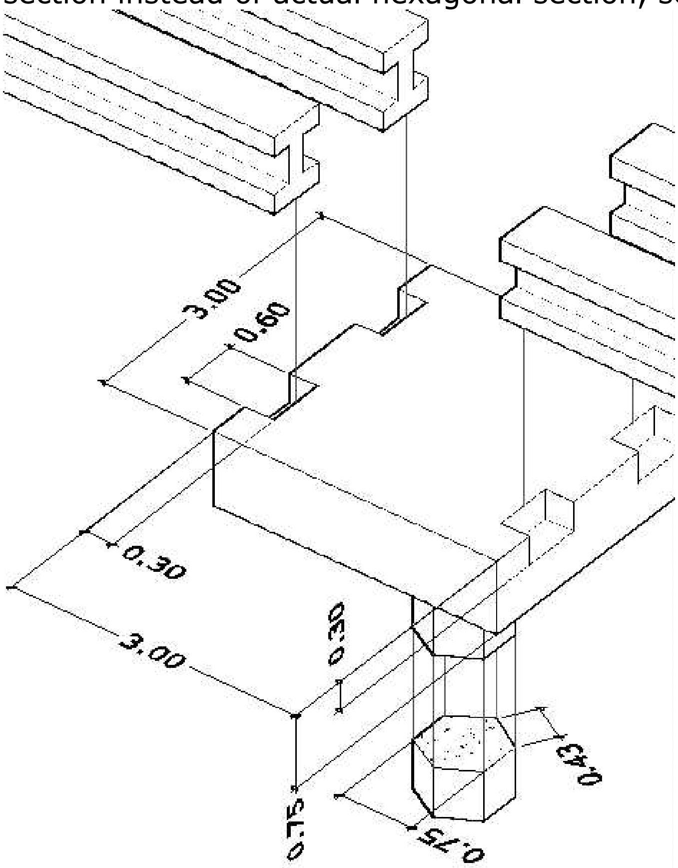


Figure 9.7-5: Bridge girders and column for Problem 9.7-4.

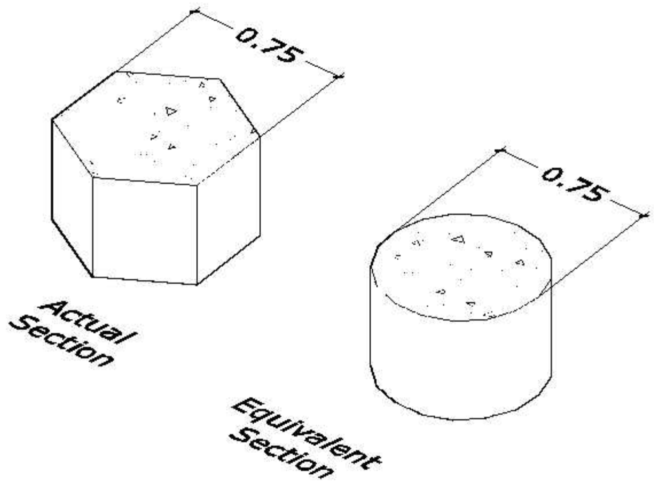


Figure 9.4-5: Transformation of an octagonal column into the equivalent circular section. Reproduced for convenience

Answers

Applied Factored Loads:

$$P_D = (150\ kN \times 4)_{Girders\ Reactions} + \left((3.0^2 \times 0.75 - 0.3^2 \times 0.6 \times 4)m^3 \times 24 \frac{kN}{m^3}\right)_{Cap\ Selfweight} + \left(\left(\frac{\pi 0.75^2}{4} \times 4\right)m^3 \times 24 \frac{kN}{m^3}\right)_{Column\ Selfweight}$$

$$P_D = (600\ )_{Girders\ Reactions} + (157)_{Cap\ Selfweight} + (42)_{Column\ Selfweight} = 799\ kN$$

As all dead loads are symmetric, then M_{Dead} is zero.

$$P_L = (100\ kN \times 2)_{Live\ Load\ Reactions\ from\ Right\ Side\ Span} = 200\ kN$$

$$M_L = P_L \times Arm\ = 200\ kN \times \left(\frac{3.0}{2} - \frac{0.3}{2}\right) m = 270\ kN.m$$

Then factored forces will be:

$$P_u = \ 1\,279\ kN, M_u = 432\ kN.m \Longrightarrow e = 0.338\ m \Longrightarrow \frac{e}{h} = 0.45$$

Longitudinal reinforcement:

- Compute γ:

  $$\gamma h = 750 - 40 \times 2 - 10 \times 2 - 25 = 625 \Longrightarrow \gamma = \frac{\gamma h}{h} = \frac{625}{750} = 0.83$$

  Say γ = 0.8.
- Based on $\frac{e}{h}$, the strength reduction factor "∅" can be taken equal to 0.75:

  $$K_n = \frac{P_u}{\emptyset f_c' A_g} = \frac{1\,279\,000\ N}{0.75 \times 28 \times \frac{750^2 \times \pi}{4}} = 0.138$$
- Based on interaction diagram shown ***Figure 9.7-6*** below, it seems that required ratio ρ_g is less than 1%, then ACI minimum reinforcement ratio should adopted:

  $$\rho_g = 0.01 \Longrightarrow A_{st} = \ 0.01 \times \frac{750^2 \times \pi}{4} = 4\,416 \text{mm}^2 \Longrightarrow No.of\ Rebars\ = \ \frac{4\,416}{510} = 8.65$$

  Then use ***9Ø25mm***.

Spiral Design:

- Spiral diameter:

  $$\because \varphi_{Spiral} = 10^{mm}\ \ \therefore Ok.$$
- Compute $\rho_{SMinimum}$:

  $$D_c = 750^{mm} - 2 \times 40^{mm} = 670^{mm}$$

  $$A_c = \frac{\pi \times 670^2}{4} = 352\,386\ mm^2, A_g = \frac{\pi \times 750^2}{4} = 441\,562\ mm^2$$

  $$\rho_{sMinimum} = 0.45 \left(\frac{A_g}{A_c} - 1\right) \frac{f_c'}{f_{sy}} = 0.45 \left(\frac{441\,562}{352\,386} - 1\right) \times \frac{28}{420} = 7.59 \times 10^{-3}$$

  $$A_{sp} = \frac{\pi \times 10^2}{4} = 78.5^{mm^2}$$

  $$S_{Required} = \frac{4A_{sp}}{D_c\ \rho_{sMinimum}} \Longrightarrow \therefore S_{Required} = \frac{4 \times 78.5^{mm^2}}{670^{mm} \times 7.59 \times 10^{-3}} = 61.7^{mm}$$

Try $\varphi 10^{mm}$ @ 60^{mm}

Use $\boldsymbol{\varphi 10^{mm}}$ @ $\boldsymbol{60^{mm}}$

- The final section of the column is shown in ***Figure 9.7-7*** below.

  $\because S_{Clear} = 50^{mm} < 80^{mm} \quad \therefore Ok.$

  $\because S_{Clear} = 50mm > 25^{mm} \quad \therefore Ok.$

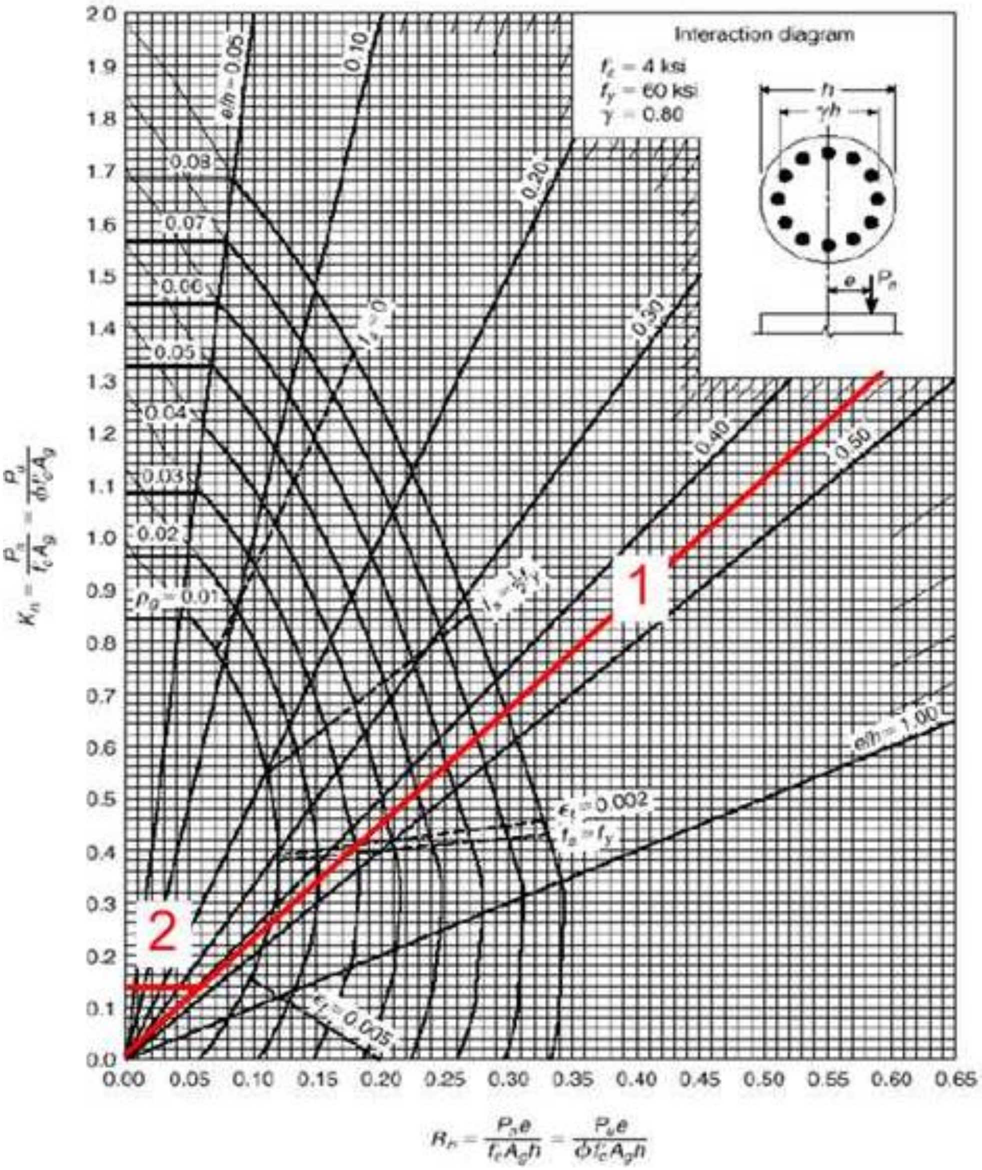


Figure 9.7-6: Inteaction diagram for the equivalent circular column of Problem 9.7-4.

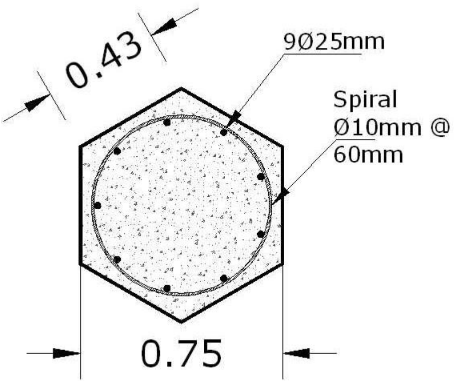


Figure 9.7-7: Final column section for the column of Problem 9.7-4.

9.8 ANALYSIS OF COLUMNS SUBJECTED TO COMPRESSION FORCE AND BIAXIAL MOMENTS

9.8.1 A Circular Column under an Axial Force and Biaxial Moments

- Circular columns have polar symmetry and thus the same ultimate capacity in all directions.
- Then if a circular column is subjected to biaxial moments, these moments can be transformed into an equivalent uniaxial bending moment that computed based on the following relations:

$$e = \sqrt{e_x^2 + e_y^2}$$

or

$$M_u = \sqrt{M_{ux}^2 + M_{uy}^2}$$

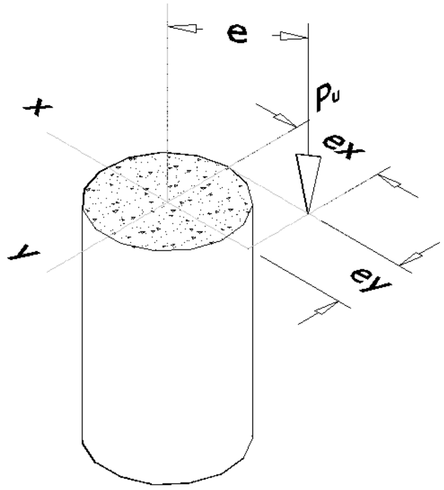


Figure 9.8-1: Circular column under axial force and biaxial moments.

Example 9.8-1

Use an appropriate interaction diagram to determine P_n value that can be supported by circular column shown in ***Figure 9.8-2*** below.
Assume that: $f_y = 420MPa$, $f_c' = 28\ MPa$ and $A_{Bar} = 645\ mm^2$.

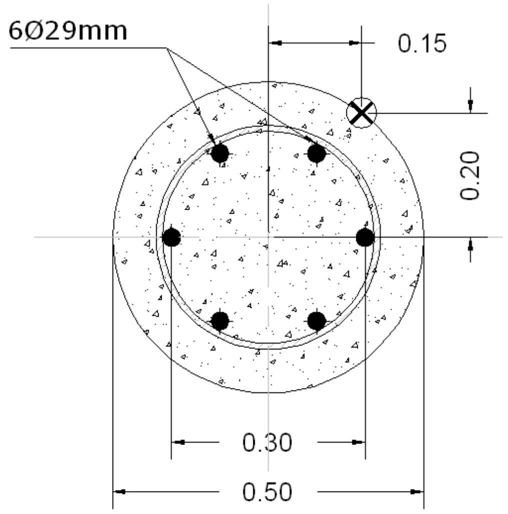


Figure 9.8-2: Circular column of Example 9.8-1.

Solution

- The equivalent eccentricity for the resultant moment can be determined based on the following relation.

$$e = \sqrt{e_x^2 + e_y^2} = \sqrt{0.15^2 + 0.20^2} = 0.25 \Longrightarrow \frac{e}{h} = \frac{0.25}{0.50} = 0.5$$

$$\gamma = \frac{0.30}{0.50} = 0.6$$

- Based on γ value and as the column is a circular column, then the interaction diagram of ***Figure 9.8-3*** below has been adopted.

$$A_{st} = 6 \times 645\, mm^2 = 3\,870\, mm^2, A_g = \frac{\pi \times 500^2}{4} = 196\,250\, mm^2 \Rightarrow \rho_g = \frac{3\,870\, mm^2}{196\,250\, mm^2} \approx 2.0\%$$

$$K_n = 0.25 = \frac{P_n}{A_g f_c'} \Rightarrow P_n = 0.25 \times 196\,250\, mm^2 \times 28 \frac{N}{mm^2} = 1\,374\, kN$$

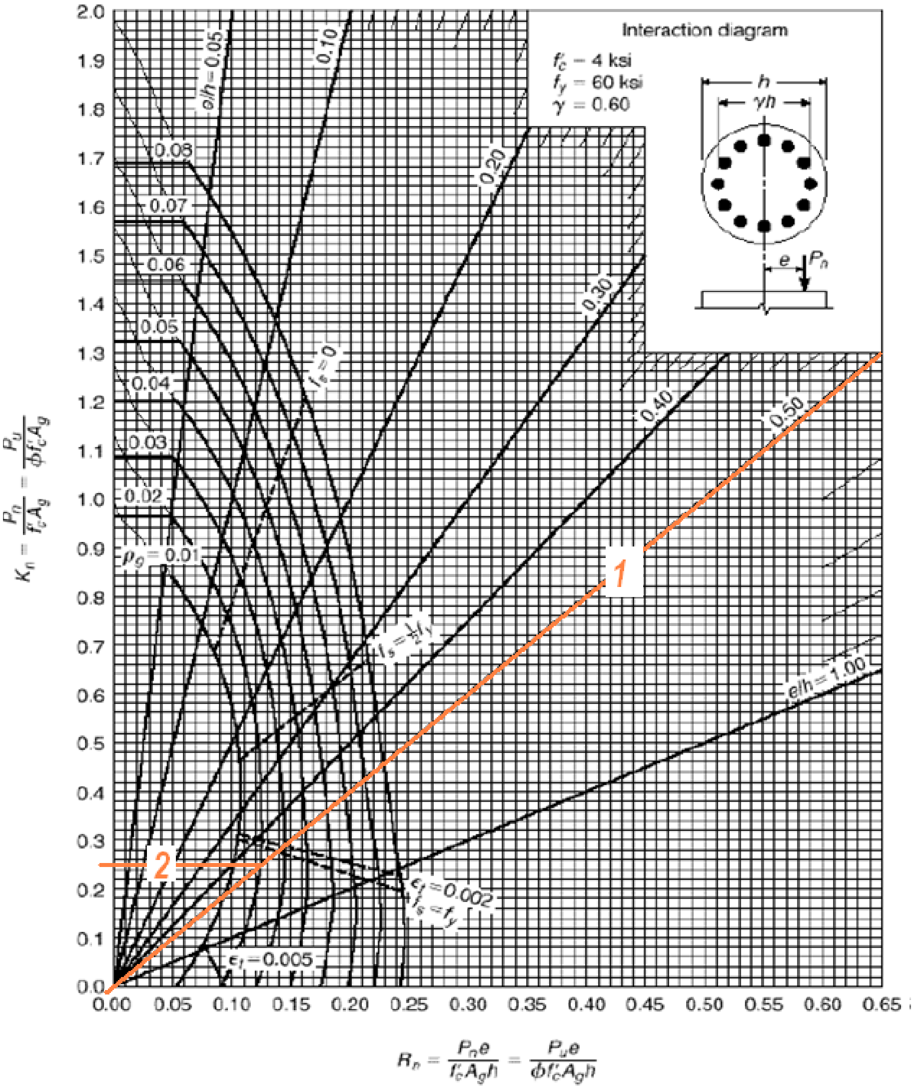


Figure 9.8-3: Interaction diagram adopted for circular column of Example 9.8-1.

9.8.2 Analysis of a Rectangular Column under an Axial Force and Biaxial Moments

9.8.2.1 Basic Concepts

- The article aims to analyze a rectangular column under a compression force and biaxial moments.
- With analysis problem, column is assumed to has known dimensions and known reinforcement and to be checked for resisting a force set consists of a compression force and biaxial moments (see ***Figure 9.8-4*** below).

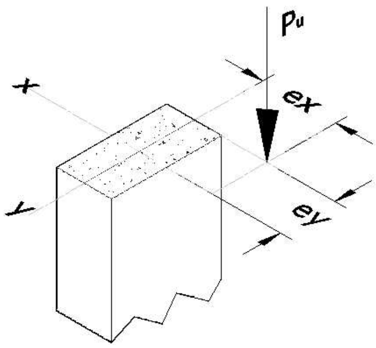


Figure 9.8-4: A Rectangular column under an axial force and biaxial moments.

- Criterion for Including or Negating the Effect of the Smaller Moment:
- According (Nilson, Design of Concrete Structures, 14th Edition, 2011), following criterion can be adopted to consider the minor bending moment into consideration: "***In general, biaxial bending should be taken into account when the estimated ratio of smaller to larger bending moments approaches or exceed 0.2***".
- As for a column with compression force and uniaxial moment, analysis of a column with a compression force and biaxial moments starts with construction of column interaction diagram.
- If a specific load set is located inside or on the interaction diagram, then this column is adequate to resist applied load set safely and vice versa.
- Typical interaction diagram for a rectangular column under a compression force and biaxial moments is shown in ***Figure 9.8-5*** below.

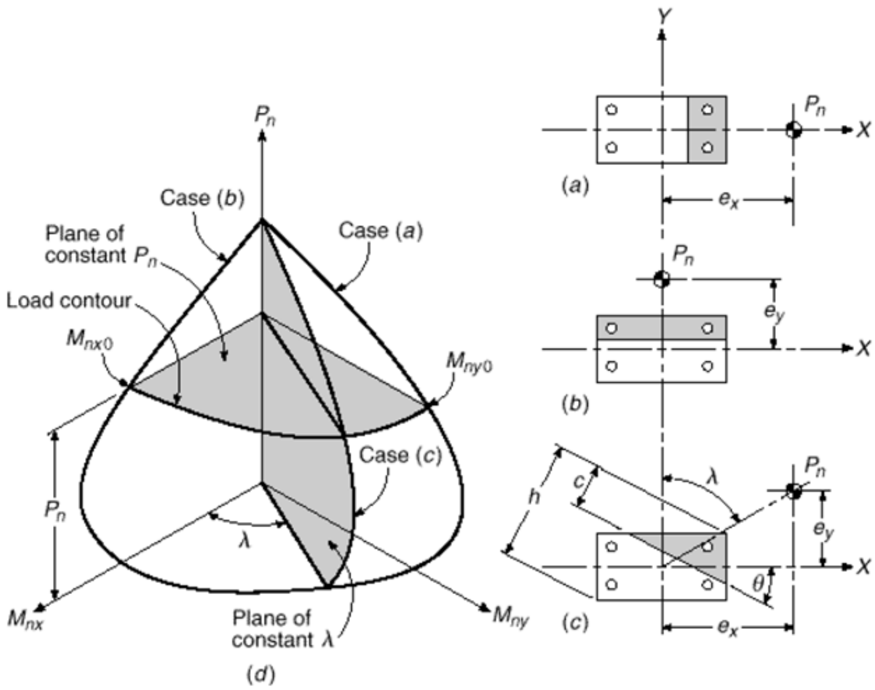


Figure 9.8-5: Interaction diagram for compression plus biaxial bending: (a) uniaxial bending about Y axis; (b) uniaxial bending about X axis; (c) biaxial bending about diagonal axis; (d) interaction surface.

- It is difficult to draw or represent of a three-dimension interaction diagram (especially without a computer program), then for practical applications curve of ***Figure 9.8-5*** is usually approximated based on one of two methods presented in Articles ***9.8.2.2*** and ***9.8.2.3*** below.

9.8.2.2 Reciprocal Load Method

- It is a simple approximate design method developed by Bresler.
- It has been satisfactorily verified by comparison with results of extensive tests and accurate calculations.
- The method can be summarized as follows:
  - Re-draw the interaction diagram in terms of ($\frac{1}{P_n}$, e_x, and e_y) instead of (P_n, M_x, and M_y) to obtain the surface "S" that shown in ***Figure 9.8-6*** below. Based on the new terms, the main unknown in an analysis problem is $\frac{1}{P_n}$.
  - Use a plane S_1 that defined by points A, B, and C to approximate the original surface S. Then the approximate value of unknown $\frac{1}{P_n}$ can be computed based on the following relation:
    $$\frac{1}{P_n} = \frac{1}{P_{nx0}} + \frac{1}{P_{ny0}} - \frac{1}{P_0}$$
    where
    P_n is approximate value of nominal load in biaxial bending with eccentricities e_x and e_y
    P_{ny0} is nominal load when only eccentricity e_x is present $(e_y = 0)$ (can be computed from a specific interaction diagram for an axial force and uniaxial bending moment).
    P_{nx0} is nominal load when only eccentricity e_y is present $(e_x = 0)$ (can be computed from a specific interaction diagram for an axial force and uniaxial bending moment).
    P_0 is nominal load for concentrically loaded column (can be computed from a specific interaction diagram for an axial force and uniaxial bending moment or may be computed based on relations given in article 2 but without factors of 0.8 for tied columns and 0.85 for spiral columns).
  - Finally, column adequacy can be checked based on the following comparison:
    If
    $P_u \leq \emptyset P_n$
    Then the column is adequate. Else the column is inadequate to support a factored applied load of P_u acting at eccentricities e_x and e_y.

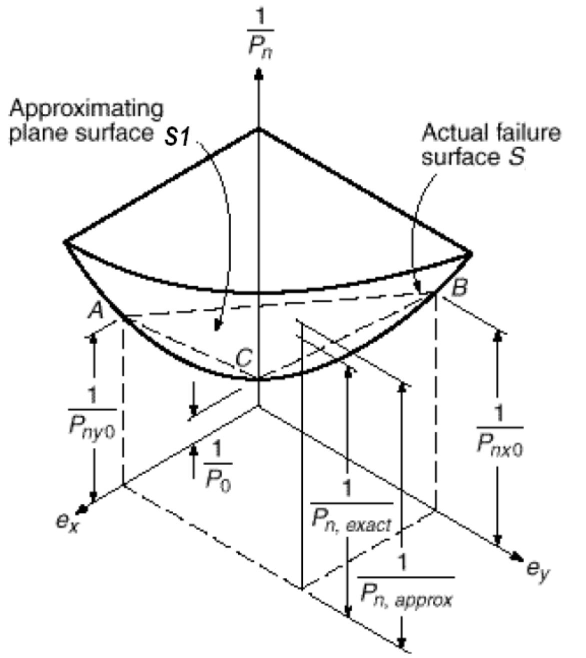


Figure 9.8-6: Interaction surfaces for the reciprocal load method.

- Notes on Reciprocal Load Method:
  The reciprocal load method is very simple to use, ***but the representation of the curved failure surface by an approximating plane is not reliable in the range of large eccentricities, where failure is initiated by steel yielding***.

Example 9.8-2

The 300 by 500mm column shown in ***Figure 9.8-7*** below is reinforced with eight No. 29 bars ($A_{Bars} = 645mm^2$) arranged around the column perimeter. A factored load P_u of $1\,134\,kN$ is to be applied with eccentricities $e_y = 75mm$ and $e_x = 150mm$. Material strengths $f_y = 420MPa$ and $f_c' = 28\,MPa$ are specified. Check the adequacy of the column using the reciprocal load method.

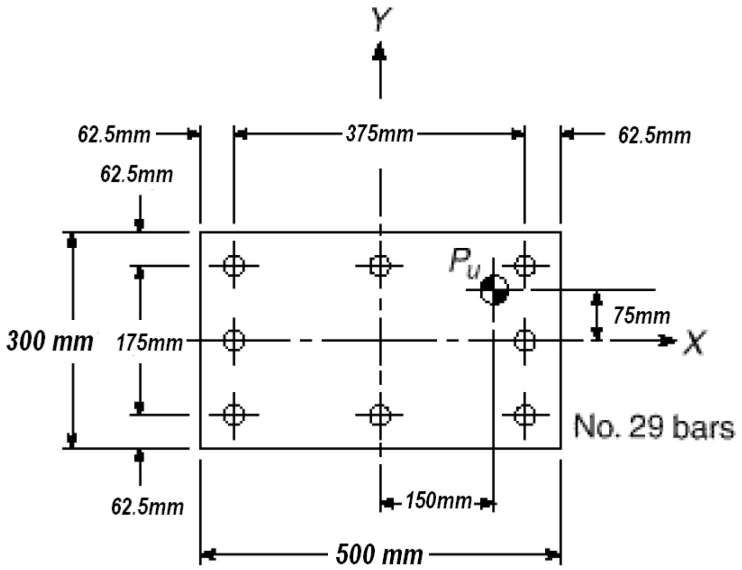


Figure 9.8-7: Column for Example 9.8-2.

Solution

- Considering the bending moment about y-axis (To compute P_{ny0}):
  $A_{st} = 8 \times 645mm^2 = 5\,160\,mm^2, A_g = 500 \times 300 = 150\,000\,mm^2 \Longrightarrow \rho_g = 3.44\,\%$
  $\frac{e}{h} = \frac{150}{500} = 0.3, \gamma = \frac{375}{500} = 0.75$
  As we don't have an interaction diagram with $\gamma = 0.75$, then we'll use the average value for $\gamma = 0.70$ and $\gamma = 0.80$, see ***Figure 9.8-8*** below.
  $K_{n\,avg.} = \frac{P_{ny0}}{A_g f_c'} = \frac{0.62 + 0.66}{2} = 0.64$
  $P_{ny0} = 0.64 \times 150\,000 \times 28 = 2\,688\,kN$
- Considering the bending moment about x-axis (To compute $\mathbf{P_{xy0}}$):
  $\frac{e}{h} = \frac{75}{300} = 0.25, \gamma = \frac{175}{300} = 0.58$
  Say $\gamma = 0.60$
  $K_n = \frac{P_{nx0}}{A_g f_c'} = 0.65$
  $P_{nx0} = 0.65 \times 150\,000 \times 28 = 2\,730\,kN$
- Consider the case of axially load column (To compute $\boldsymbol{P_0}$):
  $P_0 = 0.85 \times 28 \times (150\,000 - 5\,160) + 420 \times 5\,160 = 5\,614\,kN$
- Compute the approximate column strength when it is subjected to an axial force and biaxial moments:
  $\frac{1}{P_n} = \frac{1}{P_{nx0}} + \frac{1}{P_{ny0}} - \frac{1}{P_0} = \frac{1}{2\,730} + \frac{1}{2\,688} - \frac{1}{5\,614} = 5.60 \times 10^{-4}$
  $P_n = 1\,785\,kN\ ?\,P_{nmax} = 0.80 \times P_0 = 0.8 \times 5\,614\,kN = 4\,491\,kN$
  $P_n = 1\,785\,kN < P_{nmax} = 4\,491\,kN \quad Ok.$
- Finally, check column adequacy based on following comparison:
  $\emptyset P_n = 0.65 \times 1\,785\,kN = 1\,160\,kN > 1\,134\,kN$
  The column is adequate according to reciprocal load method.

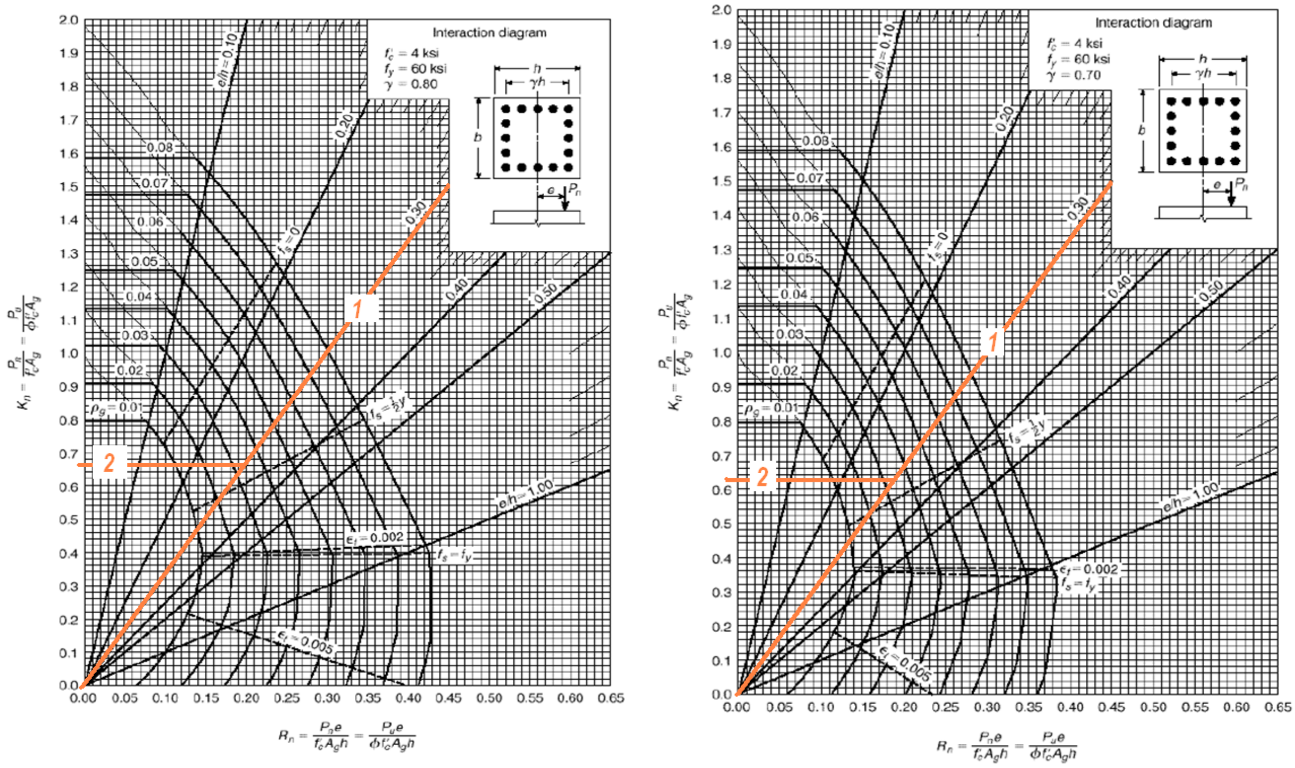


Figure 9.8-8: Interaction diagrams adopted in Example 9.8-2.

9.8.2.3 Load Contour Method

- Checking of a column adequacy for an axial force and biaxial moments (i.e., checking if the load state is inside or outside the column interaction diagram) can also be done based on checking if the load state is inside or outside the ***Load Contour*** for a plane of constant force P_n.

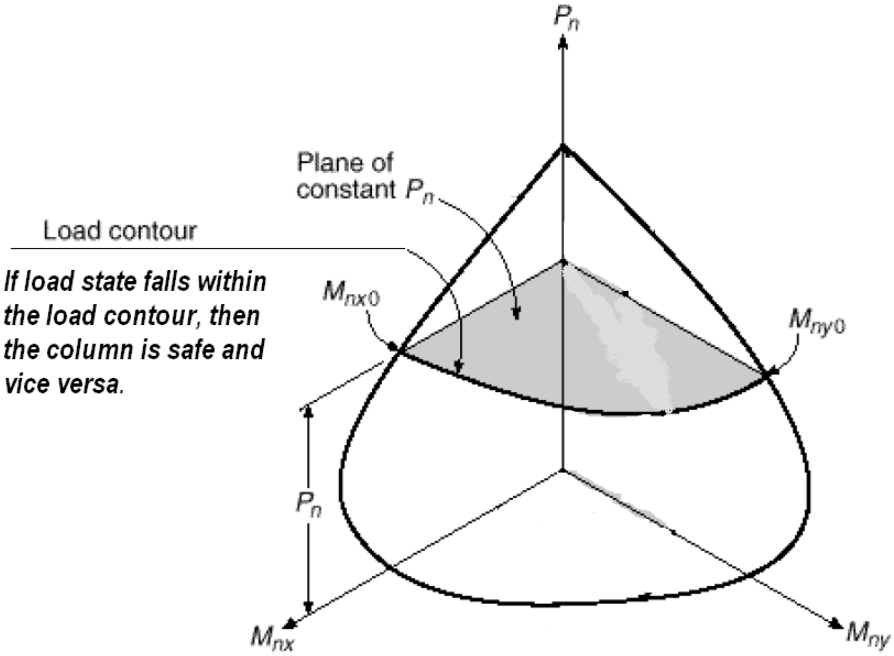


Figure 9.8-9: Interaction contours at constant P_n.

- If load state falls within the *Load Contour*, then the column is safe and vice versa.
- General form of load contour curve can be approximated by a nondimensional interaction equation:

  $$\left(\frac{M_{nx}}{M_{nx0}}\right)^{\alpha 1} + \left(\frac{M_{ny}}{M_{ny0}}\right)^{\alpha 2} = 1.0$$

  where

  $M_{nx} = P_n e_y$

  $M_{nx0} = M_{nx}$ when $M_{ny} = 0.0$

  $M_{ny0} = M_{ny}$ when $M_{nx} = 0.0$

  α_1 and α_2 are exponents depending on:
  - Column dimensions.
  - Amount and distribution of steel reinforcement.
  - Stress-strain characteristics of steel and concrete.
  - Amount of concrete cover.
  - Size of lateral ties or spiral.
- When $\alpha_1 = \alpha_2 = \alpha$, the shapes of such interaction contours are as shown in ***Figure 9.8-10*** below for specific values.
- Values of $\boldsymbol{\alpha}$:
  - α values fall in the range from 1.15 to 1.55 for square and rectangular columns.
  - Values near the lower end of that range are the more conservative.

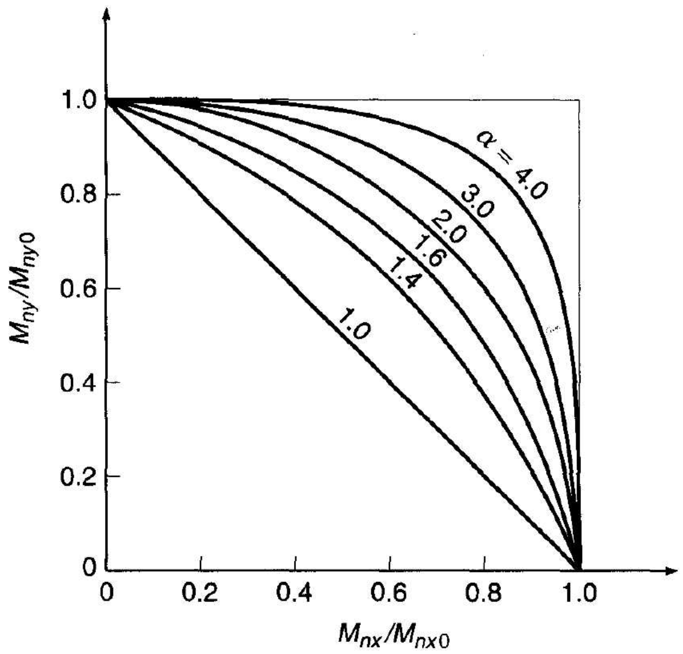


Figure 9.8-10: Interaction contours at constant P_n for varying α.

- More Useful Form of Load Contour:
    - Introducing of the ACI factors for reducing nominal axial and flexure strengths to design strength presents no difficulty. With the appropriate ϕ factors applied to P_n , M_{nx}, and M_{ny}, a new failure surface is defined:
    $$\left(\frac{\phi M_{nx}}{\phi M_{nx0}}\right)^{\alpha} + \left(\frac{\phi M_{ny}}{\phi M_{ny0}}\right)^{\alpha} = 1.0$$
    - The above equation can be rewritten in terms of applied moments:
    $$\left(\frac{M_{ux}}{\phi M_{nx0}}\right)^{\alpha} + \left(\frac{M_{uy}}{\phi M_{ny0}}\right)^{\alpha} = 1.0$$
- How to Use Load Contour:
    - In practice, the values of P_u, M_{ux}, M_{uy} are known from the analysis of the structure.
    - For a trial column section, the values of M_{nx0} and M_{ny0} corresponding to the load P_u can easily be found by the usual methods for uniaxial beading.
    - It can be confirmed that a particular combination of factored moments falls within the load contour (safe design) or outside the contour (failure), and the design modified if necessary.
- Notes on Load Contour Method:
    - Selection of the appropriate value of the exponent $\propto$ is made difficult by a number of factors relating to column shape and bar distribution.
    - For many cases, the usual assumption that $\alpha_1 = \alpha_2$ is a poor approximation.

Example 9.8-3

Re-check the column of ***Example 9.8-2*** by the Load Contour Method. Assume that the exponent ∝ conservatively taken equal to 1.15.

Solution

- Nominal Bending Strength about y-axis ($\emptyset M_{ny0}$):

  $\gamma = \frac{0.375^m}{0.5^m} = 0.75$

  $A_g = 500^{mm} \times 300^{mm} = 150\ 000\ mm^2$

  $A_{st} = 8 \times 645 mm^2 = 5\ 160\ mm^2$

  $\rho_g = 3.44\ \%$

  Based on above definition of ($\emptyset M_{ny0}$), ***one must start the solution with*** K_n ***value to compute the required*** R_n ***value based on steel reinforcement ratio***. Required R_n can't be computed based on e/h ratio as this solution will not be consistent with the definition of ($\emptyset M_{ny0}$). Based on interaction diagrams presented in ***Figure 9.8-11*** below, column strength $\emptyset M_{ny0}$ would be:

  $K_n = \frac{P_u}{\emptyset f'_c A_g} = \frac{1\ 134\ 000}{0.65 \times 28 \times 150000} = 0.41$

  $R_{n\ avg.} = \left(\frac{\emptyset M_{ny0}}{f'_c A_g h}\right)_{avg} = \frac{0.21 + 0.24}{2} = 0.22$

  $\emptyset M_{ny0} = 0.65(0.22 \times 28 \times 150\ 000 \times 500) = 300\ kN.m$

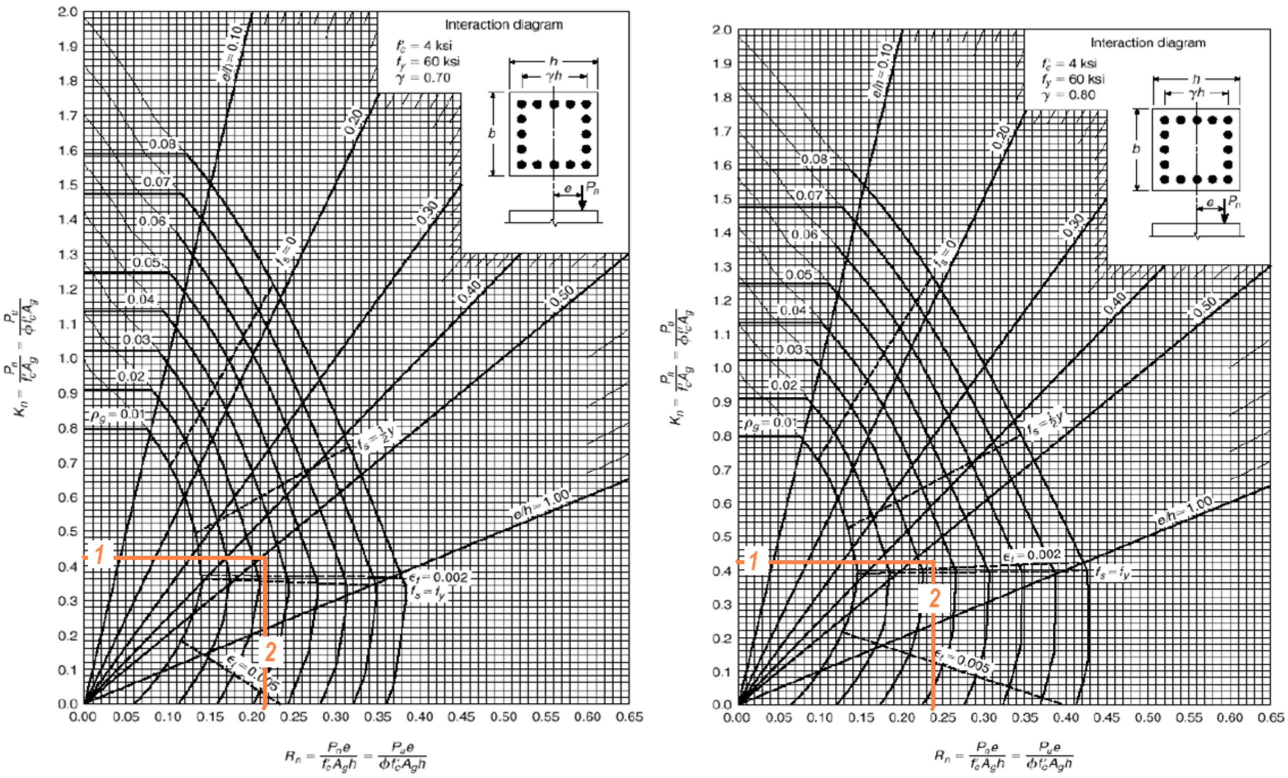


Figure 9.8-11: Interaction diagrams adopted to compute $\emptyset M_{ny0}$ of Example 9.8-3.

- Nominal Bending Strength about x-axis ($\emptyset M_{nx0}$):

  $\gamma = \frac{0.175^m}{0.3^m} = 0.58$

  Say $\gamma = 0.60$, and based on interaction diagram of ***Figure 9.8-12*** below.

  $K_n = \frac{P_u}{\emptyset f'_c A_g} = 0.41\ (as\ befor)$

  $R_n = \left(\frac{\emptyset M_{nx0}}{f'_c A_g h}\right) = 0.19$

  $\emptyset M_{nx0} = 0.65(0.19 \times 28 \times 150\ 000 \times 300) = 156\ kN.m$

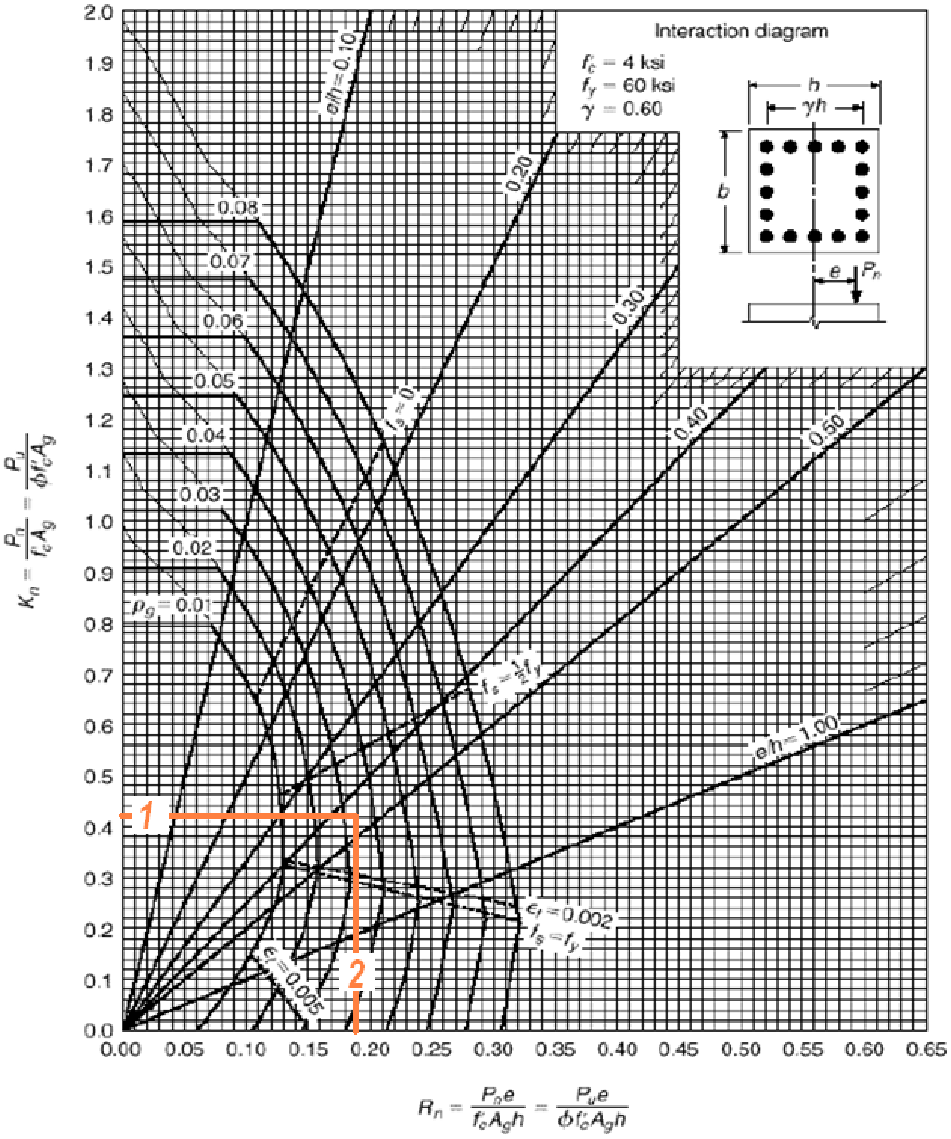


Figure 9.8-12: Interaction diagram adopted to compute $\emptyset M_{nx0}$ of Example 9.8-3.

- Check column adequacy based on Load Contour Method:

$M_{uy} = 1\,134\ kN \times 0.150m = 170\ kN.m$

$M_{ux} = 1\,134\ kN\ \times 0.075m = 85\ kN.m$

$$\left(\frac{M_{ux}}{\phi M_{nx0}}\right)^{1.15} + \left(\frac{M_{uy}}{\phi M_{ny0}}\right)^{1.15}\ ?\ 1.0$$

$$\left(\frac{85}{156}\right)^{1.15} + \left(\frac{170}{300}\right)^{1.15}\ ?\ 1.0$$

$0.548 + 0.566 = 1.1 \approx\ 1.0\quad Ok.$

The column is adequate according to Load Contour Method.

Table of Contents

10.1 INTRODUCTION AND BASIC CONCEPTS

10.1.1 Definition of Slender Columns

- A column is said to be slender if its cross-sectional dimensions are small compared with its length.
- The degree of slenderness is generally expressed in terms of the slenderness ratio ℓ_u/r, where ℓ_u is the unsupported length of the member and r is the radius of gyration of its cross section, equal to:

$$r = \sqrt{\frac{I}{A}}$$ **Eq. 10.1-1**

- According to ***ACI 6.2.5.1***, the radius of gyration r for rectangular column can be determined from ***Eq. 10.1-2***.

$$r_{For\ a\ Rectangular\ Section} = 0.3h$$ **Eq. 10.1-2**

while for circular columns it may be taken as in ***Eq. 10.1-3***.

$$r_{For\ a\ Circular\ Section} = 0.25D$$ **Eq. 10.1-3**

- It has long been known ***that a member of great slenderness will collapse under a smaller compression load than a stocky member with the same cross-sectional dimension***.

Example 10.1-1

With referring to gross homogenous sections, show that Eq. 10.1-2 and Eq. 10.1-3 are rational in nature and can be derived from definition of ***Eq. 10.1-1***.

Solution

For rectangular section:

$$r_{rectangular} = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{bh^3}{12}}{bh}} = \frac{1}{\sqrt{12}}h = 0.228h \approx 0.3h$$

For circular section:

$$r_{circular} = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi D^4}{64}}{\frac{\pi D^2}{4}}} = \frac{1}{4}D = 0.25D$$

- This article aims to discuss the effects of slenderness on:
  - The strength of axially loaded columns,
  - The strength of columns that subjected to axial force and bending moment.

10.1.2 Effect of Slenderness Ratio on Strength of Axially Loaded Columns

10.1.2.1 Basic Concepts

- Based on experimentally work, the relation between column strength and its slenderness ratio is as shown in ***Figure 10.1-1*** below.

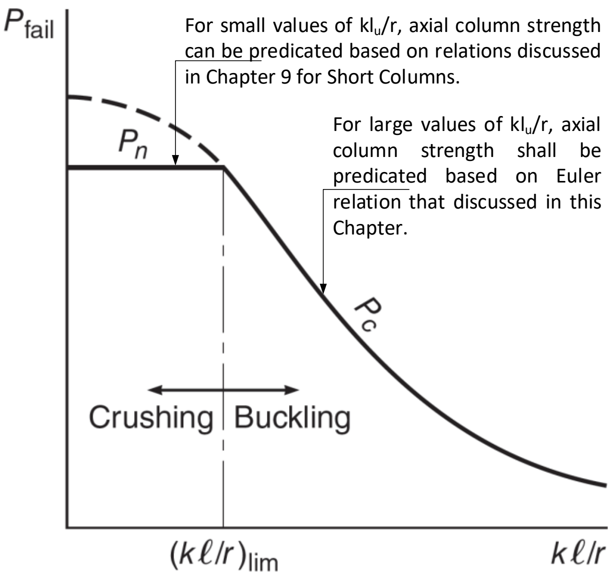


Figure 10.1-1: Effect of slenderness on strength of axially loaded columns.

- It can been shown that, for lower values of kl_u/r (values less than $(kl_u/r)_{Limit}$ in ***Figure 10.1-1*** above) column strength can be predicated by the relation derived in Chapter 9:

$$P_n = 0.85 f_c' (A_g - A_{st}) + A_{st} f_y$$ **Eq. 10.1-4**

- For larger slenderness ratio, column strength can be predicated based on the following relation that derived by Euler more than 200 years ago:

$$P_c = \frac{\pi^2 EI}{(kl_u)^2}$$ **Eq. 10.1-5**

where kl_u is defined as the ***effective length*** and it ***represents the distance between the inflection points***.

- Correspondingly, there is a limiting slenderness ratio $(kl_u/r)_{Limit}$:
  - For values smaller than $(kl_u/r)_{Limit}$ this, failure occurs by simple crushing, regardless of kl_u/r;
  - For values larger than $(kl_u/r)_{Limit}$ failure occurs by buckling, the buckling load or stress decreasing for greater slenderness.

10.1.1.1 Physical Meaning of Euler Load or Critical Load

- An axially loaded column similar to that shown below should be designed for the dominated acting force (axial force P_u in this case).
- However, this column is a part from structure that should be adequate for many decades. During that long age, this column may be subjected to a temporary lateral force due to a minor cause that can't be accounted in the design process. Then this column will be displaced laterally as shown below:

Figure 10.1-2: A column subjected to dominate axial force.

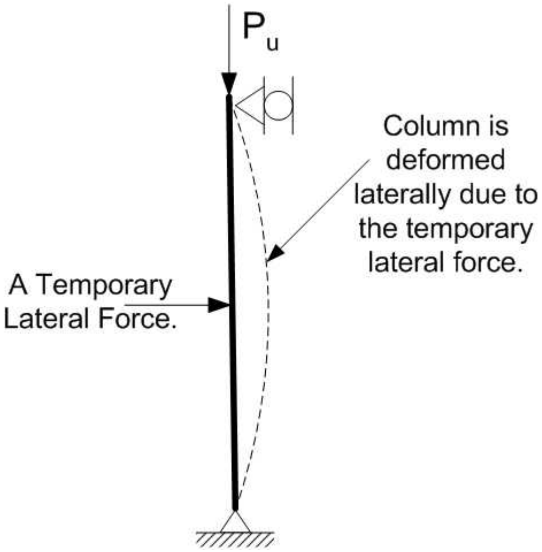


Figure 10.1-3: A column subjected to dominate axial force and to minor or temporary lateral forces.

- It has been noted experimentally, and has been approved analytically, that each column has a critical load (P_c) that when the column is loaded with an axial load P_u less than P_c and subjected to a lateral temporary force at the same time, it will return to its undeform shape when this temporary lateral load remove and vice versa.

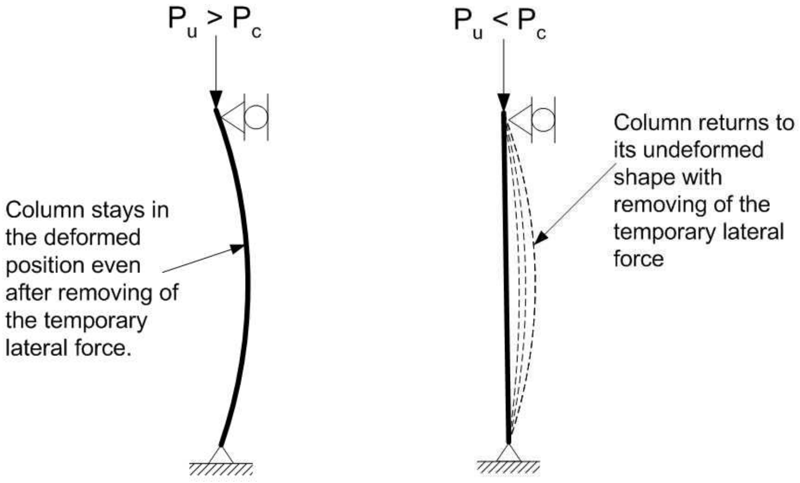


Figure 10.1-4: Behavior of axially loaded column when P_u is less or/and greater than P_c.

- As we have no control on the occurring of such temporary lateral force, then we cannot accept a column that loaded with an axial force equal to or greater than its critical load. Such column is classified as ***unstable column*** in engineering practice.
- Then critical or Euler load represents a very important limit on axial load in columns:
  - For short columns:

    $P_{crashing} < P_c$ **Eq. 10.1-6**

    then

    $P_n = P_{crashing} = 0.85f_c'(A_g - A_{st}) + A_{st}f_y$ **Eq. 10.1-7**
  - For long or slender columns:

    $P_{crashing} \geq P_c$ **Eq. 10.1-8**

    then

    $$P_n = P_c = \frac{\pi^2 EI}{(kl_u)^2}$$ **Eq. 10.1-9**

10.1.2.2 Computing of Buckling Load or Euler Load

- To compute or estimate critical load (or Euler Load) one should compute or estimate following quantities:
  1. Member stiffness or rigidity (EI)
  2. Member unsupported length (l_u).
  3. Effective length factor or k factor.
- Each one of above quantities will be discussed briefly below:

10.1.2.3 ACI Procedure for Computing (EI) to be used in Euler Formula

- In homogeneous elastic members such as steel columns, EI is easily obtained from Young's modulus and the usual moment of inertia.
- Reinforced concrete columns, however, are
  - Nonhomogeneous, since they consist of both steel and concrete,
  - Steel is substantially elastic, concrete is not and is in addition subject to creep and to cracking if tension occurs on the convex side of the column.
- All of these factors affect the effective value of (EI) for a reinforced concrete member.
- According to ***Article 6.6.4.4.4*** of the ACI code effective value of (EI) or, EI_{eff}, as called by the code, can be determined based on any one of the following relations:

  $$EI_{eff} = \frac{0.4E_cI_g}{1+\beta_{dns}}$$ **Eq. 10.1-10**

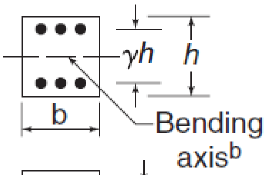
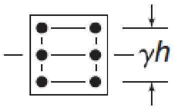
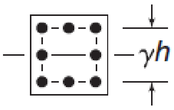
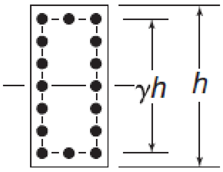
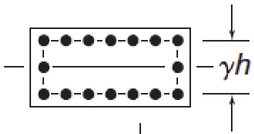
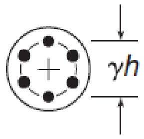
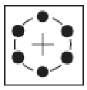
  $$EI_{eff} = \frac{0.2E_cI_g + E_sI_{se}}{1+\beta_{dns}}$$ **Eq. 10.1-11**

  $$EI_{eff} = \frac{E_c\mathrm{I}}{1+\beta_{dns}}$$ **Eq. 10.1-12**

  where E_c is modulus of elasticity of concrete.
  - I_g is moment of inertia of gross section of column.

- E_s is modulus of elasticity of steel.
- I_{se} is moment of inertia of reinforcement about centroidal axis of member cross section. According to (Wight, 2016), calculation of I_{se} can be simplified with refereeing to Table 10.1-1.

Table 10.1-1: Calculations of I_{se}, adopted from (Wight, 2016).

Type of Column	Number of Bars	I_{se}
	—	$0.25A_{st}(\gamma h)^2$
	3 bars per face 6 bars per face	$0.167A_{st}(\gamma h)^2$ $0.117A_{st}(\gamma h)^2$
	8 bars (3 per face) 12 bars (4 per face) 16 bars (5 per face)	$0.187A_{st}(\gamma h)^2$ $0.176A_{st}(\gamma h)^2$ $0.172A_{st}(\gamma h)^2$
	$h = 2b$ 16 bars as shown About strong axis	$0.128A_{st}(\gamma h)^2$
	$b = 2h$ About weak axis	$0.219A_{st}(\gamma h)^2$
	—	$0.125A_{st}(\gamma h)^2$
	—	$0.125A_{st}(\gamma h)^2$

- I is the effective moment of inertial computed based on ***Table 10.1-2*** below.
- β_{dns} is ratio of maximum factored axial sustained axial load to maximum factored axial load associated with the same load combination.

Table 10.1-2: Alternative moments of inertia for elastic analysis at factored load, *Table 6.6.3.1.1(b)* **of the ACI code.**

Member	Alternative value of I for elastic analysis		
	Minimum	I	Maximum
Columns and walls	$0.35I_g$	$\left(0.80+25\frac{A_{st}}{A_g}\right)\left(1-\frac{M_u}{P_u h}-0.5\frac{P_u}{P_o}\right)I_g$	$0.875I_g$
Beams, flat plates, and flat slabs	$0.25I_g$	$(0.10+25\rho)\left(1.2-0.2\frac{b_w}{d}\right)I_g$	$0.5I_g$

- Notes on Computing of EI_{eff}:
  - Creep due to sustained loads will increase the lateral deflections of a column and, hence, the moment magnification.

- Creep effects are approximated in design by reducing the stiffness EI_{eff} by dividing the short-term EI provided by the numerator ***Eq. 10.1-10*** through ***Eq. 10.1-12*** by $(1 + \beta_{dns})$.
- For simplification, ***it can be assumed that*** $\beta_{dns} = 0.6$***. In this case*** Eq. 10.1-10 ***becomes*** $EI_{eff} = 0.25E_cI_g$.
- In reinforced concrete columns subject to sustained loads, creep transfers some of the load from the concrete to the longitudinal reinforcement, increasing the reinforcement stresses. In the case of lightly reinforced columns, this load transfer may cause the compression reinforcement to yield prematurely, resulting in a loss in the effective EI. Accordingly, both the concrete and longitudinal reinforcement terms in ***Eq. 10.1-10*** through ***Eq. 10.1-12*** are reduced to account for creep.
- The equations in ***Table 10.1-2*** above provide more refined values of I considering:
  - Axial load,
  - Eccentricity,
  - Reinforcement ratio,
  - Concrete compressive strength.

10.1.2.4 Column Unsupported Length (ℓ_u)

- The unsupported length of a compression member, ℓ_u, shall be taken as the clear distance between floor slabs, beams, or other members capable of providing lateral support in the direction being considered.
- Where column capitals or haunches are present, ℓ_u shall be measured to the lower extremity of the capital or haunch in the plane considered."

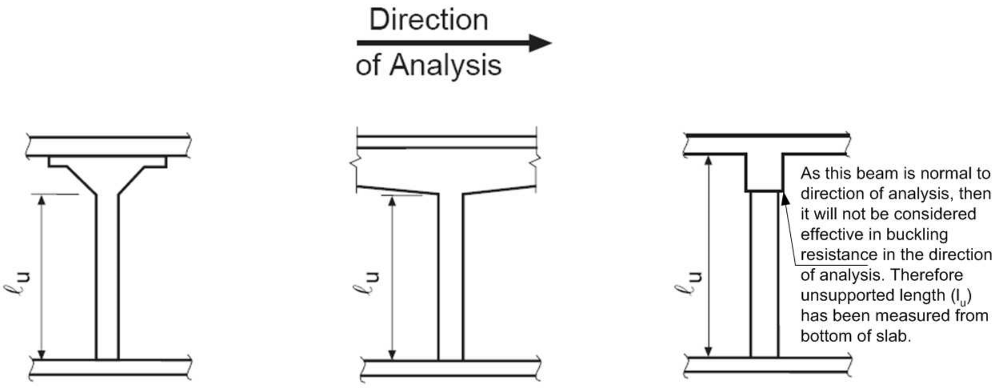


Figure 10.1-5: Unsupported column length, ℓ_u.

10.1.2.5 Computing of Effective Length Factor (or k Factor)

- Meaning of Effective Length Factor (or k Factor):
  - Above Euler relation has been derived originally for simple boundary conditions (i.e. has been derived for a column that has hinge support at both ends). For this column Euler load or (critical load) is:

    $$P_c = \frac{\pi^2 EI}{(l_u)^2} \qquad \textbf{Eq. 10.1-13}$$

  - For other boundary conditions, k factor (effective length factor) can be used to transform length of the column under consideration to a length of an equivalent column with both ends are pinned.
  - For example, assume that we intend to compute the critical load for a cantilever column that has 4m height. As cantilever has k = 2 (as will be discussed below), then form buckling analysis point of view, behavior of this cantilever column will be similar to behavior of pinned column with length equal to ($klu = 2x4 = 8m$). Based on this reasoning, one can conclude that:

    $$P_{c\ for\ Cantilever\ Column} = \frac{1}{4} of\ P_{c\ for\ the\ pinned\ column\ that\ has\ same\ Length} \qquad \textbf{Eq. 10.1-14}$$

- k Factor for a Isolated Columns with Typical Support Conditions:
  Above transformation of the actual column to an equivalent pinned column is based on the concept of extending or trimming the length of actual column until arriving to the inflection points, see ***Figure 10.1-6*** and ***Figure 10.1-7*** below.

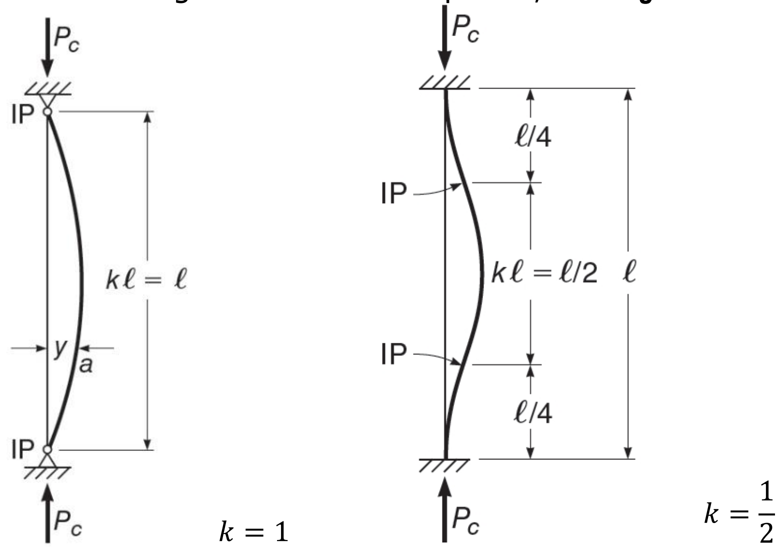


Figure 10.1-6: Effective length for isolated columns, braced columns.

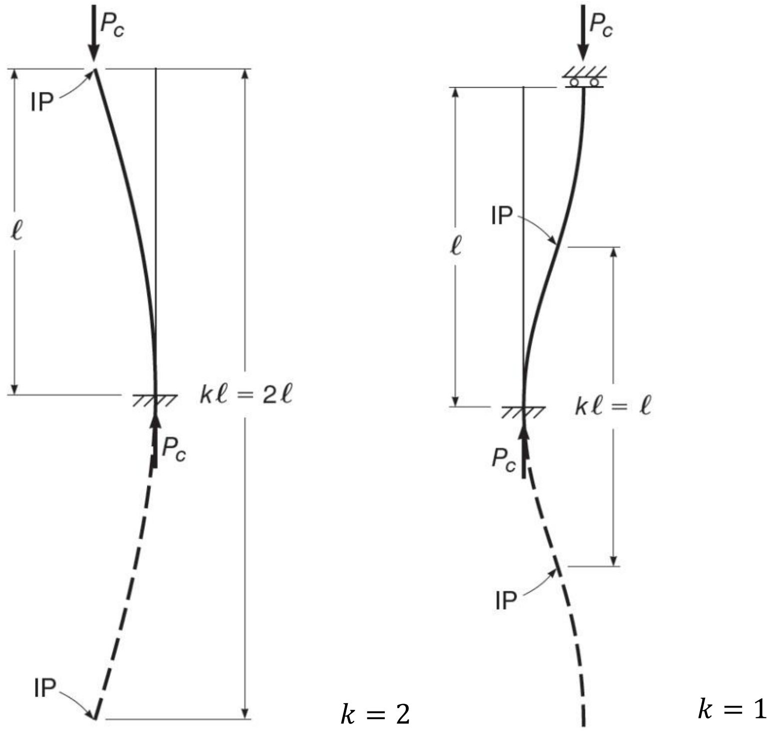
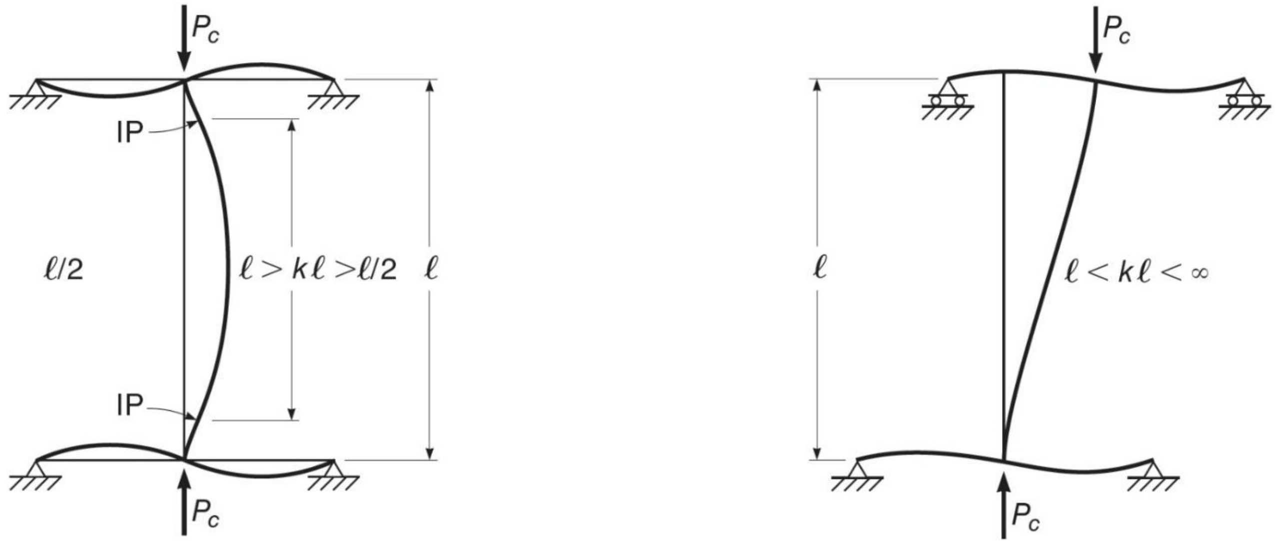


Figure 10.1-7: Effective length for isolated columns, sway columns.

- k Factor for a Column that is Part from a Structure:
  - Columns in real structures are rarely either hinged or fixed but have ends partially restrained against rotation by abutting members. Therefore, the k will be within limits shown in ***Figure 10.1-8*** below.



$\frac{1}{2} < k < 1$ $1 < k < \infty$

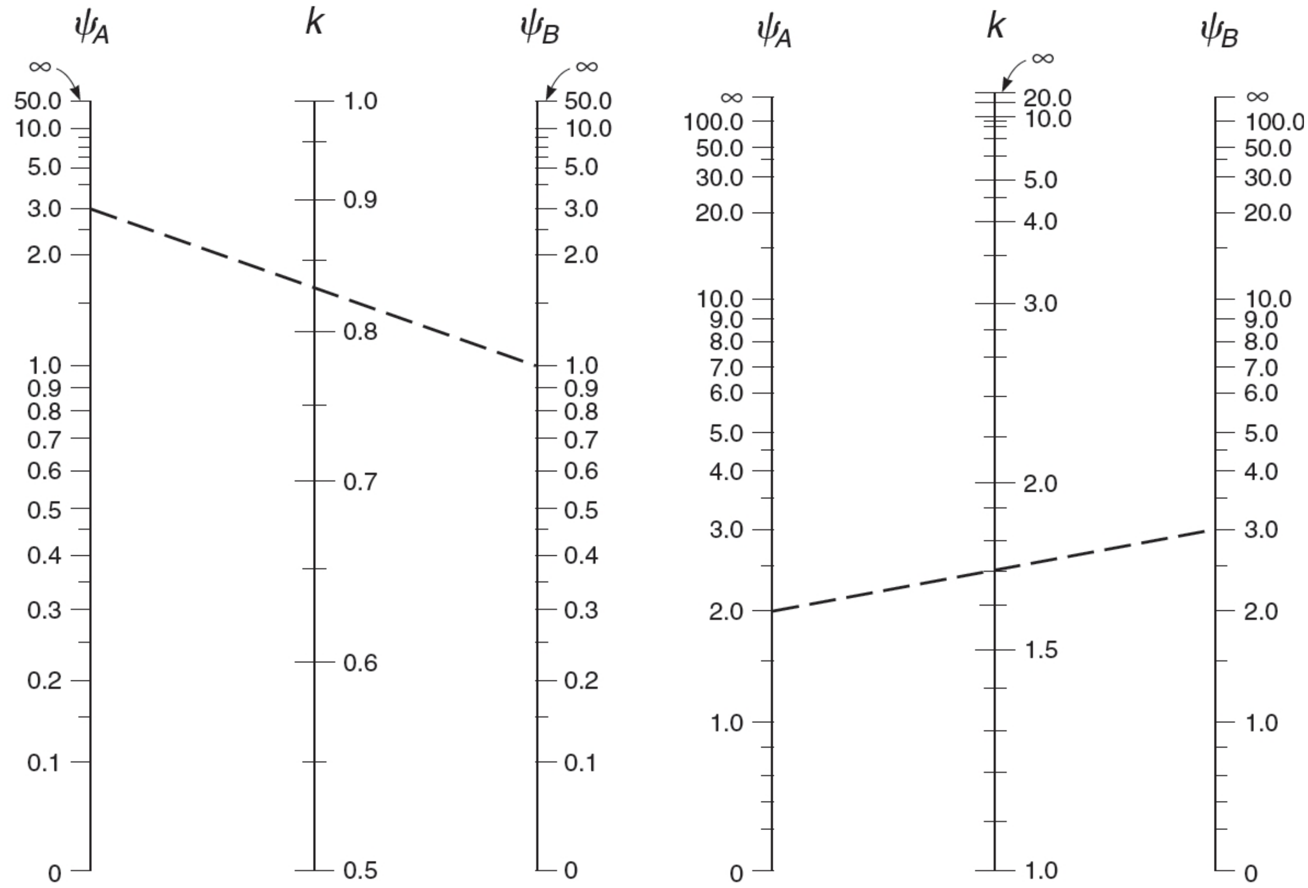
(a) Braced frames. (b) Sway frames.

Figure 10.1-8: Effective length factor for columns that are parts from frames.

  - From ***Figure 10.1-8*** above, one concludes that compression members free to buckle in a sway frame are always considerably weaker than when braced against sway.
  - An approximate but generally satisfactory way of determining (k) is by means of ***alignment charts.*** This method can be summarized as follows:
    - Compute the degree of end restraint at each end based on the following relation:

$$\psi = \frac{\sum \frac{EI}{l}_{Columns}}{\sum \frac{EI}{l}_{Beam}}$$ **Eq. 10.1-15**

    - Based on ψ and frame classification (braced against sway or not), effective length factor (k) can be computed based on alignment charts of ***Figure 10.1-9*** below.



(a) Nonsway frames. (b) Sway frames

Figure 10.1-9: Alignment charts for effective length factors k.

- ψ for Hinge Support:
  Hinge support can be understood as columns that connected to beams with zero stiffness:

$$\psi_{\text{Hinge}} = \lim_{\frac{EI}{l}_{Beam} \to 0} \frac{\sum \frac{EI}{l}_{Columns}}{\sum \frac{EI}{l}_{Beam}} = \infty$$ **Eq. 10.1-16**

- In the same approach, ψ_{Fixed} can be interpreted as columns that connected to infinitely rigid beams.

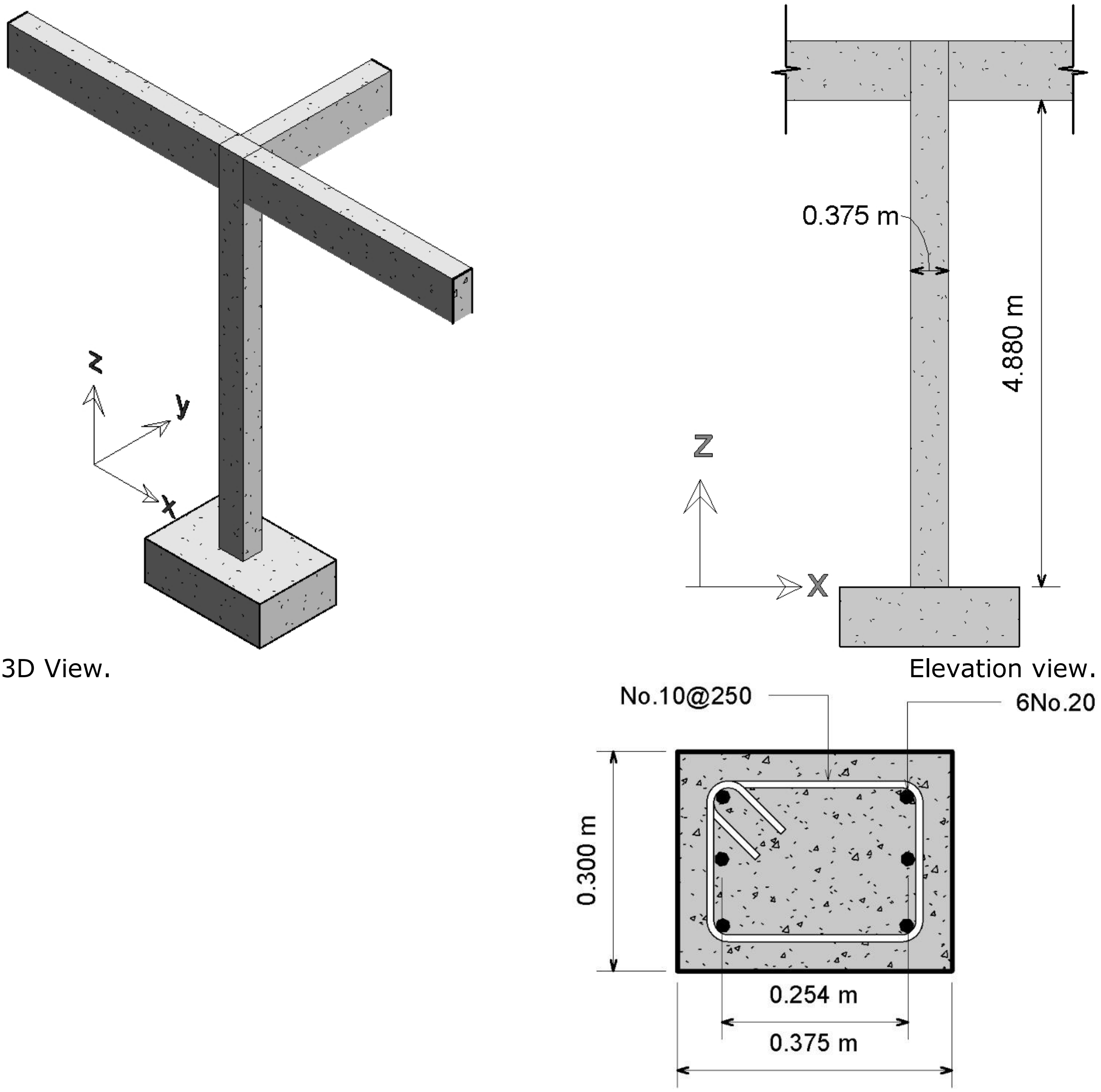
$$\psi_{\text{Fixed}} = \lim_{\frac{EI}{l}_{Beam} \to \infty} \frac{\sum \frac{EI}{l}_{Columns}}{\sum \frac{EI}{l}_{Beam}} = 0$$ **Eq. 10.1-17**

10.1.2.6 Analysis Examples for Euler Loads

Example 10.1-2

Using ***Eq. 10.1-10*** to compute critical load about major axis, i.e. buckling in x-z plane, for the column shown in ***Figure 10.1-10*** below. In your solution, assume that:

- k = 0.83.
- f_c' = 28 MPa.
- Assume the sustained load is only 32.7%.
- Column length is 4.88m.



Cross sectional of the column

Figure 10.1-10: Column for Example 10.1-2.

Solution

Critical load (or Euler load) can be computed based on following relation:

$$P_c = \frac{\pi^2 EI}{(kl_u)^2}$$

According to example statement, the column shall be adopted to compute EI_{eff}:

$$EI_{\text{eff}} = \frac{0.4E_c I_g}{1 + \beta_{dns}}$$

$$E_c = 4700\sqrt{28} = 24870 \text{ MPa}$$

$$I_g = \frac{300 \times 375^3}{12} = 1.32 \times 10^9 \text{mm}^4$$

Based on problem information, $\beta_{dns} = 0.327$.

$$EI_{\text{eff}} = \frac{0.4 \times 24870 \times 1.32 \times 10^9}{1 + 0.327} = 9.89 \times 10^{12}\ N.mm^2$$

Therefore, the critical or Euler load would be:

$$P_c = \left(\frac{\pi^2 \times 9.89 \times 10^{12}\ N.mm^2}{(0.83 \times 4880)^2\ mm^2}\right) \times \frac{1}{1000} = 5944\ \text{kN} \quad \blacksquare$$

Example 10.1-3

Resolve ***Example 10.1-2*** above but with determination of EI_{eff} based on ***Eq. 10.1-11*** to take reinforcement into account.

Solution

According to ***Eq. 10.1-11***, EI_{eff} would be:

$$EI_{eff} = \frac{0.2E_cI_g + E_sI_{se}}{1 + \beta_{dns}}$$

As discussed in ***Chapter 2***,

$E_s = 200000\ MPa$

While based on ***Parallel-Axis Theorem*** of Engineering Mechanics,

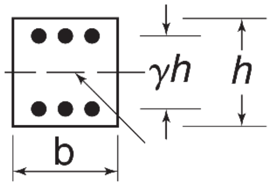
$I_y = \bar{I}_{y'} + Ad^2$

The centroidal moment of inertia for each bar, $\bar{I}_{y'}$, is so small and can be neglected in general:

$\bar{I}_{y'} \approx 0$

$$I_{se} = I_y \approx Ad^2 = \left((314 \times 3) \times \left(\frac{254}{2}\right)^2\right) \times 2 = 0.0304 \times 10^9\ mm^4$$

It can also be determined directly with refereeing to ***Table 10.1-1*** where I_{se} for the indicated case



$$I_{se} = 0.25A_{st}(\gamma h)^2 = 0.25 \times (314 \times 6) \times 254^2 = 0.0304 \times 10^9\ mm^4$$

$$EI_{\text{eff}} = \frac{(0.2 \times 24870 \times 1.32 \times 10^9) + (200000 \times 0.0304 \times 10^9)}{1 + 0.327}$$

$$= 9.53 \times 10^{12}\ N.mm^2$$

$$P_c = \left(\frac{\frac{\pi^2 \times 9.53 \times 10^{12}}{(0.83 \times 4880)^2}}{1000}\right) = 5733\ \text{kN} \quad \blacksquare$$

Example 10.1-4

For frame indicated in ***Figure 10.1-11*** below:

- Using the alignment chart of ***Figure 10.1-9***, calculate the effective length factor for column AB of the braced frame shown below.
- Compute the slenderness ratio of column AB.

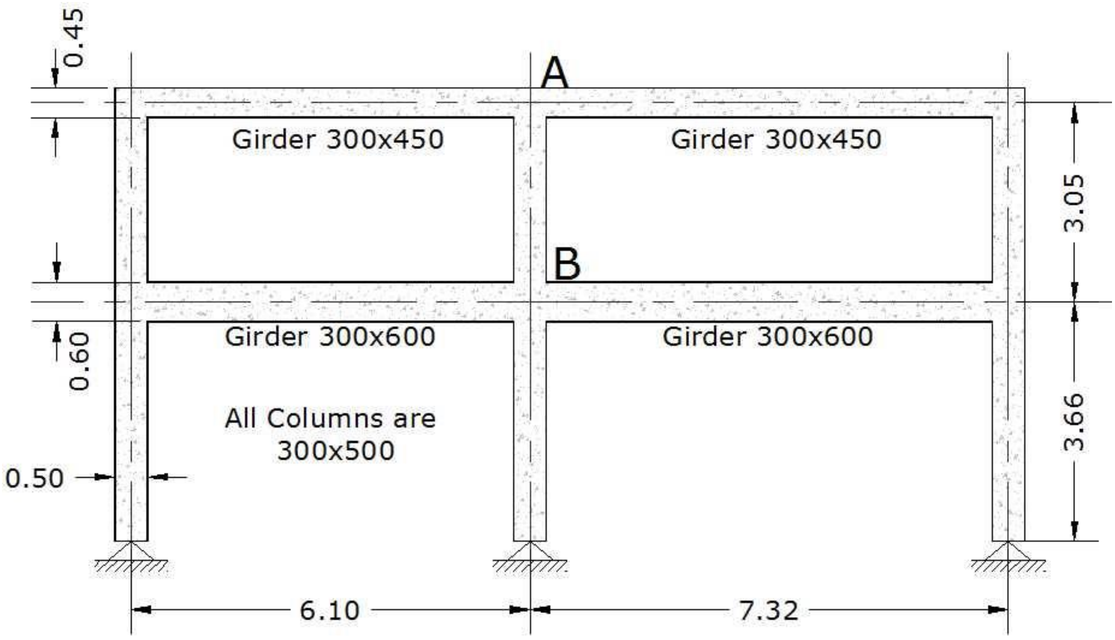


Figure 10.1-11: Frame for Example 10.1-4.

Solution

- Effective Length Factor:

$$\psi = \frac{\sum \frac{EI}{L}_{Columns}}{\sum \frac{EI}{L}_{Beams}}$$

As E_c for columns and beams can be assumed equal, then:

$$\psi_A = \frac{\sum \frac{I}{L_{Columns}}}{\sum \frac{I}{L_{Beams}}}$$

As would be discussed later, according to ACI, I for columns can be taken as $0.7I_g$ and for beams can be taken as $0.35I_g$. These reductions are mainly due to cracking in reinforced concrete.

$$\psi_A = \frac{\frac{0.7 \times \frac{300 \times 500^3}{12}}{3050}}{\frac{0.35 \times \frac{300 \times 450^3}{12}}{6100} + \frac{0.35 \times \frac{300 \times 450^3}{12}}{7320}} = 2.99$$

$$\psi_B = \frac{\frac{0.7 \times \frac{300 \times 500^3}{12}}{3050} + \frac{0.7 \times \frac{300 \times 500^3}{12}}{3660}}{\frac{0.35 \times \frac{300 \times 600^3}{12}}{6100} + \frac{0.35 \times \frac{300 \times 600^3}{12}}{7320}} =$$

$$\psi_B = \frac{1.34 \times 10^6}{0.568 \times 10^6} = 2.36$$

From braced alignment chart, k = 0.875.

- Slenderness Ratio:

$$\frac{kl_u}{r} = \frac{0.875 \times (3.05 - \frac{0.45}{2} - \frac{0.6}{2})}{0.3 \times 0.5} = 14.7$$

Example 10.1-5

With referring to ***Figure 10.1-12***, determine the buckling load for the indicated truss-supporting column when it bends about its major axis. Assume braced story and β_{dns} of 0.7.

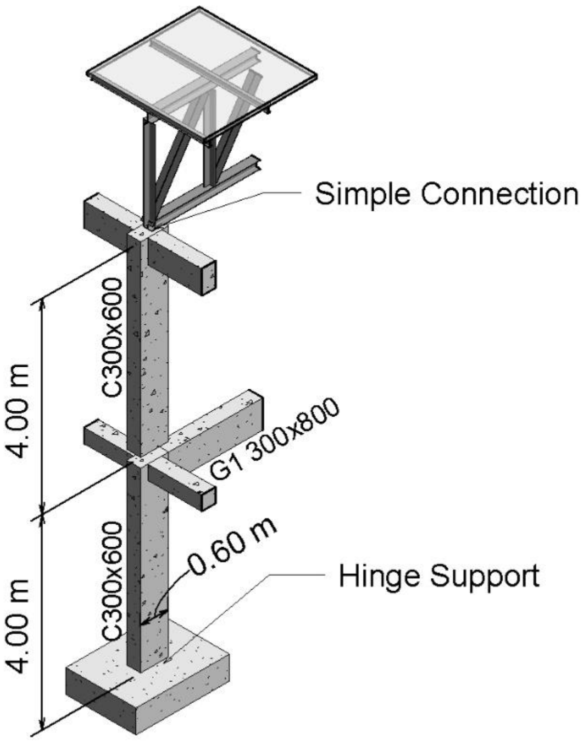


Figure 10.1-12: Truss-supporting column for Example 10.1-5.

Solution

- When column extends from the foundation to the first floor:

According to Euler formula, the buckling load would be:

$$P_c = \frac{\pi^2 EI}{(kl_u)^2}$$

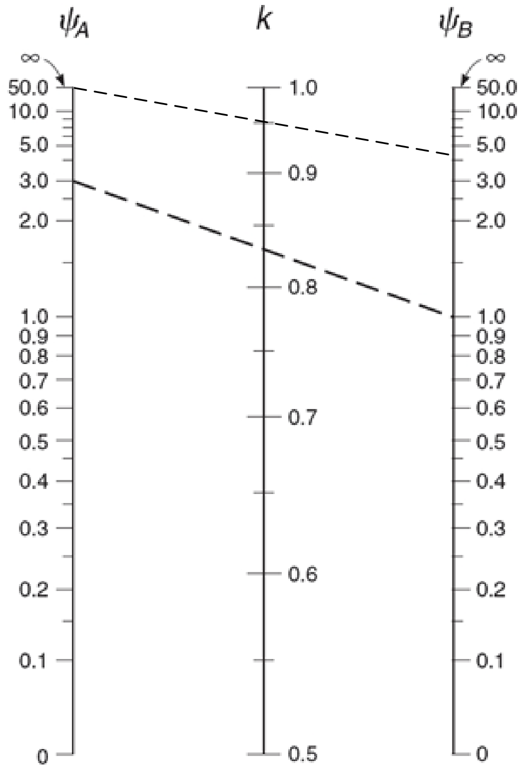
With no information regarding column reinforcement, its rigidity, EI, can be estimated based on the following relation:

$$EI_{eff} = \frac{0.4E_c I_g}{1 + \beta_{dns}}$$

As bending is about the major axis, column moment of inertia, I_g, would be as indicated in below:

$$EI_{eff} = \frac{0.4 \times \left(4700 \times \sqrt{28}\right) \times \frac{300 \times 600^3}{12}}{1 + 0.7}$$
$$= 31.5 \times 10^{12}\ N.mm^2$$

The effective length factor, k, can be determined based on the alignment chart for the braced frame:



$$\psi_{lower} = \psi_{hinge} = \lim_{\frac{EI}{l}_{Beam}\to 0} \frac{\sum \frac{EI}{l}_{Columns}}{\sum \frac{EI}{l}_{Beam}} = \infty$$

$$\psi_{upper} = \psi_A = \frac{\sum \frac{I}{L_{Columns}}}{\sum \frac{I}{L_{Beams}}} = \frac{\frac{0.7 \times \frac{300 \times 600^3}{12}}{\left(4000 - \frac{800}{2}\right)} \times 2}{\frac{0.35 \times \frac{300 \times 800^3}{12}}{9000}}$$

$$= 4.22$$

From the alignment chart, one concludes that the effective length factor is:

$k \approx 0.92$

Finally, the unsupported length, l_u, for the column would be:

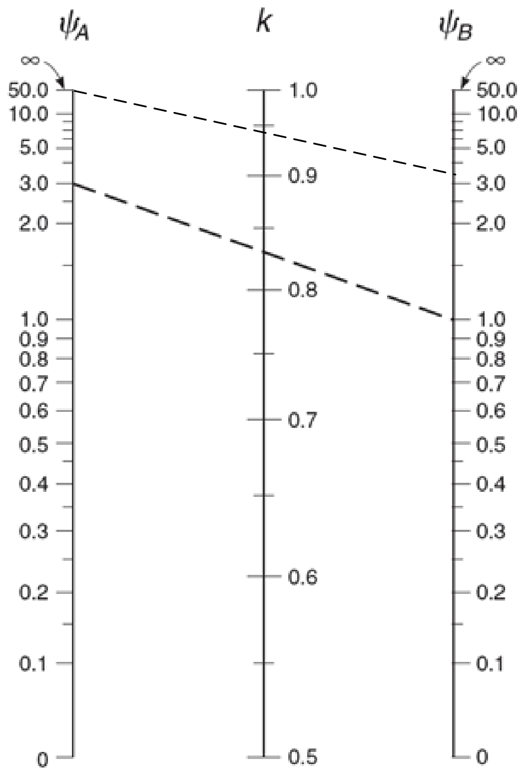
$l_u = 4000 - 800 = 3200\ mm$

Therefore, the buckling load would be:

$$P_c = \left(\frac{\pi^2 \times (31.5 \times 10^{12})}{(0.95 \times 3200)^2}\right) \times \frac{1}{1000} = 33641\ kN\ ■$$

- When column extends from to the first floor to the roof:

$$\psi_{upper} = \psi_{pin} = \lim_{\frac{EI}{l}_{Beam}\to 0} \frac{\sum \frac{EI}{l}_{Columns}}{\sum \frac{EI}{l}_{Beam}} = \infty$$



$$\psi_{lower} = \frac{\sum \frac{I}{L_{Columns}}}{\sum \frac{I}{L_{Beams}}} = \frac{\frac{0.7 \times \frac{300 \times 600^3}{12}}{\left(4000 + \frac{800}{2}\right)} \times 2}{\frac{0.35 \times \frac{300 \times 800^3}{12}}{9000}} = 3.45$$

From the alignment chart, one concludes that the effective length factor is:

$k \approx 0.94$

$l_u = 4000$ mm

$$P_c = \left(\frac{\pi^2 \times (31.5 \times 10^{12})}{(0.94 \times 4000)^2}\right) \times \frac{1}{1000} = 21990\ kN\ ■$$

Example 10.1-6

With referring to ***Figure 10.1-13***, determine the buckling load for the indicated truss-supporting column when it bends about its major axis. Assume braced story and β_{dns} of 0.7.

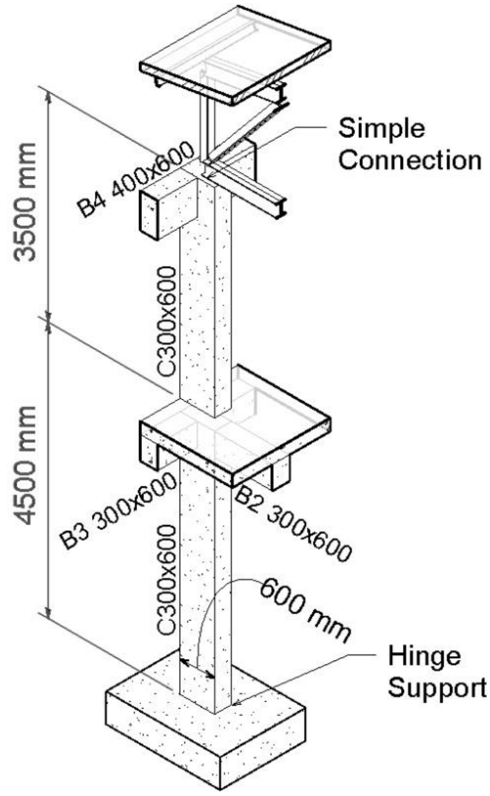


Figure 10.1-13: Truss-supporting column for Example 10.1-6.

Solution

- When column extends from the foundation to the first floor:
  According to Euler formula, the buckling load would be:
  $$P_c = \frac{\pi^2 EI}{(kl_u)^2}$$
  With no information regarding the column reinforcement, its rigidity, EI, can be estimated based on the following relation:
  $$EI_{eff} = \frac{0.4E_c I_g}{1+\beta_{dns}}$$
  As bending is about the major axis, column moment of inertia, I_g, is:
  $$EI_{eff} = \frac{0.4\times\left(4700\times\sqrt{28}\right)\times\dfrac{300\times 600^3}{12}}{1+0.7}$$
  $$= 31.5\times 10^{12}\ N.mm^2$$
  The effective length factor, k, can be determined based on the alignment chart for the braced frame:
  $$\psi_{lower} = \psi_{hinge} = \lim_{\frac{EI}{l}_{Beam}\to 0} \frac{\sum \frac{EI}{l}_{Columns}}{\sum \frac{EI}{l}_{Beam}} = \infty$$
  $$\psi_{upper} = \psi_A = \frac{\sum \frac{I}{L}_{Columns}}{\sum \frac{I}{L}_{Beams}} = \frac{\dfrac{0.7\times\frac{300\times 600^3}{12}}{\left(4500-\frac{600}{2}\right)} + \dfrac{0.7\times\frac{300\times 600^3}{12}}{\left(3500+\frac{600}{2}\right)}}{\dfrac{0.35\times\frac{300\times 600^3}{12}}{6000}} = 6.0$$
  From the alignment chart, the effective length factor is:
  $k \approx 0.96$
  Finally, the unsupported length, l_u, for the column would be:
  $l_u = 4500 - 600 = 3900\ mm$
  Therefore, the buckling load would be:
  $$P_c = \frac{\pi^2 EI}{(kl_u)^2} = \left(\frac{\pi^2\times(31.5\times 10^{12})}{(0.96\times 3900)^2}\right)\times\frac{1}{1000} = 22179\ kN\ \blacksquare$$
- When column extends from to the first floor to the roof:
  $$\psi_{upper} = \psi_{pin} = \lim_{\frac{EI}{l}_{Beam}\to 0} \frac{\sum \frac{EI}{l}_{Columns}}{\sum \frac{EI}{l}_{Beam}} = \infty$$
  $$\psi_{lower} = \frac{\sum \frac{I}{L}_{Columns}}{\sum \frac{I}{L}_{Beams}} = \frac{\dfrac{0.7\times\frac{300\times 600^3}{12}}{\left(4500-\frac{600}{2}\right)} + \dfrac{0.7\times\frac{300\times 600^3}{12}}{\left(3500+\frac{600}{2}\right)}}{\dfrac{0.35\times\frac{300\times 600^3}{12}}{6000}} = 6.0$$
  From the alignment chart, the effective length factor is:
  $k \approx 0.96$,
  $$P_c = \left(\frac{\pi^2\times(31.5\times 10^{12})}{(0.96\times 3500)^2}\right)\times\frac{1}{1000} = 27538\ kN\ \blacksquare$$

10.1.3 Effects of Slenderness on a Column Subjected to a Compression Force and a Moment

- Most reinforced concrete compression members are also subject to simultaneous flexure, caused by transverse loads or by end moments owing to continuity.
- The behavior of members subject to such combined loading also depends greatly on their slenderness.

10.1.3.1 Columns Bent into a Single Curvature

Figure 10.1-14 below shows such a member, axially loaded by P and bent by equal end moments M_e.

- If no axial load were present, the moment M_0 in the member would be constant throughout and equal to the end moments M_e. In this situation, i.e., in simple bending without axial compression, the member deflects as shown by, the dashed curve of ***Figure 10.1-14***.
- When P is applied, the moment at any point increases by an amount equal to P times its lever arm. Then the maximum bending moment due to simultaneous action of flexure and axial force will be:

  $M_{\max} = M_0 + P\Delta$ **Eq. 10.1-18**
- It can be shown that above summation can be re-written in the following multiplication:

  $$M_{\max} = M_0 \times \frac{1}{1 - \frac{P}{P_c}}$$ **Eq. 10.1-19**

  where

  $$\frac{1}{1 - \frac{P}{P_c}}$$

  is known as the ***Moment Magnification Factor***, which reflects the amount by which the moment M_0 is magnified by the presence of simultaneous axial force P.

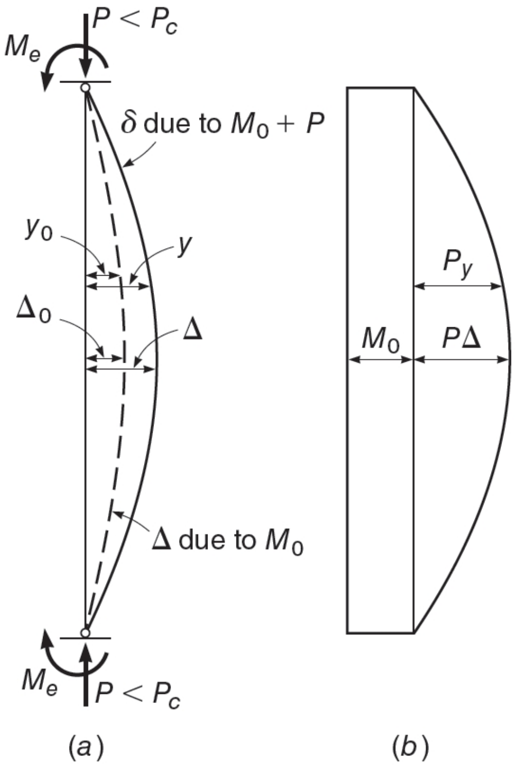


Figure 10.1-14: Moments in slender members with compression plus bending, bent in single curvature.

10.1.3.2 Columns Bent into Double Curvatures

- It clear that the simultaneous effect of the axial force P on the moment magnification for that column shown in ***Figure 10.1-15*** below that has double curvature is ***less than its effect on the moment magnification for a column that has single curvature*** (as the column shown ***Figure 10.1-14*** above).

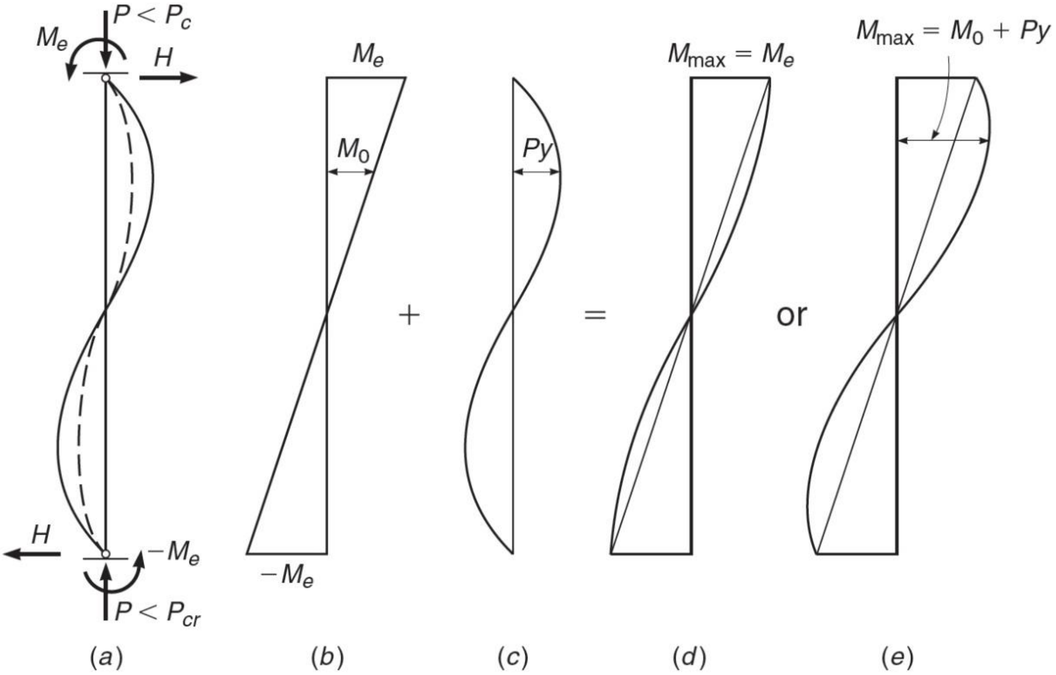


Figure 10.1-15: Moments in slender members with compression plus bending, bent in double curvature.

- Then the moment magnification factor
$$\frac{1}{1-\frac{P}{P_c}}$$
must be modified to be able to represent the difference between a column that has single curvature and a column that has double curvature.
- This can be done by including the C_m factor:
$$Moment\ Magnification\ Factor\ = \frac{C_m}{1-\frac{P}{P_c}} \qquad \textbf{Eq. 10.1-20}$$
where C_m factor can be computed as follows (***ACI Code 6.6.4.5.3***), see ***Figure 10.1-16*** below
- It is very useful to note that ***in addition to its direct effect on the strength of an axially loaded column, Euler or Bucking load*** $\mathrm{P_c}$ ***has an indirect effect on the strength of a column subjected to an axial force and bending moment through its effect on the moment magnification factor***.

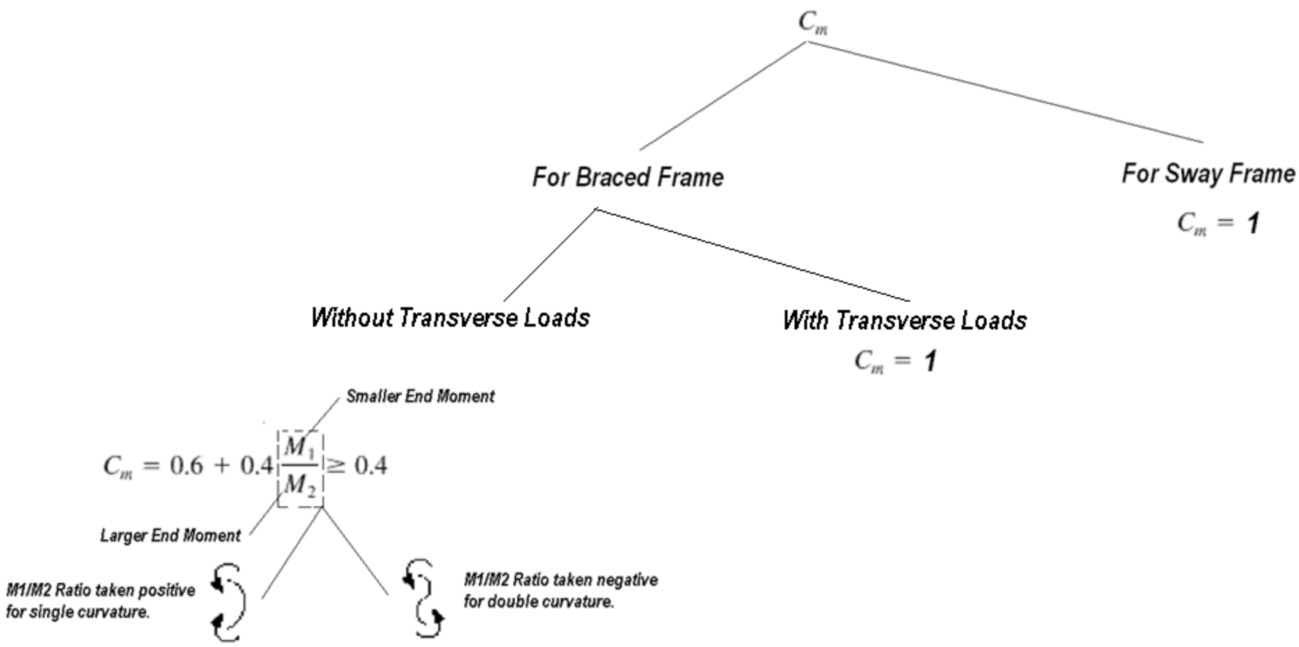


Figure 10.1-16: Computing factor C_m according to Article 6.6.4.5.3 of the code.

10.2 ACI STRATEGIES FOR SLENDER COLUMNS

This article summary ACI strategy to deal with slender columns based on following steps:

- First step in ACI strategy:
  - A criterion should be imposed to check if the effect of slenderness is important and should be included or it is minor and can be neglected (***Article 6.2.5*** of the ACI Code). See ***Article 10.2.2*** below.
  - As this criterion depends on pre-classification of building into ***sway*** or ***non-sway*** building, then this article include a general guide to classify the building into sway or non-sway, ***Article 6.6.4*** of the code or ***Article 10.2.3*** below .
- Second step in ACI strategy:
  - When the effect of slenderness is classified important according to above first step, ACI offers following ***three different methods*** indicated in ***Figure 10.2-1*** below to compute this effect.
  - ***First and second methods are out the scope of our course***. Therefore, ***the course will focus on the third method only*** (***moment magnification method***).
  - According to ACI Code (***6.2.6***), ***total moment including secondary moment shall not exceed 1.4 times the main moment that compute based on first order analysis*** (i.e., the analysis that based on undeformed shape). According to this limitation, ACI Code considers secondary moments that have values greater that 40% of the corresponding main moments as an indication on the instability of the building.

1. Elastic Second Order Analysis
   Article 6.7 of the code

2. Inelastic Second Order Analysis
   Article 6.8 of the code

In these two methods, equilibrium equations are written in terms of the deflected shape. Therefore, the computed moments already include the secondary effects and they can be adopted directly in the design process.

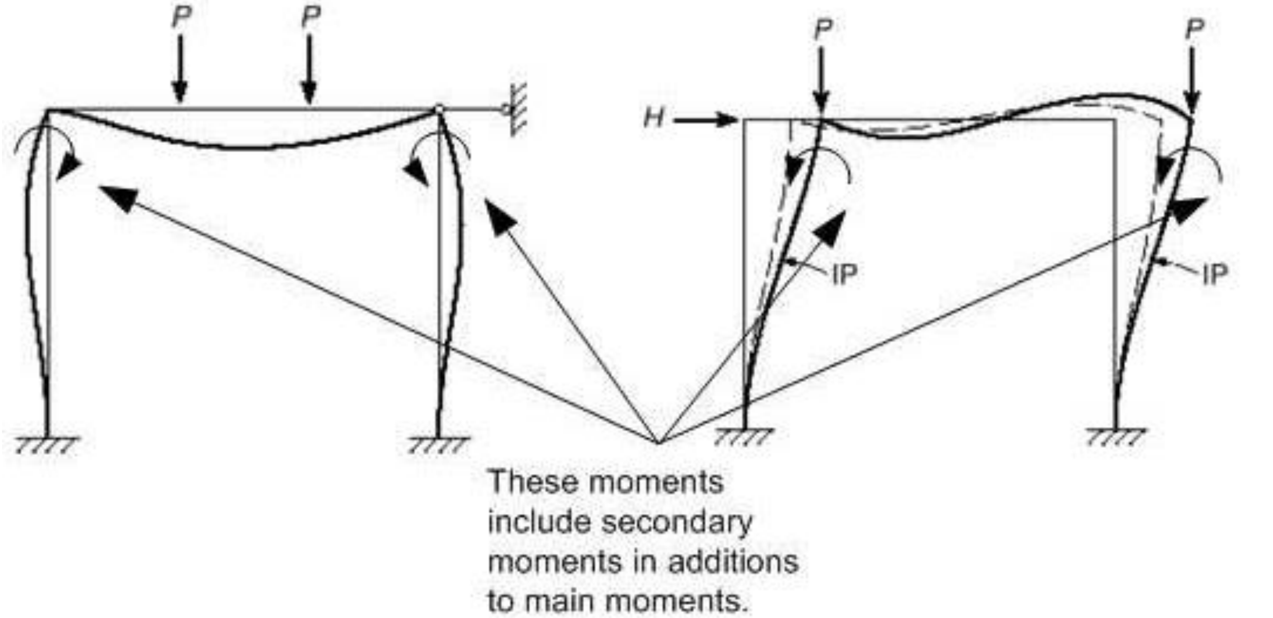


Figure 10.2-1: ACI procedures to deal with the secondary moments in slender columns.

Moment Magnification Method
Article 6.6 of the code

In this method, equilibrium equations are written in terms of the un-deflected shape. Therefore, the computed moments include the main moments only. These moment should be magnified with appropriate factors to include the secondary effects to be adopted in the design process.

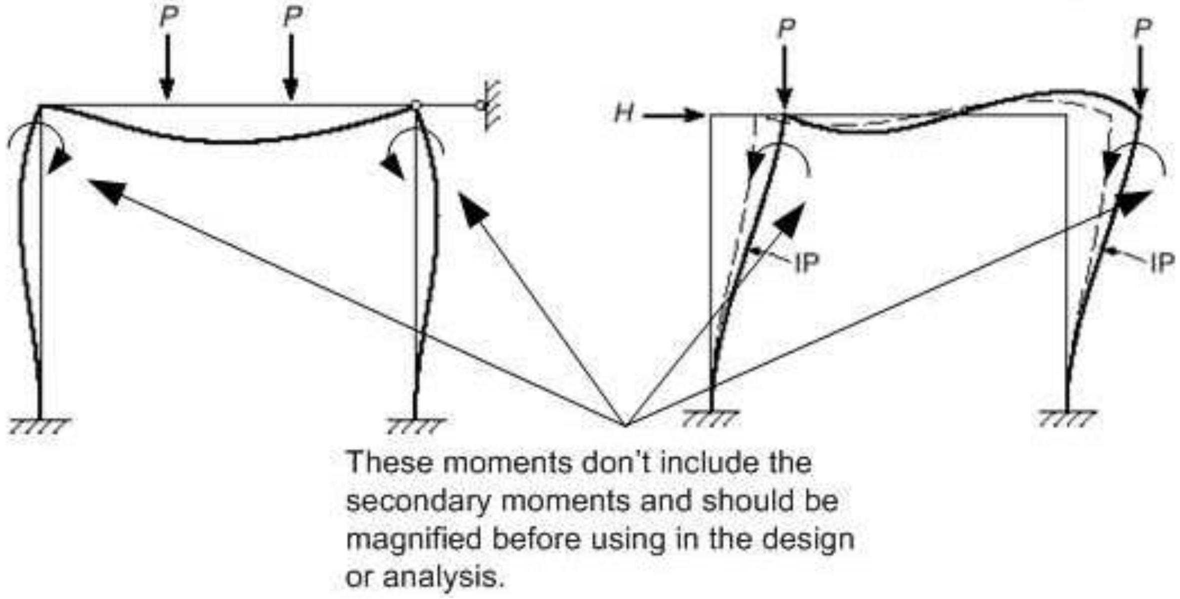


Figure 10.2-1: ACI procedures to deal with the secondary moments in slender columns. Continued.

- As it is clear from above discussions of ACI methods, ACI strategy focuses on the effects of slenderness on columns that subjected to an axial force and uniaxial moment. Columns that subjected to concentric load will be treated indirectly to predicate the effect of slenderness (this will be discussed in ***Article 10.3***).

10.2.1 ACI Procedure in a Flowchart Form

Aforementioned discussed procedures to deal with secondary effects of column slenderness have been summarized in a flowchart form as indicated in ***Figure 10.2-2*** below.

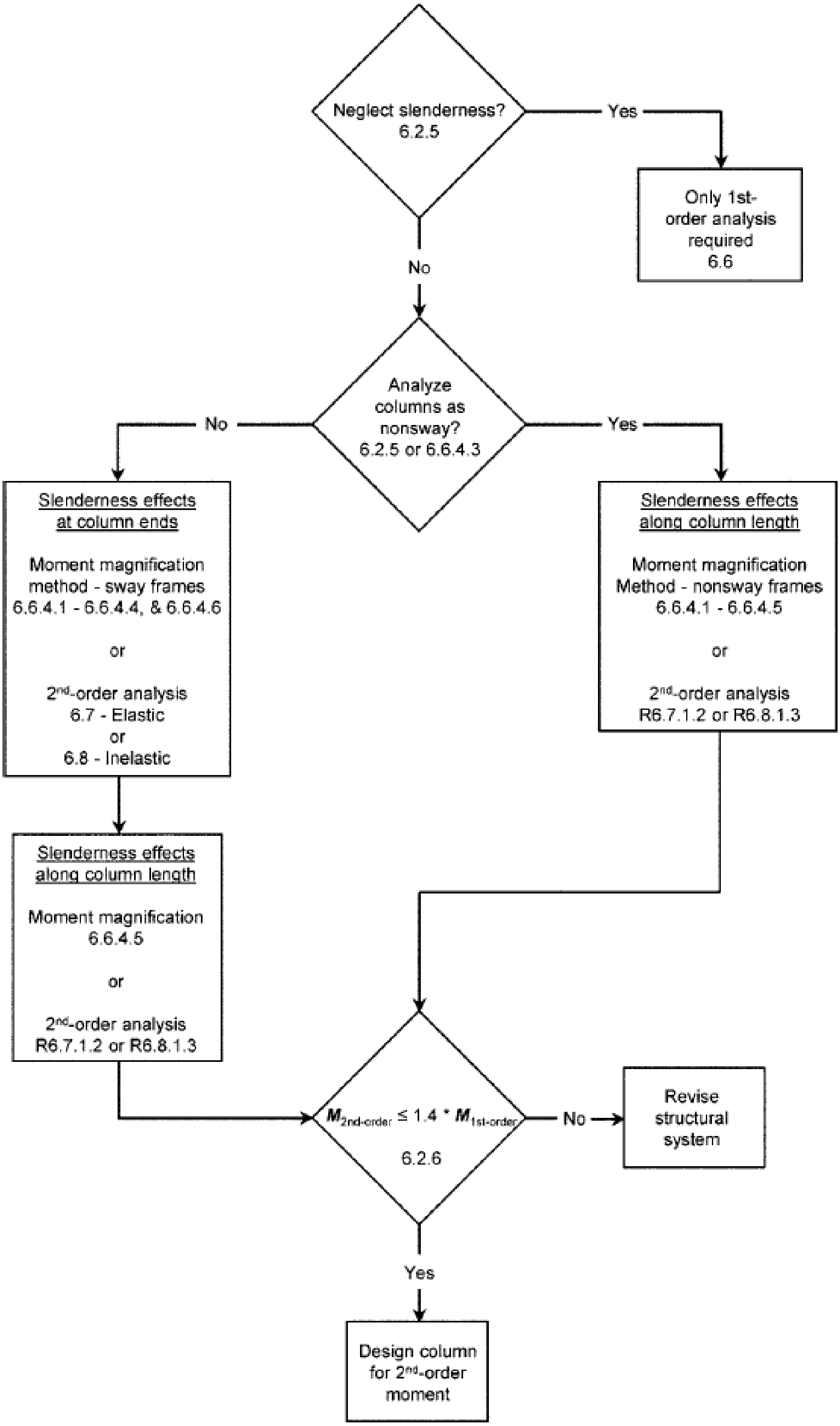


Figure 10.2-2: Flowchart for determining column slenderness effects.

10.2.2 ACI Criteria for Neglecting of Slenderness Effects

- To permit the designer to dispense with the complicated analysis required for slender column design for the ordinary cases in which the Slenderness Effect can be neglected, ***ACI Code (6.2.5)*** provides limits below which the effects of slenderness are insignificant and may be neglected.
- These limits are adjusted to result in a ***maximum unaccounted reduction in column capacity of no more than 5 percent***.
- Separate limits are applied to braced and unbraced frames. The Code provisions are as follows:
- For compression members in ***nonsway frames***, the effects of slenderness may be neglected when:

$$\frac{kl_u}{r} \leq 34 - 12\frac{M_1}{M_2} \qquad \textbf{Eq. 10.2-1}$$

where

$$34 - 12\frac{M_1}{M_2} \leq 40 \qquad \textbf{Eq. 10.2-2}$$

- For compression members in ***sway frames***, the effects of slenderness may be neglected when:

$$\frac{kl_u}{r} \leq 22 \qquad \textbf{Eq. 10.2-3}$$

- In these provisions:
  - k is the effective length factor (computed as Discusses in ***Article 10.1.2.5***).
  - l_u is the unsupported length, taken as the clear distance between floor slabs, beams, or other members providing lateral support.
  - M_1 is the smaller factored end moment on the compression member.
  - M_2 is the larger factored end moment on the compression member.
  - Sign for Ratio $\frac{M_1}{M_2}$ is determined based on ***Figure 10.2-3*** below.

Figure 10.2-3: Sign convention for the ratio M_1/M_2.

Example 10.2-1

For a hall-braced frame shown in ***Figure 10.2-4*** below, classify indicated column into short or slender column when:

- Working in xz plane.
- Working in yz plane.

In your solution, assume that all foundations are behave as perfect hinges.

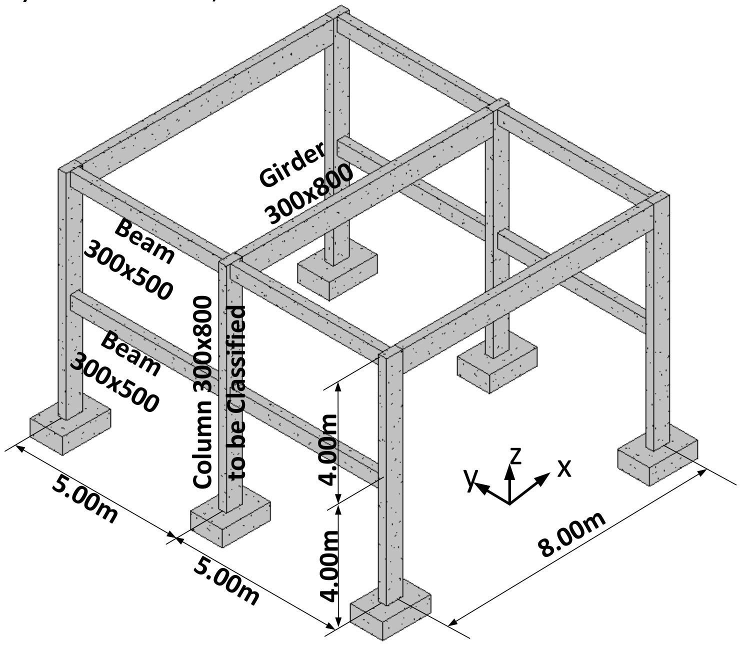


Figure 10.2-4: Hall-braced Frame of Example 10.2-1.

Solutions

Working in xz plane

Effective length factor:

$\psi_{Bottom} \approx \infty$

$$\psi_{Top} = \frac{\Sigma \frac{EI}{L}_{Columns}}{\Sigma \frac{EI}{L}_{Beams}} = \frac{\dfrac{0.7 \times \frac{0.3 \times 0.8^3}{12}}{8 - \frac{0.8}{2}}}{\dfrac{0.35 \times \frac{0.3 \times 0.8^3}{12}}{8}} = 2.1$$

Based on alignment chart for braced frame, see ***Figure 10.2-5*** below.

$k = 0.92$

Slenderness Ratio:

$$\frac{kl_u}{r} = \frac{0.92 \times (8.0 - 0.8)}{0.3 \times 0.8} = 27.6$$

ACI Classification:

$$\frac{kl_u}{r} \ ? \ 34 - 12\frac{M_1}{M_2} \leq 40$$

With hinge support, $M_1 = 0$:

$$\frac{kl_u}{r} = 27.6 < 34$$

Then column is short.

Working in yz plane:

Effective length factor:

With a lower hinge support, column lower part is more critical than upper part.

$\psi_{Bottom} \approx \infty$

$$\psi_{Top} = \frac{\Sigma \frac{EI}{L}_{Columns}}{\Sigma \frac{EI}{L}_{Beams}} = \frac{2 \times \dfrac{0.7 \times \frac{0.8 \times 0.3^3}{12}}{4}}{2 \times \dfrac{0.35 \times \frac{0.3 \times 0.5^3}{12}}{5}} = 1.44$$

Based on alignment chart for braced frame, see ***Figure 10.2-5*** below.

$k = 0.88$

<u>Slenderness Ratio:</u>

$$\frac{kl_u}{r} = \frac{0.88 \times (4.0 - 0.5)}{0.3 \times 0.3} = 34.2$$

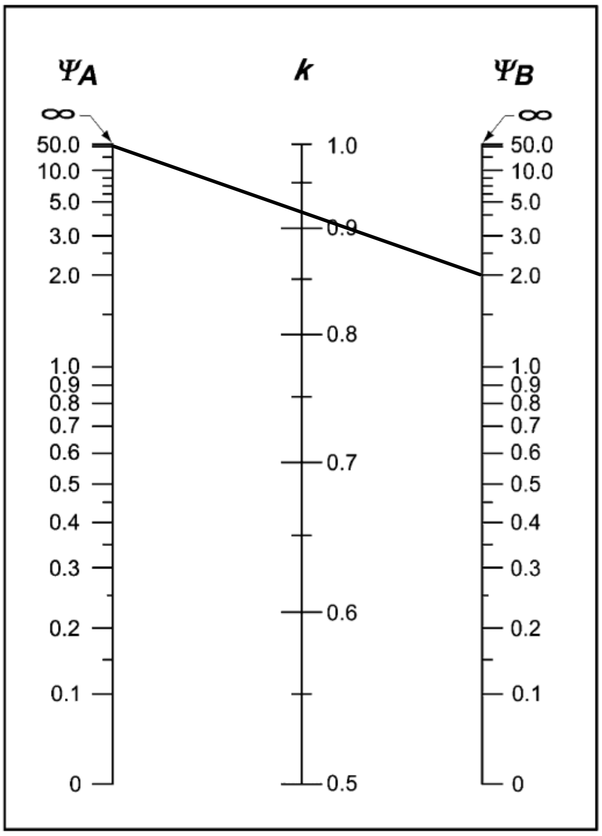
<u>ACI Classification:</u>

$$\frac{kl_u}{r} \;?\; 34 - 12\frac{M_1}{M_2} \leq 40$$

With hinge support, $M_1 = 0$:

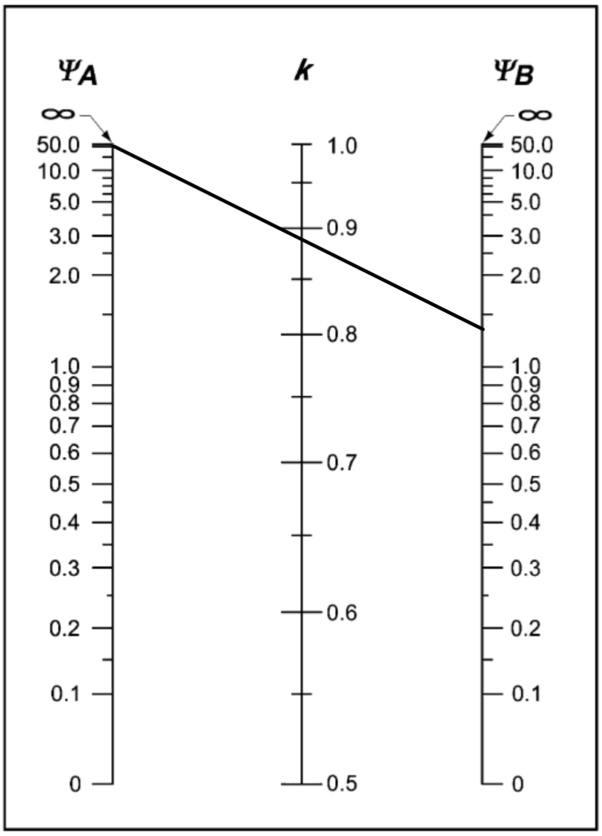
$$\frac{kl_u}{r} = 34.2 > \; 34$$

Then column is slender.



(a)
Nonsway Frames

K factor for working with xz plane.



(a)
Nonsway Frames

K factor for working with yz plane.

Figure 10.2-5: Effective length factor for Example 10.2-1.

Example 10.2-2

For the braced column presented in ***Figure 10.2-6*** below,

- Relative to plane XZ,
  - Is the column classified as short or slender?
  - What is column critical load in this plane?
- Relative to plane YZ, from foundation to first levels,
  - Is the column classified as short or slender?
  - What is column critical load in this plane?

In your solution, assume that footing behaves as a perfect hinge, sustained load is 90% of the total load, the girder has a span of 12m, and beams have spans of 6m.

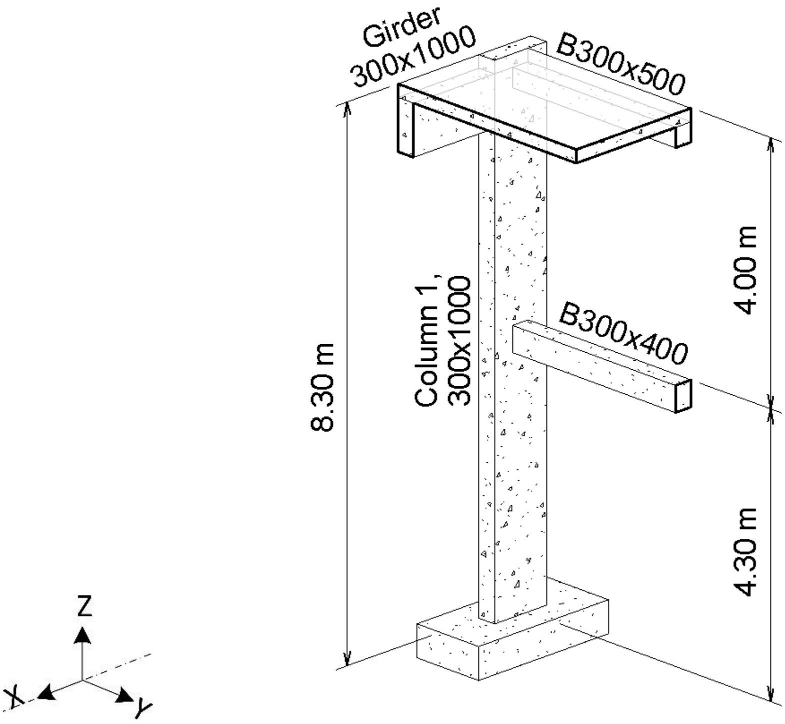


Figure 10.2-6: Braced column for Example 10.2-2.

Solution

Plane XZ;

$\psi_{A\ at\ hinge\ end} = \infty$

$$\psi_{B\ @\ top\ of\ column} = \frac{\dfrac{0.7 \times \dfrac{300 \times 1000^3}{12}}{8300 - \dfrac{1000}{2}}}{\dfrac{0.35 \times \dfrac{300 \times 1000^3}{12}}{12000}} \approx 3.0$$

Then, effective length factor, k, is, see ***Figure 10.2-7*** below:

$k = 0.94$

Column unsupported length is,

$l_u = 8000 + 300 - 1000 = 7300\ mm$

Slenderness ratio is,

$$\frac{kl_u}{r} = \frac{0.94 \times 7300}{0.3 \times 1000} = 22.9\ ?\ 34 - 12\frac{M_1}{M_2}$$

With hinge support,

$M_1 = 0$

$$\frac{kl_u}{r} = 22.9 < 34$$

Then the column could be classified as short.

Column critical load is,

$$P_{Critical} = \frac{(\pi^2 EI)}{(kl_u)^2}$$

As nothing is mentioned about reinforcement, then

$$EI = \frac{0.4E_cI_g}{1 + \beta_{dns}} = \frac{0.4 \times (4700 \times \sqrt{28}) \times \dfrac{300 \times 1000^3}{12}}{1 + 0.9} = 131 \times 10^{12}\ N.mm^4$$

$$P_{Critical} = \frac{(\pi^2 \times 131 \times 10^{12})}{(0.94 \times 7300)^2} = 27458\ kN$$

Relative to plane YZ and from foundation to first levels,

$\psi_{A\ at\ hinge\ end} = \infty$

$$\psi_{B\ @\ top\ of\ column} = \frac{\dfrac{0.7 \times \dfrac{1000 \times 300^3}{12}}{4300}}{\dfrac{0.35 \times \dfrac{300 \times 400^3}{12}}{6000}} = 3.9$$

Then, effective length factor, k, is, see ***Figure 10.2-7*** below:

$k = 0.95$

Column unsupported length is,

$l_u = 4000 + 300 - 400 = 3900\ mm$

Slenderness ratio is,

$$\frac{kl_u}{r} = \frac{0.95 \times 3900}{0.3 \times 300} = 41.2\ ?\ 34 - 12\frac{M_1}{M_2}$$

With hinge support,

$M_1 = 0$

$$\frac{kl_u}{r} = 41.2 > 34$$

Then the column could be classified as slender. Column critical load is,

$$P_{Critical} = \frac{(\pi^2 EI)}{(kl_u)^2}$$

As nothing is mentioned about reinforcement, then

$$EI\ = \frac{0.4E_cI_g}{1+\beta_{dns}} = \frac{0.4 \times (4700 \times \sqrt{28}) \times \frac{1000 \times 300^3}{12}}{1 + 0.9} = 11.8 \times 10^{12}\ N.mm^4$$

$$P_{Critical} = \frac{(\pi^2 \times 11.8 \times 10^{12})}{(0.95 \times 3900)^2} = 8481\ kN$$

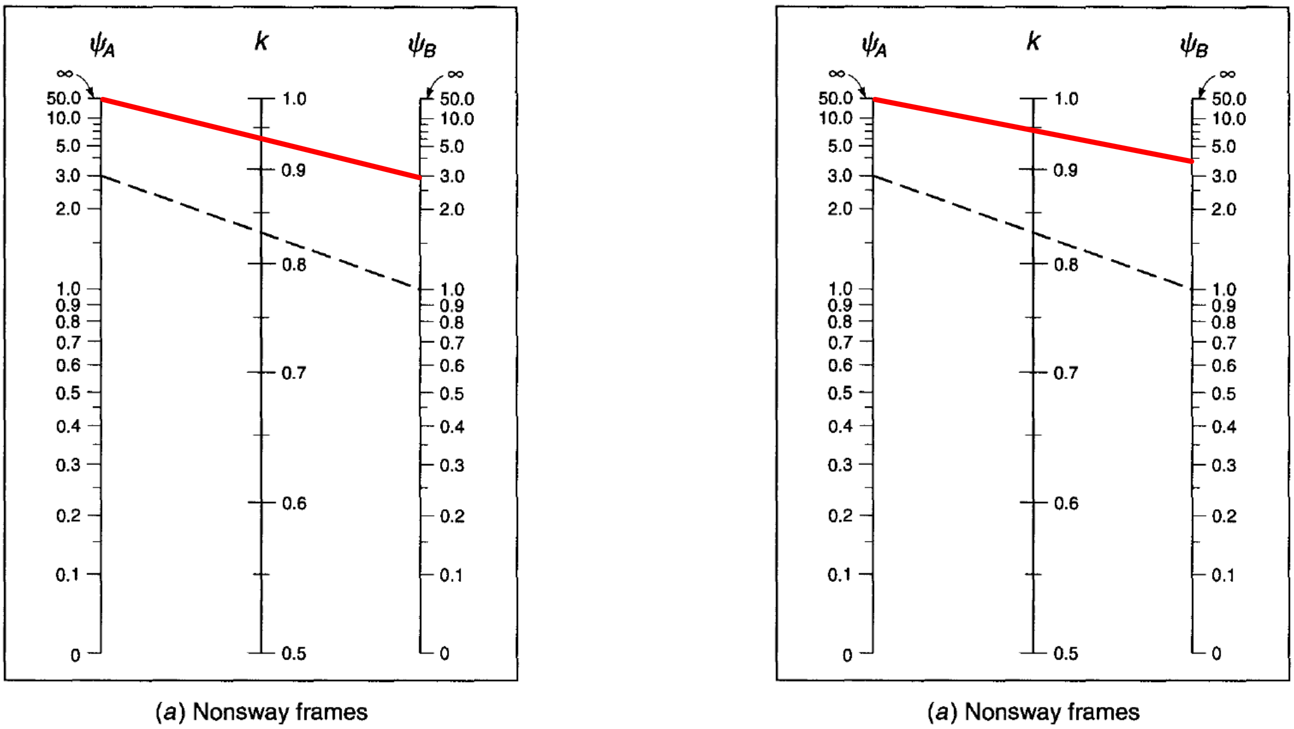


Figure 10.2-7: Alignment chart applied for Example 10.2-2.

Example 10.2-3

With referring to building of ***Figure 10.2-8*** below, is a typical interior column classified as short or slender when analyzed:

- In a plane along Sec. 1-1.
- In a plane along Sec. 2-2.

In your solution, assume braced building and assume footings behave as hinges.

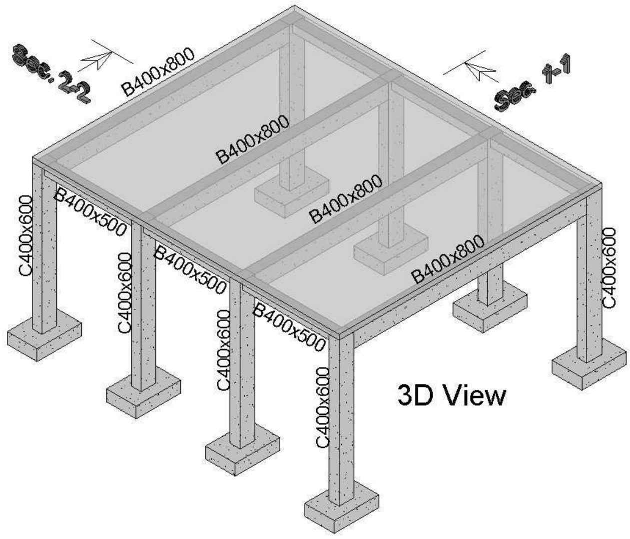
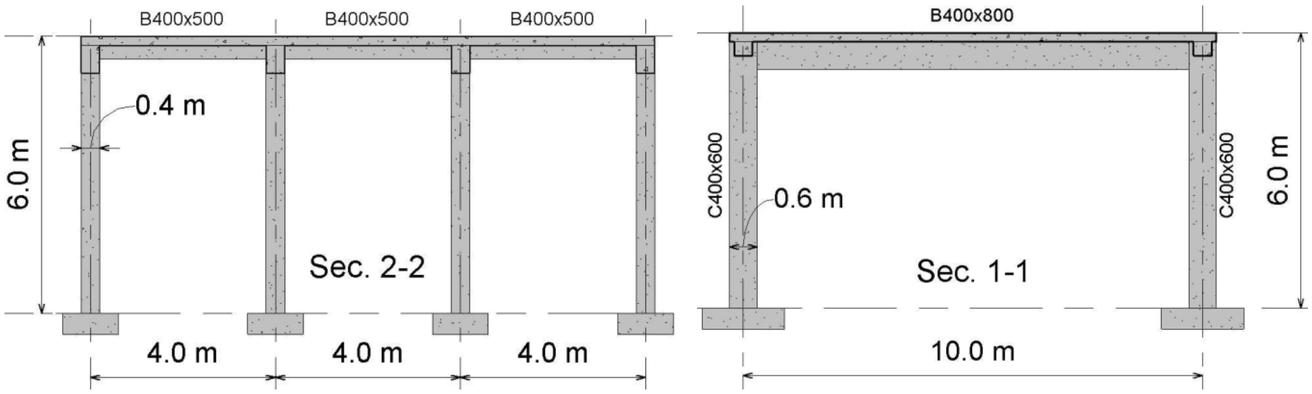


Figure 10.2-8: Building for Example 10.2-3.



SOLUTION

IN PLANE ALONG SEC. 1-1

$\psi_{A\ at\ hinge\ end} = \infty$

$$\psi_{B\ @\ top\ of\ column} = \frac{\dfrac{0.7 \times \dfrac{400 \times 600^3}{12}}{6000 - \dfrac{800}{2}}}{\dfrac{0.35 \times \dfrac{400 \times 800^3}{12}}{10000}} = 1.51$$

Then, effective length factor, k, is, see ***Figure 10.2-9*** below:

$k = 0.9$

Column unsupported length is,

$l_u = 6000 - 800 = 5200\ mm$

Slenderness ratio is,

$$\frac{kl_u}{r} = \frac{0.9 \times 5300}{0.3 \times 600} = 26.5\ ?\ 34 - 12\frac{M_1}{M_2}$$

With hinge support,

$M_1 = 0$

$$\frac{kl_u}{r} = 26.2 < 34$$

Then the column could be classified as short.

IN PLANE ALONG SEC. 2-2

$\psi_{A\ at\ hinge\ end} = \infty$

$$\psi_{B\ @\ top\ of\ column} = \frac{\dfrac{0.7 \times \dfrac{600 \times 400^3}{12}}{\left(6000 - \dfrac{500}{2}\right)}}{2 \times \left(\dfrac{0.35 \times \dfrac{400 \times 500^3}{12}}{4000}\right)} = 0.53$$

Then, effective length factor, k, is see ***Figure 10.2-9*** below:

$k = 0.82$

Column unsupported length is,

$l_u = 6000 - 500 = 5500\ mm$

Slenderness ratio is,

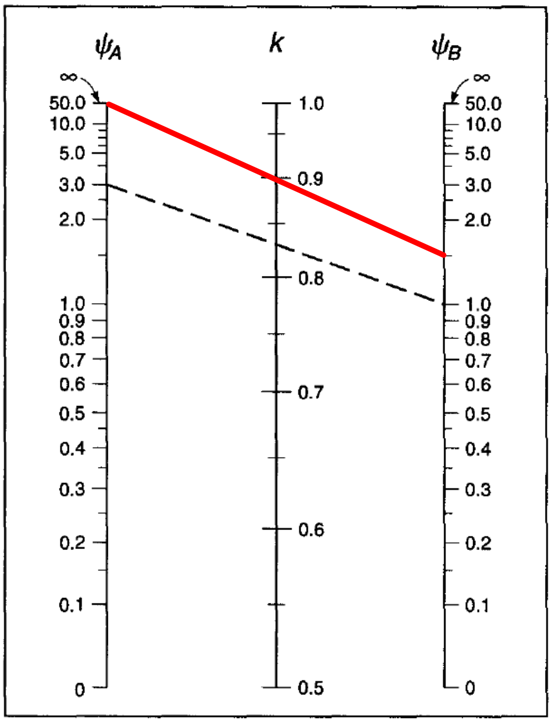
$$\frac{kl_u}{r} = \frac{0.82 \times 5500}{0.3 \times 400} = 37.6\ ?\ 34 - 12\frac{M_1}{M_2}$$

With hinge support,

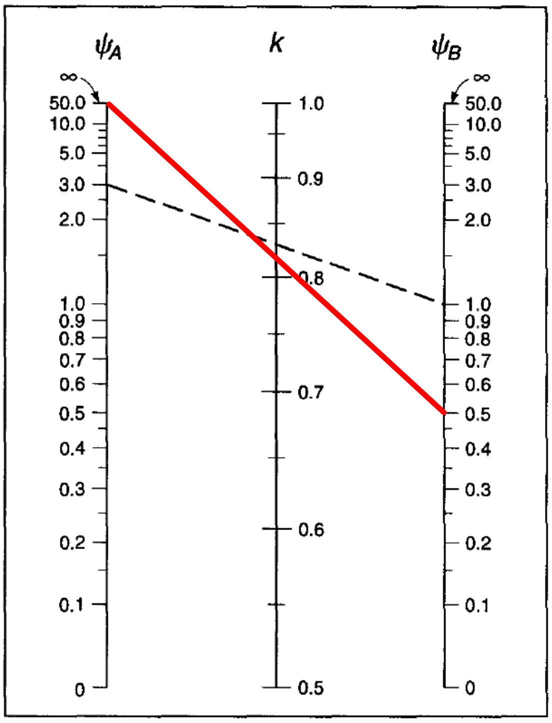
$M_1 = 0$

$$\frac{kl_u}{r} = 37.6.34$$

The column is classified as slender.



(a) Nonsway frames



(a) Nonsway frames

Figure 10.2-9: Alignment chart applied for Example 10.2-3.

10.2.3 ACI Criteria for Nonsway versus Sway Frames

- The previous discussion shows clearly significant difference between the behavior of slender columns in nonsway (braced) frames and the corresponding columns in sway (unbraced) frames.
- ACI Code provisions for the approximate design of slender columns reflect this difference and there are separate provisions for nonsway versus sway frames.
- In actual structures, a frame is seldom either completely braced or completely unbraced. It is necessary, therefore, to determine in advance if bracing provided by shear walls, elevator and utility shafts, stairwells, or other elements is adequate to restrain the frame against significant sway effects, see ***Figure 10.2-10***.

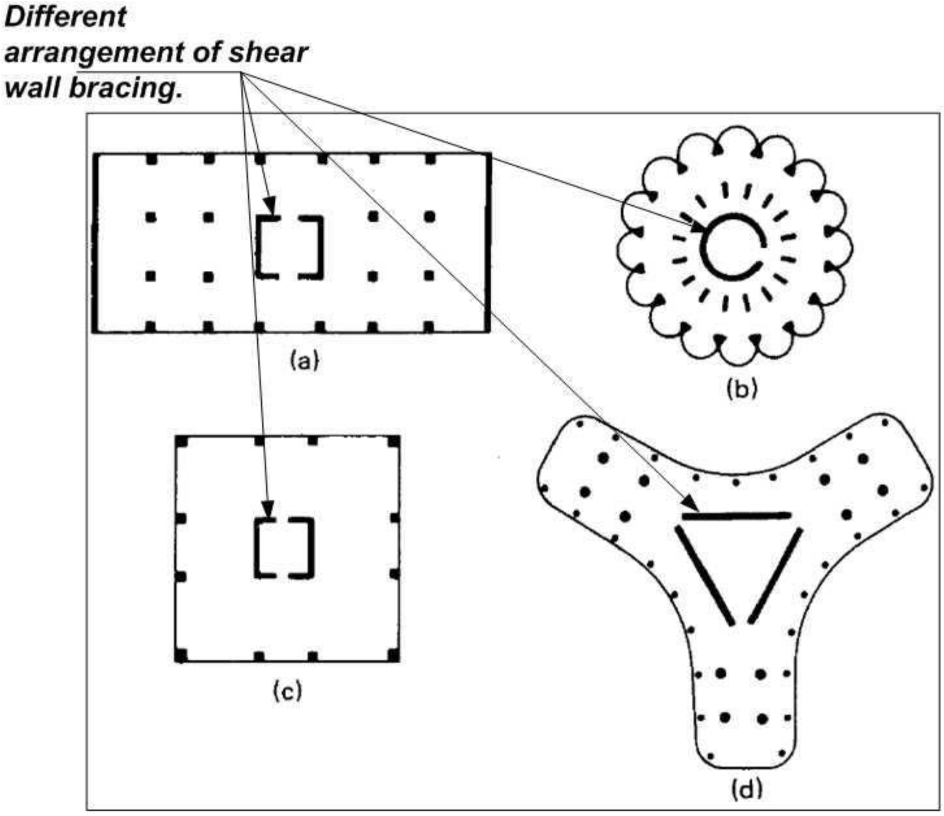


Figure 10.2-10: Different arrangement of shear walls that may provide frame bracing.

- Determination of bracing system effectiveness can be executed based any one of the following methods:
  - By Inspection (***ACI Commentary 6.6.4.1***)
    i. By engineering judgment, the engineer may decide if the stiffness of shear wall or a steel bracing system is adequate to classify the frame under consideration as a braced frame or not.
    ii. According to ***previous code provisions***, it shall be permitted to consider ***compression members braced against sidesway when bracing elements have a total stiffness, resisting lateral movement of that story, of at least 12 times the gross stiffness of the columns within the story***.
  - Based on Stability Index Concept (ACI code ***Article 6.6.4.3***)
    i. If the effectiveness of a shear wall or bracing system is questionable, a frame can be classified to a braced or nonbraced based on the concept of ***Stability Index*** which computed as follows, see ***Figure 10.2-11*** below.

$$Stability\ Index = Q = \frac{\sum P_u \Delta_0}{V_u l_c} \quad \textbf{Eq. 10.2-4}$$

where:

$\sum P_u$ and V_u are the total factored vertical load and story shear, respectively, for the story.

Δ_0 is the first-order relative deflection between the top and the bottom of the story due to V_u.

l_c is the length of the compressive member measured center-to-center of the joints in the frame.

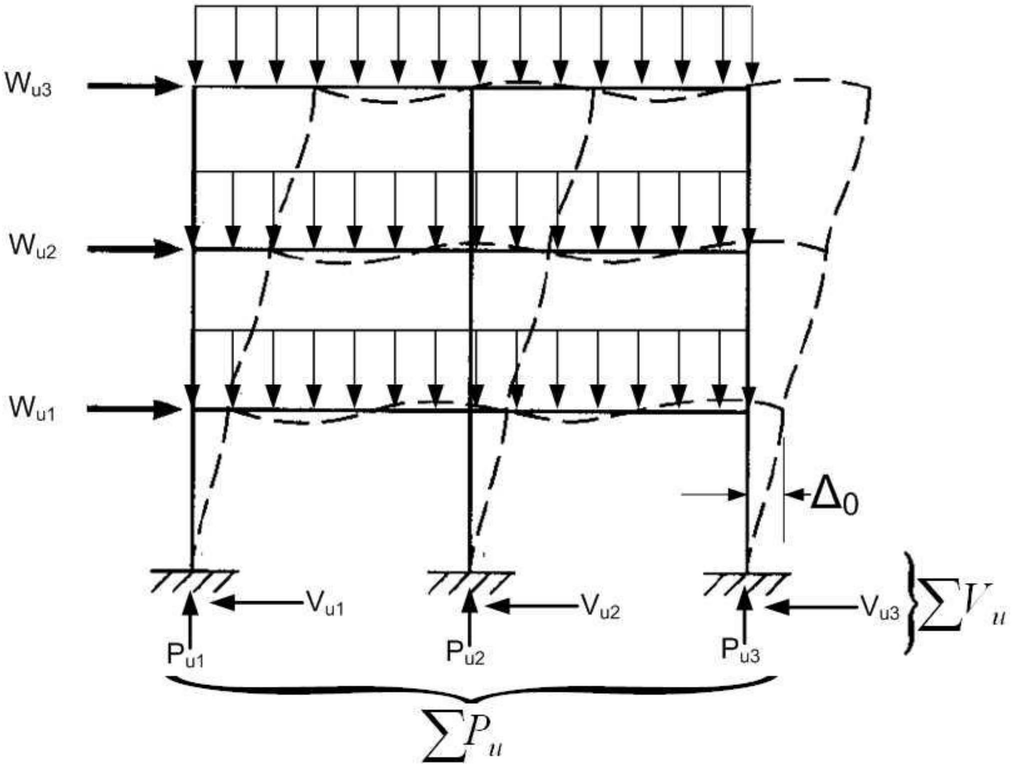


Figure 10.2-11: Parameters adopted in computing of the Stability Index.

ii. A *story* that has a ***stability index not greater 0.05 can be classified as a braced and vice versa***.

iii. According to ***ACI Code 6.6.3.1***, section properties may be represented using the modulus of elasticity, E, of:

$E = E_c$

and section properties as indicated in ***Table 10.2-1*** below.

Table 10.2-1: Moment of inertia and cross sectional area permitted for elastic analysis at factored load level, Table 6.6.3.1.1(a).

Member and condition		Moment of Inertia	Cross-sectional area
Columns		$0.70I_g$	$1.0A_g$
Walls	Uncracked	$0.70I_g$	
	Cracked	$0.35I_g$	
Beams		$0.35I_g$	
Flat plates and flat slabs		$0.25I_g$	

Example 10.2-4

For the building frame shown in ***Figure 10.2-12*** below, based on an elastic first order analysis with ACI stiffnesses of ***Table 10.2-1*** above and with neglecting of frame selfweight, lateral deflections have been computed for ground and first stories and summarized in ***Figure 10.2-13*** below. Use ACI ***stability index method*** to classify ground and first stories as braced or sway.

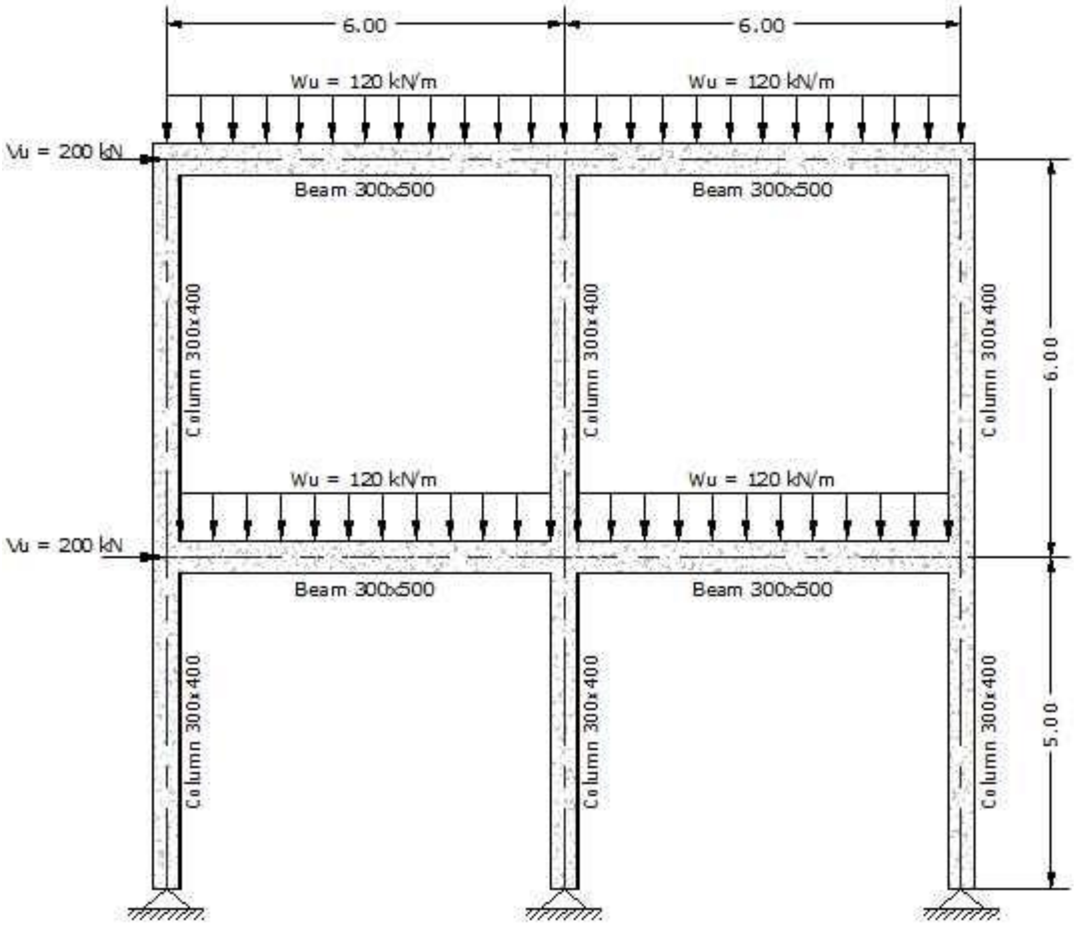


Figure 10.2-12: Frame for Example 10.2-4.

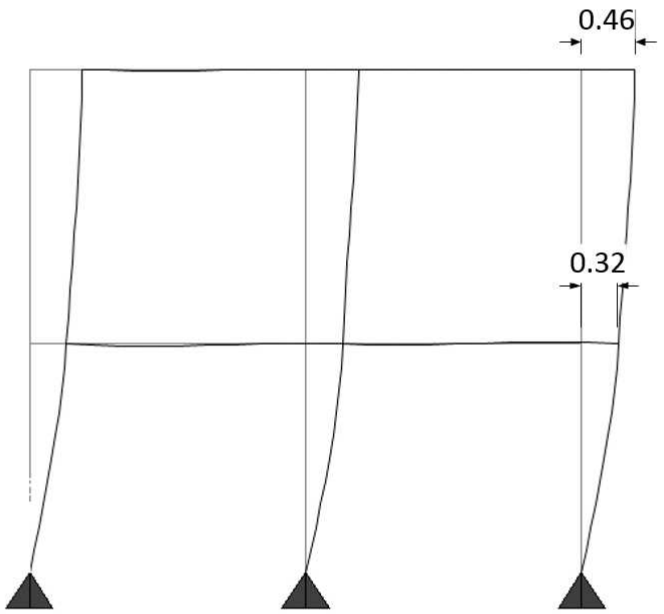


Figure 10.2-13: Lateral deflections of the ground and first stories for the frame of Example 10.2-4.

Solution

- For first story:

$$Stability\ Index = Q = \frac{\sum P_u \Delta_0}{V_u l_c}$$

$$\sum P_u = \text{the total factored vertical for the story} = 120\frac{\text{kN}}{\text{m}} \times 6\text{m} \times 2 = 1440\text{ kN}$$

$V_u =$ total factored story shear $= 200$ kN
$\Delta_0 =$ the first order relative deflection between the top and the bottom of the story due to V_u
$\Delta_0 = 0.46 - 0.32 = 0.14$ m
$l_c =$ the length of the compressive member measured center − to − center of the joints in the frame
$l_c = 6$m

$$Stability\ Index = Q = \frac{1440\ kN \times 0.14m}{200\ kN \times 6m} = 0.168 > 0.05$$

Then, the story is unbraced.

- For the ground story:

$$\sum P_u = \text{the total factored vertical for the story} = \left(120 \frac{\text{kN}}{\text{m}} \times 6\text{m} \times 2\right) \times 2 = 2880\ \text{kN}$$

$V_u =$ total factored story shear $= 200\ \text{kN} \times 2 = 400$ kN
$\Delta_0 =$ the first order relative deflection between the top and the bottom of the story due to V_u
$\Delta_0 = 0.32$ m
$l_c =$ the length of the compressive member measured center − to − center of the joints in the frame
$l_c = 5$m

$$Stability\ Index = Q = \frac{2880\ \text{kN} \times 0.32m}{400\ kN \times 5m} = 0.46 > 0.05$$

Then, the story is unbraced.

Example 10.2-5

Re-solve ***Example 10.2-4*** above, but when the building is stiffened with a shear wall that shown in ***Figure 10.2-14*** below. the deformation would be as indicated in ***Figure 10.2-15*** below.

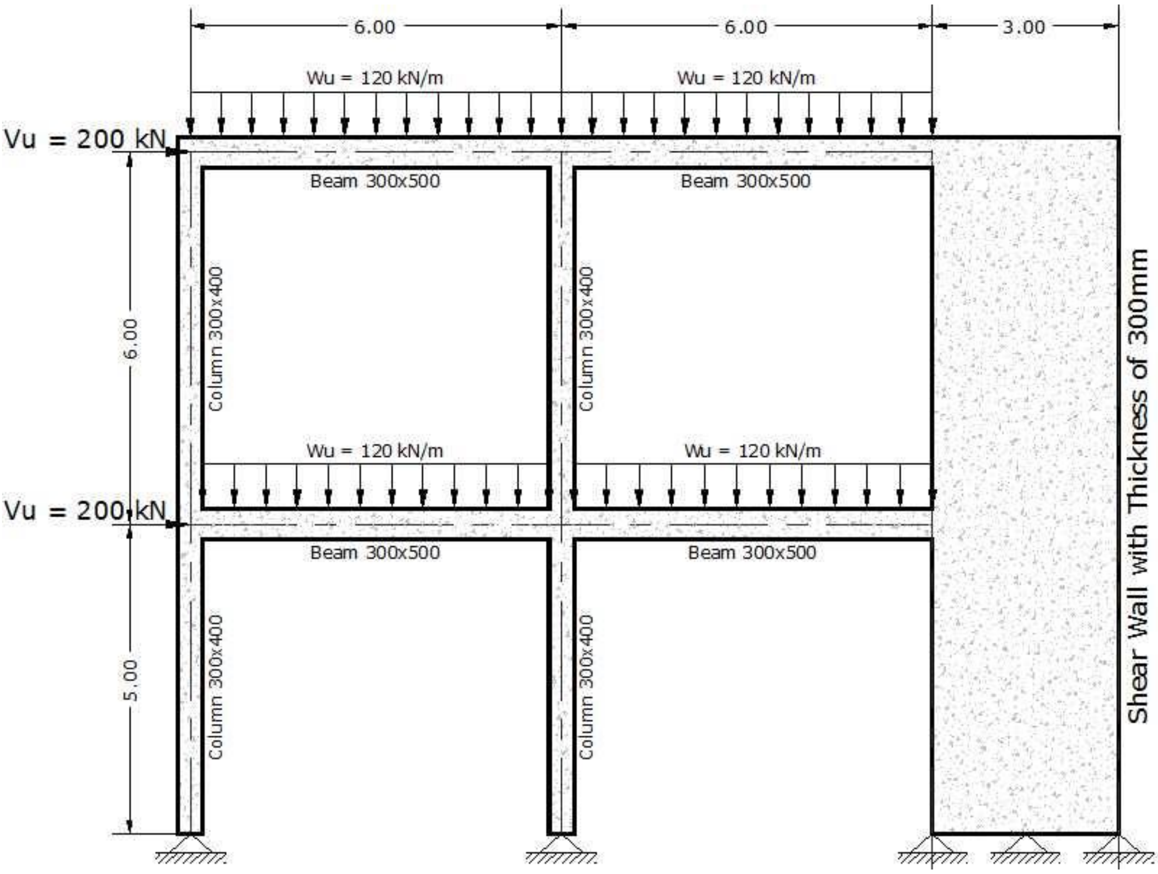


Figure 10.2-14: Frame for Example 10.2-5.

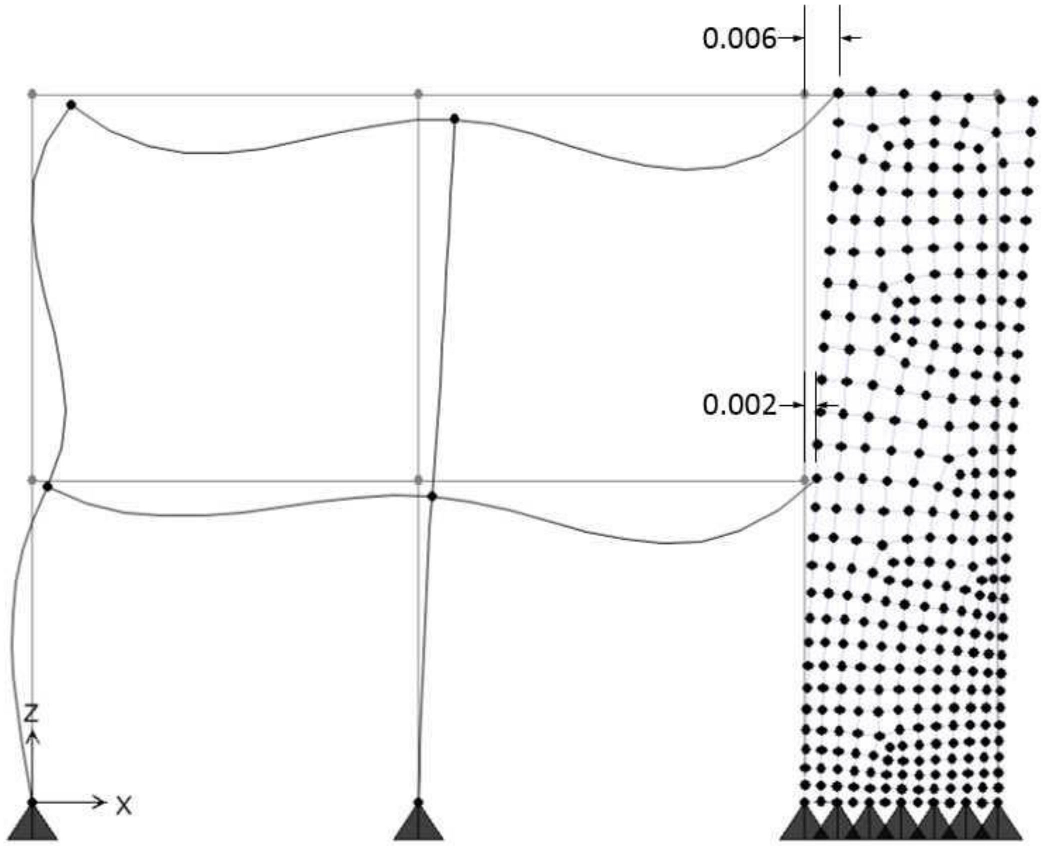


Figure 10.2-15: Lateral deflections of the ground and first stories for the frame of Example 10.2-5.

Solution

- For First Story:
  $\Delta_0 = 0.006 - 0.002 = 0.004 \text{ m}$
  $Stability\ Index = Q = \frac{\sum P_u \Delta_0}{V_u l_c} = \frac{1440\ kN \times .004m}{200\ kN \times 6m} = 0.005 < 0.05$
  Then, the story is braced.
- Ground Story:
  $\Delta_0 = 0.002 \text{ m}$
  $Stability\ Index = Q = \frac{\sum P_u \Delta_0}{V_u l_c} = \frac{2880 \text{ kN} \times 0.002m}{400\ kN \times 5m} = 0.003 < 0.05$
  Then, the story is braced.

10.3 SUMMARY OF ACI MOMENT MAGNIFIER METHOD FOR NONSWAY FRAMES

- For a column that may be a slender column in a braced story, ACI checking procedure can be summarized as follows:
  1. Select a trial column section to carry the factored axial load P_u and moment $M_u = M_2$ (where M_2 is that maximum moment that occurs at one of column two ends) that computed from a first-order frame analysis (i.e. an analysis that based on undeformed shape and that predicate main moments only), assuming short column behavior and following the procedures of ***Chapter 9***.
  2. Determine if the frame should be considered as nonsway or sway, either based on intuition or based on stability index and as discussed in ***Article 10.2.3***.
  3. Find the unsupported length l_u (as discussed in ***Article 10.1.2.4***).
  4. For the trial column, check for consideration of slenderness effects using the criteria of ***Article 10.2.2*** with k = 1.0.
  5. If slenderness is tentatively found to be important, refine the calculation of k based on the alignment chart in ***Article 10.1.2.5***.
  6. If moments from the frame analysis are small, check to determine if the following minimum moment controls.
     a. If

        $M_2 < M_{2,min} = P_u(15 + 0.03h)$ **Eq. 10.3-1**

        where 15 and h are in mm.

     b. Then use:

        $M_2 = M_{2,min} = P_u(15 + 0.03h)$ **Eq. 10.3-2**
  7. Calculate the equivalent uniform moment factor C_m (as was discussed in ***Article 10.1.3.2***.
  8. Calculate β_{dns}, EI and P_c as discussed in ***Article 10.1***.
  9. Calculate the moment magnification factor δ_{ns} and magnified moment M_c based on following relations:

     $$\delta_{ns} = \frac{C_m}{1 - \frac{P_u}{0.75P_c}} \geq 1.0$$ **Eq. 10.3-3**

     $$M_c = \delta_{ns} M_2$$ **Eq. 10.3-4**
  10. Check the adequacy of the column to resist axial load and magnified moment using the column design charts of ***Chapter 9*** in the usual way. Revise the column section and reinforcement if necessary.
  11. If column dimensions are altered, repeat the calculations for k, I_{eff} . and P_c based on the new cross section. Determine the revised moment magnification factor and check the adequacy of the new design.

Example 10.3-1

Forces and moments acting on column shown in Figure 10.3-1 have been computed based on a ***First Order Analysis*** (i.e. equilibrium relations have been written based on un-deflected shape):

- Check to see if this column is classified short or slender based on ACI criterion.
- If the column is slender, magnify the applied moment based on ACI Moment Magnification Method.

In your solution, assume that:

- Frame is braced.
- The column has an effective length factor (k) equal to 0.9.
- Sustained load is 60% of the total load.
- $f_c' = 21\ MPa$.

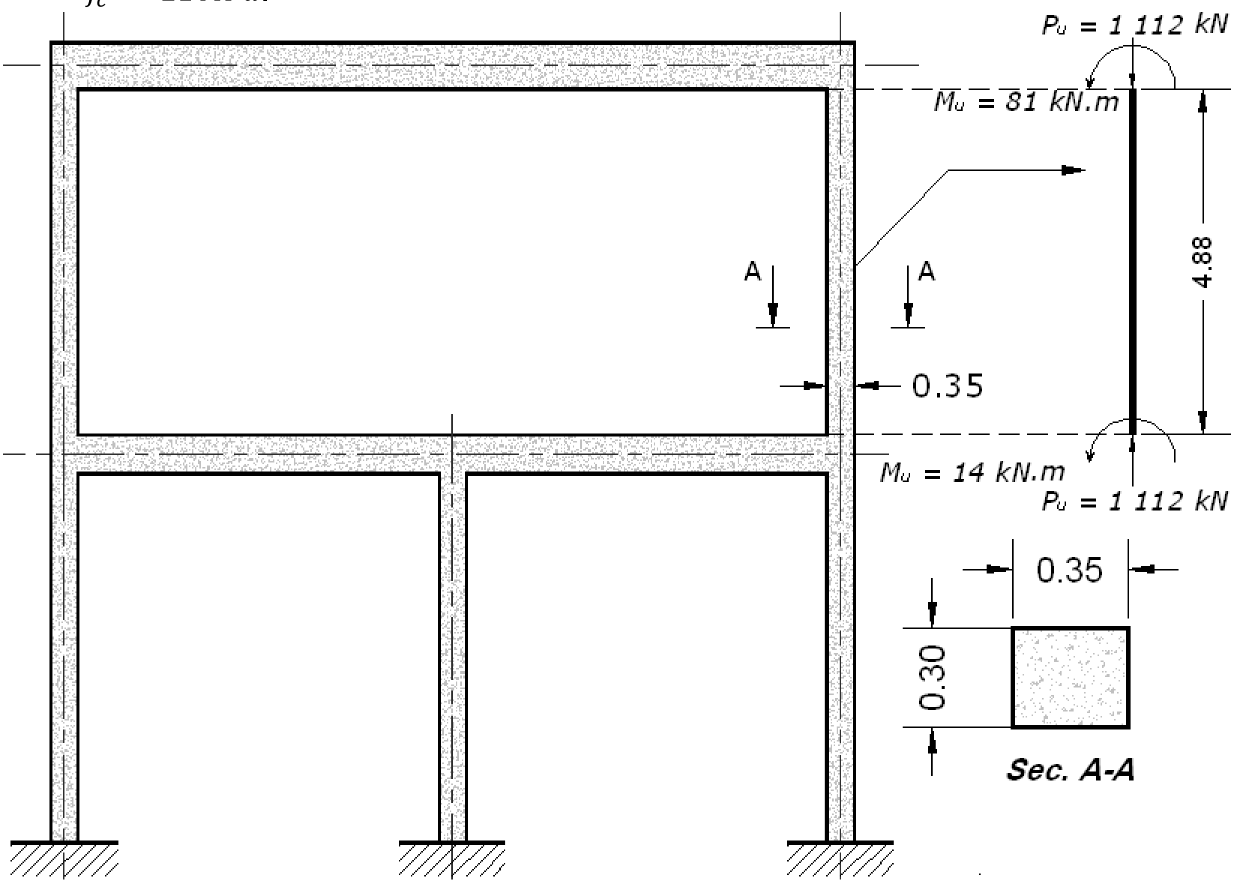


Figure 10.3-1: Frame and column forces for Example 10.3-1.

Solution

- Check to see if the column is classified short or slender according to ACI criterion:

$$\frac{kl_u}{r}\ \ ?\ \ 34 - 12\frac{M_1}{M_2} \leq 40$$

- Sign of M_1/M_2 can be conclude as follows:
  - Both ends of the column will rotate in anti-clockwise direction according to applied moments.
  - Then based on continuity principle, deflected shape will be as indicated in ***Figure 10.3-2*** below.
  - Based on above reasoning, one can conclude that the column is under double curvature and sign of M_1/M_2 ***should be negative according to ACI sign convention***.

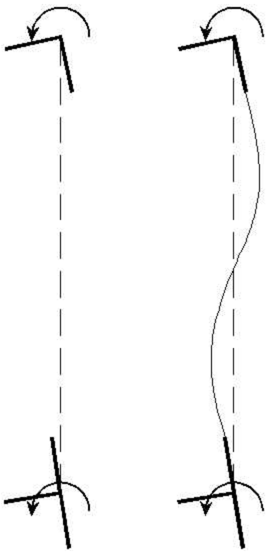


Figure 10.3-2: Deflected shape for columns of Example 10.3-1, deduced based on continuity conditions.

- Then

  $$\frac{\mathrm{kl_u}}{\mathrm{r}} = \frac{0.9 \times 4.88}{0.3 \times 0.35} \ ? \ 34 - 12\left(-\frac{14}{81}\right) \leq 40$$

  $$\frac{\mathrm{kl_u}}{\mathrm{r}} = 41.8 > (36.1 \ \leq 40\,)$$

  Therefore, ***column is classified long according to ACI criterion***.
- Moments of first order analysis should be modified to include the slenderness effect:

  $M_c = \delta_{ns} M_2$

  where:

  $$\delta_{ns} = \frac{C_m}{1 - \frac{P_u}{0.75P_c}}$$

  $$C_m = 0.6 + 0.4\frac{M_1}{M_2} = 0.6 + 0.4\left(-\frac{14}{81}\right) = 0.53$$

  $$P_c = \frac{\pi^2 EI}{(kl_u)^2}$$

  $$EI_{eff} = \frac{0.4E_c I_g}{1 + \beta_{dns}} = \frac{0.4\left(4700\sqrt{21}\right)\left(\frac{350^3 \times 300}{12}\right)}{1.6}$$

  $$EI_{eff} = \frac{0.4(21.5 \ \times 10^3)(1.07 \times 10^9)}{1.6} = 5.75 \times 10^{12}\ N \times mm^2$$

  $$P_c = \frac{\pi^2 \times 5.75 \times 10^{12}}{(0.9 \times 4\,880)^2} = 2\,939\ kN$$

  $$\delta_{ns} = \frac{0.53}{1 - \frac{1\,112}{0.75 \times 2\,939_c}} = 1.07$$
- This means that the primary moment that compute based on first order analysis should be increased by 7% to include the secondary moment.

  $M_2 = 81_{Larger\ one\ of\ end\ moments}?\ M_{2Min} = P_u(15mm + 0.03h)$

  $M_2 = 81\ kN_{Larger\ one\ of\ end\ moments}?\ M_{2Min} = 1\,112\,000\ N(15mm + 0.03 \times 350)$

  $M_2 = 81\ kN.m_{\ Larger\ one\ of\ end\ moments} > M_{2Min} = 28.4\ kN.m$

  $M_2 = 81\ kN.m$

  $M_c = 1.07 \times 81 = 86.7\ kN.m$ ■

Example 10.3-2

Figure 10.3-3 below shows an elevation view of a multistory concrete frame building. Assumptions and Data:

- 1.2m wide by 0.3 deep beams on all column lines.
- Interior columns are tentatively dimensioned at 0.45m by 0.45m.
- Exterior columns are 0.40m by 0.40m.
- The frame is effectively braced against sway by stair and elevator shafts having concrete walls that are monolithic with the floors located in the building corners (not shown in the figure).
- Live loads have been subjected based on load pattern shown. This pattern has been selected based on a first-order analysis. Full roof live load distribution on roof and upper floors has been used.
- Service internal forces (axial force and bending moments at column two ends) for the typical interior column C3 have been computed based on a first order analysis (usually done by a computer program) and as shown below:

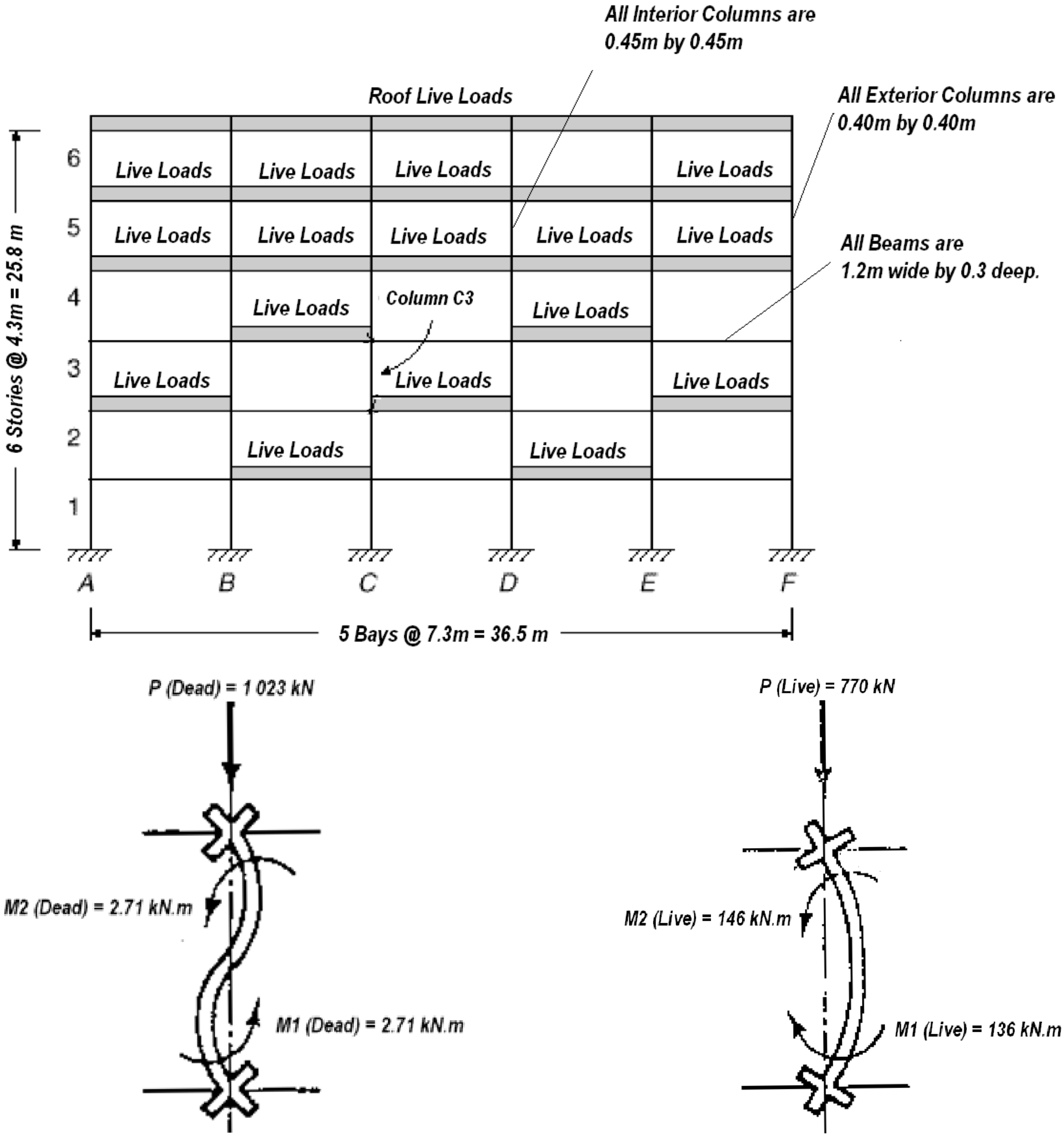


Figure 10.3-3: Frame for Example 10.3-2.

Based on above data and assumptions, design column C3 using the ACI moment magnifier method. Use $f_c' = 28MPa$ and $f_y = 420\ MPa$.

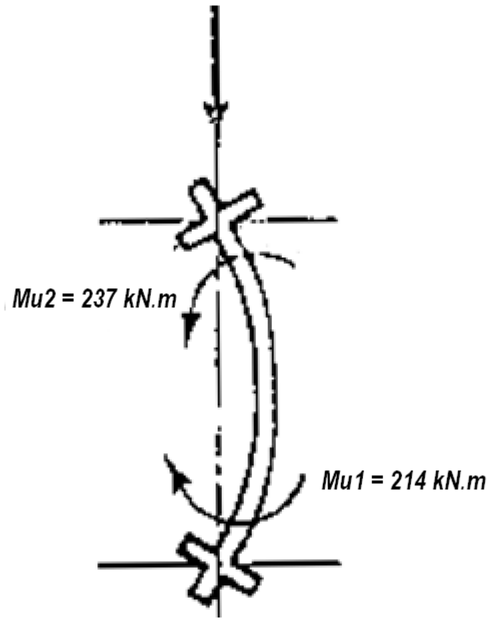
Solution

- Aim of this example is to show the direct application of ACI procedure for design a slender column in a braced frame (the column is not in the ground floor).
- Select a trial column: dimensions of a trial column have been assumed in the problem statement (0.45m by 0.45m).
- Factored axial force and bending moments on Column C3 can be computed as follows:

  $P_u = 1.2 \times 1\,023\ kN + 1.6 \times 770\ kN = 2\,460\ kN$

  $M_{u2} = 1.2 \times 2.71\ kN.m + 1.6 \times 146\ kN.m = 237\ kN.m$

  $M_{u1} = 1.2 \times (-2.71)\ kN.m + 1.6 \times 136\ kN.m = 214\ kN.m$
- Resultant of factored loads is shown in Figure below. Then column C3 will have single curvature due to factored loads.



- Determine if the frame should be considered as nonsway or sway:
  It is clear from problem statement that based on intuition the stiffness of shear wall has been assumed to be adequate to classify the building frame as a braced frame. If this decision is questionable, it can be checked based on stability index Q.
- Find the unsupported length l_u:

  $l_u = 4.3m - 0.3m = 4.0m$
- For the trial column, check for consideration of slenderness effects using the k = 1.0.

  $$\frac{kl_u}{r} = \frac{1.0 \times 4.0\ m}{0.3 \times 0.45m} = 29.6\ ?\ 34 - 12\frac{M_1}{M_2} = 34 - 12 \times \frac{214}{237} = 23.2 < 40$$

  $$\frac{kl_u}{r} = 29.6 > 34 - 12\frac{M_1}{M_2} = 23.2$$

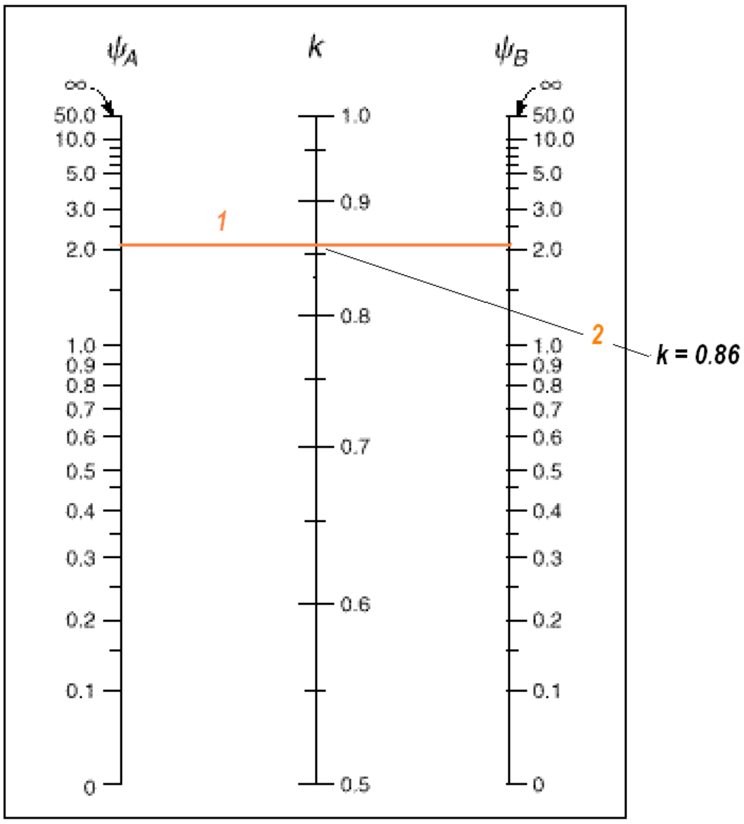
  Then, the column C3 is a slender column.
- As slenderness is tentatively found to be important, refine the calculation of k based on the alignment chart in ***Article 10.1.2.5***:

  $$\Psi = \frac{\sum \frac{EI}{l}_{Columns}}{\sum \frac{EI}{l}_{Beam}}$$

  Because E_c is the same for column and beams, it will be canceled in the stiffness calculations.

  $$\Psi = \frac{\sum \frac{I}{l}_{Columns}}{\sum \frac{I}{l}_{Beam}}$$

$$\Psi_a = \Psi_b = \frac{\dfrac{0.7 \times \dfrac{0.45^4}{12}}{4.3} \times 2}{\dfrac{0.35 \times (\dfrac{1.2 \times 0.3^3}{12} \times 2)}{7.3} \times 2} = \frac{1.112 \times 10^{-3}}{0.518 \times 10^{-3}} = 2.15$$



(a) Nonsway frames

Figure 10.3-4: Alignment chart applicable for Example 10.3-2.

$$\frac{kl_u}{r} = \frac{0.86 \times 4.0\ m}{0.3 \times 0.45m} = 25\ >\ 34 - 12\frac{M_1}{M_2} = 23.2$$

$$\frac{kl_u}{r} = 29.6 >\ 34 - 12\frac{M_1}{M_2} = 23.2$$

This is still above the limit value of 23.2, conforming that slenderness must be considered.

- If moments from the frame analysis are small, check to determine if the following minimum moment controls:

  $M_2 ?\ M_{2,min} = P_u(15 + 0.03h)$

  $M_2 = 237\ kN.m\ ?\ M_{2,min} = 2\,460\,000\ (15 + 0.03 \times 450mm) = 70.1\ kN.m$

  $M_2 = 237\ kN.m > M_{2,min} = 70.1\ kN.m\ \ Ok.$

- Calculate the equivalent uniform moment factor C_m (as was discussed in ***Article 10.1.3.2***).

  $$C_m = 0.6 + 0.4\frac{M_1}{M_2} = 0.6 + 0.4 \times \frac{214}{237} = 0.96$$

- Calculate β_{dns}, EI and P_c as discussed in ***Article 10.1***.

  $$\beta_d = \frac{\text{maximum factored axial sustained load}}{\text{maximum factored axial load associated}} = \frac{1.2\ \times 1\,023}{2\,460} = 0.50$$

  $$EI = \frac{0.4E_cI_g}{1 + \beta_d} = \frac{0.4 \times 4700 \times \sqrt{28} \times \dfrac{450^4}{12}}{1.5} = 2.27\ \times 10^{13}\ N.mm^2$$

  $$P_c = \frac{\pi^2 EI}{kl^2} = \frac{\pi^2 \times 2.27\ \times 10^{13}\ N.mm^2}{(0.86 \times 4\,000\ mm)^2} = 18\,913\ \text{kN}$$

- Calculate the moment magnification factor δ_{ns} and magnified moment M_c based on following relations:

  $$\delta_{ns} = \frac{C_m}{1 - \dfrac{P_u}{0.75P_c}} = \frac{0.96}{1 - \dfrac{2\,460}{0.75 \times 18\,913}} = 1.16\ \geq 1.0\ Ok.$$

$M_c = \delta_{ns} M_2 = 1.16 \times 237 = 275\ kN.m$

- Finally, column C3 can be designed for an axial force of $P_u = 2\,460\ kN.m$ and a bending moment of $M_u = 275\ kN.m$ and according to procedures of Chapter 9 to obtain the design results that shown in Figure below:

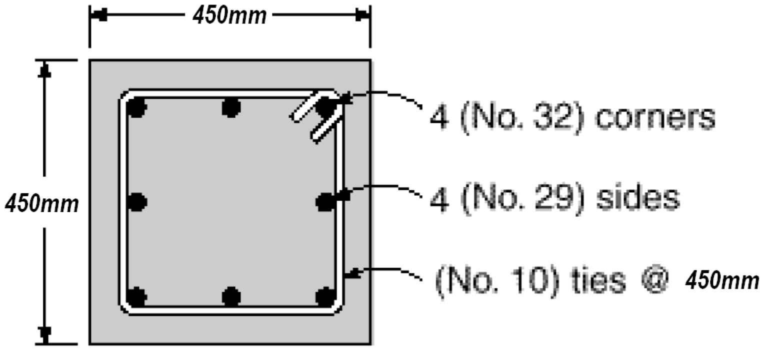


Figure 10.3-5: Final design section for Example 10.3-2.

Example 10.3-3

The frame shown in ***Figure 10.3-6*** below is a part of a building that can be considered braced by presence of stiff concrete walls surrounding the elevator shafts.

The soil under the footings is soft with a relative stiffness of $\psi = 5$[1] is considered appropriate at the base. Other values of ψ have been computed and are given in ***Figure 10.3-6*** below.

Structural factored load for column B between points 0 and 1 is $P_u = 2\,000\ kN$. The concrete modulus of elasticity is $E_c = 30\,000\ MPa$. All columns are 400 mm x 400 mm square. Assume that $\beta_{dns} = 0.5$.

- Calculate the buckling load;
- Calculate the moment that should be used to design column B in the ground floor.

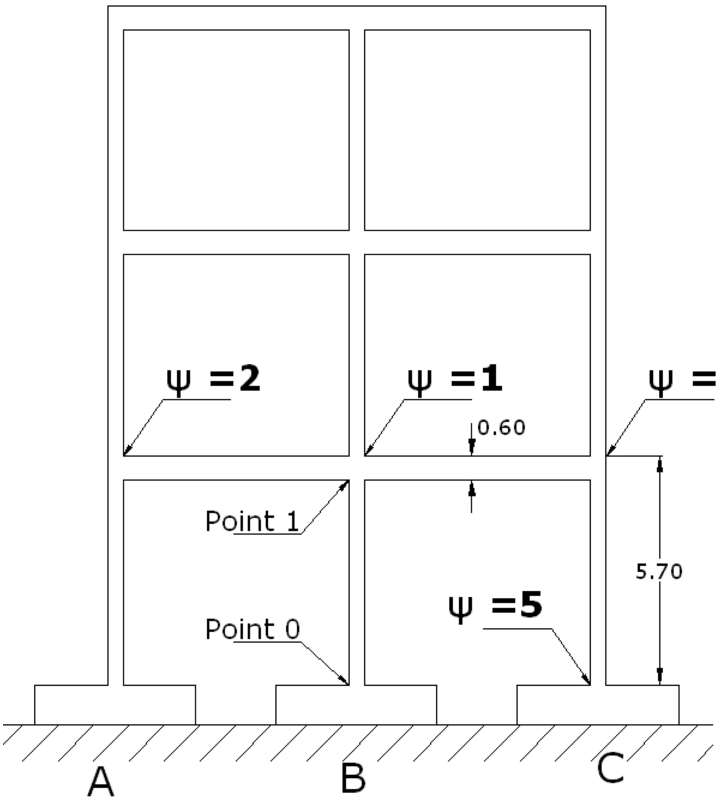


Figure 10.3-6: Frame for Example 10.3-3.

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[1] For more details about computing ψ for foundation, see "Reinforced Concrete: Mechanics and Design", 4th Edition by J. G. Macgregor (Page 568).

Solution

Practical Aspects of the Problem

- This problem simulates a very common case that faces the structural engineer in his daily work with the interior columns. Generally, moment in an interior column of a multistory building has a negligible value due to the length and load symmetry between different spans. As the approach of structural stability of the slender columns is based on the amplification of the actual moments only, then it cannot be able to simulate inverse proportionality between the strength of an axially loaded column and its slenderness ratio.
- To extend this approach to include this common and important case, the ACI Code states that (as was previously discussed) the design moment in a slender column of a braced frame must not be taken less than the following minimum value:

  $M_{2,Min} = P_u(15mm + 0.03hmm)$
- When this moment is substituted into the relation of the ACI moment magnification method, one will obtain the following relation:

  $$M_c = \frac{1}{1 - \frac{P_u}{0.75P_c}} \times [P_u(15\text{ mm} + 0.03h\text{ mm})]$$
- If the applied factored load P_u approaching the Euler Buckling Load P_c, the design moment M_c will approach the infinite. Then the ACI moment magnification method has been extended to simulate the well-known fact of the inverse proportionality between the strength of the axially loaded column and its slenderness ratio (Euler Law).

Compute the Buckling Load P_{cr}:

Compute the effective length factor from the alignment chart of braced frames, with $\psi_0 = 5$ and $\psi_1 = 1$, see ***Figure 10.3-7*** below.

$k = 0.84$

$l_u = 5.7m - 0.6m = 5.1m$

$$P_{cr} = \frac{\pi^2 EI}{(kl_u)^2} = \frac{\pi^2 \times \frac{30\,000}{1+0.5} \times 0.4 \times \frac{(400^4)}{12}}{(0.84 \times 5\,100)^2} = 9\,169\ kN$$

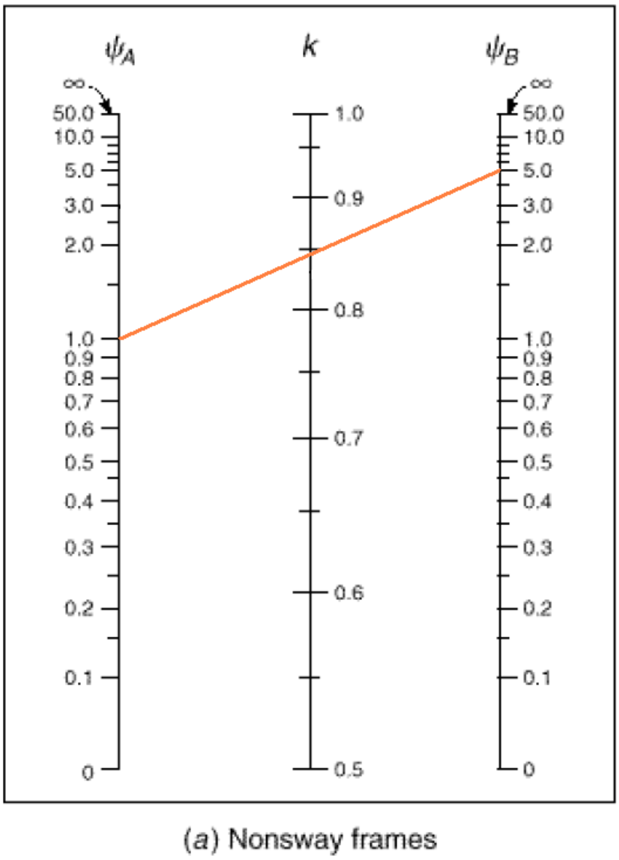

Figure 10.3-7: Alignment chart applied to Example 10.3-3.

Compute the Design Moment

- Check if the column is short or slender column:

$$\frac{kl_u}{r} = \frac{0.85 \times 5.1m}{0.3 \times 0.4m} = 36.125 \ ? \ 34 - 12\frac{M_1}{M_2}$$

As $M_1 = 0$ and $M_2 = M_{2\,Min}$, then the above ratio will be:

$$\frac{kl_u}{r} = 36.2 > 34 - 12 \times 0$$

$$\frac{kl_u}{r} = 36.2 > 34$$

Then, the column is a slender column.

$$M_c = \delta_{ns} M_{2\,Min}$$

$$\delta_{ns} = \frac{C_m}{1 - \frac{P_u}{0.75P_c}}$$

- For member in which $M_{2\,Min}$ exceeds M_2 (as for our problem), ACI states that the C_m can be either taken equal to 1.0 or shall be based on the ratio of the computed end moments M_1 and M_2. Our solution will be based on the more conservative value of $C_m = 1.0$.

Then:

$$\delta_{ns} = \frac{1}{1 - \frac{P_u}{0.75P_c}}$$

$$P_u = 2\,000\ kN$$

$$\delta_{ns} = \frac{1}{1 - \frac{2\,000}{0.75 \times 9\,169}} = 1.42$$

As the ***magnification factor is in the range of maximum value of 1.4, hence column dimensions should be revised or additional bracing should be adopted***.

- Finally:

$$M_c = 1.42 \times [2\,000 \times 10^3 \times (15\ mm + 0.03 \times 400mm)] = 1.42 \times 54\ kN.m = 76.7\ kN.m\ ■$$

Example 10.3-4

A column is loaded as shown in ***Figure 10.3-8*** below. Check if the column is short or slender, and then compute the moment that must be used in the design. Assume $f_c' = 28\ MPa$ and $\beta_{dns} = 0.4$.

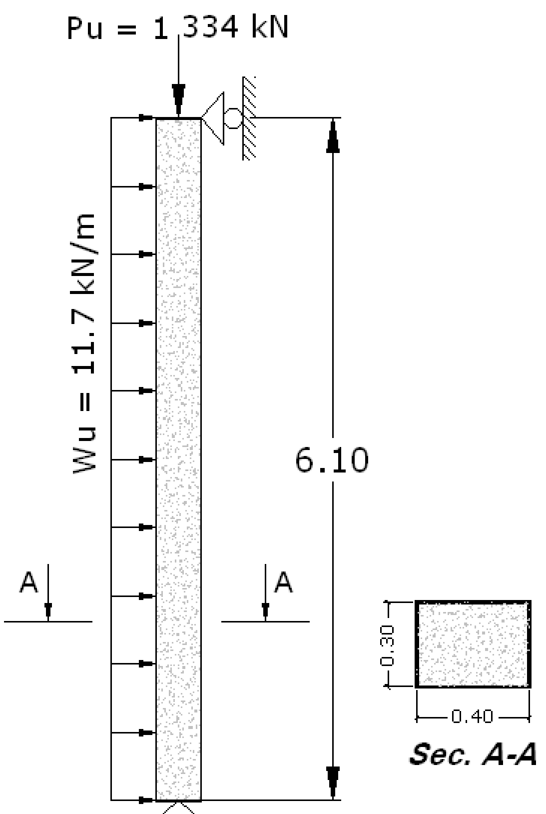

Figure 10.3-8: Column for Example 10.3-4.

Solution

- The problem shows how to use the moment magnification method in a column that has no end moments $M_1 = M_2 = 0$ but has a large mid-span bending moment.
- As ACI design procedure in written in terms of end moment M_2, then this problem requires a special consideration.
- It can be shown that the ACI for procedure for a column with mid-span moment can be rewritten in the following form[2]:
  $M_c = M_{Mid\ Span} \times \delta_{ns}$
- Checking to see if the column is short or slender:
  $$\frac{\mathrm{kl_u}}{\mathrm{r}} = \frac{1.0 \times 6.1\mathrm{m}}{0.3 \times 0.4\mathrm{m}} = 50.8 \ ?\ 34 - 12\frac{\mathrm{M_1}}{\mathrm{M_2}} = 34$$
  $$\frac{\mathrm{kl_u}}{\mathrm{r}} = 50.8 > 34$$
  Then the column is classified as a long column according to ACI Code.
- Compute the Design Moment $\boldsymbol{M_c}$:
  $$\delta_{\mathrm{ns}} = \frac{\mathrm{C_m}}{1 - \frac{\mathrm{P_u}}{0.75\mathrm{P_c}}}$$
  As the column is subjected to transverse loads, then:
  $\mathrm{C_m} = 1.0$
  $$EI = \frac{4700\sqrt{28}}{1 + 0.4} \times \left(0.4 \times \frac{400^3 \times 300}{12}\right) = 1.14 \times 10^{13}\ N.mm^2$$
  $$P_{cr} = \frac{\pi^2 EI}{(kl_u)^2} = \frac{\pi^2 \times 1.14 \times 10^{13}\ N.mm^2}{(1.0 \times 6\,100)^2\ mm^2} = 3\,021\ kN$$
  $$\delta_{\mathrm{ns}} = \frac{\mathrm{C_m}}{1 - \frac{\mathrm{P_u}}{0.75\mathrm{P_c}}} = \frac{1.0}{1 - \frac{1\,334}{0.75 \times 3\,021}} = 2.43$$
  As the ***magnification factor is significantly larger than the maximum value of 1.4, hence column dimensions should be revised or additional bracing should be adopted***.
  $$M_c = M_{Mid\ Span} \times \delta_{ns} = \frac{11.7 \times 6.1^2}{8} \times 2.43 = 132\ kN.m\ ■$$

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[2] See "Design of Reinforced Concrete" 7th Edition, by J. C. McCormac (Page 329).

10.4 Summary of ACI Moment Magnifier Method for Sway Frames

For a column that may be a slender column in a sway story, ACI checking procedure can be summarized as follows:

- Check to see if the story can be classified as sway or braced. This can be done either based on inspection or based on the concept of stability index "Q".
- Check if the column is short or slender based on the following limitation:
  $\frac{kl_u}{r} \leq 22$
- Classify the applied loads into:
  - That don't produce sway (for example dead and live loads),
  - That produces sway (for example wind load, earthquake load, and loads due to lateral earth pressure.
- Based on first order structural analysis compute the axial forces "P_u" and bending moment ("M_s" and "M_{ns}") due to sway and nonsway loads respectively.
- Compute the design moments based on the following relation:
  $M_1 = M_{1ns} + \delta_s M_{1s}$ $M_2 = M_{2ns} + \delta_s M_{2s}$
  where the moment magnification factor for a sway story can be computed based on one of the following two approaches:
  - First Method:

$$\delta_s = \frac{1}{1-Q} \geq 1.0 \quad \textbf{Eq. 10.4-1}$$

If δ_s calculated exceeds 1.4, δ_s shall be re-calculated by second method below.

  - Second Method:

$$\delta_s = \frac{1}{1-\frac{\sum P_u}{0.75 \sum P_c}} \geq 1.0 \quad \textbf{Eq. 10.4-2}$$

where

i. $\sum P_u$ is the summation for all the factored vertical loads in a story,

ii. $\sum P_c$ is the summation for all sway-resisting columns in a story,

iii. P_c is the Euler load computed as discussed in ***Article 10.1*** with k from alignment chart for sway frame and with β_{ds} defined as ***the ratio of the maximum factored sustained shear within a story to the maximum factored shear in that story***. Lateral forces due to wind or earthquake cannot be sustained loads. Then β_{ds} for these loads is always equal to zero. While lateral forces due to earth pressure can be sustained loads and β_{ds} for these loads is not equal to zero (See Figures below).

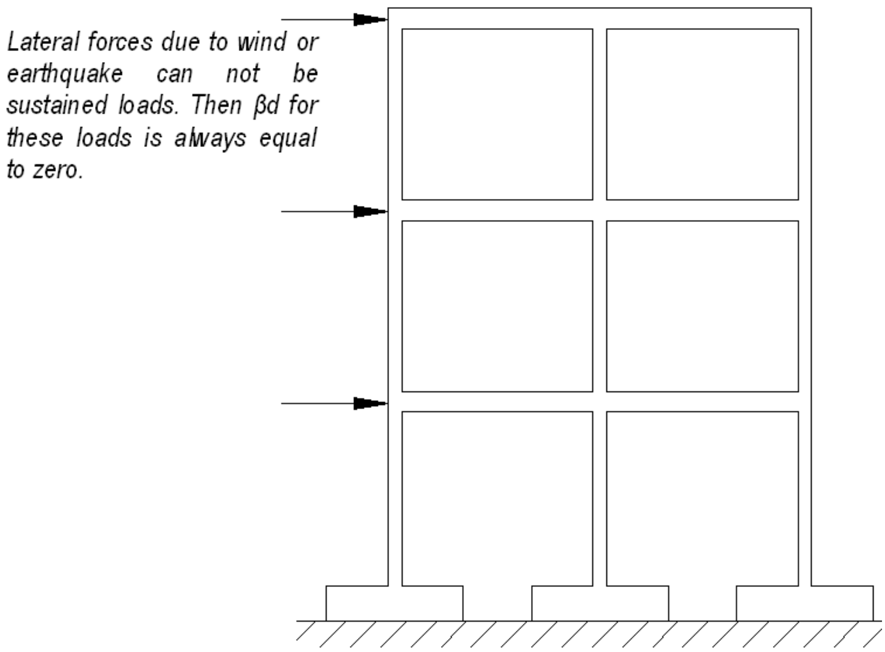


Figure 10.4-1: Transient lateral force with $\beta_{ds} = 0$.

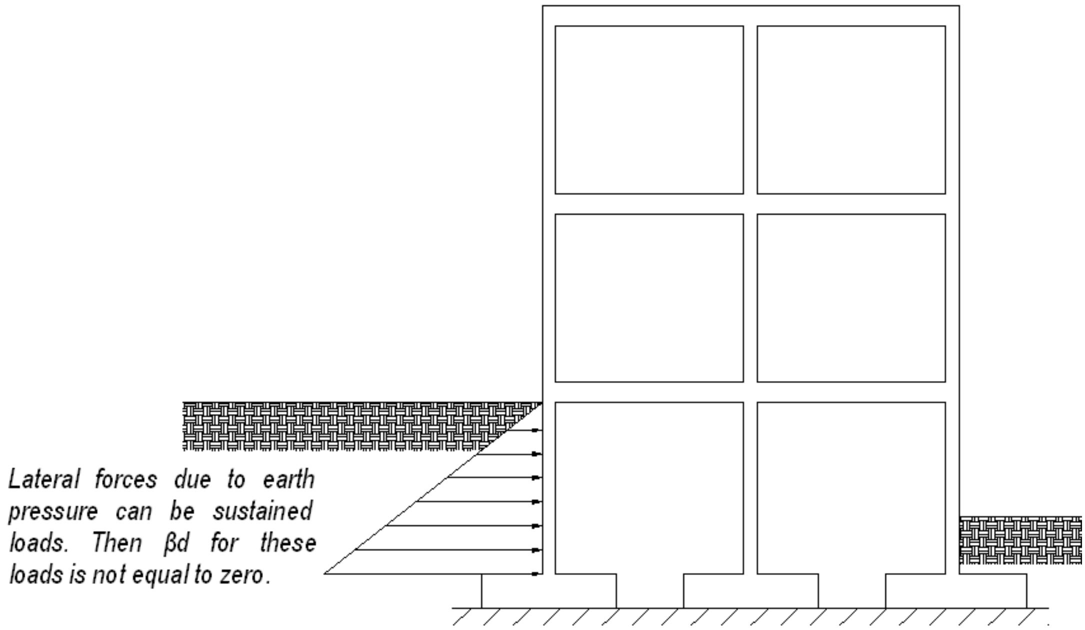


Figure 10.4-2: Sustained lateral force with $\beta_{ds} \neq 0$.

Example 10.4-1

For the sway frame shown Figure 10.4-3 below:

- Check to see if column AB is classified as short or slender according to ACI criterion?
- If column is found to be slender, use ACI ***Moment Magnification Method*** to magnify the first order moments to include the slenderness effects.

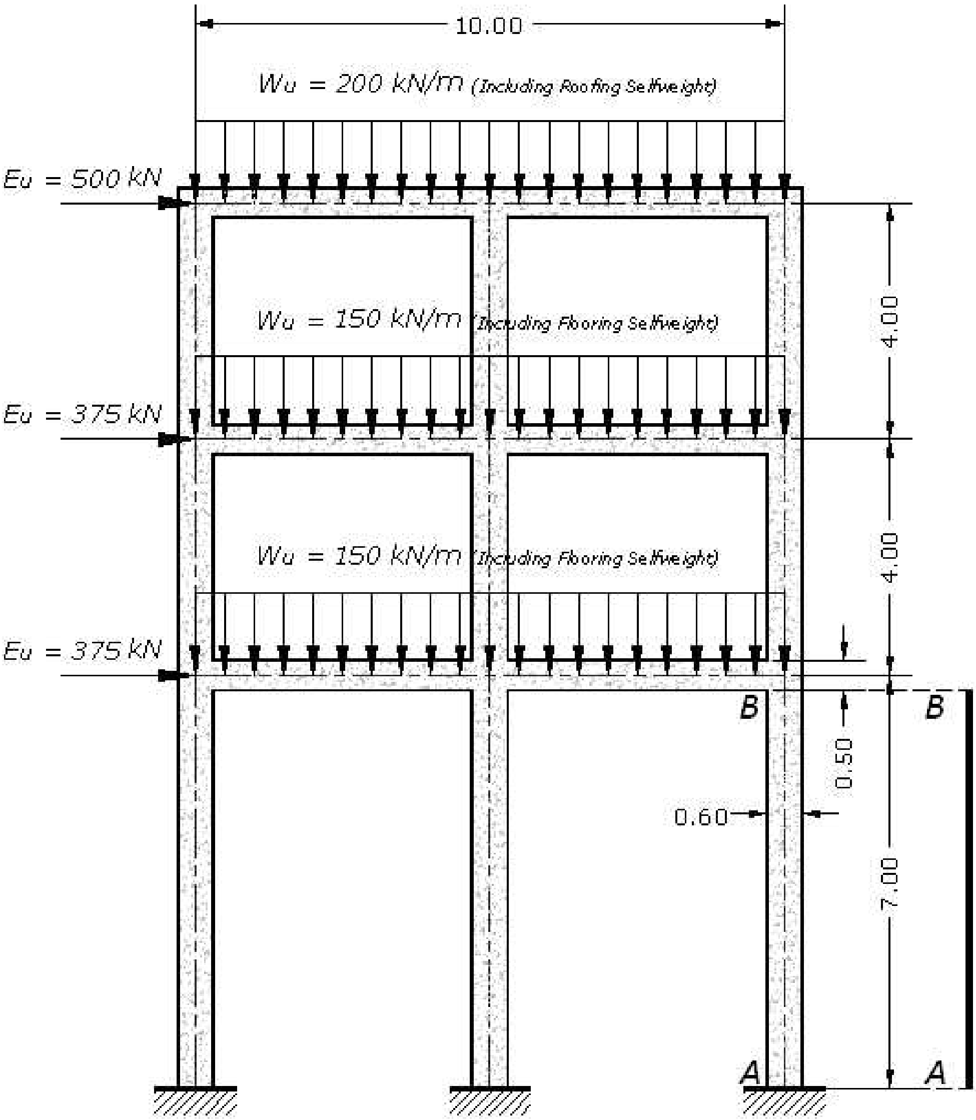


Figure 10.4-3: Frame for Example 10.4-1.

In your solution assume that:

- All columns are 0.6m by 0.3m with f_c' = 28 MPa,
- All supports are fixed,
- Columns selfweight can be neglected,
- First order moment due to gravity loads (nonsway load) and due to lateral loads (sway loads) are as indicated in ***Figure 10.4-4*** below,
- Under sway loads, end B has zero rotation (***Shear Building Assumption***).

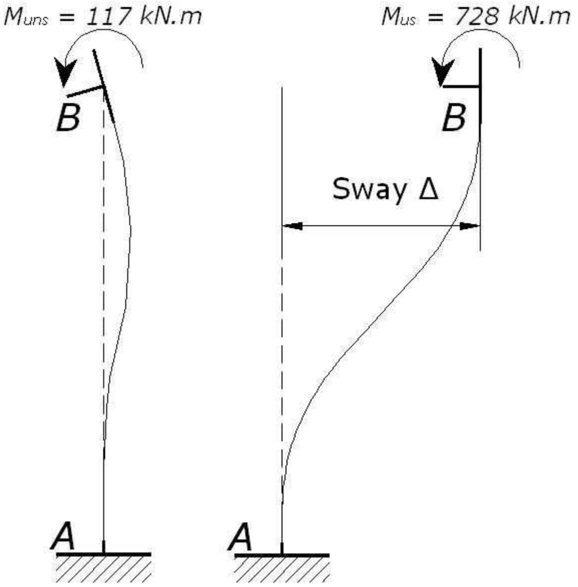


Figure 10.4-4: Moments from a first order analysis for Column AB of Example 10.4-1.

Solution

- Check to see if the story can be classified as sway or braced. This can be done either based on intuition or based on the concept of stability index "Q":
  Based on problem statement, column is part from unbraced story.
- Check if the column is short or slender based on the following limitation:
  $$\frac{kl_u}{r} \ ?\ 22$$
  $$l_u = 7.0 - \frac{0.5}{2} = 6.75m$$
- Based on assuming that end B has zero rotation, column boundary conditions will be similar to those presented in ***Figure 10.4-5*** below.

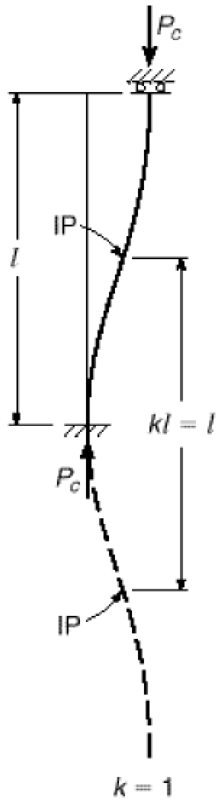


Figure 10.4-5: Column boundary conditions with shear building assumption.

Then k = 1.0

$r = 0.3h = 0.3 \times 0.6m = 0.18m$

$$\frac{kl_u}{r} = \frac{1.0 \times 6.75}{0.18} = 37.5 > 22$$

Then the column is a slender column.

- Classify the applied loads into a load that don't produce sway and that produces sway:
  Loads are already classified in the problem statement.
- Compute the design moments based on the following relation:
  $M_2 = M_{2ns} + \delta_s M_{2s}$
  As we have no information about stability index, the δ_s can only be computed based on second method:

$$\delta_s = \frac{1}{1 - \frac{\sum P_u}{0.75 \sum P_c}}$$

As selfweight of columns can be neglected according to problem statement, then resultant of vertical loads $\sum P_u$ will be computed based on factored loads acting on roof and floors:

$$\sum P_u = \left(200\frac{kN}{m} \times 10m\right)_{Roof\ Loads} + 2\left(150\frac{kN}{m} \times 10m\right)_{Floors\ Loads}$$

$$\sum P_u = 5000\ kN$$

Based on assumption of rigid flooring systems, all columns will have same boundary conditions, and as all columns have same dimensions, then they will have same critical load P_C.

$$\sum P_c = 3 \times P_{c\ for\ column\ AB}$$

$$P_{c\ \text{for column AB}} = \frac{\pi^2 EI}{(kl_u)^2}$$

$$EI = \frac{4700\sqrt{28}}{1 + 0_{\beta_{ds}\ is\ Zero\ for\ Lateral\ Loads}} \times \left(0.4 \times \frac{600^3 \times 300}{12}\right)$$

$$EI = 24.9 \times 10^3 \frac{N}{mm^2}(2.16 \times 10^9 mm^4) = 53.8 \times 10^{12}\ N.mm^2$$

$$P_{c\ \text{for column AB}} = \frac{\pi^2 \times 53.8 \times 10^{12}\ N.mm^2}{(1.0 \times 6\,750\ \text{mm})^2} = 11\,642\ kN$$

$$\sum P_c = 3 \times 11\,642\ kN = 34\,926\ kN$$

$$\delta_s = \frac{1}{1 - \frac{\sum P_u}{0.75 \sum P_c}} = \frac{1}{1 - \frac{5000}{0.75 \times 34\,926}} = 1.24 < 1.4 \therefore \text{Ok.}$$

$$M_2 = M_{2ns} + \delta_s M_{2s} = 117\ \text{kN.m} + 1.24 \times 728\ \text{kN.m} = 1\,020\ \text{kN.m}$$

Contents

CHAPTER 11

ANALYSIS OF INDETERMINATE BEAMS AND FRAMES

11.1 CONTINUITY

- Continuity in steel structures:
    - The individual members that compose a steel structure are fabricated or cut separately and joined together by rivets, bolts, welds, or nails.
    - Unless the joints are specially designed for rigidity, they are ***too flexible to transfer moments of significant magnitude from one member to another***.
- Continuity in reinforced concrete structures:
    - In contrast, in reinforced concrete structures:
        - As much of the concrete as is practical is placed in one single operation.
        - Reinforcing steel is not terminated at the ends of a member but is extended through the joints into adjacent members.
        - At construction joints, special care is taken to bond the new concrete to the old by carefully cleaning the latter, by extending the reinforcement through the joint, and by other means.

      As a result, reinforced concrete structures usually represent ***monolithic***, or ***continuous***, units.
    - A load applied at one location causes deformation and stress at all other locations.
    - Even in precast concrete construction, which resembles steel construction in that individual members are brought to the job site and joined in the field, connections are often designed to provide for the transfer of moment as well as shear and axial load, producing at least partial continuity.
- The effect of continuity:
    - In continuous beams:
        - The effect of continuity is most simply illustrated by a continuous beam, as shown in ***Figure 11.1-1a***.
        - With simple spans, such as provided in many types of steel construction, only the loaded member CD would deform, and all other members of the structure would remain straight.
        - But with continuity from one member to the next through the support regions, as in a reinforced concrete structure, the distortion caused by a load on one single span is seen to spread to all other spans, although ***the magnitude of deformation decreases with increasing distance from the loaded member***.
        - All members of the six-span structure are subject to curvature, and thus also to bending moment, as a result of loading span CD.
    - In rigid-jointed frame subjected to gravity forces:
        - Similarly, for the rigid-jointed frame of ***Figure 11.1-1b***, the distortion caused by a load on the single member GH spreads to all beams and all columns, although, as before, ***the effect decreases with increasing distance from the load***.
        - All members are subject to bending moment, even though they may carry no transverse load.

- In rigid-jointed frame subjected to horizontal forces:
  - If horizontal forces, such as forces caused by ***wind*** or ***seismic action***, act on a frame, it deforms as illustrated by ***Figure 11.1-1c***.
  - Here, too, all members of the frame distort, even though the forces act only on the left side; the amount of distortion is ***seen to be the same for all corresponding members, regardless of their distance from the points of loading, in contrast to the case of vertical loading***.
  - A member such as EH, even without a directly applied transverse load, will experience deformations and associated bending moment.

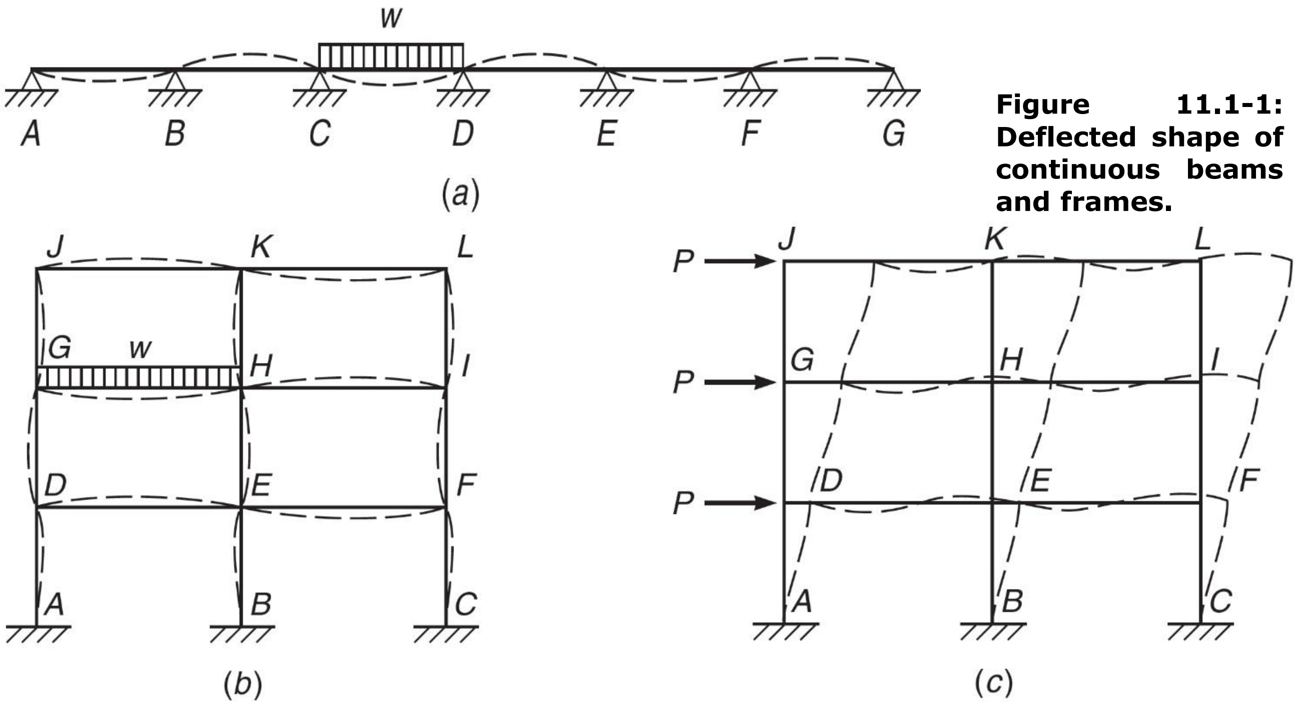


Figure 11.1-1: Deflected shape of continuous beams and frames.

- Statically determinate versus statically indeterminate structures:
  - In ***statically determinate structures***, such as simple-span beams, ***the deflected shape and the moments and shears depend only on the type and magnitude of the loads and the dimensions of the member***.
  - In contrast, inspection of the ***statically indeterminate structures*** in Figure 11.1-1 shows that ***the deflection curve of any member depends not only on the loads but also on the joint rotations, whose magnitudes in turn depend on the distortion of adjacent, rigidly connected member***s.
  - For a rigid joint such as joint H in the frame shown in Figure 11.1-1b or c, all the rotations at the near ends of all members framing into that joint must be the same.
  - For a correct design of continuous beams and frames, it is evidently necessary to determine moments, shears, and thrusts considering the effect of continuity at the joints.
- Analysis of statically indeterminate structures
  - Elastic analysis:

    The determination of these internal forces in continuously reinforced concrete structures is usually based on elastic analysis of the structure at factored loads with methods that will be described in Sections 11.2 through 11.5.
  - Prerequisite data for an elastic analysis:

    Such analysis requires knowledge of the cross-sectional dimensions of the members.

- Preliminary estimation of member dimensions for analysis purpose:
  Member dimensions are initially estimated during preliminary design, which is described in ***Section 11.6*** along with guidelines for establishing member proportions.
- Approximate methods and their usefulness:
  - For ***checking the results of more exact analysis***, the approximate methods of ***Section 11.7*** are useful.
  - ACI coefficient method:
    For many structures, ***a full elastic analysis is not justified***, and ***the ACI coefficient method of analysis described in Section 11.8 provides an adequate basis for design moments and shears***.

11.2 LOADING*

- The individual members of a structural frame must be designed for the worst combination of loads that can reasonably be expected to occur during its useful life.
- Internal moments, shears, and thrusts are brought about by the combined effect of dead and live loads, plus other loads, such as wind and earthquake, as discussed in ***Chapter 1***.
- Dead versus live loads:
  - While dead loads are constant, live loads such as floor loads from human occupancy can be placed in various ways, some of which will result in larger effects than others.
  - In addition, the various combinations of factored loads specified in ***Chapter 1*** must be used to determine the load cases that govern member design. The subject of load placement will be addressed first.

11.2.1 PLACEMENT OF LOADS

- Influence lines for maximum positive moments through imagination of deflected shape:
  - In ***Figure 11.2-1a*** only span CD is loaded by live load. The distortions of the various frame members are seen to be largest in, and immediately adjacent to, the loaded span and to decrease rapidly with increasing distance from the load.
  - Since bending moments are proportional to curvatures, the moments in more remote members are correspondingly smaller than those in, or close to, the loaded span.
  - However, the loading shown in ***Figure 11.2-1a*** does not produce the maximum possible positive moment in CD. In fact, if additional live load were placed on span AB, this span would bend down, BC would bend up, and CD itself would bend down in the same manner, although to a lesser degree, as it is bent by its own load.
  - Hence, the positive moment in CD is increased if AB and, by the same reasoning, EF are loaded simultaneously.
  - ***By expanding the same reasoning to the other members of the frame, it is easy to see that the <u>checkerboard pattern</u> of live load shown in Figure 11.2-1b produces the largest possible positive moments***, not only in CD but also in all loaded spans.
  - Hence, two such checkerboard patterns are required to obtain the maximum positive moments in all spans.

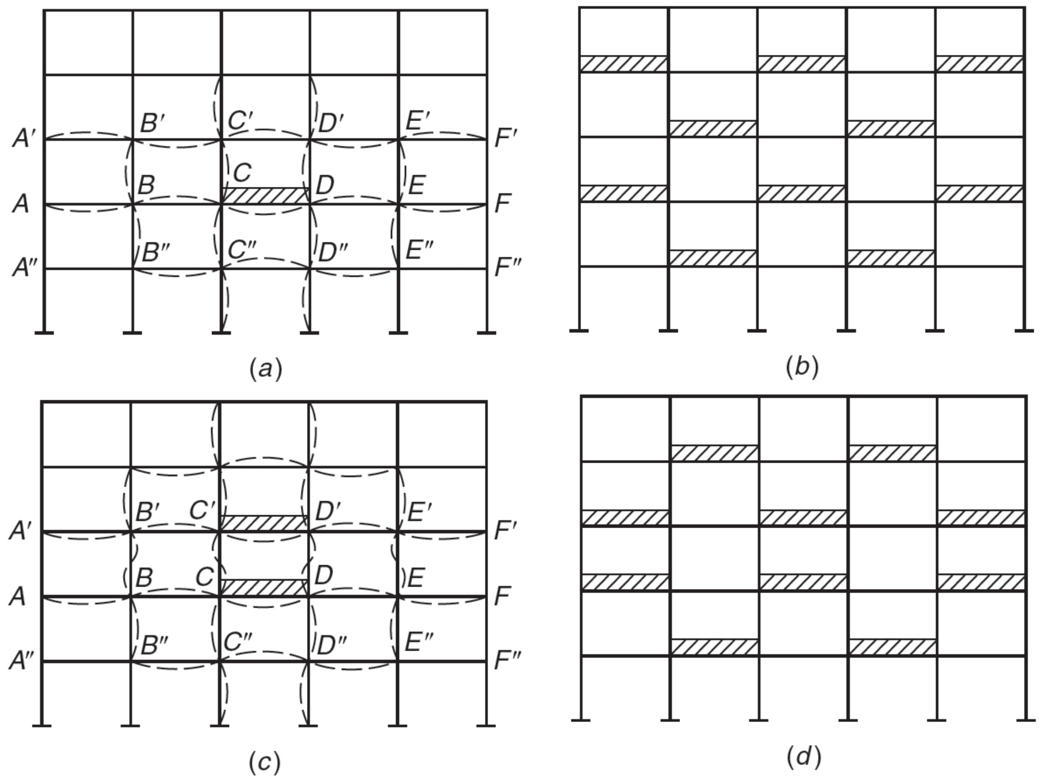


Figure 11.2-1: Alternate live loadings for maximum effects.

- Influence lines for minimum span moments:
  - In addition to maximum span moments, i***t is often necessary to investigate minimum span moments***.
  - Dead load, acting as it does on all spans, usually produces only positive span moments.
  - However, live load, placed as in ***Figure 11.2-1a***, and even more so in ***Figure 11.2-1b***, is seen to bend the unloaded spans upward, that is, to produce negative moments in the span.
  - ***If these negative live load moments are larger than the generally positive dead load moments, a given girder, depending on load position, may be subject at one time to positive span moments and at another to negative span moments***. It must be designed to withstand both types of moments; that is, it must be furnished with tensile steel at both top and bottom.
  - Thus, the loading of ***Figure 11.2-1b***, in addition to giving maximum span moments in the loaded spans, gives minimum span moments in the unloaded spans.
- Influence lines for maximum negative moments at the supports:
  - Maximum negative moments at the supports of the girders are obtained, on the other hand, if loads are ***placed on the two spans adjacent to the particular support, and in a corresponding pattern on the more remote girders***.
  - A separate loading scheme of this type is then required for each support for which maximum negative moments are to be computed.
- Influence lines for maximum moments in columns:
  - In each column, the largest moments occur at the top or bottom.
  - While the loading shown in ***Figure 11.2-1c*** results in large moments at the ends of columns CC' and DD', the reader can easily be convinced that these moments are further augmented (i.e. increased) if additional loads are placed as shown in ***Figure 11.2-1d***.

11.2.2 LOAD COMBINATIONS

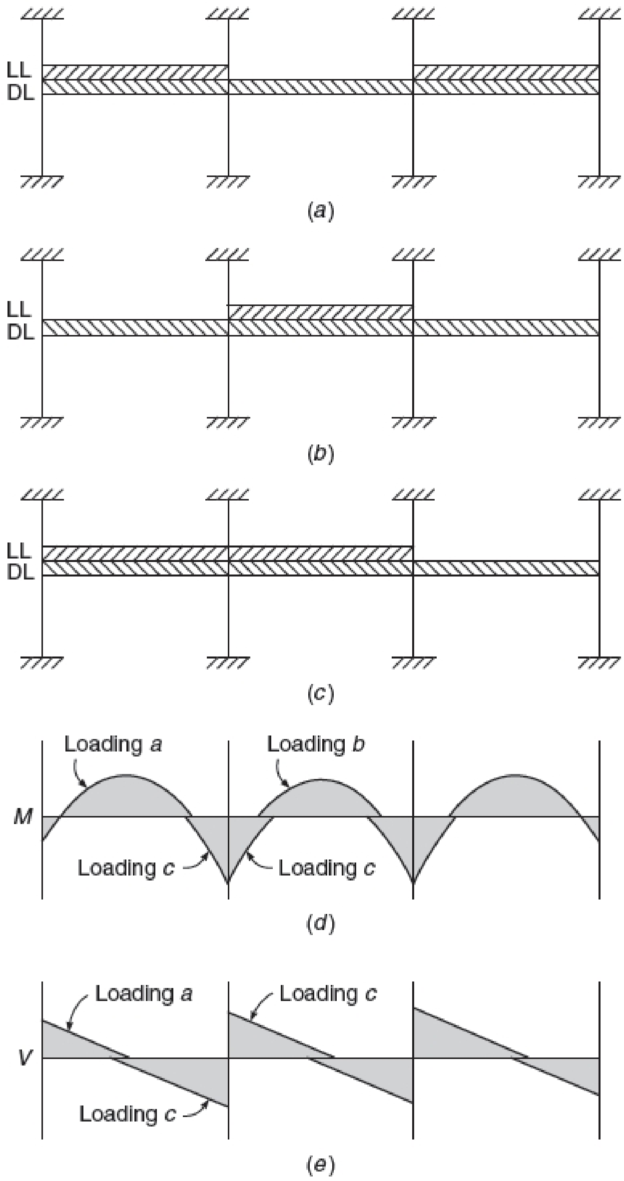
- The ACI Code requires that structures be designed for a number of load combinations, as discussed in ***Chapter 1***.
- Thus, for example, factored load combinations might include:
  - Dead plus live load;
  - Three possible combinations that include dead, live, and wind load;
  - Two combinations that include dead load, live load, and earthquake load, with some of the combinations including snow, rain, and roof live load.
- Load Case and Load Combination:
  - While each of the combinations may be considered as an individual loading condition, ***experience has shown that the most efficient technique involves separate analyses for each of the basic loads without load factors***, that is, a full analysis for unfactored dead load only, separate analyses for the various live load distributions described in Section 11.2.1, and separate analyses for each of the other loads (wind, snow, etc.).
  - Once the separate analyses are completed, it is a simple matter to combine the results using the appropriate load factor for each type of load.
  - This procedure is most advantageous because, for example, live load may require a load factor of 1.6 for one combination, a value of 1.0 for another, and a value of 0.5 for yet another. Once the forces have been calculated for each combination, the combination of loads that governs for each member can usually be identified by inspection.

11.3 SIMPLIFICATIONS IN FRAME ANALYSIS*

- Considering the complexity of many practical building frames and the need to account for the possibility of alternative loadings, there is evidently a need to simplify.
- This can be done by means of certain approximations that allow the ***determination of moments with reasonable accuracy while substantially reducing the amount of computation***.
- Simplification for Regular Frames:
  - Definition of Regular Frames:
    Regular frames can be defined as the frames that have no unusual asymmetry of loading or shape.
  - Concept of Subframe:
    - Definition of Subframe:
      In case of regular frames, moments due to vertical loads can be determined with sufficient accuracy by dividing the entire frame into simpler subframes. Each of these consists of one continuous beam, plus the top and bottom columns framing into that particular beam.
    - Load acting on Subframe:
      Placing the live loads on the beam in the most unfavorable manner permits sufficiently accurate determination of all beam moments, as well as the moments at the top ends of the bottom columns and the bottom ends of the top columns.
    - Boundary conditions for subframe:
      For this partial structure, the far ends of the columns are considered fixed, except for such first-floor or basement columns where soil and foundation conditions dictate the assumption of hinged ends.
    - Subframe in ACI code:
      Subframe approach is ***explicitly permitted by ACI Code 6.3.1.2 and 6.4***, which allow the following assumptions for floor and roof members under gravity load:
    - To calculate moments and shears in columns, beams, and slabs, the structural model may be limited to the members in the level being

considered and the columns above and below that level; the far ends of columns built integrally with the structure may be considered fixed.
- The maximum positive moment near midspan occurs with the factored live load on the span and on alternate spans, and the maximum negative moment at a support occurs with the factored live load on adjacent spans only.

- An example subframe:
  - Figure 11.3-1 demonstrates the application of the ACI Code requirements for live load on a three-span subframe.
  - The loading in ***Figure 11.3-1a*** results in:
    - The ***maximum positive moments in the exterior spans***,
    - The ***minimum positive moment in the center span***,
    - The ***maximum negative moments at the interior faces of the exterior columns***.
  - The loading shown in ***Figure 11.3-1b*** results in:
    - The ***maximum positive moment in the center span***,
    - The ***minimum positive moments in the exterior spans***.



igure 11.3-1: Subframe loading as required y ACI Code 6.4: Loading for (a) maximum ositive moments in the exterior spans, the ninimum positive moment in the center pan, and the maximum negative moments t the interior faces of the exterior columns; b) maximum positive moment in the center pan and minimum positive moments in the xterior spans; (c) maximum negative noment at both faces of the interior olumns; (d) envelope moment diagram; and e) envelope shear diagram. (DL and LL epresent factored dead and live loads, espectively).

  - The loading in ***Figure 11.3-1c*** results in:
    - The ***maximum negative moment at both faces of the interior columns***.
    - Since the structure is symmetrical, values of moment and shear obtained for the loading shown in ***Figure 11.3-1c*** apply to the right side of the structure as well as the left.

        - Due to the simplicity of this structure, joints away from the spans of interest are not treated as fixed.
    - Envelopes versus diagrams:
        - Moments and shears used for design are determined by combining the moment and shear diagrams for the individual load cases to obtain the maximum values along each span length.
        - The resulting envelope moment and shear diagrams are shown in ***Figure 11.3-1***d and ***e***, respectively.
        - Critical sections and cutoff points:

          The moment and shear envelopes (note the range of positions for points of inflection and points of zero shear) are used not only to design the critical sections but also to determine cutoff points for flexural reinforcement and requirements for shear reinforcement.
- Forces in columns:

  About columns, the ACI Code indicates:
    - The factored axial load and factored moment occurring simultaneously for each applicable factored load combination shall be considered (ACI Code 10.2.4.1).
    - For frames or continuous construction, consideration shall be given to the effect of floor and roof load patterns on the transfer of moment to exterior and interior columns and of eccentric loading due to other causes (ACI Code 6.6.2.2).
    - In computing moments in columns due to gravity loading, the far ends of columns built integrally with the structure may be considered fixed (ACI Code 6.3.1.2).
    - Floor or roof level moments shall be resisted by distributing the moment between columns immediately above and below the given floor in ***proportion to the relative column stiffnesses considering conditions of restraint*** (ACI Code 6.5.5 and 6.6.2.1).
    - Concept of Tributary Areas:
        - Although it is not addressed in the ACI Code, axial loads on columns are usually determined based on the column ***tributary areas***, which are defined based on ***the midspan of flexural members framing into each column***.
        - The axial load from ***the tributary area is used in design***, with the exception of ***first interior columns***, which are typically designed for ***an extra 10 percent axial load to account for the higher shear expected in the flexural members framing into the exterior face of first interior columns***.
        - The use of this procedure to determine axial loads due to gravity is conservative (note that the total vertical load exceeds the factored loads on the structure) and is adequately close to the values that would be obtained from a more detailed frame analysis.

11.8 ACI Moment Coefficients

11.8.1 Basic Concepts

- ***ACI Code 6.5*** includes expressions that may be used for the ***approximate calculatio***n of ***maximum moments*** and ***shears*** in:
    - Continuous beams,
    - One-way slabs.
- The expressions for moment take the form of:

  $M_u = Coefficient\ w_u \ell_n^2$ **Eq. 11.8-1**

  where:

  w_u is the total factored load per unit length on the span,

  ℓ_n is the ***clear span from face to face of supports for positive moment***, or the ***average of the two adjacent clear spans for negative moment***.
- Shear is taken equal to:

  $V_u = Coefficient\ w_u \ell_n$ **Eq. 11.8-2**
- ***Nondimensional analysis*** can be adopted to show the validity of ***Eq. 11.8-1*** and ***Eq. 11.8-2***.
- The coefficients, found in ***ACI Code 6.5.2*** and ***6.5.4***, are shown in ***Table 11.8-1*** and summarized in ***Figure 11.8-1***.

Table 11.8-1: Moment and shear values using ACI coefficient, Table 6.5.2 of ACI code.

<table>
<tr><th>Moment</th><th>Location</th><th>Condition</th><th>M_u</th></tr>
<tr><td rowspan="3">Positive</td><td rowspan="2">End span</td><td>Discontinuous end integral with support</td><td>$w_u\ell_n^2/14$</td></tr>
<tr><td>Discontinuous end unrestrained</td><td>$w_u\ell_n^2/11$</td></tr>
<tr><td>Interior spans</td><td>All</td><td>$w_u\ell_n^2/16$</td></tr>
<tr><td rowspan="6">Negative[1]</td><td rowspan="2">Interior face of exterior support</td><td>Member built integrally with supporting spandrel beam</td><td>$w_u\ell_n^2/24$</td></tr>
<tr><td>Member built integrally with supporting column</td><td>$w_u\ell_n^2/16$</td></tr>
<tr><td rowspan="2">Exterior face of first interior support</td><td>Two spans</td><td>$w_u\ell_n^2/9$</td></tr>
<tr><td>More than two spans</td><td>$w_u\ell_n^2/10$</td></tr>
<tr><td>Face of other supports</td><td>All</td><td>$w_u\ell_n^2/11$</td></tr>
<tr><td>Face of all supports satisfying (a) or (b)</td><td>(a) slabs with spans not exceeding 3 m
(b) beams where ratio of sum of column stiffnesses to beam stiffness exceeds 8 at each end of span</td><td>$w_u\ell_n^2/12$</td></tr>
</table>

[1]To calculate negative moments, ℓ_n shall be the average of the adjacent clear span lengths.

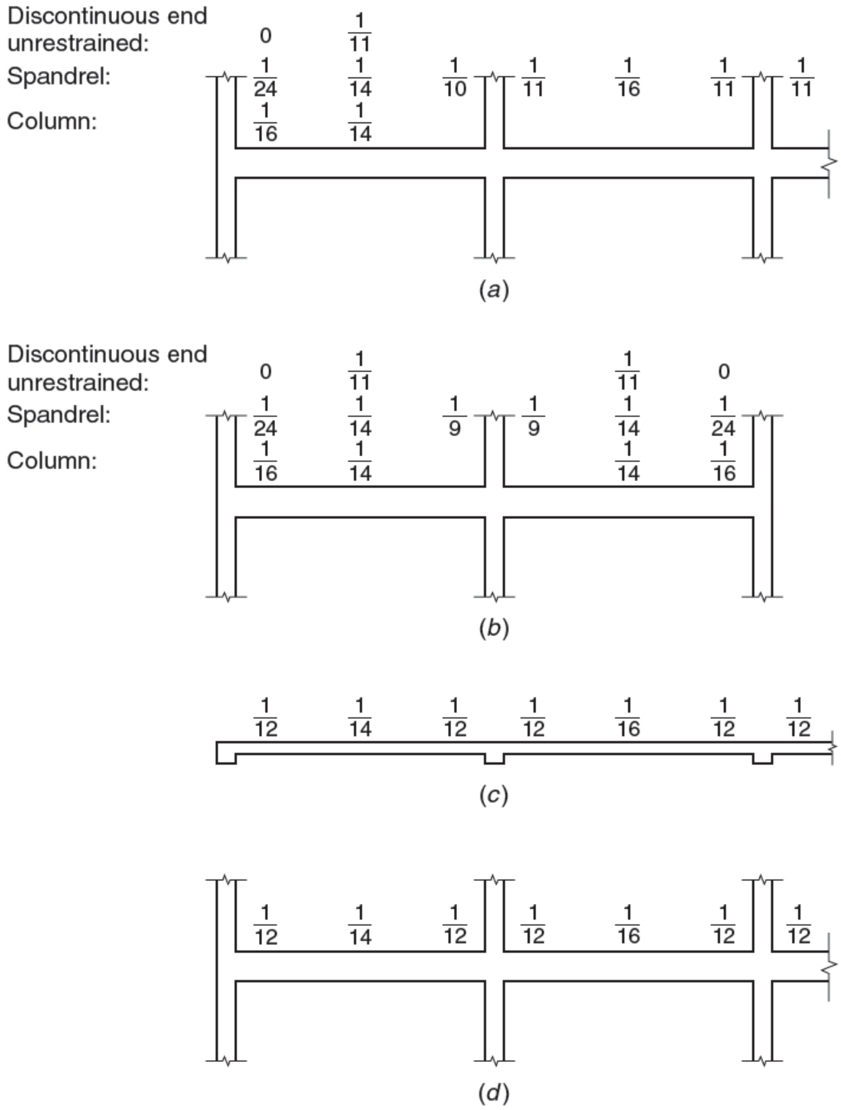


Figure 11.8-1: Summary of ACI moment coefficients: (a) beams with more than two spans; (b) beams with two spans only; (c) slabs with spans not exceeding 3m; and (d) beams in which the sum of column stiffnesses exceeds 8 times the sum of beam stiffnesses at each end of the span.

11.8.2 DIFFERENT TYPES OF DISCONTINUOUS END

Three types for discontinues supports of ***Table 11.8-1*** are presented and discussed in below.

- **Unrestrained Support:** This type of support occurs when beams or slabs are supported directly on masonry walls or concrete walls without monolithic casting.

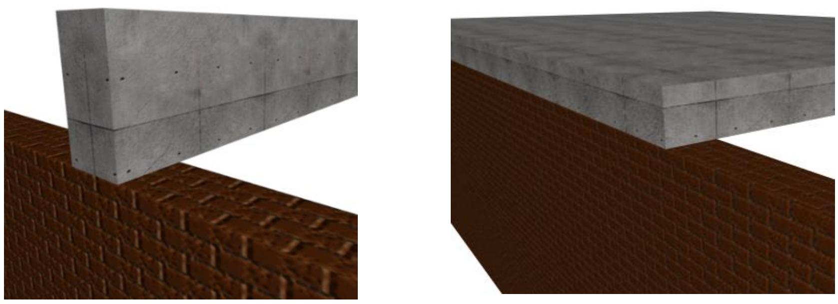


Figure 11.8-2: Unrestrained Discontinuous Ends.

- **Spandrel Support:** This type of support occurs when beams or slabs are supported on a monolithically casted edge beam or girder.

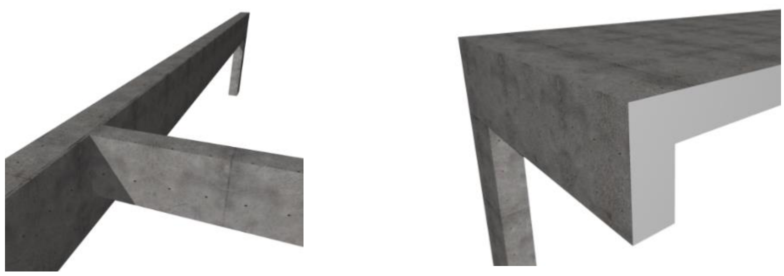


Figure 11.8-3: Discontinuous ends with spandrel member support.

- **Column Support:** This type of support occurs when beams supported directly on columns. Slabs that supported directly on columns are out the scope of our course (junior course) and will be studied thoroughly in senior course.

Figure 11.8-4: Discontinuous ends with column support.

11.8.3 BASES AND CONDITIONS OF ACI COEFFICIENTS

- The ACI coefficients were ***derived by elastic analysis***.
- It is considering ***alternative placement of live load*** to yield maximum negative or positive moments at the critical sections.
- Limitations for ACI coefficients:

  Following limitations have to be satisfied to ensure that the problem to be analyzed is ***similar to that adopted in the elastic analysis of ACI coefficients method*** and to avoid ***overestimation of moments and shear forces***.

  - Members are prismatic, see ***Figure 11.8-5***.
  - Loads are uniformly distributed.
  - The unfactored live load does not exceed 3 times the unfactored dead load.
  - There are two or more spans.
  - The longer of two adjacent spans does not exceed the shorter by more than 20 percent.

    As indicated in ***Figure 11.8-6*** when spans significantly differ, the shortest span may be subjected to negative moments.

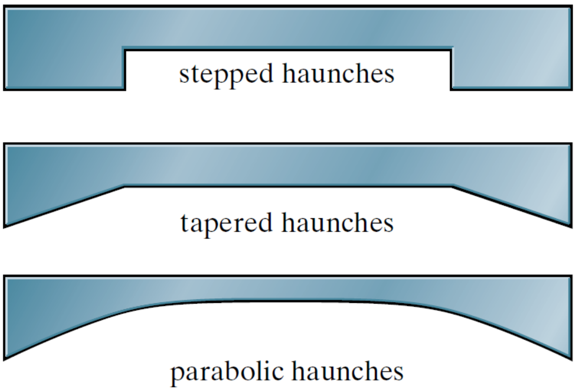


Figure 11.8-5: Non-prismatic beams.

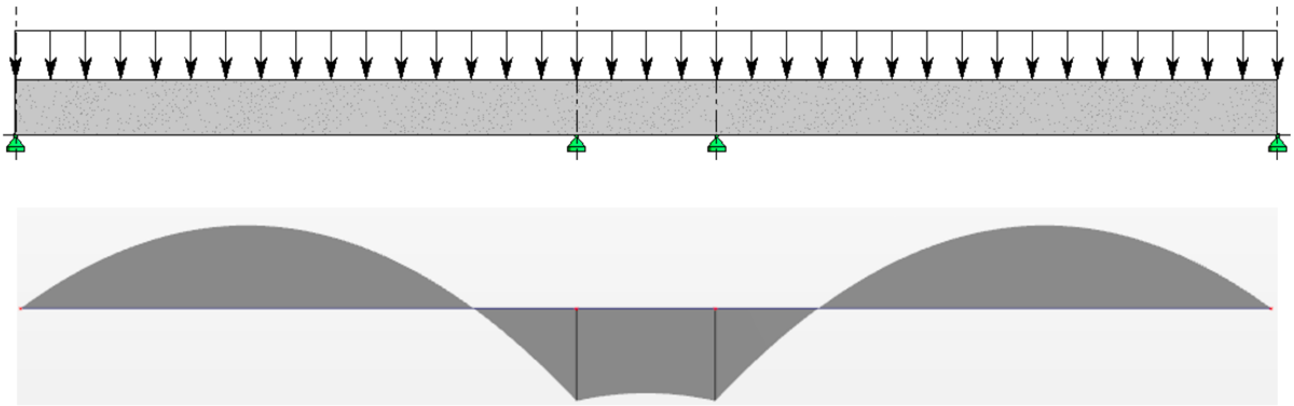


Figure 11.8-6: A beam with significantly differs spans.

11.8.4 MOMENT DIAGRAM AND MOMENT ENVELOPE

- As alternative loading patterns are considered in applying the Code moment coefficients result in an ***envelope of maximum moments***, as illustrated in ***Figure 11.8-7*** for one span of a continuous frame.
- For maximum positive moment, that span would carry dead and live loads, while adjacent spans would carry dead load only, producing the diagram of ***Figure 11.8-7 a***.
- For maximum negative moment at the left support, dead and live loads would be placed on the given span and that to the left, while the adjacent span on the right would carry only dead load, with the result shown in ***Figure 11.8-7b***.

- ***Figure 11.8-7c*** shows the corresponding results for maximum moment at the right support.
- The composite moment diagram formed from the controlling portions of those just developed (***Figure 11.8-7d***) provides the basis for design of the span.

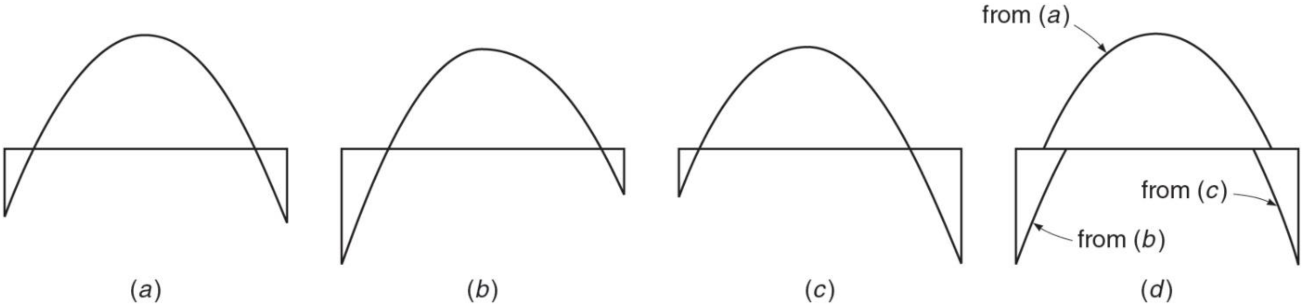


Figure 11.8-7: Maximum moment diagrams and moment envelope for a continuous beam: (a) maximum positive moment; (b) maximum negative moment at left end; (c) maximum negative moment at right end; and (d) composite moment envelope.

- Inflection Points:
  - As observed in ***Figure 11.8-7***, there is a range of positions for the points of inflection resulting from alternate loadings.
  - In the region of the inflection point, it is evident from ***Figure 11.8-7d*** that there may be a ***reversal of moments for alternative load patterns***. However, within the stated limits for use of the coefficients, there should be no reversal of moments at the critical design sections near midspan or at the support faces.

11.8.5 NOTES ON MAXIMUM SHEAR FORCE ACCORDING ACI COEFFICIENTS METHOD

- As indicated in ***Figure 11.8-8***, the shears at the ends of the spans in a continuous frame are modified from the value of $w_u \ell_n/2$ for a simply supported beam because of the usually unbalanced end moments.

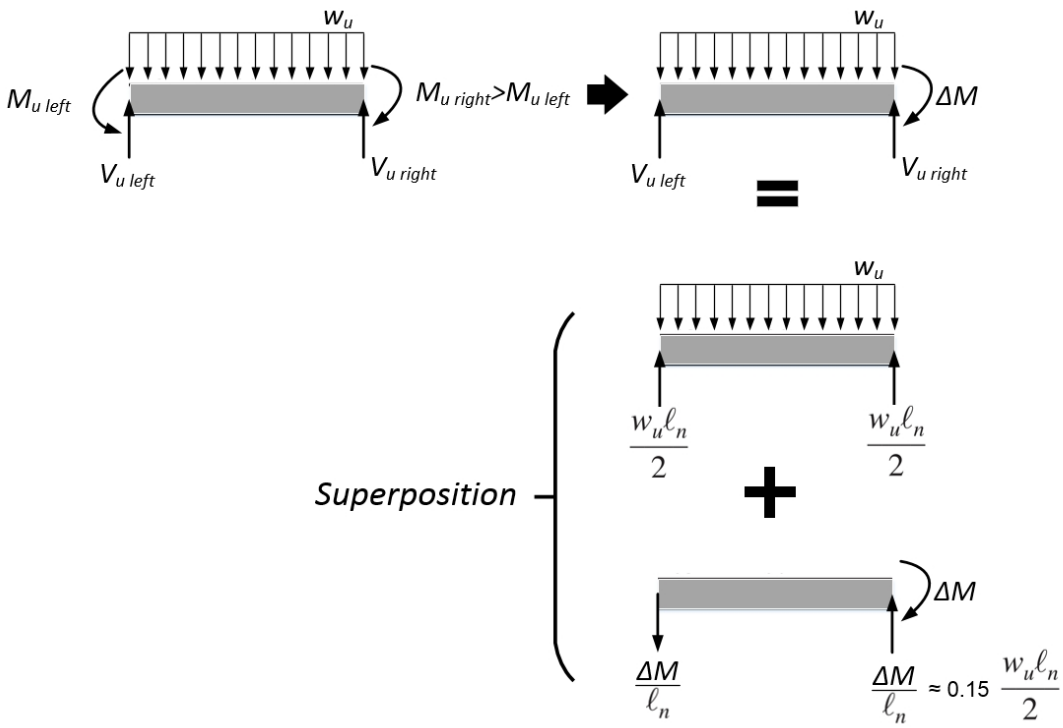


Figure 11.8-8: Effect of unbalanced moment on shear of end spans.

- For ***interior spans***, within the limits of the ACI coefficient method, ***this effect will seldom exceed about 8 percent***, and it may be ***neglected***, as suggested in ***Table 11.8-1***.
- However, for end spans, ***at the face of the first interior support***, the additional shear is significant, and a ***15 percent increase*** above the simple beam shear is indicated in ***Table 11.8-1***. The corresponding ***reduction in shear at the face of the exterior support is conservatively neglected***.

11.8.6 ACI Coefficient Method versus Closed-from Elastic Analysis

- Comparison of the moments found using the ACI coefficients with those calculated by more exact analysis will usually indicate that the ***coefficient moments are quite conservative***. Actual elastic moments may be ***considerably smaller***.
- Consequently, in many reinforced concrete structures, ***significant economy can be achieved by making a more precise analysis***. This is ***mandatory*** for beams and slabs with spans differing by more than 20 percent, sustaining loads that are not uniformly distributed, or carrying live loads greater than 3 times the dead load.

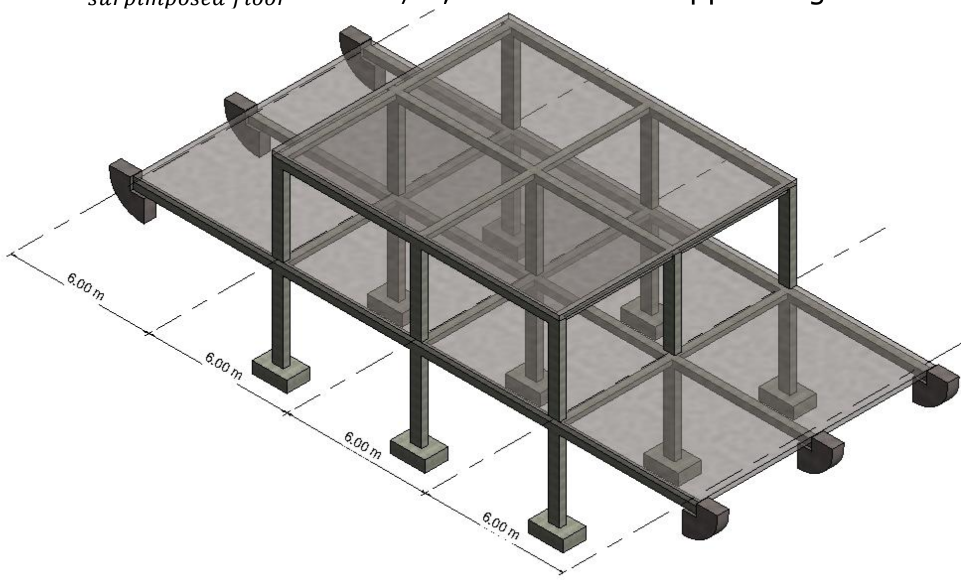
11.8.7 ACI Coefficient Method and Moments in Columns

Because the load patterns in a continuous frame that produce critical moments in the columns are different from those for maximum negative moments in the beams, column moments must be found separately.

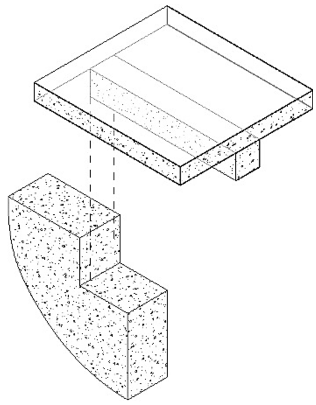
11.8.8 ANALYSIS EXAMPLES

Example 11.8-1

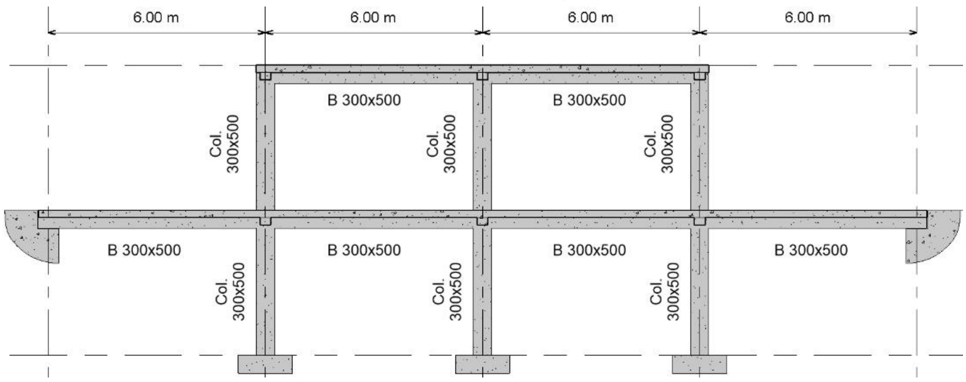
Use ACI coefficients method, if applicable, to determine the factored moments and shear for roof and floor beams of interior frame of building indicated in ***Figure 11.8-9***. In your solution assume that $W_{roof\ live} = 3\ kN/m$, $W_{floor\ live} = 10\ kN/m$, $W_{surpimposed\ roof} = 9\ kN/m$, and $W_{surpimposed\ floor} = 6\ kN/m$, and that the support region has a width of 500mm.



3D View.



Support region.



Elevation view for interior frame.

Figure 11.8-9: Building frame for Example 11.8-1.

Solution

<u>Roof Beams:</u>

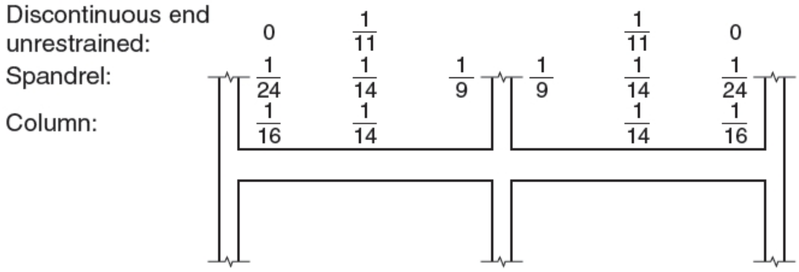
$$W_{self\ of\ beam} = 0.5 \times 0.3 \times 24 = 3.6 \frac{kN}{m} \Rightarrow W_{dead\ roof} = 9 + 3.6 = 12.6 \frac{kN}{m}$$

$$W_{u\ roof} = \max(1.4 \times 12.6, 1.2 \times 12.6 + 1.6 \times 3) \approx 20 \frac{kN}{m}$$

Check applicability of ACI coefficients method for roof beams:

- Members are prismatic, Okay.
- Loads are uniformly distributed, Okay.
- The unfactored live load does not exceed 3 times the unfactored dead load,
  $$W_{dead\ roof} = 12.6 \frac{kN}{m} > W_{roof\ live} = 3 \frac{kN}{m} \therefore Ok.$$
- There are two or more spans:
  As there are two spans, therefore okay.
- The longer of two adjacent spans does not exceed the shorter by more than 20 percent:
  Adjacent spans are equals, therefore okay.

For roof beams, moments can be determined with referring to ***Case b*** of ***Figure 11.8-1*** above.



$\ell_n = 6.0 - \frac{0.5}{2} \times 2 = 5.5\ m$

$M_{u-ve\ int.} = \frac{W_u \ell_n^2}{9} = \frac{20 \times 5.5^2}{9} = 67.2\ kN.m$

As the discontinuous end is a column support, hence positive and exterior negative moments are:

$M_{u+ve} = \frac{W_u \ell_n^2}{14} = \frac{20 \times 5.5^2}{14} = 43.2\ kN.m$

$M_{u-ve\ ext.} = \frac{W_u \ell_n^2}{16} = \frac{20 \times 5.5^2}{16} = 37.8\ kN.m$

According to ***Table 11.8-1*** above, factored shear force at exterior face of first interior support is:

$V_u = 1.15 \frac{W_u \ell_n}{2} = 1.15 \times \frac{20 \times 5.5}{2} = 63.3\ kN$

<u>Floor Beams:</u>

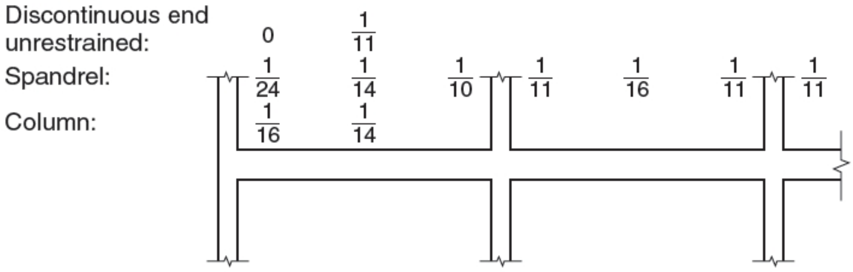
$W_{self\ of\ beam} = 0.5 \times 0.3 \times 24 = 3.6 \frac{kN}{m} \Rightarrow W_{dead\ floor} = 6 + 3.6 = 9.6 \frac{kN}{m}$

$W_{u\ floor} = \max(1.4 \times 9.6, 1.2 \times 9.6 + 1.6 \times 6) \approx 21.2 \frac{kN}{m}$

Check applicability of ACI coefficients method for floor beams:

- Members are prismatic, Okay.
- Loads are uniformly distributed, Okay.
- The unfactored live load does not exceed 3 times the unfactored dead load,
  $3W_{dead\ floor} = 3 \times \left(9.6 \frac{kN}{m}\right) > W_{floor\ live} = 6 \frac{kN}{m}\ \therefore Ok.$
- There are two or more spans:
  As there are four spans, therefore okay.
- The longer of two adjacent spans does not exceed the shorter by more than 20 percent: Adjacent spans are equals, therefore okay.

For floor beams, moments can be determined with referring to ***Case a*** of ***Figure 11.8-1*** above.



$\ell_n = 6.0 - \frac{0.5}{2} \times 2 = 5.5\ m$

For interior spans:

$M_{u-ve\ for\ interior\ span} = \frac{W_u \ell_n^2}{11} = \frac{21.2 \times 5.5^2}{11} = 58.3\ kN.m$

$M_{u+ve\ for\ interior\ span} = \frac{W_u \ell_n^2}{16} = \frac{21.2 \times 5.5^2}{16} = 40.1\ kN.m$

For exterior spans:

$M_{u-ve\ int. for\ exterior\ span} = \frac{W_u \ell_n^2}{10} = \frac{21.2 \times 5.5^2}{10} = 64.1\ kN.m$

As the discontinuous end is simple support, hence positive and exterior negative moments are:

$M_{u+ve\ for\ exterior\ span} = \frac{W_u \ell_n^2}{11} = \frac{21.2 \times 5.5^2}{11} = 58.3\ kN.m$

$M_{u-ve\ ext. for\ exterior\ span} = 0$

Finally, the shear at exterior face of first interior support is:

$V_u = 1.15 \frac{W_u \ell_n}{2} = 1.15 \times \frac{21.2 \times 5.5}{2} = 67.0\ kN$

Factored moments and shear forces for roof and floor beams are summarized in ***Figure 11.8-10*** below.

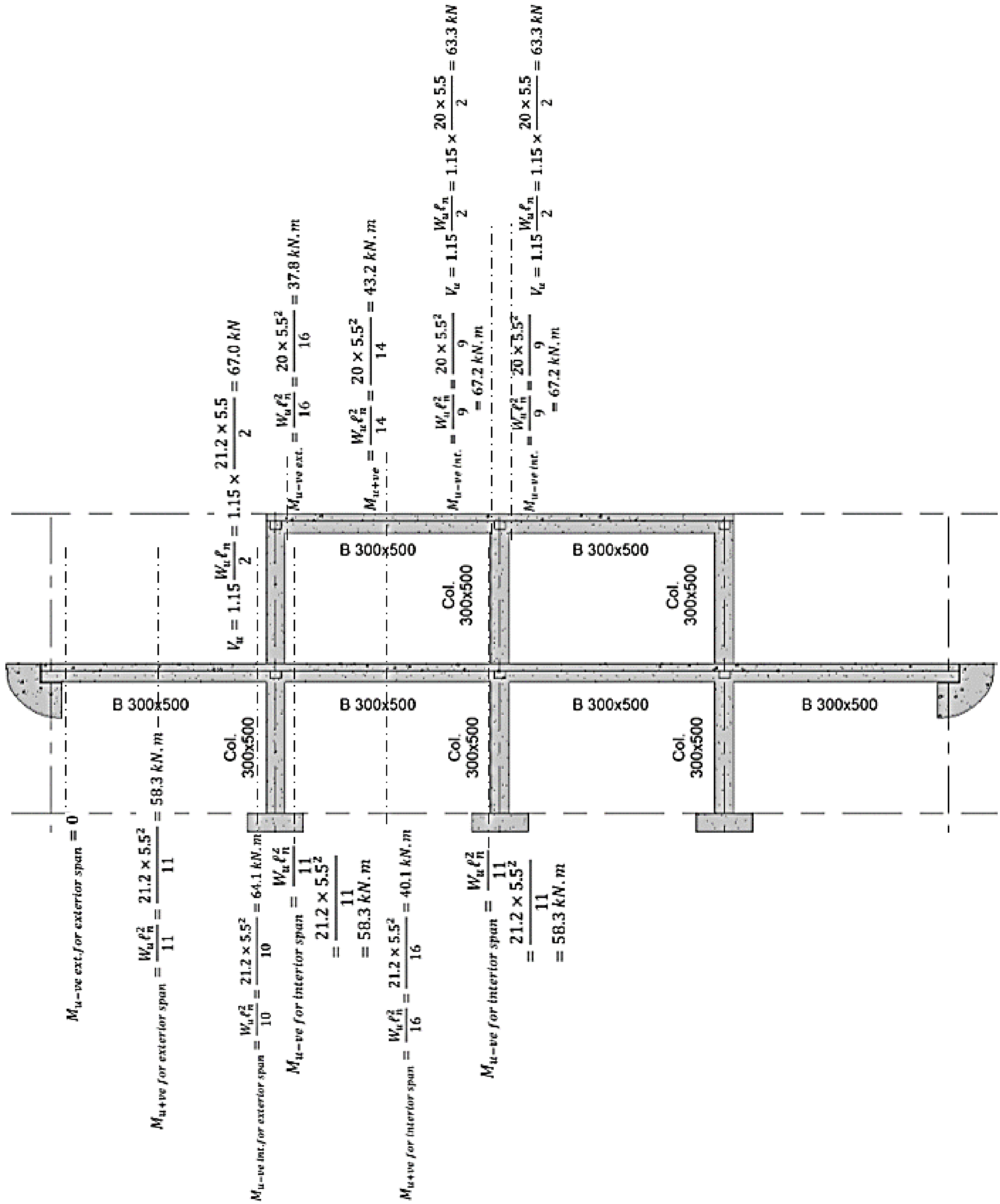
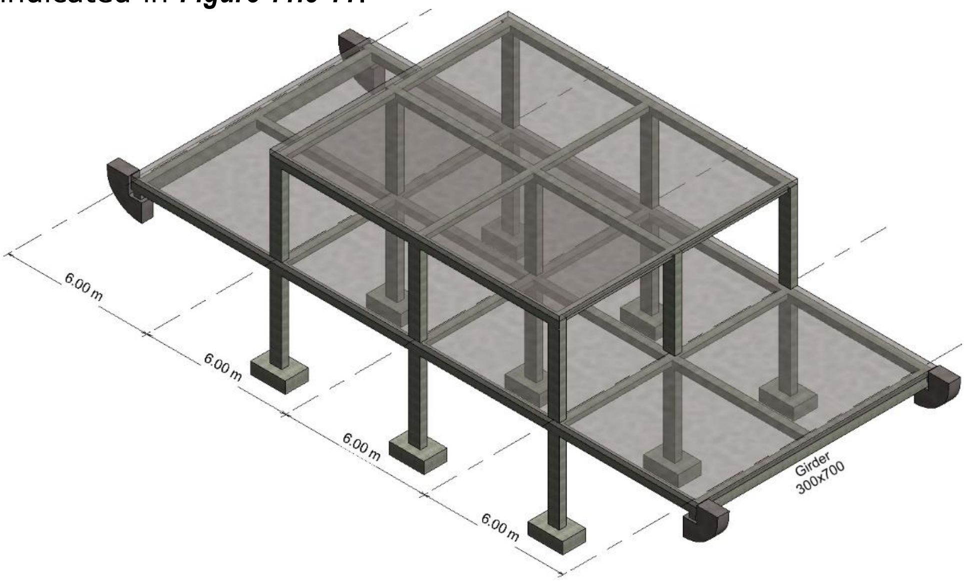


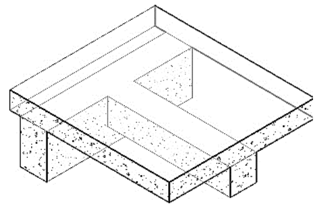
Figure 11.8-10: Summary of factored moments and shears for Example 11.8-1.

Example 11.8-2

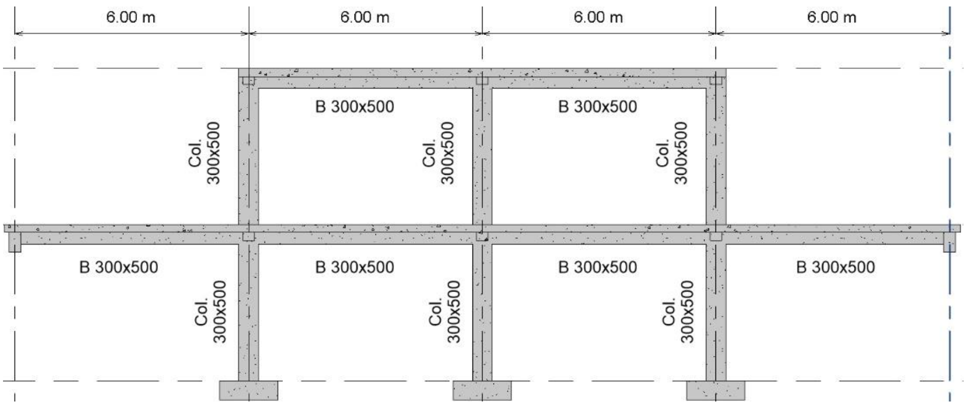
Resolve ***Example 11.8-2*** above when floor beam are supported on spandrel girder as indicated in ***Figure 11.8-11***.



3D View.



Support region.



Elevation view for interior frame.

Figure 11.8-11: Building frame for Example 11.8-2.

Solution

<u>Roof Beams:</u>

Change of exterior support condition for floor beams has no effect on factored forces of roof beams; hence, they would be as indicated in ***Figure 11.8-10*** above.

<u>Floor Beams:</u>

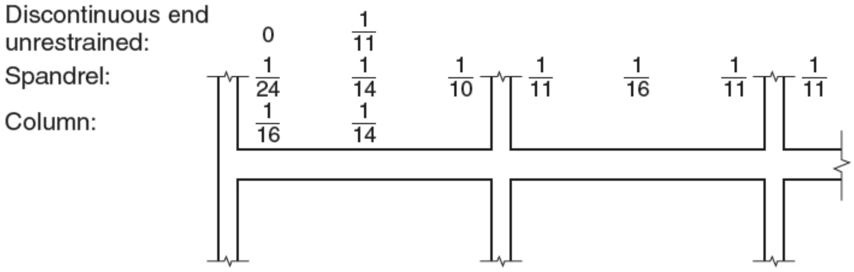
Regarding to floor beams, clear spans for exterior and interior spans are:

$\ell_{n\,interior\,span} = 6.00 - \frac{0.5}{2} \times 2 = 5.5\,m$, $\ell_{n\,exterior\,span} = 6.00 - \frac{0.5}{2} - \frac{0.3}{2} = 5.60\,m$

According to ***Table 11.8-1*** above, ℓ_n shall be average of two adjacent span for negative moment:

$$\ell_{n\,avg.} = \frac{5.5 + 5.6}{2} = 5.55\,m$$

With these clear spans, factored moments for an interior span can be determined with referring ***Case a*** of ***Figure 11.8-1*** above:



$$M_{u-ve\,for\,interior\,span} = \frac{W_u \ell_n^2}{11} = \frac{21.2 \times 5.55^2}{11} = 59.4\,kN.m$$

$$M_{u+ve\,for\,interior\,span} = \frac{W_u \ell_n^2}{16} = \frac{21.2 \times 5.5^2}{16} = 40.1\,kN.m$$

While for an exterior span with spandrel exterior support moments would be:

$$M_{u-ve\,int.for\,exterior\,span} = \frac{W_u \ell_n^2}{10} = \frac{21.2 \times 5.55^2}{10} = 65.3\,kN.m$$

$M_{u+ve\ for\ exterior\ span} = \frac{W_u \ell_n^2}{14} = \frac{21.2 \times 5.6^2}{14} = 47.5\ kN.m$

$M_{u-ve\ ext.for\ exterior\ span} = \frac{W_u \ell_n^2}{24} = \frac{21.2 \times 5.6^2}{24} = 27.7\ kN.m$

It is useful to note that $\ell_{n\ exterior\ span}$ of $5.60\ m$ is used to determine the $M_{u-ve\ ext.for\ exterior\ span}$ as it is governed by its exterior span.

Finally, the shear at exterior face of first interior support is:

$V_u = 1.15 \frac{W_u \ell_n}{2} = 1.15 \times \frac{21.2 \times 5.6}{2} = 68.3\ kN$

Factored moments and shear forces for roof and floor beams are summarized in ***Figure 11.8-12*** below.

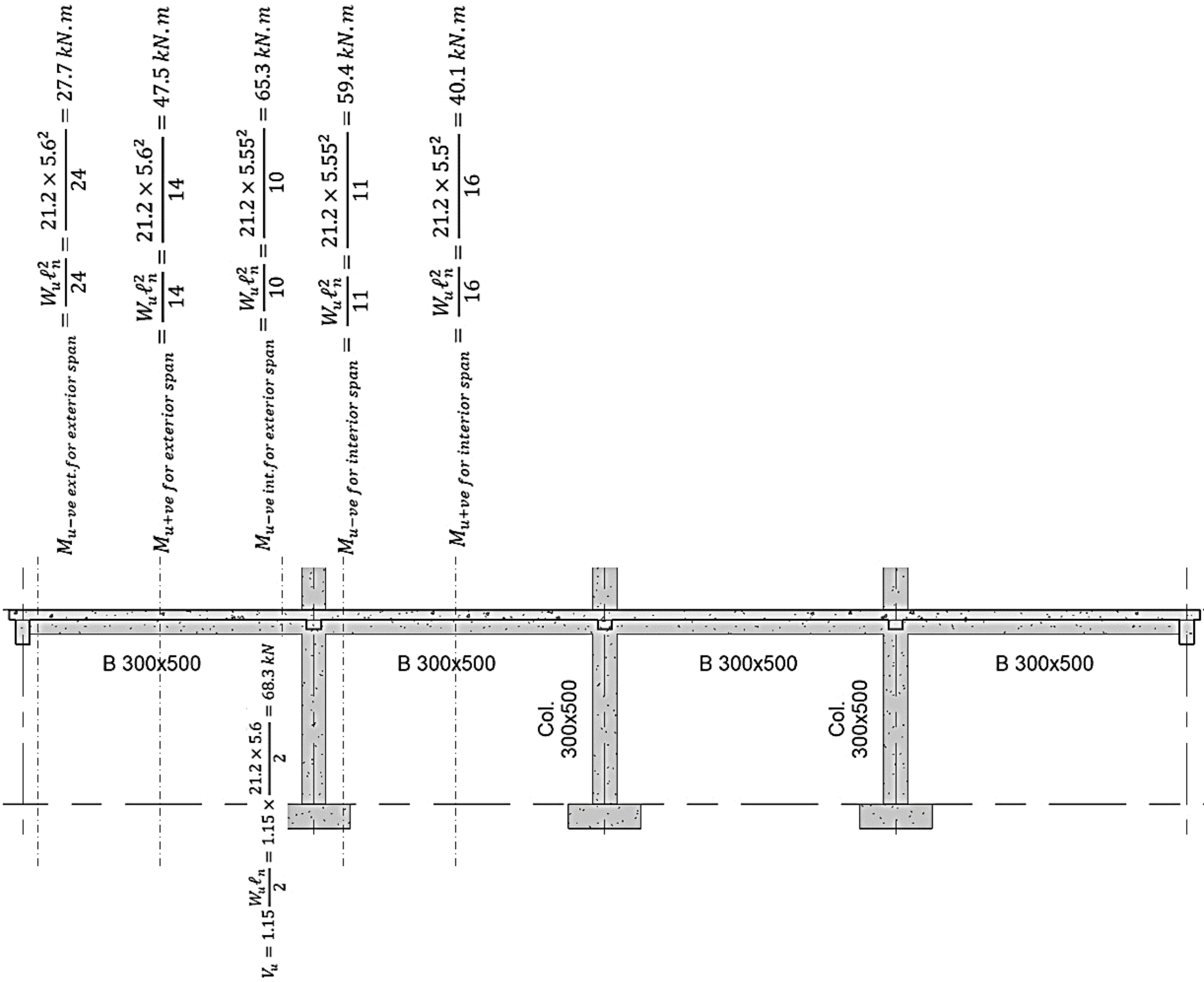
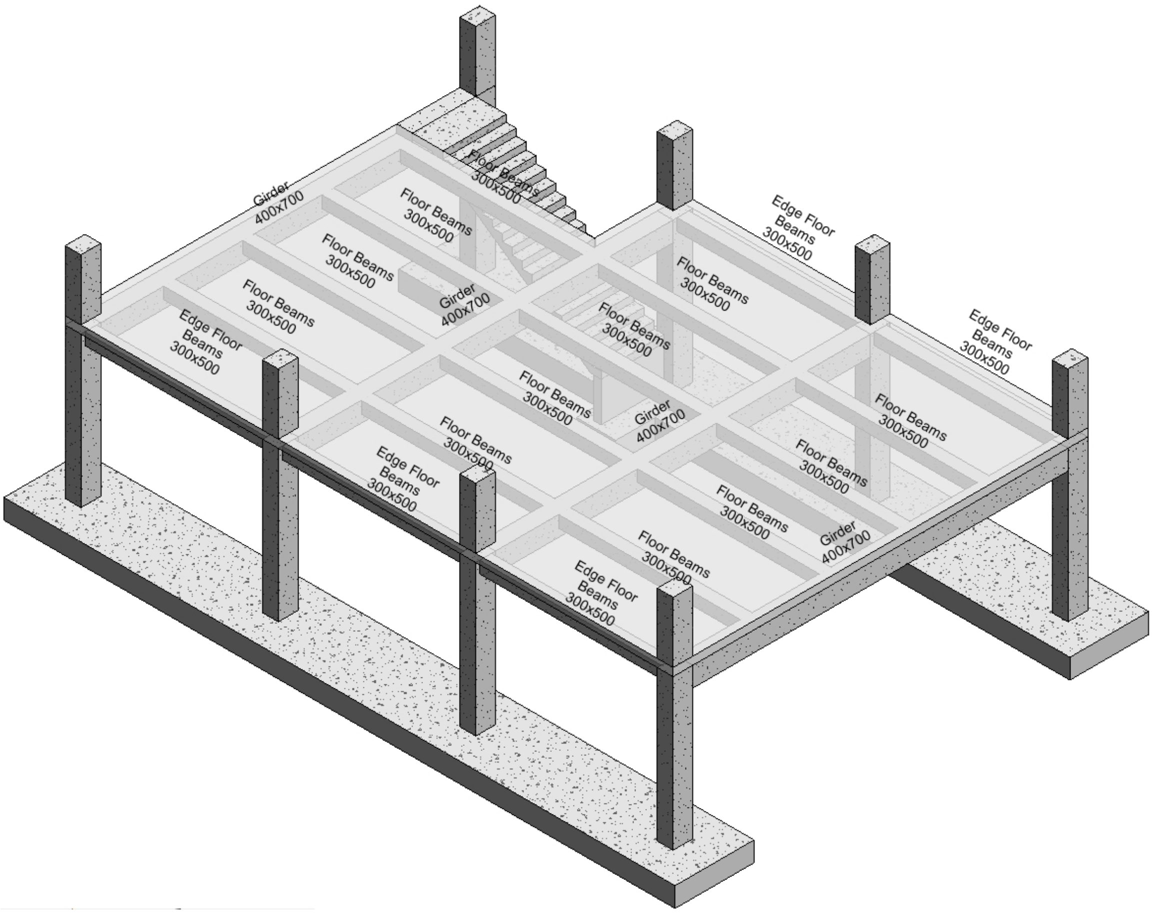


Figure 11.8-12: Summary of factored moments and shears for Example 11.8-2.

Example 11.8-3

For building frame indicated in ***Figure 11.8-13*** below, edge floor beams and floor beams are subjected to factored loads of $20\ kN/m$ and $40\ kN/m$ respectively. Check if ACI coefficients method is applicable to determine factored forces of a typical floor beam and of the edge floor beam located along stair. In your checking, assume the unfactored live load does not exceed 3 times the unfactored dead load.



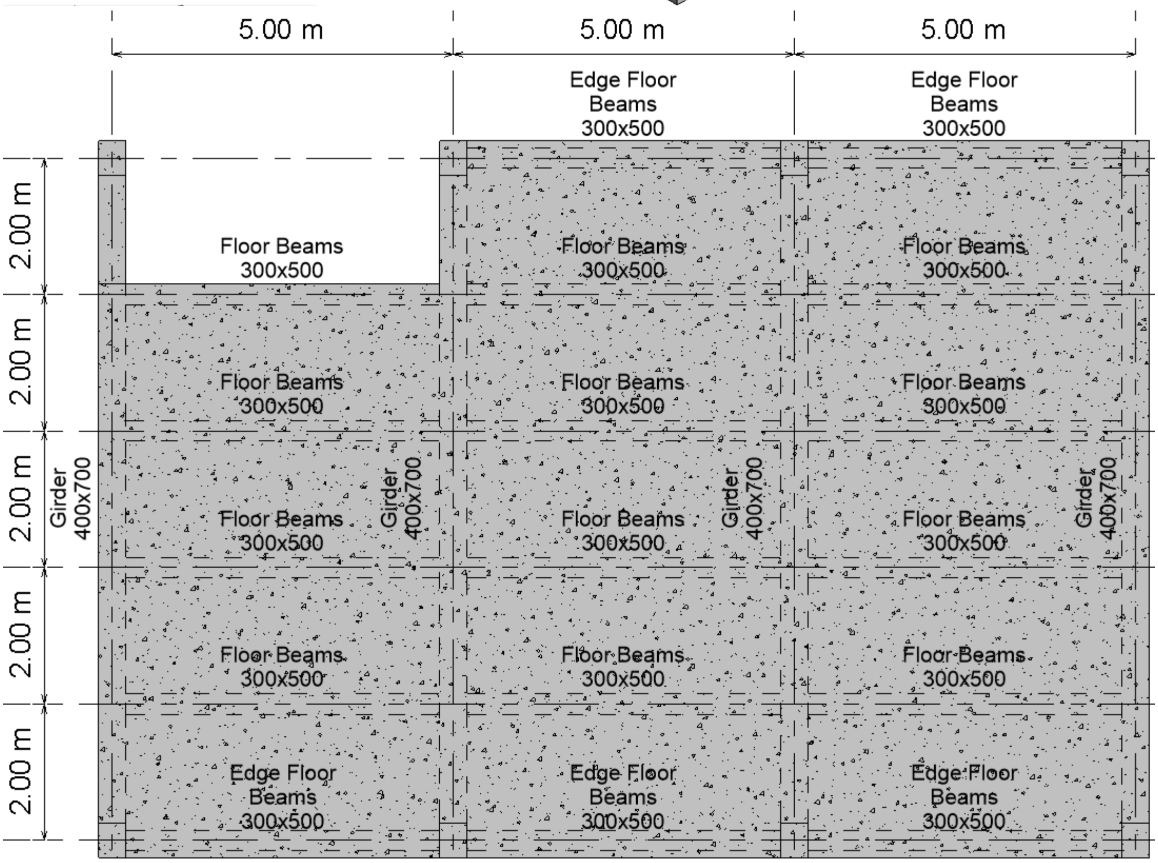


Figure 11.8-13: Frame for Example 11.8-3.

Solution

<u>Applicability of ACI coefficients method:</u>

Checking applicability of ACI coefficients method for the edge floor beams and a typical floor beam:

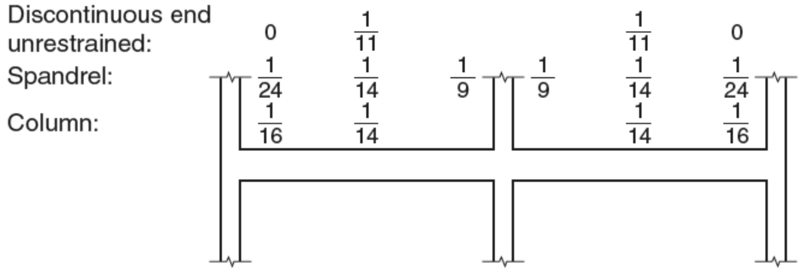
- Members are prismatic, Okay.
- Loads are uniformly distributed, Okay.
- The unfactored live load does not exceed 3 times the unfactored dead load: This condition is satisfied according to example statement.
- There are two or more spans:
  - There are two spans for the edge floor beam located along stair shaft, oaky.
  - There are three spans for a typical floor beam, okay.
- The longer of two adjacent spans does not exceed the shorter by more than 20 percent: Adjacent spans are equals, therefore okay.

<u>Factored Forces for the Edge Floor Beam:</u>

As the edge beam is directly supported on columns (that have width equal to that of the girder), therefore their clear span is:

$$\ell_{n\ for\ edge\ floor\ beams} = 5.00 - \frac{0.4}{2} \times 2 = 4.60\ m$$

With two spans and column exterior support, the factored moments can be determined with referring to ***Case b*** of ***Figure 11.8-1*** above.



$$M_{u-ve\ ext.} = \frac{W_u \ell_n^2}{16} = \frac{20 \times 4.6^2}{16} = 26.5\ kN.m$$

$$M_{u+ve} = \frac{W_u \ell_n^2}{14} = \frac{20 \times 4.6^2}{14} = 30.2\ kN.m$$

$$M_{u-ve\ int.} = \frac{W_u \ell_n^2}{9} = \frac{20 \times 4.6^2}{9} = 47.0\ kN.m$$

$$V_u = 1.15 \frac{W_u \ell_n}{2} = 1.15 \times \frac{20 \times 4.6}{2} = 52.9\ kN$$

The factored forces for edge beams are summarized in see ***Figure 11.8-14*** below.

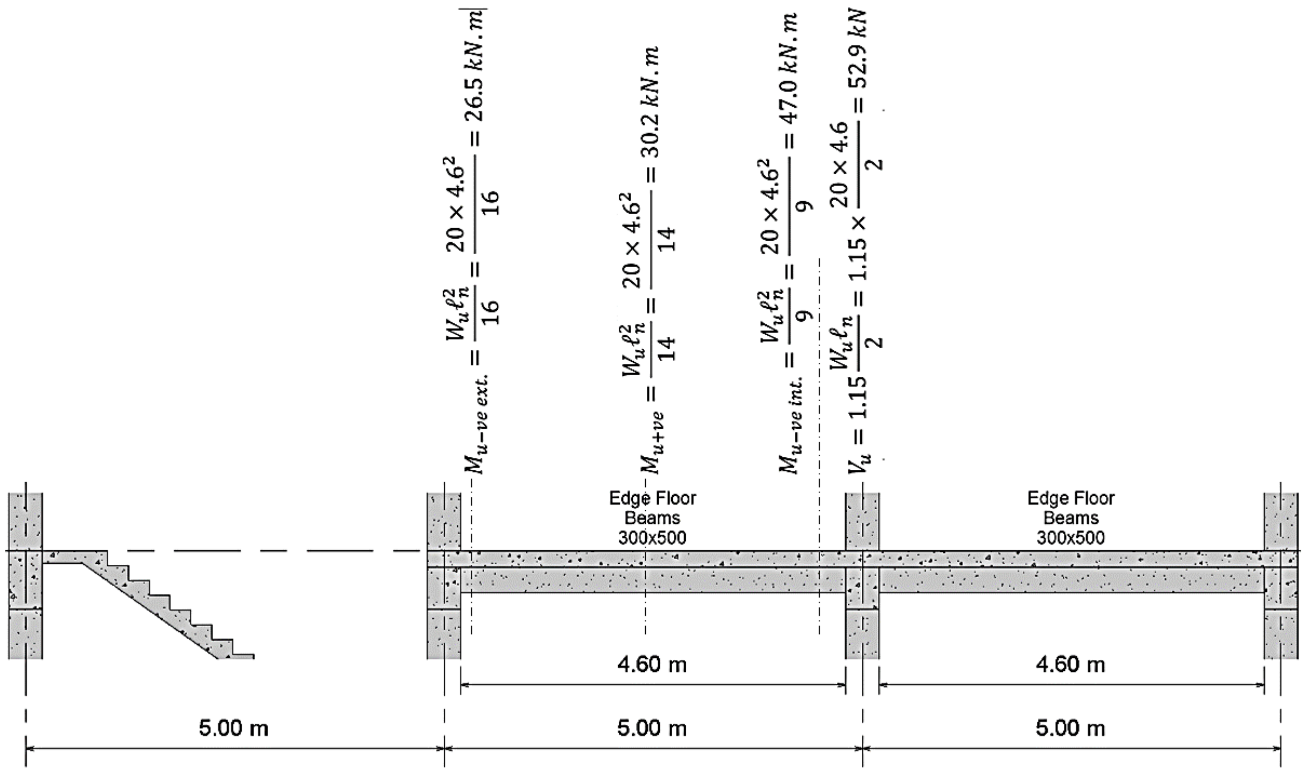
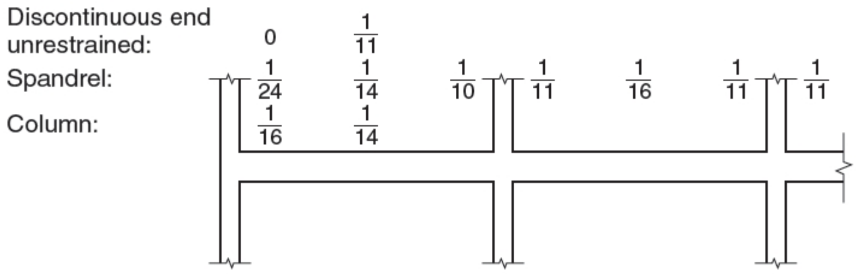


Figure 11.8-14: Summary of factored forces for edge floor beams Example 11.8-3.

Factored Forces for a Typical Floor Beam:

As a typical floor beam has three spans and at discontinuous ends, it is supported on spandrel girders, its factored forces can be determined with referring to ***Case b*** of ***Figure 11.8-1*** above.



$$\ell_{n\,for\,floor\,beams} = 5.00 - \frac{0.4}{2} \times 2 = 4.60\;m$$

$$M_{u-ve\,for\,interior\,span} = \frac{W_u \ell_n^2}{11} = \frac{40 \times 4.6^2}{11} = 76.9\;kN.m$$

$$M_{u+ve\,for\,interior\,span} = \frac{W_u \ell_n^2}{16} = \frac{40 \times 4.6^2}{16} = 52.9\;kN.m$$

$$M_{u-ve\,int.for\,exterior\,span} = \frac{W_u \ell_n^2}{10} = \frac{40 \times 4.6^2}{10} = 84.6\;kN.m$$

$$M_{u+ve\,for\,exterior\,span} = \frac{W_u \ell_n^2}{14} = \frac{40 \times 4.6^2}{14} = 60.5\;kN.m$$

$$M_{u-ve\,ext.for\,exterior\,span} = \frac{W_u \ell_n^2}{24} = \frac{40 \times 4.6^2}{24} = 35.3\;kN.m$$

$$V_u = 1.15\frac{W_u \ell_n}{2} = 1.15 \times \frac{40 \times 4.6}{2} = 106\;kN$$

The factored forces for beams are summarized in see ***Figure 11.8-15*** below.

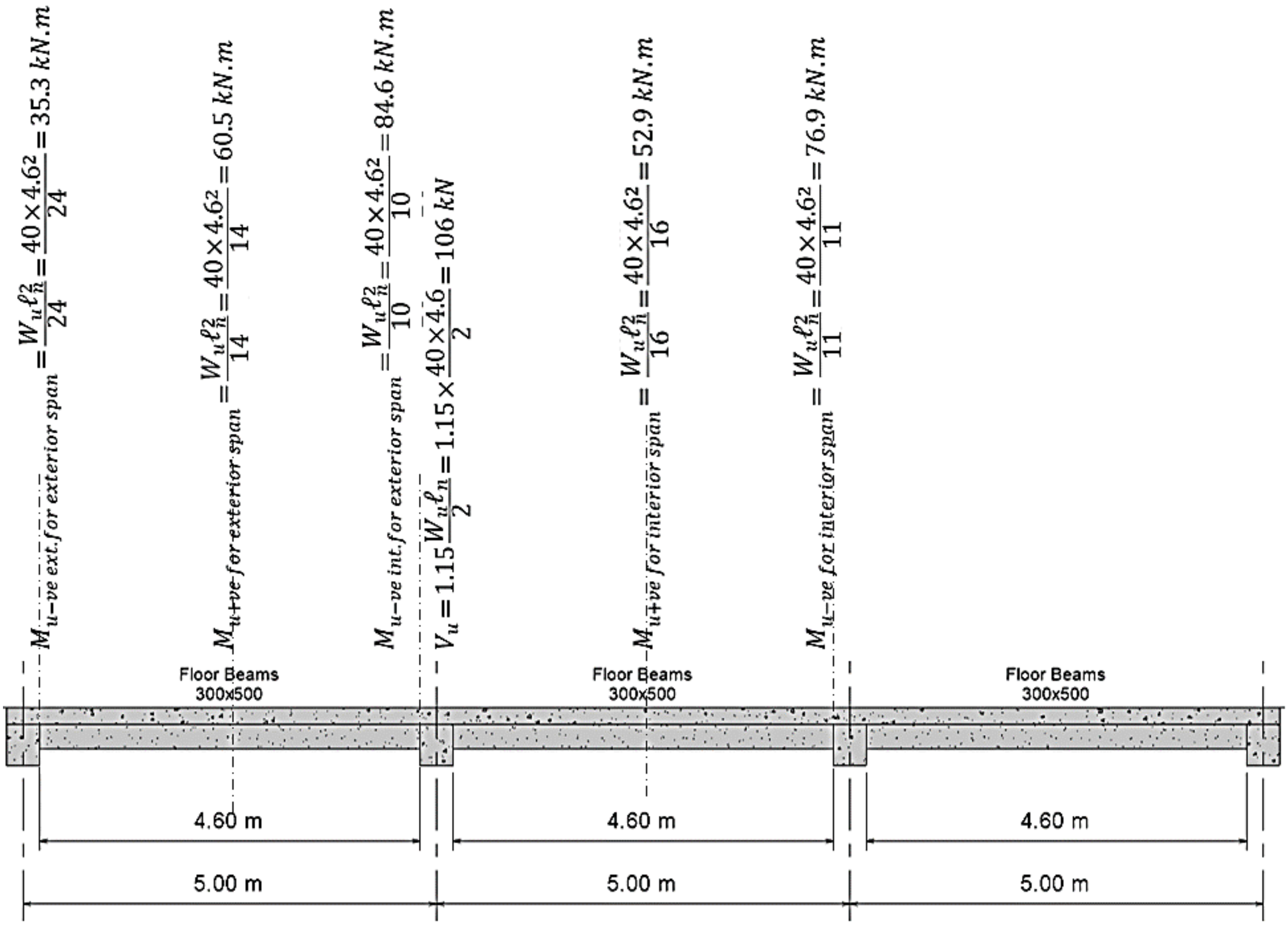


Figure 11.8-15: Summary of factored forces for floor beams Example 11.8-3.

Contents

CHAPTER 12
ANALYSIS AND DESIGN
OF
ONE-WAY SLABS

12.1 BASIC CONCEPTS OF ONE-WAY SYSTEM

12.1.1 DEFINITION OF ONE-WAY SLAB SYSTEM

- Slabs may be supported on two opposite sides only, as shown in ***Figure 12.1-1*** below, in which case the structural action of the slab is essentially one-way, the loads being carried by the slab in the direction perpendicular to the supporting beams.

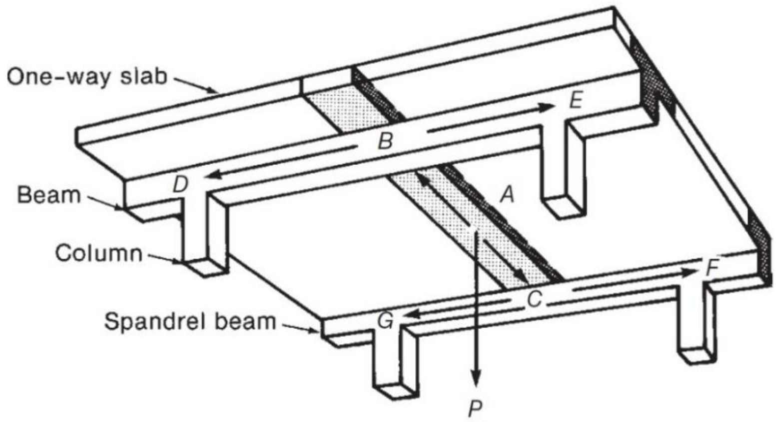


Figure 12.1-1: Slabs supported on two sides only.

- Intermediate beams, as shown in **Figure 12.1-2** below may be provided. If the ratio of length to width of one slab panel is larger than about 2, most of the load is carried in the short direction to the supporting beams and one-way action is obtained in effect, even though supports are provided on all sides [1].

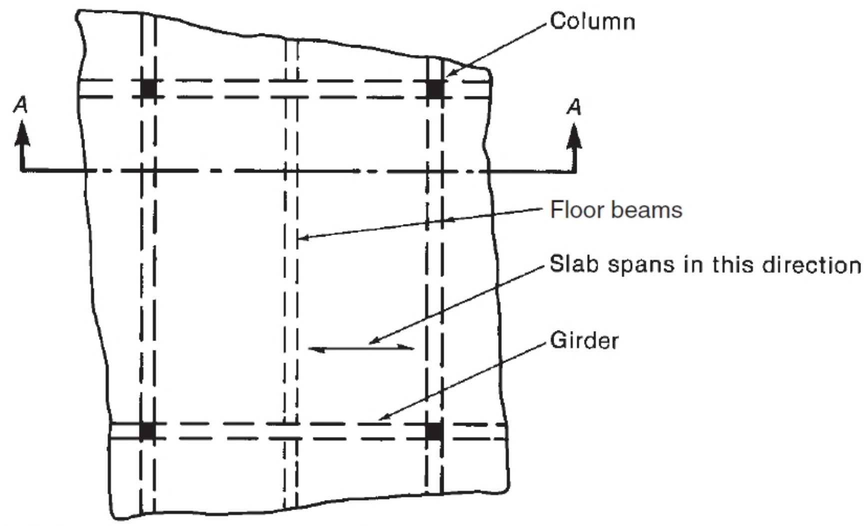


Figure 12.1-2: Slab with intermediate beams.

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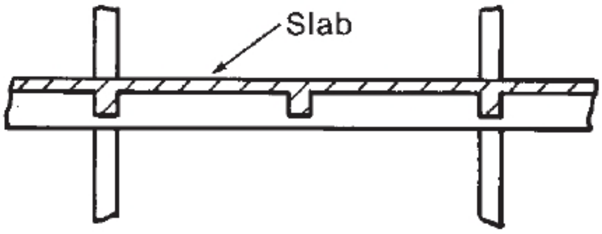
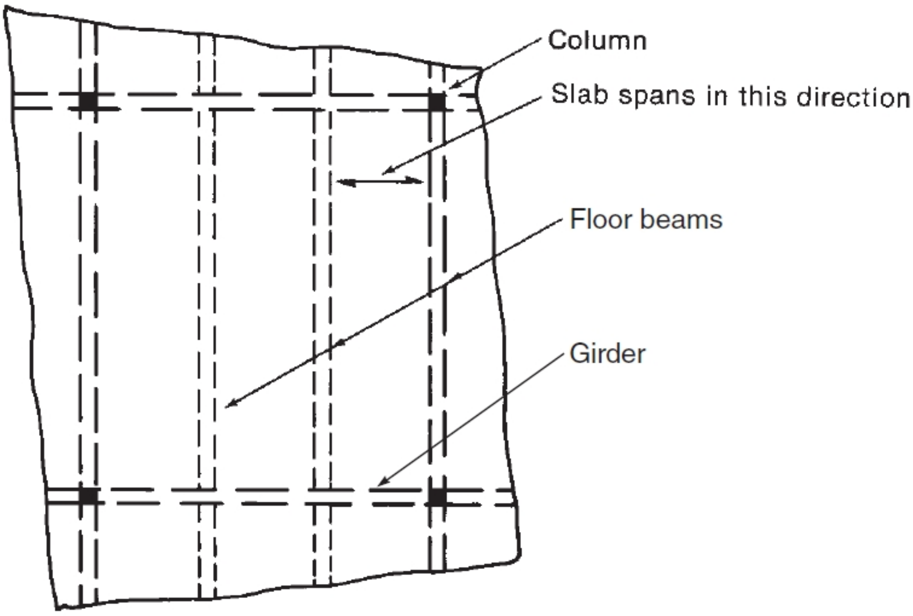
[1] According to (Reinforced Concrete: Mechanics and Design by J. G. MacGregor Page 436), this system (i.e. systems with intermediate beams) used in slabs with heavy loads and spans greater than 6m.



(b) Section A–A.



(c) Floor plan with two intermediate floor beams.

Figure 12.1-2: Slab with intermediate beams. Continued.

12.1.2 ONE-WAY SLAB BEHAVIOR

- The structural action of a one-way slab may be visualized in terms of the deformed shape of the loaded surface.
- ***Figure 12.1-3*** below shows a rectangular slab, simply supported along its two opposite long edges and free of any support along the two opposite short edges.
- If a uniformly distributed load is applied to the surface, the deflected shape will be as shown by the solid lines. Curvatures and consequently bending moments are the same in all strips (s) spanning in the short direction between supported edges, whereas there is no curvature, hence no bending moment, in the long strips (l) parallel to the supported edges. The surface is approximately cylindrical.

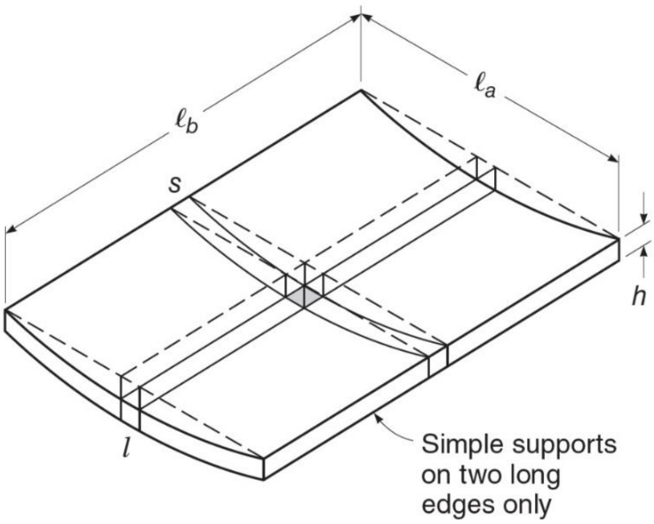


Figure 12.1-3: Cylindrical curvature of one-way slabs.

12.1.3 ANALYSIS OF ONE-WAY SLAB SYSTEM

- For purposes of analysis and design, a unit strip of such a slab cut out at right angles to the supporting beams, as shown in ***Figure 12.1-4*** below may be considered as a rectangular beam of unit width, with a depth h equal to the thickness of the slab and a span L_a equal to the distance between supported edges.
- This simplified analysis, which assumes Poisson's ratio to be zero, is slightly conservative.

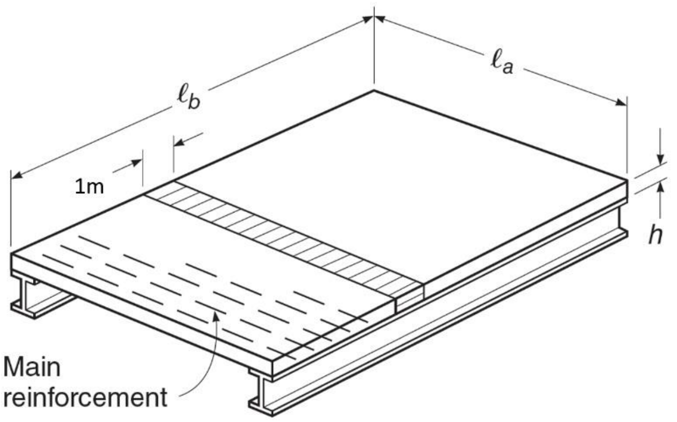


Figure 12.1-4: Strip with unit width for one-way slabs modeling.

- This strip can then be analyzed by the methods that were used for ***rectangular beams, the bending moment being computed for the strip of 1m width***. The ***load per unit area on the slab becomes load per unit length on the slab strip at right angles to these beams***. Then each ***slab strip will behave as a beam***.
- This equivalent beam may be:
  - A statically determinate beam if slab is simply supported or cantilever.
  - A statically indeterminate beam if the slab is a continuous one.
- Factored moments and shears of an equivalent statically indeterminate beam can be found either by elastic analysis or through the use of the ***Simplified method of analysis for nonprestressed continuous beams and one-way slabs*** that discussed below.
- ACI coefficients method, that discussed in ***Chapter 11*** of the text and in ***Article 6.5*** of the code, represents a typical method that usually adopted for analysis of one-way slabs when its conditions are satisfied.

12.1.4 SECTION TYPE AND FLEXURAL STRENGTH FACTOR, ϕ

- One-way slabs are normally designed with tensile reinforcement ratios well below the maximum practical value of $\rho_{0.005}$.
- Typical reinforcement ratios range from about 0.004 to 0.008.
- This is:
  - Partially for reasons of economy, because the saving in reinforcement associated with increasing the effective depth more than compensates for the cost of the additional concrete,
  - And partially because very thin slabs with high reinforcement ratios would be likely to permit large deflections.
- With these low reinforcement ratios, slab section is a tension controlled section with strength reduction factor, ϕ, of 0.9.

12.1.5 TEMPERATURE, SHRINKAGE, AND MINIMUM REINFORCEMENT

- Shrinkage:
  - It is advisable to ***minimize shrinkage*** by using ***concretes with the smallest possible amounts of water and cement*** compatible with other requirements, such as strength and workability, and by thorough moist-curing of sufficient duration.
  - However, no matter what precautions are taken, ***a certain amount of shrinkage is usually unavoidable***.
  - If a slab of moderate dimensions ***rests freely on its supports***, it can ***contract to accommodate the shortening of its length produced by shrinkage***. Usually, however, ***slabs and other members are joined rigidly to other parts of the structure and cannot contract freely***. This results in ***tension stresses known as shrinkage stresses***.
- Decrease in Temperature:
  - A decrease in temperature relative to that at which the slab was cast, particularly in outdoor structures such as bridges may have an effect similar to shrinkage. That is the slab tends to contract and if restrained from doing so becomes subject to tensile stresses.
  - Since concrete is weak in tension, these temperature and shrinkage stresses are likely to result in cracking. Cracks of this nature are not detrimental, provided their size is limited to what are known as hairline cracks. This can be achieved by placing reinforcement in the slab to counteract contraction and distribute the cracks uniformly.
- Temperature or Shrinkage Reinforcement or Distribution Reinforcement
  - In one-way slabs, the reinforcement provided for resisting the bending moments has the desired effect of reducing shrinkage and distributing cracks.
  - However, as contraction takes place equally in all directions. It is necessary to provide special reinforcement for shrinkage and temperature contraction in the direction perpendicular to the main reinforcement.
  - This added steel is known as ***Temperature or Shrinkage Reinforcement or Distribution Reinforcement.***

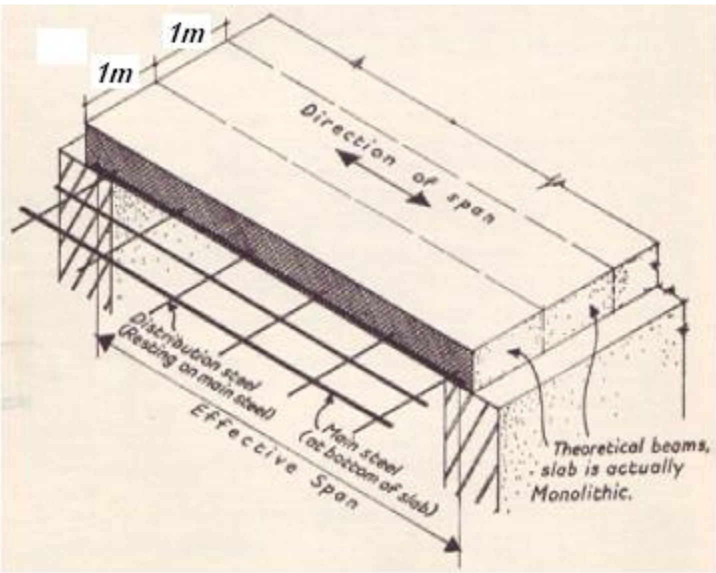


Figure 12.1-5: Temperature and structural reinforcement in one-way slabs.

- Minimum flexural reinforcement in nonprestressed slabs
  - The required area of reinforcement used as minimum flexural reinforcement is the same as provided for shrinkage and temperature.
  - However, whereas shrinkage and temperature reinforcement is permitted to be distributed between the ***two faces of the slab as deemed appropriate for specific conditions***, ***minimum flexural reinforcement should be placed as close as practicable to the face of the concrete in tension due to applied loads***.

- According to (ACI318M, 2014), ***Article 7.6.1***, area of shrinkage, temperature, and minimum reinforcement shall be determined from ***Table 12.1-1*** below.

Table 12.1-1: As,min for nonprestressed one-way slabs, Table 7.6.1.1 of (ACI318M, 2014).

Reinforcement type	f_y, MPa	$A_{s,min}$	
Deformed bars	< 420	$0.0020A_g$	
Deformed bars or welded wire reinforcement	≥ 420	Greater of:	$\frac{0.0018\times 420}{f_y}A_g$
			$0.0014A_g$

12.1.6 SLAB THICKNESS

12.1.6.1 Slab thickness for Deflection Control

- Except for very heavily loaded slabs, such as slabs supporting several meters of earth, the slab thickness is chosen so that deflections will not be a problem.
- According to (ACI318M, 2014), ***Article 7.3.1.1***, ***for solid nonprestressed slabs not supporting or attached to partitions or other construction likely to be damaged by large deflections***, overall slab thickness h shall not be less than the limits in ***Table 12.1-2***, unless the calculated deflection limits of 7.3.2 are satisfied.

Table 12.1-2: Minimum thickness of solid nonprestressed one-way slabs (Table 7.3.1.1 of (ACI318M, 2014)).

Support condition	Minimum h[1]
Simply supported	$\ell/20$
One end continuous	$\ell/24$
Both ends continuous	$\ell/28$
Cantilever	$\ell/10$

[1]Expression applicable for normalweight concrete and f_y = 420 MPa. For other cases, minimum h shall be modified in accordance with 7.3.1.1.1 through 7.3.1.1.3, as appropriate.

- According to (ACI318M, 2014), ***Article 7.3.1.1.1***, for f_y other than 420 MPa, the expressions in Table 12.1-2 above shall be multiplied by $(0.4 + f_y/700)$.
- According to (ACI318M, 2014), ***Article 7.3.1.1.2*** For nonprestressed slabs made of lightweight concrete having w_c in the range of 1440 to 1840 kg/m^3, the expressions in Table 12.1-2 above shall be multiplied by the greater of (a) and (b):
  - (a) $1.65 - 0.0003w_c$
  - (b) 1.09
- Slab span, ℓ:
  - According to Chapter 2, Notation and Terminology, of the ACI code, ℓ ***is the span length of the one-way slab; clear projection of cantilever***.
  - According to (Darwin, Dolan, & Nilson, 2016), clear span is adopted for ℓ in continuous spans. More conservative center to center span is adopted according to (Wight, 2016), (Wang, Salmon, & Pincheira, 2007), and (Leet, 1997).
  - Clear span is adopted in this course.
- Notes on Table 12.1-2:
  - The main practical concern in the use of ***Table 12.1-2*** above is the condition of ***members not supporting or attached to partitions or other construction likely to be damaged by large deflections***.

- According to (Darwin, Dolan, & Nilson, 2016), this table first appeared in the 1963 ACI Code and ***may not fully reflect current loads and design practice, and he recommends a check of all member deflections***.
- If traditional finishing, surfacing, and brick partitions that used in Iraq are considered to be damaged in the above meaning, then this table will have no practical applications. To skip this confusion, any one of following two proposals may be used:
  - J. G. MacGregor in his book "***Reinforced Concrete: Mechanics and Design 4th Page 436***" recommends the ***Table 12.1-3*** below to be used for minimum thickness of one-way slabs that do and do not support such partitions. Values of ***Table 12.1-3*** below seems so high specially when used for slab designed.
  - Based on a native experience, values of ACI Table (after little rounding to higher values) seem adequate when used in traditional buildings with bricks partitions.

Table 12.1-3: Minimum Thickness of Non-prestressed Beams or One-way Slabs Unless Deflections are Computed (Adopted from Reinforced Concrete: Mechanics and Design 4th, by J. G. MacGregor Page 436).

		Minimum Thickness, h				
Exposure	Member	Simply Supported	One End Continuous	Both Ends Continuous	Cantilever	Source
Not supporting or attached to partitions or other construction likely to be damaged by large deflections	Solid one-way slabs	*L/20*	*L/24*	*L/28*	*L/10*	ACI Table 9.5(a)
	Beams or ribbed one-way slabs	*L/16*	*L/18.5*	*L/21*	*L/8*	
Supporting or attached to partitions or other construction likely to be damaged by large deflections	All members: $\omega \le 0.12^a$ and $\frac{\text{sustained load}}{\text{total load}} < 0.5$	*L/10*	*L/13*	*L/16*	*L/4*	*Ref. Reinforced Concrete: Mechanics and Design 4th by J. G. MacGregor*
	All members: $\frac{\text{sustained load}}{\text{total load}} > 0.5$	*L/6*	*L/8*	*L/10*	*L/3*	

$^a\omega = \rho f_y / f'_c$

12.1.6.2 Fire Rating Requirements

- In addition to above deflection requirements, slab thickness may be based on firefighting requirements to avoid danger of heat transmission during a fire.
- For this criterion, slab thickness can be computed based on ***Table 12.1-4*** below , where fire rating of a floor is ***the number of hours necessary for temperature of the unexposed surface to rise by 250 °F*** (J. G. MacGregor "Reinforced Concrete: Mechanics and Design 4th Page 436").

Table 12.1-4: Fire Rating for Slabs (Adopted from Reinforced Concrete: Mechanics and Design 4th, by J. G. MacGregor Page 436).

	Slab Thickness (mm)	Corresponding Fire Rating (hours)
1.	90	1.0
2.	125	2.0
3.	160	3.0

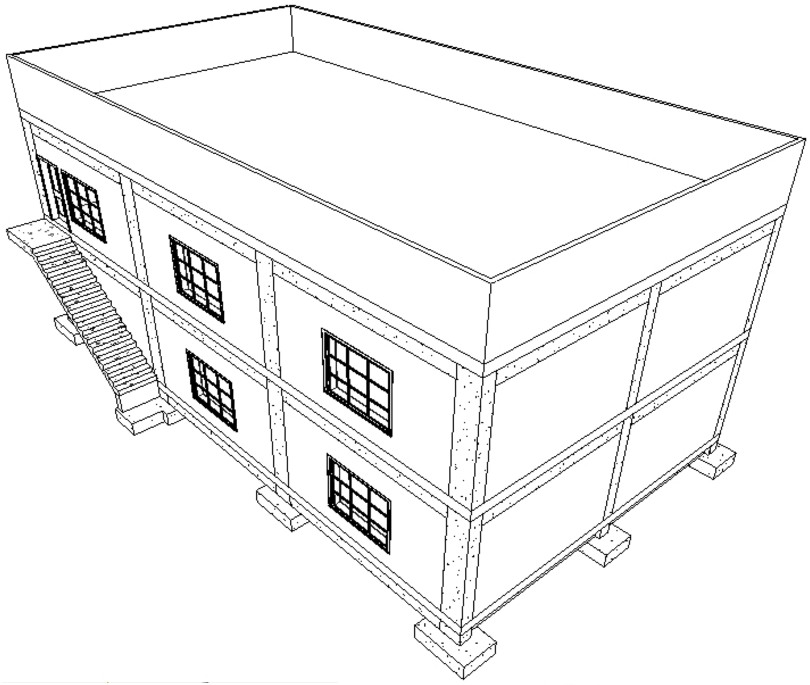
12.1.6.3 Rounding of Slab Thickness

- The total slab thickness h is usually rounded to the next higher 25mm.
- Best economy is often achieved when the slab thickness is selected to match nominal lumber dimensions.

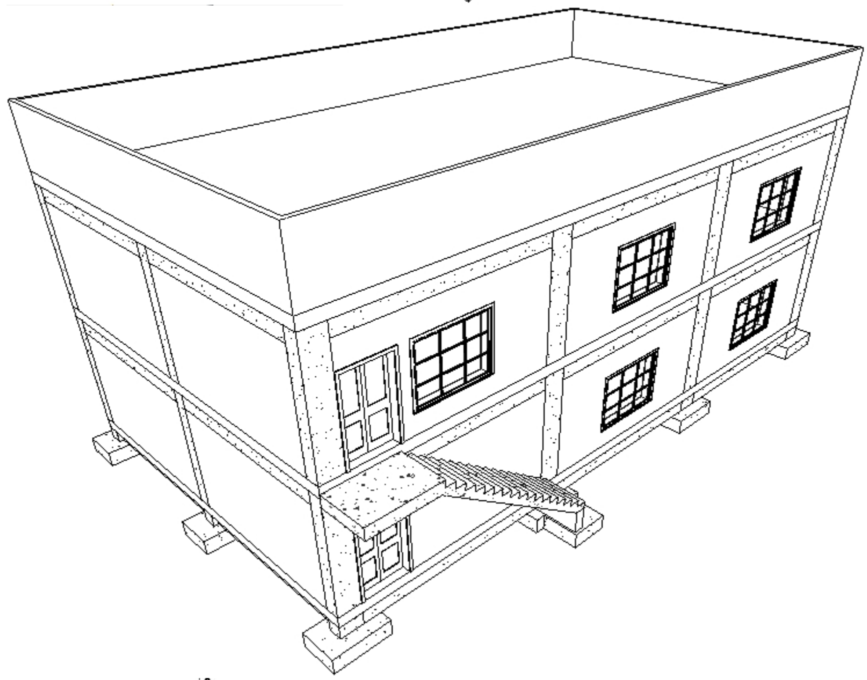
12.2 Design Examples of One-Way Slab Systems

Example 12.2-1

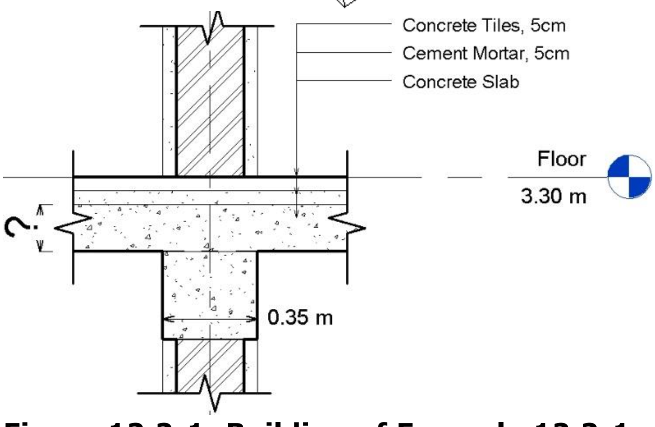
Design floor slab for building shown in ***Figure 12.2-1*** below and its supporting beam (Beam A). The service live load is 4.8 kN/m^2, and weight of suspended ceiling is 0.5 kN/m^2. Material properties are f_c' = 21MPa and f_y = 420 MPa.



Front 3D View

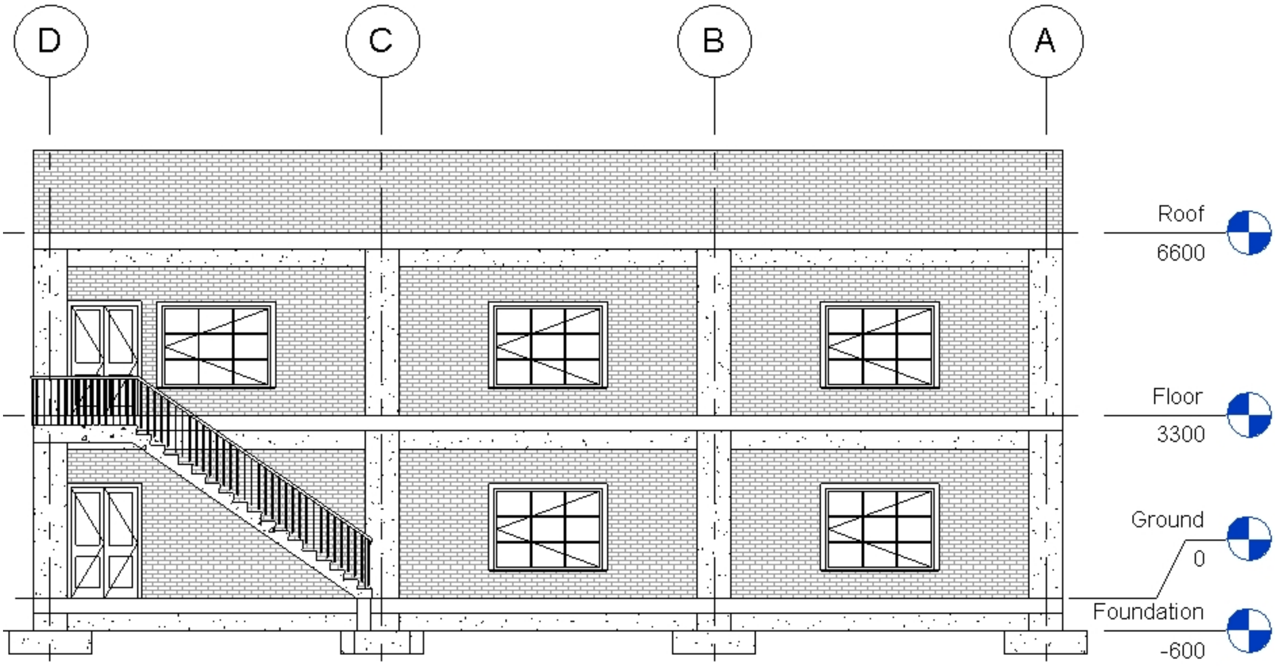


Back 3D View

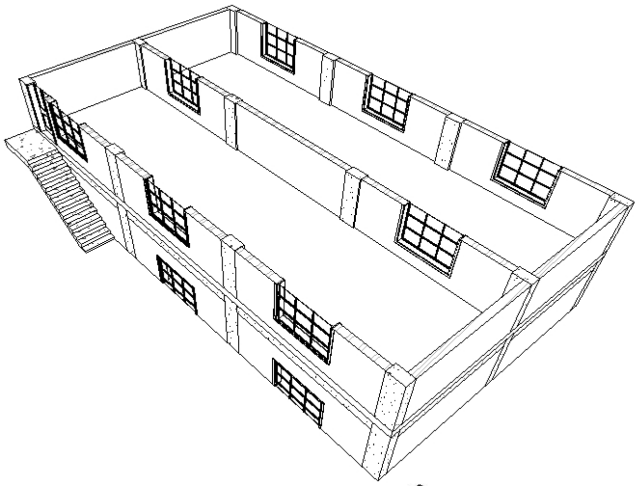


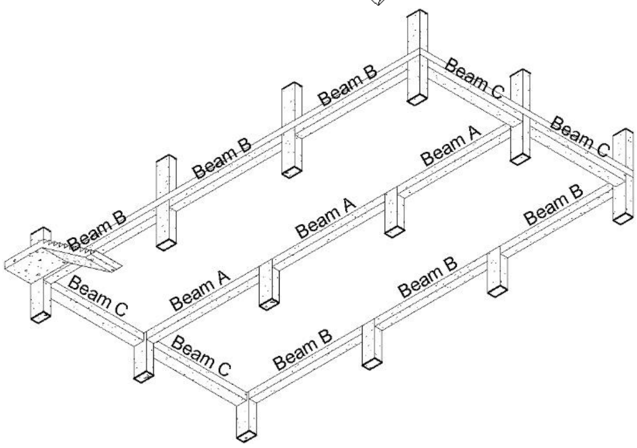
Floor Details

Figure 12.2-1: Building of Example 12.2-1.

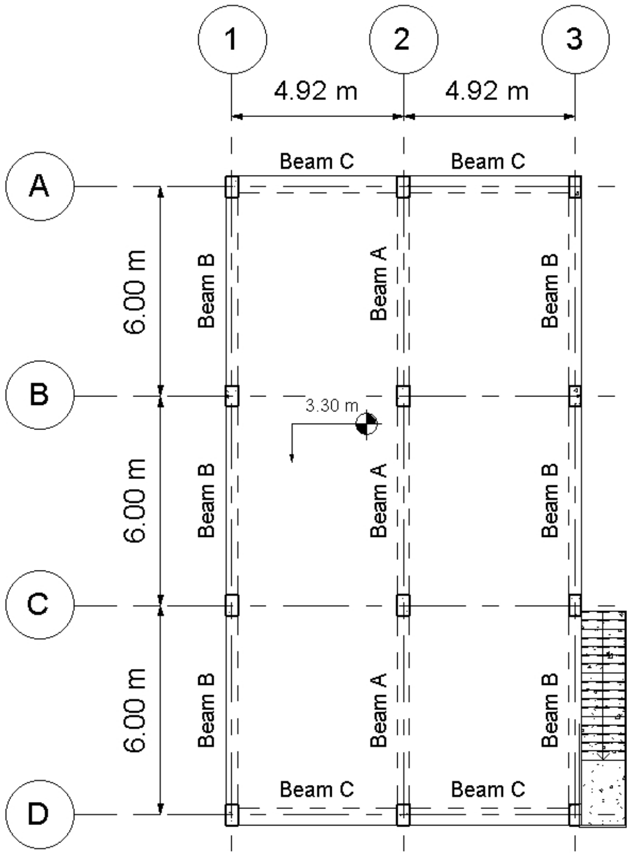


Elevation view.

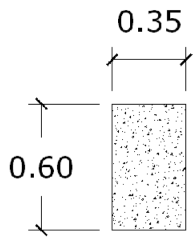




Floor system, 3D view.

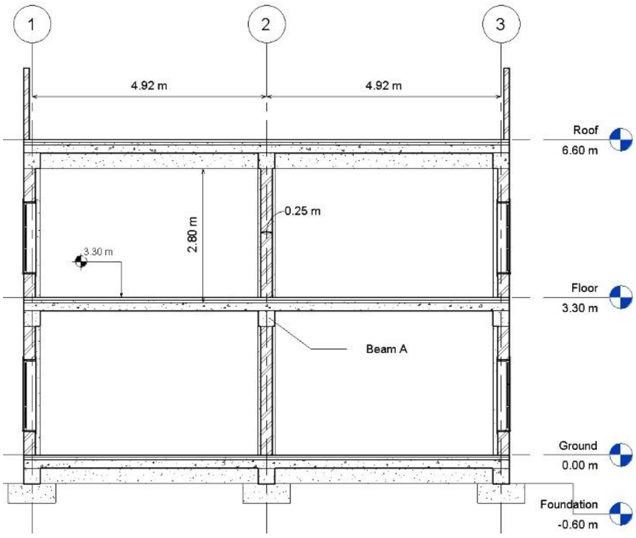


Floor system, a plan view

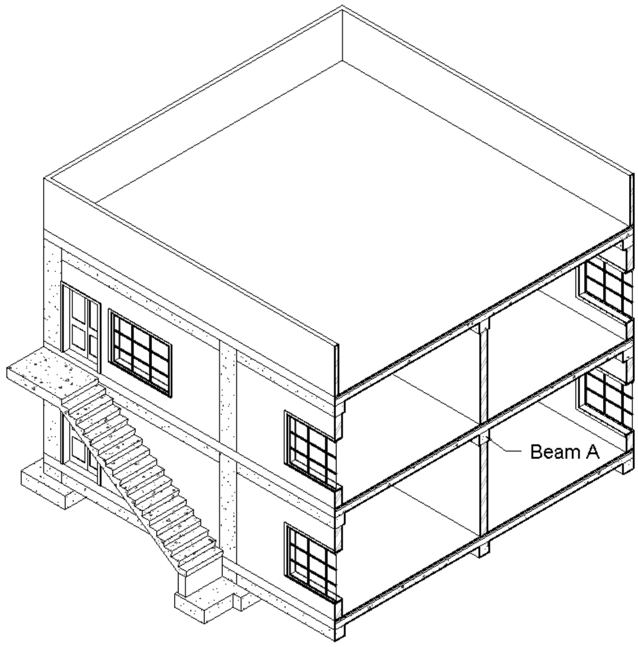


Typical column section

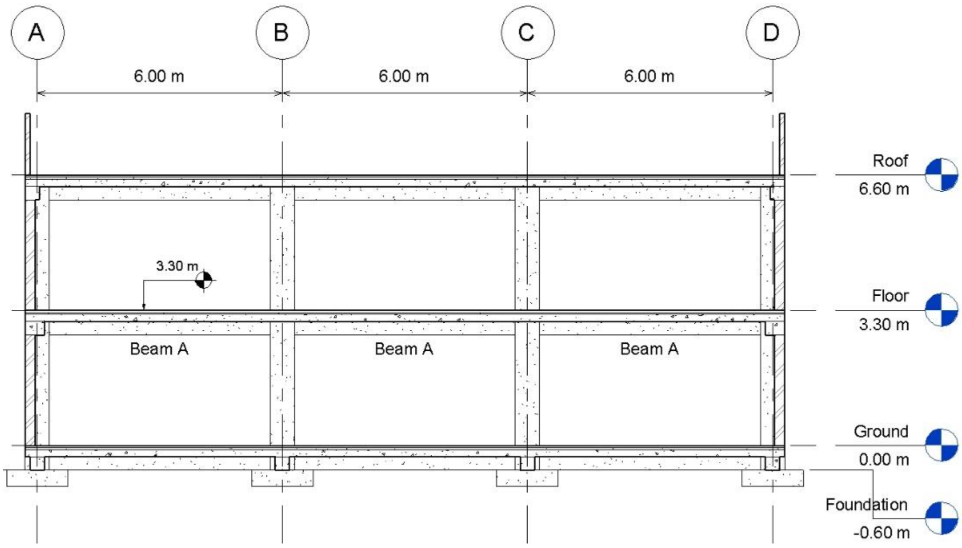
Figure 12.2-1: Building of Example 12.2-1. Continued.



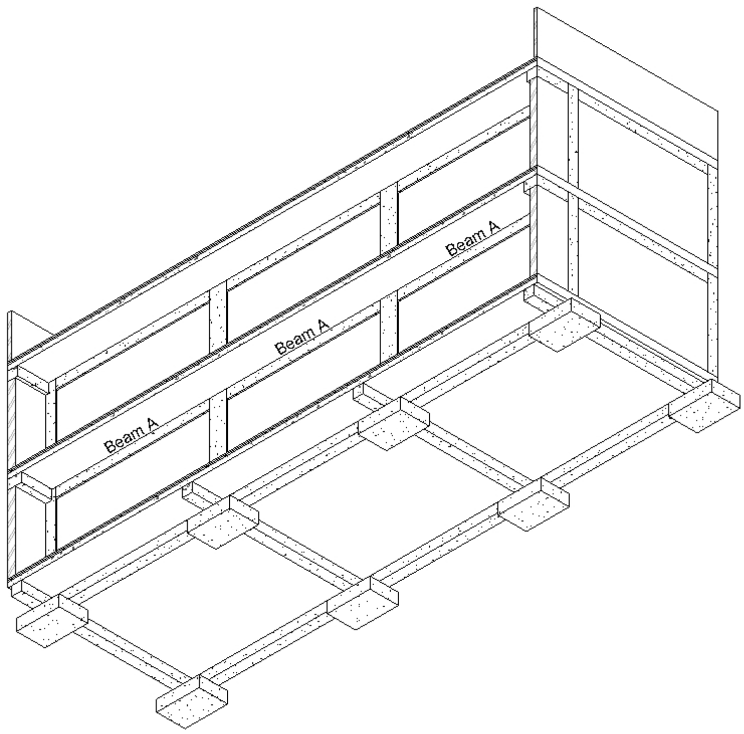
Cross sectional 2D view



Cross sectional 3D view



Longitudinal sectional 2D view



Longitudinal sectional 3D view

Figure 12.2-1: Building of Example 12.2-1. Continued.

Solution

Slab Design

- System Classification

$$Slab\ Aspect\ Ratio\ = \frac{L}{S} = \frac{18}{4.92} = 3.65 > 2$$

Then, the system is a one-way system.

- Slab Thickness

The slab thickness for deflection control can be determined based on Table 12.1-2 of the (ACI318M, 2014). As the slab built integrally with the spandrel beam, then the slab thickness can be computed based on the assumption of two ends continuous.

$$h_{min} = \frac{l}{28} = \frac{4920 - \frac{350}{2} \times 2}{28} = \frac{4\,570^{mm}}{28} = 163^{mm}$$

Try $h = 175^{mm}$.

- Loads Acting on Slab
  - Dead Loads

$$W_{Dead} = \left(0.175^{m} \times 24\frac{kN}{m^3}\right)_{Slab\ Selfweight} + (0.05 + 0.05)m \times 20\frac{kN}{m^3}_{Slab\ Surfacing} + \left(0.5\frac{kN}{m^2}\right)_{Suspended\ Ceiling} = 4.2\frac{kN}{m^2} + 2.5\frac{kN}{m^2} = 6.70\frac{kN}{m^2}$$

  - Live Loads

$$W_{Live} = 4.8\frac{kN}{m^2}$$

  - Factored Loads

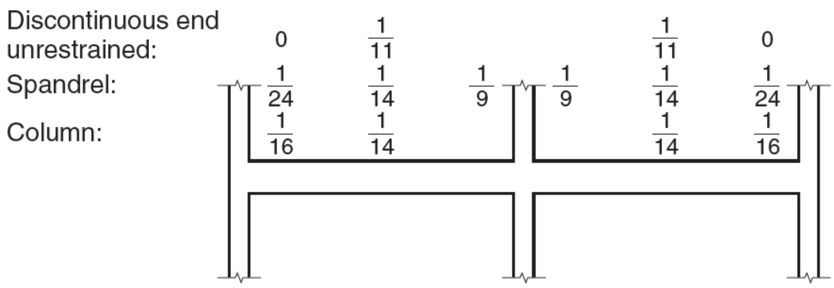
$$W_u = maximum\ of\ [1.4\ W_{Dead}\ or\ 1.2\ W_{Dead} + 1.6\ W_{Live}]$$

$$W_u = maximum\ of\ [1.4 \times 6.70\ or\ 1.2 \times 6.70 + 1.6 \times 4.8] = maximum\ of\ [9.38\ or\ 15.7]$$

$$W_u = 15.7\frac{kN}{m^2}$$

- Design Moments of Slab

Factored design moments will be computed by ACI Coefficients Method:



$$M_{u\ Negative\ Exterior} = \frac{W_u L_n^2}{24} = \frac{15.7 \times 4.57^2}{24} = 13.7\ kN.m\ per\ m$$

$$M_{u\ Positive} = \frac{W_u L_n^2}{14} = \frac{15.7 \times 4.57^2}{14} = 23.4\ kN.m\ per\ m$$

$$M_{u\ Negative\ Interior} = \frac{W_u L_n^2}{9} = \frac{15.7 \times 4.57^2}{9} = 36.4\ kN.m\ per\ m$$

- Flexure Reinforcement in Structural Direction

Try $\emptyset\ 13mm$ for flexure reinforcement. Then the effective slab depth will be:

$$d = 175mm - 20mm_{Cover} - \frac{13mm}{2} = 148mm$$

- ACI Maximum Reinforcement

$$\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} = 0.85^2 \times \frac{21}{420} \times \frac{0.003}{0.007} = 15.5 \times 10^{-3}$$

$$A_{smax} = 15.5 \times 10^{-3} \times 1000mm \times 148mm = 2\,294\ mm^2\ per\ m$$

- Minimum, Temperature, and Shrinkage Reinforcement

According to (ACI318M, 2014), ***Article 7.6.1.1***, A minimum area of flexural reinforcement, As,min, shall be provided in accordance with ***Table 12.1-1*** above:

Reinforcement type	f_y, MPa	$A_{s,min}$	
Deformed bars	< 420	$0.0020A_g$	
Deformed bars or welded wire reinforcement	≥ 420	Greater of:	$\frac{0.0018\times 420}{f_y}A_g$
			$0.0014A_g$

$A_{S\ Minimum\ or\ temp} = 0.0018 \times 1000mm \times 175mm = 315\ mm^2 per\ m$

- Summary of Required Reinforcement in Structural Direction

  Assume that the strength reduction moment $\emptyset = 0.9$ (as the steel ratios of slabs are so small, then this assumption is generally satisfied). Try $\emptyset 13mm$:

  $A_{Bar} = \frac{\pi \times 13^2}{4} = 133\ mm^2$

  Required reinforcements according to requirements of ACI Code are summarized in ***Table 12.2-1*** below.

- Minimum, Temperature Reinforcement in Other Direction

  $A_{S\ Temperature} = 0.0018 \times 1000mm \times 175mm = 315\ mm^2 per\ m$

  $S_{Required}\ (mm) = \frac{A_{Bar}}{A_{S\ Temperature}} \times 1000 = \frac{133}{315} \times 1000 = 422\ mm$

  $S_{maximum\ for\ Other\ Direction} = minimum\ (5 \times t\ or\ 450mm) = 450mm$

  Use $\boldsymbol{\emptyset 13mm\ @\ 400mm}$ for Temperature Reinforcement.

- Checking for Shear

  $V_u = \frac{1.15(W_u l_n)}{2} = 1.15 \times \frac{15.7 \times 4.57}{2} = 41.3\ kN\ per\ m$

  $\emptyset V_n = 0.75 \times \left(0.17\sqrt{f_c'}\ bd\right) = 0.75 \times 0.17 \times \sqrt{21} \times 1000 \times 148 = 86.5\ kN\ per\ m > V_u\ Ok.$

Table 12.2-1: Slab Reinforcement Design of Example 12.2-1.

	Exterior Negative	***Positive Moment***	***Interior Negative***
$M_u (kN.m\ per\ m)$	13.7	23.4	36.4
$M_n (kN.m\ per\ m)$	15.2	26.0	40.4
$\rho_{Required} = \frac{1-\sqrt{1-2.36\frac{M_n}{f_c' bd^2}}}{1.18 \times \frac{f_y}{f_c'}}$	1.68 x 10^{-3}	2.93 x 10^{-3}	4.60 x 10^{-3}
$A_{S\ Theoretical}\ (mm^2 per\ m) = \rho_{Required} \times b \times d$	249	434	681
$\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004}$	$0.85^2 \times \frac{21}{420} \times \frac{0.003}{0.007} = 15.5 \times 10^{-3}$		
$A_{smax}(mm^2\ per\ m) = \rho_{max} \times b \times d$	$15.5 \times 10^{-3} \times 1000mm \times 148mm = 2\,294$		
$A_{S\ Minimum}\ mm^2 per\ m$	$0.0018 \times 1000mm \times 175mm = 315$		
$A_{S\ Required} mm^2 per\ m$	315	434	681
$S_{Theoretical}(mm) = \frac{A_{Bar}}{A_{S\ Required}} \times 1000$	422	306	195
$S_{maximum} = minimum\ (3 \times t\ or\ 450mm)$	$= minimum\ (3 \times 175\ or\ 450mm) = 450$		
Final Main Reinforcement	***Use*** $\boldsymbol{\emptyset 13mm\ @\ 400mm}$	***Use*** $\boldsymbol{\emptyset 13mm\ @\ 300mm}$	***Use*** $\boldsymbol{\emptyset 13mm\ @\ 175mm}$

- Reinforcement Cutoff and Bend Points:
  - The steel provided should be varied according to the bending moment diagram, from bottom reinforcement for positive moment at mid-span to top reinforcement for negative moment at face of supports and vice versa. This can be done either through bend or through cutoff of the reinforcements.

- Because the determination of cutoff or bend points may be rather tedious, many designers specify that bars be cut off or bent at more or less arbitrarily defined points that experience has proven to be safe.
- For nearly equal spans, uniformly loaded, in which not more than about one-half the tensile steel is to be cut off or bent, the locations shown in ***Figure 12.2-2*** below are satisfactory.

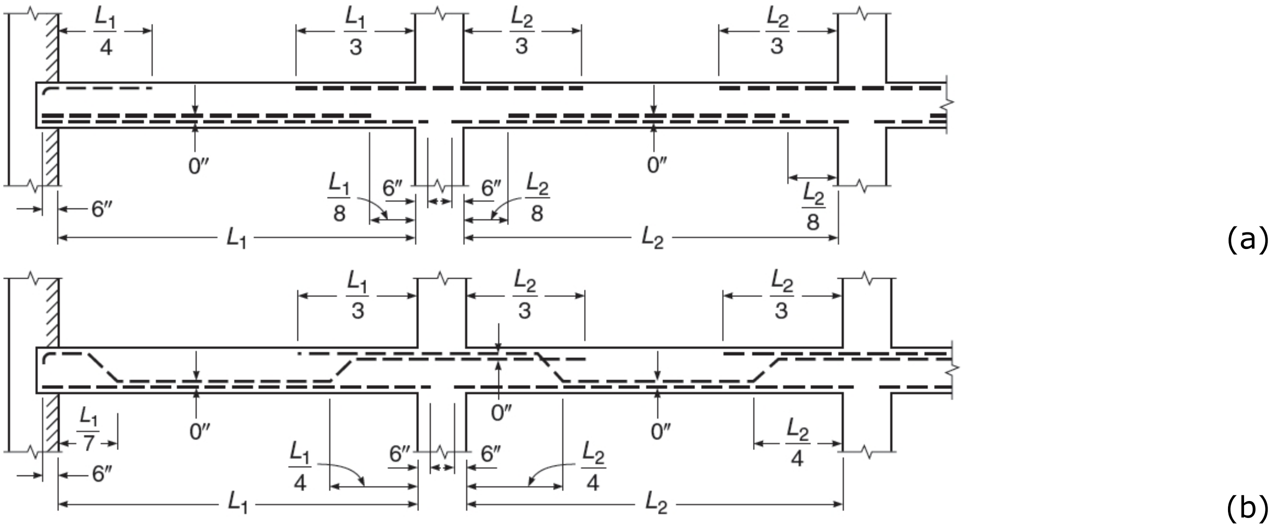
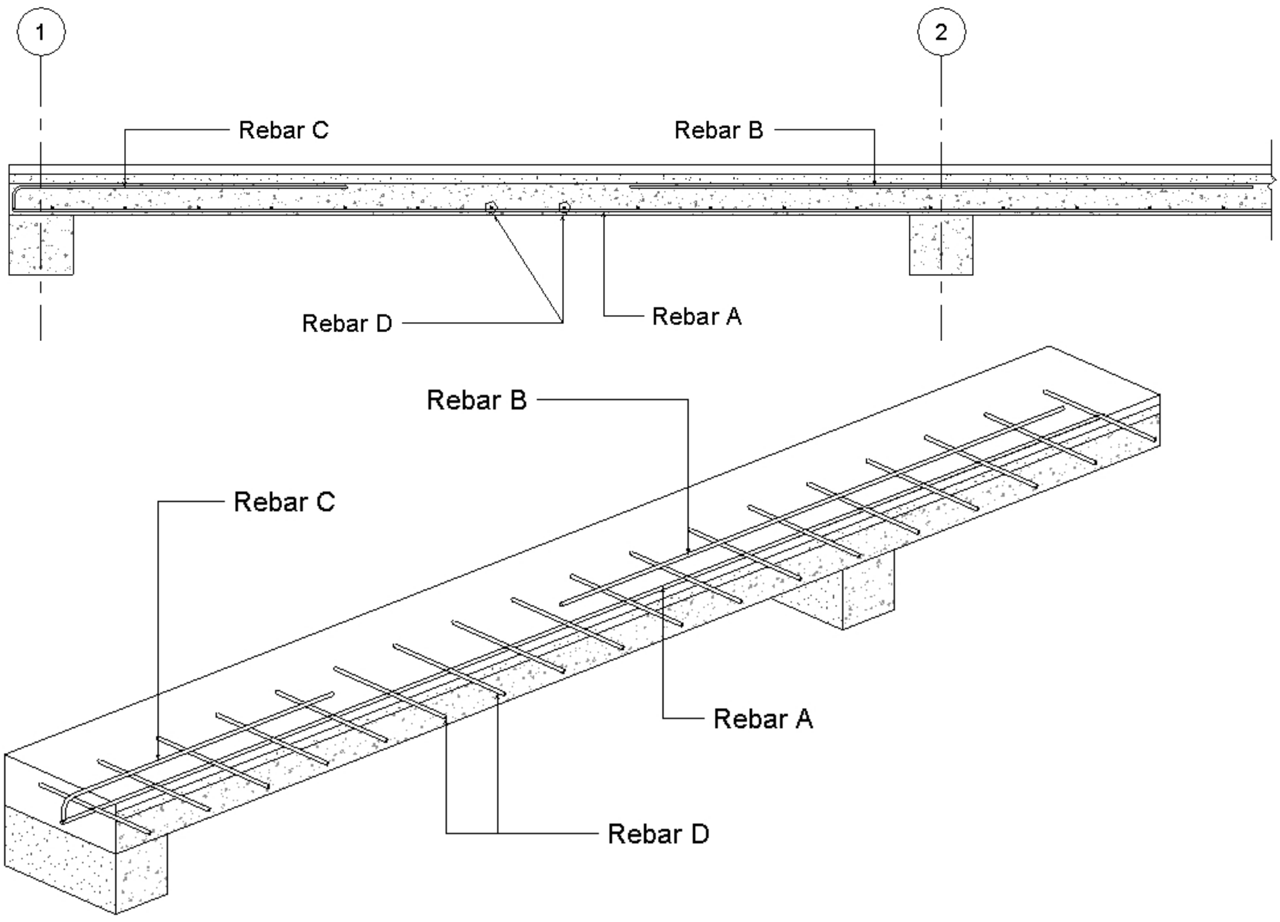
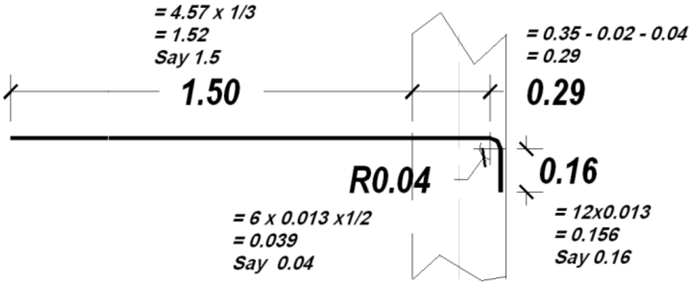


Figure 12.2-2: Cutoff or bend points for bars in approximately equal spans with uniformly distributed loads.

- Note, in ***Figure 12.2-2*** above, that the beam at the exterior support at the left is shown to be simply supported. If the beam is monolithic with exterior columns or with a concrete wall at that end details for a typical interior span could be used for the end span as well.
- In modern design, straight bars are usually used instead of bent bars.
- If straight bars have been used in our design, then the slab reinforcement will be as shown below:





Details of rebar C

= (38 x 13) x 1.3 = 642 Say 650

This Value is only Approximated for fc' = 21 MPa

= 4.57 x 2 + 0.35 + 0.15 x 2 = 9.79
Say 9.8

Details of Slab Reinforcement

Bar Designation Name	Location	Diameter (mm)	Spacing (mm)	Length (m)	Splices (mm)
A	**Bottom**	**13**	**300**	9.8	650
B	**Top**	**13**	**175**	3.4	
C	**Top**	**13**	**400**		
D	**Bottom**	**13**	**400**	18.2	650
E	**Top**	**13**	**300**		
Slab Thickness = 175 mm					

= 4.57 x 1/3 x 2 + 0.35 = 3.397
Say 3.4

= 6 x 3 + 0.3 x 2 - 0.35 x 2 + 0.15 x 2
= 18.2

Figure 12.2-3: Slab reinforcement for Example 12.2-1.

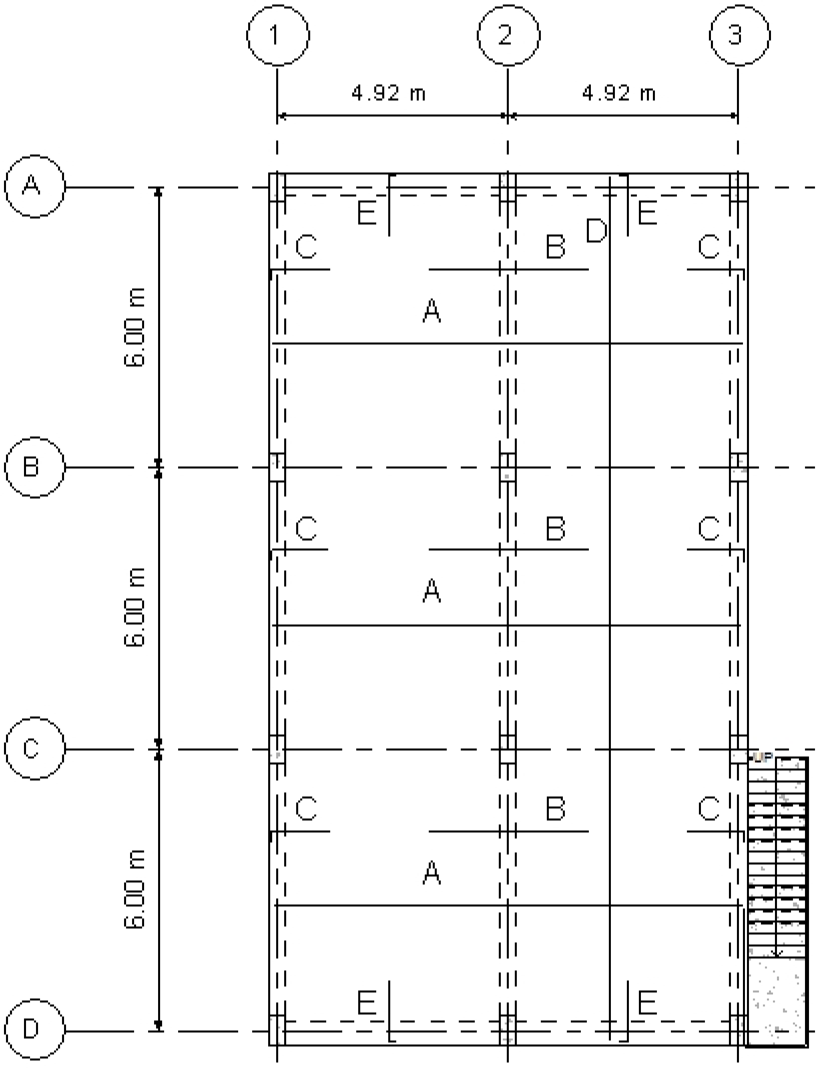


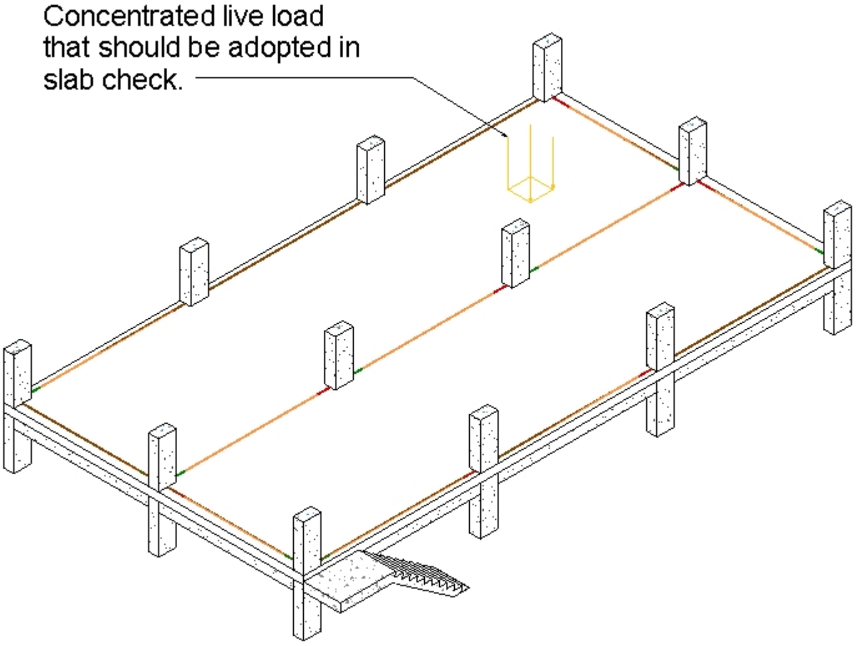
Figure 12.2-3: Slab reinforcement for Example 12.2-1. Continued.

Slab Checking for Concentrated Loads

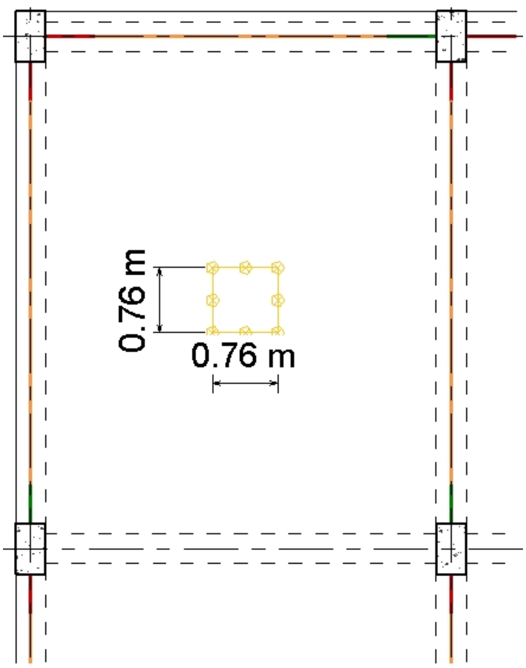
- It has been discussed in ***Chapter 1*** that according to ***ASCE 7***, slab should be checked for concentrated live load corresponding to the uniformly distributed live load that has been adopted in the design process.
- For a uniformly distributed live load of ***4.8 kPa***, the corresponding concentrated live load would be:

  $P_L = 8.9\ kN$
- According to ASCE 7, unless otherwise specified, the indicated concentration shall be assumed to be uniformly distributed over an area ***762 mm*** by ***762 mm***.

  $q_L = \frac{8.9}{0.762^2} = 15.3\ kPa$
- This load shall be located as indicated in ***Figure 12.2-4*** below to produce the maximum moment effects in the slab.
- Many designers intuitively assume that this load is resisted with main and distributed reinforcements.
- In general, numerical finite element model should be adopted to determine the resulting shear force and bending moment due to this concentrated load.



3D view



Plan view

Figure 12.2-4: Concentrated live load that should be considered in slab checked.

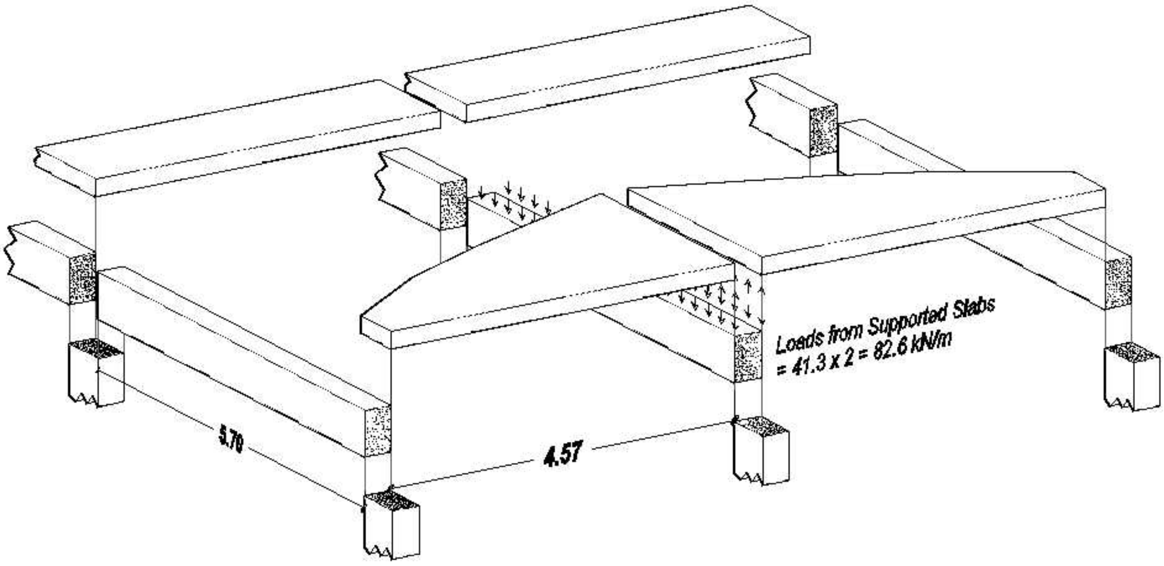
<u>Design of Supporting Beam A</u>

- Load on Supporting Beam
  - Load on supporting beam is equal to the shear forces transfer from the supported slab, weight of brick partitions, and beam selfweight:

    $W_u = 41.3 \frac{kN}{m} \times 2 + 1.2 \times W_{selfweight} + 1.2 \times W_{Brick\ Partition}$
  - Assume that, the beam has a depth equal to 0.5m (this assumption will be checked later):

    $W_u = 41.3 \frac{kN}{m} \times 2 + 1.2 \times \left(0.5m \times 0.35m \times 24 \frac{kN}{m^3}\right) + 1.2\ (2.8m \times 0.25m \times 19 \frac{kN}{m^3})$

    $W_u = 82.6 + 5.04 + 16.0 = 104 \frac{kN}{m}$



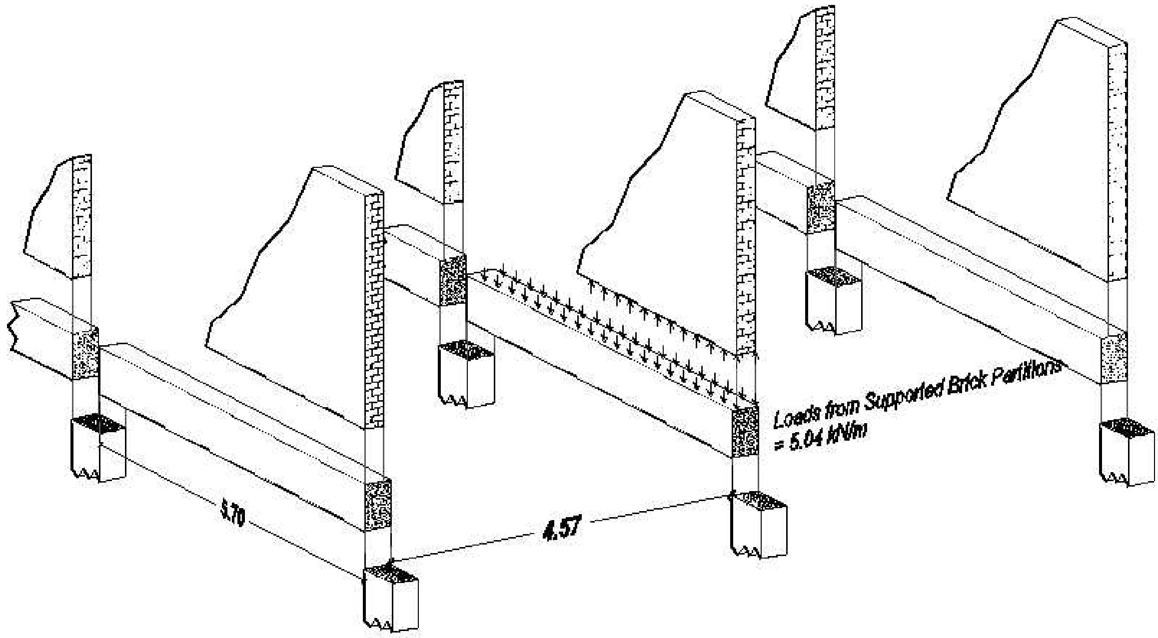
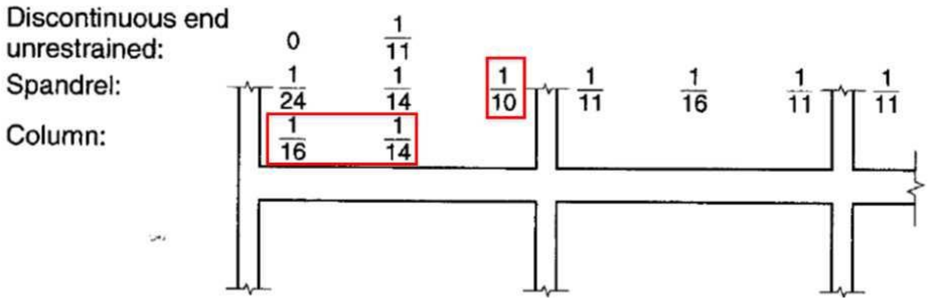


Figure 12.2-5: Forces acting on beam "A" of Example 12.2-1.

- Design Moments and Shears for Beam A
  - Beam factored moments and shears can be computed by ACI Coefficients Method.
  - As beam A extends over three spans, then moment will be computed based coefficients for more than two spans.
  - For spans that are nearly equals, structural designers usually use same top and bottom reinforcement along exterior and interior spans. These common reinforcements should then be computed based on moments for the exterior

span as it represents the critical span. Based on this practice, moments and shears have been computed for the exterior span only:



$$M_{u\,Exterior\,Negative} = \frac{W_u l_n^2}{16} = \frac{104 \times 5.4^2}{16} = 190\ kN.m$$

$$M_{u\,Positive} = \frac{W_u l_n^2}{14} = \frac{104 \times 5.4^2}{14} = 217\ kN.m$$

$$M_{u\,Interior\,Negative} = \frac{W_u l_n^2}{10} = \frac{104 \times 5.4^2}{10} = 303\ kN.m$$

$$V_u = 1.15\,\frac{W_u l_n}{2} = 1.15\ \times \frac{104\frac{kN}{m} \times 5.4m}{2} = 323\ kN$$

- After computing of beam effective depth, V_u at distance d from face of support can be computed and used in later shear design.

- Beam Design
  - If there are no previous limitations on beam dimensions either based on architectural requirements, or based on functional requirements, then the design will be classified as Design with non Pre-specified Dimensions. Beam design of this example is under this category.
  - As was discussed in ***Chapter 3***, beam dimensions are either determined based on economical requirements that based on steel ratio of $0.5\rho_{max}$ to $0.75\rho_{max}$ or based on deflection requirements that based on steel ratio not greater than $0.18f_c'/f_y$.
  - As the exterior face of first interior support has a maximum moment value compared with moments at mid span and at faces of other supports, and as beam behaves as rectangular section in negative regions, then dimensions of a continuous beam should be determined based on requirements at exterior face of first interior support to be adequate for other regions.

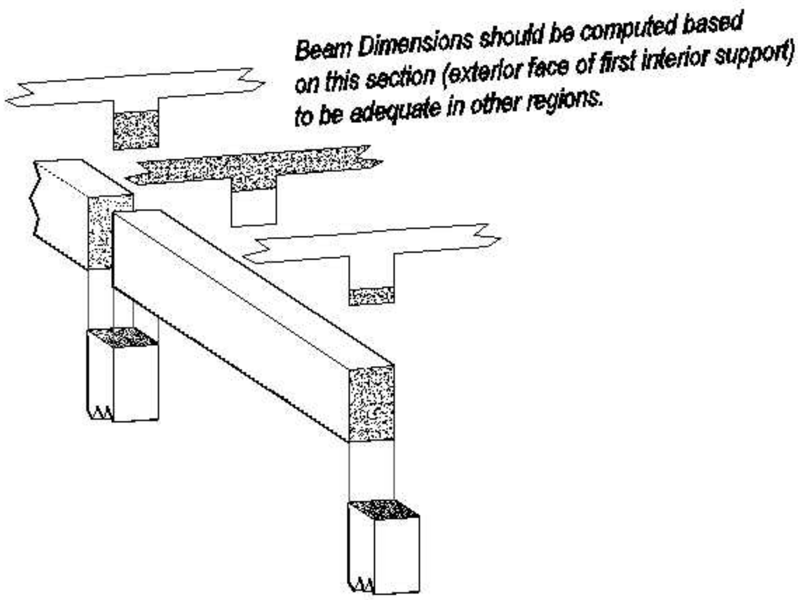


Figure 12.2-6: Sections behavior for beam "A" of Example 12.2-1.

Example 12.2-2

Circular slab indicated in Figure 12.2-7 has been supported on steel beams and reinforced as indicated. Use one-way behavior approximation for a preliminary checking of the proposed slab thickness and slab reinforcements. In your checking assume $f_c' = 28\,MPa$ and $f_y = 420\,MPa$.

Solution

This example aims to show how ideal one-way behavior can be adopted for a preliminary checking for critical regions of irregular slabs. With referring to Table 12.2-2 below, the minimum slab thicknesses for cantilever and continuous span are:

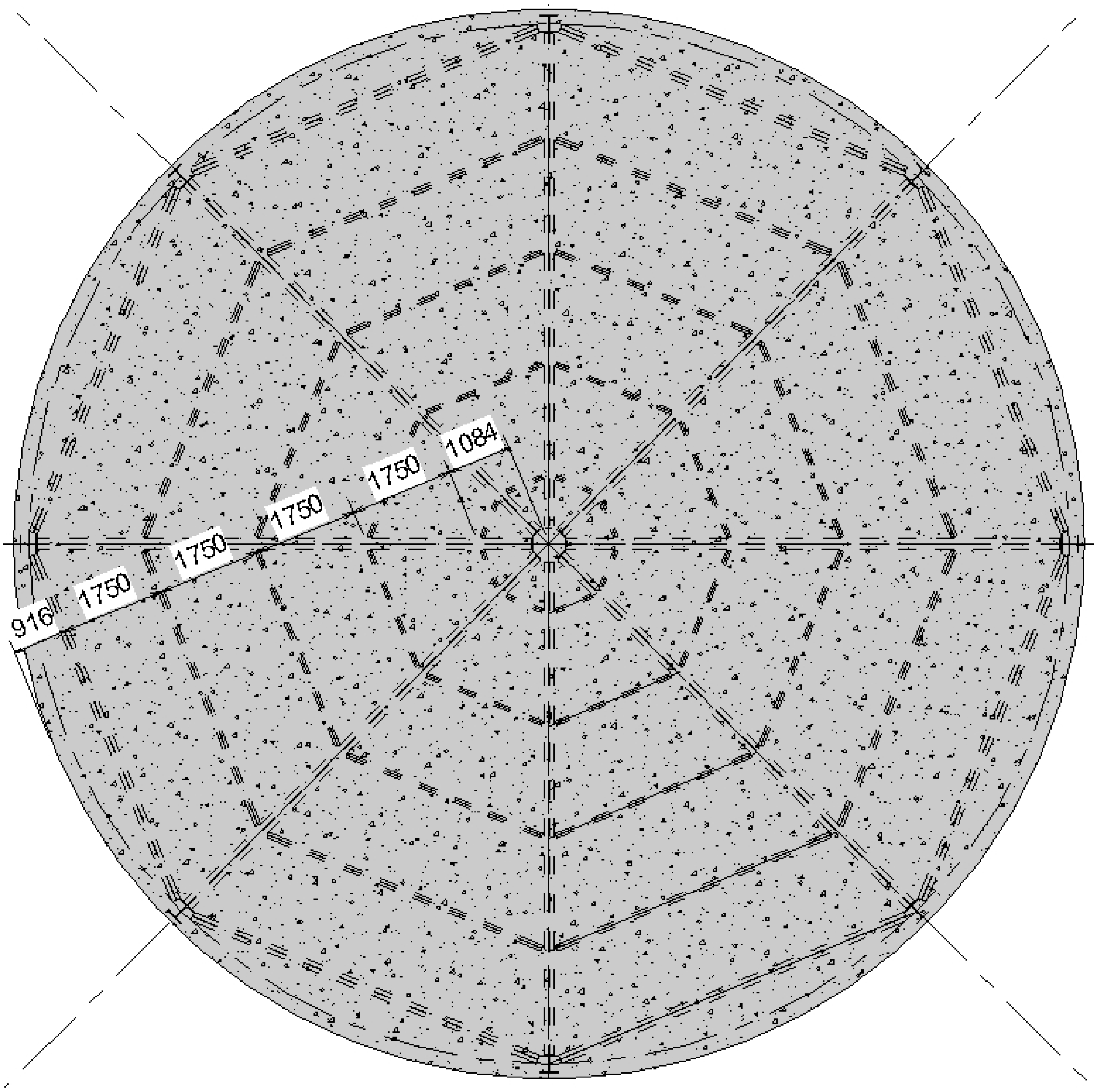
$$h_{minimum\,for\,cantilver} = \frac{\ell}{10} = \frac{916}{10} = 91.6\,mm < h_{proposed} \therefore Ok$$ **Eq. 12.2-1**

$$h_{minimum\,for\,\text{continuous span}} = \frac{\ell}{28} = \frac{1750}{28} = 62.5\,mm < h_{proposed} \therefore Ok$$ **Eq. 12.2-2**

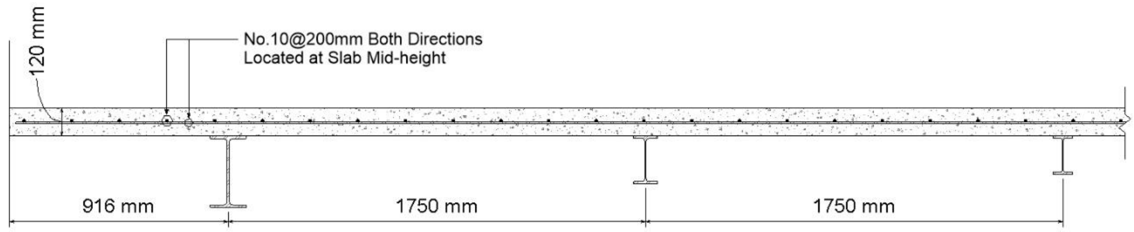
With referring to applied loads of Figure 12.2-7 below, the factored load would be:

$$W_D = 0.12 \times 24 + 3 = 5.88\,kPa$$ **Eq. 12.2-3**

$$W_u = \max(1.4 \times 5.88, 1.2 \times 5.88 + 1.6 \times 1.0) = 8.66\,kPa$$ **Eq. 12.2-4**



Plan view



Sectional View

Figure 12.2-7: Circular roof slab for Example 12.2-2.

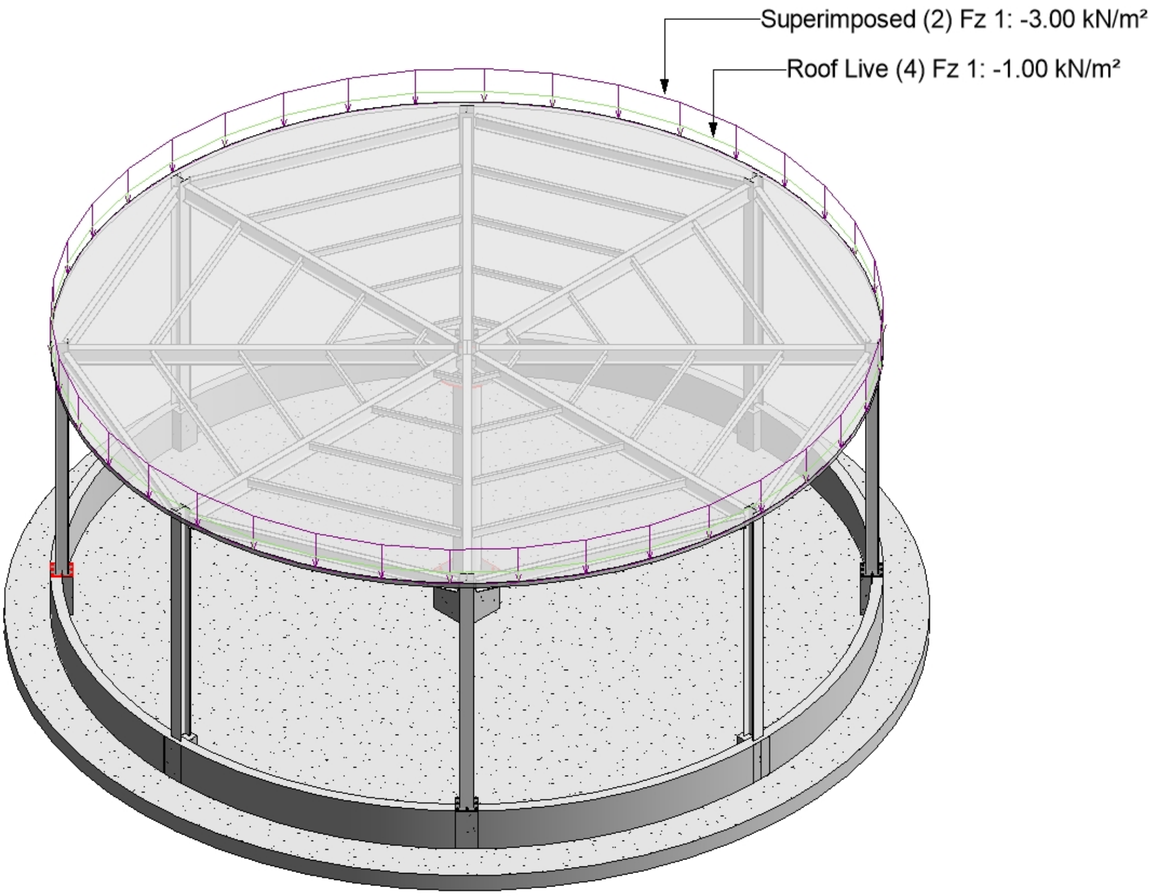


Figure 12.2-7: Circular roof slab for Example 12.2-2. Continue.

Table 12.2-2: Minimum thickness of solid nonprestressed one-way slabs (Table 7.3.1.1 of (ACI318M, 2014)).

Support condition	Minimum h[1]
Simply supported	$\ell/20$
One end continuous	$\ell/24$
Both ends continuous	$\ell/28$
Cantilever	$\ell/10$

[1]Expression applicable for normalweight concrete and f_y = 420 MPa. For other cases, minimum h shall be modified in accordance with 7.3.1.1.1 through 7.3.1.1.3, as appropriate.

As nothing has been mentioned about floor beam section, c/c span has been conservatively adopted. The factored negative moment at cantilever support is:

$$M_{-ve\,u\,cantilever}=\frac{W_u\ell_n^2}{2}=\frac{8.66\times0.916^2}{2}=3.63\,kN.m\ per\ m \quad \textbf{Eq. 12.2-5}$$

The negative moment for continuous span can be preliminary estimated from ACI coefficient method with assumption of more than two spans:

$$M_{-ve\,\text{continuous}}=\frac{W_u\ell_n^2}{10}=\frac{8.66\times1.750^2}{10}=2.65\,kN.m\ per\ m \quad \textbf{Eq. 12.2-6}$$

As rebars are located at slab mid-height, the effective depth would be:

$$d=\frac{120}{2}=60mm$$

$$\Sigma F_x=0\Rightarrow a=\frac{\left(\frac{\pi\times10^2}{4}\times\frac{1000}{200}\right)\times420}{0.85\times28\times1000}=6.92mm \quad \textbf{Eq. 12.2-7}$$

$$\phi M_n=\frac{0.9\left(\left(\frac{\pi\times10^2}{4}\times\frac{1000}{200}\right)\times420\right)\times\left(60-\frac{6.92}{2}\right)}{10^6}=8.39\,kN.m>M_u\therefore Ok. \quad \textbf{Eq. 12.2-8}$$

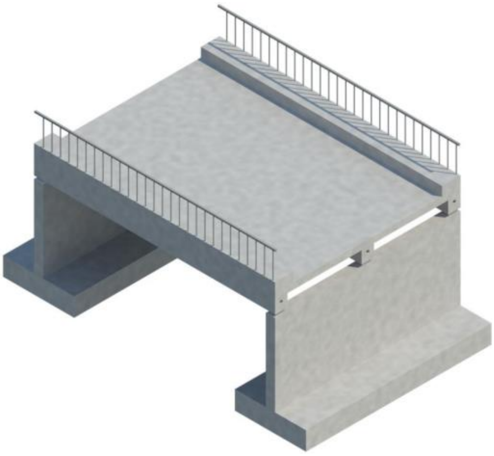
Example 12.2-3

For pedestrian bridge indicated in ***Figure 12.2-8*** below, structural designer includes sleeves for communication and electrical cables. For this bridge:

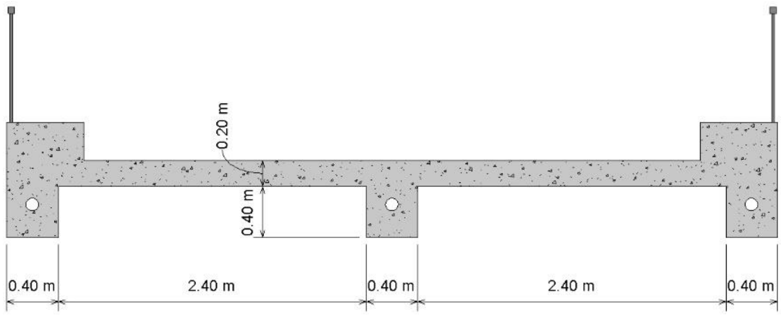
- Classify slab system into one-way system or two-way system.
- Determine the factored load that acting on slab.
- Check proposed slab thickness for deflection control.
- Check proposed slab thickness for shear.
- Determined slab factored moments and design it for flexure.
- Determine the load share that supported by each beam and determined factored moment accordingly.

In your solution, assume that:

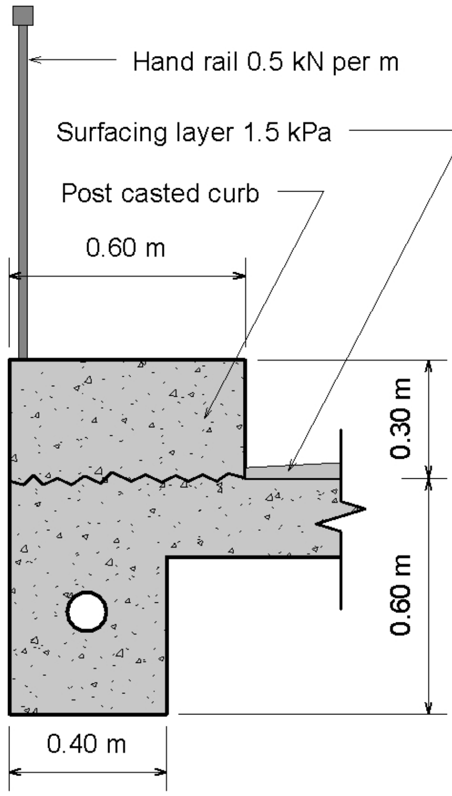
- Materials strength are $f_c' = 28\ MPa$ and $f_y = 420\ MPa$.
- Try $No. 10$ for slab reinforcement.
- Live load of $W_{Live} = 4.0\ kPa$.



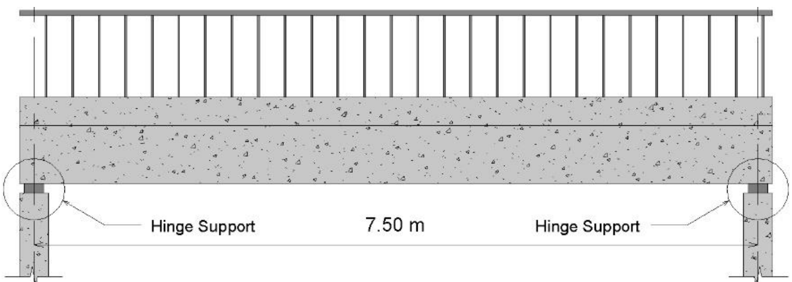
3D view



Cross sectional view



Callout view for

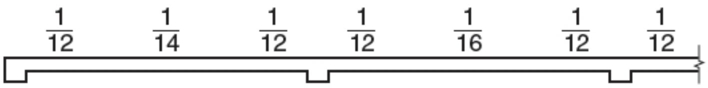


Elevation view

Figure 12.2-8: A pedestrian bridge.

Solution

- Slab system classification:
  As the slab panels are supported on two parallel sides only, therefore they are classified as one-way irrespective of their aspect ratios.
- Factored load acting on slab:
  $W_D = 0.2 \times 24 + 1.5 = \ 6.3\ kPa, W_L = 4.0\ kPa$
  $W_u = \max(1.4 \times 6.3, 1.2 \times 6.3 + 1.6 \times 4.0) = 13.96 \approx 14\ kPa$
- Checking of proposed thickness for deflection control:
  According to ***Table 12.1-2*** above, the minimum slab thickness for deflection control is:
  $$h_{minimum} = \frac{\ell}{28} = \frac{2400}{28} \approx 86\ mm < h_{proposed} \therefore Ok.$$
- Checking of proposed thickness for shear:
  According to ACI coefficients method, the shear at exterior face of first interior support is:
  $$V_u = 1.15 \frac{(W_u \ell_n)}{2} = 1.15 \times \left(\frac{14 \times 2.4}{2}\right) = 19.3\ kN\ per\ m$$
  $$d = 200 - 20 - \frac{10}{2} \approx 175\ mm$$
  $$\phi V_c = \phi\left(0.17\lambda\sqrt{f_c'}bd\right) = \frac{0.75 \times (0.17 \times 1.0 \times \sqrt{28} \times 1000 \times 175)}{1000} = 118\ kN\ per\ m > V_u \therefore Ok.$$
- Slab factored moment:
  According to ACI coefficients method, moments for a slab that has a span not exceeding 3m can be determined using indicated coefficients.



  $$M_{u+ve} = \frac{W_u \ell_n^2}{14} = \frac{14 \times 2.4^2}{14} = 5.76\ kN.m\ per\ m$$
  $$M_{u-ve} = \frac{W_u \ell_n^2}{12} = \frac{14 \times 2.4^2}{12} = 6.72\ kN.m\ per\ m$$
- Design for flexure:
  Slab flexural design has been prepared with referring to ***Table 12.2-3*** and reinforcement details are presented in ***Figure 12.2-9***.

Table 12.2-3: Slab Reinforcement Design of Example 12.2-3.

	Exterior Negative	***Positive Moment***	***Interior Negative***
$M_u (kN.m\ per\ m)$	6.72	5.76	6.72
$M_n (kN.m\ per\ m)$	7.47	6.40	7.47
$\rho_{Required} = \frac{1 - \sqrt{1 - 2.36 \frac{M_n}{f_c' b d^2}}}{1.18 \times \frac{f_y}{f_c'}}$	$\frac{1 - \sqrt{1 - 2.36 \times \frac{7.47 \times 10^6}{28 \times 1000 \times 175^2}}}{1.18 \times \frac{420}{28}} = 0.584 \times 10^{-3}$	$\frac{1 - \sqrt{1 - 2.36 \times \frac{6.40 \times 10^6}{28 \times 1000 \times 175^2}}}{1.18 \times \frac{420}{28}} = 0.500 \times 10^{-3}$	$\frac{1 - \sqrt{1 - 2.36 \times \frac{7.47 \times 10^6}{28 \times 1000 \times 175^2}}}{1.18 \times \frac{420}{28}} = 0.584 \times 10^{-3}$
$A_{S\,Theoretical}\ (mm^2 per\ m) = \rho_{Required} \times b \times d$	$0.584 \times 10^{-3} \times 1000 \times 175 = 102\ mm^2\ per\ m$	$0.500 \times 10^{-3} \times 1000 \times 175 = 87.5\ mm^2\ per\ m$	$0.584 \times 10^{-3} \times 1000 \times 175 = 102\ mm^2\ per\ m$
$\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004}$	$0.85^2 \times \frac{28}{420} \times \frac{0.003}{0.007} = 20.6 \times 10^{-3}$		
$A_{smax} (mm^2\ per\ m) = \rho_{max} \times b \times d$	$20.6 \times 10^{-3}\ \times 1000 \times 175 = 3605\ mm^2\ per\ m$		
$A_{S\,Minimum}\ mm^2 per\ m$	$0.0018 \times 1000 \times 200 = 360\ mm^2\ per\ m$		
$A_{S\,Required} mm^2 per\ m$	360	360	360
$S_{Theoretical} (mm) = \frac{A_{Bar}}{A_{S\,Required}} \times 1000$	$= \frac{\left(\frac{\pi \times 10^2}{4}\right)}{360} \times 1000 = 218\ mm$	$= \frac{\left(\frac{\pi \times 10^2}{4}\right)}{360} \times 1000 = 218\ mm$	$= \frac{\left(\frac{\pi \times 10^2}{4}\right)}{360} \times 1000 = 218\ mm$
$S_{maximum} = \ minimum\ (3 \times t\ or\ 450mm)$	$= \ minimum\ (3 \times 200\ or\ 450mm) = 450\ mm$		
Final Main Reinforcement	***Use*** $\emptyset 10mm\ @\ 200mm$	***Use*** $\emptyset 10mm\ @\ 200mm$	***Use*** $\emptyset 10mm\ @\ 200mm$

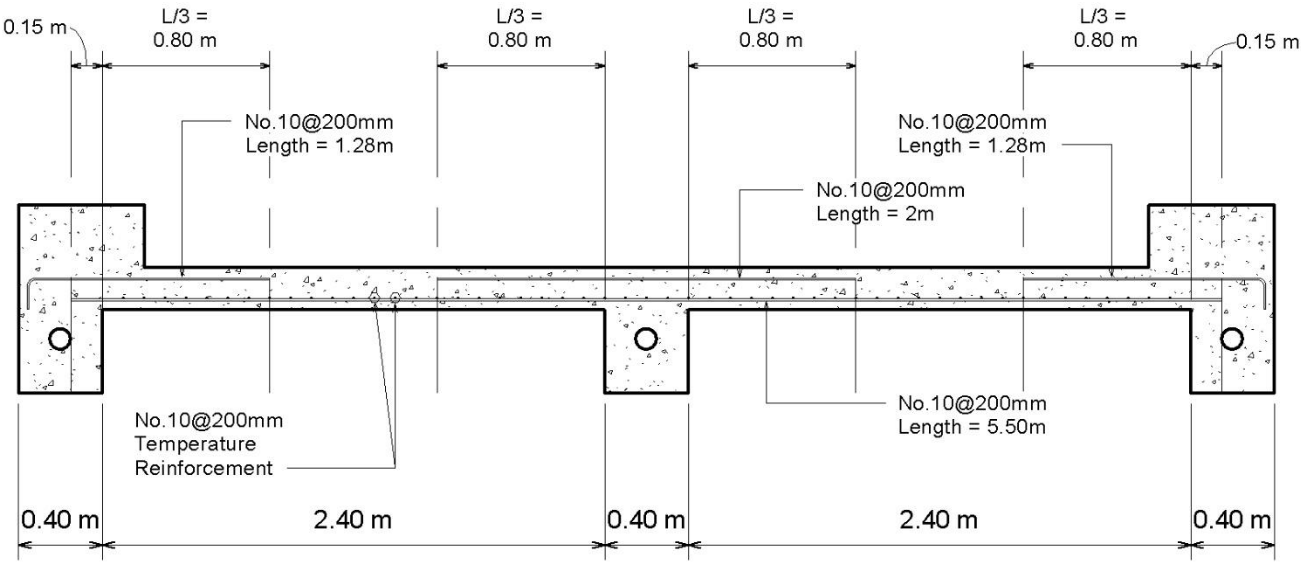


Figure 12.2-9: Slab reinforcement details for Example 12.2-3.

- Load share supported by the interior beam and its factored moment:
  With neglecting selfweight reduction due to sleeves, factored load share supported by the interior beam is:
  $W_{u\,for\,the\,interior\,beam} = 2 \times (1.15 \times 14 \times 2.4/2) + 1.2 \times (0.4 \times 0.6 \times 24) = 45.6\ kN/m$
  As the beam is simply supported, therefore it is factored moment can be determined directly based on equilibrium conditions:
  $M_{u\,for\,the\,interior\,beam} = (45.6 \times$ 〖7.50〗 ^2)/8 $= 321\ kN.m$
- Load share supported by an exterior beam and its factored moment:

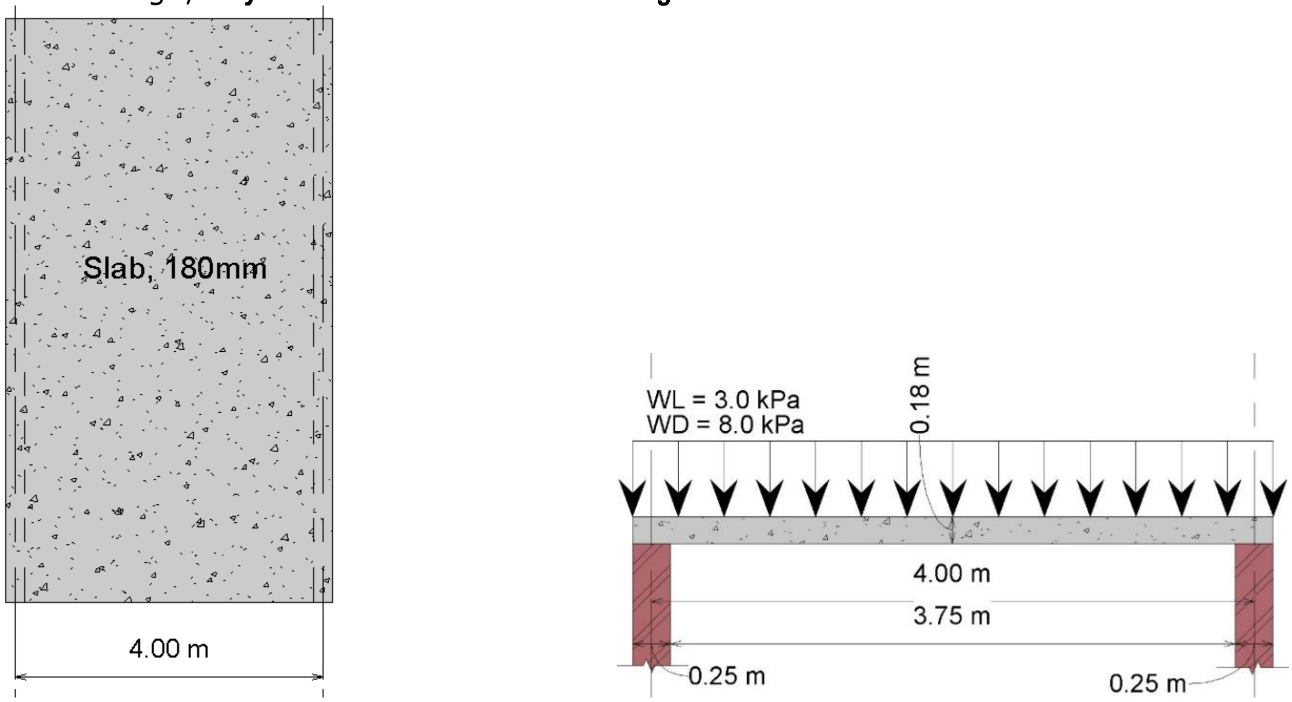
$$W_{u\,for\,an\,exterior\,beam} = \left(14 \times \frac{2.4}{2}\right) + 1.2 \times (0.4 \times 0.6 \times 24) + 1.2(0.6 \times 0.3 \times 24) + 1.2 \times 0.5 = 29.5\frac{kN}{m}$$

$$M_{u\,for\,an\,exterior\,beam} = \frac{29.5 \times 7.50^2}{8} = 207\ kN.m$$

Example 12.2-4

For one-way slab presented in ***Figure 12.2-11*** below:

- With neglecting slab selfweight, compute the factored load W_u that acting on slab.
- Check the proposed slab thickness for deflection control requirements.
- Assuming an effective depth, d, of 155mm, check the proposed slab thickness for shear strength requirements.
- Determine the factored positive moment, M_{u+ve}, that should be adopted in the slab design, ***only determine the moment not design the slab***.



Plan view **Sectional View**

Figure 12.2-10: One-way slab for Example 12.2-4.

Solution

- Factored Load W_u

$$W_u = Maximum\ (1.4W_D\ or\ 1.2W_D + 1.6W_L)$$
$$W_u = Maximum\ (1.4 \times 8.0\ or\ 1.2 \times 8.0 + 1.6 \times 3.0)$$
$$W_u = Maximum\ (11.2\ or\ 14.4) = 14.4 \frac{kN}{m^2}\ \blacksquare$$

- Checking of the Proposed Slab Thickness for Deflection Requirements.

With referring to ***Table 12.1-2*** above, the minimum slab thickness for deflection control would be:

$$h_{Minimum} = \frac{l}{20} = \frac{4000}{20} = 200\ mm > h_{Proposed} = 180mm \therefore Not\ Ok.$$

- Checking of the Proposed Slab Thickness for Shear Strength:

$$V_u = \frac{W_u l_n}{2} = \left(\frac{14.4 \times 3.75}{2}\right) = 27\ kN\ per\ m$$

$$\phi V_c = 0.75 \times \left(0.17\lambda\sqrt{f_c'}bd\right) = \frac{0.75 \times \left(0.17 \times 1.0 \times \sqrt{28} \times 1000 \times 155\right)}{1000} = 105\ kN\ per\ m > V_u$$
$$\therefore Ok.$$

- The Factored Positive Moment:

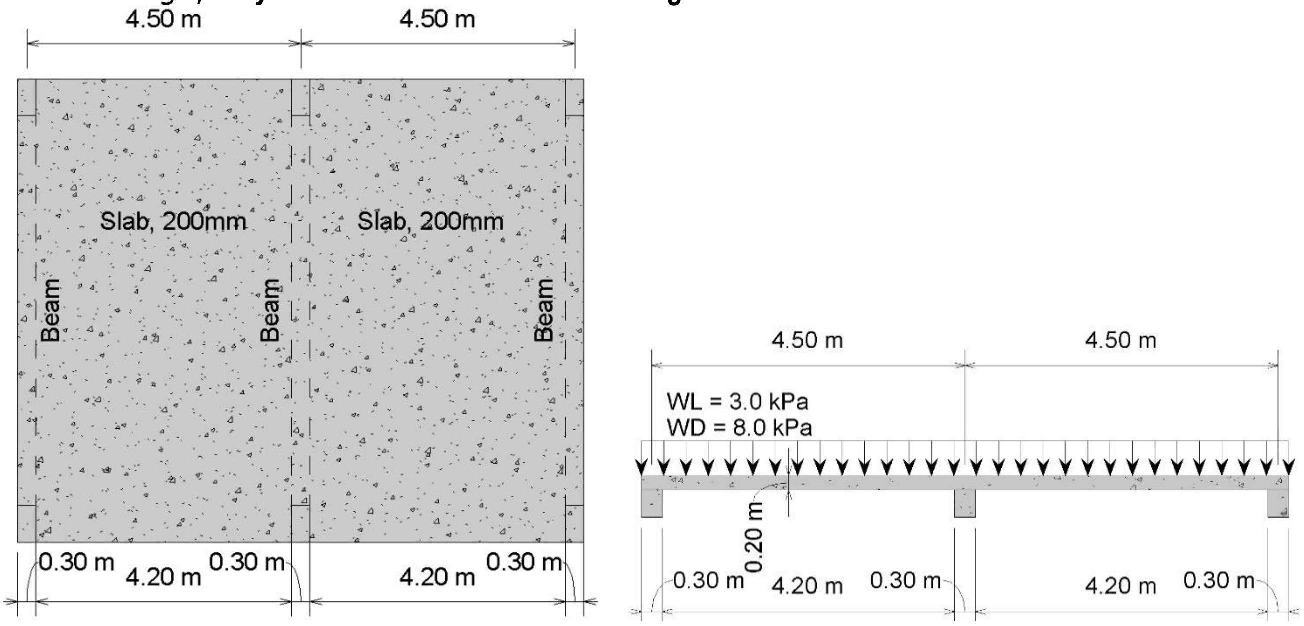
As the slab has a single simply supported span, hence it is a statically determinate span and its moment can be determined directly from equilibrium conditions:

$$M_{u+ve} = \frac{W_u \ell^2}{8} = \frac{14.4 \times 4.0^2}{8} = 28.8\ kN.m\ per\ m\ \blacksquare$$

Example 12.2-5

For one-way slab presented in ***Figure 12.2-11*** below:

- With neglecting slab selfweight, compute the factored load W_u that acting on slab.
- Check the proposed slab thickness for deflection control requirements.
- Assuming an effective depth, d, of 175mm, check the proposed slab thickness for shear strength requirements.
- Determine the factored positive moment, M_{u+ve}, that should be adopted in the slab design, ***only determine the moment not design the slab***.



Plan view **Sectional View**

Figure 12.2-11: One-way slab for Example 12.2-5.

Solution

- Factored Load W_u

$W_u = Maximum\ (1.4W_D\ or\ 1.2W_D + 1.6W_L)$

$W_u = Maximum\ (1.4 \times 8.0\ or\ 1.2 \times 8.0 + 1.6 \times 3.0)$

$W_u = Maximum\ (11.2\ or\ 14.4) = 14.4 \frac{kN}{m^2}$ ■

- Checking of the Proposed Slab Thickness for Deflection Requirements.

With referring to ***Table 12.1-2*** above, the minimum slab thickness for deflection control would be:

$$h_{Minimum} = \frac{l}{28} = \frac{4200}{28} = 150\ mm < h_{Proposed} = 200mm \therefore Ok.$$

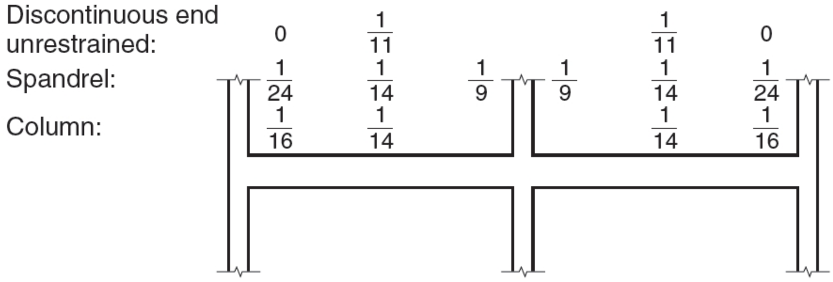
- Checking of the Proposed Slab Thickness for Shear Strength:

$$V_u = 1.15 \frac{W_u l_n}{2} = 1.15 \times \left(\frac{14.4 \times 4.2}{2}\right) = 34.8\ kN\ per\ m$$

$$\phi V_c = 0.75 \times \left(0.17\sqrt{f_c{}'}bd\right) = \frac{0.75 \times \left(0.17\sqrt{28} \times 1000 \times 175\right)}{1000} = 116\ kN\ per\ m > V_u \therefore Ok.$$

- The Factored Positive Moment:

As the slab has two spans only and with referring to ACI coefficients method with adopting spandrel beam discontinuous end, the factored positive moment would be:



$$M_{u+ve} = \frac{W_u l_n^2}{14} = \frac{14.4 \times 4.2^2}{14} = 18.1\ kN.m\ per\ m\ ■$$

12.3 ONE-WAY RIBBED SLABS*

12.3.1 DEFINITION

- A one-way joist floor consists of a series of small, closely spaced reinforced concrete T beams, called ***joists***, framing into monolithically cast concrete girders, which are in turn carried by the building columns.

12.3.2 HOW JOIST FORMED

- The joists are formed by creating void spaces in what otherwise would be a solid slab. Usually these voids are formed using special steel pans, as shown in ***Figure 12.3-1***.
- Concrete is cast between the forms to create ribs and placed to a depth over the top of the forms so as to create a thin monolithic slab that becomes the T beam flange.

Figure 12.3-1: Steel forms for one-way joist floor.

12.3.3 ECONOMICAL ASPECTS OF THE SYSTEM

- Since the strength of concrete in tension is small and is commonly neglected in design, elimination of much of the tension concrete in a slab by the use of pan forms results in a saving of weight with little change in the structural characteristics of the slab.
- Ribbed floors are economical:
  - For buildings, such as apartment houses, hotels, and hospitals,
  - The live loads are fairly small,
  - The spans comparatively long.
- Where ribbed floors are not suitable:
  They are not suitable for ***heavy construction*** such as in ***warehouses***, ***printing plants***, and ***heavy manufacturing buildings***.

12.3.4 STANDARD FORMS FOR THE VOID SPACES

- Standard forms for the void spaces for the void spaces between ribs are either ***500 or 750mm wide*** and ***200, 250, 300, 350, 400, or 500 mm deep***.
- They are ***tapered in cross section***, as shown in ***Figure 12.3-2***, generally at a slope of 1 to 12, to ***facilitate removal***.
- Any joist width can be obtained by varying the spacing between pans. Tapered end pans are used where it is desired to obtain a wider joist near the end supports, such as may be required for high shear or negative bending moment.
- After the concrete has hardened, ***the steel pans are removed for reuse***.

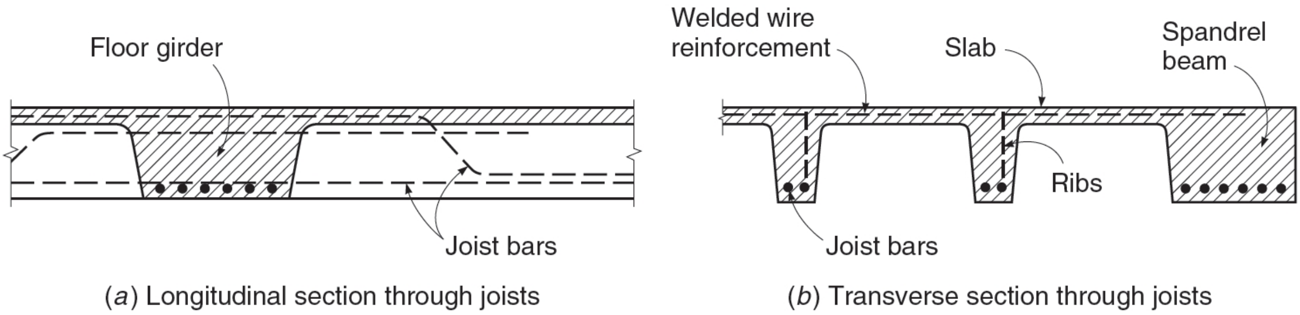


Figure 12.3-2: One-way joist floor cross sections: (a) cross section through supporting girder showing ends of joists and (b) cross section through typical joists.

12.3.5 NOMINAL DIMENSIONS OF JOIST

- Joist thickness for deflection control:
  - According to (ACI318M, 2014), minimum beam thickness for deflection control can be determined with referring to Table 12.3-1.
  - According to previous revisions of code, Table 12.3-1 can also be applied for one-way ribbed slabs.

Table 12.3-1: Minimum depth of nonprestressed beams.

Support condition	Minimum h[1]
Simply supported	$\ell/16$
One end continuous	$\ell/18.5$
Both ends continuous	$\ell/21$
Cantilever	$\ell/8$

[1]Expressions applicable for normalweight concrete and Grade 420 reinforcement. For other cases, minimum h shall be modified in accordance with 9.3.1.1.1 through 9.3.1.1.3, as appropriate.

- According to ***ACI Code 9.8.1***,
  - Ribs must not be less than 100mm wide and may not have a depth greater than 3.5 times the minimum web width. (For easier bar placement and placement of concrete, a minimum web width of 125mm is desirable.)
  - The clear spacing between ribs (determined by the pan width) must not exceed 750mm.
- ***ACI Code 9.8.3***
  The slab thickness over the top of the pans must not be less than one-twelfth of the clear distance between ribs, nor less than 50mm, according to ACI Code

12.3.6 JOIST REINFORCEMENT AND COVER

- Reinforcement for the joists usually consists of ***two bars in the positive bending region***, with ***one bar discontinued where no longer needed*** or ***bent up to provide a part of the negative steel requirement over the supporting girders***.
- Straight top bars are added over the support to provide for the negative bending moment.

- According to ***ACI Code 9.8.1***, ***at least one bottom bar must be continuous over the support, or at noncontinuous supports***, ***terminated with a standard hook or headed bar***, as a measure to ***improve structural integrity in the event of major structural damage***.
- ***ACI Code 20.6.1.3*** permits a reduced concrete cover of 20mm to be used for joist construction, just as for slabs.

12.3.7 Nominal Slab Thickness

- In one-way joist floors, the thickness of the slab is often controlled by fire resistance requirements.
- For a rating of 2 hours, for example, the slab must be about 100mm thick.

12.3.8 Slab Mesh Reinforcement

- The thin slab (top flange) is reinforced ***mainly for temperature and shrinkage stresses***, using ***welded wire reinforcement*** or ***small bars placed at right angles to the joists***.
- The area of this reinforcement is usually 0.18 percent of the gross cross section of the concrete slab.

12.3.9 Shear Strength for One-way Joist Systems

- One-way joists are generally proportioned with the concrete providing all of the shear strength, ***with no stirrups used***.
- A 10 percent increase in V_c above the value given by Eq. 12.3-1 below is permitted for joist construction, according to ***ACI Code 9.8.1***, based on ***the possibility of redistribution of local overload to adjacent joists***.
- Tests have shown that while local redistribution does occur, the shear strength of the full system (all joists acting together) is enhanced by less than 10 percent.

$$V_c = 0.17\lambda\sqrt{f'_c}\, b_w d \qquad \text{Eq. 12.3-1}$$

12.3.10 Weight of one-way Joist Floor Systems

Table 12.3-2 gives unit weights, in terms of psf of floor surface, for common combinations of joist width and depth, slab thickness, and form width.

Table 12.3-2: Weight of one-way joist floor systems

3 in. Top Slab			$4\frac{1}{2}$ in. Top Slab		
Depth of Pan Form, in.	Width of Joist + Pan Form, in.	Weight, psf	Depth of Pan Form, in.	Width of Joist + Pan Form, in.	Weight, psf
8	5 + 20	60	8	5 + 20	79
8	5 + 30	54	8	5 + 30	72
10	5 + 20	67	10	5 + 20	85
10	5 + 30	58	10	5 + 30	77
12	5 + 20	74	12	5 + 20	92
12	5 + 30	63	12	5 + 30	82
14	5 + 30	68	14	5 + 30	87
14	6 + 30	72	14	6 + 30	91
16	6 + 30	78	16	6 + 30	97
16	7 + 30	83	16	7 + 30	101
20	6 + 30	91	20	6 + 30	109
20	7 + 30	96	20	7 + 30	115

12.3.11 The Joists and the Supporting Girders

- The joists and ***the supporting girders are placed monolithically***.
- Like the joists, ***the girders are designed as T beams***.
- The shape of the girder cross section depends on the shape of the end pans that form the joists, as shown in ***Figure 12.3-2a***.

- If the girders are deeper than the joists, the thin concrete slab directly over the top of the pans is often neglected in the girder design.
- The girder width can be adjusted, as needed, by varying the placement of the end pans. The width of the web below the bottom of the joists must be at least 75mm. narrower than the flange (on either side) to allow for pan removal.
- Joist-band System:
  - A type of one-way joist floor system has evolved known as a ***joist-band system*** in which ***the joists are supported by broad girders having the same total depth as the joists***, as illustrated in Figure 12.3-2.
  - Separate beam forms are eliminated, and ***the same deck forms the soffit of both the joists and the girders***.
  - The ***simplified formwork***, ***faster construction***, ***level ceiling with no obstructing beams***, and ***reduced overall height of walls, columns, and vertical*** utilities combine to ***achieve an overall reduction in cost in most cases***.

12.3.12 WIDE MODULE JOIST SYSTEM

- If 500 or 750mm pan forms are used, ***slab span is small and slab strength is underutilized***. This has led to what is known as the ***wide module joist system***, or ***skip joist system***, as shown in ***Figure 12.3-3***.
- Such floors generally have 150 to 200mm wide ribs that are 1.5 to 1.8m on centers, with a 100mm top slab.
- These floors not only provide ***more efficient use of concrete in the slab*** but ***also require less formwork labor***.
- By ***ACI Code 9.8.1.8***, wide ***module joist ribs must be designed as ordinary T beams***, because the clear spacing between ribs exceeds the ***750mm***. maximum for joist construction, and the ***special ACI Code provisions for joists do not apply***:
  - Concrete cover for reinforcement is as required for beams, not joists,
  - The 10 percent increase in V_c.
- Often the joists in wide module systems are carried by wide beams on the column lines, the depth of which is the same as that of the joists, to form a joist-band system equivalent to that described earlier.

Figure 12.3-3: Skip joist system showing wide spacing between ribs.

12.3.13 DESIGN TABLES AND REINFORCEMENT DETAILS

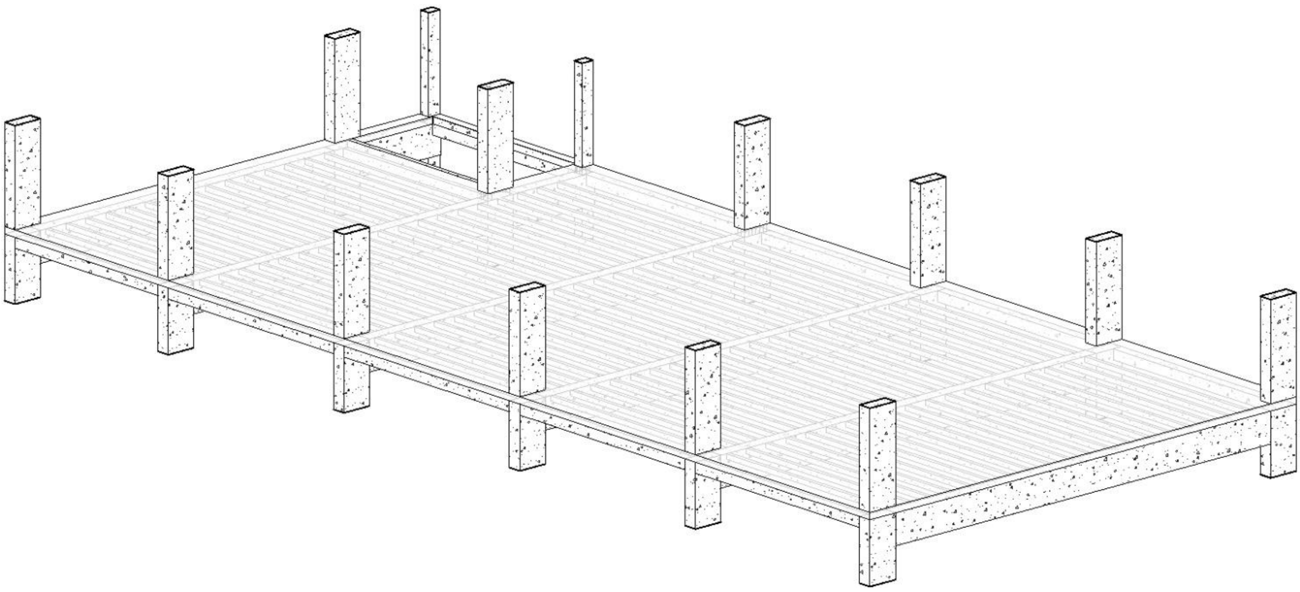
- Useful design information pertaining to one-way joist floors, including extensive load tables, will be found in the ***CRSI Design Handbook***.
- Suggested bar details and typical design drawings are found in the ***ACI Detailing Manual***.

12.3.14 DESIGN EXAMPLES

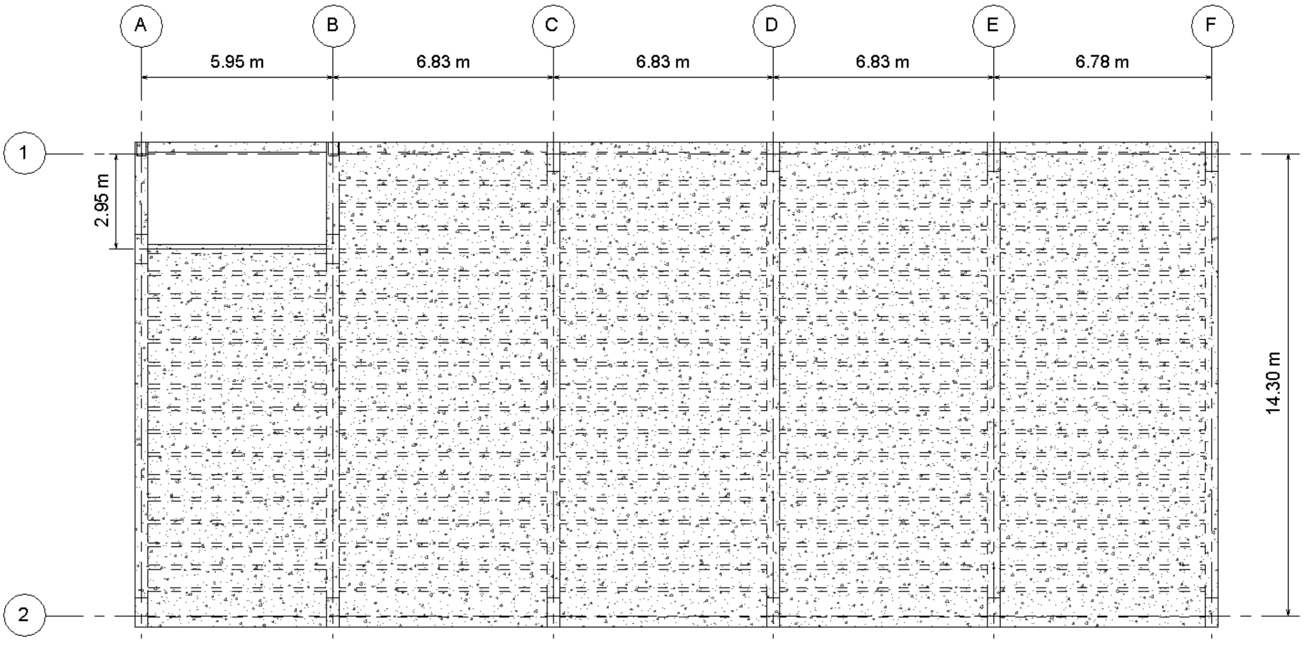
Example 12.3-1

Check the adequacy of one-way ribbed slab system indicated in Figure 12.3-4. In your checking assume that:

- $W_{Superimposed\ on\ Floors} = 2.1\ kPa$,
- $W_{Floor\ Live} = 5.3\ kPa$,
- $f_c' = 25\ MPa$ and $f_y = 420\ MPa$,
- Rib supporting girders have width of 400mm,
- Specific weight for polystyrene blocks is 0.8.

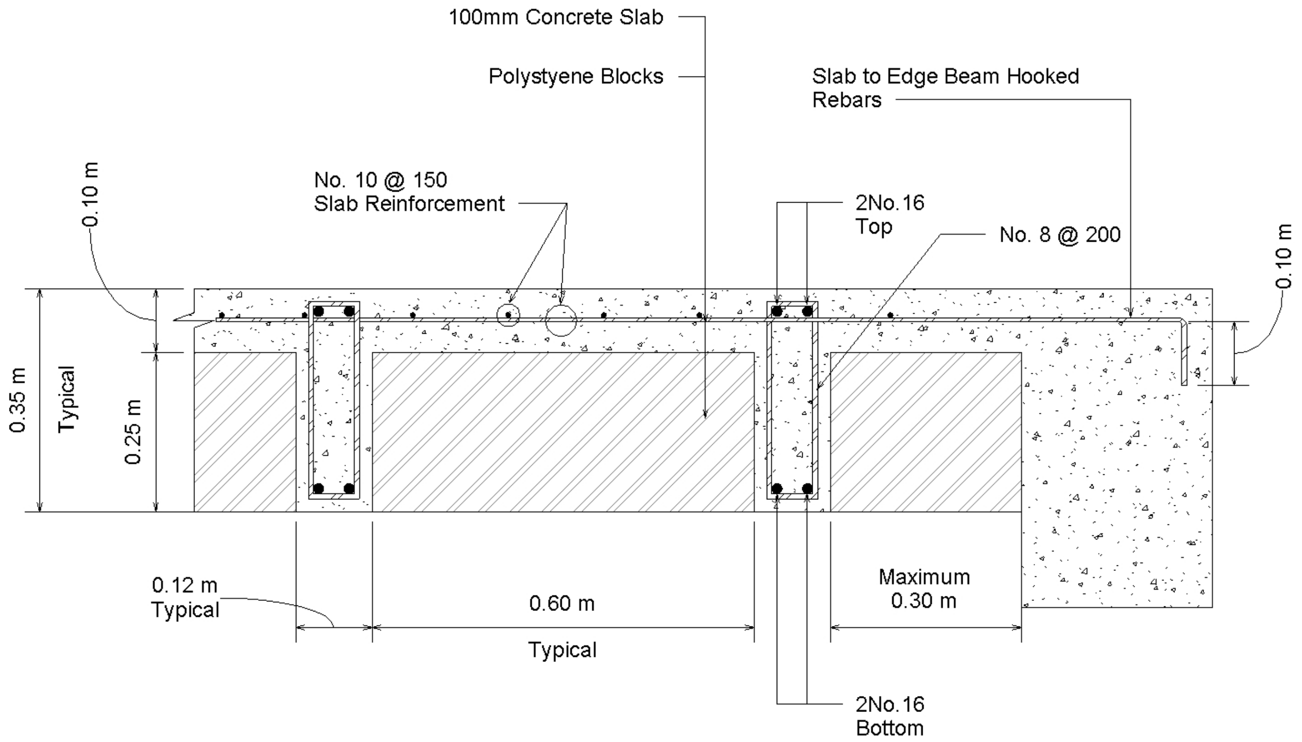


3D view



Plan View

Figure 12.3-4: One-way floor for Example 12.3-1.



Typical proposed section

Figure 12.3-4: One-way floor for Example 12.3-1. Continued.

Solution

Checking for General Code Requirements

- According to ***ACI Code 9.8.1***, ribs shall be not less than 100 mm in width, and shall have a depth of not more than 3.5 times the minimum width of rib.

  $b_w = 120mm > 100mm\ Ok.$

  $h = 350mm < 3.5 \times 120 = 420mm\ Ok.$

- According to ***ACI Code 9.8.1***, clear spacing between ribs shall not exceed 750 mm.

  $S_{proposed\ between\ ribs} = 600mm < 750mm\ Ok.$

- According to ***ACI Code 9.8.3***, the slab thickness shall be not less than one-twelfth the clear distance between ribs, nor less than 50 mm:

  $$h_f = 100mm > minimum\ \left(50mm\ or \frac{1}{12} \times (600)mm\right) \therefore Ok.$$

- Slab Reinforcement:

  Normal to the ribs shall be provided in the slab as required for flexure, considering load concentrations, if any, but not less than temperature and shrinkage reinforcement that required by ***Table 12.1-1***:

  $$A_{s\ provided} = \frac{\pi \times 10^2}{4} \times \frac{1000}{150} = 523\ mm^2\ per\ m$$

  $$A_{s\ temp.} = 0.0018 \times 100 \times 1000 = 180\ mm^2\ per\ m < A_{s\ provided} \therefore Ok.$$

Deflection Checking

- Minimum thickness for deflection control of one-way ribbed slab can be determined from Table 12.1-2 for solid one-way slabs:

  $$h_{minimum} = \frac{\ell}{28} = \frac{6780 - 400}{28} = 227 < 350 \therefore Ok$$

Flexural Checking

- Loads acting on a typical rib can be determined as follows:

  $$W_{self} = \left(0.1 \times \left(\frac{0.6}{2} \times 2 + 0.12\right) + 0.25 \times 0.12\right) \times 24 + \left(\frac{0.6}{2} \times 0.25 \times 0.8 \times 10\right) \times 2$$

  $$= 3.65 \frac{kN}{m}\ per\ rib$$

  $$W_{Superimposed} = 2.1 \times \left(\frac{0.6}{2} \times 2 + 0.12\right) = 1.51 \frac{kN}{m}\ per\ rib$$

$$W_{Live} = 5.3 \times \left(\frac{0.6}{2} \times 2 + 0.12\right) = 3.82 \frac{kN}{m}\ per\ rib$$

$$W_u = \max(1.4 \times (3.65 + 1.51), 1.2 \times (3.65 + 1.51) + 1.6 \times 3.82) = 12.3 \frac{kN}{m}\ per\ rib$$

- Factored moments for ribs:

  As rib spans are almost equals, therefore their factored moments can be determined by ACI coefficients methods. With more than two spans, the maximum negative and positive moments would be:

$$\ell_{n\ avg. for-ve\ moment} = \frac{1}{2}\left((6.83 - 0.4) + (6.78 - 0.4)\right) = 6.41\ m$$

$$M_{u-ve} = \frac{W_u \ell_{n\ avg.}^2}{10} = \frac{12.3 \times 6.41^2}{10} = 50.5\ kN.m\ per\ rib$$

$$\ell_{n\ exterior\ for+ve\ moment} = 6.78 - 0.4 = 6.38\ m$$

$$\ell_{n\ interior\ for+ve\ moment} = 6.83 - 0.4 = 6.43\ m$$

$$M_{u+ve} = \max\left(\frac{W_u \ell_{n\ ext.}^2}{14}, \frac{W_u \ell_{n\ int.}^2}{16}\right) = \max\left(\frac{12.3 \times 6.38^2}{14}, \frac{12.3 \times 6.43^2}{16}\right) = 35.8\ kN.m\ per\ rib$$

- Rib flexural strength:

  For the proposed section the positive flexural strength would be:

  Let $a < h_f$

$$a = \frac{\left(\frac{\pi \times 16^2}{4} \times 2\right) \times 420}{0.85 \times 25 \times 720} = 11.0\ mm < h_f\ \therefore Ok.$$

$$d = 350 - 20 - 8 - \frac{16}{2} \approx 310\ mm$$

$$\phi M_{n+ve} = \left(\frac{0.9 \times \left(\frac{\pi \times 16^2}{4} \times 2 \times 420\right) \times \left(310 - \frac{11.0}{2}\right)}{10^6}\right) = 46.3\ kN.m\ per\ rib > M_{u+ve}\ \therefore Ok.$$

  While negative flexural strength where the section would behave as rectangular section:

$$a = \frac{\left(\frac{\pi \times 16^2}{4} \times 2\right) \times 420}{0.85 \times 25 \times 120} = 66.2\ mm$$

$$d = 350 - 20 - 8 - \frac{16}{2} \approx 310\ mm$$

$$\phi M_{n-ve} = \left(\frac{0.9 \times \left(\frac{\pi \times 16^2}{4} \times 2 \times 420\right) \times \left(310 - \frac{66.2}{2}\right)}{10^6}\right) = 42.1\ kN.m\ per\ m < M_{u-ve}$$

$$\therefore \text{Not } Ok.$$

  Therefore, proposed top reinforcement is inadequate, Use $2No. 20$.

<u>Shear Checking</u>

- Factored Shear Forces:

  Based on ACI coefficient method, maximum factored shear force is:

$$V_u = 1.15 \frac{W_u \ell_{n\ ext}}{2} = 1.15 \times \frac{12.3 \times 6.38}{2} = 45.1\ kN\ per\ rib$$

- Shear Strength

  With neglecting of the stirrups, as bar diameter of 8mm is not supported by ASTM specification, and with increasing by 10% as permitted by ***ACI Code 9.8.1***, shear strength for ribbed slab would be as indicated in below:

$$\phi V_c = 0.75 \times \left(\frac{1.1 \times 0.17 \times \sqrt{25} \times 120 \times 310}{1000}\right) = 26.1\ kN\ per\ rib < V_u\ \therefore Not\ Ok.$$

When stirrups diameter is increased to $10mm$, it will be accepted according to American standards and design shear strength will be:

$$\phi V_n = \frac{0.75\left(\left(1.1 \times 0.17 \times \sqrt{25} \times 120 \times 310\right) + \left(\frac{\pi \times 10^2}{4} \times 2 \times 420\right) \times \frac{310}{150}\right)}{1000} = 128\ kN\ per\ rib$$

$$> V_u \therefore Ok.$$

Checking of Slab between Ribs

- As discussed previously, according to ASCE-7 in addition to uniformly distributed live loads that adopted above, slabs should be checked for possible concentrated loads with values determined based on corresponding value of the uniform loads.
- These concentrated loads are critical in checking for possible local failure as in the case of slab between ribs.
- According to ASCE-7, for floors with uniform live load in the range of ***5.3 kPa***, the corresponding concentrated live load would be in the range of ***8.90 kN*** and acting on an area of ***762mm*** by ***762mm***. The point load shall be located so as to produce the maximum load effects in the structural member.

$$W_{Live\ due\ to\ concentrated\ loads} = \frac{8.90}{0.762^2} = 15.3\ kPa$$

$$W_{self} = 0.1 \times 24 = 2.4\ kPa$$

$$W_{Superimposed\ on\ Floors} = 2.1\ kPa$$

$$W_u = 1.2(2.4 + 2.1) + 1.6 \times 15.3 \approx 30\ kPa$$

- Bending moments due to aforementioned concentrated loads can be determined in an approximated form based on ACI coefficient method for slabs with spans not exceeding 3m:

$$M = \frac{W_u l_n^2}{14} = \frac{30 \times 0.6^2}{14} = 0.77\ kN.m\ per\ m$$

- As indicated in the proposed section, the grid of $\phi 10@150$ is located at mid slab thickness, then the section strength would be:

$$d = \frac{h}{2} = \frac{100}{2} = 50\ mm$$

$$a = \frac{\frac{\pi \times 10^2}{4} \times \frac{1000}{150} \times 420}{0.85 \times 25 \times 1000} = 10.3\ mm$$

$$\phi M_n = \frac{0.9 \times \left(\frac{\pi \times 10^2}{4} \times \frac{1000}{150} \times 420\right) \times \left(50 - \frac{10.3}{2}\right)}{10^6} = 8.88\ kN.m\ per\ m \gg M_u \therefore Ok.$$

Example 12.3-2

A designer intends to use the one-way ribbed slab section of ***Example 12.3-1***, see ***Figure 12.3-4***, for the restaurant hotel floor presented in ***Figure 12.3-5***. The ribs extend in long direction normal to the supporting beams. The floor is subjected to loads presented in ***Figure 12.3-6***. As the floor layout is irregular in shape, therefore a finite element analysis [2] has been adopted to determine the factored shear force and bending moment in the ribbed slab. Finite element results that:

$V_{u\,for\,interior\,spans} = 48\; kN\; per\; m$

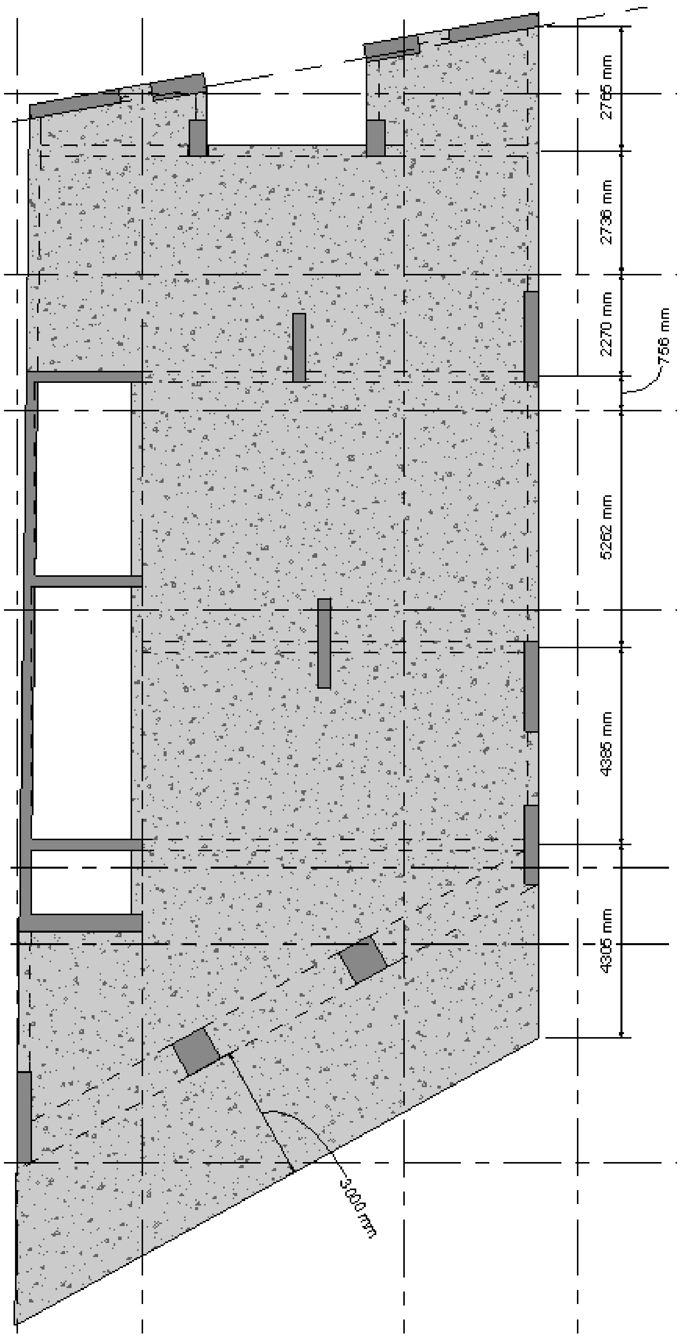
$M_{u+ve} = 30\; kN.m\; per\; m$

$M_{u-ve\,for\,interior\,spans} = 50\; kN.m\; per\; m$

For the cantilever part, the shear force and bending moment can be approximated from statics.

Check the proposed section for:

- Deflection control requirements,
- Nominal ACI requirements,
- Shear strength requirements,
- Flexural strength requirements.



Plan view.

Figure 12.3-5: One-way ribbed floor for hotel building of Example 12.3-2.

Solution

- Checking proposed thickness for deflection control

  According to Table 12.3-1, the minimum thickness for deflection control for the longest interior span is [3]:

$$h_{minimum} = \frac{\ell}{21} = \frac{5262}{21} = 250\; mm < h_{Proposed} \therefore Ok.$$

  For the cantilever span:

$$h_{minimum} = \frac{\ell}{8} = \frac{3000}{8} = 375\; mm > h_{Proposed} \therefore Not\; Ok.$$

  Hence, try to increase thickness into 380mm.

- Checking for nominal code requirements:

  The checking for nominal code requirements can be achieved as discussed in Example 12.3-1. Regrading to the increase in the rib depth:

  $h = 380mm < 3.5 \times 120 = 420mm\; Ok.$

---

[2] In commercial software, one-way ribbed slab is analyzed using orthotropic shell element where shell flexural and membrane stiffnesses are modified to simulate rib existing.

[3] Center to center span is conservatively adopted in the deflection control as there is no information about the width of the interior supporting beams. This is common in practice when the designer has no final decision regarding the supporting beam as he/she is working on the design of the ribs.

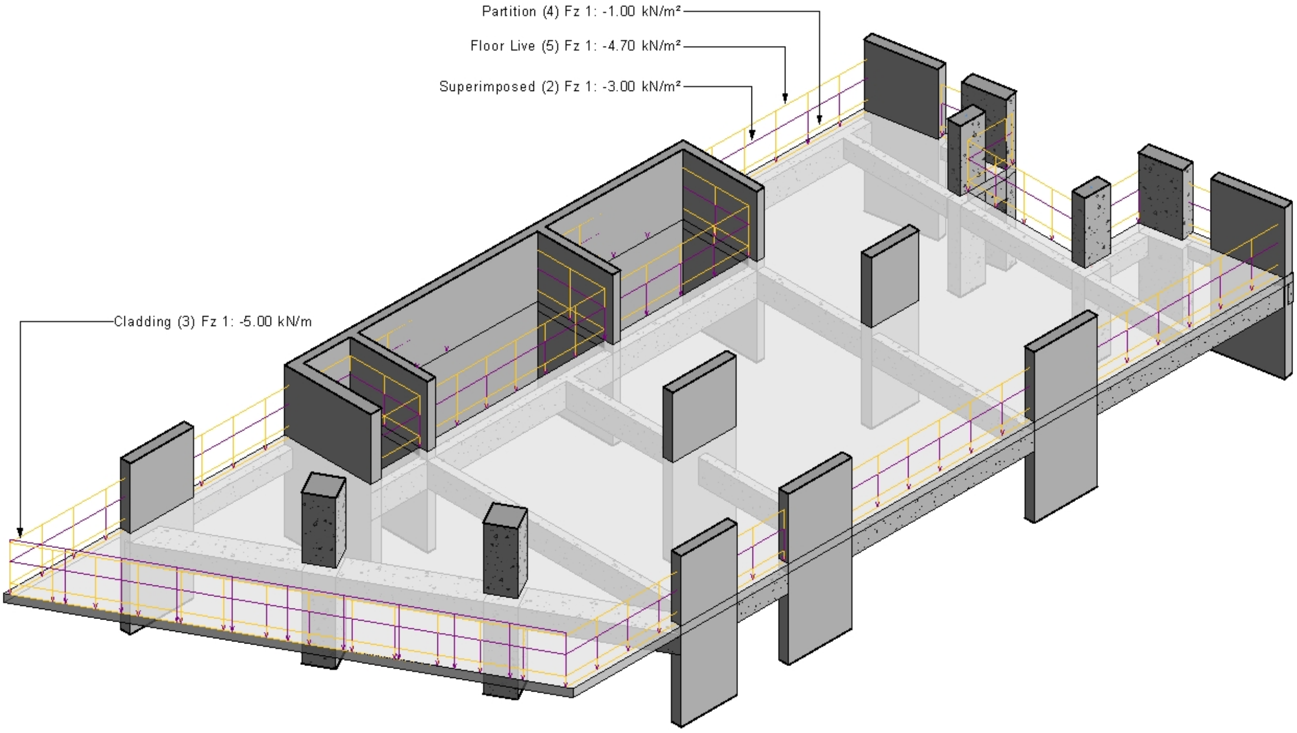


Figure 12.3-6: Loads act on the one-way ribbed floor for hotel building of Example 12.3-2.

- Computing the shear force and the negative moment at face of support for the cantilever:

  The selfweight of the ribbed slab is:

  $$W_{self} = \frac{(0.38 \times 0.12 + 0.1 \times (0.72 - 0.12)) \times 24}{0.72} = 3.52\ kPa$$

  Say:

  $$W_{self} \approx 4\ kPa$$

  to include the own weight of the polystyrene blocks.

  $$W_D = 4 + 3 = 7.0\ kPa$$

  The weight of movable partitions can conservatory considered as live load to have a total live load of:

  $$W_L = 4.7 + 1.0 = 5.7\ kPa$$

  As the cladding is relatively stationary, therefore their own weight is simulated as a dead load:

  $$P_D = 5 \frac{kN}{m}$$

  The moments at face of support are[4]:

  $$V_D = (W_D \ell + P_D) = (7 \times 3 + 5) = 26\ kN\ per\ m$$

  $$V_L = W_L \ell = 5.7 \times 3 = 17.1\ kN\ per\ m$$

  $$V_u = \max(1.4 \times 26, 1.2 \times 26 + 1.6 \times 17.1) = 58.6\ kN\ per\ m$$

  $$M_D = \frac{W_D \ell^2}{2} + P_D \ell = \frac{7 \times 3^2}{2} + 5 \times 3 = 46.5\ kN.m\ per\ m$$

  $$M_L = \frac{W_L \ell^2}{2} = \frac{5.7 \times 3^2}{2} = 25.6\ kN.m\ per\ m$$

  $$M_u = \max(1.4 \times 46.5, 1.2 \times 46.5 + 1.6 \times 25.6) = 97\ kN.m\ per\ m$$

---

[4] These relations are approximated in nature as the ribs are not normal to the supporting beam.

- Checking for shear:

  With neglecting of the stirrups, as bar diameter of 8mm is not supported by ASTM specification, and with increasing by 10% as permitted by ***ACI Code 9.8.1***, shear strength for ribbed slab would be as indicated in below:

$$d = 380 - 20 - 8 - \frac{16}{2} \approx 344\, mm$$

$$\phi V_c = 0.75 \times \left(\frac{1.1 \times 0.17 \times \sqrt{25} \times 120 \times 344}{1000}\right) = 28.9\, kN\, per\, rib = \frac{28.9}{0.72} = 40.1\, kN\, per\, m < V_u \therefore Not\, Ok.$$

  When stirrups diameter is increased to $10mm$, it will be accepted according to American standards and design shear strength will be:

$$\phi V_n = \frac{0.75\left(\left(1.1 \times 0.17 \times \sqrt{25} \times 120 \times 344\right) + \left(\frac{\pi \times 10^2}{4} \times 2 \times 420\right) \times \frac{344}{150}\right)}{1000} = 142\, kN\, per\, rib$$

$$= \frac{142}{0.72} = 197\, kN.m\, per\, m > V_u \therefore Ok.$$

- Checking for flexure:

  For the proposed section the positive flexural strength would be:

  Let $a < h_f$

$$a = \frac{\left(\frac{\pi \times 16^2}{4} \times 2\right) \times 420}{0.85 \times 25 \times 720} = 11.0\, mm < h_f \therefore Ok.$$

$$d = 344\, mm$$

$$\phi M_{n+ve} = \left(\frac{0.9 \times \left(\frac{\pi \times 16^2}{4} \times 2 \times 420\right) \times \left(344 - \frac{11.0}{2}\right)}{10^6}\right) = 51.4\, kN.m\, per\, rib = \frac{51.4}{0.72}$$

$$= 71.4\, kN.m\, per\, m > M_{u+ve} \therefore Ok.$$

  While negative flexural strength where the section would behave as rectangular section:

$$a = \frac{\left(\frac{\pi \times 16^2}{4} \times 2\right) \times 420}{0.85 \times 25 \times 120} = 66.2\, mm$$

$$d = 344\, mm$$

$$\phi M_{n-ve} = \left(\frac{0.9 \times \left(\frac{\pi \times 16^2}{4} \times 2 \times 420\right) \times \left(344 - \frac{66.2}{2}\right)}{10^6}\right) = 47.3\, kN.m\, per\, m = \frac{47.3}{0.72}$$

$$= 65.7\, \text{kN.m per m} < M_{u-ve\, for\, cantilever} \therefore \text{Not}\, Ok.$$

  Therefore proposed top reinforcement is inadequate, Use $2No.20$:

$$a = \frac{\left(\frac{\pi \times 20^2}{4} \times 2\right) \times 420}{0.85 \times 25 \times 120} = 103\, mm$$

$$\phi M_{n-ve} = \left(\frac{0.9 \times \left(\frac{\pi \times 20^2}{4} \times 2 \times 420\right) \times \left(344 - \frac{103}{2}\right)}{10^6}\right) = 69.5\, kN.m\, per\, m = \frac{69.5}{0.72}$$

$$= 96.5\, \text{kN.m per m} \approx M_{u-ve\, for\, cantilever} \therefore Ok.$$

12.4 Using STAAD Pro for Analysis and Design of Buildings with One-way Systems

12.4.1 Basic Concepts

In practical applications, STAAD Pro is usually adopted in two different methods:

- In the first methods, slabs panels are analyzed and design manually using code approximated methods, then their loads are distributed to the supporting beams using ***STAAD Floor Command***.
- In the second method, the whole structures including slab panels, beams, and As columns are simulated in the STAAD Pro. Space frame element is used to simulate beams and columns whereas shell element is used for slabs.
- First method is presented in ***Example 12.4-1*** while the second method is illustrated in ***Example 12.4-2***.
- Both methods are presented in this article with referring to one-way building of ***Example 12.2-1***.

12.4.2 Examples

Example 12.4-1

Use STAAD Pro software to analysis and design of the supporting frame, i.e. beams and columns only, for the one-way building of ***Example 12.2-1*** above. In addition to indicated loads, assume that the roof slab is subjected into a superimposed dead load of 3.5 kPa and a live load of 1.0 kPa.

Solution

This example presents a common design practice that is usually adopted by many designers, where STAAD Pro is used for analysis and design of the supporting system while traditional methods, e.g. the ACI coefficients method, is for analysis and design of the slabs. If this practice is adopted, then we should already have detailed information of the slab and we shall focus on analysis and design of the supporting beams and columns.

<u>Modeling of Building Geometry</u>

- Starting with selecting a connecting point between column and spread footing at grids A-1 to represent the original point of the STAAD model. Then using lower and upper node to draw the column located at A-1, see ***Figure 12.4-1***
- Use ***Translation Repeat***, , icon to duplicate the column at A-1 along x-axis and as indicated in ***Figure 12.4-2*** below.
- Use ***Translation Repeat***, , command again to duplicate the columns along y-axis. The columns for first floor would be as indicated in ***Figure 12.4-3***.
- Use ***Add Beam*** icon, , to draw the beam along ***Gridline 1*** and between ***Gridlines A-B*** and
- Then use ***Translation Repeat***, , icon two times, one to duplicate the beam along x-axis and the other to duplicate the beams along z-axis.
- Finally use The main beam would be as indicated in ***Add Beam*** icon, , to draw secondary beams along ***gridlines A*** and ***D***. The beams for first floor would be as indicated in ***Figure 12.4-4***.

Figure 12.4-1: Column located at A-1.

- Use ***Translation Repeat***, , icon to generate beams and columns for the roof from those of the first floor, see ***Figure 12.4-5***.

- With using ***Translation Repeat***, icon, columns for the roof would have a height of 3.9m. As the actual height of these columns is 3.3m only, hence nodes for roof should be moved by 0.6m in downward directions. This can be achieved though using ***Move*** command as indicated in ***Figure 12.4-6***.

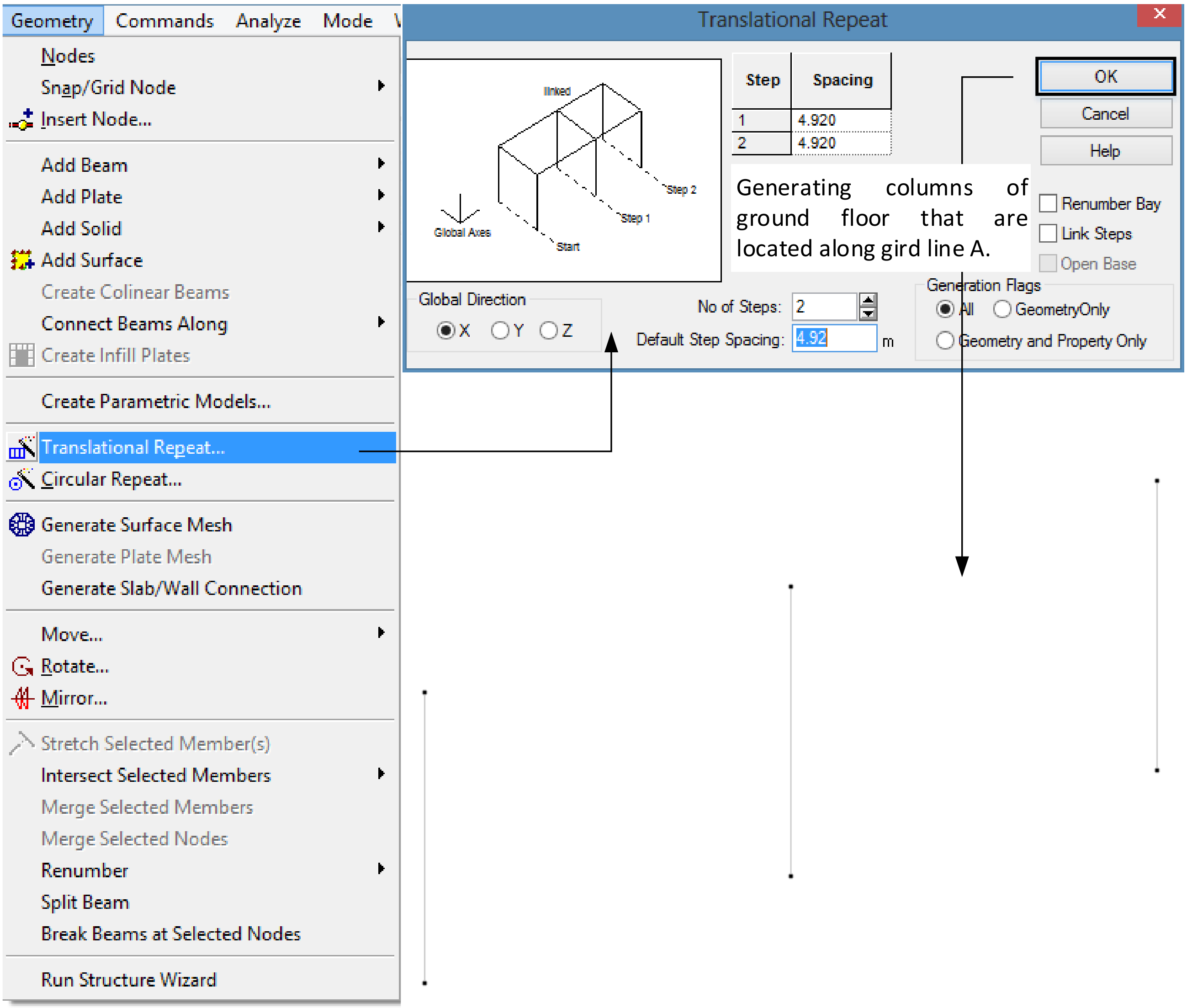


Figure 12.4-2: Using Translation Repeat command to generate the columns for ground floor located along gridline A.

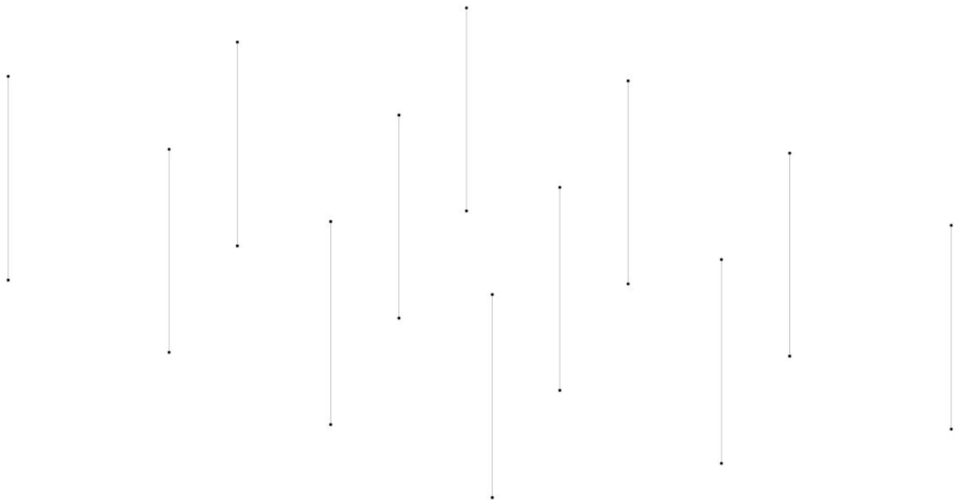


Figure 12.4-3: First floor columns in STAAD model.

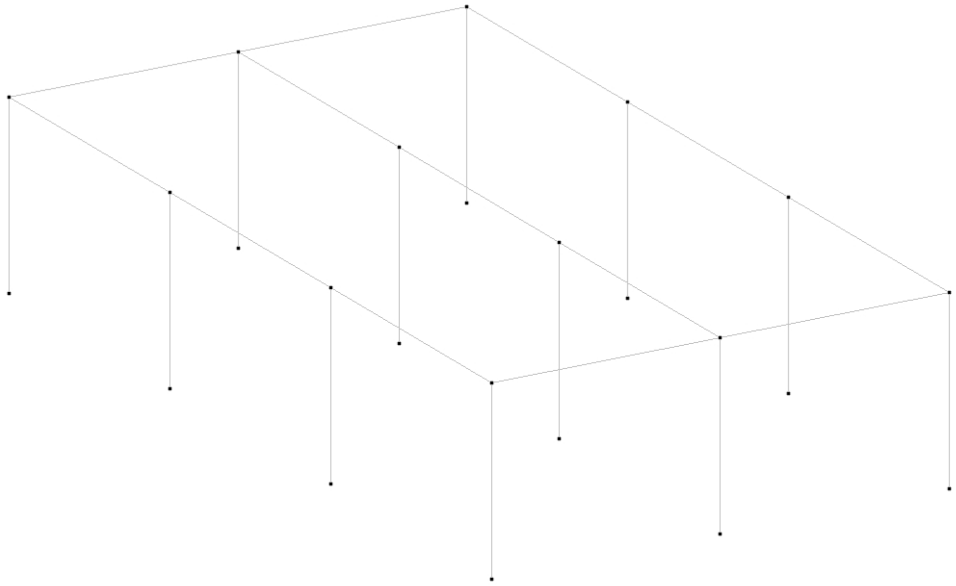


Figure 12.4-4: First floor columns and beams in STAAD model.

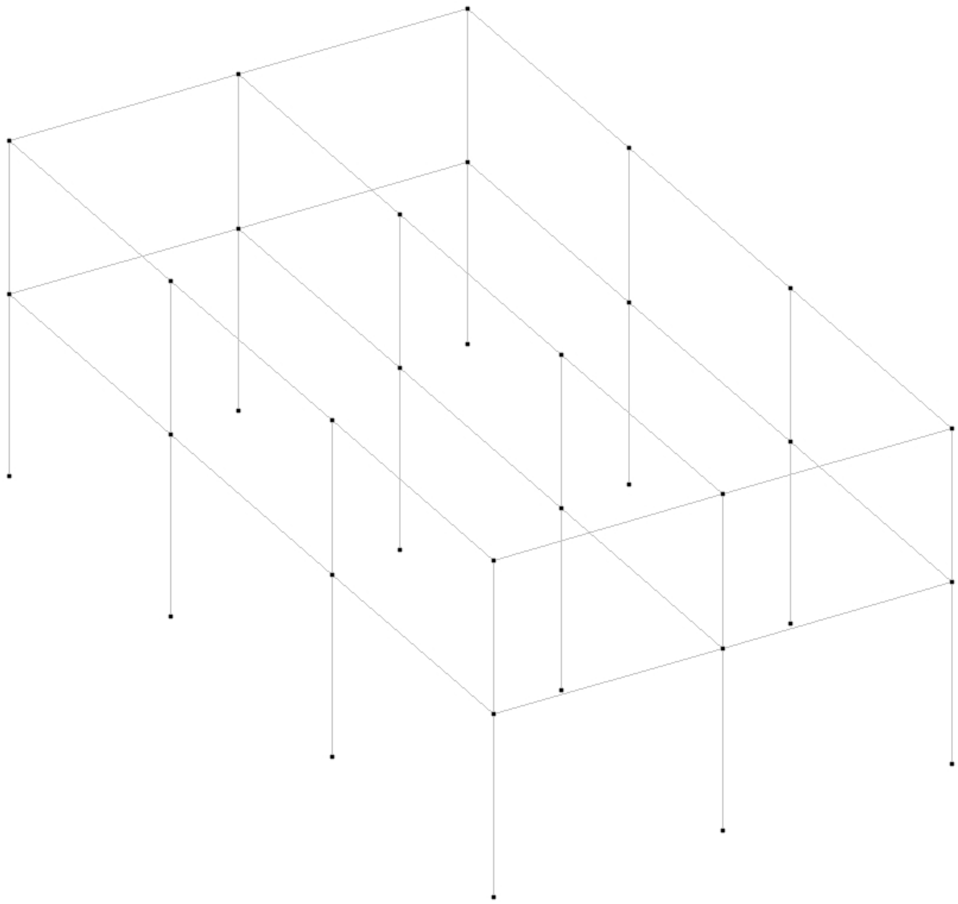


Figure 12.4-5: Beams and columns in STAAD model.

Definition and Assignment of Sections for Beams and Columns

- Assuming a depth of 600mm for beams, beams and columns sections can be defined and assigned as discussed in previous chapters.
- During assignment of beams section, they can be isolated from whole structure using steps presented in ***Figure 12.4-7***. When assign columns section, they can be isolated using same procedure.
- As indicated in the rendering view of ***Figure 12.4-8*** STAAD default ***Beta Angle*** leads to columns orientations other than the correct one where the columns depth should be along the main beams.

When a ***Beta Angle*** of ***90^o*** is assigned to columns their orientation will be corrected to that indicated in ***Figure 12.4-9***.

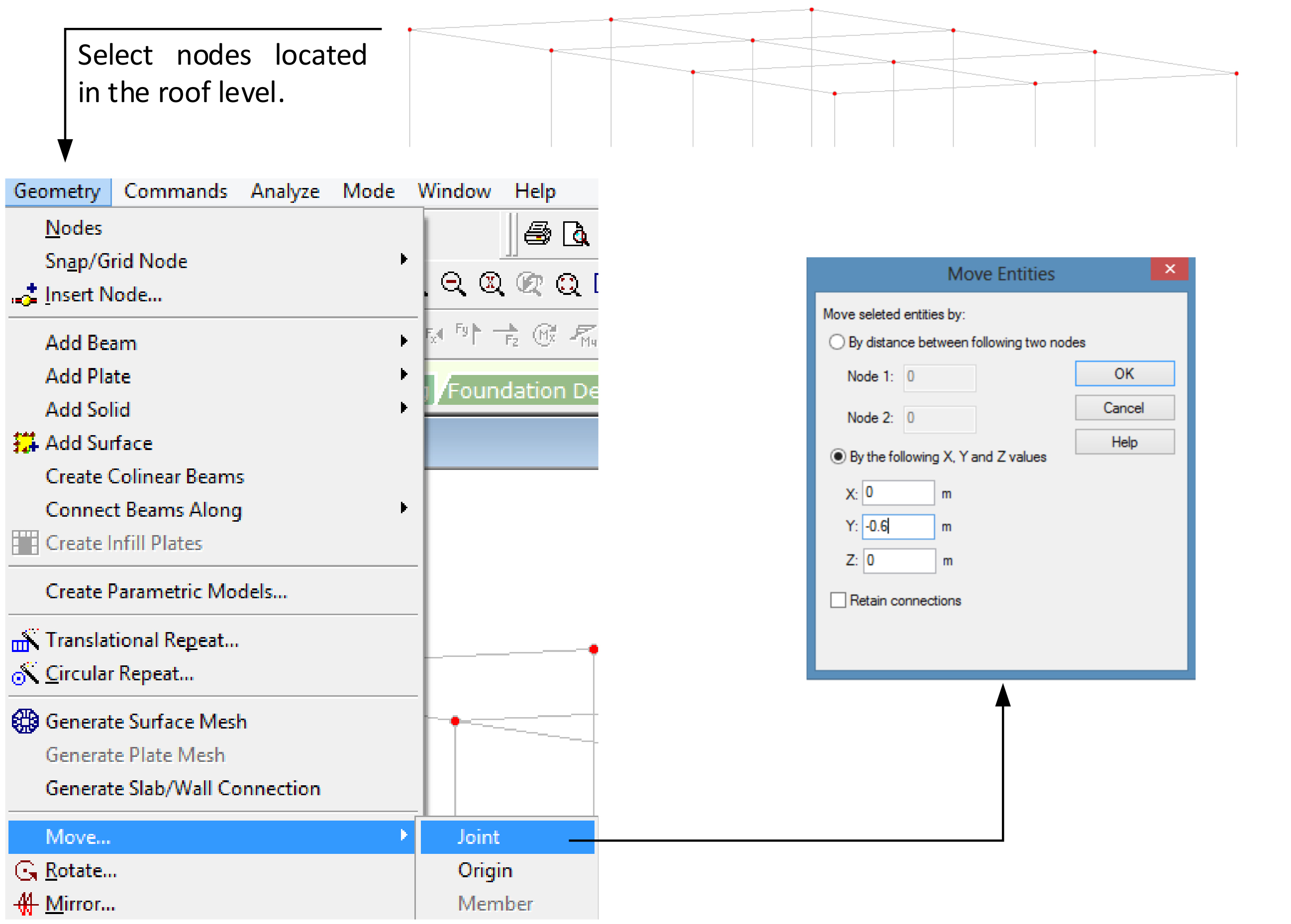


Figure 12.4-6: Use Move command to reduce height for roof columns from 3.9m into 3.3m.

Supports Modeling

- It may be suitable to use shell element for footings and frame element for the tie beams. Moreover, for this introductory example, hinges are used for approximate simulation of the foundation systems.
- Using hinges leads to a conservative columns design where column moment is concentrated in its top, instead of its distribution along column height as in the actual physical behavior.
- Hinges can be defined and assigned as discussed in previous chapters. With hinge supports, the structure would be as indicated in ***Figure 12.4-10***.

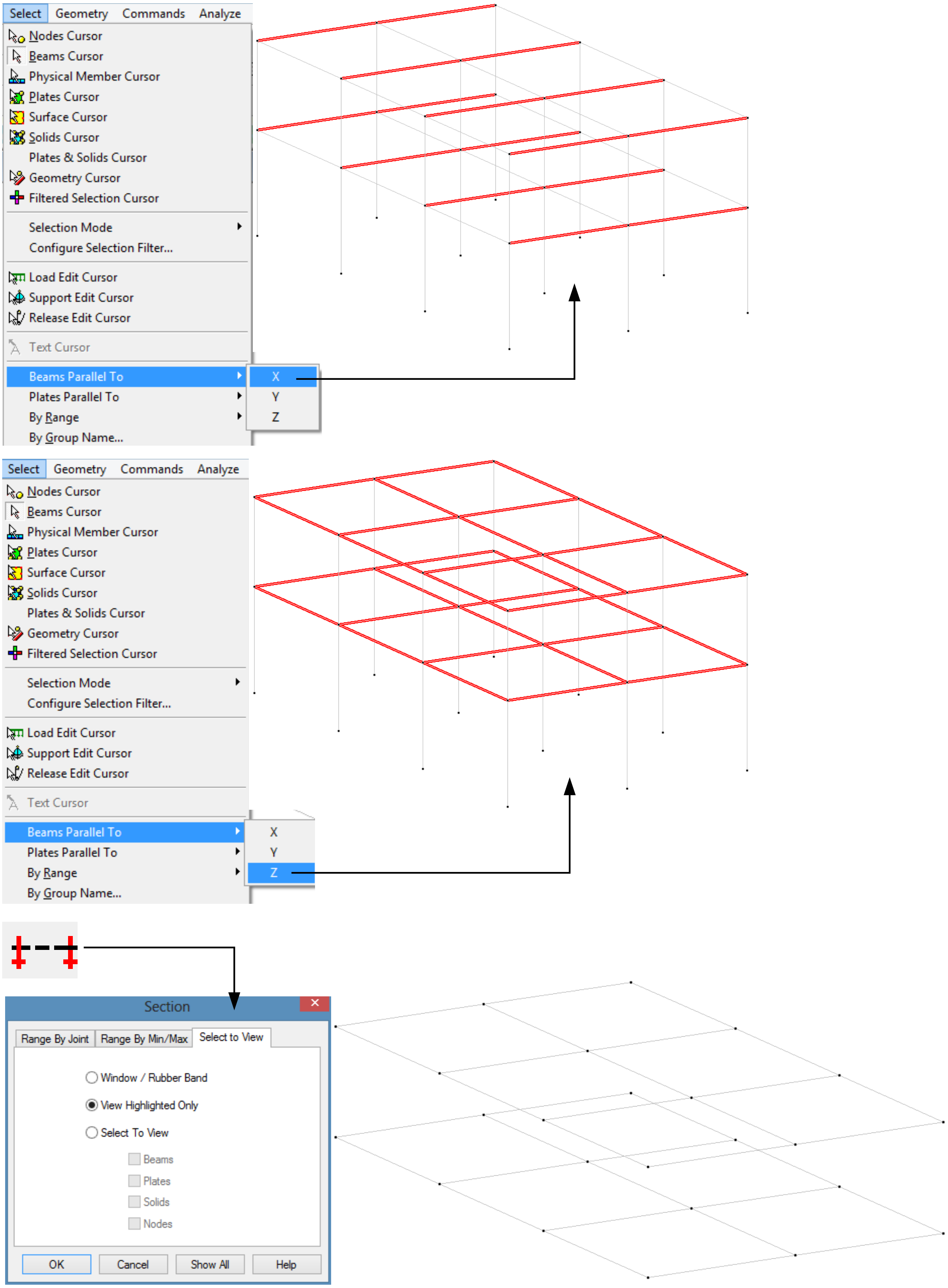


Figure 12.4-7: Procedure to isolate beams from the whole model in STAAD environment.

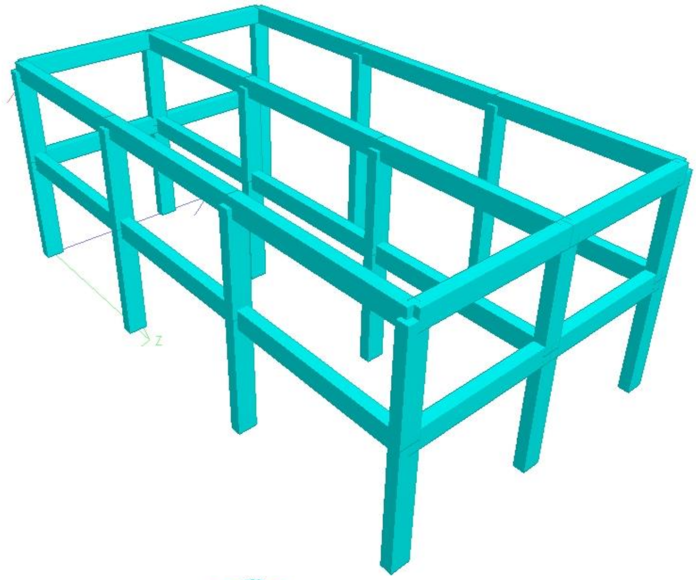


Figure 12.4-8: Render view according STAAD default Beta Angle.

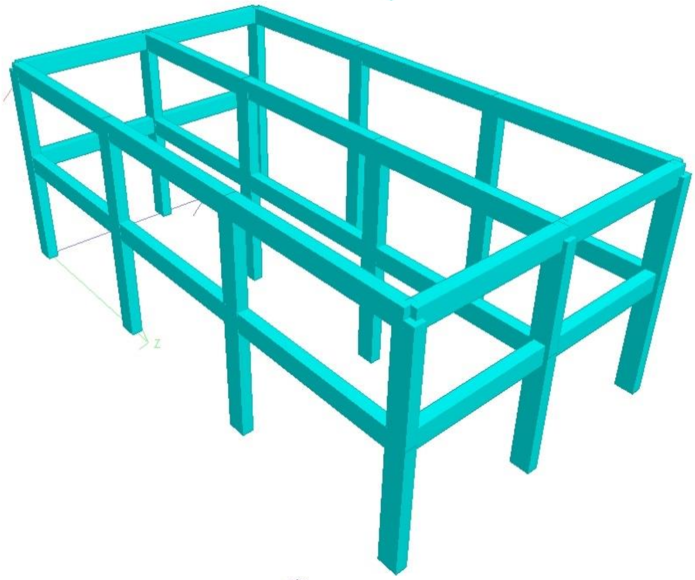


Figure 12.4-9: Render view when a Beta Angle of 90º is assigned to columns.

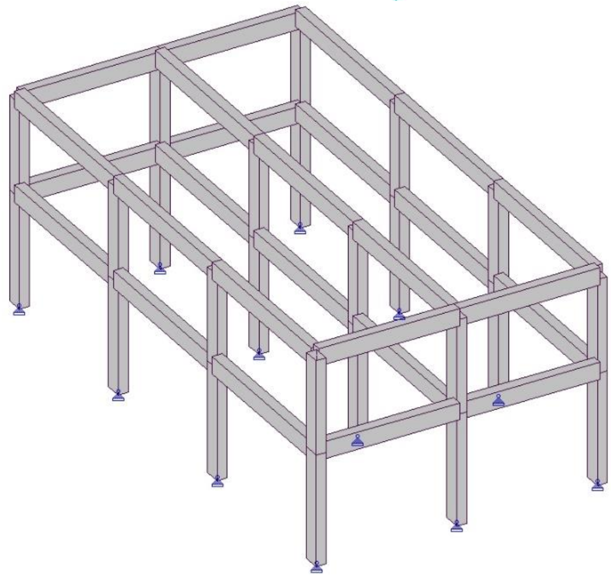


Figure 12.4-10: Structure with assigned hinge supports.

Definition of Basic Load Combinations

Figure 12.4-11: Basic load cases adopted in STAAD model.

- Basic load cases for dead, live, and roof live loads can be defined in same way discussed previously.
- To obtain a more readable model, it is a good practice to separate dead load into two cases namely ***Selfweight Case*** and ***Superimposed Case*** as indicated in ***Figure 12.4-11***.
- As discussed in ***Chapter 1***, American codes and specifications diagnose between live load acting on floors and those acting on roof(s).
- Selfweight for beams and columns can be determined automatically by STAAD software based on proposed sections and material density. However, as no slab element is adopted in the model, therefore its selfweight does not include.
- Assuming a slab thickness of 175mm, slab own weight would be:

  $W_{selfwt.of\ slab} = 0.175 \times 24 = 4.2\ kPa$
- This load can be applied through ***Floor command*** as indicated in ***Figure 12.4-12***. When it is subjected to the Floor loads, the structure would be as indicated in ***Figure 12.4-13***.

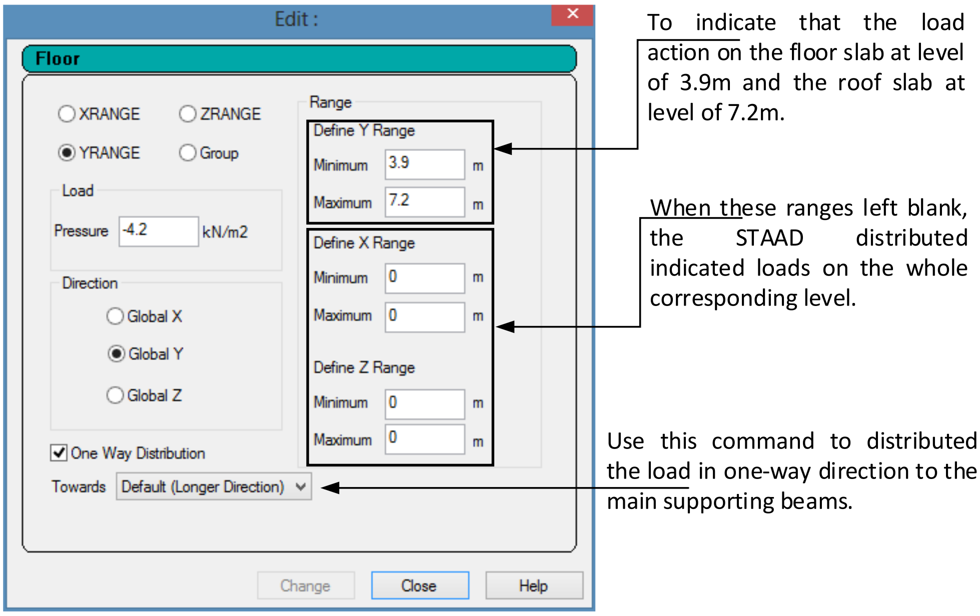


Figure 12.4-12: Use Floor command to distribute slab own weight between the supporting main beams.

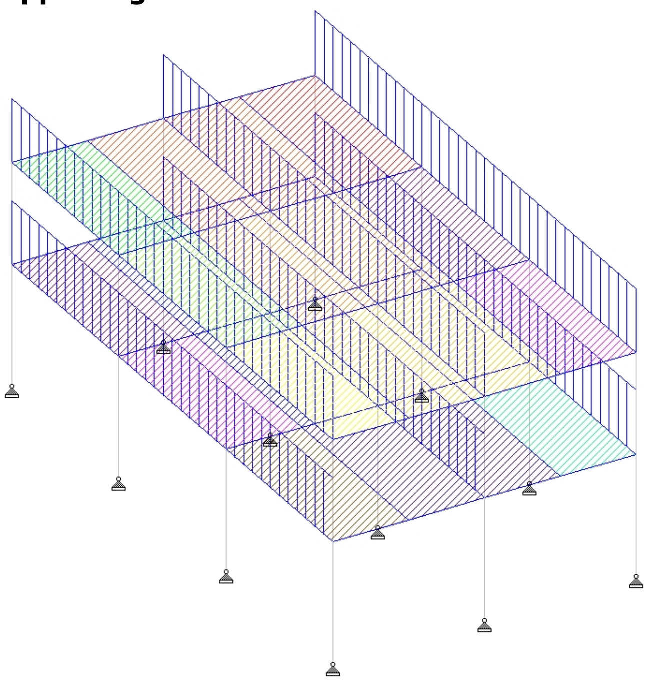


Figure 12.4-13: Structure with assigned Floor loads to simulate slabs own weight.

- In same approach, superimposed dead, floor live, and roof live loads can be applied using Floor command. It is useful to note that selfweight of partitions supported by floor is simulated as line loads with value of:

  $$W_{\text{self wt.of partition}} = (2.8 \times 0.25 \times 19) = 13.3\frac{\text{kN}}{\text{m}}$$

  and imposed to the supporting beams.
- Load combinations can be generated automatically by STAAD software as discussed in previous chapter.

```
LOAD 1 LOADTYPE Dead  TITLE SELFWEIGHT
SELFWEIGHT Y -1
ONEWAY LOAD
YRANGE 3.9 7.2 ONE -4.2 GY
LOAD 2 LOADTYPE Dead  TITLE SUPERIMPOSED DEAD
ONEWAY LOAD
YRANGE 3.9 3.9 ONE -2.5 GY
YRANGE 7.2 7.2 ONE -3.5 GY
MEMBER LOAD
13 TO 23 28 29 UNI GY -13.3
LOAD 3 LOADTYPE Live  TITLE FLOOR LIVE
ONEWAY LOAD
YRANGE 3.9 3.9 ONE -4.8 GY
LOAD 4 LOADTYPE Roof Live  TITLE ROOF LIVE
ONEWAY LOAD
YRANGE 7.2 7.2 ONE -1 GY
```

Basic load case

```
LOAD COMB 5 Generated ACI Table1 1
1 1.4 2 1.4
LOAD COMB 6 Generated ACI Table1 2
1 1.2 2 1.2 3 1.6 4 0.5
LOAD COMB 7 Generated ACI Table1 3
1 1.2 2 1.2 3 1.6
LOAD COMB 8 Generated ACI Table1 4
1 1.2 2 1.2 3 1.0 4 1.6
LOAD COMB 9 Generated ACI Table1 5
1 1.2 2 1.2 4 1.6
LOAD COMB 10 Generated ACI Table1 6
1 1.2 2 1.2 3 1.0
LOAD COMB 11 Generated ACI Table1 7
1 1.2 2 1.2
LOAD COMB 12 Generated ACI Table1 8
1 1.2 2 1.2 3 1.0 4 0.5
LOAD COMB 13 Generated ACI Table1 9
1 0.9 2 0.9
```

Automatically generated load combinations.

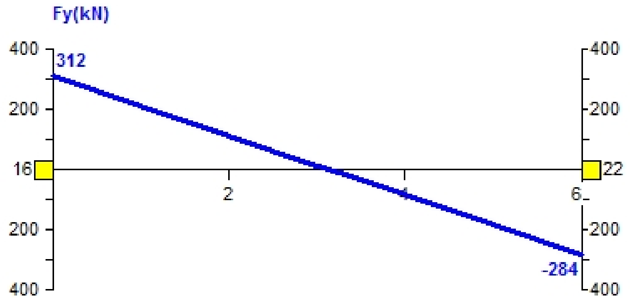
Figure 12.4-14: Basic load cases and load combinations adopted in STAAD model.

Analysis

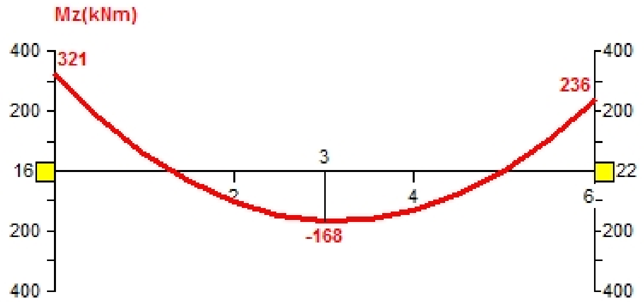
Perform Analysis command can be added and analysis and it can be executed in same approach that adopted in the previous chapters.

Factored Moment, Shear Forces for Exterior Span Beam A

- Factored shear force and bending moment diagrams from ***LOAD COMB 6*** are drawn with STAAD post processing facilities and presented in ***Figure 12.4-15***.



Shear force.



Bending moment.

Figure 12.4-15: Shear force and bending moment diagrams for Beam A from STAAD analysis.

- Comparison with following factored moments and shear forces of ACI coefficients method:

$$M_{u\,Exterior\,Negative} = \frac{W_u l_n^2}{16} = \frac{104 \times 5.4^2}{16} = 190\ kN.m$$

$$M_{u\,Positive} = \frac{W_u l_n^2}{14} = \frac{104 \times 5.4^2}{14} = 217\ kN.m$$

$$M_{u\,Interior\,Negative} = \frac{W_u l_n^2}{10} = \frac{104 \times 5.4^2}{10} = 303\ kN.m$$

$$V_u = 1.15\frac{W_u l_n}{2} = 1.15 \times \frac{104\frac{kN}{m} \times 5.4m}{2} = 323\ kN$$

Aforementioned results indicate that positive moment from ACI coefficients method, $217\ kN.m$, is significantly greater than those determined from STAAD analysis, $168\ kN.m$. This difference may be partially due to load patterns that have been considered in ACI coefficients method while they are neglected in STAAD analysis.

As shear forces and negative moments are determined in terms of clear span, ℓ_n, in ACI coefficients method, therefore they are implicitly measured at face of supports and cannot be compared directly with STAAD results that are determined at centerline of columns. To transform STAAD forces to face of support, double click on target beam to have the interactive box indicated in ***Figure 12.4-16***, from Shear Bending ribbon enter one-half of support width, $0.6/2 = 0.3\text{m}$, and select suitable load combination, ***LOAD COMB 6***, in this to have shear force and bending moment at face of support:

$$V_{u\,@\,face\,of\,support\,from\,STAAD} \approx 282\ kN$$

$$M_{u-ve\,@\,face\,of\,support\,from\,STAAD} \approx 234\ kN.m$$

These results indicate that even with transformation to face of supports, ACI values still relative conservative and significant saving can be achieved through adopted analytical values from STAAD analysis.

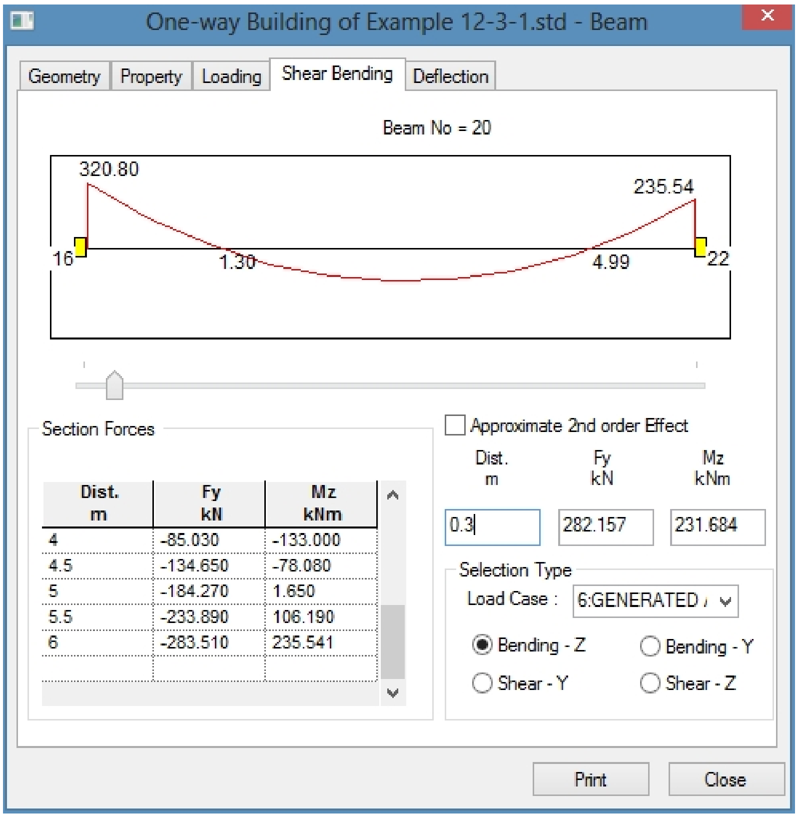
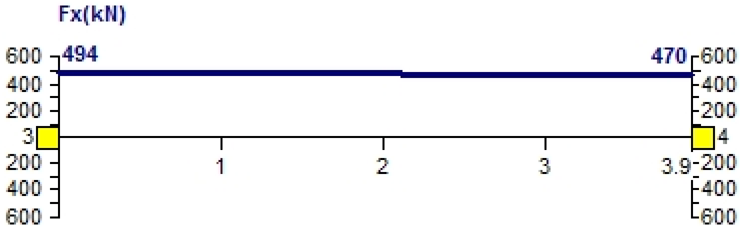
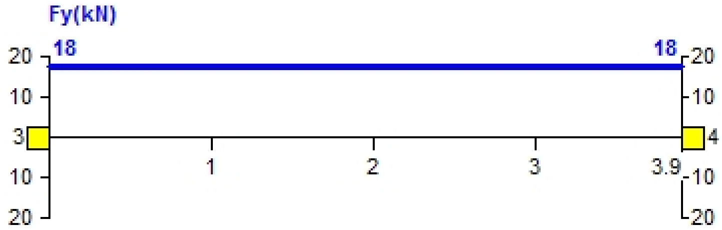


Figure 12.4-16: STAAD interactive box to transform shear force and negative bending moment from column centerline to column support.

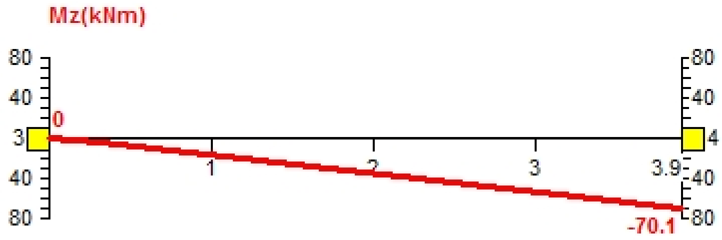
- Factored forces axial force, shear force, and bending moment for an exterior column located at gridline A-2 or D-2 are presented in ***Figure 12.4-17***.



Axial force.



Shear force.



Bending moment.

Figure 12.4-17: Axial force, shear force, and bending moment diagrams for exterior columns located at gridlines of A-2 or D-2.

Contents

CHAPTER 13

ANALYSIS AND DESIGN OF TWO-WAY SLABS

13.1 BASIC CONCEPTS

13.1.1 DEFINITION

Two-way slab is a slab that ***has curvatures in both directions*** and then ***has bending moments in both directions***.

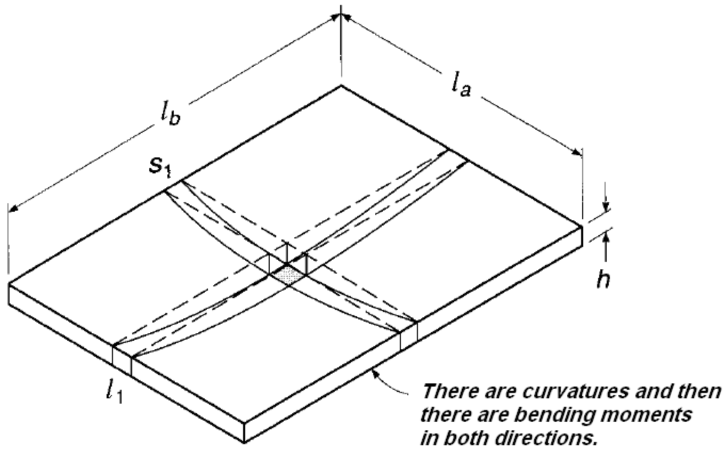


Figure 13.1-1: Curvatures of a two-way system.

13.1.2 TYPES OF TWO-WAY SYSTEMS

Based on analysis methods, and how they interact with supporting beams, if any, two-way systems can be classified into different types and as indicated in below.

13.1.2.1 Edge-Supported Slabs

- A concrete slab supported on four sides either by stiff[1] beams or by walls and has a length to width ratio equal to or less than 2 ($\frac{Length}{Width} \leq 2.$) is classified as Edge Supported Two-way Slab, see ***Figure 13.1-2***.

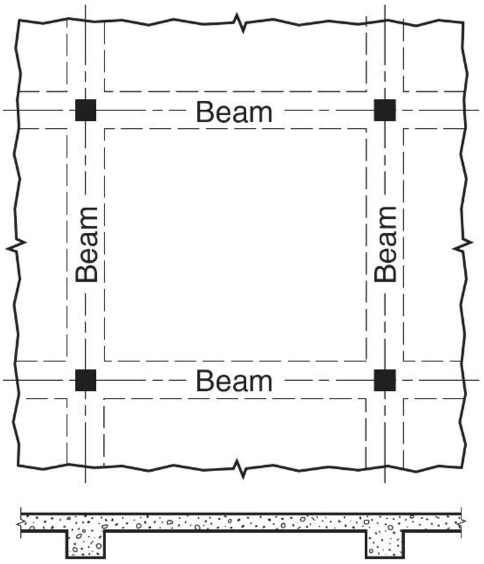


Figure 13.1-2: Edge supported two-way slab system.

[1] Slab supported on flexible beams behaves similar to beamless slab systems (flat plate and flat slab systems).

- According to (Dunham, 1966) the waffle slab with beams shown in ***Figure 13.1-3*** below can be classified as an edge supported slab.

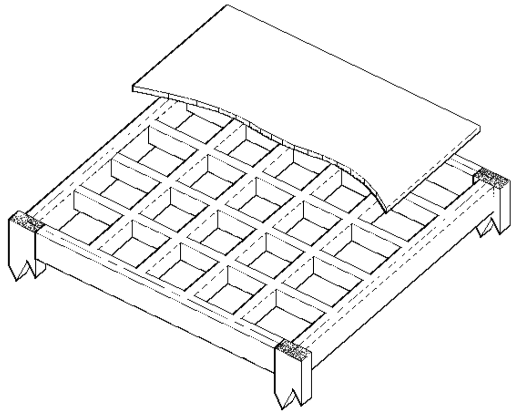


Figure 13.1-3: Edge supported waffle slab system.

13.1.2.2 Column Supported Slabs

- A concrete slab supported directly on columns and without supporting beams will deflect as indicated in ***Figure 13.1-4*** below.

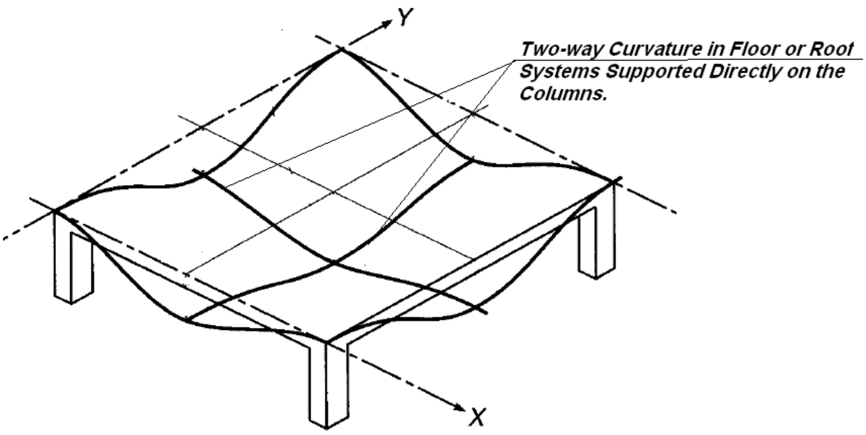


Figure 13.1-4: Deflected shape for column supported slab.

- As there are curvatures in both directions, then the slab is classified as two-way system.
- Column supported slabs may be subdivide into:
  - Flat plate slabs indicated in ***Figure 13.1-5*** which are commonly used where ***spans are not large*** and ***loads not particularly heavy***.
  - Flat slab system indicated in ***Figure 13.1-6*** is also beamless but incorporates a ***thickened slab region in the vicinity of the column*** and often employs ***flared column tops***. They are referred to as ***drop panels*** and ***column capitals***, respectively. Both devices increase the shear capacity around columns. Drop panels increase bending capacity, as well. Shear caps, a smaller version of drop panels, can be used to increase shear but not bending capacity.
  - Waffle slab without seams indicated in ***Figure 13.1-7*** is a closely related to the flat plate slab. It is also known as the ***two-way joist slab***. To reduce the ***dead load of solid-slab construction***, voids are formed in a rectilinear pattern through use of metal or fiberglass form inserts. Usually form inserts are omitted near the columns, so a solid slab is available to resist the higher moments and shears in these areas. According to "***Structural Concrete: Theory and Design, by M. N. Hassoun and A. AlManasser, Page 641***", waffle slab shown below could be analyzed and designed as a solid flat plate slab.

13.1.2.3 Slab on Grades

- In addition to the column-supported types of construction shown in above, many slabs are ***supported continuously on the ground***, as for ***highways***, ***airport runways***, and ***warehouse floors***.
- In such cases, a well-compacted layer of crushed stone or gravel is usually provided to ensure uniform support and to allow for proper subgrade drainage, see ***Figure 13.1-8***.

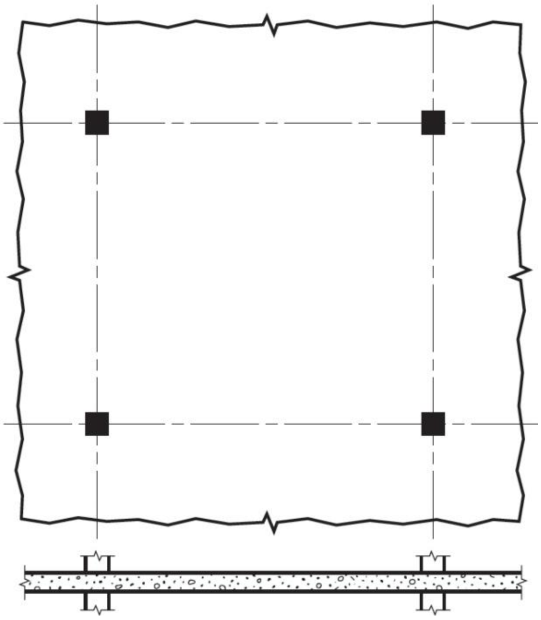


Figure 13.1-5: Flat plat system.

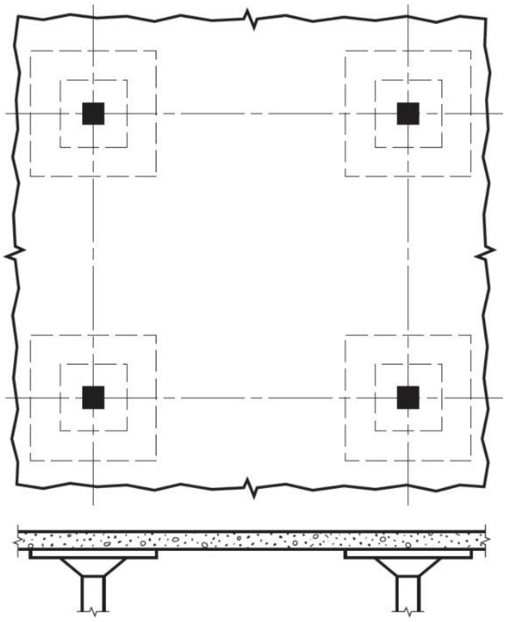


Figure 13.1-6: Flat slab system.

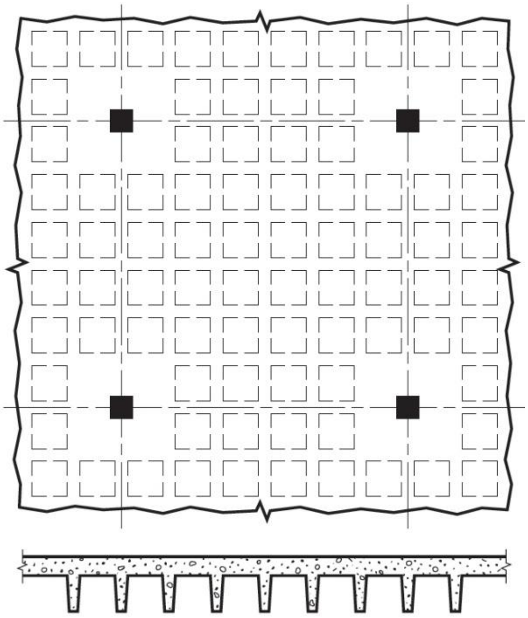


Figure 13.1-7: Column supported waffle slab.

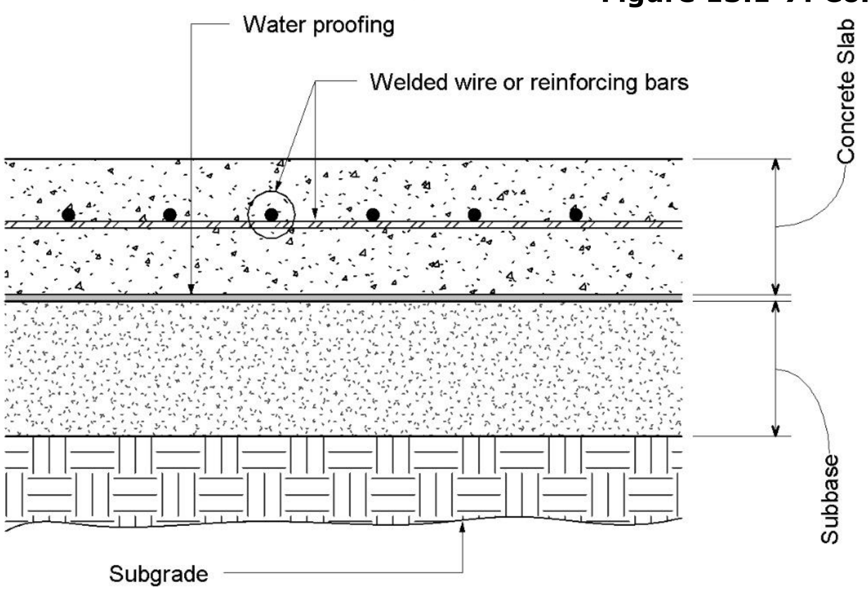


Figure 13.1-8: Typical slab on grade.

13.1.3 ECONOMICAL CHOICE OF CONCRETE FLOOR SYSTEM

- The choice of an adequate and economic floor system depends on:
  - The type of building (as the values of applied loads depend on the type of building).
  - Architectural layout.
  - Aesthetic features.
  - Span length between columns.
- Based on past experience and theoretical analysis, a general guide for the economical use of floor system is presented in ***Table 13.1-1***.

Table 13.1-1: Guide to select type of floor system.

Type of Floor System		Span (m)	Live Loads ($\frac{kN}{m^2}$)
1.	One-way slabs on beams.	3-6	4-5
2.	Two-way slab on beams.	6-9	3-6
3.	Flat slab.	6-9	4-7
4.	Flat plate.	6-8	3-5
5.	Waffle slab.	6-9	4-6

13.1.4 SCOPE

Junior course or third year course includes only edge supported two-way systems. Other systems are discussed in the senior course (fourth year course).

13.2 ANALYSIS AND DESIGN OF EDGE-SUPPORTED SLABS

13.2.1 BEHAVIOR OF TWO-WAY EDGE-SUPPORTED SLABS

- As discussed in ***Chapter 12***, an edge-supported rectangular slab that has length per width ratio larger than two deforms under load into an approximately cylindrical surface with neglected two-way action.
- The main structural action is one-way in such cases, in the direction normal to supports that relatively closed to each other.
- In many cases, however, rectangular slabs are of such proportions that length to width ratio is equal or less than two. When loaded, such slabs ***bend into a dished surface rather than a cylindrical one***. This means that at any point the slab is ***curved in both principal directions***, and since ***bending moments are proportional to curvatures, moments also exist in both directions***.
- To resist these moments, the slab must be reinforced in both directions, by at least two layers of bars perpendicular, respectively, to two pairs of edges. The slab must be designed to take a proportionate share of the load in each direction.
- Deflected shape for an edge-supported two-way slab is shown in ***Figure 13.2-1a***.

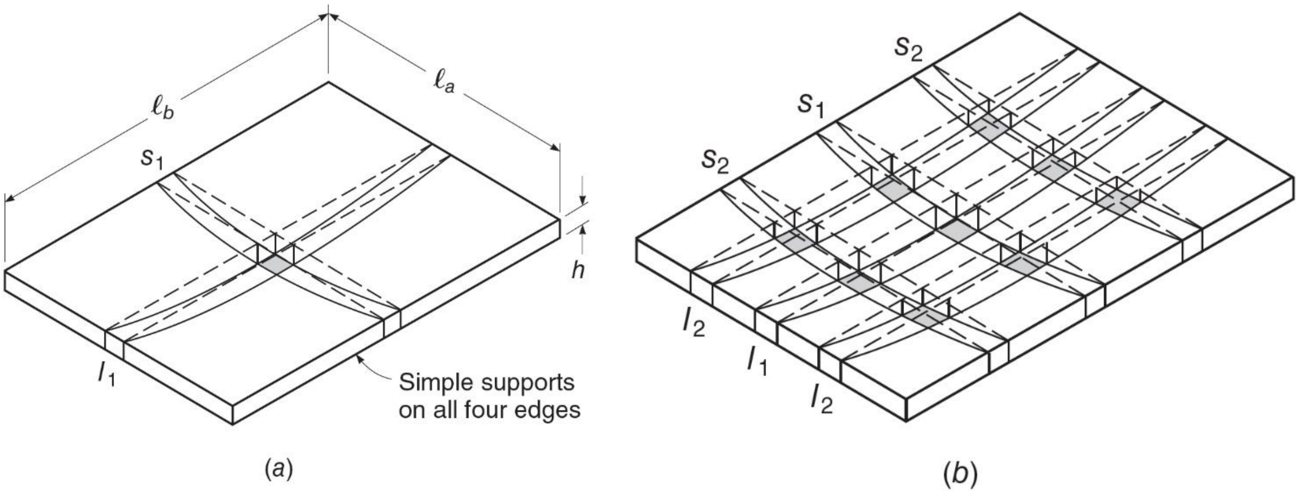


Figure 13.2-1: Two-way slab on simple edge supports: (a) bending of center strips of slab and (b) grid model of slab.

- Approximated load shares in each direction:
  - To visualize its flexural performance, it is ***convenient to think of it as consisting of two sets of parallel strips***, in each of the ***two directions***, intersecting each other. Evidently, part of the load is carried by one set and transmitted to one pair of edge supports, and the remainder by the other.
  - ***Figure 13.1a*** shows the two center strips of a rectangular plate with short span ℓ_a and long span ℓ_b. If the uniform load is q per square meter of slab, each of the two strips acts approximately as a simple beam, uniformly loaded by its share of q.
  - Because these imaginary strips actually are part of the same monolithic slab, ***their deflections at the intersection point must be the same***. Equating the center deflections of the short and long strips gives:
  $$\frac{5q_a\ell_a^2}{384EI}=\frac{5q_b\ell_b^2}{384EI} \quad \textbf{Eq. 13.2-1}$$
  where q_a is the share of the load q carried in the short direction and q_b is the share of the load q carried in the long direction.
  - Solve ***Eq. 13.2-1*** for q_a/q_b:
  $$\frac{q_a}{q_b}=\frac{\ell_b^4}{\ell_a^4} \quad \textbf{Eq. 13.2-2}$$
  - From ***Eq. 13.2-2***, one sees that the ***larger share of the load is carried in the short direction***, the ***ratio of the two portions of the total load being inversely proportional to the fourth power of the ratio of the spans***.

- This result is ***approximate because the actual behavior of a slab is more complex than that of the two intersecting strips***.
- An understanding of the behavior of the slab itself can be gained from ***Figure 13.2-1b***, which shows a slab model consisting of two sets of three strips each. It is seen that the two central strips s_1 and ℓ_1 bend in a manner similar to that shown in ***Figure 13.2-1a***. The outer strips s_2 and ℓ_2 , however, are ***not only bent but also twisted***.
- This twisting results in torsional stresses and torsional moments that are seen to be most pronounced near the corners. Consequently, ***the total load on the slab is carried not only by the bending moments in two directions but also by the twisting moments***. For this reason, bending moments in elastic slabs are smaller than would be computed for sets of unconnected strips loaded by q_a and q_b.
- An example for approximation order:

  For instance, for a ***simply supported square slab***:

  $q_a = q_b$ **Eq. 13.2-3**

  If only bending were present, the maximum moment in each strip would be:

  $$M_a = M_b = \frac{\left(\frac{q}{2}\right)\ell^2}{8} = 0.0625q\ell^2$$ **Eq. 13.2-4**

  The exact theory of bending of elastic plates shows that actually the maximum moment in such a square slab is only:

  $$M_{a\ exact} = M_{b\ exact} = 0.048q\ell^2$$ **Eq. 13.2-5**

  so that in this case the ***twisting moments relieve the bending moments by about 25 percent***.
- Inelastic Redistribution of Moments:
  - The largest moment occurs where the curvature is sharpest. ***Figure 13.2-1b*** shows this to be the case at midspan of the short strip s_1.
  - Suppose the load is increased until this location is overstressed, so that the steel at the middle of strip s_1 is yielding. ***If the strip were an isolated beam, it would now fail***.
  - Considering the slab as a whole, however, one sees that no immediate failure will occur. The neighboring strips (those parallel as well as those perpendicular to s_1), being actually monolithic with it, will take over any additional load that strip s_1 can no longer carry until they, in turn, start yielding.
  - ***This inelastic redistribution will continue until in a rather large area in the central portion of the slab all the steel in both directions is yielding***. Only ***then will the entire slab fail***.
  - From this reasoning, which is ***confirmed by tests***, it follows that ***slabs need not be designed for the absolute maximum moment in each of the two directions*** (such as $0.048q\ell^2$ of ***Eq. 13.2-5***), but only for a smaller average moment in each of the two directions in the central portion of the slab.
  - For instance, ***one of the several analytical methods in general use permits a square slab to be designed for a moment of*** $0.036q\ell^2$.
  - By comparison with the actual elastic maximum moment $0.048q\ell^2$, ***it is seen that, owing to inelastic redistribution, a moment reduction of 25 percent is provided***.
- Moments' variation and the concept of middle and column strip:
  - The largest moment in the slab occurs at midspan of the short strip s_1 of ***Figure 13.2-1b***.
  - It is evident that the curvature, and hence the moment, in the short strip s_2 is less than at the corresponding location of strip s_1. Consequently, ***a variation of short-span moment occurs in the long direction of the span***. This variation is shown qualitatively in ***Figure 13.2-2***.
  - The short-span moment diagram in ***Figure 13.2-2a*** is valid only along the center strip at 1-1. Elsewhere, the maximum-moment value is less, as shown. Other moment ordinates are reduced proportionately.

- Similarly, the long-span moment diagram in ***Figure 13.2-2*** applies only at the longitudinal centerline of the slab; elsewhere, ordinates are reduced according to the variation shown.
- These variations in maximum moment across the width and length of a rectangular slab are ***accounted for in an approximate way in most practical design methods by designing for a reduced moment in the outer quarters of the slab span in each direction***, the concept of ***column strip*** and ***middle strip***.

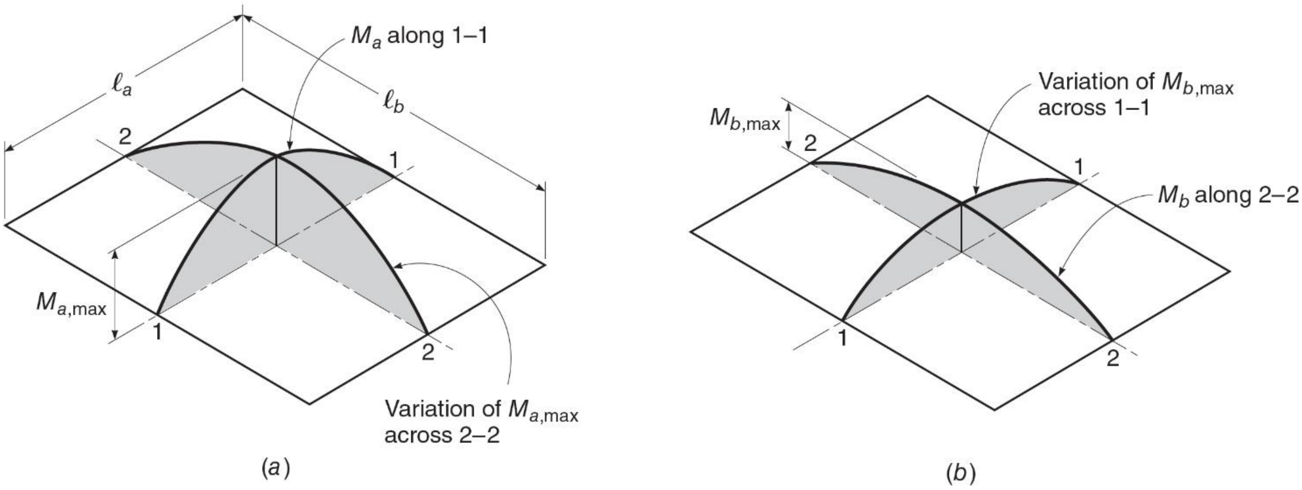


Figure 13.2-2: Moments and moment variations in a uniformly loaded slab with simple supports on four sides.

- Criterion aspect ratio ℓ_b/ℓ_a for two-way slabs:
  - It should be noted that only slabs with ***side ratios less than about 2 need be treated as two-way slabs***.
  - From ***Eq. 13.2-2*** above, it is seen that for a slab of this proportion, ***the share of the load carried in the long direction is only on the order of one-sixteenth of that in the short direction***. Such a slab acts almost as if it were spanning in the short direction only.
  - Consequently, ***rectangular slab panels with an aspect ratio of more than 2***, $\ell_b/\ell_a > 2$, ***may be reinforced for one-way action***, with the ***main steel perpendicular to the long edges***.

13.2.2 REINFORCEMENT DETAILS

- Reinforcement placing in two-way slabs:
  - Orthogonal reinforcement pattern:
    Consistent with the assumptions of the analysis of two-way edge-supported slabs, the main flexural reinforcement is placed in an orthogonal pattern, with reinforcing bars parallel and perpendicular to the supported edges.
  - Different effective depths for positive reinforcement:
    As the positive steel is placed in two layers, the effective depth d for the upper layer is smaller than that for the lower layer by one bar diameter.
  - Which positive layer should be put firstly?
    Because the moments in the long direction are the smaller ones, it is economical to ***place the steel in that direction on top of the bars in the short direction***.
    The stacking problem does not exist for negative reinforcement perpendicular to the supporting edge beams except at the corners, where moments are small.

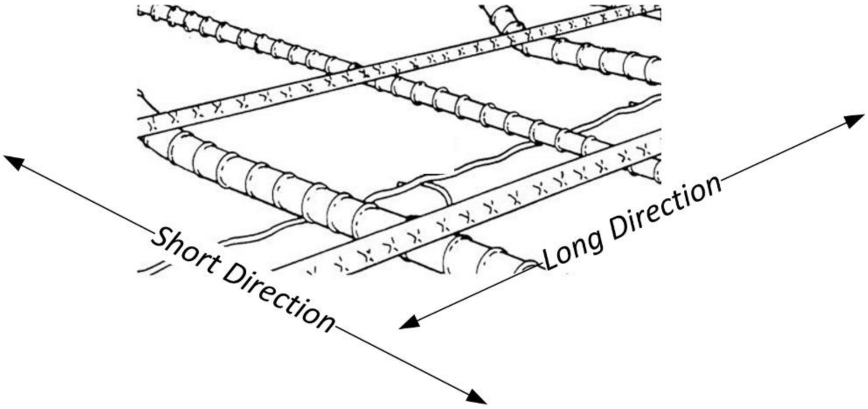


Figure 13.2-3: Positive reinforcement of a two-way slab.

- Cutoff and bent points for reinforcement:
  - Either straight bars, ***cut off*** where they are no longer required, or ***bent bars*** may be used for ***two-way slabs***. But economy of ***bar fabrication and placement will generally favor all straight bars***.
  - Precise locations of inflection points:
    The precise locations of inflection points (or lines of inflection) are not easily determined, because they depend upon:
    - The side ratio,
    - the ratio of live to dead load,
    - and continuity conditions at the edges.
  - Standard cutoff and bend points:
    The standard cutoff and bend points for beams, one-way slab summarized in ***Figure 13.2-4***, may be used for ***<u>edge-supported</u> slabs*** as well.

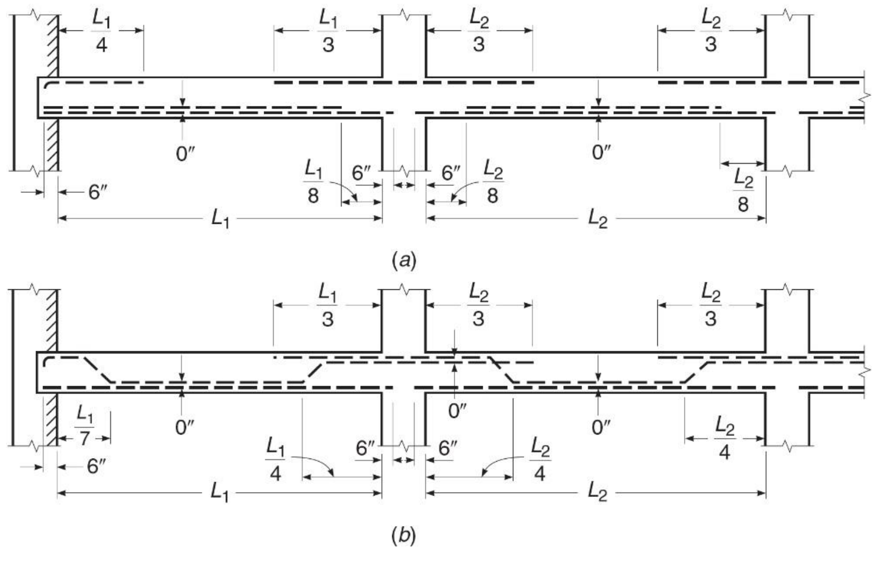


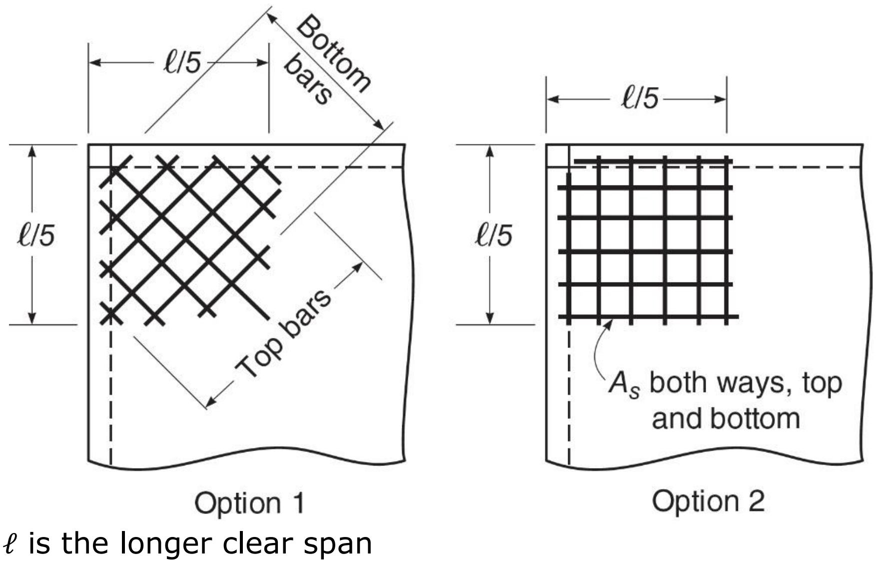
Figure 13.2-4: Cutoff or bend points for bars in approximately equal spans with uniformly distributed loads.

- Minimum reinforcement:
  According to ***ACI Code 8.6.1***, the minimum reinforcement near the tension face in each direction for two-way slabs is that required for shrinkage and temperature crack control, as given in ***Table 13.2-1***.

Table 13.2-1: As,min for nonprestressed two-way slabs, Table 8.6.1.1 of ACI

Reinforcement type	f_y, MPa	$A_{s,min}$, mm^2	
Deformed bars	< 420	$0.0020A_g$	
Deformed bars or welded wire reinforcement	≥ 420	Greater of:	$\frac{0.0018\times 420}{f_y}A_g$
			$0.0014A_g$

- Maximum spacing between reinforcement:
  For two-way systems, the spacing of flexural reinforcement at critical sections must not exceed 2 times the slab thickness h.
  $s \leq 2h$ **Eq. 13.2-6**
- Corner reinforcement, ***ACI Code 8.7.3***:
  - The twisting moments discussed earlier are usually of consequence only at exterior corners of a two-way slab system, where they tend to crack the slab at the bottom along the panel diagonal, and at the top perpendicular to the panel diagonal.
  - Special reinforcement should be provided at ***exterior corners in both the bottom and the top of the slab***.
  - Length of corner reinforcement:
    Corner reinforcement shall be provided for a ***distance in each direction from the corner equal*** to ***one-fifth the longer span of the corner panel***, as shown in ***Figure 13.2-5***.
  - Two placing alternatives:
    The reinforcement at the top of the slab should be parallel to the diagonal from the corner, while that at the bottom should be perpendicular to the diagonal. Alternatively, either layer of steel may be placed in two bands parallel to the sides of the slab.
  - Amount and spacing of reinforcement:
    The positive and negative reinforcement, in any case, should be of ***a size and spacing equivalent to that required for the maximum positive moment (per foot of width) in the panel***.



ℓ is the longer clear span

Figure 13.2-5: Special reinforcement at exterior corners of a beam supported two-way slab.

13.2.3 MOMENTS AND SHEAR FORCES IN EDGE-SUPPORTED TWO-WAY SLAB

13.2.3.1 Methods for Analysis of Two-way Slabs

- According to general and intuitive discussion of ***Section 13.2.1***, one notice that the behavior of edge supported two-way system is highly indeterminate and highly complicated as it contains two flexural moments that interact with shear forces and torsional moments.
- In general, methods for slab analysis can be classified into following three categories:
  - Analytical methods:
    These methods adopted a sophisticated mathematic to formulate the governing differential equation for slab then use Fourier series to solve it. These methods are so complicated to be adopted in design practice. See (Timoshenko & Woiowsky-Krieger, 1959) for more details about analytical analysis of plates and slabs.
  - Numerical methods:
    In numerical methods including finite difference and finite element methods the governing partial differential equation for slab is approximately transformed into a set of simultaneous equations. Finite element method is the most powerful numerical method that adopted in almost all of current commercial software like STAAD Pro, SAP 2000, ETASBS, SAFE, and Robot. STAAD application for analysis of two-way slabs is considered later in this chapter.
  - Semi-empirical methods:
    - These methods are approximated methods that usually adopted by design codes for practical analysis and design of beams.
    - In 1963, ACI code offered two different sets of approximated methods for analysis of two-way slabs. The first set includes three equivalent methods; namely ***Method 1***, ***Method 2***, and ***Method 3***; that can be in analysis of edge supported slabs only. These methods were included in ***Appendix A*** of the code.
      The second set include two methods, namely the ***Elastic Method*** and the ***Empirical Method***, in Chapter 21 of the code for analysis of flat slabs.
    - In later versions; Method 1, Method 2, and Method 3 were deleted from the code while the Elastic Method and the Empirical Method have been respectively upgraded to the ***Equivalent Frame Method*** and the ***Direct Design Method*** that are suitable for analysis of two-way slabs with and without beams.
    - In spite of their deletion of Method 1, Method 2, and Method 3 from ACI code, they still appealing for many engineers. Method 3, in particular, is still included in our curriculum and it has been included in the draft version of Iraqi code for concrete design.

13.2.3.2 Method 3 for Analysis of Two-way Edge-supported Slab

- Limitations:
  - According to ACI code for 1963, Method 3 can be used for a slab supported at four edges by ***walls***, ***stiff steel*** or ***concrete beams***
  - According to Nilson in the 10th edition, a ***stiff beam is a beam having total depth equal to or greater than 3 times slab thickness***. Generally, this is a definition related to beam rigidity instead of beam stiffness as it should be.
- Middle and Column Strips:
  - In Method 3, moment variations of ***Figure 13.2-2*** are approximated through dividing each panel in both directions into a ***Middle Strip*** and ***Column Strip*** (shown in ***Figure 13.2-6***) and through assuming that the entire middle strip to be designed for full moment. In the column strips, this moment is assumed to decrease from its full value at the edge of middle strip to one-third of this value at the edge of the panel.

- In engineering practice, same reinforcements are usually used through slab width and these reinforcements are computed based on full moments in the middle strips. Then designers rarely need the moments in the column strips in their practical works.

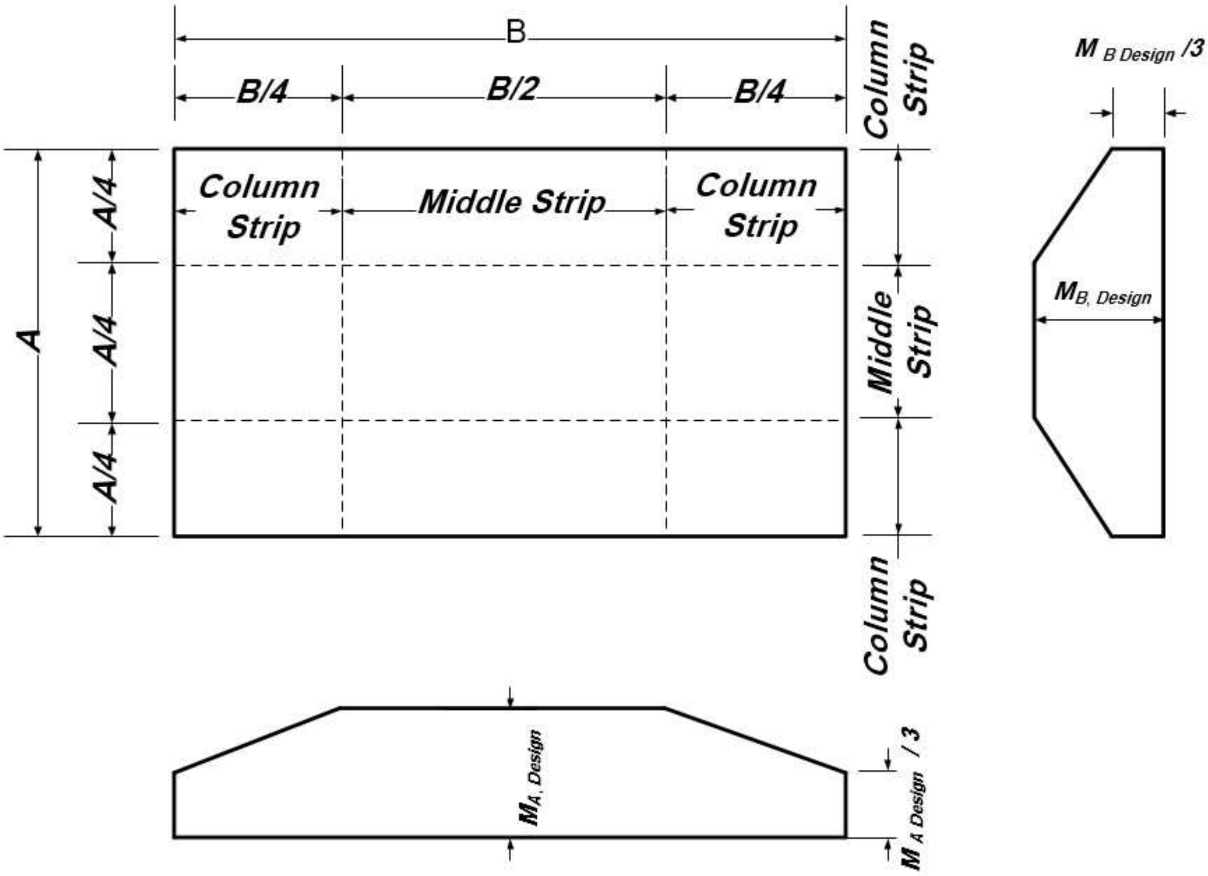


Figure 13.2-6: Definition of middle strip and column strip according to ACI code.

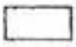
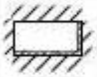
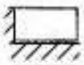
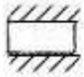
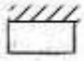
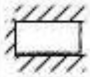
- Moments Tables:
  According to Method 3, moments in middle strips of an edge supported two-way slabs could be computed based on the following equations:
  - Moments in Short Direction:
    $M_{a\ Negative} = C_{a\ from\ Table\ 1}\ W_u l_a^2$
    $M_{a\ Positive} = [C_{a\ from\ Table\ 2}\ W_{u\ DL} + C_{a\ from\ Table\ 3}\ W_{u\ LL}\ ]l_a^2$
  - Moments in Long Direction:
    $M_{b\ Negative} = C_{b\ from\ Table\ 1}\ W_u l_b^2$
    $M_{b\ Positive} = [C_{b\ from\ Table\ 2}\ W_{u\ DL} + C_{b\ from\ Table\ 3}\ W_{u\ LL}\ ]l_b^2$
  - Negative Moment at Discontinuous Edge:
    $M_{\ Negative\ Moment\ at\ Discontinuous\ Edge} = \frac{1}{3}\ M_{Positive}$

  The C_a and C_b can be computed from ***Table1***, ***Table 2***, and ***Table 3*** of ACI code 1963 that shown in ***Table 13.2-2***, ***Table 13.2-3***, and ***Table 13.2-4*** below.

Table 13.2-2 Coefficients for negative moments in slabs (Table 1 according ACI code 1963).

$M_{a,neg} = C_{a,neg} w l_a^2$ where w = total uniform dead plus live load

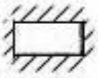
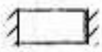
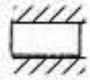
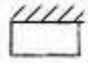
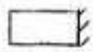
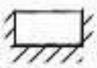
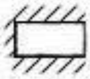
$M_{b,neg} = C_{b,neg} w l_b^2$

Ratio $m = \frac{l_a}{l_b}$		Case 1 	Case 2 	Case 3 	Case 4 	Case 5 	Case 6 	Case 7 	Case 8 	Case 9 
1.00	$C_{a,neg}$		0.045		0.050	0.075	0.071		0.033	0.061
	$C_{b,neg}$		0.045	0.076	0.050			0.071	0.061	0.033
0.95	$C_{a,neg}$		0.050		0.055	0.079	0.075		0.038	0.065
	$C_{b,neg}$		0.041	0.072	0.045			0.067	0.056	0.029
0.90	$C_{a,neg}$		0.055		0.060	0.080	0.079		0.043	0.068
	$C_{b,neg}$		0.037	0.070	0.040			0.062	0.052	0.025
0.85	$C_{a,neg}$		0.060		0.066	0.082	0.083		0.049	0.072
	$C_{b,neg}$		0.031	0.065	0.034			0.057	0.046	0.021
0.80	$C_{a,neg}$		0.065		0.071	0.083	0.086		0.055	0.075
	$C_{b,neg}$		0.027	0.061	0.029			0.051	0.041	0.017
0.75	$C_{a,neg}$		0.069		0.076	0.085	0.088		0.061	0.078
	$C_{b,neg}$		0.022	0.056	0.024			0.044	0.036	0.014
0.70	$C_{a,neg}$		0.074		0.081	0.086	0.091		0.068	0.081
	$C_{b,neg}$		0.017	0.050	0.019			0.038	0.029	0.011
0.65	$C_{a,neg}$		0.077		0.085	0.087	0.093		0.074	0.083
	$C_{b,neg}$		0.014	0.043	0.015			0.031	0.024	0.008
0.60	$C_{a,neg}$		0.081		0.089	0.088	0.095		0.080	0.085
	$C_{b,neg}$		0.010	0.035	0.011			0.024	0.018	0.006
0.55	$C_{a,neg}$		0.084		0.092	0.089	0.096		0.085	0.086
	$C_{b,neg}$		0.007	0.028	0.008			0.019	0.014	0.005
0.50	$C_{a,neg}$		0.086		0.094	0.090	0.097		0.089	0.088
	$C_{b,neg}$		0.006	0.022	0.006			0.014	0.010	0.003

[a] A crosshatched edge indicates that the slab continues across, or is fixed at, the support; an unmarked edge indicates a support at which torsional resistance is negligible.

Table 13.2-3 Coefficients for dead load positive moments in slabs (Table 2 according ACI code 1963).

$M_{a,pos,dl} = C_{a,dl} w l_a^2$
$M_{b,pos,dl} = C_{b,dl} w l_b^2$ where w = total uniform dead load

Ratio $m = \frac{l_a}{l_b}$		Case 1 	Case 2 	Case 3 	Case 4 	Case 5 	Case 6 	Case 7 	Case 8 	Case 9 
1.00	$C_{a,dl}$	0.036	0.018	0.018	0.027	0.027	0.033	0.027	0.020	0.023
	$C_{b,dl}$	0.036	0.018	0.027	0.027	0.018	0.027	0.033	0.023	0.020
0.95	$C_{a,dl}$	0.040	0.020	0.021	0.030	0.028	0.036	0.031	0.022	0.024
	$C_{b,dl}$	0.033	0.016	0.025	0.024	0.015	0.024	0.031	0.021	0.017
0.90	$C_{a,dl}$	0.045	0.022	0.025	0.033	0.029	0.039	0.035	0.025	0.026
	$C_{b,dl}$	0.029	0.014	0.024	0.022	0.013	0.021	0.028	0.019	0.015
0.85	$C_{a,dl}$	0.050	0.024	0.029	0.036	0.031	0.042	0.040	0.029	0.028
	$C_{b,dl}$	0.026	0.012	0.022	0.019	0.011	0.017	0.025	0.017	0.013
0.80	$C_{a,dl}$	0.056	0.026	0.034	0.039	0.032	0.045	0.045	0.032	0.029
	$C_{b,dl}$	0.023	0.011	0.020	0.016	0.009	0.015	0.022	0.015	0.010
0.75	$C_{a,dl}$	0.061	0.028	0.040	0.043	0.033	0.048	0.051	0.036	0.031
	$C_{b,dl}$	0.019	0.009	0.018	0.013	0.007	0.012	0.020	0.013	0.007
0.70	$C_{a,dl}$	0.068	0.030	0.046	0.046	0.035	0.051	0.058	0.040	0.033
	$C_{b,dl}$	0.016	0.007	0.016	0.011	0.005	0.009	0.017	0.011	0.006
0.65	$C_{a,dl}$	0.074	0.032	0.054	0.050	0.036	0.054	0.065	0.044	0.034
	$C_{b,dl}$	0.013	0.006	0.014	0.009	0.004	0.007	0.014	0.009	0.005
0.60	$C_{a,dl}$	0.081	0.034	0.062	0.053	0.037	0.056	0.073	0.048	0.036
	$C_{b,dl}$	0.010	0.004	0.011	0.007	0.003	0.006	0.012	0.007	0.004
0.55	$C_{a,dl}$	0.088	0.035	0.071	0.056	0.038	0.058	0.081	0.052	0.037
	$C_{b,dl}$	0.008	0.003	0.009	0.005	0.002	0.004	0.009	0.005	0.003
0.50	$C_{a,dl}$	0.095	0.037	0.080	0.059	0.039	0.061	0.089	0.056	0.038
	$C_{b,dl}$	0.006	0.002	0.007	0.004	0.001	0.003	0.007	0.004	0.002

[a] A crosshatched edge indicates that the slab continues across, or is fixed at, the support; an unmarked edge indicates a support at which torsional resistance is negligible.

Table 13.2-4 Coefficients for live load positive moments in slabs (Table 3 according ACI code 1963).

$M_{a,pos,ll} = C_{a,ll} w l_a^2$
$M_{b,pos,ll} = C_{b,ll} w l_b^2$ where w = total uniform live load

Ratio $m = \frac{l_a}{l_b}$		Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
1.00	$C_{a,ll}$	0.036	0.027	0.027	0.032	0.032	0.035	0.032	0.028	0.030
	$C_{b,ll}$	0.036	0.027	0.032	0.032	0.027	0.032	0.035	0.030	0.028
0.95	$C_{a,ll}$	0.040	0.030	0.031	0.035	0.034	0.038	0.036	0.031	0.032
	$C_{b,ll}$	0.033	0.025	0.029	0.029	0.024	0.029	0.032	0.027	0.025
0.90	$C_{a,ll}$	0.045	0.034	0.035	0.039	0.037	0.042	0.040	0.035	0.036
	$C_{b,ll}$	0.029	0.022	0.027	0.026	0.021	0.025	0.029	0.024	0.022
0.85	$C_{a,ll}$	0.050	0.037	0.040	0.043	0.041	0.046	0.045	0.040	0.039
	$C_{b,ll}$	0.026	0.019	0.024	0.023	0.019	0.022	0.026	0.022	0.020
0.80	$C_{a,ll}$	0.056	0.041	0.045	0.048	0.044	0.051	0.051	0.044	0.042
	$C_{b,ll}$	0.023	0.017	0.022	0.020	0.016	0.019	0.023	0.019	0.017
0.75	$C_{a,ll}$	0.061	0.045	0.051	0.052	0.047	0.055	0.056	0.049	0.046
	$C_{b,ll}$	0.019	0.014	0.019	0.016	0.013	0.016	0.020	0.016	0.013
0.70	$C_{a,ll}$	0.068	0.049	0.057	0.057	0.051	0.060	0.063	0.054	0.050
	$C_{b,ll}$	0.016	0.012	0.016	0.014	0.011	0.013	0.017	0.014	0.011
0.65	$C_{a,ll}$	0.074	0.053	0.064	0.062	0.055	0.064	0.070	0.059	0.054
	$C_{b,ll}$	0.013	0.010	0.014	0.011	0.009	0.010	0.014	0.011	0.009
0.60	$C_{a,ll}$	0.081	0.058	0.071	0.067	0.059	0.068	0.077	0.065	0.059
	$C_{b,ll}$	0.010	0.007	0.011	0.009	0.007	0.008	0.011	0.009	0.007
0.55	$C_{a,ll}$	0.088	0.062	0.080	0.072	0.063	0.073	0.085	0.070	0.063
	$C_{b,ll}$	0.008	0.006	0.009	0.007	0.005	0.006	0.009	0.007	0.006
0.50	$C_{a,ll}$	0.095	0.066	0.088	0.077	0.067	0.078	0.092	0.076	0.067
	$C_{b,ll}$	0.006	0.004	0.007	0.005	0.004	0.005	0.007	0.005	0.004

[a] A crosshatched edge indicates that the slab continues across, or is fixed at, the support; an unmarked edge indicates a support at which torsional resistance is negligible.

- Notes on Moment Tables:
  - One may ask why there are two tables for positive moment (one is for dead load and the other is for live load) while there is only one table for negative moment (for dead and live loads together)?
  - To answer this question, it is useful to note that coefficients C_a and C_b have been derived with including load pattern effects for live loads. Then there are three load cases:
    - First Load case is for dead loads, where all panels have been loaded with full dead load, see ***Figure 13.2-7***.
    - Second Load Case is for live load pattern that produce maximum negative moments in support regions. Negative maximum moment due to live load increases as the support conditions approaching fixed conditions. These conditions could be approached through loading the adjacent panels in addition to panel under consideration, see ***Figure 13.2-8***.

- Third Load Case is for live load pattern that produce maximum positive moment. Maximum positive moment due to live load increases as the support conditions approaching hinge conditions. These conditions could be approached through loading the panel under consideration only, see ***Figure 13.2-9***.

- As load pattern for negative moment due to live load is similar to that for dead load, then it is natural to be treated based on same table (Table 13.2-2). While as load pattern for positive moment due to live load is differ from that for dead load, then it is natural to use a different table for each one (Table 13.2-3, and Table 13.2-4).

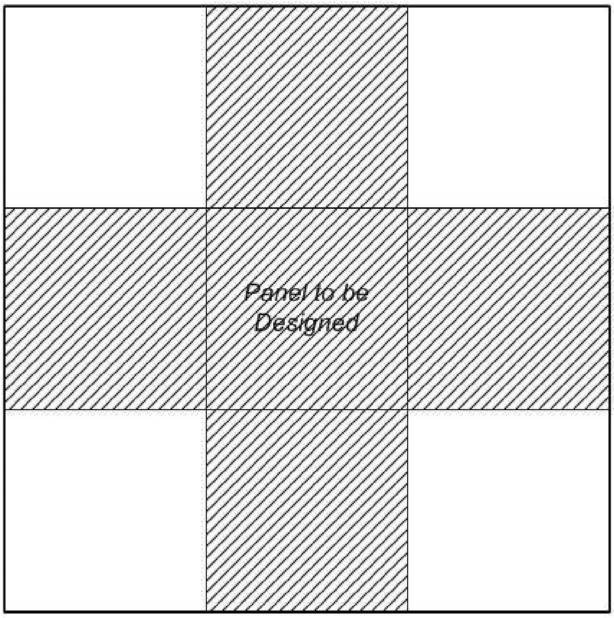


Figure 13.2-7: Load pattern for dead load of a typical interior panel.

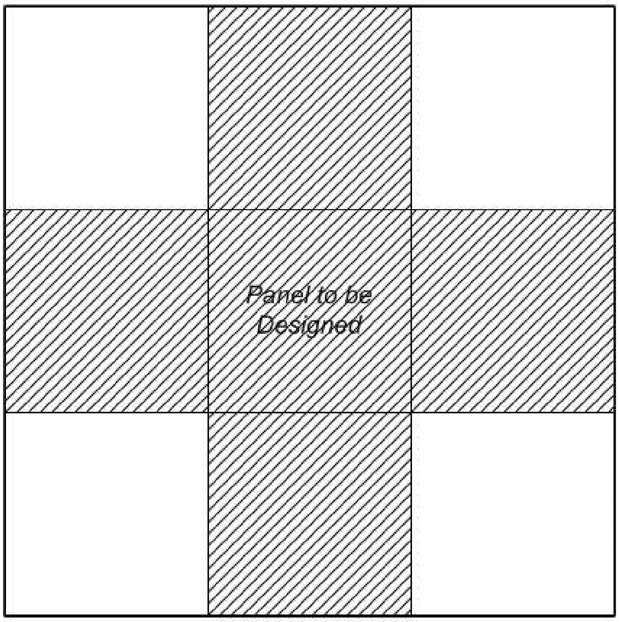


Figure 13.2-8: Load pattern for live load maximum negative moment of a typical interior panel.

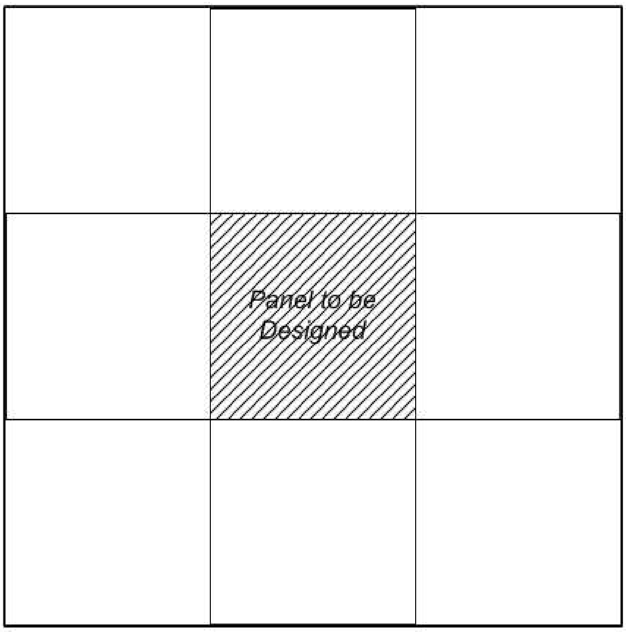
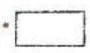
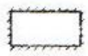
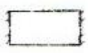
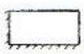
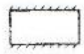
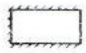


Figure 13.2-9: Load pattern for live load maximum positive moment of a typical interior panel.

- Table for Shear Forces and Load Shares:
  For computing shear in the two-way slab and loads on supporting beams, Table 4 of ACI code 1963, shown in ***Table 13.2-5***, gives the fractions of the total load W_u which are transmitted in the two directions.

Table 13.2-5: Ratio of load W in l_a and l_b directions for shear in slab and load on supports (Table 4 according ACI code 1963).

Ratio		Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
$m = \frac{l_a}{l_b}$										
1.00	W_a	0.50	0.50	0.17	0.50	0.83	0.71	0.29	0.33	0.67
	W_b	0.50	0.50	0.83	0.50	0.17	0.29	0.71	0.67	0.33
0.95	W_a	0.55	0.55	0.20	0.55	0.86	0.75	0.33	0.38	0.71
	W_b	0.45	0.45	0.80	0.45	0.14	0.25	0.67	0.62	0.29
0.90	W_a	0.60	0.60	0.23	0.60	0.88	0.79	0.38	0.43	0.75
	W_b	0.40	0.40	0.77	0.40	0.12	0.21	0.62	0.57	0.25
0.85	W_a	0.66	0.66	0.28	0.66	0.90	0.83	0.43	0.49	0.79
	W_b	0.34	0.34	0.72	0.34	0.10	0.17	0.57	0.51	0.21
0.80	W_a	0.71	0.71	0.33	0.71	0.92	0.86	0.49	0.55	0.83
	W_b	0.29	0.29	0.67	0.29	0.08	0.14	0.51	0.45	0.17
0.75	W_a	0.76	0.76	0.39	0.76	0.94	0.88	0.56	0.61	0.86
	W_b	0.24	0.24	0.61	0.24	0.06	0.12	0.44	0.39	0.14
0.70	W_a	0.81	0.81	0.45	0.81	0.95	0.91	0.62	0.68	0.89
	W_b	0.19	0.19	0.55	0.19	0.05	0.09	0.38	0.32	0.11
0.65	W_a	0.85	0.85	0.53	0.85	0.96	0.93	0.69	0.74	0.92
	W_b	0.15	0.15	0.47	0.15	0.04	0.07	0.31	0.26	0.08
0.60	W_a	0.89	0.89	0.61	0.89	0.97	0.95	0.76	0.80	0.94
	W_b	0.11	0.11	0.39	0.11	0.03	0.05	0.24	0.20	0.06
0.55	W_a	0.92	0.92	0.69	0.92	0.98	0.96	0.81	0.85	0.95
	W_b	0.08	0.08	0.31	0.08	0.02	0.04	0.19	0.15	0.05
0.50	W_a	0.94	0.94	0.76	0.94	0.99	0.97	0.86	0.89	0.97
	W_b	0.06	0.06	0.24	0.06	0.01	0.03	0.14	0.11	0.03

[a] A crosshatched edge indicates that the slab continues across, or is fixed at, the support; an unmarked edge indicates a support at which torsional resistance is negligible.

13.2.4 SLAB THICKNESS

- To limit deflection, minimum thickness of ***Eq. 13.2-7*** is proposed by ACI code 1963.

$$h_{min} = \text{Maximum}\left[90^{mm}, \frac{\text{Panel Perimeter}}{180}\right] \qquad \textbf{Eq. 13.2-7}$$

- Current ACI codes includes more accurate expressions for h_{min} taking stiffness of supporting beams and slab aspect ratio into consideration.
- To be consistent with methods adopted for computing of moments and shear forces, approximate ***Eq. 13.2-7*** is adopted in this section.

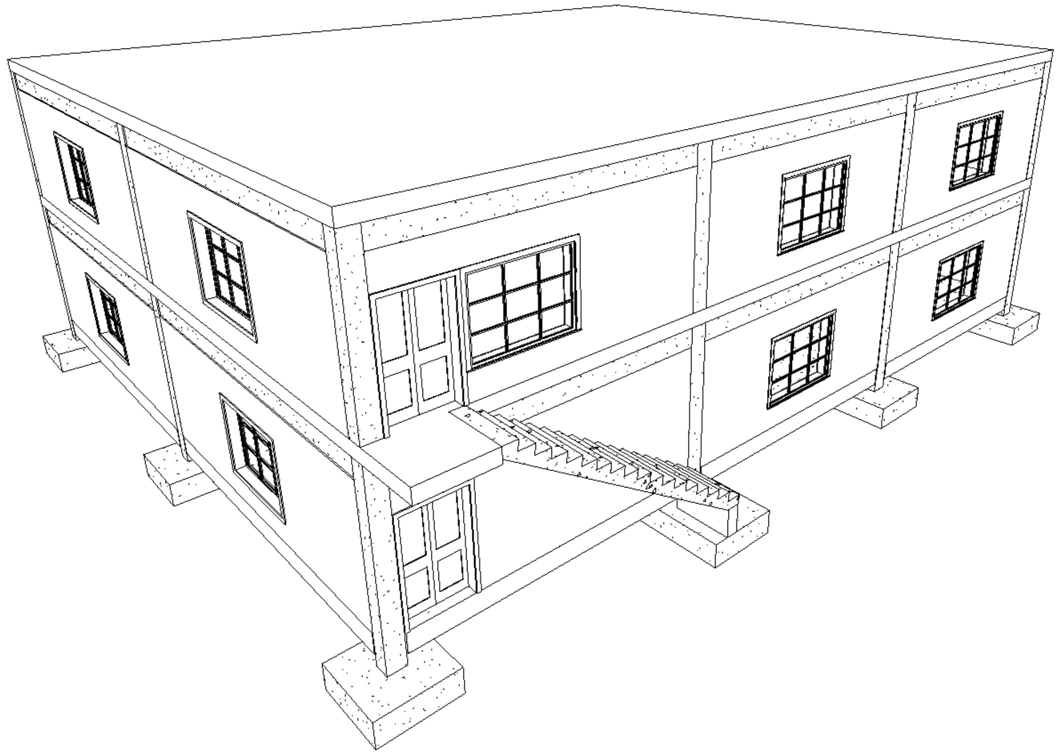
13.2.5 DESIGN EXAMPLE

Example 13.2-1

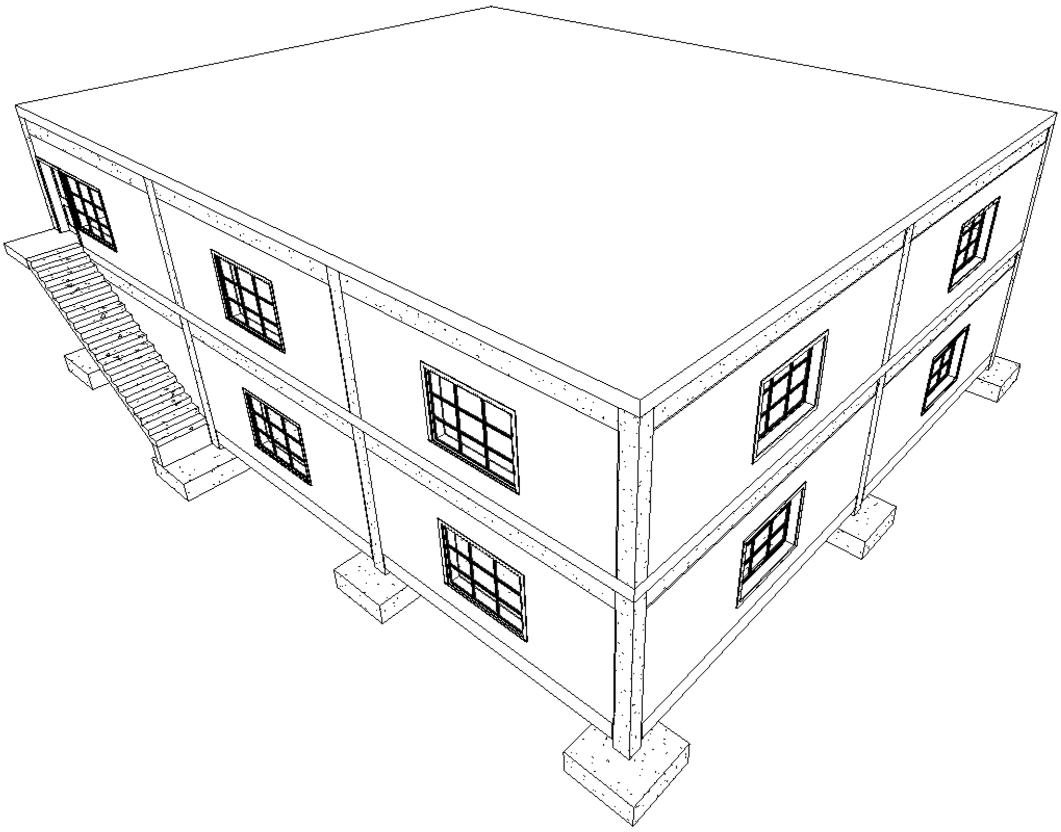
Design the corner floor panel for the building shown in views, elevations, plan, and sections of ***Figure 13.2-10*** below and its supporting beam "Beam A".

Assume that:

- $f_c' = 21\ MPa$,
- $f_y = 420\ MPa$,
- $W_{Live} = 5.0\ kN/m^2$ (has been determined based on functional requirements of the building),
- Suspending slabs of $0.5\ kN/m^2$.

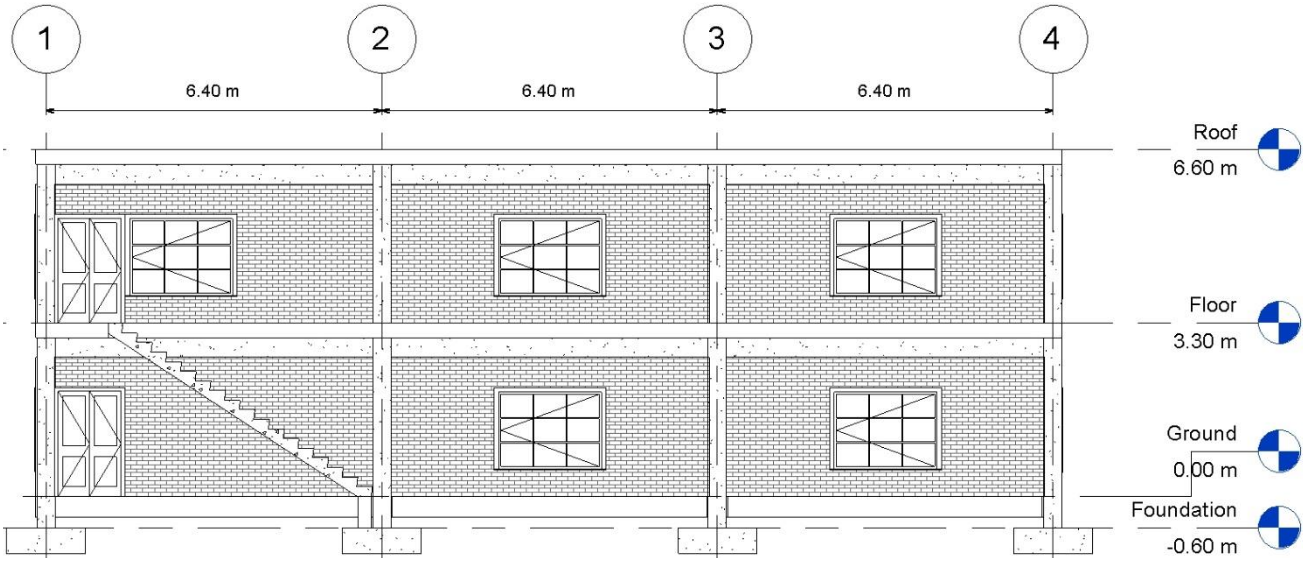


Front 3D view.

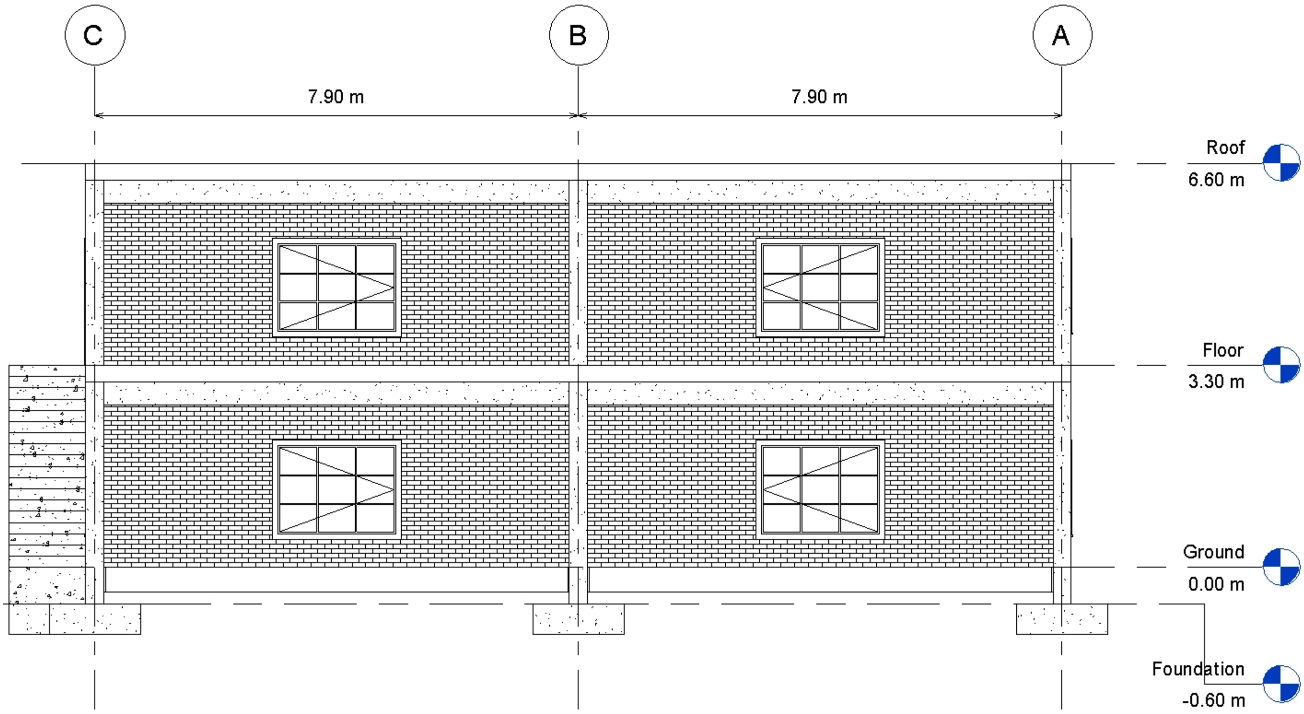


Back 3D view.

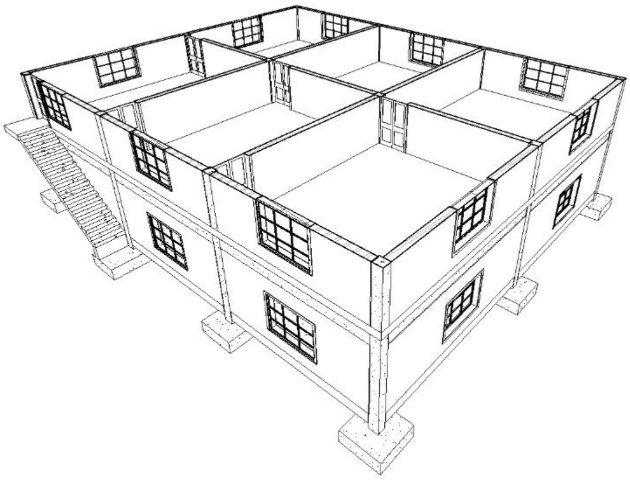
Figure 13.2-10: Building for Example 13.2-1



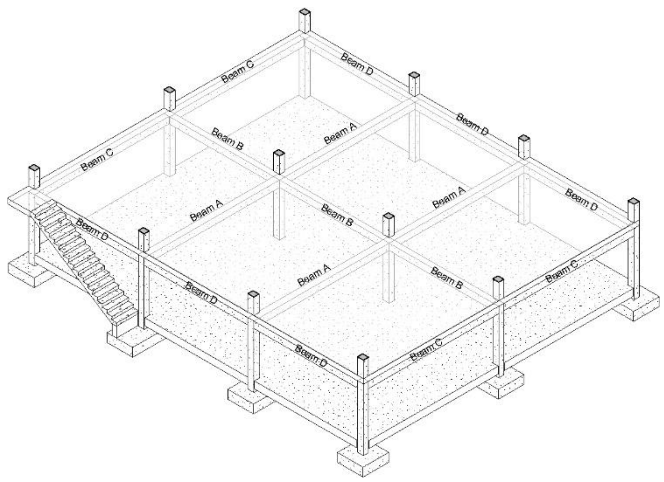
Longitudinal elevation view.



Transverse elevation view.

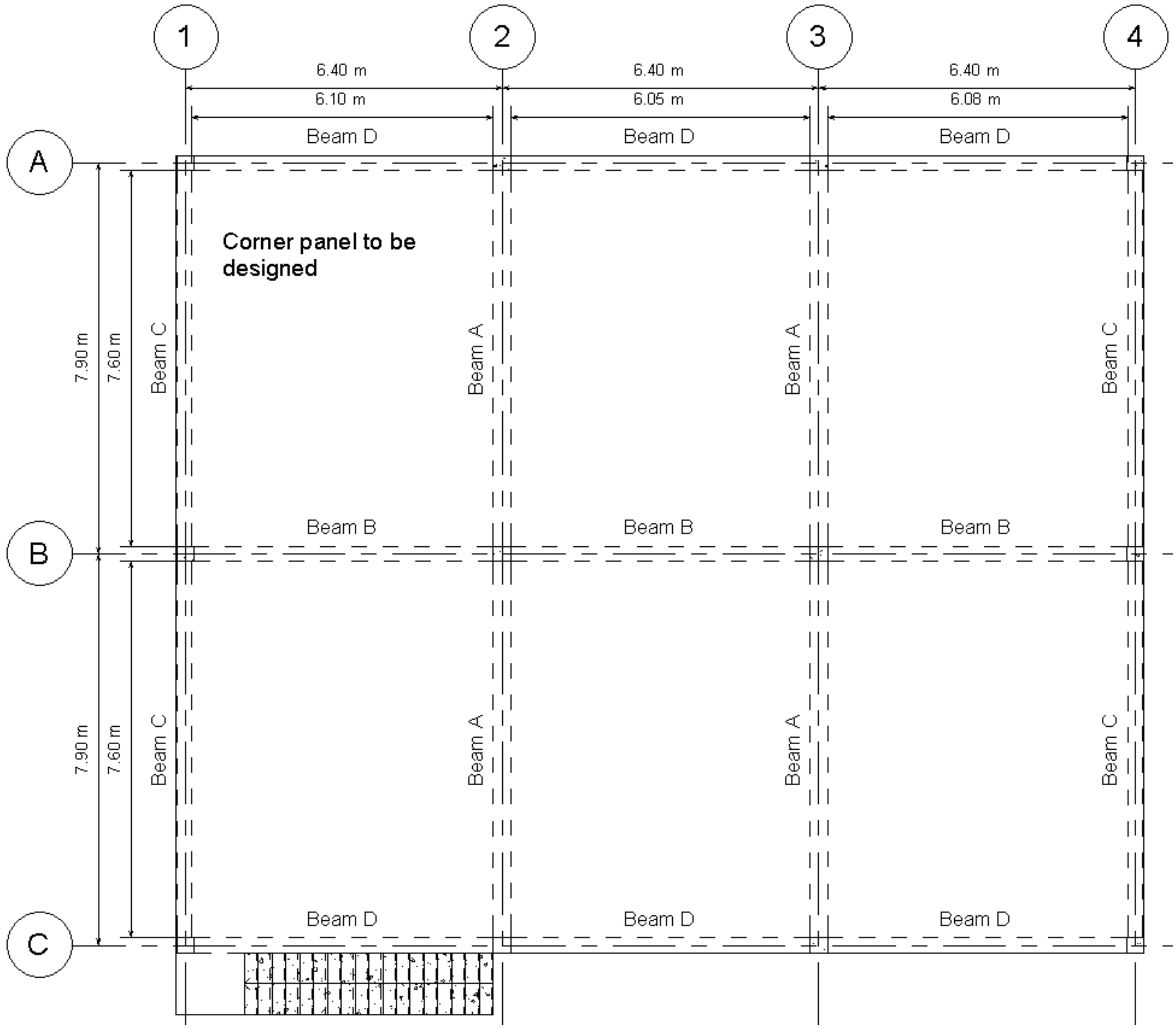


Floor system, 3D view.

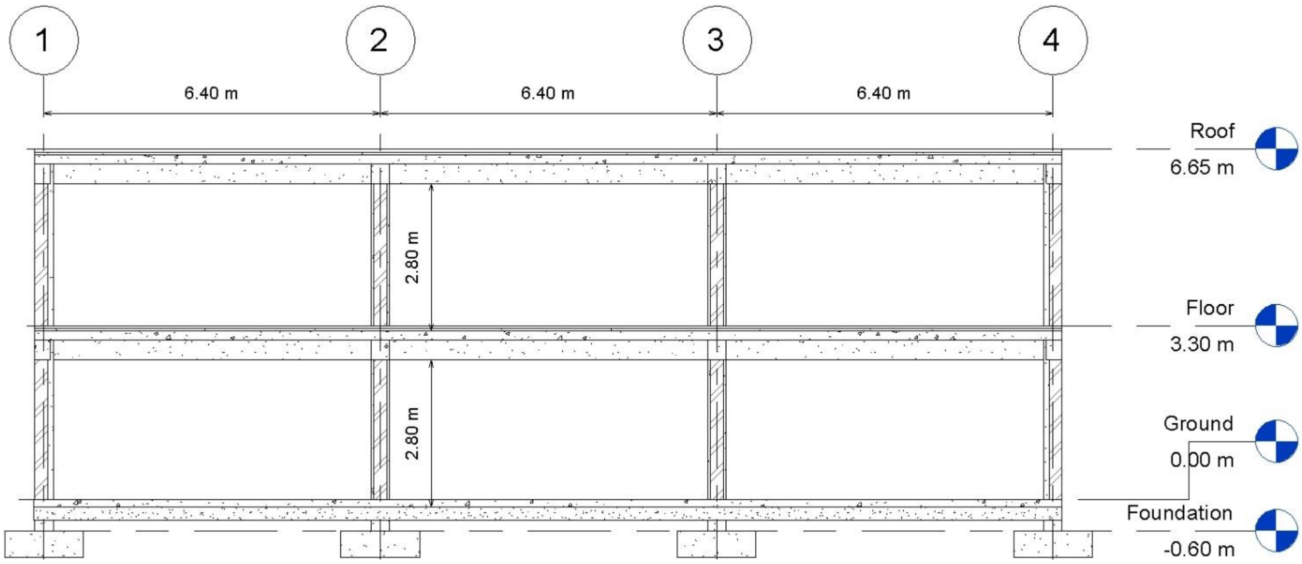


Structural system, 3D view.

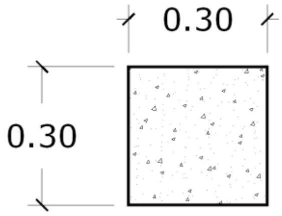
Figure 13.2-10: Building for Example 13.2-1. Continued.



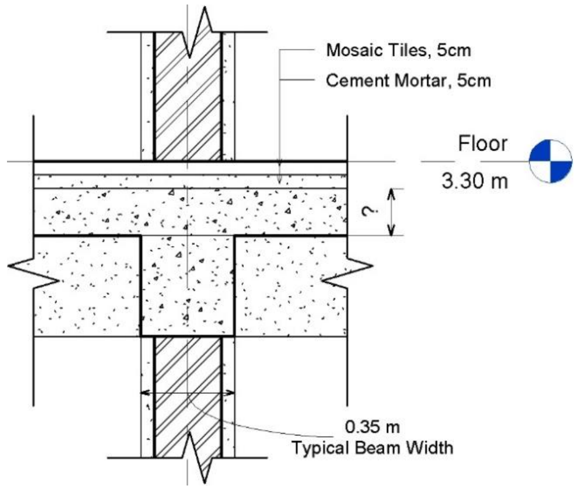
Floor plan view.



Transverse sectional view.
Figure 13.2-10: Building for Example 13.2-1. Continued.



Typical column section.



Floor surfacing details.

Figure 13.2-10: Building for Example 13.2-1. Continued.

Solution

Slab Design

- System Classification
  As the slab is edge-supported on four sides, then it can be classified into one-way or two-way depends on its aspect ratio.
  Let
  l_b is the length of clear span in long direction.
  l_a is the length of clear span in short direction.
  Then:
  $$\because \frac{l_b}{l_a} = \frac{7.6^m}{6.1^m} = 1.25 < 2$$
  hence the slab is considered as two-way edge supported slab.
- Slab Thickness
  With refereeing to ***Eq. 13.2-7*** that adopted from ACI Code (1963) (Article 2002 Page 84), minimum slab thickness for deflection control is:
  $$h_{min} = \text{Maximum}\left(90^{mm}, \frac{\text{Panel Perimeter}}{180}\right) = \text{Maximum}\left(90^{mm}, \frac{2(7\,600^{mm} + 6\,100^{mm})}{180}\right)$$
  $$= \text{Maximum}\,(90^{mm}, 152^{mm}) = 152^{mm}$$
  Try
  $h = 175^{mm} > h_{min} \therefore \text{Ok.}$
- Loads Acting on Slab
  - Dead Loads
    $$W_{Dead} = \left(0.175^m \times 24\frac{kN}{m^3}\right)_{Slab\ Selfweight} + (0.05 + 0.05)m \times 20\frac{kN}{m^3}_{Slab\ Surfacing} + 0.5\frac{kN}{m^2}_{Suspended\ Ceiling} = 4.2\frac{kN}{m^2} + 2.5\frac{kN}{m^2} = 6.70\frac{kN}{m^2}$$
  - Live Loads
    $$W_{Live} = 5.0\frac{kN}{m^2}$$
  - Factored Loads
    $$W_u = maximum\ of\ [1.4\ W_{Dead}\ or\ 1.2\ W_{Dead} + 1.6\ W_{Live}]$$
    $$= maximum\ of\ [1.4 \times 6.70\ or\ 1.2 \times 6.70 + 1.6 \times 5.0]$$
    $$= maximum\ of\ [9.38\ or\ 8.0 + 8.0] = maximum\ of\ [9.38\ or\ 16.0]$$
    $$= 16.0\frac{kN}{m^2}$$
- Support conditions according to Method 3:
  According to its continuity and for purpose of computing moment coefficients C_a and C_b from Table1, Table 2, and Table 3 (see ***Table 13.2-2***, ***Table 13.2-3***, and ***Table 13.2-4***), a typical corner slab panel can be classified as ***Case 4*** with aspect ratio, m, of:

$$\frac{l_a}{l_b} = \frac{6.1}{7.6} = 0.8$$

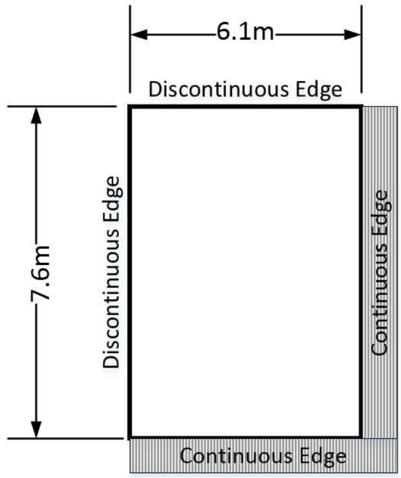


Figure 13.2-11: Support continuity for a typical corner panel of floor slab for Example 13.2-1.

- Moments in Short Direction:
  With referring to C_a from Table 1, Table 2, and Table 3 (see ***Table 13.2-2***, ***Table 13.2-3***, and ***Table 13.2-4***), moments in short direction are:

$$\mathrm{M_{a\ Negative}} = \mathrm{C_{a\ from\ Table\ 1}} \mathrm{W_u} l_a^2 = 0.071 \times 16 \frac{\mathrm{kN}}{\mathrm{m^2}} \times 6.1^2 = 42.3\ \mathrm{kN.m\ per\ m}$$

$$\mathrm{M_{a\ Positive}} = [\mathrm{C_{a\ from\ Table\ 2}}\ \mathrm{W_{u\ DL}} + \mathrm{C_{a\ from\ Table\ 3}}\ \mathrm{W_{u\ LL}}]\mathrm{l_a^2}$$

$$= \left[\ 0.039 \times 8.0 \frac{\mathrm{kN}}{\mathrm{m^2}} + 0.048 \times\ 8.0 \frac{\mathrm{kN}}{\mathrm{m^2}} \right] \times 6.1^2 \mathrm{m^2} = 25.9\ \mathrm{kN.m\ per\ m}$$

$$\mathrm{M_{\ a\ Negative\ Moment\ at\ Discontinuous\ Edge}} = \frac{1}{3} \times 25.9\ \mathrm{kN.m\ per\ m} = 8.63\ \mathrm{kN.m\ per\ m}$$

  Graphical summary for moments in short direction ***Figure 13.2-12*** below.

- Moments in Long Direction:
  In same approach, moments in long direction are:

$$\mathrm{M_{b\ Negative}} = 0.029 \times 16.0 \frac{\mathrm{kN}}{\mathrm{m^2}} \times 7.6^2 \mathrm{m^2} = 26.8\ \mathrm{kN.m\ per\ m}$$

$$\mathrm{M_{b\ Positive}} = \left[\ 0.016 \times 8.0 \frac{\mathrm{kN}}{\mathrm{m^2}} + 0.02 \times\ 8.0 \frac{\mathrm{kN}}{\mathrm{m^2}} \right] \times 7.6^2 \mathrm{m^2} = 16.6\ \ \mathrm{kN.m\ per\ m}$$

$$\mathrm{M_{\ b\ Negative\ Moment\ at\ Discontinuous\ Edge}} = \frac{1}{3} \times 16.6\ \mathrm{kN.m\ per\ m} = 5.53\ \ \mathrm{kN.m\ per\ m}$$

  Graphical summary for moments in long direction ***Figure 13.2-13*** below.

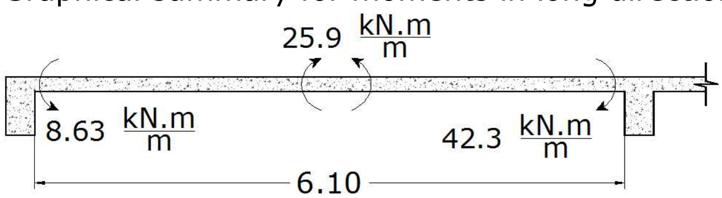


Figure 13.2-12: Graphical summary for moments in short direction of Example 13.2-1.

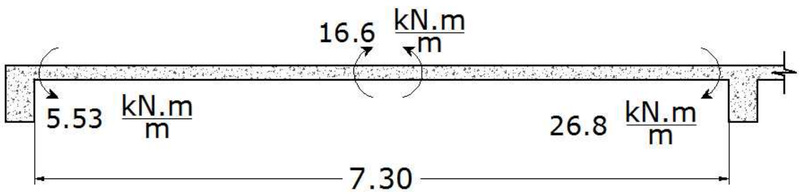


Figure 13.2-13: Graphical summary for moments in long direction of Example 13.2-1.

- Design of Main Reinforcement
  Try bar ***13mm*** for slab flexure reinforcement in both directions. With referring to reinforcement details of ***Figure 13.2-3*** that indicates that positive reinforcement in short direction should be placed below and before the corresponding reinforcement in long direction, the effective depth "d" will be:

$$d_{for\ Positive\ Reinforcement\ in\ Short\ Direction} = 175^{mm} - 20^{mm} - \frac{13^{mm}}{2} = 148\ ^{mm}$$

$$d_{for\ Positive\ Reinforcement\ in\ Long\ Direction} = 175^{mm} - 20^{mm} - 1.5 \times 13^{mm} = 135^{mm}$$

$$d_{for\ Negative\ Reinforcement} = 175^{mm} - 20^{mm} - \frac{13^{mm}}{2} = 148^{mm}$$

Details for flexural design in short and long directions have been summarized in ***Table 13.2-6*** and ***Table 13.2-7*** respectively.

- Shear Checking of the Slab

Use Table 4 of ACI code 1963 (see ***Table 13.2-5***) with referring to ***Figure 13.2-14*** to compute shear in the panel and to distribute loads among supporting beams:

$$V_{u\ Along\ Long\ Direction} = \left[\left(16.0\frac{kN}{m^2} \times 7.6m \times 6.1m\right)_{Load\ Resultant} \times 0.71_{Shear\ Share\ in\ Short\ Direction\ from\ Table\ 4} \times \frac{1}{2}_{Shear\ on\ Both\ Sides}\right] \times \frac{1}{7.6m} = 34.6\ kN\ per\ m$$

$$V_{u\ Along\ Short\ Direction} = \left[\left(16.0\frac{kN}{m^2} \times 7.6m \times 6.1m\right)_{Load\ Resultant} \times 0.29_{Shear\ Share\ in\ Long\ Direction\ from\ Table\ 4} \times \frac{1}{2}_{Shear\ on\ Both\ Sides}\right] \times \frac{1}{6.1m} = 17.6\ kN\ per\ m$$

$$\emptyset V_c = 0.75 \times \left(0.17 \times \sqrt{21} \times 1000 \times 148\right) = 86.5\ kN\ per\ m > V_u\ \ Ok.$$

Table 13.2-6: Flexural design details for reinforcement in short direction of Example 13.2-1.

	Exterior Negative	***Positive Moment***	***Interior Negative***
$M_u (kN.m\ per\ m)$	8.63	25.9	42.3
$M_n (kN.m\ per\ m)$	9.59	28.7	47.0
$\rho_{Required} = \dfrac{1-\sqrt{1-2.36\frac{M_n}{f_c' bd^2}}}{1.18 \times \frac{f_y}{f_c'}}$	1.06×10^{-3}	3.24×10^{-3}	5.42×10^{-3}
$A_{S\ Theoretical}\ (mm^2 per\ m) = \rho_{Required} \times b \times d$	157	479	802
$\rho_{max} = 0.85\beta_1\ \dfrac{f_c'}{f_y}\ \dfrac{\epsilon_u}{\epsilon_u + 0.004}$	$0.85^2 \times \frac{21}{420} \times \frac{0.003}{0.007} = 15.5 \times 10^{-3}$		
$A_{smax} (mm^2\ per\ m) = \rho_{max} \times b \times d$	$15.5 \times 10^{-3}\ \times 1000mm \times 148mm = 2\ 294$		
$A_{S\ Minimum}\ mm^2 per\ m$	$0.0018 \times 1000mm \times 175mm = 315$		
$A_{S\ Required} mm^2 per\ m$	315	479	802
$S_{Theoretical} (mm) = \dfrac{A_{Bar}}{A_{S\ Required}} \times 1000$	422	278	165
$S_{maximum} = 2 \times t$	350		
Final Main Reinforcement	***Use*** $\emptyset 13mm\ @\ 350mm$	***Use*** $\emptyset 13mm\ @\ 275mm$	***Use*** $\emptyset 13mm\ @\ 150mm$

Table 13.2-7: Flexural design details for reinforcement in long direction of Example 13.2-1.

	Exterior Negative	***Positive Moment***	***Interior Negative***
$M_u(kN.m\ per\ m)$	5.53	16.6	26.8
$M_n(kN.m\ per\ m)$	6.14	18.4	29.8
Effective Depth "d"	148	135	148
$\rho_{Required} = \frac{1-\sqrt{1-2.36\frac{M_n}{f_c' bd^2}}}{1.18\times\frac{f_y}{f_c'}}$	0.673×10^{-3}	2.48×10^{-3}	3.37×10^{-3}
$A_{S\ Theoretical}\ (mm^2 per\ m) = \rho_{Required}\times b\times d$	99.6	334	499
$\rho_{max} = 0.85\beta_1\ \frac{f_c'}{f_y}\ \frac{\epsilon_u}{\epsilon_u+0.004}$	$0.85^2\times\frac{21}{420}\times\frac{0.003}{0.007} = 15.5\times10^{-3}$		
$A_{smax}(mm^2\ per\ m) = \rho_{max}\times b\times d$	$15.5\times10^{-3}\ \times 1000mm\times 148mm = 2\,294$		
$A_{S\ Minimum}\ mm^2 per\ m$	$0.0018\times1000mm\times175mm = 315$		
$A_{S\ Required} mm^2 per\ m$	315	334	499
$S_{Theoretical}(mm) = \frac{A_{Bar}}{A_{S\ Required}}\times1000$	422	398	267
$S_{maximum} = 2\times t$	350		
Final Main Reinforcement	***Use*** $\boldsymbol{\emptyset 13mm\ @\ 350mm}$	***Use*** $\boldsymbol{\emptyset 13mm\ @\ 350mm}$	***Use*** $\boldsymbol{\emptyset 13mm\ @\ 250mm}$

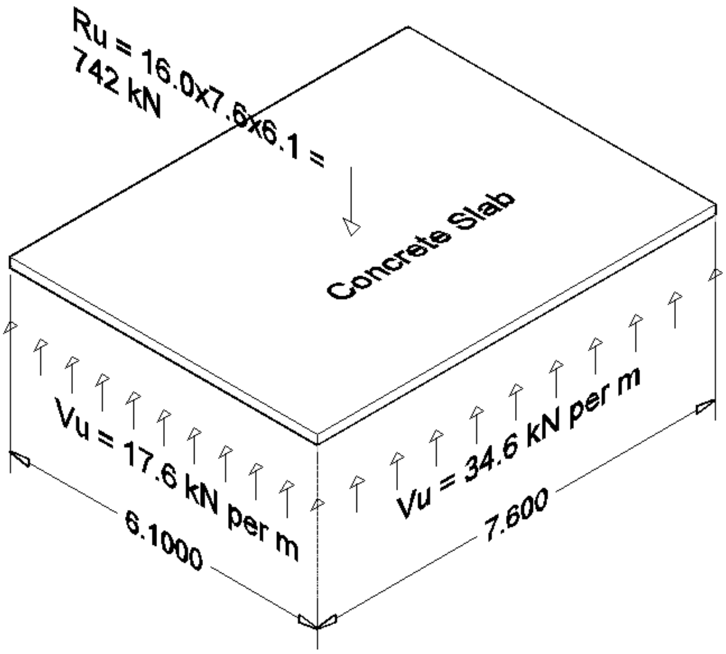


Figure 13.2-14: Load sharing and shear forces of Example 13.2-1.

- Reinforcement Details
  With adopting typical cutoff points of ***Figure 13.2-4***, and design results of ***Table 13.2-6*** and ***Table 13.2-7***, floor slab reinforcement would be as indicated in ***Figure 13.2-15*** and ***Table 13.2-8***.
- Corner Reinforcement
  Adopting ***Option 2*** of ***Figure 13.2-5*** and other recommendation of ACI code, corner slab reinforcement would be as indicated in ***Figure 13.2-16***.

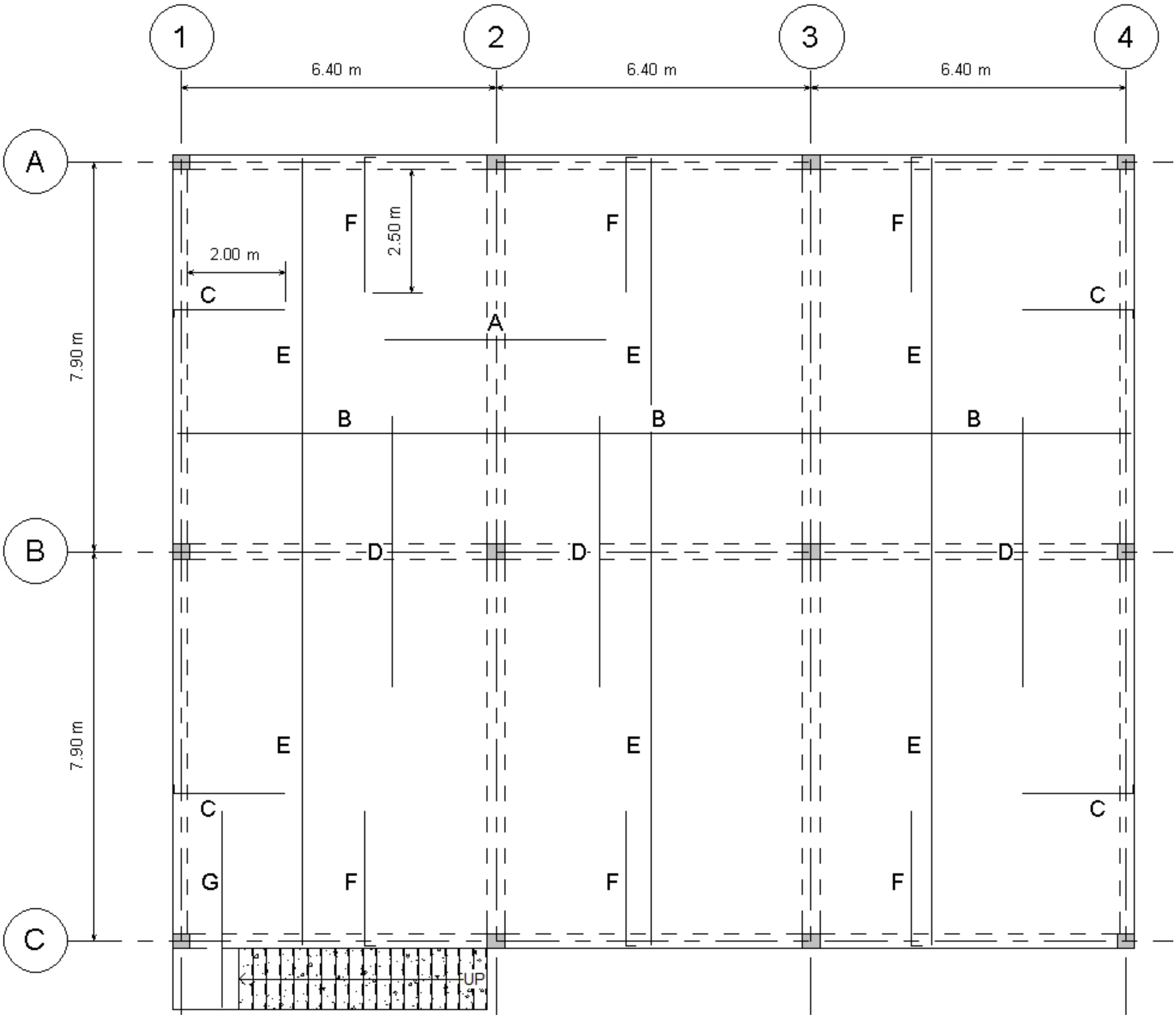


Figure 13.2-15: Floor slab reinforcement of Example 13.2-1.

Table 13.2-8: Floor slab bar schedule of Example 13.2-1.

Designation Name	Location	Bar No.	Spacing, mm	Length, m	Splice, mm
Short direction					
A	Top	13	150	4.5	-
B	Bottom	13	275	-	676
C	Top	13	350	-	-
Long direction					
D	Top	13	250	5.5	-
E	Bottom	13	350	-	676
F	Top	13	350	-	-
Slab thickness is 175mm.					

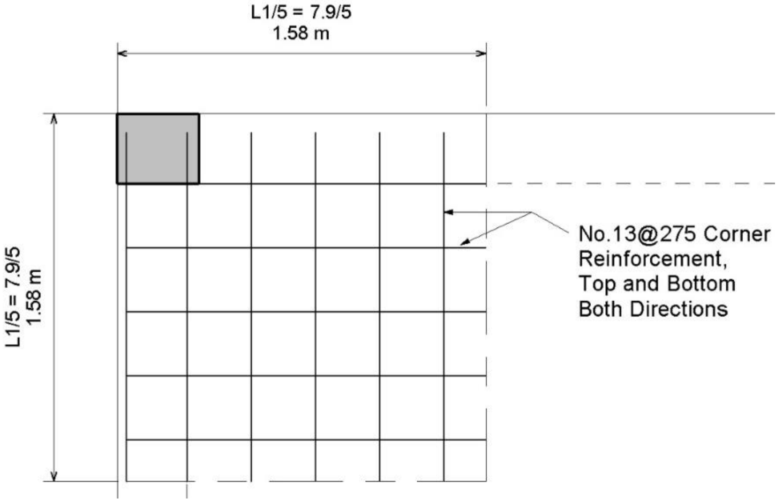


Figure 13.2-16: Slab corner reinforcement of Example 13.2-1.

Design of Supporting Beam "A"

- Loads Acting on Supporting Beam "A":

  Load on supporting beam is equal to the shear forces transfer from the supported slab, plus supported brick partition, plus beam selfweight (see ***Figure 13.2-17***):

  $$W_u = Shear\ from\ Slabs\ \frac{kN}{m} \times 2 + 1.2 \times W_{selfweight} + 1.2 \times W_{Brick\ Partition}$$

  Assume that, the beam has a depth equal to 0.65m (this assumption will be checked later):

  $$W_u = 34.6\frac{kN}{m} \times 2 + 1.2 \times \left(0.65m \times 0.30m \times 24\frac{kN}{m^3}\right) + 1.2\ (2.8m \times 0.25m \times 19\frac{kN}{m^3}) =$$

  $$W_u = 69.2 + 5.62 + 16.0 = 90.8\frac{kN}{m}$$

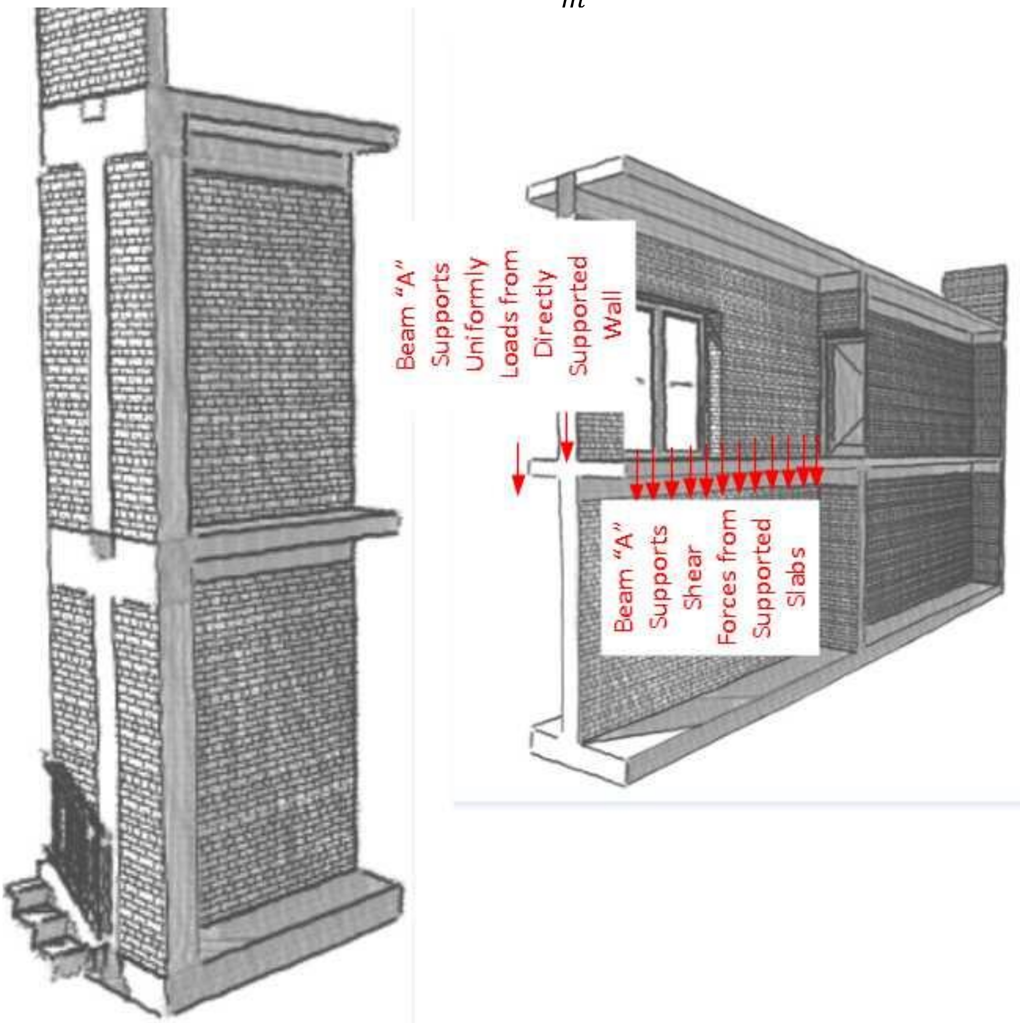
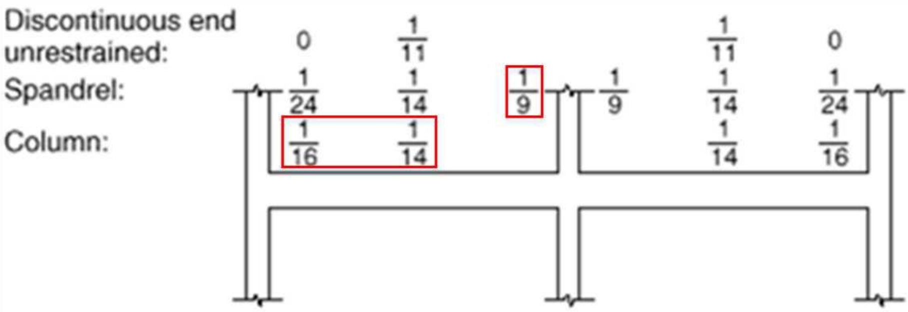


Figure 13.2-17: Loads acting on Beam A of Example 13.2-1.

- Design Moments and Shears for Beam A
  - Factored design moments and shears can be computed by ACI Coefficients Method, the exterior span for Beam A is the critical span that can be used to compute the design moments:



$$M_{u\ Exterior\ Negative} = \frac{W_u l_n^2}{16} = \frac{90.8 \times 7.6^2}{16} = 328\ kN.m$$

$$M_{u\ Positive} = \frac{W_u l_n^2}{14} = \frac{90.8 \times 7.6^2}{14} = 375\ kN.m$$

$$M_{u\,Interior\,Negative} = \frac{W_u l_n^2}{9} = \frac{90.8 \times 7.6^2}{9} = 582\ kN.m$$

$$V_u = 1.15\ \frac{W_u l_n}{2} = 1.15\ \times \frac{90.8 \frac{kN}{m} \times 7.6m}{2} = 397\ kN$$

- After computing of beam effective depth, V_u at distance d from face of support can be computed and used in later shear design.

- Beam Design
  - As was discussed previously, beam design usually starts with design for the maximum bending moment that occurs at exterior face of first interior support.
  - Beam design for interior negative moment
    - Depth is selected from design of a beam with non-specified dimensions for a factored moment of ***582 kN.m*** (as was discussed in details in Chapter 4).
    - Use 300mm by 800mm beam with 5 ∅ 25mm rebars.
    - Lesser depth and larger steel ration can be used provided that section to be checked for deflection.
  - Beam design for positive moment:
    - Beam can be designed as a T beam to resists the factored positive moment of 375 kN.m. Effective flange width of T section "b" will be:

$$b = minimum\ \left[c\ to\ c\ spacing\ between\ parallel\ beams, \frac{Beam\ Span}{4}, b_w +\ 16h_f\right]$$

$$b = minimum\ \left[6.4,\quad \frac{7.9}{4},\quad 0.3 +\ 16 \times 0.175\right] =\ 1.97m$$

    - Required positive reinforcement would be $\mathbf{3\ \emptyset\ 25mm}$ rebars.
  - Design for exterior negative moment:
  - Answers for beam negative reinforcement:
    Finally, the beam can be design as a rectangular beam with pre-specified dimensions to resists a factored negative moment of 328 kN.m.
    $\mathbf{3\ \emptyset\ 25mm}$ rebars.
  - Design for Shear Force:

$$d\ =\ 800 - 40 - 10 - 25 - \frac{25}{2} = 712\ mm$$

$$V_{u\ @\ d\ from\ Face\ of\ Support}\ =\ 397 -\ 90.8 \times 0.712 = 332\ kN$$

$\mathbf{\emptyset\ 10mm\ @\ \ 150\ mm}$ Stirrups.

Example 13.2-2

For two-way slab presented in ***Figure 13.2-18*** below that subjected to a factored load, W_u, of $16\ kN/m^2$ including its own weight:

- Check the proposed slab thickness for deflection control requirements.
- Assuming effective depth, d, of 142mm, check the proposed slab thickness for shear strength requirements.
- Determine the maximum factored negative moment, M_{u-ve} , that should be adopted in the slab design, ***only determine the moment not design the slab***.

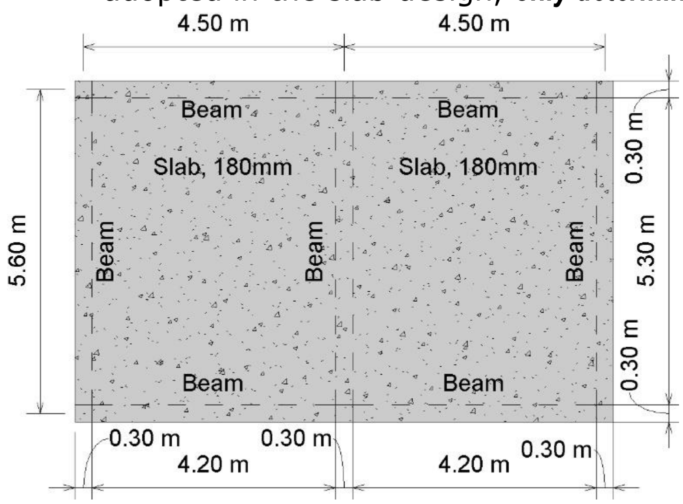


Figure 13.2-18: Two-way slab for Example 13.2-2.

Solution

- Checking of Slab Thickness for Deflection Control:
  With refereeing to ***Eq. 13.2-7*** that adopted from ACI Code (1963) (Article 2002 Page 84), minimum slab thickness for deflection control is:
  $$\mathrm{h_{min}} = \mathrm{Maximum}\ \left[90, \frac{\text{Panel Perimeter}}{180}\right] = \mathrm{Maximum}\ \left[90, \frac{(5300+4200)\times 2}{180}\right]$$
  $$= \mathrm{Maximum}\ [90, 105mm] = 105mm < h_{proposed} = 180\ mm \therefore Ok.$$
- Check Proposed Slab Thickness For Shear Strength:
  Due to continuity conditions, a slab panel can be classified as ***Case 6*** according to *Error! Reference source not found.* through ***Table 13.2-5*** with a ratio, m, of:
  $$m = \frac{l_a}{l_b} = \frac{4.5}{5.6} \approx 0.80$$
  The load shear and corresponding shear force in the short direction would be, see ***Table 13.2-5***:
  $$V_{u\ along\ long\ beam} = \frac{(16\times 5.3\times 4.2)\times 0.86\times \frac{1}{2}}{5.3} = 28.9\ kN\ per\ m$$
  $$\phi V_c = 0.75\times\left(0.17\sqrt{f_c{}'}bd\right) = \frac{0.75\times\left(0.17\sqrt{28}\times 1000\times 142\right)}{1000} = 93.9\ kN\ per\ m > V_u \therefore Ok.$$
- Maximum Factored Negative Moment:
  $$M_{u\ a,-ve} = C_{a,neg} W_u l_a^2$$
- With Case 6 continuity condition and with aspect ratio, m, of 0.8, according to ***Table 13.2-2***, the coefficient $C_{a,neg}$ would be:
  $$C_{a,neg} = 0.086$$
  Therefore, the negative moment in short direction would be:
  $$M_{u\ a,-ve} = C_{a,neg} W_u l_a^2 = 0.086\times 16\times 4.2^2 = 24.3\ kN.m\ per\ m$$

13.3 TWO-WAY RIBBED SLABS*

13.3.1 NOMINAL REQUIREMENTS OF TWO-WAY JOIST SYSTEMS

According to ***Section 8.8*** of the code, the two-way joist system has to satisfy nominal requirements similar to those imposed for one-way joist system.

13.3.2 ANALYSIS APPROACH

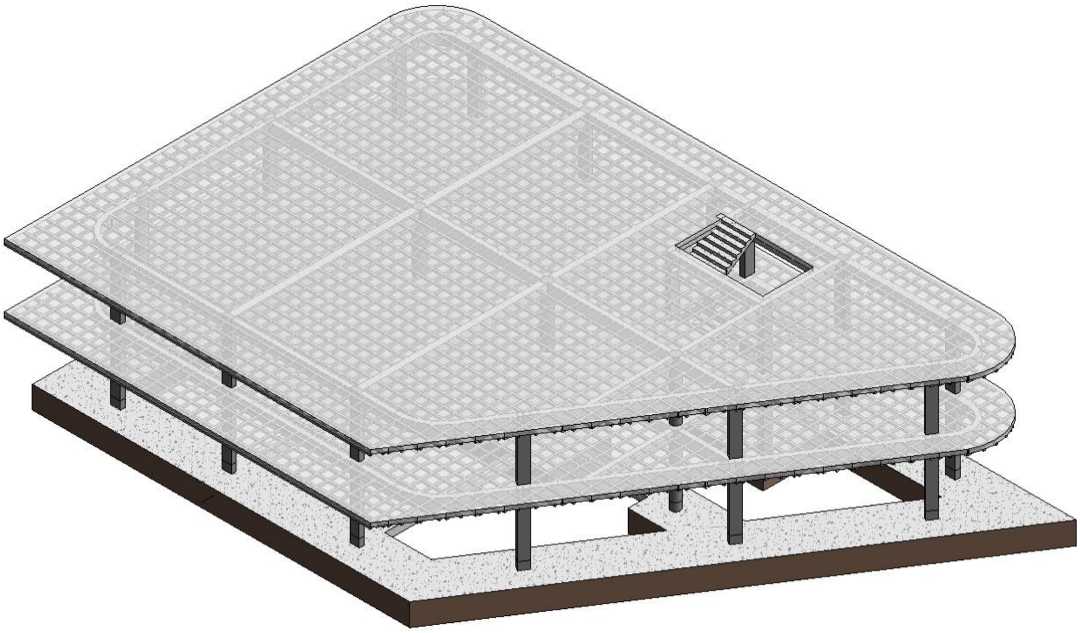
- As discussed in Section 13.1.2.1, according to (Dunham, 1966), the two-way joist systems with beams along columns centerlines can be analyzed using ACI methods for edge-supported two-way slabs.
- To simply construction process, edge beams are usually wide with a depth equal to rib depth. This practice leads to a flexible system with a significant interaction between the slab and edge-supporting beams. Therefore, more general approaches that consider slab-beam interaction seem more suitable for this system.

13.3.3 DESIGN EXAMPLES

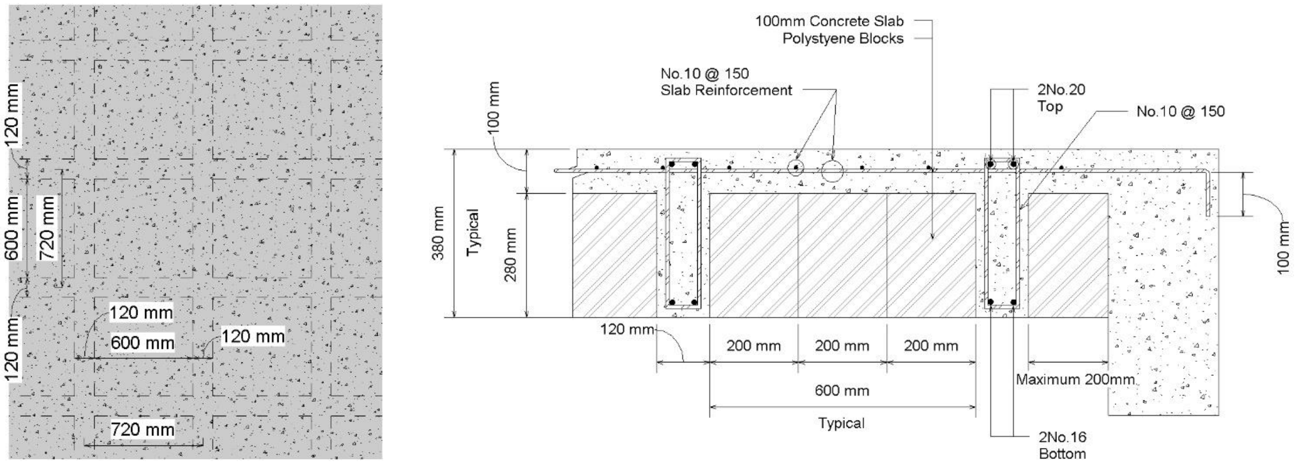
Example 13.3-1

In addition to its own weight, the café building presented in Figure 13.3-1 supports superimposed loads of 2 kPa for floor and 4 kPa for the roof and floor live of 4.8 kPa and roof live load of 1.0 kPa. Check the proposed section for nominal code requirements and then for shear and flexural adequacy. In your analysis assume that:

- The largest panel has clear dimensions of 9m by 10m,
- All supporting beams have dimensions of 400x900mm,
- Concrete compressive strength, f_c', of 28MPa and $f_y = f_{yt} = 420\ MPa$.



3D view.



Details.

Figure 13.3-1: Café building for Example 13.3-1.

Solution

As the ribbed slab attaches to relatively stiff edge beams, therefore it can be analyzed using Method 3 of 1963 ACI code.

Slab thickness for deflection control

The largest panel has plan dimensions of:

$\ell_a = 9000\,mm \quad \ell_b = 10000\,mm$

Therefore, according to Eq. 13.2-7 the minimum slab thickness for deflection control is:

$$\mathrm{h_{min}} = \mathrm{Maximum}\left[90^{\mathrm{mm}}, \frac{\text{Panel Perimeter}}{180}\right] = \max\left(90, \left(\frac{2(9000+10000)}{180}\right)\right) = 211\,mm < h_{poposed} = 380\,mm$$

$\therefore Ok.$

Checking for nominal code requirements:

- According to ***ACI Code 9.8.1***, ribs shall be not less than 100 mm in width, and shall have a depth of not more than 3.5 times the minimum width of rib.

  $b_w = 120mm > 100mm\ Ok. \quad h = 380mm < 3.5 \times 120 = 420mm\ Ok.$
- According to ***ACI Code 9.8.1***, clear spacing between ribs shall not exceed 750 mm.

  $S_{proposed\ between\ ribs} = 600mm < 750mm\ Ok.$
- According to ***ACI Code 9.8.3***, the slab thickness shall be not less than one-twelfth the clear distance between ribs, nor less than 50 mm:

  $$h_f = 100mm > minimum\left(50mm\ or\ \frac{1}{12} \times (600)mm\right) \therefore Ok.$$
- Slab Reinforcement:

  Normal to the ribs shall be provided in the slab as required for flexure, considering load concentrations, if any, but not less than temperature and shrinkage reinforcement that required by *Error! Reference source not found.*:

  $$A_{s\ temp.} = 0.0018 \times 100 \times 1000 = 180\ mm^2\ per\ m < A_{s\ provided} = \frac{\pi \times 10^2}{4} \times \frac{1000}{150} = 523\ mm^2\ per\ m$$

  $\therefore Ok.$

Boundary case and aspect ratio:

With overhang cantilevers, the panel is almost continuous on four edges and Case 2 of the Method 3 can be adopted. The panel aspect ratio is:

$$\frac{\ell_a}{\ell_b} = \frac{9}{10} = 0.9$$

Factored loads

Assuming specific weight of 0.8 for the polystyrene blocks, the self-weight for the ribbed slab is:

$$W_{self} = \frac{0.38 \times 0.72^2 \times 24 - 0.28 \times 0.6^2 \times (24 - 0.8 \times 10)}{0.72^2} \approx 6.00\ kPa$$

$W_{u\,Floor} = \max(1.4(6+4), 1.2(6+4) + 1.6 \times 1) = 14kPa$

$W_{u\,Floor} = \max(1.4(6+2), 1.2(6+2) + 1.6 \times 4.8) = 9.6 + 7.7 = 17.3\ kPa$

Since the floor is more critical, so it is adopted in the subsequent checking process.

Factored moments

According to Table 13.2-2 the negative factored moments are:

$M_{u\,a-ve} = 0.055 W_u \ell_a^2 = 0.055 \times 17.3 \times 9^2 = 77\ kN.m\ per\ m$

$M_{u\,b-ve} = 0.037 W_u \ell_b^2 = 0.037 \times 17.3 \times 10^2 = 64\ kN.m\ per\ m$

According to Table 13.2-3 and Table 13.2-4 the positive factored moments are:

$M_{u\,a+ve} = (0.022 \times 9.6 + 0.034 \times 7.7) \times 9^2 = 38.3\ kN.m\ per\ m$

$M_{u\,b+ve} = (0.014 \times 9.6 + 0.022 \times 7.7) \times 10^2 = 30.4\ kN.m\ per\ m$

Critical moments per each rib:

As the ribs have typical spacing of 0.72m, therefore the factored moments per each rib will be:

$M_{u-ve} = 77 \times 0.72 = 55.4\ kN.m\ per\ rib$

$M_{u+ve} = 38.3 \times 0.72 = 27{,}6\ kN.m\ per\ rib$

Flexural strength of the proposed section:

For the proposed section the positive flexural strength would be:

Let $a < h_f$

$$a = \frac{\left(\frac{\pi \times 16^2}{4} \times 2\right) \times 420}{0.85 \times 28 \times 720} = 9.86\ mm < h_f \therefore Ok. \quad d = 380 - 20 - 10 - \frac{16}{2} \approx 342\ mm$$

$$\phi M_{n+ve} = \left(\frac{0.9 \times \left(\frac{\pi \times 16^2}{4} \times 2 \times 420\right) \times \left(342 - \frac{9.86}{2}\right)}{10^6}\right) = 51.2\ kN.m\ per\ rib > M_{u+ve} \therefore Ok.$$

While negative flexural strength where the section would behave as rectangular section:

$$a = \frac{\left(\frac{\pi \times 20^2}{4} \times 2\right) \times 420}{0.85 \times 28 \times 120} = 92.4\ mm \quad d = 380 - 20 - 10 - \frac{20}{2} = 340\ mm$$

$$\phi M_{n-ve} = \left(\frac{0.9 \times \left(\frac{\pi \times 20^2}{4} \times 2 \times 420\right) \times \left(340 - \frac{92.4}{2}\right)}{10^6}\right) = 69.8\ kN.m\ per\ m > M_{u-ve} \therefore Ok.$$

<u>Factored shear force:</u>

According to Table 13.2-5, the factored shear force is:

$$V_{u\ Along\ Long\ Direction} = \left((17.3\ \times 10 \times 9) \times 0.6 \times \frac{1}{2}\right) \times \frac{1}{10} = 46.7\ kN\ per\ m$$

<u>Shear strength:</u>

With a diameter of $10mm$, stirrups are accepted according to code in resistance for shear. Including an increasing of 10% as per code, rib shear strength is:

$$\phi V_n = \frac{0.75\left(\left(1.1 \times 0.17 \times \sqrt{28} \times 120 \times 340\right) + \left(\frac{\pi \times 10^2}{4} \times 2 \times 420\right) \times \frac{340}{150}\right)}{1000} = 142\ kN\ per\ rib > V_u \therefore Ok.$$

Notes on the analysis approach:

As discussed above, Method 3 is suitable for edge-supported slabs with no slab-beam interaction. To assess the accuracy of the method when it is used to analyze a two-way ribbed slab including slab-beam interaction, an exaggerated deflected shape of Figure 13.3-2 has been generated using a finite element analysis. The shape indicates that the slab-beam interaction is significant and cannot be neglected in general.

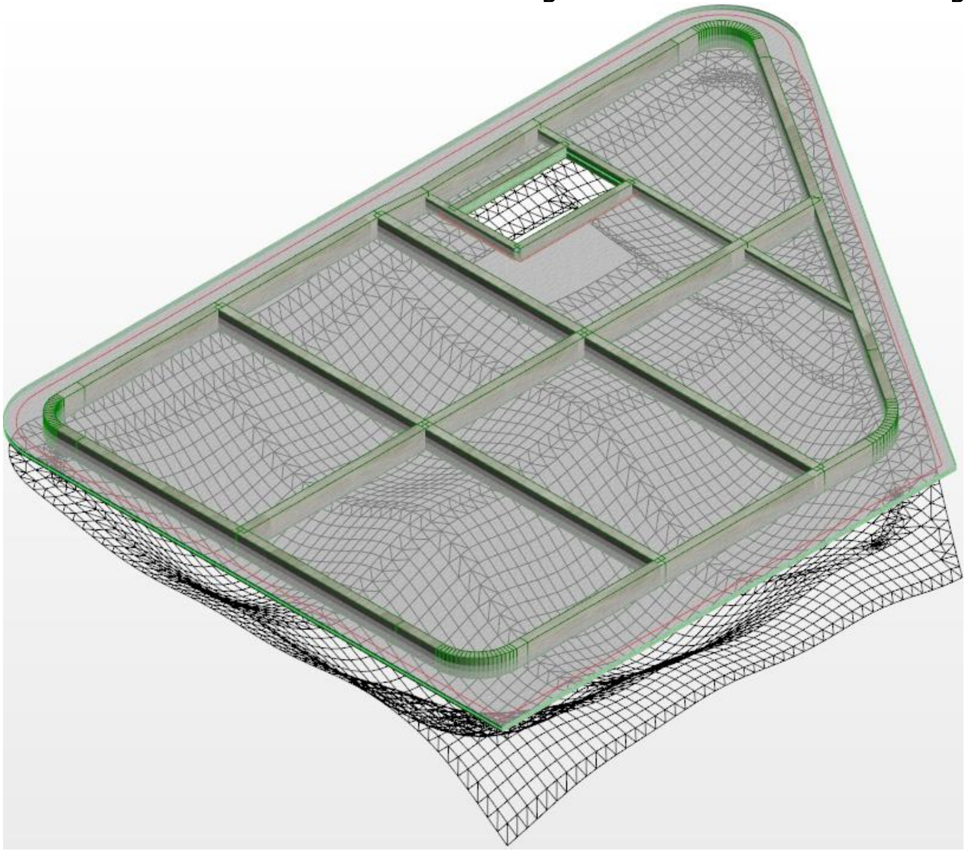


Figure 13.3-2: Exaggerated deflected shape using finite element method for the floor of Example 13.3-1.

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