Computer Engineering Department

Digital Signal Processing

Basic concepts of Digital Signal Processing

Digital Signal Processing (DSP) is an area of science and engineering that is applied in different fields. It is concerned with the numerical manipulations of signals and data in sampled form. It is applied in various applications such as noise filtering, speech and audio enhancement, biomedical signal processing, oil exploration, detection of nuclear explosions, and image processing. DSP provides another way to process the analog signal efficiently.

The scientific definition of Digital Signal Processing:

Digital: the word "digital" has different meaning based on the area of interest. For example, if the talking is about signals or data, the word digital represents as series of the digits 0 and 1 for the values of a physical quantity such as voltage. If the talking is about the digital system, this means systems with discrete inputs that produces outputs in the form of numbers.

Signal: is a function that carry information from one point to another point. It can be described by a function of one or more independent variable. The value of the function (i.e., the dependent variable) can be a real- valued scalar quantity, a complex-valued quantity, or perhaps a vector.

- Signals can have one independent variable such as audio, or two independent variable such as the image or three variable such as the video. In general, if the signal is a function of a single independent variable, the signal is called a one-dimensional signal. On the other hand, a signal is called M-dimensional if its value is a function of M independent variables.in the same concepts, there are single channel and multi- channel signals based on the generation source of the signal. For example, the color TV picture is a three-channel, three-dimensional signal.

Processing: Extraction of useful information to be used. In general, signal processing can be done in two way as shown:

Signal processing → Analog signal processing (ASP)

Signal processing → Digital signal processing (DSP)

As a conclusion: DSP extracts the desired signal from the undesired signal based on the general three topics:

1- Smoothing
2- Filtering.
3- Predicting.

These topics lead to estimate the wanted signal, which are called the estimation processes. The mathematical representation of the estimation process for the signal $x(t)$ to obtain the estimated signal $\hat{x}(t)$ is:

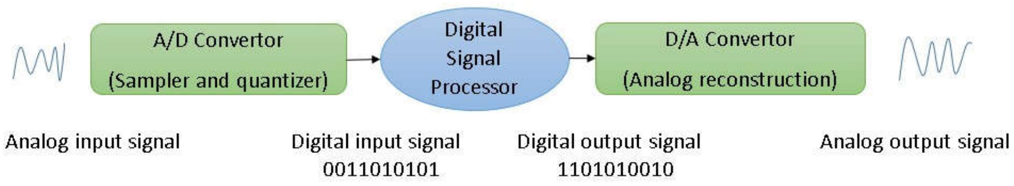
$$x(t) \xrightarrow{estimation} \hat{x}(t)$$

<u>Digital signal processing is implemented in three basic steps:</u>

1. The analog signal is *digitized*, that is, it is *sampled* and each sample *quantized* to a finite number of bits. This process is implemented using an interface between the analog signal and digital signal processor, which is called Analog- to- digital (A/D) convertor. It needs an analog filter prior ADC to determine the frequency range of the signal before the sampling process.

2. The digitized samples are processed by a digital signal processor.

3. The resulting digital output samples are converted back into analog form using an analog reconstruct, that is the Digital- to- Analog convertor (D/A). An analog filter is also needed after the D/A to remove the sharp transitions from the output of the DAC.



A typical digital signal processing system

Advantages of DSP over ASP

1. A digital programmable system allows flexibility in reconfiguring the digital signal processing operations simply by changing the program.

2. Signals and data are increasingly stored in digital computers and magnetic media, and transmitted from one place to another in digital form then processing them digitally. Moreover, the digital signal processor can be programmed to perform a variety of signal processing operations, such as filtering, spectrum estimation, and other DSP algorithms. Beside, depending on the speed and computational requirements of the application, the digital signal processor can be realized more easily.

3. Accuracy considerations also play an important role in determining the form of the signal processor. Tolerances in analog circuit components make it extremely difficult for the system designer to control the accuracy of an analog signal processing system .On the other hand, a digital system provides much better control of accuracy requirements.

However, digital implementation has its limitations, such as the limitation in the speed of operation of A/D converters and digital signal processors. For example, signals having extremely wide bandwidths require fast-sampling-rate A\D conversion and fast digital signal processor.

Some Applications of DSP

Magnetic cards, remote controls, digital entertainment systems, personal computers/networking, copiers/laser printers, new telecommunications systems (e.g., cellular phones, high-speed modems for Internet connection, video teleconferencing systems), health care apparatus, industrial control systems, automotive electronics, computerized billing/banking systems, and voice recognition/

synthesis. All above systems affect the way we live. Each of these applications has developed a deep DSP technology, with its own algorithms, mathematics, and specialized techniques. DSP has revolutionized many areas in science and engineering.

Introduction to signals and systems

Definition of signals

A signal is defined as any physical quantity with time, space, or any other independent variables. It is defined mathmatically as a function of one or more independent variables as mentioned previously. For instance, the following two functions:

$$x(t) = \frac{4}{5}t + 1$$

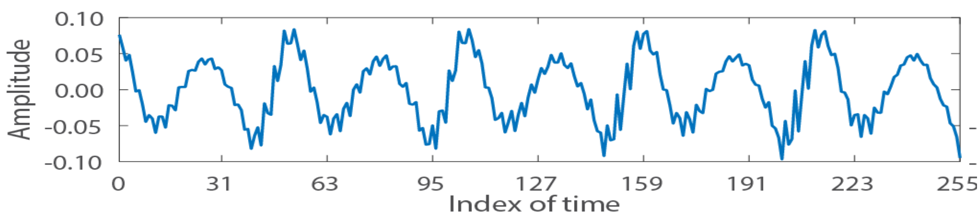
and

$$x(t) = 21t^2$$

The first equation is varing linearly with time (the independet variable) and the second equation changed quardratically with time. Another function with two independent variables is represented by:

$$f(x, y) = 5x + 6xy + 12y^2$$

These two variables could be the two spatial coordinated in the plane. There are some cases where the functional relationship is unknown or too highly complicated to be of any practical use such as speech signal.



Example of speech signal

Associated with natural signals such signals are generated. For example, speech signal is generated by forcing air through the vocal cords. Images are obtained by exposing a photographic film to a scene or an object. Thus signal generation is usually associated with a system that responds to a stimulus or force. For speech signal, the system consists of the vocal cords and the vocal tract, that is the vocal cavity. The stimulus in combination with the system is called a signal source. Thus we have speech source, images source, and other types of signal sources.

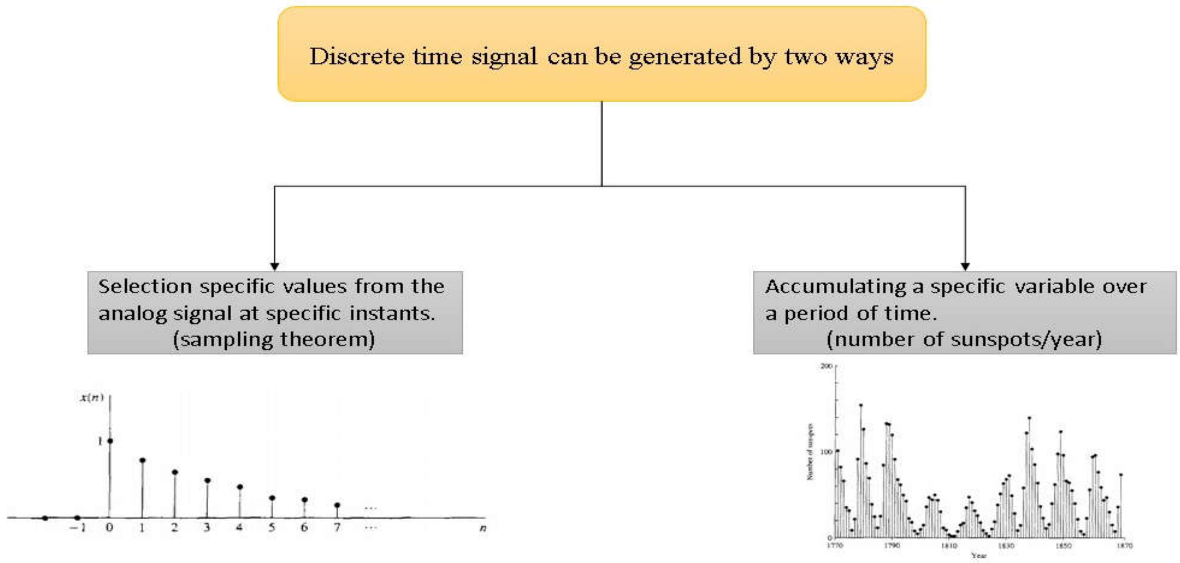
<u>**Classifications of Signals:**</u> Signals can be classified based on different aspects as follows:

1- Continuous-Time vs. Discrete-Time signals.

This classification is determined by whether or not the time axis (x-axis) is discrete (countable) or continuous. Continuous-time signals have values over the whole real numbers along the x-axis (specifically, time axis). While, discrete-time signals are generated via the sampling theorem to produce the sampling signal, which have values at equally spaced intervals. Therefore,these signals are defined only at specific values of time.

For example, the signal $x(t_n) = e^{-|t_n|}, n = 0, \pm 1, \pm 2, \ldots$ is a discrete-time signal. If we use the index n (or k) of the discrete-time instants as the independent variable, the signal value becomes a function of an integer variable (i.e., a sequence of numbers). Thus a discrete-time signal can be represented mathematically by a sequence of real or comlex numbers. Therefore, the discrete-time signal will denote as $x(n)$ instead of $x(t)$. If the time instants t are equally spaced i.e, $(t_n = n\,T)$, the notation $x(\,n\,T)$ is used, where T is the sampling period. So, $x(n) \equiv x(\,n\,T) = x(t)|_{t=nT}$

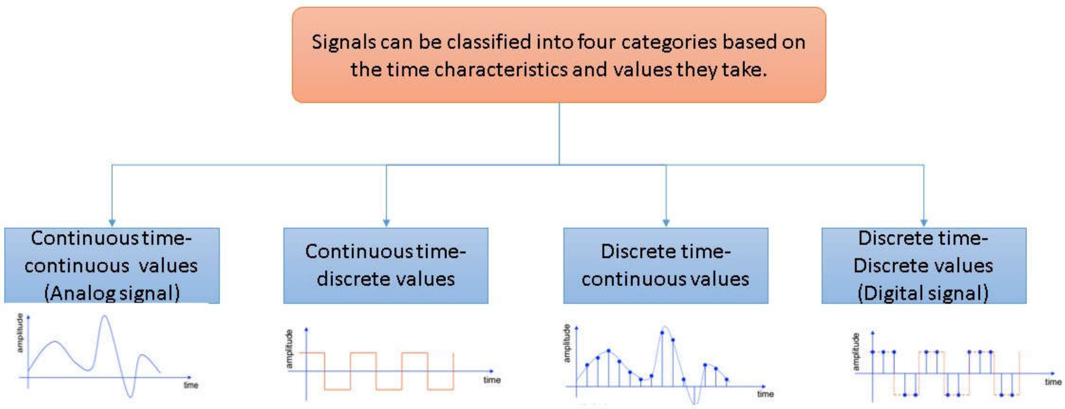
and $x(n) = \{x(0),\ x(1), x(2), \ldots\}$. $x(t) = \{x(0), x(T),\ X(2T), \ldots\}$, For example, $x(2T) = x(t)|_{t=2T} \equiv x(2)$.



2- Continuous-Valued Versus Discrete-Valued Signals

The values of a continuous-time or discrete-time signal can be continuous or discrete. If the signal takes all possible values on a finite or an infinite *range*, then this signal is termed as a continuous-valued signal which corresponds to a continuous y-axis. Otherwise , if the signal takes values from a finite *set of possible values*, then this signal is termed as a discrete-valued signal which corresponds to a discrete y-axis. To make signals processed digitally, they should be discrete in time and their values should be discrete also, which is called **digital signal**. However, to make signals processed in analog form, they need to be sampled and quantized.

Based on two above classification, there are fout other types of classification as follows:



3- Periodic vs. Aperiodic signals

A periodic signals repeat itself with a specific period (time with fixed length), while aperiodic, or nonperiodic, signal does n't repeat itself but it changed over its interval. Mathematically, a periodic function can be written as below:

$$f\ (t)\ =\ f\ (T\ +\ t) = \ f\ (mT\ +\ t)$$

where, (T) is a constant number and m is an integer.

For discrete-time signal $f(n)$, it is periodic with period N if there is a positive integer N that:

$$f[n+N] = f[n] = f[n+mN] \quad for\ all\ n$$

4- **Causal, Anticausal and Noncausal signals**

Causal signals have a zero values for all negative time, whereas anitcausal signals have zero values in the positive time. Finally, noncausal signals have nonzero values for the positive time and negative time.

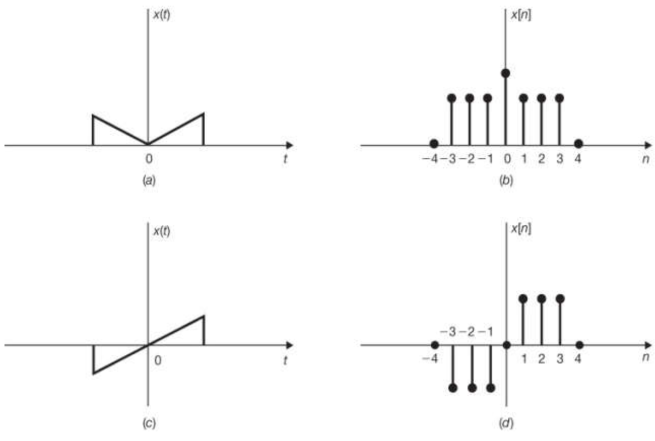
5- **Even vs. Odd functions functions**

The even signal can be defined as any signal x that achives the equation $x\,(t) = x(-t)$. Even signals have graphs that are symmetric around the y-axis like a cosine function. While, odd function must achives the equation $x\,(t) = -\,(x\,(-t))$.

Any signal can be defined as a grouping of the even and odd equations. Which means that, any signal has decomposition of the odd-even equations. In general, the signal can be written as:

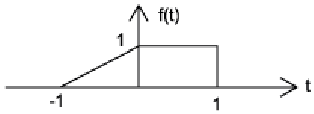
$$x\,(t) = \frac{1}{2}(x(t) + x(-t)) + \frac{1}{2}(x(t) - x(-t))$$

Where, the even part is equal to $x_e(t) = \frac{1}{2}(x(t) + x(-t))$, and the odd part of the signal is equal to $x_o(t) = \frac{1}{2}(x(t) - x(-t))$. An examples of even and odd functions are shown below.

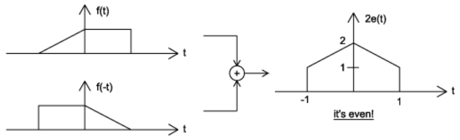
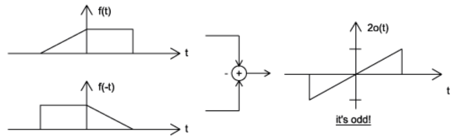


(a and b) Even signals, (c and d) Odd signals

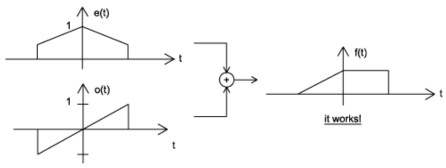
Examples: Draw the even and odd parts for the signal below. Then check your solution.



The solution:

Even part	Odd part
	

The checking:



Note that, the product of any two even signals or of any two odd signals leads to an even signal. On the other hand, the product of any even signals and an odd signals leads to odd signals. Generally, this is applied for both types: continuous or discrete time signals.

6- **Energy signals and Power Signals:**

Signal can also be classified into energy signals and power signals. Energy signal must satisfies the condition $0 < E < \infty$. The energy in continuous time systems is:

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, \mathrm{d}t \, .$$

While, for discrete time systems, the energy is:

$$E = \sum_{-\infty}^{\infty} |x[n]|^2$$

On the other hand, the power signal must satisfy the condition $0 < P < \infty$. To calculate the power for continuous time system, the equation is:

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} | x(t)|^2 \, dt .$$

And the power equation for the discrete systems is:

$$P = \underset{N \to \infty}{Lt} \frac{1}{2N+1} \sum_{-N}^{N} |x[n]|^2$$

Note that the power for an energy signal is zero $(P = 0)$ and that the energy for a power signal is infinite $(E = \infty)$. Some signals are neither energy nor power signals. Note that a periodic signal is a power signal if its energy content per period is finite, and then the average power of this signal need only be calculated over a period

<u>Example</u>: Find if the signal below is energy signal or power signal.

$$x(t) = e^{-t} u(t)$$

<u>Solution:</u>

The energy for the following signal is:

$$E = \int_{-\infty}^{\infty} \left| e^{-t} u(t) \right|^2 dt = \int_{0}^{\infty} e^{-2t} \, dt = -\frac{1}{2} e^{-2t} \Big|_{0}^{\infty} = \frac{1}{2}$$

Therefore, it is energy signal. Since the energy is finite the signal power is zero $(p = 0)$.

There are other classification of the signalssuch as: deterministic vs. Random signals, perodic and non perodic, finite signal and Infinite signal length, … …etc.

<u>References</u>

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