UNIVERSITY OF BAGHDAD
COLLEGE OF ENGINEERING
CIVIL ENGINEERING DEPARTMENT

STRUCTURAL ANALYSIS

JUNIOR COURSE 2016-2017

Instructor

Dr. HAYDER AMER AHMED AL-BAGHDADI

Faculty Member, Civil Engineering Dept., College of Engineering, University of Baghdad
B.Sc. in Civil Engineering, M.Sc., Ph.D. in Structural Engineering
P.E., Member, ASCE
dr.hayder.baghdadi@gmail.com
www.facebook.com/ubstructuralanalysis/

SYLLABUS

1. TYPES OF STRUCTURES AND LOADS.
2. IDEALIZATION AND MODELING OF STRUCTURES.
3. DEFINITIONS AND CONCEPTS.
4. STABILITY AND DETERMINACY.
5. ANALYSIS OF STATICALLY DETERMINATE STRUCTURES.
6. INTERNAL FORCES DEVELOPED IN STRUCTURAL MEMBERS.
7. ANALYSIS OF STATICALLY DETERMINATE TRUSSES.
8. INFLUENCE LINES FOR STATICALLY DETERMINATE STRUCTURES.
9. APPROXIMATE ANALYSIS OF STATICALLY INDETERMINATE STRUCTURES.
10. ELASTIC DEFORMATION OF STATICALLY DETERMINATE STRUCTURES.
11. ANALYSIS OF STATICALLY INDETERMINATE STRUCTURES BY THE FORCE METHOD.
12. ANALYSIS OF STATICALLY INDETERMINATE STRUCTURES BY THE DISPLACEMENT METHOD.
13. MATRIX STRUCTURAL ANALYSIS

RECOMMENDED TEXTBOOK

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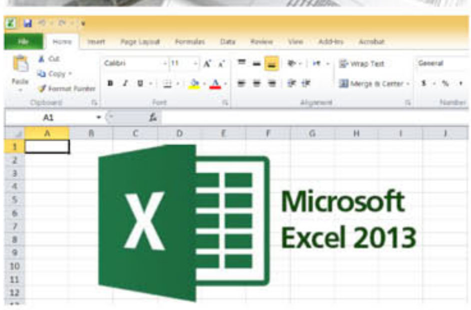
SOFTWARE AND TOOLS

1. **STAAD.PRO®** software is the powerful modeling, analysis and design tool. It is capable of analyzing any structure exposed to static loading, a dynamic response, soil-structure interaction, wind, earthquake, and moving loads.
2. **MATLAB** and **MICROSOFT® EXCEL** are required to perform general algorithms for the modeling and analysis of structural systems.
3. Scientific Calculator is required for arithmetic manipulations.



TYPES OF STRUCTURES AND LOADS 1

1.1 GENERAL

A structure refers to a system of connected parts used to support a load.

Important examples related to civil engineering include:

1. Buildings.
2. Bridges
3. Towers

   Other branches of engineering like,
4. Ship and aircraft frames.
5. Tanks and pressure vessels.
6. mechanical systems and electrical supporting structures

When designing a structure to serve a specified function for public use, the engineer must account for its:

1. Safety (Strength).
2. Esthetics.
3. Serviceability.
4. Durability.
5. Economic.
6. Environmental constraints (Sustainability).

Often this requires several independent studies of different solutions before final judgment can be made as to which structural form is most appropriate.

This design process is both **creative** and **technical** and requires a **fundamental knowledge of material properties** and the **laws** of **mechanics** which govern material response.

Once a preliminary design of a structure is proposed, the structure must then be **analyzed** to ensure that it has its required **stiffness** and **strength**.

To analyze a structure properly, certain idealizations must be made as to how the members are **supported** and **connected together**.

The loadings are determined from

- **Codes**.
- **Local specifications**.

From the theory of structural analysis, the following can be found:

- Forces in the members.
- Displacements.

The results of this analysis then can be used to **design the structure**, accounting for a more accurate determination of the weight of the members and their size.

Referring to Building Information Modeling (BIM) (Figure 1.1) structural analysis is an important part in the detail design and construction phase of the building process.

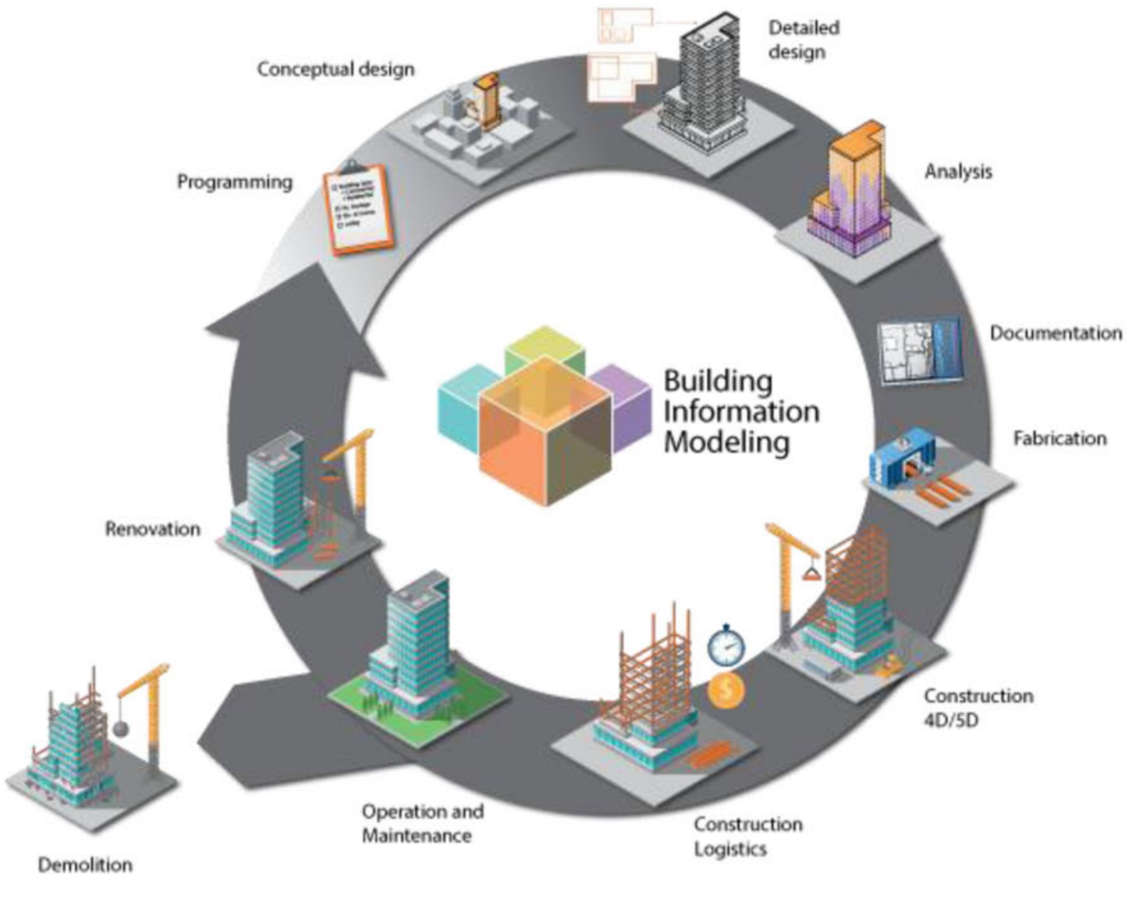


Figure 1.1: Modeling, Analysis and Design Cycle.

1.2 CLASSIFICATION OF STRUCTURES

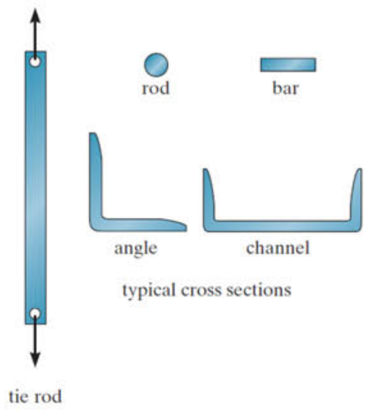
It is important for a structural engineer to recognize the various types of elements composing a structure and to be able to classify structures as to their **form** and **function**..

1.2.1 STRUCTURAL ELEMENTS

Some of the more common elements from which structures are composed are as follows:

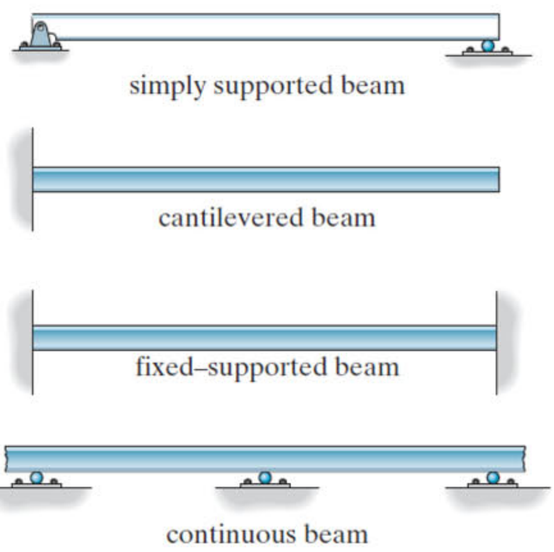
Tie Rods

Structural members subjected to a tensile force are often referred to as tie rods or bracing struts. Due to the nature of this load, these members are rather slender, and are often chosen from rods, bars, angles, or channels.

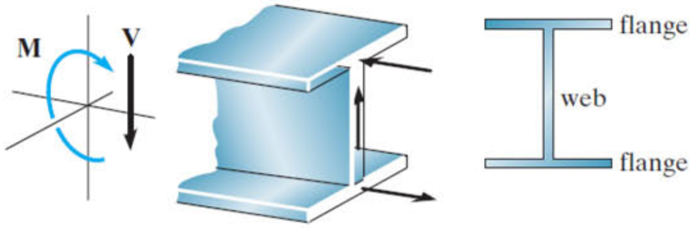


Beams

Beams are usually straight horizontal members used primarily to carry vertical loads. Quite often they are classified according to the way they are supported. In particular, when the cross section varies the beam is referred to as **tapered** or **haunched**.



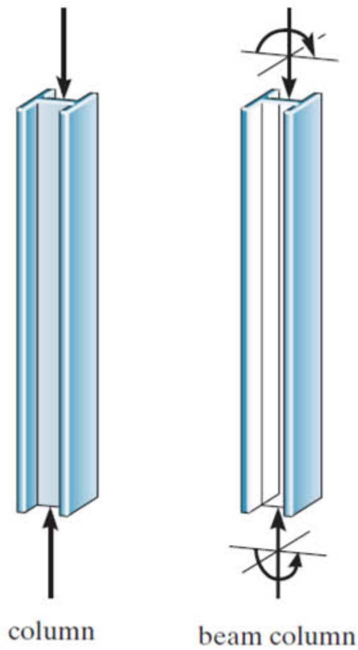
Beam cross sections may also be "**built up**"by adding plates to their top and bottom.



Beams are primarily designed to resist bending **moment**; however, if they are short and carry large loads, the internal **shear** force may become quite large and this force may govern their design.

Columns

Members that are generally **vertical** and resist **axial compressive** loads are referred to as columns. Tubes and wide-flange cross sections are often used for metal columns, and circular and square cross sections with reinforcing rods are used for those made of concrete. Occasionally, columns are subjected to both an **axial** load and a **bending** moment. These members are referred to as **beam-columns**.



1.2.2 TYPES OF STRUCTURES

The combination of structural elements and the materials from which they are composed is referred to as a **structural system**. Each system is constructed of one or more of four basic types of structures. Ranked in order of complexity of their force analysis, they are as follows:

Trusses

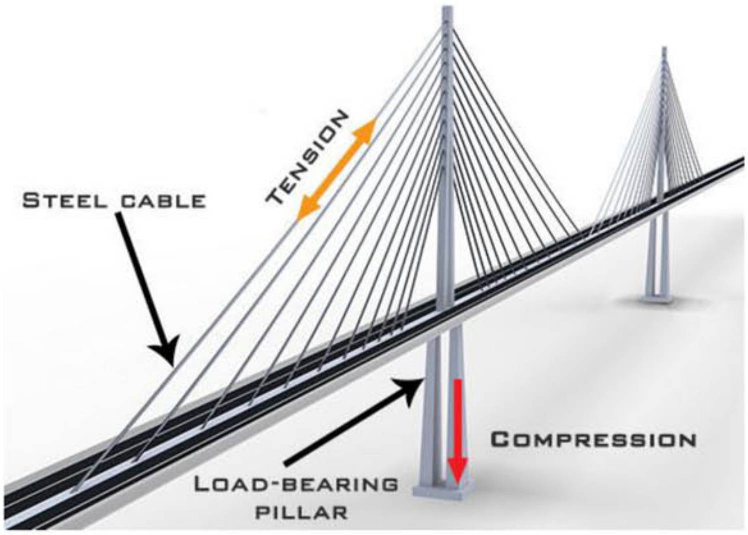
When the span of a structure is required to be large and its depth is not an important criterion for design, a truss may be selected. Trusses consist of **slender elements**, usually arranged in **triangular** fashion. **Planar trusses** are composed of members that lie in the same plane and are frequently used for bridge and roof support, whereas **space trusses** have members extending in three dimensions and are suitable for derricks and towers.

Due to the geometric arrangement of its members, loads that cause the entire truss to bend are converted into **tensile** or **compressive** forces in the members. Because of this, one of the primary advantages of a truss, compared to a beam, is that it uses less material to support a given load.

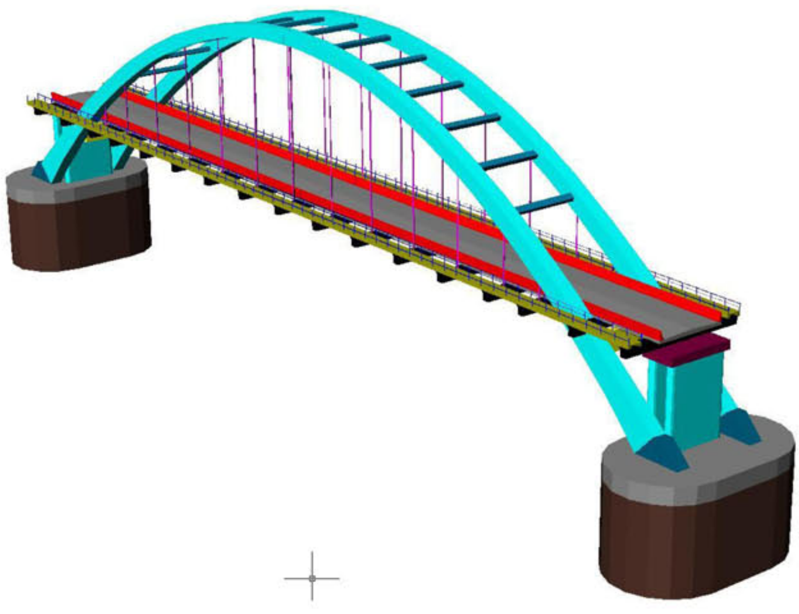
Cables and Arches

Two other forms of structures used to **span long distances** are the **cable** and the **arch**.

Cables are usually flexible and carry their loads in **tension**. They are commonly used to support bridges, and building roofs.

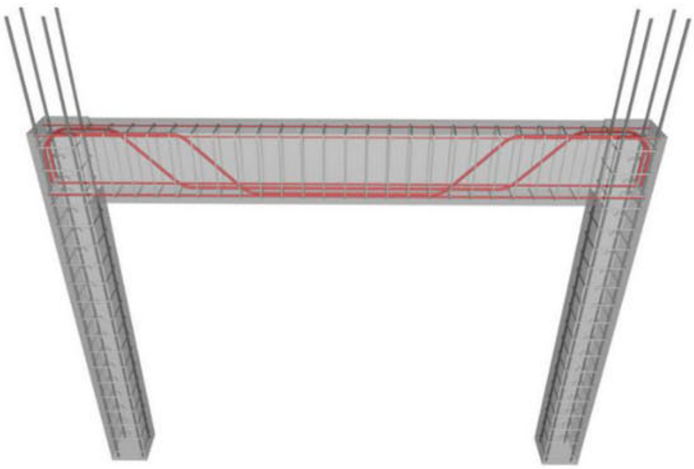


The **arch** achieves its strength in **compression**, since it has a reverse curvature to that of the cable. The arch must be rigid, however, in order to maintain its shape, and this results in secondary loadings involving shear and moment, which must be considered in its design.



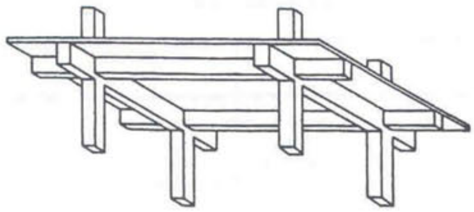
Frames

Frames are often used in buildings and are composed of **beams** and **columns** and also **trusses** that are either **pin** or **fixed** connected. Like trusses, frames extend in two or three dimensions. The loading on a frame causes **bending** of its members, and if it has rigid joint connections, this structure is generally "**indeterminate**" from a standpoint of analysis. The strength of such a frame is derived from the moment interactions between the beams and the columns at the rigid joints.

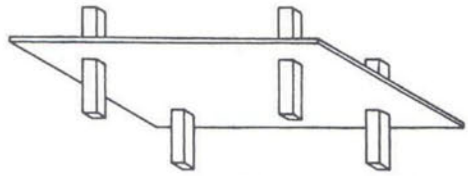


Surface Structures

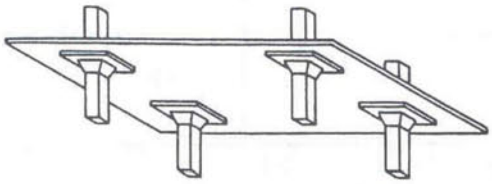
A surface structure is made from a material having a **very small thickness** compared to its other dimensions. Sometimes this material is very flexible and can take the form of a tent or air-inflated structure. In both cases the material acts as a **membrane** that is subjected to pure **tension**. Surface structures may also be made of rigid material such as **reinforced concrete**. As such they may be shaped as folded **plates**, **cylinders**, or **hyperbolic paraboloids**, and are referred to as thin **plates** or **shells**.



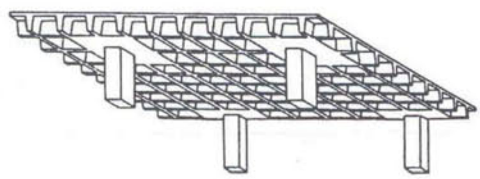
Two-Way Slab



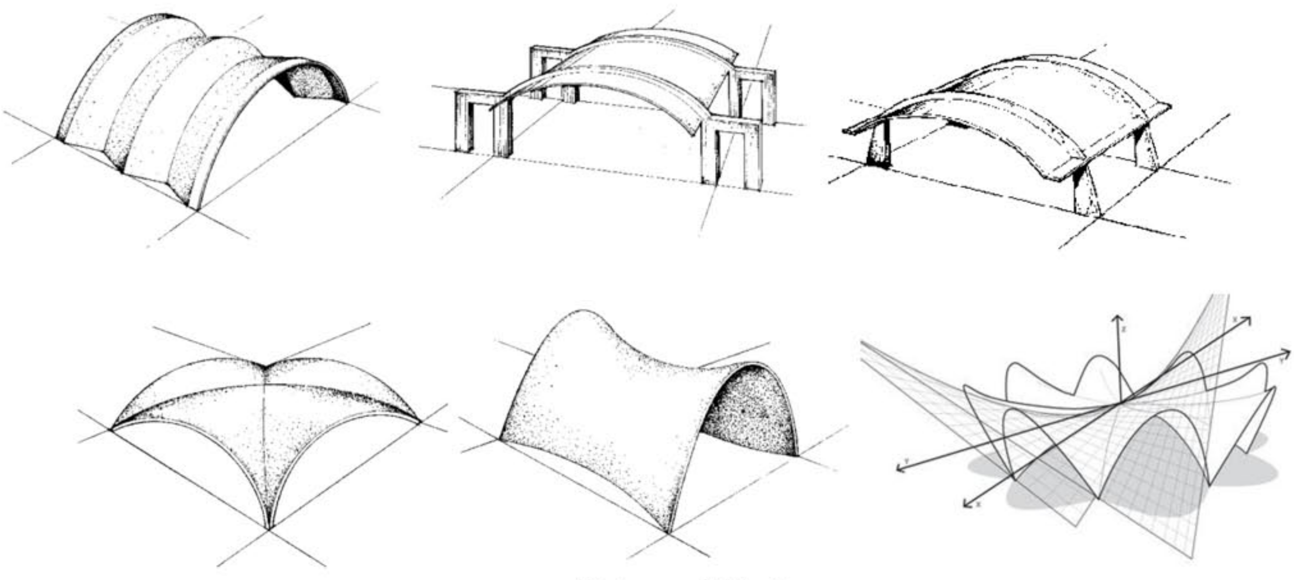
Flat Plate



Flat Slab



Waffle Slab



Plates and Shells

1.3 CODES OF PRACTICE AND SPECIFICATIONS

The design engineer is usually **guided by specifications** called the **codes of practice**.

Engineering specifications are set up by various organizations to represent the **minimum requirements** necessary for the safety of the public, although they are not necessarily for the purpose of restricting engineers.

Most codes specify the followings:

1. Design loads.
2. Allowable stresses.
3. Material quality.
4. Construction types.
5. Other requirements for building construction.

Codes of practice can be summarized as follows:

IBC

Other codes of practice and material specifications in the United States include the **I**nternational **B**uilding **C**ode (**IBC**). The International Building Code (IBC), which was first published in 2000 by the International Code Council, has consolidated the three regional building codes (Building Officials and Code Administrators, International Conference of Building Officials, and Southern Building Code Congress International) into one national document.

The IBC Code is **updated every three years** and refers to the most recent edition of ACI 318 for most of its provisions related to reinforced concrete design, with only a few modifications. It is expected that IBC 2015 will refer to ACI 318-14 for most of its reinforced concrete provisions. The ACI 318 Code is widely accepted in many countries and has had tremendous influence on the concrete codes of all countries throughout the world.

ASCE 7

The **A**merican **S**ociety of **C**ivil **E**ngineers standard ASCE 7. Minimum Design Loads for Buildings and Other Structures, ASCE/SEI 7-10, provides requirements for general structural design and includes means for determining dead, live, soil, flood, snow, rain, atmospheric ice, earthquake, and wind loads, as well as their combinations, which are suitable for inclusion in building codes and other documents.

ACI

The most significant standard for structural concrete design in the United States is the Building Code Requirements for Structural Concrete, **ACI 318**, or the ACI (**A**merican **C**oncrete **I**nstitute) Code.

American Concrete Institute

The ACI Code is not in itself a legally enforceable document. It is merely a statement of current good practice in reinforced concrete design. It is, however, written in the form of a code or law so that various public bodies, such as city councils, can easily vote it into their local building codes, and then it becomes legally enforceable in that area.

ASTM

Specifications issued by the **A**merican **S**ociety for **T**esting and **M**aterials (ASTM). ASTM International is an international standards organization that develops and publishes voluntary consensus technical standards for a wide range of materials, products, systems, and services. ASTM, founded in 1898 as the American Section of the International Association for Testing and Materials, predates other standards organizations such as BSI (1901), DIN (1917), ANSI (1918), AFNOR (1926), and ISO (1947).

ASTM INTERNATIONAL
Standards Worldwide

AASHTO

The **A**merican **A**ssociation of **S**tate **H**ighway and **T**ransportation **O**fficials (AASHTO) specifications. AASHTO provides the general requirements for design and construction of the roads, highways and bridges.

AREA

American **R**ailway **E**ngineering **A**ssociation (AREA), and the Bureau of Reclamation, Department of the Interior. The American Railway Engineering and Maintenance-of-Way Association (AREMA) or the name was shortened to the American Railway Engineering Association (AREA) is a North American railway industry group. It publishes recommended practices for the design, construction and maintenance of railway infrastructure, which are requirements in the United States and Canada.

AISC

The American Institute of Steel Construction, often abbreviated AISC, is a not-for-profit technical institute and trade association for the use of structural steel in the construction industry of the United States. It is headquartered in Chicago. Their mission is to make structural steel the material of choice for new structures. They supply specifications, codes, technical assistance, quality certification, standardization, and market development for its members.

AISC publishes the AISC 360 Specification for Structural Steel Buildings, an authoritative reference in the USA for steel building structure design.

1.4 UNITS OF MEASUREMENT

Two units of measurement are commonly used in the design of structural concrete:

U.S. Customary System

This system is lying mostly in its human scale and its ingenious use of simple numerical proportions.

SI (Syst`eme International d'Unit´es), or **Metric System**,

The metric system is planned to be in universal use within the coming few years. The United States is committed to change to SI units. Great Britain, Canada, Australia, and other countries have been using SI

units for several years. The base units in the SI system are the units of **length**, **mass**, and **time**, which are the **meter** (m), the **kilogram** (kg), and the **second** (s), respectively.
The unit of force, a derived unit called the **newton** (N), is defined as the force that gives the acceleration of one meter per second ($1\ m/s^2$) to a mass of one kilogram, or $1N = 1kg \times m/s^2$.

The weight of a body, W, which is equal to the mass, m, multiplied by the local gravitational acceleration, g ($9.81\ m/s^2$), is expressed in newtons (N).

The weight of a body of $1\ kg$ mass is $W = mg = 1kg \times 9.81\ m/s^2 = 9.81\ N.$

Multiples and submultiples

Multiples and submultiples of the base SI units can be expressed through the use of prefixes.

The prefixes most frequently used in structural calculations are:

$kilo\ (k)$	1000
$mega\ (M)$	$1000{,}000$
$milli\ (m)$	0.001
$micro\ (\mu)$	$0.000\ 001$

For example,

$$1\ km = 1000\ m \qquad 1\ mm = 0.001\ m \qquad 1\mu m = 10^{-6}\ m$$
$$1\ kN = 1000\ N \qquad 1\ Mg = 1000\ kg = 10^6\ g$$

Multiples and sub-multiples

The names of the multiples and sub-multiples of units are formed by means of the prefixes shown in this table.

Factor by which the unit is multiplied	Prefix	Symbol
$1\ 000\ 000\ 000\ 000 = 10^{12}$	tera	T
$1\ 000\ 000\ 000 = 10^9$	giga	G
$1\ 000\ 000 = 10^6$	mega	M
$1\ 000 = 10^3$	kilo	k
$100 = 10^2$	hecto	h
$10 = 10^1$	deca	da
$0.1 = 10^{-1}$	deci	d
$0.01 = 10^{-2}$	centi	c
$0.001 = 10^{-3}$	milli	m
$0.000\ 001 = 10^{-6}$	micro	µ
$0.000\ 000\ 001 = 10^{-9}$	nano	n

1.5 LOADS

Structural members must be designed to support specific loads. Loads are those forces for which a given structure should be proportioned. In general, loads may be classified as follows:

1. Dead Loads.
2. Live Loads.
3. Environmental Loads.

1.5.1 DEAD LOAD

Dead loads include the weight of the structure (its **self-weight**) and any permanent material placed on the structure, such as tiles, roofing materials, and walls (i.e, **superimposed dead load**). Dead loads can be determined with a **high degree of accuracy** from the dimensions of the elements and the unit weight of materials (Table 1.1).

$$Dead\ Load\ = Selfweight\ +\ Superimposed\ dead\ load$$

Table 1.1: Density and Specific Gravity of Various Materials.

Material	Density		Specific Gravity
	lb/ft³	kg/m³	
Building materials			
Bricks	120	1,924	1.8–2.0
Cement, portland, loose	90	1,443	—
Cement, portland, set	183	2,933	2.7–3.2
Earth, dry, packed	95	1,523	—
Sand or gravel, dry, packed	100–120	1,600–1,924	—
Sand or gravel, wet	118–120	1,892–1,924	—
Liquids			
Oils	58	930	0.9–0.94
Water (at 4°C)	62.4	1,000	1.0
Ice	56	898	0.88–0.92
Metals and minerals			
Aluminum	165	2,645	2.55–2.75
Copper	556	8,913	9.0
Iron	450	7,214	7.2
Lead	710	11,380	11.38
Steel, rolled	490	7,855	7.85
Limestone or marble	165	2,645	2.5–2.8
Sandstone	147	2,356	2.2–2.5
Shale or slate	175	2,805	2.7–2.9
Normal-weight concrete			
Plain	145	2,324	2.2–2.4
Reinforced or prestressed	150	2,405	2.3–2.5

1.5.2 LIVE LOADS

Live loads are loads that can change in magnitude and position. They include occupancy loads, warehouse materials, construction loads, overhead service cranes, equipment operating loads, and many others. In general, they are **induced** by **gravity**.

Some typical floor live loads that act on building structures are presented in Table 1.2. These loads, which are taken from **Table 4-1 in ASCE 7-10 (Pages 17 to 19)** act downward and are distributed uniformly over an entire floor.

Table 1.2: Typical Uniformly Distributed Design Loads.

		Design Live Load	
Occupancy	**Contents**	**lb/ft^2**	**kN/m^2**
Assembly hall	Fixed seats	60	2.9
	Movable seats	100	4.8
Hospital	Operating rooms	60	2.9
	Private rooms	40	1.9
Hotel	Guest rooms	40	1.9
	Public rooms	100	4.8
	Balconies	100	4.8
Housing	Private houses and apartments	40	1.9
	Public rooms	100	4.8
Institution	Classrooms	40	1.9
	Corridors	100	4.8
Library	Reading rooms	60	2.9
	Stack rooms	150	7.2
Office building	Offices	50	2.4
	Lobbies	100	4.8
Stairs (or balconies)		100	4.8
Storage warehouses	Light	100	4.8
	Heavy	250	12.0
Yards and terraces		100	4.8

ASCE/SEI
7-10

Table 4-1 Minimum Uniformly Distributed Live Loads, L_o, and Minimum Concentrated Live Loads

Occupancy or Use	Uniform psf (kN/m²)	Conc. lb (kN)
Apartments (see Residential)		
Access floor systems		
Office use	50 (2.4)	2,000 (8.9)
Computer use	100 (4.79)	2,000 (8.9)
Armories and drill rooms	150 (7.18)[a]	
Assembly areas and theaters		
Fixed seats (fastened to floor)	60 (2.87)[a]	
Lobbies	100 (4.79)[a]	
Movable seats	100 (4.79)[a]	
Platforms (assembly)	100 (4.79)[a]	
Stage floors	150 (7.18)[a]	
Balconies and decks	1.5 times the live load for the occupancy served. Not required to exceed 100 psf (4.79 kN/m²)	
Catwalks for maintenance access	40 (1.92)	300 (1.33)
Corridors		
⋮		
Stores		
Retail		
First floor	100 (4.79)	1,000 (4.45)
Upper floors	75 (3.59)	1,000 (4.45)
Wholesale, all floors	125 (6.00)[a]	1,000 (4.45)
Vehicle barriers	See Section 4.5	
Walkways and elevated platforms (other than exit ways)	60 (2.87)	
Yards and terraces, pedestrian	100 (4.79)[a]	

[a]Live load reduction for this use is not permitted by Section 4.7 unless specific exceptions apply.

[b]Floors in garages or portions of a building used for the storage of motor vehicles shall be designed for the uniformly distributed live loads of Table 4-1 or the following concentrated load: (1) for garages restricted to passenger vehicles accommodating not more than nine passengers, 3,000 lb (13.35 kN) acting on an area of 4.5 in. by 4.5 in. (114 mm by 114 mm); and (2) for mechanical parking structures without slab or deck that are used for storing passenger vehicles only, 2,250 lb (10 kN) per wheel.

[c]Design for trucks and buses shall be per AASHTO LRFD Bridge Design Specifications; however, provisions for fatigue and dynamic load allowance are not required to be applied.

[d]Uniform load shall be 40 psf (1.92 kN/m²) where the design basis helicopter has a maximum take-off weight of 3,000 lbs (13.35 kN) or less. This load shall not be reduced.

[e]Labeling of helicopter capacity shall be as required by the authority having jurisdiction.

[f]Two single concentrated loads, 8 ft (2.44 m) apart shall be applied on the landing area (representing the helicopter's two main landing gear, whether skid type or wheeled type), each having a magnitude of 0.75 times the maximum take-off weight of the helicopter and located to produce the maximum load effect on the structural elements under consideration. The concentrated loads shall be applied over an area of 8 in. by 8 in. (200 mm by 200 mm) and shall not be concurrent with other uniform or concentrated live loads.

[g]A single concentrated load of 3,000 lbs (13.35 kN) shall be applied over an area 4.5 in. by 4.5 in. (114 mm by 114 mm), located so as to produce the maximum load effects on the structural elements under consideration. The concentrated load need not be assumed to act concurrently with other uniform or concentrated live loads.

Among the many other types of live loads are:

1. **Traffic loads for bridges**

   Bridges are subjected to series of concentrated loads of varying magnitude caused by groups of truck or train wheels.

2. **Impact loads**

   Impact loads are caused by the vibration of moving or movable loads. It is obvious that a crate dropped on the floor of a warehouse or a truck bouncing on uneven pavement of a bridge causes greater forces than would occur if the loads were applied gently and gradually. Impact loads are equal to the difference between the magnitude of the loads actually caused and the magnitude of the loads had they been dead loads.

3. **Longitudinal loads**

   Longitudinal loads also need to be considered in designing some structures. Stopping a train on a railroad bridge or a truck on a highway bridge causes longitudinal forces to be applied. It is not difficult to imagine the tremendous longitudinal force developed when the driver of a $40\ ton$ trailer truck traveling at $100\ km/h$ suddenly has to apply the brakes while crossing a highway bridge. There are other longitudinal load situations, such as ships running into docks and the movement of traveling cranes that are supported by building frames.

4. **Miscellaneous loads**

   Among the other types of live loads with which the structural designer will have to contend are soil pressures (such as the exertion of lateral earth pressures on walls or upward pressures on foundations), hydrostatic pressures (such as water pressure on dams, inertia forces of large bodies of water during earthquakes, and uplift pressures on tanks and basement structures), blast loads (caused by explosions, sonic booms, and military weapons), and centrifugal forces (such as those caused on curved bridges by trucks and trains or similar effects on roller coasters).

1.5.3 ENVIRONMENTAL LOADS

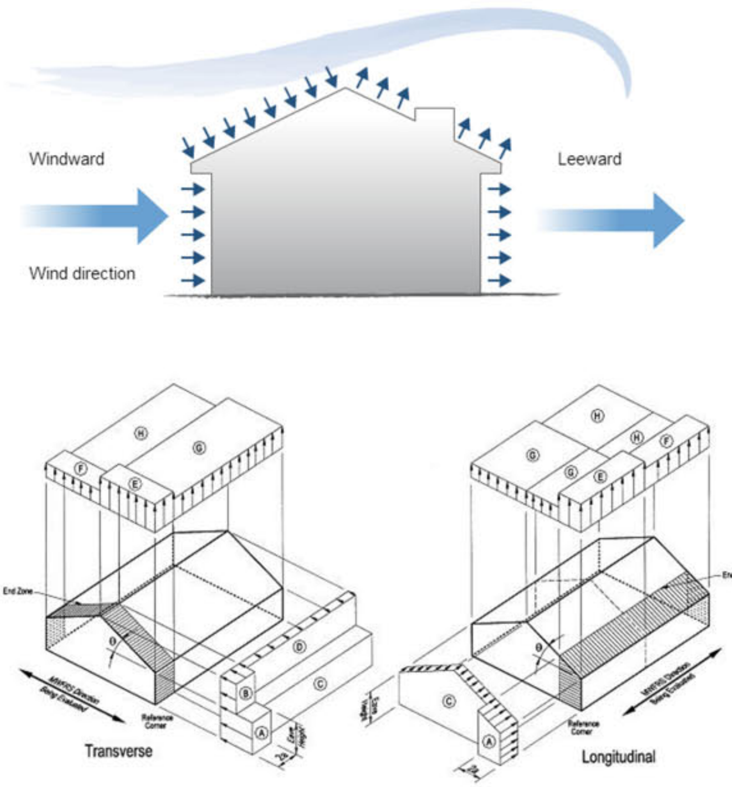
Environmental loads are loads caused by the environment in which the structure is located. For buildings, they are caused by **rain**, **snow**, **wind**, **temperature** change, and **earthquake**. Strictly speaking, these are also live loads, but they are the result of the environment in which the structure is located. Although they do vary with time, they are not all caused by gravity or operating conditions, as is typical with other live loads.

1. Wind Load (Chapters 26 to 31, ASCE 7-10)

The magnitude and duration of wind loads vary with the followings:

1. Geographical locations.
2. The heights of structures aboveground.
3. The types of terrain around the structures.
4. The proximity of other buildings.
5. The location within the structure, and the character of the wind itself.

Chapters 26 to 31 of the ASCE 7-10 specification provide a rather lengthy procedure for estimating the wind pressures applied to buildings. It is worthwhile to mention that in **Iraq** the basic wind speed varies from $120\ km/h$ to $160\ km/h$.



Effect of Wind Loads

2. Seismic Load (Chapters 11 to 23, ASCE 7-10)

Recent earthquakes have clearly shown that the average building or bridge that has not been designed for earthquake forces can be destroyed by an earthquake that is not particularly severe. Most structures can be economically designed and constructed to withstand the forces caused during most earthquakes. The cost of providing seismic resistance to existing structures (called retrofitting), however, can be extremely high.

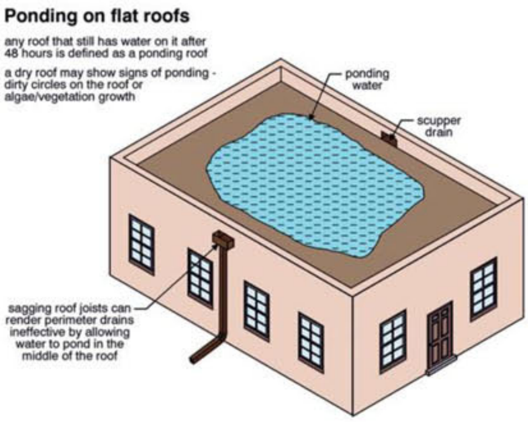
Some engineers seem to think that the seismic loads to be used in design are merely percentage increases of the wind loads. This assumption is incorrect, however, as seismic loads are different in their action and are not proportional to the exposed area of the building but rather are proportional to the distribution of the mass of the building above the particular level being considered.

Another factor to be considered in seismic design is the soil condition. It is well to understand that earthquakes load structures in an indirect fashion. The ground is displaced, and because the structures are connected to the ground, they are also displaced and vibrated. As a result, various deformations and stresses are caused throughout the structures.

Effect of Seismic Loads

3. Rain Load (Chapter 8, ASCE 7-10)

If water on a flat roof accumulates faster than it runs off, the result is called ponding because the increased load causes the roof to deflect into a dish shape that can hold more water, which causes greater deflections, and so on. This process continues until equilibrium is reached or until collapse occurs.



Effect of Rain Load

4. Snow and Ice Loads (Chapter 7, ASCE 7-10)

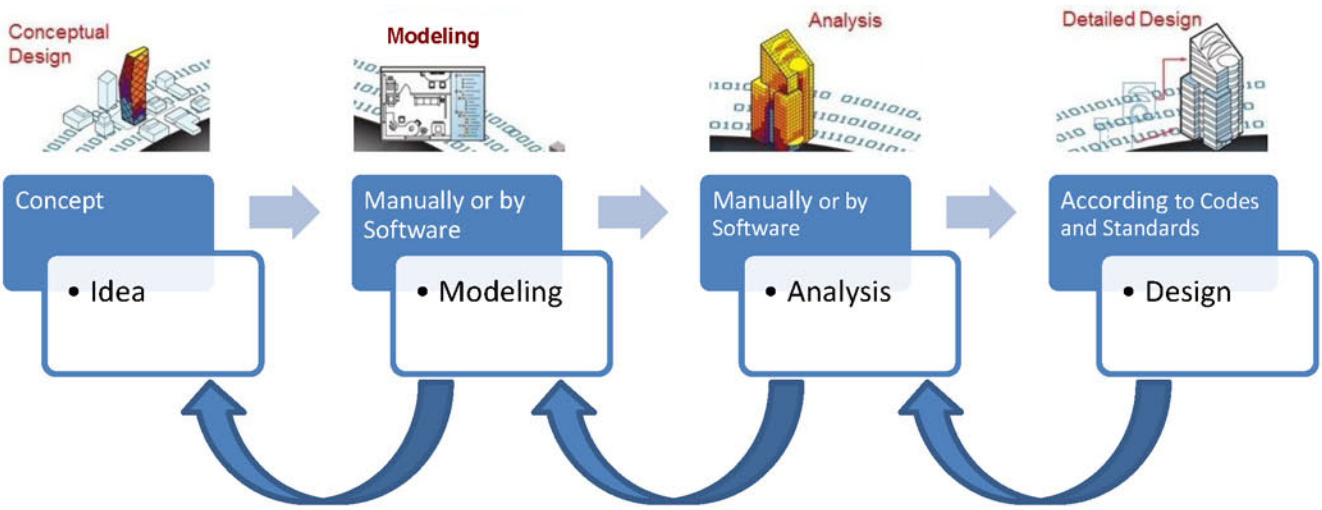
In the colder states, snow and ice loads are often quite important. For example $25\ mm$ of snow is equivalent to approximately $0.024\ kN/m^2$, but it may be higher at lower elevations where snow is denser. For roof designs, snow loads of from $0.48\ kN/m^2$ to $2.0\ kN/m^2$ are used, the magnitude depending primarily on the slope of the roof and to a lesser degree on the character of the roof surface. The larger values are used for flat roofs, the smaller ones for sloped roofs. Snow tends to slide off sloped roofs, particularly those with metal or slate surfaces. A load of approximately $0.48\ kN/m^2$might be used for 45^o slopes, and a $2.0\ kN/m^2$ load might be used for flat roofs.

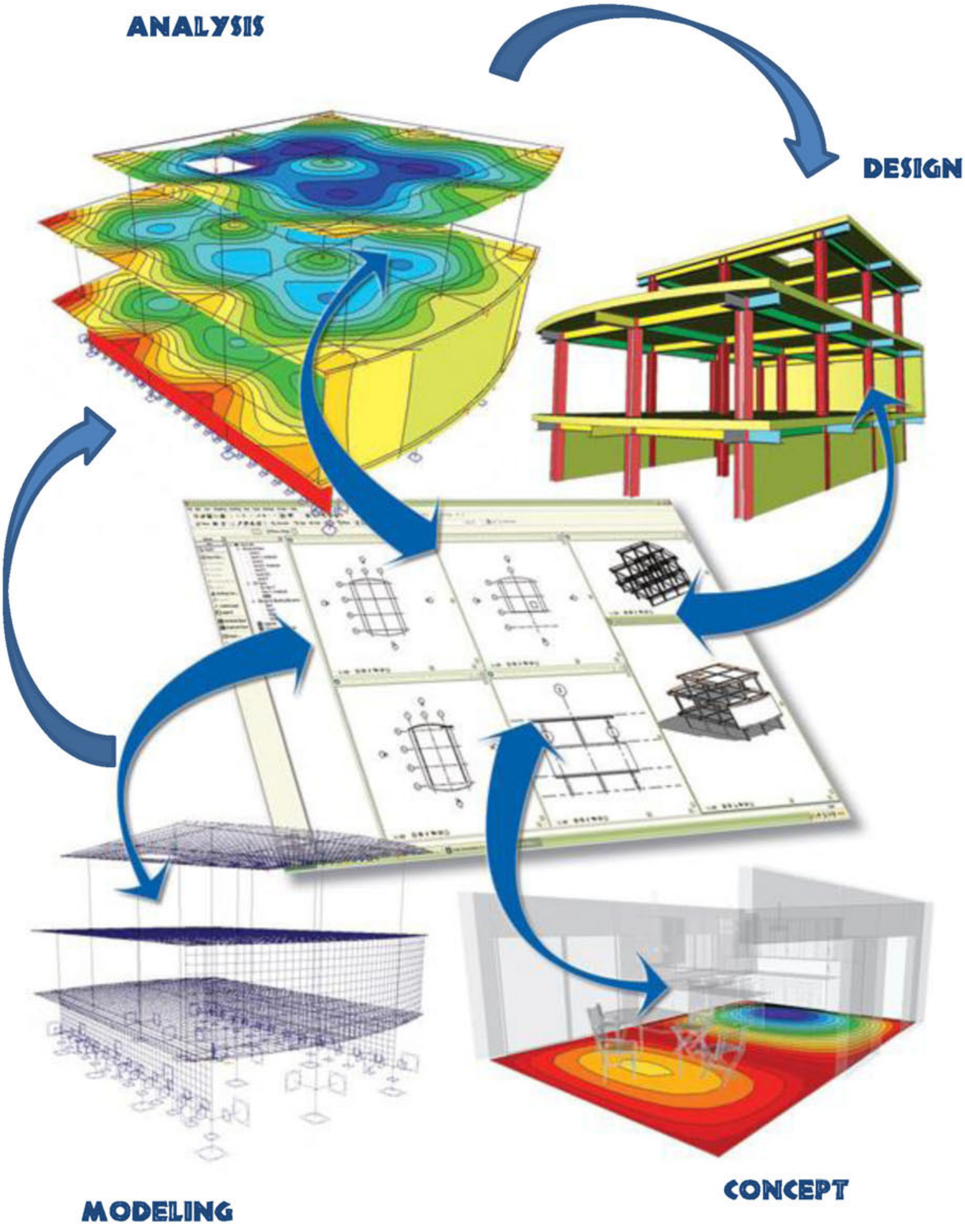
Effect of Snow Load

1.6 STRUCTURAL DESIGN

The first step in the design of a building is the general planning carried out by the **architect** to determine the layout of each floor of the building to meet the owner's requirements. Once the architectural plans are **approved**, the structural engineer then determines the most adequate structural system to ensure the **safety** and **stability** of the building. Different structural options must be considered to determine the most economical solution based on the materials available and the soil condition. This result is normally achieved by:

1. **Idealizing** the building into a structural model of load-bearing frames and elements.
2. Estimating the different **types of loads** acting on the building.
3. Performing the **structural analysis** using computer or manual calculations to determine the maximum **moments**, **shear**, **torsional** forces, **axial** loads, and other forces.
4. Proportioning the different **structural elements** (**design of structural elements**).
5. Producing **structural drawings** and **specifications** with enough details to enable the contractor to construct the building properly.




ANALYSIS
DESIGN
MODELING
CONCEPT

Whenever a structure is designed, it is important to give consideration to:

- **Material**
- **Load uncertainties.**

These **uncertainties** include a possible variability in:

1. Material properties.
2. Residual stress in materials.
3. Intended measurements being different from fabricated sizes.
4. Loadings due to vibration or impact.
5. Material corrosion or decay.

1.6.1 ASD

Allowable-Stress Design (ASD) methods include both the **material** and **load uncertainties** into a **single factor of safety**. For allowable-stress design the computed **elastic stress** in the material must not exceed the **allowable stress** for each of various load combinations.

$$\text{elastic stress} \leq \text{allowable stress}$$

Typical load combinations as specified by the ASCE 7-10 Standard include:

- $Dead\ load$
- $0.6 \times\ (Dead\ Load) + 0.6 \times\ (Wind\ Load)$
- $0.6 \times\ (Dead\ Load) + 0.7 \times\ (Earthquake\ load)$

1.6.2 LRFD.

Since **uncertainty** can be considered using **probability theory**, there has been an increasing trend to separate material uncertainty from load uncertainty. This method is called **Strength Design** or **LRFD** (**Load and Resistance Factor Design**). For example, to account for the uncertainty of loads, this method uses **load factors** applied to the **loads** or **combinations** of loads. According to the ASCE 7-10 Standard, some of the load factors and combinations are:

- $1.4\ \times (Dead\ Load)$
- $1.2 \times\ (Dead\ Load) +\ 1.6\ \times (Live\ Load) +\ 0.5\ \times (Snow\ Load)$
- $0.9\ \times (Dead\ Load) +\ 1.0 \times\ (Wind\ Load)$
- $0.9\ \times (Dead\ Load) +\ 1.0 \times\ (Earthquake\ Load)$

EXAMPLES

Example 1.1 — Calculation of Loads

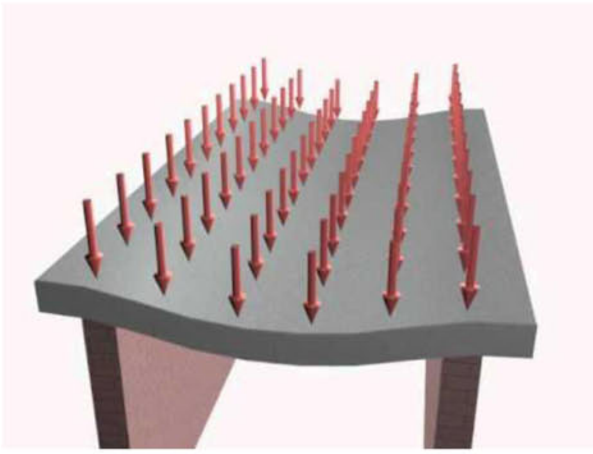
The floor of the office building is made of 150 mm thick normal-weight concrete. If the office floor is a slab having a length of 6 m and width of 4.5 m, determine the resultant force caused by the dead load and the live load.

Solution

From Table 1.1, $\rho_{Concrete} = 2400\ kg/m^3$ and $\gamma_{Concrete} = 24\ kN/m^3$

$DL\ (Force/Area) = \gamma_{Concrete} \times t_{slab} = 24 \times 0.15 = 3.6\ kN/m^2$

$DL(Force) = 3.6 \times 6 \times 4.5 = 97.2\ kN = 9.72\ ton$ ■



$LL(Force/Area) = 2.4\ kN/m^2$ from Table 1.2

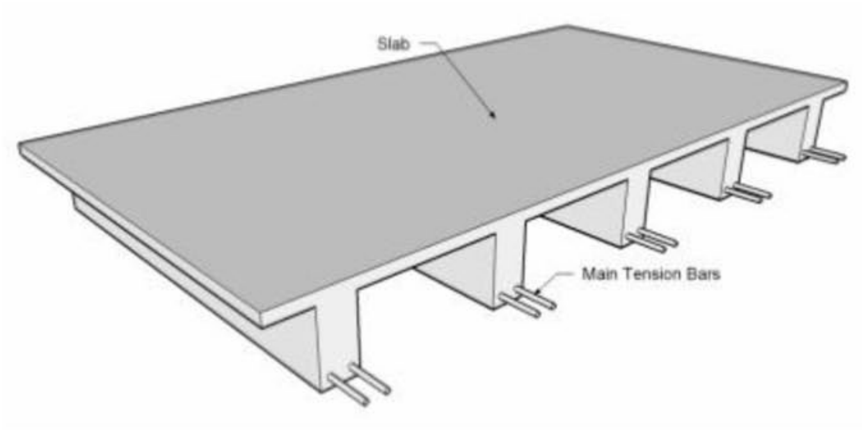
$LL(Force) = 2.4 \times 6 \times 4.5 = 64.8\ kN = 6.48\ ton$ ■

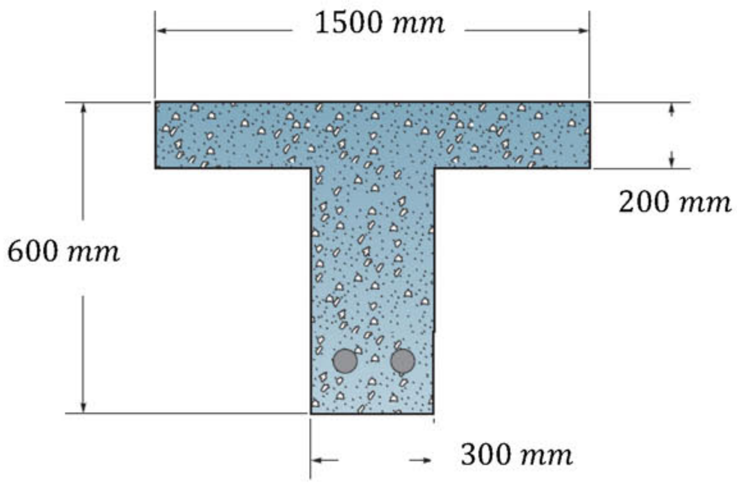
Then the service total load is

$SL = 97.2 + 64.8 = 162\ kN = 16.2\ ton$ ■

Example 1.2 — Calculation of Loads

The T-beam is made from concrete having a specific weight of 24 kN/m^3. Determine the dead load per meter length of beam and the total weight of T-beam if the length is 6 m. Neglect the weight of the steel reinforcement.





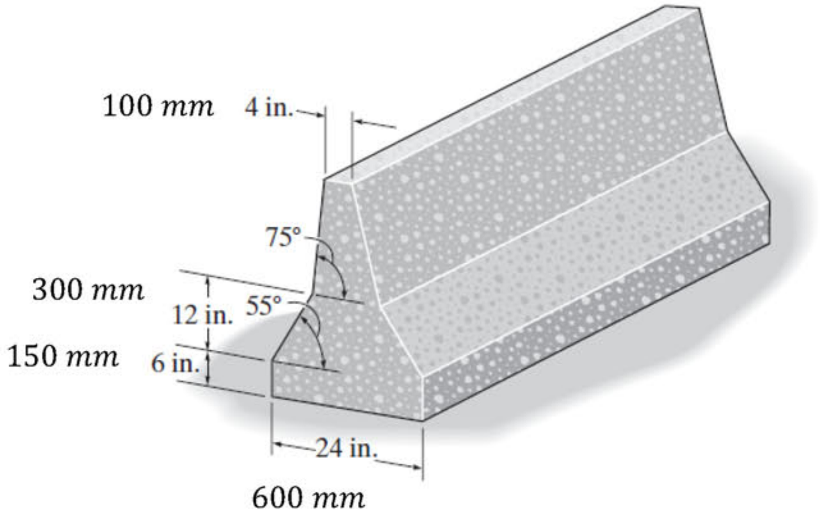
Solution

$$DL\ (Force/Length) = 24 \times [(1.5 \times 0.2) + (0.6 - 0.2) \times (0.3)] = 10.08\ kN/m\ ■$$

$$DL\ (Force) = 10.08 \times 6 = 60.48\ kN = 6.05\ ton\ ■$$

Example 1.3 — Calculation of Loads

The "New Jersey" barrier is commonly used during highway construction. Determine its weight per meter of length if it is made from plain stone concrete.



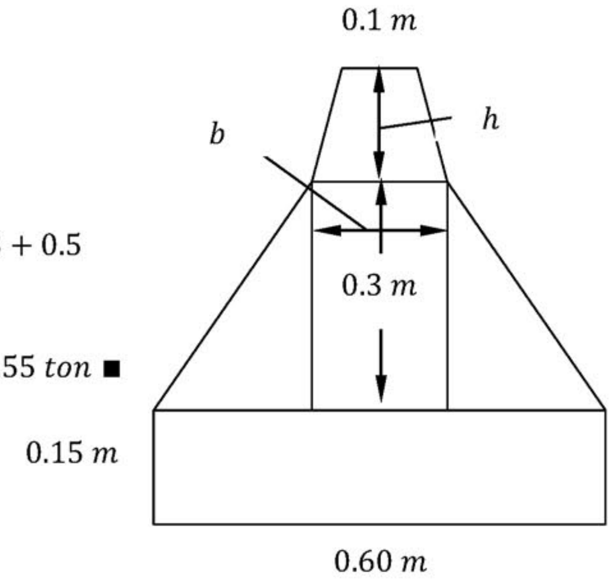
Solution

$b = 0.6 - 2 \times [0.3/\tan(55)] = 0.18\, m$

$h = \dfrac{0.18 - 0.1}{2} \tan(75) = 0.149\, m$

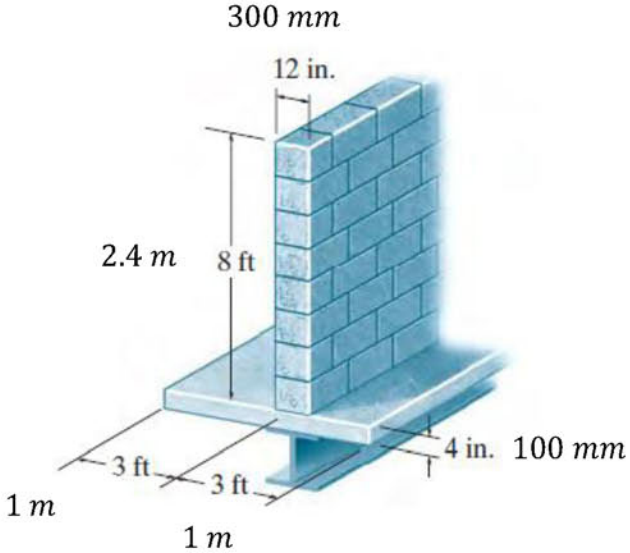
$Cross - sectional\ area$
$= 0.15 \times 0.6 + 0.5 \times (0.18 + 0.6) \times 0.3 + 0.5 \times (0.3 + 0.1) \times 0.149 = 0.237\, m^2$

$DL\ (Force/Length) = 23 \times 0.237 = 5.45\, kN/m = 0.55\, ton$ ■



Example 1.4 — Calculation of Loads

The floor beam shown is used to support the $2\ m$ width of a normal-weight concrete slab having a thickness of $100\ mm$. The slab serves as a portion of the ceiling for the floor below, and therefore its bottom is coated with plaster (assume the plaster load is $0.24\ kPa$). Furthermore, an $2.4\ m$-high, $300\ mm$-thick lightweight solid concrete block wall is directly over the top flange of the beam. Determine the loading on the beam measured per meter of length of the beam.



Solution

Concrete slab	$24\ kN/m^3 \times 2\ m \times 0.1\ m$	$= 4.8\ kN/m$
Plaster ceiling	$0.24\ kN/m^2 \times 2\ m$	$= 0.48\ kN/m$
Block wall	$23\ kN/m^3 \times 2.4\ m \times 0.3\ m$	$= 16.56\ kN/m$
Total load		$21.84\ kN/m$ ■

HOME WORKS

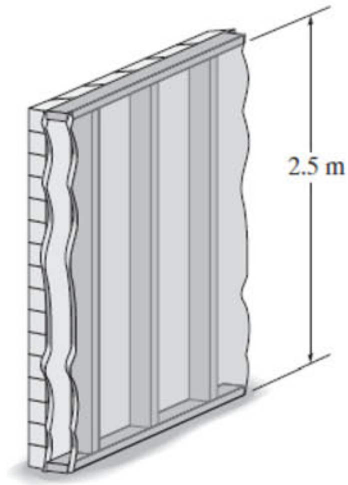
H.W 1.1 — Calculation of Loads

The wall is 2.5 m high and consists of 51 mm × 102 mm studs plastered on one side. On the other side is 13 mm fiberboard, and 102 mm clay brick. Determine the average load in kN/m of length of wall that the wall exerts on the floor. For dead load calculations, use the following table.

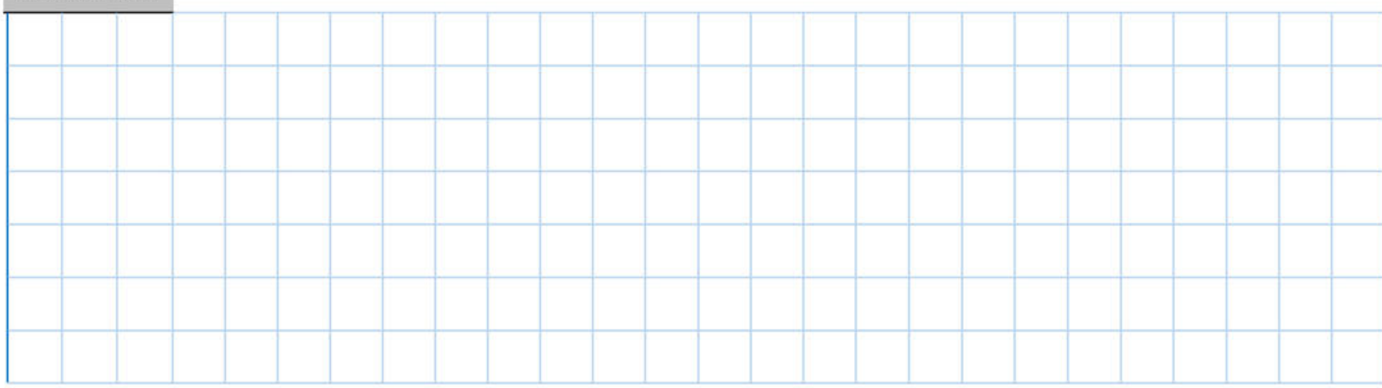
TABLE 1–3 Minimum Design Dead Loads*

Walls	psf	kN/m^2
4-in. (102 mm) clay brick	39	1.87
8-in. (203 mm) clay brick	79	3.78
12-in. (305 mm) clay brick	115	5.51
Frame Partitions and Walls		
Exterior stud walls with brick veneer	48	2.30
Windows, glass, frame and sash	8	0.38
Wood studs 2 × 4 in., (51 × 102 mm) unplastered	4	0.19
Wood studs 2 × 4 in., (51 × 102 mm) plastered one side	12	0.57
Wood studs 2 × 4 in., (51 × 102 mm) plastered two sides	20	0.96
Floor Fill		
Cinder concrete, per inch (mm)	9	0.017
Lightweight concrete, plain, per inch (mm)	8	0.015
Stone concrete, per inch (mm)	12	0.023
Ceilings		
Acoustical fiberboard	1	0.05
Plaster on tile or concrete	5	0.24
Suspended metal lath and gypsum plaster	10	0.48
Asphalt shingles	2	0.10
Fiberboard, $\frac{1}{2}$-in. (13 mm)	0.75	0.04

*Reproduced with permission from American Society of Civil Engineers *Minimum Design Loads for Buildings and Other Structures*, ASCE/SEI 7-10.

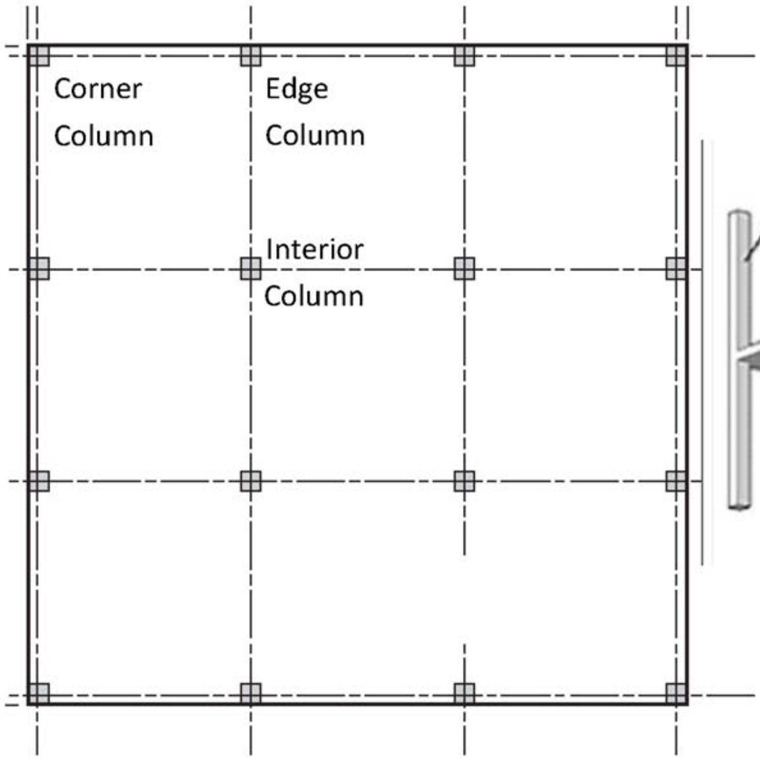


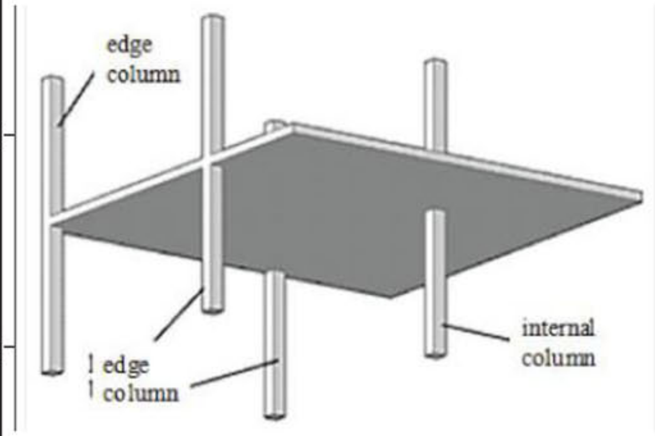
Solution



H.W. 1.2 — Calculation of Loads

A two-story office building shown in the figure has interior, exterior and edge columns that are spaced $6.5\ m$ apart in two perpendicular directions. If the flat slab thickness is $250\ mm$, determine the service load (dead load plus live load) supported by a typical interior, edge and corner column located at ground level.

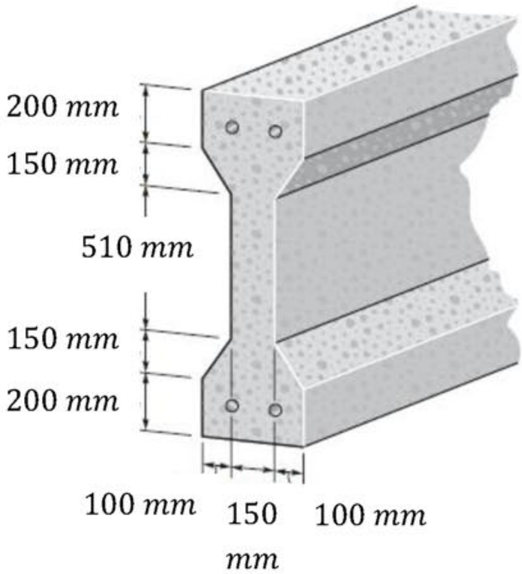




Solution

H.W. 1.3 — Calculation of Loads

For the prestressed concrete girder shown. Determine the dead weight of the girder per meter of its length.



Solution

IDEALIZATION AND MODELING OF STRUCTURES

2

2.1 GENERAL

An exact analysis of a structure can never be carried out, since estimates always have to be made of the loadings and the strength of the materials composing the structure.

Furthermore, points of application for the loadings must also be estimated. It is important, therefore, that the **structural engineer** develop the ability to **model** or **idealize** a structure so that he or she can perform a practical force analysis of the members.

2.2 SUPPORT CONNECTIONS

Structural members are joined together in various ways depending on the intent of the designer.

The three types of joints most often specified:

1. The pin connection.
2. The roller support.
3. The fixed joint.

Examples of these joints, fashioned in **metal** and **concrete**, are shown in Figure 2.3 and Figure 2.2, respectively.

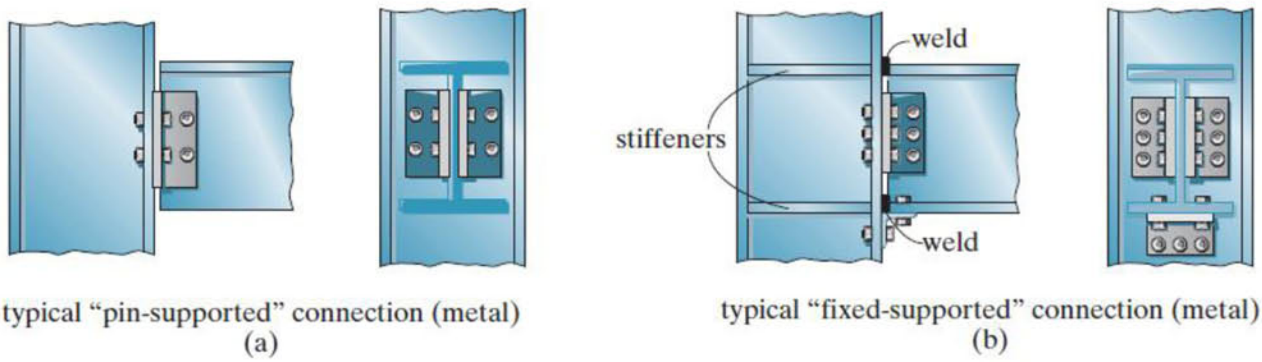
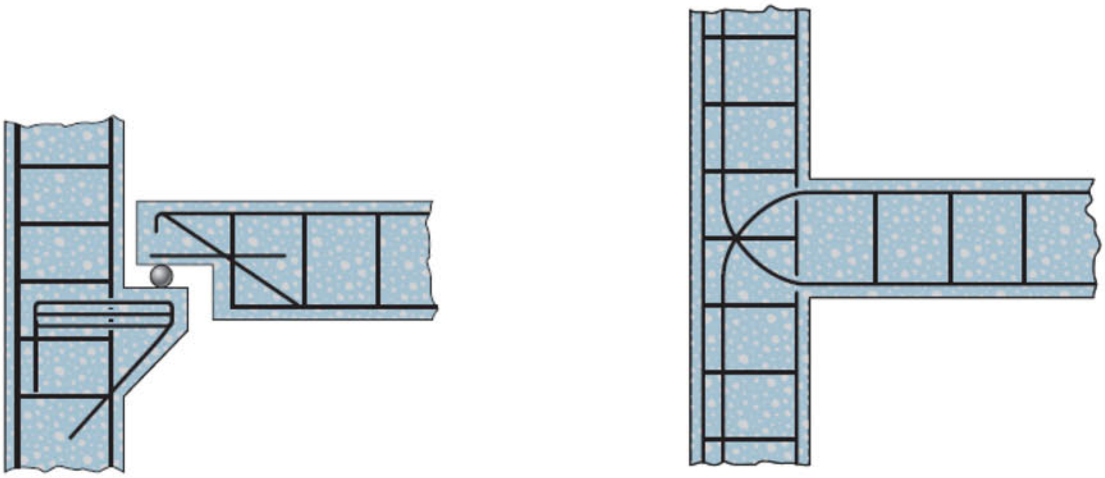


Figure 2.1: Common Real Connections in Metal.



typical "roller-supported" connection (concrete)
(a)

typical "fixed-supported" connection (concrete)
(b)

Figure 2.2: Common Real Connections in Concrete.

In reality, however, **all connections exhibit some stiffness toward joint rotations**, owing to friction and material behavior. In this case a more appropriate model for a support or joint might be that shown in ***Error! Reference source not found.***.If the torsional spring constant the joint is a pin, and if the joint is fixed.

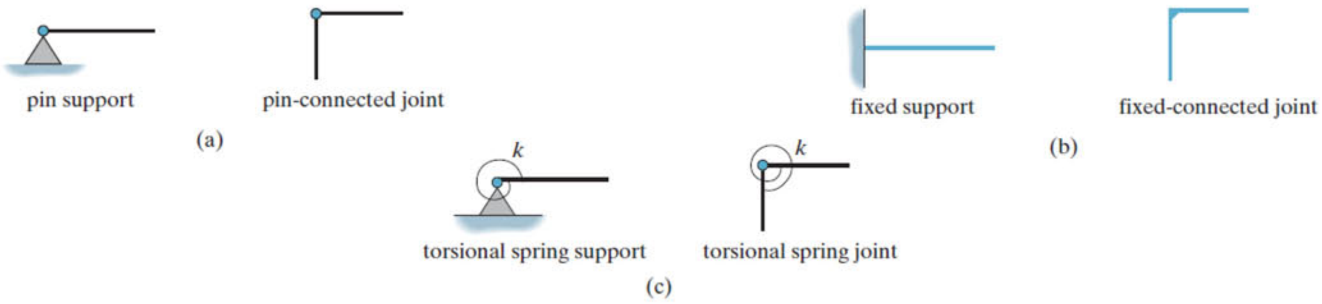


Figure 2.3: Modeling of Supports and Connections.

When selecting a particular model for each support or joint, the engineer must be aware of how the assumptions **will affect** the **actual performance** of the member and whether the assumptions are reasonable for the structural design.

For example, consider the beam shown in Figure 2.4a, which is used to support a concentrated load P.

- The angle connection at support A can be idealized as a typical pin support.
- Furthermore, the support at B provides an approximate point of smooth contact and so it can be idealized as a roller.
- The beam's thickness can be neglected since it is small in comparison to the beam's length, and therefore the idealized model of the beam is as shown in Figure 2.4b.
- The analysis of the loadings in this beam should give results that closely approximate the loadings in the actual beam.

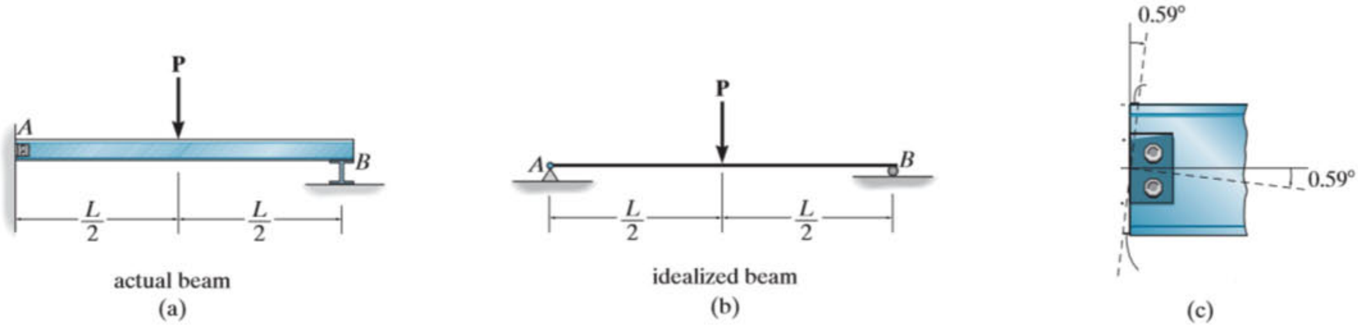


Figure 2.4: Idealization of Simple Beam.

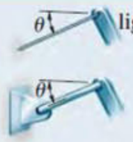
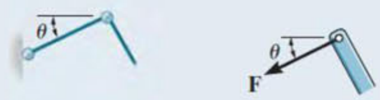
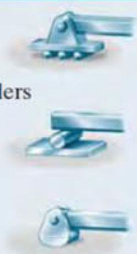
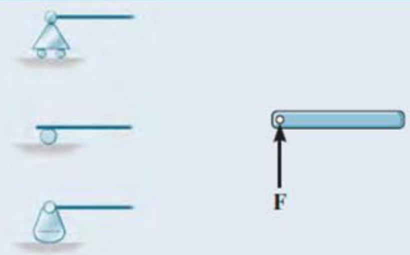
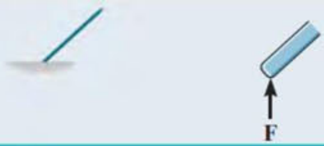
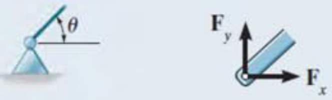
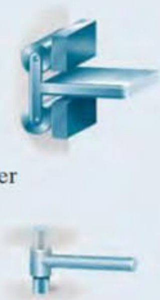
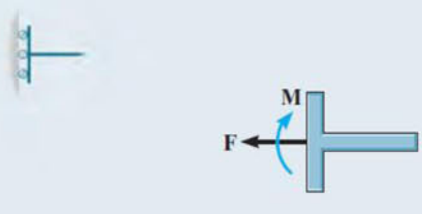
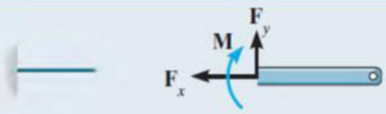
Figure 2.5: Shear Supports in Reality.

Figure 2.6: Shear Connections in Reality.

Figure 2.7: Shear and Moment Connections in Reality.

Other types of connections most commonly encountered on coplanar structures are given in Table 2–1.

TABLE 2–1 Supports for Coplanar Structures

Type of Connection	Idealized Symbol	Reaction	Number of Unknowns
(1)  light cable; weightless link			One unknown. The reaction is a force that acts in the direction of the cable or link.
(2)  rollers; rocker			One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.
(3)  smooth contacting surface			One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.
(4)  smooth pin-connected collar			One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.
(5)  smooth pin or hinge			Two unknowns. The reactions are two force components.
(6)  slider; fixed-connected collar			Two unknowns. The reactions are a force and a moment.
(7)  fixed support			Three unknowns. The reactions are the moment and the two force components.

2.3 IDEALIZATION OF STRUCTURES

2.3.1 MATHEMATICAL MODELING

The purpose of mathematical modelling is to predict structural behavior in terms of loads, stresses and deformations under any specified, externally applied force system.

Since actual structures are **physical, three-dimensional entities** it is necessary to create an idealized model which is representative of:

- The materials used.
- The geometry of the structure.
- The physical constraints (e.g. the support conditions and the externally applied force system).

The precise idealization adopted in a particular case is dependent on the complexity of the structure and the level of the required accuracy of the final results. The idealization can range from **simple 2-Dimensional** 'beam-type' and 'plate' elements for pin-jointed or rigid jointed plane frames and space frames to more **sophisticated 3-Dimensional** elements such as those used in grillages or finite element analyses adopted when analyzing for example bridge decks, floor-plates or shell roofs. It is essential to recognize that irrespective of how advanced the analysis method is, it is always an approximate solution to the real behavior of a structure.

In some cases the approximation reflects very closely the actual behavior in terms of both stresses and deformations whilst in others, only one of these parameters may be accurately modeled or indeed the model may be inadequate in both respects resulting in the need for the physical testing of scaled models.

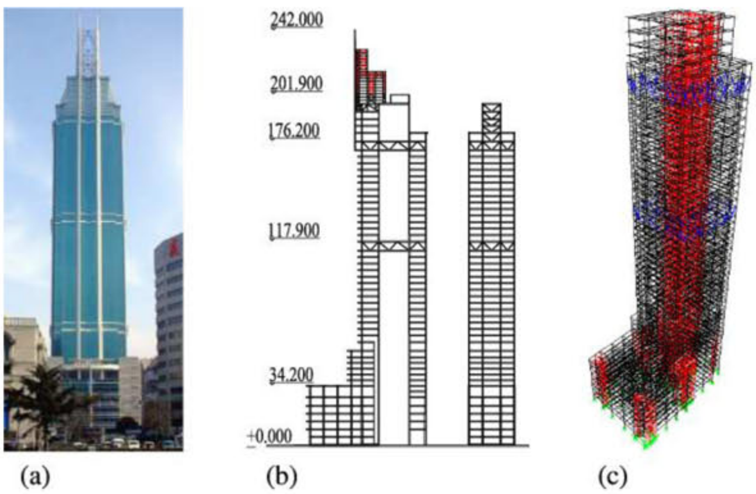


Figure 2.8: Modeling of Multi-Story Building, (a) Reality, (b) 2D Model, (c) 3D Model.

2.3.2 LINE DIAGRAMS

When modeling it is necessary to represent the form of an actual structure in terms of idealized structural members, e.g. in the case of plane frames as beam elements, in which the beams, columns, slabs etc. are indicated by line diagrams.

The lines normally coincide with one of the following lines of the members:

- Center Line.
- Centroid.
- Center of gravity.
- Neutral axis.

A number of such line diagrams for a variety of typical plane structures is shown in Figure 2.9 to Figure 2.18 In some cases it is sufficient to consider a section of the structure and carry out an approximate analysis on a **sub-frame** as indicated in Figure 2.19 and Figure 2.20.

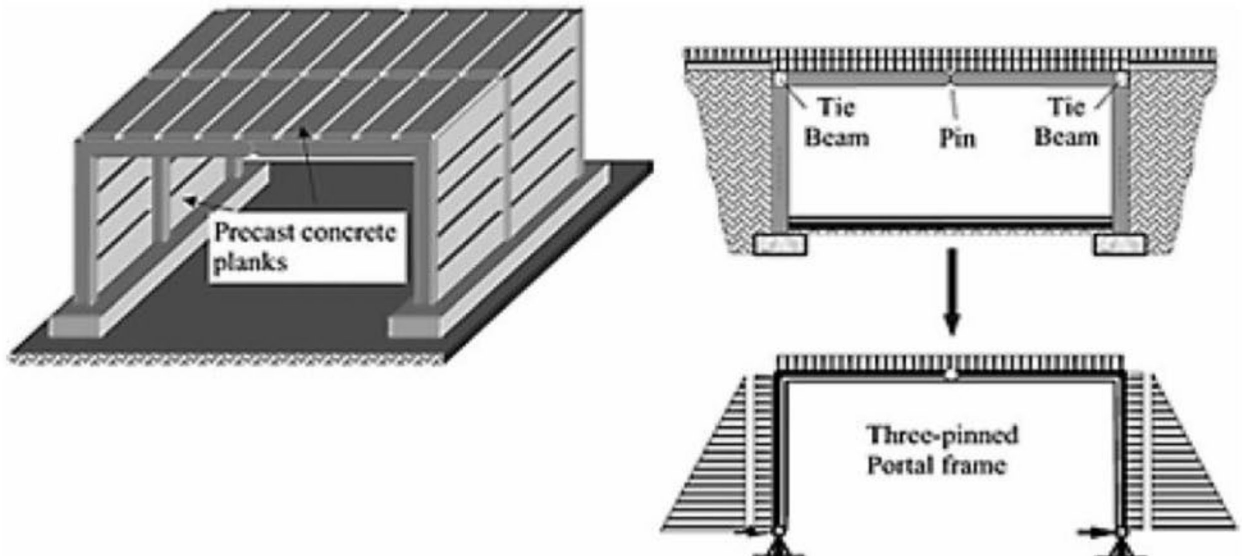


Figure 2.9: Modeling Underground Tunnel.

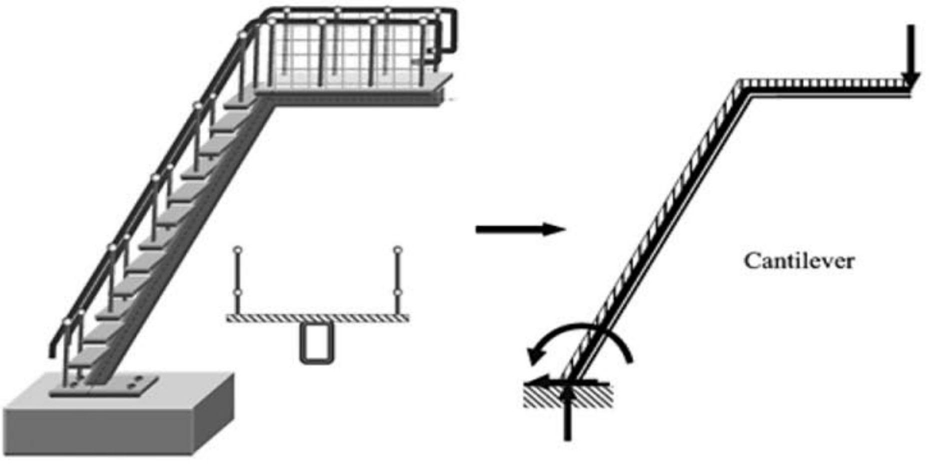


Figure 2.10: Cantilever Staircase in a Swimming pool.

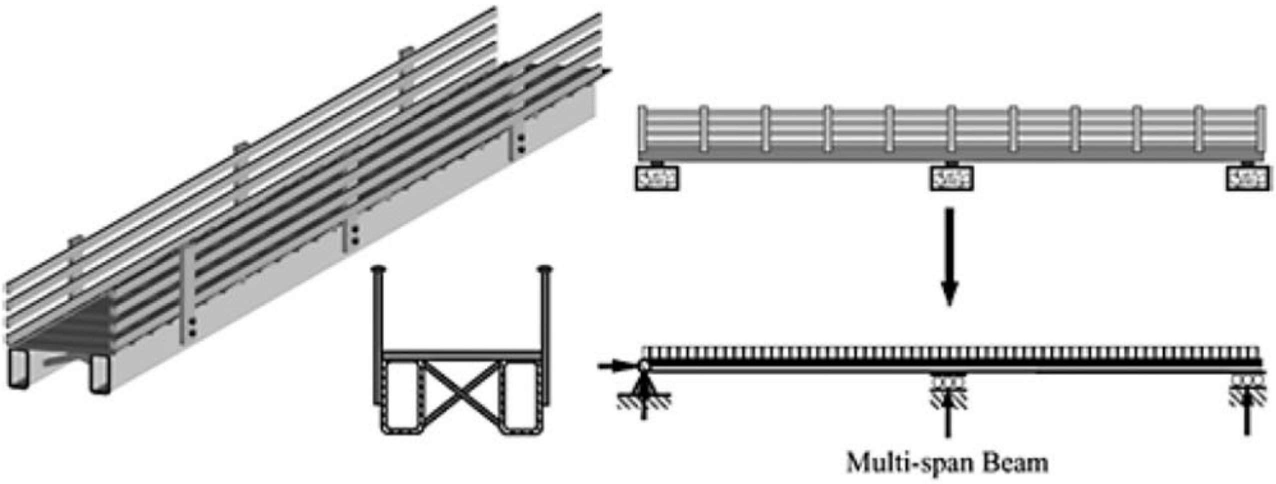


Figure 2.11: Idealization of a Pedestrian Bridge.

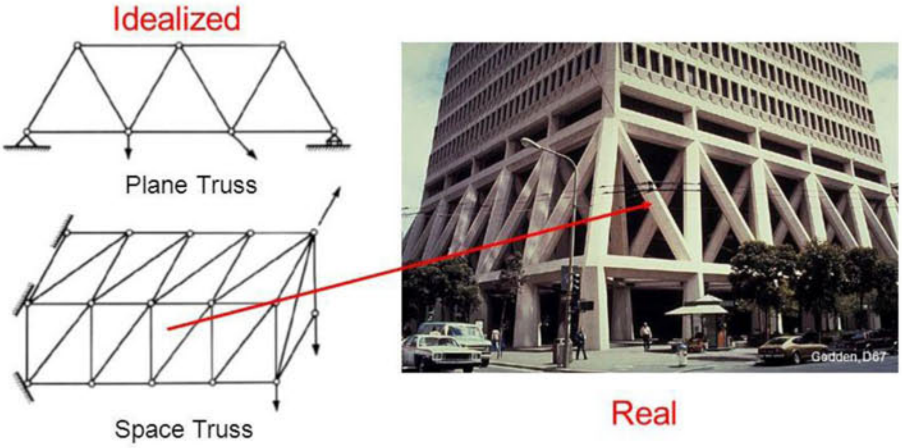


Figure 2.12: Idealization of a Truss in a Building.

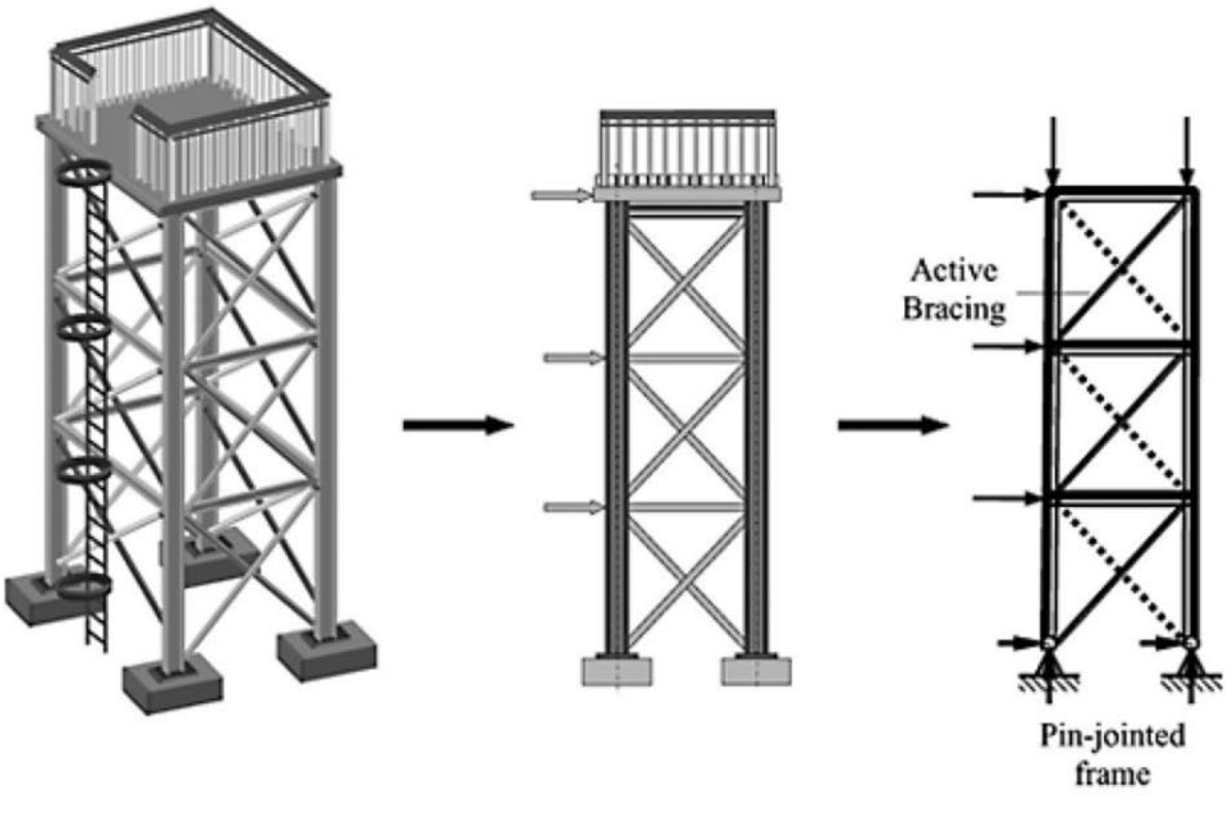


Figure 2.13: Idealization of a Steel Observation Tower.

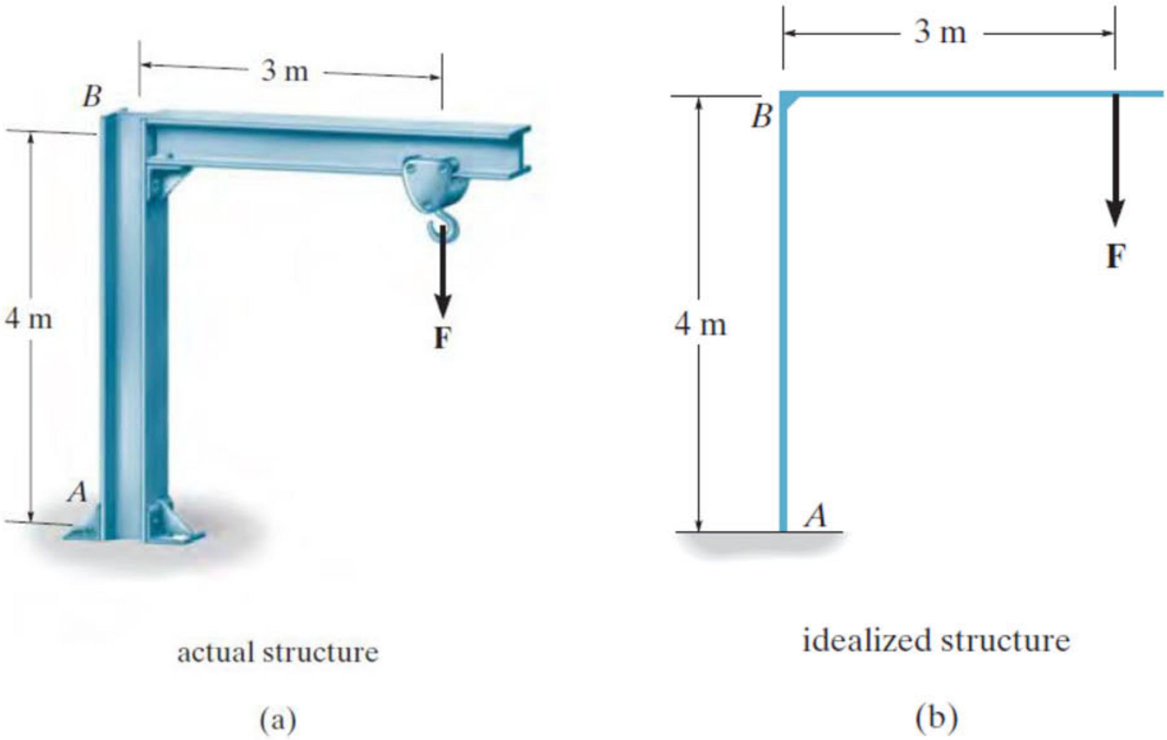


Figure 2.14: Idealization of a Steel Jib.

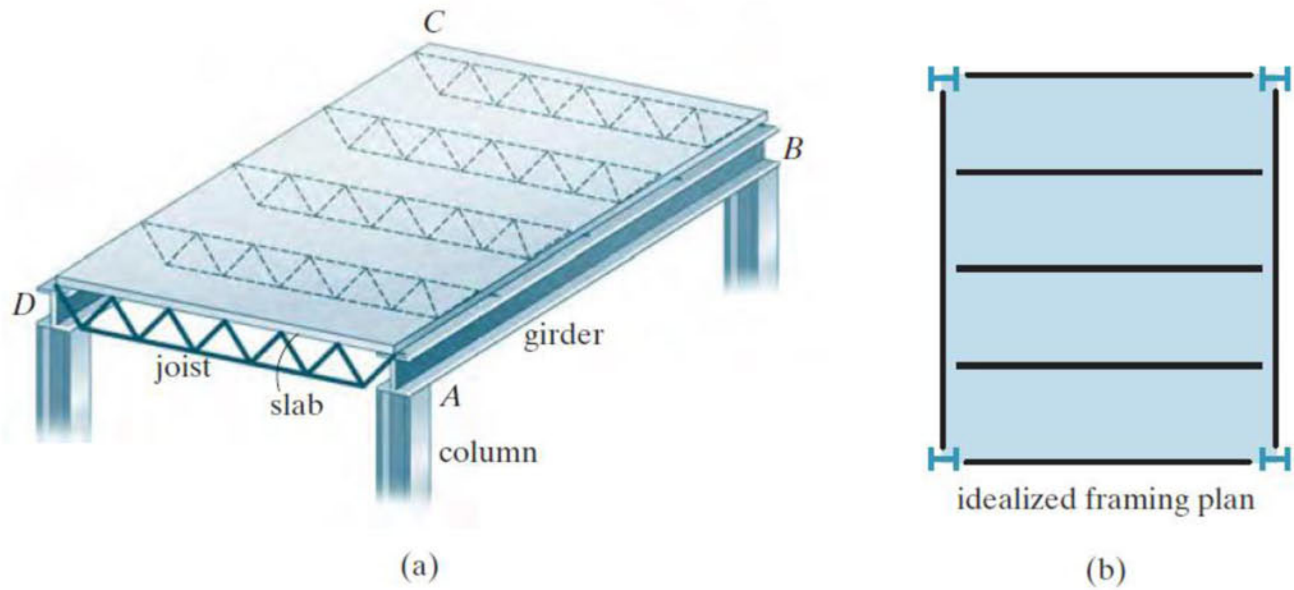


Figure 2.15: Idealization of a Typical Floor Slab in a Steel Building.

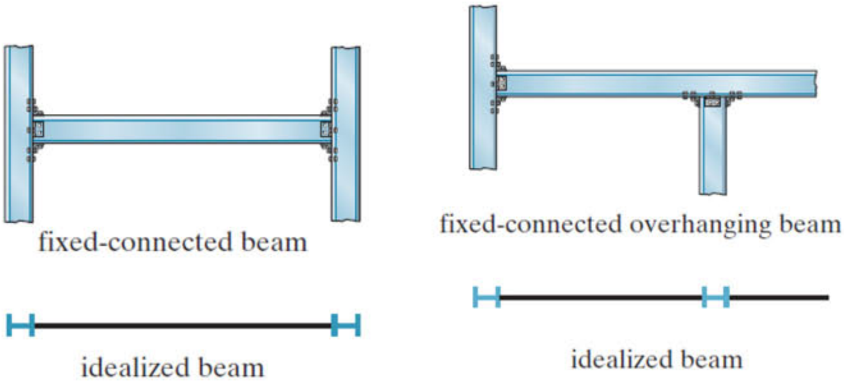


Figure 2.16: Idealization of a Steel Beam.

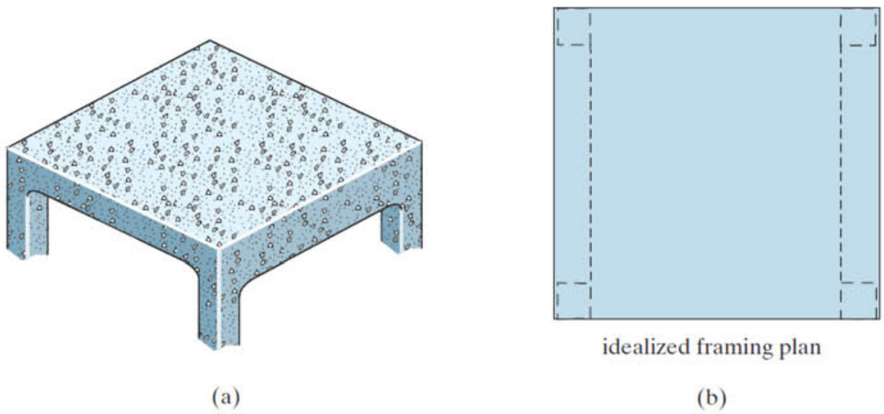


Figure 2.17: Idealization of a Concrete Frame.

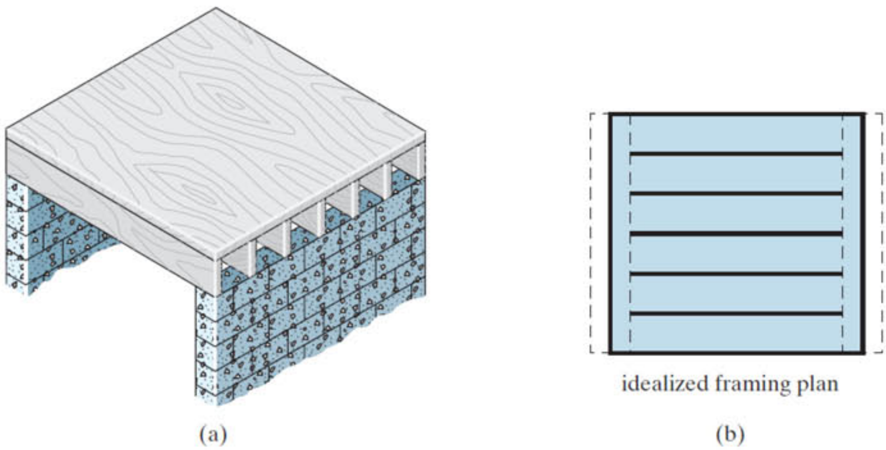


Figure 2.18: Idealization of a Wood Floor Slab.

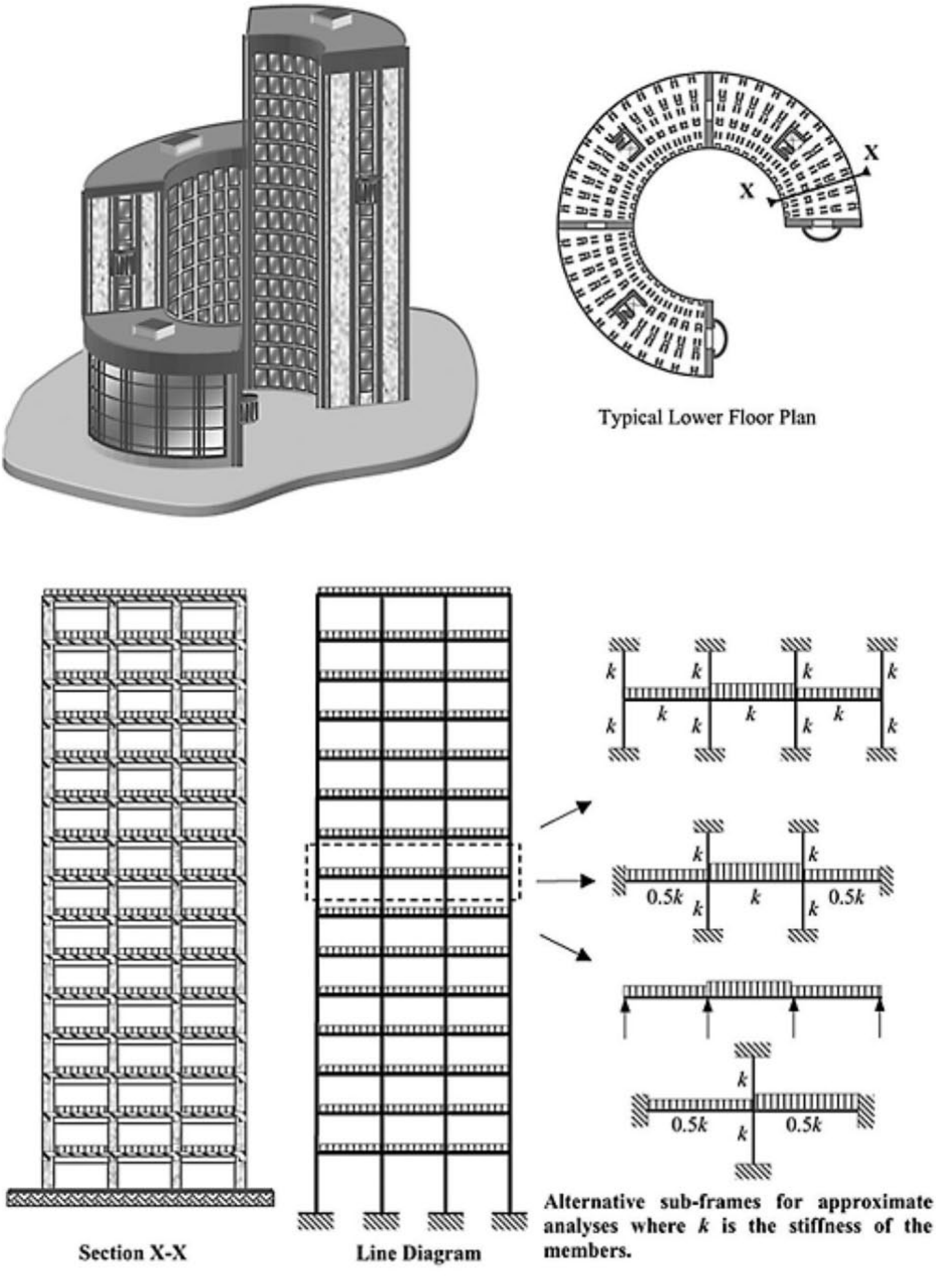


Figure 2.19: Idealization of a Multi-Story Building.

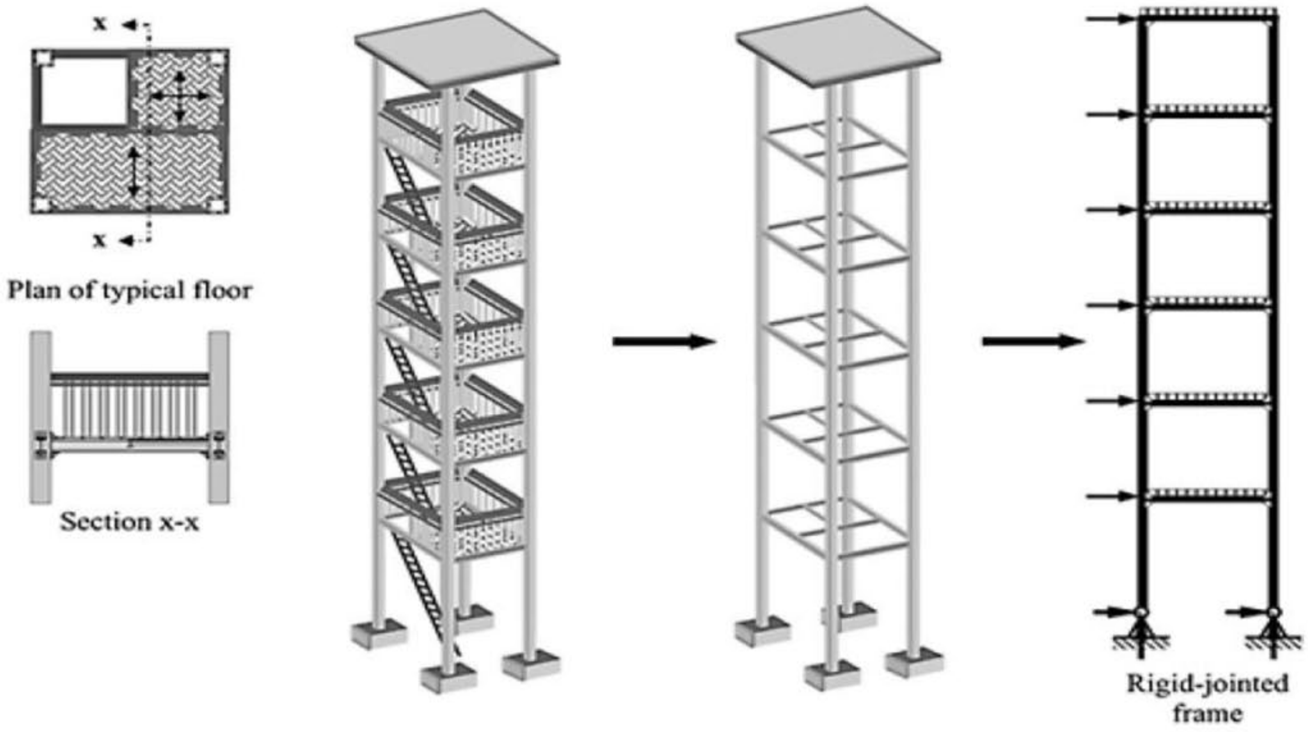


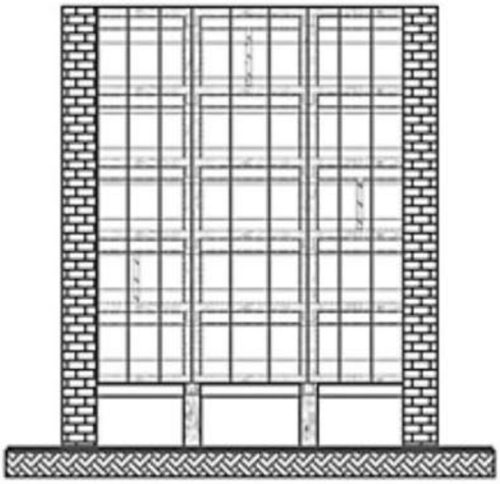
Figure 2.20: Idealization of a Steel Observation Tower (Sub-Fame Model)

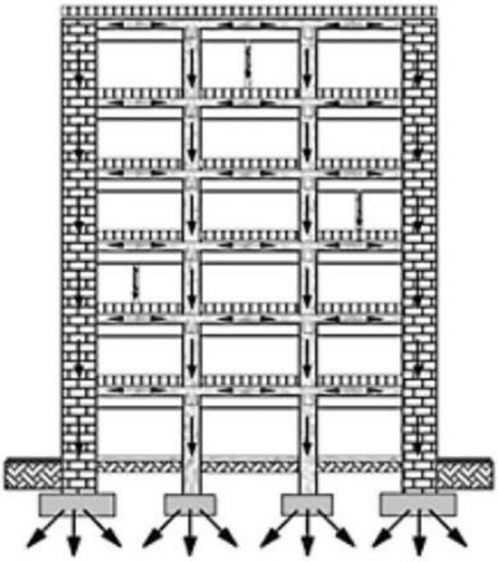
2.3.3 LOAD PATH

The support reactions for structures relate to the restraint conditions against linear and rotational movement. Every structural element and structure must be supported in order to transfer the applied loading to the foundations where it is dissipated through the ground. For example beams and floor slabs may be supported by other beams, columns or walls which are supported on foundations which subsequently transfer the loads to the ground. It is important to trace the load path of any applied loading on a structure to ensure that there is no interruption in the flow as shown in Figure 2.21.

The loads are transferred between structural members at the joints using either simple or rigid connections (i.e. moment connections). In the case of simple connections axial and/or shear forces are transmitted whilst in the case of rigid connections in addition to axial and shear effects, moments are also transferred.

The type of connections used will influence the degree-of-indeterminacy and the method of analysis required (e.g. determinate, indeterminate, pin jointed frame, rigid-jointed frame). Connection design, reflecting the assumptions made in the analysis, is an essential element in achieving an effective load path.





Load path for a typical frame

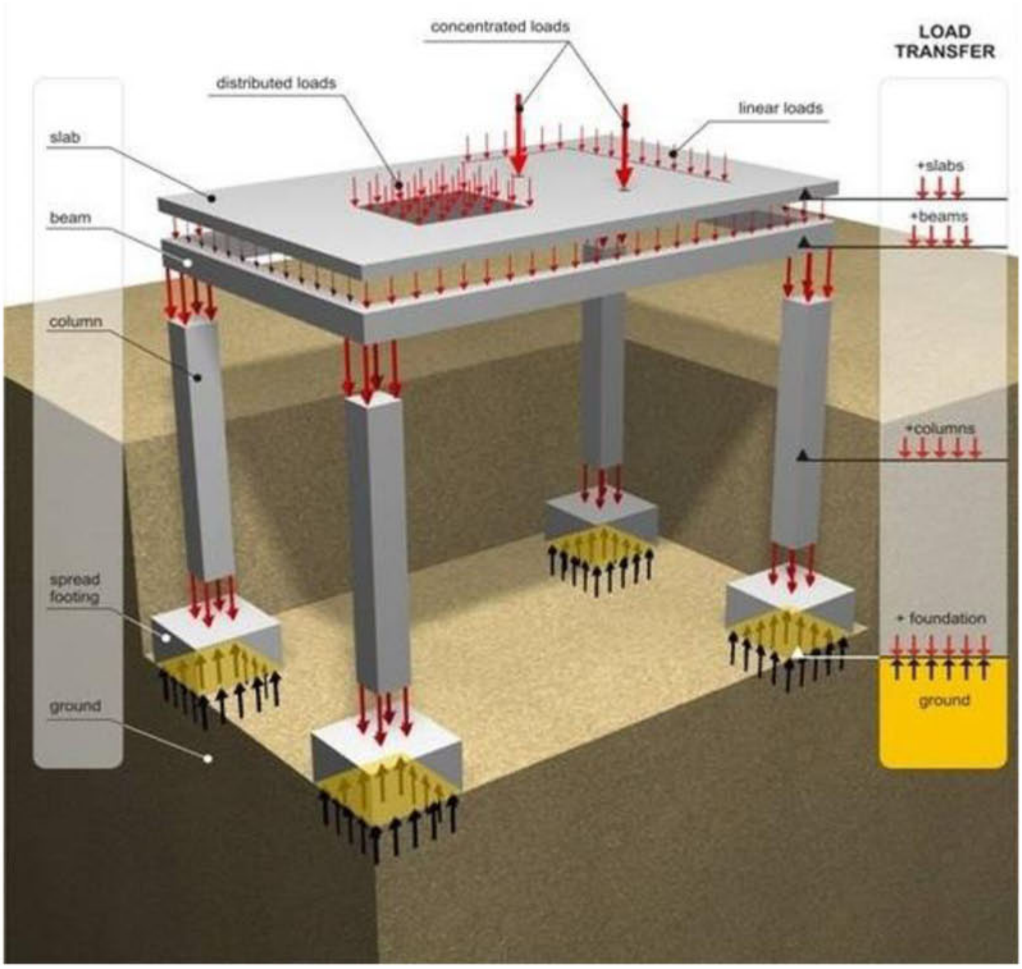


Figure 2.21: Load Path.

2.3.4 FOUNDATIONS

The primary function of all structural members/frames is to transfer the applied dead and imposed loading, from whichever source, to the foundations and subsequently to the ground. The type of foundation required in any particular circumstance is dependent on a number of factors such as the magnitude and type of applied loading, the pressure which the ground can safely support, the acceptable levels of settlement and the location and proximity of adjacent structures. In addition to purpose made pinned and roller supports the most common types of foundation currently used are indicated Figure 2.22. The support reactions in a structure depend on the types of foundation provided and the resistance to lateral and rotational movement.

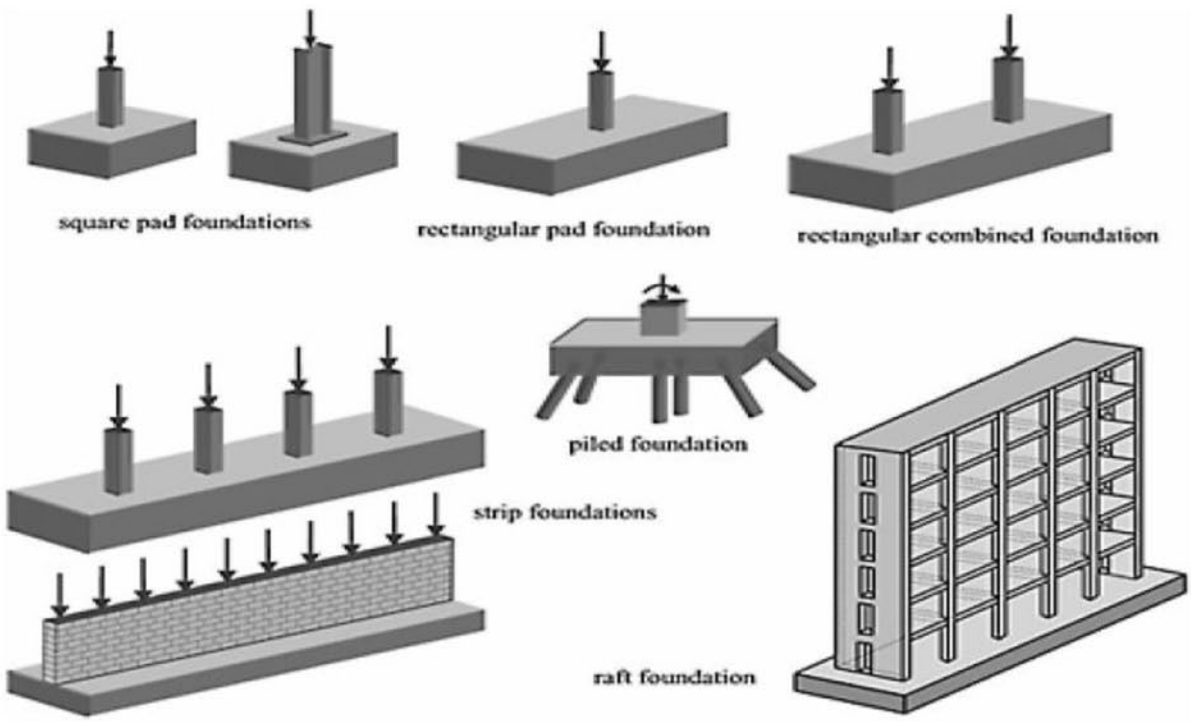


Figure 2.22: Foundations.

2.4 STRUCTURAL LOADING

All structures are subjected to loading from various sources.

In additional to the main categories of loading some circumstances there may be other loading types which should be considered, such as:

- Settlement.
- Fatigue.
- Temperature effects.
- Dynamic loading.
- Impact effects (e.g. when designing bridge decks, cranegantry girders or maritime structures).

The primary objective of structural analysis is to determine the distribution of internal moments and forces throughout a structure such that they are in equilibrium with the applied design loads.

Mathematical models which can be used to idealize structural behavior include: two- and three-dimensional elastic behavior, elastic behavior considering a redistribution of moments, plastic behavior and non-linear behavior.

2.4.1 FLOOR LOADS AND TRIBUTARY LOADINGS

When **flat surfaces** such as walls, floors, or roofs are supported by a structural frame, it is necessary to determine how the load on these surfaces is transmitted to the various structural elements used for their support.

There are generally two ways in which this can be done. The choice depends on the geometry of the structural system, the material from which it is made, and the method of its construction.

One-Way Floor System

A slab or deck that is supported such that it delivers its load to the supporting members by one-way action, is often referred to as a one-way slab (Figure 2.23).

An example of one-way slab construction of a steel frame building having a poured concrete floor on a corrugated metal deck. The load on the floor is considered to be transmitted to the beams, not the girders.

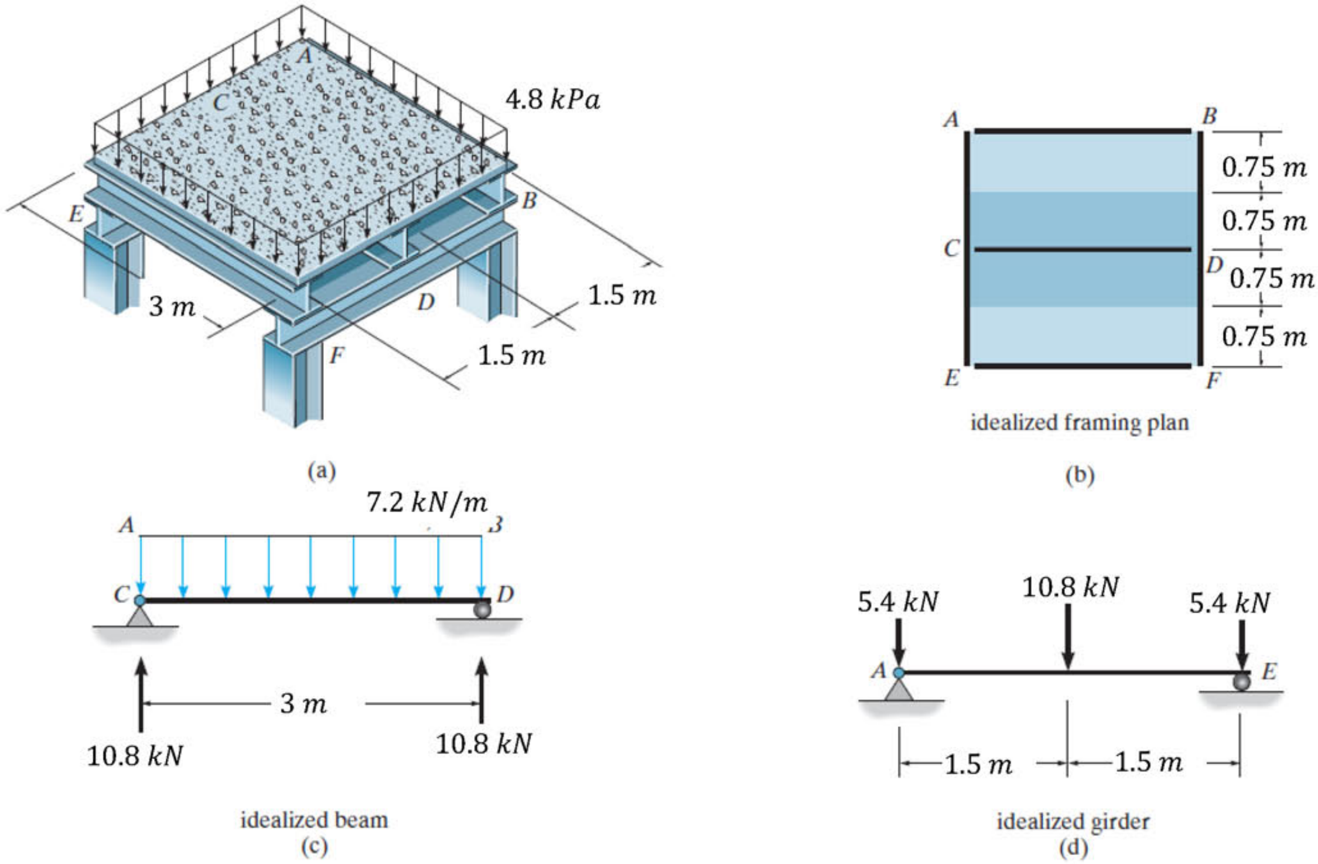


Figure 2.23: One-Way Floor System (Multi-Level Connection).

For some floor systems the beams and girders are connected to the columns at the **same elevation**, as in (Figure 2.24a). If this is the case, the slab can in some cases also be considered a "one-way slab." For example, if the slab is reinforced concrete with reinforcement in only one direction, or the concrete is poured on a corrugated metal deck, as in the above photo, then one-way action of load transmission can be assumed.

On the other hand, if the slab is flat on top and bottom and is reinforced in two directions, then consideration must be given to the possibility of the load being transmitted to the supporting members from either one or two directions. For example, consider the slab and framing plan in Figure 2.24b.

According to the American Concrete Institute, ACI 318 code,

If $L_2 > L_1$ and if the span ratio $L_2/L_1 > 2$ the slab will behave as a one-way slab,

since as becomes smaller, the beams AB, CD, and EF provide the greater stiffness to carry the load.

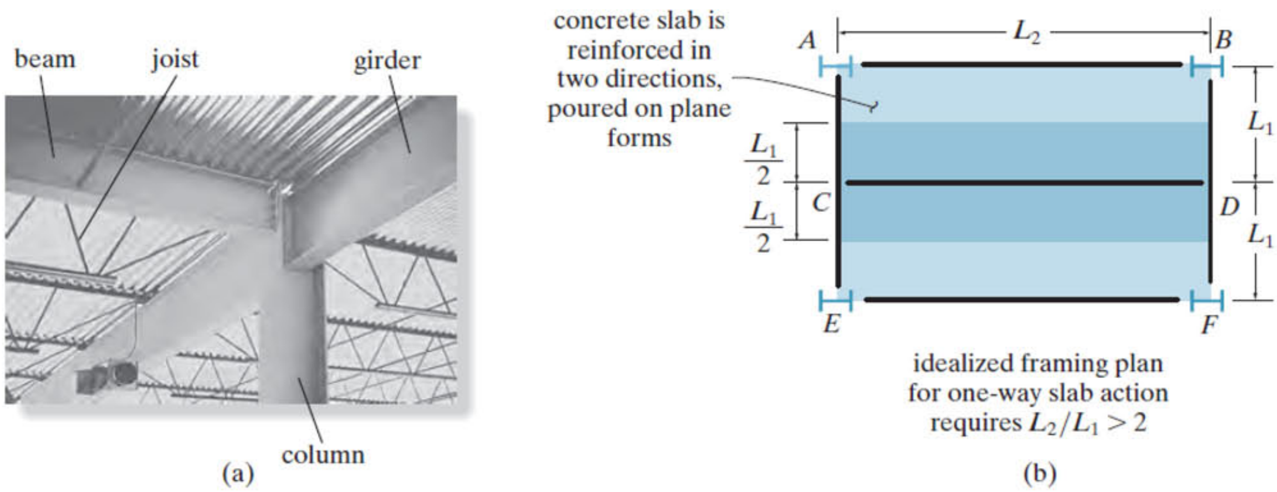


Figure 2.24: One-Way Floor System (One-Level Connection).

Two-Way Floor System

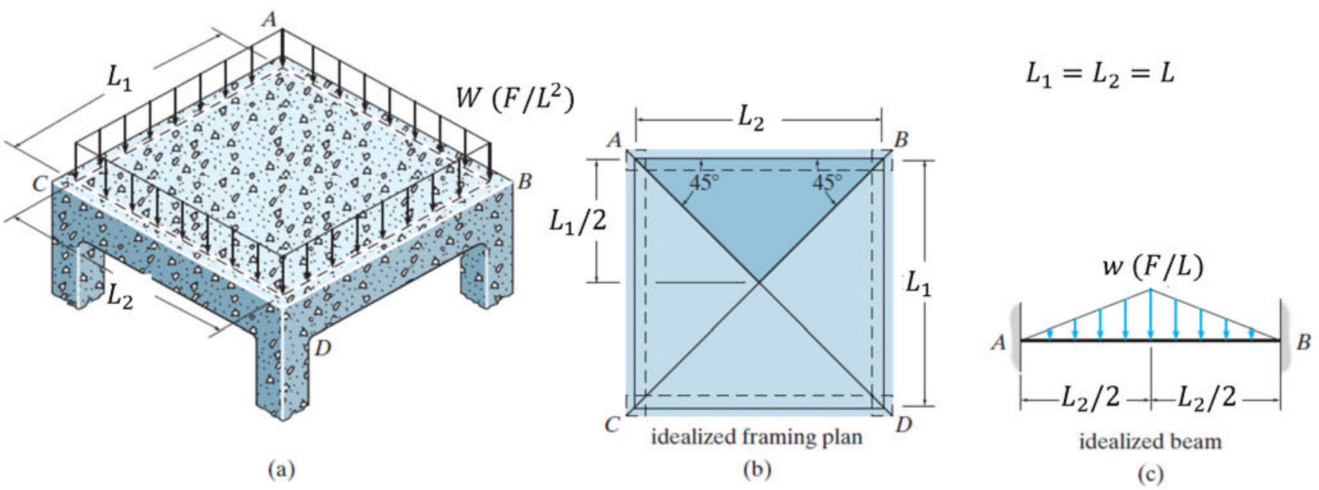
According to the ACI 318 concrete code

If $L_2 > L_1$ and if the span ratio $L_2/L_1 \leq 2$ the slab will behave as a two-way slab,

the support ratio in Figure 2.24b is the load is assumed to be delivered to the supporting beams and girders in two directions.

When this is the case the slab is referred to as a **two-way slab**.

Case 1: $L_1 = L_2$



$$w\left(\frac{F}{L}\right) = W\left(\frac{F}{L^2}\right) \times \frac{L}{2} = \frac{WL}{2}$$

Proof

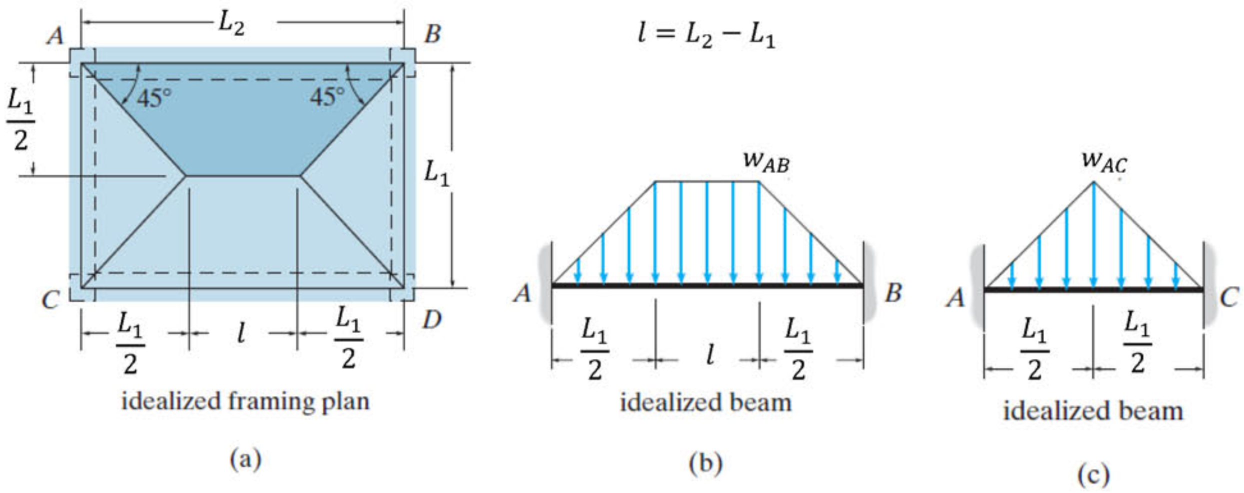
The resultant for one triangular tributary area is

$$R_{AB} = W \times \frac{1}{2} L \times \frac{L}{2} = \frac{WL^2}{4}$$

So w can be calculated as

$$w \times \frac{L}{2} = R_{AB} = \frac{WL^2}{4} \Rightarrow w = \frac{WL}{2}$$

Case 1: $L_1 \neq L_2$



The loading placed on the slab will then produce **trapezoidal** and **triangular** distributed loads on members AB and AC, respectively.

$$w_{AC}\left(\frac{F}{L}\right) = W\left(\frac{F}{L^2}\right) \times \frac{L_1}{2} = \frac{WL_1}{2}$$

$$w_{AC}\left(\frac{F}{L}\right) = W\left(\frac{F}{L^2}\right) \times \frac{L_1}{2} = \frac{WL_1}{2}$$

Proof

The resultant for AB triangular tributary area is

$$R_{AB} = W \times \frac{l + L_2}{2} \times \frac{L_1}{2} = W \times \frac{L_2 - L_1 + L_2}{2} \times \frac{L_1}{2} = \frac{WL^2}{4}$$

So w can be calculated as

$$w \times \frac{L}{2} = R_{AB} = \frac{WL^2}{4} \Rightarrow w = \frac{WL}{2}$$

INFLUENCE LINES FOR STATICALLY DETERMINATE STRUCTURES

8

8.1 INTRODUCTION

Influence lines have important application for the **design of structures** that resist **large live loads**. The theory is applied to:

1. Structures subjected to a **distributed load**.
2. Structures subjected to a **series of concentrated forces**.
3. Specific applications to **floor girders** and **bridge trusses**.
4. The determination of the **absolute maximum live shear** and **moment** in a member.

If a structure is subjected to a **live** or **moving load**, however, the variation of the **shear** and **bending moment** in the member is best described using the **influence line.**

An influence line represents the variation of either the reaction, shear, moment, or deflection at a specific point in a member as a concentrated force moves over the member.

Furthermore, the **magnitude** of the associated **reaction**, **shear**, **moment**, or **deflection** at the point **can then be calculated** from the **ordinates** of the **influence-line diagram**.

For these reasons, influence lines play an important part in the design of bridges, industrial crane rails, conveyors, and other structures where loads move across their span.

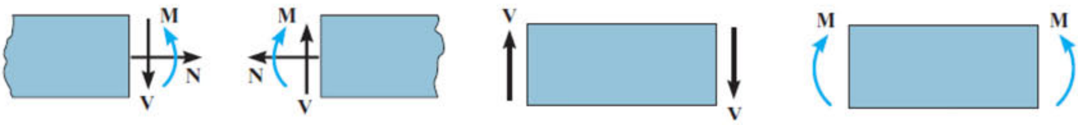
It is worthwhile to mention that **influence lines** represent the effect of a **moving load only** at a **specified point** on a member, whereas **shear** and **moment diagrams** represent the effect of **fixed loads** at **all points** along the axis of the member.

Procedure for Analysis

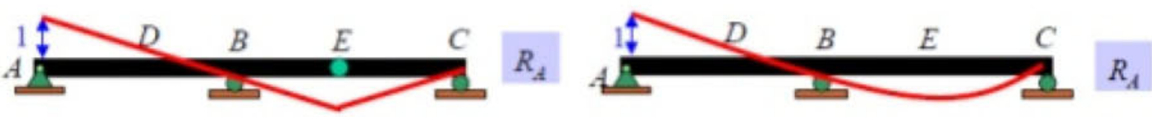
There are two procedures to construct the influence line at a specific point P in a member for any function (**reaction**, **shear**, or **moment**). For both of these procedures we will choose the moving force to have a **dimensionless magnitude of unity**.

Tabulate Values

- Place a **unit load** at various locations, x, along the member, and at each location use statics to determine the value of the function (**reaction**, **shear**, or **moment**) at the specified point.
- If the influence line for a vertical force reaction at a point on a beam is to be constructed, consider the **reaction** to be **positive** at the point when it acts **upward** on the beam.
- If a shear or moment **influence line** is to be drawn for a point, take the **shear** or **moment** at the point as **positive** according to the same **sign convention** used for drawing **shear** and **moment diagrams**.



- All **statically determinate** beams will have influence lines that consist of **straight line segments**.



Statically Determinate Beam   Statically Indeterminate Beam

- To avoid errors, it is recommended that one first construct a table, listing “$unit\ load\ at\ x$” versus the corresponding value of the function calculated at the specific point; that is, “$reaction\ R$,” “$shear\ V$,” or “$moment\ M$.” Once the load has been placed at various points along the span of the member, the tabulated values can be plotted and the influence-line segments constructed.

Influence-Line Equations

- The influence line can also be constructed by placing the **unit load** at a **variable position** x on the member and then computing the **value** of R, V, or M at the point as a **function** of x. In this manner, the equations of the various line segments composing the influence line can be determined and plotted.

8.2 INFLUENCE LINES FOR BEAMS

Since beams (or girders) often form the main load-carrying elements of a floor system or bridge deck, it is important to be able to construct the influence lines for the reactions, shear, or moment at any specified point in a beam.

LOADINGS

Once the influence line for a function (reaction, shear, or moment) has been constructed, it will then be possible to position the live loads on the beam which will produce the maximum value of the function. Two types of loadings will now be considered.

Concentrated Force

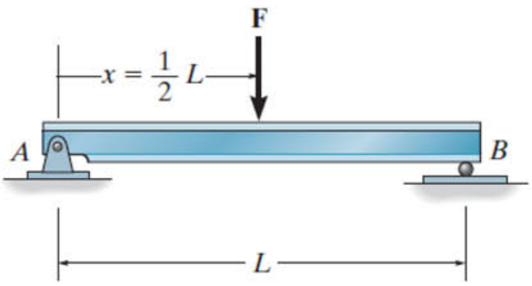
Since the numerical values of a function for an influence line are determined using a **dimensionless unit load**, then for any concentrated force $\boldsymbol{F}$ acting on the beam at any position x, the value of the function can be found by **multiplying** the **ordinate** of the influence line at the position x by the magnitude of $\boldsymbol{F}$.

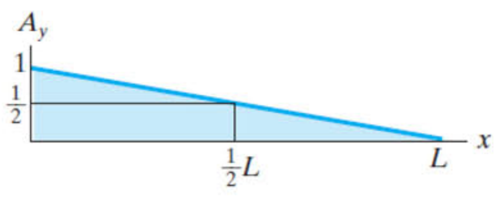
For example, consider the influence line for the reaction at A for the beam AB, shown in the figure. If the **unit load** is at $x = L/2$ the reaction at A is $A_y = \frac{1}{2}$ as indicated from the influence line. Hence, if the force F $(kN\ or\ lb)$ is at this same point, the reaction is

$$A_y = \left(\frac{1}{2}\right)(F), \qquad kN\ or\ lb$$

Obviously, the **maximum influence** caused by F occurs when it is placed on the beam at the same location as the **peak** of the influence line—in this case at $x = 0$ where the reaction would be

$$A_y = (1)(F), \qquad kN\ or\ lb$$

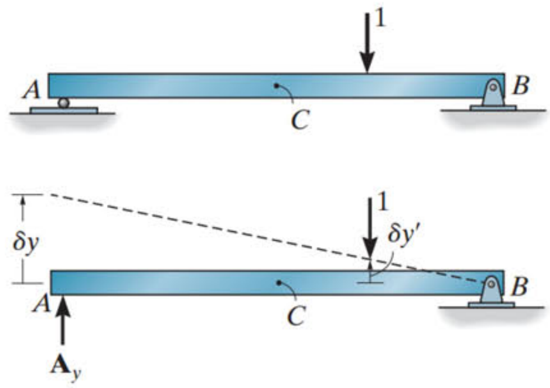




influence line for A_y

2. When the **positive** (**upward**) force is then applied at A, the beam deflects to the dashed position which represents the general shape of the influence line for A_y.
3. Since the beam is in equilibrium and therefore does not actually move, the virtual work sums to zero, i.e.,

$$A_y \delta y - 1\delta y' = 0$$



If $\boldsymbol{\delta y} = \mathbf{1}$, then

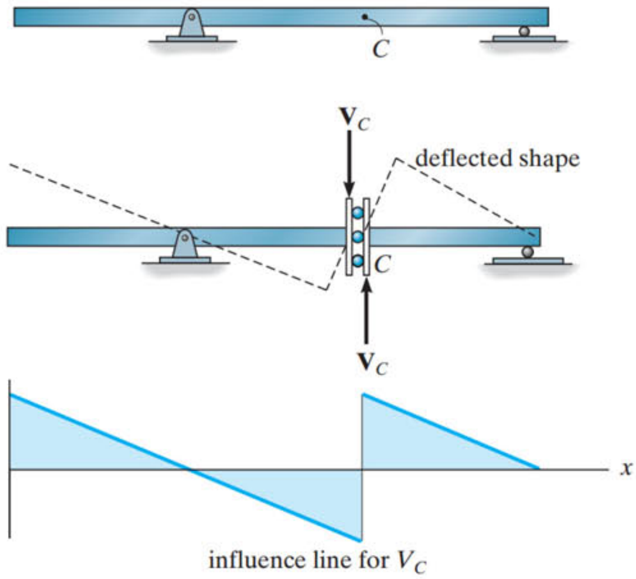
$$A_y = \delta y'$$

In other words, the value of represents the ordinate of the influence line at the position of the unit load. Since this value is equivalent to the displacement at the position of the unit load, it shows that the shape of the influence line for the reaction at A has been established. This proves the Müller-Breslau principle for reactions.

8.3.2 THE INFLUENCE LINE FOR THE SHEAR

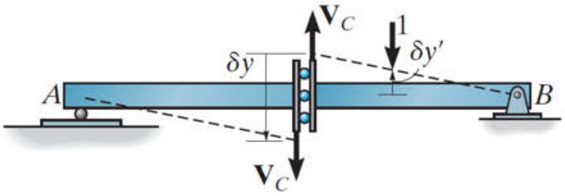
1. The **shear** at any section or connection (e.g., at C) may be symbolized by a **Guided Roller**.
2. Applying a **positive shear** (virtual linear displacement is left downward and right upward) force to the beam at C and allowing the beam to deflect to the dashed position, we find the influence-line shape.
3. Since the beam is in equilibrium and therefore does not actually move, the virtual work sums to zero, i.e.,

$$V_C \delta y - 1\delta y' = 0$$



If $\boldsymbol{\delta y} = \mathbf{1}$, then

$$V_C = \delta y'$$



8.3.3 THE INFLUENCE LINE FOR THE MOMENT

4. The **moment** at any section or connection (e.g., at C) may be symbolized by an **Internal Hinge** or **Internal Pin**.
5. Applying a **positive moment** (virtual rotational displacement is left counterclockwise and right clockwise) to the beam at C and allowing the beam to deflect to the dashed position, we find the influence-line shape.

Since the beam is in equilibrium and therefore does not actually move, the virtual work sums to zero. If a virtual rotation is introduced at the pin, virtual work will be done only by the internal moment and the unit load. So .

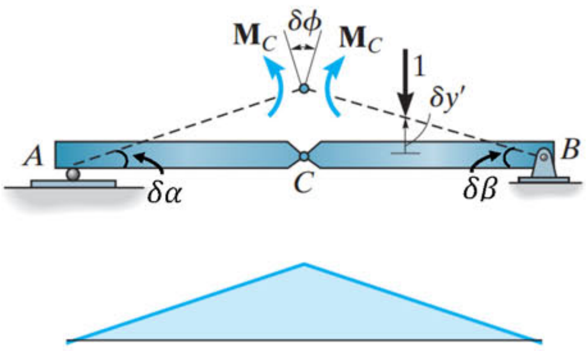
$$M_C \delta\emptyset - 1\delta y' = 0$$

where

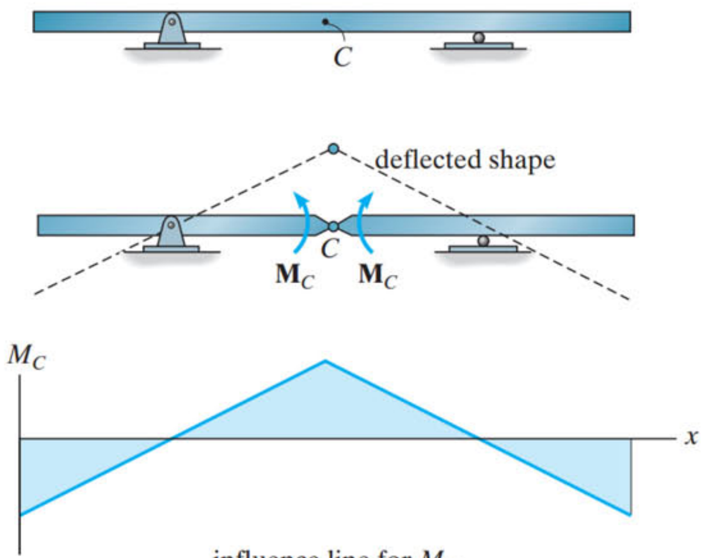
$$\delta\emptyset = \delta\alpha + \delta\beta$$

If $\boldsymbol{\delta\emptyset = 1}$, then

$$M_C = \delta y'$$



influence line for M_C

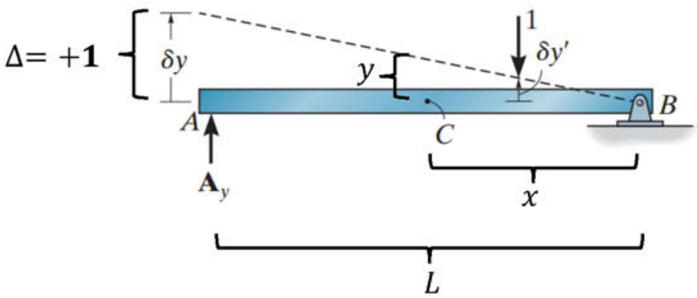


influence line for M_C

8.4 QUANTITATIVE INFLUENCE LINES

8.4.1 THE INFLUENCE LINE FOR THE REACTION

From the figure shown



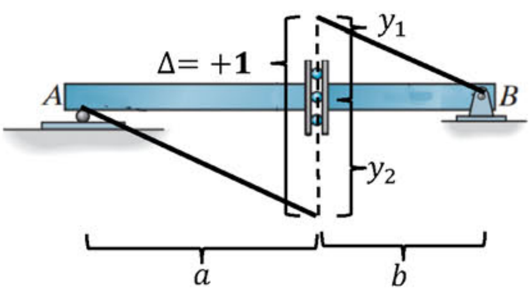
$$\frac{\Delta}{L} = \frac{y}{x} \Rightarrow y = x\frac{\Delta}{L}$$

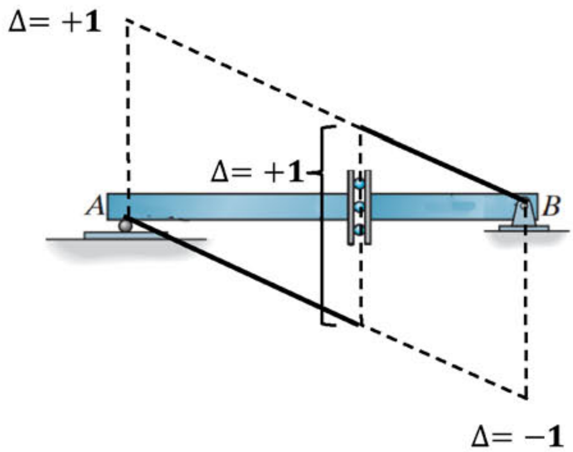
For $\Delta = +1$ then

$$y = \frac{x}{L}$$

8.4.2 THE INFLUENCE LINE FOR THE SHEAR

From the figure shown





$$\frac{y_1}{b} = \frac{y_2}{a} \quad and \quad y_1 + y_2 = 1$$

from

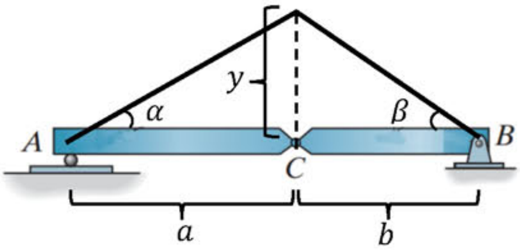
$$y_2 = \frac{a}{b}y_1 \Rightarrow y_1 + \frac{a}{b}y_1 = 1 \Rightarrow y_1\left(1 + \frac{a}{b}\right) = 1 \Rightarrow y_1\left(\frac{a+b}{b}\right) = 1$$

then

$$y_1 = \frac{b}{a+b} \quad and \quad y_2 = \frac{a}{a+b}$$

8.4.3 THE INFLUENCE LINE FOR THE MOMENT

For the figure shown



$$\alpha + \beta = 1$$

For very small angles

$$\tan\alpha \approx \alpha \quad and \quad \tan\beta \approx \beta$$

Then

$$\tan\alpha + \tan\beta = 1$$

$$\tan\alpha = \frac{y}{a} \quad and \quad \tan\beta = \frac{y}{b}$$

$$\frac{y}{a} + \frac{y}{b} = 1$$

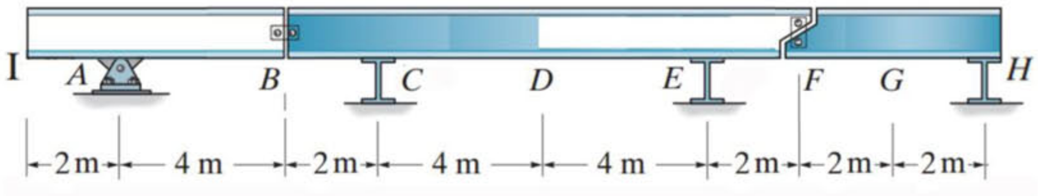
$$by + ay = ab \quad \Rightarrow \quad y(a + b) = ab$$

then

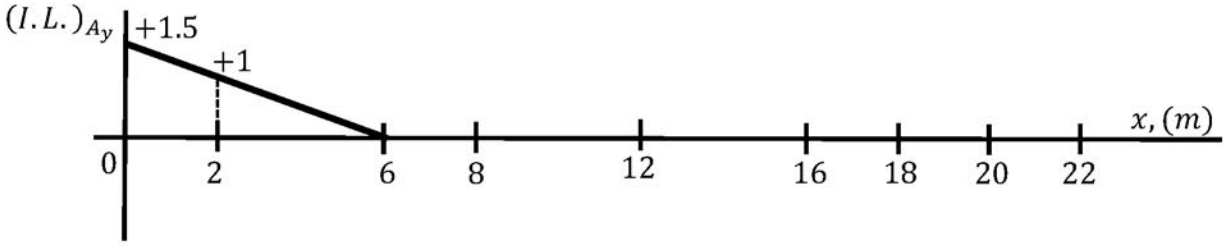
$$\boxed{y = \frac{ab}{a + b}}$$

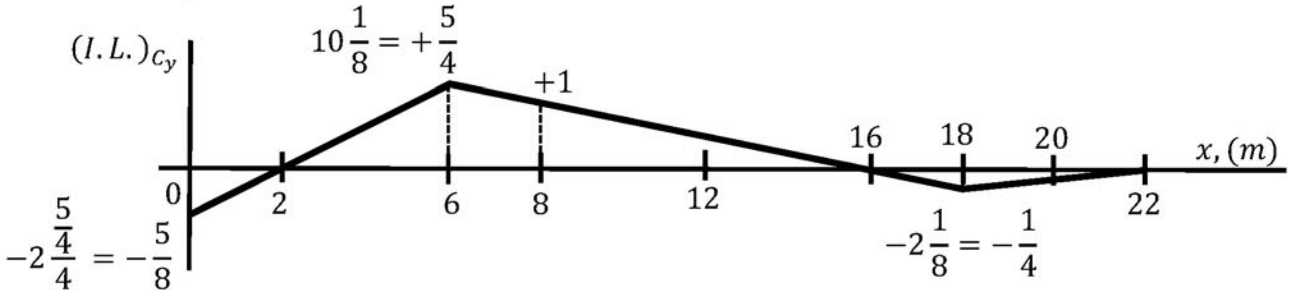
Example 8.16 — **Quantitative Influence Lines, Müller-Breslau Principle**

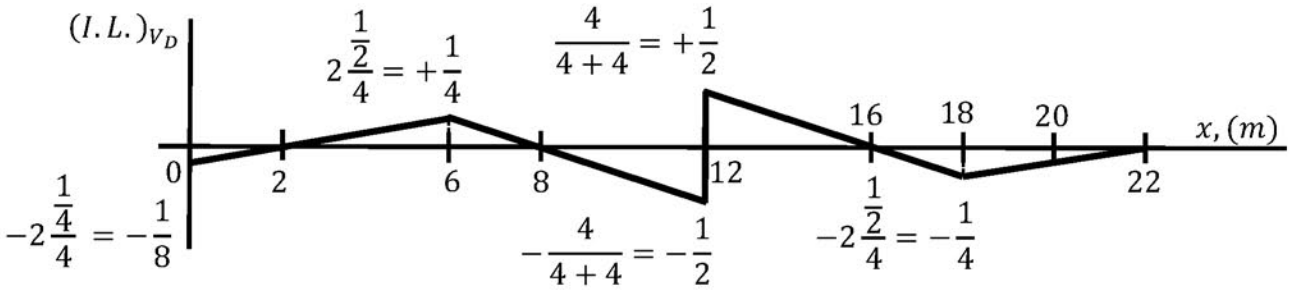
Draw the influence lines for $A_y, C_y, V_D, V_{E^-}, V_{E^+}, M_C, M_D, M_G$ and M_A.

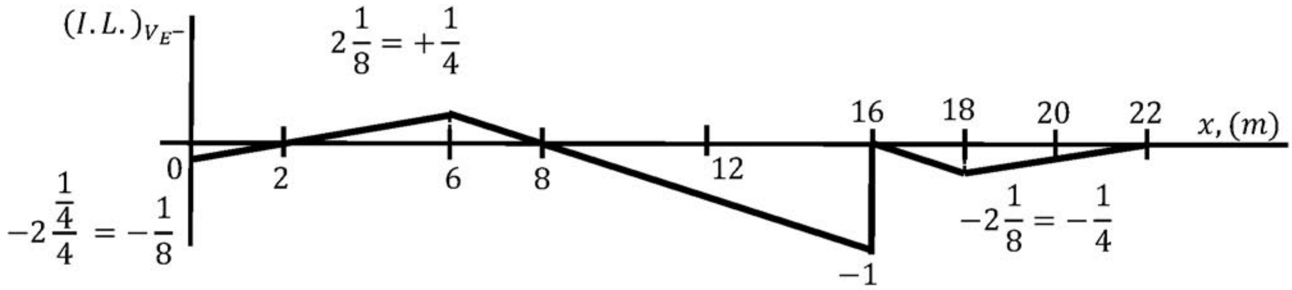


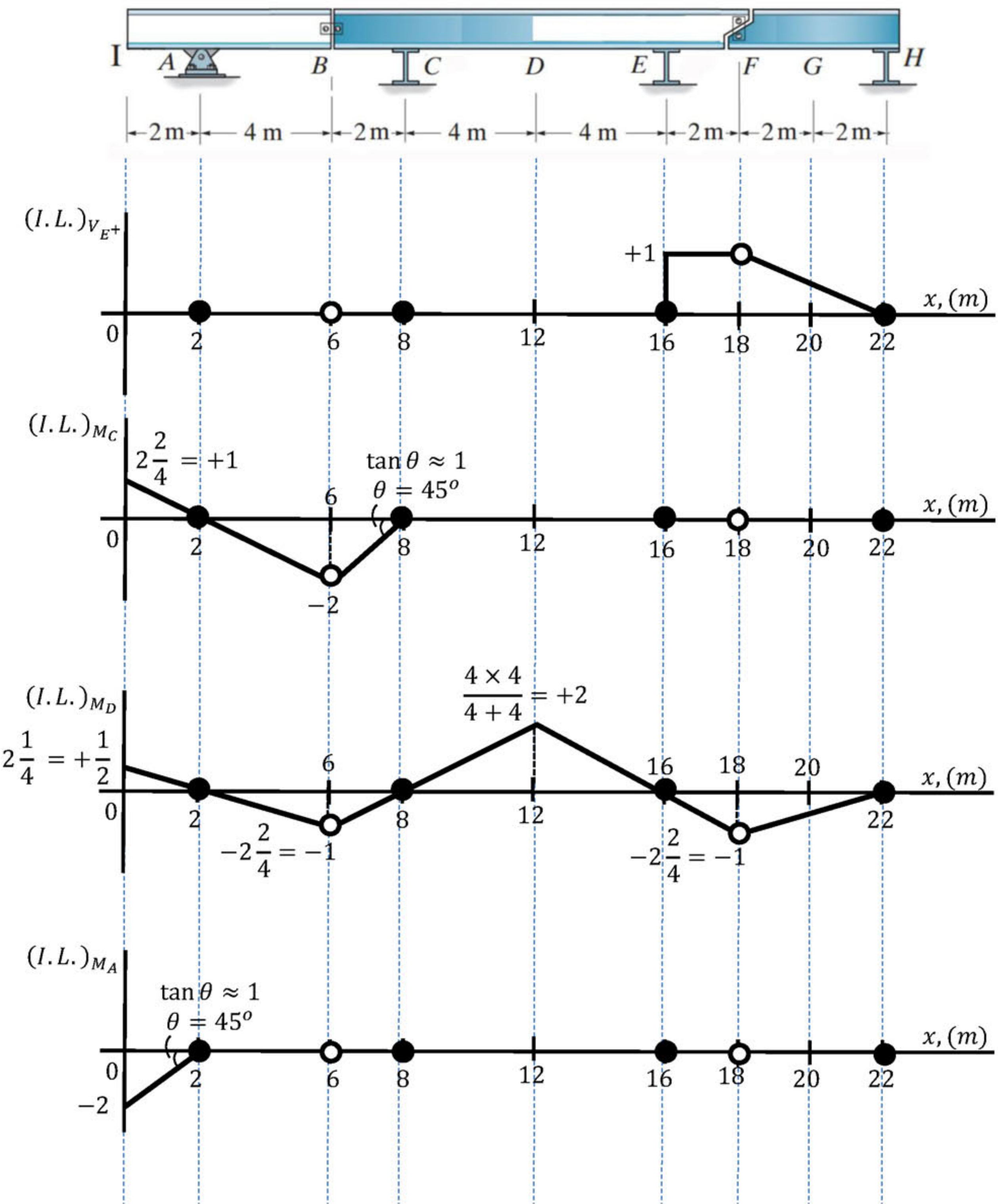
Solution





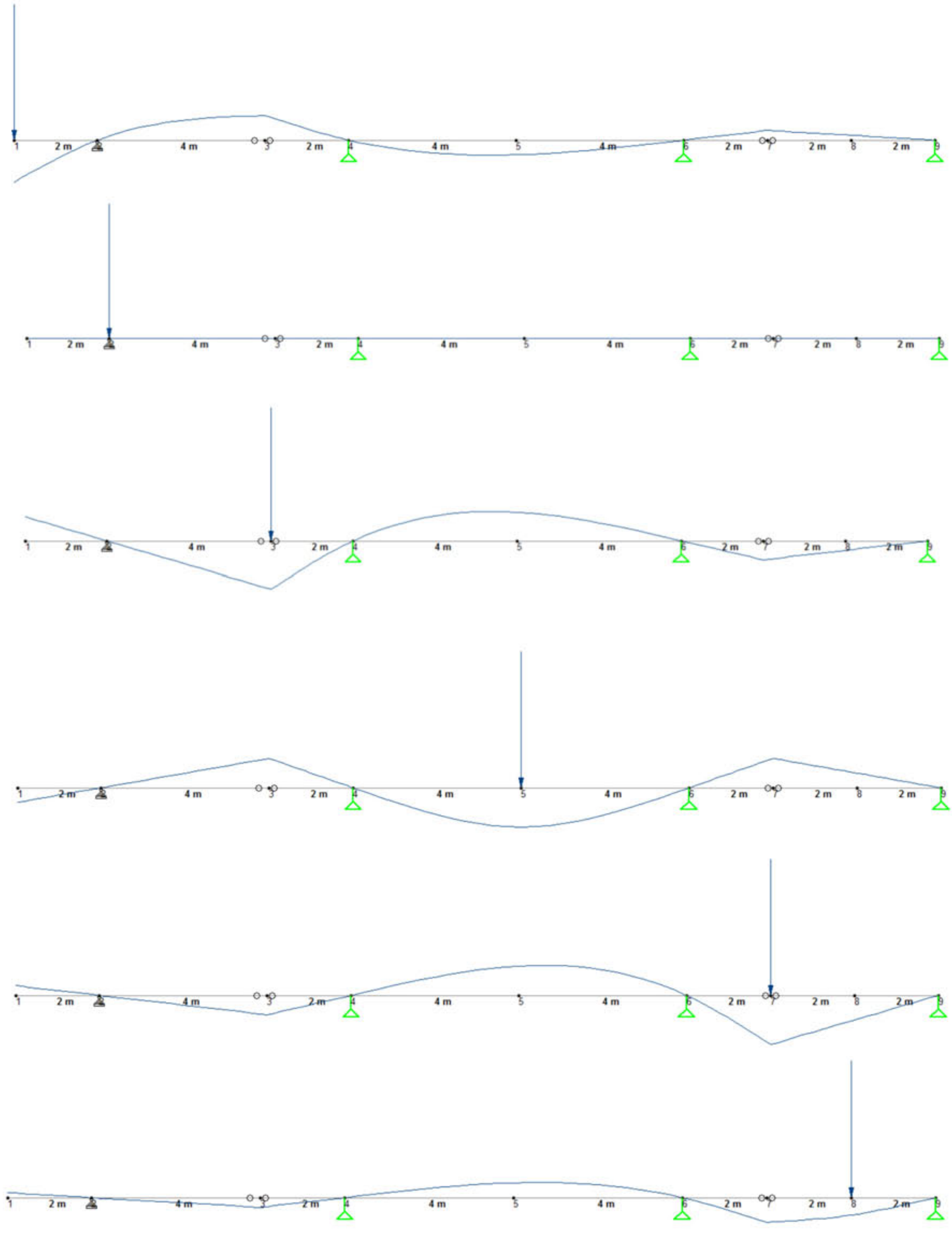




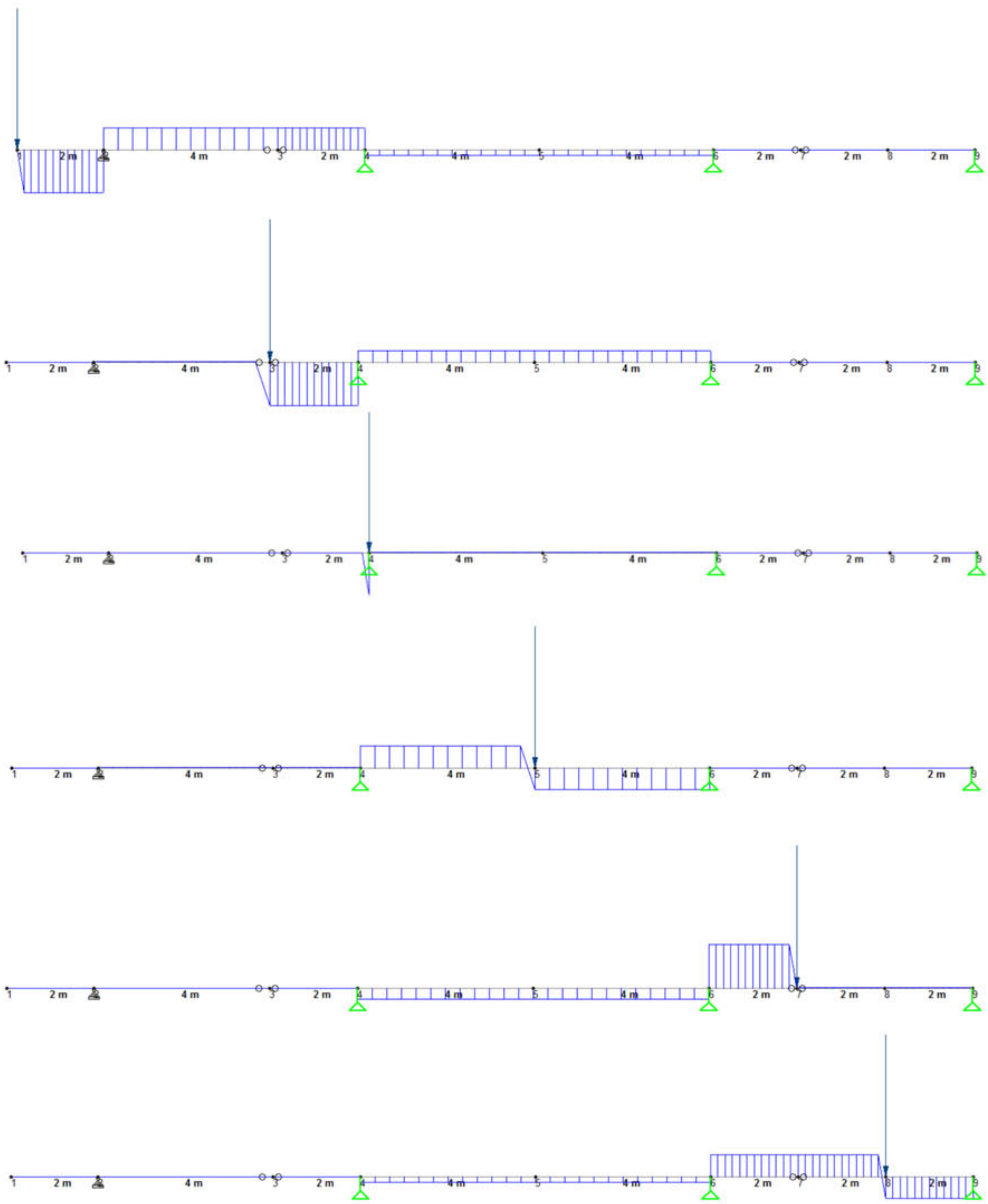

I A B C D E F G H
2m 4 m 2m 4 m 4 m 2m 2m 2m
$(I.L.)_{V_{E^+}}$
+1
$x,(m)$
0 2 6 8 12 16 18 20 22
$(I.L.)_{M_C}$
$2\frac{2}{4}=+1$
$\tan\theta\approx 1$
$\theta=45^o$
−2
$(I.L.)_{M_D}$
$\frac{4\times 4}{4+4}=+2$
$2\frac{1}{4}=+\frac{1}{2}$
$-2\frac{2}{4}=-1$
$-2\frac{2}{4}=-1$
$(I.L.)_{M_A}$
$\tan\theta\approx 1$
$\theta=45^o$
−2

STAAD.Pro

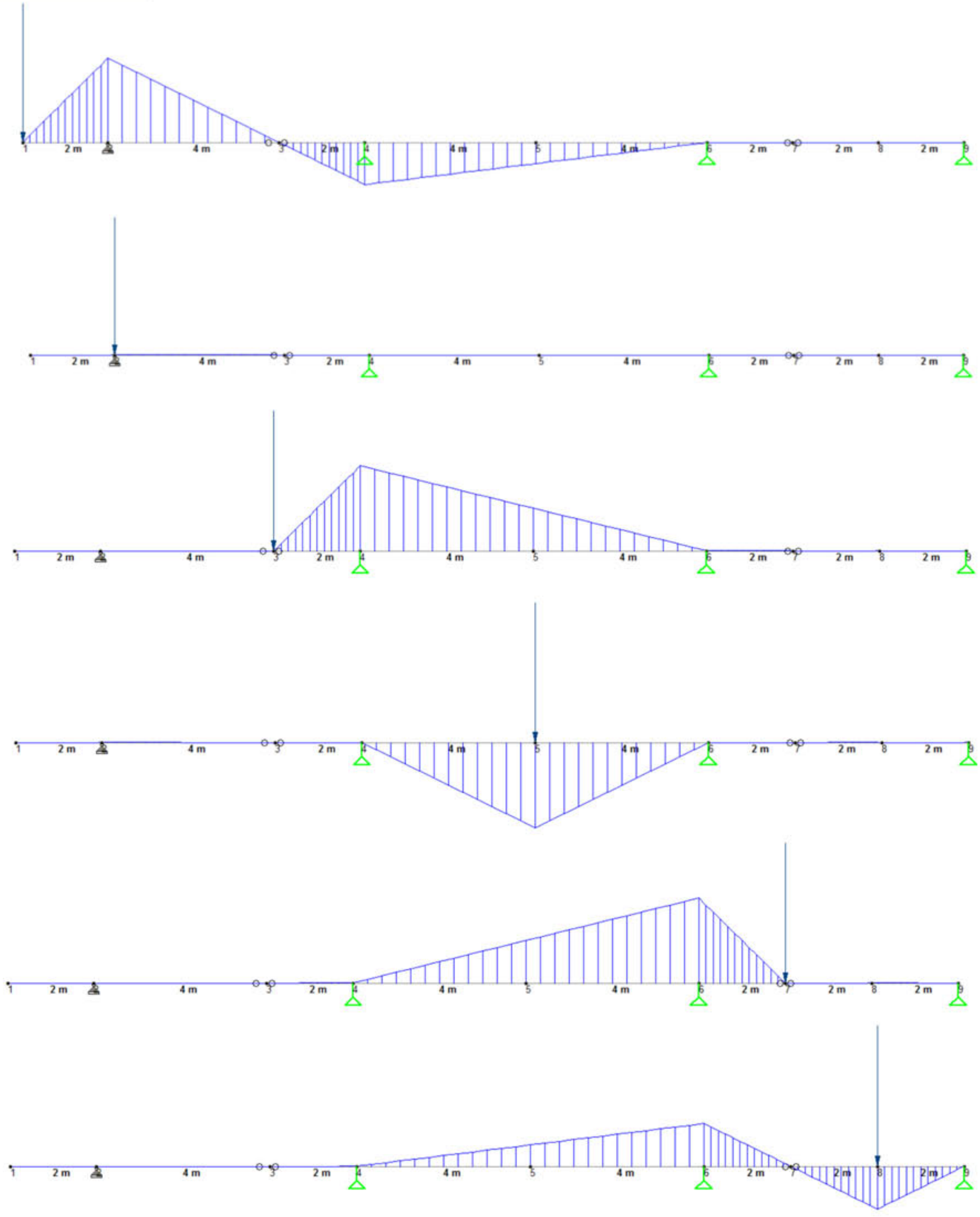
Deflected Shapes of Example 6.16 (1 Unit Moving Force)



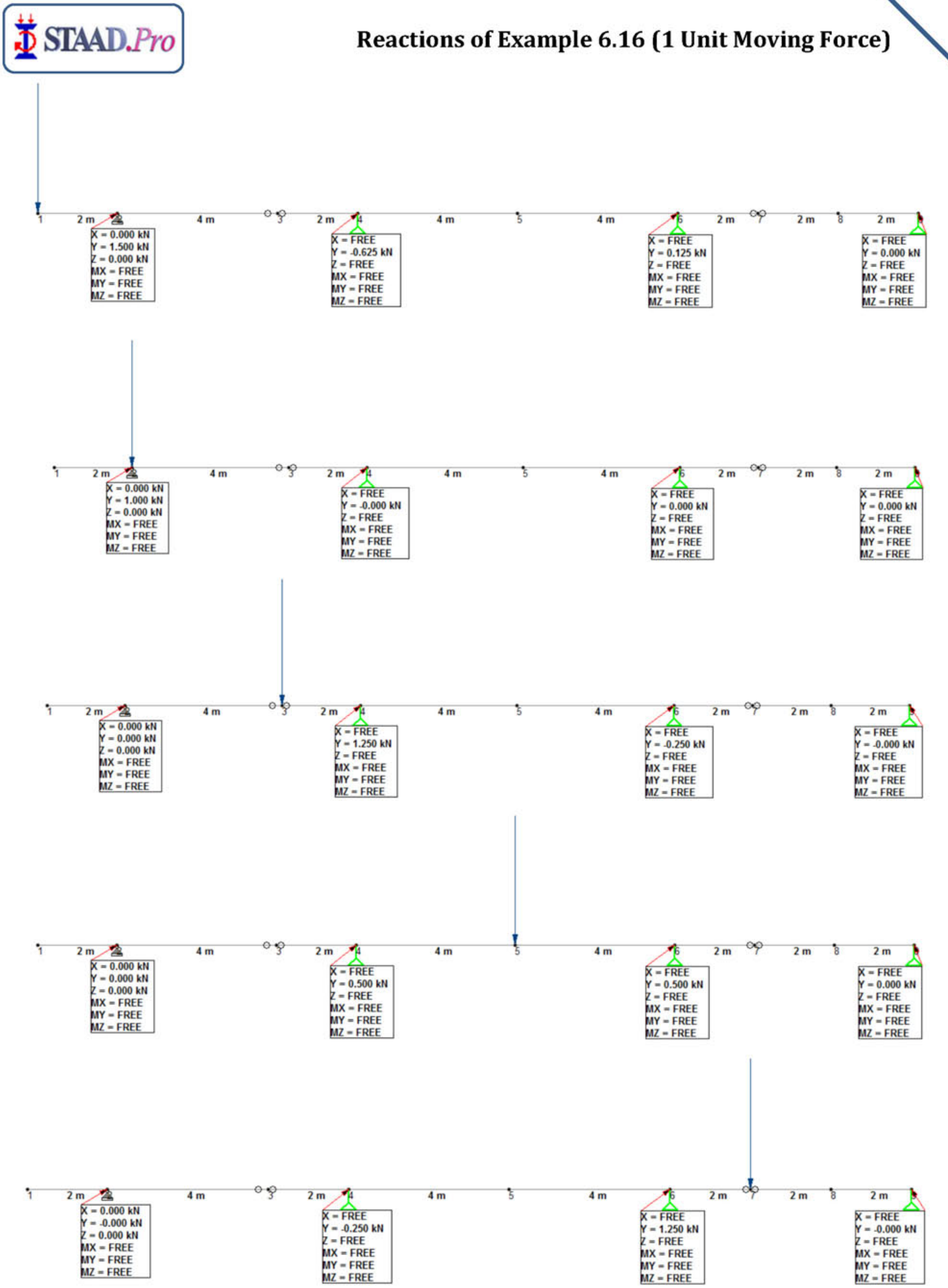
Shear Force Diagrams of Example 6.16 (1 Unit Moving Force)



Bending M. Diagrams of Example 6.16 (1 Unit Moving Force)



Reactions of Example 6.16 (1 Unit Moving Force)

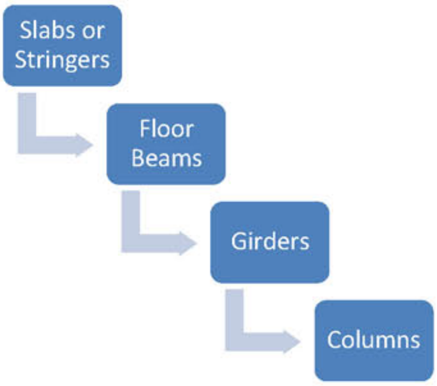


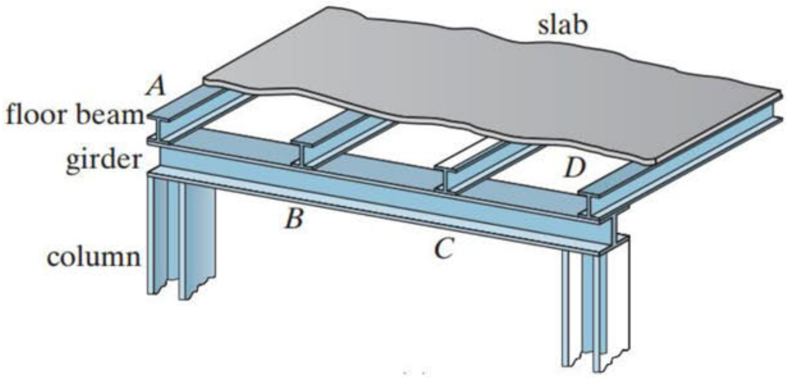
STAAD Command File of Example 6.16 (1 Unit Moving Force)

```
STAAD PLANE
START JOB INFORMATION
ENGINEER DATE 18-Dec-15
END JOB INFORMATION
INPUT WIDTH 79
UNIT METER KN
JOINT COORDINATES
1 0 0 0; 2 2 0 0; 3 6 0 0; 4 8 0 0; 5 12 0 0; 6 16 0 0; 7 18 0
0; 8 20 0 0;
9 22 0 0;
MEMBER INCIDENCES
1 1 2; 2 2 3; 3 3 4; 4 4 5; 5 5 6; 6 6 7; 7 7 8; 8 8 9;
SUPPORTS
2 PINNED
4 6 9 FIXED BUT FX FZ MX MY MZ
MEMBER RELEASE
2 6 END MX MY MZ
3 7 START MX MY MZ
DEFINE MATERIAL START
ISOTROPIC CONCRETE
E 2.17185e+007
POISSON 0.17
DENSITY 23.5616
ALPHA 1e-005
DAMP 0.05
END DEFINE MATERIAL
MEMBER PROPERTY
1 TO 8 PRIS YD 1.0 ZD 0.3
CONSTANTS
MATERIAL CONCRETE ALL
DEFINE MOVING LOAD
TYPE 1 LOAD 1
DIST 0
LOAD GENERATION 45
TYPE 1 0 0 0 XINC 0.5
PERFORM ANALYSIS
FINISH
```

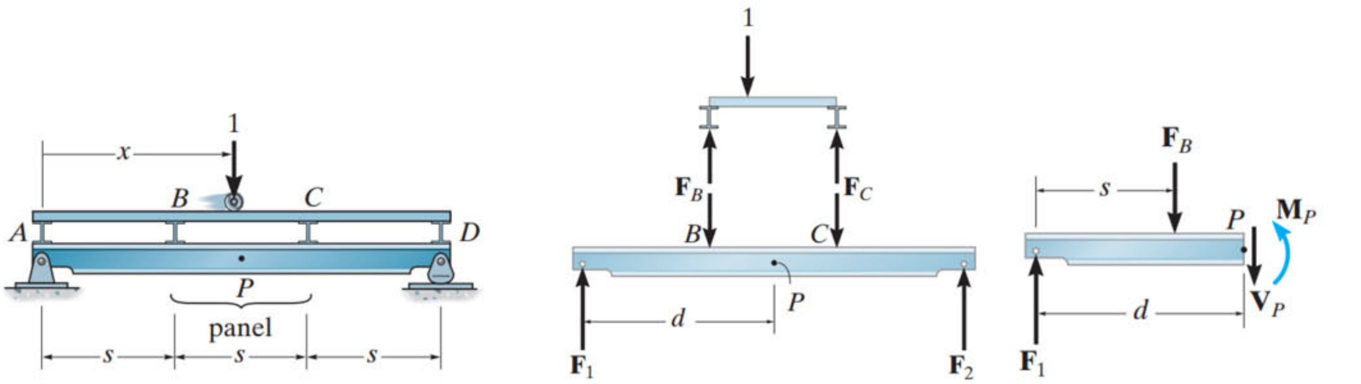
8.5 INFLUENCE LINES FOR FLOOR GIRDERS

Occasionally, floor systems are constructed as shown in the figure, where it can be seen that floor loads are transmitted from **slabs** (or **stringers**) to **floor beams**, then to side **girders**, and finally supporting **columns**.





An idealized model of this system is shown in the following plane view,

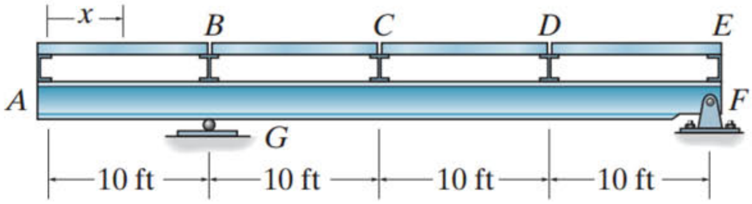


Here the **slab** is assumed to be a **one-way slab** and is **segmented** into simply supported spans resting on the floor beams.

Furthermore, the girder is simply supported on the columns. Since the girders are main load-carrying members in this system, it is sometimes necessary to construct their **shear** and **moment influence lines**. This is especially true for industrial buildings subjected to heavy concentrated loads. In this regard, notice that a **unit load** on the floor slab **is transferred** to the girder only at points where it is in contact with the floor beams, i.e., points A, B, C, and D. These points are called **panel points**, and the region between these points is called a **panel**, such as BC shown in the figure.

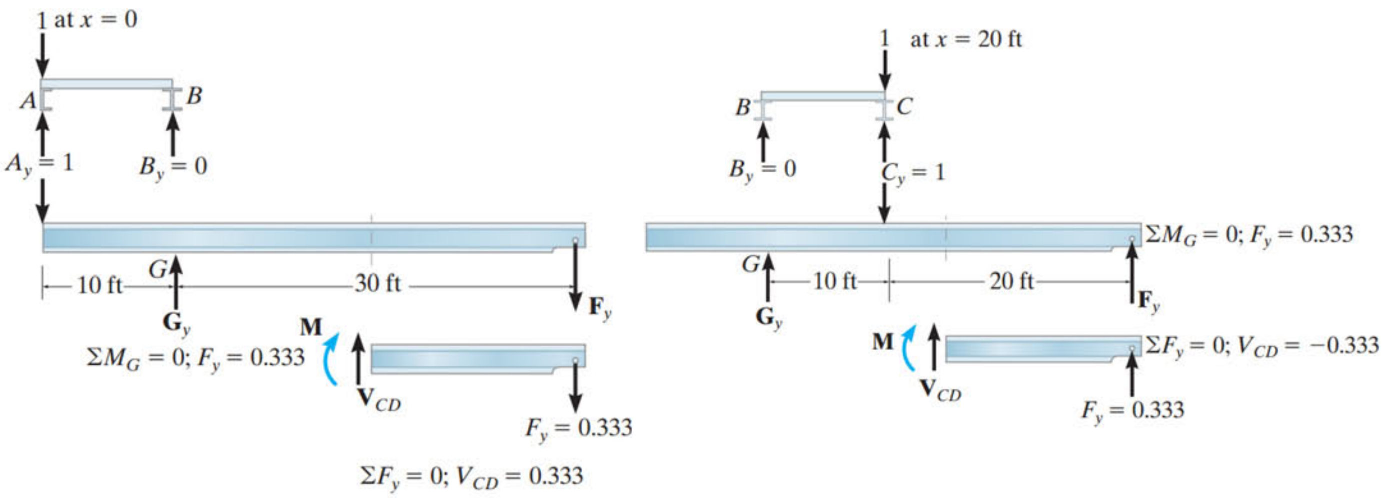
Example 8.17 — Influence Lines for Floor Girders

Draw the influence line for the shear in panel CD of the floor girder shown in the figure.

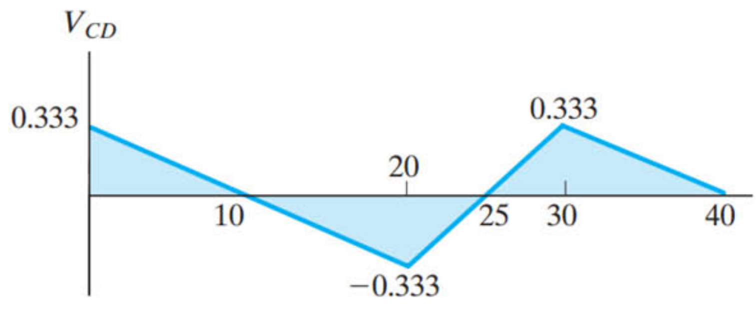


Solution

Tabulate Values Approach

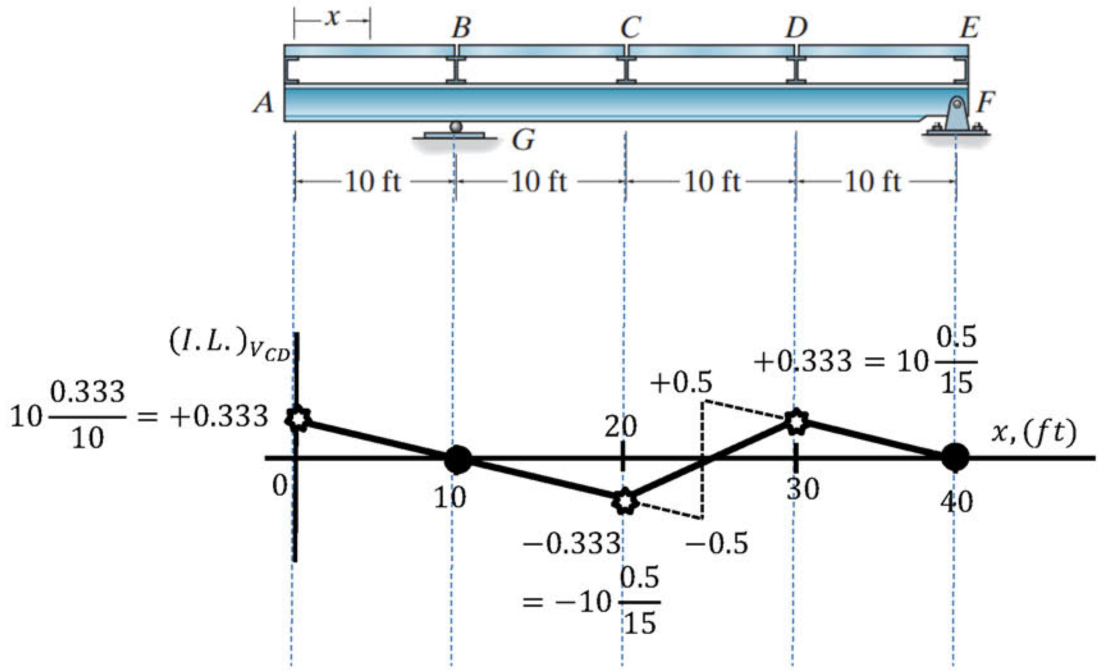


x	V_{CD}
0	0.333
10	0
20	−0.333
30	0.333
40	0



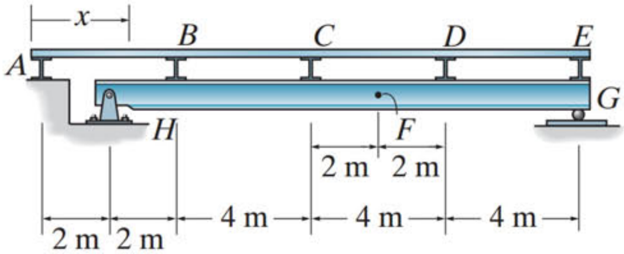
influence line for V_{CD}

Hybrid Approach



Example 8.18 — Influence Lines for Floor Girders

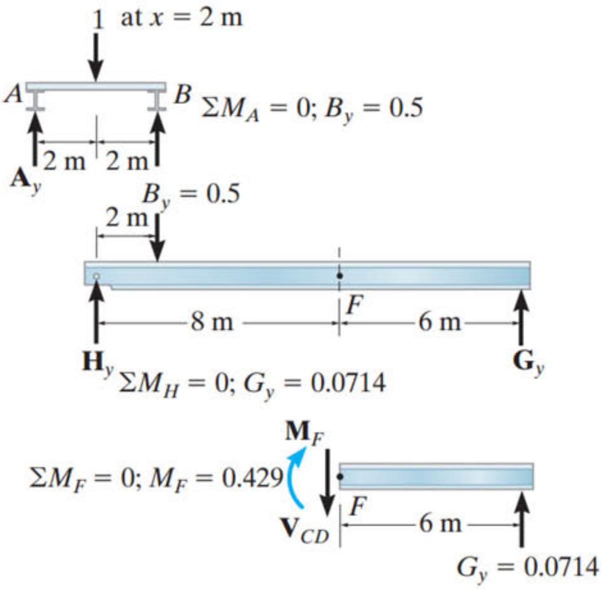
Draw the influence line for the moment at point F for the floor girder shown.



Solution

Tabulate Values Approach

x	M_F
0	0
2	0.429
4	0.857
8	2.571
10	2.429
12	2.286
16	0



ANALYSIS OF STATICALLY INDETERMINATE STRUCTURES BY THE FORCE METHOD

11

11.1 STATICALLY INDETERMINATE STRUCTURES

A structure of any type is classified as **statically indeterminate** when the number of unknown reactions or internal forces exceeds the number of equilibrium equations available for its analysis.

$$Number\ of\ Unknown\ Reactions\ or\ Internal\ Forces\ >\ Number\ of\ Equilibrium\ Equations$$

Realize that most of the structures designed today are statically indeterminate. This indeterminacy may arise as a result of added **supports** or **members**, or by the general form of the structure. For example, **reinforced concrete buildings** are almost always **statically indeterminate** since the columns and beams are poured as continuous members through the joints and over supports.

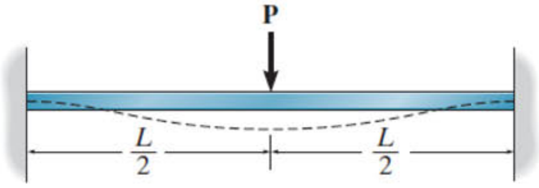
11.2 ADVANTAGES AND DISADVANTAGES OF INDETERMINATE STRUCTURES

Although the analysis of a statically indeterminate structure is more involved than that of a statically determinate one, there are usually several very important reasons for choosing this type of structure for design.

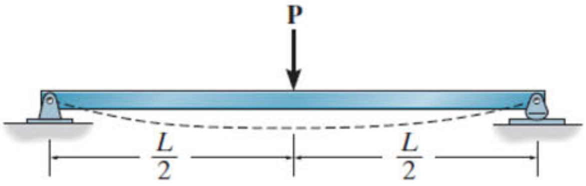
Advantages

1. Most important, for a given loading the **maximum stress and deflection of an indeterminate structure are generally smaller than those of its statically determinate counterpart**.

For example, for the beam shown in the figure below



(a)



(b)

- The statically indeterminate, fixed-supported beam in Figure (a) will be subjected to a maximum moment of

$$M_{\text{fixed-supported}}\Big)_{max} = \frac{PL}{8}$$

- The statically determinate, simply supported beam in Figure (b), will be subjected to **twice** the **moment**, that is,

$$M_{\text{simply-supported}}\Big)_{max} = \frac{PL}{4}$$

- As a result, the fixed-supported beam has **one fourth** the **deflection** and one half the stress at its center of the one that is simply supported.

$$\Delta_{\text{fixed-supported}}\Big)_{max} = \frac{PL^3}{192EI}$$

$$\Delta_{\text{simply-supported}}\Big)_{max} = \frac{PL^3}{48EI}$$

Therefore,

$$M_{\text{simply-supported}}\Big)_{max} = 2\times M_{\text{fixed-supported}}\Big)_{max}$$

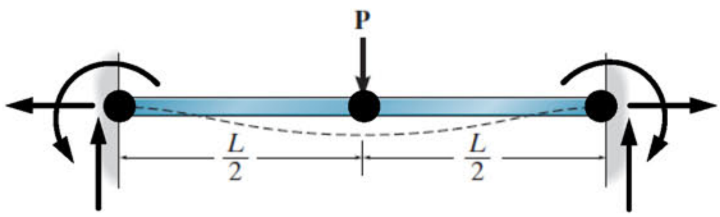
$$\Delta_{\text{fixed-supported}}\Big)_{max} = \frac{1}{4}\times \Delta_{\text{simply-supported}}\Big)_{max}$$

2. Another important reason for selecting a statically indeterminate structure is because it has a tendency to **redistribute** its load to its redundant supports in cases where faulty design or overloading occurs. In these cases, **the structure maintains its stability and collapse is prevented**.

- This is particularly important when **sudden lateral loads, such as wind or earthquake, are imposed on the structure**.

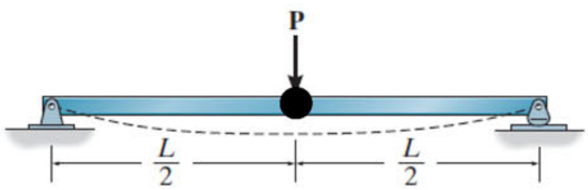
To illustrate, consider again the fixed-end beam.

- As P is increased, the beam's material at the walls and at the center of the beam begins to yield and forms localized "plastic hinges," which causes the beam to deflect as if it were hinged or pin connected at these points. Although the deflection becomes large, the walls will develop horizontal force and moment reactions that will hold the beam and thus prevent it from totally collapsing.



Plastic hinges in the indeterminate beam (horizontal force and moment reactions that will hold the beam and thus prevent it from totally collapsing)

- In the case of the simply supported beam, an excessive load P will cause the "plastic hinge" to form only at the center of the beam, and due to the large vertical deflection, the supports will not develop the horizontal force and moment reactions that may be necessary to prevent total collapse.



Plastic hinges in the determinate beam (the supports will not develop the horizontal force and moment reactions that may be necessary to prevent total collapse)

- Although statically indeterminate structures can support a loading with thinner members and with increased stability compared to their statically determinate counterparts.

Disadvantages

- The cost savings in material must be compared with the added cost necessary to fabricate the structure, since oftentimes it becomes **more costly** to **construct** the **supports** and **joints** of an indeterminate structure compared to one that is determinate.

- More important, though, because statically indeterminate structures have redundant support reactions, one has to be very careful to prevent **differential displacement of the supports**, since this effect will introduce **internal stress** in the structure.

- For example, if the wall at one end of the fixed-end beam were to settle, stress would be developed in the beam because of this "forced" deformation. On the other hand, if the beam were simply supported or statically determinate, then any settlement of its end would not cause the beam to deform, and therefore no stress would be developed in the beam.

- In general, then, any deformation, such as that caused by relative support displacement, or changes in member lengths caused by **temperature** or **fabrication errors**, will introduce additional stresses in the structure, which must be considered when designing indeterminate structures.

11.3 METHODS OF ANALYSIS

When analyzing any indeterminate structure, it is necessary to satisfy the following requirements for the structure:

1. Equilibrium.
2. Compatibility.
3. Force-displacement.

Equilibrium is satisfied when the reactive forces hold the structure at rest.

Compatibility is satisfied when the various segments of the structure fit together without intentional breaks or overlaps.

Force-displacement requirements depend upon the way the material responds; in our course, we have assumed linear elastic response.

In general there are two different ways to satisfy these requirements when analyzing a statically indeterminate structure:

1. The force or **flexibility** method.
2. The **displacement** or **stiffness** method.

11.3.1 FORCE METHOD

The force method was originally developed by James Clerk **Maxwell** in 1864 and later refined by Otto **Mohr** and Heinrich **Müller-Breslau**. This method was one of the first available for the analysis of statically indeterminate structures.

Since compatibility forms the basis for this method, it has sometimes been referred to as the **compatibility method** or the **method of consistent displacements** (or **method of consistent deformation**).

This method consists of **writing equations** that satisfy the **compatibility** and force-displacement requirements for the structure in order to determine the redundant forces.

Method of Consistent Deformation

Once these forces have been determined, the remaining reactive forces on the structure are determined by satisfying the equilibrium requirements.

11.3.2 DISPLACEMENT METHOD

The displacement method of analysis is based on first **writing force-displacement** relations for the members and then satisfying the **equilibrium** requirements for the structure.

In this case the **unknowns** in the equations are **displacements**.

Once the displacements are obtained, the forces are determined from the compatibility and force displacement equations.

Displacement Method

Method of Consistent Deformation vs Displacement Method

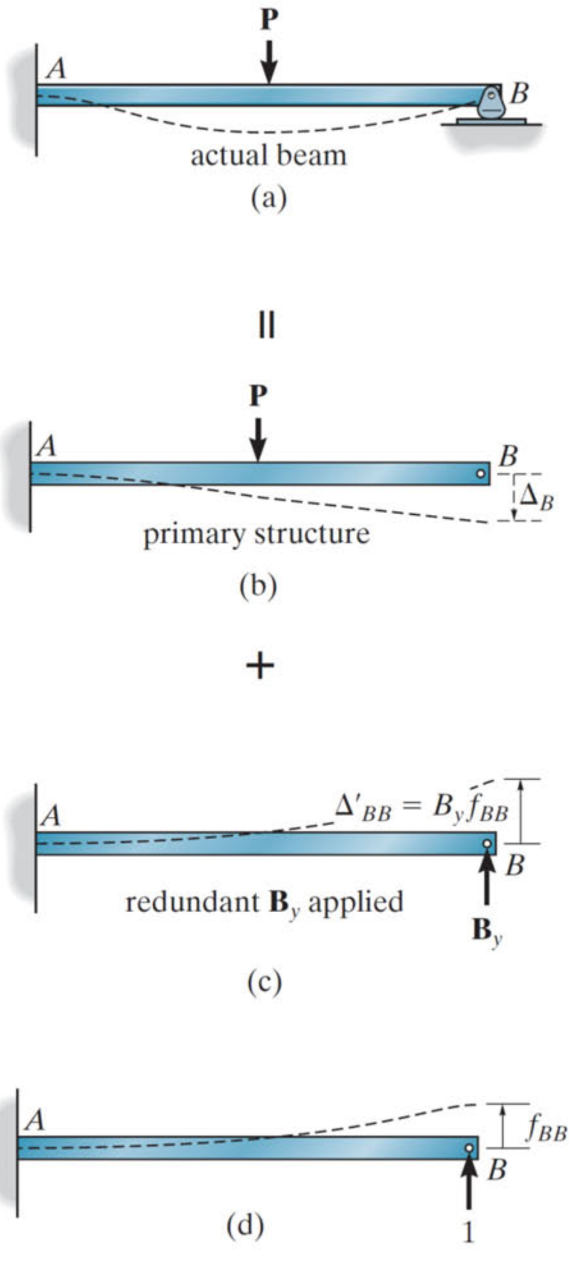
	Unknowns	Equations Used for Solution	Coefficients of the Unknowns
Force Method	Forces	Compatibility and Force Displacement	Flexibility Coefficients
Displacement Method	Displacements	Equilibrium and Force Displacement	Stiffness Coefficients

11.4 FORCE METHOD OF ANALYSIS: GENERAL PROCEDURES FOR BEAMS AND FRAMES

For the beam shown in the figure. If its free-body diagram were drawn, there would be **four unknown support reactions**; and since **three equilibrium equations** are available for solution, the beam is **indeterminate to the first degree**.

Consequently, one additional equation is necessary for solution. To obtain this equation, we will use the **principle of superposition** and consider the compatibility of displacement at one of the supports. This is done by choosing one of the support reactions as "**redundant**" and temporarily removing its effect on the beam so that the beam then becomes **statically determinate** and **stable**.

This beam is referred to as the **primary structure**. Here we will remove the restraining action of the rocker at B. As a result, the load P will cause B to be displaced downward by an amount Δ_B as shown in the figure (b). By superposition, however, the unknown reaction at B, i.e., B_y causes the beam at B to be displaced Δ'_{BB} upward, Figure (c).



Here the first letter in this double-subscript notation refers to the point (B) where the deflection is specified, and the second letter refers to the point (B) where the unknown reaction acts.

$$\Delta'_{BB} = \Delta'_{(Deflection\ Location)(Load\ Location)}$$

Assuming positive displacements act upward, we can write the necessary compatibility equation at the rocker as

$$(+\uparrow) \qquad -\Delta_B + \Delta'_{BB} = 0$$

Let us now denote the displacement at B caused by a unit load acting in the direction of B_y as

the **linear flexibility coefficient** f_{BB}

Using the same scheme for this double-subscript notation as above, is the deflection at B caused by a unit load at B. Since the material behaves in a linear-elastic manner, a force of acting at B, instead of the unit load, will cause a proportionate increase in f_{BB}. Thus we can write

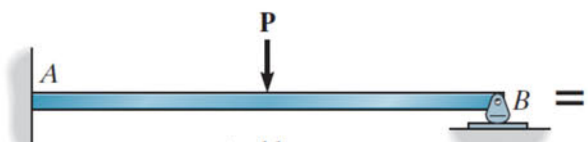
$$\Delta'_{BB} = B_y f_{BB}$$

When written in this format, it can be seen that the **linear flexibility coefficient** is a **measure of the deflection per unit force**, and so its units are $m/N, ft/lb$ etc. The compatibility equation above can therefore be written in terms of the unknown as

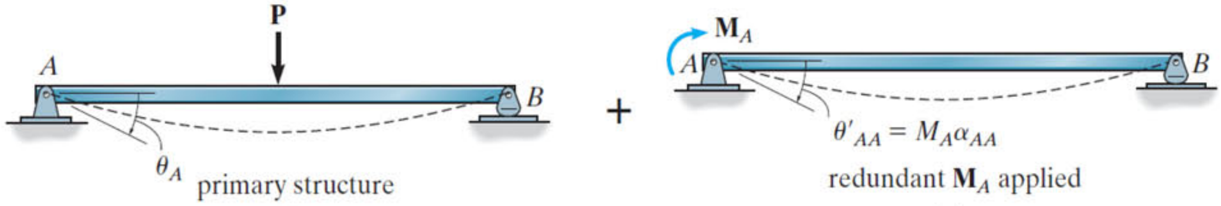
$$(+\uparrow) \qquad -\Delta_B + B_y f_{BB} = 0$$

$$f_{BB} = f_{(Deflection\ Location)(Load\ Location)}$$

A second example, for the beam shown below



actual beam
(a)



primary structure
(b)

redundant $\mathbf{M}_A$ applied
(c)

(d)

the moment at A, can be determined directly by removing the capacity of the beam to support a moment at A, that is, by replacing the fixed support by a pin.

As shown in Figure (b), the rotation at A caused by the load P is θ_A, and the rotation at A caused by the redundant at A is M_A, Figure (c). If we denote an **angular flexibility coefficient** α_{AA} as the **angular displacement** at A caused by a **unit couple moment** applied to A, Figure (d), then

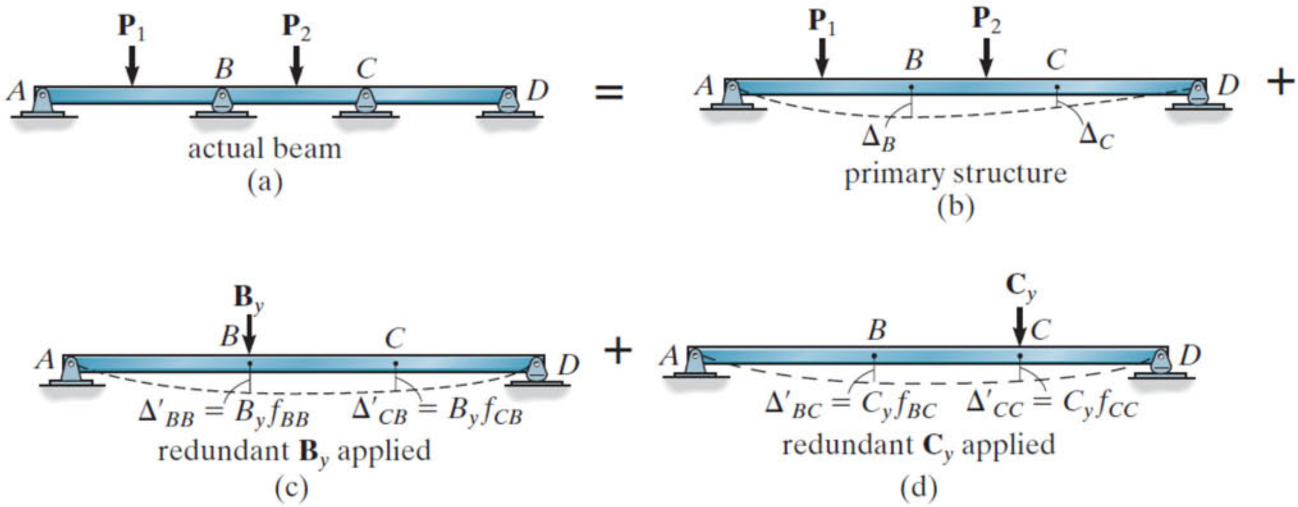
$$\theta'_{AA} = M_A \alpha_{AA}$$

Thus, the angular flexibility coefficient measures the **angular displacement** per **unit couple moment**, and therefore it has units of $rad/N.m$ or $rad/lb.ft$ etc. The compatibility equation for rotation at A therefore requires

$$(+\circlearrowleft) \qquad \theta_A + M_A \alpha_{AA} = 0$$

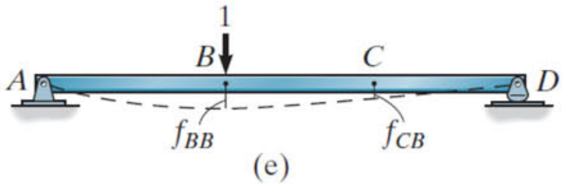
In this case, $M_A = -\theta_{AA}/\alpha_{AA}$, a negative value, which simply means that M_A acts in the opposite direction to the unit couple moment.

A third example

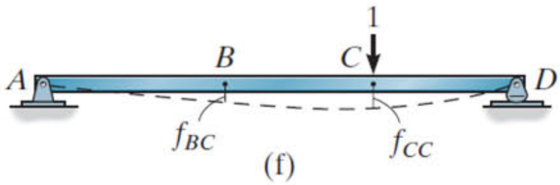


(+↓) $\Delta_B + B_y f_{BB} + C_y f_{BC} = 0$

(+↓) $\Delta_C + B_y f_{CB} + C_y f_{CC} = 0$



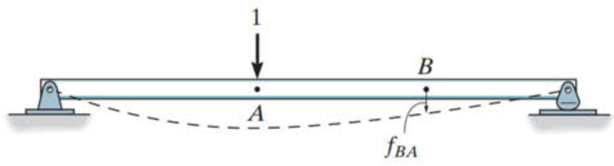
For all these cases, however, realize that since the method depends on superposition of displacements, it is necessary that the **material remain linear elastic** when loaded.



11.5 MAXWELL'S THEOREM OF RECIPROCAL DISPLACEMENTS; BETTI'S LAW

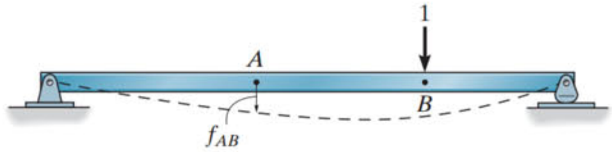
When Maxwell developed the force method of analysis, he also published a theorem that relates the flexibility coefficients of any two points on an elastic structure—be it a truss, a beam, or a frame. This theorem is referred to as the theorem of reciprocal displacements and may be stated as follows:

The displacement of a point B on a structure due to a unit load acting at point A is equal to the displacement of point A when the unit load is acting at point B, that is, .



$$f_{AB} = f_{BA}$$

$$f_{AB} = \int \frac{m_A m_B}{EI} dx \qquad f_{BA} = \int \frac{m_B m_A}{EI} dx$$



Procedure for Analysis — Method of Consistent Deformation

The following procedure provides a general method for determining the reactions or internal loadings of statically indeterminate structures using the force or flexibility method of analysis.

Principle of Superposition

- Determine the number of degrees n to which the structure is indeterminate.
- Then specify the n **unknown redundant forces** or **moments** that must be removed from the structure in order to make it **statically determinate** and **stable**.
- Using the principle of superposition, draw the statically indeterminate structure and show it to be equal to a **series** of corresponding **statically determinate structures**.
- The **primary structure** supports the same external loads as the statically indeterminate structure, and each of the other structures added to the primary structure shows the structure loaded with a separate redundant force or moment.
- Sketch the elastic curve on each structure and indicate symbolically the displacement or rotation at the point of each redundant force or moment.

Compatibility Equations

- Write a **compatibility equation** for the displacement or rotation at each point where there is a redundant force or moment.

$$(+\uparrow) \qquad \Delta_B + B_y f_{BB} + C_y f_{BC} + \cdots = 0$$

$$(+\uparrow) \qquad \Delta_C + B_y f_{CB} + C_y f_{CC} + \cdots = 0$$

$$\vdots$$

$$(+\circlearrowleft) \qquad \theta_A + M_A\alpha_{AA} + M_B\alpha_{AB} + \cdots = 0$$

$$(+\circlearrowleft) \qquad \theta_B + M_A\alpha_{BA} + M_B\alpha_{BB} + \cdots = 0$$

$$\vdots$$

- These equations should be expressed in terms of the **unknown redundants** and their corresponding **flexibility coefficients** obtained from **unit loads** or **unit couple moments** that are collinear with the redundant forces or moments.

$$\Delta = \int \frac{Mm_\Delta}{EI} dx$$

$$\theta = \int \frac{Mm_\theta}{EI} dx$$

$$f_{ii} = \int \frac{m_\Delta^i m_\Delta^i}{EI} dx = \int \frac{(m_\Delta)^2}{EI} dx \qquad \alpha_{ii} = \int \frac{m_\theta^i m_\theta^i}{EI} dx = \int \frac{(m_\theta)^2}{EI} dx$$

$$f_{ij} = \int \frac{m_\Delta^i m_\Delta^j}{EI} dx \qquad \alpha_{ij} = \int \frac{m_\theta^i m_\theta^j}{EI} dx$$

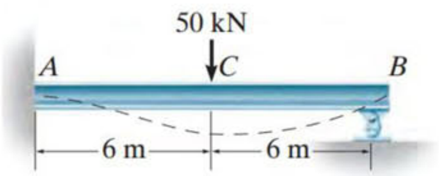
- Determine all the deflections and **flexibility coefficients**. Substitute these load-displacement relations into the compatibility equations and solve for the unknown redundants.
- In particular, if a numerical value for a redundant is **negative**, it indicates the redundant acts **opposite** to its corresponding unit force or unit couple moment.

Equilibrium Equations

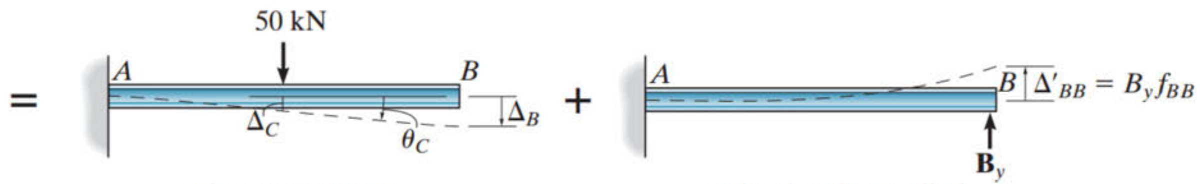
- Draw a free-body diagram of the structure.
- Since the redundant forces and/or moments have been calculated, the remaining unknown reactions can be determined from the equations of equilibrium.
- It should be realized that once all the support reactions have been obtained, the shear and moment diagrams can then be drawn, and the deflection at any point on the structure can be determined using the same methods outlined previously for statically determinate structures.

Example 11.1 — Method of Consistent Deformation, Beams

Determine the reaction at the roller support B of the beam shown in the figure. EI is constant.



actual beam
(a)



primary structure

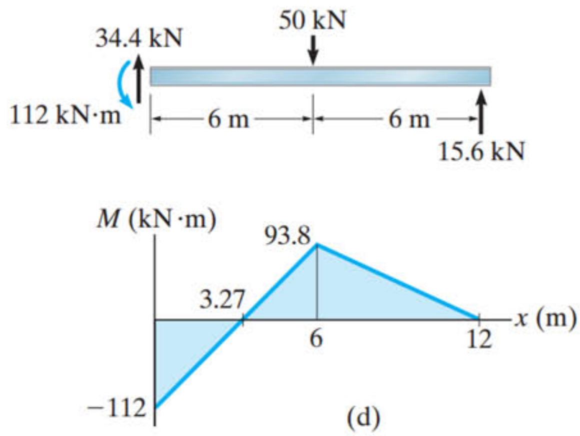
redundant B_y applied

(b)

Solution

Principle of Superposition

By inspection, the beam is statically indeterminate to the first degree. The redundant will be taken as B_y so that this force can be determined directly.



(d)

Compatibility Equation

Taking positive displacement as upward,

$$(+\uparrow) \qquad -\Delta_B + B_y f_{BB} = 0$$

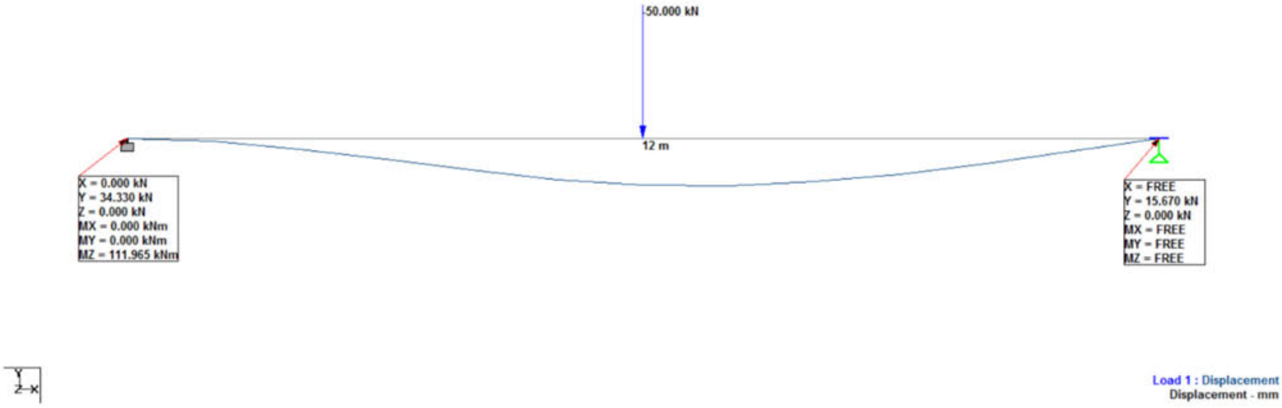
$$\Delta_B = \int_{x_A}^{x_C} \frac{M m_\Delta}{EI} dx = \int_0^6 \frac{(50x - 300)(x - 12)}{EI} dx = \frac{9000}{EI}\ kN.m^3 \downarrow$$

$$f_{BB} = \int \frac{m^2}{EI} dx = \int_0^{12} \frac{(x)^2}{EI} dx = \frac{576}{EI}\ m^3 \uparrow$$

$$-\frac{9000}{EI} + \frac{576}{EI} B_y = 0 \Rightarrow B_y = 15.6\ kN \quad \boldsymbol{Ans.}$$

STAAD.Pro

Reactions of Example 11.1 (Consistent Deformation Method)



Example 11.1.std - Support Reactions:

All | Summary | Envelope

		Horizontal	Vertical	Horizontal	Moment		
Node	L/C	Fx kN	Fy kN	Fz kN	Mx kNm	My kNm	Mz kNm
1	1 IMPOSED L	0.000	34.330	0.000	0.000	0.000	111.965
3	1 IMPOSED L	0.000	15.670	0.000	0.000	0.000	0.000

Example 11.1.std - Statics Check Results

L/C		Fx kN	Fy kN	Fz kN	Mx kNm	My kNm	Mz kNm
1	Loads	0.000	-11.240	0.000	0.000	0.000	-2655.224
	Reactions	0.000	11.240	0.000	0.000	0.000	2655.224
	Difference	0.000	0.000	0.000	0.000	0.000	0.000

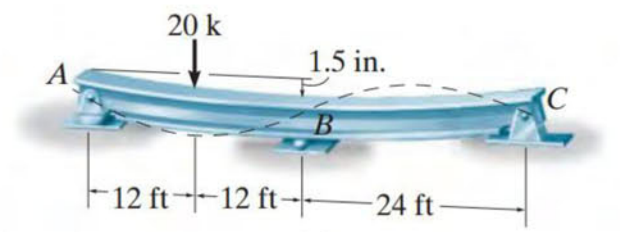
STAAD.Pro

STAAD Command File of Example 11.1

```
STAAD FLOOR
START JOB INFORMATION
ENGINEER DATE 14-Mar-17
END JOB INFORMATION
INPUT WIDTH 79
UNIT METER KN
JOINT COORDINATES
1 0 0 0; 3 12 0 0;
MEMBER INCIDENCES
1 1 3;
DEFINE MATERIAL START
ISOTROPIC STEEL
E 2.05e+008
POISSON 0.3
DENSITY 76.8195
ALPHA 1.2e-005
DAMP 0.03
ISOTROPIC CONCRETE
E 2.17185e+007
POISSON 0.17
DENSITY 23.5616
ALPHA 1e-005
DAMP 0.05
END DEFINE MATERIAL
MEMBER PROPERTY
1 PRIS YD 1 ZD 0.4
CONSTANTS
MATERIAL CONCRETE ALL
SUPPORTS
1 FIXED
3 FIXED BUT FX MX MY MZ
LOAD 1 LOADTYPE None  TITLE IMPOSED LOAD
MEMBER LOAD
1 CON GY -50 6
PERFORM ANALYSIS
FINISH
```

Example 11.2 — Method of Consistent Deformation, Beams

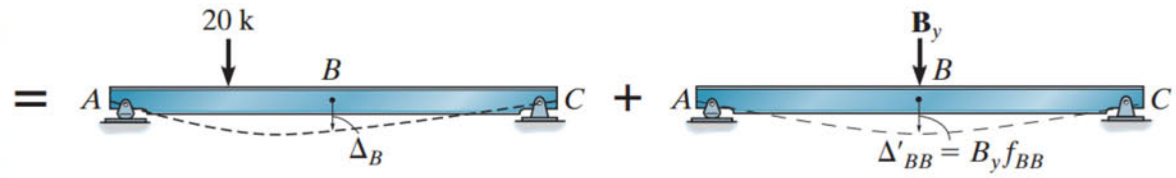
Draw the shear and moment diagrams for the beam shown in the figure. The support at B settles 1.5 $in.$ Take $= 29 \times 10^3\ ksi,\ I\ =\ 750\ in^4$.



actual beam

(a)

Solution



primary structure   redundant $\mathbf{B}_y$ applied

(b)

Principle of Superposition

By inspection, the beam is indeterminate to the first degree.

The center support B will be chosen as the redundant, so that the roller at B is removed. Here B_y is assumed to act downward on the beam.

Compatibility Equation

With reference to point B, using units of inches, we require

$$(+\downarrow) \qquad \Delta_B + B_y f_{BB} = 1.5\ in$$

$$\Delta_B = \int_{x_A=0}^{x_{20\,k}=12} \frac{Mm_\Delta}{EI} dx + \int_{x_B=0}^{x_{20\,k}=12} \frac{Mm_\Delta}{EI} dx + \int_{x_C=0}^{x_B=24} \frac{Mm_\Delta}{EI} dx$$

$$\Delta_B = \int_0^{12} \frac{(15x)(0.5x)}{EI} dx + \int_0^{12} \frac{\big(5(x+24)\big)(0.5(x+24)-x)}{EI} dx + \int_0^{24} \frac{(5x)(0.5x)}{EI} dx$$

$$\Delta_B = 4{,}320 + 15{,}840 + 11{,}520 = \frac{31{,}680}{EI}\ k.ft^3$$

$$f_{BB} = 2\int \frac{m^2}{EI} dx = 2\int_0^{24} \frac{(0.5x)^2}{EI} dx = \frac{2304}{EI}\ k.ft^3$$

$$\frac{31{,}680(12)^3}{21.75\times 10^6} + B_y \frac{2304(12)^3}{21.75\times 10^6} = 1.5\ in$$

$$31{,}680(12)^3 + 2304(12)^3 B_y = 1.5\ in\ \times 21.75\times 10^6$$

$$\therefore B_y = -5.56\ k = 5.56\ k\ \uparrow$$

The negative sign indicates that acts B_y upward on the beam

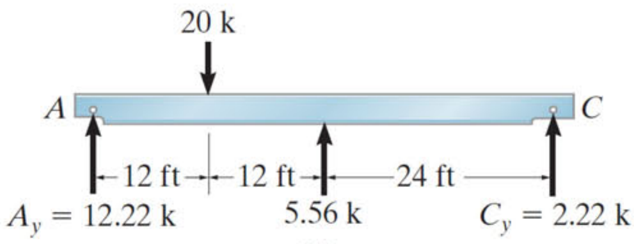
Equilibrium Equations

$\circlearrowleft + \Sigma M_A = 0;$ $\quad -20(12) + 5.56(24) + C_y(48) = 0$

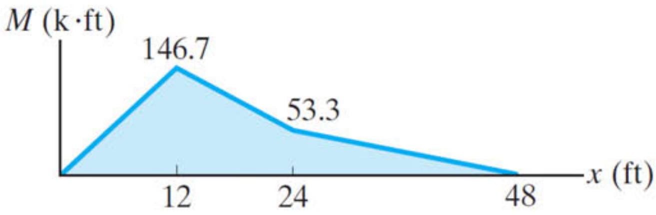
$C_y = 2.22$ k

$+\uparrow \Sigma F_y = 0;$ $\quad A_y - 20 + 5.56 + 2.22 = 0$

$A_y = 12.22$ k

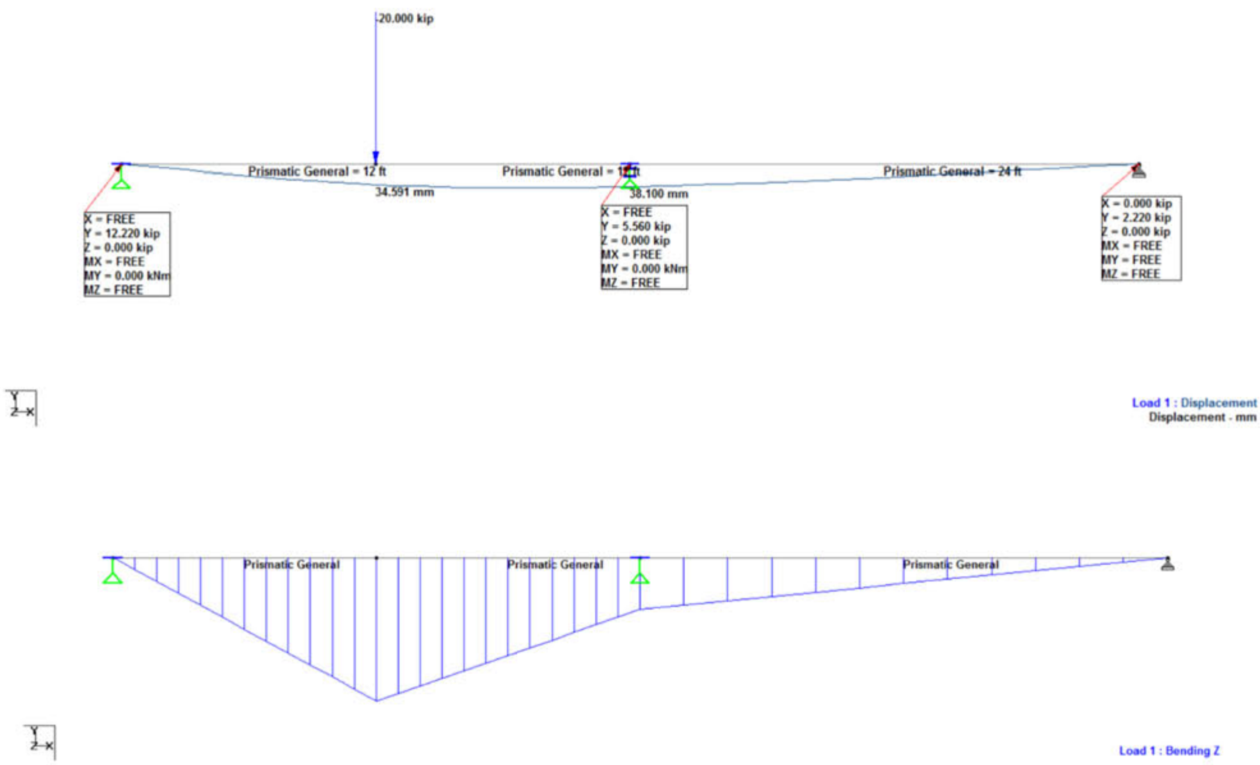


(c)



(d)

BMD of Example 11.2 (Consistent Deformation Method)



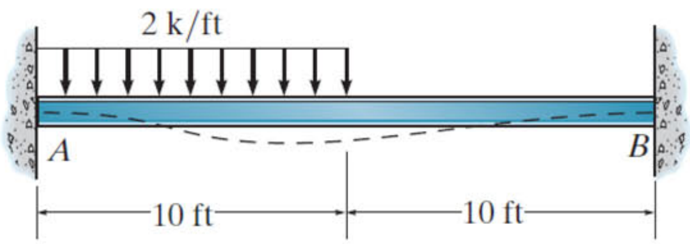
STAAD.Pro

STAAD Command File of Example 11.2

```
STAAD PLANE
START JOB INFORMATION
ENGINEER DATE 15-Mar-17
END JOB INFORMATION
INPUT WIDTH 79
UNIT FEET KIP
JOINT COORDINATES
1 0 0 0; 2 12 0 0; 3 24 0 0; 4 48 0 0;
MEMBER INCIDENCES
1 1 2; 2 2 3; 3 3 4;
UNIT INCHES KIP
DEFINE MATERIAL START
ISOTROPIC STEEL
E 29000
POISSON 0.3
DENSITY 0.000283
ALPHA 1.2e-005
DAMP 0.03
END DEFINE MATERIAL
MEMBER PROPERTY AMERICAN
1 TO 3 PRIS IZ 750
CONSTANTS
MATERIAL STEEL ALL
SUPPORTS
4 PINNED
1 3 FIXED BUT FX MX MY MZ
LOAD 1 LOADTYPE None  TITLE Imposed
JOINT LOAD
2 FY -20
SUPPORT DISPLACEMENT LOAD
3 FY -1.5
PERFORM ANALYSIS PRINT ALL
FINISH
```

Example 11.3 — Method of Consistent Deformation, Beams

Draw the shear and moment diagrams for the beam shown in the figure. EI is constant. Neglect the effects of axial load.



Solution

Principle of Superposition

Since axial load is neglected, the beam is indeterminate to the second degree. The two end moments at A and B will be considered as the redundants. The beam's capacity to resist these moments is removed by placing a pin at A and a rocker at B.

Compatibility Equations

Reference to points A and B.

$$(+\circlearrowright) \qquad \theta_A + M_A\alpha_{AA} + M_B\alpha_{AB} = 0 \qquad (1)$$

$$(+\circlearrowleft) \qquad \theta_B + M_A\alpha_{BA} + M_B\alpha_{BB} = 0 \qquad (2)$$

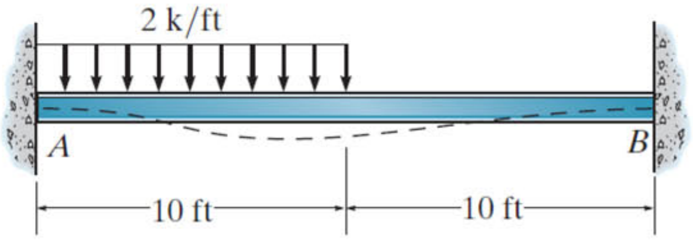
$Segment$	$Origin$	$Limit$	M	m_θ^A	m_θ^B
AC	A	$0 \to 10$	$(15x - x^2)$	$\left(1 - \frac{x}{20}\right)$	$\left(\frac{x}{20}\right)$
BC	B	$0 \to 10$	$(5x)$	$\left(\frac{x}{20}\right)$	$\left(1 - \frac{x}{20}\right)$

$$\theta_A = \int_0^{10} \frac{(15x - x^2)\left(1 - \frac{x}{20}\right)}{EI} dx + \int_0^{10} \frac{(5x)\left(\frac{x}{20}\right)}{EI} dx = \frac{375}{EI}$$

$$\theta_B = \int_0^{10} \frac{(15x - x^2)\left(\frac{x}{20}\right)}{EI} dx + \int_0^{10} \frac{(5x)\left(1 - \frac{x}{20}\right)}{EI} dx = \frac{291.67}{EI}$$

$Segment$	$Origin$	$Limit$	m_θ^A	m_θ^B
AB	A	$0 \to 20$	$\left(1 - \frac{x}{20}\right)$	$\left(\frac{x}{20}\right)$
AB	B	$0 \to 20$	$\left(\frac{x}{20}\right)$	$\left(1 - \frac{x}{20}\right)$

$$\alpha_{AA} = \int_0^{20} \frac{m_\theta^A m_\theta^A}{EI} dx$$
$$= \int_0^{20} \frac{\left(1 - \frac{x}{20}\right)^2}{EI} dx$$
$$= \frac{6.67}{EI}$$



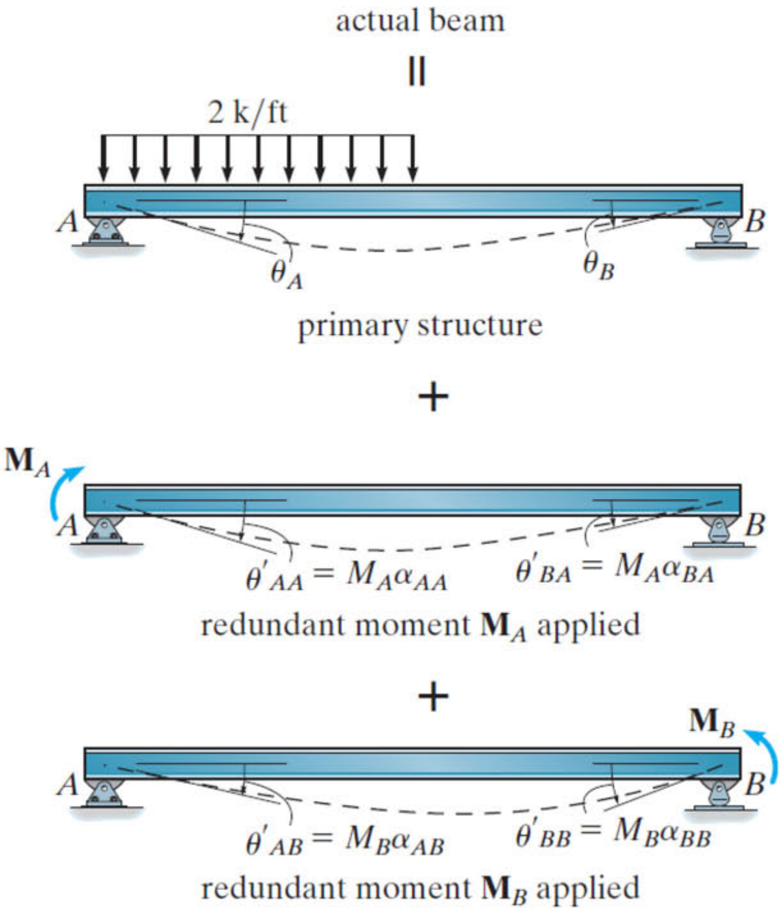
(a)

$$\alpha_{BB} = \int_0^{20} \frac{m_\theta^B m_\theta^B}{EI} dx = \int_0^{20} \frac{\left(\frac{x}{20}\right)^2}{EI} dx$$
$$= \frac{6.67}{EI}$$

$$\alpha_{AB} = \alpha_{BA} = \int_0^{20} \frac{m_\theta^A m_\theta^B}{EI} dx$$

$$= \int_0^{20} \frac{\left(1 - \frac{x}{20}\right)\left(\frac{x}{20}\right)}{EI} dx = \frac{3.33}{EI}$$

Note that $\alpha_{AB} = \alpha_{BA}$,a consequence of Maxwell's theorem of reciprocal displacements



(b)

$$(+\circlearrowright) \qquad \frac{375}{EI} + M_A\left(\frac{6.67}{EI}\right) + M_B\left(\frac{3.33}{EI}\right) = 0 \qquad (1)$$

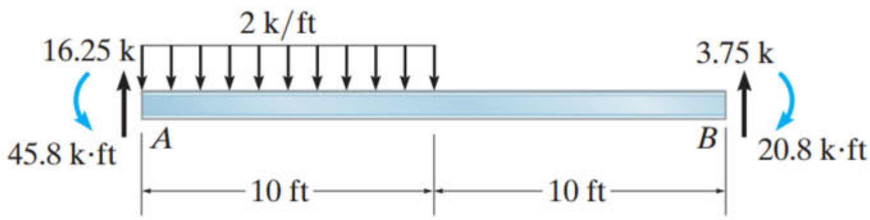
$$(+\circlearrowleft) \qquad \frac{291.7}{EI} + M_A\left(\frac{3.33}{EI}\right) + M_B\left(\frac{6.67}{EI}\right) = 0 \qquad (2)$$

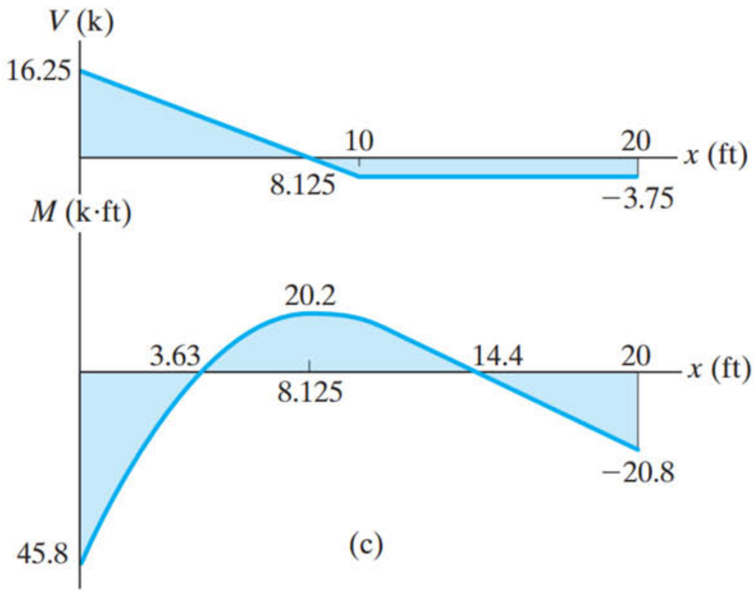
Canceling EI and solving these equations simultaneously, we have

$$M_A = -45.8\ k.ft = 45.8\ k.ft \circlearrowleft \quad \textbf{\textit{Ans}}.$$

$$M_B = -20.8\ k.ft = 20.8\ k.ft \circlearrowright \quad \textbf{\textit{Ans}}.$$

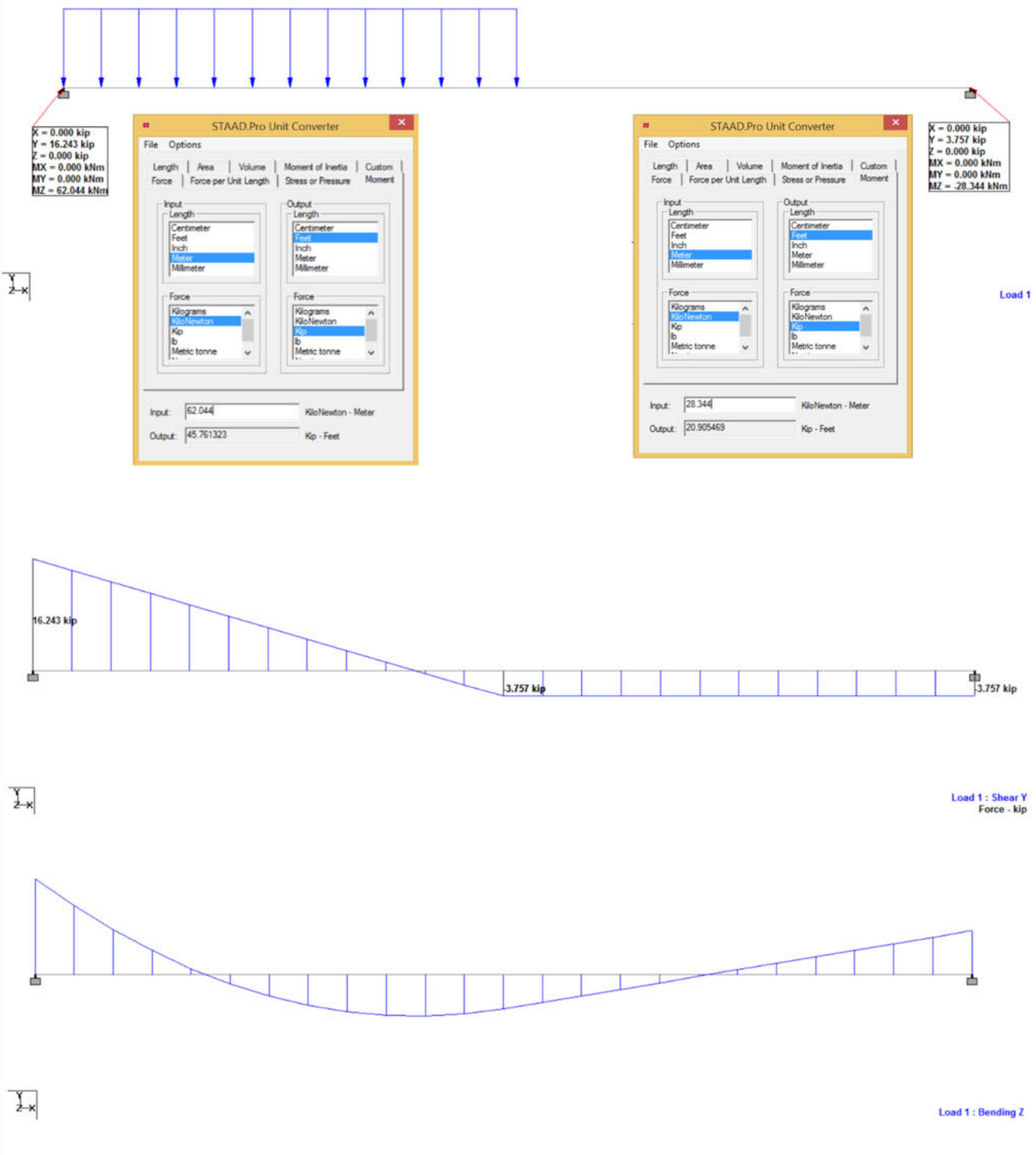
Equilibrium





(c)

BMD and SFD of Example 11.3 (Consistent Deformation)



STAAD.Pro

STAAD Command File of Example 11.3

```
STAAD floor
START JOB INFORMATION
ENGINEER DATE 15-Mar-17
END JOB INFORMATION
INPUT WIDTH 79
UNIT FEET KIP
JOINT COORDINATES
1 0 0 0; 2 10 0 0; 3 20 0 0;
MEMBER INCIDENCES
1 1 2; 2 2 3;
UNIT INCHES KIP
DEFINE MATERIAL START
ISOTROPIC STEEL
E 29000
POISSON 0.3
DENSITY 0.000283
ALPHA 1.2e-005
DAMP 0.03
END DEFINE MATERIAL
MEMBER PROPERTY AMERICAN
1 2 PRIS IZ 750
CONSTANTS
MATERIAL STEEL ALL
SUPPORTS
1 3 FIXED
LOAD 1 LOADTYPE None  TITLE IMPOSED
UNIT FEET KIP
MEMBER LOAD
1 UNI GY -2
UNIT INCHES KIP
PERFORM ANALYSIS PRINT ALL
FINISH
```

ANALYSIS OF STATICALLY INDETERMINATE STRUCTURES BY THE MOMENT DISTRIBUTION METHOD

12

12.1 GENERAL

- The method of analyzing beams and frames using moment distribution was developed by **Hardy Cross**, in **1930**.
- At the time this method was first published it attracted immediate attention, and it has been recognized as **one of the most notable advances** in structural analysis during the **twentieth century**.
- The moment distribution is a method of **successive approximations** that may be carried out to any desired degree of accuracy.
- Essentially, the method begins by assuming each joint of a **structure is fixed**. Then, by **unlocking** and **locking** each joint in succession, the internal moments at the joints are "**distributed**" and balanced until the joints have rotated to their final or nearly final positions. It will be found that this process of calculation is both **repetitive** and **easy to apply**..

12.2 SIGN CONVENTION

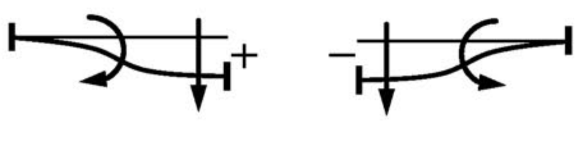
The sign convention is as that established for the slope-deflection equations.

Internal end moments M_{AB} and M_{BA}

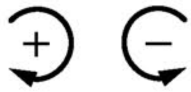
in terms of its three degrees of freedom, namely,

Angular displacements θ_A and θ_B

Linear displacement Δ

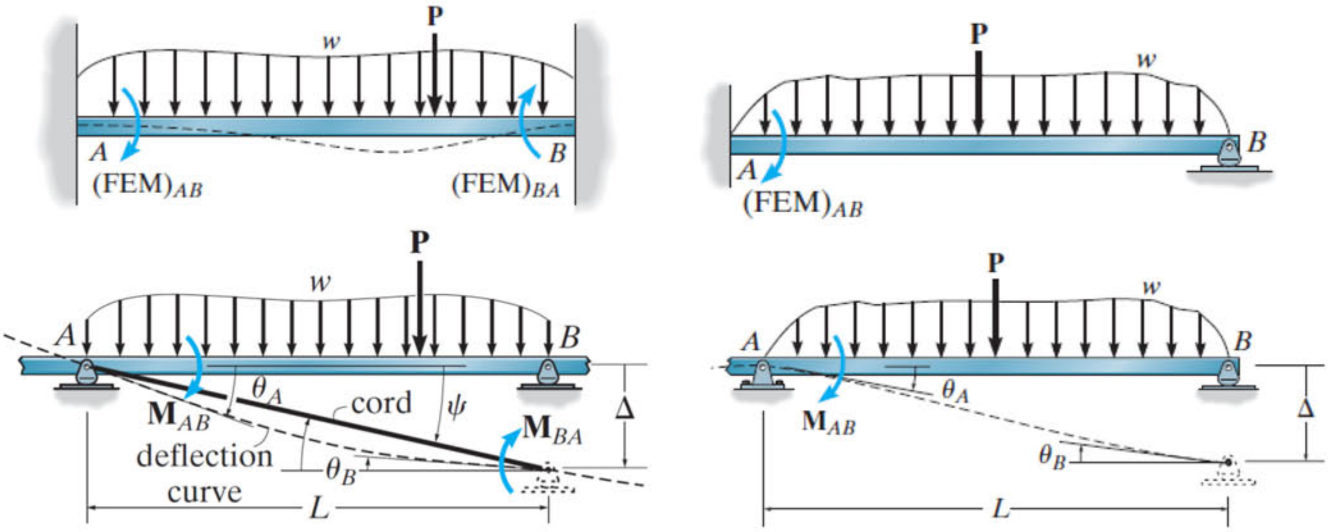


Span Chord Angle $\psi = (\Delta/L)$

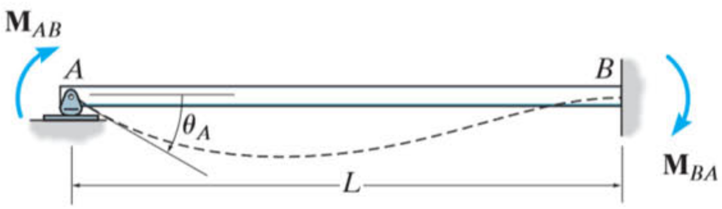


12.3 FIXED-END MOMENTS (FEMS)

The moments at the "**walls**" or fixed joints of a loaded member are called **fixed-end moments.**



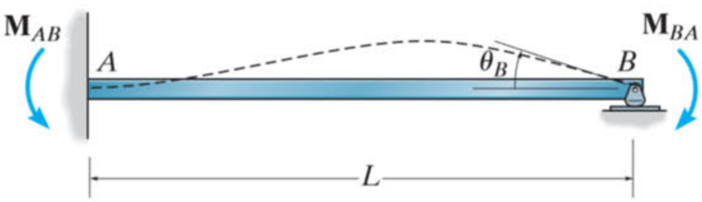
12.4 ANGULAR DISPLACEMENTS AT A, θ_A



$$M_{AB} = \frac{4EI}{L}\theta_A$$

$$M_{BA} = \frac{2EI}{L}\theta_A$$

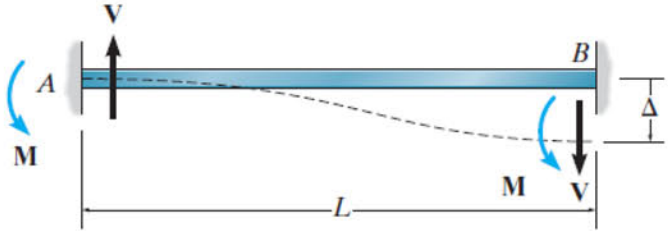
12.5 ANGULAR DISPLACEMENTS AT B, θ_B



$$M_{BA} = \frac{4EI}{L}\theta_B$$

$$M_{AB} = \frac{2EI}{L}\theta_B$$

12.6 LINEAR DISPLACEMENT Δ



$$M_{AB} = M_{BA} = M = \frac{-6EI}{L^2}\Delta$$

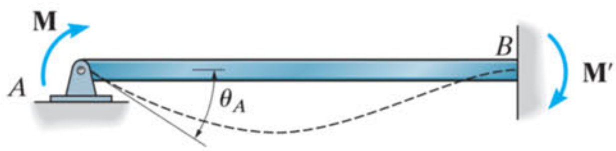
12.7 MEMBER STIFFNESS FACTOR

Flexural Stiffness referred to as the stiffness factor at A and can be defined as the amount of moment M required to rotate the end A of the beam $\theta_A = 1$ rad.

12.7.1 FAR END IS FIXED OR CONTIUOUS

$$K = \frac{4EI}{L}$$

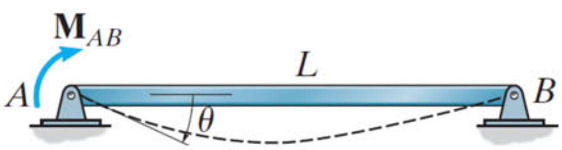
Far End Fixed



12.7.2 FAR END IS HINGE OR ROLLER

$$K = \frac{3EI}{L}$$

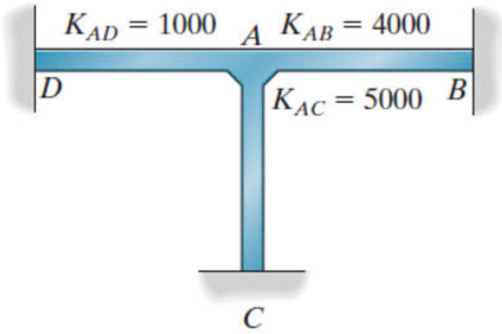
Far End Pinned
or Roller Supported



12.8 JOINT STIFFNESS FACTOR

If several members are fixed connected to a joint and each of their far ends is fixed, then by the principle of superposition, the total stiffness factor at the joint is the sum of the member stiffness factors at the joint, that is,

$$K_T = \sum K_i$$



For example, consider the frame joint A in the figure shown. The numerical value of each member stiffness factor is determined as follows

$$K_T = 1000 + 4000 + 5000 = 10{,}000$$

This value represents the amount of moment needed to rotate the joint through an angle of 1 rad

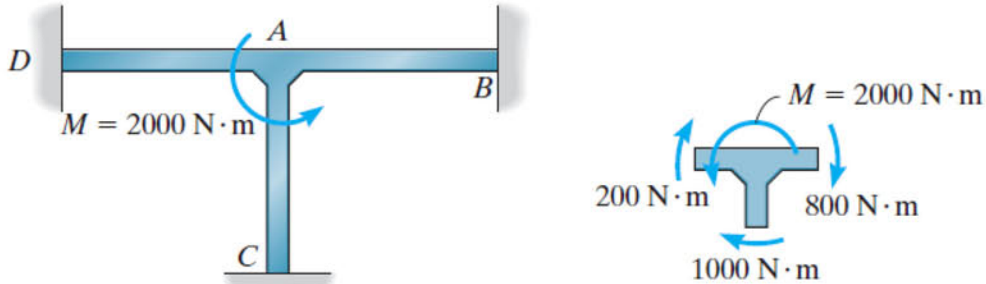
12.9 DISTRIBUTION FACTOR (DF)

That fraction of the total resisting moment supplied by the member is called the **distribution factor (DF)**.

$$\text{DF} = \frac{K}{\Sigma K}$$

$$\sum DF_i = 1$$

For example, the distribution factors for members AB, AC, and AD at joint A, are



$DF_{AB} = 4000/10\,000 = 0.4$
$DF_{AC} = 5000/10\,000 = 0.5$
$DF_{AD} = 1000/10\,000 = 0.1$

As a result, if $M = 2000\ N.m$ acts at joint A, the equilibrium moments exerted by the members on the joint, are

$M_{AB} = 0.4(2000) = 800\ \text{N}\cdot\text{m}$
$M_{AC} = 0.5(2000) = 1000\ \text{N}\cdot\text{m}$
$M_{AD} = 0.1(2000) = 200\ \text{N}\cdot\text{m}$

12.10 SPECIAL CASES FOR DISTRIBUTION FACTOR

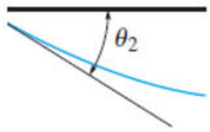
12.10.1 FIXED END

$$K_{Fixed} = \infty, \qquad K_{Member} = K_m, \qquad DF_{Member} = \frac{K_m}{\infty + K_m} = 0$$

$$DF = 0$$

12.10.2 CANTILEVER

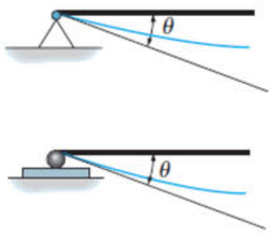
$$DF = 0$$



12.10.3 HINGE OR ROLLER

$$K_{Hinge} = 0, \qquad K_{Member} = K_m, \qquad DF_{Member} = \frac{K_m}{0 + K_m} = 1$$

$$DF = 1$$



12.11 CARRY-OVER FACTOR (CO)

Consider the beam shown in the figure.

$$M_A = M = \frac{4EI}{L}\theta_A$$

And

$$M_B = M' = \frac{2EI}{L}\theta_A$$

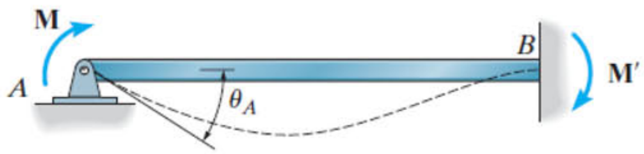
Then, the moment M at the pin induces a moment of $M' = \frac{1}{2}M$ at the wall.

$$M' = \frac{1}{2}M$$

The **carry-over factor** represents the fraction of M that is "carried over" from the pin to the wall. Hence, in the case of a beam with the far end fixed, the carry-over factor is

$$CO = +\frac{1}{2}$$

The plus sign indicates both moments act in the same direction.

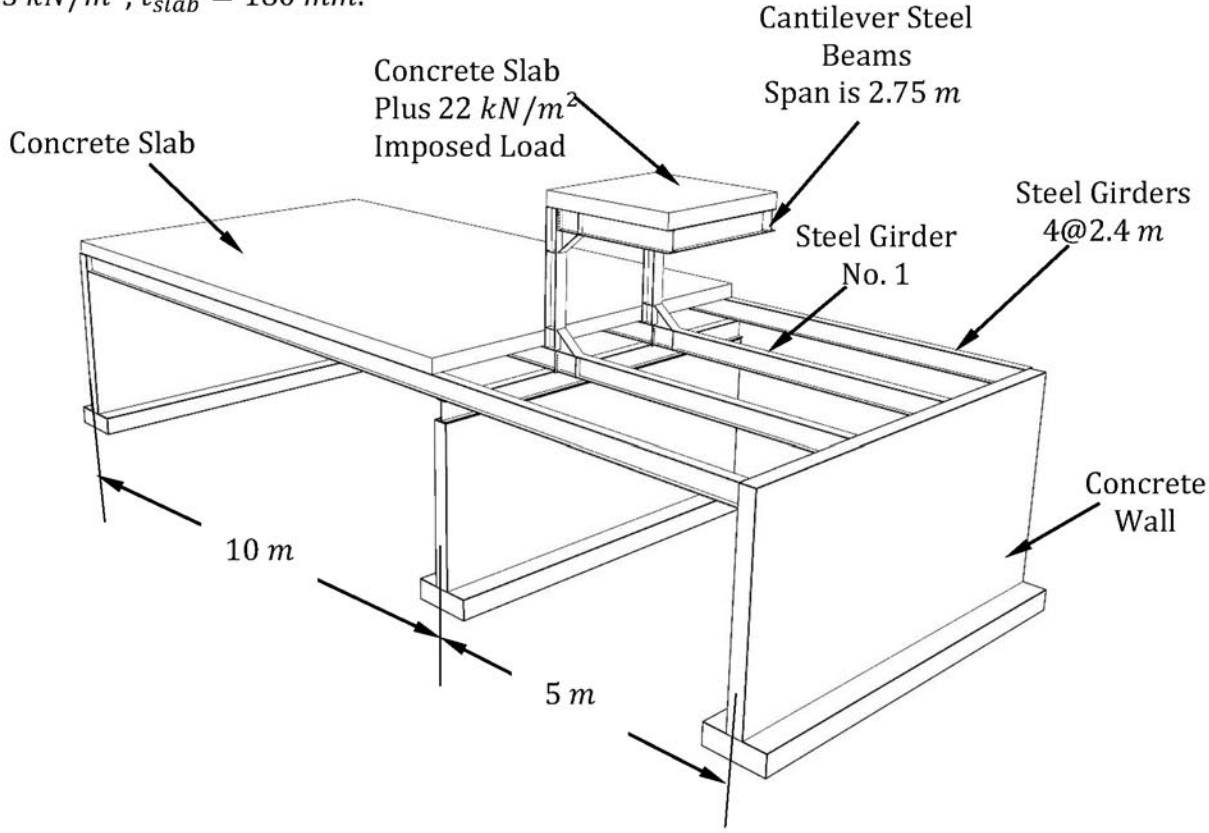


12.12 MOMENT DISTRIBUTION FOR BEAMS

Example 12.1 — **Moment Distribution Method, Beams**

Analyze Steel Girder No. 1 in the structure, shown in the figure, due to imposed load and weight of slab, by using the Moment Destitution Method. (Note: EI is constant; the steel girders are perfectly fixed to the end walls; and neglect the self-weight of the steel frame). Assume the following data:

$\gamma_{Conc.} = 25\ kN/m^3$, $t_{slab} = 180\ mm.$



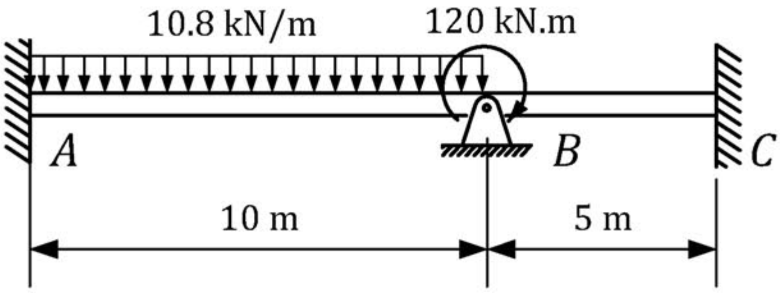
Solution

Modeling

$$W_{Girder} = \gamma_{Conc.} \times t_{slab} \times b = 25 \times 0.18 \times 2.4 = 10.8\ kN/m \downarrow$$

$$M_{Girder} = F \times d = \frac{(\gamma_{Conc.} \times t_{slab} \times b \times w + L_{Imposed} \times Area)}{2} \times d$$

$$= \frac{(25 \times 0.18 \times 2.4 \times 2.75 + 22 \times 2.4 \times 2.75)}{2} \times \frac{2.75}{2} = 120\ kN.m \circlearrowleft = +120\ kN.m$$



Fixed-End Moments (FEM)

$$(FEM)_{AB} = -\frac{wL^2}{12} = -\frac{10.8(10)^2}{12} = -90\ kN.m$$

$$(FEM)_{BA} = +\frac{wL^2}{12} = +\frac{10.8(10)^2}{12} = +90\ kN.m$$

$$(FEM)_{BC} = (FEM)_{CB} = 0$$

Distribution Factors (DF)

Joint *B*

$$K_{BA} = \frac{4EI}{10} = 0.4EI$$

$$K_{BC} = \frac{4EI}{5} = 0.8EI$$

$$\sum K_B = 1.2EI$$

$$DF_{BA} = \frac{0.4EI}{1.2EI} = \frac{1}{3}$$

$$DF_{BA} = \frac{0.8EI}{1.2EI} = \frac{2}{3}$$

$$\sum DF_i = \frac{1}{3} + \frac{2}{3} = 1$$

Fixed Ends

$$DF_{AB} = DF_{CB} = 0$$

Moment Distribution Table

Joint	A	B			C
Member	AB (Fixed)	BA		BC	CB (Fixed)
DF	0	1/3		2/3	0
FEM	−90	+90	−120	0	0
Dist.	0	+10		+20	0
CO	+5	0		0	+10
Dist.	0	0		0	0
$\sum M$	−85	+100		+20	+10

Then

$M_{AB} = -85\ kN.m \quad Ans.$

$M_{BA} = +100\ kN.m \quad Ans.$

$M_{BC} = +20\ kN.m \quad Ans.$

$M_{CB} = +10\ kN.m \quad Ans.$

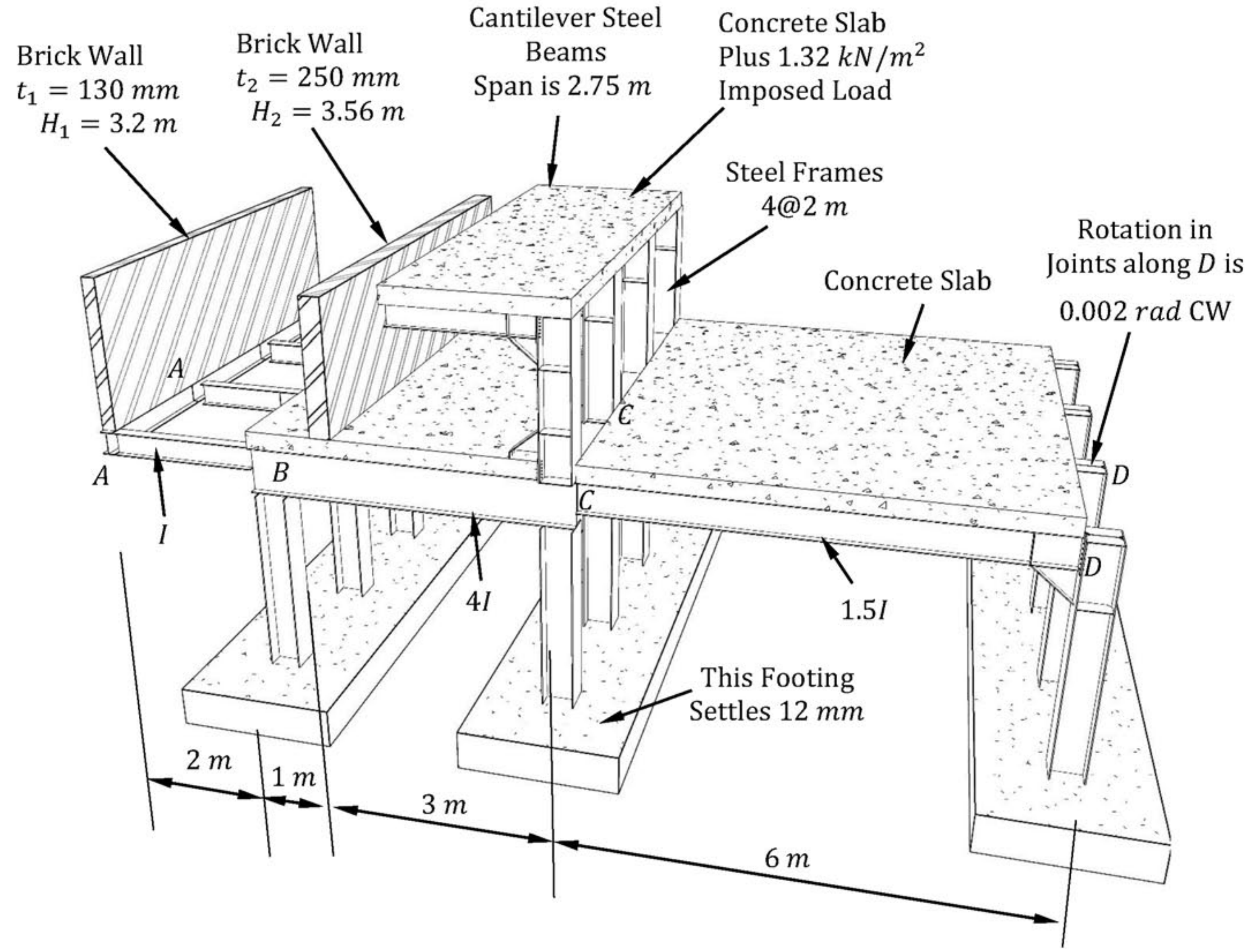
Example 12.2 — Moment Distribution Method, Beams

Analyze the interior steel girder $ABCD$ in the structure, shown in the figure, due to imposed load and weight of nonbearing walls, by using the Moment Destitution Method. (Note: $I = 10^4 kN.m^2$; the steel girders are perfectly fixed to the columns along D; and neglect the self-weight of the steel frame).

Assume the following data:

$\gamma_{Conc.} = 24\ kN/m^3$, $t_{slab} = 125\ mm$.

$\gamma_{Brick} = 18\ kN/m^3$.



Solution

Modeling

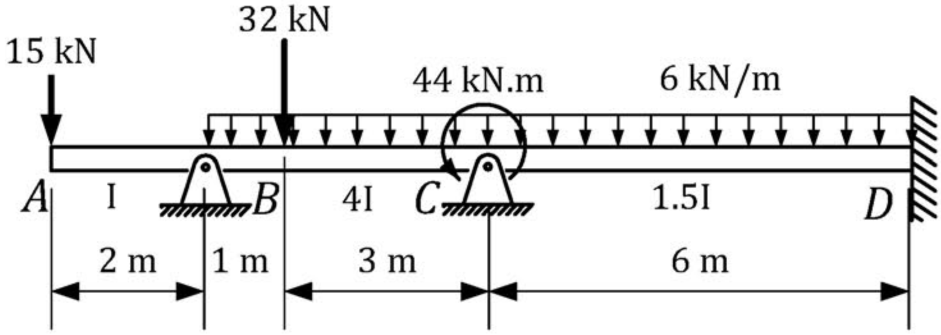
$$W_{Girder} = \gamma_{Conc.} \times t_{slab} \times b = 24 \times 0.125 \times 2.0 = 6\ kN/m \downarrow$$

$$M_{Girder} = F \times d = \left(\gamma_{Conc.} \times t_{slab} \times b \times w + L_{Imposed} \times Area\right) \times d$$

$$= (25 \times 0.18 \times 2.0 \times 2.75 + 1.32 \times 2.0 \times 2.75) \times \frac{2.75}{2} = 44\ kN.m \circlearrowleft = -44\ kN.m$$

$$F1_{Girder} = \gamma_{Brick} \times t_{Wall\ 1} \times H_1 \times b = 18 \times 0.13 \times 3.2 \times 2 = 15\ kN \downarrow$$

$$F2_{Girder} = \gamma_{Brick} \times t_{Wall\ 2} \times H_2 \times b = 18 \times 0.25 \times 3.56 \times 2 = 32\ kN \downarrow$$



Fixed-End Moments (FEM)

$$(FEM)_{BA} = +15 \times 2 = +30\ kN.m$$

$$(FEM)_{BC} = -\frac{wL^2}{12} - \frac{Pab^2}{L^2} - \frac{6EI\Delta}{L^2} = -\frac{6(4)^2}{12} - \frac{32(1)(3)^2}{4^2} - \frac{6(4 \times 10^4)(0.012)}{4^2} = -206\ kN.m$$

$$(FEM)_{CB} = +\frac{wL^2}{12} + \frac{Pa^2b}{L^2} - \frac{6EI\Delta}{L^2} = +\frac{6(4)^2}{12} + \frac{32(1)^2(3)}{4^2} - \frac{6(4 \times 10^4)(0.012)}{4^2} = -166\ kN.m$$

$$(FEM)_{CD} = -\frac{wL^2}{12} + \frac{6EI\Delta}{L^2} - \frac{2EI\theta}{L} = -\frac{6(6)^2}{12} + \frac{6(1.5 \times 10^4)(0.012)}{6^2} - \frac{2(1.5 \times 10^4)(0.002)}{6}$$
$$= +2\ kN.m$$

$$(FEM)_{DC} = +\frac{wL^2}{12} + \frac{6EI\Delta}{L^2} - \frac{4EI\theta}{L} = +\frac{6(6)^2}{12} + \frac{6(1.5 \times 10^4)(0.012)}{6^2} - \frac{4(1.5 \times 10^4)(0.002)}{6}$$
$$= +28\ kN.m$$

Distribution Factors (DF)

Joint *C*

$$K_{CB} = \frac{3E(4I)}{4} = 3EI$$

$$K_{CD} = \frac{4E(4I)}{6} = 1EI$$

$$\sum K_C = 4EI$$

$$DF_{CB} = \frac{3EI}{4EI} = 0.75$$

$$DF_{CD} = \frac{1EI}{4EI} = 0.25$$

$$\sum DF_i = 0.75 + 0.25 = 1$$

Free End *A*

$$DF_{BA} = 0$$

Hinge End *B*

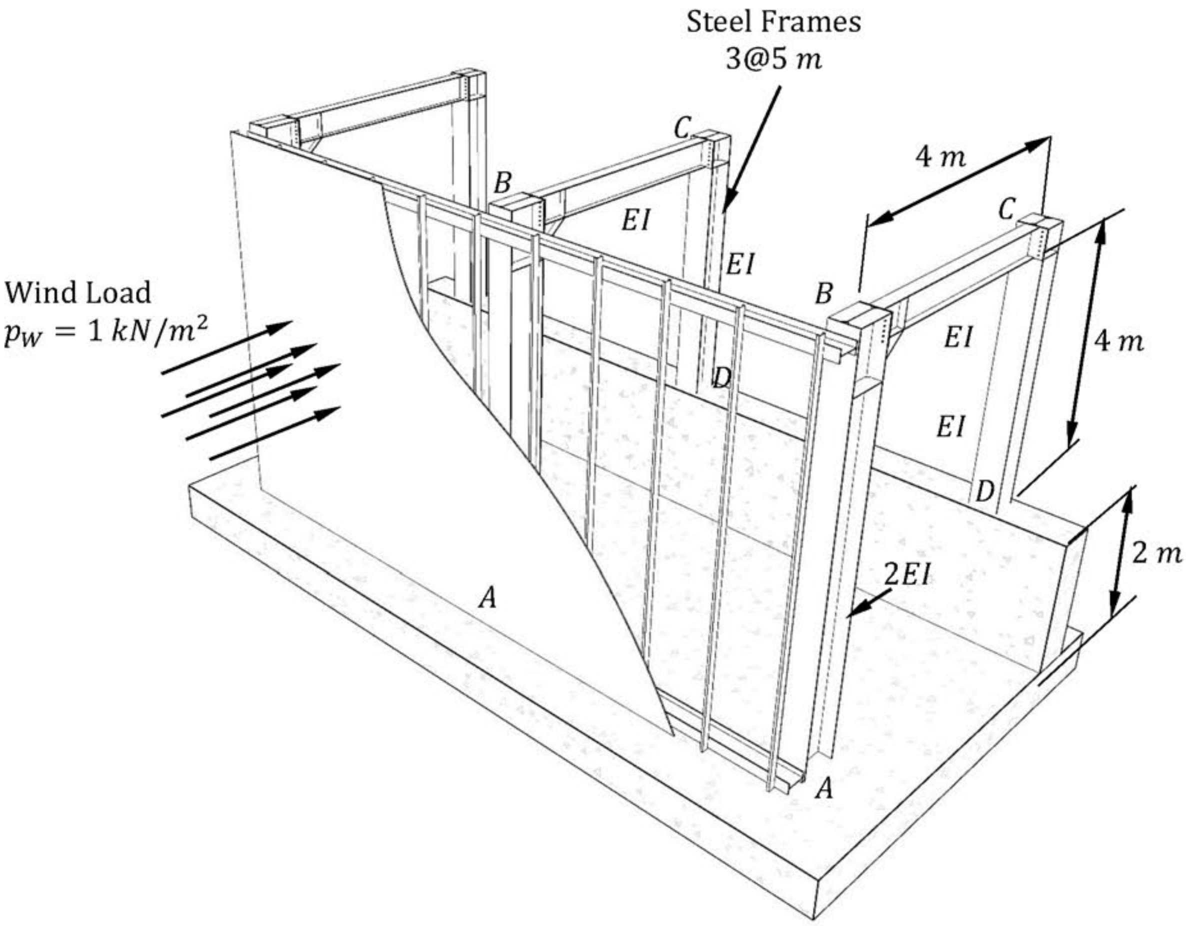
$$DF_{BC} = 1$$

Fixed End *D*

$$DF_{DC} = 0$$

Example 12.6 — Moment Distribution Method, Frames (Sidesway)

Analyze the interior steel frame $ABCD$ in the structure, shown in the figure, due to wind load only, by using the Moment Destitution Method. (Note: EI is constant; the steel columns are perfectly fixed to the foundation; and neglect the self-weight of the steel frame).



Solution

Modeling

$$P_A = P_B = \frac{(p_W \times W \times H)}{2} = \frac{(1 \times 5 \times 6)}{2} = 15\ kN\ \rightarrow$$