REI-1

3rd year 07-08

PE 300: Petroleum Reservoir Engineering I ((8 Units))

Syllabus: -

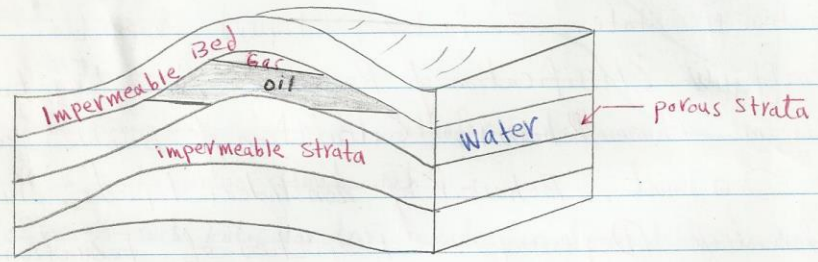
Types of Traps; Fluids Distribution; Porosity; Compressibility; permeability; Darcy's law (piston like, leaky piston); Gas Flow Equations; Radial Flow, Productivity Equation; Radial Flow of Gases; Average Permeability for Stratified Reservoir; Klinkenberge Effect; Flow Through Channels and Fractures; Saturation; Capillary pressure; Wettability, Multiphase Flow Through Porous Media; Effective and Relative Permeability; Calculation of Relative Permeability, Fractional flow Equation; Buckley-Leverett Equation; Gas Properties (Boyle and Charles laws); Avogadro law; Dalton law, Equation of State; Z-factor, liquid Properties: Viscosity; Classification of Reservoir's According to P-T Diagram; Phase Behavoir; Calculation of Bubble-point and Dew point; Behavoir of non-ideal liquids, Flash and Differential Degassing; Properties of Formation Water Volumetric Calculation of reserve, Material Balance

Classification of Petroleum Reservoir-Forming Traps

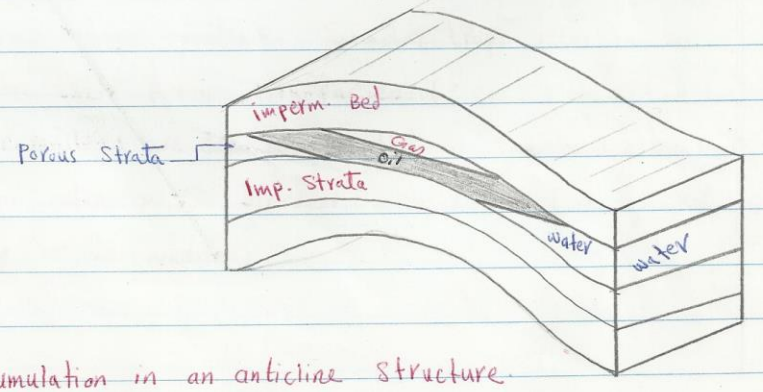
In broad terms, one may distinguish between Structural traps (related to tectonic structure) and Stratigraphic traps (" " sealing effect of unconformities and rock-type, or lithofacies changes)

Domes and Anticlines

Domes and anticlines are structures formed by the tectonic uplift and/or folding of sedimentary rocks. When viewed from above, a dome is circular in shape, whereas an anticline is an elongated fold.

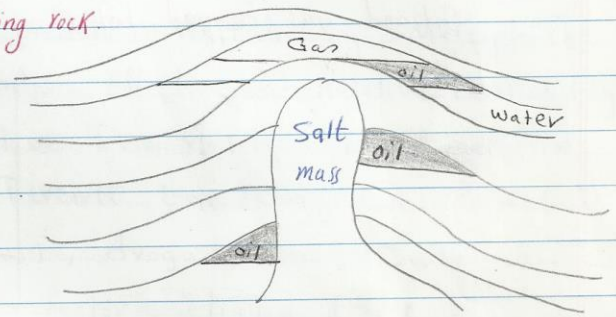


Oil and Gas Accumulation in a dome structure.



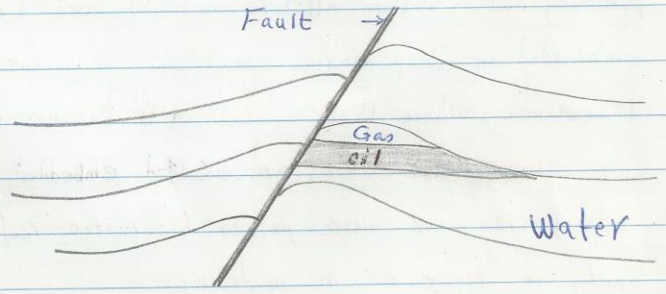
Oil and Gas Accumulation in an anticline structure.

Salt Domes:- This type of geological structure is caused by the upward intrusion of a diapiric body of salt, volcanic rock, or serpentine. In pushing up or piercing through the overlaying sedimentary rocks, the diapir may cause the formation of numerous traps on its flanks, in which petroleum may accumulate. Some salt domes may be highly elongated, rather than cylindrical, and are called salt walls [e.g. Southern North Sea region]. Salt itself is a perfect sealing rock.



Hydrocarbon accumulation associated with a salt dome

Faut Structure:- Many petroleum traps are related to faults, which commonly displace permeable rock against the impreviouls one.



Hydrocarbon Accumulation related to a fault

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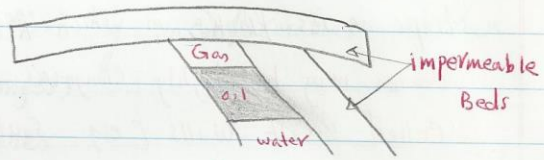
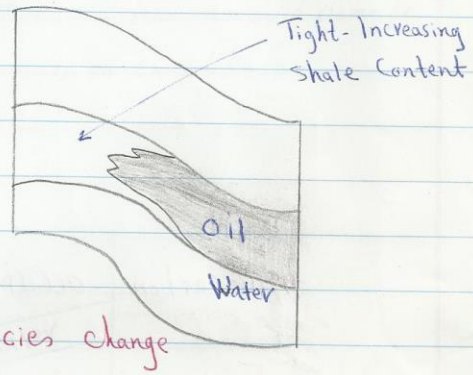
Lenticular Traps :- Oil and gas may accumulate in traps formed by the bodies of porous lithofacies (rock types) embeded in impermeable lithofacies, or by the pinch-outs of porous lithofacies within impermeable ones.



Oil and gas trapped below an unconformity



Petroleum trap formed by lithofacies change
((Sand-Stone pinch-out))

Examples of such lenticular traps include: fluvial sandstone bodies embedded in floodbasin mudrocks, deltaic or mouth-bar sandstone wedges pinching out within offshore mudrocks and turbiditic sandstone lobes embedded in deep marine mudrocks. Similar traps occur in various limestones, where their porous lithofacies (e.g. oolithic limestone or other Calcarenites) are ~~bounded~~ embedded in impermeable massive lithofacies; or where porous bioclastic reefal limestones pinch out in marls or in mudrocks.

Remarks:

1- upto 75% of world's petroleum accumulation are associated with Structural Traps (Major → Anticlines, minor fault)

2- 25% of oil traps are of Stratigraphic nature or Combination traps. // Unconformity, Reef ... //

Basic Concepts and Definitions in Reservoir Engineering

Porosity // ϕ //

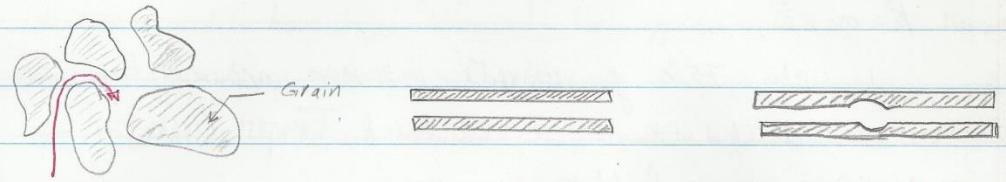
The porosity (ϕ) Can be defined as follows:-

- The fluid - storage Capacity of a porous medium, which means the part of the rock's total volume that is not occupied by solid particles.
- A measure of the pore space available for the storage of fluids in rock.

Genetically, the following types of porosity Can be distinguished

- Intergranular porosity
- Fracture "
- Micro porosity
- Vugular "

Rock media having both fracture & intergranular pores are called double-porous or fracture-porous media



pore space — Fracture — Vugs & Fracture

Intergranular porosity — void space

(primary porosity) — Secondary porosity

Note:- As the sediments were deposited and the rock were being formed during past geological times, some void space that developed became isolated from the other void space by excessive Cementation. Thus, many of the void space are interconnected while some of the pore spaces are Completely isolated. This leads to two distinct types of porosity, namely:-

- Absolute porosity
- Effective porosity

Absolute porosity (ϕ_a): The absolute porosity is defined as the ratio of total pore space in the rock to that of the bulk volume. A rock may have considerable absolute porosity and yet have no Conductivity to fluid for lack of pore interconnection. The absolute porosity is generally expressed mathematically by the following relation ships.

$$\phi_a = \frac{\text{total pore volume}}{\text{bulk volume}}$$

$$\phi = \frac{\text{bulk volume} - \text{grain volume}}{\text{bulk volume}}$$

or:-

$$\phi_a = \frac{V_b - V_G}{V_b} \equiv \frac{V_P}{V_b}$$

where :-

ϕ_a = Absolute porosity

V_b = Bulk volume (L^3)

V_G = Grain Volume (L^3) [or ≡ Matrix volume]

V_P = pore volume (L^3)

The effective porosity: The effective porosity is the percentage of interconnected pore-space with respect to the bulk volume, or

$$\phi = \frac{\text{interconnected pore volume}}{\text{bulk volume}}$$

where:

ϕ = Effective porosity

The effective porosity is the value that is used in all reservoir engineering calculations because it represents the interconnected pore space that contains the recoverable hydrocarbon fluids.

Hint:

$$\phi_A = \phi + \phi_r$$

where:

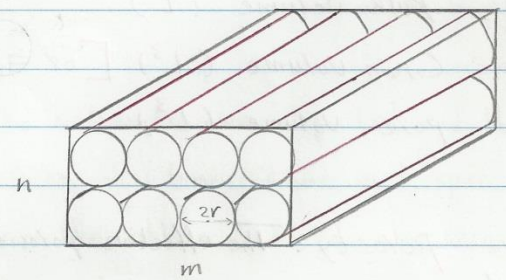
ϕ_r = Residual porosity $\qquad \phi_r = \dfrac{\text{Isolated Pore space}}{\text{Bulk volume}}$

Models of Porous Media

The geometric character of rock's permeable pore space is in reality quite complicated, and may vary greatly from one rock type to another. In practice, it is impossible to counter the pore-system geometry in a detailed and faithful way. Therefore, several

idealized models have been developed to approximate porous rock media and their varied characteristics.

1 - Idealised Porous Medium Represented by Parallel Cylindrical Pores.



$$\phi = \frac{V_p}{V_b} = \frac{(\pi r^2)(n)(m)}{(2rn)(2rm)} = \frac{\pi}{4} \Rightarrow \phi = 0.785 \text{ or } 78.5\%$$

where:-

r = The pipe radius

n and m = The number of cylinders contained in the bulk volume [It is obvious that rocks do not have pores like this and that this model gives a unrealistically high porosity value. This model may though, be used in some situations where fluid flow under simplified conditions is modelled.

2 - Idealised Porous Medium Represented by Regular Cubic-Packed Spheres

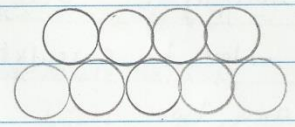
Cubic packing equal grain size

The estimated value of porosity according to this model

$$\phi = 0.476 \text{ or } 47.6\%$$

3- Idealised Porous Medium Represented by Regular Rhombohedral- Packed spheres

The estimated value of porosity according to this model is $\phi = 0.26$ or 26%



Rhombohedral. Packing

«الرص المعين»

4- Idealised porous Medium Represented by irregular- Packed spheres with Different Radii

Some real values of measured Porosities

1- Sandstone : 10%- 40% ; depending upon the nature of Cement and their nature of Consolidation.

2- Limestone and Dolomite 5% - 25%

3- Clays : 20% - 45% depends upon origin and depth.

It is generally Said that the porosity is

a- Negligable if $\phi < 5\%$

b- Low if $5\% < \phi < 10\%$

c- Good if $10\% < \phi < 20\%$

d- Very Good if $\phi > 20\%$

Remark: Table of matrix densities

Lithology	ρ_m (g/cm^3)
Quartz	2.65
limestone	2.71
Dolomite	2.87

Measurement of porosity

- Laboratory Measurement ((Conventional Core Analysis ; Routine core Analysis Conducted on a- Full-Diameter Core Analysis b- plug or c- Sidewall plug))

In order to calculate the value of porosity measure any two

- Bulk volume, V_b
- Matrix Volume, V_m
- Pore Volume, V_p

Bulk volume measurements :-

V_b calculated either a- from dimensions or 2- by displacement methods

/ Displacement Methods :- [1/11/2007]

- Drop the plug (or sample) into a liquid and observe volume change of liquid ; you must prevent test liquid from entering pore space of Sample by:
  - Coat with paraffin
  - Presaturate sample with test liquid
  - Use Mercury [Hg] as test liquid

EX :- A core Sample coated with paraffin was immersed in a Russell tube. The dry sample weighed 20 gm. The dry sample coated with paraffine weighed 20.9 gm. The coated sample displaced 10 cc of liquid. Assume the density of solid paraffine is 0.9 gm/cc. what is the bulk volume of the sample.

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Solution:

Weight of paraffine Coating = 20.9 − 20 = 0.9 gm

Volume " " " = 0.9/0.9 = 1 cc

∴ Bulk volume of sample = 10.0 − 1 = 9.0 cc

Matrix Volume Measurements

- Assume grain density ((Matrix density))

$$\text{Matrix Volume} = \frac{\text{Dry weight}}{\text{matrix density}}$$

- Displacement method :- Reduce Sample to particle size*, then measure :-
  - The volume of grain
  - " weight " "

Ex:- If the core sample of previous example was stripped of the paraffin Coat. crushed to grain size and immersed in a Russell tube. The volume of the grain was 7.7 cc. what was the porosity of the sample. Is this effective or total porosity.

Solution:-

Bulk volume = 9.0 cc

Matrix " = 7.7 cc

$$\phi = \frac{V_b - V_m}{V_b} \Rightarrow \phi = \frac{9.0 - 7.7}{9.0} \Rightarrow \phi = 14.22\%$$

here ϕ is total (absolute porosity)

* By crushing

Ex:- Calculate the porosity of a core sample when the following information is available. Dry weight of sample = 427.3 gm, weight of sample when saturated with water = 448.6 gm. Density of water = 1 gm/cc. Weight of water saturated sample immersed in water = 269.6 gm

Solution:-

$$V_p = \frac{\text{Saturated core weight in air} - \text{Dry core weight}}{\text{density of water}}$$

$$V_p = \frac{448.6 - 427.3}{1} \Rightarrow V_p = 21.3 \text{ cc}$$

$$V_b = \frac{\text{Saturated core weight in air} - \text{Saturated core weight in water}}{\text{density of water}}$$

$$V_b = \frac{448.6 - 269.6}{1} \Rightarrow V_b = 179 \text{ cc}$$

" قاعدة ارخميدس : الجسم المغمور في سائل يفقد من وزنه بقدر وزن السائل الذي يزيحه "

$$\phi = \frac{V_p}{V_b} \Rightarrow \phi = \frac{21.3}{179} \Rightarrow \phi = 0.119$$

$$\therefore \phi = 11.9\ \%$$ [[Effective porosity]]

Determination of lithology

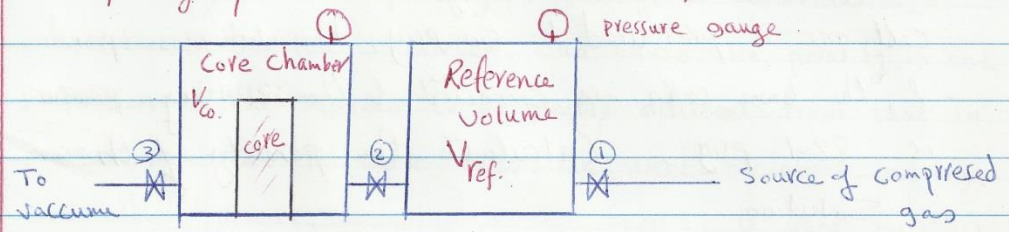
$$V_m = V_b - V_p \Rightarrow V_m = 179 - 21.3 \Rightarrow V_m = 157.7 \text{ cc}$$

$$\rho_m = \frac{\text{wt. of dry sample}}{\text{matrix volume}} = \frac{427.3}{157.7} \Rightarrow \rho_m = 2.71$$

[[limestone]]

Boyle's law Porosimeter

The gas Transfer technique involves the injection and decompression of gas into the pores of a fluid-free (Vacuum), dry core sample. Either the pore volume or the grain volume can be determined depending upon the instrumentation and procedures used.



To perform the laboratory measurement, Helium gas is often used due to its following properties

- The very small size of Helium molecules makes the gas rapidly penetrate small pores.
- Helium is an inert gas.

[N_2 or CO_2 may be used].

١) يقفل الصمام ① ويوضع اللباب في الحجرة المخصصة يفتح الصمام ② و ③ ونجري عملية تفريغ من الهواء للحجرتين بواسطة مضخة فراغية Vaccume pump يقفل الصمام ② و ③ يفتح الصمام ① ويسمح للغاز المضغوط بالدخول الى الحجرة الاولى المعلومة الحجم وبعد استقرار مقياس الضغط يسجل الضغط الابتدائي P_1 . يقفل الصمام ① ويفتح الصمام ② حيث سينتقل الغاز من الحجرة (الاولى) الى الحجرة الثانية وسيدخل المسامات الموجودة في اللباب

وبتطبيق Boyle's law

$$P_1 V_{ref} = P_2 [V_{ref} + V_{co.} - V_g]$$

solving for grain volume:

$$V_g = \frac{P_2 V_{ref} + P_2 V_{co.} - P_1 V_{ref.}}{P_2}$$ or.

$$V_g = (V_{ref.} + V_{co.}) - \frac{P_1}{P_2}(V_{ref.}) \equiv V_m$$

Ex:- A full diameter core ((D = 3", L = 6")) is placed in a core chamber of Boyle's law device. Each cell has a volume of 1000 cc. After evacuation cell one is pressured to 50 psig, the value is opend & the two cells are connected & the resulting pressure is 28.1 psig ; Calculate the porosity of the core

Solution:-

$$V_b = \frac{\pi}{4} D^2 L \qquad [1'' = 2.54\ cm]$$

$$V_b = \frac{\pi}{4}(3)^2(6)(2.54)^3$$

$\therefore$ The Bulk Volume = 695 cc

$P_1 = 50 + 14.7 = 64.7$ psia

$P_2 = 28.1 + 14.7 = 42.8$ psia

Calculate the matrix volume :-

$$V_m = (1000 + 1000) - \left(\frac{64.7}{42.8}\right)(1000)$$

$$V_m = 488.32 \text{ cc}$$

$$\therefore \%\phi = \left(\frac{695 - 488.32}{695}\right)(100) = 29.74\ \%$$

or

$$\phi = 0.2974$$

Sun. 4/11/2007

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Note: The reservoir rock may generally show large variation in porosity vertically but does not show very great variations in porosity parallel to the bedding planes. In this case, the arithmetic average porosity or the thickness-weighted average porosity is used to describe the average reservoir porosity. A change in sedimentation or depositional conditions, however, can cause the porosity in one portion of the reservoir to be greatly different from that in another area. In such cases, the areal-weighted average or the volume-weighted average porosity is used to characterize the average rock porosity; Mathematically:

Arithmetic average $\phi = \sum \phi_i / n$

Thickness-Weighted average $\phi = \sum \phi_i h_i / \sum h_i$

Areal- " " $\phi = \sum \phi_i A_i / \sum A_i$

Volumetric- " " $\phi = \sum \phi_i A_i h_i / \sum A_i h_i$

where:-

n = total number of core sample

h_i = thickness of core sample i, or reservoir area i

ϕ_i = porosity of core sample i, or reservoir area i

A_i = reservoir area i

Ex :- Calculate the arithmetic average and thickness_weighted average from the following measurements

Sampl	Thickness, ft	porosity %
1	1.0	10
2	1.5	12
3	1.0	11
4	2.0	13
5	2.1	14
6	1.1	10

Arithmetic Average

$\phi = \frac{70}{6}$

$= 11.67\ \%$

Thickness_weighted average

$\phi = \frac{105.4}{8.7}$

$= 12.11\ \%$

(n = 6 sample)

Solution:

- Arithmetic average ; here $n = 6$

$\phi = (10 + 12 + 11 + 13 + 14 + 10)/6 \Rightarrow \phi = 11.67\%$

- Thickness_weighted average

$\phi = [(1)(10) + (1.5)(1.5) + (1)(1) + (2)(13) + (2.1)(2.1) + (1.1)(1.1)] / [1 + 1.5 + 1 + 2 + 2.1 + 1.1]$

$\phi = 12.11\%$

Sample	Thickness, h_i, f	Porosity ϕ_i, %	$\phi_i h_i$
1	1	10	10
2	1.5	12	18
3	1.0	11	11
4	2	13	26
5	2.1	14	29.4
6	1.1	10	11
$\boxed{n=6}$	$\Sigma h_i = 8.7$ ft	$\Sigma \phi_i = 70$	$\Sigma \phi_i h_i = 105.4$ ft.

Porosity Estimation from well logs

porosity of reservoir rock can be estimated not only by using methods, as has been described above, but also from geophysical well logs, often called wireline logs. This method of porosity evaluati is not very accurate, but has advantage of providing continous porosity data. Porosity can be estimated from

- Formation resistivity factor (F)
- Microresistivity log (from which F can be derived
- Neutron - gamma log
- Density log
- Acoustic (Sonic) log

Saturation

Let us consider a representative elementary volume of the reservoir with the pores filled with oil, gas and water. In volumetric term

$$V_p = V_g + V_o + V_w$$

where

V_p = pore volume (L^3)

V_g = gas volume

V_o = oil "

V_w = water "

Which leads to the definition of saturation, S, as a fraction of the pore volume occupied by a particular fluid:

$$S_i = \frac{V_i}{V_p}, \quad i = 1, \ldots, n$$

where n denotes the total number of fluid phase present

in the porous medium. thus

$$S_g = \frac{V_g}{V_p} \quad , \quad S_o = \frac{V_o}{V_p} \quad , \quad S_w = \frac{V_w}{V_p}$$

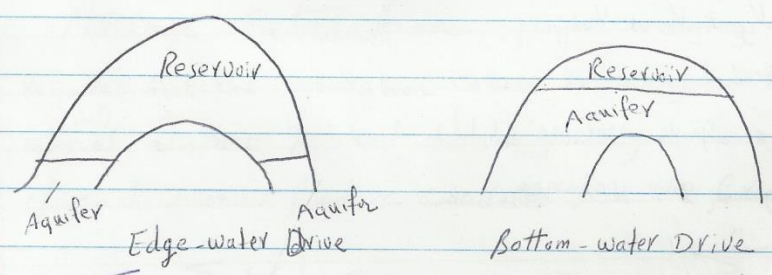
or Consequently:

$$\sum_{i=1}^{n} S_i = 1 \quad \text{[in primary recovery } n=3\text{]}$$

i.e $S_g + S_o + S_w = 1.0$ [Summation of Saturations is Constant].

Notes: • The saturation of each individual phase ranges between zero to 100%.

• The fluids in most reservoirs are believed to have reached a state of equilibrium and, therefore, will have become Separated according to their density i.e oil overlain by gas and underlain by water. In addition to the bottom or (edge) water, there will be Connate water distributed through-out the gas and oil zone →



← The water in these zones will have reduced to some irreducible minimum (S_{w_i}) ; The force retaining the water in the oil and gas zones are referred to as Capillarity forces, because they are important only in pores spaces of Capillary size.

Connate وادي ، وراثي ، خلقي ، فطري

Connate (Interstitial) water saturation S_{wc} is important primarily because it reduces the amount of space available between oil and gas. It is generally not uniformly distributed throughout the reservoir but varies with permeability, lithology, and height above the free water level.

Another particular phase saturation of interest is called the Critical saturation and it is associated with each reservoir fluid. The difinition and the significance of the critical saturation for each phase is described below.

Critical gas Saturation, S_{gc}: As the reservoir pressure declines below the bubble point pressure, gas evolves from the oil phase and consequently the saturation of the gas increases as the reservoir pressure declines. The gas phase remains immobile until its saturation exceeds a certain saturation, called Critical gas saturation, above which gas begins to move.

Critical oil saturation, S_{oc}: For the oil phase to flow, the saturation of the oil must exceed a certain value which is termed Critical oil saturation. At this particular saturation, the oil remains in the pores and, for all practical purposes, will not flow.

Residual oil Saturation: During the displacing process of the crude oil system from the porous media by water or gas injection (or encroachment) there will be some remaining oil left that is quantitatively characterized by a saturation value is called the ~~critical~~ Residual oil saturation, S_{or}, the term residual saturation is usually associated with the non-wetting

phase when it is being displaced by a wetting phase.

Movable oil Saturation, S_{om} :-

Movable oil " , S_{om} is another saturation of interest and is defined as the fraction of pore volume occupied by movable oil as expressed by the following equation:

$$S_{o_m} = 1 - S_{w_c} - S_{o_c}$$

where:

S_{w_c} = Connate water Saturation

S_{oc} = Critical oil Saturation

Critical water saturation, S_{w_c}

The Critical water saturation, Connate water saturation, and irreducible water Saturation are extensively used interchangeably to define the maximum water saturation at which the water phase will remain immobile.

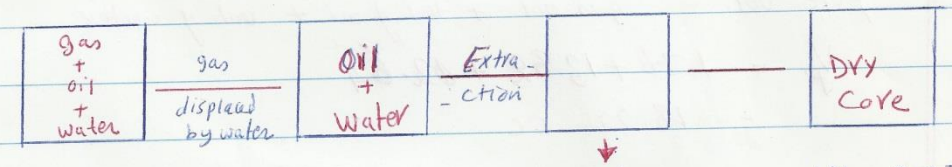
Average Saturation : proper averaging of saturation data requires that the saturation values be weighted by both the interval (الفترة) thickness h_i and interval porosity. The average saturation of each reservoir fluid is calculated from the following equations

$$S_o = \frac{\sum_{i=1}^{n} \phi_i h_i S_{o_i}}{\sum_{i=1}^{n} \phi_i h_i} \quad , \quad S_w = \frac{\sum_{i=1}^{n} \phi_i h_i S_{w_i}}{\sum_{i=1}^{n} \phi_i h_i}$$

$$S_g = \frac{\sum_{i=1}^{n} \phi_i h_i S_{g_i}}{\sum_{i=1}^{n} \phi_i h_i}$$

Ex:- A core sample is saturated with oil, gas and water. The initial weight of the sample is 224.14 gm. After the gas is displaced by water, the weight is increased to 225.9 gm. The sample is then placed in a Soxhlet distillation apparatus and 4.4 cm^3 of water is extracted (استخلصت). After drying the core sample, the weight is now 209.75 gm. The sample bulk volume is 95 cm^3. Find the porosity, gas saturation, Oil saturation and water saturation, $\rho_o = 0.85$ gm/cc, $\rho_w = 1$ gm/cc

Solution:-



$w_t = 224.14$ gm $w_t = 225.9$ gm 4.4 cc of water $w_t = 209.75$ gm, $V_b = 95\ cm^3$

wt of water that displaces gas = 225.9 − 224.14 = 1.76 gm

vol. of water = 1.76/1 = 1.76 cc

since water displaces gas

∴ Vol. of gas = 1.76 cc

Vol. of extracted water = Vol. of water intially saturated the core + Vol. of water displaced the gas

∴ Vol. of water initially saturate the core = 4.4 − 1.76

= 2.64 cc

∴ Wt. of water " " " " = 2.64 gm

wt. of oil + water = wt. of core saturated with oil and water − wt. of dry core.

wt. of oil + water = 225.9 − 209.75

= 16.15 gm

Vol. of extracted water = 4.4 cc

wt. " " " = 4.4 gm [ρ_w = 1 gm/cc]

then:-

Wt. of oil = 16.15 − 4.4 = 11.75 gm

$$\text{Vol. of oil} = \frac{11.75}{0.85} = 13.82 \text{ cc}$$

pore vol. = gas vol. + vol. of oil + vol. of water

V_p = 1.76 + 13.82 + 02.64

= 18.22 cc

$$\phi = \frac{V_p}{V_b} = \frac{18.22}{95}(100) = 19.2\ \% \quad \text{or } \phi = 0.192$$

$$S_g = \frac{1.76}{18.22}(100) = 9.66\%$$

$$S_o = \frac{13.82}{18.22}(100) = 75.85\ \%$$

$$S_w = \frac{2.64}{18.22}(100) = 14.49\ \%$$

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Ex.: The following measurements on pay zone are available:

Sample	Thickness, ft	ϕ_i, %	S_{o_i}, %
1	2	12	75
2	3	16	74
3	1	10	73
4	4	14	76
5	2	15	75
6	2	15	72

Calculate the average oil and connate water saturation

Solution:

$$S_{w_c} = 1 - S_{o_i}$$

Make a table

Sample	h_i, ft	ϕ_i, %	$\phi_i h_i$	S_{o_i}	$S_{o_i} h_i \phi_i$	S_{w_c}	$S_{w_c} \phi_i h_i$
1	2	12	24	75	1800	25	600
2	3	16	48	74	3552	26	1248
3	1	10	10	73	730	27	270
4	4	14	56	76	4256	24	1344
5	2	15	30	75	2250	25	750
6	2	15	30	72	2160	28	840
			$\Sigma = 198$		$\Sigma = 14748$		$\Sigma = 5052$

$$S_o = \frac{14748}{198} = 74.5\%$$

$$S_{w_c} = \frac{5052}{198} = 25.5\%$$

Laboratory Determination of Residual oil and water saturation

Two Laboratory ~~retort~~ techniques are commonly used for determining the residual oil and water saturation

- a high temperature retort distillation method and *
- the Dean-stark method.

[Prob. 6 p.60 should be given]

Permeability

8-11-2007

Permeability is a property of the porous medium that measures the capacity and ability of the formation to transmit fluids. The rock permeability, K, is very important rock property because it controls the directional movement and the flow rate of the reservoir fluids in the formation. This rock ~~proper~~ characterization was first defined mathematically by Henri Darcy in 1856. The equation that defines permeability in term of measurable quantities is called Darcy's Law.

A porous medium of one Darcy permeability will transmit a fluid of one centipoise viscosity through a cross sectional area of one cm^2 @ the rate of one cm^3/sec under pressure gradient of one atmospher / cm

For horizontal bed, the Darcy's law is *

$$Q = \frac{AK}{\mu} \frac{\Delta P}{L}$$ [D.F $v = -\frac{K}{\mu}\frac{dP}{dL}$, should be disc]

Q: cc/sec, A = cross sectional area cm^2, K = perm. Darcy

μ = viscosity c.P ; ΔP : pressure drop (decline) (atm)

L = length, cm.

* see Zolot. P.55

* [illegible]

07-08

Dimensions of permeability:-

$Q = \frac{L^3}{T}$, $A = L^2$, $P = \frac{M}{LT}$ (press = $\frac{\text{Mass} \times \text{acceleration}}{\text{Area}}$

$\frac{m}{L^2} \cdot \frac{L}{T^2}$] , $\mu = \frac{M}{L \cdot T}$

$$Q = A \frac{K}{\mu} \cdot \frac{\Delta P}{L}$$

or

$$K = \frac{Q \cdot \mu \cdot L}{A \cdot \Delta P}$$

$$= \frac{\frac{L^3}{T} \cdot \frac{M}{L \cdot T} \cdot L}{L^2 \cdot \frac{M}{L^2} \cdot \frac{L}{T^2}} \Rightarrow K = L^2$$ [~~unite~~

~~of area~~ , [Dimensions of area].

Units of permeability:

$Q = cm^3/sec$, $\mu = 1\,cp = 10^{-2}$ poise $= 10^{-2} \frac{dyn.sec}{cm^2}$

$A = 1\ cm^2$, $\Delta P = 1\,atm = 1.013 \times 10^6 \frac{dyn}{cm^2}$

$$\therefore K(1\,Darcy) = \frac{\left(\frac{cm^3}{sec}\right) \cdot \left(10^{-2} \frac{dyn.sec}{cm^2}\right) (cm)}{(cm^2)\left(1.013 \times 10^6 \frac{dyn}{cm}\right)}$$

$\therefore$ 1 Darcy $= 9.87 \times 10^{-9}\ cm^2$

1 millidarcy $= \frac{1}{1000}$ Darcy [1D = 1000 m.d]

$\therefore K = 1\,m.d = 9.87 \times 10^{-12}\ cm^2$

" معظم الحالات النفطية لها نفاذية أقل من 1 Darcy "

Again to Darcy law

$$Q = A \frac{K}{\mu} \frac{\Delta P}{L}$$ in term of velocity

$$V = \frac{K}{\mu} \frac{\Delta P}{L}$$ ($\frac{K}{\mu}$ = Mobility $\frac{md}{c.p}$)

here

V = Superficial velocity السرعة الظاهرية

The real velocity of fluid flow in the pores is called interstitial (True) velocity V_{pore} and is higher than superficial $\left[V_{por.} = \frac{V_{sup.}}{\phi} \right]$ { should be discussed }

permeability's Terminology

1- Absolute permeability: The absolute permeability of a given porous material is the ability to pass a fluid through its interconnected pore and/or fracture network, provided that the fluid 100% saturates the effective pores. [K]

2- Effective permeability: effective permeability to a given fluid ~~porous material is the~~ at a saturation less than 100% is the fluid conductivity of the porous media at a given saturation where other fluids are present. In reservoir engineering three types of effective permeability are considered:

1- Effective permeability to gas K_g

2- " " " oil K_o

3- " " " water K_w

The ratio of effective permeability and the viscosity of the fluid is called the mobility λ

Gas Mobility $\lambda_g = \frac{K_g}{\mu_g}$, oil Mobility $\lambda_o = \frac{K_o}{\mu_o}$ and water Mobility $\lambda_w = \frac{K_w}{\mu_w}$

In summary, it should be evident that the effective permeability values of K_g, K_o, K_w may vary from zero upto the absolute permeability.

$$0 \leq K_g, K_o, K_w \leq K$$

3 - Relative permeability : Relative permeability is a dimension-less quantity representing the ratio of effective permeability to absolute permeability , it is a fraction varies between zero and one

$$K_{rg} = \frac{K_g}{K} \quad ; \quad K_{r_o} = \frac{K_o}{K} \quad , \quad K_{rw} = \frac{K_w}{K}$$

$\therefore$ $K_r = f(S)$ [Relative permeability is a function of saturation]

$\therefore$ $K_{rg} = 0$ if $S_g \leq S_{g_c}$

$K_{ro} = 0$ if $S_o \leq S_{or}$

$K_{rw} = 0$ if $S_w \leq S_{wi}$

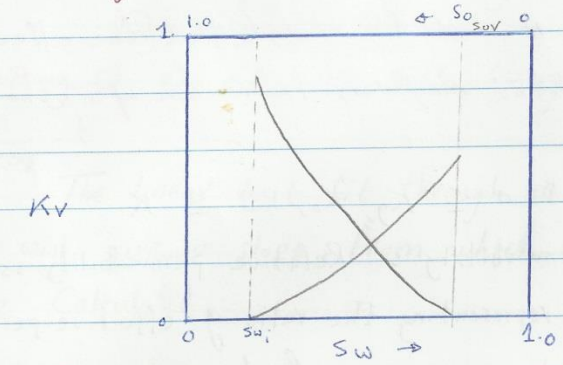
Remark:

In a vast number of laboratory experiments it has been observed that a sum of effective permeabilities is less than the total or absolute permeability

$$\sum_{j=1}^{n} K_{rj} < K \qquad [j = \text{fluid type}$$

in res. Eng $j = 3$]

In the presence of two coexisting phases, the typical curves of relative permeability are as shown in figure below:-



Sun. 11/11/2007

Ex: A core is 2 ft. long and has a cross-sectional area of 1 ft^2. The core is oriented so fluid flows horizontally through it. Pressure gradient is kept @ 50 psi/ft. when the core is 100% saturated with $\mu_o = 1.2$ cP viscosity oil, the flow rate is 13.29 ft^3/day. Determine a- absolute permeability b- effective permeability to oil c. The rate at which water will flow through the core when it is 100% saturated with $\mu_w = 1$ cP viscosity water. d. The effective & relative permeabilities to oil and water when both oil and water are flowing in the core @ rate of 9 ft^3/day and 5 ft^3/day respectively.

Solution:

In field unit: $Q = 1.127 \times 10^{-3} A \frac{K}{\mu} \frac{\Delta P}{\Delta L}$ [Q in bbl/day]

1 bbl = 5.615 ft^3

$\therefore$ $Q = 6.328 \times 10^{-3} A \frac{K}{\mu} \cdot \frac{\Delta P}{\Delta L}$ [Q: ft^3/day, A: ft^2, K: md, μ: cP, $\Delta P/\Delta L$: psi/ft]

a- Since $S_o = 100\%$ ∴ K is absolute permeability

$$13.29 = 6.328\times10^{-3}(1)\frac{K}{1.2}(50) \Rightarrow K = 50.4 \text{ md}$$

b- here effective permeability = absolute permeability [$S_o = 100\%$]

∴ $K_o = 50.4$ md.

c- here $S_w = 100\%$ ∴ $K_w = K$

$$Q_w = 6.328\times10^{-3}\frac{50.4}{1}(50) \Rightarrow Q_w = 15.95 \text{ ft}^3/\text{day}$$

d-

$Q_o = 9$ ft³/day ⇒ calculate effective oil permeability (K_o)

$$9 = (1)\frac{K_o}{1.2}(50)(6.328\times10^{-3}) \Rightarrow K_o = 34.13 \text{ md}$$

$$K_{r_o} = \frac{K_o}{K} \Rightarrow K_{r_o} = \frac{34.13}{50.4} \Rightarrow K_{r_o} = 0.68$$

$Q_w = 5$ ft³/day ⇒ calculate effective water permeability

$$5 = 6.328\times10^{-3}(1)\frac{K_w}{1}(50) \Rightarrow K_w = 15.8 \text{ md}$$

$$K_{r_w} = \frac{K_w}{K} = \frac{15.8}{50.4} \Rightarrow K_{r_w} = 0.31$$

Permeability Measurements

a- liquid permeability Measurements : In laboratory, the permeability of a core samples is measured using Darcy's law for horizontal flow. In these Tests some important conditions have to be ~~established~~ satisfied before permeability could be estimated from the measured data. These Conditions are the following:-

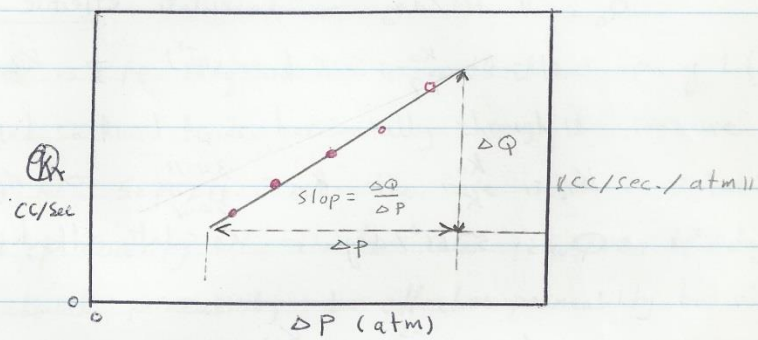
1- Incompressible fluid 2- 100% fluid saturation in the porous media 3- Constant cross-section in flow direction 4- Laminar flow 5- No chemical exchange or reaction between fluid and porous media.

A different flow rates are measured for different pressure gradient ; for viscous flow :-

$$Q = \frac{AK}{\mu \Delta L} \Delta P$$

i.e plotting Q vs ΔP will give straight line with slop $= \frac{A}{\mu \Delta L} \cdot K$

So, The linear best fit through all experimental data points will give a slope, from which the permeability can be Calculated.



$$\text{Slop } \frac{\Delta Q}{\Delta P} = \frac{A}{\mu \Delta L} \boxed{K} \Rightarrow \text{find } K \text{ in Darcy}$$

[[The importance of linear representation of the measured data is the advantage of visual inspection, which may reveal non-linear effects in the data, e.g. at high or low flow rates, or uncertainty in lab. measurements]]

Gas Permeability Measurements

Due to certain interactions between the liquid and the porous media (rock) absolute permeability is routinely measured in the laboratory by flowing gas (usually inert gas) through the core sample. Because gas is a highly Compressible substances i.e the gas rate is pressure dependent, the darcy's law may not be utilized directly. Assuming the used gases follow the ideal gas behavoir (@ low pressure), the following

relationship apply: $P_1 V_1 = P_2 V_2 = P_m V_m$, In terms of the flow rate $Q\left(\frac{vol.}{time}\right)$, the above equation can be equivalently expressed as

$$P_1 Q_1 = P_2 Q_2 = P_m Q_m$$

with the mean pressure P_m expressed as:-

$$P_m = \frac{P_1 + P_2}{2}$$

where:

P_1 = Inlet pressure, atm

P_2 = Outlet " , atm

P_m = Mean " , atm

Q_1 = inlet gas flow rate cc/sec

Q_2 = outlet " " " cc/sec

Q_m = mean " " " cc/sec

vol. ↑ as P↓ so Q↑

The gas flow rate is usually measured @ base (atmospheric) pressure P_b, and therefore, the term Q_{gsc} is introduced to above equations

$$Q_{gsc} P_b = Q_m P_m$$

$$Q_{gsc} P_b = A \frac{K}{\mu_g} \frac{(P_1 - P_2)}{L} \left(\frac{P_1 + P_2}{2}\right) \Rightarrow Q_{gsc} = \frac{AK}{\mu_g} \cdot \frac{P_m}{P_b} \cdot \left(\frac{P_1 - P_2}{L}\right)$$

or

$$Q_{gsc} = \frac{A}{2} \frac{K}{\mu_g P_b} \left(\frac{P_1^2 - P_2^2}{L}\right)$$

where:

K = absolute permeability, Darcy

* μ_g = Gas viscosity, cP

P_1 = inlet (upstream) pressure, atm

P_2 = outlet (downstream) " , "

L = length of the core, cm

A = cross-sectional area, cm^2

Q_{gsc} = Gas flow rate @ standard condition cm^3/sec

** جداول خاصة بعلاقة لزوم الغاز مع الضغط ودرجة الحرارة **

in field unit:

$$Q_{s.c} = 6.33 \times 10^{-3} A \frac{K}{\mu g} \cdot \frac{P_m}{P_b} \cdot \left(\frac{P_1 - P_2}{L}\right)$$

where:

$Q_{s.c}$ = Gas flow rate scf / day

A = Area ft^2

K = permeability = m.d.

μ = Gas viscosity c.P

P_1 = inlet pressure psia

P_2 = out let pressure psia

P_b = base pressure (usually 14.7 psia)

L = bed (or Porous Media) Length, ft.

or in terms of pressure squared

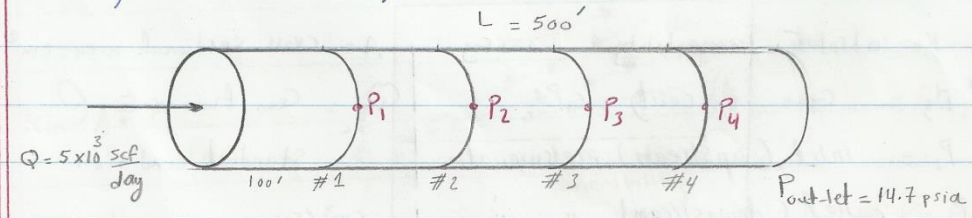
$$Q_{s.c} = 3.17 \times 10^{-3} A \cdot \frac{k}{\mu g} \cdot \frac{1}{P_b} \cdot (P_1^2 - P_2^2) / L$$

EX:-

يوضح بالعربي

An ideal gas of 0.02 cp is flowing at a rate of 5 Mscf/day through the core shown in figure below, having a uniform cross-sectional area of $1 ft^2$ and 500 ft length under overall pressure difference of 1455 psig

a- Calculate the average permeability b- Calculate the nodal pressure @ 100' interval, Hint: out let pressure is ~~atmospheric pressure~~ = 300 psia the flow is isothermal., Base Pressure = 14.7 psia



Solution:-

a- Since the pressure difference = 1455

$\Delta P = P_{in.} - P_{out} \Rightarrow 1455 = P_{in.} - 30$

∴ Inlet pressure = 1485 psia

$$Q_{s.c} = 3.17 \times 10^{-3} A \frac{K}{\mu_g} \cdot \frac{1}{P_b} \left(\frac{P_1^2 - P_2^2}{L} \right)$$

$$5000 = 3.17 \times 10^{-3} (1) \frac{K}{(0.02)} \cdot \frac{1}{(14.7)} \left(\frac{1485^2 - 30^2}{500} \right)$$

∴ $K = 105.2$ md [average absolute porosity]

b- Since $Q_{s.c}$ is constant & K_g, μ_g are constant

∴

$$\frac{P_{inlet}^2 - P_{outlet}^2}{500} = \frac{P_{inlet}^2 - P_1^2}{100} = \frac{P_1^2 - P_2^2}{100} = \frac{P_2^2 - P_3^2}{100} = \frac{P_3^2 - P_4^2}{100}$$

$$\frac{1485^2 - 30^2}{500} = \frac{1485^2 - P_1^2}{100} \Rightarrow P_1 = 1328.3 \text{ psia}$$

similarly

$$1485^2 - 1328.3^2 = 1328.3^2 - P_2^2 \qquad [\Delta L = 100' \text{ same}]$$

$$P_2 = 1150.45 \text{ psia}$$

$$1328.3^2 - 1150.45^2 = 1150.45^2 - P_3^2 \Rightarrow P_3 = 939.52 \text{ psia}$$

$$1150.45^2 - 939.52^2 = 939.52^2 - P_4^2 \Rightarrow P_4 = 664.73 \text{ psia}$$

The Klinkenberg Effect

It has been observed that at low average pressure, measurements of gas permeability give erroneously high result, as compared to the non-reactive liquid permeability measurements (Absolute permeability). This effect is known as the gas slippage effect or as the Klinkenberg effect, investigated by

Klinkenberg in 1941 }} Klinkenberg postulated, on the basis of his lab. experiments, that liquid had a zero velocity @ the sand grain surface while gases exhibited some finite velocity " " " "
i.e the gases exhibited slippage @ the sand grain surface, this slippage resulted in a higher flow rate for the gas @ a given pressure differential }}: one of the conditions for the validity of Darcy's law, is the requirement of laminar flow, @ low gas pressure in combination with small pore channels, this condition is broken.
[@ low $\bar{P}$, gas molecules are often so far apart, that they slip through the pore channels almost without interaction (no friction loss) & hence, yield an increased flow velocity or flow rate.

So @ low pressure difference

$$q_{Klink.} > q_{laminar}$$

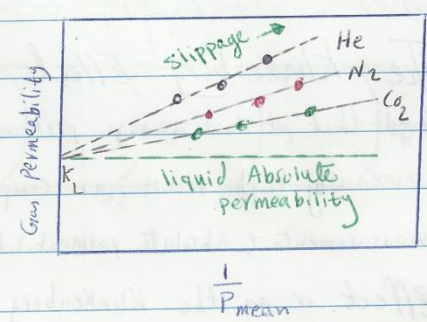
[At higher pressure the gas molecules are closer to each other and interact more strongly as liquid molecules]

Experiments show that when gas permeability is plotted versus $\frac{1}{P_m}$, a straight line can be fitted through the data points. Extrapolation of this line to infinite mean pressure, i.e when $\frac{1}{P_{mean}} \to 0$ gives the absolute (liquid) permeability. The measured permeability K_m and liquid permeability K_L is correlated by $K_m = K_L \left(1 + \frac{b}{P_{mean}}\right)$

or

$$K_m = K_L + C \frac{1}{P_{mean}}$$

$$C = b\, K_L$$



As M_w increases the mean free path of Mole. increases.
also MFP = f(P)

Mean Free Path MFP

K_m = measured permeability , K_L = liquid permeability

C = slop of the line. $\Rightarrow$ $C = K_L b$ [b : atm]

Klinkenberg suggested that the slop C is a function of the following factors:-

1- Absolute permeability
2- Type of gas used
3- Average pore radius

b = Constant which depend on the size of the pore openings & is inversely proportional to radius of capillaries.

Jones (1972) correlated the parameter b with K_L by the following expression

$$b = 6.9 K_L^{-0.36}$$

In most lab measurements of K_g, it is safe to neglect the Klinkenberg effect if the gas pressure is higher than 10 bar. In reservoirs the pressure will be much higher & consequently, the significance of the Klinkenberge effect of no importance.

H.W:- Calculate the equivalent liquid permeability from the following data

Mean pressure P_{mean} (atm) :	1.192	2.517	4.571	9.484
Measured permeability (md) :	3.76	3.04	2.76	2.54

also find the value of the parameter b.

Solution:

$\frac{1}{P_{mean}}$ (atm⁻¹) :	0.840	0.4	0.22	0.105
K_m (md) :	3.76	3.04	2.76	2.54

plot K_m vs $\frac{1}{P}$

plot K_g vs. $\frac{1}{P_m}$, construct the best straight line, then extrapolate toward $\frac{1}{P_m} = 0$ [i.e $P_m \to \infty$]

$\therefore$ Equivalent liquid permeability $K_L = 2.4$ md.

Since $K_g = K_L \left(1 + \frac{b}{P_{mean}}\right)$

Sub. for $K_g = 3.04$ md. $P_m = 2.517$ atm then

$3.04 = 2.4 (1 + 0.4b] \Rightarrow b = 0.667$ atm.

By Jones Correlation $b = 6.9\, K_L^{-0.36}$

$b = 6.9 (2.4)^{-0.36} \Rightarrow b = 5.03$ atm.

Sun. 18-11-2007 "الاحد"

Simple linear Regression

Suppose that $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$ are obtained experimentally, It is required to derive a curve of specified shap to pass these points with minimum errors { Error = Calculated - observed }; The most popular relationships are

1- Straight line $y = a + bx$
2- Exponential curve $y = a b^x$
3- Logarithmic " $y = a + b \ln x$
4- Power curve $y = a x^b$

Straight line linear regression, Method of least squars.

if we have n pair of (x, y), to find the value of a & b. find the following sums:

السبت ١٠/١١/٢٠٠٧ محاضرات جمة للميكانيكيين عراقيين في الساعة ٨:٢٥ صباحاً

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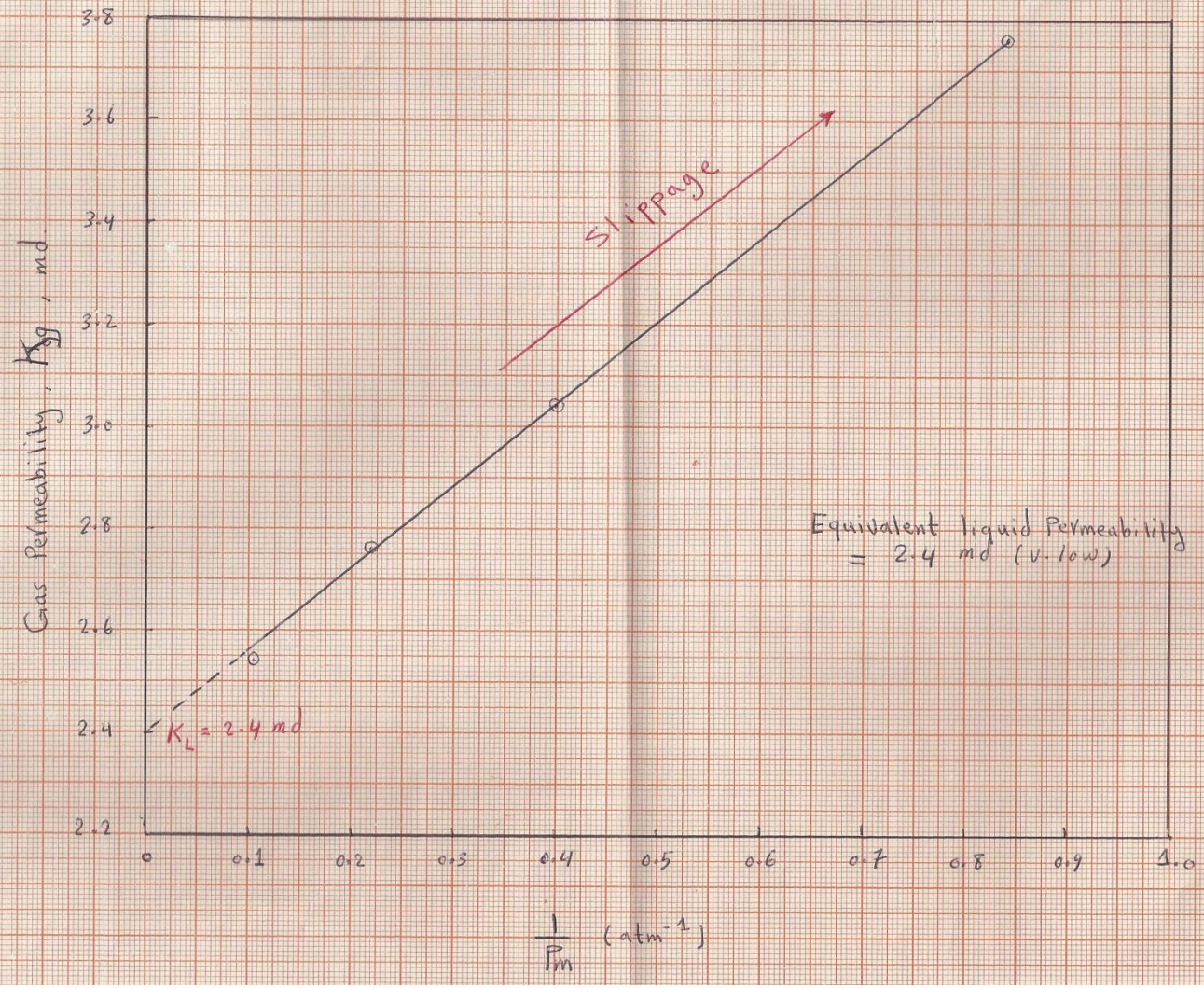

Gas Permeability, K_g, md
3.8
3.6
3.4
3.2
3.0
2.8
2.6
2.4
2.2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
$\frac{1}{P_m}$ (atm^{-1})
Slippage
$K_L = 2.4$ md
Equivalent liquid Permeability
= 2.4 md (v. low)

Σx ; $(\Sigma x)^2$, Σx^2 , Σy , Σxy

$\therefore$

$$b = \frac{(\Sigma x)(\Sigma y) - n\,\Sigma xy}{(\Sigma x)^2 - n\,\Sigma x^2}$$

n = No. of points [no. of observations]

$$a = \frac{1}{n}\left[\Sigma y - b\,\Sigma x\right]$$

or

$$a = \bar{y} - b\,\bar{x} \quad ; \quad \bar{y} = \frac{\Sigma y}{n} , \quad \bar{x} = \frac{\Sigma x}{n}$$

$\therefore$ You obtain the following eq.

$$y = a + bx$$

[The method used to obtain a: intercept & b: the slop is called the method of least squars]

Ex: Apply the least squar Method to find the best relation

X :	1	2	3.5	5	7	9
Y :	2.5	2	1.5	0.5	-0.5	-1.5

Sol.

$$y = 3 - 0.5x$$

Sol.: $\Sigma x = 27$, $(\Sigma x)^2 = 729$, $\Sigma x^2 = 169$ $\quad \bar{x} = 4.5 \quad [n=6]$

$\Sigma y = 4.5 \quad \bar{y} = 0.75 \quad \Sigma xy = -3.5$

$$\therefore \quad b = \frac{(27)(4.5) - 6(-3.5)}{(729) - 6(169)} \Rightarrow b = -0.5$$

$$a = 0.75 - (-0.5)(4.5) \Rightarrow a = 3$$

$$\therefore \quad \boxed{y = 3 - 0.5x}$$

الجمعة ١٦/١١/٢٠٠٧ عند قراءة كتيب على باب علي موسى ٢:٠٠ ظهراً !

Wettability and Capillary pressure

Surface and Interfacial Tension [ST and IFT]

An interface is known as the boundary region between two adjacent bulk phases, The equilibrium bulk phases can be:

- Liquid - vapor (LV)
- Liquid - liquid (LL)
- Liquid - Solid (LS)
- Solid - Vapor (SV)

Gases are basically miscible and thus no interfacial tension is observed between gases.

Considering water and oil, fluids commonly found in petroleum reservoirs, it is found that an interfacial tension always exists between the fluids. A water molecules which is remot from the interface is surrounded by other water molecules, thus having a resulting net attractive force on the molecules of zero. However, a molecule at the interface has a force acting upon it from the oil lying immediately above the interface & water molecules lying below the interface. The resulting forces are unbalanced and give rise to interfacial tension. The underbalanced attractive force between the molecules creates a membranlike surface. A certain amount of work is required to move a water molecule from within the body of the liquid through the interface. This work is frequently referred to as the free surface energy of the liquid.

IFT and ST are commonly expressed in dynes/cm

[air-water $\sigma = 72.8$ dyn/cm

air-Mercury $\sigma = 476$

* The surface tension between a pure liquid and its vapour phase is usually in the range of 10 – 80 dyn/cm.

** The stronger the intermolecular attractions in the liquid, the larger is the IFT

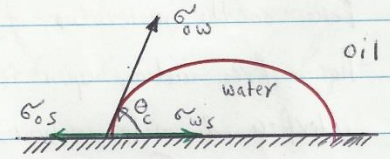
*** If IFT = 0, the two liquids are truly Miscible.

Rock Wettability

Wettability can be defined as the tendency of one fluid to spread on or adher to a solid surface in the presence of other immiscible fluid. Applying this term to reservoir engineering, the solid surface is the reservoir rocks (S.S, L.S or dolomite). The fluid that exist in the rock pore space are gas, oil and water.

Consider a typical water-oil-solid system. The surface energies in such a system are related by the Young-Dupre equation

$$\sigma_{os} - \sigma_{ws} = \sigma_{ow} \cos\theta_c$$



where:

σ_{os} = IFT between the oil and solid, dyn/cm

σ_{ws} = " " " water " " "

σ_{ow} = " " " oil " " "

θ_c = Angle ~~between~~ @ the oil-water-solid interface measured through the water [or through the denes fluid]

θ_c Some-time Called the Contact angle.

The difference $\sigma_{os} - \sigma_{ws}$ is called the adhesion tension

$$A_T = \sigma_{os} - \sigma_{ws}$$

R.E.I 20

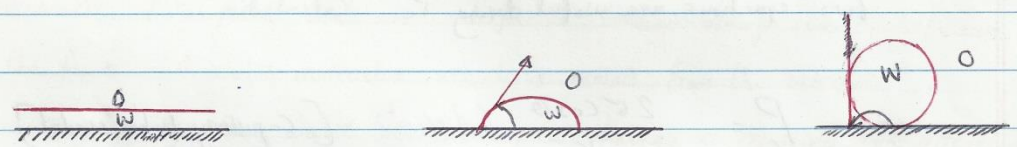
07-08

A positive adhesion tension indicates that the denser phase preferentially wet the solid surface. If the adhesion tension is large, as the denser phase will readily spread and tend to coat the surface

A_T is max when $\theta_c = 0$ where $\cos\theta = 1$, max.

Wettability preference expressed by contact angle

Value of θ_c	wettability preference
0 - 30	Strongly water wet
30 - 90	preferentially water wet
90	Neutral wettability
90 - 150	preferentially oil wet
150 - 180	Strongly oil wet



$\theta_c = 0°$ $\qquad$ $\theta_c < 90°$ $\qquad$ $\theta \sim 180°$

A_T Max $\qquad$ A_T Min

يتم تحديد درجة البتلليه للكامن اما بشكل نوعي أو بشكل كمي Qualitative or Quantitative

الطرق الكميه

أ- قياس زاويه التماس Contact angle

ب- طريقة Amott

جـ- مؤشر البتلليه Wettability Index USBM

الطرق النوعيه

أ- طريقة التشرب Imbibition Method

ب- النفاذيه النسبيه

Tarq. p.210
Fig → ex.

Definitions:

1- Imbibition: Flow resulting in an increase in wetting phase saturation. " كلمة - تتمكن في المكان :- التشرب الماء "

2- Drainage: Flow resulting in a decrease in the wetting phase saturation.

Capillary Pressure

Sun 25-11-2007

In reservoir engineering the capillary pressure P_c is defined as the difference between the nonwetting phase pressure and wetting phase pressure or

$$P_c = P_o - P_w \qquad \text{(water-wet)}$$

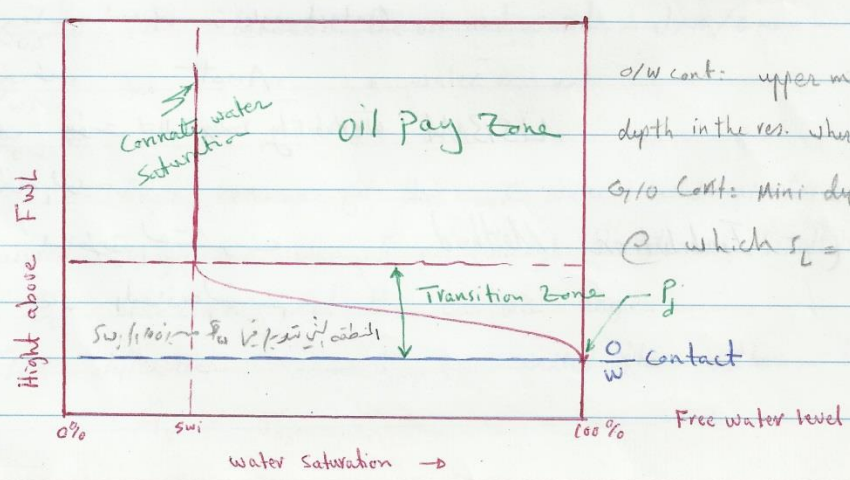
two equations are useful during P_c Calculations:

$$P_c = \frac{2\sigma \cos\theta}{r} \quad (dyn/cm^2) \quad \text{[Capillary tube model]}$$

$P_c = \frac{\sigma}{R_m}$, R_m = Mean radius of curvature of o/w interface.

or $P_c = 0.433\, \Delta\gamma h \qquad \Delta\gamma = \gamma_w - \gamma_o$, h: hight

Water Saturation Profile



o/w cont: upper most depth in the res. where $S_w = 100\%$

G/O Cont: Mini depth which $S_L = 100\%$

Free water level

Three pricipal region of saturation are designated.

1. The 100% water saturation @ P_c below the initial displacement pressure.

* P_d = Threshold pressure : The minimum capillary pressure required to introduce non wetting phase into a rock completely saturated with the wetting phase, since $P_c = \frac{2\sigma \cos\theta}{r}$ ∴ displacement corresponds to the largest diameter capillary in the rock available for entry of non-wetting phase.

2. Transition Zone "funicular region" w+o إرباء المخروطي في الطبقة تتم

3. Pendular region

Ex:- *Depth to free water level = 5230 ft, formation thickness = 130'

σ_{o-w} = 30 dyn/cm, oil gravity = 42 °API, water density = 1.1 gm/cc

Calculate 1- The depth @ which P_c = 9.25 psi

2- Calculate the mean radius of curvature of o/w interface @ that depth

Solution:-

$$\gamma_o = \frac{141.5}{API+131.5} = \frac{141.5}{42+131.5} \Rightarrow \gamma_o = 0.815$$

$$P_c = 0.433\, \Delta\gamma\, h \Rightarrow 9.25 = 0.433(1.1-0.815)\, h$$

∴ h = 75 ft above free water level [FWL]

∴ The depth = 5230 − 75 = 5155 ft

P_c = 9.25 psi convert to dyn/cm²

$$P_c = 9.25 \frac{1\ dyn/cm^2}{1.41\times10^{-5}\ psi} \Rightarrow P_c = 6.38\times10^{5}\ dyn/cm^2$$

but

$$P_c = \frac{\sigma}{R_m} \Rightarrow 6.38\times10^{5} = \frac{30}{R_m}$$

$$R_m = 4\times10^{-5}\ cm.$$

Exam.: height above free w.l & owc.

Capillary Pressure Measurements are made on small core samples. to obtain a representative Capillary pressure curve for an entire reservoir, some technique for averaging is necessary, since different cores have different permeability and hence different Capillary pressure behavoir, to approach this aim we will use the following Correlating function [leverett or J - function]

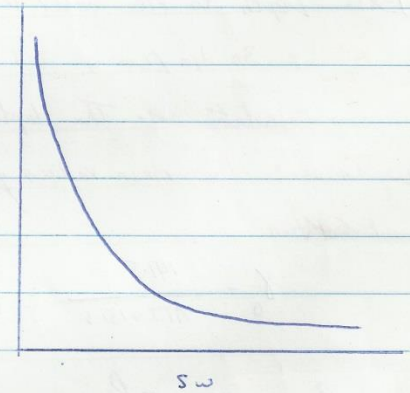
$$J(sw) = \frac{P_c}{\sigma \cos\theta} \left(\frac{K}{\phi} \right)^{\frac{1}{2}}$$

J (sw) = leverete function dimensionless. This function is plotted against water saturation (Sw) to obtain a plot similar inshape to a Capillary pressure.

Assuming $\theta = 0^\circ$

then

$$J(sw) = 0.21645 \frac{P_c}{\sigma} \sqrt{\frac{K}{\phi}}$$



P_c = Capillary pressure (psi)

σ = IFT dyn/cm

K = perm., md.

ϕ = porosity.

Ex:- A laboratory P_c test was Conducted on a core Sample. The core sample has a porosity of 0.16 and permeability of 80md, respectively; the capillary pressure- saturation data are given as follows

Sw	1.	0.8	0.6	0.4	0.2
P_c (Psi)	0.5	0.6	0.75	1.05	1.75

$\sigma = 50$ dyn/cm. Further reservoir engineering analysis indicated that the reservoir is better described @ $\phi = 0.19$ & $K = 120$; generate the P_c data for the reservoir

Solution:-

step 1. Calculate the J-function using the measured Capillary pressure data

$$J(S_w) = 0.21645 \left(\frac{P_c}{50}\right)\left(\frac{80}{0.16}\right)^{0.5}$$

$$J(S_w) = 0.0967994\ P_c$$

S_w	1.0	0.8	0.6	0.4	0.2
P_c (PSI)	0.5	0.6	0.75	1.05	1.75
$J(S_w)$	0.048	0.058	0.073	0.102	0.169

step2. using the new porosity and permeability values Calculate P_c

$$P_c = J(S_w)\,\sigma \Big/ \left[0.21645\sqrt{\frac{K}{\phi}}\right]$$

$$P_c = J(S_w)(50) \Big/ \left[0.21645\sqrt{\frac{120}{0.19}}\right]$$

$$P_c = 9.192\ J(S_w)$$

S_w	1.0	0.8	0.6	0.4	0.2
P_c	0.441	0.533	0.671	0.938	1.553

* 2nd exp

REI 231

Sun 2/12/2007 07-08

Mechanics of fluid static and flow

Fluid static and flow problems are concerned with four kinds of forces, potential or energy, namely:

1. Gravitational
2. Pressure
3. Capillary [is of minor importance]
4. Kinetic [Negligible importance in reservoir fluid mechanics]

Gravity Potential: The gravitational force acting on a body with a mass m is:

$$F_g = mg \quad \text{—— } \boxed{1}$$

where

F_g: gravitational force ; m = mass , g = acceleration due to gravity ($\frac{L}{T^2}$) ∴ $\{\{F = \frac{m.L}{T^2}\}\}$

The direction of this force is vertical (Z- Coordinate). if movement takes place under the influence of the force F, ∴ the change in gravitational energy or work is given by:

$$dE_g = F_g\, dZ = mg\, dZ \quad \text{... (2)}$$

" change in gravitational energy (work) " , " $E_g = \frac{m.L^2}{T^2}$ "

The total change in gravitational energy is represented by integral

$$\int_{E_0}^{E} dE_g = mg \int_{Z_0}^{Z} dZ \quad \text{—— (3)}$$

or

$$E - E_0 = mg\,(Z - Z_0) \quad \text{—— (4)}$$

At the zero reference level Z_0, the gravitational energy is E_0,

for example, a good reference $z = 0$ would be the sea level where also $E_0 = 0$

$$\therefore \quad E_g = mgz \quad \text{—— (5)}$$

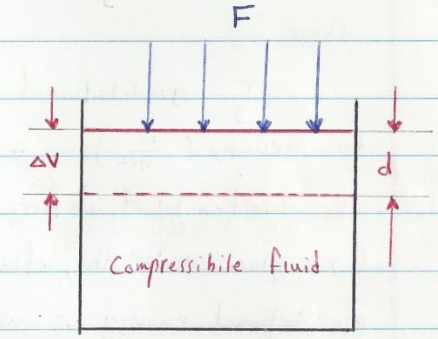
E_g: gravitational energy or work

Note that mg is the force necessary to lift a body z distance above a reference level, this work done provides stored energy ($lb_f \cdot ft$)

The energy per unit mass is called the potential Φ; the gravitational potential Φ_g is then expressed

$$\Phi_g = \frac{E_g}{m} = \frac{mgz}{m} = gz \qquad \left[\frac{L^2}{T^2}\right] \quad \text{—— } \boxed{6}$$

Pressure Potential



$$P = \frac{F}{A}$$

Recall the work equation in fluid displacement

Work = Force x displacement

i.e $\quad W = F \cdot d \quad$ —— (1)

$$W = P \cdot Ad \quad \text{—— (2)}$$

where W = work, F: work, d: displacement;

but Force = pressure x Area

Since: Change in volume $\Delta V = Ad$

$$\therefore \quad \Delta W = P \, \Delta V \quad \text{—— (3)}$$

[work = pressure x Change in volume]

in differential form

$$dw = P \, dV \quad \text{—— (4)}$$

The energy available in a compressed fluid is equivalent to the work done in compressing this fluid to the existing state.

$V = f(P)$ $\quad E = \int f(x)\,dx = \int v\,dP$

The available energy may be derived with respect to pressure as:

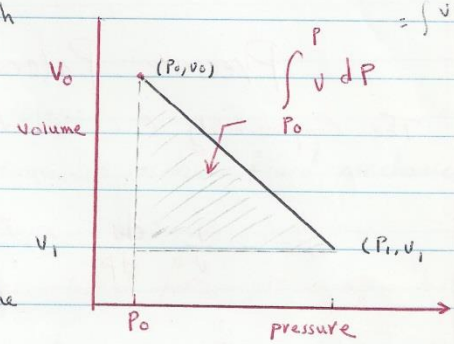


Fig. 1

$$dE_p = v\,dp \quad \text{--- (5)}$$

as pressure change from $P_0 \rightarrow P$ the energy change would be found in the integral

$$\int_{E_0}^{E} dE_p = \int_{P_0}^{P} v\,dp \quad \text{--- (6)}$$

The R.H.S of integral would be evaluated numerically from P.V relationship measured in PVT Lab as illustrated in above figure.

Alternatively the energy change can be expressed in term of fluid density ρ, noting that $\rho = \frac{m}{V}$ $\therefore$

$$E_p - E_0 = m \int_{P_0}^{P} \frac{dP}{\rho} \quad \text{--- (7)}$$

with reference to atmospheric pressure, where $E_0 = 0$, eq. (7) becomes

$$E_p = m \int_{P_{atm}}^{P} \frac{dP}{\rho} \quad \text{--- (8)}$$

The pressure potential, which is the pressure energy per unit of mass may be expressed as follows:-

$$\Phi_P = E_P / m \Rightarrow \Phi_P = \int_{P_{atm.}}^{P} \frac{dP}{\rho} \quad \text{— } \boxed{9}$$

"The above equation is a general form of pressure potential of fluids"

Pressure Potential for liquids

The Compressibility C is defined by.

$$C = -\frac{1}{V}\frac{dV}{dP} \quad \text{— (11) } [\text{as } P\uparrow;\ V\downarrow]$$

but

$$V = \frac{m}{\rho}$$

$$\therefore\ C = -\frac{\rho}{m}\cdot\frac{d}{dP}\left(\frac{m}{\rho}\right)$$

but m constant

$$C = -\rho\frac{d}{dP}\left(\frac{1}{\rho}\right) \Rightarrow C = -\rho\left(-\frac{1}{\rho^2}\right)\left(\frac{d\rho}{dP}\right)$$

$$C = \frac{1}{\rho}\cdot\frac{d\rho}{dP} \quad \text{— (12) } [\text{as } P\uparrow,\ \rho\uparrow\ (+)]$$

rearrange above equation,

$$dP = \frac{d\rho}{C\rho} \quad \text{— (13)}$$

sub. eq. (13) in (9)

$$\Phi_P = \int_{\rho_{atm.}}^{\rho}\left(\frac{1}{\rho}\cdot\frac{d\rho}{C\rho}\right) \quad \text{— (14)}$$

07-08

$$\Phi_P = \frac{1}{c}\int_{\rho_{atm}}^{\rho} \rho^{-2}\, d\rho \Rightarrow \Phi_P = \frac{1}{c}\left[\frac{1}{\rho}\right]_{\rho}^{\rho_{atm}}$$ "change the order of integration"

$$\Phi_P = \frac{1}{c}\left[\frac{1}{\rho_{atm}} - \frac{1}{\rho}\right] \quad \text{------} \quad (15)$$

The pressure potential in term of density [For liquid only]

If we wish to express the pressure-potential in term of pressure difference $(P - P_{atm})$, we will make use of the following treatment.

$$C = \frac{1}{\rho}\frac{d\rho}{dP}$$

separate the variable and integrate both sides

$$c\int_{P_{atm}}^{P} dP = \int_{\rho_{atm}}^{\rho}\frac{d\rho}{\rho} \Rightarrow C(P - P_{atm}) = \ln\frac{\rho}{\rho_{atm}}$$

or

$$\rho = \rho_0\, e^{c(P - P_{atm})} \quad \text{---} (16) \quad [\text{as } P\uparrow, \rho\uparrow]$$

Sub eq. (16) in eq. (15).

$$\Phi_P = \frac{1}{c}\left[\frac{1}{\rho_{atm}} - \frac{1}{\rho_{atm}\, e^{c(P - P_{atm})}}\right]$$

$$\Phi_P = \frac{1}{c\rho_{atm}}\left[1 - e^{-c(P - P_{atm})}\right] \quad \text{---} (17)$$

if the pressure difference is small then

$$e^{-c(P - P_{atm})} \approx 1 - c(P - P_{am}) \quad \left[e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} \cdots\right.$$

$$\therefore \quad \Phi_P = \frac{P - P_{atm.}}{\rho_{atm.}} \quad \text{------} \quad \boxed{18}$$

pressure potential for liquid.

Conclusion: The value of liquid compressibility is small which provides very littel workenergy per unit of mass.

Pressure potential for Gases

Mon 3/12/2007

Equation of state for ideal gases suggests that

$$PV = nRT \quad \text{---} \quad (1) \qquad [Z = 1.0]$$

or

$$PV = \frac{m}{M_W} \cdot R \cdot T \qquad \text{but} \quad \rho = \frac{m}{V} \quad \text{or} \quad V = \frac{m}{\rho}$$

$\therefore$ For ideal gas

$$\rho = \frac{P \cdot M_W}{RT} \quad \text{--} (2) \qquad \text{and} \quad \frac{P}{\rho} = \frac{RT}{M_W}$$

Boyles law states that $P_1 V_1 = P_2 V_2$

$$\therefore \quad P_1 \cdot \frac{m}{\rho_1} = P_2 \cdot \frac{m}{\rho_2}$$

or

$$\frac{P_1}{\rho_1} = \frac{P_2}{\rho_2} = \frac{P_{atm}}{\rho_{atm}} = \text{---} = \frac{P}{\rho}$$

in otherword $\rho = \frac{\rho_{atm}}{P_{atm.}} \cdot P$ ---- (3)

Recall the equation of pressure potential

$$\Phi_P = \int_{P_{atm}}^{P} \frac{dP}{\rho} \qquad \therefore \text{ make use of eq. (3)}$$

$$\Phi_P = \int_{P_{atm}}^{P} \frac{\rho_{atm}}{P_{atm}} \frac{dP}{P} \quad , \text{ but } \rho_{atm}, P_{atm} \text{ are constants}$$

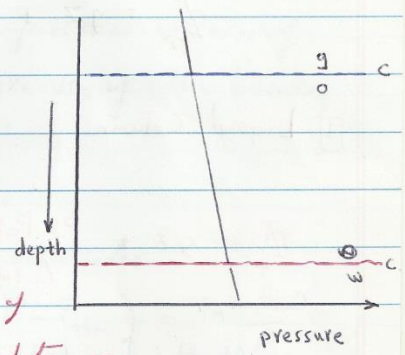
$$\therefore \Phi_p = \frac{P_{atm}}{\rho_{atm}} \ln \frac{P}{P_{atm}} \quad [\text{For gas only}]$$

$$\text{"}\Phi_p = \frac{L^2}{T^2}\text{"} \qquad \left[\Phi_p = \frac{\frac{m \cdot L}{L^2 T^2}}{\frac{m}{L^3}} = \frac{L^2}{T^2}\right]$$

$$[\text{when } P = P_{atm} \quad \Phi_p = 0]$$

"Considerable energy per unit mass or "Potential" is available from compressed gas and gas released form solution in oil below the bubble point pressure"

"Pressure Potential and Pressure Gradients in Static fluid Columns"



Some practical applications of gravity and pressure potentials related to static fluid column in wellbore and in reservoir system.

1- with a bottomhole pressure gauge, downhole gradient can be measured by taking a series of measurements at various depths, providing data on fluid densities "gas, oil and water"

2- Such gradients permit extrapolation to obtain pressure at a datum not reached by pressure gauge. In gas wells, bottomhole pressure may be estimated from density data and tubing-head pressure

3- within the reservoir measurements of a pressure at a few datum levels in one or more wells, the fluid contacts may be located "g/o and o/w contacts"

4- In surveys concerning hydrodynamic or movements of ground water, a fluid level in water well serves as a manometer to indicate the hydraulic head, measuring the total potential Φ which is the sum of

of gravitational and pressure potential @ the point penetrated in an aquifer. $[\Phi_T = \Phi_g + \Phi_P]$

Since the static column are at rest, ∴ the total potential should be constant throughout the column. or

$$\Phi_T = \Phi_g + \Phi_P = \text{Constant}$$

or more generally

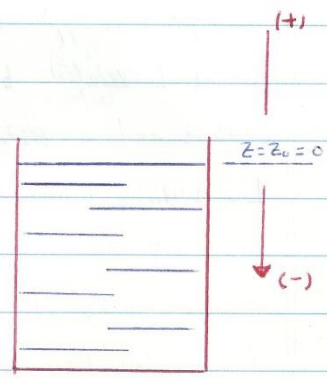
$$\Phi_T = gz + \frac{P - P_{atm}}{\rho_{atm}} = \text{Constant} \quad [\text{liquid}]$$

and

$$\Phi_T = gz + \frac{P_{atm}}{\rho_{atm}} \ln \frac{P}{P_{atm}} = \text{Constant} \quad [\text{Gas}]$$

} Static Column

1. Liquid Column



$$\Phi_T = gz + \frac{P - P_{atm}}{\rho_{atm}} = \text{Constant, for static}$$

Column,

At the top of liquid column, the pressure $= P_{atm}$

∴ $\Phi_P = 0$ also @ the top of liquid column $z = 0$

∴ $\Phi_T = \Phi_g + \Phi_P = 0 \quad [@\ z = z_0 = 0]$

but Φ_T is constant throughout the column

∴ for any other points in the column $\Phi_T = 0$ or

$$\frac{P - P_{atm}}{\rho} + gz = 0 \Rightarrow P = P_{atm} - g\rho z,$$

the z-coordinates are all negative, so that $P > P_{atm}$, usually this pressure-depth relation is written in terms of depth "D" below the zero level $[D = -z$ or Depth $= -$ elevation$]$

$\therefore P = P_{atm} + g\rho D$

i.e the pressure is a linear function of depth

In oil field unit:

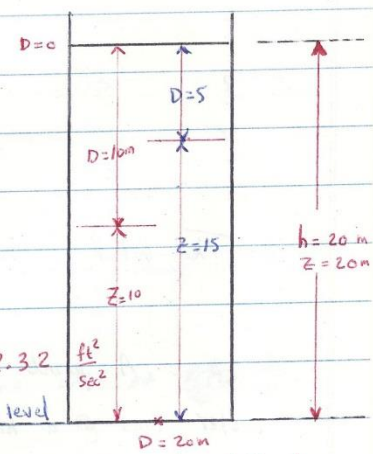
P in psi, D in ft and $g\rho$ in psi/ft, for fresh water $g\rho = 0.433$ psi/ft, the gradient of salt water is upto 0.465 psi/ft [$\gamma_w = 1.073$], oil column gradients (generally) range from 0.2 to 0.35 psi/ft

$$P = P_{atm} + 0.433\,\gamma D \qquad [\gamma = \frac{\rho}{\rho_w}]$$

A pressure survey in a well might indicate three fluid gradients (Water, oil and gas & two contacts)

Ex:- A hydrostatic column of water of hight of 20m was placed @ the sea level Calculate the Total potential at a- The top of the Column b- at a point of a hight of 15m c- at a point of a hight of 10m d- at the bottom of the Column.

Solution:



a- At the top of the Column

$P = P_{atm}$ & $\Phi_p = 0$

$Z = 20m$ or $Z = (20)(3.28) = 65.6$ ft

& $\Phi_\pi = \Phi_g$

$$\Phi_\pi = 32.2 \frac{ft}{sec^2} \times 65.6\,ft \Rightarrow \Phi_\pi = 2112.32 \frac{ft^2}{sec^2}$$

b- At a point of hight 15m, here $Z = 49.2$ ft, Depth to the point $D = 20 - 15 \Rightarrow D = 5m \Rightarrow D = 16.4$ ft

$$\Phi_p = \frac{P - P_{atm}}{\rho} \qquad \text{but } P = P_{atm} + g\rho D$$

$$\therefore \Phi_P = \frac{P_{atm} + \rho g D - P_{atm}}{\rho}$$

$$\Phi_P = gD \qquad \left(\left(\frac{L}{T}\right)^2\right)$$

or

$$\Phi_T = g(Z+D)$$

$\therefore$

$$\Phi_T = 32.2\,(5+15)(3.28)$$
$$= 2112.32 \ \frac{ft^2}{sec^2}$$

C- At a point of high 10m , $Z = 10m$, $D = 20-10 = 10m$

$\therefore \Phi_T = (32.2)(10+10)(3.28) \Rightarrow \Phi_T = 2112.32 \left(\frac{ft}{sec.}\right)^2$

D- At the bottom of the column $Z=0$ $D = 20m$

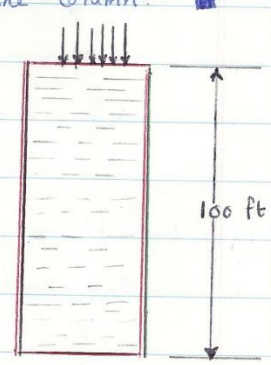
$\Phi_T = 32.2\,(0+20)(3.28) \Rightarrow \Phi_T = 2112.32 (ft/sec^2)$

Summary of Calculations

Depth	Total potential Φ_T $\left(\frac{ft^2}{sec^2}\right)$
0	2112.32
5	2112.32
10	2112.32
20	2112.32

$\therefore$ The total potential is Constant throughout the Column. ■

Ex.2 : A point in a static Column of fresh water was @ a hight of 100 ft above the sea level and a pressure of 100 psia; Calculate the total potential at that point also Calculate the total potential at the bottom of the Column.



Solution: $\Phi_T = \Phi_p + \Phi_g$

1- pressure potential

$$\Phi_p = \frac{P - P_{atm}}{\rho_w} = \frac{(100 - 14.7)\frac{lb}{in^2} \cdot \left(\frac{144\ in^2}{ft^2}\right)}{62.4\ \frac{lb}{ft^3}}$$

$$\Phi_p = 196.85 \quad ft^2/sec^2$$

2- Gravitational potential

$\Phi_g = gZ$ here $Z = 100$ ft

$\Phi_g = (32.2)(100) \Rightarrow \Phi_g = 3220\ (ft/sec)^2$

Therefore

$$\Phi_T = 196.85 + 3220 = 3416.85 \left(\frac{ft}{sec}\right)^2$$

At the bottom of the Column :-

$Z = 0$ & $\Phi_g = 0$

pressure potential

$$\Phi_p = \frac{P - P_{atm}}{\rho_w} + gD \qquad [\text{here } P \text{ in gauge pressure}]$$

$$= \frac{(100.0 - 14.7)(144)}{62.4} + (32.2)(100)$$

& $\Phi_p = 3416.85 \ \frac{ft^2}{sec^2}$

$$\Phi_T = \Phi_g + \Phi_p \Rightarrow \Phi_T = 3416.85 \ \frac{ft^2}{sec}$$

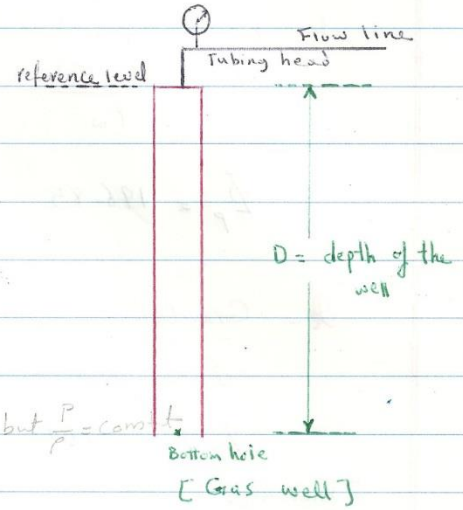
& The total potential is Constant throughout the Column.

أنجم ٢٤/١١/٢٠٠٧

Mon 10-12/2007

Gas Column - Calculation of static bottom-hole pressure [shut-in bottom hole pressure SBHP, SIBHP]

The pressure ~~gradient~~ potential Φ_P of gases is:



$$\Phi_P = \frac{P_{atm}}{\rho_{atm}} \ln \frac{P}{P_{atm}}$$

The gravitational potential $\Phi_g = gZ$

∴ Total potential Φ_T is:

$$\Phi_T = \frac{P_{atm}}{\rho_{atm}} \ln \frac{P}{P_{atm}} + gZ \quad \text{--- } \boxed{1} \quad \text{but } \frac{P}{\rho} = \text{const}$$

$$\Phi_T = \frac{P_{th}}{\rho_{th}} \ln \frac{P}{P_{th}} + gZ$$

If we take the tubing head pressure (P_{th}) as a reference pressure (P_o) and the tubing head as a reference level

$$\therefore \quad \Phi_T = \frac{P_{th}}{\rho_{th}} \ln \frac{P}{P_{th}} + gZ \quad \text{--- } \boxed{2}$$

ρ_{th}: gas density evaluated @ $P = P_{th}$

but $Z = 0$ @ tubing head [since Tubing head is the reference level]

~~∴ $\Phi_T = \frac{P_{th}}{\rho_{th}} \ln \frac{P_{th}}{P_{th}}$ or $\Phi_T = \Phi_P$~~

∴ @ Tubing head

$$\Phi_T = \frac{P_{th}}{\rho_{th}} \ln \frac{P_{th}}{P_{th}} + g(0) \quad \text{or} \quad \Phi_T = 0$$

[static Column]

at any point in the gas Column $\Phi_T = 0$

or

$$\frac{P_{th}}{\rho_{th}} \ln \frac{P}{P_{atm}} + gZ = 0 \quad \text{or} \quad \frac{P_{th}}{\rho_{th}} \ln \frac{P}{P_{atm}} = -gZ$$

Since $D = -Z$ [where D is the depth of the well.

$$\ln \frac{P}{P_{th}} = g D \frac{\rho_{th}}{P_{th}} \quad \text{--- } \boxed{3}$$

but $\frac{P}{\rho} = \frac{P_{atm}}{\rho_{atm}}$ = Constant

then eq. 3 can be written as:

$$\ln \frac{P}{P_{th}} = g D \frac{\rho_{atm}}{P_{atm}} \quad \text{--- } \boxed{4}$$ [$g\rho$: pressure gradient)

by difinition the gas gravity $\gamma_g = \frac{\rho_{gas}}{\rho_{air}}$ (air = 1.0)

and for ideal gases $(\rho_{gas})_{atm} = \gamma_g (\rho_{air})_{atm}$ --- $\boxed{5}$

$\rho = \frac{P \cdot M_w}{RT}$; ρ: gas density lb/ft³, P: pressure psia

M_w: Molecular weight ; R: Universal gas Constant 10.73

T: Absolute Temperature °R (°R = °F + 460)

@ standard Condition the air density is

$$(\rho_{air})_{atm} = \frac{(14.7)(28.97)}{(10.73)(520)}$$; S.C; P = 14.7 psia, T = 520 °R

$(M_w)_{air}$ = 28.97 lb./lb.mole

$\therefore (\rho_{air})_{atm} = 0.07632$ lb/ft³

$\therefore (\rho_{gas})_{atm} = 0.07632\, \gamma_g$ lb/ft³

or

$$(\rho_{gas})_{atm} = \frac{0.07632\gamma_g}{144} \text{ lb}_m/\text{in}^2/\text{ft} \quad \text{---- } 5$$ [$g\rho$ gradient]

$lb_f = (lb_m)(g)$

~~$= \frac{0.07632\gamma_g}{144}$ psi/ft ⑤ [gas gradient]~~

Sub. eq. 5 in eq. 4

$$\ln \frac{P}{P_{th}} = \frac{0.07632\,\gamma_g}{144} \frac{D}{P_{atm}}$$

A fairly good approximation for bottom hole pressure [where D = L] relative to tubing head pressure (P_{th}) which takes into account average Column temperature and average gas deviation factor

$$P_{SIBHP} = P_{th}\, e^{\frac{0.01875\, \gamma_g L}{T_{avg} \cdot Z_{avg}}}$$

$$\left[\left[P_{SIBHP} = P_{th}\, e^{\frac{0.0765}{144(14.7)} \cdot \frac{T_{atm}}{T} \cdot \frac{1}{Z_{avg}} \cdot \gamma_g L} \right]\right]$$

Ex:- Calculate the static bottom hole pressure under the following Condition, $P_{th} = 2600$ psia, $\gamma_g = 0.744$ (air = 1.00) $L = 7500$ ft. $T_{avg} = 152.5\,^{\circ}F$ (612.5 °R) ; $Z_{avg} = 0.82$

$$P_{SIBHP} = 2600\, e^{\frac{(0.01875)(0.744)(7500)}{(612.5)(0.82)}}$$

$$P_{SIBHP} = 3202 \text{ psia}$$

∴ The pressure exerted by gas Column = 3202 − 2600

= 602 psia

$$\text{pressure gradient} = \frac{602}{7500} =$$

07-08

Total Potential Expressed as "Hydraulic Head"*

Fig. (1) illustrates the flow of water down structure through formation. The potential Φ_1 at point [1] must be greater than the potential Φ_2 @ point [2] to cause flow. If manometer (i.e wells) could be placed at these points, water would rise to H_1 and H_2, both hights measured from the datum level $Z=0$ corresponding to point [2]. By difinition

H_1 = Hydraulic Head of the fluid at point [1] «ft. of water»

H_2 = Hydraulic Head of the fluid @ point [2] «ft of water»

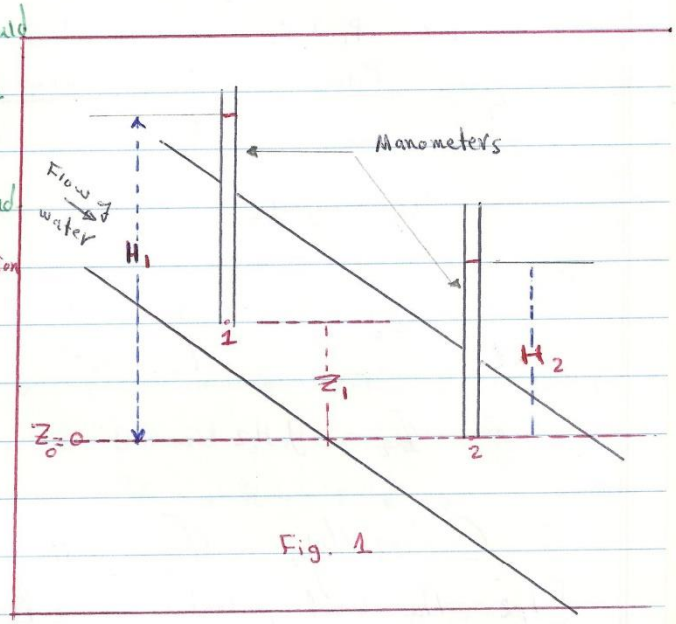


Fig. 1

The relation between these hydraulic heads and the total potential Φ_1 and Φ_2 can be shown as follows:-

Well No. 1: at the bottom of the well [i.e @ point [1]]

$$\Phi_1 = \frac{P_1 - P_{atm}}{\rho_{atm}} + g Z_1 \quad \text{--- (1)}$$

Well No. 2: at the bottom of the well [point 2, where $Z=0$]

$$\Phi_2 = \frac{P_2 - P_{atm}}{\rho_{atm}} \quad \text{--- (2)}$$

The pressures P_1 and P_2 can be related to the liquid column (hights) in the wells as follows:-

* ارتفاع أو كلّو هيدروليكي

Well No. 1 : The height of water Column $= H_1 - Z_1$

$\therefore \quad P_1 = P_{atm} + \rho g (H_1 - Z_1) \quad \cdots (3)$

Well No. 2 : The height of water Column $= H_2$

$\therefore \quad P_2 = P_{atm} + \rho g H_2$

Sub. for P_1 & P_2 in eqs. (1) & (2)

$$\Phi_1 = \frac{P_{atm} + \rho g (H_1 - Z_1) - P_{atm}}{\rho_{alr}} + g Z_1$$

$$\therefore \quad \Phi_1 = g H_1 \qquad \left[\frac{L^2}{T^2} \right]$$

and

$$\Phi_2 = \frac{P_{atm.} + \rho g H_2 - P_{atm}}{\rho_{alr}}$$

$$\therefore \quad \Phi_2 = g H_2 \qquad [\Phi = g(z+0)]$$

Generally $\Phi_T = g H$

[i.e The total potential is a function of Hydraulic head]

The hydraulic head is therefore a direct measure of the total potential at any point in the reservoir. In fig. 1 flow is downward or downstructure because $\Phi_1 > \Phi_2$ (i.e $H_1 > H_2$)

Note: P_1 need not be greater than P_2 to have flow in the direction indicated if the elevation Z_1 is greater enough.

The hydraulic head can always be calculated if the pressure and elevation Z above a datum level are known.

Since $\Phi_T = g H$ and $\Phi_T = \frac{P - P_{atm}}{\rho} + g Z$

then

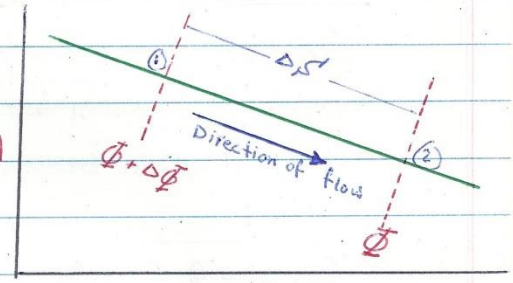
$$\frac{P - P_{atm}}{\rho} + g Z = g H \quad ; \quad P - P_{atm} = P_{gauge}$$

$$\therefore \quad H = \frac{P_{gauge}}{g\rho} + Z$$

Hydraulic head (H) = pressure head (h) + Elevation (Z)

(($P = \rho g h$ or $h = \frac{P}{g\rho}$ « pressure head »))

Fluid Flow Through Reservoir Rocks



In examining fluid flow through reservoir rock, various attitude or vectors must be considered, Horizontal, vertical and sloping beds. The motivation is potential difference ((analogus to voltage drop in electric circuits). Consider flow between point ① and ② in fig. above:-

where: $\Delta s'$: Distance between the two points.

$\Delta\Phi$: Losses in total potential due to flow from point ① to point ②. The losses in total potential $\Delta\Phi$, has been found proportional to the distance $\Delta s'$, the flow velocity V, the Kinematic viscosity $\frac{\mu}{\rho}$ and inversely proportional to a rock permeability (K) so the difference $\Delta\Phi$ may be expressed as:-

$$-\Delta\Phi = \Delta s' \, V \cdot \frac{\mu}{\rho} \cdot \frac{1}{K} \quad \ldots \boxed{1}$$ rearrange for velocity

$$V = -\frac{K\rho}{\mu} \cdot \frac{\Delta\Phi}{\Delta s'}$$ and in differential form:-

$$V = -K \cdot \frac{\rho}{\mu} \cdot \frac{d\Phi}{ds'} \quad \ldots \boxed{2}$$

where $d\Phi/ds'$: The gradient of total potential.

Equation ② is the general form of Darcy's law for flow of a single fluid through a porous medium in any direction [horizontal, vertical or slope]. Throug a cross section of area A, in any direction, the total rate of flow

$$Q = VA \quad \text{or}$$

$$\boxed{Q = -A \frac{K}{\mu} \cdot \rho \frac{d\Phi}{ds} \quad \text{---} \quad ③}$$

The total potential gradient $\frac{d\Phi}{ds}$ can be described by hydraulic Hed (H) since $\Phi = gH \Rightarrow d\Phi = g\,dH$

Then

$$\boxed{Q = -A \frac{K}{\mu} \rho g \frac{dH}{ds} \quad \text{---} \quad ④}$$

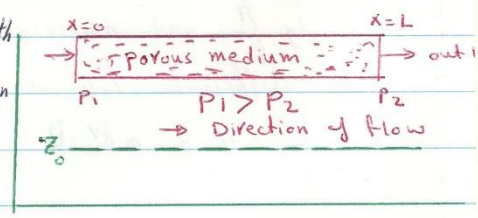
" $\rho g \frac{dH}{ds}$ = pressure gradient "

" المعادله ③ الشكل العام لقانون دارسي بدلاله الجهد الكلي اما المعادله ④ فهي الشكل العام لقانون دارسي بدلاله الارتفاع الهيدروليكي "

Flow Directions

1. Horizontal Flow" Flow in X- direction i.e $X \equiv s$

Since $H = \frac{P}{\rho g} + z$, differentiat both sides with respect to [WRT] flow direction X then



$$\frac{dH}{dX} = \frac{1}{\rho g}\frac{dP}{dX} + 0 \quad \text{---} \quad ⑤$$

sub. ⑤ in ④

$$Q = -\frac{AK}{\mu} \cdot \frac{dP}{dX} \quad \text{---} \quad ⑥ \quad \text{integrate both sides}$$

$$Q \int_0^L dX = -\frac{AK}{\mu} \int_{P_1}^{P_2} dP$$

$Q = A \frac{K}{\mu} \cdot \frac{P_1 - P_2}{L}$, Darcy's law for Horizontal flow

Q: cc/sec. , A = cm², K: Darcy, μ = c.p, P: atm, L: cm.

In oil field units & for flow of oil

$$Q_o = 1.127 \times 10^{-3} A \frac{K_o}{\mu_o B_o} \cdot \frac{P_e - P_{wf}}{L} \quad \text{—} \boxed{7}$$

where:

Q_o = Oil flow rate (production rate) STB/day

A = Area, ft², K_o = effective permeability (to oil), md.

μ_o = oil viscosity cp, P_e = static reservoir pressure psi.

P_{wf} = Bottom hole flowing pressure. psi.

B_o = Oil Formation volume factor, RB/STB

L = Bed Length. ft.

2- Vertical Flow

تعطى بدون التفاصيل

A- Vertical Flow (upward) without head where $Z \equiv S'$

Here $Z = S'$ (Direction of flow)

since

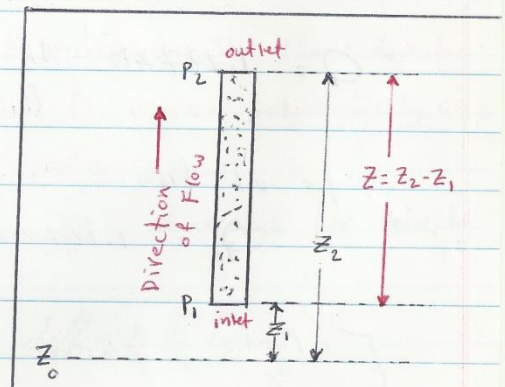
$H = \frac{P}{\rho g} + Z$; differentiate both sides with respect to Z

$$\frac{dH}{dZ} = \frac{1}{\rho g} \cdot \frac{dP}{dZ} + 1$$

so The flow equation can be written as:-

$$Q = -\frac{AK}{\mu} \cdot \rho g \left[\frac{1}{\rho g} \cdot \frac{dP}{dZ} + 1 \right]$$

$$Q = -\frac{AK}{\mu} \left[\frac{dP}{dZ} + \rho g \right]$$



$P_1 > P_2$

[vertical Flow - upward]

vertical Movement of water from Aquifer to res. (bottom water)

@ $Z = Z_1$ $P = P_1$, @ $Z = Z_2$ $P = P_2$ [$Z_2 - Z_1 = Z$ elevation]

Separate the variable

$$\left[Q + \frac{AK}{\mu} \cdot \rho g\right] \int_{z_1}^{z_2} dz = -\frac{AK}{\mu} \int_{P_1}^{P_2} dP$$

$$\therefore \quad Q = \frac{AK}{\mu}\left[\frac{P_1 - P_2}{z} - \rho g\right]$$

In Darcy's unit

$$Q = \frac{AK}{\mu}\left[\frac{P_1 - P_2}{z} - \frac{\rho g}{1.0133 \times 10^6}\right]$$

$Q = cc/sec$, $A = cm^2$, K md , $\mu = c.P$

$P = atm$, $z = cm$, $\rho = gm/cc$. $g = 980\ cm/sec^2$

$1.0133 \times 10^6\ dyn/cm^2 = 1\ atm$

In oil Field units:

$$Q = 1.127 \times 10^{-3} \frac{AK}{\mu}\left[\frac{P_1 - P_2}{z} - 0.433\gamma\right]$$

for oil flow

$$Q_o = 1.127 \times 10^{-3} \frac{AK_o}{\mu_o}\left[\frac{P_1 - P_2}{z} - 0.433\gamma_o\right]$$

[$\frac{P_1 - P_2}{z}$ = pressure Gradient , $0.433\gamma_o$ = Gravity gradient

here the gravity gradient act against press. grad. i.e. retard the flow rate]

Sat 15-12-2007 11:10 P.M.

* Vertical Flow تقطر بدون تفاصيل 07 - 08

b - Up-ward vertical flow under head.

1- Point ①

$h_t = h + x + L$

L = length of porous media

$P_1 = \rho g h_t$

$P_1 = \rho g (h + x + L)$ --- [1]

2- Point ②

$P_2 = \rho g x$

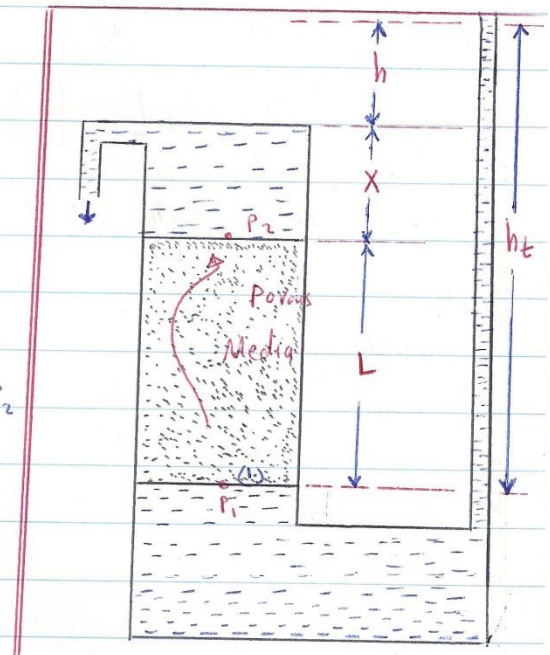
upward flow occure when $P_1 > P_2$

$\therefore \Delta P = P_2 - P_1$

$= \rho g x - \rho g (h + x + L)$

$\Delta P = -\rho g (h + L)$ -- [2]

$\frac{\Delta P}{L}$ = pressure Giradient



$\frac{\Delta P}{L} = -\frac{\rho g h}{L} - \rho g$ --- [3]

here $Z = s'$

$H = \frac{P}{\rho g} + Z$

$\therefore \frac{dH}{dZ} = \frac{1}{\rho g} \cdot \frac{dP}{dZ} + 1$ -- [4]

but $\frac{dP}{dz} = \frac{\Delta P}{L}$; Make use of eq. [3]

$\therefore \frac{dH}{dz} = \frac{1}{\rho g}\left[-\frac{\rho g h}{L} - \rho g\right] + 1$

$\therefore \frac{dH}{dz} = -\frac{h}{L}$

X = height of liquid above the top of Porous media.

L = length of Porous media

recall general form of Darcy's law

$$Q = -\frac{Ak}{\mu} \rho g \frac{dH}{ds}$$

$$Q = -\frac{Ak}{\mu} \rho g \left[-\frac{h}{L} \right]$$

$$Q = \frac{Ak}{\mu} \frac{\rho g h}{L}$$

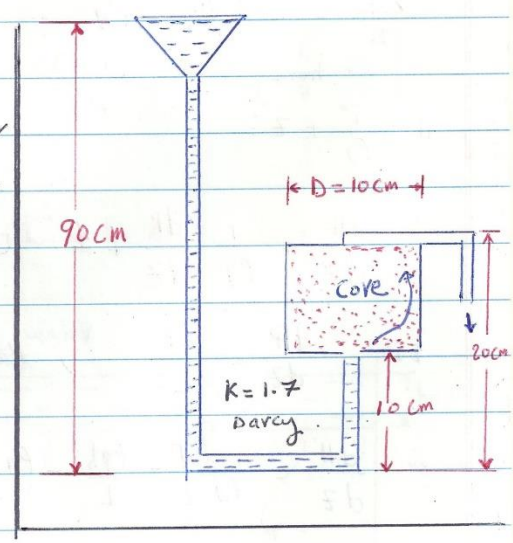
In Darcy's unit

$$Q = \frac{Ak}{\mu} \frac{\rho g h}{L} \left(\frac{1}{1.0133 \times 10^6} \right) \quad \text{// } \rho g h / L \text{ : pres. grad. //}$$

in oil field unit

$$Q_o = 1.127 \times 10^{-3} \frac{Ak}{\mu B_o} \left[0.433 \gamma_o \frac{h}{L} \right]$$

Ex: A laboratory experiment is arranged as in figure, to test the flow of water through core sample. The funnel is kept full of water. How much water flows through the core in one hour.



Solution:

Core length (L) = 20 - 10 = 10 cm

$A = \frac{\pi}{4} (10)^2 = 78.54 \text{ cm}^2$

$h = 90 - 20 = 70 \text{ cm}$

$$\therefore Q = 78.54 \frac{1.7}{1.0} \frac{(1)(980)(70)}{L} \left(\frac{1}{1.0133 \times 10^6} \right)$$

$Q = 0.904 \text{ cc/sec}$

Volume of water $= Q \times t$

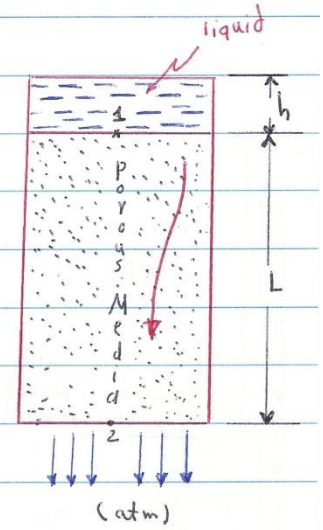
$= 0.904 \frac{cc}{sec} \cdot \frac{3600 sec}{1 hr}$

$= 3254$ cc/hr.

C- Vertical down ward, Flow under head.

here $s' = -z$

i.e $\frac{dz}{ds'} = -1$



$P_1 = P_{atm} + \rho g h$

$P_2 = P_{atm}$

$\therefore \Delta P = -\rho g h$

i.e $\frac{\Delta P}{L} = -\frac{\rho g h}{L}$

since

$\Phi_{T'} = \frac{P}{\rho} + gz$

((محطا = تصفيه الماء))

$\therefore \frac{d\Phi}{ds'} = \frac{1}{\rho} \cdot \frac{dP}{ds'} + g \frac{dz}{ds'}$

but $\frac{dP}{ds'} = \frac{\Delta P}{L} = -\rho g h$

and

$Q = -A \frac{K}{\mu} \rho \frac{d\Phi}{ds'}$

$\therefore Q = -A \frac{K}{\mu} \rho \left[\frac{1}{\rho} \cdot \left(-\frac{\rho g h}{L}\right) + g(-1) \right]$

$Q = \frac{AK}{\mu} \rho g \left[\frac{h}{L} + 1 \right]$

In Darcy's unit

$Q = \frac{AK}{\mu} \rho g \left(\frac{h}{L} + 1 \right) \left(\frac{1}{1.10133 \times 10^6} \right)$

[The gravity force enhance the flow].

$\boxed{\frac{dz}{ds'} = -1}$

Ex:- In a city water-filtring plant, it was desired to filter 5000 gal of water / hr through a sand filter bed to remove all the suspended matter and solids from water. A vertical cross-sectional view of the unit is shown in fig. below. Data: Thickness of filter bed = 4ft. $K = 1.2$ Darcy, $A = 1800\ ft^2$, $\mu_w = 1$ cp, Barometric pressure $= 14.7$ psi, Calculate the level of water that must be kept in a point above the sand filter inorder to filter 5000 gal / hr.

Solution:

In Oil Field unit:

$$Q = 1.127 \times 10^{-3}\ \frac{AK}{\mu} \cdot (0.433\gamma) \left[\frac{h}{L} + 1 \right]$$

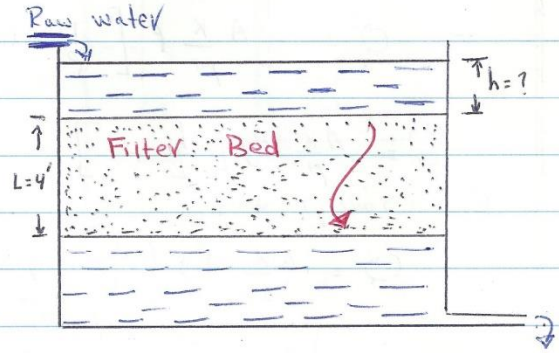
$$Q = \frac{5000\ gal \times \frac{bbl}{42\ gal}}{hr \times \frac{day}{24\ hr}} = 2857.1429\ bbl/day$$

$\therefore$

$$2857.1429 = 1.127 \times 10^{-3} \frac{(1800)(1.2 \times 10^3)}{(1.0)} \left[\frac{h}{4} + 1 \right]$$

Solve for $h = 6.825$ ft.

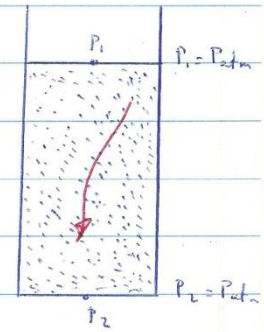
$\therefore$ Water head of 6.825 ft should be kept above sand filter



D- Vertical Down-ward (free flow) :

here the pressures at the inlet and outlet are equal to P_{atm}.

Then:- $s' = -z \qquad \frac{dz}{ds'} = -1$



$$\Phi = \frac{P_{gauge}}{\rho} + gZ$$

$$\frac{dP}{ds'} = \frac{dP}{dz} = 0 \Rightarrow \frac{d\Phi}{ds'} = -g$$

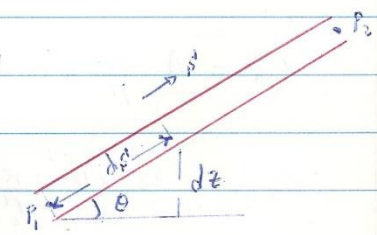
$$\therefore \quad Q = -\frac{AK\rho}{\mu}[-g]$$

$$Q = \frac{AK}{\mu}\left(\frac{\rho g}{1.0133\times10^{6}}\right) \quad \text{Darcy's unit}$$

$$Q = 1.127\times10^{-3}\,\frac{AK}{\mu}\,(0.433\,\gamma) \quad \text{Field Unit.}$$

3- Directional Flow

θ = Dip angle (Angle between the bed and horizontal)



Recall Darcy's law

$$Q = -A\frac{K}{\mu}\cdot\rho\cdot\frac{d\Phi}{ds'}$$

and

$$\Phi = \frac{P - P_{atm}}{\rho} + gZ \quad ; \quad \text{differentiat both sides with respect to } s'$$

$$\frac{d\Phi}{ds'} = \frac{1}{\rho}\frac{dP}{ds'} + g\frac{dZ}{ds'}$$

$$\therefore \quad Q = -\frac{AK}{\mu}\cdot\left[\frac{dP}{ds'} + \rho g\frac{dz}{ds'}\right]$$

, ρg = pressure gradient

$\frac{dz}{ds'} = \sin\theta$

[1] Flow up dip ; here $P_1 > P_2$

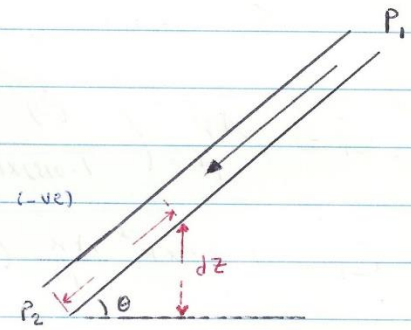
$\therefore \frac{dP}{ds'} = -ve$

$$\left\{ \frac{dP}{ds} = \frac{P_2 - P_1}{s_2 - s_1} \quad \text{or} \quad \frac{dP}{ds} = -\frac{(P_1 - P_2)}{ds'} \right\}$$

$$\therefore Q = \frac{AK}{\mu}\left[\frac{P_1 - P_2}{\Delta s'} - \rho g \frac{\sin\theta}{1.0133 \times 10^6}\right] \quad \text{Darcy's unit}$$

$$Q = 1.127 \times 10^{-3} A \frac{K}{\mu}\left[\frac{P_1 - P_2}{\Delta s'} - 0.433\,\gamma \sin\theta\right]$$

[2] Down Dip Flow



here :

$\frac{ds'}{dz} = -ve \qquad \theta =$ dip angle (-ve)

[a] if $P_1 = P_2$

$\frac{dP}{ds} = 0$

$$Q = A \cdot \frac{K}{\mu} \rho g \frac{\sin|\theta|}{1.0133 \times 10^6} \quad [\text{Darcy's unit}]$$

$$Q_o = 1.127 \times 10^{-3} \frac{K}{\mu_o B_o}\left[0.433\,\gamma_o \sin|\theta|\right]$$

[b] if $P_1 \neq P_2$

$$Q = \frac{AK}{\mu}\left[-\frac{dP}{ds'} + \rho g \frac{\sin|\theta|}{1.0133 \times 10^6}\right], \quad [dP = P_2 - P_1]$$

In oil field

$$Q_o = 1.127 \times 10^{-3} A \frac{K_o}{\mu_o B_o}\left[-\frac{dP}{ds'} + 0.433\,\gamma_o \sin|\theta|\right]$$

* if $P_1 > P_2$, $\frac{dP}{ds} = -ve$ $\therefore$ flow increases

** " $P_1 < P_2$, $dP/ds' = +ve$ $\therefore$ Flow decreases.

A

35-A

Mon. 31/12/2007

REI 07-08

Radial Flow

الجريان الشعاعي

1- Steady State Radial Flow of Incompressible Fluid (liquid)

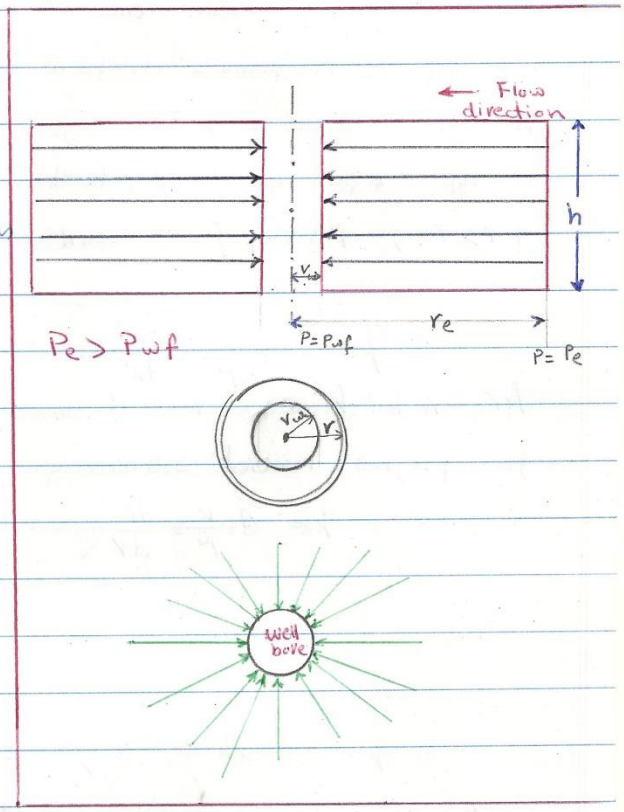
P_{wf} = Bottom-hole flowing pressure (Sandface pressure)

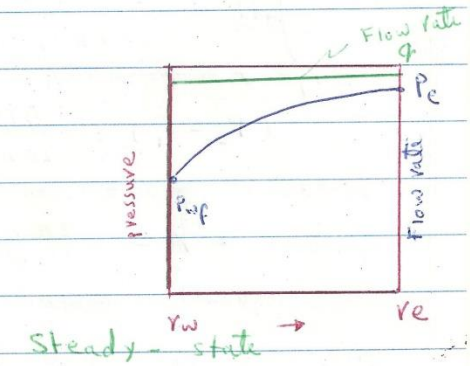
P_e = Static ~~reservoir~~ reservoir pressure.

r_w = well radius, r = any radius

r_e = Drainage Radius

h = Net pay thickness (reservoir thickness)





where $C = 141.243 \dfrac{q_o B_o \mu_o}{Kh}$

eq. [9] represent an equation of straight line with slop = C & intercept = P_{wf} [$P \equiv y$, $\ln r/r_w \equiv x$]

eq. [9] Can be written in term of $\log (r/r_w)$

$$P = P_{wf} + 2.303\, C \log (r/r_w)$$

[Mathematically $\ln(a) = 2.303 \log(a)$]

or in more Compact form

$$P = P_{wf} + m \log (r/r_w) \quad \text{---} \quad [10]$$

where:

$$m = 325.3 \frac{q_o B_o \mu_o}{Kh} \quad \text{---} \quad [11]$$

" المعادلة [10] تمثل معادلة خط مستقيم

plot P vs $\log (r/r_w)$, Constract the best straigh line with slop = m & intercept = P_{wf}

التمثيل العلاقة باستخدام ورقة Semi-log ، حيث يكون المحور P على المحور العمودي وبمقياس خطي linear scale اما (r/r_w) فترسم على مقياس لوغاريتمي log. scale

العلاقة الخطية عندما تتوفر شروط تطبيق قانون دارسي "

Ex:- An oil well is producing 3000 STB/day; from a reservoir survey P_{wf} = 1890 psi , K = 78 md , h = 30 ft , μ = 0.9 cP B_o = 1.22 RB/STB r_w = 0.4 ft . Calculate the pressure at radii 4', 40', 400 & 4000' .

Solution: -

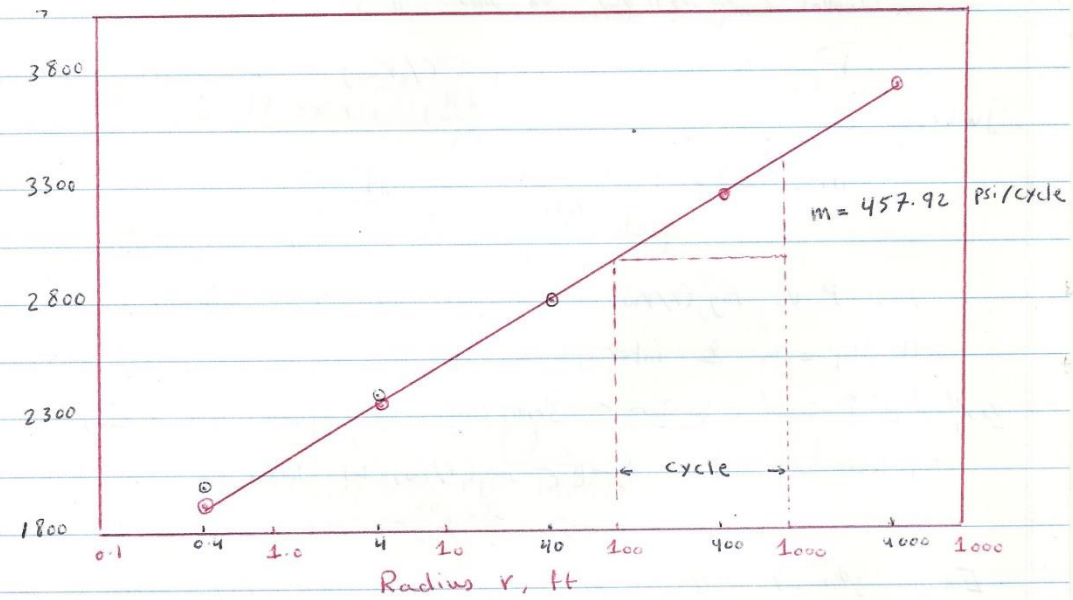
$$P = P_{wf} + m \log (r/r_w)$$

$$m = 325.3 \frac{(3000)(1.22)(0.9)}{(78)(30)} \Rightarrow m = 457.92$$

$$\therefore P = 1890 + 457.92 \log(2.5 r) \qquad [\ \tfrac{1}{0.4} =]$$

Radius r (ft)	Pressure P (Psi)
0.4	1890
4	2348
40	2806
400	3264
4000	3722

تناقص لنتائج
باستعاضه

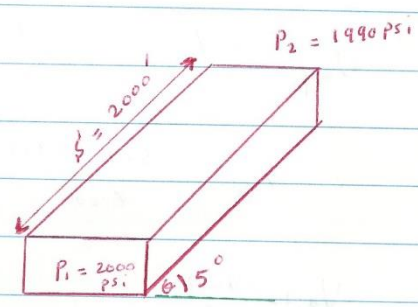


07-08

Ex: An Incompressible fluid flows in a tilted porous media with a dip angle $\theta = 5°$ as shown in figure below; the fluid has a density = 42 lb/ft³. Find the direction of flow, the flow rate, apparent & actual fluid velocity. if length of the bed = 2000 ft. h = 20 ft, width = 300 ft, K = 100 md $\phi = 0.15$, $\mu = 2$ cp, $P_1 = 2000$ psi, $P_2 = 1990$ psi

Solution:

$$\sin\theta = \frac{dz}{ds'} = \frac{z}{s'}$$



$$\therefore \quad Z = (2000)(\sin 5)$$
$$= 174.3 \text{ ft}$$

let the refrence level z_0 passes through Point [1]

Calculate the total potential @ Point ① & Point ②.

$$\Phi_{\eta} = \Phi_p + \Phi_g$$

$$(\Phi_{\eta})_1 = \frac{P_1 - P_{atm}}{\rho} + gZ$$

$$= \frac{(2000 - 14.7)\ \frac{lb}{in^2} \cdot \frac{ft^2}{144\ in^2}}{42} + (32.2)(0)$$

$$(\Phi_{\eta})_1 = 6807\ \frac{ft^2}{sec^2}$$

$$(\Phi_{\eta})_2 = \frac{(1990 - 14.7)(144)}{42} + (32.2)(174.3)$$
$$= 12385\ ft^2/sec \qquad [\Phi_2 > \Phi_1]$$

$\therefore$ The fluid flows down ward

So P_1 will be = 1990 & P_2 = 2000

السبت ٢٩/١٠/٢٠٠٧ مضى يومين على انتقال بنازير بوتو ،

$$\gamma_o = \frac{42}{62.4} = 0.673$$

$$Q_o = 1.127 \times 10^{-3} \frac{(20)(300)}{} \left(\frac{100}{2}\right) \left[- \frac{(2000-1990)}{2000} + (0.433)(0.673)(\sin 5^\circ) \right]$$

$$Q_o = 6.9 \text{ bbl/day}$$

$$V_{app.} = \frac{Q_o}{A} \quad ; \quad A = (h)(w) \Rightarrow A = (20)(300) = 6000 \text{ ft}^2$$

$$V_{app} = \left\{ \left(6.9 \text{ bbl} \times \frac{5.615 \text{ ft}^3}{\text{bbl}} \right) \Big/ \left(\text{day} \times \frac{86400 \text{ sec}}{\text{day}} \right) \right\} \Big/ 6000$$

$$\therefore V_{app.} \quad \frac{6.9 \times 5.615}{6000} = 0.00646 \text{ ft/day}$$

$$V_{act} = \frac{V_{app}}{\phi}$$

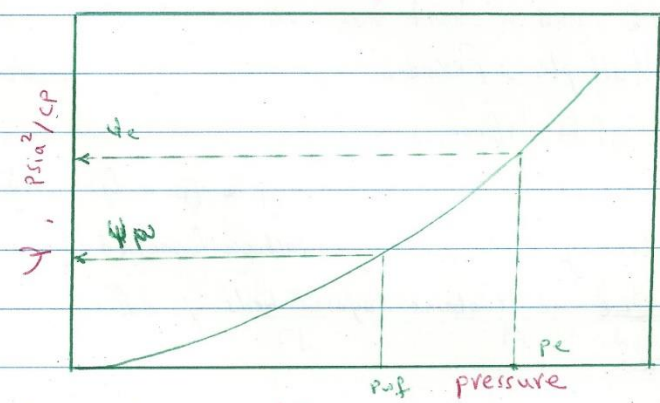
$$= \frac{0.00646}{0.15} = 0.043 \text{ ft/day}$$

Mathematically:

$$\int_{P_{wf}}^{P} \frac{2P}{\mu_g Z}\,dP = \int_{0}^{P} \frac{2P}{\mu_g Z}\,dP - \int_{0}^{P_{wf}} \frac{2P}{\mu_g Z}\,dP$$

The integral $\int_0^P \frac{2P}{\mu_g Z}\,dP$ is called the real gas potential or real gas pseudopressure & it is usually represented by $m(P)$ or Ψ thus

$$m(P) = \Psi = \int_0^P \left(\frac{2P}{\mu_g Z}\right) dP$$



In term of $m(P)$ or Ψ:-

$$Q_g = \frac{Kh\,(\Psi_e - \Psi_w)}{1422\,T_f \ln r_e/r_w}$$

where:-

Q_g = gas flow rate, Mscf/day

Ψ_e = real gas potential as evaluated form 0 to P_e ($Psia^2/cp$)

Ψ_w = " " " " " " 0 to P_{wf} (")

Ref.-

1- Tarek Ahmed " Reservoir Engineering Hand Book"
Copyright 2001 by GPP [Gulf Professional Publishing]
CH.4 Fundamentals of rock properties
CH.6 " " Reservoir Fluid Flow.
2- Djebbar Tiab and Erle C Donaldson " petrophysics theory and practice of measuring reservoir rock and fluid transport properties: Copyright 2004 by GPP . 2nd edition
CH.3: Porosity and permeability
CH.7: Application of Darcy's law

Multiple - Permeability Rocks

The foregoing fluid flow equations were developed on the assumption that the reservoir is homogeneous. In reality, homogeneous reservoirs ~~is~~ are seldom. Practically every producing clastic formation is stratified to some extent i.e it contains layered beds of differing petrophysical rock properties

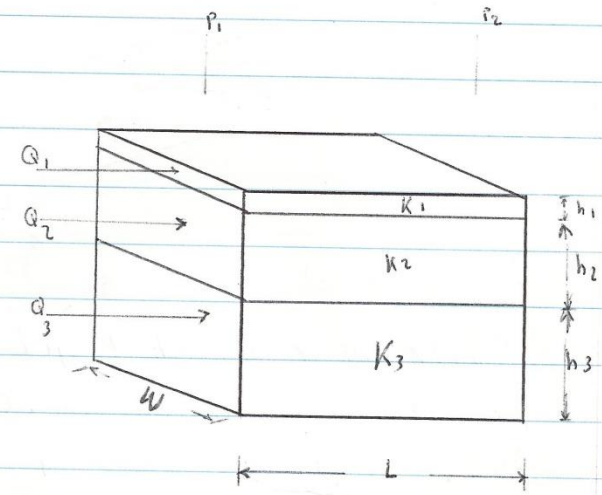
Layered Reservoirs with Crossflow

frequently, overlying reservoir beds, which have different thicknesses and petrophysical properties (such as K and Ø) are hydrodynamically communicating at the contact plane.

Layered Reservoirs without Crossflow

In many oil and gas pools, the reservoir rocks are interbedded with impermeable shales beds (الصجر) and silt (الزبل) laminations (صفائح), such that there is no crossflow between the oil- and gas saturated beds.

a- linear Flow in Parallel Beds



Consider two or more beds of equal length but unequal cross-section and Permeability flowing the Same fluid in linear flow under the Same pressure drop $(P_1 - P_2)$

The total flow rate is the sum of individual flows or:

$$q_t = Q_1 + Q_2 + Q_3$$

Apply Darcys law

$$\frac{K_{avg} A_t}{\mu L_{avg}} (P_1 - P_2) = \frac{K_1 A_1 (P_1 - P_2)}{\mu_1 L_1} + \frac{A_2 K_2}{\mu_2} \cdot \frac{P_1 - P_2}{L_2} + \frac{A_3 K_3}{\mu_3} \frac{P_1 - P_2}{L_3}$$

but ① $\mu_1 = \mu_2 = \mu_3 = \mu_{avg}$

② $L_{avg} = L_1 = L_2 = L_3$

$A_t = (h_t)(w)$ and $h_t = h_1 + h_2 + h_3$

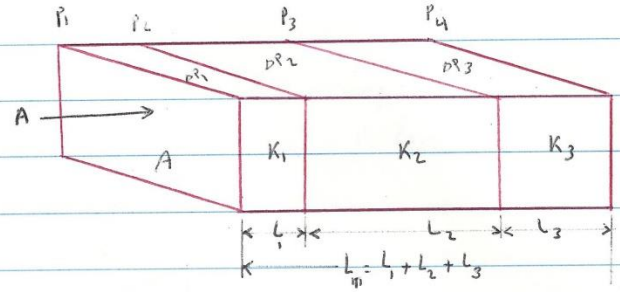
$A_1 = h_1 w \quad A_2 = h_2 w \quad A_3 = h_3 w$

Sub. in above equation

$$(K_{avg})(h_t) = K_1 h_1 + K_2 h_2 + K_3 h_3$$

or $$K_{avg} = \frac{\sum_{i=1}^{n} K_i h_i}{\sum_{i=1}^{n} h_i}$$

linear beds in Series :-



$$L_m = \sum_{i=1}^{n} L_i$$

Consider two or more beds of equal cross-section but unequal lengths & Permeabilities in which the same flow rate exist (i.e assuming incompressible fluid, obviously the pressure drop are additive and :-

$$P_1 - P_4 = (P_1 - P_2) + (P_2 - P_3) + (P_3 - P_4)$$

Apply Darcys law $Q = \frac{AK}{\mu}\frac{\Delta P}{L} \Rightarrow \Delta P = \frac{Q\mu L}{AK}$

$$\cancel{\frac{A K_{avg}}{\mu_{avg}}} \quad \therefore \quad \frac{Q_t \mu_{avg} L_t}{A_{avg} K_{avg}} = \frac{Q_1 \mu_1 L_1}{A_1 K_1} + \frac{Q_2 \mu_2 L_2}{A_2 K_2} + \frac{Q_3 \mu_3 L_3}{A_3 K_3}$$

but $Q_t = Q_1 = Q_2 = Q_3$

$\mu_{avg} = \mu_1 = \mu_2 = \mu_3$

$A_{avg} = A_1 = A_2 = A_3$ & $L_t = \sum_{i=1}^{n} L_i$ (here $L_t = L_1 + L_2 + L_3$)

$$\frac{L_t}{K_{avg}} = \frac{L_1}{K_1} + \frac{L_2}{K_2} + \frac{L_3}{K_3}$$

or $\sum_{i=1}^{n} L_i / K_{avg} = \sum_{i=1}^{n} \left(\frac{L_i}{K_i}\right) \Rightarrow K_{avg} = \frac{\sum_{i=1}^{n} L_i}{\sum_{i=1}^{n} \left(\frac{L_i}{K_i}\right)}$

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Ex:- Find the average permeability of a bed consists of three layer have the same cross section placed in series

L(ft)	K(m.D)	L/K
6	10	0.6
18	50	0.36
40	1000	0.04

$\Sigma L_i = 64$ ft $\qquad \Sigma \frac{L_i}{K_i} = 1$ ft/md

$$K_{avg} = \frac{\Sigma L_i}{\Sigma(\frac{L_i}{K_i})} = \frac{64}{1} \Rightarrow K_{avg} = 64 \text{ md}.$$

Ex: Find the equivalent linear permeability of four parallel beds having equal widths and lengths under the following conditions.

Bed	Pay thickness (ft)	K_h (m.D)	Kh (md.ft)
1	20	100	2000
2	15	200	3000
3	10	300	3000
4	5	400	2000
	$\Sigma h_i = 50$ ft		$\Sigma h_i K_i = 10.000$

$$\therefore K_{avg} = \frac{\Sigma h_i K_i}{\Sigma h_i} = \frac{10\,000}{50} = 200 \text{ md}.$$

Permeability Variation in Radial flow

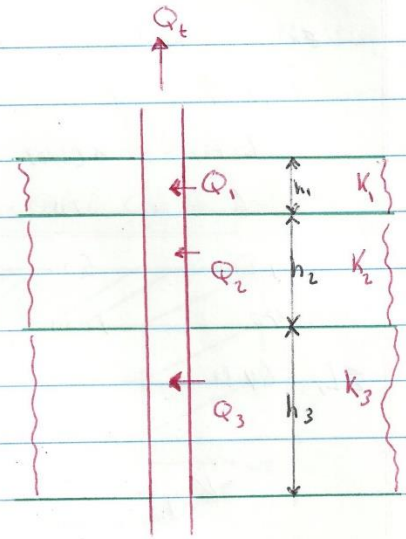
Many producing ~~well~~ formations are composed of strata or stringers which may vary widely in permeability and thickness, if these strata producing fluid to a common wellbore under the same drawdown and from the same drainage radius (r_e)

انا تكامل بقرين احدها سهله والاخر صعبه (توضيح عبد الله)

Here $Q_t = Q_1 + Q_2 + Q_3 + \cdots$

According to Darcy's law

$$Q = \frac{2\pi K h}{\mu} \frac{P_e - P_{wf}}{\ln r_e/r_w}$$



$\therefore$

$$2\pi \frac{K_{avg}}{\mu} h_t \frac{P_e - P_{wf}}{\ln r_e/r_w} = 2\pi \frac{K_1}{\mu} h_1 \frac{P_e - P_{wf}}{\ln r_e/r_w}$$

$$+ 2\pi \frac{K_2}{\mu} h_2 \frac{P_e - P_{wf}}{\ln r_e/r_w} + 2\pi \frac{K_3}{\mu} h_3 \frac{P_e - P_{wf}}{\ln r_e/r_w}$$

or

$$K_{avg} h_t = K_1 h_1 + K_2 h_2 + K_3 h_3$$

$$\therefore K_{avg} = \frac{\sum K_i h_i}{\sum h_i} = \frac{\sum K_i h_i}{h_t}$$

which is same as for // flow in linear beds with the same bed width.

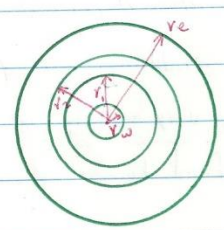
Note: The product Kh (md-ft) is called the capacity of the bed.

Radial Flow; Bed in series {Same thickness}

here

$$P_e - P_{wf} = (P_1 - P_{wf}) + (P_2 - P_1) + (P_e - P_2)$$

and $Q_t = Q_1 = Q_2 = Q_3$



Apply Darcy's law

$$\frac{\ln(r_e/r_w)}{K_{avg}} = \frac{\ln(r_1/r_w)}{K_1} + \frac{\ln(r_2/r_1)}{K_2} + \frac{\ln(r_e/r_2)}{K_3}$$

$$\frac{\ln(r_e/r_w)}{K_{avg.}} = \frac{\ln(r_1/r_w)}{K_1} + \frac{\ln(r_2/r_1)}{K_2} + \frac{\ln(r_e/r_2)}{K_3}$$

or

$$\frac{\ln(r_e/r_w)}{K_{avg}} = \sum_{i=1}^{n} \frac{\ln r_i/r_{i-1}}{K_i}$$

$$K_{avg} = \frac{\ln r_e/r_w}{\sum_{i=1}^{n} \frac{\ln(r_i/r_{i-1})}{K_i}}$$

with

$$\boxed{r_0 \equiv r_w} \qquad \boxed{r_n = r_e}$$

Ex:- Calculate the average permeability of a bed connected in series if the flow system is radial; all beds have the same thickness well bore radius $r_w = 6$ in.

Bed	Radius of the bed (ft)	Permeability (K_h); m.D
1	250	25
2	500	50
3	1000	100
4	2000 (r_e)	200

Solution:-

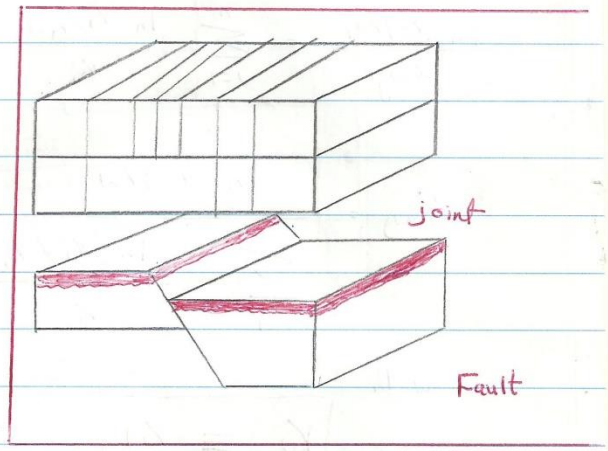
$$K_{avg} = \frac{\ln(2000/0.5)}{\frac{\ln(250/0.5)}{25} + \frac{\ln(500/250)}{50} + \frac{\ln(1000/500)}{100} + \frac{\ln(2000/1000)}{200}}$$

$\therefore \bar{K} = 30.4$ md.

40-A

Naturally Fractured Reservoir

* From a strictly geo-mechanical point of view a fracture is the surface in which a loss of Cohesion has taken place, a fracture in which relative displacement has occured can be defined as a fault, while a fracture in which no noticeable displacement has occurred can be defined as a joint, a fracture can also be defined, in a more general way, as the discontinuity which breaks the rock beds into blocks along cracks fissures, joints and along which there is no displacement parallel with the planes of discontinuity.



The fractures represent mechanical failure of the rock strength to natural geological stresses such as a tectonic movement, lithostatic pressure change, thermal stresses, high fluid pressure. Fracture may apear as a microfissures with an extension of only several micrometers or as Contenental fractures with an extension of several thousand kilometers.

Naturally fractured rocks can be ~~genera~~ geologically Categorized into three main types, based on their porosity systems:

1. Intercrystalline - intergranular [Umm Farud in libya]
2. Fracture - Matrix [Kirkuk field, Ain Zala in Iraq, Dukhan field in Qatar & Masjidi - Sulaiman and Haft - Gol field in Iran]
3. vagular - solution

Aguilera, Saidi & Nelson reviewed many of the approaches

Aguilera: Natural Fracture Reservoir, Saidi: Reservoir Engineering of fractured Reservoir, Nelson: SPET April _ pp. 407 - 414

used to detecte and analyze naturally fractured reservoir. Some of these methods are as follows:-

1- Loss of circulating fluids and an increase in penetration rate during drilling.

2. Fractures and Solution channels in Cores.

3- Down hole direct and indirect viewing System.

4- Very high Productivity index [500 STB/D/PSi or higher] Some wells were reported a P.I of 10 000 STB/D/PSi, in these well ~~Som~~ flow is through fractures.

5- Because of the high permeability of the fractures, the horizontal pressure gradient is typically small near the wellbore as well as through the reservoir.

Porosity and Permeability of fracture-system

Fracture porosity

In a fractured reservoir the total porosity ϕ_t is the result of the simple addition of the primary and secondary porosity

$$\phi_t = \phi_1 + \phi_2$$

where:-

ϕ_1 = Matrix pore volume / total bulk volume

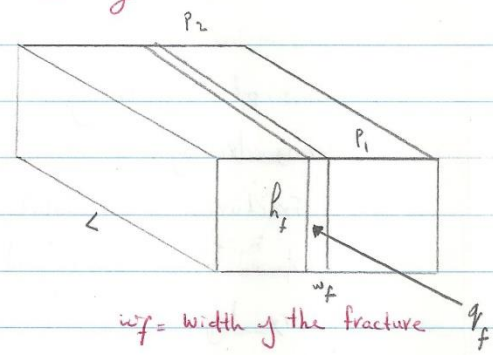
ϕ_2 = fracture " " / " " "

* The fracture porosity is Considerably less than the matrix porosity [The range of fracture porosity is 0.1 to 5%, depending on the degree of solution Channeling & on the fracture width and spacing.

Mathematically:-

$$\phi_t = \phi_f + \phi_m = \frac{V_{pf}}{V_{bt}} + \frac{V_{pm}}{V_{bt}}$$ [for discussion only]

Fracture Permeability



w_f = Width of the fracture

L = Fracture length

h = Lateral extent of the fracture

$A = w_f h_f$

An equation for the viscous flow of wetting fluids through smoth fracture of constant width is given by:

$$Q_f = \frac{W_f^2 A (P_1 - P_2)}{12 \mu L} \quad \text{---} \quad \boxed{1} \quad \left[\text{Hagen- Poiseiulle equation}\right]$$

Q_f = cc/sec, $A = w_f h_f$ (cm^2), ΔP: dynes/cm^2, L: cm, μ = Poise

Darcy's law for linear flow:

$$Q = A \frac{K}{\mu} \frac{\Delta P}{L} \quad \text{---} \quad \boxed{2}$$

Eq. (1) may be Combined with Darcy's law & solving for Permeability

$$K_f = 84.4 \times 10^5 \; w_f^2 \quad [K: \text{Darcy} ; \; w_f = \text{cm}]$$

Ex:- Consider a cube of reservoir rock 1 ft on side and having a matrix permeability of 10 md. if a liquid of 1 cp viscosity flow linearly through the rock under pressure gradient of 1 psi/ft. Find the total flow rate also the contribution of the fracture as a % of the total flow rate if the fracture width = 0.01 in., fracture lateral ext. = 1 ft, L = 1 ft

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Solution:-

1- Flow through matrix

$A = 1 \times 1 \times (12 \times 2.54)^2 = 929 \text{ cm}^2$

$K = \frac{10}{1000} = 0.01 \text{ Darcy}$; $\frac{\Delta P}{\Delta L} = \frac{1 \text{ psi} \cdot \frac{1 \text{ atm}}{14.7}}{1 \text{ ft} \times \frac{30.48}{\text{ft}}} = 2.23 \times 10^{-3} \frac{\text{atm}}{\text{cm}}$

$\therefore Q_m = (929) \frac{(0.01)}{1} (2.23 \times 10^{-3}) \Rightarrow Q_m = 0.0207 \text{ cc/sec.}$

2- Flow through Fracture

$w_f = 0.01 \times 2.54 = 0.0254 \text{ cm}$

$A = (0.0254)(30.48) = 0.774 \text{ cm}^2$

$K_f = (84.4)(10^5)(0.0254)^2 \Rightarrow K_f = 5445.2 \text{ Darcy}$

$Q_f = (0.774) \frac{(5445.2)}{1} (2.23 \times 10^{-3}) \Rightarrow Q_f = 9.4 \text{ cc/sec.}$

$Q_t = Q_m + Q_f \Rightarrow Q_t = 0.0207 + 9.4 = 9.4207 \text{ cc/sec}$

% of frac. Contr. $= \frac{9.4}{9.4207} (100) = 99.8\%$

Rock Compressibility

The pressure difference between overburden pressure & internal pore pressure is referred to as the effective overburden pressure. During pressure depletion operations, the internal pore pressure decreases and, therefore the effective overburden pressure increases. This increase cause the following effect:

- The bulk volume of the reservoir rock is reduced.
- Sand grains within the pore spaces expand. These two volume changes tend to reduce the pore space and, therefor, the porosity of the rock. Geertsma (1957) points out that there are three different types of compressibility that must be distinguished in rocks:

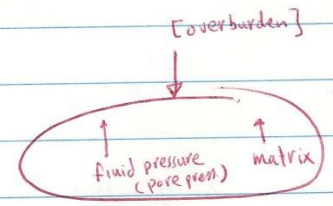
1. Rock-Matrix Compressibility, C_r

$$C_r = -\frac{1}{V_r}\left(\frac{dV_r}{dP}\right)_T \quad \text{--- } \boxed{1} \quad [\text{isothermal Condition}]$$

C_r: rock-Matrix Compressibility, psi^{-1}

V_r = volume of solid

$C_r = 0$ [ignored]



2. Rock-bulk Compressibility, C_B

$$C_B = -\frac{1}{V_B}\left(\frac{\partial V_B}{\partial P}\right)_T \quad \text{--- } \boxed{2}$$

$P_{ov.b.} = 1$ psi/ft

$P_f = 0.465$ psi/ft

where:

C_B = rock-bulk compressibility coefficient, psi^{-1}

V_B = Bulk volume

3. Pore Compressibility, C_p

$$C_p = -\frac{1}{V_p}\left(\frac{\partial V_p}{\partial P}\right)_T \quad \text{--- } \boxed{3}$$

* Trans. AIME 1957 pp: 331-340

Equation [3] can be expressed in terms of porosity ϕ by noting that ϕ increases with the increased in the pore pressure; or

$$C_p = \frac{1}{\phi} \cdot \frac{\partial \phi}{\partial P} \quad \text{--- [4]}$$

For more petroleum reservoir, the rock and bulk compressibility are considered small in comparison with the pore compressibility C_p. The formation compressibility C_f is the term commonly used to describe the total compressibility of the formation & is set equal to C_p i.e:

$$C_f = C_p = \frac{1}{\phi} \cdot \frac{\partial \phi}{\partial P} \quad \text{--- [5]}$$

Typical values for the formation compressibility range from 3×10^{-6} to 25×10^{-6} psi^{-1} ; in term of volume chang

$$C_f = \frac{1}{V_p} \cdot \frac{\Delta V_p}{\Delta P} \quad \text{or} \quad \Delta V_p = C_f \, V_p \, \Delta P$$

where:

ΔV_p and ΔP are the change in the pore volume and pore pressure respectively.

Geertsma suggested that the bulk compressibility C_B is related to the pore compressibility C_p by the following expression

$$C_B \cong C_p \, \phi$$

Ex:- Calculate the reduction in the pore volume of a reservoir due to a pressure drop of 10 psi. The reservoir original pore volume is one million barrels with an estimated formation compressibility of 10×10^{-6} psi^{-1}

Solution:-

$$\Delta V_p = (10 \times 10^{-6})(1 \times 10^{6})(10) = 100 \text{ bbl}$$

The reduction in the pore volume due to pressure decline can also be expressed in term of the changes in the reservoir porosity. since

$$C_f = \frac{1}{\phi} \cdot \frac{\partial \phi}{\partial P}$$

Separate the variable & integrate

$$C_f \int_{P_o}^{P} \partial P = \int_{\phi_o}^{\phi} \frac{d\phi}{\phi}$$

$$C_f (P - P_o) = \ln \left(\frac{\phi}{\phi_o} \right)$$

or

$$\phi = \phi_o e^{C_f (P - P_o)}$$

where:

P_o = Original pressure, psi

ϕ_o = " porosity

P = Current pressure

ϕ = porosity @ pressure P

Apply the expansion series.-

$$\phi = \phi_o \left[1 + C_f (P - P_o) \right]$$

Ex: Given the following data

$C_f = 10 \times 10^{-6}$ psi^{-1}, original pressure = 5000 psi, original porosity = 0.18 current pressure = 4500 psi

Solution:-

$$\phi = 0.18 \left[1 + 10 \times 10^{-6} (4500 - 5000) \right] = 0.179$$

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Multiphase flow through Porous Media

In our consideration of the initial saturation distribution we may assume that the initial saturations are uniform through out the water zone, oil zone or gas cap of a reservoir, also we may assume that as production take place in a reservoir, the gas cap expands or the water encroaches & the saturation in the invaded portions of the reservoir becomes uniform. Such an assumption is often referred to as piston like displacement

عندما تحل ازاحة النفط بالماء فإن الماء والنفط هما سائلين غير ممتزجين في هذه الحالة تتكون جبهة Front فاصلة بين المنطقة التي يجري فيها الماء والمنطقة التي يجري فيها النفط هذا النوع من الازاحة يسمى بالازاحة الغير ممتزجة immiscible disp هناك نظريتان لتمثيل الازاحة الغير ممتزجة هما

- الازاحة المكبسية piston like drive
- ازاحة المكبس الناضح leaky piston drive

الازاحة المكبسية Piston-like drive

تفترض هذه النظرية ما يلي: عند ازاحة النفط بالماء تتكون جبهة فاصلة أمام الجبهة تبلغ درجة تشبع النفط 100% أي أن النفط فقط هو الذي يجري أما خلف الجبهة فإن تشبع النفط $S_o = 0$ وأما تشبع الماء فهو 100% أي أن الماء هو الطور الوحيد الذي يجري خلف الجبهة

$Q_w \rightarrow$	$S_w = 100\%$, $S_o = 0$	$S_o = 100\%$, $S_w = 0$	$\rightarrow Q_o$

عند حقن حجم من الماء مساوي لحجم المسامية (1 PV)

تعمل الجبهة المكبسية النموذج وتنتهي هذه الحالة بالاختراقات Break through حيث بعدها يكون إنتاج الماء فقط من الضروري التنويه بأن هذه النظرية تنطبق على نظام الازاحة الخطية linear

اشتقاق المعادلة: لنفترض لدينا نموذج صخري طوله L ومساحة المقطع العرضي للنموذج A مسامية النموذج متجانسة أي أن قيمة النفاذية ثابتة

ابتداءً يكون النموذج مشبعاً كلياً بالنفط $S_o = 100\%$ أي أن التشبع المائي $S_w = 0\%$

عندما تبدأ عملية الازاحة نحقن كمية من الماء مقدارها Q_w يخلق ضغط كلي مقداره ΔP_T

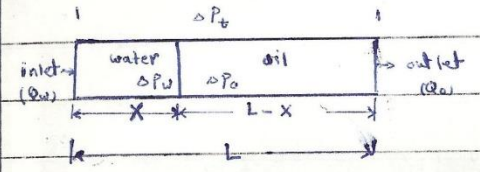
وبما أننا نفترض أن الازاحة مكبسية فتوقع مقدار ما ينتج من النفط مساوي لمقدار الماء المحقون

$$Q_o = Q_w$$

بعد مرور فترة زمنية مقدارها (t) نلاحظ تقدم Front بمسافة فعلية مقدارها (X)

مقاسة من نقطة دخول الماء (inlet)

وفي هذه الحالة يكون طول المنطقة النفطية (L-x)



يمكن حساب المسافة التي تقطعها الجبهة بالمعادلة التالية

$$X = \frac{Q \cdot t}{A \cdot \phi}$$

حيث Q هو معدل الحقن ، t الزمن ، A : مساحة المقطع العرضي ، ϕ النفاذية

[ان البسط $A \times \phi$ هي في الحقيقة الحجم المحقون بعد مضي فترة (t) من بدء الحقن]

بما ان لزوجة النفط لا تساوي لزوجة الماء كذلك هنالك تغير في طول المنطقة النفطية اذن توجد تدرجات ضغط مختلفة . يمكن استخدام قانون دارسي للاتجاه الخطي . في حساب معدل جريان الماء و النفط

1- water flow rate

$$Q_w = \frac{1.127 \times 10^{-3} A K_w}{\mu_w} \left(\frac{\Delta P_w}{x}\right) \quad \text{--- [1]} \quad \text{oil field units}$$

$$\therefore \Delta P_w = \frac{887.3\, Q_w \mu_w x}{A K_w}$$

2- oil flow rate

$$Q_o = \frac{1.127 \times 10^{-3} A K_o}{\mu_o} \left(\frac{\Delta P_o}{L-x}\right)$$

or

$$\Delta P_o = \frac{887.3\, Q_o \mu_o (L-x)}{A K_o}$$

Total pressure drop $\Delta P_T = P_{inlet} - P_{outlet}$

$$\Delta P_T = \Delta P_w + \Delta P_o$$

$$\Delta P_t = \frac{887.3\, Q_w \mu_w x}{A K_w} + \frac{887.3\, Q_o \mu_o (L-x)}{A K_o}$$

Since the flow is ~~steady state~~ assumed to be piston like drive $\therefore Q_w = Q_o =$ injection rate Q , also since the medium is homogen and the oil flowing only ahead the front and water flow only behind the front $\therefore K_w = K_o = K$

$$\Delta P_T = \frac{887.3 Q}{AK}\left[\mu_w x + \mu_o (L-x)\right] = \frac{887.3 Q}{AK}\left[\mu_o L + x(\mu_w - \mu_o)\right] \quad \text{--- [2]}$$

oil field unit

Eq 2 indicates that the total pressure drop ΔP_T is function of 1- Position of the front (X) and 2- The viscosity of water and oil.

∴ we may have three cases

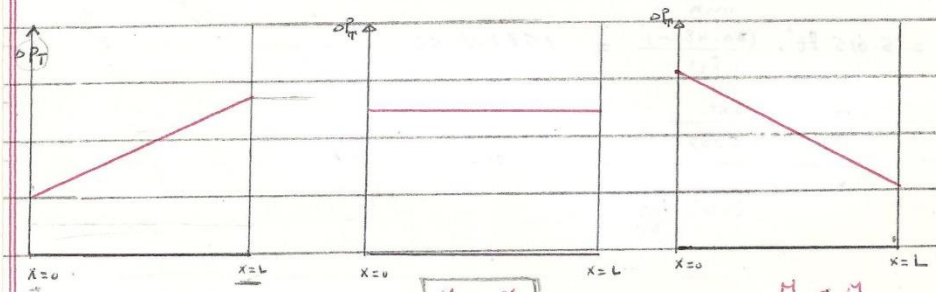
1- water viscosity is greater than oil visci.

in the case $\mu_w - \mu_o = +ve$ ∴ ΔP_T increases as the front advance i.e ΔP_T is maximum at Break through

2- water viscosity = oil viscosity, here ΔP_T remain constant i.e ΔP_T is not a function of the position of interface

3- water viscosity is less than that of oil i.e $\mu_w - \mu_o = -ve$

∴ ΔP_T decreases continously and has a minimum value at Break through



Case 1: $\mu_w > \mu_o$ | $\boxed{\mu_w = \mu_o}$ | $\mu_w < \mu_o$

لزاج سوائل خفيف | لزاج سوائل ثقيل

For all Cases

$$(\Delta P_T)_{initial} = 887.3 \frac{Q_o \mu_o L}{A K}$$

pressure drop at the start of injection [Flooding]

Oil Field unite

$$(\Delta P_T)_{B.T} = 887.3 \frac{Q_w \mu_w L}{A K}$$

pressure drop at break through

[26 - A]

EX:- A core sample is saturated with oil. The oil is being displaced by water at a rate of 100 cc/min. other data are $\phi = 0.2$, $\mu_o = 2.5$ cp, $\mu_w = 1.2$ cp, $K = 150$ md, length of the core = 2 ft, cross-sectional area = 1.5 ft^2, calculate the inlet pressure at start of injection, after, 0.5, 2, and 3 hrs. Also calculate the time of break through. Assume piston like displacement in all calculation also assume that the out let pressure is atmospheric pressure

solution:

$$(\Delta P_T)_{initial} = 887.3 \frac{Q \mu_o L}{A K}$$ oil field unite

show that 1 bbl/day = 1 cc/sec

given

$$\text{injection rate} = \frac{100\ cc}{min}$$

$$1\ bbl = 5.615\ ft^3 \cdot \frac{(30.48\ cm)^3}{ft^3} = 158999\ cc$$

$$Q = \frac{100\ cc \times \frac{1\ bbl}{158999\ cc}}{min \times \frac{day}{24\ hr \times \frac{60\ min}{hr}}} = 0.906\ bbl/day$$

ΔP_F

1- $$\Delta P_{initial} = 887.3 \frac{(0.906)(2.5)(2)}{(1.5)(150)} = 17.86\ psi$$

$\Delta P_T = P_{inlet} - P_{outlet}$; since $P_{outlet} = 14.7$ psi

$\therefore P_{inlet} = 17.86 + 14.7 = 32.6$ psi

2- $$\Delta P_{II} = \frac{887.3\, Q}{AK} \left[\mu_w x + \mu_o (L - x) \right]$$

$$\Delta P_{II} = \frac{(887.3)(0.906)}{(1.5)(150)} \left[1.2x + 2.5(2 - x) \right]$$

$$\Delta P_{II} = 3.573 \left[1.2x + 2.5(2 - x) \right]$$

44-c

07-08

27!

C

50

Let us determine the position of front after 0.5, 2, 3 hrs

since $X = \frac{Q \cdot t}{A \phi}$

$$Q = 0.906 \frac{bbl}{day} = \frac{0.906 \ bbl \times \frac{5.615 \ ft^3}{bbl}}{day \times \frac{24 \ hr}{day}} =$$

$$Q = 0.212 \frac{ft^3}{hr}$$

$$\therefore X = \frac{0.212}{(1.5)(0.2)} t = 0.707 t \quad (ft)$$

$$X|_{0.5hr} = 0.353 \ ft$$

$$X|_{1hr} = 0.707 \ ft$$

$$\boxed{X|_{3hr} = 2.12 \ ft}$$

1. $\therefore \Delta P_T|_{0.5hr} = 3.573 \left[(1.2)(0.353) + 2.5(2 - 0.353) \right] = 16.23 \ psi$

$\therefore P_{inlet}|_{0.5hr} = 16.23 + 14.7 = 30.93$

2. $\Delta P_T|_{1hr} = 3.573 \left[(1.2)(0.707) + 2.5(2 - 0.707) \right] = 14.6 \ psi$

$P_{inlet}|_{1hr} = 29.3 \ psi$

3. After 3hr $X > L$ i.e the breakthrough is took place

$\Delta P_T|_{3hr} = (3.573)(1.2)(2) = 8.6 \ psi$

$P_{inlet}|_{3hr} = 8.6 + 14.7 = 23.3 \ psi$

Since the less viscous fluid displace the more viscous fluid Total pressure drop decrease as the injection continue!

27 - A1

Time to BT = $\frac{\text{the Core Pore volume}}{\text{injection rate}}$

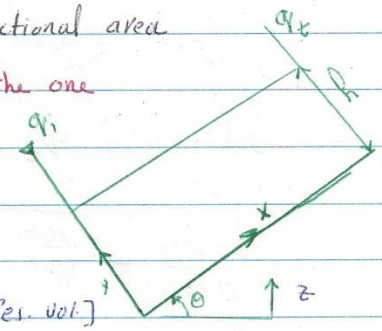
$$P.V = AL\phi = (1.5)(2)(0.2) = 0.6 \ ft^3$$

injection rate $Q = 0.212 \ ft^3/hr$

$$\therefore t_{BT} = \frac{0.6}{0.212} = 2.83 \ hr$$

The Fractional Flow Equation

Consider the oil being displaced by water, in a tilted reservoir block has a uniform cross sectional area



Applying Darcy's law, for linear flow, the one dimensional equation for the simultaneous flow of oil and water :-

$$q_o = -A \frac{K K_{ro}}{\mu_o} \rho_o \frac{\partial \Phi_o}{\partial x} \quad [\text{Res. vol.}]$$

$$q_o = -A \frac{K K_{ro}}{\mu_o} \left[\frac{\partial P_o}{\partial x} + \rho_o g \sin\theta \right] \quad \boxed{1} ; \qquad \frac{\partial z}{\partial x} = \sin\theta$$

also

$$q_w = -A \frac{K K_{rw}}{\mu_w} \left[\frac{\partial P_w}{\partial x} + \rho_w g \sin\theta \right] \quad \boxed{2}$$

** The total Production rate $q_t = q_o + q_w$

or

$$q_o = q_t - q_w$$

$$-A \frac{K K_{ro}}{\mu_o B_o} \left[\frac{\partial P_o}{\partial x} + \rho_o g \sin\theta \right] = q_t + \frac{A K K_{rw}}{B_w \mu_w} \left[\frac{\partial P_w}{\partial x} + \rho_w g \sin\theta \right]$$

$$\frac{q_o \mu_o}{K K_{ro}} = -A \left[\frac{\partial P_o}{\partial x} + \rho_o g \sin\theta \right] \quad \text{--- } 1\text{-A}$$

$$\frac{q_w \mu_w}{K K_{rw}} = -A \left[\frac{\partial P_w}{\partial x} + \rho_w g \sin\theta \right] \quad \text{--- } 2\text{-A}$$

Sub 1-A from 2-A

$$\frac{q_w \mu_w}{K K_{rw}} - \frac{q_o \mu_o}{K K_{ro}} = -A \left[\frac{\partial P_w}{\partial x} + \rho_w g \sin\theta \right] + A \left[\frac{\partial P_o}{\partial x} + \rho_o g \sin\theta \right]$$

$$= +A \left[\frac{\partial P_o}{\partial x} - \frac{\partial P_w}{\partial x} + \rho_o g \sin\theta - \rho_w g \sin\theta \right]$$

$$\frac{q_w \mu_w}{K K_{rw}} - \frac{q_o \mu_o}{K K_{ro}} = A\left[\frac{\partial P_c}{\partial x} - \Delta\rho g \sin\theta\right]$$

where:

$$\frac{\partial P_c}{\partial x} = \frac{\partial P_o}{\partial x} - \frac{\partial P_w}{\partial x}$$

but $q_o = q_t - q_w$

$\therefore$

$$\frac{q_w \mu_w}{K K_{rw}} - \frac{\mu_o}{K K_{ro}}\left[q_t - q_w\right] = A\left[\frac{\partial P_c}{\partial x} - \Delta\rho g \sin\theta\right]$$

$$q_w\left[\frac{\mu_w}{K K_{rw}} + \frac{\mu_o}{K K_{ro}}\right] = \frac{q_t \mu_o}{K K_{ro}} + A\left[\frac{\partial P_c}{\partial x} - \Delta\rho g \sin\theta\right]$$

$$\frac{q_w}{q_t} = \frac{\frac{\mu_o}{K K_{ro}} + \frac{A}{q_t}\left[\frac{\partial P_c}{\partial x} - \Delta\rho g \sin\theta\right]}{\frac{\mu_w}{K K_{rw}} + \frac{\mu_o}{K K_{ro}}}$$

But, The fractional flow f_w of water @ any point in the reservoir is defined as.

$$f_w = \frac{q_w}{q_o + q_w} \quad \text{or} \quad f_w = \frac{q_w}{q_t}$$

$$\therefore f_w = \frac{1 + \frac{K K_{ro}}{q_t \mu_o} A\left[\frac{\partial P_c}{\partial x} - \Delta\rho g \sin\theta\right]}{1 + \frac{\mu_w}{\mu_o} \cdot \frac{K_{ro}}{K_{rw}}}$$

In field unit:

$$f_w = \frac{1 + 1.127\times10^{-3} \frac{A K K_{ro}}{q_t \mu_o}\left[\frac{\partial P_c}{\partial x} - \cancel{\Delta\rho g \sin\theta}\ 0.433\,\Delta\gamma \sin\theta\right]}{1 + \frac{\mu_w}{\mu_o} \cdot \frac{K_{ro}}{K_{rw}}}$$

[q_t bbl/day , $K K_{r_o}$ = md

μ_o : cp. $\quad \frac{\partial P_c}{\partial x}$: Capillary pressure gradient psi/ft

$\Delta\gamma = \gamma_w - \gamma_o$

if the value of $\frac{\partial P_c}{\partial x}$ considered very small

Then $$f_w = \frac{1 - 1.127\times10^{-3}\,\frac{A K K_{ro}}{q_t \mu_o}\left(-0.4335\,\Delta\gamma \sin\theta\right)}{1 + \frac{\mu_w}{\mu_o}\cdot\frac{K_{ro}}{K_{rw}}}$$ *

special case $\theta = 0$

$$f_w = \frac{1}{1 + \frac{\mu_w}{\mu_o}\cdot\frac{K_{ro}}{K_{rw}}}$$

* نقاط للاختبار

1- $\Delta\gamma$ تأثير

2- dip angle

3- viscosity , is $f_w = f(S_w)$!

plot of f_w vs. S_w