

Use of Pressure Derivative in Well Test Analysis

Modern Analysis has been greatly enhanced by the use of derivative plot introduced by Bourdet et al. [Bourdet et al. "A New set of type curves simplifies well test analysis". World oil May 1983 pp. 95-106]. also discussed by Bourdet et al. [Bourdet et al. "Use of pressure derivative in well test interpretation" SPE Formation Evaluation June 1989 P.p. 293-302].

The derivative plot provides a simultaneous presentation of $\log \Delta P$ vs $\log t$ and $\log t \cdot \frac{dP}{dt}$. Vs. st, the advantage of the derivative plot is that it is able to display in a single graph many separate characteristics that would otherwise require different plots (as will be shown latter).

Figure 6.2 is the composite Bourdet - Gringarten type Curve (p. 5-A)

Properties of The Derivative

[a.] When the well bore storage effect prevails, dimensionless pressure is expressed by

$$P_D = \frac{t_0}{c_D} \quad \text{--- [1]}$$

differential both sides of eq. [1] with respect to $(\frac{t_0}{c_D})$

$$\frac{d}{d(\frac{t_0}{c_D})} (P_D) = \frac{d}{d(\frac{t_0}{c_D})} \left(\frac{t_0}{c_D} \right) \quad \text{--- [2]}$$

- * Agarwal et al. "An investigation of wellbore storage and skin effect in unsteady liquid flow. Analytical treatment" SPEJ, Sep. 1970 P.p. 279-290, Trans. AIME 249

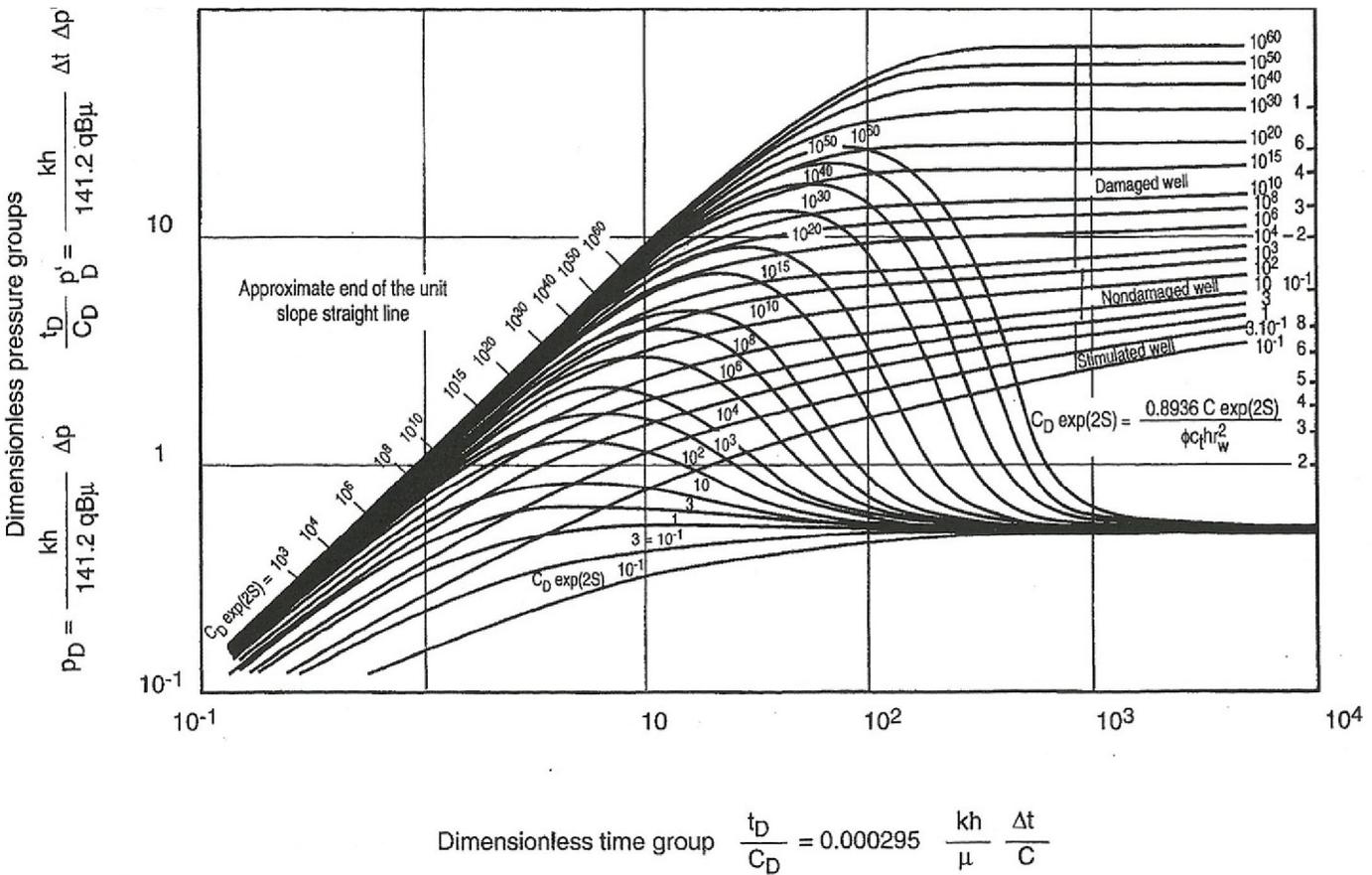


Fig. 6.2 Type curves for a well with wellbore storage and skin (infinite acting homogeneous reservoir)

or

$$\frac{d}{d\left(\frac{t_D}{c_D}\right)} (P_D) = 1 \quad \text{--- [3]}$$

Let $P_D' = \frac{d}{d\left(\frac{t_D}{c_D}\right)} (P_D)$, then

$$P_D' = 1 \quad \text{--- [4]}$$

also differentiate eq. [1] with respect to logarithm of dimensionless time function $\left(\frac{t_D}{c_D}\right)$

$$\frac{d}{d\left(\ln \frac{t_D}{c_D}\right)} (P_D) = \frac{d}{d\left(\ln \frac{t_D}{c_D}\right)} \cdot \left(\frac{t_D}{c_D}\right)$$

$$\frac{t_D}{c_D} \frac{dP_D}{d\left(\frac{t_D}{c_D}\right)} = \frac{t_D}{c_D} \cdot \frac{d}{d\left(\frac{t_D}{c_D}\right)} \cdot \left(\frac{t_D}{c_D}\right)$$

$$\therefore \left(\frac{t_D}{c_D} P_D'\right) = \frac{t_D}{c_D} \quad \text{--- [5]}$$

\therefore Plotting $\left(\frac{t_D}{c_D} P_D'\right)$ vs. $\left(\frac{t_D}{c_D}\right)$, on log-log scale will result in a straight line with slope = 1 passes through the origin of the coordinates on a log-log graph \ll USL \gg

[b] - During radial flow the dimensionless pressure is expressed by

$$P_D = \frac{1}{2} \left[\ln t_D + 0.80907 + 2s' \right] \quad \text{--- [6]}$$

add and subtract $\ln c_D$

$$P_D = \frac{1}{2} \left[\ln t_D + \ln c_D e^{2s'} + 0.80907 \right] \quad \text{--- [7]}$$

differentiate eq. 7 [wrt] the radial flow function $\left(\ln \frac{t_D}{c_D}\right)$

$$P_D = \frac{d}{d(\ln \frac{t_D}{c_D})} \left[\frac{1}{2} \left(\ln \frac{t_D}{c_D} + \ln c_D e^{2s} + 0.80907 \right) \right]$$

$$\lim_{t_D \rightarrow \infty} \frac{t_D}{c_D} P'_D = \frac{1}{2} \quad \boxed{8}$$

Therefore, All type curves has 0.5 ordinate horizontal line as an asymptote during radial flow \ll Re-examine Composite type-Curve \gg

\therefore Bourdet's Type Curves have the following Properties

1. They have the «USL» passing through the origin of the coordinate as an asymptote as long as the wellbore storage effect dominates.
2. They have the « $\frac{1}{2}$ » ordinate horizontal line as an asymptote when the wellbore storage effect is over.
3. The curves corresponding to value of $c_D e^{2s} > 1.0$, exhibit a maximum. The ones corresponding to value ≤ 1.0 or equal to one increases continually.

4. Between the maximum of the curve and the beginning of horizontal straight line \approx $1\frac{1}{2}$ cycle \ll $1\frac{1}{2}$ Cycle Rule \gg

All above notes are shown in figure 6.1 (P.7-A)

(also see fig. 6.2 p.

Chapter 6 • THE DERIVATIVE

The pressure derivative is expressed by:

$$p_D' = \frac{dp_D}{d \ln \frac{t_D}{C_D}} = \frac{t_D}{C_D} \frac{dp_D}{d \frac{t_D}{C_D}} \quad (6.8)$$

$$p_D' = \frac{t_D}{C_D} \quad (6.9)$$

Therefore, the derivative like the pressure has as an asymptote the unit slope straight line that passes through the origin of the coordinates on a log-log graph (Fig. 6.1)

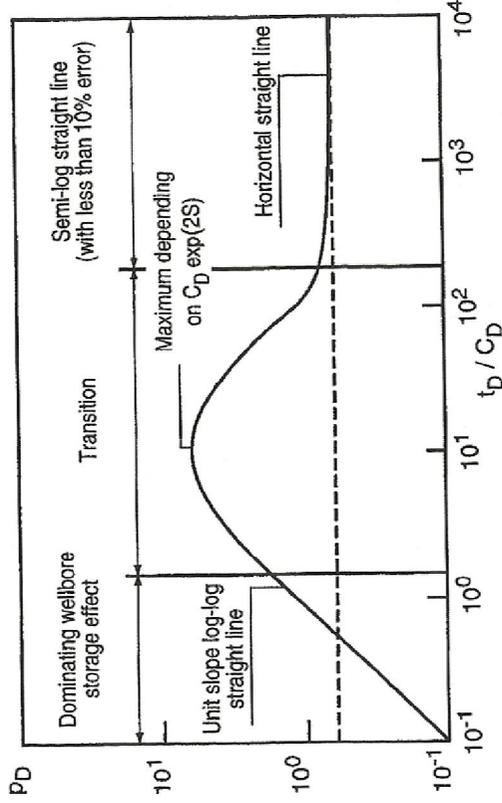


Fig. 6.1

6.3 THE DERIVATIVE AS DIAGNOSTIC TOOL

- Flows with a power function equation:

Generally speaking, whenever a flow presents pressure variations of the type:

$$p_D = a \left(\frac{t_D}{C_D} \right)^n + b \quad (6.10)$$