UNIVERSITY OF BAGHDAD
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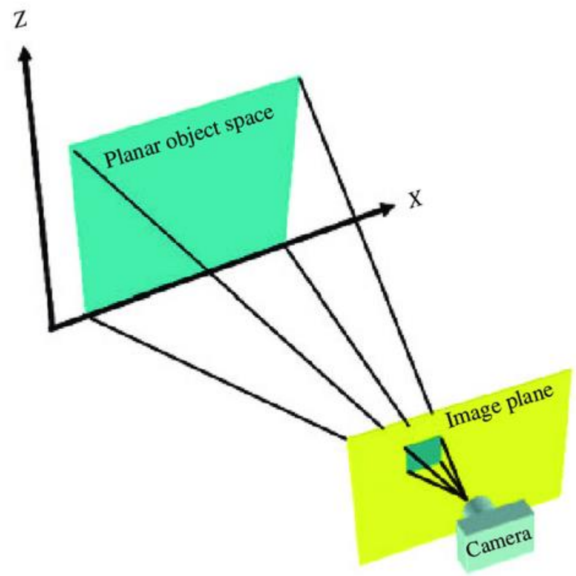

Close-Range Photogrammetry- DLT Solution

BSc - 4th Stage

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Lecture 6

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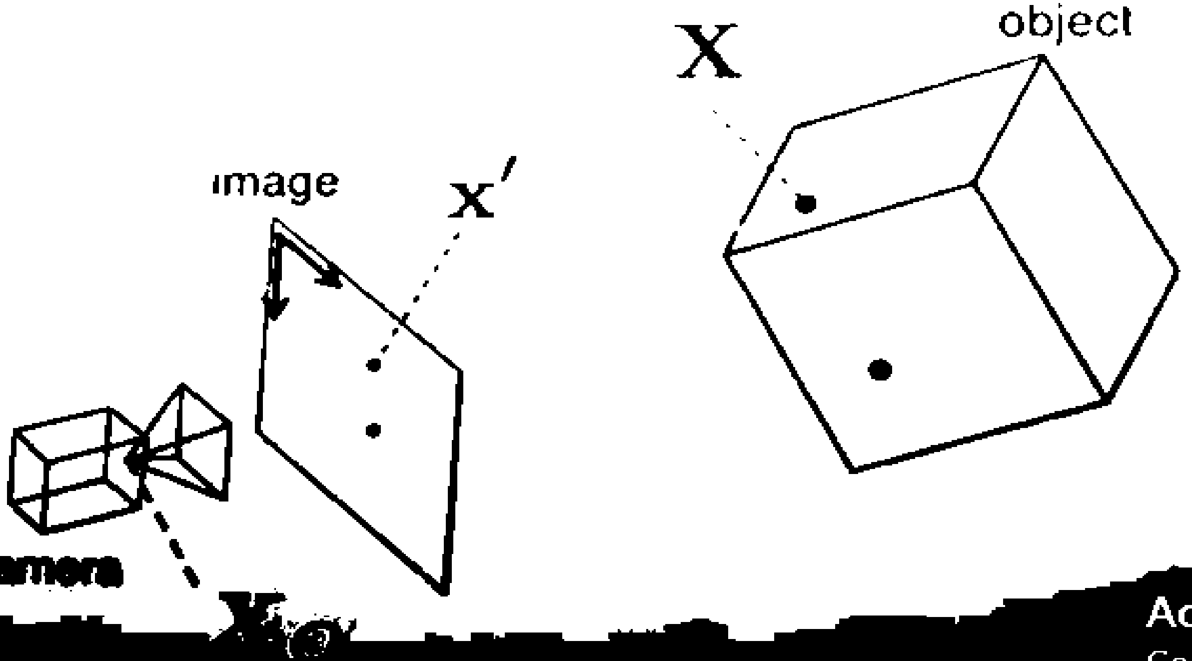
Introduction

- **Direct linear transformation** (**DLT**) is an algorithm which transforms the comparator co-ordinates to the terrain co-ordinates, avoiding the interior, relative and absolute orientation steps!
- It is a linear solution and therefore has an advantage over the solution with the collinearity condition equations.
- For this reason it is useful to summarise the whole photogrammetry in very few analytical passages for the handling of the non metric images.

Direct linear transform (DLT) maps any object point $\mathbf{X}$ to the image point $\mathbf{x}'$

$$\begin{aligned}\mathbf{x}' &= \mathsf{K}R[I_3 | -\mathbf{X}_O]\mathbf{X} \\ &= \mathsf{P}\,\mathbf{X}\end{aligned}$$



The Direct Linear Transformation (DLT)

- The direct linear transformation (DLT) is developed, and simplifies the collinearity solution.
- The (DLT) expresses a linear transformation from image coordinates system into object-space coordinates system in the following form:

$$x + \frac{L_1 X + L_2 Y + L_3 Z + L_4}{L_9 X + L_{10} Y + L_{11} Z + 1} = 0 \quad \ldots\ldots\ldots\ldots$$

$$y + \frac{L_5 X + L_6 Y + L_7 Z + L_8}{L_9 X + L_{10} Y + L_{11} Z + 1} = 0 \quad \ldots\ldots\ldots\ldots$$

- Where x and y are the image coordinates of an image point.

X, Y, and Z are object space coordinates of the point.

L1, L2, ……, L11 are transformation parameters.

The Direct Linear Transformation (DLT)

- A large amount of control points is required.
- The parameters of the I.O.P and of the E.O.P are implicit in the transformation parameters.
- 11 “basic” parameters are included within this solution (L_1 to L_{11}).
- Minimum number of G.C.Ps to find the solution are 6! (each point delivers 2 equations).
- When adding camera distortion parameters the solution will need to solve for 16 parameters in total.

Direct Linear Transform (DLT)

Compute the **11 intrinsic and extrinsic parameters (I.O. and E.O.)**

$$\mathbf{x}' = KR[I_3 | -X_O]\mathbf{X}$$

observed image points

c, s, m, x_H, y_H

3 rotations

3 translations

control point coordinates (given)

Spatial Resection vs. DLT

- **Calibrated camera**
  - 6 unknowns
  - We need at least **3 points**
  - Problem solved by **spatial resection**
- **Uncalibrated camera**
  - 11 unknowns
  - We need at least **6 points**
  - Assuming the model of an **affine camera**
  - Problem solved by **DLT**

DLT: Direct Linear Transform

Computing the Orientation of an Uncalibrated Camera Using $\geq$ 6 Known Points

DLT in Stereo-Photogrammetry

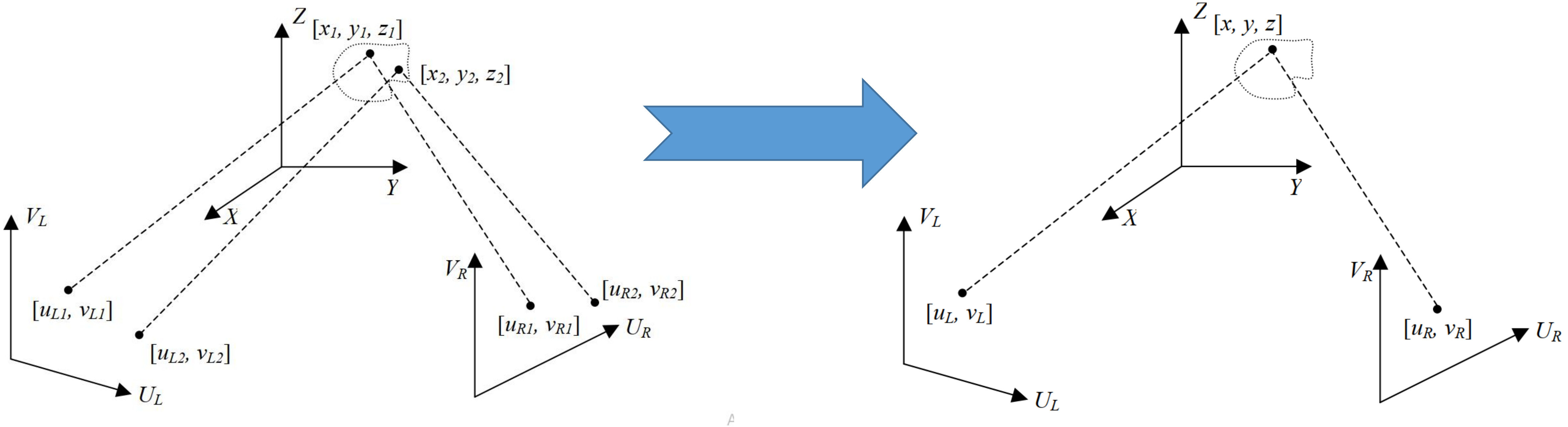
Two cameras

+ The object can occupy the full image in each camera, thereby yielding a lot of pixels for high resolution.
+ Easy to adjust the angle between the cameras and the object for optimal viewing.
– Synchronization of the cameras can be difficult, and will usually involve separate (and often expensive) hardware when imaging a moving object.

One camera, two separate images

+ Very simple; only requires one camera.
+ As in the two camera approach, object can occupy the full image in each camera.
– Object must be stationary.
– Each image pair must be calibrated.

DLT in Stereo-Photogrammetry



DLT Solution Procedures

- DLT Equations can be put in the following linearized form:

$$x_i + L1X_i + L2Y_i + L3Z_i + L4 + L9x_iX_i + L10x_iY_i + L11x_iZ_i = 0 \ldots\ldots\ldots$$
$$y_i + L5X_i + L6Y_i + L7Z_i + L8 + L9y_iX_i + L10y_iY_i + L11y_iZ_i = 0 \ldots\ldots\ldots$$

- Each point (i) observed on the left and right images can give two equations of this form.
- To solve for the 11 transformation parameters (L1 to L11), 6 points have to be measured generating 12 equations.
- Observing six or more points gives redundant observations that require least squares solutions.
- After solving the equations for the 11 transformation parameters for the left and right photos, the object space coordinates Xi, Yi, and Zi can be obtained by solving the following four equations:

$$(L_1 + \bar{x}_i L_9)X_i + (L_2 + \bar{x}_i L_{10})Y_i + (L_3 + \bar{x}_i L_{11})Z_i + \bar{x}_i + L_4 = 0$$
$$(L_5 + \bar{y}_i L_9)X_i + (L_6 + \bar{y}_i L_{10})Y_i + (L_7 + \bar{y}_i L_{11})Z_i + \bar{y}_i + L_8 = 0$$

…. (3-16)

$$(\bar{L}_1 + \bar{\bar{x}}_i \bar{L}_9)X_i + (\bar{L}_2 + \bar{\bar{x}}_i \bar{L}_{10})Y_i + (\bar{L}_3 + \bar{\bar{x}}_i \bar{L}_{11})Z_i + \bar{\bar{x}}_i + \bar{L}_4 = 0$$
$$(\bar{L}_5 + \bar{\bar{y}}_i \bar{L}_9)X_i + (\bar{L}_6 + \bar{\bar{y}}_i \bar{L}_{10})Y_i + (\bar{L}_7 + \bar{\bar{y}}_i \bar{L}_{11})Z_i + \bar{\bar{y}}_i + \bar{L}_8 = 0$$

Where:

L_1 ….. L_{11} are the coefficients for the left image.

$\bar{L}_1$….. $\bar{L}_{11}$ are the coefficients for the right image.

X_i, Y_i, and Z_i are object space coordinates of point i.

$\bar{x}_i$ and $\bar{y}_i$ are the left image coordinates of point i.

$\bar{\bar{x}}_i$ and $\bar{\bar{y}}_i$ are the right image coordinates of point i.

DLT Solution Procedures

- Obviously, the number of unknowns in this system of equation is three (X, Y& Z). This means that a least squares adjustment has to be used for the solution.
- After solving for the 11 transformation parameters, the system of equations must be repeatedly solved for every point whose image coordinates are observed at both photos.
- The estimation of the position of the principal point location (x0, y0) and the principal distance (C) are derived as by-product of the DLT solution based on the eleven parameters (L1…L11). However, E.O.P.s can be also derived as follows:

Deriving camera calibration and E.O.P. s

$$L = -\frac{1}{(m_{31}X_L + m_{32}Y_L + m_{33}Z_L)}$$

$$L_1 = L(x_o m_{31} - c_x m_{11})$$

$$L_2 = L(x_o m_{32} - c_x m_{12})$$

$$L_3 = L(x_o m_{33} - c_x m_{13})$$

$$L_4 = x_o + L\,c_x(m_{11}X_L + m_{12}Y_L + m_{13}Z_L)$$

$$L_5 = L(y_o m_{31} - c_y m_{21})$$

$$L_6 = L(y_o m_{32} - c_y m_{22})$$

$$L_7 = L(y_o m_{33} - c_y m_{23})$$

$$L_8 = y_o + L\,c_y(m_{21}X_L + m_{22}Y_L + m_{23}Z_L)$$

$$L_9 = L\,m_{31}$$

$$L_{10} = L\,m_{32}$$

$$L_{11} = L\,m_{33}$$

$$x_o = (L_1L_9 + L_1L_{10} + L_3L_{11})/(L_9^{\,2} + L_{10}^{\,2} + L_{11}^{\,2})$$

$$y_o = (L_5L_9 + L_6L_{10} + L_7L_{11})/(L_9^{\,2} + L_{10}^{\,2} + L_{11}^{\,2})$$

$$c_x = \sqrt{(L_1^{\,2} + L_2^{\,2} + L_3^{\,2})/(L_9^{\,2} + L_{10}^{\,2} + L_{11}^{\,2}) - x_o^{\,2}}$$

$$c_y = \sqrt{(L_5^{\,2} + L_6^{\,2} + L_7^{\,2})/(L_9^{\,2} + L_{10}^{\,2} + L_{11}^{\,2}) - y_o^{\,2}}$$

$$c = (c_x + c_y)/2$$

$$\begin{bmatrix} X_L \\ Y_L \\ Z_L \end{bmatrix} = -\begin{bmatrix} L_1 & L_2 & L_3 \\ L_5 & L_6 & L_7 \\ L_9 & L_{10} & L_{11} \end{bmatrix}^{-1} \begin{bmatrix} L_4 \\ L_8 \\ 1 \end{bmatrix}$$

$$\omega = \tan^{-1}(-L_{10}/L_{11})$$

$$\varphi = \sin^{-1}\left(-L_9\Big/\sqrt{L_9^{\,2} + L_{10}^{\,2} + L_{11}^{\,2}}\right)$$

$$\kappa = \cos^{-1}\left((L_1 - x_o L_9)\Big/\left(c.\cos\varphi.\sqrt{L_9^{\,2} + L_{10}^{\,2} + L_{11}^{\,2}}\right)\right)$$

$$x_o = (L_1L_9 + L_2L_{10} + L_3L_{11})L^2$$

$$y_o = (L_5L_9 + L_6L_{10} + L_7L_{11})L^2$$

Where:

$$L^2 = \frac{1}{L_9^2 + L_{10}^2 + L_{11}^2}$$

$$C_x^2 = -x_o^2 + L^2(L_1^2 + L_2^2 + L_3^2)$$

$$C_y^2 = -y_o^2 + L^2(L_5^2 + L_6^2 + L_7^2)$$

And the principal distance C:

$C = \frac{1}{2}(C_x + C_y)$ .... (

DLT Advantages

1- Image coordinates can be used directly in the equation. This advantage can be appreciated if non metric camera is used.
2- It is a linear system of equations which produce results directly after least squares adjustment, i.e. there is no need for initial approximate values for the knowns and the solution is not iterative.
3- There is no need to have a prior knowledge of the inner orientation parameters such as the principal distance or the location of the principal point.
4- The solution is easy to program and the algorithm has less equations and terms to deal with.
5- It provides sufficient accuracy for most of the engineering applications. However, the accuracy of the method is sensitive to the number and location of object space control points.
6- Using part of the original image (a sub-image crop from the original image) produces the same results as if the measurements are made on the original image. In any case there is no need to reduce the measured image coordinates to have the origin at the principal point.

Exercise:

- Analytical photogrammetry text book, example 3-5, p. 210.

Acknowledgments

- *Photogrammetry I Course, Chapter: DLT and Camera Calibration - Part 1, Cyrill Stachniss at the University of Bonn, Germany in the summer term 2015.*
- *Direct Linear Transformation (DLT), Elementary Instrumentation, Dr. Scott Thomson.*
- *Analytical Photogrammetry, Bashar S. Al-Jubori, Fanar M. Abed, Maitham M. Al-Bakri, 2009.*