

# Photogrammetry II

## 3<sup>rd</sup> Stage

# Stereoscopic Viewing – lecture 2

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# Stereoscopic parallax - Review

- **Parallax**: is the apparent displacement in the position of object caused by a shift in the position of observation.
- The change in position of an image from one photograph to the next caused by the aircraft's motion is termed **stereoscopic parallax, x parallax, or simply parallax**.
- The parallax of any point is directly related to its **elevation**.
- Parallax is **greater** for high points than for low points.

**NOTE:**

**Differentiate** parallax (x parallax) from y parallax!

# Stereoscopic parallax

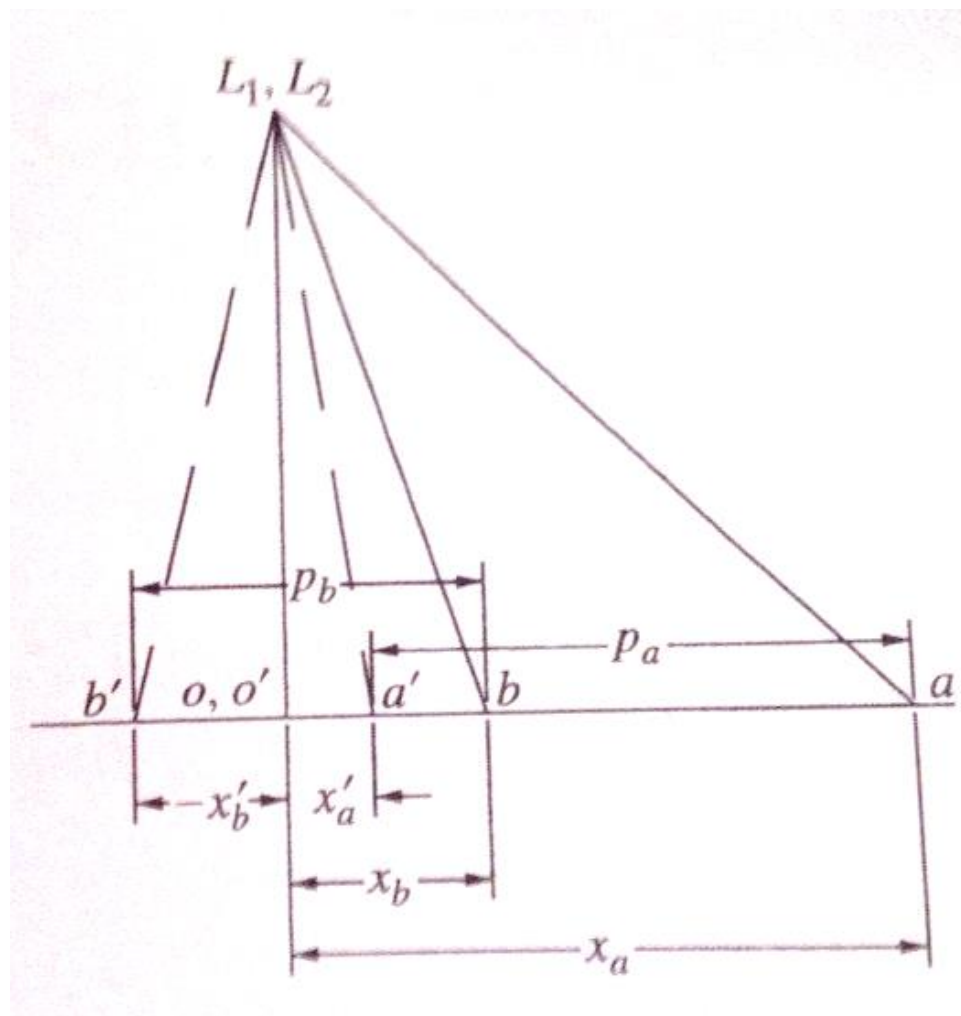
- Parallaxes of point A and B in the Fig. shown are  $p_a$  and  $p_b$  respectively.
- Stereoscopic parallax for any point such as A whose images appear on two photos of a stereo-pair can be expressed as follow:

$$P_a = x_a - x'_a$$

$x_a$  is the measured photo coordinates of image a on the left photo

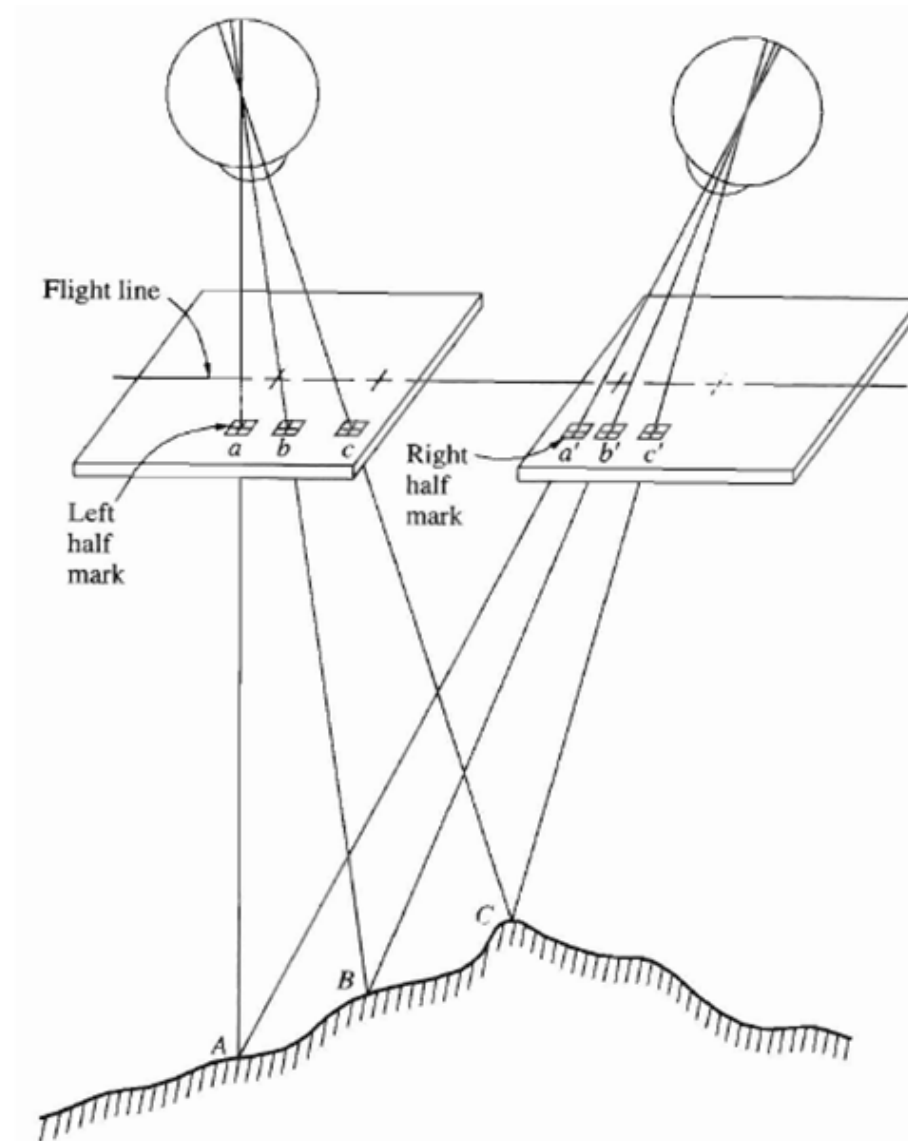
$x'_a$  is measured photo coordinates of image a' on the right photo

**Note:** These photo coords. Are **NOT** measured with respect to principal point rather they are measured with respect to flight-line axis system.



# Principal of the floating mark

- The measuring index in stereo is called a floating mark.
- A dot, or half a dot, that when rests on a location in the stereo model, the observer sees one mark instead of two
- The apparent height ( $Z$ ) of the mark is related to the  $x$  separation of the point in photos, its parallax.



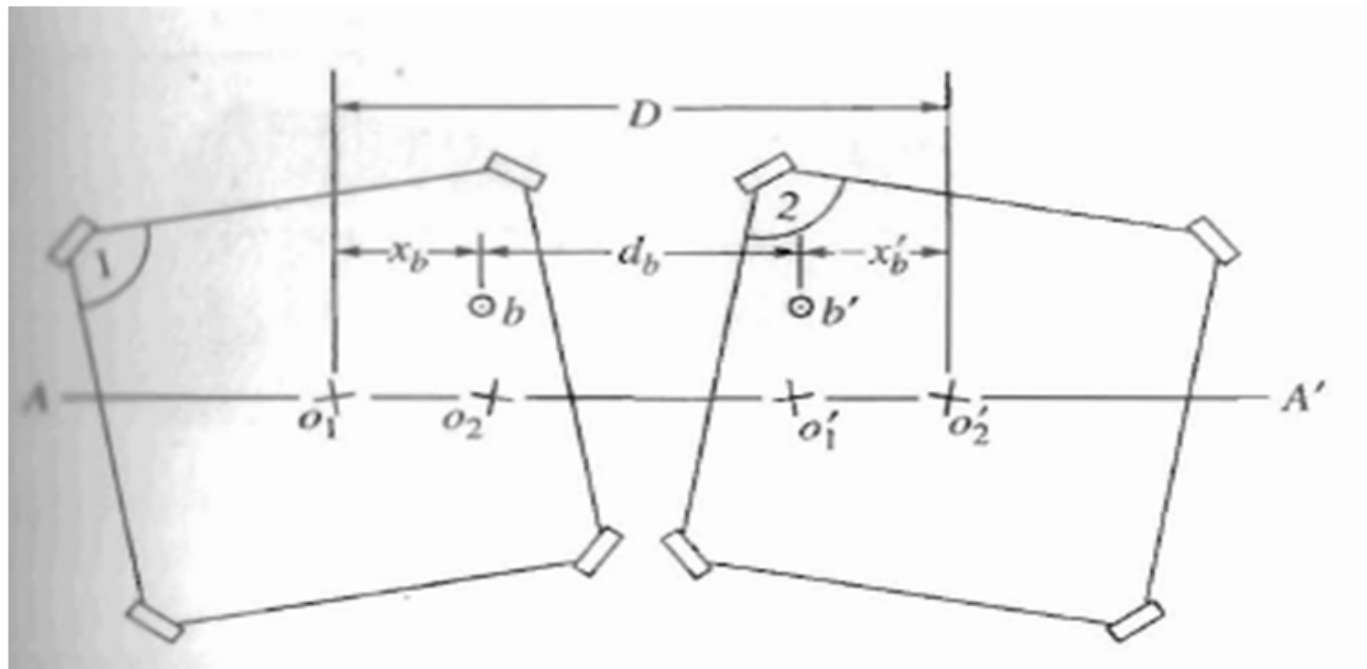
# Monoscopic method of parallax measurements

- Since x parallax is a function of relief or heights of points above datum, elevations of points can be computed by measuring their parallax.
- Other values can also be derived if unknown such as the flying height, or air base.
- To measure parallax of a point you can :
  - a) Either measure in mono: measure the value of the x coordinate of the point on each photograph and subtract the left from the right value. For example, if point a appears on Left and right photos then:

$$p_a = x_{aL} - x_{ar}$$

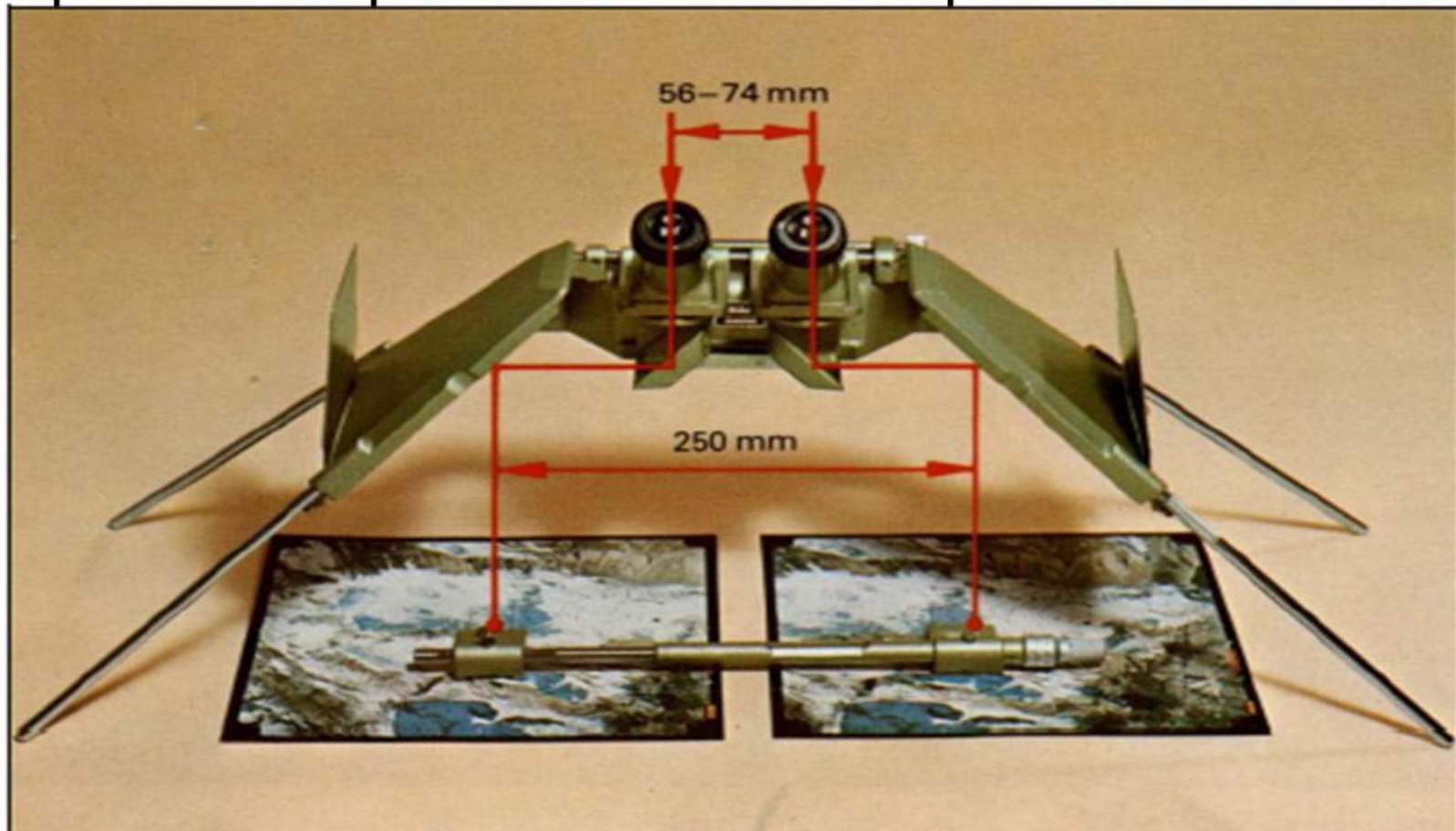
# Monoscopic method of parallax measurements

- b) Or measure in stereo using a parallax bar. Put the photos on a table under a stereoscope, move them right and left up and down until you see comfortably in stereo, in this case: air base B as photo base b:  $o_1$   $o_2$ , and the center of the lenses of the device must be in the same direction



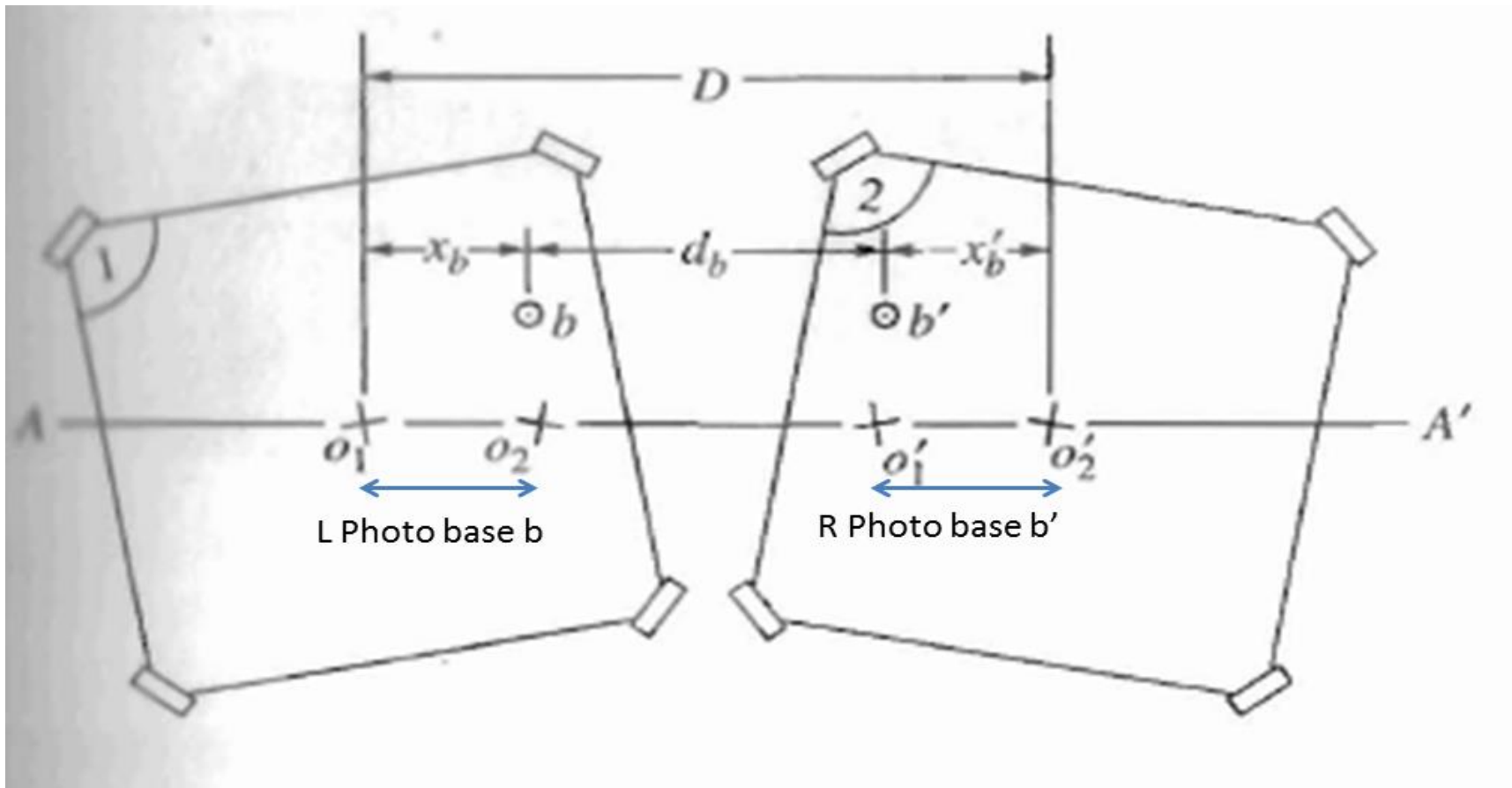
# Measuring parallax with parallax bar

The parallax bar is a measuring device to measure the x parallax of points in a fast and precise manner.





# Measuring parallax with parallax bar

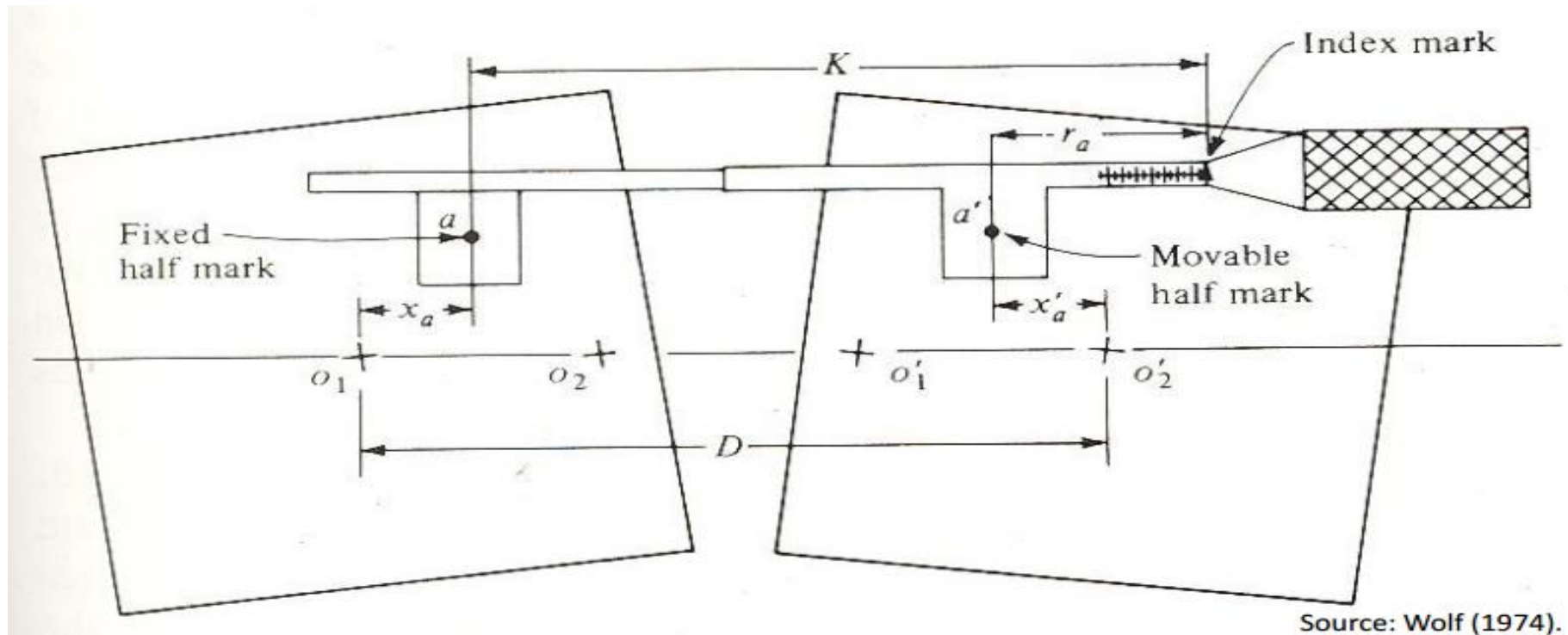


The image of each photo nadir, P.P in a vertical photo ( $o_1$  and  $o'_2$ ) appears on the other photo

Parallax of  $b = D - d_b = x_b - x'_b$  (note that  $x'_b$  is negative)



# Measuring parallax with parallax bar



$p_a$  – parallax for point  $a$

$$p_a = x_a - x'_a = D - (K - r_a) = (D - K) + r_a$$

The term  $D - K$  is  $C$  -- the parallax constant for the setup.

$$p_a = C + r_a$$

# Measuring parallax with parallax bar

- To calculate the parallax of a point, you measure the value  $r$  for it with a parallax bar and Algebraic addition with the constant  $c$  you can do that in stereo for a set of points in few minutes.
- Now, how to compute the constant  $c$  ?
- It is computed only once by measuring the parallax of two point monoscopically by measuring their  $x$  coordinate on each photo and subtract:  $p = x - x_1$
- Now measure the value ( $r$ ) for each point and apply the equation:  $c = p - r$
- You get two values for  $c$ , take the average.
- You can use photo centers for this process since the  $x$  value of each one on its own photograph is 0, you just need to measure its  $x$  value on the other photo

# Measuring parallax with parallax bar

- Summary of computation of parallax using a parallax bar:
- Without the bar, using a precise ruler measure the values of  $x$  of two points  $a$  and  $b$  on both photos, well distinctive points
- Calculate the parallax of the two points, where:  $p_a = x_a - x'_a$  and  $p_b = x_b - x'_b$
- Use the parallax bar to measure  $r_a$  and  $r_b$  from the same points.
- Compute  $c_1 = p_a - r_a$  and  $c_2 = p_b - r_b$
- $c$  is the average =  $(c_1 + c_2)/2$
- For any point measure  $r$  with the bar and | Algebraic addition  
With  $c$  get the parallax

# Parallax equations

- Parallax  $p = x - x_1$

-  $LL_1 a' a_1'$  and  $A' L L_1$  are similar triangles, then

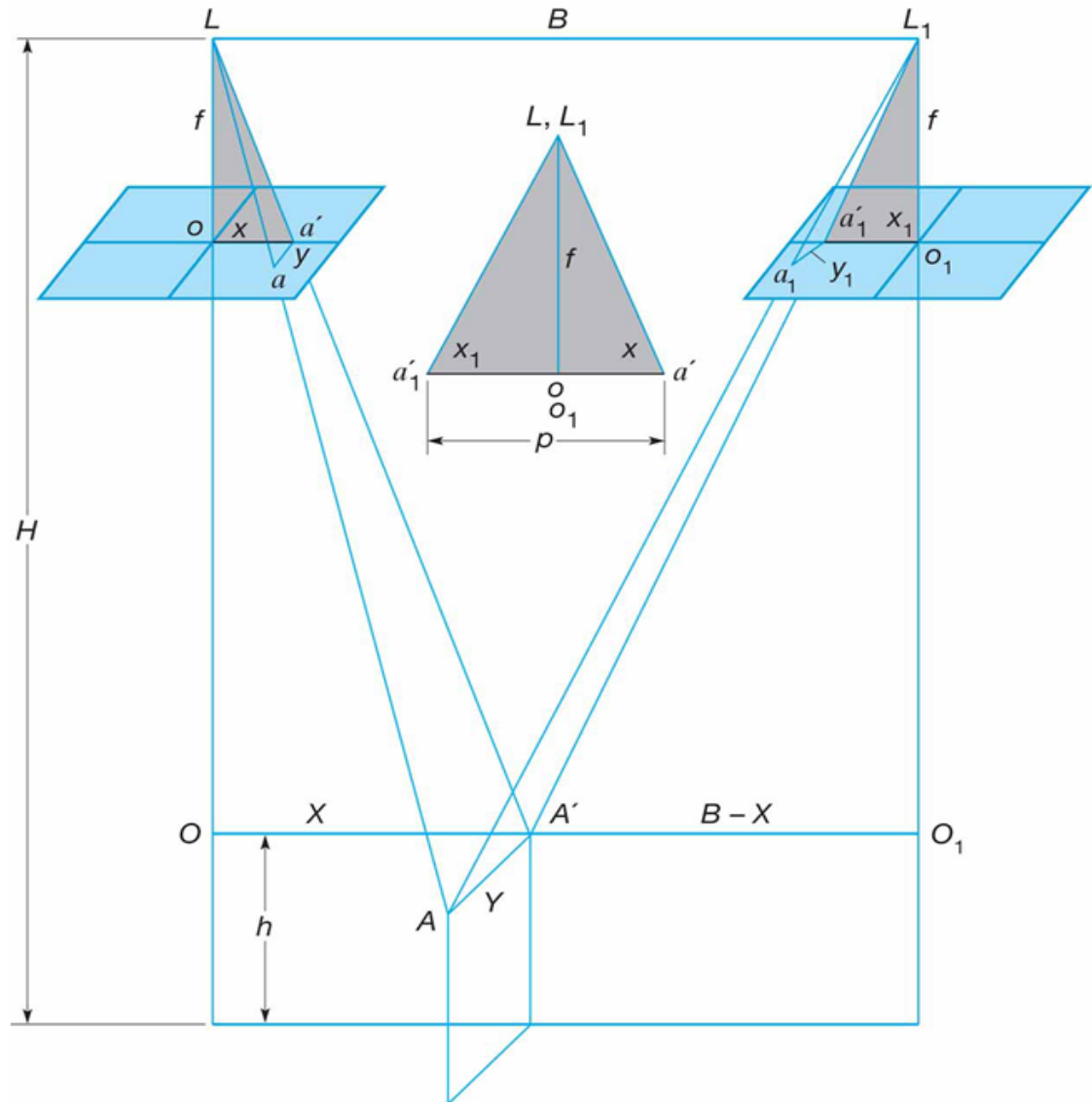
$P / B = f / (H-h)$ , or

$$P = f B / (H-h)$$

Also, since the scale =  $f / (H-h)$

then.,

$$\text{Scale} = P / B$$



# Parallax equations

$$\text{Parallax} = p = x_{\text{left}} - x_{\text{right}} = x - x_1$$

$$H - h = \frac{Bf}{p}$$

get the flying height  
or point elevation

$$X = \frac{B}{p} x \quad Y = \frac{B}{p} y$$

get ground coordinates

**Therefore,**

$$h = H - \frac{Bf}{p}$$

The bigger the parallax, the higher the point

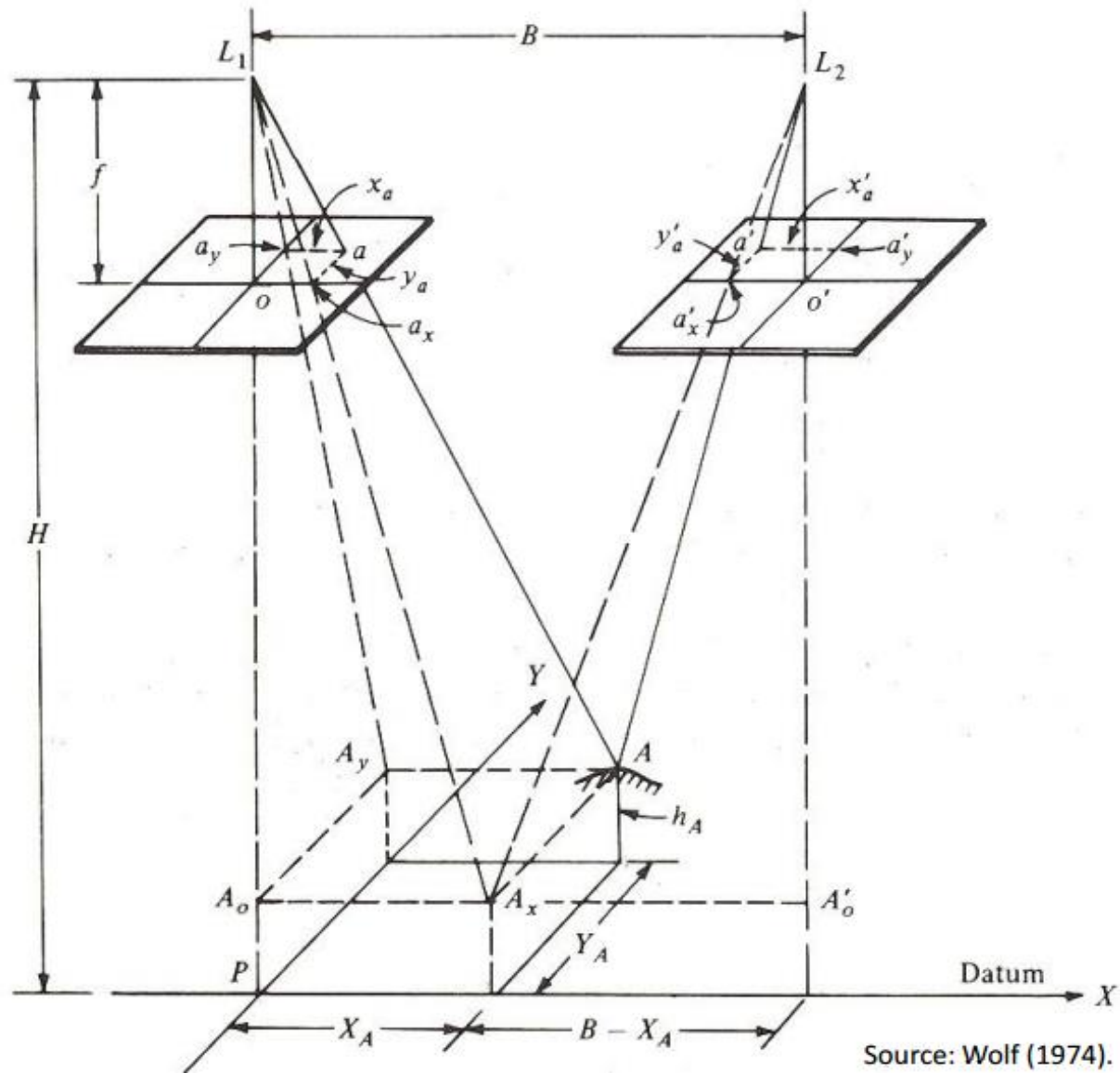
# Parallax equations

Ground x, y, and z coordinates can be calculated by using a parallax.

$$h_A = H - Bf/p_a$$

$$X_A = B * x_a/p_a$$

$$Y_A = B * y_a/p_a$$



Source: Wolf (1974).

# Parallax equations

- If we have two points (A & C), we can deliver the formula from the parallax difference between them as follow:

$$h_A = h_C + \frac{\Delta p (H - h_C)}{p_a}$$

- $H = h + \frac{Bf}{p}$
- $B = (H - h) \frac{p}{f}$



## Example:

A pair of overlapping vertical photographs were taken from a flying height of 4045 feet with a 152.4 mm focal length camera. The air base was 1280 feet and parallax bar readings of 12.57 and 13.04 mm were obtained with the floating mark set on principal points ( $o_1$ ) and ( $o_2$ ) respectively. If  $b$  and  $b'$  (left and right photo bases) were measured as 93.73 and 93.30 mm. Parallax bar readings of 10.96 and 15.27 mm were taken on points A and B. Also the  $x$  and  $y$  photo coordinates of points A and B were  $X_a = 53.41$  mm,  $Y_a = 50.84$  mm,  $X_b = 88.92$  mm and  $Y_b = -46.69$  mm. Calculate the elevations of points A and B and the horizontal distance between them.

## Solution:

Parallax of Point  $O_1 = P_{o1} = b' = 93.30\text{mm}$ .

Parallax of Point  $O_2 = P_{o2} = b = 93.73\text{mm}$

Equation of parallax bar:  $P = c + r \Rightarrow c = P - r$

For point  $O_1$ :  $c_1 = P_{o1} - r_{o1} = 93.30 - 12.57 = 80.73\text{mm}$

For Point  $O_2$ :  $c_2 = P_{o2} - r_{o2} = 93.73 - 13.04 = 80.69\text{mm}$

$$C_{\text{average}} = \frac{c_1 + c_2}{2} = \frac{80.73 + 80.69}{2} = 80.71\text{mm}.$$

For point a:  $P_a = c + r_a = 80.71 + 10.96 = 91.67\text{mm}$ .

For point b:  $P_b = c + r_b = 80.71 + 15.27 = 95.98\text{mm}$ ,

# Elevations

$$h_A = H - \frac{f \cdot B}{P_a} = 4045 - \frac{1280 \times 152.4}{91.67} = 1917.02 \text{ feet}$$

$$h_B = H - \frac{f \cdot B}{P_b} = 4045 - \frac{1280 \times 152.4}{95.67} = 2012.576 \text{ feet}$$

$$X = x \cdot \frac{B}{p}$$

$$Y = y \cdot \frac{B}{p}$$

$$X_A = 53.4 \times \frac{1280}{91.67} = 745.77 \text{ feet,}$$

$$Y_A = 50.84 \times \frac{1280}{91.67} = 709.885 \text{ feet.}$$

$$X_B = 88.92 \times \frac{1280}{95.98} = 1185.85 \text{ feet,}$$

$$Y_B = -46.69 \times \frac{1280}{95.98} = -622.663 \text{ feet.}$$

Horizontal ground distance AB:

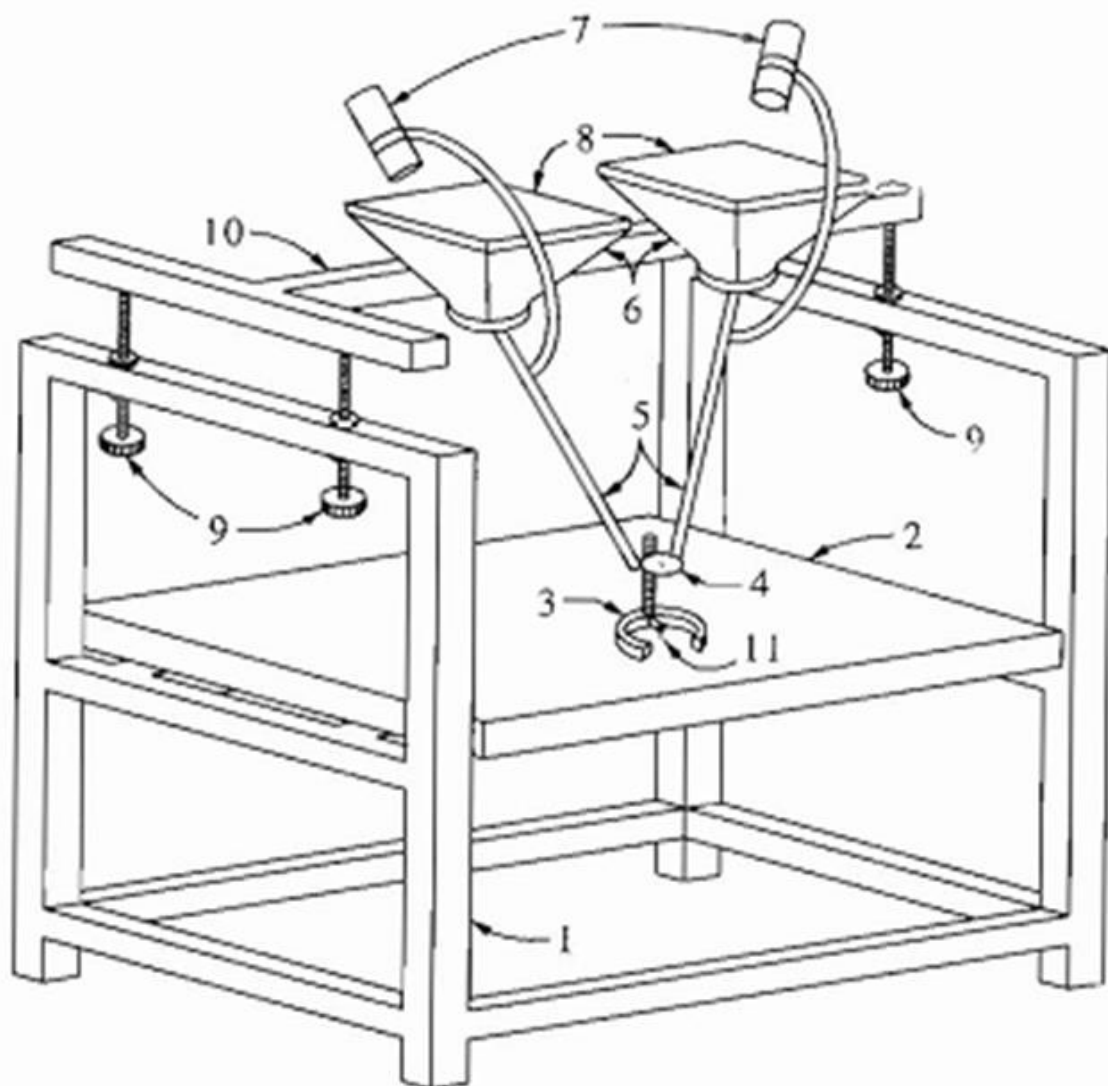
$$AB = \sqrt{(X_A - X_B)^2 + (Y_A - Y_B)^2} = 1403.336 \text{ feet}$$

# Stereoscopic plotters

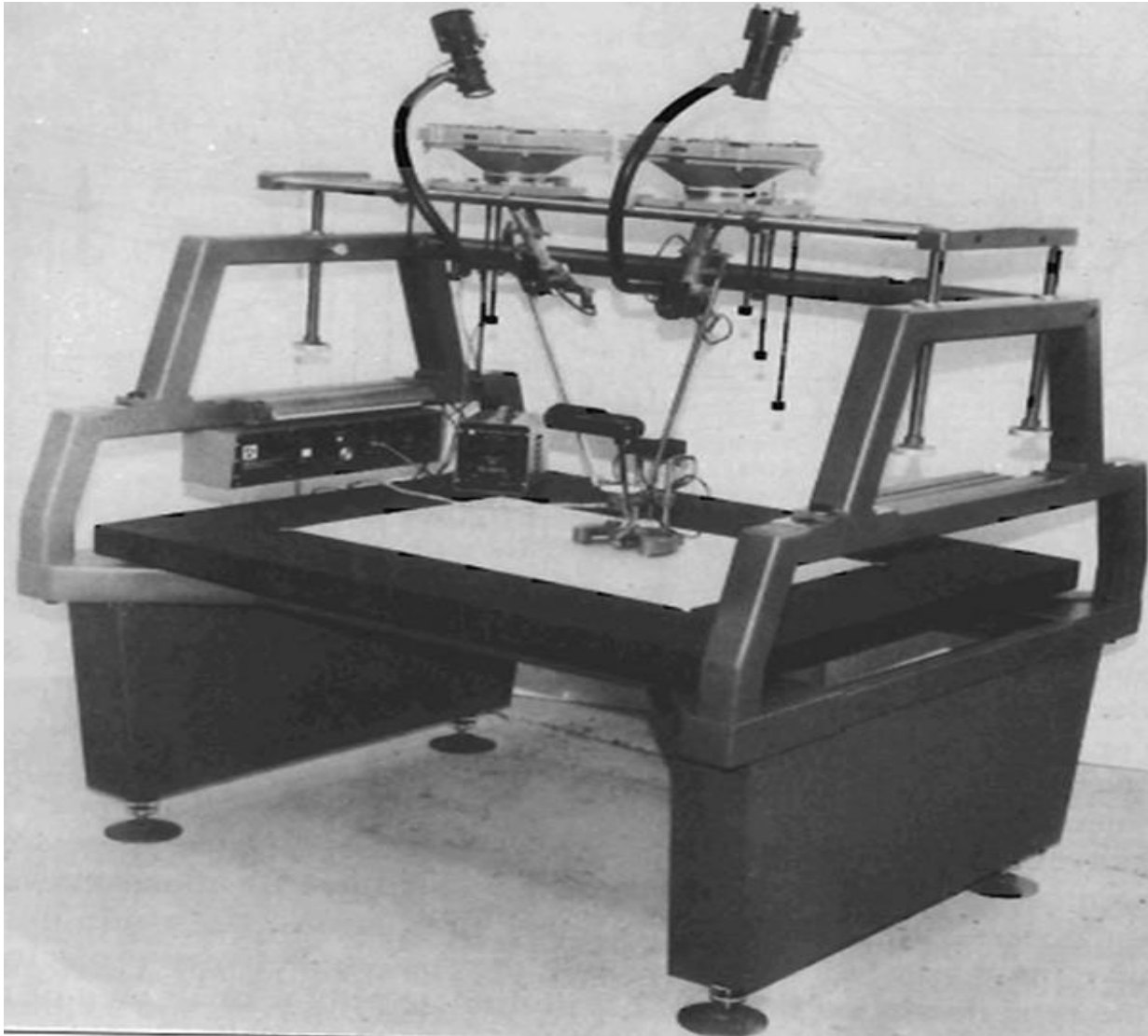
- Stereoscopic plotters are instruments used to measure ground coordinates of points in the overlapped area by measuring their photo coordinates in stereo mode.
- Two overlapping photos are used, either placed in projectors as the case in early models, or displayed in digital format as current technology.
- Such plotters allow for the removal of the Y parallax, the effect of different of scale between the photos, tilt of photos,

Stereo plotters may be classified into three categories:

## 1- Analogue Optical



# Analogue stereoplotter



# Analogue stereoplotter

- Film or transparent photos are positioned on the projectors, light is projected through them, their relative positions are adjusted to form a model to scale.
- Points are measured as they are traced on the tracing table (4) in the graph above, a pencil at (11) will draw a map as the tracing table is moved.
- *Such a device is used to illustrate the idea, but not for production today.*



# Analytical stereoplotter

- Still uses photographs, but the model is mathematical,
- Two comparators are used to precisely measure photo coordinates, which are recorded digitally.
- The stereo-model is seen through optics as a computer adjusts the photos for stereo viewing and measurements as the mouse is moved by driving servo motors.
- A point is digitized by clicking a mouse when the floating mark rests on it to store the coordinates.
- The digital output is stored and a CAD system can be used to produce a map, on the fly if needed

# Analytical stereoplotter



# Softcopy photogrammetric workstation

- Softcopy workstations employ digital images, a software, a stereovision system, installed on a powerful computer.
- The output is totally digital, and many operations are automated.
- Softcopy is the current technology used for photogrammetric measurements.
- Images are captured by a digital camera, or scanning photographs.



# Photogrammetric workstation





# Credits

- <http://www.ccrs.nrcan.gc.ca/ccrs/learn/tutorials/stereosc/chap4/>
- **Engineering Geology**, ETH Zurich
- HU faculty research directory