Photogrammetry II

3rd Stage

Single Photo Resection and Orientation(S.P.R.O) Case 1

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Rotation Matrix

- $M=\begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix}$

- M_{11}= cos Ø cos Ķ
- M_{12}= sin Ψ sin Ø cos Ķ + cos Ψ sin Ķ
- M_{13}= -cos Ψ sin Ø cos Ķ + sin Ψ sin Ķ
- M_{21}= -cos Ø sin Ķ
- M_{22}= - sin Ψ sin Ø sin Ķ + cos Ψ cos Ķ
- M_{23}= cos Ψ sin Ø sin Ķ + sin Ψ cos Ķ
- M_{31}= sin Ø
- M_{32}= -sin Ψ cos Ø
- $M_{33=}$ cos Ψ cos Ø

Collinearity Equation

- $x = -f\left[\frac{m_{11}(X-X_L)+m_{12}(Y-Y_L)+m_{13}(Z-Z_L)}{m_{31}(X-X_L)+m_{32}(Y-Y_L)+m_{33}(Z-Z_L)}\right]$

- $y = -f\left[\frac{m_{21}(X-X_L)+m_{22}(Y-Y_L)+m_{23}(Z-Z_L)}{m_{31}(X-X_L)+m_{32}(Y-Y_L)+m_{33}(Z-Z_L)}\right]$

Observation Equations

- AV + $B\,\Delta\, + F = 0$
- V= Matrix of Residuals for observations
- Δ= Matrix of Corrections for unknowns
- F= Matrix of Absolute Values

Case 1

- Known : - observed Photo Coordinate (x,y)
- - Approximate value for unknowns
  (Exterior Orientation Parameters $(X_L, Y_L, Z_L, \mathrm{W}, \mathrm{Q}, \mathrm{K})$)
- - fixed Ground Coordinate for GCPs

- Unknown: - Exterior Orientation Parameters $(X_L, Y_L, Z_L, \mathrm{W}, \mathrm{Q}, \mathrm{K})$

- Note : (3 points) minimum number of point require to compute (Exterior Orientation Parameters $(\mathrm{X_L}, \mathrm{Y_L}, \mathrm{Z_L}, \mathrm{W}, \mathrm{Q}, \mathrm{K})$) in Case 1

Case 1(Observation Equations)

- AV+$B^e \Delta^e + F = 0$
- $A_j = \begin{matrix} \\ x \\ y \end{matrix} \begin{matrix} \overset{x}{} \\ \\ \end{matrix} \begin{matrix} \overset{y}{} \\ \\ \end{matrix} \begin{bmatrix} \frac{\partial f_j}{\partial x_j} & \frac{\partial f_j}{\partial y_j} \\ \frac{\partial f_j}{\partial x_j} & \frac{\partial f_j}{\partial y_j} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = [I]$

Case 1(B matrix)

n= number of points

- $B_{(2n*6)} =$

	$\partial \mathrm{W}$	$\partial \mathrm{\omega}$	$\partial \mathrm{K}$	∂X_L	∂Y_L	∂Z_L
x_1	b_{11}	b_{12}	b_{13}	$-b_{14}$	$-b_{15}$	$-b_{16}$
y_1	b_{21}	b_{22}	b_{23}	$-b_{24}$	$-b_{25}$	$-b_{26}$
x_2	b_{11}	b_{12}	b_{13}	$-b_{14}$	$-b_{15}$	$-b_{16}$
y_2	b_{21}	b_{22}	b_{23}	$-b_{24}$	$-b_{25}$	$-b_{26}$
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x_n	b_{11}	b_{12}	b_{13}	$-b_{14}$	$-b_{15}$	$-b_{16}$
y_n	b_{21}	b_{22}	b_{23}	$-b_{24}$	$-b_{25}$	$-b_{26}$

Case 1

- $F_{ip} = \begin{bmatrix} (Jp)_i \\ (Kp)_i \end{bmatrix}$ OR $F_{(2n*1)} = \begin{bmatrix} F(x)_i \\ F(y)_i \end{bmatrix}$

- $F_{(x)} = x_{comp} - x_{obs}$
- $F_{(y)} = y_{comp} - y_{obs}$
- Where as:
- x_{obs} & y_{obs} =Observed Photo Coordinate for points
- x_{comp} & y_{comp} = Compute Photo Coordinate for points by collinearity equation

Case 1

- $\Delta_{(6*1)} = \begin{bmatrix} \partial \mathrm{W} \\ \partial \mathrm{\omega} \\ \partial \mathrm{K} \\ \partial X_L \\ \partial Y_L \\ dZ_L \end{bmatrix}$ & $V_{(2n*1)} = \begin{bmatrix} Vx_1 \\ V_{y1} \\ V_{x2} \\ V_{y2} \\ . \\ . \\ . \\ . \\ V_{xn} \\ V_{yn} \end{bmatrix}$

Case 1(Normal Equation)

- Δ= $(B^{et}WB)^{-1}\ (B^{et}WF)$
- let N^{-1}= $(B^{et}WB)^{-1}$
- U= $(B^{et}WF)$
- Δ=$N^{-1}U$ Normal Equation
- $N_{(6*6)}$
- $U_{(6*1)}$

Case 1 (Adjusted Values for Unknown)

- $a_{\mathrm{W}} = \mathrm{W}_{oo} + \partial_{\mathrm{W}}$
- $a_{\mathrm{Q}} = \mathrm{Q}_{oo} + \partial_{\mathrm{Q}}$
- $a_{\mathrm{K}} = \mathrm{K}_{oo} + \partial_{\mathrm{K}}$
- $a_{XL} = X_{Loo} + \partial X_L$
- $a_{YL} = Y_{Loo} + \partial Y_L$
- $a_{ZL} = Z_{Loo} + \partial Z_L$

Case 1(Adjusted Values for Observation)

V=$B^e \Delta^e - F$

$\overline{x_1} = x_1 + V_{x1}$

$\overline{y_1} = y_1 + V_{y1}$

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$\overline{x_n} = x_n + V_{xn}$

$\overline{y_n} = y_n + V_{yn}$

Case 1(Variance Covariance Matrix)

- $\sum xx = {\sigma_o}^2 N^{-1}$
- ${\sigma_o}^2 = \frac{V^t W V}{r}$
- r= n-n_o
- r= 2n-6

Linearization of Collinearity Equation

- q = $m_{31}(X_A - X_L) + m_{32}(Y_A - Y_L) + m_{33}(Z_A - Z_L)$
- r = $m_{11}(X_A - X_L) + m_{12}(Y_A - Y_L) + m_{13}(Z_A - Z_L)$
- S = $m_{21}(X_A - X_L) + m_{22}(Y_A - Y_L) + m_{23}(Z_A - Z_L)$

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- $b_{11} = \frac{f}{q^2}$ [r$(-m_{33}\Delta Y + m_{32}\ \Delta$Z)_ q$(-m_{13}\Delta Y + m_{12}\ \Delta$Z)]
- $b_{12} = \frac{f}{q^2}$[r(cos ϕΔX + sin Ψ sin ϕΔY - cos Ψ sin ϕ $\Delta Z) - q(-$ sin ϕ cos Ķ ΔX +sin Ψ cos ϕ cos Ķ ΔY - cos Ψ cos ϕ cos Ķ ΔZ)]
- $b_{13} = \frac{-f}{q}(m_{21}\ \Delta$X + $m_{22}\ \Delta$Y + $m_{23}\ \Delta Z)$
- b_{14} = $\frac{f}{q^2}$(r$m_{31} - qm_{11}$)
- $b_{15} = \frac{f}{q^2}$(rm_{32} _ qm_{12})

B matrix

- $b_{16} = \frac{f}{q^2}(\mathrm{r}m_{33} \_ qm_{13})$
- $b_{21} = \frac{f}{q^2}[\mathrm{s}(-m_{33}\,\Delta Y + m_{32}\,\Delta Z) - q\,(-m_{23}\,\Delta Y + m_{22}\,\Delta Z)$
- $b_{22} = \frac{f}{q^2}[\mathrm{s}\,(\cos\phi\Delta X + \sin W \sin\phi\Delta Y - \cos W \sin\phi\,\Delta Z) - q(-\sin\phi\cos К\,\Delta X + \sin W\cos\phi\cos К\,\Delta Y - \cos W\cos\phi\cos К\,\Delta Z)]$
- $b_{23} = \frac{f}{q}(m_{11}\Delta X + m_{12}\Delta Y + m_{13}\Delta Z)$
- $b_{24} = \frac{f}{q^2}(\,sm_{31} \_ qm_{21})$
- $b_{25} = \frac{f}{q^2}(sm_{32} \_ qm_{22})$
- $b_{26} = \frac{f}{q^2}(\,sm_{33} \_ qm_{23})$

References

- Wolf, Paul.R. and Dewitt, Bon A.,Elements of Photogrammetry with applications in GIS, 3rd ed., McGraw-Hill,New York, 2000

- بشار سليم عباس، فنار منصور ، ميثم البكري ، المسح التصويري التحليلي، الطبعة الاولى ، اثراء للنشر والتوزيع ، الاردن2009