Photogrammetry II

3rd Stage

Single Photo Resection and Orientation(S.P.R.O) Case(2-3-4)

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Observation Equations

- AV + $B\,\Delta + F = 0$
- V= Matrix of Residuals for observations
- Δ= Matrix of Corrections for unknowns
- F= Matrix of Absolute Values

Case 2

- Known : - Observed Photo Coordinate (x,y)
- - Observed value for unknowns
  (Exterior Orientation Parameters $(X_L, Y_L, Z_L, \omega, \phi, \kappa)$)
- - Ground Coordinate for GCPs

- Unknown: - Exterior Orientation Parameters $(X_L, Y_L, Z_L, \omega, \phi, \kappa)$

Equation for case (2)

- $F_{(x)}=$ x +f$\left[\frac{m_{11}(X-X_L)+m_{12}(Y-Y_L)+m_{13}(Z-Z_L)}{m_{31}(X-X_L)+m_{32}(Y-Y_L)+m_{33}(Z-Z_L)}\right]$=0
- $F_{(y)}=$ y+ f $\left[\frac{m_{21}(X-X_L)+m_{22}(Y-Y_L)+m_{23}(Z-Z_L)}{m_{31}(X-X_L)+m_{32}(Y-Y_L)+m_{33}(Z-Z_L)}\right]$=0

- $F_{(W)} = (W_0 - W_a) = 0$
- $F_{(\varphi)} = (\varphi_0 - \varphi_a) = 0$
- $F_{(K)} = (K_0 - K_a) = 0$
- $F_{(X_L)} = (X_{L0} - X_{La}) = 0$
- $F_{(Y_L)} = (Y_{L0} - Y_{La}) = 0$
- $F_{(Z_L)} = (Z_{L0} - Z_{La}) = 0$

Case 2

- $Ɯ_0 + V_{Ɯ} = Ɯ_{00} + \delta_{Ɯ}$
- $Ꞷ_0 + V_{Ꞷ} = Ꞷ_{00} + \delta_{Ꞷ}$
- $Ķ_0 + V_{Ķ} = Ķ_{00} + \delta_{Ķ}$
- $X_{0L} + V_{XL} = X_{L00} + \delta_{XL}$
- $Y_{0L} + V_{YL} = Y_{L00} + \delta_{YL}$
- $Z_{0L} + V_{ZL} = Z_{L00} + \delta_{ZL}$

Case 2

- $V_{\text{Ɯ}} + (\text{Ɯ}_0 - \text{Ɯ}_{00}) - \delta_{\text{Ɯ}} = 0$
- $V_{\text{Ꞷ}} + (\text{Ꞷ}_0 - \text{Ꞷ}_{00}) - \delta_{\text{Ꞷ}} = 0$
- $V_{\text{Қ}} + (\text{Қ}_0 - \text{Қ}_{00}) - \delta_{\text{Қ}} = 0$
- $V_{XL} + (X_{L0} - X_{L00}) - \delta_{XL} = 0$
- $V_{YL} + (Y_{L0} - Y_{L00}) - \delta_{YL} = 0$
- $V_{ZL} + (Z_{L0} - Z_{L00}) - \delta_{ZL} = 0$

- $V^e - \Delta^e + F^e = 0$

Observation Equations for Case(2)

- $V_{(2n*1)} + {B^e}_{(2n*6)} {\Delta^e}_{(6*1)} + F_{(2n*1)} = 0$
- ${V^e}_{(6*1)} - {\Delta^e}_{(6*1)} + {F^e}_{(6*1)} = 0$

$$\bar{V}_{(2n+6)*1} + \bar{B}_{(2n+6)*6}\, {\Delta^e}_{(6*1)} + \bar{F}_{(2n+6)*1} = 0$$

Matrices for Case(2)

Whereas :

$\bar{V} = \begin{bmatrix} V \\ V^e \end{bmatrix}$, $\bar{B} = \begin{bmatrix} B^e \\ -I \end{bmatrix}$, $\bar{F} = \begin{bmatrix} F \\ F^e \end{bmatrix}$

$\bar{W}_{(2n+6)*(2n+6)} = \begin{bmatrix} W_{(2n*2n)} & 0 \\ 0 & W^e{}_{(6*6)} \end{bmatrix}$

Normal Equation:

$(\overline{B^T}_{6*(2n+6)} \bar{W}_{(2n+6)*(2n+6)} \bar{B}_{(2n+6)*6}) \bar{\Delta}_{(6*1)}$
$= \overline{B^T}_{6*(2n+6)} \bar{W}_{(2n+6)*(2n+6)} \bar{F}_{(2n+6)*1}$

Case(2)

- $\begin{bmatrix} B^{e^T} & -I \end{bmatrix} \begin{bmatrix} W & 0 \\ 0 & W^e \end{bmatrix} \begin{bmatrix} B^e \\ -I \end{bmatrix} \Delta^e + \begin{bmatrix} B^{e^T} & -I \end{bmatrix} \begin{bmatrix} W & 0 \\ 0 & W^e \end{bmatrix} \begin{bmatrix} F \\ F^e \end{bmatrix} = 0$

- Normal Equation:
- $\left(B^{e^T} W^e B^e + W^e\right) \Delta^e + \left(B^{e^T} WF - W^e F^e\right) = 0$
- $\bar{N} \Delta^e + \bar{U} = 0$
- $X_{00} = X_0$
- $\Delta^e = \overline{-N}^{-1} \bar{U}$
- $X_a = X_{00} + \Delta^e$

Case 3

- Known : - Observed Photo Coordinate (x,y)
- - Observed value for unknowns
  (Exterior Orientation Parameters $(X_L, Y_L, Z_L, \omega, \phi, \kappa)$)
- - Observed Ground Coordinate for GCPs

- Unknown: - Exterior Orientation Parameters $(X_L, Y_L, Z_L, \omega, \phi, \kappa)$
  - Ground Coordinate for GCPs

Equation for case (3)

- $F_{(x)}=$ x +f$\left[\frac{m_{11}(X-X_L\ )+m_{12}(Y-Y_L)+m_{13}(Z-Z_L)}{m_{31}(X-X_L\ )+m_{32}(Y-Y_L)+m_{33}(Z-Z_L)}\right]$=0
- $F_{(y)}$= y+ f $\left[\frac{m_{21}(X-X_L\ )+m_{22}(Y-Y_L)+m_{23}(Z-Z_L)}{m_{31}(X-X_L\ )+m_{32}(Y-Y_L)+m_{33}(Z-Z_L)}\right]$=0

- $F_{(W)} = (W_0 - W_a) = 0$
- $F_{(\phi)} = (\phi_0 - \phi_a) = 0$
- $F_{(K)} = (K_0 - K_a) = 0$
- $F_{(X_L)} = (X_{L0} - X_{La}) = 0$
- $F_{(Y_L)} = (Y_{L0} - Y_{La}) = 0$
- $F_{(Z_L)} = (Z_{L0} - Z_{La}) = 0$
- F$(X_j) = X_{j0} - X_{ja} = 0$
- F$(Y_j) = Y_{j0} - Y_{ja} = 0$
- F$(Z_j) = Z_{j0} - Z_{ja} = 0$

Observation Equations for Case(3)

- $V^e{}_{(6*1)}$ - $\Delta^e{}_{(6*1)}$ + $F^e{}_{(6*1)} = 0$
- $V^s{}_{(3n*1)}$- $\Delta^s{}_{(3n*1)}$ + $F^s{}_{(3n*1)} = 0$
- $V_{(2n*1)}$ + $B^e{}_{(2n*6)}\Delta^e{}_{(6*1)}$ + $B^s{}_{(2n*3n)}$ $\Delta^s{}_{(3n*1)}$ + $F_{(2n*1)}$= 0

- $\bar{V}$ = $\begin{bmatrix} V \\ V^e \\ V^s \end{bmatrix}$, $\bar{B}$ = $\begin{bmatrix} B^e\, B^s \\ -I \;\; 0 \\ 0 \;\; -I \end{bmatrix}$, $\bar{\Delta}$= $\begin{bmatrix} \Delta^e \\ \Delta^s \end{bmatrix}$, $\bar{F}$ = $\begin{bmatrix} F \\ F^e \\ F^s \end{bmatrix}$

B^e Matrix

- $$B^{e}{}_{(2n*6)} = \begin{array}{cc} & \begin{array}{cccccc} \partial\omega & \partial\varphi & \partial\kappa & \partial X_L & \partial Y_L & \partial Z_L \end{array} \\ \begin{array}{c} x_1 \\ y_1 \\ x_2 \\ y_2 \\ . \\ . \\ . \\ . \\ . \\ . \\ x_n \\ y_n \end{array} & \begin{bmatrix} b_{11} & b_{12} & b_{13} & -b_{14} & -b_{15} & -b_{16} \\ b_{21} & b_{22} & b_{23} & -b_{24} & -b_{25} & -b_{26} \\ b_{11} & b_{12} & b_{13} & -b_{14} & -b_{15} & -b_{16} \\ b_{21} & b_{22} & b_{23} & -b_{24} & -b_{25} & b_{26} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ b_{11} & b_{12} & b_{13} & -b_{14} & -b_{15} & -b_{16} \\ b_{21} & b_{22} & b_{23} & -b_{24} & -b_{25} & -b_{26} \end{bmatrix} \end{array}$$

B^s Matrix

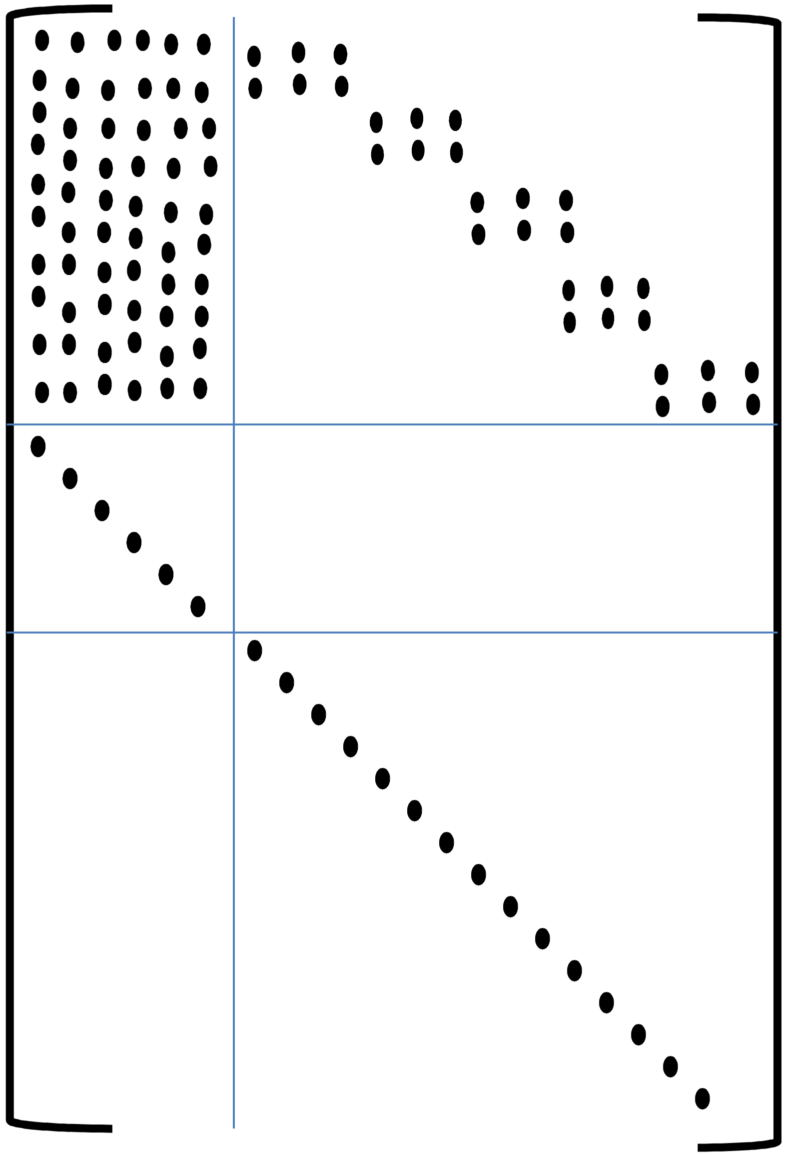
- $B^s{}_{(2n*3n)} =$

$$
\begin{array}{cc}
 & \begin{array}{cccccccccccc} \partial X_1 & \partial Y_1 & \partial Z_1 & \partial X_2 & \partial Y_2 & \partial Z_2 & \partial X_j & \partial Y_j & \partial Z_j & \partial X_n & \partial Y_n & \partial Z_n \end{array} \\
\begin{array}{c} x_1 \\ y_1 \\ x_2 \\ y_2 \\ . \\ . \\ . \\ . \\ . \\ . \\ x_n \\ y_n \end{array} &
\begin{bmatrix}
b_{14} & b_{15} & b_{16} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
b_{24} & b_{25} & b_{26} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & b_{14} & b_{15} & b_{16} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & b_{24} & b_{25} & b_{26} & 0 & 0 & 0 & 0 & 0 & 0 \\
. & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{14} & b_{15} & b_{16} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{24} & b_{25} & b_{26}
\end{bmatrix}
\end{array}
$$

$\bar{B}$ Matrix

B^{e}(2n*6)	B^{s}(2n*3n)
I(6*6)	0(6*3n)
0(3n*6)	I(3n*3n)

$\bar{B}$ Matrix



Matrices for Case(3)

- $V^{e}{}_{6*1} = \begin{bmatrix} V_{\omega} \\ V_{\varphi} \\ V_{\text{Ķ}} \\ V_{XL} \\ V_{YL} \\ V_{ZL} \end{bmatrix}$ $V^{s}{}_{3n*1} = \begin{bmatrix} V_{X1} \\ V_{Y1} \\ V_{Z1} \\ V_{X2} \\ . \\ . \\ . \\ V_{Zn} \end{bmatrix}$ $V\ {}_{2n*1} = \begin{bmatrix} V_{X1} \\ V_{Y1} \\ V_{X2} \\ . \\ . \\ . \\ V_{xn} \\ V_{yn} \end{bmatrix}$

Matrices for Case(3)

$$F_{(2n*1)}=\begin{bmatrix} F_{(xj)} \\ F_{(yj)} \end{bmatrix} \quad F^{e}{}_{(6*1)}=\begin{bmatrix} F_{(\omega)} \\ F_{(\varphi)} \\ F_{(\text{Ϗ})} \\ F_{(XL)} \\ F_{(YL)} \\ F_{(ZL)} \end{bmatrix} \quad F^{s}{}_{(3n*1)}=\begin{bmatrix} F_{(X1)} \\ F_{(Y1)} \\ F_{(Z1)} \\ . \\ . \\ F_{(Xn)} \\ F_{(Yn)} \\ F_{(Zn)} \end{bmatrix}$$

Matrices for Case(3)

- $\Delta^{e}{}_{(6*1)} = \begin{bmatrix} \delta\omega \\ \delta\varphi \\ \delta\mathrm{K} \\ \delta X_L \\ \delta Y_L \\ \delta Z_L \end{bmatrix}$ $\Delta^{s}{}_{(3n*1)} = \begin{bmatrix} \delta_{X1} \\ \delta_{Y1} \\ \delta_{Z1} \\ . \\ . \\ \delta_{Xn} \\ \delta_{Yn} \\ \delta_{Zn} \end{bmatrix}$

Matrices for Case(3)

$$B^{e}{}_{(2n*6)} = \begin{bmatrix} \dfrac{\partial F_{(x1)}}{\partial(\omega,\varphi,\text{Қ},X_{L,}Y_{L},Z_{L})} \\ \dfrac{\partial F_{(Y1)}}{\partial(\omega,\varphi,\text{Қ},X_{L,}Y_{L},Z_{L})} \\ . \\ . \\ \dfrac{\partial F_{(xn)}}{\partial(\omega,\varphi,\text{Қ},X_{L,}Y_{L},Z_{L})} \\ \dfrac{\partial F_{(yn)}}{\partial(\omega,\varphi,\text{Қ},X_{L,}Y_{L},Z_{L})} \end{bmatrix} \quad B^{s}{}_{(2n*3n)} = \begin{bmatrix} \dfrac{\partial F_{(x1)}}{\partial(X_1,Y_1,Z_1,\ldots,X_n,Y_n,Z_n)} \\ \dfrac{\partial F_{(y1)}}{\partial(X_1,Y_1,Z_1,\ldots,X_n,Y_n,Z_n)} \\ \dfrac{\partial F_{(z1)}}{\partial(X_1,Y_1,Z_1,\ldots,X_n,Y_n,Z_n)} \\ . \\ . \\ \dfrac{\partial F_{(Zn)}}{\partial(X_1,Y_1,Z_1,\ldots,X_n,Y_n,Z_n)} \end{bmatrix}$$

Matrices for Case(3)

- $$\begin{bmatrix} V_{(2n*1)} \\ {V^e}_{(6*1)} \\ {V^s}_{(3n*1)} \end{bmatrix} + \begin{bmatrix} {B^e}_{(2n*6)} & {B^s}_{(2n*3n)} \\ -I_{(6*6)} & 0 \\ 0 & -I_{(3n*3n)} \end{bmatrix}$$

- $$\begin{bmatrix} {\Delta^e}_{(6*1)} \\ {\Delta^s}_{(3n*1)} \end{bmatrix} + \begin{bmatrix} F_{(2n*1)} \\ {F^e}_{(6*1)} \\ {F^s}_{(3n*1)} \end{bmatrix} = 0$$

Matrices for Case(3)

- $\bar{V} + \bar{B}\overline{\Delta} + \bar{F} = 0$

- $\overline{W} = \begin{bmatrix} W_{(2n*2n)} & & 0 \\ & {W^{e}}_{(6*6)} & \\ 0 & & {W^{s}}_{3n*3n} \end{bmatrix}$
- $\overline{N}\overline{\Delta} + \overline{U} = 0$
- $\overline{\Delta} = \overline{-N}^{-1}\overline{U}$

Normal Equation for Case(3)

- $\begin{bmatrix} {B^e}^T\, W\, B^e + W^e & {B^e}^T\, W\, B^s \\ {B^s}^T\, W\, B^e & {B^s}^T\, W\, B^s + W^s \end{bmatrix} \begin{bmatrix} \Delta^e{}_{(6*1)} \\ \Delta^s{}_{(3n*1)} \end{bmatrix} + \begin{bmatrix} {B^e}^T\, W\, F - W^e F^e \\ {B^s}^T\, W\, F - W^s F s \end{bmatrix}$ = 0
- $\bar{N}\ \ \bar{\Delta}+ \bar{\mathrm{U}} = 0$
- $\bar{\Delta} = \overline{-N}^{-1}\bar{\mathrm{U}}$

Case 4

- Known : - Observed Photo Coordinate (x,y)
- - Observed value for unknowns
  (Exterior Orientation Parameters $(X_L, Y_L, Z_L, \omega, \varphi, \kappa)$)
- - Observed Ground Coordinate for GCPs

- Unknown: - Exterior Orientation Parameters $(X_L, Y_L, Z_L, \omega, \varphi, \kappa)$
  - Ground Coordinate for GCPs
  - Interior orientation parameters

Case 4

- x =- f$\left[\frac{m_{11}(X-X_L)+m_{12}(Y-Y_L)+m_{13}(Z-Z_L)}{m_{31}(X-X_L)+m_{32}(Y-Y_L)+m_{33}(Z-Z_L)}\right]+\Delta X$
- Y=- f $\left[\frac{m_{21}(X-X_L)+m_{22}(Y-Y_L)+m_{23}(Z-Z_L)}{m_{31}(X-X_L)+m_{32}(Y-Y_L)+m_{33}(Z-Z_L)}\right]+\Delta Y$

References

- Wolf, Paul.R. and Dewitt, Bon A.,Elements of Photogrammetry with applications in GIS, 3rd ed., McGraw-Hill,New York, 2000

- بشار سليم عباس، فنار منصور ، ميثم البكري ، المسح التصويري التحليلي، الطبعة الاولى ، اثراء للنشر والتوزيع ، الاردن 2009