Photogrammetry II

3rd Stage

The Geometry of Aerial Stereo-Pair

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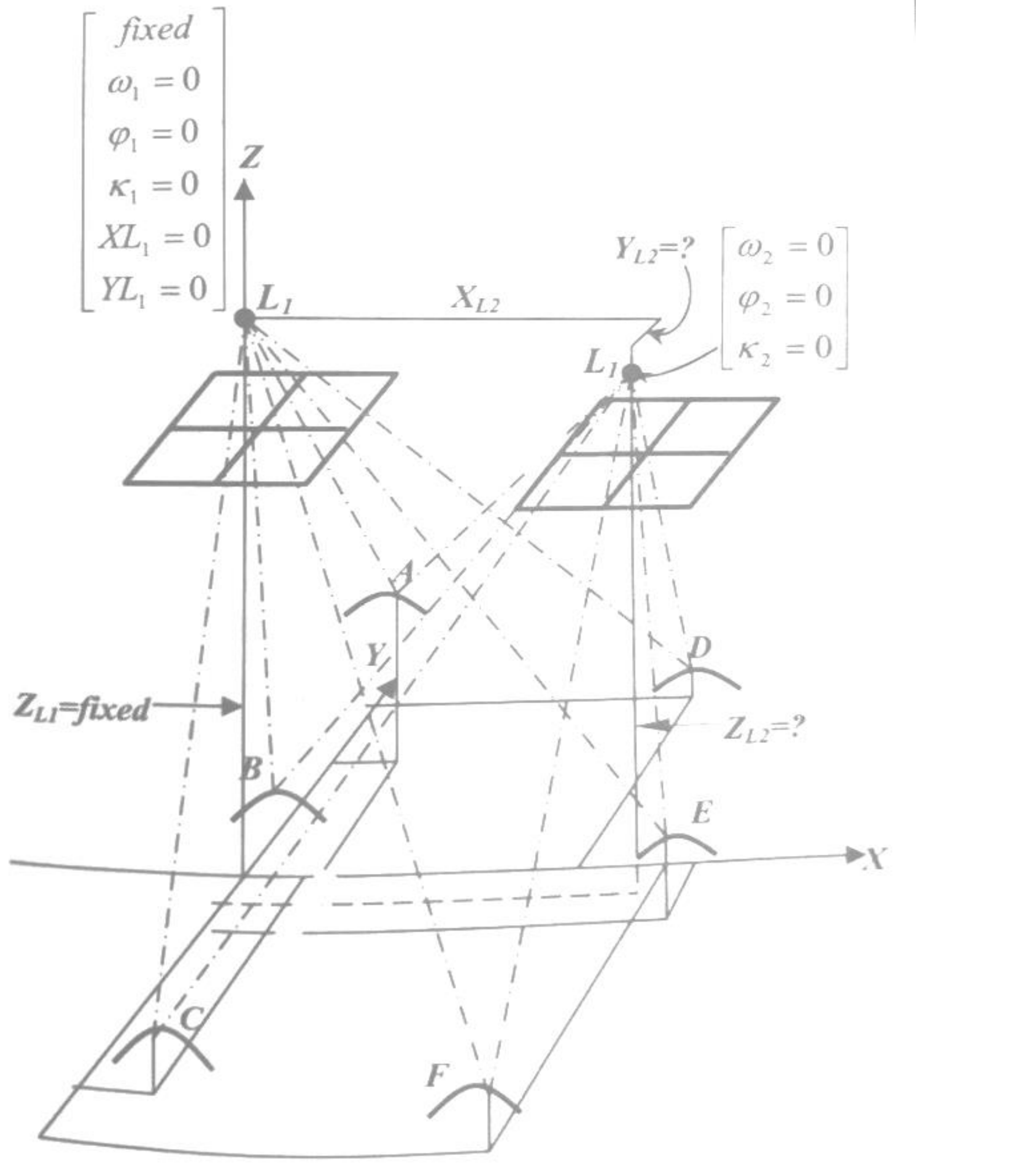
The Geometry of Aerial Stereo-Pair

- Analytical Relative Orientation
- Analytical Absolute Orientation
- Space Intersection

Analytical Relative Orientation



Analytical Relative Orientation

- Collinearity Equation
- $x = -f\left[\frac{m_{11}(X-X_L\ )+m_{12}(Y-Y_L)+m_{13}(Z-Z_L)}{m_{31}(X-X_L\ )+m_{32}(Y-Y_L)+m_{33}(Z-Z_L)}\right]$
- $y = -f\left[\frac{m_{21}(X-X_L\ )+m_{22}(Y-Y_L)+m_{23}(Z-Z_L)}{m_{31}(X-X_L\ )+m_{32}(Y-Y_L)+m_{33}(Z-Z_L)}\right]$
- Note: (5 points) minimum number of point require to do Analytical Relative Orientation

Analytical Relative Orientation

- Observation Equations
- AV - $B\,\Delta\ + F = 0$
- V= Matrix of Residuals for observations
- Δ= Matrix of Corrections for unknowns
- F= Matrix of Absolute Values

Analytical Relative Orientation

- $$B_{(4n*(5+3n)} = \begin{bmatrix} {}_{2}0^{5} & {}_{2}B^{s^{3}}_{1a} & {}_{2}0^{3} & {}_{2}0^{3} & {}_{2}0^{3} & {}_{2}0^{3} & {}_{2}0^{3} \\ {}_{2}0^{5} & {}_{2}0^{3} & {}_{2}B^{s^{3}}_{1b} & {}_{2}0^{3} & {}_{2}0^{3} & {}_{2}0^{3} & {}_{2}0^{3} \\ {}_{2}0^{5} & {}_{2}0^{3} & {}_{2}0^{3} & {}_{2}B^{s^{3}}_{1c} & {}_{2}0^{3} & {}_{2}0^{3} & {}_{2}0^{3} \\ {}_{2}0^{5} & {}_{2}0^{3} & {}_{2}0^{3} & {}_{2}0^{3} & {}_{2}B^{s^{3}}_{1d} & {}_{2}0^{3} & {}_{2}0^{3} \\ {}_{2}0^{5} & {}_{2}0^{3} & {}_{2}0^{3} & {}_{2}0^{3} & {}_{2}0^{3} & {}_{2}B^{s^{3}}_{1e} & {}_{2}0^{3} \\ {}_{2}0^{5} & {}_{2}0^{3} & {}_{2}0^{3} & {}_{2}0^{3} & {}_{2}0^{3} & {}_{2}0^{3} & {}_{2}B^{s^{3}}_{1f} \\ {}_{2}B^{e^{5}}_{2a} & {}_{2}B^{s^{3}}_{2a} & {}_{2}0^{3} & {}_{2}0^{3} & {}_{2}0^{3} & {}_{2}0^{3} & {}_{2}0^{3} \\ {}_{2}B^{e^{5}}_{2b} & {}_{2}0^{3} & {}_{2}B^{s^{3}}_{2b} & {}_{2}0^{3} & {}_{2}0^{3} & {}_{2}0^{3} & {}_{2}0^{3} \\ {}_{2}B^{e^{5}}_{2c} & {}_{2}0^{3} & {}_{2}0^{3} & {}_{2}B^{s^{3}}_{2c} & {}_{2}0^{3} & {}_{2}0^{3} & {}_{2}0^{3} \\ {}_{2}B^{e^{5}}_{2d} & {}_{2}0^{3} & {}_{2}0^{3} & {}_{2}0^{3} & {}_{2}B^{s^{3}}_{2d} & {}_{2}0^{3} & {}_{2}0^{3} \\ {}_{2}B^{e^{5}}_{2e} & {}_{2}0^{3} & {}_{2}0^{3} & {}_{2}0^{3} & {}_{2}0^{3} & {}_{2}B^{s^{3}}_{2e} & {}_{2}0^{3} \\ {}_{2}B^{e^{5}}_{2f} & {}_{2}0^{3} & {}_{2}0^{3} & {}_{2}0^{3} & {}_{2}0^{3} & {}_{2}0^{3} & {}_{2}B^{s^{3}}_{2f} \end{bmatrix}$$

Analytical Relative Orientation

- $\Delta = \begin{bmatrix} \Delta^{e}{}_{2} \\ \Delta^{s}{}_{A} \\ \Delta^{s}{}_{B} \\ \Delta^{s}{}_{C} \\ \Delta^{s}{}_{D} \\ \Delta^{s}{}_{E} \\ \Delta^{s}{}_{F} \end{bmatrix}$ V = $\begin{bmatrix} V_{1a} \\ V_{1b} \\ V_{1c} \\ V_{1d} \\ V_{1e} \\ V_{1f} \\ V_{2a} \\ V_{2b} \\ V_{2c} \\ V_{2d} \\ V_{2e} \\ V_{2f} \end{bmatrix}$ F = $\begin{bmatrix} F_{1a} \\ F_{1b} \\ F_{1c} \\ F_{1d} \\ F_{1e} \\ F_{1f} \\ F_{2a} \\ F_{2b} \\ F_{2c} \\ F_{2d} \\ F_{2e} \\ F_{2f} \end{bmatrix}$

Analytical Relative Orientation

- $_2 0^5 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ $_2 0^3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

- $_2 {B^{e}}^{5}_{2p} = \begin{bmatrix} (b_{p11})_2 & (b_{p12})_2 & (b_{p13})_2 & (-b_{p15})_2 & (-b_{p16})_2 \\ (b_{p21})_2 & (b_{p22})_2 & (b_{p23})_2 & (-b_{p25})_2 & (-b_{p26})_2 \end{bmatrix}$

${B^{s}}^{3}_{ip} = \begin{bmatrix} (b_{p14})_i & (b_{p15})_i & (b_{p16})_i \\ (b_{p24})_i & (b_{p25})_i & (b_{p26})_i \end{bmatrix}$

Analytical Relative Orientation

- $\Delta^{e}_{2} = \begin{bmatrix} d\omega_2 \\ d\varphi_2 \\ d\kappa_2 \\ dY_{L2} \\ dZ_{L2} \end{bmatrix} \Delta^{s}_{p} = \begin{bmatrix} dX_p \\ dY_p \\ dZ_p \end{bmatrix} \quad F_{ip} = \begin{bmatrix} (J_p)_i \\ (K_p)_i \end{bmatrix} V_{ip} = \begin{bmatrix} (v_{xp})_i \\ (v_{yp})_i \end{bmatrix}$

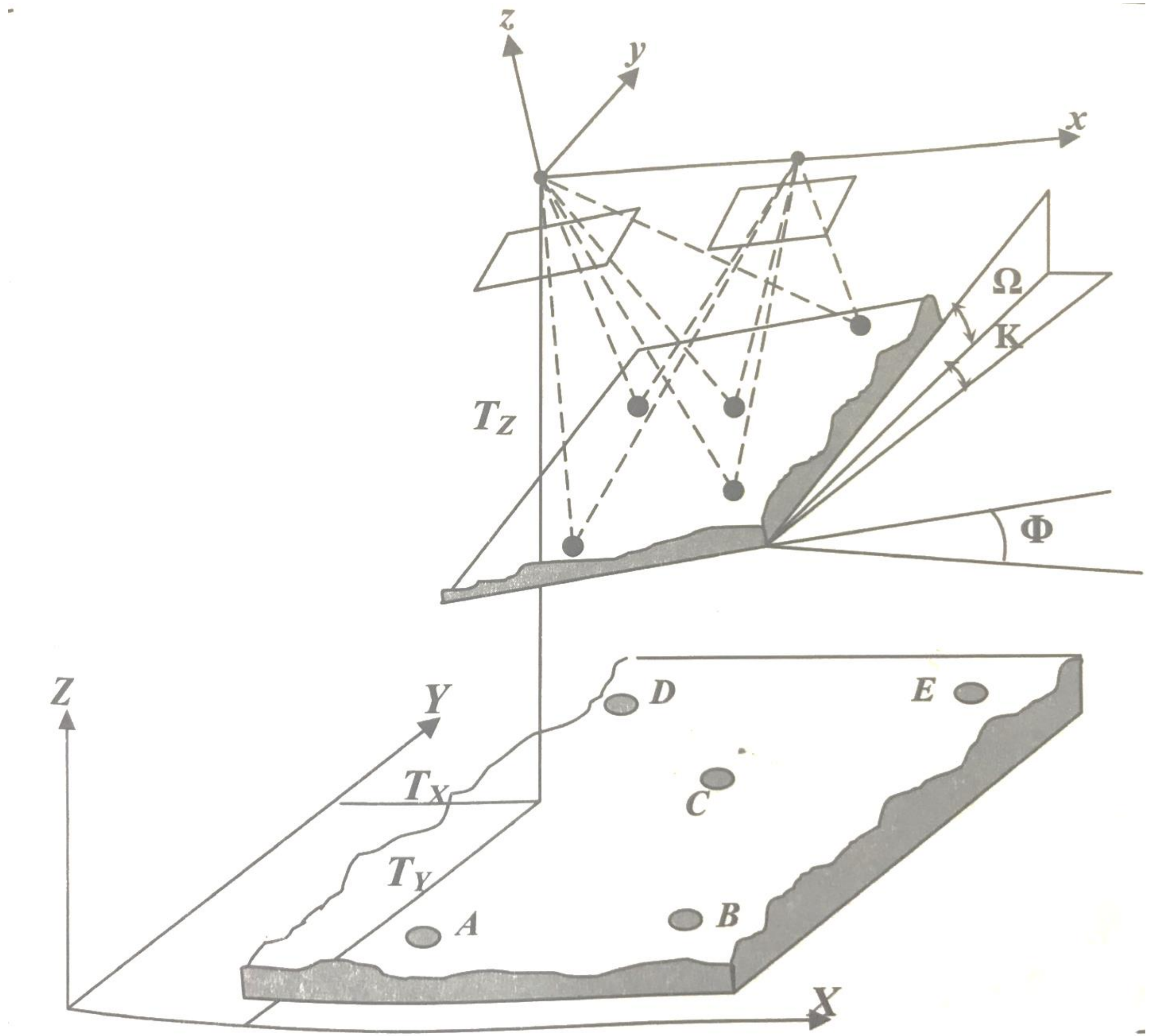
Analytical Relative Orientation

- $$\begin{bmatrix} N^{e}{}_{2} & \bar{N}_a & \bar{N}_b & \bar{N}_c & \bar{N}_d & \bar{N}_e & \bar{N}_f \\ \bar{N}_a{}^{T} & N^{s}{}_{a} & {}_{3}0^{3} & {}_{3}0^{3} & {}_{3}0^{3} & {}_{3}0^{3} & {}_{3}0^{3} \\ \bar{N}_b{}^{T} & {}_{3}0^{3} & N^{s}{}_{b} & {}_{3}0^{3} & {}_{3}0^{3} & {}_{3}0^{3} & {}_{3}0^{3} \\ \bar{N}_c{}^{T} & {}_{3}0^{3} & {}_{3}0^{3} & N^{s}{}_{c} & {}_{3}0^{3} & {}_{3}0^{3} & {}_{3}0^{3} \\ \bar{N}_d{}^{T} & {}_{3}0^{3} & {}_{3}0^{3} & {}_{3}0^{3} & N^{s}{}_{d} & {}_{3}0^{3} & {}_{3}0^{3} \\ \bar{N}_e{}^{T} & {}_{3}0^{3} & {}_{3}0^{3} & {}_{3}0^{3} & {}_{3}0^{3} & N^{s}{}_{e} & {}_{3}0^{3} \\ \bar{N}_f{}^{T} & {}_{3}0^{3} & {}_{3}0^{3} & {}_{3}0^{3} & {}_{3}0^{3} & {}_{3}0^{3} & N^{s}{}_{f} \end{bmatrix} \bullet \begin{bmatrix} \Delta^{e}{}_{2} \\ \Delta^{s}{}_{A} \\ \Delta^{s}{}_{B} \\ \Delta^{s}{}_{C} \\ \Delta^{s}{}_{D} \\ \Delta^{s}{}_{E} \\ \Delta^{s}{}_{F} \end{bmatrix} = \begin{bmatrix} U^{e}{}_{2} \\ U^{s}{}_{A} \\ U^{s}{}_{B} \\ U^{s}{}_{C} \\ U^{s}{}_{D} \\ U^{s}{}_{E} \\ U^{s}{}_{F} \end{bmatrix}$$

Analytical Relative Orientation

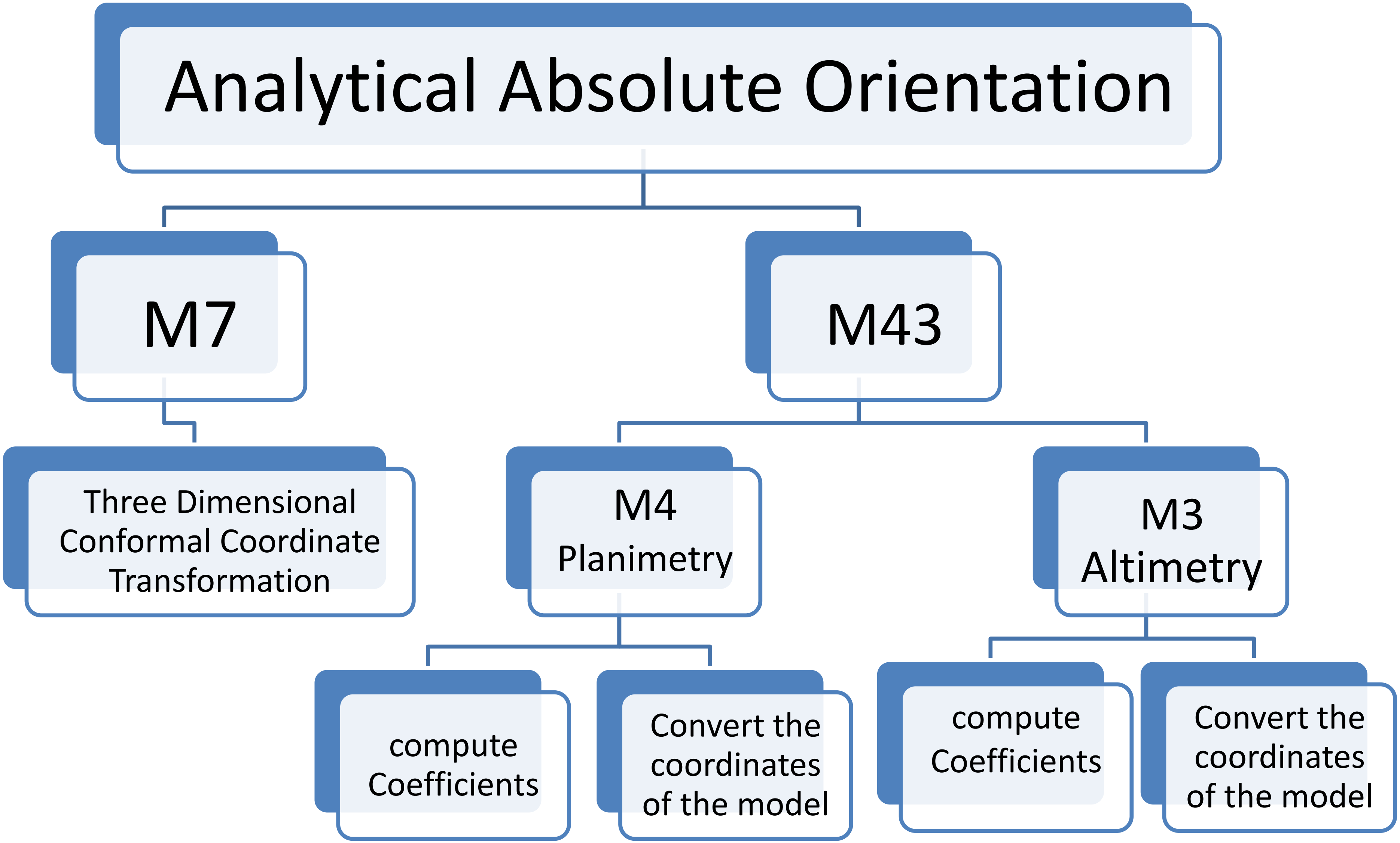
- $N^{e}{}_{2} = \sum_{p=a}^{f} (B^{e^T}{}_{2p} \; B^{e}{}_{2p})$ (5*5) sub matrix
- $\overline{N}_{P} = (B^{e^T}{}_{2p} B^{s}{}_{2p})$ (5*3) sub matrix
- $N^{s}{}_{P} = (B^{s^T}{}_{1p} B^{e}{}_{1p}) + (B^{s^T}{}_{2p} B^{e}{}_{2p})$ (3*5) sub matrix
- $U^{e}{}_{2} = \sum_{p=a}^{f} (B^{e^T}{}_{2p} F_{2P})$ (5*1) sub matrix
- $U^{s}{}_{P} = (B^{s^T}{}_{1p} F_{1p}) + (B^{s^T}{}_{2p} F_{2p})$ (3*1) sub matrix
- $_{3}0^{3} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Analytical Absolute Orientation



Analytical Absolute Orientation

- $\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = s \ * \mathrm{M}(\Omega, \phi, \text{Ҁ})^* \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix}$
- Whereas :-
- X,Y,Z :Ground Coordinate
- x,y,z :Model Coordinate
- $\mathrm{M}(\Omega, \phi, \text{Ҁ})$:Rotation Matrix
- $T_x T_y T_z$:Translation Distances
- E = s . M . X + E_0
- Note: (2 Horizontal G.C.P&3 Vertical G.C.P) minimum number of point require to do Analytical Absolute Orientation



Analytical Absolute Orientation
M7
M43
Three Dimensional Conformal Coordinate Transformation
M4 Planimetry
M3 Altimetry
compute Coefficients
Convert the coordinates of the model
compute Coefficients
Convert the coordinates of the model

Analytical Absolute Orientation M4 Planimetry

- X = xq - yλ + T_x
- Y = yq + xλ + T_y
- We need min.(2 Horizontal G.C.P)
- Whereas :-
- q = s $\cos K$
- λ = s $\sin K$
- S =$(q^2 + \lambda^2)^{\frac{1}{2}}$
- K = $\tan^{-1}$(λ/q)

Analytical Absolute Orientation M4 Planimetry

- $$\begin{bmatrix} V_{x1} \\ V_{y1} \\ . \\ . \\ . \\ V_{xn} \\ V_{yn} \end{bmatrix} = \begin{bmatrix} x_1 & -y_1 & 1 & 0 \\ y_1 & x_1 & 0 & 1 \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ x_n & -y_n & 1 & 0 \\ y_n & x_n & 0 & 1 \end{bmatrix} \begin{bmatrix} q \\ \lambda \\ T_x \\ T_y \end{bmatrix} - \begin{bmatrix} X_1 \\ Y_1 \\ . \\ . \\ . \\ X_n \\ Y_n \end{bmatrix}$$

Analytical Absolute Orientation M3 Altimetry

$Z\text{-}z = y\ \Omega - x\ \Phi + T_z$ (We need min. 3 Vertical G.C.P)

- $$\begin{bmatrix} V_{z1} \\ V_{z1} \\ . \\ . \\ . \\ V_{zn} \end{bmatrix} = \begin{bmatrix} y_1 & -x_1 & 1 \\ y_2 & -x_2 & 1 \\ . & . & . \\ . & . & . \\ . & . & . \\ y_n & -x_n & 1 \end{bmatrix} \begin{bmatrix} \Omega \\ \Phi \\ T_z \end{bmatrix} - \begin{bmatrix} Z_1 & - & z_1 \\ Z_2 & - & z_2 \\ . & . & . \\ . & . & . \\ Z_n & - & z_n \end{bmatrix}$$

References

- Wolf, Paul.R. and Dewitt, Bon A.,Elements of Photogrammetry with applications in GIS, 3rd ed., McGraw-Hill,New York, 2000

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