Photogrammetry II

3rd Stage

Two dimensional transformation

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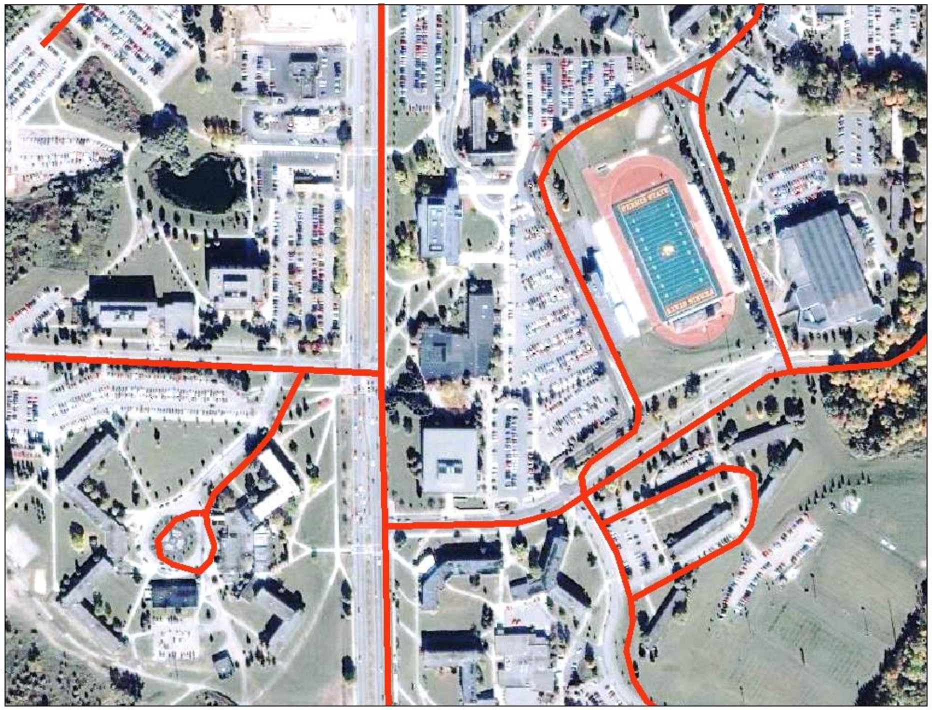
2D Coordinate Transformations

- Why?
- Integrating of maps and spatial data in local coordinate system into a world database system.



2D Spatial Transformations

- **Three different transformation primitives for the Similarity transformation:**

- **Translation-** origin is moved, axes do not rotate i.e.:

$$Xn = X_0 \pm DX_0 \qquad Y_n = Y_0 \pm DY_0$$

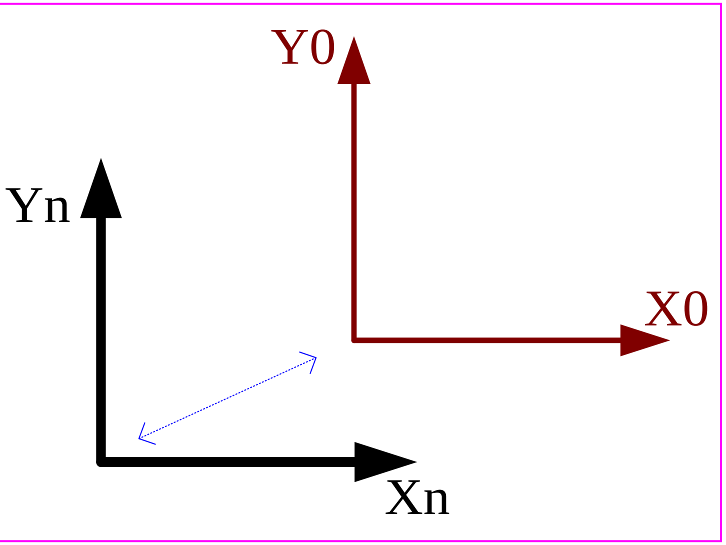
- **Scaling** - both origin and axes are fixed, scale change

$$Xn = s_X \times X_0 \qquad Y_n = s_Y \times Y_0$$

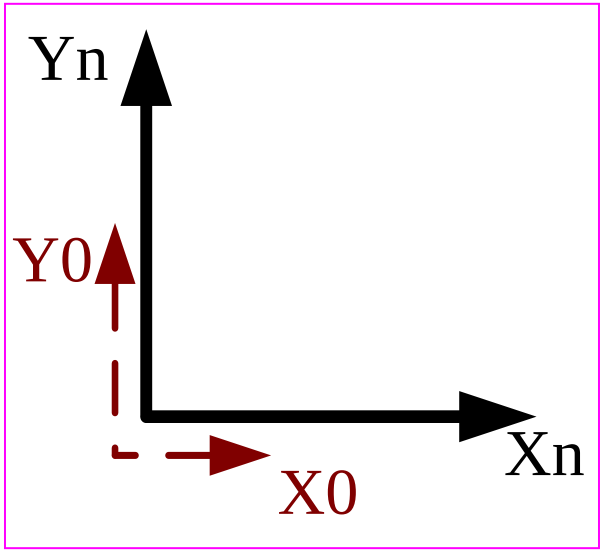
- **Rotation** - origin fixed, axes move (rotate about origin)

$$Xn = X_0 \cdot \cos(\alpha) + Y_0 \cdot \sin(\alpha); \qquad Y_n = - X_0 \cdot \sin(\alpha) + Y_0 \cdot \cos(\alpha)$$

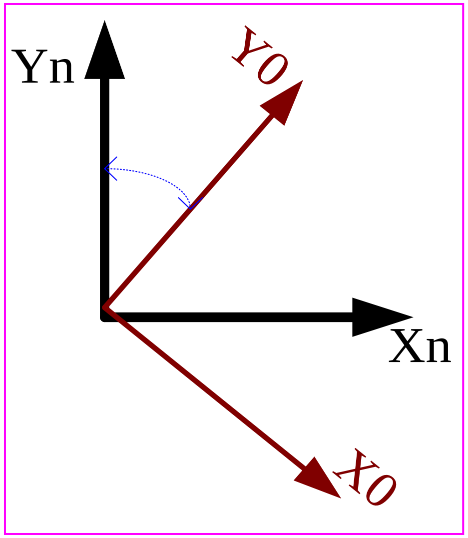
Two-Dimensional Geographic Transformations



Translation



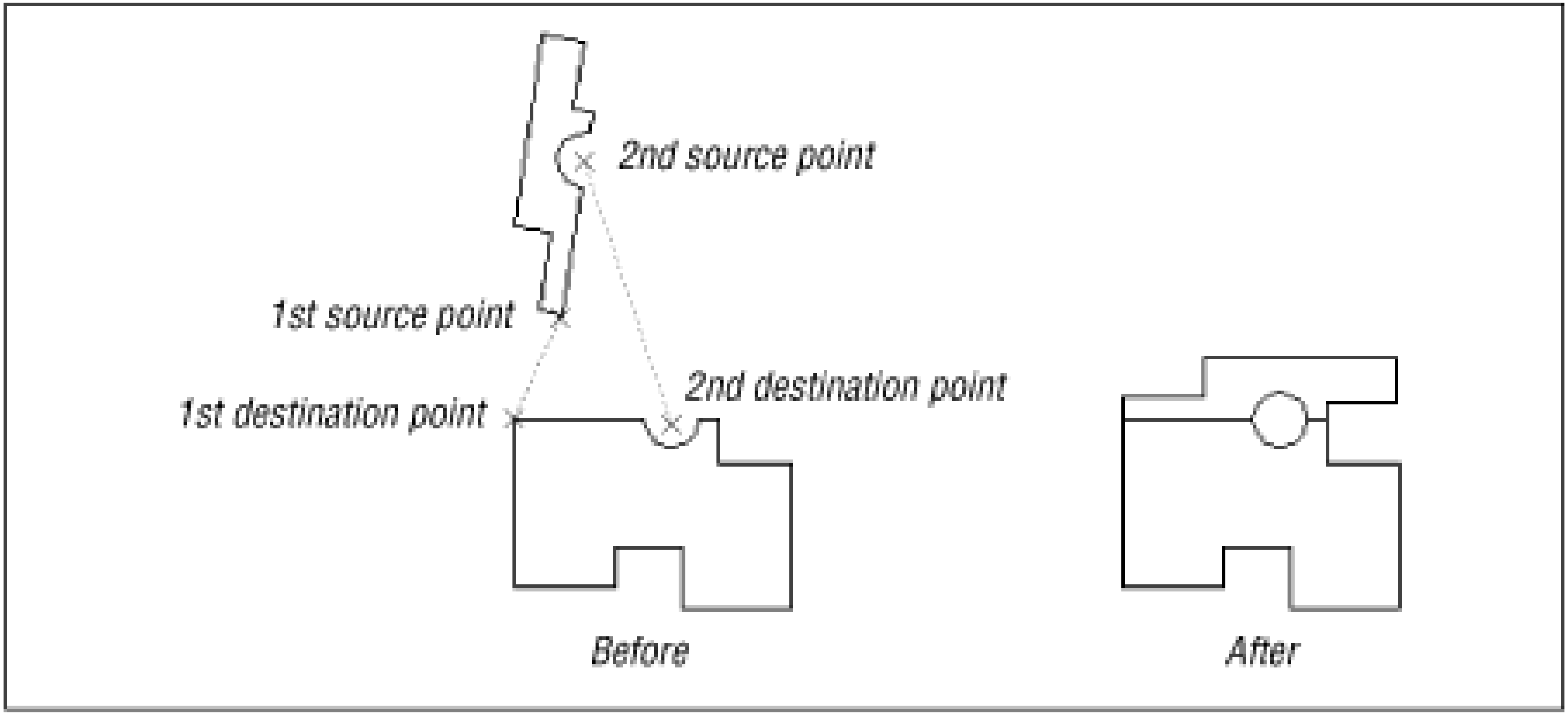
Scaling



Rotation

Conformal Transformation

- Moves and rotates objects in 2D and 3D space. Additionally, you can scale the objects based on alignment points when using the 2D option.



Isogonal Affine Transformation or Conformal/Similarity Transformation

- Isogonal: having equal angles
- Impose additional condition of equal scale ($S = C_x = C_y$) yielding <u>4 parameters</u>: S, α, DX_0, DY_0
- Moves and rotates and scale objects in 2D space.

$$\begin{bmatrix} Xn \\ Yn \end{bmatrix} = \begin{bmatrix} a & b \\ -b & a \end{bmatrix} * \begin{bmatrix} Xo \\ Yo \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix} = S * \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} * \begin{bmatrix} Xo \\ Yo \end{bmatrix} + \begin{bmatrix} DXo \\ DYo \end{bmatrix}$$

<u>4 parameters</u>: S –scale, α rotation, DX0, DY0 shifts in X and Y.

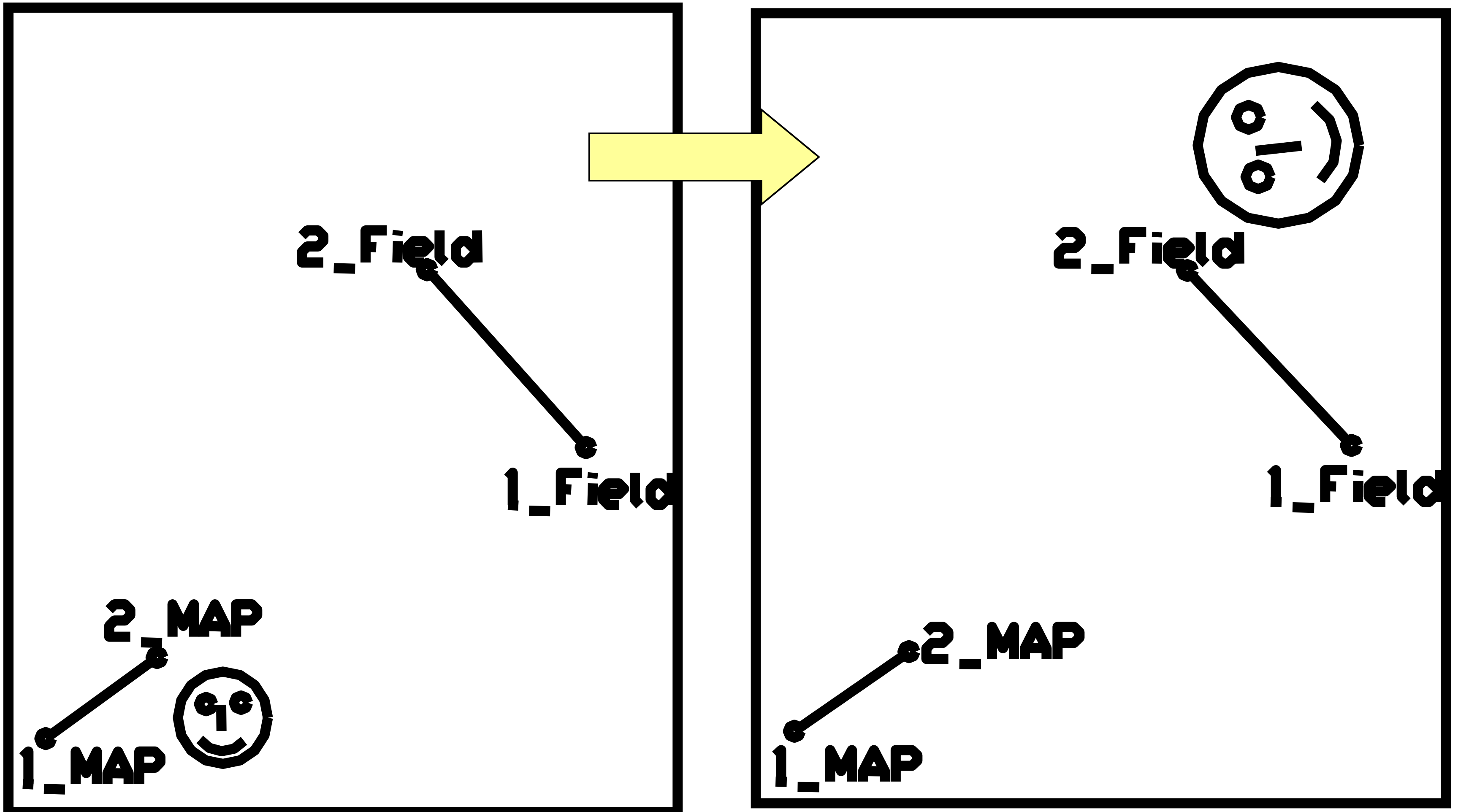
Xn,Yn are the transformed coordinates.

Xo, Yo are the original coordinates.

Two given points are required (X1,Y1 and X2,Y2)

Conformal Transformation

- Moves and rotates and scale objects in 2D space.
- Often called similarity Transformation since the basic shape remain similar after the transformation



Conformal Transformation

- The formulas can be written in different forms

1. To compute the parameters given the coordinates
2. To compute the new coordinates given the parameters

$$Xn = S \cdot \cos\alpha \cdot X_0 + S \cdot \sin\alpha \cdot Y_0 + DX_0$$

$$Yn = -S \cdot \sin(\alpha) \cdot X_0 + S \cdot \cos(\alpha) \cdot Y_0 + DY_0$$

$$Xn = a \cdot X_0 + b \cdot Y_0 + c$$

$$Yn = -b \cdot X_0 + a \cdot Y_0 + d$$

$$\begin{bmatrix} Xn \\ Yn \end{bmatrix} = \begin{bmatrix} X_0 & Y_0 & 1 & 0 \\ Y_0 & -X_0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

Conformal/Similarity Transformation

- The formulas can be written in different forms

3. To compute the old coordinates given the parameters and new coordinates (back substitution)

$$Xn = a \cdot X_0 + b \cdot Y_0 + c$$
$$Yn = -b \cdot X_0 + a \cdot Y_0 + d$$

$$(Xn - c) = a \cdot X_0 + b \cdot Y_0$$
$$(Yn - d) = -b \cdot X_0 + a \cdot Y_0$$

Multiply Eq. 1 by a, Eq 2 by b

$$a(Xn - c) = a^2 \cdot X_0 + ab \cdot Y_0$$
$$-b(Yn - d) = b^2 \cdot X_0 - ab \cdot Y_0$$

Add equations to get X_0

$$X_0 = \frac{a(Xn - c) - b(Yn - d)}{a^2 + b^2}$$

$$Y_0 = \frac{b(Xn - c) + a(Yn - d)}{a^2 + b^2}$$

The same operation to obtain Y_0

Conformal Transformation: Example

- Moves, rotates and scale objects in 2D space.

No	X_o (map)	Y_o (map)	X_n (ground)	Y_n (ground)
1	10	10	350	190
2	80	60	250	300

$$\begin{bmatrix} Xn \\ Yn \end{bmatrix} = \begin{bmatrix} a & b \\ -b & a \end{bmatrix} * \begin{bmatrix} Xo \\ Yo \end{bmatrix} + \begin{bmatrix} Cx \\ Cy \end{bmatrix}$$

$$\begin{aligned} 350 &= a \cdot 10 + b \cdot 10 + C_x \\ 190 &= -b \cdot 10 + a \cdot 10 + Cy \\ 250 &= a \cdot 80 + b \cdot 60 + C_x \\ 300 &= -b \cdot 80 + a \cdot 60 + C_y \end{aligned} \quad \Rightarrow \quad \begin{bmatrix} 350 \\ 190 \\ 250 \\ 300 \end{bmatrix} = \begin{bmatrix} 10 & 10 & 1 & 0 \\ 10 & -10 & 0 & 1 \\ 80 & 60 & 1 & 0 \\ 60 & -80 & 0 & 1 \end{bmatrix} * \begin{bmatrix} a \\ b \\ C_x \\ C_y \end{bmatrix}$$

Conformal Transformation: Example

$$\begin{bmatrix} a \\ b \\ C_x \\ C_y \end{bmatrix} = \begin{bmatrix} 10 & 10 & 1 & 0 \\ 10 & -10 & 0 & 1 \\ 80 & 60 & 1 & 0 \\ 60 & -80 & 0 & 1 \end{bmatrix}^{-1} * \begin{bmatrix} 350 \\ 190 \\ 250 \\ 300 \end{bmatrix} = \begin{bmatrix} -0.203 \\ -1.716 \\ 369.189 \\ 174.865 \end{bmatrix}$$

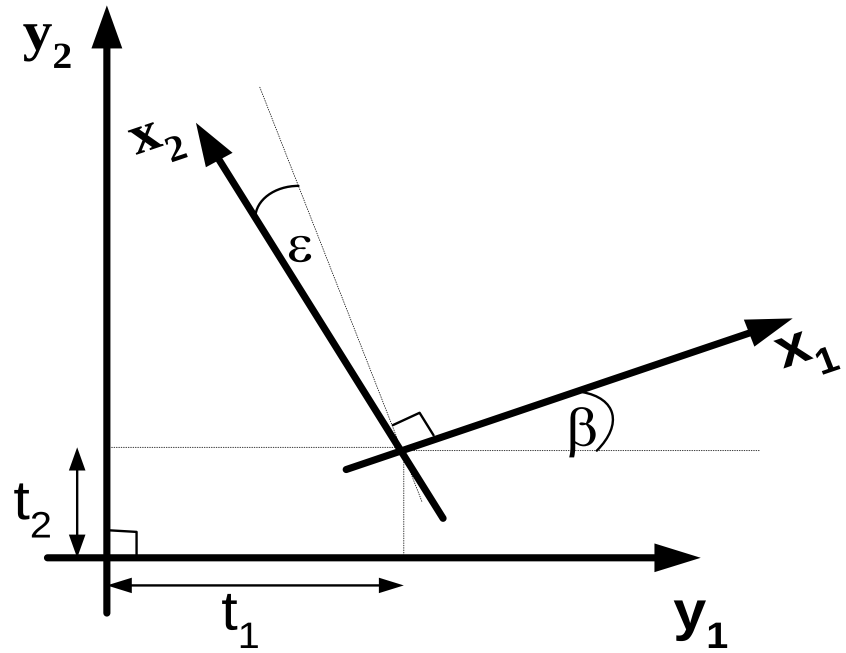
- Transform a given point X_0=121.48, Y_0=22.78

$$\begin{bmatrix} Xn \\ Yn \end{bmatrix} = \begin{bmatrix} -0.203 & -1.716 \\ 1.716 & -0.203 \end{bmatrix} * \begin{bmatrix} 121.48 \\ 22.78 \end{bmatrix} + \begin{bmatrix} 369.189 \\ 174.865 \end{bmatrix} = \begin{bmatrix} 310.59 \\ 373.6773 \end{bmatrix}$$

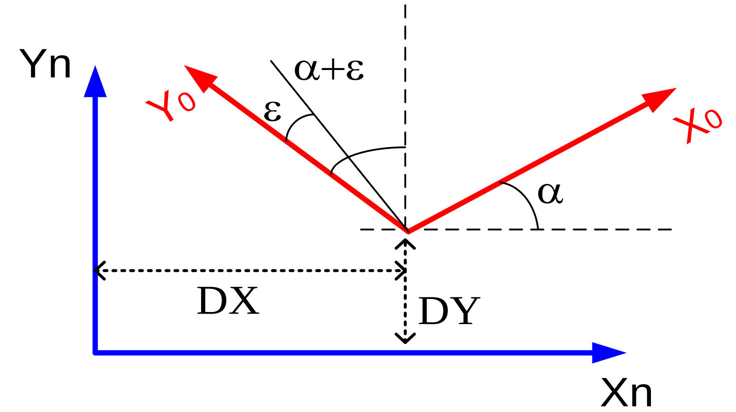
- The point is transformed to Xn=310.59, Yn=373.6773

Affine Transformation

- Used in photogrammetry for:
  - Transform comparator coordinates to photo coordinates and used for correcting film distortion
  - Transform model coordinates to survey coordinates
- Property
  - Carry parallel lines into parallel lines
  - Does not have to preserve orthogonality



Affine Transformation



Physical interpretation:

$$Xn = C_x \cdot (X_0) \cdot \cos(\alpha) - C_y \cdot (Y_0) \cdot \sin(\alpha + \varepsilon) + DX_0$$

$$Yn = C_x \cdot (X_0) \cdot \sin(\alpha) + C_y \cdot (Y_0) \cdot \cos(\alpha + \varepsilon) + DY_0$$

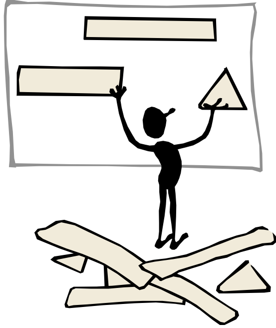
6 parameters: Cx, Cy, α, ε, Dx_0, $Dy_{0,}$ and in linear form:

$$a = C_x \cos\alpha \qquad d = C_x \sin(\alpha)$$

$$b = -C_y \sin(\alpha + \varepsilon) \qquad e = C_y \cos(\alpha + \varepsilon)$$

$$c = Dx_0 \qquad f = Dy_0$$

$$\begin{bmatrix} Xn \\ Yn \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} * \begin{bmatrix} Xo \\ Yo \end{bmatrix} + \begin{bmatrix} Cx \\ Cy \end{bmatrix}$$



2-D Affine Transformation

- The formulas for an affine transformation:

$$\begin{bmatrix} x_T \\ y_T \end{bmatrix} = \begin{bmatrix} a_A & b_A \\ d_A & e_A \end{bmatrix} \cdot \begin{bmatrix} x_s \\ y_s \end{bmatrix} + \begin{bmatrix} c_A \\ f_A \end{bmatrix}$$

- If n control points are measured, this Equation is reorganized as follows:

$$\underbrace{\begin{bmatrix} x_{T1} \\ y_{T1} \\ \vdots \\ x_{Tn} \\ y_{Tn} \end{bmatrix}}_{y} \approx \underbrace{\begin{bmatrix} x_{s1} & y_{s1} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_{s1} & y_{s1} & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_{sn} & y_{sn} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_{sn} & y_{sn} & 1 \end{bmatrix}}_{A} \cdot \underbrace{\begin{bmatrix} a_A \\ b_A \\ c_A \\ d_A \\ e_A \\ f_A \end{bmatrix}}_{\xi}$$

Credits

- University of Texas
- University of Florida
- University of Calgary