Communication Electronics

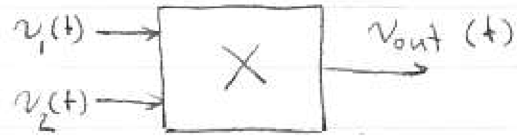
Fourth Class

Electronics and Communications Engineering Department

Analog Multipliers:

The analog multiplier is a circuit where the o/p voltage V_{out} is proportional to the product of the two i/p v_1 and v_2,

where $V_{out} = K v_1 v_2$



K: is the gain constant of the multiplier.

In practical multipliers, the o/p is related to any one of the inputs v_1 & v_2 by a generalized expression of the form

$$v_{out} = \underbrace{K v_1 v_2}_{\text{ideal o/p}} + \underbrace{[K_1 v_2 + K_2 v_1 + K_o]}_{\text{offset term}} + \underbrace{f(v_1, v_2)}_{\text{nonlinearity}}$$

K_o, K_1, and K_2 define the amount of deviation from ideal case

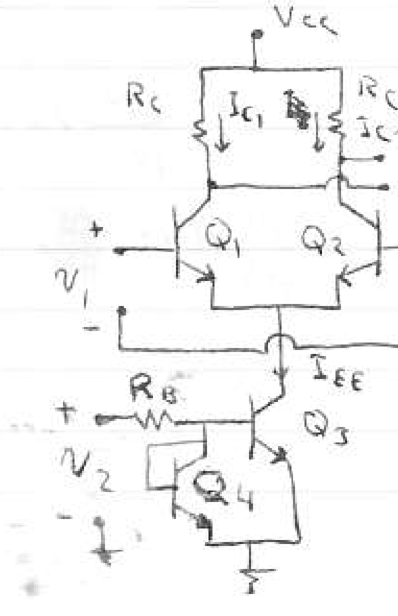
K_o: is the measure of the offset voltage due to the o/p with both v_1 and $v_2 = 0$.

K_1 & K_2: the measure of the offset voltages associated with v_2 & v_1

$f(v_1, v_2)$: represents the amount of deviation from linearity at the o/p which can not be reduced under any combination of I/p values or offset adjusments.

Variable - Transconductance Multiplier

To obtain an o/p proportional to the product of the two I/p signals (v_1, v_2), we will use the cct shown



$$I_{C1} = I_{S1} \exp \frac{V_{BE1}}{V_T}$$

$I_{E_2} = I_{S2} \exp \frac{V_{BE2}}{V_T}$ where $V_T = KT/q$

I_s : saturation current

for $I_{S_1} = I_{S_2}$

$$\frac{I_{C1}}{I_{E_2}} = \exp\left[\frac{V_{BE_1} - V_{BE_2}}{V_T}\right]$$

$$V_{BE_1} - V_{BE_2} = V_1$$

$$\Rightarrow \frac{I_{C1}}{I_{E2}} = \exp \frac{V_1}{V_T} \quad ----(1)$$

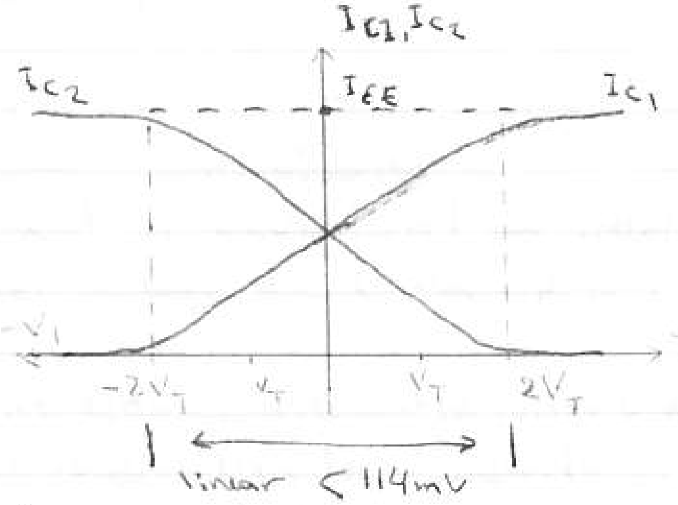
$$I_{C_1} + I_{C_2} \simeq I_{E_1} + I_{E_2} = I_{EE} \quad -----(2)$$

From (1) and (2)

$$I_{C1} = \frac{I_{EE}}{1 + \exp\left(\frac{-V_1}{V_T}\right)}$$

and

$$I_{C2} = \frac{I_{EE}}{1 + \exp\left(\frac{V_1}{V_T}\right)}$$



$$\Delta I_C = I_{C_1} - I_{C_2} = I_{EE} \tanh\left(\frac{V_1}{2V_T}\right) \quad ----(3)$$

where $\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

$$\Delta V_O = V_{O_1} - V_{O2} = \Delta I_C R_C = R_C I_{EE} \tanh\left(\frac{V_1}{2V_T}\right)$$

For linear operation $V_1 \ll V_T$ so that $\tanh \frac{V_1}{2V_T} \simeq \frac{V_1}{2V_T}$

$\Rightarrow V_1$ few millivolts

From (2) and (3)

$$\Rightarrow I_{C_1} \simeq \frac{I_{EE}}{2}\left(1 + \tanh \frac{V_1}{2V_T}\right) \simeq \frac{I_{EE}}{2}\left(1 + \frac{V_1}{2V_T}\right)$$

$$I_{C_2} = \frac{I_{EE}}{2}\left(1 - \tanh \frac{V_1}{2V_T}\right) \simeq \frac{I_{EE}}{2}\left(1 - \frac{V_1}{2V_T}\right)$$

$$\Rightarrow \Delta I_C = \frac{I_{EE} V_1}{2V_T}$$

Note that Q_3 and Q_4 is a current mirror

$$\Rightarrow I_{R_B} \simeq I_{EE} \quad \text{for } \beta >> 2$$

$$\Rightarrow I_{EE} = \frac{V_2 - V_{BE}}{R_B}$$

for $V_2 >> V_{BE}$ $\Rightarrow$ $I_{EE} = \dfrac{V_2}{R_B}$

$$\Rightarrow \Delta I_C = V_1 V_2 \frac{1}{2 R_B V_T}$$

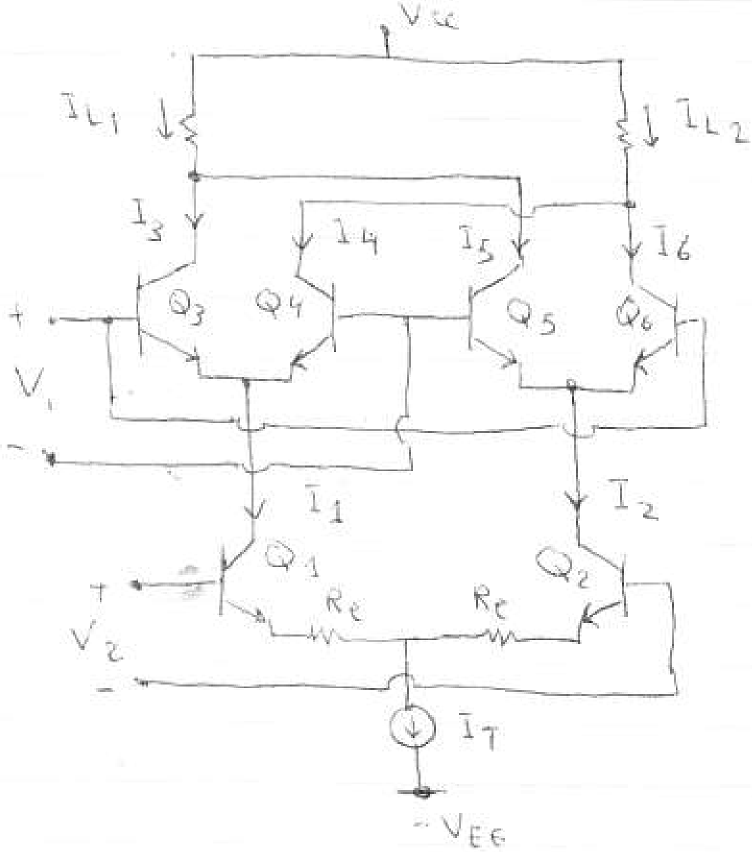
<u>Disadvantages :</u>

1) The cct operates in two quadrants since polarity of V_2 can not be reversed (because the current mirror cct will not operate)
2) The I/p dynamic range of V_1 is extremely small for linear operation

<u>Four - Quadrant Multiplier</u>

The drawbacks of the two - quadrant multiplier can be solved by using two differential stages in parallel and cross-coupling their o/p's. The cct can be operated in four quadrant since both V_1 and V_2 are fully differential and can reverse polarity.

Assuming that all devices are well matched, then



$$I_T = I_1 + I_2$$

$$I_1 = I_3 + I_4$$

$$I_2 = I_5 + I_6$$

the o/p current is

$$\Delta I_L = I_{L_1} - I_{L2}$$

$$= (I_3 + I_5) - (I_4 + I_6)$$
$$= (I_3 - I_4) - (I_6 - I_5)$$

using eqn (3) then

$$I_3 - I_4 = I_1 \tanh\left(\frac{V_1}{2V_T}\right)$$

$$I_6 - I_5 = I_2 \tanh\left(\frac{V_1}{2V_T}\right)$$

$$\Rightarrow \Delta I_L = (I_1 - I_2) \tanh\left(\frac{V_1}{2V_T}\right)$$

for Q_1 & Q_2

$$V_2 - V_{BE_1} + V_{BE_2} - I_1 Re + I_2 R_E = 0$$

$$\Rightarrow V_2 - V_{BE_1} + V_{BE_2} - I_{od} R_E = 0$$

$$I_{C_1} = \frac{I_T}{1+\exp\left(\frac{-V_2 + I_{od} R_e}{V_T}\right)} \quad , \quad I_{C_2} = \frac{I_T}{1+\exp\left(\frac{V_2 - I_{od} R_e}{V_T}\right)}$$

$$\Rightarrow I_{od} = \Delta I_c = I_1 - I_2 = I_T \tanh\left(\frac{V_2 - I_{od} R_e}{2V_T}\right)$$

For $(V_2 - I_{od} R_e) < 2V_T$

$$\Rightarrow I_{od} = I_T \frac{V_2 - I_{od} R_e}{2V_T}$$

if R_e is large so that $I_{od} R_e \gg 2V_T$

$$\Rightarrow I_{od} = \frac{I_T \frac{V_2}{2V_T}}{1 + I_T \frac{R_e}{2V_T}} \simeq I_T \frac{V_2}{I_T R_e}$$

$\therefore$ Q_1 and Q_2 functions as a voltage to current conver. and if $V_1 \ll V_T$ then

$$\Delta I_L = V_1 V_2 \frac{1}{2 R_e V_T}$$

<u>Disadvantages</u>:-

For linear operation V_1 must be kept very small (few millivolts)

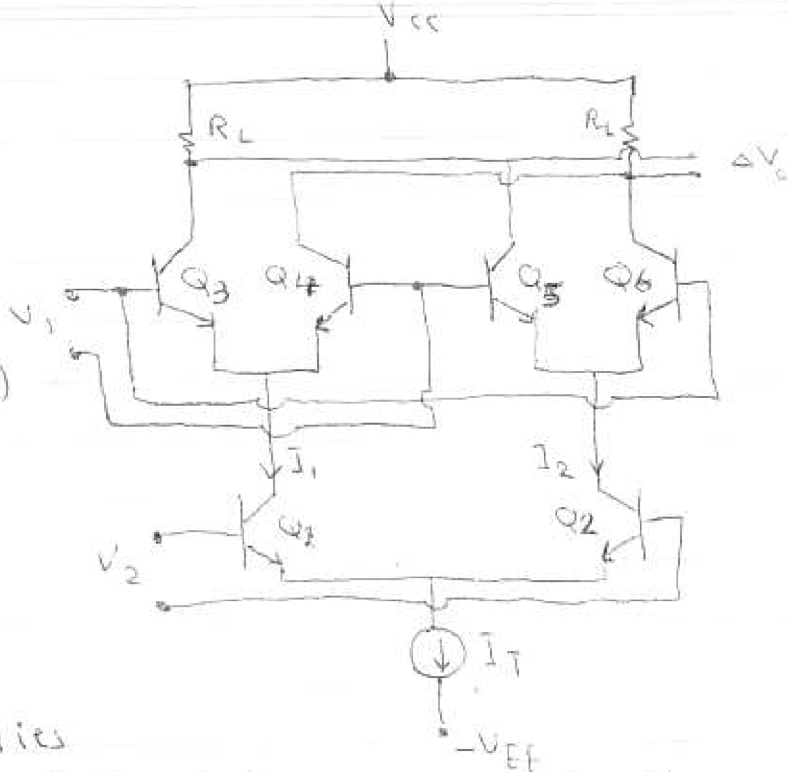
* The 4Q Modulator (Balanced Modulator)

For the cct shown below the term for the ΔI_L is the same as the previouse cct that is:

$$\Delta I_L = (I_1 - I_2) \tanh\left(\frac{V_1}{2V_T}\right)$$

for Q_1 & Q_2

$$I_1 - I_2 = I_T \tanh\left(\frac{V_2}{2V_T}\right)$$



$$\Rightarrow \Delta I_L = I_T \tanh\left(\frac{V_2}{2V_T}\right) \tanh\left(\frac{V_1}{2V_T}\right)$$

for V_1 & V_2 very small

$$\Delta V_o = R_L I_T \left(\frac{V_1}{2V_T}\right)\left(\frac{V_2}{2V_T}\right)$$

This cct has three categories depending on the magnitude of V_1 and V_2 compared to V_T

1) if V_1 & $V_2 < V_T$ $\Rightarrow$ linear operation and the cct behaves as multiplier developing the product of V_1 & V_2

2) if V_1 or $V_2 > V_T$, this will case the transistors to which that signal is applied to behave as switches This effectively multiplies the applied small signal by a square wave and the cct acts as a modulator

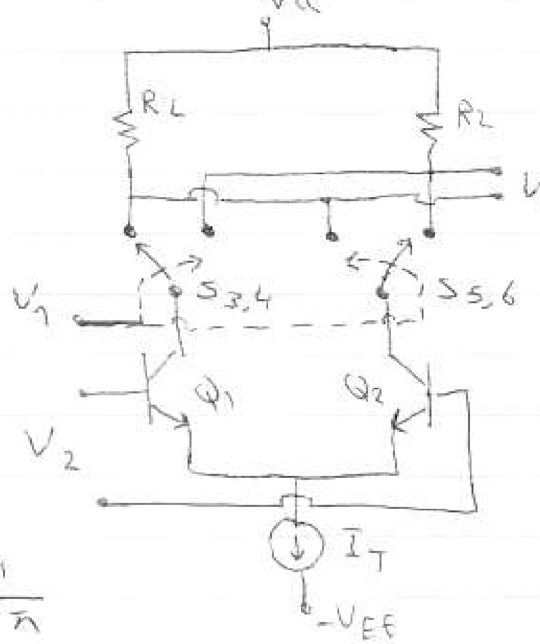
3) if V_1 and $V_2 > V_T$, this mode of operation is useful for the detection of phase differences between two signals.

Chopper Modulator (switching Modulator)

for $\frac{V_1}{2V_T} >> 1$ and $\frac{V_2}{2V_T} << 1$

~~$\Rightarrow V_2 = V_m(t) = V_m \cos \omega_m t$, $V_1 = V_c(t)$~~

the transistors pairs (3,4) and (5,6) function as synchronous switches which turns on & off depending on the polarity of V_1



and if $\omega_1 >> \omega_2$

$$\Rightarrow V_1 = V_c(t) = \sum_{n=1}^{\infty} A_n \cos(n\omega_c t)$$

$$\text{where } A_n = \frac{\sin(\frac{n\pi}{2})}{\frac{n\pi}{4}} = \frac{4}{\pi}, \frac{-4}{3\pi}, \frac{+4}{5\pi}$$

$$V_2 = V_m(t) = V_m \cos(\omega_m t)$$

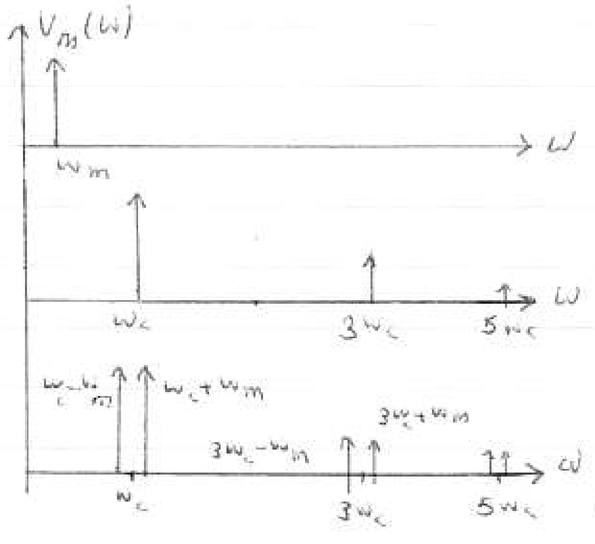
$$\Rightarrow V_o(t) = R_L I_T \tanh\left(\frac{V_1}{2V_T}\right) \tanh\left(\frac{V_2}{2V_T}\right)$$

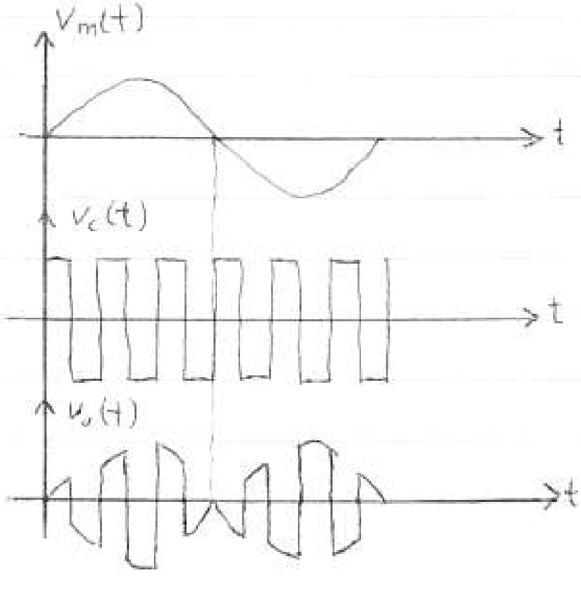
$$V_o(t) = \frac{R_L I_T}{2V_T} V_2 \sum_{n=1}^{\infty} A_n \cos n\omega_c t$$

$$= K \sum_{n=1}^{\infty} A_n V_m \cos(\omega_m t) \cos(n\omega_c t)$$

$$= K \sum_{n=1}^{\infty} \frac{A_n V_m}{2} \left[\cos(n\omega_c + \omega_m)t + \cos(n\omega_c - \omega_m)t\right]$$

where k, is the magnitude of the gain of the multiplier from the small-signal I/p to the o/p.





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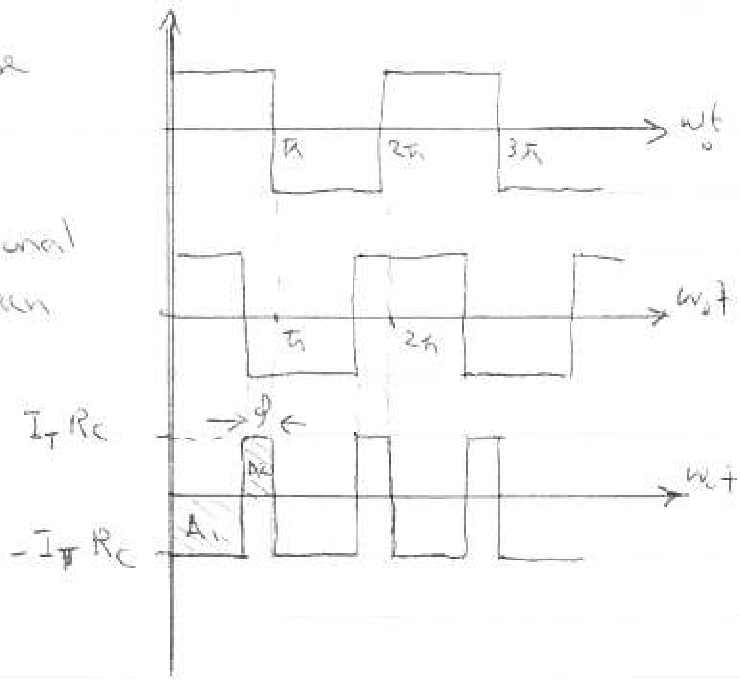
Phase detector using 4QM

for $\frac{V_1}{2V_T} >> 1$ and $\frac{V_2}{2V_T} >> 1$

all the transistors in the cct behave as switches and if $w_1 = w_2$

The cct will work as a phase detector when the o/p of the cct will consists of a DC component which is proportional to the phase difference between the two i/ps



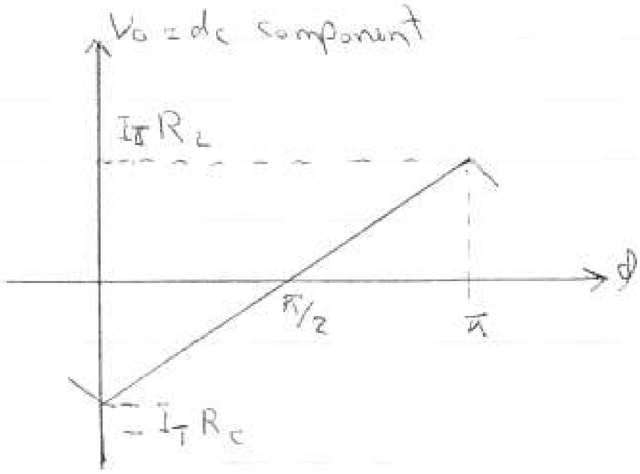
$$V_{av} = \frac{1}{2\pi} \int_0^{2\pi} V_o(t) \, d(w_o t)$$

$$= \frac{-1}{\pi} (A_1 - A_2)$$

$$V_{av} = -\left[I_T R_L \frac{(\pi - \phi)}{\pi} - I_T R_L \frac{\phi}{\pi} \right]$$

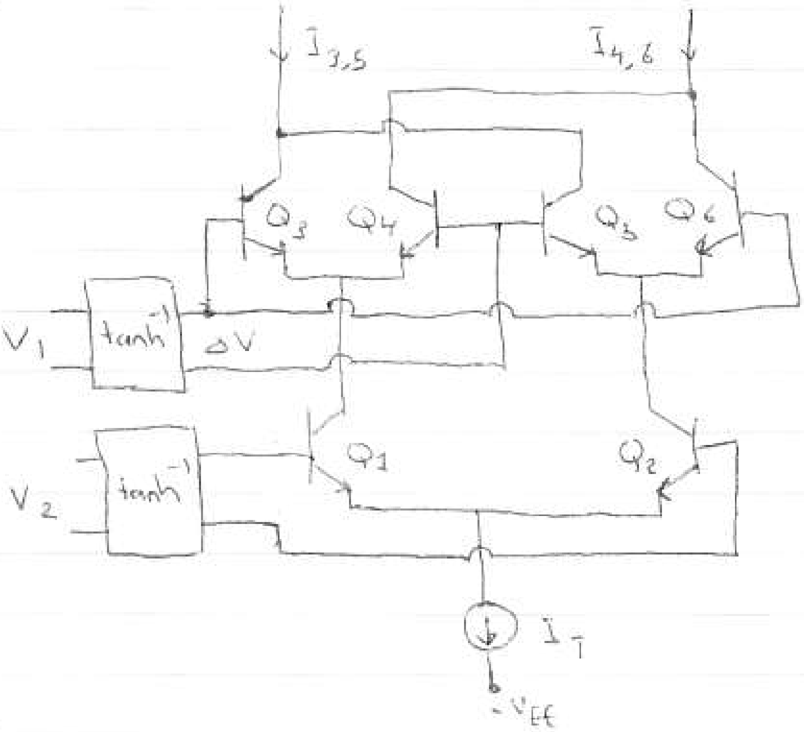
$$= I_T R_c \left(\frac{2\phi}{\pi} - 1 \right)$$

This phase relationship is plotted here



4QM with wide Dynamic Range:

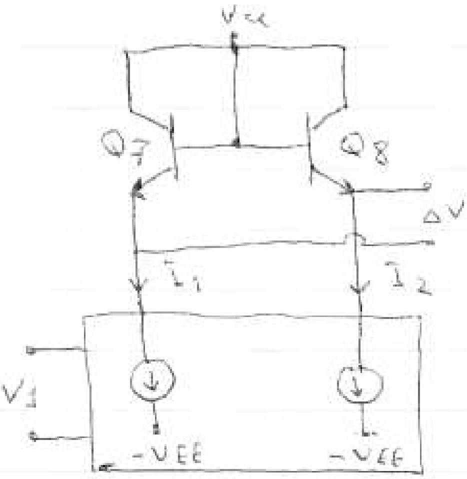
To increase the dynamic range of the multiplier cct the transfer characteristics of a differential gain stage should be linearized. To obtain linear multiplication over a wide dynamic range V_1 is processed in a non linear manner to obtain ΔV which is a new signal logarithmically related to V_1, then ΔV is applied into the cct in place of V_1 and so the exponential relation of the multiplier would be linearized. using the cct shown below



$$I_1 = I_{o1} + k_1 V_1$$

$$I_2 = I_{o1} - k_1 V_1$$

assume that the cct in the box develops a differential o/p current that is linearly related to the i/p V_1 and I_{o1} is the DC current that flows in each transistor when $V_1 = 0$ and k_1 is the transconductance of the voltage-to-current converter.



Voltage-Current converter

$$I_c = I_s \exp\left(\frac{V}{V_T}\right) \Rightarrow V = V_T \ln\left(\frac{I_c}{I_s}\right)$$

$$\Delta V = V_T \ln\left(\frac{I_{o1}+k_1V_1}{I_S}\right) - V_T \ln\left(\frac{I_{o1}-k_1V_1}{I_S}\right)$$

$$= V_T \ln\left(\frac{I_{o1}+k_1V_1}{I_{o1}-k_1V_1}\right)$$

knowing that $\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$

$$\Rightarrow \quad \Delta V = 2V_T \tanh^{-1}\left(\frac{k_1V_1}{I_{o1}}\right)$$

also previously we see that $\Delta I = I_1 - I_2 = I_T \tanh\left(\frac{V_d}{2V_T}\right)$

$$\Rightarrow \Delta I = I_T \left(\frac{k_1V_1}{I_{o1}}\right)\left(\frac{k_2V_2}{I_{o2}}\right)$$

where I_{o2} and k_2 are the parameters of the block of V_2

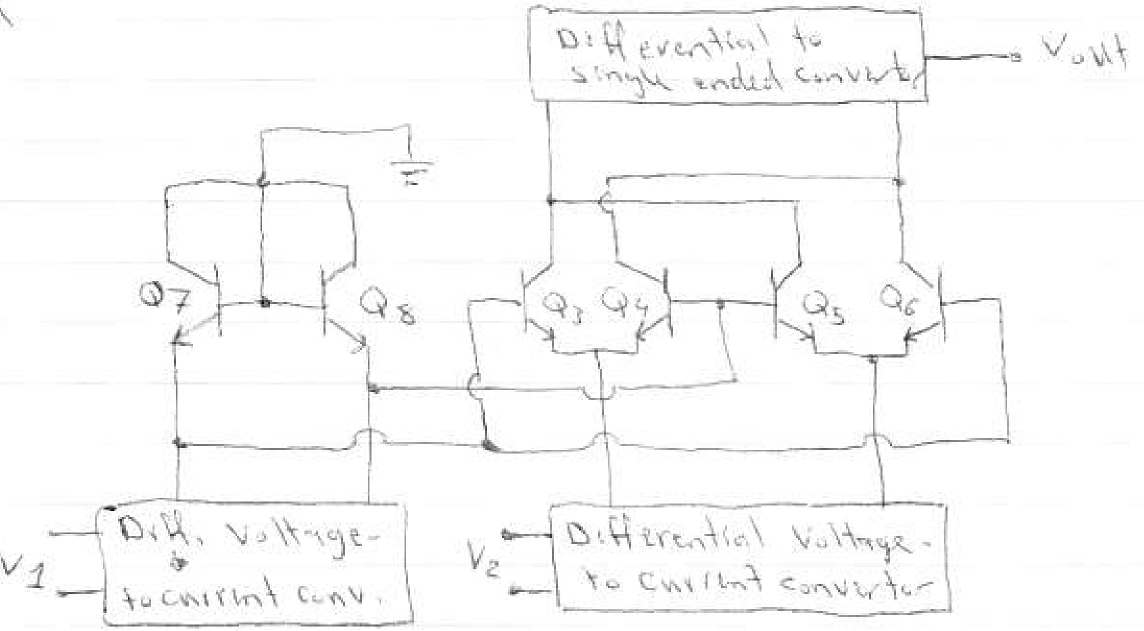
The o/p current is directly proportional to the product of V_1 and V_2 and this relation holds for all values of V_1 & V_2 for which the two o/p current of the differential voltage-to-current converters are positive (I_1 & I_2)

$$\Rightarrow -\frac{I_{o1}}{k_1} < V_1 < \frac{I_{o1}}{k_1} \quad , \quad -\frac{I_{o2}}{k_2} < V_2 < \frac{I_{o2}}{k_2}$$

Complete Analog Multiplier

The cct below shows the complete Analog Multiplier. Q_1 and Q_2 has been removed because the o/p current of the voltage to current converter on the V_2 i/p can be fed directly into the emitters of the Q_3-Q_4 and Q_5-Q_6 pairs with exactly the same results.

The transistors Q_3, Q_4, Q_5, Q_6, Q_7 and Q_8 are referred to as the Multiplier Core and produce a differential current o/p that must be amplified and converted to a signal referenced to the ground



The complete multiplier consists of two voltage-current converters, the core transistors, and an o/p current to voltage amplifier. The core transistors are common to most 4QM, the rest of the circuitry can be realized in a variety of ways

The most common configrations used for the voltage-current converters are emitter-coupled pairs with emitter degeneration Resistors and the differential to single ended convertor is often realized with op-amp cct.

If the o/p amplifier has

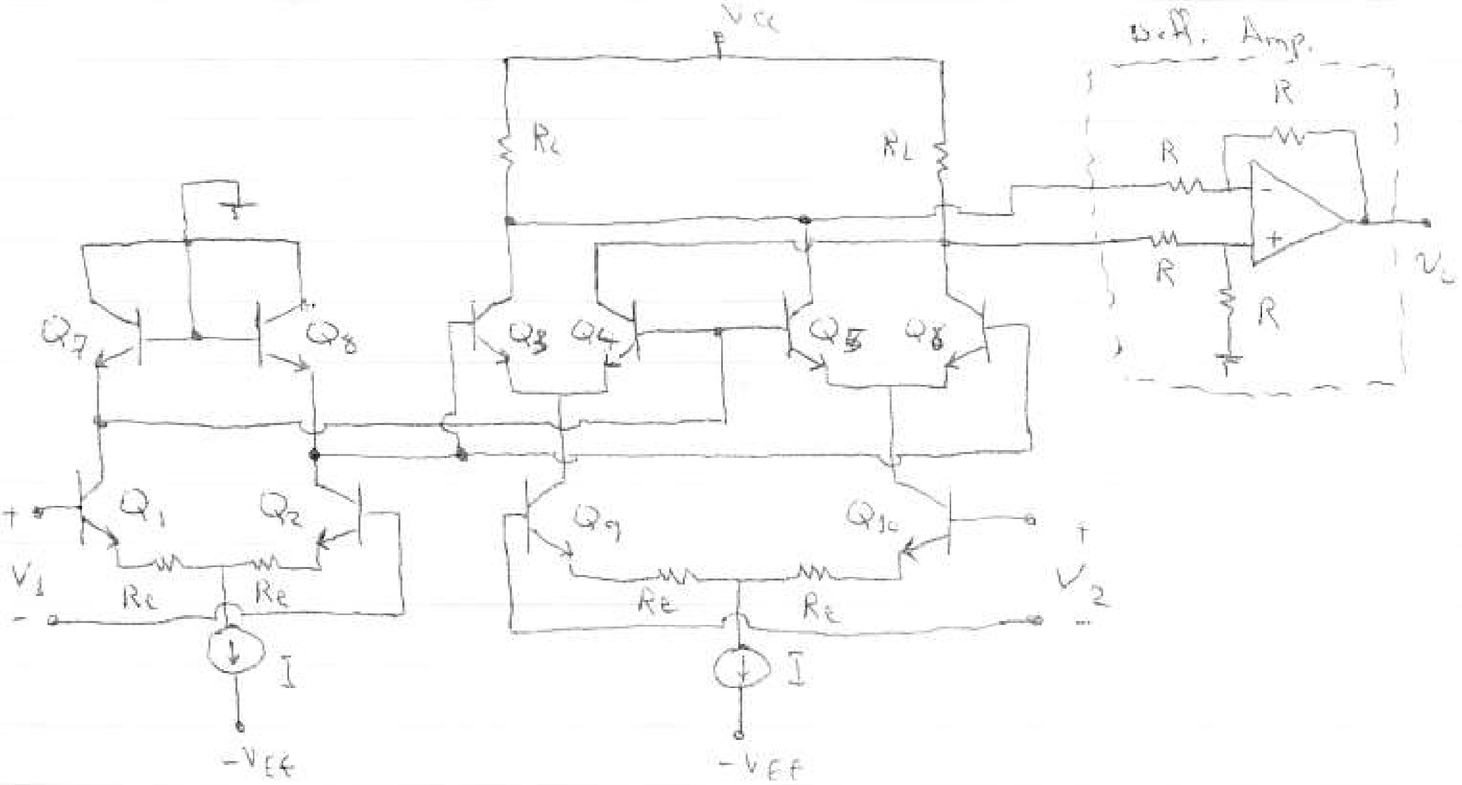
$$\frac{V_{out}}{\Delta I} = K_3$$

$$\Rightarrow V_{out} = I_T K_3 \frac{K_1}{I_{c1}} \frac{K_2}{I_{c2}} V_1 V_2$$

The constants are usually chosen so that

$V_{out} = 0.1 V_1 V_2$

and all voltages have $\pm 10V$ range



AM Transmitters

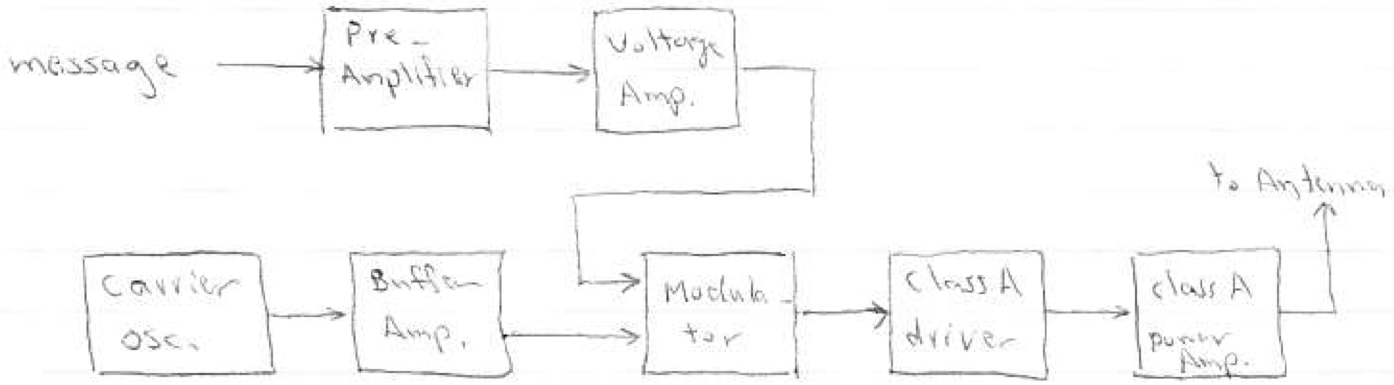
A transmitter is an electronic system that will convert a massage signal into a form that can be sent via an antenna system to the surrounding atmosphere. it must be able to;

1. Produce a carrier signal
2. Code the carrier signal with the message signal
3. Supply enough power to the coded carrier to travel the distance betwten transmitter and receiver.

There are two basic types of AM transmitters according to where the modulation takes place, high level and low level modulation.

Low Level Modulation

In this type, the Message signal controls the amplitude of the carrier signal at some location within the chain of carrier amplifiers. The control may take place at the base of the final power amplifier or at any previous point in the system, beyond the buffer amplifier.



low-level AM transmitter block diagram

Advantage:

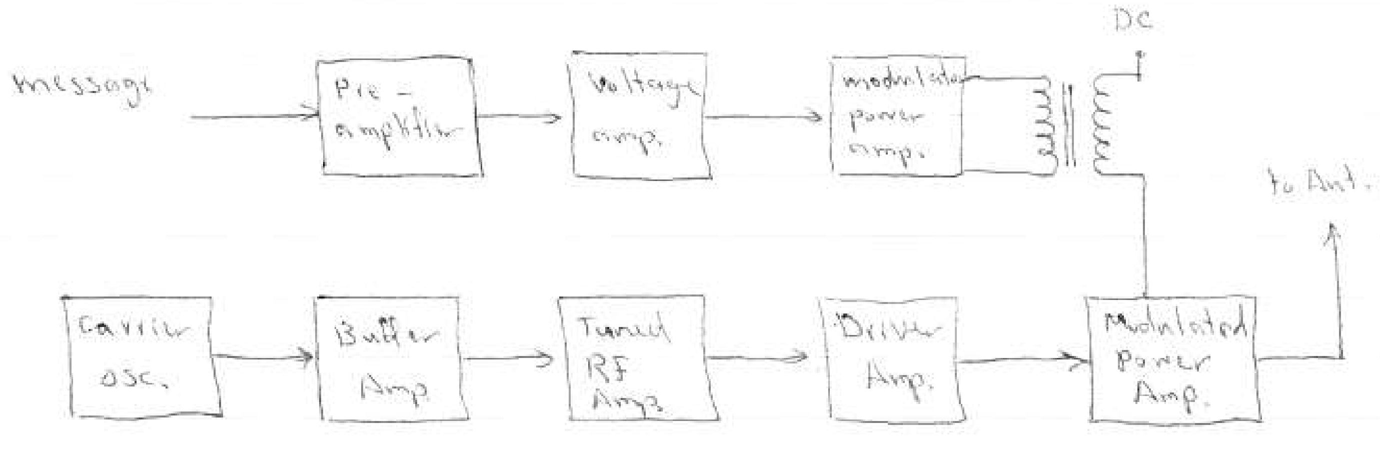
small amount of message power will fully modulate the carrier since modulation takes place at parts of the system where voltage is the prime consideration.

Disadvantage:

class (A) amplifiers, should be used in the stages following the modulator for linearty purposes. these are low efficiency amplifiers.

High level Modulation

In this type the message signal controls the amplitude of the carrier signal by delivering to the final power Amp. an AC voltage that is in series with DC power supply.



High-level AM transmitter block diagram

Advantages:
all the amplifiers following the carrier signal buffer can be operated as class C amp. for max. efficiency

Disadvantages:
high level of message power are required to fully modulate the carrier.

Amplitude Modulation Techniques:

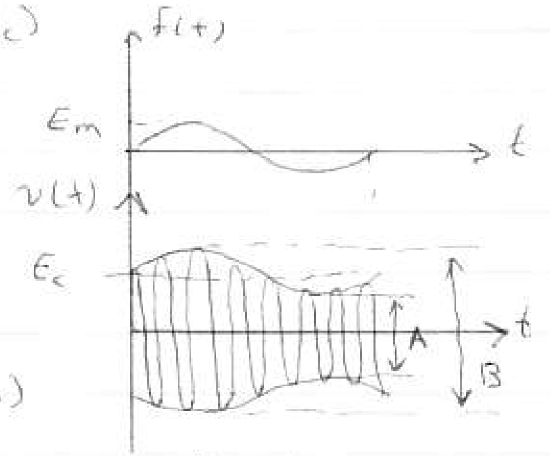
There are four basic methods of AM

1. Analog Multiplication
2. Chopper Modulation
3. Non linear device modulation
4. Direct tuned - circuit modulation ← high level

(items 1–3: low level modulation)

A typical amplitude modulated carrier has the form

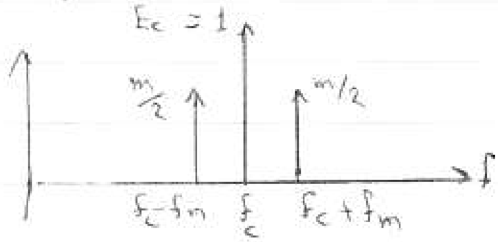
$$v(t) = E_c\,[1 + m\,f(t)]\,\cos(w_c t + \theta_c)$$



where

E_c = Carrier Amplitude
m = modulation index
$f(t)$ = modulating signal
w_c = carrier freq.
θ_c = Carrier phase angle (usually zero)

For normal AM the modulation index is



$$m = \frac{B - A}{B + A}$$

where B is the max. peak to peak value of $v(t)$
A is the min. peak to peak value of $v(t)$

the total power transmitted is

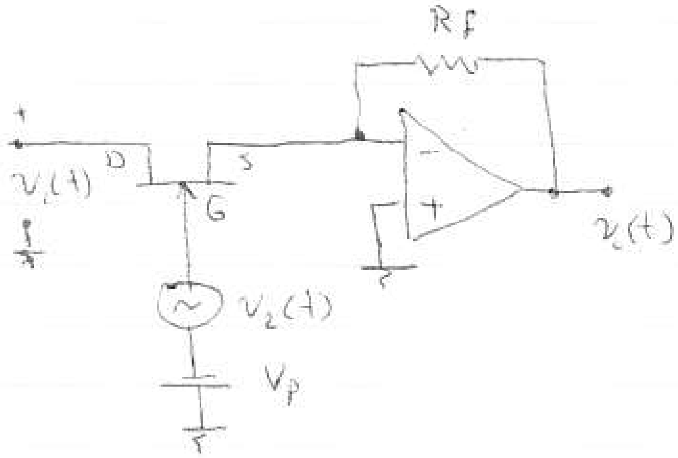
$$P_t = P_c\left(1 + \frac{m^2}{2}\right)$$

where P_t = total power transmitted of sidebands and carrier
P_c = carrier power
m = modulation index

1) Analog Multipliers:

we can use the four quadrant differential-pair as discussed before

FET multiplier (DSB)



An N-channel JFET with small drain to source voltage may be closely modelled as a voltage controlled conductance

$$g_{DS} = \frac{i_d}{v_{ds}} \simeq \frac{2 I_{DSS}}{-V_p}\left(1 - \frac{V_{GS}}{V_p}\right)$$

with $|V_{DS}| < 100\ mV$

and $|V_{GS}| \leq |V_p|$

the o/p $V_o(t)$ is given by: $v_o(t) = -g_{DS} R_f\, v_1(t)$

where $v_1(t) = V_{DS}$, $V_{GS} = v_2(t) + V_p$

$$g_{DS} = \frac{2 I_{DSS}}{V_p}\left(\frac{v_2(t)}{V_p}\right)$$

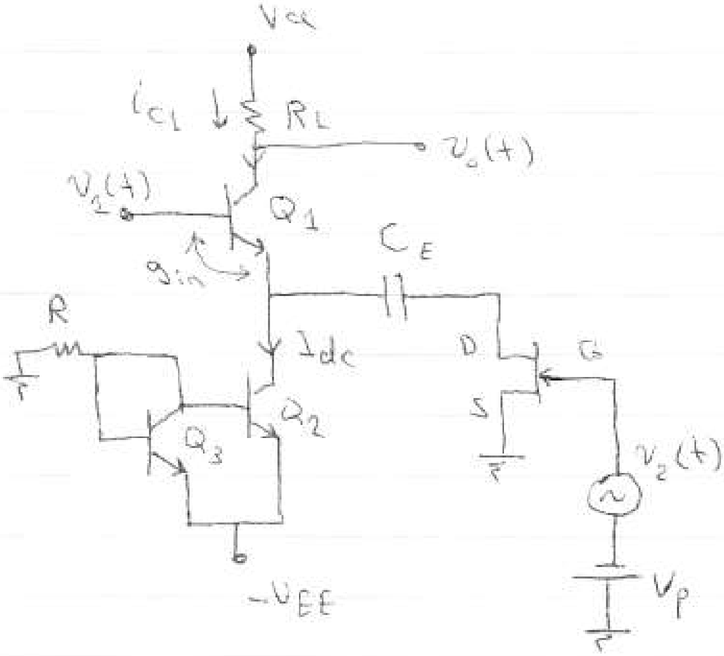
$$\Rightarrow \quad v_o(t) = -R_f \frac{2 I_{DSS}}{V_p}\left(\frac{V_2(t)}{V_p}\right) v_1(t)$$

this cct has $V_1(t) = V_{DS} < 100\,mV$
and

$V_2(t)$ must be +ve

$v_2(t) < |V_p| + 0.7$ for Si FET

A more simpler multiplier may be obtained by replacing the Op-Amp by a single transistor as shown in this cct



C_E is sufficiently large to be short cct for the freq. of $v_1(t)$. I_{dc} is large so that

(for diode $r_{in} = \frac{V_T}{I_{EQ}}$)

$$g_{in} = \frac{I_{dc}}{V_T} \gg g_{DS(max)}$$

so that $v_1(t)$ appears directly across the FET then

$$v_{DS} = v_1(t)$$

$$i_D(t) = g_{DS} v_1(t)$$

$$i_{C1}(t) \simeq I_{dc} + i_D(t)$$

$$\Rightarrow v_o(t) = V_{cc} - i_{C1}(t) R_L = V_{cc} - I_{dc} R_L - g_{DS} R_L v_1(t)$$

$$= \underbrace{V_{cc} - I_{dc} R_L}_{D.C} - \underbrace{\frac{2 R_L I_{DSS}}{V_p} v_1(t) \frac{v_2(t)}{V_p}}_{AC}$$

$$v_o(t)_{a.c} = \frac{-2 R_L I_{DSS}}{V_p^2} v_1(t) v_2(t)$$

$\therefore v_o(t)$ is proportional to $v_1(t) v_2(t)$

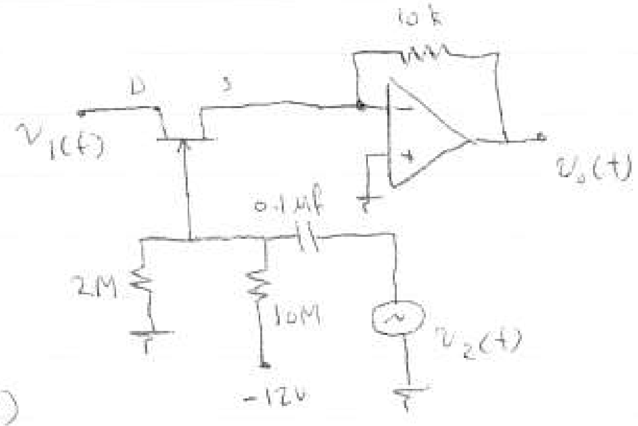
$|v_1(t)| \lesssim 100\,mV$

V_2 +ve values less than $(|V_p| + 0.7)$ volt.

Ex: for the cct shown find $v_o(t)$ and the modulation index if $I_{DSS} = 6\,mA$, $V_p = -4V$

$v_1(t) = (50\,mV)\cos 10^7 t$

$v_2(t) = (1V)\, f(t)$



Sol:

$$V_G = V_{GS} = \frac{-12 \times 2}{2+10} = -2V \quad \text{(DC part)}$$

$$v_1 < 100\,mV \Rightarrow g_{DS} = \frac{-2 I_{DSS}}{V_p}\left(1 - \frac{V_{gs}}{V_p}\right)$$

$$g_{DS} = \frac{2 \times 6 \times 10^{-3}}{4}\left(1 - \frac{-2 + v_2(t)}{-4}\right) = \frac{3 \times 10^{-3}}{4}\left[2 + f(t)\right]$$

$$v_o(t) = -g_{DS}\, R_f\, v_1(t) = \frac{-30}{4}\left[2 + f(t)\right] v_1(t)$$

$$= -750\,mV\left[1 + \frac{1}{2} f(t)\right]\cos 10^7 t \quad \text{normal AM equ.}$$

$\therefore$ the modulation index $m = \frac{1}{2}$

Ex: for the cct shown

$I_{DSS} = 4\,mA$, $V_p = -4V$, $\beta = 100$

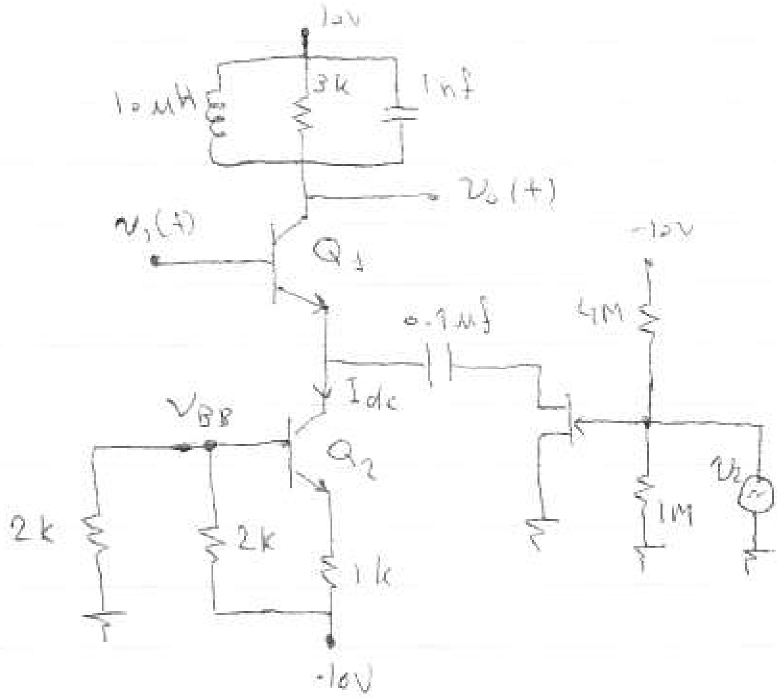
$V_{BE} = 0.7V$

$v_1(t) = (100\,mV)\cos 10^7 t$

$v_2(t) = 2V\, f(t)$

$f(t) = \cos 1.6 \times 10^5 t$

find $v_o(t)$ & w_c for the filter

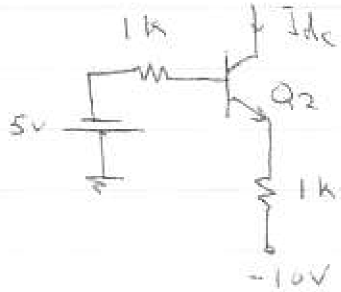


$$I_{dc} = I_{C_2} \simeq I_{EQ_2} = I_{EQ_1}$$

$$-V_{BB} = -10\,\frac{2}{2+2} = -5V$$

$$R_{B_2} = 2 // 2 = 1\,k\Omega$$

$$I_{dc} = I_{EQ_2} = \frac{V_{EE} - V_{BE_2} - V_{BB}}{R_E + \frac{R_B}{1+\beta}}$$



$$= 4.25 \text{ mA}$$

$$r_{in} = \frac{V_T}{I_{EQ}} = \frac{25}{4.25} = 6.11 \ \Omega$$

$$\Rightarrow g_{in} = 0.163 \ \mho$$

$$V_{GS} = -10 \frac{1}{5} = -2 V$$

$$g_{DS_{max}} = \frac{2 I_{DSS}}{-V_P} \left(1 - \frac{V_{GS} + V_{2max}}{V_P}\right) = 2 m\mho$$

$$\therefore g_{in} >> g_{DS_{max}}$$

$$i_{c_1}(t) = I_{EQ_1} + i_D(t) \qquad , i_D(t) = g_{DS} V_1(t)$$

$$v_o(t) = V_{cc} - i_{c_1}(t) R_L = 10 - R_L g_{DS} V_1(t)$$, $R_L I_{EQ_1}$ shorted by the coil

$$v_o(t) = 10 - 3\times10^3 * 100\times10^{-3} \cos 10^7 t \left[\frac{2\times4\times10^{-3}}{4}\left(1 - \frac{-2+2f(t)}{-4}\right)\right.$$

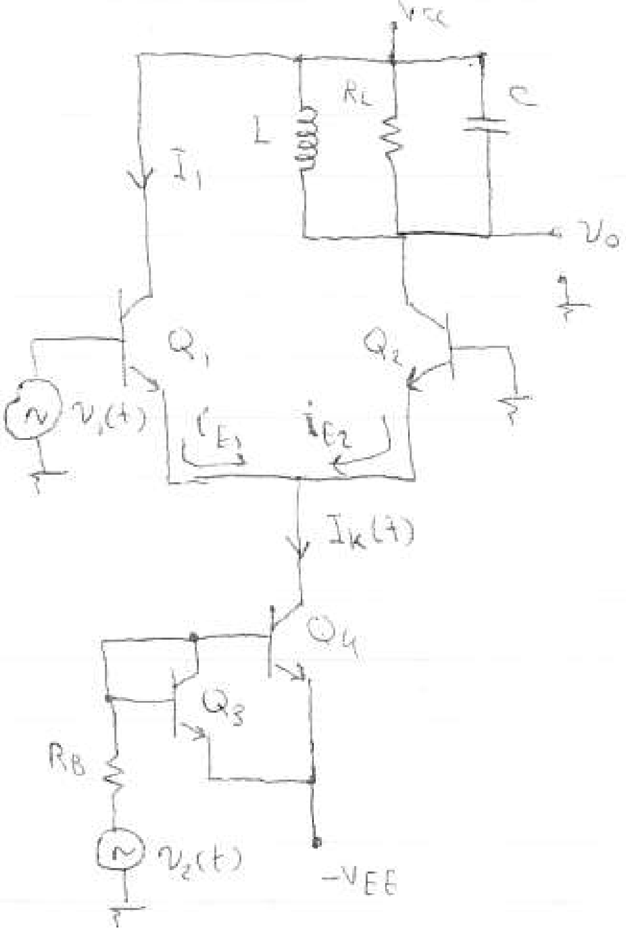
$$= 10 - 0.3 [1 + f(t)] \cos 10^7 t$$

Filter cut-off frqn $w_c = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-5} \times 10^{-9}}} = 10^7$ rad/sec.

Filter band width $= 1/R_L C = \frac{1}{3\times10^3 * 10^{-9}} = 3.33 * 10^5$ rad/sec $= 2w_m$

The filter cct is tuned at w_c and pass the modulation

3) Analog multipliers with one nonlinear I/p channel:



Amplitude modulation can be accomplished by applying the carrier to the nonlinear input channel and the modulation to the linear I/p channel and placing the o/p through a bandpass filter centered about the carrier freq.

$$I_k(t) = I_{k_{dc}} + i_{k_{ac}}$$

$$= I_{dc}[1 + m f(t)]$$

$$I_{k_{dc}} = \frac{V_{EE} - V_{BE}}{R_B}$$

$$i_{k_{ac}} = \frac{v_2(t)}{R_B}$$

$$v_2(t) = V_2 f(t)$$

$$I_k(t) = \frac{V_{EE} - V_{BE}}{R_B}\left[1 + \frac{V_2}{V_{EE} - V_{BE}} f(t)\right]$$

$$m = \frac{i_{ac}}{I_{dc}} = \frac{V_2}{V_{EE} - V_{BE}}$$

assume that $I_{ES_3} = I_{ES_4}$

$$i_{E_2} = \frac{I_k}{2}\left[1 - \tanh\left(\frac{V_1(t)}{2V_T}\right)\right] \quad , \quad V_1(t) = V_1 \cos \omega_0 t$$

i_{E_2} can be expanded to

$$i_{E_2} = I_k(t)\left[\frac{1}{2} - \sum_{n=1,3,5,\dots}^{\infty} a_n(x)\cos n w_o t\right]$$

where $a_n(x) = \frac{1}{\pi}\int_{-\pi}^{\pi}\left[\frac{1}{2}\tanh\left(\frac{x}{2}\cos\theta\right)\right]\cos n\theta\, d\theta$

where $x = \frac{V_1}{V_T}$, and n odd only

$$\Rightarrow a_n(x) = \frac{I_n}{I_{kdc}}$$

$$i_{E_2} = I_k(t)\left[\frac{1}{2} - a_1(x)\cos w_o t - a_3(x)\cos 3w_o t - \dots\right]$$

Then, the fundamental component of the collector current of Q_2 is given by:

$$i_{E_2} = -I_{k_{dc}}\, a_1(x)\,[1+mf(t)]\cos w_o t$$

(note that $a_n(x)$ is found using tables)

$$v_o(t) = V_{cc} + I_{k_{dc}}\, a_1(x)\left[(1+mf(t))\cos w_o t\right] Z_{11}(t)$$

$Z_{11}(t)$ is the impulse response of the o/p tuned cct. The tuned cct. has a sufficiently high impedance to remove the low freq. and high freq. harmonics components of $i_{C_2}(t)$ then:

$v_o(t)$ can be simplified to

$$v_o(t) = V_{cc} + I_{k_{dc}} R_L\, a_1(x)\,[1+mf(t)]\cos w_o t$$

This is an AM signal

Ex: For the cct shown previously let $V_{cc} = 12V$, $V_{EE} = -12V$, $V_1 = 156mV$, $V_2 = 5.63V$, $R_B = 6k\Omega$, $R_L = 5k\Omega$, $C = 1nF$, $L = 10\mu H$
$w_o = 10^7$ rad/sec, and $f(t) = \cos 10^5 t$, $V_{BE} = 0.75$
Find $v_o(t)$:

Sol:
$$I_{kdc} = \frac{V_{EE} - V_{BE}}{R_B} = \frac{11.25}{6k} = 1.875 \, mA$$

$$x = \frac{V_1}{V_T} = \frac{156}{26} = 6$$

$$m = \frac{V_2}{V_{EE} - V_{BE}} = \frac{5.63}{11.25} = \frac{1}{2}$$

From the table $a_1(6) = 0.6$

$$i_{c_2}(t) = -1.875 * 0.6 * 10^{-3} \, (1 + m \cos 10^5 t) \cos 10^7 t$$

$$v_o(t) = 12 + (5.63)(1 + \frac{1}{2} \cos 10^5 t) \cos 10^7 t$$

for the o/p filter $\quad w_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-9} * 10^{-5}}} = 10^7$ rad/sec

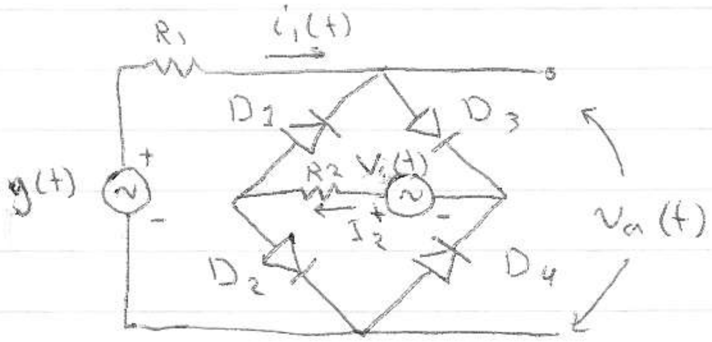
$$BW = \frac{1}{R_L C} = \frac{1}{5 * 10^3 * 10^{-9}} = 2 * 10^5 \text{ rad/sec} = 2 w_m$$

2- Chopper Modulators

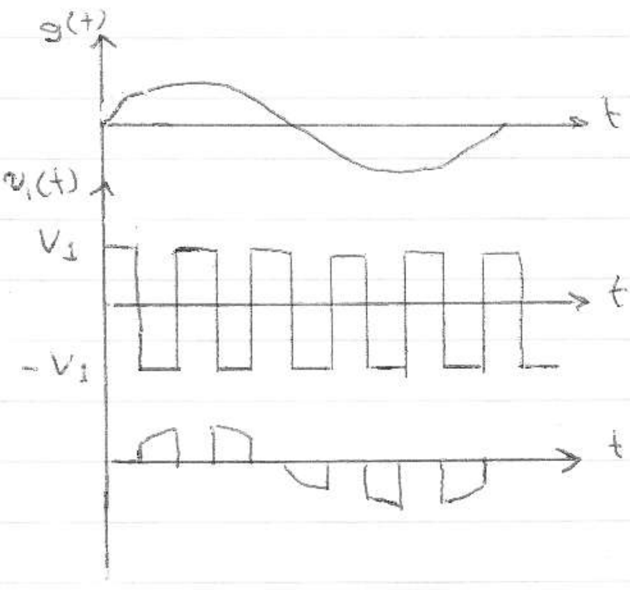
A) Diode Bridge Modulator

The diode bridge is a voltage controlled SPST switch used in chopper modulators

- When $v_1(t)$ is positive all Diodes conduct and $v_a(t) \simeq 0$



- When $v_1(t)$ is negative all Diodes are reverse biased and $v_a(t)$ Follows $g(t)$



- To ensure that all the bridge diodes remain reverse-biased for $v_1 = -V_1$ it is required that

$$V_1 > g(t) - 2V_D$$

- When $v_1(t) = V_1$, V_1 must be sufficiently large to keep all diodes forword-biase

Since all diodes are integrated on a single chip and ~~with~~ identical, the bridge is balanced and $i_1(t)$ and I_2 split equally between the two bridge arms

$$i_{D_1}(t) = \frac{I_2 - i(t)}{2} = i_{D4}(t)$$

$$i_{D_2}(t) = \frac{I_2 + i_1(t)}{2} = i_{D_3}(t)$$

and $\quad i_D = I_s \exp\left(\frac{v_D}{V_T}\right)$

$$\Rightarrow v_D = V_T \ln(I_D / I_s)$$

then $\quad v_a = v_{D_2} - v_{D_1} = V_T \left[\ln \frac{I_2 + i_1(t)}{2 I_s} - \ln \frac{I_2 - i_1(t)}{2 I_s} \right]$

$$v_a = V_T \ln \left[\frac{1 + \frac{i_1(t)}{I_2}}{1 - \frac{i_1(t)}{I_2}} \right]$$

$v_a(t)$ can be expanded using MacLauren series in $\frac{i_1}{I_2}$

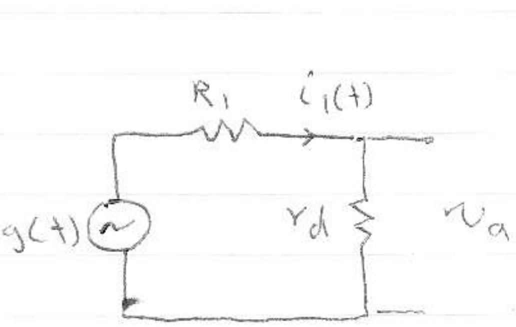
then $\quad v_a(t) = r_d\, i_1 \left(1 + \frac{i_1^2}{3 I_2^2} + \frac{i_1^4}{5 I_2^4} + \cdots \right)$

where $\quad r_d = \frac{V_T}{I_2/2} = \frac{2V_T}{I_2}$

r_d: small signal diode forward resistance

To avoid envelope distortion in $v_o(t)$ nonlinear components of i_1 should be neglected i.e.:

$$\frac{i_1^2}{3 I_2^2} \ll 1$$



$$\Rightarrow v_a(t) = r_d\, i_1(t)$$

from the above cct

$$i_1(t) = \frac{g(t)}{R_1 + r_d}$$

and
$$I_2 = \frac{V_1 - 2V_D}{R_2}$$

$$r_d = \frac{2V_T R_2}{V_1 - 2V_D}$$

to keep $\frac{i_1^2}{3 I_2^2} < 0.01$

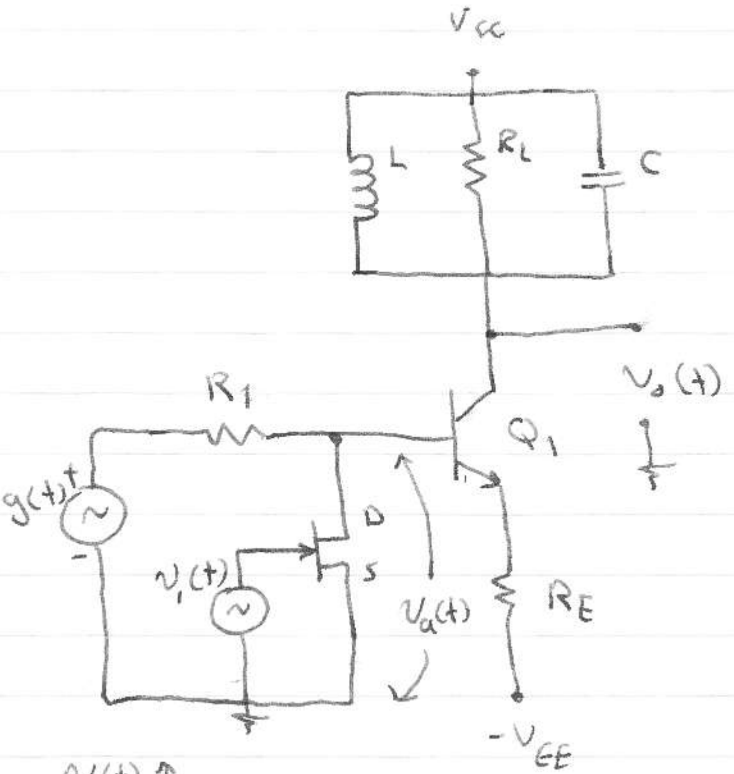
then $V_1 > \frac{5.8\, g(t) R_2}{R_1 + r_d} + 2V_D$

Then $V_a(t)$ functions as a switching fun. that varies between $g(t)$ and $r_d\, i(t)$

B) FET modulator

when $v_1(t) = -V < V_p$

the FET opens and $v_a(t) \simeq g(t)$

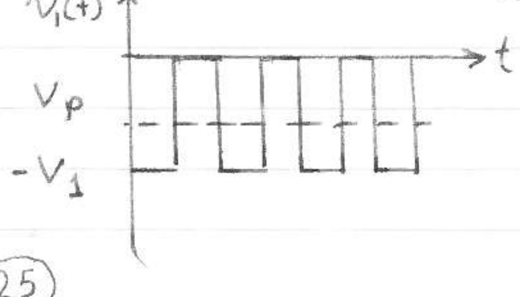


when $v_1(t) = 0$, Then the FET functions as an ohmic conductance g_{DSS} where

$$g_{DSS} = \frac{2 I_{DSS}}{-V_p}\left(1 - \frac{V_{GS}}{V_p}\right)$$

where $V_{gs} = V_1(t) = 0$

$$\Rightarrow g_{DSS} = \frac{2 I_{DSS}}{-V_p}$$



also $|V_{DS}| = |V_a| < 100\,mV$

The FET can be modeled as an ideal voltage controlled switch in series with a resistance r_{DSS}

$$r_{DSS} = \frac{1}{g_{DSS}}$$ (for typical JFET, r_{DSS} varies from several ohms to several kΩ)

with $V_1(t) = 0$

$$\Rightarrow V_a(t) = V_{DS} = \frac{g(t)\, r_{DSS}}{R_1 + r_{DSS}} < 100\,mV \text{ for all } t$$

$$\Rightarrow R_1 > r_{DSS}\left(\frac{|g(t)|_{max}}{100\,mV} - 1\right)$$

R_1 should not be chosen too much greater than the value given by the above relation to avoid loading by the o/p transistor when the FET is reverse biased.

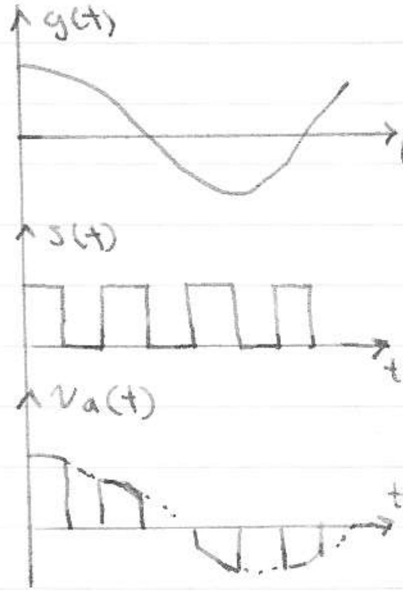
If R_1 is chosen to be less than the value of the above eqn, $V_a(t)$ exceeds 100 mV and r_{DSS} becomes nonlinear and $V_a(t)$ is no longer a linear fun. of $g(t)$

R_E is sufficiently large so that the transistor loading is neglected then

$$V_o(t) = \frac{2\, g(t)}{\pi R_E} \; \frac{R_1 R_L}{R_1 + r_{DSS}} \cos w_o t + V_{cc}$$

this equation can be derived as follows;

$$s(t) = \begin{cases} 1 & \cos \omega_0 t \geq 0 \quad , V_1 = -V_p \\ 0 & \cos \omega_0 t < 0 \quad , V_1 = 0 \end{cases}$$



s(t) can be expanded in fourier series

$$s(t) = \frac{1}{2} + \frac{2}{\pi}\cos \omega_0 t - \frac{2}{3\pi}\cos 3\omega_0 t + \cdots$$

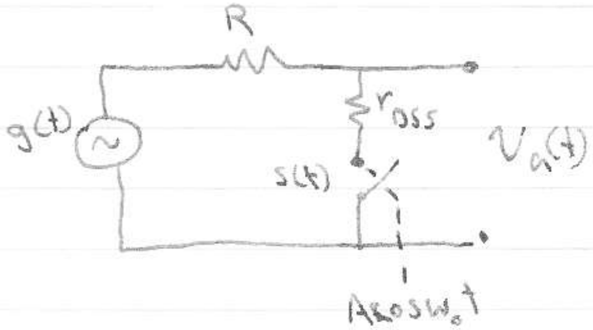
$v_a(t)$ canbe written as

$$v_a(t) = g(t)\, s(t)$$

then $$v_a(t) = \frac{g(t)}{2} + \frac{2g(t)}{\pi}\cos \omega_0 t - \frac{2g(t)}{3\pi}\cos 3\omega_0 t + \cdots$$

For the FET, the switch has a resistance r_{DSS} in series with it

This will prevent complete attenuation of g(t) when the swith is closed.



$$\therefore \quad v_a(t) = \begin{cases} g(t)\dfrac{r_{DSS}}{R_1 + r_{DSS}} \\ g(t) \end{cases}$$

$$v_a(t) = g(t)\, s(t) + g(t)\frac{[1 - s(t)]\, r_{DSS}}{r_{DSS} + R_1}$$

$$= g(t)\, s(t)\frac{R_1}{R_1 + r_{DSS}} + g(t)\frac{r_{DSS}}{r_{DSS} + R_1}$$

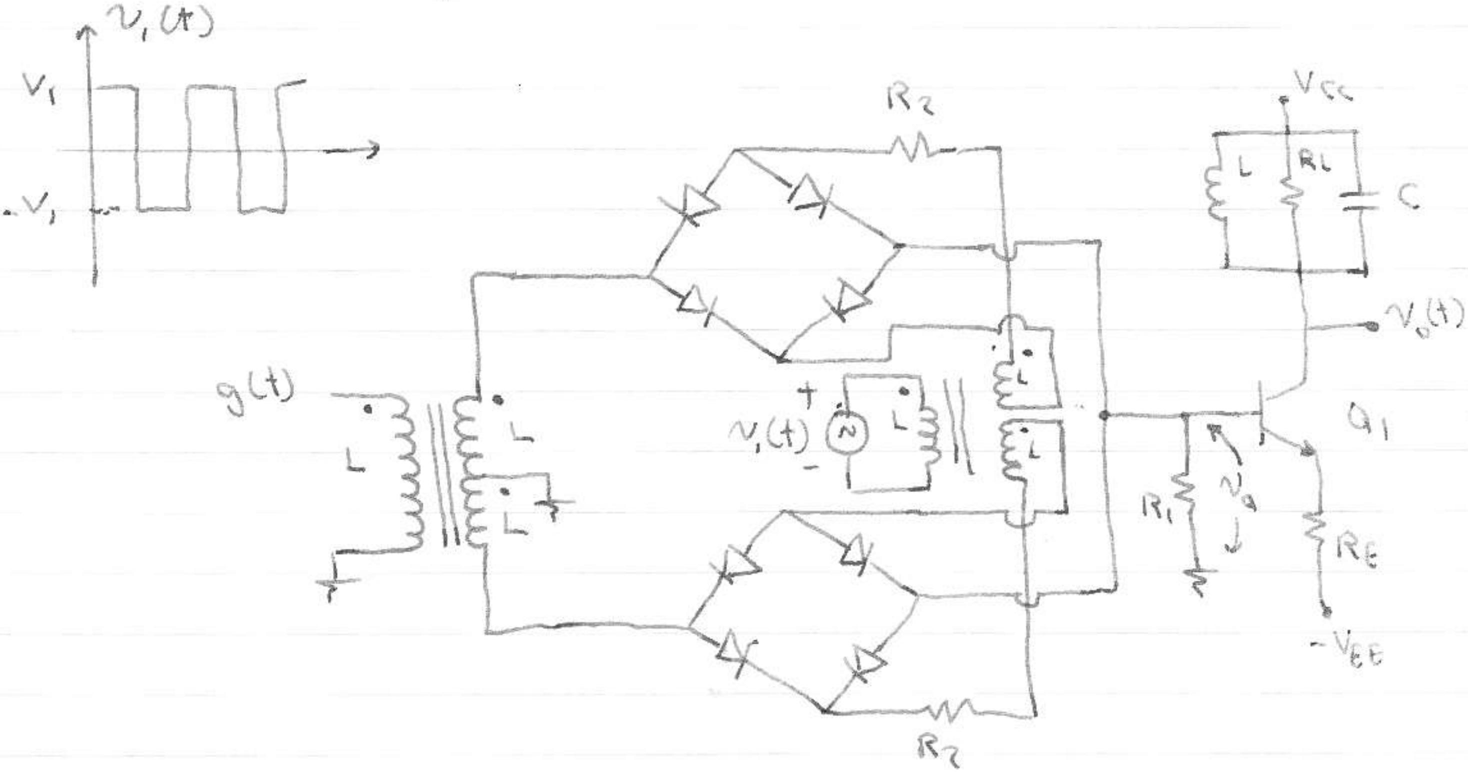
The o/p LC filter will select the fundamental component of $v_a(t)$

$$\Rightarrow v_a(t) = \frac{2\, g(t)}{\pi}\frac{R_1}{r_{DSS} + R_1}\cos \omega_0 t$$

$$i_{E_1} = i_{C_1} \simeq \frac{v_a(t)}{R_E} \quad \text{(for large } R_E \text{)}$$

$$\Rightarrow v_o(t) = V_{cc} + \frac{2g(t)}{\pi R_E} \left(\frac{R_1 R_L}{r_{DSS} + R_1} \right) \cos w_o t$$

c) Balanced chopper modulator using diode bridge
(Ring Modulator)



$g(t)$ and $-g(t)$ are alternately applied across R_1 to produce the effect of a reversing switch

The closed bridge is unaffected by the open bridge.
then

$$V_1 > \frac{5.8\, g(t) R_2}{R_1 + r_d} + 2V_D$$

required value of V_1 to ensure that the bridge remains closed ~~for~~ all t, The closed bridge affects the open bridge, it increases the voltage across the open bridge to

$$g(t)\left[1+\frac{R_1}{R_1+r_d}\right],$$ then the required value of

V_1 that ensures that all diodes in the open bridge remain reverse biased is given by;

$$V_1 > g(t)\left[1+\frac{R_1}{R_1+r_d}\right] - 2V_D \quad \text{for all } t,$$

For large R_E $\quad v_a(t) = \dfrac{g(t)\,R_1}{R_1+r_d}\; s(t)$

where $s(t) = \begin{cases} 1 & \cos w_o t \geq 0 \\ -1 & \cos w_o t < 0 \end{cases}$

and $i_c \simeq \dfrac{v_a(t)}{R_E}$

and for the o/p filter to be tuned at w_o

$$\Rightarrow v_o(t) = V_{cc} - \frac{4R_L}{\pi R_E}\,\frac{R_1}{R_1+r_d}\,g(t)\cos w_o t \,.$$

3) Square low modulators (non-linear device modulator)

This type is used as an AM modulator when a low index modulator required.

In the cct ~~below~~ shown the FET is assumed to operate in it's saturation region, and that R_L is much less than the output impedance of the FET.

$$i_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_p}\right)^2$$

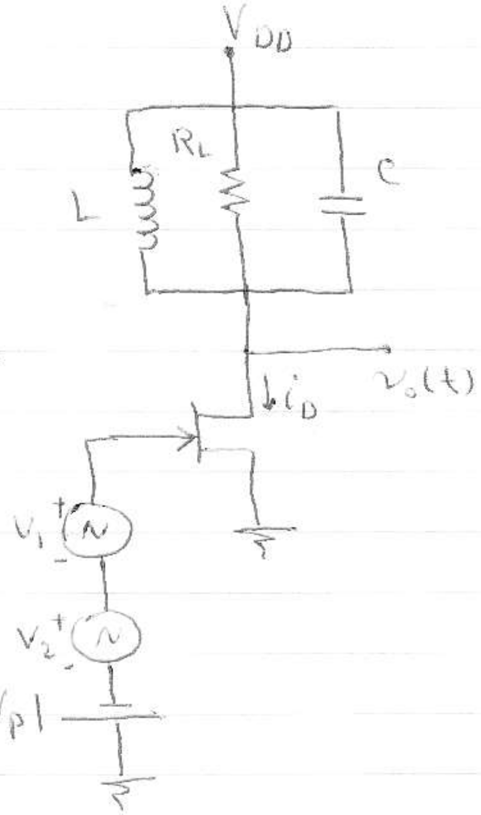
$$V_{GS} = V_1 + V_2 + V_p$$

where

$v_1 = V_1 \cos w_o t$

$v_2 = A[1 + mf(t)]$

V_p = pinch-off voltage of the FET



$$\Rightarrow i_D(t) = I_{DSS}\left(\frac{v_1 + v_2}{V_p}\right)^2$$

$$i_D(t) = I_{DSS}\left[\frac{V_1 \cos w_o t + A\{1 + mf(t)\}}{V_p}\right]^2$$

For the o/p filter to be centred about w_o and have a flat band over the band of freq's occupied by the AM signal

$$\Rightarrow v_o(t) = V_{DD} - \frac{2 I_{DSS} V_1 A R_L}{V_p^2}\{1 + mf(t)\}\cos w_o t$$

to ensure that operation remains within saturation region

$v_o(t) > -|V_p|$ for all time

and $|v_o(t)|_{max} < V_{DD} + V_p$

$V_p \leq V_{GS} \leq V_D$

$0 \leq V_{GS} \leq V_D + |V_p|$

$V_{GS} = V_1 + V_2$

$v_1(t) + v_2(t) \geq 0$ to keep FET from being cutoff.

$v_1(t) + v_2(t) \leq |V_p| + V_D$ to keep the Gate-source diode from turning on.

when $v_1(t)_{min} = V_1(-1) = -V_1$

$v_1(t)_{max} = V_1(+1) = V_1$

$$\Rightarrow A(1-m) - V_1 \geq 0 \quad \text{--- } ①, \quad A(1+m) + V_1 \leq V_p' \quad \text{--- } ②$$

where $V_p' = |V_p| + V_D$

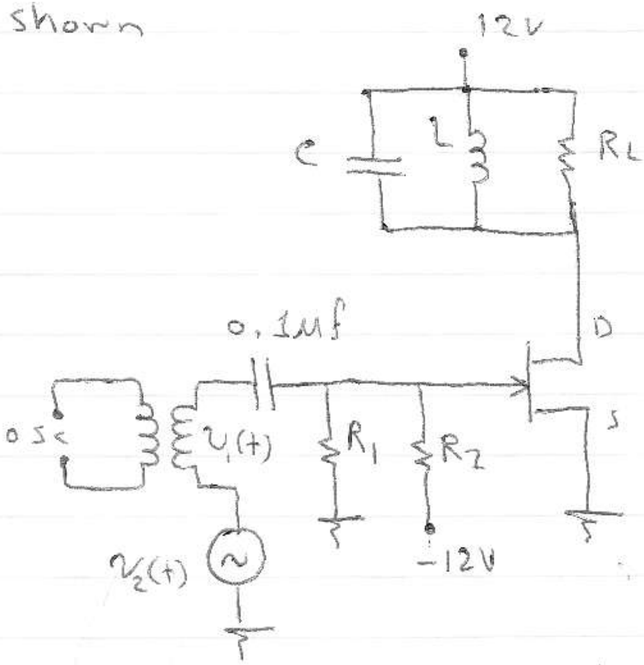
The largest possible value of the output Voltage with smallest value of R_L ($R_L << r_o$) is obtained when
from (1) and (2)

$$\Rightarrow \quad A = \frac{V_p'}{2}, \text{ and } V_1 = \frac{V_p'}{2}(1-m)$$

and the o/p becomes

$$v_o(t) = (1-m)\frac{I_{DSS} R_L}{2}\left(\frac{|V_p| + V_D}{V_p}\right)^2 [1 + mf(t)] \cos w_o t$$

Ex: For the FET modulator shown
$V_p = -4V$, $V_D = 0.7V$
$I_{DSS} = 4mA$, $V_1(t) = V_1 \cos 10^7 t$
$v_2(t) = A[1 + m\cos(2.5 \times 10^5 t)]$
Design the cct to achive maximum o/p voltage with $m = \frac{1}{2}$.



Sol:

$$v_o(t) = (1-m)\frac{I_{DSS} R_L}{2}\left(\frac{|V_p| + V_D}{V_p}\right)^2$$
$$* [1 + mF(t)] \cos w_o t$$

$|v_o(t)|_{max} < V_{DD} + V_p$ to ensure saturation region operation

$$\left(1 - \frac{1}{2}\right)\left(\frac{4 \times 10^{-3} \times R_L}{2}\right)\left(\frac{4 + 0.7}{-4}\right)\left(1 + \frac{1}{2}\right) < 12 - 4$$

$\Rightarrow R_L < 3.8 \, k\Omega \quad << r_o$ (typically 100k)

$w_m = 2.5 * 10^5$ rad/sec
$\Rightarrow B.W = 5 \times 10^5$ rad/sec

$$B.W = \frac{1}{R_L C}$$

take $R_L = 3.6\,k$ for practical value

$$C = \frac{1}{5 \times 10^5 \times 3.6 \times 10^3} = 566\ pf$$

$$w_0^2 = \frac{1}{LC} = 10^{14}\ rad/sec$$

$$L = \frac{1}{10^{14} \times 566 \times 10^{-12}} = 18\ \mu H$$

$$A = \frac{V_p'}{2} = \frac{4.7}{2} = 2.35V$$

$$V_1 = \frac{V_p'}{2}(1-m) = 1.175\ V$$

$$v_o(t) = (1-0.5)\frac{4 \times 3.6}{2}\left(\frac{4+0.7}{-4}\right)^2 [1+0.5\ f(t)] \cos w_o t$$

$$= 4.96\ [1 + \frac{1}{2} f(t)] \cos w_o t$$

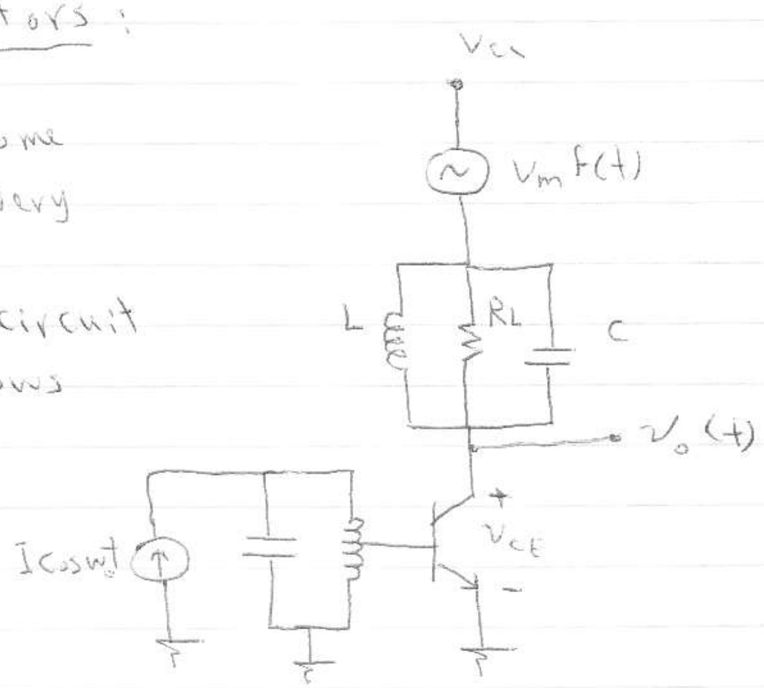
$$v_{GS} = V_p + v_1 + v_2$$

$$v_{GS} = V_p + A[1 + mf(t)] + V_1 \cos w_o t$$
$$= -1.65 + 1.175\ f(t) + 1.175 \cos 10^7 t$$

$$V_{GS} = -1.65 = -12 \frac{R_1}{R_1+R_2}$$

let $R_1 = 1\ M\Omega \Rightarrow R_2 = \frac{12}{1.65} \times 1 \times 10^6 - 1 \times 10^6 = 6.3\ M\Omega$

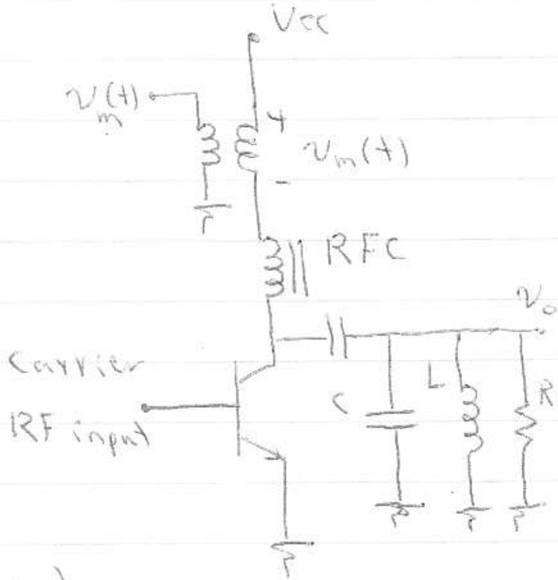
4) Tuned circuit modulators :



The Transistor must become saturated at the peak of every driving carrier cycle. As a result, the o/p tuned-circuit Voltage amplitude follows the variations in f(t), and the loading in the base circuit increases causing the I/p driving voltage to follow the variation in f(t)

$$m = \frac{V_m}{V_{cc} - V_{CE\,sat}}.$$

AM signal at high power level :



In this cct, which is used to generate AM signal at high power level the transistor can be operated as either class B or C, the o/p is tuned to the carrier freq and has a bandwidth equal to twice that of the modulating signal.

The total low freq. supply voltage for the transistor is $V = V_{cc} + V_m(t)$

$V = V_{cc}(1 + m\cos w_m t)$

where $V_m(t) = mV_{cc}\cos w_m t$

For class C power Amplifiers the Amplitude of the o/p signal under saturation-limited coditions equals the power supply Voltage. Therefore, changing the transistor supply Voltage modulates the o/p signal amplitude, and the o/p voltage becomes

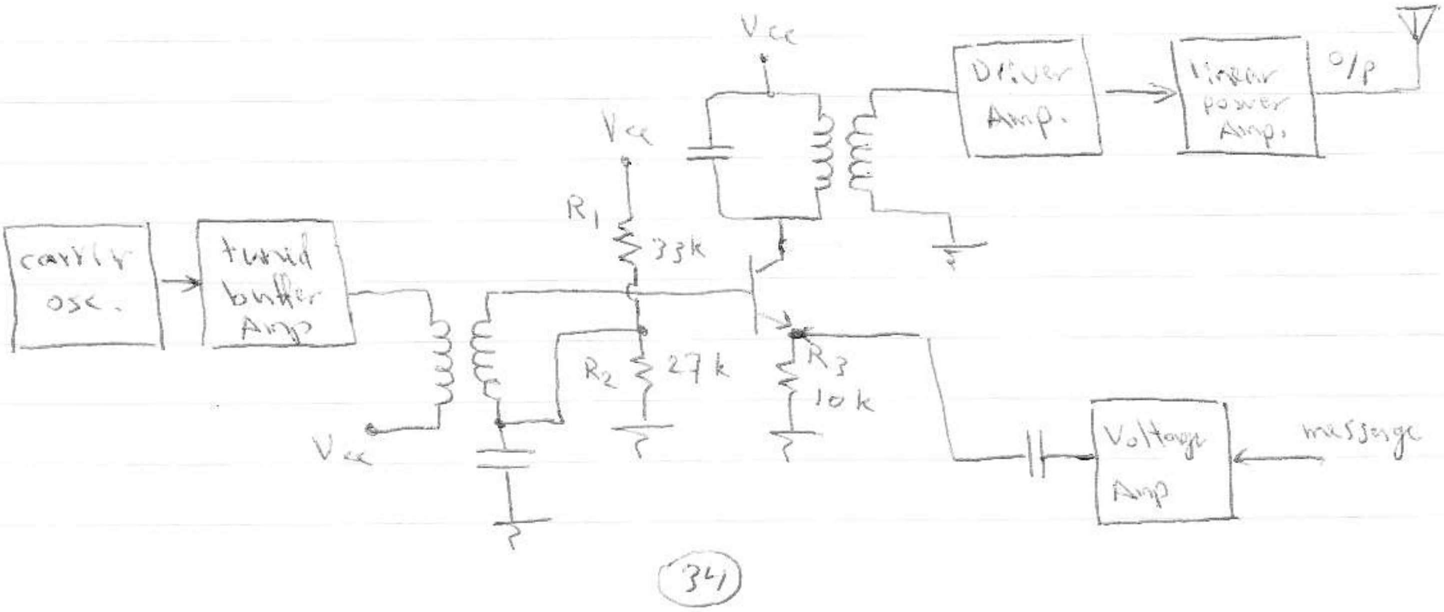
$$v_o = V_{cc}(1 + m\cos w_m t)\cos w_o t$$

The carrier signal has an amplitude of V_{cc}. For 100% modulation the peak voltage of $V_m(t) = V_{cc}$

Then the o/p power $P_{out} = P_c\left(1 + \frac{m^2}{2}\right) = \frac{3}{2} P_c$

$$= \frac{3}{2}\left(\frac{V_{cc}^2}{2R_L}\right)$$

Low-level amplitude modulation



RF carrier is transformer coupled to the base of the modulator
Modulating signal is capacitively coupled to the emitter

For $V_{cc} = 24\,V$

The base voltage $(V_B) = 24 \dfrac{R_2}{R_2 + R_1} = 10.2V$

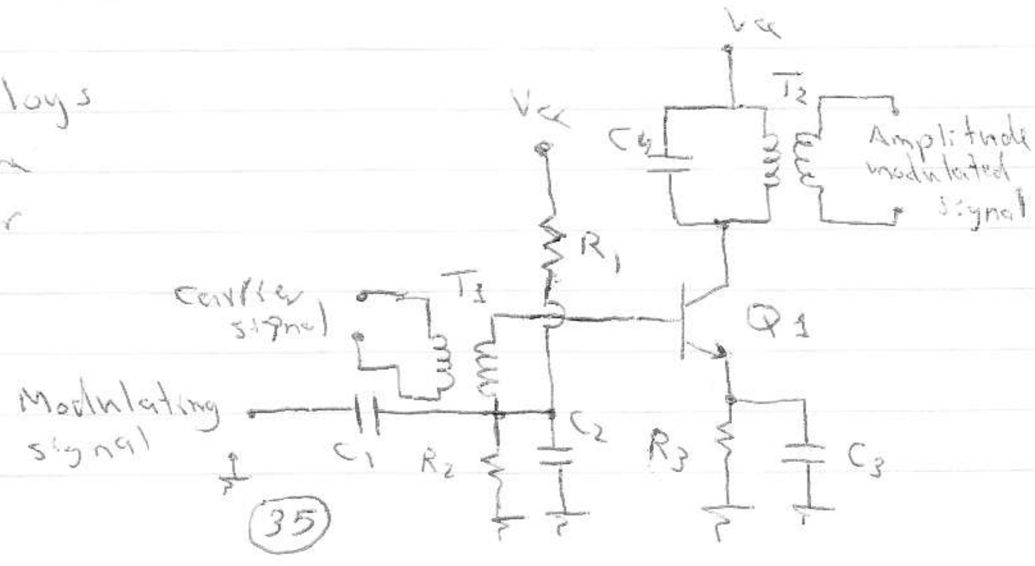
$R_3 = 10k\Omega$ larger enough to hold transistor slightly in conduction
$\Rightarrow I_{EQ} \simeq 1mA$ (for $\beta \simeq 100$)

and $V_E = 10V$, then $V_{BE} = 0.2V$

If the emitter Voltage is changed, the gain of the amplifier changes, then the message signal is fed to the emitter. This has same effect as to change the gain by changing the emitter resistance. The tuned collector cct passes only the modulated carrier and its sidebands.

Amplitude modulated transistor amplifier with base injection

This cct employs a transistor in the common-emitter configration.



The carrier signal is transformer coupled to the base of Q_1, and the modulating signal is coupled to the base of Q_1 by a capacitor C_1.

R_1 and R_2 establish the quiescent forward bias for the transistor.

The modulating signal causes the base bias to increase and decrease with variations in the modulating signal. As a result the collector current amplitude changes with the base current resulting in amplitude modulated signal at the collector cct.

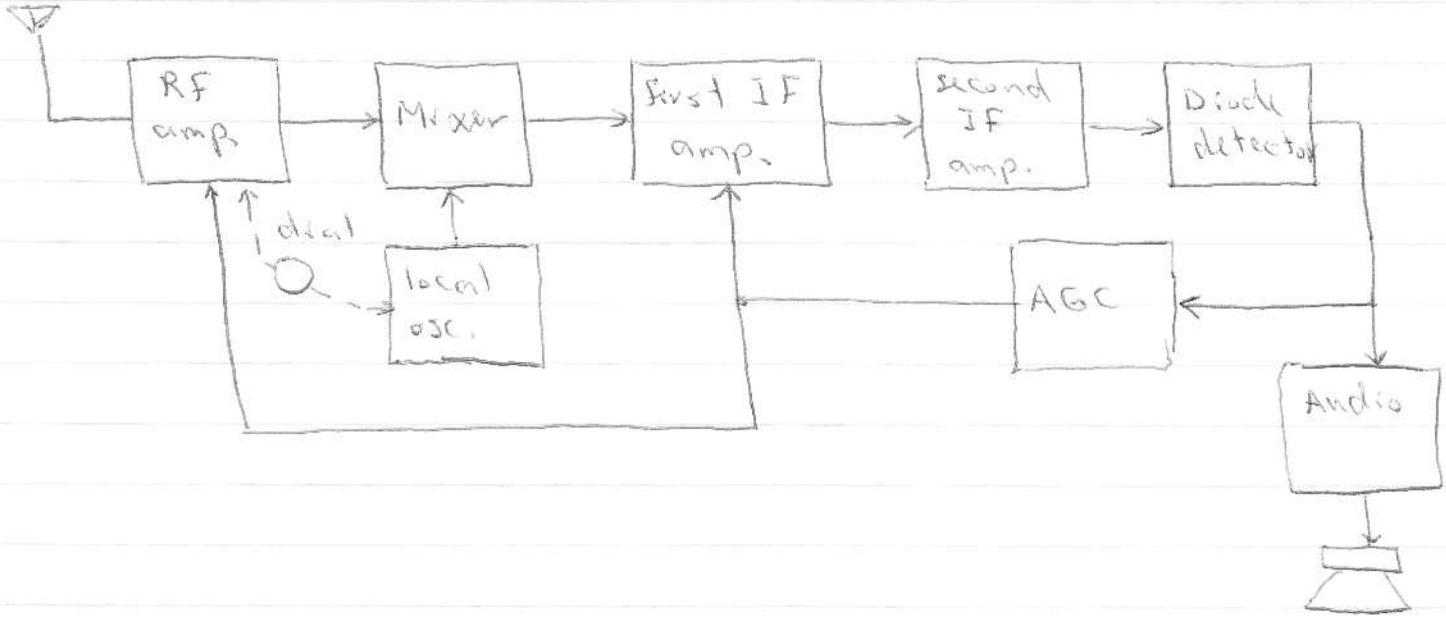
The primary cct of the o/p transformer T_2 is tuned by capacitor C_4 to the carrier signal freqn

AM Receivers :-

The simplest receiver contains the four blocks which are

1. the Antenna
2. tuned circuit
3. Diode rectifier cct
4. reproducing device to convert the electrical impulses to a work function.

A typical block diagram of an AM radio receiver is shown below, this is called superhetrodyne receiver.



The RF amp. :

It is used to provide gain at the point of lowest noise in the system. It also participates in selecting the signal to be processed through the receiver. The RF amplifier is often a tuned-I/p, tuned o/p Amplifier

The mixer / oscillator:

The mixer is a true modulator. It has two I/p's, one coming from the RF amplifier and the second from the local oscillator. The mixer downshift's the carrier signal to an intermediate frequency (IF). The IF for low carrier frequency AM receivers is an industry standard of 455 kHz.

The local oscillator in low frequency system is always operat at a freq. higher than the incoming RF signal by an amount equal to the IF. In higher-freq. service bands, the local oscillator may be operated above or below the freq. of the incoming RF signal

A tuning dial assembly is used to link the RF amp. and the local osc. tuned cct. so that to keep the difference between the two signals always constant when the station selection is changed.

The intermediate freq. Amplifiers:

The IF amplifiers are tuned cct's that select the difference freq. (from the o/p signal of the mixer) only. The IF cct may have 1, 2 or 3 seperated amplifiers all tuned to the same freq. (455 KHz)

The Detector:

There are several cct's for detection of the AM wave, all use diode rectifier and low pass filter. The diode detector recovers one half cycle of the modulated signal

Automatic Gain Control (AGC):

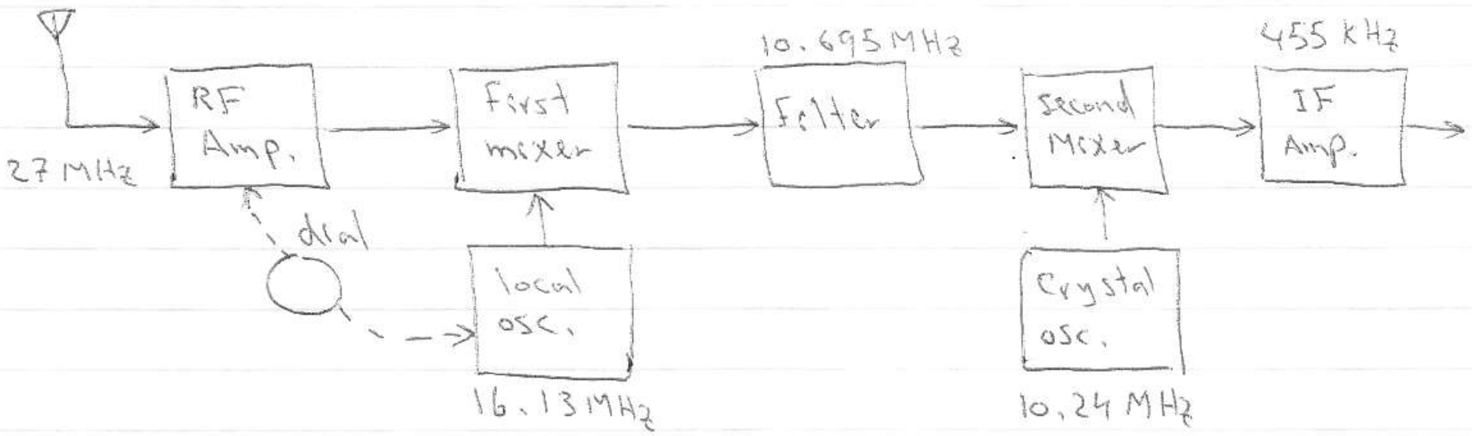
The AGC cct is used to feed back a bias voltage (taken from the DC component of the wave recovered by the detector) to the first IF amplifier to modify the gain of the amplifier relative to the signal strength at the antenna

Output reproducer:

For radio receivers the o/p reproducer is an audio cct connected to a speaker to change the message signal into sound.

Double _conversion Receivers

Receivers in the high-freq. RF range are more likely to use double_conversion front-end circuitry.



system Advantages:

1) RF signal is reduced to a very low IF (in two steps) which causes less problems and provides gain bandwidth product features better suited to stable cct.

2) Double conversion systems have a better image rejection ratio than single_conversion cct.

system disadvantages:

higher cost and more complecated system

* Definitions

1) selectivity:
It describes the ability of the receiver to accept the

desired signal freq. while rejecting all closely adjacent signals

2) Dynamic Range :
It is the difference in I/p level, in dB's, between the minimum detectable signal and the over load condition

3) Image Freq. Rejection:
An image is an unwanted RF signal that is the same distance from the local oscillator freq. as the desired signal but in the opposite direction

Image freq. = RF + 2IF For osc. operates at a higher freq. than the desired RF signal

Image freq. = RF - 2IF For osc. operates at a lower freq. than the desired RF signal

The rejection of an image signal depends on the ratio of wanted to unwanted signal frequencies (F) and on the total quality factor (Q_t) of the resonant cts. that precede the mixer

$$\text{Image rejection (dB)} = 20 \log \sqrt{Q_t^2 F^2}$$

$$F = \frac{\text{image freq.}}{\text{RF freq.}} - \frac{\text{RF Freq.}}{\text{image freq.}}$$ (local osc. freq. is higher than RF)

and

$$F = \frac{\text{RF freq.}}{\text{image freq.}} - \frac{\text{image freq.}}{\text{RF freq.}}$$ (For local osc. freq. lower than RF freq.)

4) Signal to noise ratio (S/N) :

$$S/N\ (dB) = 10 \log \frac{\text{signal Power}}{\text{noise power}}$$

5) Sensitivity :-

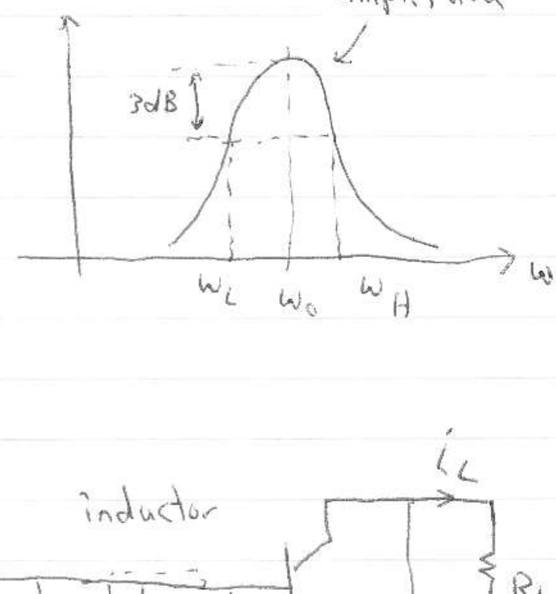
It is the ability of the receiver to amplify weak signals. It identifies the minimum I/p at the Antenna that will result in a signal 20 dB greater than the noise of detector o/p.

Tuned Amplifiers

Tuned amplifiers are used to amplify signals within a narrow frequency band centered about a frequency ω_0. These amplifiers are used in almost all communications equipment like radio receiver.

pass band $BW = \omega_H - \omega_L$

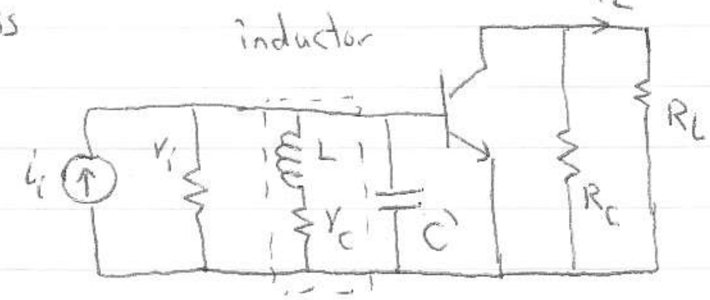
when tuning a radio ω_0 is varied, while the pass band is kept constant.



Single - Tuned Amplifiers :-

To obtain a tuned bandpass amplifier, a parallel tuned cct is added to the ordinary common - emitter Amp.

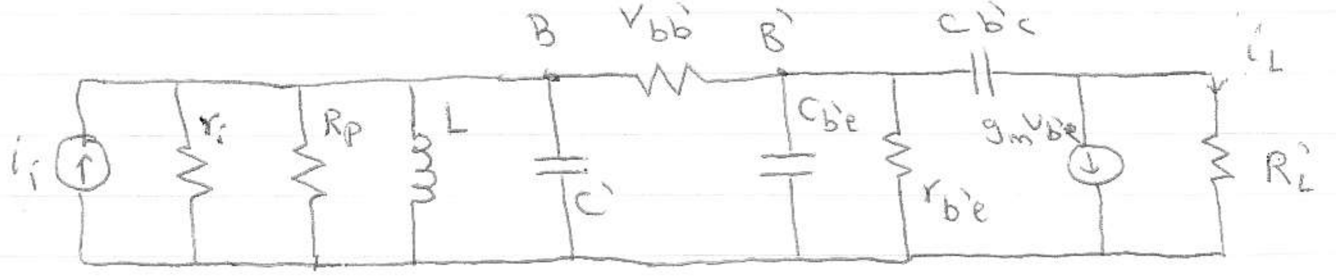
C : external capacitor

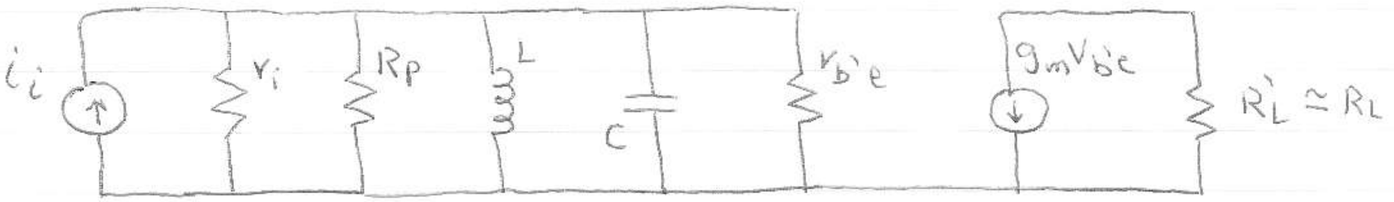


Assume $R_L << R_c$ then $R_L' \simeq R_L$

and $r_{bb'} \simeq 0$

Then the simplified equivalent cct will be



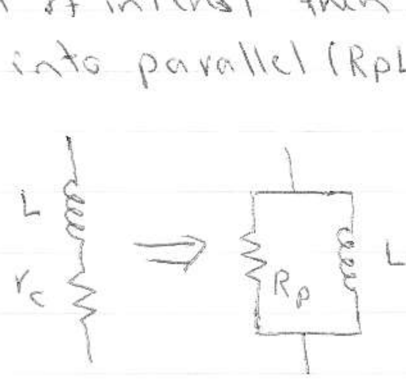


$$C = C' + C_{b'e} + C_{b'c}(1 + g_m R_L)$$

If the coil has high quality factor Q_c, i.e. coil ~~has~~ losses are low over the freq. band of interest then the series ($r_c L$) cct can be converted into parallel ($R_p L$) cct.

$$R_p = Q_c^2 r_c = \omega_0 L Q_c \quad \left(\text{where } Q_c = \frac{\omega_0 L}{r_c}\right)$$

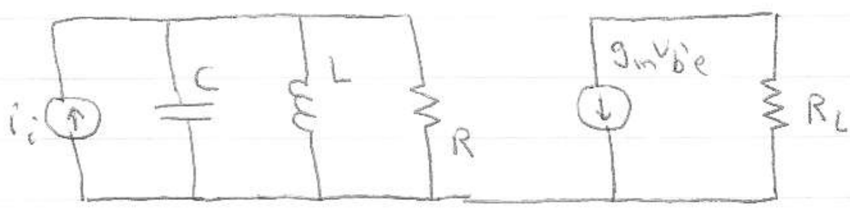
$$Y_p = Y_s$$



$$R_p + \frac{1}{j\omega L} = \frac{1}{r_c + j\omega L}$$

$$\Rightarrow R_p = \frac{\omega^2 L^2}{r_c}$$

$$R = R_p \,//\, r_i \,//\, r_{b'e}$$



The current gain of the Amplifier is

$$A_i = \frac{i_L}{i_i} = \frac{-g_m V_{b'e}}{V_{b'e}(j\omega C + \frac{1}{j\omega L} + \frac{1}{R})} = \frac{-g_m R}{1 + j(\omega RC - \frac{R}{\omega L})}$$

$$= \frac{-g_m R}{1 + j\omega_o RC\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)} \quad \text{----} (1)$$

where $\omega_o^2 = \frac{1}{LC}$ center freq. or resonant freq.

The quality factor (Q) of the tuned input cct. at the resonant freq. ω_o can be defined to be

$$Q_i = \frac{R}{\omega_o L} = \omega_o RC \quad \text{----} (2)$$

substituting eq. (2) into eq. (1), then eq. (1) becomes

$$A_i = \frac{-g_m R}{1 + jQ_i\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)} \quad \text{----} (3)$$

when $\omega = \omega_o$ the gain is maximum

$$A_i = -g_m R$$

at $\omega = \omega_L$ or $\omega = \omega_H$ the gain is $|A_i| = \frac{g_m R}{\sqrt{2}}$ --(5)

from eq. (3) and eq. (5)

$$\frac{g_m^2 R^2}{2} = \frac{g_m^2 R^2}{1 + Q_i^2\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)^2}$$

~~This is ω_H~~

$$\Rightarrow 1 + Q_i^2\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)^2 = 2$$

This is quadratic equation and has two positive solution ω_H and ω_L

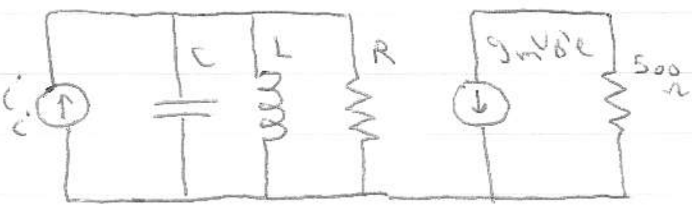
The 3-dB bandwidth is

$$B.W. = f_H - f_L = \frac{\omega_o}{2\pi Q_i} = \frac{1}{2\pi RC}$$

Ex: Design a single-tuned Amp. to operate at a centre freq. of 455 KHz with a bandwidth of 10 KHz. The transistor has the parameters $g_m = 0.04\,S$, $h_{fe} = 100$, $C_{b'e} = 1000\,pf$ and $C_{b'c} = 10\,pf$. The bias network and I/p resistance are adjusted so that $r_i = 5\,k\Omega$ and $R_L = 500\,\Omega$

Sol:-

$$BW = \frac{1}{2\pi RC} = 10\,k$$



$$\Rightarrow RC = \frac{10^{-4}}{2\pi}$$

$$R = r_i // R_p // r_{b'e}$$

$$r_{b'e} = \frac{h_{fe}}{g_m} = 2.5\,k\Omega$$

$$R_p = Q_c\,\omega_o L = \frac{Q_c}{\omega_o C} \qquad \left[\text{where } \omega_o^2 = \frac{1}{LC} \Rightarrow \omega_o L = \frac{1}{\omega_o C}\right]$$

$$R = 5 // 2.5 // \frac{Q_c}{\omega_o C}$$

$$\frac{1}{R} = \frac{1}{5k} + \frac{1}{2.5k} + \frac{\omega_o C}{Q_c}$$

$$C = \frac{1}{2\pi\,B.W.\,R} = \frac{10^{-4}}{2\pi}\left[\frac{10^{-3}}{5} + \frac{10^{-3}}{2.5} + \frac{2\pi(455\times10^{3})C}{Q_c}\right]$$

$$C = \frac{3\times10^{-7}}{10\pi} + \frac{45.5\,C}{Q_c}$$

$$C\left(1 - \frac{45.5}{Q_c}\right) = 0.95\times10^{-8}$$

$$\Rightarrow C = \frac{0.95 \times 10^{-8}}{1 - \frac{45.5}{Q_c}}$$

The total I/p capacitance is :

$$C = C' + C_{b'e} + C_{b'c}(1 + g_m R_L)$$

$$= C' + 1000 + 10(1 + 0.04 \times 500)$$

$$= C' + 1200\ pf = \frac{0.95 \times 10^{-8}}{1 - \frac{45.5}{Q_c}}$$

Q_c must be grater than 45.5 so that the capacitance be positive. At 455 KHz, a typical range of practical values of Q_c lies between 10 and 150, so let $Q_c = 100$ then:

$$\therefore C = \frac{0.95 \times 10^{-8}}{1 - \frac{45.5}{100}} = 1.74 \times 10^{-8} = 0.0174\ \mu f$$

$\therefore C' \simeq 0.016\ \mu f$ (practical value)

$$w_o^2 = \frac{1}{LC} \Rightarrow L = \frac{1}{w_o^2 C}$$

$$L = \frac{1}{(2\pi * 455)^2 \times 0.017\ \mu f} = 6.9\ \mu H$$

$$R_p = Q_c w_o L = 100 \times 2\pi \times 455 \times 10^3 \times 6.9 \times 10^{-6} = 2\,k\Omega$$

$$r_c = \frac{Q_c^2}{R_p} = 0.2\,\Omega$$

$$R = r_i // R_p // r_{b'e} = 0.91\ k\Omega$$

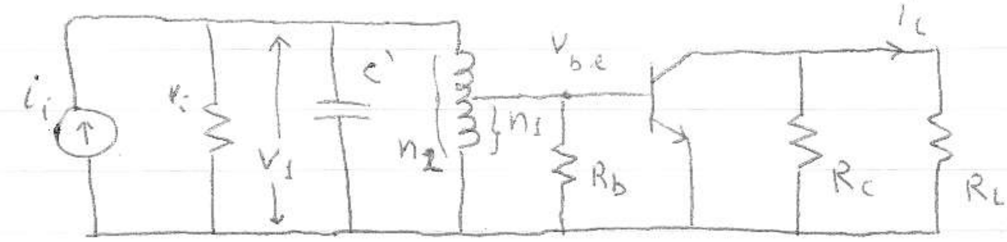
The mid freq gain $A_{im} = -g_m R = -0.04 \times 910 = -36.4$

Impedance Matching to Improve Gain:

The single tuned amp. has the following drawbacks:

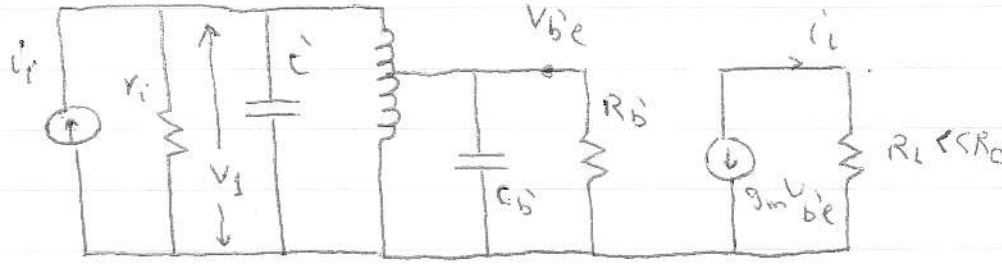
1) Impractical element values and low gain because of the low effective resistance (R) in the base circuit of the transistor.
2) low resistance implies low Qi and therefore, it is difficult to achive narrow bandwidth in this parallel tuned Amp.

To solve this problem a tapped inductor (like an auto transformer) is used. Which serves to transform the low resistance into more reasonable value.



The turns ratio of the ideal auto transformer is

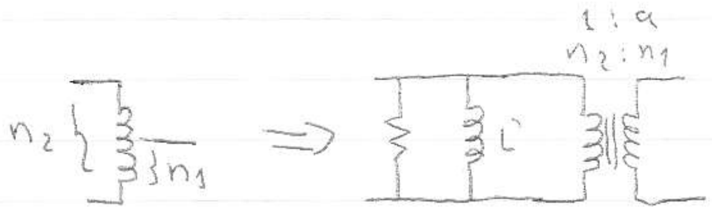
$$a = \frac{n_1}{n_2} = \frac{V_{b'e}}{V_1} < 1$$



$$R_{b'} = R_b \, // \, r_{b'e}$$

$$C_{b'} = C_{b'e} + C_{b'c}(1 + g_m R_L)$$

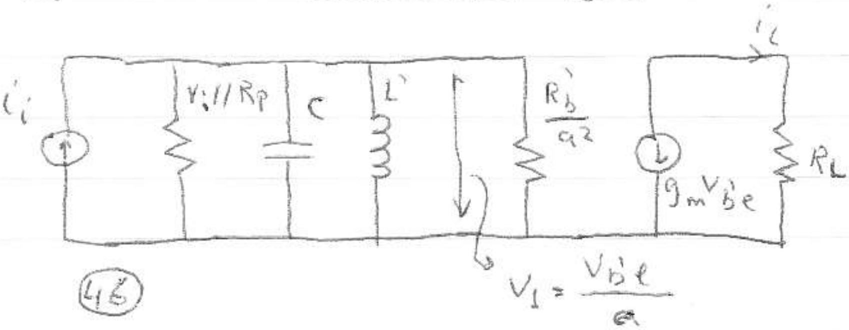
The current gain is



$$A_i = \frac{i_L}{i_i} = \frac{-g_m V_{b'e}}{V_1\left(\frac{1}{R} + j\omega C + \frac{1}{j\omega L'}\right)}$$

$$R = r_i \, // \, R_p \, // \, \frac{R_{b'}}{a^2}$$

$$C = C' + a^2 C_{b'}$$



$$V_1 = \frac{V_{b'e}}{a}$$

$$V_1 = \frac{V_{b'e}}{a}$$

$$\therefore A_i = \frac{-a g_m R}{1 + j\omega C R\left(1 - \frac{1}{\omega^2 C L'}\right)}$$

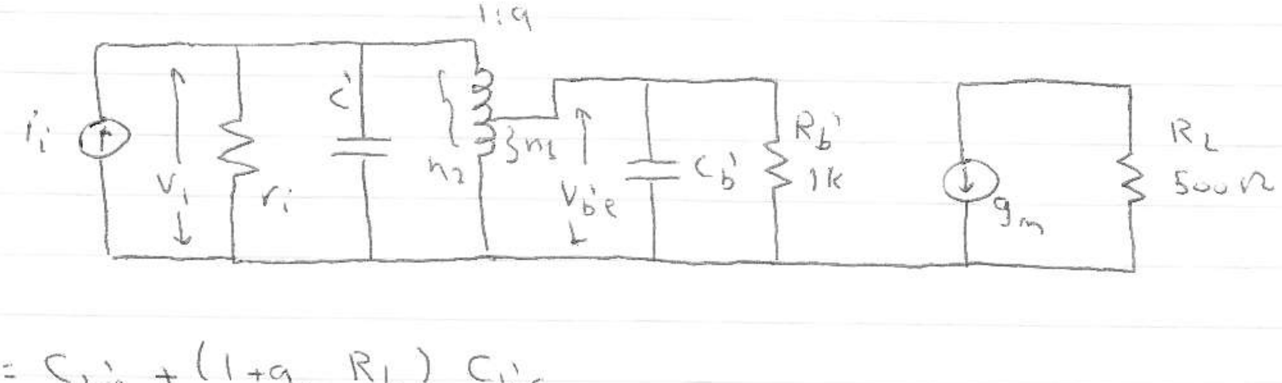
$$= \frac{-a g_m R}{1 + jQ_i\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)} \qquad \text{where } Q_i = \omega_o R C$$

and $\omega_o^2 = \frac{1}{L'C} = \frac{1}{L'(C' + a^2 C_{b'})} \simeq \frac{1}{L'C'}$ since $C' >> a^2 C_{b'}$

The midfreqn gain $= -a g_m R = A_{i\,max}$

The 3dB bandwidth $= BW = f_H - f_L = \frac{1}{2\pi RC}$

Ex: A single tuned Amp. operates at $f_o = 455$ KHz and is to have a bandwidth of 10kHz, $L' = 6.9$ MH, $r_{b'e} = R_{b'} = 1k\Omega$, $r_i = 5k\Omega$, $R_p = 2k\Omega$, $C_{b'e} = 1000$ pf, $g_m = 0.15$, $R_L = 500\Omega$ and $C_{b'c} = 4$Pf. find the required turns ratio (a) and the midfrequen. Current gain



$$C_{b'} = C_{b'e} + (1 + g_m R_L)\, C_{b'c}$$

$$C_{b'} = 1000 + (1 + 0.1 \times 500) * 4$$
$$= 1200 \text{ pf}$$
$$C = C' + a^2 C_{b'} = C' + a^2 \times 1200 p$$

$R = r_i \,//\, R_p \,//\, \frac{R_b'}{a^2}$, $R_b' = R_b \,//\, r_{b'e}$

$$\frac{1}{R} = \frac{1}{r_i} + \frac{1}{R_p} + \frac{a^2}{R_b'}$$

$$= \frac{10^{-3}}{5} + \frac{10^{-3}}{2} + a^2 \times 10^{-3} = (7 + 10a^2)\, 10^{-4}$$

$$W_o^2 = \frac{1}{L'C} = (2\pi f_o)^2 = 4\pi^2 * (455 \times 10^3)^2 = 8.17 \times 10^{12}$$

$C \simeq 0.0177\, uf = C' + 1.2\, a^2 \times 10^{-9}$

since $a \leq 1 \Rightarrow a^2 \leq 1$ then $C \approx C' = 0.017\, uf$

$$B.W = \frac{1}{2\pi RC} \approx \frac{1}{2\pi RC'} = 10^4$$

$$\therefore R = \frac{10^{-4}}{2\pi * 0.017 \times 10^{-6}} = 900\, \Omega$$

$$\frac{1}{R} = (7 + 10a^2)\, 10^{-4} = \frac{1}{900}$$

$\Rightarrow a^2 = 0.414 \Rightarrow a = 0.64$

The midfreq gain is $A_{im} = -a g_m R = -0.64 \times 0.1 \times 900$

$$A_{im} = -58$$

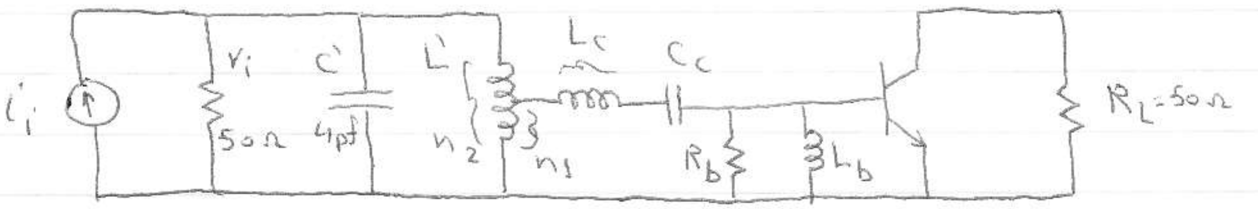
Note: as compared with the tuned Amp. in the previous example a higher gain (−58) is obtained (compared to −36.4) for the same bandwidth (10 kHz) and the same I/p impedance R.

If the auto transformer were not used, the same midband gain would obtained but the bandwidth would be larger as R changes due to the factor $\frac{r_{b'e}}{a^2}$

Series _ Resonant Circuits:-

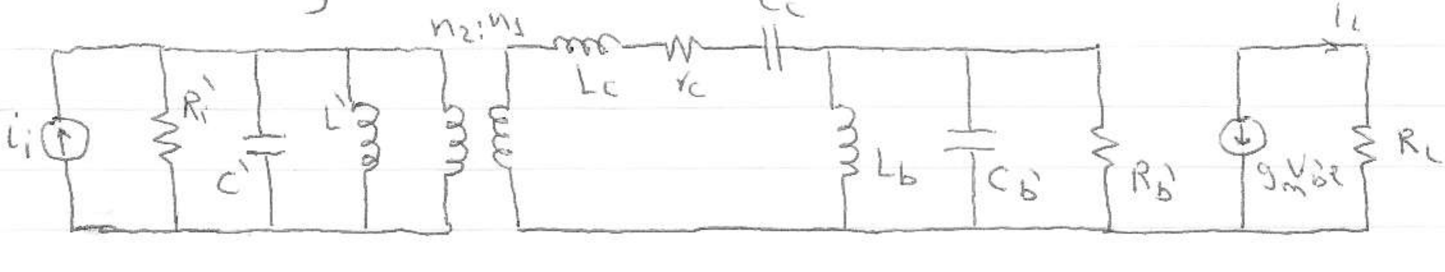
At high freq's a series - resonant circuit can be employed to provide a very high , Q , for the circuit with resonable inductance values. since at very high frequencies (f_o > 50MHz) parallel tuning results in very low , Q , for the circuits and hence wide band width.

Ex: The Amp. shown is to have a 3dB bandwidth of 2MHz and resonant freq. of 100 MHz. The transistor employed has the parameters $r_{b'e} = 50\Omega$, $g_m = 0.1S$, $C_{b'e} = 10\,pf$, and $C_{b'c} = 1\,pf$, $R_b = 1k\Omega$. The I/p circuit consists of 50Ω resistance in parallel with a 4pf capacitor, $R_L = 50\Omega$. Find L, L_b, C_c and the turns ratio (a)



Sol:-

This Amp. is designed so that the circuit , Q , is determined by the series - resonant circuit. the parallel RLC circuit at the I/p and the base are each designed to have a low , Q , . Quite often in practice the two ccts are not carefully tuned



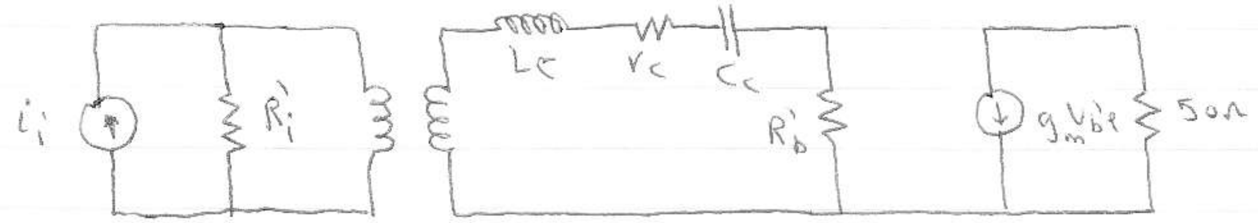
$R_i' = r_i \,//\, R_p'$ (effective res. of L')

$R_b' = R_b \,//\, r_{b'e} \,//\, R_p$ (effective res. of L_b)

$$C_b' = C_{b'e} + C_{b'c}(1 - A_V)$$

$$w_o^2 = \frac{1}{L'C'} = \frac{1}{L_cC_c} = \frac{1}{L_bC_b'}$$

The cct Q, is determined from the simplified equivalent cct shown below



In this equivalent cct, it is assumed that the quality factors of the parallel cct are very small.

The cct Q, is the same as that of the series resonant cct, Q_c, and is given by:

$$Q_c = \frac{w_o L_c}{R_b' + r_c + a^2 R_i'}$$

$$w_o^2 = \frac{1}{L'C'} = 4\pi^2 * 10^{16} \Rightarrow L' = \frac{1}{4\pi^2 * 10^{16} * 4 * 10^{-12}} = 0.63\,\mu H$$

Let Q': the quality factor of the transformer, be 100, which is a value that can be easily obtained at $100\,MHz$, then

$$Q' = \frac{R_p'}{w_o L'} = 100 \Rightarrow R_p' = 100(2\pi * 10^8)(0.63 * 10^{-6}) \simeq 40\,k\Omega$$

$$r_i = 50\,\Omega \Rightarrow R_i' = r_i \parallel R_p' \simeq 50\,\Omega$$

For the base cct:- $\quad w_o^2 = \dfrac{1}{L_b C_b'} = (2\pi * 10^8)^2$

$$C_b' = C_{b'e} + C_{b'c}(1 + g_m R_L) = 16\,pf$$

then $L_b = \dfrac{1}{(2\pi * 10^8)^2 * 16 * 10^{-12}} = 0.16\ \mu H$

Assuming $Q_b = 100 = \dfrac{R_p}{w_o L_b}$

$\Rightarrow R_p = 100\ (2\pi * 10^8)(0.16 * 10^{-6}) \simeq 10k\Omega$

$R_b' = R_p // r_{be} // R_b = 10k // 10k // 50\Omega \simeq 50\Omega$

$\Rightarrow$ The I/p cct Quality factor $Q_i \simeq w_o R_i' C_i$
$= 0.12$

The base cct Quality factor $Q_b' = w_o R_b' C_b'$
$= 0.5$

The required cct $Q = \dfrac{f_c}{B.W} = \dfrac{10^8}{2 \times 10^6} = 50 \gg Q_i$ or Q_b'

The circuit $Q_c = \dfrac{w_o L_c}{R_b' + r_c + a^2 R_i'} = 50$

or $Q_c = 50 = \dfrac{1}{w_o C_c (R_b' + r_c + a^2 R_i')}$

$w_o L_c = 50(50 + r_c + 50a^2)$ ---- (1)

For the overall cct Q_c to equal 50, the Q of the inductor L_c must be greater than 50,
let $Q_{inductor} = 250$, then

$Q_{ind.} = \dfrac{w_o L_c}{r_c} = 250 \Rightarrow w_o L_c = 250\, r_c$ ---- (2)

from (1) & (2)

$w_o L_c = 50\ (50 + \dfrac{w_o L_c}{250} + 50a^2)$

$\omega_o L_c = (1.25)(50)^2(1+a^2)$

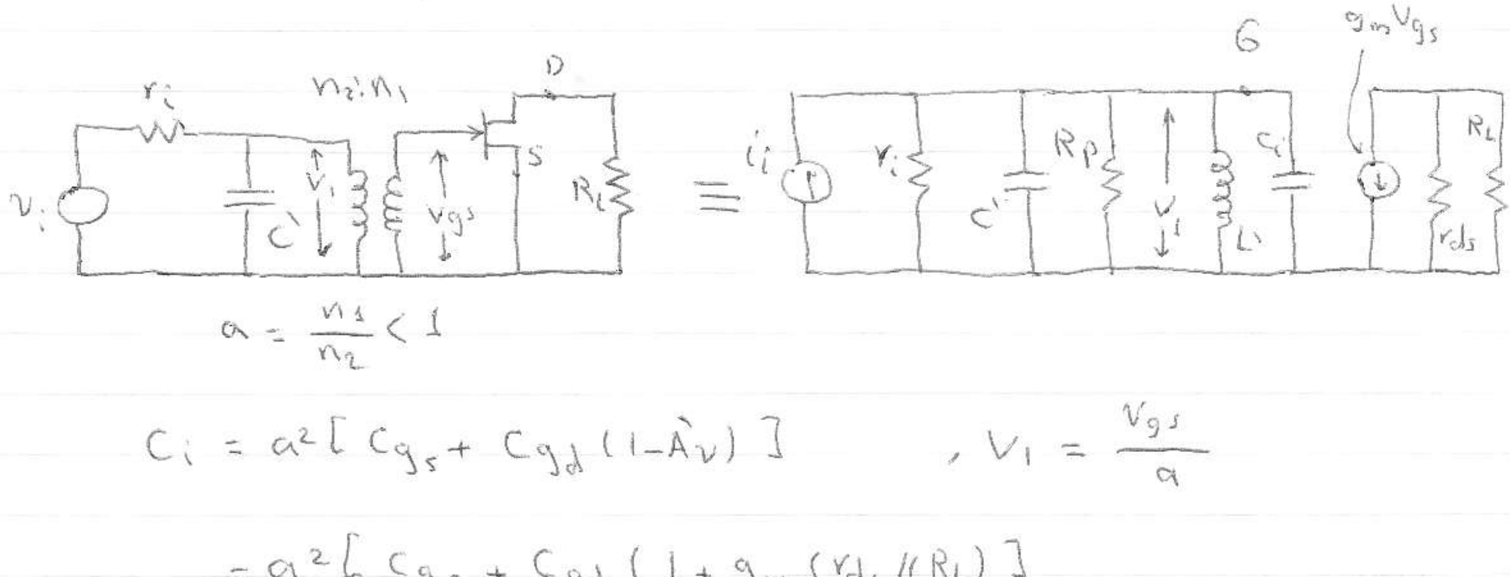
set $a^2 = 0.1$

$\Rightarrow L_c = 5.5\,\mu H$, and $C_c = \dfrac{1}{\omega_o^2 L_c} \simeq 0.45\,pf$

The Synchronously Tuned Amplifiers

Synchronous tuning means cascading of tuned amplifiers in order to achieve high gain and a bandwidth which is narrower than the bandwidth of each stage.

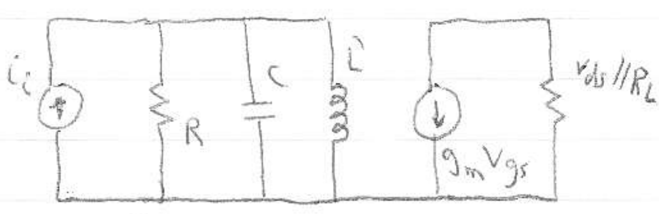
For the single tuned FET amplifier shown:



$a = \dfrac{n_1}{n_2} < 1$

$C_i = a^2[C_{gs} + C_{gd}(1 - A_v)]$, $V_1 = \dfrac{V_{gs}}{a}$

$= a^2[C_{gs} + C_{gd}(1 + g_m(r_{ds} // R_L)]$



$$A_v = \frac{v_o}{v_i} = \frac{v_o}{i_i r_i} = \frac{v_o}{v_1 Y_{in} r_i}$$

$$= \frac{-g_m V_{gs}(r_{ds} // R_L)}{\frac{V_{gs}}{a} r_i \left[\frac{1}{R} + j\omega C + \frac{1}{j\omega L'}\right]}$$

$C = C' + C_i$

$R = r_i // R_p$

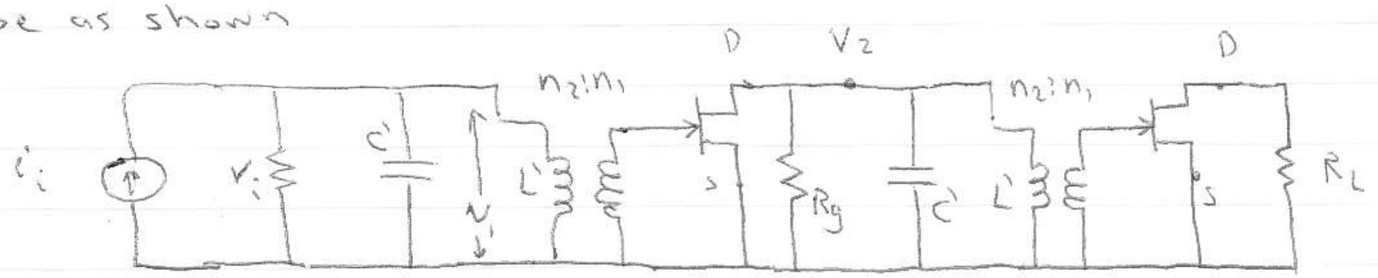
$$A_v = \frac{-a g_m (R/r_i)(r_{ds} // R_L)}{1 + j Q_i \left[\frac{w}{w_o} - \frac{w_o}{w}\right]}$$

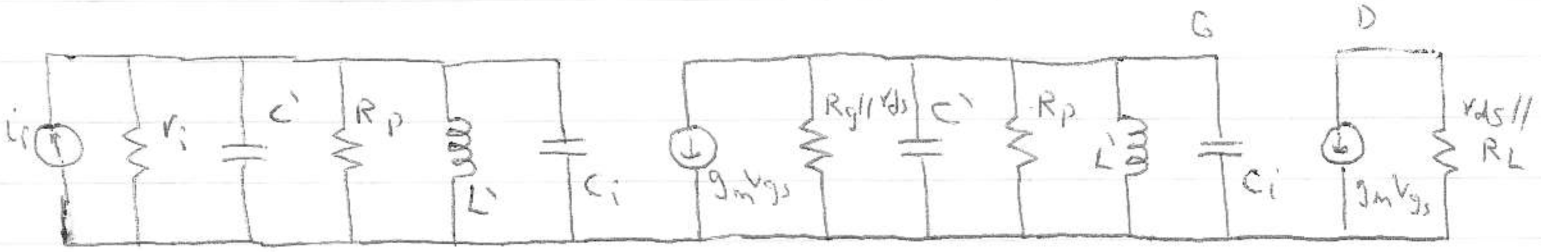
$$Q_i = w_o R C \quad , \quad w_o^2 = \frac{1}{L' C}$$

The center freq gain is $A_v = -a g_m (r_{ds} // R_L) \dfrac{R_p}{R_p + r_i}$

The 3dB bandwidth is $BW = \dfrac{1}{2\pi (r_i // R_p)(C' + C_i)} = \dfrac{1}{2\pi RC}$

If two identical tuned FET are cascaded, then the synchvonously tuned amplifier and its equivalent cct will be as shown





In a synchrounously tuned Amplifier each resonant cct is tuned to the same freq and has the same bandwidth, Hence,

$$w_o^2 = \frac{1}{L'(C' + C_i)}$$

$$R = r_i // R_p = r_{ds} // R_g // R_p = r_{ds} // R_L$$

$$C_i = a^2 [C_{gs} + C_{gd}(1 - A_v)]$$

The voltage gain of the amplifier is

$$A_v = \frac{v_L}{v_i} = \frac{(a g_m R)^2 (R/r_i)}{[1 + jQ_i(\frac{w}{w_o} - \frac{w_o}{w})]^2}$$

where

$$Q_i = w_o R (C' + C_i) \quad \& \quad A_{v_{\text{mid freq.}}} = (a g_m R)^2 \frac{R_p}{R_p + r_i}$$

The 3dB B.W of the two-stage amplifier is obtained from:

$$[1 + Q_i^2 (\frac{w}{w_o} - \frac{w_o}{w})^2]^2 = 2$$

$$B.w = \frac{w_o}{2\pi Q_i} \sqrt{\sqrt{2} - 1} = 0.643$$

$$\frac{f_o}{Q_i} = 0.643 \frac{1}{2\pi (C' + C_i) R}$$

Ex: Given that $g_m = 5\,m\mho$, $r_i = r_{ds} = 10\,k\Omega$, $R_p = R_L = 100\,k\Omega$, $R_g = 910\,k\Omega$, $a = \frac{1}{2}$, $C' = 0$, $C_{gs} = 10\,pf$, $C_{gd} = 0.1\,pf$, and $L' = 0.25\,mH$, calculate w_o, A_v, and the bandwidth assuming ① a single-tuned stage, ② two synchronously ~~tuned~~ tuned stage

Sol: let $a = \frac{1}{2}$

The resonant freq. w_o is

$$w_o = \frac{1}{\sqrt{L'C}} = \left[\frac{1}{L' a^2 \{C_{gs} + C_{gd}[1 + g_m (r_{ds} // R_L)]\}}\right]^{0.5}$$

$$w_o = \left[\frac{1}{(0.25 * 10^{-6})(0.5)^2 (15 * 10^{-12})}\right]^{0.5} = 10^9 \text{ rad/sec}$$

a) single tuned stage

The center freq gain for one stage is:

$$A_v = -a g_m (r_{ds} // R_L) \frac{R_p}{r_i + R_p} = -\frac{1}{2}(5\times10^{-3})(10^4 // 10^5)\frac{100}{110}$$

$$\simeq -25$$

$$BW = \frac{1}{2\pi (r_i // R_p)(C' + C_i)}$$

$$C_i = a^2 [C_{gs} + C_{gd}(1 + g_m r_{ds} // R_L)]$$

$$= (0.5)^2 [15 \times 10^{-12}] = 3.75 \times 10^{-12}\ F$$

$$BW = 4.1\ MHz$$

2) For two stages

$$A_v \simeq (a g_m R)^2 \frac{R_p}{R_p + r_i} = \left(\frac{5}{2}\times 10\right)^2 \frac{100}{110} \simeq 625$$

$$BW = 0.643 \frac{1}{2\pi R (C' + C_i)} \simeq 2.6\ MHz$$

Tuned Input tuned output Amp:-

Ex: for the RF amplifier shown, Find the DC currents I_C, I_E and I_B and the DC voltages V_C, V_E, and V_B then calculate the gain at the centre freq.

Sol:

for the RF amplifier used in an AM radio receiver then it must be operated in the 540 KHz to 1600 KHz range of freq's

Communication Electronics

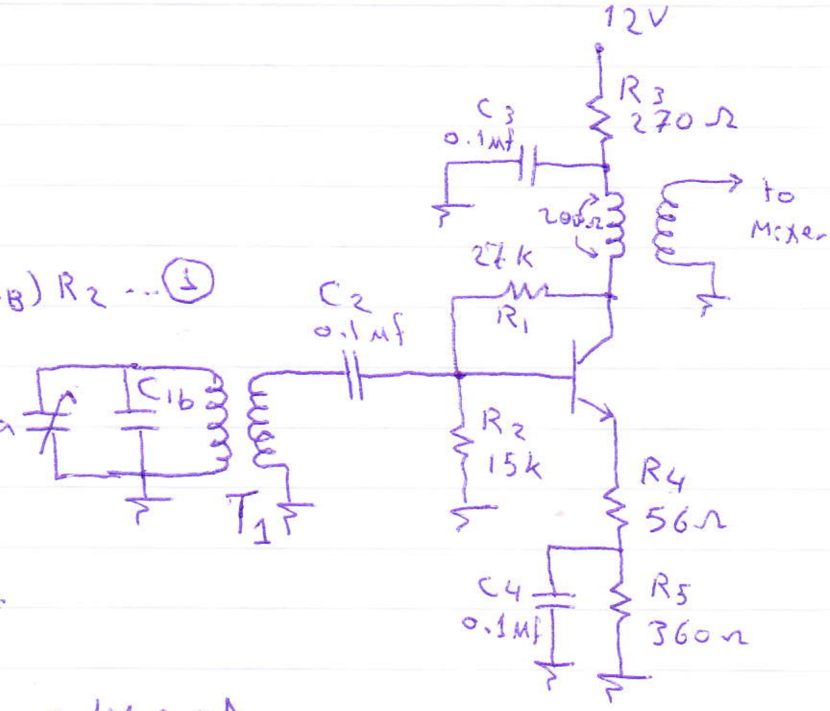
Fourth Class

Part II

Electronics and Communications Engineering
Department

DC analysis:

For the DC equivalent cct shown below



$$12 = (I_c + I_1) R_3 + I_1 R_1 + (I_1 - I_B) R_2 \quad \cdots (1)$$

$$(I_1 - I_B) R_2 = V_{BE} + I_E R_E \quad \cdots (2)$$

From eq. (1) and (2) and taking $\beta = 150$ at low freq. and $\beta = 50$ for High-freq.

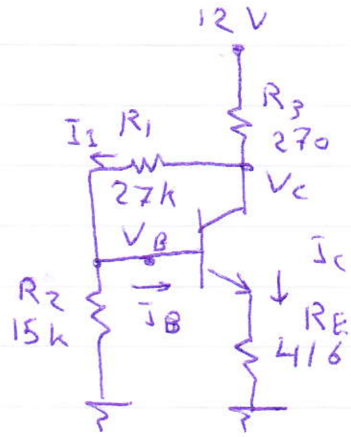
then $I_c \simeq I_E = 6.2\ mA$, $I_B \simeq 41.3\ \mu A$

then $V_E \simeq 2.6\ V$, and $V_B = 3.3\ V$

and $V_c = V_{cc} - (I_1 + I_c) R_3$

$$I_1 = \frac{V_B}{R_2} + I_b = 0.26\ mA$$

$$V_c = 10.27\ V$$



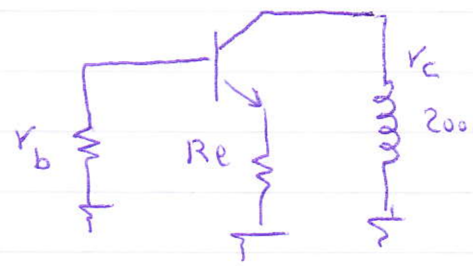
AC analysis:

At resonance

$$v_o = -i_c r_c$$

$$v_{in} = i_b (r_{b'e} + \beta R_e) = \beta i_b (r_e + R_e)$$

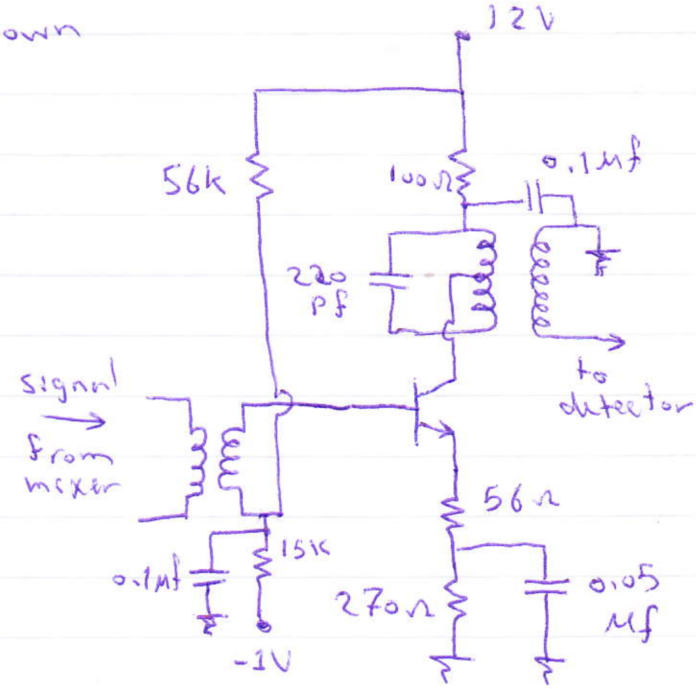
$$\Rightarrow v_i = i_e R_e \qquad \text{where} \quad r_e = \frac{V_T}{I_{EQ}} = \frac{26}{6.2} \simeq 4\Omega \ll R_e$$



$$|A_v|_{max} = \left| \frac{v_o}{v_{in}} \right| \simeq \left| \frac{-r_c}{R_e} \right| = \frac{200}{56} = 3.57 = 11\ dB$$

The I/p cct is tunable by the variable capacitor (C_{1a}) across the antenna tank cct over the frequency range of 540 kHz to 1600 KHz. The tuning of the o/p is through the inductance of the output transformer.

Ex: For the IF amplifier shown find the DC voltages V_B, V_C, V_E and DC currents I_B, I_C and I_E.
Also find the centre freq. and B.W. knowing that $\beta = 150$ and the total primary resistance of the o/p transformer is $55\ \Omega$



DC analysis

$R_B = 15 // 56k = 11.83\ k\Omega$

$V_{BB} = 12 \frac{15}{15+56} - 1 \frac{56}{56+15}$ (super position)

$V_{BB} = 1.746\ V$

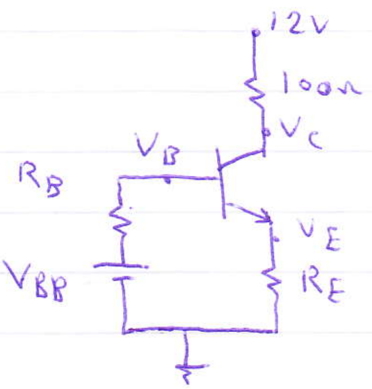
$I_B = \frac{V_{BB} - V_{BE}}{R_B + (1+\beta)R_E}$

$I_B = 17.14\ \mu A$

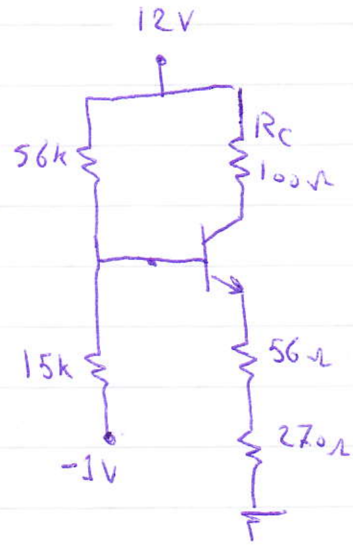
$I_E = (1+\beta) I_B$

$= 2.58\ mA$

$I_C = 2.57\ mA$



⇐



$V_C = V_{CC} - I_C R_C = 12 - 2.57 \times 0.1 = 11.743V$

$V_B = V_E + 0.7 = 1.543\ V$

A.C. analysis:

The collector is tied to a tap on the transformer primary to match the o/p impedance of the transistor, to the impedance of the transformer at the tap.

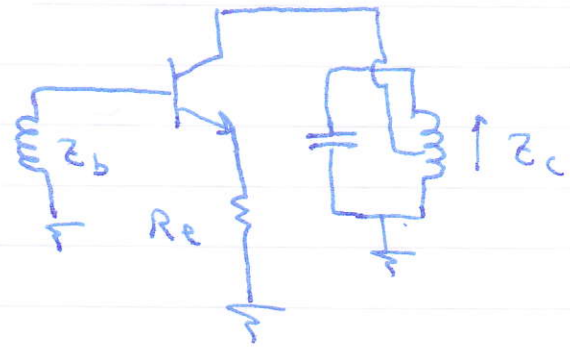
The O/p impedance of the transistor

$$Z_o = \frac{V_{CE}}{I_E} = \frac{11.743 - 0.843}{2.58} = 4.225\ k\Omega \quad (\text{at Q-point})$$

$$v_b = v_{in} = i_b(r_{be}' + \beta R_e) = \beta i_b(r_e + R_e) \simeq i_c(r_e + R_e)$$

$$A_{v_{max}} = \frac{v_o}{v_{in}} = \frac{-i_c Z_o}{i_e(r_e + R_e)} \simeq \frac{-Z_o}{R_e} = \frac{-4.223k}{56} = -75.45$$

dB gain $= 20 \log 75.45$
$= 37.54$ dB



When the transformer is tuned to 455 kHz, with C = 220 pf

$$\text{then } L_p = \frac{1}{(2\pi * 455 * 10^3)^2 * 220 * 10^{-12}} = 556.15\ \mu H$$

The loaded Q of the primary

$$Q_{loaded} = \frac{X_{Lp}}{R} = \frac{2\pi * 455 * 556.15 * 10^{-3}}{55} \simeq 29$$

$$B.W = \frac{f_o}{Q} = \frac{455}{29} = 15.69\ kHz$$

Note that two transformers are used in the cct. with the same centre freq. and Bandwidth, then the amplifier bandwidth shrinks to

$$B.W = BW\sqrt{\sqrt{2}-1}$$

$$= (15.69) \times (0.643) = 10\ kHz$$

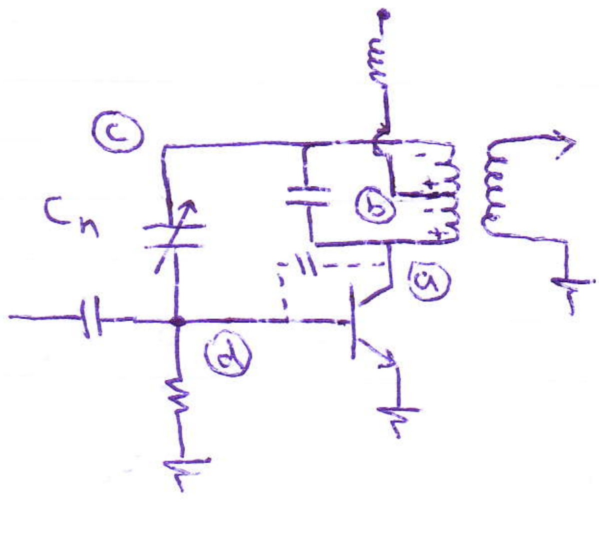
Neutralization

Neutralization of a circuit means cancelling the effects of the interelectrode capacitances. The most effective capacitance is that between the base and collector (for BJT). Most amplifiers have 180° phase shift, from base to collector, but an additional phase shift can occur through the reactance of the small value of base - collector capacitance ($C_{B'C}$), allowing a feedback signal to the base that is close enough in phase to cause parasitic oscillations. Commonly, these oscillations are at freq. higher than the RF signal being amplified. parasitic oscillations drain power from the amplifier and cause over heating and distortion.

Most of the amplifiers include a neutralizing feedback capacitor to cancel the interelectrode capacitance. The following methods are used for neutralization :-

1) Hazeltine neutralization

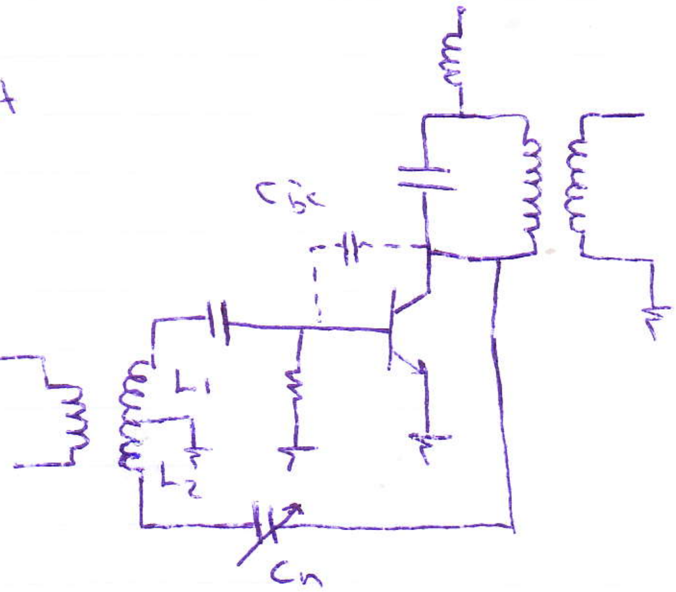
If point ⓐ is +ve with respect to ⓑ which is at ac ground then ⓒ is -ve with respect to



(b). C_n is adjusted to equal C_{BC}. the feedback Voltages are equal and opposite.

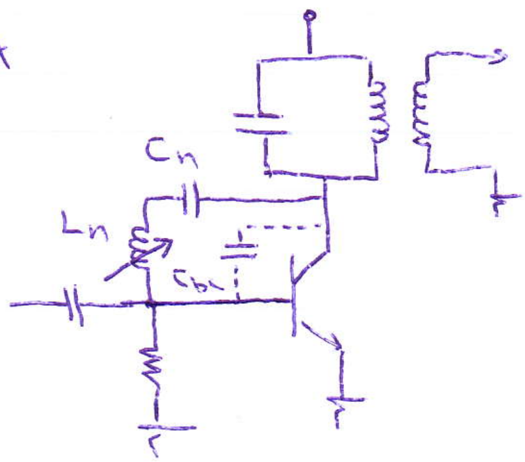
2) Base neutralization

The signal voltage at one end is of opposite polarity of that at the other end of the inductor



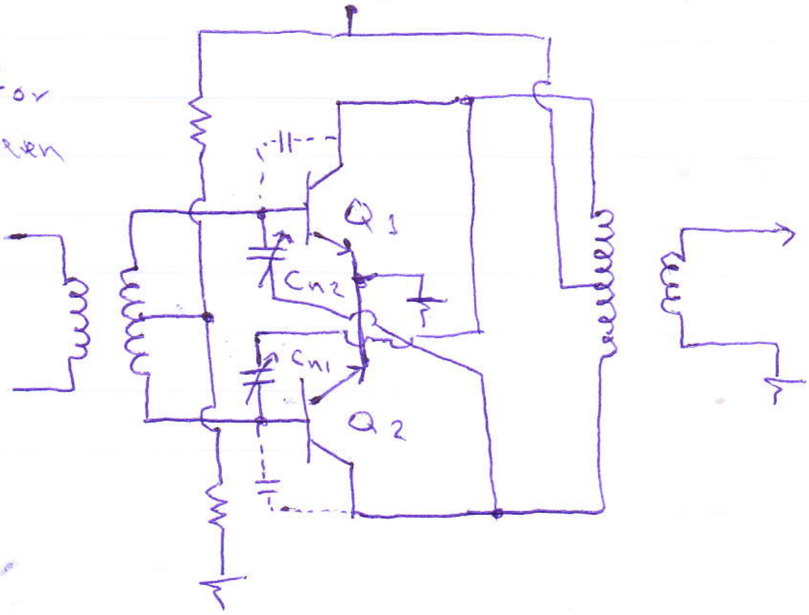
3) Direct or inductive neutralization

C_n has a short cct. effect at RF freq. but isolate the collector DC voltage from affecting the base DC voltage.
L_n and C_{bc} would form a resonant cct. and a high impedance at resonance



4) Cross neutralization of push-pull RF Amplifier

Direct connection for the capacitor between the collector of one transistor to the base of the other



* Mixers:

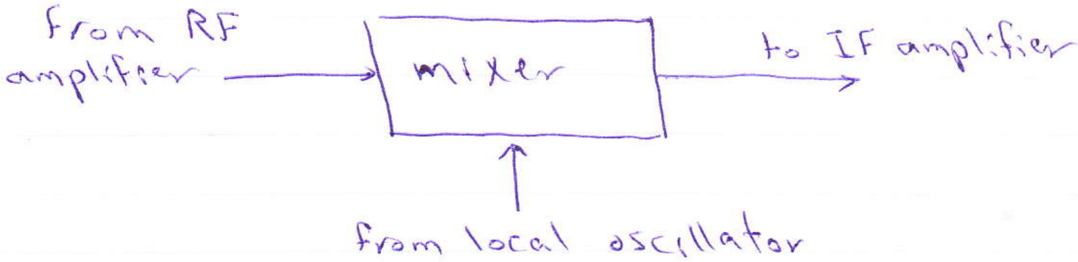
The mixer utilizes the trigonometric identity that expands the product of two cosine terms into sum and difference frequencies.

If $a(t)\cos At$ is the desired incoming signal
and $b(t)\cos Bt$ is the constant amplitude local oscillator signal

then

$$[a(t)\cos At][b(t)\cos Bt] = \frac{a(t)b(t)}{2}[\cos(A-B)t + \cos(A+B)t]$$

If the IF amplifier following the mixer is tuned to a radian freq. $(A-B)$, then the IF output will be a freq. translated version of the incoming signal and is modulation insensitive.



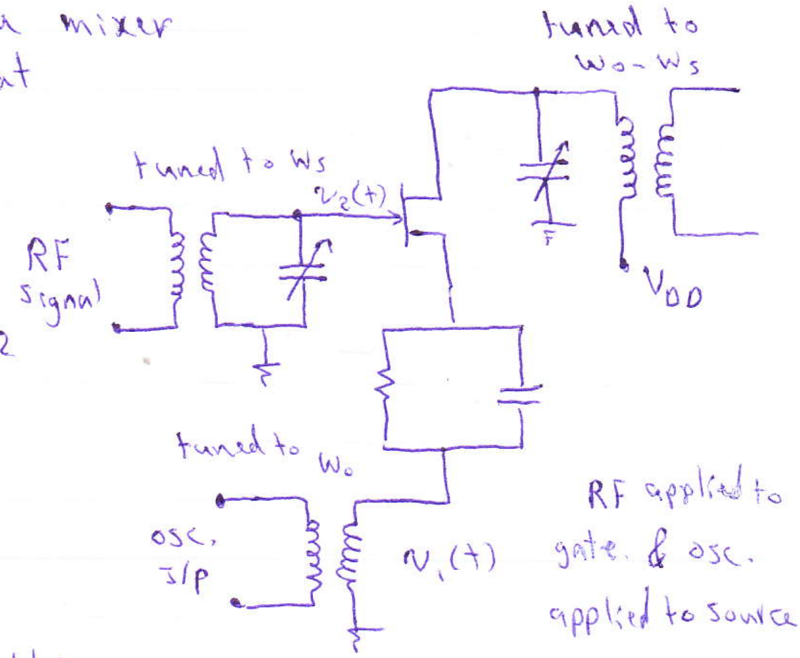
FET mixers :

If the JFET used in the mixer cct shown is biased so that it operates in the constant current region for all time then

$$i_D = I_{DSS}\left[1 - \frac{v_1(t) + v_2(t) + V_{GS}}{V_p}\right]^2$$

where

$$v_{GS} = v_1(t) + v_2(t) + V_{GS}$$



If a MOSFET is used in the cct then

for MOSFET $i_D = \frac{\beta}{2}\left[v_{GS} - V_{Th}\right]^2$

$$i_D = \frac{\beta}{2}\left[v_1(t) + v_2(t) + V_{GS} - V_{Th}\right]^2$$

(β depends on the transistor dimensions and other thinks)

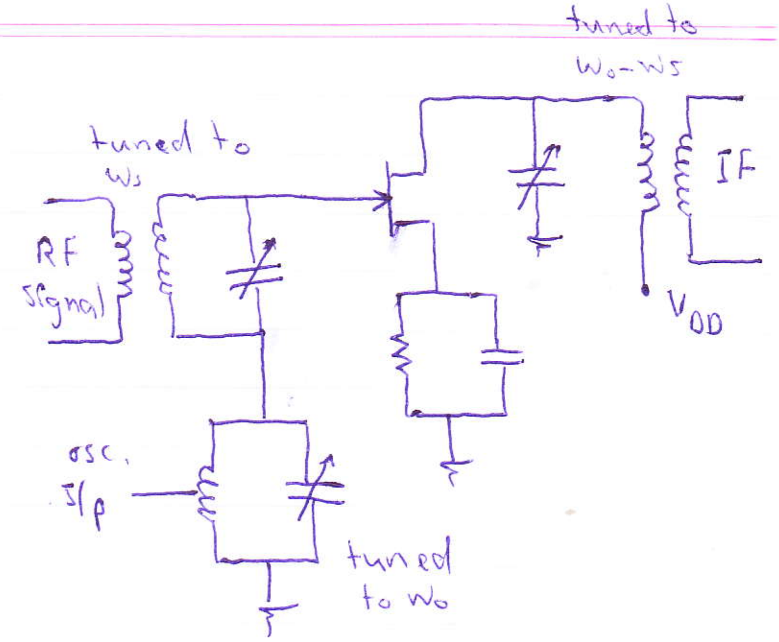
For MOSFET

$$i_D = \beta\left[v_1(t)\,v_2(t) + \frac{[v_1(t)]^2}{2} + \frac{[v_2(t)]^2}{2} + \frac{(V_{GS} - V_{Th})^2}{2} + (V_{GS} - V_{Th})(v_1(t) + v_2(t))\right]$$

If $v_1(t) = V_1 \cos w_o t$
and $v_2(t) = V_s(t) \cos w_s(t)$
and if the IF filtering at the o/p is tuned to $w_o - w_s$
then:

$$G_c = \frac{\beta V_1}{2}$$

$$G_m = \beta(V_{GS} - V_{Th}) = g_m$$

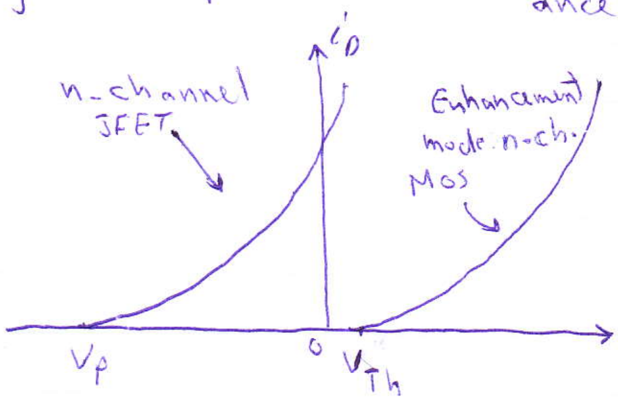


FET Mixer with both signals (RF and local osc.) applied to the gate

where G_c is a large signal conversion transconductance and it is the envelope of the o/p current at the desired freq. divided by the envelope of the I/p signal voltage (RF signal) and G_m the ordinary large-signal amplifier transconductance

$$\frac{G_c}{G_m} = \frac{V_1}{2(V_{GS} - V_{Th})}$$

For the operation to remain in the square law region



$$v_s(t) + V_1 < V_{GS} - V_{Th}$$

$V_{GS} > V_{Th} \Rightarrow v_1(t) + v_2(t) + V_{GS} > V_{Th}$
$\therefore -V_1 - v_s(t) + V_{GS} > V_{Th}$
$\therefore V_{GS} - V_{Th} > v_s(t) + V_1$

and $G_c < \frac{1}{2} G_m$

If V_1 is constant, then distortionless conversion results.
Similarly, for the JFET:

$$G_c = \frac{I_{DSS} V_1}{V_p^2} \quad , G_m = \frac{-2 I_{DSS}}{V_p^2} (V_p - V_{GS})$$

If $V_1 = \frac{V_p}{2}$ and $V_{GS} = 0$ then $G_c = \frac{I_{DSS}}{2V_p}$ and

$G_m = \left. \frac{-2 I_{DSS}}{V_p} \right|_{V_{GS}=0} = g_m$ and $\frac{G_c}{G_m} = \frac{1}{4}$ (This choice minimizes distortion)

Ex: A JFET with $I_{DSS} = 40\,mA$ and $g_m = 14\,ms$ at $V_{GS} = 0$ is to be used in a mixer. Estimate the conversion gain if a 50Ω load is to be used.

Sol:

the transconductance $g_m = \frac{-2 I_{DSS}}{V_p} \left(1 - \frac{V_{GS}}{V_p}\right)$

$$\left. g_m \right|_{V_{gs}=0} = \frac{-2 I_{DSS}}{V_p} = 14 * 10^{-3}$$

$$\therefore V_p = \frac{-2 * 40}{14} = -5.7\,V$$

If the local oscillator peak Voltage $V_1 = \frac{|V_p|}{2}$ (to minimize distortion) then the conversion transconductance

$$G_c = \frac{I_{DSS} V_1}{V_p^2} = \frac{I_{DSS}}{2|V_p|} = \frac{40 \times 10^{-3}}{2 \times 5.7} = 3.5 * 10^{-3}\,s$$

The magnitude of the Voltage gain is

$$A_v \approx G_c R_L = 1.75$$

Ex.: A silicon FET is used in the mixer shown in Fig(a) with $I_{DSS} = 4mA$, $V_p = -4V$ and $R_s = 2k\Omega$. The oscillator peak voltage is 1.8 V, and the RF input is sinusoidal signal of 1mV peak amplitude. Evaluate the conversion and amplifier transconductance, the drain current components at ω_s, ω_o and $\omega_o - \omega_s$

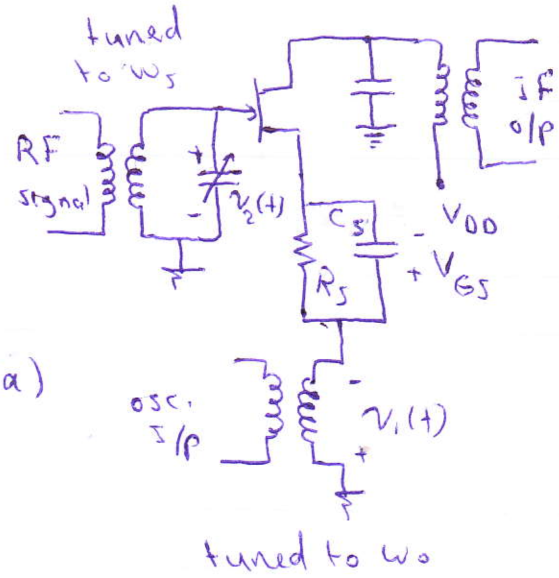


Fig (a)

Sol:

The conversion transconductance

$$G_c = \frac{I_{DSS} V_1}{V_p^2} = \frac{4 * 1.8}{16}$$

$= 0.45 \; m℧$

The Amplifier transconductance

$$G_m = \frac{2 I_{DSS} (V_p - V_{GS})}{V_p^2}$$

The dc voltage V_{GS} is due to clampped voltage across the capacitor C_s.

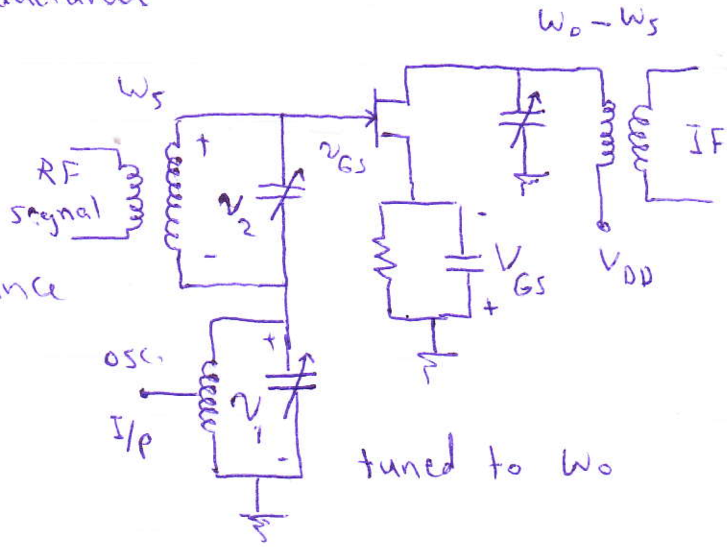


Fig (b)

Cs will clamp the peak value of the oscillator signal $v_1(t)$

$$I_D = I_s = \frac{1.8}{2k} = 0.9 \; mA = \frac{I_{DSS}}{V_p^2}(V_p - V_{GS})^2$$

$$0.9 \; mA = \frac{4 * 10^{-3}(-4 - V_{GS})^2}{16} \Rightarrow V_{GS} = -4 + 1.89 = -2.103 V$$

$G_m = 0.948 \; m℧$

$$i_d = I_{DSS}\left(1 - \frac{v_{GS}}{V_p}\right)^2 = \frac{I_{DSS}}{V_p^2}\left[V_p - (v_1(t) + v_2(t) + V_{GS})\right]^2$$

$$= \frac{I_{DSS}}{V_p^2}\left[(V_p - V_{GS})^2 + 2\{v_1(t) + v_2(t)\}\{V_{GS} - V_p\} + (v_1(t))^2 + 2v_1(t)v_2(t) + (v_2(t))^2\right]$$

$$v_1(t) = V_1 \cos w_o t \quad , \quad V_2(t) = v_s(t) \cos w_s t$$

$$2 v_1(t) v_2(t) = V_1 v_s(t)\left[\cos(w_o - w_s)t + \cos(w_o + w_s)t\right]$$

$$i_d(w_o - w_s) = \frac{I_{DSS}}{V_p^2} V_1 V_s(t) \cos(w_o - w_s)t$$

$$= \frac{4m}{16} * 1.8 * 1 * 10^{-3} \cos(w_o - w_s)t$$

$I_d(w_o - w_s) = 450\ nA$ peak value of the current component at the difference freq.

$$i_d(w_o) = \frac{2 I_{DSS}}{V_p^2} v_1(t)(V_{GS} - V_p)$$

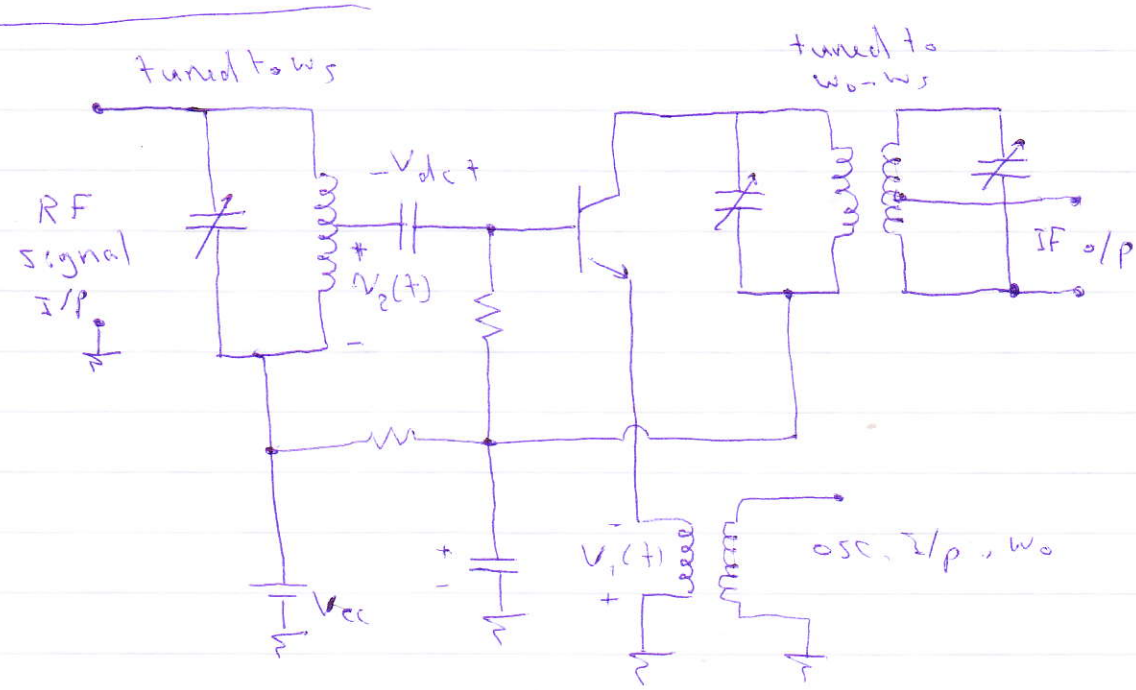
$$I_d(w_o) = \frac{2 I_{DSS}}{V_p^2}(V_{GS} - V_p) V_1 = \frac{2 \times 4}{16}(-2.103 + 4) * 1.8$$

$$= 1.7\ mA$$

$$i_d(w_s) = \frac{2 I_{DSS}}{V_p^2}(V_{GS} - V_p) v_2(t)$$

$$I_d(w_s) = \frac{2 \times 4}{16}(-2.103 + 4)(1 mV) = 0.95\ \mu A$$

Bipolar transistor Mixers



For the BJT mixer shown:

if $v_1(t) = V_1 \cos w_o t$

$v_2(t) = V_s(t) \cos w_s t$

then

$$i_E(t) = I_{ES}\left(e^{\frac{V_{dc}}{V_T}}\right)\left(e^{\frac{v_1(t)}{V_T}}\right)\left(e^{\frac{v_2(t)}{V_T}}\right)$$

Then, the current can be expanded in a series of modified Bessel functions

$$i_E(t) = I_{ES}\, e^{V_{dc}/V_T}\left\{I_0(x) + 2\sum_1^{\infty} I_n(x)\cos n w_o t\right\}\left[I_0(y) + 2\sum_1^{\infty} I_n(y)\cos n w_s t\right]$$

where $y(t) = v_s(t)/V_T$

$$x = V_1/V_T\,, \quad V_T = \frac{kT}{q}$$

I_{Es}: emitter saturation current, I_n: nth order modified Bessel function

Cross-multiplying the terms yields

$$i_E(t) = [I_{E_S} e^{V_{dc}/V_T}][I_o(x) I_o(y) + 2 I_o(x) I_1(y) \cos w_s t + 2 I_o(y) I_1(x) \cos w_o t + 4 I_1(x) I_1(y) \cos(w_o t)\cos(w_s t) + 4 I_1(x) I_2(y) \cos(w_o t)\cos(2 w_s t) + \cdots] \quad \text{---- (1)}$$

from eq (1)

then $I_{dc} = I_{E_S} e^{V_{dc}/V_T} I_o(x) I_o(y)$ ---- (2)

$$I_{w_s} = 2 I_{dc} \frac{I_1(y)}{I_o(y)} , \quad I_{w_o} = 2 I_{dc} \frac{I_1(x)}{I_o(x)} \quad \text{---- (3)}$$

$$I_{w_o - w_s} = 2 \frac{I_1(x) I_1(y)}{I_o(x) I_o(y)} I_{dc} \quad \text{---- (4)}$$

To have a linear mixer, $I_{w_o - w_s}$ must be linearly proportional to y and then to $V_s(t)$.

If V_1 and x are constant

and $V_1 >> |V_s|$ (variations in V_s donot effect Idc)

$$\lim_{y \to 0} I_o(y) = 1$$

then $I_{w_o - w_s}$ will vary with $\frac{I_1(y)}{I_o(y)}$ and hence with V_s

$$\frac{I_1(y)}{I_o(y)} \approx \frac{y}{2}\left(1 - \frac{y^2}{8} + \frac{y^4}{16}\right) \quad \text{---- (5)}$$

using modified Bessel fun. with small arguments

For $\frac{I_1(y)}{I_o(y)}$ and $I_{w_o - w_s}$ to be linear with respect to (y). To be within 2% then $\frac{y^2}{8} \leq 0.02$ or $y \leq 0.4$

$\frac{I_1(y)}{I_o(y)}$

$$\Rightarrow \frac{V_s}{V_T} \leq 0.4 \quad \Longrightarrow |V_s| \leq 10.4\,mV$$

For linearty to be $\frac{I_1(y)}{I_0(y)}$ with in 0.5 % then $y \leq 0.2$

$\Rightarrow |v_s| \leq 5.2\ mV$

For the case of 2 % linearty that is $|v_s| \leq 10.4\ mV$

and $\frac{I_1(y)}{I_0(y)} \simeq \frac{y}{2}$ and from eq. 4,

$$\frac{I_{\omega_c-\omega_s}}{V_s} = G_c = \frac{I_1(X)}{I_0(X)} g_m \quad \text{Conversion transconductance}$$

$$\frac{I_{\omega_c}}{V_1} = G_m = 2\frac{I_1(X)}{X I_0(X)} g_m \quad \text{amplifier transconductance (large signal)}$$

$$\frac{I_{\omega_s}}{V_s} = G_s = g_m$$

$$\frac{G_c}{G_m} = \frac{x}{2} \ , \quad \frac{G_c}{G_s} = \frac{I_1(x)}{I_0(x)}$$

where $g_m = \frac{I_{dc}}{V_T}$ and $v_s(t) = V_s$ (the resultant conductances are independent of V_s)

Note that

$$\lim_{x\to\infty} \frac{I_1(x)}{I_0(x)} = 1 \quad \text{and} \quad \frac{I_1(x)}{I_0(x)} \simeq 0.86 \text{ for } X=4$$

and $I_{\omega_c-\omega_s} \simeq g_m V_s$

Ex: $I_{dc} = 1\ mA$, $V_1 = 100\ mV$, $V_s = 2\ mV$ Determine G_c and G_m, I_{ω_s}, I_{ω_c}, $I_{\omega_c-\omega_s}$

Sol.,
$$X = \frac{V_1}{V_T} = \frac{100}{26} = 3.84$$

$$y = \frac{V_s}{V_T} = \frac{2}{26} = 0.069 \ll 1 \quad \Rightarrow \quad \frac{I_1(y)}{I_0(y)} = \frac{y}{2}$$

$$I_{w_o} = I_{dc} \frac{2I_1(3.84)}{I_0(3.84)} = 1.72 \text{ mA}$$ (using tables of Bessel fun.)

$$I_{w_s} = 2 I_{dc} \frac{I_1(0.069)}{I_0(0.069)} , \quad \frac{I_1(0.069)}{I_0(0.069)} \simeq \frac{y}{2}$$

$$I_{w_s} = 0.069 \text{ mA}$$

$$I_{w_o - w} = 2 \frac{I_1(x)}{I_0(x)} \frac{I_1(y)}{I_0(y)} I_{dc} \simeq 2 \frac{I_1(3.84)}{I_0(3.84} \frac{y}{2} (1 \text{ mA})$$

$$= 66 \ \mu A$$

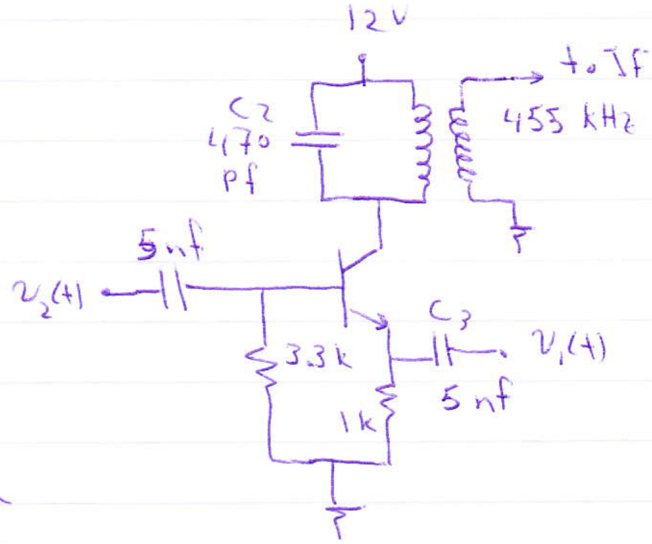
$$G_c = \frac{I_{w_o - w_s}}{V_s} = \frac{66 \times 10^{-6}}{2 \times 10^{-3}} = 33 \text{ m℧}$$

$$G_m = \frac{I_{w_o}}{V_1} = \frac{1.72 \times 10^{-3}}{100 \times 10^{-3}} = 17 \text{ m℧}$$

$$G_s = \frac{I_{w_s}}{V_s} = \frac{1}{26} \ ℧$$

Ex:

Analyze the performance of the mixer cct shown, The series resistance of the inductor $R = 15.5 \ \Omega$



Sol:

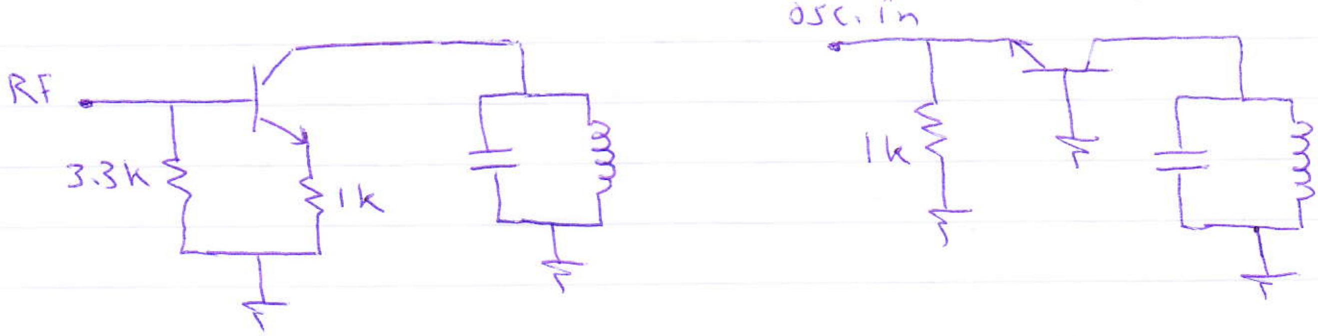
The cct performs as a common-emitter Amplifier for the RF signal and as a common-base Amplifier for the oscillator signal.

There is no DC voltage applied to the base of the amplifier, then with both signals removed, the DC voltage are $V_B = 0$, $V_E = 0$, and $V_c = V_{cc}$.

when an RF signal is presented, the base goes negative due to base leak bias capacitor C_1 (class C cut-off).

The RF signal in the base will cause current pulles in the collector cct. each time the I/p signal is sufficiently positive to turn the transistor on.

when the osc signal goes negative it will drive the mixer into conduction



AC cct common emitter for RF

AC cct. common-base for the osc.

The required inductance at the output is

$$L = \frac{1}{\omega_{IF}^2 C} = \frac{1}{(2\pi * 455k)^2 * 470p} = 260 \mu H$$

with $R = 15.5 \Omega$ the collector impedance at resonence

$v_o = -i_c Z_c$

$v_{in} = i_b(r_{be} + \beta R_e)$

$= \beta i_b(r_e + R_e)$

$\approx i_c R_e \Rightarrow A_v = |\frac{v_o}{v_{in}}|$

$\Rightarrow A_v = |\frac{-Z_c}{R_e}|$

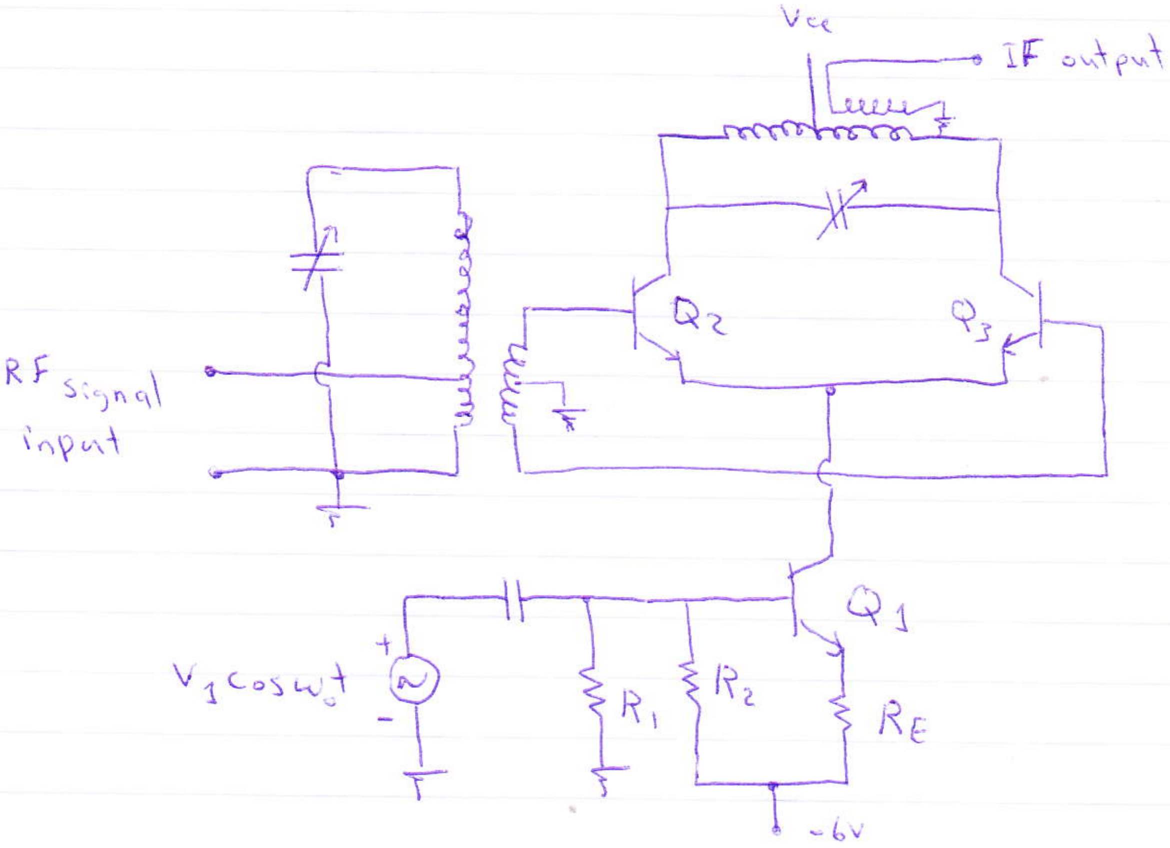
$$Z_c = \frac{L}{RC} = 35.7 K\Omega = R_p = Q^2 R$$

conduction for half cycle

$$\text{Then } A_v = (\frac{Z_c}{R_e}) = (\frac{35.7k}{1k\Omega}) = 35.7$$

This is the gain relative with the RF signal where the amplifier conducts for one half cycle or less. This is the same as the gain relative with the osc. signal. thus the two signals are treated exactly alike

Differential _pair Mixers



For the differential pair mixer, if the differential Pair transistors Q2 and Q3 are identical and the output and input transformers are balanced, then:

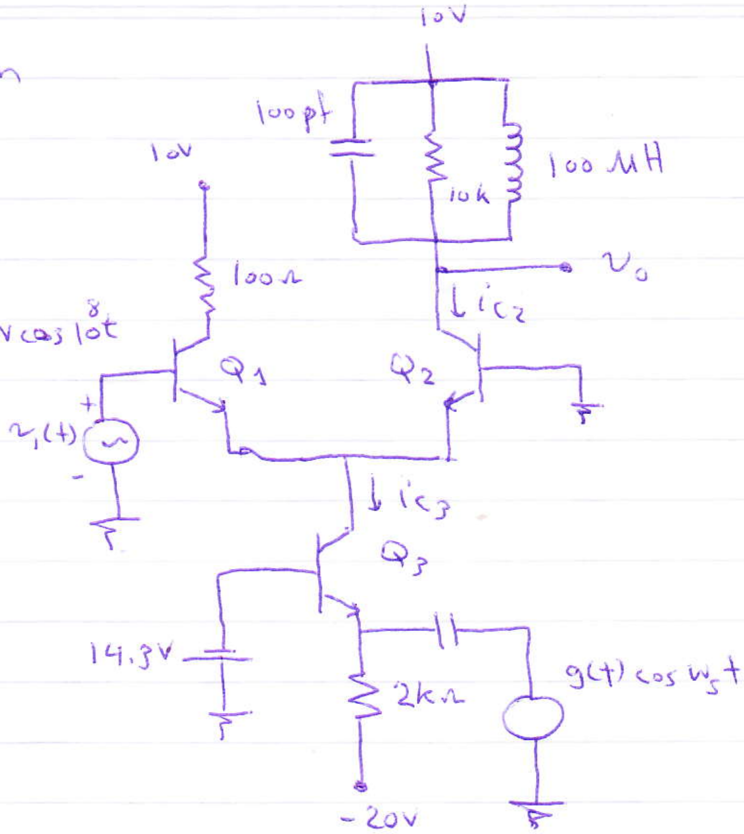
1- No oscillator voltage will appear across either the input or the output transformer.
2- Second harmonic component is not generated in the cct.

Ex: Determine an expression for $V_o(t)$ for the case where $g(t) = 1mV\,[1 + mf(t)]$, $\omega_s = 9 \times 10^7$ rad/sec, and the output tuned circuit has sufficient bandwidth to pass modulation. $\alpha \simeq 1$, Q_1, Q_2 and Q_3 are identical

The tuned cct. is tuned to $\omega_{IF} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{100M * 100p}}$
$= 10^7$ rad/sec

The tank cct. has a bandwidth of $B.W = \frac{1}{RC}$

$$= \frac{1}{10^4 \times 10^{-10}} = 10^6 \text{ rad/sec.}$$



this bandwidth is sufficient to pass modulation ($10^6 > 2W_m$)

$$i_{c_3} \simeq i_{E_3} = I_{dc} + i_{ac}$$

$$I_{dc} = \frac{20 - 14.3 - 0.7}{2k} = 2.5 \text{ mA}$$

$$i_{ac} = \frac{g(t)}{2k} \cos w_s t$$

$$= 0.5 \text{ mA} [1 + mf(t)] \cos w_s t$$

$$i_{E_2} = \frac{i_{c_3}}{2} \left(1 - \tanh \frac{v_1(t)}{2V_T}\right) \quad \text{where } v_1(t) = 90\text{mV} \cos 10^8 t$$

$$i_{E_2} = i_{c_3} \left(\frac{1}{2} - \sum_{n=1,2,3}^{\infty} a_{(2n-1)}(x) \cos (2n-1) w_o t \right)$$

where $a_n(x) = \frac{1}{\pi} \int_{-\pi}^{\pi} \left[\frac{1}{2} \tanh \frac{x}{2} (\cos \theta) \right] \cos(n\theta) d\theta$

$$= \frac{I_n}{I_{dc}}$$

and $X = \frac{V_1}{V_T} = \frac{90}{26} = 3.5$, $a_1(3.5) = 0.525$

$$i_{E_2} = i_{c_3} \left[\frac{1}{2} - a_1(x) \cos w_o t - a_3(x) \cos 3 w_o t - \ldots \right]$$

$$= [2.5 \text{ mA} + 0.5 \text{ mA} (1 + mf(t)) \cos w_s t] \left[\frac{1}{2} - a_1(x) \cos w_o t - a_3(x) \cos 3w_o t \ldots \right]$$

at resonance

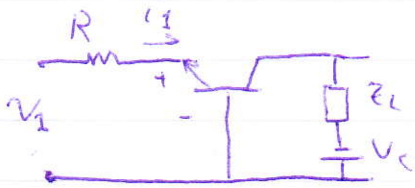
$$i_{c_2} = -0.5\,MA\,[1+mf(t)]\,a_1(3.5)\,[\tfrac{1}{2}\cos(w_o - w_s)t)$$

$$= -0.13\,[1+mf(t)]\cos 10^7 t$$

$$v_o = V_{cc} - i_{c_2} R_L = 10 + 1.3\,mV\,[1+mf(t)]\cos 10^7 t$$

* Series resistance in mixers :

Adding a resistor in series with the emitter of a BJT or the source of an FET, or with the emitters of a differential pair tends to linearize the characteristics and reduces the conversion transconductance.



$$i_E = I_{ES}\exp\left(\frac{v_{EB}}{V_T}\right) = i_1 \quad \text{-----} \quad (1)$$

$$v_1 = i_1 R + v_{BE} = i_1 R + V_T \ln \frac{i_1}{I_{ES}} \quad \text{-----} \quad (2)$$

For small signal operation, the incremental input resistance for the cct is

$$r'_{in} = \frac{\partial v_1}{\partial i_1}\Big|_{i_1 = I_{dc}} = R + r_{in}$$

where $r_{in} = \frac{1}{g_{in}} = \frac{V_T}{I_{dc}}$, I_{dc} is the DC value of i_1

(Note that $g_m = \alpha\, g_{in} \simeq g_{in}$)

then $g'_m = \frac{\alpha}{r'_{in}} = \frac{\alpha\, g_{in}}{1 + g_{in} R}$ ----- (3)

thus g'_m decreases as R increases

For $R = 0 \Rightarrow g'_m = \alpha g_{in} = g_m$

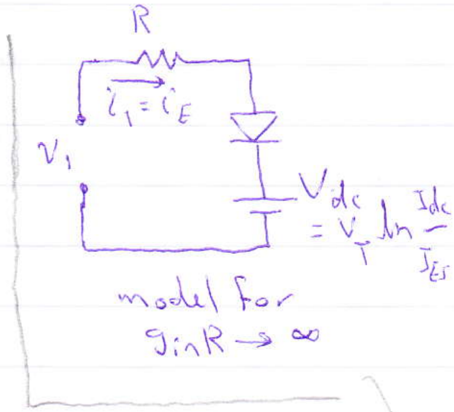
For $g_{in} R >> 1$ then $g'_m \simeq \frac{\alpha}{R}$ independent of the change in bias current of the transistor

Rearranging eq (2) in the form

$$v_1 - V_{dc} = i_1 R + V_T \ln \frac{i_1}{I_{dc}}$$

where I_{dc} the quiescent value of i_1 and V_{dc} is the quiescent value of v_1 and

$$V_{dc} = V_T \ln \frac{I_{dc}}{I_{ES}}$$



let $V_{co} = I_{dc}(r_{in} + R) = \frac{kT}{q}(1 + g_{in} R)$

then $\frac{v_1 - V_{dc}}{V_{co}} = \left(\frac{g_{in} R}{1 + g_{in} R}\right) \frac{i_1}{I_{dc}} + \frac{\ln(i_1 / I_{dc})}{1 + g_{in} R}$

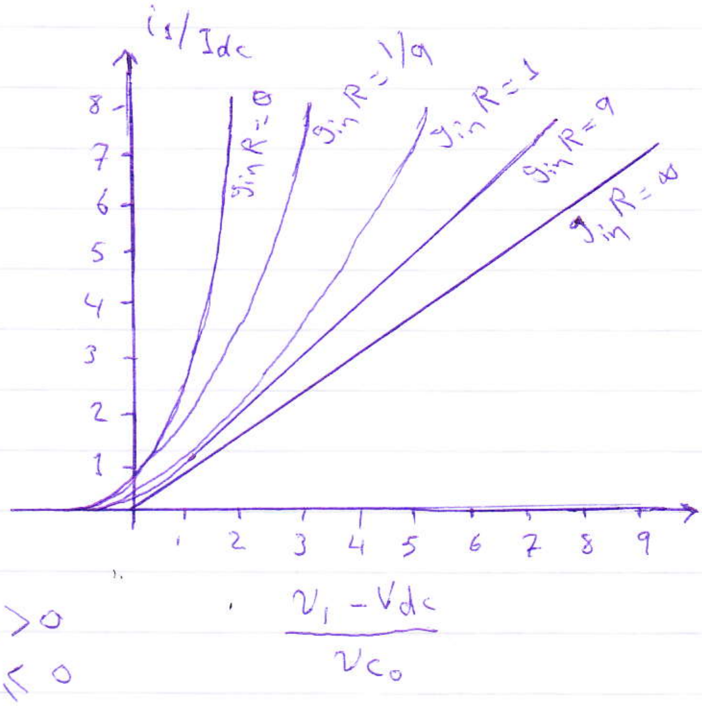
V_{co} is the decrease in v_1 from V_{dc} which would be required to reduce i_1 to zero (to cut off the transistor)

It is seen from the plot that the series resistance of the cct. tends to linearize the c/cs (while the mixer should have nonlinearities).

For $g_{in} R << 1$, the cct can be modeled by the transistor characteristic alone

for $g_{in} R >> 1$ the cct. can be modeled by

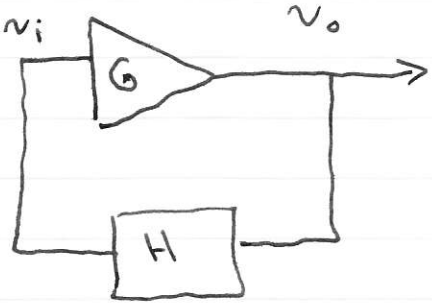
$$i_1 = \begin{cases} \frac{v_1 - V_{dc}}{R} & v_1 > 0 \\ 0 & v_1 < 0 \end{cases}$$



Oscillators :

~~Q1. Explain the~~

Positive feedback forms the basis for many of the more commonly used osc. cct. The Fig. below shows the general form of the feedback amp. cct.



The conditions for osc. are:

1) The net gain around the F.B. loop must be no less than 1,
2) The net phase shift around the loop must be a positive integer multiple of 2π.

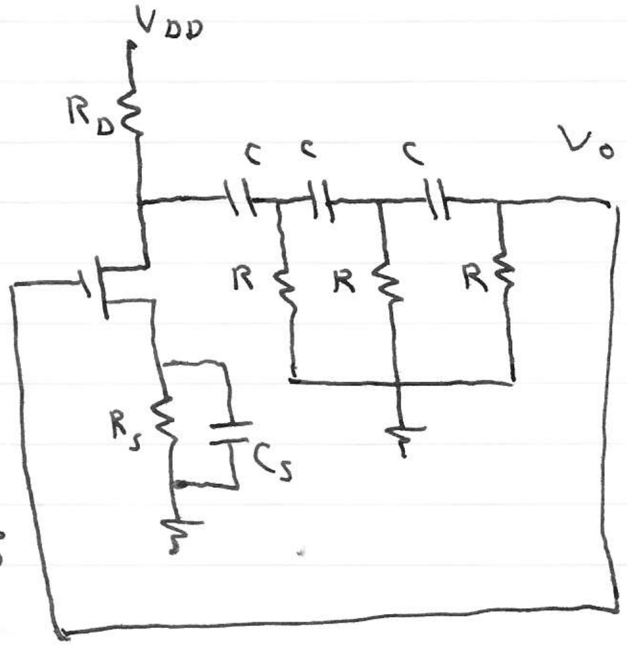
$$\text{Loop Gain} = |GH| \angle \theta$$

where $\theta = n2\pi$, and $n = 0, 1, 2 \ldots$

The over all gain with F.B. is expressed as

$$A_v = \frac{G}{1 - GH}$$

RC Phase-shift osc.

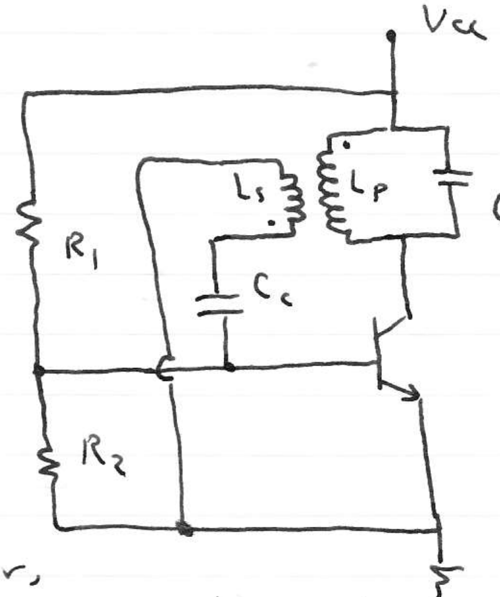


This cct shows the arrangement of the RC phase shift osc.. The cct has Three RC lead sections in cascade. each with the same time constant. at freq of osc. each section provides about one-third of the total required phase shift of 180°

at the freq. of osc. the losses of the N.W. reduce the o/p voltage V_o to 1/29 of the I/p voltage V_i and the freq. is given by

$$f = \frac{1}{2\pi\sqrt{6}\,RC} \quad , \quad \left|\frac{V_o}{V_i}\right| = \frac{1}{29}$$

Tuned LC oscillator

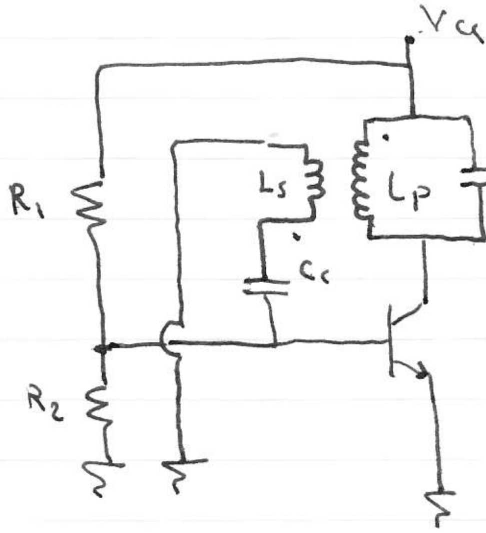


The gain in this cct is provided by a common emitter amp.. The transformer provides ~~a convenient~~ the 180° phase shift necessary for positive F.B. AT the parallel resonant freq. of the tank cct made up of the transformer primary inductance and the capacitor, the net susptance of the tank is zero, and the tank cct looks like a pure resistance. Under these conditions, the Phase shift in the amp. is 180° and the transformer provides a further 180° shift. Since the transformer can provide a turns ratio greater than unity, it is possible to make an oscillation using an amp. with a voltage gain less than unity.

The resonant freq. of the osc. along with the gain required, is given by

$$f_o = \frac{1}{2\pi\sqrt{LC}} \quad , \quad |A_v| \geq \frac{L_P}{M}$$

Ex: The cct shown uses a transistor with $\beta = 100$, $g_m = 20\,mS$, $r_o = 50\,k\Omega$. The transformer has a mutual inductance of 25 MH, a primary inductance of 100 MH, and Q of 100 at the resonant freq. The tuning capacitor is 100 pf. Find
1) freq of osc.
2) whether the cct will osc. or not.



1) $$f_o = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{6.28 \times \sqrt{200\,MH \times 100\,pf}}$$

$$= 1.59\ MHz$$

2) $$Q_P = R_P / X_L$$

and $R_P = R_D$

The amplifier load resistance $(R_L') = r_o // R_D$

$$R_L' = r_o // Q_P X_L$$

$$= 50\,k\Omega // (100 \times 2\pi \times 1.59\,MHz \times 100\,MH)$$

$$= 33.3\ K\Omega$$

$$A_v = -g_m R_L'$$

$$= -20 \times 33.3$$

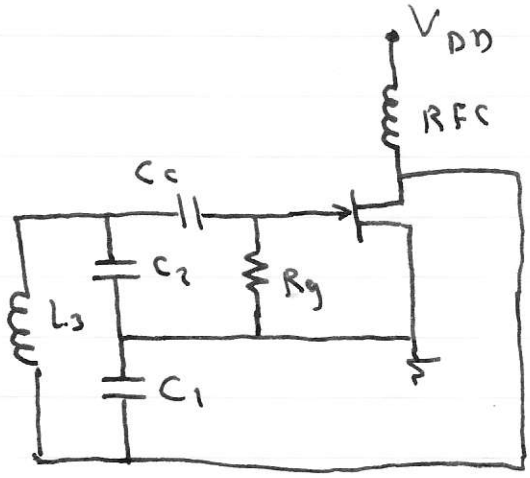
$$= -676$$

Hence

$$\frac{L_P}{M} = \frac{100}{25} = 4 << |A_v|$$

Tuned LC oscillators

* Colpitts Oscillators



this cct shows the field-effect transistor Colpitts osc. . Z_1 and Z_2 are capacitors, and Z_3 is an inductor. The RF choke provides a low resistance DC path for the collector current while blocking the signal. The coupling capacitor C_c and the base resistor R_g act to bias the transistor. ~~into the~~

$$f_o = \frac{1}{2\pi\sqrt{L_3 C_{eq}}}$$

where $C_{eq} = \dfrac{C_1 C_2}{C_1 + C_2}$

The ~~max~~ minimum gain required of the amplifier to maintain osc. is

$$|A_{v_o}| \geqslant \frac{C_2}{C_1}$$

where

$$A_{v_o} = -g_m r_d$$

and $A_{v\,loop} = -A_{v_o}\dfrac{C_1}{C_2} \geqslant 1$

Ex: ~~The~~ For the Colpitts osc. that uses an n-channel junction FET biased to give an initial $g_m = 2\,mS$ and an $r_d = 10\,k\Omega$. Find the freq of osc. and whether or not osc. will start.

Sol.

$$C_{eq} = \frac{C_1 C_2}{C_1+C_2} = \frac{500 \times 50}{500+50} = 45.5\ pf$$

$$f_o = \frac{1}{2\pi\sqrt{L_3 C_{eq}}}$$

$$= \frac{1}{2\pi\sqrt{75 \times 10^{-6} \times 45.5 \times 10^{-12}}}$$

The open cct gain of the Amp.

$$A_{v_o} = -g_m r_d = -2ms * 10k\Omega = -20$$

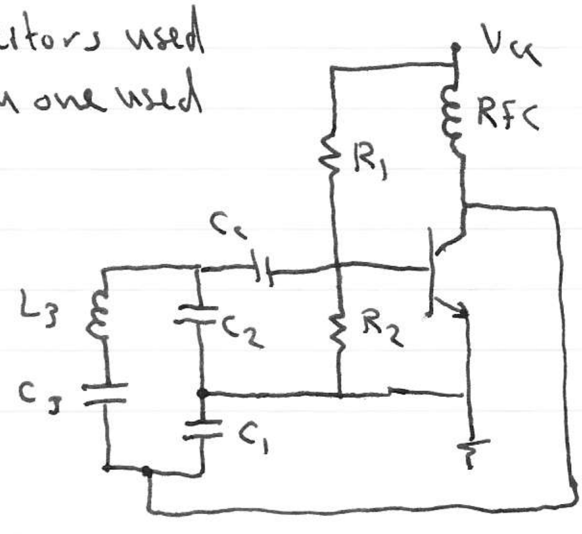
The Total loop gain

$$A_{v(Loop)} = -A_{v_o}\left(\frac{C_1}{C_2}\right) = 20 * \frac{50}{500} = +2 \quad > 1$$

Therefore, osc. will start.

* Clapp oscillator

The clapp osc. is a particular version of the Colpitts cct the difference between the two is that X_3 is comprised of an inductor and capacitor in series, such that the net reactance at resonance is inductive. The capacitors used for X_1 and X_2 are much larger than the one used in X_3. So the series resonant freq. is controlled mainly by C_3 (added for stability of osc.)



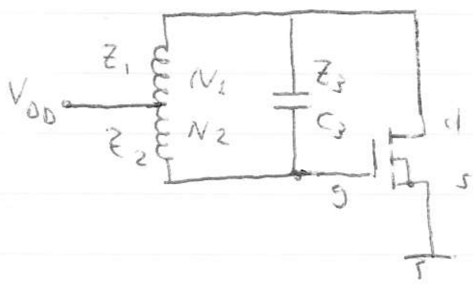
$$C_{eq} = \frac{1}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}}$$

$$f_o = \frac{1}{2\pi\sqrt{L_3 C_{eq.}}}$$

$$|A_{v_o}| \geq \frac{C_2}{C_1}$$ Condition for osc.

Hartley Oscillator

The cct of this osc. is the same as the Colpitts but Z_1 and Z_2 are inductors and Z_3 is a capacitor



In this fig. The tapped coil forms a mutually inductive coupled cct for which ~~for which~~ the coefficient of coupling, k, is Unity, assuming coil resistance is zero then

$$\frac{Z_2}{Z_1} = \frac{X_2}{X_1} = \frac{L_2 + M}{L_1 + M} = \frac{N_1}{N_2}$$

The resonant freq. is given by

$$f_o = \frac{1}{2\pi\sqrt{L_t C_3}}$$

where $L_t = L_1 + L_2 + 2M$ and $M = \sqrt{L_1 L_2}$

and the gain loop is $A_v = -A_{v_o} \frac{N_2}{N_1} \geq 1$

Ex: The Hartley osc. is biased to give $g_m = 1.5$ mS and a drain resistance of $10 k\Omega$. The coil is tapped to give a turns ratio $(N_1/N_2) = 3$ with $L_t = 64\ \mu H$ and $C_3 = 39$ pf calculate the freq. of osc. and determine whether osc. will self-start.

$$f_o = \frac{1}{2\pi\sqrt{64 \times 39}} = 3.19 \text{ MHz}$$

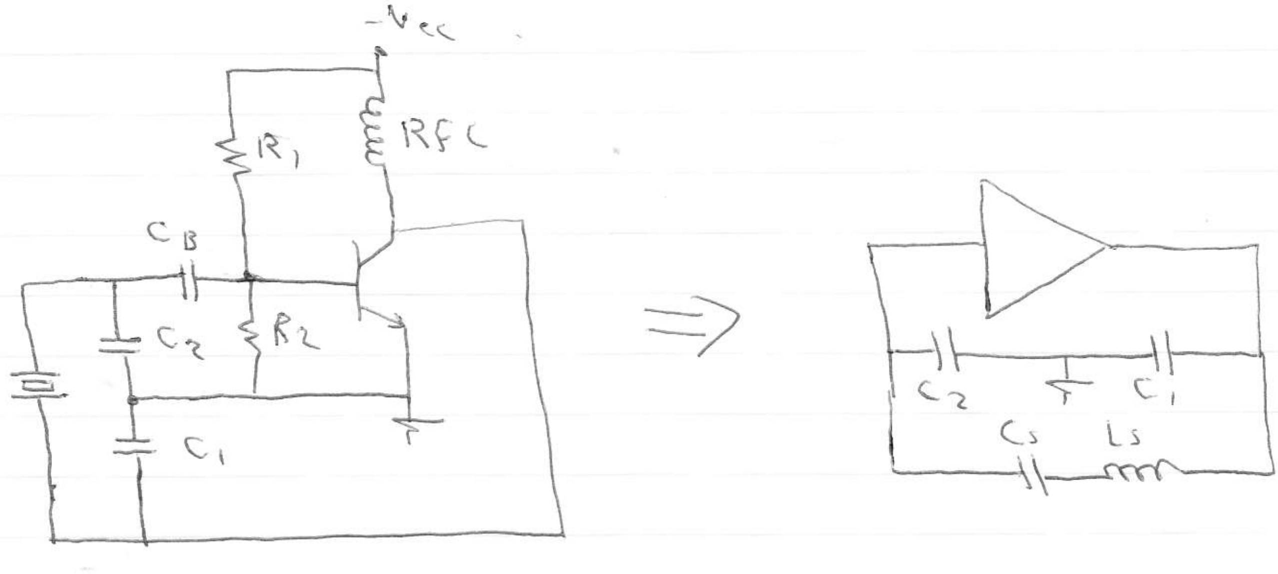
$$A_v = -(-1.5 \times 10) \times \frac{1}{3} = 5 > 1$$

the osc. will start

Crystal Oscillators

Crystals can be used to control any of the tuned LC oscillators discussed previously by appropriate connection. The crystal may be used to replace an entire LC tank circuit or it may be used to replace one of the reactances in a tank cct.

The Pierce oscillator, like the Clapp osc., is basically a Colpitts osc. in which the inductor is replaced by the crystal. The cct is shown below with its equivalent, in which the crystal has been replaced by its equivalent cct, C_s and L_s which connected in serial



The resonant freq. of the cct is determined by the series resonance of the cct mach up of C_1, C_2, C_s and L_s. C_1 and C_2 are both very much larger than C_s, so the resonance freq. is almost entirely dependent on the value of C_s.

Ex: For the Pierce crystal osc. cct. a 5MHz quartz crystal connected to a transistor with $g_m = 10\ mS$, $\beta = 100$, $r_o = 50k\Omega$, $C_2 = 5000pf$; the cct is coupled to a $12.5k\Omega$ load resistance. Find the value of C_1 necessary to give a 20-dB

Sol.

$$A_{v_o} = -g_m (r_o // R_L)$$

$$= -10 (50 \times 10^3 // 12.5 \times 10^3)$$

$$= -100 \quad (40\ dB$$

$$A_{v_{loop}} = \frac{-A_{v_o} C_1}{C_2} = +10 \quad \text{(from the 20 dB requirement)}$$

$$\Rightarrow \quad C_1 = \frac{10\, C_2}{|A_{v_o}|} = \frac{10 \times 5000}{100} = 500pf$$

Angle Modulation

Information can be transmitted by modulating the phase and frequency of a sinewave carrier (PM and FM).

FM has the following advantages, as compared to AM:

1. almost total immunity from noise interference
2. eliminates the problem of fading, and
3. the power in the transmitted FM signal is constant for both the modulated and unmodulated conditions of the carrier

Disadvantage: wider B.W is required for FM transmission as compared to AM

An angle-modulated waveform can be written as

$$E(t) = A(t) \sin [W_c t + \theta(t)]$$

where W_c: carrier radian freq.

$A(t)$: is the amplitude modulation, ideally $A(t)$ is constant, and $A(t) = A$, the peak value of the original carrier

$\theta(t)$: represents the Angle modulation and

$$\theta(t) = m_p \sin W_i t \quad \text{For PM}$$

and

$$\theta(t) = m_f \sin W_i t \quad \text{For FM}$$

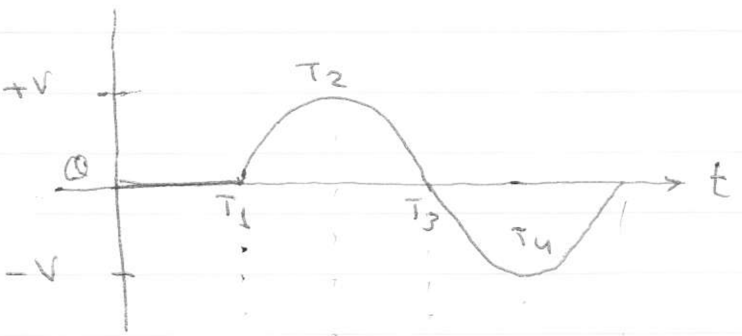
where W_i: modulating signal (angular) freq. in rad/sec

m_p: maximum phase shift caused by the intelligence (modulating) signal (in radian) or the modulation index:

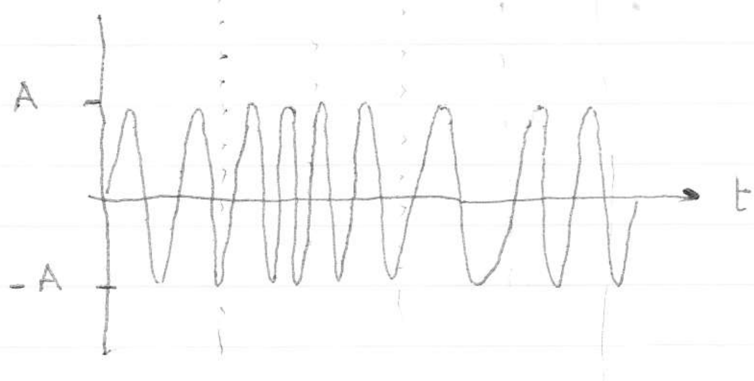
m_f: the modulation index for FM

$$m_f = \frac{\Delta f}{f_i}$$

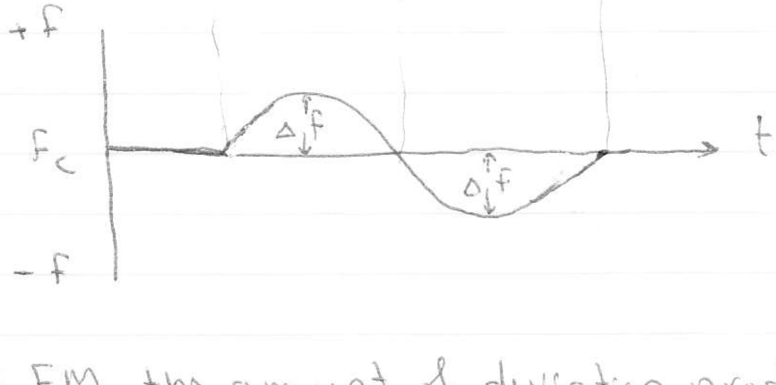
Δf: maximum freq. shift caused by the modulating signal
f_i: frequency of the modulating signal



intelligence signal

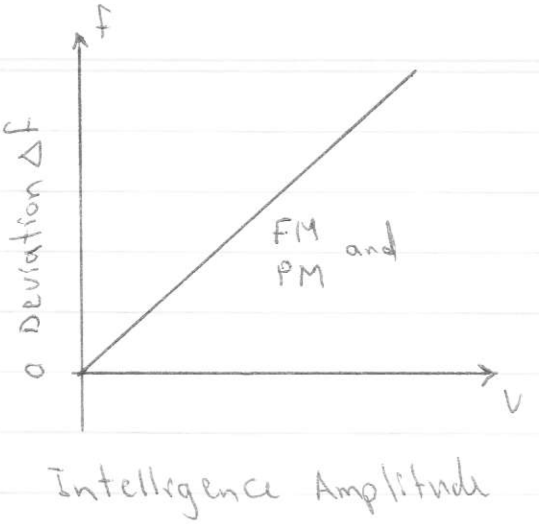


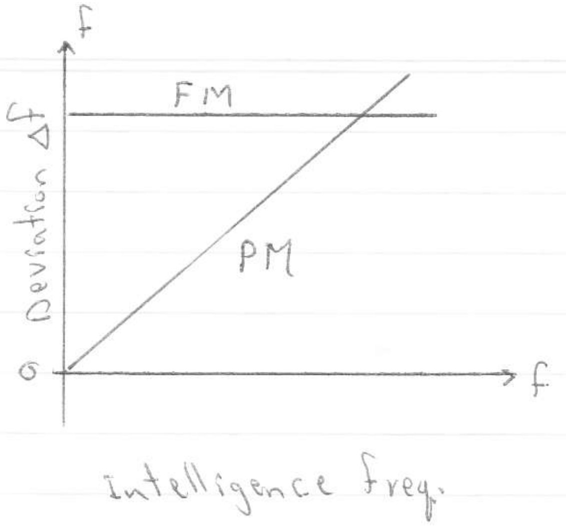
frequency modulated signal



freq. versus time in FM

In FM the amount of deviation produced is not dependent on the modulating frequency as it is for PM.





The amount of deviation is proportional to the modulating signal amplitude for both PM and FM.

FM Mathematical Solution (frequency Spectrum)

To obtain the frequency spectrum of an FM signal, Bessel functions are used. They show that frequency-modulating a carrier with a pure sine wave actually generates an infinite number of sidebands spaced at multiples of the intelligence frequency, f_i, above and below the carrier. The amplitude of the sidebands approaches a negligible level the farther away they are from the carrier, which allows FM transmission within finite bandwidths.

The Bessel function solution to the FM equation is

$$f_c(t) = J_0(m_f)\cos w_c t - J_1(m_f)\left[\cos(w_c - w_i)t - \cos(w_c + w_i)\right]$$
$$+ J_2(m_f)\left[\cos(w_c - 2w_i)t + \cos(w_c + 2w_i)t\right]$$
$$- J_3(m_f)\left[\cos(w_c - 3w_i)t - \cos(w_c + 3w_i)t\right] + \cdots$$

where $f_c(t)$ = FM frequency components
$J_0(m_f)\cos w_c t$: carrier component.

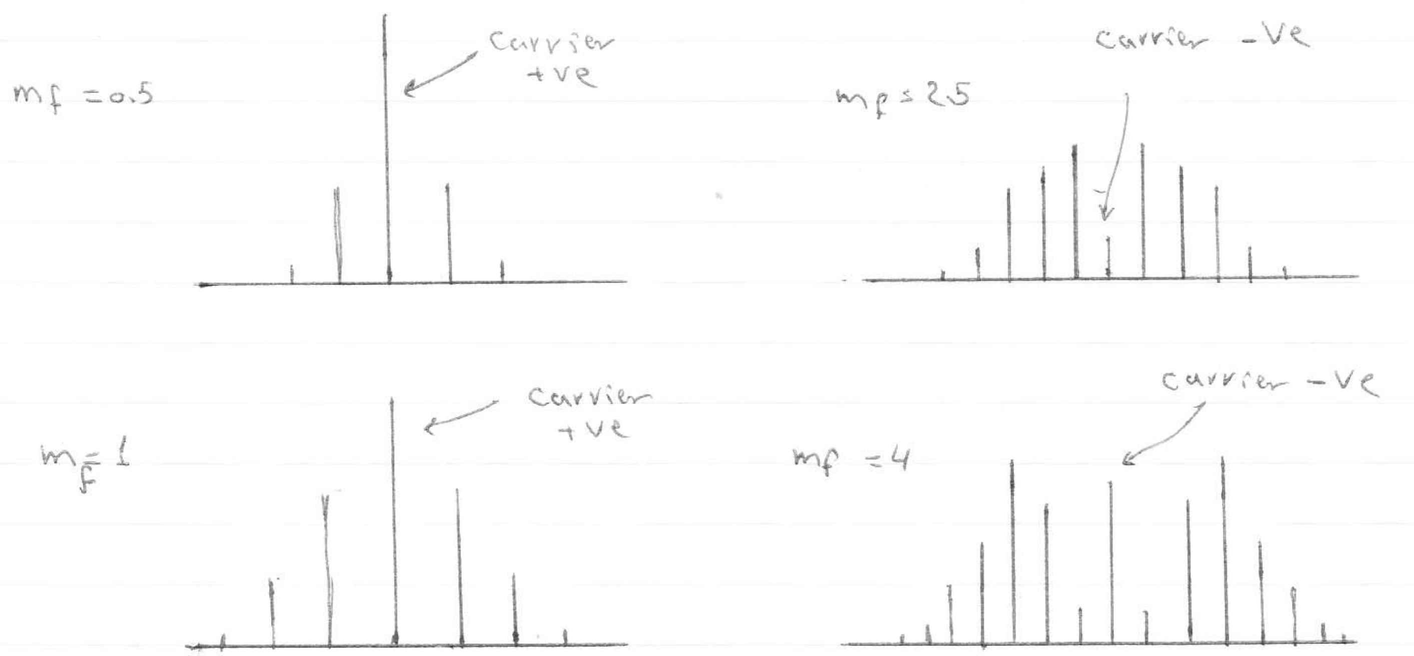
To solve for the amplitude of any side-frequency component, J_n, the following equation should be applied

$$J_n(m_f) = \left(\frac{m_f}{2}\right)^n \left[\frac{1}{n!} - \frac{(m_f/2)^2}{1!(n+1)!} + \frac{(m_f/2)^4}{2!(n+2)!} - \frac{(m_f/2)^6}{3!(n+3)!} + \cdots\right]$$

The bandwidth necessary for an FM signal is given approximately, by

$$BW \cong 2(\Delta f_{max} + f_{i\,max}) \quad \text{(using earson's rule)}$$

This Bandwidth includes 98% of the total power

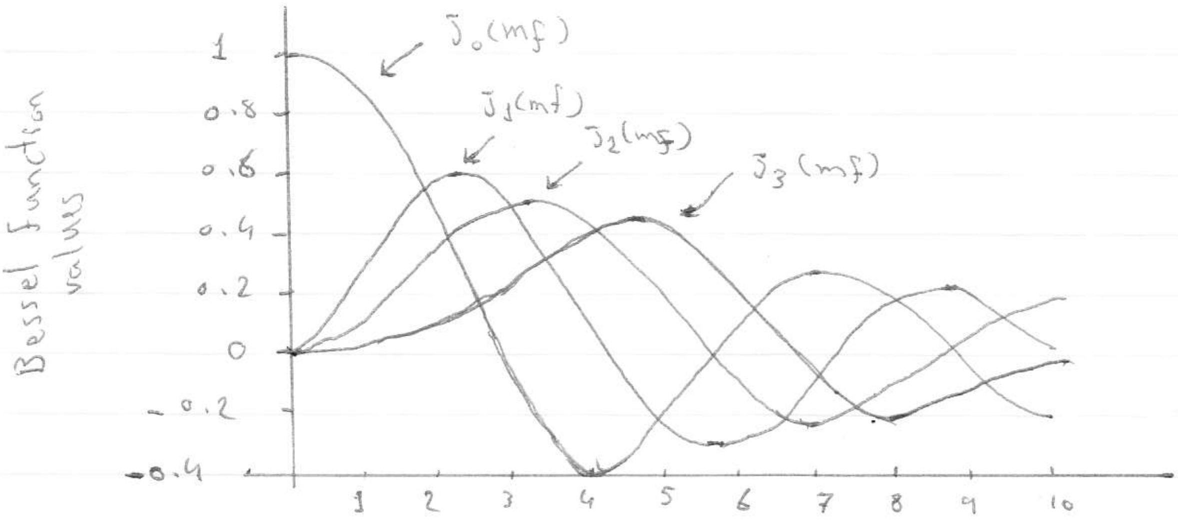


freq. spectrum for FM (constant modulating frequency variable deviation)

Notice

* The carrier in FM is not redundant as in AM, since its amplitude is dependent on the intelligence signal.
* The frequency spectrum for PM signal is the same as that for FM, for the same values of m_f and m_p

* The carrier in FM has zero amplitude at $m_f = 2.4$, and all energy is contained in the side frequencies, This also occurs at $m_f = 5.5, 8.65, ---$



Value of Bessel functions as a function of the modulation index m_f

Direct FM Generation

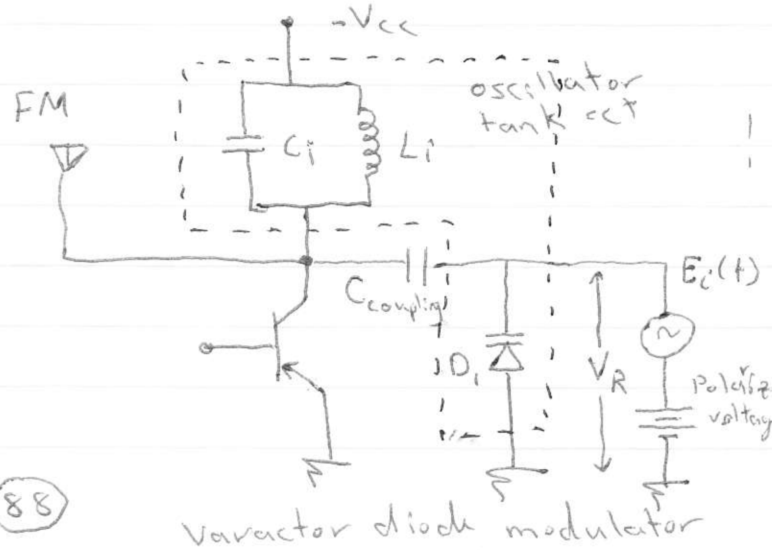
* FM Generation using Varactor Diode

A varactor diode may be used to generate FM directly

A varactor diode is a reverse-biased diode that exhibits a junction

* with no modulating signal applied:

the parallel combination of C_1, L_1 and D_1's capacitance



Varactor diode modulator

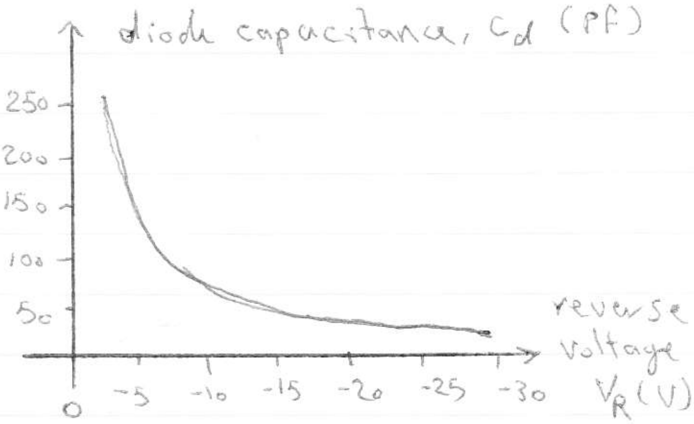
forms the resonant carrier frequency.

The coupling capacitor $C_{coupling}$ isolates the DC levels and intelligence signal and acts like a short cct. to the high-frequency carrier

* When the intelligence signal is applied to the varactor diode its reverse bias is varied which causes the diode junction capacitance to vary in step with E_i. The oscillator frequency is varied as required for FM

* The osc. freq.

$$\omega = \frac{1}{\sqrt{L_1 (C_1 + C_d)}}$$



* The amount of capacitance exhibited by a reverse-biased silicon diode, Cd, can be approximated as

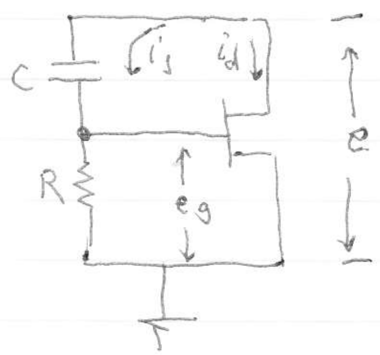
$$C_d = \frac{C_0}{\sqrt{1+2V_R}}$$

where

C_0 : diode capacitance at zero biase

2. Reactance FM modulator

This is a very popular means of FM generation. In reactance modulators, the active device acts like a reactance.



Assuming the gate current to be zero

then $i_1 = \dfrac{e}{R - jX_c}$

$$X_c = \frac{1}{\omega C}$$

$$e_g = i_1 R \quad , \quad e_g = \frac{eR}{R - jX_c}$$

The drain current, $i_d = g_m e_g = \dfrac{g_m R e}{R - jX_c}$

The impedance seen from the drain to ground will be

$$Z = \frac{e}{i_d} = e \, \frac{R - jX_c}{g_m R e}$$

$$= \frac{R - jX_c}{g_m R} = \frac{1}{g_m} - j \frac{X_c}{g_m R}$$

If the values of R and C are chossen such that $R \ll X_c$, then

$$Z \simeq -\frac{jX_c}{g_m R} = \frac{-j}{2\pi f C_{eq}}$$

where

$$C_{eq} = g_m R C$$ (The impedance Z has been made to look like a capacitance)

and $g_m = g_{m_o}\left(1 - \dfrac{v_{GS}}{V_p}\right)$, $v_{GS} = e_g$

The capacitance C_{eq} may be varied by varying the JFET transconductance, g_m when the applied gate Voltage (e_g) is varied.

Interchanging the position of R and C causes the variable impedance to look like an inductor with

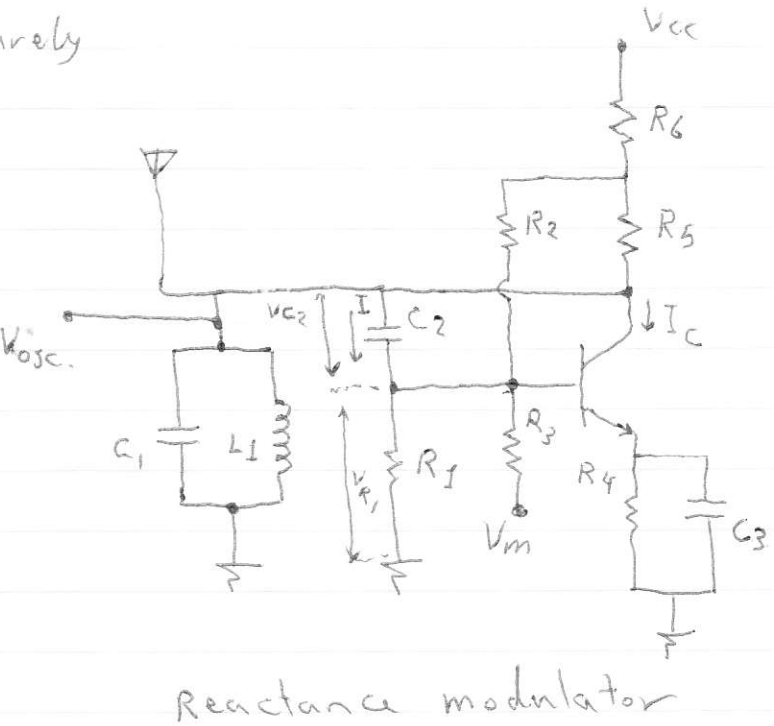
$$L_{eq} = \frac{RC}{g_m}$$

BJT as a reactance FM modulator

The RC circuit appears purely capacitive, since X_{C_2} should be larger than the value of R_1 by five to ten times at the oscillator frequency.
$(X_{C_2} >> R_1)$

Thus the current I leads the voltage V_{C_2} by $90°$.

$I = jwC_2 V_{C_2}$, $V_{C_2} = V_{osc}$.



Reactance modulator
V_m: modulating signal i/p (audio)

The same leading current flows through R_1 where I and V_{R_1} are inphase, so the voltage across the resistor V_{R_1} leads the Oscillator signal voltage V_{osc} by $90°$

$$V_{R_1} = I R_1 = I R_1 = J w C_2 R_1 V_{C_2}$$

The transistor collector current I_c is inphase with the base voltage V_{R_1}, then

I_c leads V_{osc} by $90°$ (the current through the transistor leads the voltage accross it by $90°$ making the cct. look like acapacitor)

$$Z_{out} = \frac{V_{osc}}{I_c} \quad , \quad I_c = g_m V R_1$$

$$= \frac{1}{g_m (j w C_2 R_1)} = - \frac{j}{w g_m C_2 R_1}$$

This reactance cct. looks capacitve to the oscillator by an amount equal to:

$$C_{eq} = \frac{RC}{Z_{in}} \text{ farad.} \quad , \quad Z_{in} = r_e = \frac{1}{g_m}$$

If R_1 and C_2 are interchanged, the reactance cct. looks inductive to the oscillator with:

$$L_{eq} = \frac{R C Z_{in}}{\beta} \quad , \quad Z_{in} = r_{b'e}$$