What is digital logic:

It is the basis for digital computing and provides a fundamental understanding on how circuits and hardware communicate within a computer.

Digital logic is typically embedded into most electronic devices like calculators , computer, watches.

Topics:

- Number systems and codes
- Logic gate and Boolean algebra combination logic circuits
- Digital arithmetic : operation and circuits
- Decoder, Encoder and Multiplexer
- Flip-Flop
- Counters and registers

Number system:

Many number systems are used in digital technology the most common use are the decimal , binary , octal and hexadecimal system

1- Decimal number system:

This system is composed of ten symbols or digits (0 - 9), it is also called based 10

Note: the number of digits used in the system is known as its base or radix

Decimal used 10 digits → base 10

Binary used 2 digits → base 2

Octal used 8 digits → base 8

Hexadecimal used 16 digits → base 16

ex: $(327)_{10}$

↑ This mean base is 10 which is mean decimal number system

This system is a positional – valued system ; which means when we write numbers, the position or place of each digit is important ex: in the number $(327)_{10}$

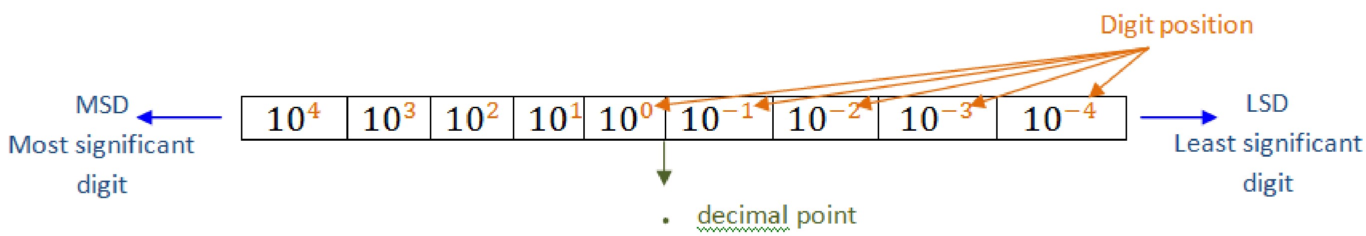
The 7 is in the ones position = 7

The 2 is in the tens position = 20

The 3 is in the hundred position = 300

327 = 300+20+7

= $10^2 \times 3 + 10^1 \times 2 + 10^0 \times 7$



ex: 327.4 = 300+20+7+0.4

= $10^2 \times 3 + 10^1 \times 2 + 10^0 \times 7 + 10^{-1} \times 4$

10^2	10^1	10^0	10^{-1}
3	2	7	.4

Note: base 10 mean each digit is multiplied by an appropriate power of (10) depending on it's position

2- Binary number system:

The system uses two digits 0 and 1

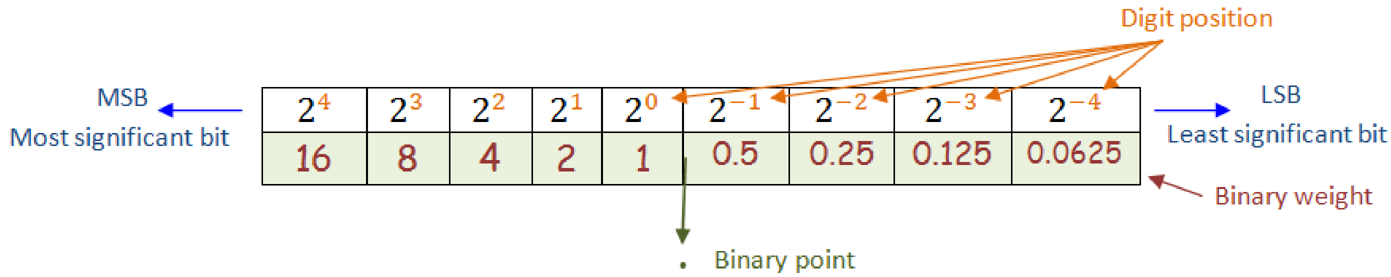
each digit referred to as a bit

This system has a base or radix of 2

This system is a positional – valued system

ex:

110110 → this number has 6 bits



Binary to decimal conversion:

ex: convert the binary number $(1101101)_2$ To decimal $(\quad)_{10}$

$= 2^6\times1+2^5\times1+2^4\times0+2^3\times1+2^2\times1+2^1\times0+2^0\times1$

$= 64+32+0+8+4+0+1$

$= (109)_{10}$

ex: convert the fractional binary number 0.1011 To decimal

$= 2^{-1}\times1+2^{-2}\times0+2^{-3}\times1+2^{-4}\times1$

$= 0.5+0+0.125+0.0625$

$= (0.6875)_{10}$

ex: Convert $(1011.1101)_2$ To $(\quad)_{10}$

$= 2^3\times1+2^2\times0+2^1\times1+2^0\times1+2^{-1}\times1+2^{-2}\times1+2^{-3}\times0+2^{-4}\times1$

$= 8+0+2+1+0.5+0.25+0+0.0625$

$= (11.8125)$

H.W: convert the binary numbers to decimal

a- 1101.011 b- 10111101.110 c- 001 d- 0101.0 e- 1111111 f- 0011011

Decimal to binary conversion:

To convert a decimal integer to the equivalent in any radix (base) . Divide the integer repeatedly by the radix successive remainders giving the required number.

ex: convert $(53)_{10}$ To $(\quad)_2$

	q	R	
	53		
2	26	1	← LSB
2	13	0	
2	6	1	
2	3	0	
2	1	1	
2	0	1	← MSB

$(53)_{10} \rightarrow (110101)_2$

MSB ↗ ↖ LSB

Note: whenever you arrive quotient of 0 with a remainder of 1 conversion is finished.

ex: convert $(342)_{10} \rightarrow (\quad)_2$

		R	
	342		
2	171	0	← LSB
2	85	1	
2	42	1	
2	21	0	
2	10	1	
2	5	0	
2	2	1	
2	1	0	
2	0	1	← MSB

$= (101010110)_2$

Decimal fraction converts to binary

ex: convert $(0.6)_{10} \rightarrow (\quad)_2$

$0.6 \times 2 = 1.2 \rightarrow 1$ ← MSB after binary point

$0.2 \times 2 = 0.4 \rightarrow 0$

$0.4 \times 2 = 0.8 \rightarrow 0$

$0.8 \times 2 = 1.6 \rightarrow 1$

$0.6 \times 2 = 1.2 \rightarrow 1$ ← LSB

stop when the number is repeated

$\therefore (0.6)_{10} \rightarrow (0.10011)_2$

ex: convert $(0.3125)_{10} \rightarrow (\quad)_2$

$0.3125 \times 2 = 0.625$ → 0 ← MSB after point
$0.625 \times 2 = 1.25$ → 1
$0.25 \times 2 = 0.5$ → 0
$0.5 \times 2 = 1.0$ → 1 ← LSB

stop when the Fractional part is all zero

$(0.3125)_{10} \rightarrow (0.0101)_2$

ex: $(0.35)_{10} \rightarrow (\quad)_2$

$0.35 \times 2 = 0.7$ 0 ← MSB after point
$0.7 \times 2 = 1.4$ 1
$0.4 \times 2 = 0.8$ 0
$0.8 \times 2 = 1.6$ 1
$0.6 \times 2 = 1.2$ 1
$0.2 \times 2 = 0.4$ 0
$0.4 \times 2 = 0.8$ 0 LSB

stop is repeated $= (0.0101100)_2$

ex: $(15.8)_{10} \rightarrow (\quad)_2$

a- convert integer $(15)_{10} \rightarrow (\quad)_2$

	15	
2	7	1 ← LSB
2	3	1
2	1	1
2	0	1 ← MSB

$\Rightarrow (1111)_2$

b- convert Fraction $(0.8)_{10} \rightarrow (\quad)_2$

$0.8 \times 2 = 1.6$ 1 ← MSB
$0.6 \times 2 = 1.2$ 1
$0.2 \times 2 = 0.4$ 0 $\Rightarrow (0.11001)_2$
$0.4 \times 2 = 0.8$ 0
$0.8 \times 2 = 1.6$ 1 ← LSB

$\therefore (15.8)_{10} \rightarrow (1111.11001)_2$

H.W: convert the decimal number to binary

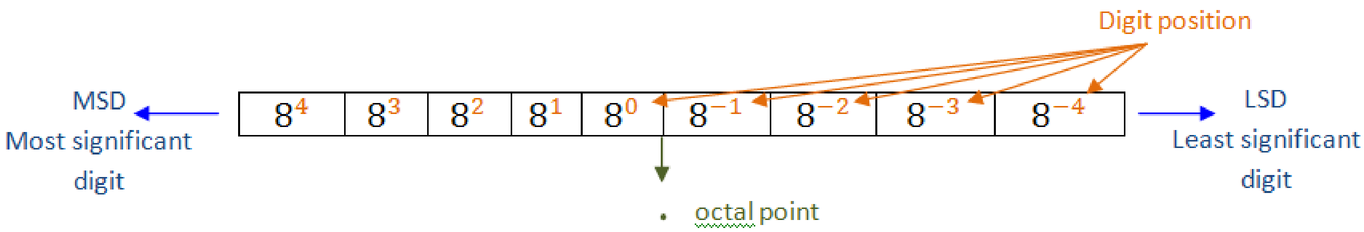
a- 14.5   b- 21   c- 0.375   d- 255.0625

3- Octal number system

This system has eight digits (0 – 7)

This system has a base of 8

This system is a positional – valued system



Conversion from octal to decimal

ex: $(147.3)_8 \rightarrow (\quad)_{10}$

$= 8^2 \times 1 + 8^1 \times 4 + 8^0 \times 7 + 8^{-1} \times 3$

$= 64 + 32 + 7 + 0.375$

$= (103.375)_{10}$

ex: $(632)_8 \rightarrow (\quad)_{10}$

$= 8^2 \times 6 + 8^1 \times 3 + 8^0 \times 2$

$= 384 + 24 + 2$

$= (410)_{10}$

ex: $(23.6)_8 \rightarrow (\quad)_{10}$

$= 8^1 \times 2 + 8^0 \times 3 + 8^{-1} \times 6$

$= 16 + 3 + 0.75$

$= (19.75)_{10}$

Conversion from decimal to octal

ex: $(342)_{10} \rightarrow (\quad)_8$

	Q	R	
	342		
8	42	6	← LSB
8	5	2	
8	0	5	← MSD

$\frac{342}{8} = 42.75$

$Q = 42$

$R = 0.75$

$0.75 \times 8 = 6$

يجب عدد صحيح

$= (526)_8$

ex: $(0.31)_{10} \rightarrow (\quad)_8$

$0.31 \times 8 = 2.48 \rightarrow 2$ ← MSB after point

$0.48 \times 8 = 3.84 \rightarrow 3$

$0.84 \times 8 = 6.72 \rightarrow 6$

$0.72 \times 8 = 5.76 \rightarrow 5$

$0.31 \times 8 = 2.48$ ← LSB

STOP

$= (0.2365 \ldots)_8$

Octal to binary conversion:

octal	binary
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

This conversion is performed by converting each octal digits to it's 3 bit binary equivalent ex: $(634)_8 \longrightarrow (\quad)_2$

$$\underbrace{6}_{110}\ \underbrace{3}_{011}\ \underbrace{4}_{100}$$

$= (110\ 011\ 100)_2$

Binary to octal conversion

ex: $(1011.01101)_2 \rightarrow (\quad)_8$

$$= \underbrace{001}_{1}\ \underbrace{011}_{3} . \underbrace{011}_{3}\ \underbrace{010}_{2}$$

$= (13.32)_8$

Note: starting at the binary point group the bits in threes then convert each group of threes to it's octal equivalent.

H.W:

1- convert the octal numbers to binary

a- 0.3 b- 7 c- 0 d- 7642

2- convert the binary numbers to octal

a- 101 b- 110 c- 010 d- 111000101 e- 1011000111

3-convert the decimal numbers to octal

a-359 b- 98 c- 163

4- Hexadecimal number system

It has 16 symbols or digits (0 – F)

0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F

It has a base of 16

It's positional – valued system

decimal	binary	octal	hexa
0	0000	0	0
1	0001	1	1
2	0010	2	2
3	0011	3	3
4	0100	4	4
5	0101	5	5
6	0110	6	6
7	0111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

Hexa to decimal conversion:

ex: convert $(2C6E)_{16} \rightarrow (\quad)_{10}$

$= 16^3 \times 2 + 16^2 \times C + 16^1 \times 6 + 16^0 \times E$

$= 8192 + 3072 + 96 + 14$

$= (11374)_{10}$

Decimal to hexa conversion:

ex: convert $(15797)_{10} \to (\quad)_{16}$

		R	
	15797		
16	987	5	← LSD
16	61	B	
16	3	D	
16	0	3	← MSD

$= (3DB5)_{16}$

Note: any reminder that > 9 are represented by the character (A-F)

Hexa to binary conversion:

ex: convert $(9EF0) \to (\quad)_2$

9 E F 0

1001 1110 1111 0000

$= (1001\ 1110\ 1111\ 0000)_2$

Binary to Hexa conversion:

ex: $(001110011000.1110)_2 \to (\quad)_{16}$

$\underbrace{0011}_{3}\ \underbrace{1001}_{9}\ \underbrace{1000}_{8}\ .\ \underbrace{1110}_{E}$

$= (398.E)_{16}$

Note: to convert any system to any other system it's advisable to

1- Convert to decimal

2- Convert to new system

Except for conversion from binary to octal or binary to hexa and vice versa.

ex: convert $(231.3)_4 \to (\quad)_7$

1- convert $(231.3)_4 \to (\quad)_{10}$

$= 4^2 \times 2 + 4^1 \times 3 + 4^0 \times 1 + 4^{-1} \times 3$

$= 32 + 12 + 1 + 0.75 = (45.75)_{10}$

2- convert $(45.75)_{10} \to (\quad)_7$

a- $(45)_{10} \to (\quad)_7$

		R	
	45		
7	6	3	← LSD
7	0	6	← MSD

$= (63)_7$

6- $(0.75)_{10} \to (\quad)_7$

$0.75 \times 7 = \underline{5}.25 \to 5$ ← MSD after point

$0.25 \times 7 = \underline{1}.75 \to 1$

$0.75 \times 7 = \underline{5}.25 \to 5$ ← LSD

$(0.75)_{10} \to (.515)_7$

$\Rightarrow (45.75)_{10} \to (63.515)_7$

$\therefore (231.3)_4 \to (63.515)_7$

Note: to count decimal number with N decimal numbers we can count from 0 to 10^N-1

N=2 → count is from 0 – 99

For binary the count is from 0 up to 2^N-1

N= 2 → count is from 0 – (2^N-1) → 0-3

decimal	binary
0	0
1	1
2	10
3	11