Boolean Algebra:

It is a set of lows, rules and theorems used to equate and manipulate variable quantities which are only allowed to make either one of two state (1 or 0) and if it can be used to help analyze a logic circuit and express it's operation mathematically , the operation of Boolean algebra are the logical operation (AND , OR, NOT)

Boolean Addition:

It's equivalent to OR operation

$0 + 0 = 0$
$0 + 1 = 1$
$1 + 0 = 1$
$1 + 1 = 1$

Note: A sum term is equal to 1 when one or more of literals in the term are 1
A sum term is equal to 0 only if each of the literals is 0.

Boolean Multiplication:

It's equivalent to AND operation

$0 . 0 = 0$
$0 . 1 = 0$
$1 . 0 = 0$
$1 . 1 = 1$

Note: A product term is equal to 1 only if each of the literals in the term is 1
A product term is equal 0 when one or more of literals are 0 .

ex: determine the value of A, B , C and D that make the sum $A + \bar{B} + C + \bar{D} = 0$

Sol: for the sum term to be 0, each of literal in the term must be 0 therefore

A = 0 , B = 1 $\rightarrow \bar{B} = 0$, C = 0 and D = 1 $\rightarrow \bar{D} = 0$

$$A + \bar{B} + C + \bar{D} = 0$$
$$0 + \bar{1} + 0 + \bar{1} = 0$$
$$0 + 0 + 0 + 0 = 0$$

∴ A = 0 , B = 1 , C = 0 and D = 1

ex: determine the value of A and B that make $\bar{A} + B = 0$

Sol: $\bar{A} + B = 0$

$\bar{1} + 0 = 0 \rightarrow 0 + 0 = 0$

∴ A = 1 , B = 0

ex: determine the value of A, B , C and D from $A\bar{B}C\bar{D} = 1$

Sol: $A\bar{B}C\bar{D} = 1$

$1 \cdot \bar{0} \cdot 1 \cdot \bar{0} = 1$

$1 \cdot 1 \cdot 1 \cdot 1 = 1$

∴ A = 1 , B = 0 , C = 1 , D = 0

OR: $A\bar{B}C\bar{D} = 1$

$1 \cdot 1 \cdot 1 \cdot 1 = 1$

A = 1, $\bar{B} = 1$, C = 1, $\bar{D} = 1$ → D = 0

∴ B = 0

H.W

1- if A=0 , what does $\bar{A}$ equal ?

2- determined the value A, B and C from $\bar{A} + \bar{B} + C = 0$

3- determined the value A, B and C from $A\bar{B}C = 1$

Laws of Boolean algebra:

1- Commutative laws:

$A + B = B + A$

$AB = BA$

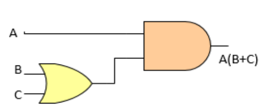
2- Associative laws:

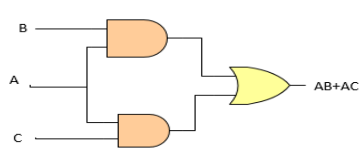
$A + (B + C) = (A + B) + C$

$A(BC) = (AB)C$

3- Distributive laws:

$A(B + C) = AB + AC$



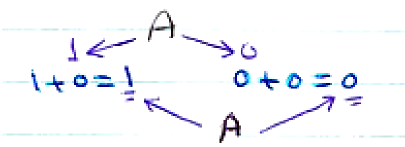


Rules of Boolean algebra:

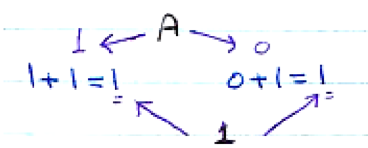
The 12 basic rules are shown bellow:

1- $A + 0 = A$
2- $A + 1 = 1$
3- $A \cdot 0 = 0$
4- $A \cdot 1 = A$
5- $A + A = A$
6- $A + \overline{A} = 1$
7- $A \cdot A = A$
8- $A \cdot \overline{A} = 0$
9- $\overline{\overline{A}} = A$
10- $A + AB = A$
11- $A + \overline{A}B = A + B$
12- $(A + B)(A + C) = A + BC$

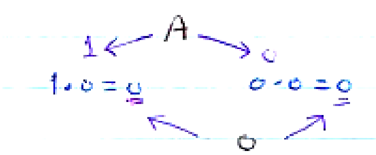
Rule 1: A+ 0 = A



Rule 2: A+ 1 = 1

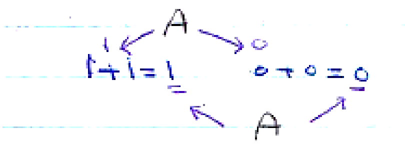


Rule 3: A . 0 = 0

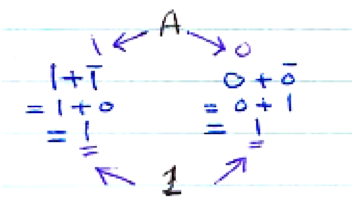


Rule 4: A . 1 = A

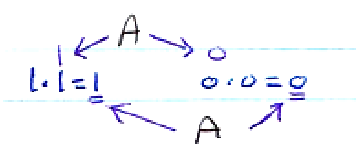
Rule 5: $A + A = A$



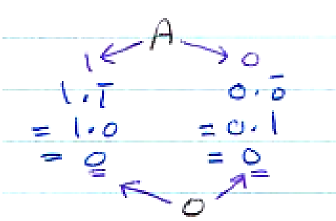
Rule 6: $A + \overline{A} = 1$



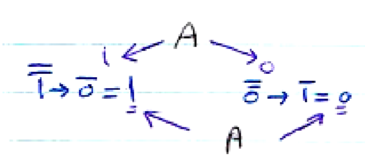
Rule 7: $A \,.\, A = A$



Rule 8: $A \,.\, \overline{A} = 0$



Rule 9: $\overline{\overline{A}} = A$



Rule 10: **$A + AB = A$**

Prove that:

$A + AB$

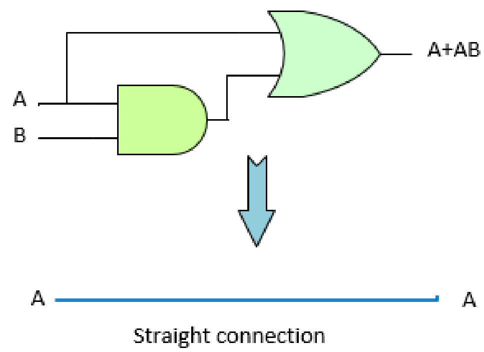
$= A\,(1 + B)$; $(1+B) = 1$ rule 2

$= A \cdot 1$; $A \cdot 1 = 1$ rule 4

$= A$

A B	A.B	A+AB
0 0	0	0
0 1	0	0
1 0	0	1
1 1	1	1

equal



Rule 11: **$A + \overline{A}B = A + B$**

Prove that:

$A + \overline{A}B$

$= A + AB + \overline{A}B$; $A + AB$ rule 10

$= A + B\,(A + \overline{A})$; $A + \overline{A} = 1$ rule 4

$= A + B \cdot 1$

$= A + B$

OR

$A + \overline{A}B$

$= A + AB + \overline{A}B$; $A + AB$ rule 10

$= AA + AB + \overline{A}B$; $A = AA$ rule 7

$= AA + AB + \overline{A}B + A\overline{A}$; $\overline{A} \cdot A = 0$ rule 8

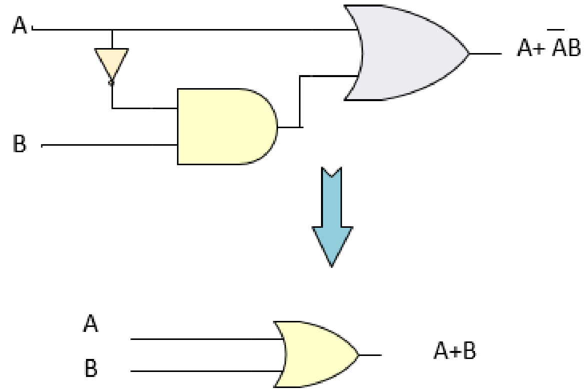
$= (A + \overline{A})\,(A + B)$

$= 1 \cdot (A + B)$; $A + \overline{A} = 1$ rule 6

$= A + B$

A B	$\bar{A}$	$\bar{A}B$	$A+\bar{A}B$	A+B
0 0	1	0	0	0
0 1	1	1	1	1
1 0	0	0	1	1
1 1	0	0	1	1

equal



Rule 12:(A + B) (A+C) = A + BC

Prove that

$(A + B)(A + C)$

$= AA + AB + AC + BC$; $A \cdot A = 1$ rule 7

$= A + AB + AC + BC$

$= A(1 + C) + AB + BC$; $A+1 = 1$ rule 2

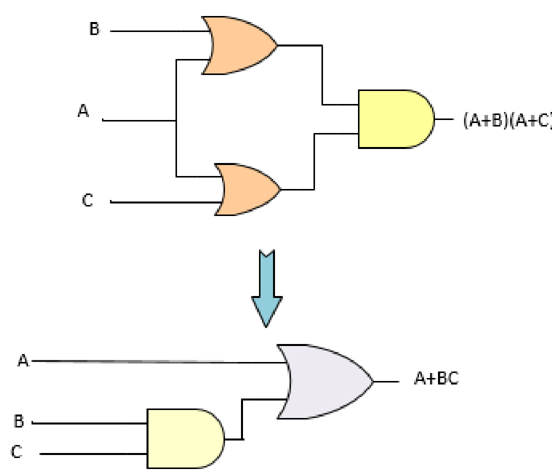
$= A + AB + BC$

$= A(1 + B) + BC$; $A+1 = 1$ rule 2

$= A + BC$

A B C	A+B	A+C	(A+B) (A+C)	BC	A+BC
0 0 0	0	0	0	0	0
0 0 1	0	1	0	0	0
0 1 0	1	0	0	0	0
0 1 1	1	1	1	1	1
1 0 0	1	1	1	0	1
1 0 1	1	1	1	0	1
1 1 0	1	1	1	0	1
1 1 1	1	1	1	1	1

equal



ex: simplify this expression

AB + A(B+C) + B(B+C)

Sol:

=AB + AB + AC + BB + BC

=AB + AC + B + BC

=AB + AC + B (1+C)

=AB + AC + B .1

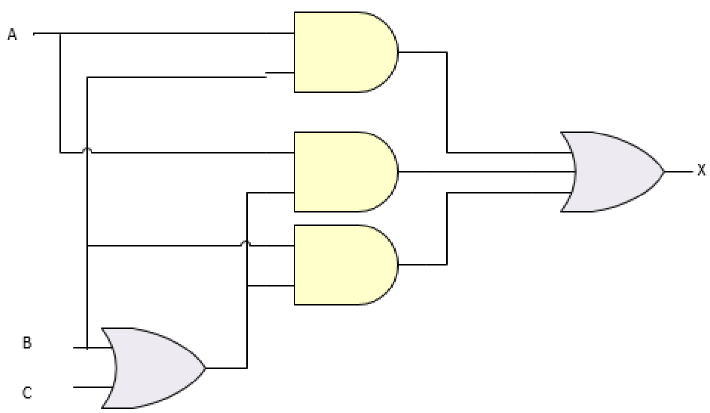
=AB + AC + B

=B (A+1) + AC

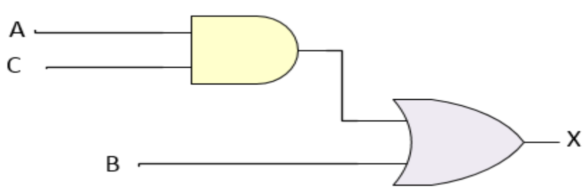
=B . 1 + AC

=B + AC

The circuit of X=AB + A(B+C) + B(B+C)



Simplify to the circuit of X = B + AC



These two cct.s are equivalent

ex: simplify this expression

$$(A\bar{B}(C+BD)+\bar{A}\bar{B})C$$

Sol:

= $(A\bar{B}C+A\bar{B}BD+\bar{A}\bar{B})C$

= $(A\bar{B}C+\bar{A}\bar{B})C$

= $A\bar{B}CC+\bar{A}\bar{B}C$

= $A\bar{B}C+\bar{A}\bar{B}C$

=$\bar{B}C(A+\bar{A})$

=$\bar{B}C$

ex: simplify this expression

$$\bar{A}BC+A\bar{B}\bar{C}+\bar{A}\bar{B}\bar{C}+A\bar{B}C+ABC$$

Sol:

$\bar{A}BC+A\bar{B}\bar{C}+\bar{A}\bar{B}\bar{C}+A\bar{B}C+ABC$

=$BC(\bar{A}+A)+A\bar{B}(\bar{C}+C)+\bar{A}\bar{B}\bar{C}$

= $BC+A\bar{B}+\bar{A}\bar{B}\bar{C}$

= $BC+\bar{B}(A+\bar{A}\bar{C})$

= $BC+\bar{B}(A+\bar{C})$=

= $BC+\bar{B}A+\bar{B}\bar{C}$

DE Morgan's theorems:

1- The complement of a product of variables is equal to the sum of the complements of the variables

$$\overline{X\cdot Y}=\bar{X}+\bar{Y}$$

NAND = negative – OR

2- The complement of a sum of variables is equal to the product of the complements of the variables

$$\overline{X+Y}=\bar{X}\cdot\bar{Y}$$

NOR = negative – AND

ex: apply DE Morgan's theorems of

1- $\overline{X \cdot Y \cdot Z}$

$= \bar{X} + \bar{Y} + \bar{Z}$

2- $\overline{X + \bar{Y} + Z}$

$= \bar{X} \cdot \bar{\bar{Y}} \cdot \bar{Z} = \bar{X} \cdot Y \cdot \bar{Z}$

ex: find the compliment of F

$F = (\bar{A} + B)\bar{C}$

Sol:

$F = \overline{(\bar{A} + B)\bar{C}}$

$= \overline{(\bar{A} + B)} + \bar{\bar{C}}$

$= \bar{\bar{A}} \cdot \bar{B} + C$

$= A\bar{B} + C$

ex: find the complement of F

$F = (A\bar{B} + C) \cdot \bar{D} + E$

Sol:

$F = \overline{(A\bar{B} + C) \cdot \bar{D} + E}$

$= \overline{(A\bar{B} + C) \cdot \bar{D}} \cdot \bar{E}$

$= (\overline{(A\bar{B} + C)} + \bar{\bar{D}}) \cdot \bar{E}$

$= ((\overline{A\bar{B}} \cdot \bar{C}) + \bar{\bar{D}}) \cdot \bar{E}$

$= (((\bar{A} + \bar{\bar{B}}) \cdot \bar{C}) + D) \cdot \bar{E}$

$= (((\bar{A} + B) \cdot \bar{C}) + D) \cdot \bar{E}$

$= (\bar{A}\bar{C} + B\bar{C} + D)\bar{E}$

$= \bar{A}\bar{C}\bar{E} + B\bar{C}\bar{E} + D\bar{E}$

ex: apply DE Morgan's theorem and Boolean algebra for this expressions:

1- $\overline{\overline{A + B\bar{C}} + D\overline{(E + \bar{F})}}$

Sol:

$= \overline{\overline{A + B\bar{C}}} \cdot \overline{D\overline{(E + \bar{F})}}$

$= (A + B\bar{C}) \cdot \bar{D} + \overline{\overline{E + \bar{F}}}$

$= (A + B\bar{C}) \cdot (\bar{D} + E + \bar{F})$

2- $\overline{A\bar{B} + \bar{C}D + EF}$

Sol:

$= \overline{A\bar{B}} \cdot \overline{\bar{C}D} \cdot \overline{EF}$

$= (\bar{A} + \bar{\bar{B}}) \cdot (\bar{\bar{C}} + \bar{D}) \cdot (\bar{E} + \bar{F})$

$= (\bar{A} + B) \cdot (C + \bar{D}) \cdot (\bar{E} + \bar{F})$

3- $\overline{AB + AC} + \bar{A}\bar{B}C$

Sol:

$= \overline{AB} \cdot \overline{AC} + \bar{A}\bar{B}C$

$= (\bar{A} + \bar{B}) \cdot (\bar{A} + \bar{C}) + \bar{A}\bar{B}C$

$= \bar{A}\bar{A} + \bar{A}\bar{B} + \bar{A}\bar{C} + \bar{B}\bar{C} + \bar{A}\bar{B}C$

$= \bar{A} + \bar{A}\bar{B} + \bar{A}\bar{C} + \bar{B}\bar{C} + \bar{A}\bar{B}C$

$= \bar{A} + \bar{A}\bar{B}(1 + C) + \bar{A}\bar{C} + \bar{B}\bar{C}$

$= \bar{A} + \bar{A}\bar{B} + \bar{A}\bar{C} + \bar{B}\bar{C}$

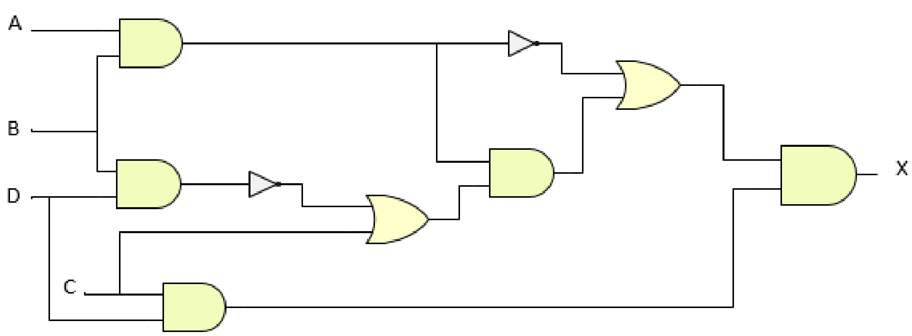
$= \bar{A}(1 + \bar{C}) + \bar{A}\bar{B} + \bar{B}\bar{C}$

$= \bar{A} + \bar{A}\bar{B} + \bar{B}\bar{C}$

$= \bar{A}(1 + \bar{B}) + \bar{B}\bar{C}$

$= \bar{A} + \bar{B}\bar{C}$

ex: simplify this cct. Shown



Sol:

$X = (\overline{AB} + AB(C + \overline{DB})) \cdot CD$

$= (\bar{A} + \bar{B} + AB(C + \overline{DB})) \cdot CD$

$= (\bar{A} + \bar{B} + AB(C + \bar{D} + \bar{B})) \cdot CD$

$= (\bar{A} + \bar{B} + ABC + AB\bar{D} + AB\bar{B}) \cdot CD$

$= \bar{A}\,CD + \bar{B}CD + ABCCD + AB\bar{D}CD$

$= \bar{A}\,CD + \bar{B}CD + ABCD$

$= CD(AB + \bar{A}\,) + \bar{B}CD$

$= CD(B+\bar{A}) + \bar{B}CD$

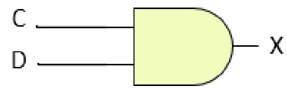
$= CDB + CD\bar{A} + \bar{B}CD$

$= CD\,(B+\bar{B}) + CD\bar{A}$

$= CD + CD\bar{A}$

$= CD\,(1+\bar{A})$

$= CD$



this is the simplest cct.

H.W

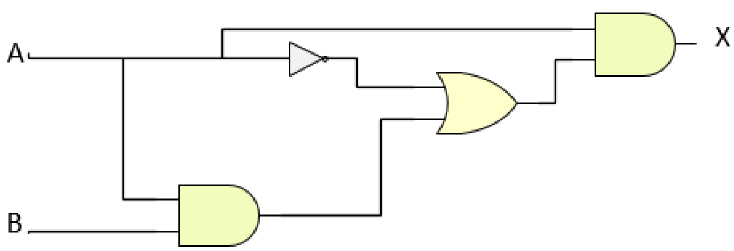
1 - Simplify this expression

1) $\overline{AB} + \overline{AC} + \bar{A}\bar{B}\bar{C}$

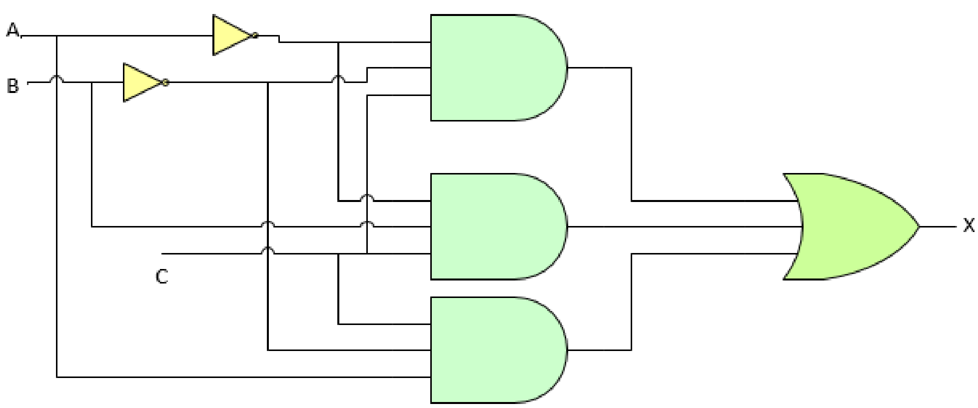
2) $AB\bar{C} + \bar{A}\bar{B}C + \bar{A}BC + \bar{A}\bar{B}\bar{C}$

2 –Simplify the cct. show

1)-



2)-



3 - prove that

1)- $\overline{A\bar{B}(A+C)} + \bar{A}B \cdot \overline{A+\bar{B}+\bar{C}} = \bar{A} + B$

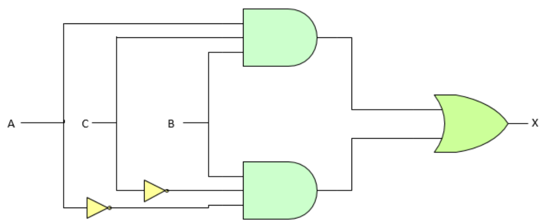
2)- $\overline{A + \overline{B\bar{C}} + CD} + BC = \bar{A}B + BC$

Designing of combination logic circuit:

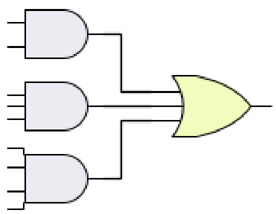
1- Sum of product SOP (minterm expression)

A SUM of product expression is two or more AND function ORed together.

ex: $ABC + \bar{A}B\bar{C}$



ex: $AB + CDB + ABCD$



An SOP is equal to 1 only if one or more of the product term is equal to 1

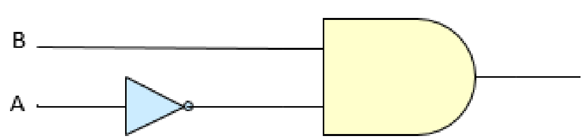
Note: in SOP expression a single overbar cannot extend over more than one variable SOP can have the term $\bar{A}\bar{B}\bar{C}$ BUT not $\overline{ABC}$.

ex: design a cct. That has the following function X=1 only if A=0 and B=1

Sol:

AB	X	SOP minterm
00	0	M_0
01	1	$M_1 = \bar{A}B$
10	0	M_2
11	0	M_3

$M_1 = \bar{A}B$ this term is only 1 if A=0 , B=1



ex: write the expression x

AB	X	SOP minterm
00	0	M_0
01	1	$M_1 = \bar{A}B$
10	1	$M_2 = A\bar{B}$
11	0	M_3

Sol: $X = M_1 + M_2 = \bar{A}B + A\bar{B}$

ex: find X expression

ABC	X	SOP minterm
000	0	M_0
001	0	M_1
010	1	$M_2 = \bar{A}B\bar{C}$
011	1	$M_3 = \bar{A}BC$
100	0	M_4
101	0	M_5
110	0	M_6
111	1	$M_7 = ABC$

Sol:

$X = M_2 + M_3 + M_7$

$= \bar{A}B\bar{C} + \bar{A}BC + ABC$

$= \bar{A}B(\bar{C} + C) + ABC$

$= \bar{A}B + ABC$

$= B\,(\bar{A} + AC)$

$= B\,(\bar{A} + C)$

ex: design a logic circuit that has three input A , B , C and whose output will be high only when a majority of input is high

Sol:

ABC	X	SOP minterm
000	0	M_0
001	0	M_1
010	0	M_2
011	1	$M_3 = \bar{A}BC$
100	0	M_4
101	1	$M_5 = A\bar{B}C$
110	1	$M_6 = AB\bar{C}$
111	1	$M_7 = ABC$

$X = M_3 + M_5 + M_6 + M_7$

$= \bar{A}BC + A\bar{B}C + AB\bar{C} + ABC$

$= \bar{A}BC + A\bar{B}C + AB\,(\bar{C} + C)$

$= \bar{A}BC + A\bar{B}C + AB$

$= \bar{A}BC + A(\bar{B}C + B)$

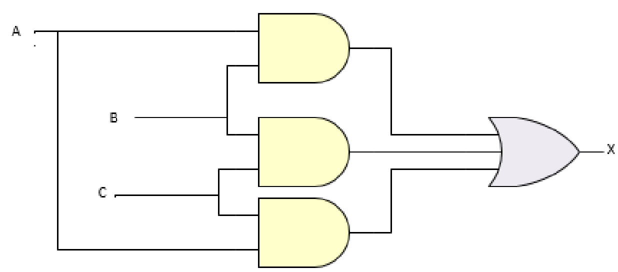
$= \bar{A}BC + A(C + B)$

$= \bar{A}BC + AC + AB$

$= B(\bar{A}C + A) + AC$

$= B(C + A) + AC$

$= BC + AB + AC$

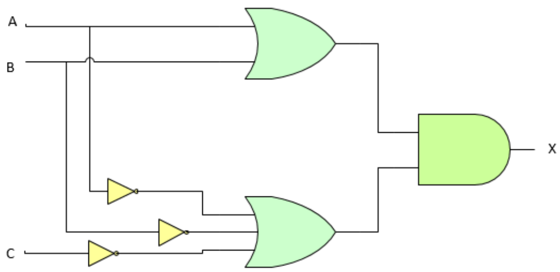


The logic cct.

2- Product of Sum POS (maxterm expression)

A product of sum expression is two or more OR function ANDed together.

ex: $(A + B) \cdot (\bar{A} + \bar{B} + \bar{C})$



ex: $(C + D + \bar{B} + \bar{A}) \cdot (A + \bar{B} + C)$

A POS expression is equal to 0 only if at least one of the sum term is equal to 0.

Note: in a POS a single overbar cannot extend over more than one variable

POS can have the term $(\bar{A} + \bar{B} + \bar{C})$ but not $(\overline{A + B + C})$

Note: each maxterm is the complement of the corresponding minterm

ex:

AB	X	SOP minterm	POS maxterm
00	1	$M_0 = \bar{A}\bar{B}$	M_0
01	1	$M_1 = \bar{A}B$	M_1
10	0	M_2	$M_2 = \bar{A} + B$
11	0	M_3	$M_3 = \bar{A} + \bar{B}$

Note: A Boolean function may be expressed algebraically from a given truth table by forming:

- Maxterm for each combination of variable which produce a 0 in the function and then form AND of those maxterm

Or

- Minterm for each combination of variable which produce a 1 in the function , and then OR of those minterm.

Sol:

POS

$$X = M_2 \cdot M_3$$
$$= (\bar{A} + B) \cdot (\bar{A} + \bar{B})$$
$$= \bar{A}\bar{A} + \bar{A}\bar{B} + \bar{A}B + B\bar{B}$$
$$= \bar{A} + \bar{A}(\bar{B} + B)$$
$$= \bar{A} + \bar{A} = \bar{A}$$

SOP

$$X = M_0 \cdot M_1$$
$$= \bar{A}\bar{B} + \bar{A}B$$
$$= \bar{A}(\bar{B} + B)$$
$$= \bar{A}$$

ex:

AB	X	POS maxterm
00	1	M_0
01	0	$M_1 = A + \bar{B}$
10	0	$M_2 = \bar{A} + B$
11	0	$M_3 = \bar{A} + \bar{B}$

Sol:

$$X = M_1 \cdot M_2 \cdot M_3$$
$$= (A + \bar{B}) \cdot (\bar{A} + B) \cdot (\bar{A} + \bar{B})$$
$$= (A\bar{A} + AB + \bar{A}\bar{B} + B\bar{B}) \cdot (\bar{A} + \bar{B})$$
$$= (AB + \bar{A}\bar{B}) \cdot (\bar{A} + \bar{B})$$
$$= A\bar{A}B + \bar{A}\bar{A}\bar{B} + AB\bar{B} + \bar{A}\bar{B}B$$
$$= \bar{A}\bar{B} + \bar{A}\bar{B} = \bar{A}\bar{B}$$