the Karnaugh map (k-map):

is a method to simplify a Boolean expression, the k-map is similar to a truth table, it is just another way of presenting truth table.

The k-map is an array of cells.

The cells are ordered in Gray code.

Each cell position represents one combination of input variables.

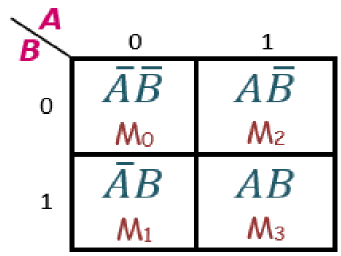
Each cell value represents the corresponding output value.

The number of cells in k-map is equal to 2^n, where n is the number of input variable.

The k-map can be used with 2,3,4 and 5 variable.

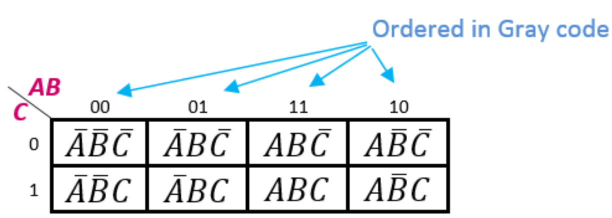
Two variable k-map:

AB	SOP minterm
00	$M_0 = \bar{A}\bar{B}$
01	$M_1 = \bar{A}B$
10	$M_2 = A\bar{B}$
11	$M_3 = AB$



Three variable k-map:

ABC	SOP minterm
000	$M_0 = \bar{A}\bar{B}\bar{C}$
001	$M_1 = \bar{A}\bar{B}C$
010	$M_2 = \bar{A}B\bar{C}$
011	$M_3 = \bar{A}BC$
100	$M_4 = A\bar{B}\bar{C}$
101	$M_5 = A\bar{B}C$
110	$M_6 = AB\bar{C}$
111	$M_7 = ABC$



Four variable k-map:

ABCD	SOP minterm
0000	$M_0 = \bar{A}\bar{B}\bar{C}\bar{D}$
0001	$M_1 = \bar{A}\bar{B}\bar{C}D$
0010	$M_2 = \bar{A}\bar{B}C\bar{D}$
0011	$M_3 = \bar{A}\bar{B}CD$
0100	$M_4 = \bar{A}B\bar{C}\bar{D}$
0101	$M_5 = \bar{A}B\bar{C}D$
0110	$M_6 = \bar{A}BC\bar{D}$
0111	$M_7 = \bar{A}BCD$
1000	$M_8 = A\bar{B}\bar{C}\bar{D}$
1001	$M_9 = A\bar{B}\bar{C}D$
1010	$M_{10} = A\bar{B}C\bar{D}$
1011	$M_{11} = A\bar{B}CD$
1100	$M_{12} = AB\bar{C}\bar{D}$
1101	$M_{13} = AB\bar{C}D$
1110	$M_{14} = ABC\bar{D}$
1111	$M_{15} = ABCD$

CD \ AB	00	01	11	10
00	$\bar{A}\bar{B}\bar{C}\bar{D}$	$\bar{A}B\bar{C}\bar{D}$	$AB\bar{C}\bar{D}$	$A\bar{B}\bar{C}\bar{D}$
01	$\bar{A}\bar{B}\bar{C}D$	$\bar{A}B\bar{C}D$	$AB\bar{C}D$	$A\bar{B}\bar{C}D$
11	$\bar{A}\bar{B}CD$	$\bar{A}BCD$	$ABCD$	$A\bar{B}CD$
10	$\bar{A}\bar{B}C\bar{D}$	$\bar{A}BC\bar{D}$	$ABC\bar{D}$	$A\bar{B}C\bar{D}$

ex: represent the truth table in k-map

AB	X
00	1
01	0
10	0
11	1

Sol:

B \ A	0	1
0	1	0
1	0	1

ex:

ABC	X
000	1
001	1
010	1
011	0
100	0
101	0
110	1
111	0

Sol:

C \ AB	00	01	11	10
0	1	1	1	0
1	1	0	0	0

ex:

ABCD	X
0000	0
0001	1
0010	0
0011	0
0100	0
0101	1
0110	0
0111	0
1000	0
1001	0
1010	0
1011	0
1100	0
1101	1
1110	0
1111	1

Sol:

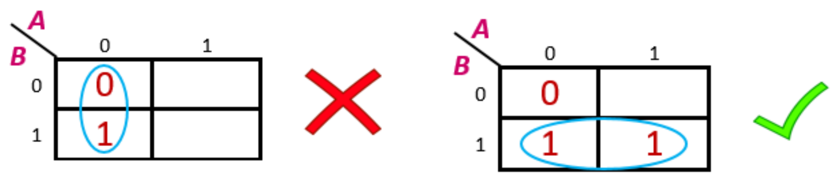
CD \ AB	00	01	11	10
00	0	0	0	0
01	1	1	1	0
11	0	0	1	0
10	0	0	0	0

Note: once the k-map has been filled with 0's and 1's, the sum of product expression for the output can be obtained by ORing together cells that contain a 1

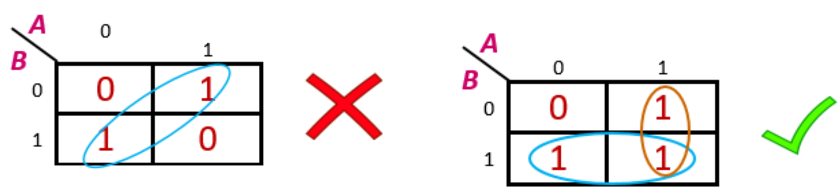
k-map Rules of simplification:

the k-map uses the following rules for the simplification of expression by grouping together adjacent cells containing ones

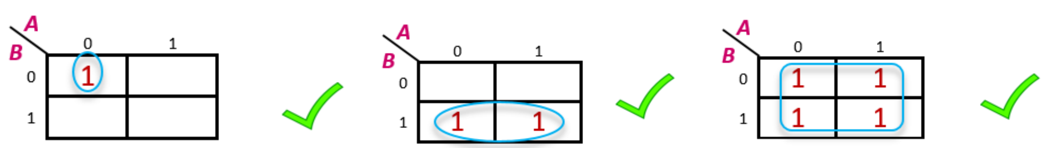
1- No zero allowed



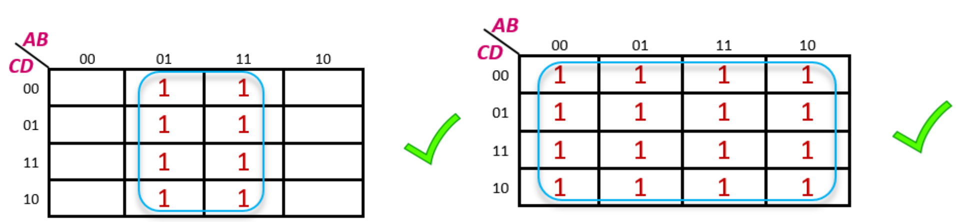
2- No diagonals



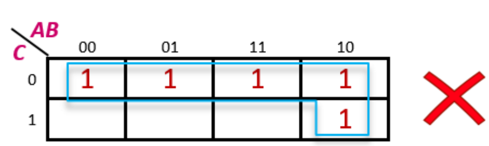
3- Only power of 2 number of cells in each group



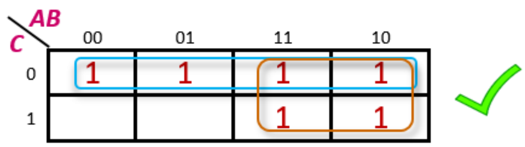
Group of $2^0 = 1$ Group of $2^1 = 2$ Group of $2^2 = 4$

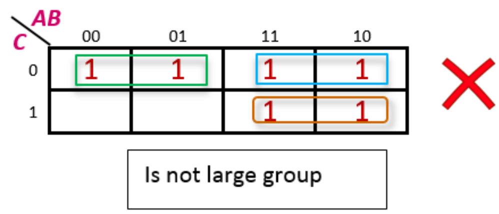


Group of $2^3 = 8$ Group of $2^4 = 16$

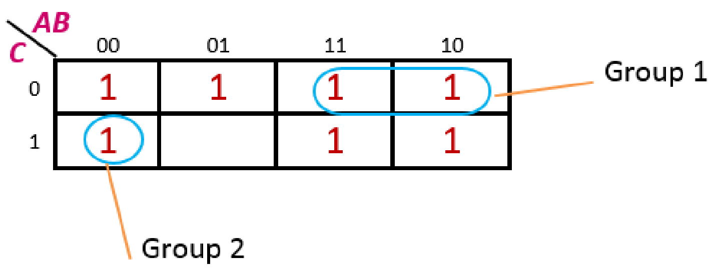


4- Group should be as large as possible

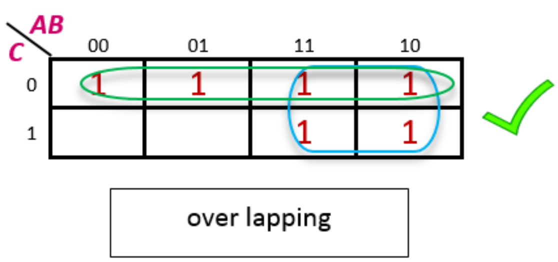


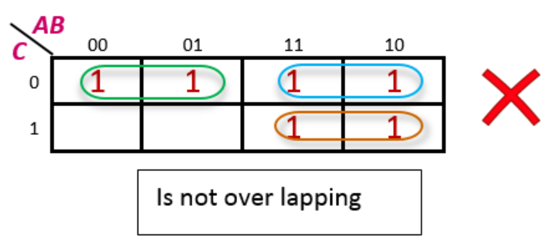


5- Every one must be in at least one group



6- Overlapping allowed

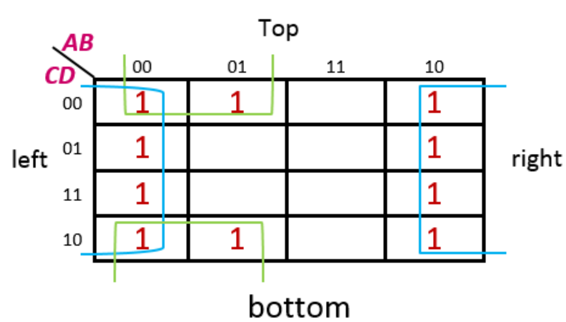




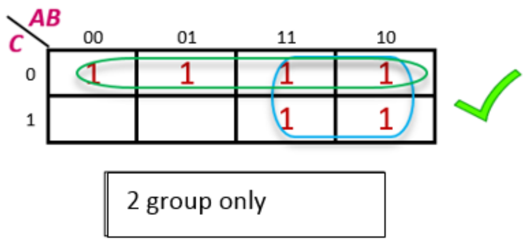
7- Wrap around allowed

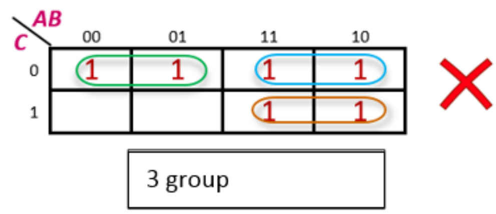
Left most cells grouped with right most cells

Top cells group with bottom cells



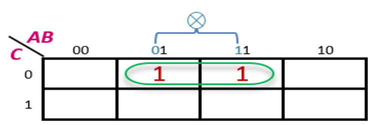
8- Fewest number of group possible



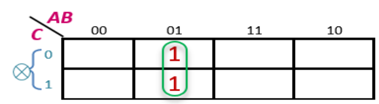


ex:

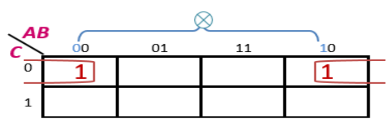
ABC	Y1	Y2	Y3
000	0	0	1
001	0	0	0
010	1	1	0
011	0	1	0
100	0	0	1
101	0	0	0
110	1	0	0
111	0	0	0



$Y1 = B\bar{C}$



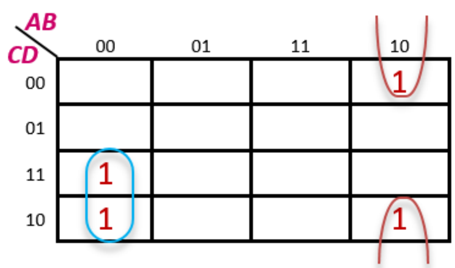
$Y2 = \bar{A}B$



$Y3 = \bar{B}\bar{C}$

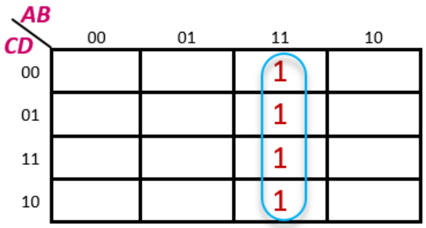
ex: find the expression X from these k-map

1-



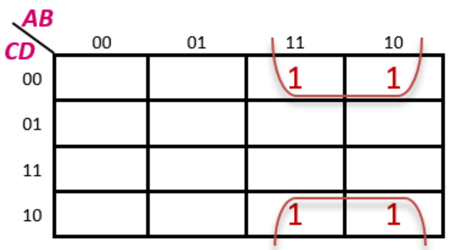
$$X = \bar{A}\bar{B}C + A\bar{B}\bar{D}$$

2-



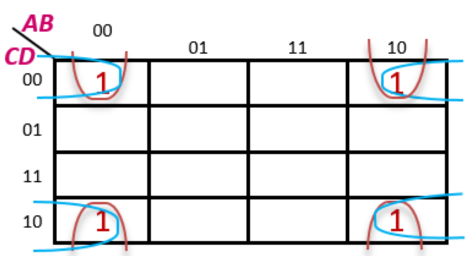
$X = AB$

3-



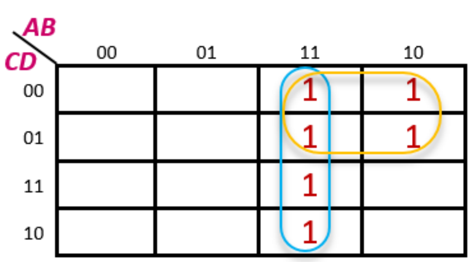
$X = A\overline{D}$

4-



$X = \overline{B}\overline{D}$

5-



$X = AB + A\bar{C}$

6-

CD \ AB	00	01	11	10
00		1	1	
01		1	1	
11		1	1	
10		1	1	

$$X = B$$

7-

CD \ AB	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11				
10				

$$X = \bar{C}$$

8-

CD \ AB	00	01	11	10
00	1			1
01	1			1
11	1			1
10	1			1

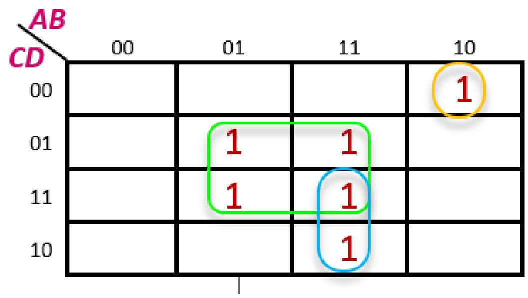
$$X = \overline{B}$$

9-

CD \ AB	00	01	11	10
00	1	1	1	1
01				
11				
10	1	1	1	1

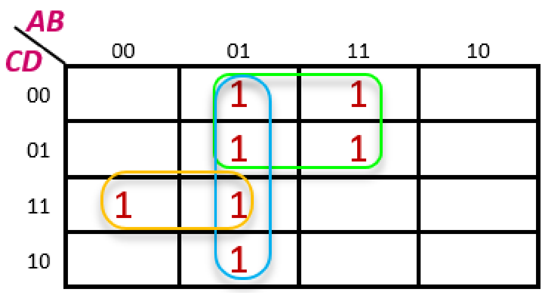
$$X = \overline{D}$$

10-



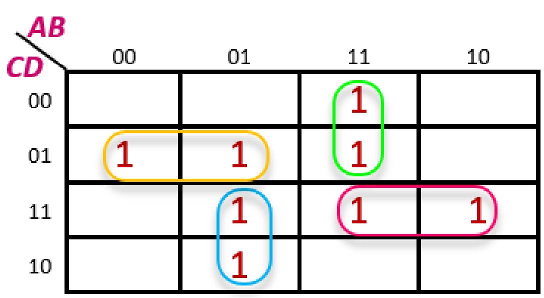
$X = A\bar{B}\bar{C}\bar{D} + BD + ABC$

11-



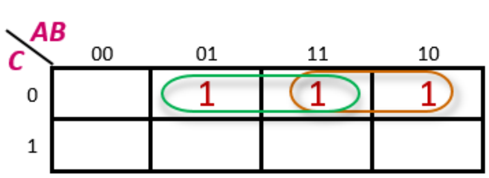
$X = B\bar{C} + \bar{A}B + \bar{A}CD$

12-



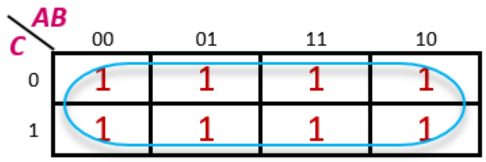
$X = \bar{A}\bar{C}D + AB\bar{C} + \bar{A}BC + ACD$

13-



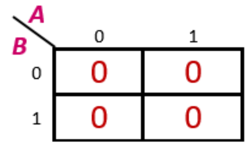
$X = B\bar{C} + A\bar{C}$

14-



$X = 1$

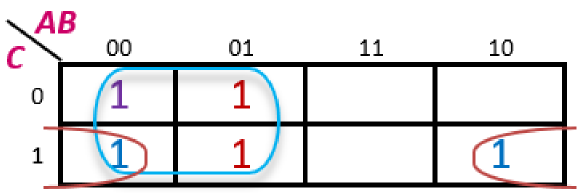
15-



$X = 0$

ex: using the k-map to simplify the expression

$$X = \bar{A}\bar{B}\bar{C} + \bar{B}C + \bar{A}B$$



$$X = \bar{A} + \bar{B}C$$

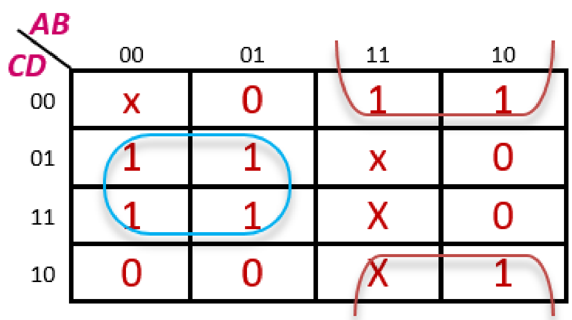
Note: enter 1 in the $\bar{A}\bar{B}\bar{C}$ cell and enter 1 in each cell that contain $\bar{B}C$ and enter 1 in each cell that contain $\bar{A}B$ then simplify.

Don't care condition:

Is a certain combinations of input levels where we don't care whether the output in high or low.
The don't care terms can be used to advantage on the k-map.

ex:

ABCD	Y
0000	X
0001	1
0010	0
0011	1
0100	0
0101	1
0110	0
0111	1
1000	1
1001	0
1010	1
1011	0
1100	1
1101	X
1110	X
1111	X



$$Y = \bar{A}D + A\bar{D}$$
$$Y = A \oplus D$$

ex:

ABC	Z
000	0
001	0
010	0
011	X
100	X
101	1
110	1
111	1

Five input k-map:

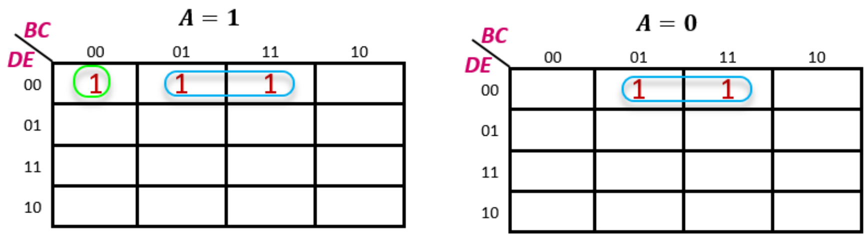
ex:

ABCDE
00000
00001
00010
00011
00100
00101
00110
00111
01000
01001
01010
01011
01100
01101
01110
01111
10000
10001
10010

.

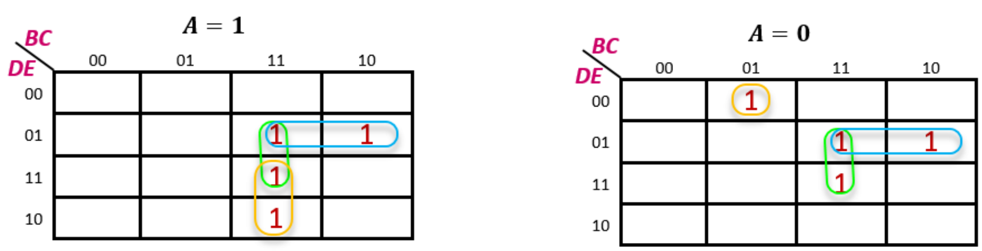
.

.



$$X = A\bar{B}\bar{C}\bar{D}\bar{E} + C\bar{D}\bar{E}$$

ex:



$$X = \bar{A}\bar{B}C\bar{D}\bar{E} + ABCD + B\bar{D}E + BCE$$

ex: simplify the following Boolean function in

1- Sum of product (SOP)
2- Product of sum (POS)

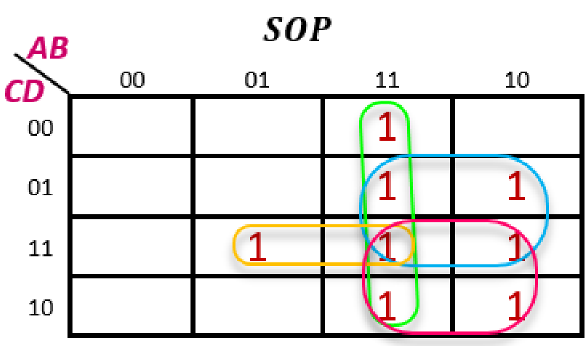
Used

1- NAND gates only
2- NOR getes only

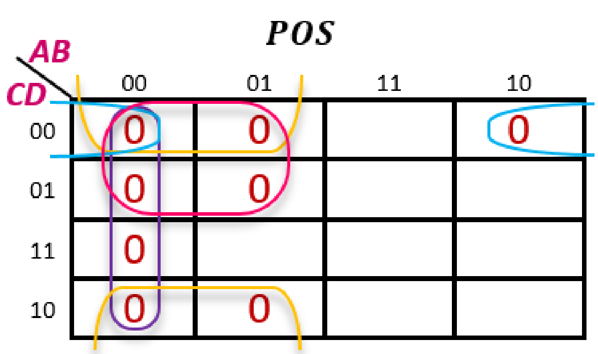
$$F = \bar{A}BCD + A\bar{B}\bar{C}D + A\bar{B}C\bar{D} + A\bar{B}CD + AB\bar{C}\bar{D} + AB\bar{C}D + ABC\bar{D} + ABCD$$

Sol:

1-



$$F = AC + AD + AB + BCD$$

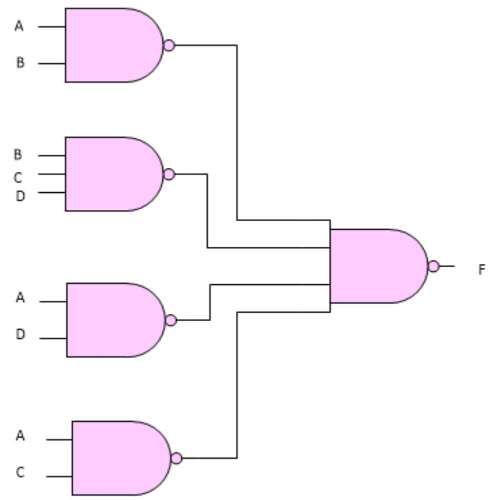


$$F = (A + B) \cdot (A + D) \cdot (A + C) \cdot (B + C + D)$$

1- NAND gate only

Used SOP expression, and complement twice

$$F = \overline{\overline{AC + AD + AB + BCD}}$$
$$F = \overline{\overline{AC} \cdot \overline{AD} \cdot \overline{AB} \cdot \overline{BCD}}$$



2- NOR gate only, used POS and complement twice

$$F = \overline{\overline{(A+B) \cdot (A+D) \cdot (A+C) \cdot (B+C+D)}}$$

$$F = \overline{(\overline{A+B}) + (\overline{A+D}) + (\overline{A+C}) + (\overline{B+C+D})}$$