

Microwave Engineering: Course Book

2017/2018

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Prerequisites

The prerequisites of this course include (but not limited to) the following:

- An understanding of electromagnetic theory and propagation principles (Maxwell's equations, plane waves, ..etc.).
- A general knowledge of transmission lines theory.
- An understanding of basic calculus (integration, matrix algebra, differential equations, ..etc.).
- An intermediate level in English so the student can understand the lectures and their handouts. Arabic glossary for some difficult words may be provided when they appear in the lecture notes for the first time.

Learning Objectives

This course aims to help the students understanding the basics of Microwave theory and techniques. It also intends to introduce the applications of Microwave Engineering in the modern communication and radar systems.

By the end of this course, the students should be able to understand basic Microwave electromagnetic structures, analyze Microwave networks, and design simple passive and active Microwave components.

Course Contents

Subject	Details	Duration (Hours)
Introduction to Microwave Engineering	Introducing the concept of Microwave Engineering and its difference from other fields of science. In addition, various applications of devices and systems realized by applying Microwave Engineering principles will be explored.	2
Review of Electromagnetic Theory	This review will focus on different theoretical and mathematical aspects of electromagnetic theory that are relevant to Microwave Engineering.	2
Microwave Rectangular and Circular Waveguides	Introducing waveguides and their usage in microwave systems, solving Helmholtz equation in rectangular and cylindrical coordinates, applying the solution of Helmholtz equations to waveguides with rectangular and circular cross section. Solving for TE and TM modes and their propagation parameters, description of power flow in waveguides.	6
Microwave and RF Transmission Lines	Introduction to transmission lines, equivalent circuit representation of transmission lines, transmission line equation, traveling voltage and current waves, lossless transmission line model, planar transmission lines (microstrip and striplines), terminated lossless transmission lines, special termination conditions.	6
The Smith Chart	The relation between reflection coefficient and load impedance, impedance and admittance transformation, parallel and series connections.	2
Impedance Matching	Matching with lumped elements, single stub tuning, double stub tuning, quarter wave transformer.	4
Microwave Network Analysis	Impedance and equivalent voltages and currents, impedance and admittance matrices, the scattering matrix, ABCD network representation, practical measurements of S-parameters.	6
Microwave Resonators	Series and parallel resonant circuits, transmission line resonators, rectangular and circular waveguide resonators.	4
Microwave and RF Passive Components	Power dividers, directional couplers, waveguide T-junctions, the quarter (90°) hybrid.	4
Microwave and RF Active Components	RF Diodes, BJTs, RF FETs, High Electron Mobility Transistors (HEMTs).	2
Modeling of Active Components	Large and small-signal BJT and FET models, measurement of active RF and microwave devices.	2

Biasing Active Components	Amplifier classes of operation, BJT biasing, FET biasing.	2
Microwave and RF Amplifier Design	Characteristics of amplifiers, stability considerations, two port power gain, single stage amplifiers, multistage amplifiers.	6

Marks Distribution

The marking of this course will be distributed as follows:

- **28%** Calculated from the average of different tests taken during the course.
- **2%** Attendance.
- **70%** Final Exam.

Homework assignments will be given throughout the course. Extra marks will be given to the **first five** students who **submit a correctly answered assignment** via email. The **first** student with correct assignment will receive **3 marks**, the **second** student will receive **2 marks**. **Third, fourth and fifth** students will receive **1 mark** each. No answered assignment will be considered after receiving the first five correct ones.

By the end of the course, the **average** homework score will be added to the total mark for each student.

Textbooks and References

1. Pozar, David M. *Microwave engineering*. John Wiley & Sons, 2009.
2. Collin, Robert E. *Foundations for microwave engineering*. John Wiley & Sons, 2007.
3. White, Joseph F. *High Frequency Techniques: An Introduction to RF and Microwave Design and Computer Simulation*. John Wiley & Sons, 2004.
4. Golio, Mike, ed. *The RF and microwave handbook*. CRC press, 2000.
5. Misra, Devendra. *Radio-frequency and microwave communication circuits: analysis and design*. John Wiley & Sons, 2004.
6. Liao, Samuel Y. *Microwave devices and circuits*. Englewood Cliffs: Prentice-Hall, 1985.
7. Baru, Grard. *Microwave engineering: land & space radiocommunications*. Vol. 9. John Wiley & Sons, 2008.

Microwave Engineering: Lecture #1

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Definition of Microwaves

In general, the term *Microwaves* refers to the Alternating Current (AC) signals with frequency range between 300 MHz and 300 GHz. This frequency range corresponds to a wavelength ($\lambda = \frac{c}{f} = \frac{3 \times 10^8}{f}$) between 1 m and 1 mm. Figure (1) illustrates the location of the microwave frequencies within the electromagnetic spectrum. Microwave frequencies are further divided into bands, each having a designated letter as illustrated in Table (1).

Reasons for Studying Microwave Engineering

The main reason for studying microwave theory and technique as a dedicated branch of Electrical engineering is the relationship between the wavelength of operation (λ) and the dimensions of the circuit elements.

Generally speaking, Maxwell's equations are the universal tool that can be used to analyze any circuit regardless of its dimensions in terms of its electromagnetic fields. However,

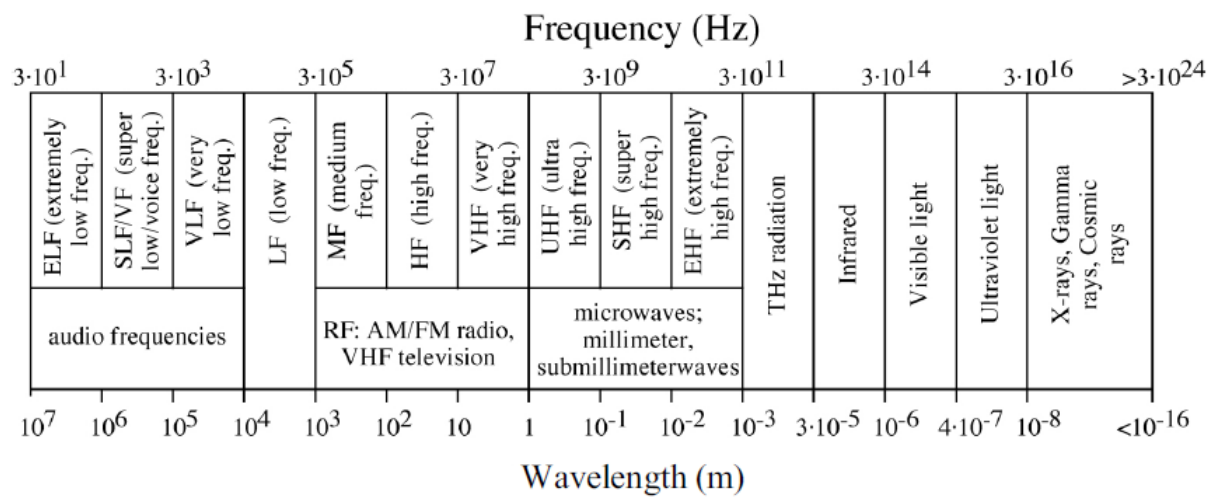


Figure 1: The location of the microwave frequency range within the electromagnetic spectrum

Table 1: Microwave frequency bands

Band Designator	Frequency (GHz)
UHF	0.3 to 1
L band	1 to 2
S band	2 to 4
C band	4 to 8
X band	8 to 12
Ku band	12 to 18
K band	18 to 27
Ka band	27 to 40
V band	40 to 75
W band	75 to 110
mm-Wave band	110 to 300

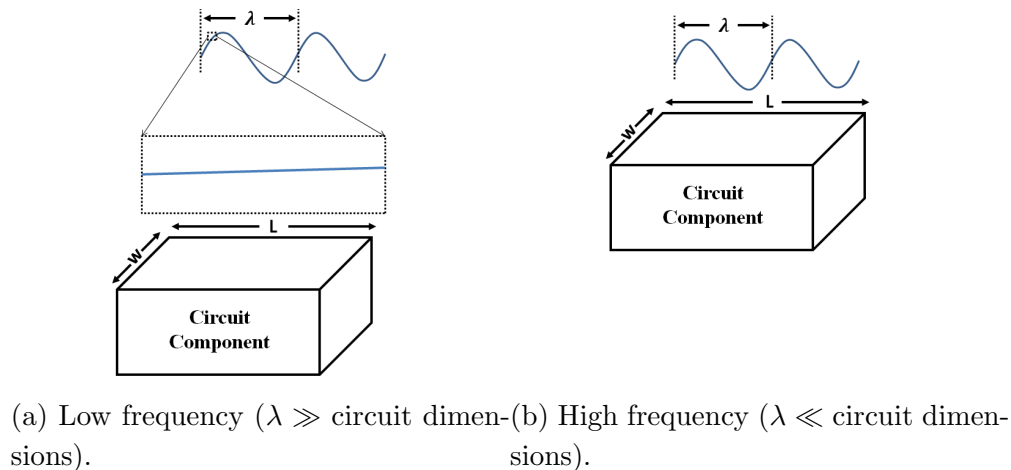


Figure 2: A comparison between the wavelength of a microwave signal and the dimensions of an arbitrary circuit component

solving Maxwell's equations is an extremely hard task, and it takes experts very long time to provide a solution for any electrical circuit based on the solution of Maxwell's equations. Hence, a number of approximations had to be introduced to simplify circuit analysis and provide a fast, easy to use methodologies to study and design electrical circuits.

In the case of circuits operating with wavelengths much larger than their own dimensions, the time taken for the electromagnetic field to propagate from one point of the circuit to another is a small fraction of its period. Hence, the simple *lumped circuit theory* is applied, which represents a simplification of Maxwell's equations.

In the opposite case, i.e., when the objects of the circuit elements are much larger than the wavelength, *optical laws*, which are another type of simplification of Maxwell's equations, are applied. In this approximation, electromagnetic fields are treated as optical traces and the circuit components are no longer treated as lumped elements.

The microwave range corresponds to those cases when the wavelengths are *of the same order* (roughly, from one-tenth to 10 times) as the circuit dimensions (as shown in Figure (2b)), so that neither one nor the other approximation is permissible: Maxwell's equations must be solved in their entirety.

For microwave frequencies, the finite propagation velocity of electromagnetic waves can no longer be neglected. For these frequencies, the time delay associated with signal propagation from one end of a component to the other is an appreciable fraction of the signal period, and thus lumped-element description are no longer adequate to describe electrical behavior. The time delay associated with finite wave propagation velocity that gives rise to the *distributed circuit effects* is a distinguishing feature of the mindset of Microwave Engineering.

In addition to the above, other physical effects that are negligible at lower frequencies be-

come increasingly important at high frequencies. Two of these effects are the *skin effect* and *radiation losses*. The skin effect is a function of frequency; the depth of penetration is given by $\delta_s = \frac{1}{\sqrt{\omega\mu\sigma}}$ where μ is the permeability, f is the frequency and σ is the conductivity of the material. As the expression indicates, δ_s decreases with increasing frequency, and so the electromagnetic fields are confined to the regions increasingly near the surface as the frequency increases. This results in the microwave currents flowing exclusively along the surface of the conductor, significantly increasing the effective resistance (and thus the loss) of metallic interconnections. *Radiation loss* also become increasingly important as the signal wavelengths approach the component and interconnection dimensions. For conductors and other components of comparable size to the signal wavelengths, standing waves caused by reflection of the electromagnetic waves from the boundaries of the components can greatly enhance the radiation of electromagnetic energy. These standing waves can be easily established either intentionally (in the case of antennae and resonant structures) or unintentionally (in the case of abrupt transitions, poor circuit layout, or other imperfections). Careful attention to transmission line geometry, placement relative to other components, transmission lines, and ground planes, as well as circuit packaging is essential for avoiding excessive signal attenuation and unintended coupling due to radiation effects.

A Brief History of Microwave Engineering

The foundations of modern electromagnetic theory were formulated in 1873 by the Scottish scientist James Clerk Maxwell. All of the practical applications of electromagnetic theory owe their existence to the theoretical work of Maxwell.

During the 1880s, German scientist Heinrich Hertz has conducted experiments that proved Maxwell's theories. For the first time, Hertz has used operation frequencies around 500 MHz, where the physical dimensions of the experimental apparatus were a significant fraction of the wavelength.

In 1894, Guglielmo Marconi began experiments in Italy sending a signal using Morse code ¹. Later, Marconi received Nobel prize after successfully sending a message across the Atlantic Ocean from England to USA in 1901.

In the beginning of the 1900s, most of the radio technology was in the high frequency (HF) and very high frequency (VHF) range due to the lack of components working at the microwave frequency range. However, in 1932, Robert Watson-Watt came up with the idea of Radio Direction Finding (RDF) technique, which was later called RAdio Detection And Ranging, or RADAR. During the 1940s, the development of radar for the use in the World War II gave Microwave theory and technology substantial interest.

The successful record of radar systems WWII urged many scientist in the United States to start researching in the field of microwave technology. This led to establishing the Radiation

¹Morse code: Is a system of sending messages that uses long and short sounds or dots and dashes to represent letters and numbers. The video [here](#) will show you the history of this system.

Laboratory (or RadLab) at Massachusetts Institute of Technology (MIT). By the end of the 1904s, scientist at Radlab were able to develop the radar technology by improving the efficiency of the microwave sources.

During that period, Percy Spencer, one of the scientists at RadLab, noticed that a chocolate bar in his pocket melted when he walked in front of a high-power microwave source. This led to the invention of the microwave oven in 1949.

During the 1950s and the 1960s, communications systems using the microwave technology began to be developed, benefiting from the work originally made for the radar systems. By the 1970s, Computer-Aided design industry began to help engineers designing and optimizing microwave passive and active circuits.

In 1975, Ray Pengelly proposed a new class of microwave circuits called the Microwave Monolithic Integrated Circuits, or MMIC. This technology combines various semiconductor and metallization layers on a single wafer of gallium arsenide or silicocon to provide a circuit in a very small area. This led to a huge reduction in the cost of microwave circuit fabrication, and marked the beginning of the modern satellite and mobile communications revolution.

Applications of Microwave Engineering

The use of microwave frequencies is spurred by a number of advantages. One of the major advantages is the reduced dimensions of the circuit components. Antennas, for example, can be miniaturized when using higher frequencies since the antenna size is inversely proportional to the frequency of operation. In addition, the speed of data transmission depends on the frequency band employed. Hence, the use of higher frequencies allows one to increase the channel capacity and thus to increase the data transmission rate.

Line-of-sight propagation is another advantage of microwave signals, which makes satellite and terrestrial communication links with very high capacities possible, with frequency reuse at minimally distant locations.

Microwave applications can be divided into five main sectors, as follows:

1. **Communications:** This includes satellite and mobile communications, Wireless Local Area Networks (WLAN), Local Multipoint Distribution Systems (LMDS) and Multipoint Multichannel Distributed System (MMDS).
2. **Radar:** Radar systems can be divided into two sectors: the first is dedicated for Military applications, such as surveillance, navigation and guidance of weapons. The second sector is used for civilian purposes. Such applications include air traffic control, ship traffic control, remote sensing and weather forecasting.
3. **Industrial:** Microwave technology can be used for process control, monitoring, sensing and food processing.

4. **Biomedical:** Microwave imaging is used for investigation and diagnosis of different types of diseases. Microwave hyperthermia is also used for cancer treatment.
5. **Scientific:** Microwaves are utilized in many scientific fields, such as radio astronomy for space exploration, and for remotely monitoring water resources.

Microwave Engineering: Lecture #2

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1 A Review of Some Fundamentals

We will start this course by reviewing some fundamental concepts that have to be fully understood before studying microwave electromagnetic analysis. These concepts include the representation of waves with time and distance, and the representation of fields as vectors.

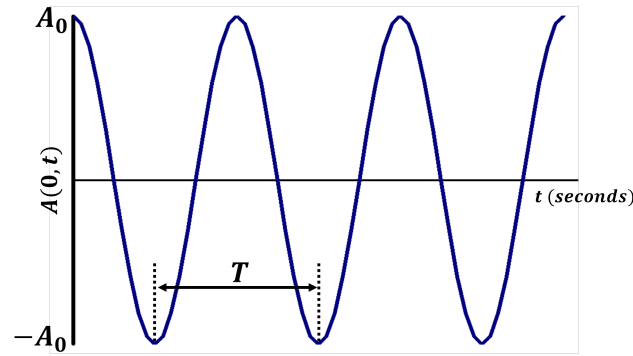
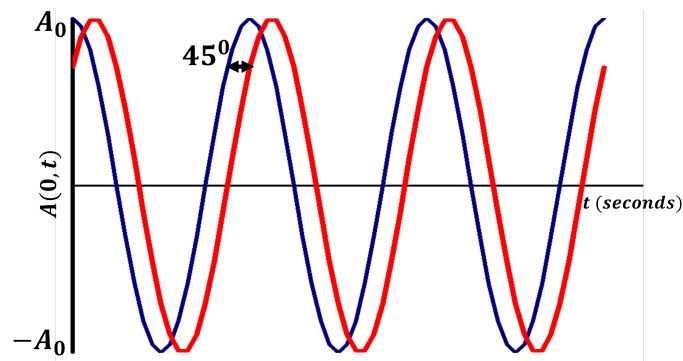
1.1 Representation of Waves

Let's consider a sinusoidal wave traveling in the z-direction. A general equation for such a wave is:

$$A(z, t) = A_0 e^{-\alpha z} \cos(\omega t - \beta z + \phi) \quad (1)$$

Here we let $A(z, t)$ represent the value of the wave at some point (z) at a specific time (t). The amplitude, $A_0 e^{-\alpha z}$, is made up of the initial amplitude at $z = 0$ (A_0), and an exponential term to account for attenuation of the wave as it propagates.

The phase inside the cosine argument consists of three parts: ωt , where ω is the angular frequency ($\omega = 2\pi f$), with units of radians per second (rad/s); βz , where β is the phase constant in radians per meter (rad/m); and a phase shift (ϕ).

Figure 1: A plot of $A(0, t)$ versus time at $z = 0$ Figure 2: A plot of $A(0, t)$ versus time at $z = 0$ with $\phi = -45^\circ$ compared to the same function without phase shift.

Let's assume that the phase shift (ϕ) is zero, we can look at the wave versus time when $z = 0$. We have the equation:

$$A(0, t) = A_0 e^{j0} \cos(\omega t - \beta \times 0 + 0) = A_0 \cos(\omega t) \quad (2)$$

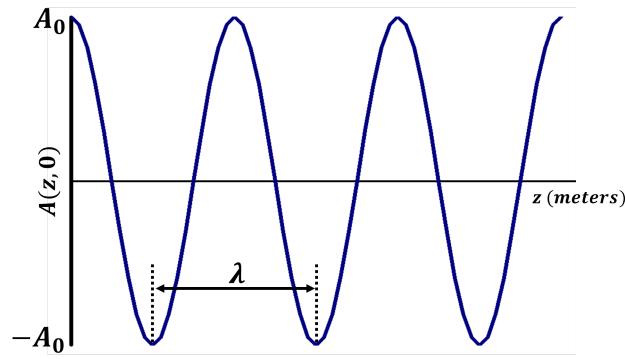
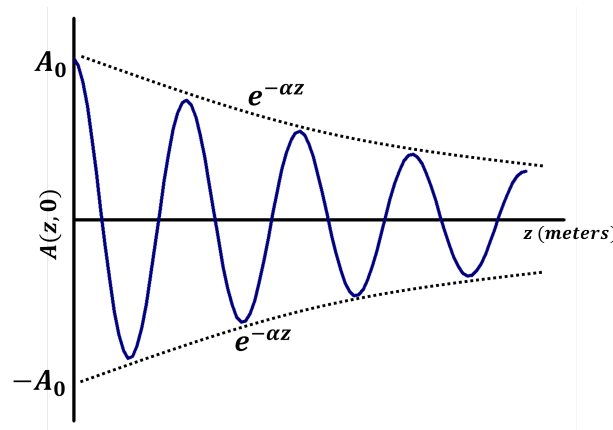
This wave is plotted in Figure (1). A characteristic of a sine or cosine wave is that it repeats every 2π radians (or 360°). So, we have $\cos(\omega t) = 1$ for $\omega t = n \times 2\pi$, where $n = 0, 1, 2, \dots$. The period T is the time elapsed for one cycle, or $\omega T = 1 \times 2\pi$. Hence, we can see that:

$$T = \frac{2\pi}{\omega} = \frac{1}{f} \quad (3)$$

Now, assuming a phase shift of $\phi = -45^\circ$, the new trace will **lead** the original one by 45° , as shown in Figure (2). Now, let's look at the wave versus **position** (z) when the time (t) is zero (which is called the *pattern* of the wave). Assuming a lossless medium ($\alpha = 0$), we have:

$$A(z, 0) = A_0 e^{j0} \cos(\omega \times 0 - \beta z + 0) = A_0 \cos(-\beta z) \quad (4)$$

This wave is plotted in Figure (3). We again have the 2π radians per cycle, or $\cos(-\beta z) =$

Figure 3: A plot of $A(z, 0)$ versus distance (z) at $t = 0$ Figure 4: A plot of $A(z, 0)$ versus distance (z) at $t = 0$

$\cos(\beta z) = 1$ when $\beta z = n \times 2\pi$. One cycle is a **wavelength** long, or $\beta\lambda = 1 \times 2\pi$. This is rearranged to give the relation between wavelength and phase constant¹

$$\beta = \frac{2\pi}{\lambda} \quad (5)$$

Now, if we insert the attenuation to the expression in equation (4), we will get:

$$A(z, 0) = A_0 e^{-\alpha z} \cos(-\beta z) \quad (6)$$

As plotted in Figure (4), the amplitude is decreasing with increasing z . The rate of decrease is given by the *attenuation constant* (α), in nepers per meter (Np/m), where the neper is a dimensionless unit.

Now, let's consider a traveling wave, i.e., varying in both time and distance. Considering a lossless medium ($\alpha = 0$), Figure (5) shows the wave amplitude versus position at progressive values of time using:

$$A(z, t) = A_0 e^{0 \times z} \cos(\omega t - \beta z) \quad (7)$$

¹The reciprocal of the wavelength $1/\lambda$ is the spatial frequency, and it is sometimes called the *wavenumber*.

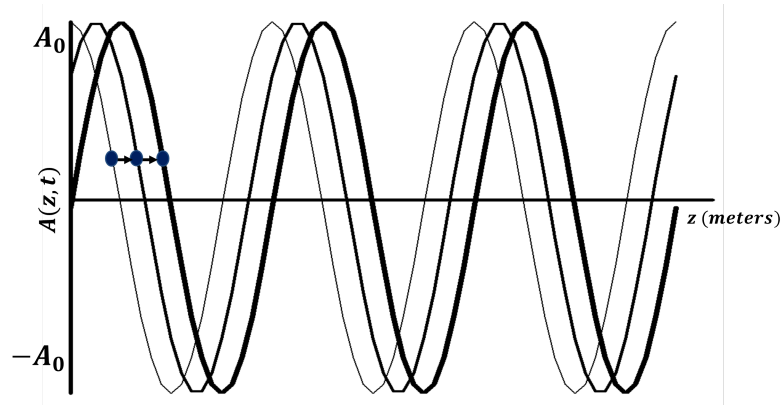


Figure 5: A plot of $A(z, t)$ versus distance (z) at progressive times showing wave travel. The dots indicate arbitrary phase points.

For each trace, a dot has been added representing a constant phase point on the wave. We see that as time increases, this phase point is moving in the $+z$ -direction, so we call this **$+z$ traveling wave**. The phase point moves like a surfer riding a wave. To find the *velocity* of this phase point, let's consider the phase:

$$\omega t - \beta z = P \quad (8)$$

where P is an arbitrary constant representing a constant phase point such as the dots in Figure (5). If we take the time derivative of both side of the above expression, we have:

$$\omega - \beta \frac{\partial z}{\partial t} = 0 \quad (9)$$

which can be rearranged as:

$$\nu_p = \frac{\partial z}{\partial t} = \frac{\omega}{\beta} = \lambda f \quad (10)$$

The **phase velocity** (or, the *propagation velocity*) of the wave is (ν_p). This is determined by the permeability and permittivity of the medium, as will be shown later.

1.2 Vectors in the Cartesian Coordinate System

A scalar field represents the mapping of a scalar quantity as it varies with position. In electromagnetics, we usually deal with *vector* quantities, which have both a magnitude and a direction. For instance, at a point in space relative to earth, there is a gravity vector that decreases in magnitude as you move further out into space, but it always points toward the center of the earth. A *vector field* can represent a mapping of these vectors in space.

Determining the precise location of a point in a three-dimensional space can be accomplish using the Cartesian coordinate system. In this course, we will mostly work with the Cartesian system. However, we might review other systems (such as cylindrical and spherical) when they are needed. This system is a right-handed system of orthogonal coordinates axes: x, y

and z . The term *orthogonal* means that each axis is mutually perpendicular to the other two. These coordinates are right-handed because if the fingers of the right hand point in the x -direction and are curled toward the y -direction, the thumb will point in the z -direction. In general, a vector in the Cartesian coordinate system is written as:

$$\bar{A} = A_x \mathbf{a}_x + A_y \mathbf{a}_y + A_z \mathbf{a}_z \quad (11)$$

Even though in most books, a vector is annotated in boldface letter (such as \mathbf{A}). In this course we will write it as a letter with a bar (as in \bar{A}).

The magnitude of this vector is found by extending the Pythagorean theorem to three dimensions:

$$A = |\bar{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2} \quad (12)$$

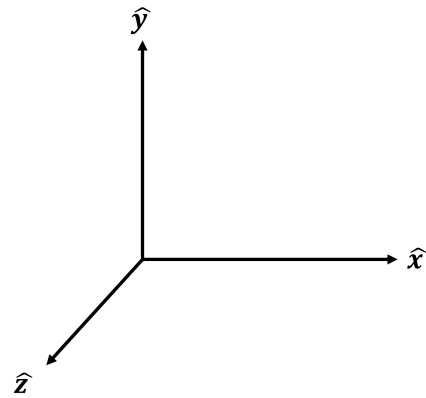


Figure 6: Cartesian coordinate system.

There are many operations that can be performed on a vector. The most important of these operations are the follows:

- **Divergence:** The divergence is a dot product between the derivative operator (∇) and a vector, and it produces a scalar.

$$\nabla \cdot \bar{A} = \frac{\partial}{\partial x} A_x + \frac{\partial}{\partial y} A_y + \frac{\partial}{\partial z} A_z \quad (13)$$

- **Gradient:** The Gradient is an operation on a scalar that produces a vector.

$$\nabla V = \frac{\partial V}{\partial x} \mathbf{a}_x + \frac{\partial V}{\partial y} \mathbf{a}_y + \frac{\partial V}{\partial z} \mathbf{a}_z \quad (14)$$

- **Curl:** It is a cross product between the differential operator (∇) and a vector. This operation produces a vector as well.

$$\nabla \times \bar{A} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} \quad (15)$$

- **Laplacian:** This operation is applied to a scalar and produces another scalar:

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} \quad (16)$$

Laplacian can also be applied to a vector to produce another vector:

$$\nabla^2 \bar{A} = \frac{\partial^2 A_x}{\partial x^2} \mathbf{a}_x + \frac{\partial^2 A_y}{\partial y^2} \mathbf{a}_y + \frac{\partial^2 A_z}{\partial z^2} \mathbf{a}_z \quad (17)$$

2 Static Electric and Magnetic Fields

Microwaves are characterized by electric and magnetic fields which are, generally, perpendicular to each other. We will first review the concepts of **static** electric and magnetic fields. These fields are **constant with time**, which mean they *don't change in intensity or direction over time*.

2.1 Static Electric Field

Many ancient civilizations have recorded many observations on static electricity. around 600 BCE, the Greek philosopher Thales noted that amber (كهرمان) after being rubbed on silk (حرير), could pick up pieces of straw (القش). This was one of the earliest documented accounts of static electricity². In the late 18th century, the French engineer Charles Augustus Coulomb found that the force between charges is proportional to the product of the two charges, inversely proportional to the square of the distance between the charges, and acts in a line containing them. The proportionality constant was found to be $\frac{1}{4\pi\epsilon_0}$, where ϵ_0 is the free space permittivity $= 8.85 \times 10^{-12} F/m$. This *Coulomb* force is expressed by:

$$\vec{F} = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{12}^2} \vec{a}_{12} \quad (18)$$

If we consider the charge Q_2 to be a test charge, **the electric field intensity**, \vec{E} , is defined as the coulombs force due to a fixed charge (Q_1) on a unit test charge (Q_2), or

$$\vec{E} = \frac{\vec{F}}{Q_2} \quad (19)$$

Hence, electric field intensity due to a certain charge in the free space is expressed by:

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 R^2} \vec{a}_R \quad (20)$$

where Q is the value of the charge (in Coulombs), and R is the distance between the charge and the testing point (in meters). Here, \vec{a}_R is a **unit vector** directed *from the charge to the testing point*, which is the direction of the electric field intensity vector \vec{E} . The units of the electric field intensity are Volts per meters, or V/m .

Now, let's consider the two metallic spheres shown in Figure (7). The inner sphere is charged with a positive charge, and then it was placed inside a *neutral* (not charged) outer sphere with about 2 cm of dielectric material between them. A negative charge will be induced on the outer sphere with magnitude equal to the charge on the inside sphere *regardless* of the

²The term (Electric) originates from the Latin word (electricus), which means (like amber). Similarly, the Arabic word (كهرباء) comes from the same origin (كهرمان).

dielectric material that separates the two spheres. This relationship between the two spheres that is independent on the medium is called the **electric flux**, or Ψ , and it is measured in Coulombs.

Let's look again at Figure (7), at the surface of the inner sphere, Ψ coulombs of electric flux are produced by the charge Q coulombs distributed uniformly over a surface having an area of $4\pi a^2$. The *density* of the flux at this surface is $\Psi/4\pi a^2$. This is an important quantity called the **Electric flux density**, or \bar{D} , and it is measured in coulombs per square meters.

Now, if we have a single charge (Q) in the free space, the flux density at any point R meters away from the charge will be:

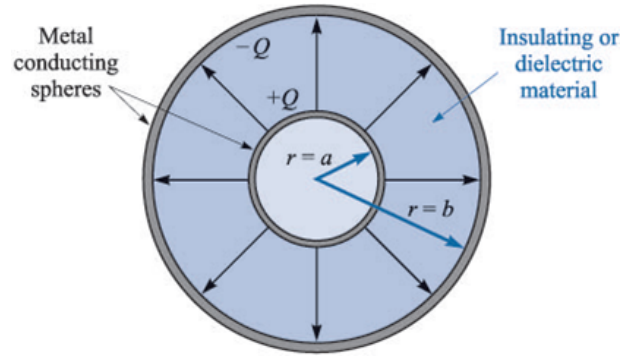


Figure 7: Electric flux in the region between two charged spheres.

$$\bar{D} = \frac{Q}{4\pi R^2} \bar{a}_R \quad (21)$$

By comparing this expression to that in equation (20), we can conclude that:

$$\bar{D} = \epsilon_0 \bar{E} \quad (22)$$

This relation between \bar{D} and \bar{E} is only valid in free space. For general media, as we will see later, the relation becomes $\bar{D} = \epsilon \bar{E}$, where ϵ is the material's permittivity.

The advantage of using electric flux density (\bar{D}) rather than electric field intensity (\bar{E}) is that the former relates to the number of flux lines emanating from one set of charge and terminating on the other, **independent of the media**.

We can conclude from the experiments with concentric spheres shown in Figure (7) that the electric flux passing through any imaginary sphere between the two metallic spheres is equal to the charge enclosed by the surface of that imaginary sphere. This concept can be generalized by saying that *the electric flux passing through any closed surface is equal to the total charge enclosed by that surface*. This is known as **Gauss's law**. In mathematical form, this law is written as:

$$\oint \bar{D} \cdot d\bar{S} = Q_{enc.} \quad (23)$$

where the circle on the integral indicates the integration is performed over a closed surface. Equation (23) is called the *integral form* of Gauss's law. There is another form of this law, which is the *differential form*, which uses the Divergence operation as follows:

$$\nabla \cdot \bar{D} = \rho_v \quad (24)$$

Here, ρ_v is the *volume charge density* in Coulomb per cubic meters, or C/m^3 . Equation (24) is the first of Maxwell's four equations.

2.2 Static Magnetic Field:

More than 4500 years ago, the Chinese discovered that certain types of iron could attract each other and certain metals. Later, Greeks found similar materials near a city called Magnesia, in what is present-day Turkey. Magnetism and electricity were considered distinct phenomena until 1820, when the Danish physicist Hans Christian Orsted showed that a current-carrying wire can produce magnetic field.

Static magnetic field can be produced by permanent magnets, steady electric currents and time-varying electric currents. The field from a permanent magnet arises from quantum mechanical electron spinning, which is outside the scope of this course. It is more important to focus on the steady electric current as the origin of static magnetic fields. We will deal with the time-varying current as a source of magnetic field in the next lecture.

After Oersted's discovery that currents produce magnetic fields, Jean Baptiste Biot (1774-1862) and Felix Savart (1791-1841) came up with a mathematical relation between the magnetic field intensity and the current:

$$\vec{H} = \oint \frac{Id\vec{L} \times \vec{a}_R}{4\pi R^2} \quad (25)$$

where the integration \oint is for all the current elements (IdL), R is the distance between the current element and the point of measurement, I is the current, $d\vec{L}$ is the length of the current element. The cross product in equation (25) above indicates that the direction of the magnetic field intensity (\vec{H}) is normal to both the direction of the current (\vec{L}) and the unit vector directed to the wire (\vec{a}_R). The units of the magnetic field intensity are Ampere per meter, or A/m .

For static electric fields, we found that Gauss's law can be used to find the electric flux density around an electric field source. A similar concept exists for the static magnetic field, which is known as *Ampere's circuital law*. This law states that the line integral of \vec{H} about any closed path is exactly equal to the direct current enclosed by that path, or:

$$\oint \vec{H} \cdot d\vec{L} = I \quad (26)$$

Figure (8) shows a circular wire carrying a direct current I , three line integrals of \vec{H} about the closed paths lettered a and b result in an answer of I . However, the integral about the closed path c gives an answer less than I since it passes through the conductor.

Ampere's circuital law has another form, which is called the *point form* or the *curl form*:

$$\nabla \times \vec{H} = \vec{J} \quad (27)$$

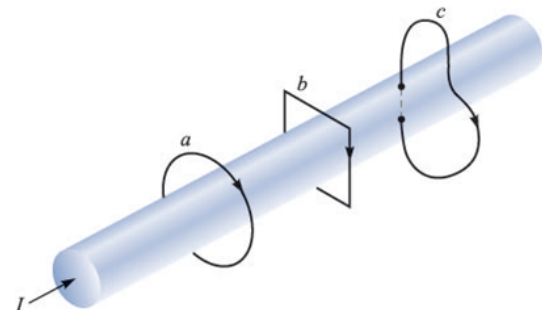


Figure 8: A circular wire carrying a direct current with three line integrals.

where \bar{J} is the current density (current per unit area) and it is measured by Amperes per square meters (A/m^2). Unlike Gauss's law, which uses the divergence operation and produces a scalar (ρ_v), Ampere's law uses the curl operation and produces a vector \bar{J} . Equation (27) is another one of Maxwell's equations when applied to static fields.

When applying a similar concept to the static electric field, the line integral of \bar{E} about any closed path is always *equal to zero*:

$$\oint \bar{E} \cdot d\bar{L} = 0 \quad (28)$$

This equation is a generalized form of Kirchhoff's voltage law, and it can also be written as:

$$\nabla \times \bar{E} = 0 \quad (29)$$

This is another one of Maxwell's equations for static electric field.

As it was sometimes more convenient to deal with electric flux (\bar{D}) lines than with static electric fields, it is also possible to work with *magnetic flux density*, or \bar{B} when dealing with static magnetic fields. The magnetic flux density is related to the magnetic field intensity in free space by:

$$\bar{B} = \mu_0 \bar{H} \quad (30)$$

where μ_0 is the *free space permeability*, given in units of henrys per meter (H/m), and equal to $4\pi \times 10^{-7} H/m$.

The units of \bar{B} are webers per square meter, or Wb/m^2 .

A fundamental feature of magnetic fields that distinguishes them from electric fields is that the field lines form closed loops. Figure (9) shows the magnetic field lines in a bar magnet, where the lines are shown extending through the bar forming closed loops. This is quite different from electric field lines, which start on positive charge and terminate on negative charge.

From this characteristic of magnetic fields, it is easy to see that the net magnetic flux passing through a Gaussian surface (a closed surface as shown in Figure (9)) must be zero. What goes into the surface must come back out. Thus we have *Gauss's law for static magnetic field*:

$$\oint \bar{B} \cdot d\bar{S} = 0 \quad (31)$$

Another form of this law is written in terms of a divergence operation as follows:

$$\nabla \cdot \bar{B} = 0 \quad (32)$$

which is another one of Maxwell's equations.

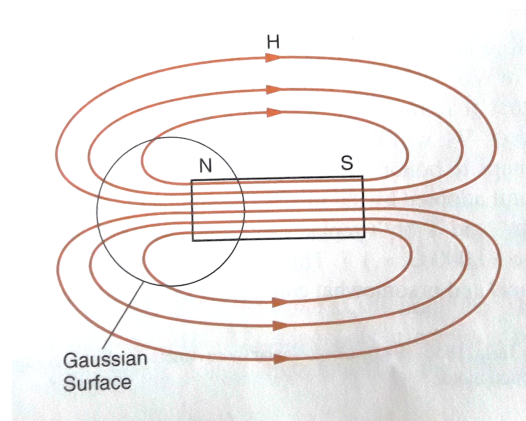


Figure 9: Magnetic field lines form closed loops, so the net flux through a Gaussian surface is zero.

3 Maxwell's Equations for Static Fields

Now we can present all the four of Maxwell's equations for static fields in *differential* (or *point*) form:

$$\nabla \cdot \bar{D} = \rho_v, \quad (33)$$

$$\nabla \times \bar{E} = 0, \quad (34)$$

$$\nabla \times \bar{H} = \bar{J}, \quad (35)$$

$$\nabla \cdot \bar{B} = 0 \quad (36)$$

In the next lecture, we will explore the same set of equations for *dynamic* fields, i.e., when the fields are changing with time. The similarities and differences between the two sets of equations will also be discussed.

End of Lecture #2

Microwave Engineering: Lecture #3

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1 Dynamic Electric and Magnetic Fields

We have so far assumed no time variation in our electric and magnetic fields. This was done to build a sense of comfort in working with vectors and coordinate systems to get a first understanding of Maxwell's equations. Now we turn our attention to the dynamic case, where electric and magnetic fields do change with time.

1.1 Faraday's Law

Following Oersted's discovery, Michael Faraday thought that if a current in a wire can produce a magnetic field, then perhaps a magnetic field can produce a current in a wire. However, he observed later that current can *only* be induced in a circuit if the magnetic flux linking the circuit *changed with time*.

Figure (1a) shows a conductive loop in a plane normal to a magnetic field that increases with time. Since the magnetic flux through the area bounded by the loop is changing, a current I_{ind} is induced in the loop as indicated by the ammeter. Note the direction of the induced

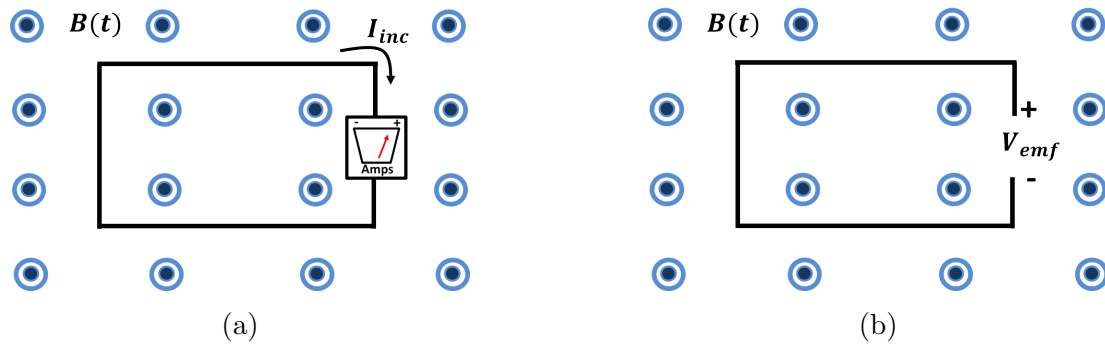


Figure 1: An increasing magnetic field out of the page induces a current in (a) or an emf in (b).

current. The flux produced by the induced current acts to oppose the *change* in flux. This statement is called *Lenz's law*.

By taking the ammeter out of the loop in Figure (1a), the current induced in the loop establishes a potential difference across the terminal known as the *electromotive force*. This electromotive force, V_{emf} (or just *emf*), is related to the change of flux linking a circuit by *Faraday's law*:

$$V_{emf} = -\frac{d\phi}{dt} \quad (1)$$

The negative sign in the equation is a consequence of Lenz's law. If we consider a single loop, Faraday's law can be written as:

$$V_{emf} = -\frac{d\phi}{dt} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{S} \quad (2)$$

Generating emf requires a time-varying magnetic flux linking the circuit. This occurs if the magnetic field changes with time (called *transformer emf*, or if the surface containing the flux changes with time (called *motional emf*).

The emf is measured around the closed path enclosing the area through which the flux is passing, and it can also be written as:

$$V_{emf} = \oint \vec{E} \cdot d\vec{L} \quad (3)$$

This is a big contrast with the static field case, where $\oint \vec{E} \cdot d\vec{L} = 0$. Using equation (3), we can rewrite Faraday's law as:

$$V_{emf} = \oint \vec{E} \cdot d\vec{L} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{S} \quad (4)$$

In this equation, the direction of the circulation integral is related to the differential surface vector direction via right hand rule. For instance, in Figure (1b) we choose $d\vec{S}$ pointing into

the page, and so the path for the circulation integral is in the same direction as that assumed for the induced current, then a V_{emf} with the shown polarity will result.

If the field is varying with time and the surface stays constant, we can pull the time derivative in equation (4) inside the integral in the right side as follows:

$$V_{emf} = - \int \frac{\partial \bar{B}}{\partial t} \cdot d\bar{S} \quad (5)$$

Equation (4) can be written in another form as:

$$\nabla \times \bar{E} = - \frac{\partial \bar{B}}{\partial t} \quad (6)$$

this is called the point form of Faraday's law, and it is one of Maxwell's equations for dynamic fields.

1.2 Displacement Current

Recall Ampere's law from the last lecture:

$$\nabla \times \bar{H} = \bar{J}_c \quad (7)$$

Here, the c subscript in \bar{J}_c has been added to the current density term to identify it as the conduction current density, which is related to the electric field by Ohm's law:

$$\bar{J}_c = \sigma \bar{E} \quad (8)$$

The current is a result of the drift of charge carriers in response to electric field. In a vacuum where $\sigma = 0$, $\bar{J}_c = 0$.

For dynamic fields, Maxwell's has found out that an additional term should be added to Ampere's circuital law in (7) as:

$$\nabla \times \bar{H} = \bar{J}_c + \bar{J}_d \quad (9)$$

The additional term, called *displacement current density*, is the rate of change of the electric flux density:

$$\bar{J}_d = \frac{\partial \bar{D}}{\partial t} \quad (10)$$

Thus, ampere's law for dynamic fields can be written as:

$$\nabla \times \bar{H} = \bar{J}_c + \frac{\partial \bar{D}}{\partial t} \quad (11)$$

Although the displacement current density doesn't represent current in the conventional sense of charge flow, it does allow a time-varying electric field to be an additional source of magnetic field.

2 Maxwell's Equations

After his inclusion of displacement current in Ampere's circuital law, Maxwell was able to unify all of the theories of electricity and magnetism into one concise set of formulas known as *Maxwell's equations*. Table (1) illustrates all these equations alongside the constitutive relations for material media.

Table 1: Maxwell's equations in point form

Maxwell's Equation	Expression (point form)
Gauss's Law	$\nabla \cdot \bar{D} = \rho_v$
Gauss's Law for Magnetic Field	$\nabla \cdot \bar{B} = 0$
Faraday's Law	$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$
Ampere's Circuital Law	$\nabla \times \bar{H} = \bar{J}_c + \frac{\partial \bar{D}}{\partial t}$
Constitutive relations	$\bar{D} = \varepsilon \bar{E}$
	$\bar{B} = \mu \bar{H}$
	$\bar{J}_c = \sigma \bar{E}$

where $\varepsilon = \epsilon_0 \varepsilon_r$, and ε_r is the relative permittivity of the medium. Similarly, $\mu = \mu_0 \mu_r$, where μ_r is the relative permeability of the medium¹.

2.1 Maxwell's Equations in Phasor Form

A general time-harmonic field is a function of position (x, y, z) and time (t) and can be written in *instantaneous form* as follows:

$$\bar{\mathcal{E}}(x, y, z, t) = \bar{E}(x, y, z) \cos(\omega t + \phi) \quad (12)$$

Now, using Euler's identity: $e^{j\theta} = \cos\theta + j\sin\theta$, we can also write this as:

$$\bar{\mathcal{E}}(x, y, z, t) = \text{Re} \left[\bar{E}(x, y, z) e^{j(\omega t + \phi)} \right] \quad (13)$$

This can be reorganized and written as:

$$\bar{\mathcal{E}}(x, y, z, t) = \text{Re} \left[\bar{E}(x, y, z) e^{j\omega t} e^{j\phi} \right] \quad (14)$$

where the *phasor form* of the field is:

$$\bar{E} = \bar{E}(x, y, z) e^{j\phi} \quad (15)$$

¹Both ε_r and μ_r are unitless quantities.

where the time dependence term $e^{j\omega t}$ is omitted for brevity.

For the rest of this course, we will write **all the fields in phasor form**, assuming them to have $e^{j\omega t}$ time dependency. However, we might need to include the time dependency in some cases and convert the fields back to the **instantaneous form** using the aforementioned relations.

Knowing that $\frac{\partial}{\partial t}e^{j\omega t}$ is $j\omega$, we can write Maxwell's equations in phasor form, as illustrated in Table (2).

Table 2: Maxwell's equations in phasor form

Maxwell's Equation	Expression (point form)
Gauss's Law	$\nabla \cdot \bar{D} = \rho_v$
Gauss's Law for Magnetic Field	$\nabla \cdot \bar{B} = 0$
Faraday's Law	$\nabla \times \bar{E} = -j\omega \bar{B}$
Ampere's Circuital Law	$\nabla \times \bar{H} = \bar{J}_c + j\omega \bar{D}$

3 Plane Waves

In free space, the constitutive parameters are $\sigma = 0$, $\mu_r = 1$ and $\varepsilon_r = 1$, so Ampere's circuital law and Faraday's law equations become:

$$\nabla \times \bar{H} = j\omega\epsilon_0\bar{E} \quad (16)$$

$$\nabla \times \bar{E} = -j\omega\mu_0\bar{H} \quad (17)$$

If we consider that at some point in such space, there is a source of time-varying electric field, then by Ampere's circuital law we know that a magnetic field is induced in the surrounding region. As this magnetic field is also changing with time, by Faraday's law it in turn induces an electric field. Energy is passed back and forth between \bar{E} and \bar{H} fields as they radiate away from the source point at the speed of light. These fields constitute waves of electromagnetic energy radiating away from the source point.

The waves radiate spherically, but at a remote distance away from the source they resemble *uniform plane wave*, as depicted in Figure (2). In a uniform plane wave, the \bar{E} and \bar{H} are orthogonal (or transverse), to the direction of propagation.

The magnitude and phase of the field vectors in a uniform plane wave is equal at every point on the *wavefront*, a plane transverse to the direction of propagation.

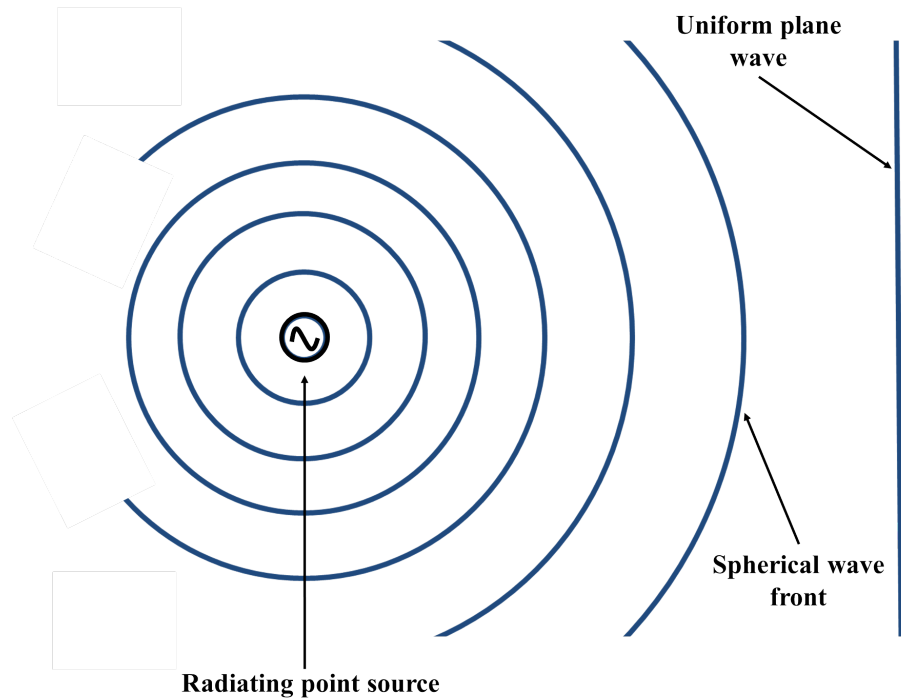


Figure 2: At a remote distance away from a point source, the waves appear to be planar.

3.1 General Wave Equation

Here, we will use Maxwell's equations to derive formulas governing electromagnetic wave propagation. We'll consider that the medium is free of any charge, or:

$$\nabla \cdot \bar{D} = 0 \quad (18)$$

Then, Maxwell's equations can be written as:

$$\nabla \times \bar{H} = \sigma \bar{E} + j\omega\epsilon \bar{E} \quad (19)$$

$$\nabla \times \bar{E} = -j\omega\mu \bar{H} \quad (20)$$

$$\nabla \cdot \bar{E} = 0 \quad (21)$$

$$\nabla \cdot \bar{H} = 0 \quad (22)$$

To begin, let's take the curl of both sides of Faraday's law in equation (47):

$$\nabla \times (\nabla \times \bar{E}) = \nabla \times (-j\omega\mu \bar{H}) = -j\omega\mu (\nabla \times \bar{H}) \quad (23)$$

Exchanging Ampere's circuital law equivalence for the curl of \bar{H} in equation (19) gives us:

$$\nabla \times (\nabla \times \bar{E}) = -j\omega\mu(\sigma\bar{E} + j\omega\varepsilon\bar{E}) = -j\omega\mu\sigma\bar{E} + \omega^2\mu\varepsilon\bar{E} \quad (24)$$

We can now manipulate the left side of the equation by invoking a vector identity, relating the curl of a curl of any vector field (\bar{A}) to a divergence and a Laplacian of the field as:

$$\nabla \times \nabla \times \bar{A} = \nabla \cdot \bar{A} - \nabla^2 \bar{A} \quad (25)$$

So applying the above identity to equation (24), we now have:

$$\nabla \cdot \bar{E} - \nabla^2 \bar{E} = -j\omega\mu\sigma\bar{E} + \omega^2\mu\varepsilon\bar{E} \quad (26)$$

Because our medium is charge-free, the divergence of \bar{E} is zero ($\nabla \cdot \bar{E} = 0$), and equation (26) becomes:

$$\nabla^2 \bar{E} = j\omega\mu\sigma\bar{E} - \omega^2\mu\varepsilon\bar{E} \quad (27)$$

This is the *Helmholtz wave equation* for \bar{E} . A similar expression can be found for \bar{H} as follows:

$$\nabla^2 \bar{H} = j\omega\mu\sigma\bar{H} - \omega^2\mu\varepsilon\bar{H} \quad (28)$$

Helmholtz equation is usually written in the form:

$$\nabla^2 \bar{E} - \gamma^2 \bar{E} = 0 \quad (29)$$

where γ is the *propagation constant*, defined as:

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)} \quad (30)$$

Since the square root of a complex number is itself complex, γ is equal to a real part (the *attenuation*, α , in nepers per meter) and an imaginary part (the *phase constant*, or β , in radians per meter). So we have:

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)} = \alpha + j\beta \quad (31)$$

For magnetic field we have:

$$\nabla^2 \bar{H} - \gamma^2 \bar{H} = 0 \quad (32)$$

To make use of Helmholtz equations, let's consider an x -polarized plane wave traveling in the z -direction. Our electric field can be expressed by:

$$\bar{E} = E_x \bar{a}_x \quad (33)$$

Since the uniform plane wave do not change in the transverse direction (i.e., the direction transverse to the direction of propagation, which is in this case the $x-y$ plane), the electric

field in equation (33) is a function of z only. The Laplacian of \bar{E} becomes a straightforward second derivative with respect to z , and Helmholtz equation for \bar{E} becomes:

$$\frac{d^2 E_x}{dz^2} - \gamma^2 E_x = 0 \quad (34)$$

This is a second-order, linear, homogeneous differential equation. A possible solution for this equation is:

$$E_x = Ae^{\lambda z} \quad (35)$$

where A and λ are arbitrary constants. It is then easy to show that:

$$\frac{d^2 E_x}{dz^2} = \lambda^2 Ae^{\lambda z} \quad (36)$$

and equation (35) becomes:

$$\lambda^2 Ae^{\lambda z} - \gamma^2 Ae^{\lambda z} = 0 \quad (37)$$

or

$$\lambda^2 - \gamma^2 = 0 \quad (38)$$

This is easily factored as:

$$(\lambda + \gamma)(\lambda - \gamma) = 0 \quad (39)$$

The first solution of this equation is $\lambda = -\gamma$, which gives:

$$E_x = Ae^{-\gamma z} \quad (40)$$

To examine this solution in its instantaneous form we substitute $\alpha + j\beta$ for γ , multiply by $e^{j\omega t}$, apply Euler's identity, and take the real part to find:

$$\mathcal{E}_x(z, t) = Ae^{-\alpha z} \cos(\omega t - \beta z) \quad (41)$$

We can substitute for A the more informative constant E_0^+ , which represents the electric field amplitude of the $+z$ traveling wave at $z = 0$. Reinserting the vector and position and time dependencies, equation (41) becomes:

$$\bar{\mathcal{E}}(z, t) = E_0^+ e^{-\alpha z} \cos(\omega t - \beta z) \mathbf{a}_x \quad (42)$$

Hence for this first solution of equation (39) we have a wave propagating and attenuating in the $+z$ direction with an amplitude E_0^+ at $z = 0$.

If we had chosen the second solution of equation (39), $\lambda = +\gamma$, we would have come to the solution:

$$E_x = E_0^- e^{+\gamma z} \quad (43)$$

or

$$\bar{\mathcal{E}}(z, t) = E_0^- e^{-\alpha z} \cos(\omega t + \beta z) a_x \quad (44)$$

This represents a wave propagating and attenuating in the $-z$ direction with an amplitude E_0^- at $z = 0$. The general solution for E_x is the linear superposition of the two solutions:

$$\bar{E} = (E_0^+ e^{-\gamma z} + E_0^- e^{+\gamma z}) a_x \quad (45)$$

and then the general instantaneous solution is:

$$\bar{\mathcal{E}} = E_0^+ e^{-\alpha z} \cos(\omega t - \beta z) a_x + E_0^- e^{+\alpha z} \cos(\omega t - \beta z) a_x \quad (46)$$

The magnetic field can be found by applying Faraday's law.

$$\nabla \times \bar{E} = -j\omega\mu\bar{H} \quad (47)$$

to equation (45). Evaluating the curl for \bar{E} we have:

$$\nabla \times \bar{E} = (-\gamma E_0^+ e^{-\gamma z} + \gamma E_0^- e^{+\gamma z}) a_y \quad (48)$$

Dividing equation (48) by $-j\omega\mu$ we can solve for \bar{H} :

$$\bar{H} = \left(\frac{\gamma E_0^+}{j\omega\mu} e^{-\gamma z} - \frac{\gamma E_0^-}{j\omega\mu} e^{+\gamma z} \right) a_y \quad (49)$$

this can be expressed as:

$$\bar{H} = (H_0^+ e^{-\gamma z} + H_0^- e^{+\gamma z}) a_y \quad (50)$$

Comparing equations (45) and (50), we can find a relationship between E_0^+ and H_0^+ (and E_0^- and H_0^-). Let's define the *intrinsic impedance* η of the medium as being the ratio of E_0^+ and H_0^+ , that is:

$$\eta = \frac{E_0^+}{H_0^+} = \frac{j\omega\mu}{\gamma} \quad (51)$$

Since the units of E_0^+ and H_0^+ are volt per meter and amperes per meter, respectively, we see that the units of η are ohms. Inserting the expression for γ from equation (30), we find:

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} \quad (52)$$

Further inspection of (45) and (50) reveals that

$$\eta = -\frac{E_0^-}{H_0^-} \quad (53)$$

The intrinsic impedance is a useful parameter for relating the electric and magnetic field amplitudes. Like the propagation constant, it is calculated from the operating frequency ($\omega = 2\pi f$) and the medium's constitutive parameters (σ , ε and μ).

End of Lecture #3

Microwave Engineering: Lecture #4

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1 Rectangular Waveguides

One of the early milestones in microwave engineering was the development of waveguide and other transmission lines of low-loss transmission of microwave power.

A rectangular waveguide is a hollow metallic tube with a rectangular cross section. The conducting walls of the guide confine the electromagnetic fields and thereby guide the electromagnetic waves. When the waves travel longitudinally down the guide, the plane waves are reflected from wall to wall. This process results in a component of either electric or magnetic field in the direction of propagation of the resultant wave; therefore the wave is no longer a *transverse electromagnetic* (TEM) wave.

It is possible to propagate several modes of electromagnetic waves within a waveguide. These modes correspond to solutions of Maxwell's equations for particular waveguides. A given waveguide has a definite cutoff frequency for each allowed mode. If the frequency of the impressed signal is above the cutoff frequency for a given mode, the electromagnetic energy can be transmitted through the guide for that particular mode without attenuation. Otherwise the electromagnetic energy with a frequency below the cutoff frequency for that particular mode will be attenuated to a negligible value in a relatively short distance. The *dominant mode* in a particular waveguide is the mode having the *lowest cutoff frequency*. It

is advisable to choose the dimensions of a guide in such a way that, for a given input signal, only the energy of the dominant mode can be transmitted through the guide.

1.1 Solution of Wave Equation in Rectangular Coordinates

A rectangular (cartesian) coordinate system is shown in Figure (1), where a metallic tube with a rectangular cross section is filled with a material of constitutive parameters ϵ and μ .

As explained earlier, the electric and magnetic wave equations can be written as:

$$\nabla^2 \bar{E} = \gamma^2 \bar{E} \quad (1)$$

$$\nabla^2 \bar{H} = \gamma^2 \bar{H} \quad (2)$$

where $\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = \alpha + j\beta$. These are called the *vector wave equations*.

Rectangular coordinates are the usual right-hand system. The rectangular components of \bar{E} or \bar{H} satisfy the complex scalar wave equation:

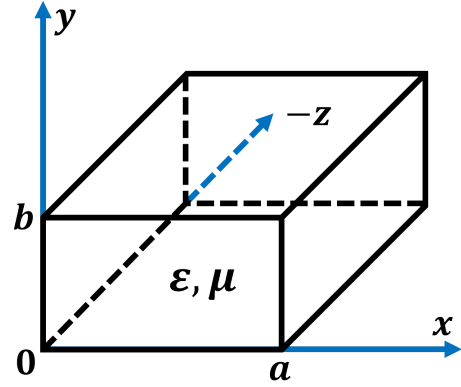


Figure 1: Rectangular coordinates.

$$\nabla^2 \psi = \gamma^2 \psi \quad (3)$$

The scalar wave equation can be written in rectangular coordinates as:

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = \gamma^2 \psi \quad (4)$$

This is a linear and inhomogeneous partial differential equation in three dimensions. By the method of separation of variables, the solution is assumed in the form of:

$$\psi = X(x)Y(y)Z(z) \quad (5)$$

where $X(x)$ = a function of the x coordinates only, $Y(y)$ = a function of the y coordinates only, and $Z(z)$ = a function of the z coordinates only.

Substitution of equation (5) in equation (4) and division of the resultant by equation (5) yields:

$$\frac{1}{X} \frac{d^2 X}{dx^2} + \frac{1}{Y} \frac{d^2 Y}{dy^2} + \frac{1}{Z} \frac{d^2 Z}{dz^2} = \gamma^2 \quad (6)$$

Since the sum of the three terms on the left hand side is a constant and each term is independently variable, it follows that each term must be equal to a constant.

Let the three terms be k_x^2 , k_y^2 and k_z^2 , respectively; then the separation equation is given by:

$$-k_x^2 - k_y^2 - k_z^2 = \gamma^2 \quad (7)$$

The general solution of each differential equation in (6) is:

$$\frac{d^2 X}{dx^2} = -k_x^2 X \quad (8)$$

$$\frac{d^2 Y}{dy^2} = -k_y^2 Y \quad (9)$$

$$\frac{d^2 Z}{dz^2} = -k_z^2 Z \quad (10)$$

will be in the form of:

$$X = A \sin(k_x x) + B \cos(k_x x) \quad (11)$$

$$Y = C \sin(k_y y) + D \cos(k_y y) \quad (12)$$

$$Z = E \sin(k_z z) + F \cos(k_z z) \quad (13)$$

The total solution of the scalar wave equation in rectangular coordinates is:

$$\psi = [A \sin(k_x x) + B \cos(k_x x)] [C \sin(k_y y) + D \cos(k_y y)] [E \sin(k_z z) + F \cos(k_z z)] \quad (14)$$

The propagation of the wave in the guide is conventionally assumed in the positive z direction. It should be noted that the propagation constant γ_g in the guide differs from the intrinsic propagation constant γ of the dielectric, and it is assumed to be equal to $-k_z$. Hence:

$$\gamma_g^2 = \gamma^2 + k_x^2 + k_y^2 = \gamma^2 + k_c^2 \quad (15)$$

where $k_c = \sqrt{k_x^2 + k_y^2}$ is usually called the *cutoff wavenumber*. For a lossless dielectric, $\gamma^2 = -\omega^2 \mu \epsilon$. Then:

$$\gamma_g = \pm j \sqrt{\omega^2 \mu \epsilon - k_c^2} \quad (16)$$

There are three cases for the propagation constant γ_g in the waveguide.

- **Case I.** There will be no wave propagation (evanescent) in the guide if $\omega_c^2 \mu \epsilon = k_c^2$ and $\gamma_g = 0$. This is the critical condition for cutoff propagation. The cutoff frequency is expressed as:

$$f_c = \frac{1}{2\pi \sqrt{\mu \epsilon}} \sqrt{k_x^2 + k_y^2} \quad (17)$$

- **Case II.** The wave will be attenuated if $\omega^2 \mu \epsilon < k_c^2$ and:

$$\gamma_g = \pm \alpha_g = \pm \omega \sqrt{\mu \epsilon} \sqrt{\left(\frac{f_c}{f}\right)^2 - 1} \quad (18)$$

This means that if the operating frequency is below the cutoff frequency, the wave will decay exponentially with respect to a factor of $-\alpha_g z$ and there will be no wave propagation because the propagation constant is real quantity.

- **Case III.** The wave will be propagating in the guide if $\omega^2\mu\varepsilon > k_c^2$ and:

$$\gamma_g = \pm j\beta_g = \pm j\omega\sqrt{\mu\varepsilon}\sqrt{1 - \left(\frac{f_c}{f}\right)^2} \quad (19)$$

This means that the operating frequency must be above the cutoff frequency in order for a wave to propagate in the guide. Therefore the solution to the Helmholtz equation in Cartesian coordinates is given by:

$$\psi = [A\sin(k_x x) + B\cos(k_x x)] [C\sin(k_y y) + D\cos(k_y y)] e^{-j\beta_g z} \quad (20)$$

1.2 TE Modes in Rectangular Waveguides

It has been previously assumed that the waves are propagating in the positive z -direction in the waveguide, where the considered coordinates are shown in Figure (1).

The TE_{mn} modes in a rectangular waveguide are characterized by $E_z = 0$. In other words, the z -component of the magnetic field, H_z , must **exist** in order to have energy transmission in the guide. Consequently, from a given Helmholtz equation,

$$\nabla^2 H_z = \gamma^2 H_z \quad (21)$$

a solution is in the form of:

$$H_z = \left[A_m \sin\left(\frac{m\pi x}{a}\right) + B_m \cos\left(\frac{m\pi x}{a}\right) \right] \times \left[C_n \sin\left(\frac{n\pi y}{b}\right) + D_n \cos\left(\frac{n\pi y}{b}\right) \right] e^{-j\beta_g z} \quad (22)$$

will be determined in accordance with the given boundary conditions, where $k_x = m\pi/a$ and $k_y = n\pi/b$ are replaced. For a lossless dielectric, and assuming $e^{j\omega t}$ time dependence, Maxwell's curl equations are:

$$\nabla \times \bar{E} = -j\omega\mu\bar{H} \quad (23)$$

$$\nabla \times \bar{H} = j\omega\varepsilon\bar{E} \quad (24)$$

In Cartesian coordinates, their components are:

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -j\omega\mu H_x \quad (25)$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y \quad (26)$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z \quad (27)$$

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = j\omega\varepsilon E_x \quad (28)$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = j\omega\varepsilon E_y \quad (29)$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\varepsilon E_z \quad (30)$$

With the substitution $\partial/\partial z = -j\beta_g$ and $E_z = 0$, the foregoing equations are simplified to:

$$\beta_g E_y = -\omega\mu H_x \quad (31)$$

$$\beta_g E_x = \omega\mu H_y \quad (32)$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z \quad (33)$$

$$\frac{\partial H_z}{\partial y} + j\beta_g H_y = j\omega\varepsilon E_x \quad (34)$$

$$-j\beta_g H_x - \frac{\partial H_z}{\partial x} = j\omega\varepsilon E_y \quad (35)$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = 0 \quad (36)$$

Solving these six equations for E_x , E_y , H_x , and H_y will give the TE-mode field equations in rectangular waveguides as:

$$E_x = \frac{-j\omega\mu}{k_c^2} \frac{\partial H_z}{\partial y} \quad (37)$$

$$E_y = \frac{j\omega\mu}{k_c^2} \frac{\partial H_z}{\partial x} \quad (38)$$

$$E_z = 0 \quad (39)$$

$$H_x = \frac{-j\beta_g}{k_c^2} \frac{\partial H_z}{\partial x} \quad (40)$$

$$H_y = \frac{-j\beta_g}{k_c^2} \frac{\partial H_z}{\partial y} \quad (41)$$

$$H_z = \text{equation (22)} \quad (42)$$

where k_c^2 has been replaced by $\omega^2\mu\varepsilon - \beta_g^2$.

Differentiating equation (22) with respect to x and z and then substituting the results in equations (37) through (41) yield a set of field equations. The boundary conditions are

applied to the newly found field in such a manner that either the tangent \bar{E} field or the normal \bar{H} field vanishes at the surface of the conductor. Since $E_x = 0$, then $\partial H_z / \partial y = 0$ at $y = 0, b$. Hence $C_n = 0$. Since $E_y = 0$, then $\partial H_z / \partial x = 0$ at $x = 0, a$. Hence $A_m = 0$.

It is generally concluded that the normal derivative of H_z must vanish at the conducting surfaces-that is,

$$\frac{\partial H_z}{\partial n} = 0 \quad (43)$$

at the guide walls. Therefore the magnetic field in the positive z -direction is given by:

$$H_z = H_{0z} \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-j\beta_g z} \quad (44)$$

where H_{0z} is the amplitude constant.

Substituting equation (44) into equations (37) through (41) yields the TE_{mn} field equations in rectangular waveguides as:

$$E_x = E_{0x} \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-j\beta_g z} \quad (45)$$

$$E_y = E_{0y} \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-j\beta_g z} \quad (46)$$

$$E_z = 0 \quad (47)$$

$$H_x = H_{0x} \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-j\beta_g z} \quad (48)$$

$$H_y = H_{0y} \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-j\beta_g z} \quad (49)$$

where $m = 0, 1, 2, \dots, n = 0, 1, 2, \dots$, and $m = n = 0$ case is excepted.

For TE₁₀, it can be seen from the above equations that E_y is the only electric field component available in the waveguide. From equation (46), E_y can be plotted as shown in Figure (2-a). As for the magnetic field, both H_x and H_z are available for this mode, as shown in Figure (2-b) and Figure (3).

The cutoff wavenumber k_c , as defined by equation (15) for the TE_{mn} modes, is given by:

$$k_c = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} = \omega_c \sqrt{\mu\epsilon} \quad (50)$$

where a and b are in meters. The cutoff frequency, as defined in equation (17) for the TE_{mn} modes, is:

$$f_c = \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2}} \quad (51)$$

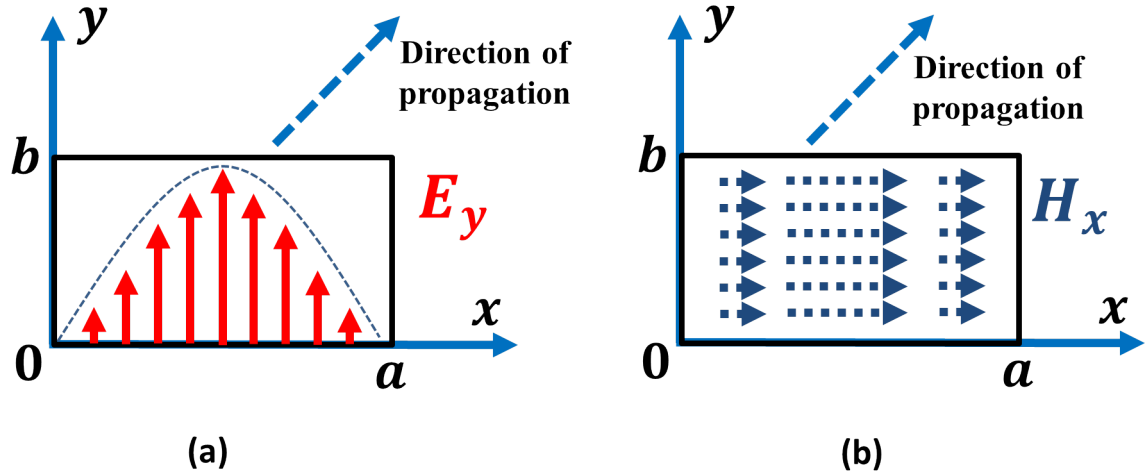


Figure 2: Tangential components of (a) electric and (b) magnetic fields distribution for TE_{10} mode across the waveguide

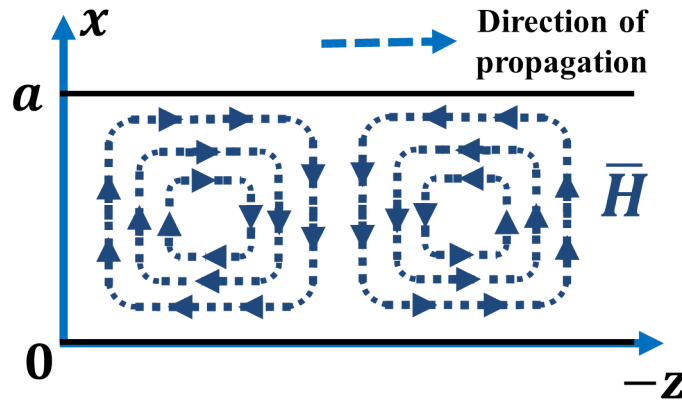


Figure 3: Magnetic field distribution for TE_{10} mode along the waveguide

The propagation constant (or the phase constant here) β_g , as defined in equation (16) is expressed by:

$$\beta_g = \omega \sqrt{\mu \epsilon} \sqrt{1 - \left(\frac{f_c}{f}\right)^2} \quad (52)$$

The phase velocity in the positive z -direction for the TE_{mn} modes is shown as:

$$\nu_g = \frac{\omega}{\beta_g} = \frac{\nu_p}{\sqrt{1 - (f_c/f)^2}} \quad (53)$$

where $\nu_p = 1/\sqrt{\mu \epsilon}$ is the phase velocity in an unbounded dielectric.

The characteristic wave impedance of TE_{mn} modes in the guide can be derived from equations (31) and (32):

$$Z_g = \frac{E_x}{H_y} = -\frac{E_y}{H_x} = \frac{\omega \mu}{\beta_g} = \frac{\eta}{\sqrt{1 - (f_c/f)^2}} \quad (54)$$

where $\eta = \sqrt{\mu/\epsilon}$ is the intrinsic impedance in an unbounded dielectric. The wavelength λ_g in the guide for the TE_{mn} modes is given by:

$$\lambda_g = \frac{\lambda}{\sqrt{1 - (f_c/f)^2}} \quad (55)$$

where $\lambda = \nu_p/f$ is the wavelength in an unbounded dielectric.

Since the cutoff frequency shown in equation (51) is a function of the modes and guide dimensions, the physical size of the waveguide will determine the propagation of the mode.

Whenever two or more modes have the same cutoff frequency, they are said to be *degenerate modes*. In a rectangular guide the corresponding TE_{mn} and TM_{mn} modes are always degenerate. In a square guide the TE_{mn} , TE_{nm} , TM_{mn} , and TM_{nm} modes form a foursome of degeneracy. Rectangular guides ordinarily have dimensions of $a = 2b$ ratio. The mode with the lowest cutoff frequency in particular guide is called the *dominant mode*. The dominant mode in a rectangular guide with $a > b$ is the TE_{10} mode. Each mode has a specific mode pattern (or field pattern).

It is normal for all modes to exist simultaneously in a given waveguide. The situation is not very serious, however. Actually, only the dominant mode propagates, and the higher modes near the sources or discontinuities decay very fast.

End of Lecture #4

Microwave Engineering: Lecture #5-A

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1 Examples about Rectangular Waveguides

Example 1:

An air-filled rectangular waveguide of inside dimensions 7×3.5 cm operates in the dominant TE_{10} mode as shown in Figure (1). (a) Find the cutoff frequency.

(b) Determine the phase velocity of the wave in the guide at a frequency of 3.5 GHz.

(c) Determine the guided wavelength at the same frequency.

Solution:

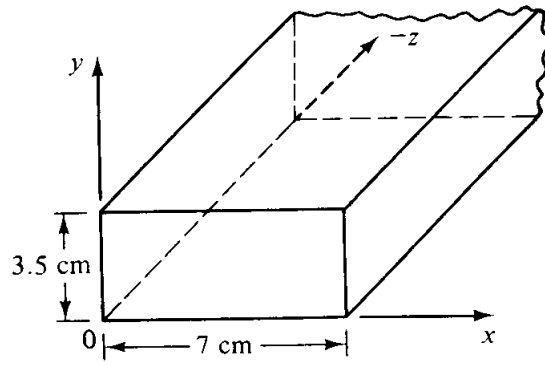


Figure 1

solution (continued)

Example 2:

Consider a length of Teflon-filled ($\varepsilon_r = 2.08$) rectangular waveguide, having dimensions $a = 1.07$ cm and $b = 0.43$ cm. Find the cutoff frequencies of the first five propagating modes. If the operating frequency is 15 GHz , and the peak electric field E_o is 1000 V/m , find the average power transmitted through the waveguide.

Solution:

Example 3:

An air-filled waveguide with a cross section 2×1 cm transports energy in the TE_{10} mode at the rate of 373 Watt. The impressed frequency is 30 GHz . What is the peak value of the electric field occurring in the waveguide?

Solution:

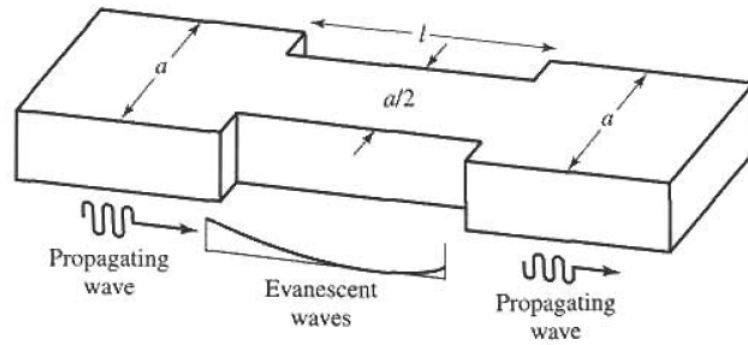


Figure 2

Example 4:

An attenuator can be made using a section of waveguide operating below cutoff, as shown in Figure (2). If $a = 2.286$ cm and the operating frequency is 12 GHz, determine the required length of the below-cutoff section of waveguide to achieve an attenuation of 100 dB between the input and output guides. Ignore the effect of reflections at the step discontinuities.

Solution:

solution (continued)

End of Lecture #5-A