D.C. motors

The construction of a d.c. motor is the same as a d.c. generator. The only difference is that in a generator the generated e.m.f. is greater than the terminal voltage, where as in a motor the generated e.m.f. is less than the terminal voltage.

D.C. motors are often used in power stations to drive emergency stand-by pump systems which come into operation to protect essential equipment and plant should the normal a.c. supplies or pumps fail.

Back e.m.f.

When a d.c. motor rotates, an e.m.f. is induced in the armature conductors. By Lenz's law this induced e.m.f. E opposes the supply voltage V and is called a **back e.m.f.**, and the supply voltage, V is given by:

$\boldsymbol{V = E + I_a R_a}$ or $\boldsymbol{E = V - I_a R_a}$ **………………**(5)

Problem 18. A d.c. motor operates from a 240V supply. The armature resistance is 0.2Ω. Determine the back e.m.f. when the armature current is 50A.

SOL.:

For a motor, $V = E + I_a R_a$ hence back e.m.f.,

$E = V - I_a R_a = 240 - (50)(0.2) = 240 - 10 =$ **230 volts**

Problem 19. The armature of a d.c. machine has a resistance of 0.25Ω and is connected to a 300V supply. Calculate the e.m.f. generated when it is running: (a) as a generator giving 100A , and (b) as a motor taking 80A.

Sol.:

(a) As a generator, generated e.m.f.,

$E = V + I_a R_a$, from Equation (3),

$= 300 + (100)(0.25) = 300 + 25 =$ **325volts**

(b) As a motor, generated e.m.f. (or back e.m.f.),

$E = V - I_aR_a$, from Equation (5),

$= 300 - (80)(0.25) =$ **280 volts**

H.W:

1. A d.c. motor operates from a 350V supply. If the armature resistance is 0.4Ω determine the back e.m.f. when the armature current is 60A. [326 volts]

2. The armature of a d.c. machine has a resistance of 0.5Ω and is connected to a 200V supply. Calculate the e.m.f. generated when it is running (a) as a motor taking 50A, and (b) as a generator giving 70A [(a) 175 volts (b) 235 volts]

3. Determine the generated e.m.f. of a d.c. machine if the armature resistance is $0.1\Omega$and it (a) is running as a motor connected to a 230V supply, the armature current being 60A, and (b) is running as a generator with a terminal voltage of 230V, the armature current being 80A. [(a) 224V (b) 238

Torque of a d.c. motor

From Equation (5), for a d.c. motor, the supply voltage V is given by

$V = E + I_aR_a$

Multiplying each term by current I_a gives:

$VI_a = EI_a + I_a^2 R_a$

The term VI_a is the total electrical power supplied to the armature, the term $I_a^2R_a$ is the loss due to armature resistance, and the term EI_a is the mechanical power developed by the armature. If T is the torque, in newton metres, then the mechanical power developed is given by $T\omega$ watts

Hence $T\omega = 2\pi nT = EI_a$ from which,

torque $T = EI_a/2\pi n$ newton metres ……………….(6)

Equation (1), the e.m.f. E generated is given by $E = 2p\varnothing nZ/c$

Hence $2\pi nT = EI_a = (2p\emptyset nZ/c)I_a$

Hence torque $T = [[(2p\emptyset nZ)/c]/2\pi n]I_a$

i.e. $T = p\emptyset ZI_a/\pi c$ newton metres …………….(7)

For a given machine, Z, c and p are fixed values Hence torque, $T \propto \emptyset I_a$ ……….(8)

Problem 20. An 8-pole d.c. motor has a wave-wound armature with 900 conductors. The useful flux per pole is 25mWb. Determine the torque exerted when a current of 30A flows in each armature conductor.

Sol.:

$p=4$, $c=2$ for a wave winding, $\emptyset=25\times10^{-3}$ Wb, $Z=900$ and $I_a=30$A. From Equation (7), **torque**, $T = p\emptyset ZI_a/\pi c$

$= (4)(25 \times 10^{-3})(900)(30)/\pi(2)$ = **429.7Nm**

Problem 21. Determine the torque developed by a 350V d.c. motor having an armature resistance of 0.5Ω and running at 15 rev/s. The armature current is 60A.

Sol.:

$V=350$V, $R_a=0.5\Omega$, $n=15$ rev/s and $I_a=60$A.

Back e.m.f. $E = V - I_aR_a = 350-(60)(0.5) = 320$V.

From Equation (6),

torque, $T = EI_a/2\pi n$

$= (320)(60)/2\pi(15)$ = **203.7Nm**

Problem 22. A six-pole lap-wound motor is connected to a 250V d.c. supply. The armature has 500 conductors and a resistance of 1Ω. The flux per pole is 20mWb. Calculate (a) the speed and (b) the torque developed when the armature current is 40A.

Sol.:

V =250V, Z =500, Ra =1Ω, =20 × 10^{-3} Wb, Ia =40A and c=2p for a lap winding

(a) Back e.m.f. $E = V - I_a R_a$ =250−(40)(1)=210V

E.m.f. $E = 2p\varnothing nZ/c$

i.e. $210 = 2p(20 \times 10^{-3})n(500)/2p = 10n$

Hence **speed n** = 210/10

= **21 rev/s** or (21 × 60) = **1260 rev/min**

(b) **Torque T** = $EI_a/2\pi n$= (210)(40)/2π(21)= **63.66Nm**

Problem 23. The shaft torque of a diesel motor driving a 100V d.c. shunt-wound generator is 25 Nm. The armature current of the generator is 16A at this value of torque. If the shunt field regulator is adjusted so that the flux is reduced by 15 per cent, the torque increases to 35 Nm. Determine the armature current at this new value of torque.

Sol.:

From Equation (8), the shaft torque T of a generator is proportional to Ø Ia, where Ø is the flux and Ia is the armature current, or, $T = k \varnothing I_a$, where k is a constant.

The torque at flux $\varnothing_1$ and armature current I_{a1} is $T1 = k \varnothing_1 I_{a1}$ Similarly, $T2 = k \varnothing_2 I_{a2}$

By division $T1/T2 = k \varnothing_1 I_{a1} / k \varnothing_2 I_{a2} = \varnothing_1 I_{a1} / \varnothing_2 I_{a2}$

$\varnothing_2 = 0.85 \varnothing_1$

Hence $25/35 = \varnothing_1 \times 16 / 0.85 \varnothing_1 \times I_{a2}$

i.e. I_{a2} = 16 × 35/0.85 × 25= 26.35 A

That is, **the armature current at the new value of torque is 26.35A**

Problem 24. A 100V d.c. generator supplies a current of 15A when running at 1500 rev/min. If the torque on the shaft driving the generator is 12 Nm, determine (a) the efficiency of the generator and (b) the power loss in the generator.

Sol.:

(a) the efficiency of a generator= output power/input power × 100 percent. The

output power is the electrical output, i.e. VI watts.

The input power to a generator is the mechanical power in the shaft driving the generator, i.e. $T\omega$ or $T(2\pi n)$ watts, where T is the torque in Nm and n is speed of rotation in rev/s. Hence, for a generator, efficiency, $\eta = VI/T(2\pi n) \times 100\%$

=[(100)(15)/(12)(2π)(1500/60)]*100%

i.e. **efficiency=79.6%**

(b) The input power=output power + losses

Hence, $T(2\pi n) = VI$ + losses

i.e. losses = $T(2\pi n) - VI$=[[(12)(2π)(1500/60)]– [(100)(15)]

i.e. **power loss** = 1885 – 1500 = **385W**

H.W:

1. The shaft torque required to drive a d.c. generator is 18.7Nm when it is running at 1250 rev/min. If its efficiency is 87 per cent under these conditions and the armature current is 17.3A, determine the voltage at the terminals of the generator [123.1V]

2. A 220V, d.c. generator supplies a load of 37.5A and runs at 1550 rev/min. Determine the shaft torque of the diesel motor driving the generator, if the generator efficiency is 78 per cent [65.2 Nm]

3. A 4-pole d.c. motor has a wave-wound armature with 800 conductors. The useful flux per pole is 20mWb. Calculate the torque exerted when a current of 40A flows in each armature conductor [203.7 Nm]

4. Calculate the torque developed by a 240V d.c. motor whose armature current is 50A, armature resistance is 0.6Ω and is running at 10 rev/s [167.1 Nm]

5. An 8-pole lap-wound d.c. motor has a 200V supply. The armature has 800 conductors and a resistance of 0.8_. If the useful flux per pole is 40mWb and the armature current is 30A, calculate (a) the speed and (b) the torque developed [(a) 5.5 rev/s or 330 rev/min (b) 152.8 Nm]

6. A 150V d.c. generator supplies a current of 25A when running at 1200 rev/min. If the torque on the shaft driving the generator is 35.8 Nm, determine (a) the efficiency of the generator, and (b) the power loss in the generator [(a) 83.4 per cent (b) 748.8W]

Types of d.c. motor

(a) Shunt wound motor

In the shunt wound motor the field winding is in parallel with the armature across the supply as shown in Fig. 8.16.

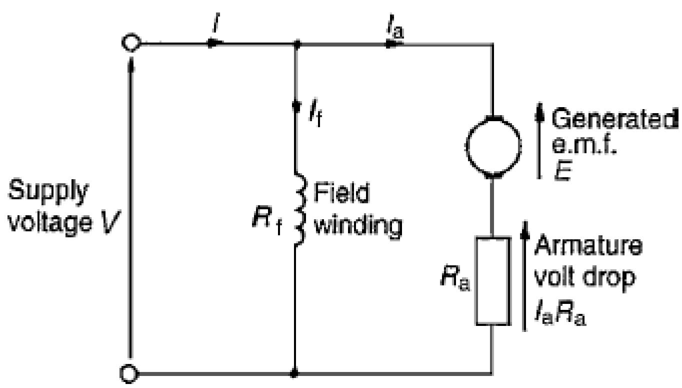


Fig 8.16

Supply voltage, $V=E+I_aR_a$ or generated e.m.f., $E=V-I_aR_a$

Supply current, $I=I_a+I_f$ from Kirchhoff's current law

Problem 25. A 240V shunt motor takes a total current of 30A. If the field winding resistance R_f =150Ω and the armature resistance R_a =0.4 determine (a) the current in the armature, and (b) the back e.m.f.

Sol.:

(a) Field current $I_f = V/R_f = 240/150 = 1.6A$

Supply current $I = I_a + I_f$

Hence armature current, $I_a = I - I_f = 30 - 1.6 =$ **28.4A**

(b) Back e.m.f. $E = V - I_aR_a = 240 - (28.4)(0.4) =$ **228.64 volts**

Problem 26. A 200V, d.c. shunt-wound motor has an armature resistance of 0.4Ω and at a certain load has an armature current of 30A and runs at 1350 rev/min. If the load on the shaft of the motor is increased so that the armature current increases to 45A, determine the speed of the motor, assuming the flux remains constant.

Sol.:

The relationship $E \propto \O n$ applies to both generators and motors. For a motor,

$E = V - I_aR_a$, (see equation (5))

Hence $E_1 = V - I_{a1} R_a = 200 - 30 \times 0.4 = 188V$

and $E_2 = V - I_{a2} R_a = 200 - 45 \times 0.4 = 182V$

The relationship: $E_1/E_2 = \O_1 n_1/\O_2 n_2$

applies to both generators and motors. Since the flux is constant, $\O_1 = \O_2$. Hence

$188/182 = [\O_1 \times (1350/60)]/\O_1 \times n_2$

i.e. $n_2 = 22.5 \times 182/188$

$= 21.78$ rev/s

Thus the speed of the motor when the armature current is 45A is 21.78 × 60 rev/min i.e. 1307 rev/min.

(b) Series-wound motor

In the series-wound motor the field winding is in series with the armature across the supply as shown in figure 8.17

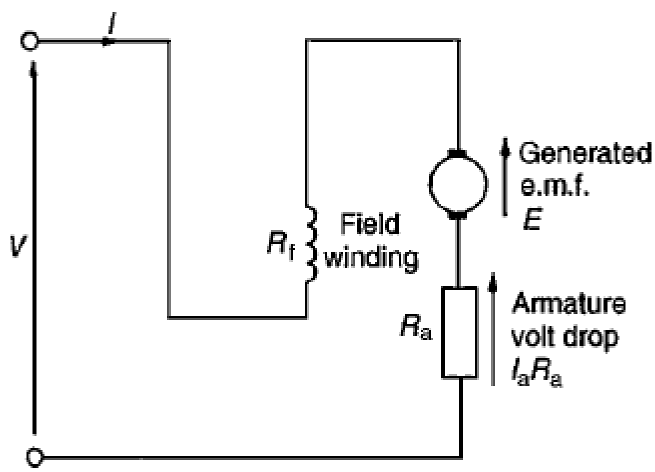


Fig. 8.17.

For the series motor shown in Fig. 8.17,

Supply voltage $V = E + I(R_a + R_f)$

or generated e.m.f. $E = V - I(R_a + R_f)$

Problem 28. A series motor has an armature resistance of 0.2Ω and a series field resistance of 0.3Ω. It is connected to a 240V supply and at a particular load runs at 24 rev/s when drawing 15A from the supply. (a) Determine the generated e.m.f. at this load (b) Calculate the speed of the motor when the load is changed such that the current is increased to 30A. Assume that this causes a doubling of the flux.

Sol.:

(a) With reference to Fig. 8.17, generated e.m.f., E_1 at initial load, is given by

$\boldsymbol{E}1 = V - I_1(Ra + Rf) = 240 - (15)(0.2 + 0.3) = 240 - 7.5 =$ **232.5 volts**

(b) When the current is increased to 30A, the generated e.m.f. is given by:

$E2 = V - I_2(Ra + Rf) = 240 - (30)(0.2 + 0.3) = 240 - 15 = 225$ volts

Now e.m.f. $E \propto \boldsymbol{\O n}$ thus

$E1/E2 = \O 1n1/\O 2n2$

i.e. $232.5/225 = \O 1(24)/(2\O 1)n2$

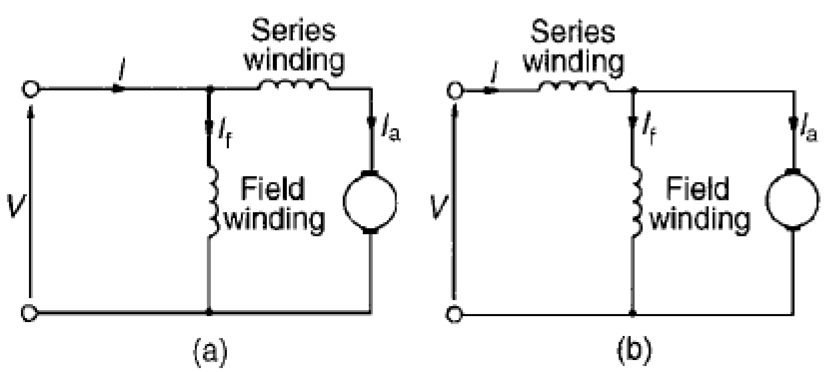
since $\O 2 = 2\O 1$

Hence **speed of motor, $\boldsymbol{n2}$** $= (24)(225)/(232.5)(2) =$ **11.6 rev/s**

As the current has been increased from 15A to 30A, the speed has decreased from 24 rev/s to 11.6 rev/s.

(c) Compound wound motor

Figure 8.18 (a) shows a **long-shunt** compound motor and Fig. 8.18(b) a **short-shunt** compound motor.



The efficiency of a d.c. motor

The efficiency of a d.c. machine is given by:

efficiency, η = output power/input power × 100%

Also, the total losses = $I_a^2 R_a + I_f V + C$ (for a shunt motor) where C is the sum of the iron, friction and windage losses.

For a motor,

the input power = VI and the output power = VI − losses= $VI - I_a^2 R_a - I_f V - C$

Hence **efficiency,** $\boldsymbol{\eta = VI - I_a^2 R_a - I_f V - C / VI \times 100\%}$ **…………**(10)

The **efficiency of a motor is a maximum** when the load is such that:

$\boldsymbol{I_a^2 R_a = I_f V + C}$

Problem 29. A 320V shunt motor takes a total current of 80A and runs at 1000 rev/min. If the iron, friction and windage losses amount to 1.5kW, the shunt field resistance is 40Ω and the armature resistance is 0.2Ω, determine the overall efficiency of the motor.

Sol:

The circuit is shown in Fig. 8.19.

Field current, $I_f = V/R_f = 320/40 = 8A$.

Armature current $I_a = I - I_f = 80 - 8 = 72A$.

C =iron, friction and windage losses=1500W.

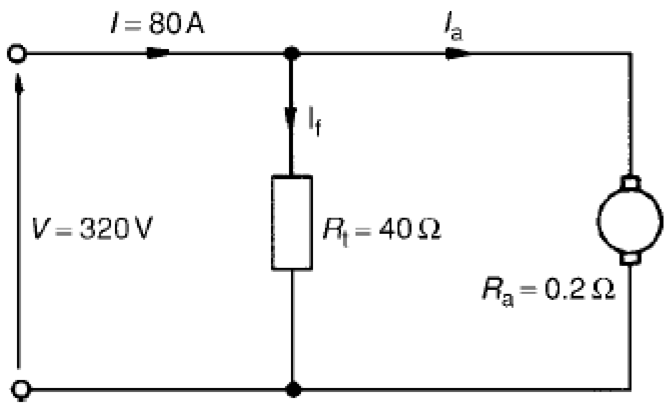


Figure 8.19

Efficiency, $\eta = [VI - I_a{}^2 R_a - I_f V - C]/VI \times 100\%$

$= [(320)(80) - (72)^2(0.2) - (8)(320) - 1500]/(320)(80) * 100\%$

$= 25\,600 - 1036.8 - 2560 - 1500/25\,600 \times 100\%$

$= 20\,503.2/25\,600 \times 100\% =$ **80.1%**

Problem 30. A 250V series motor draws a current of 40A. The armature resistance is 0.15Ω and the field resistance is 0.05Ω. Determine the maximum efficiency of the motor.

Sol.:

The circuit is as shown in Fig. 8.20

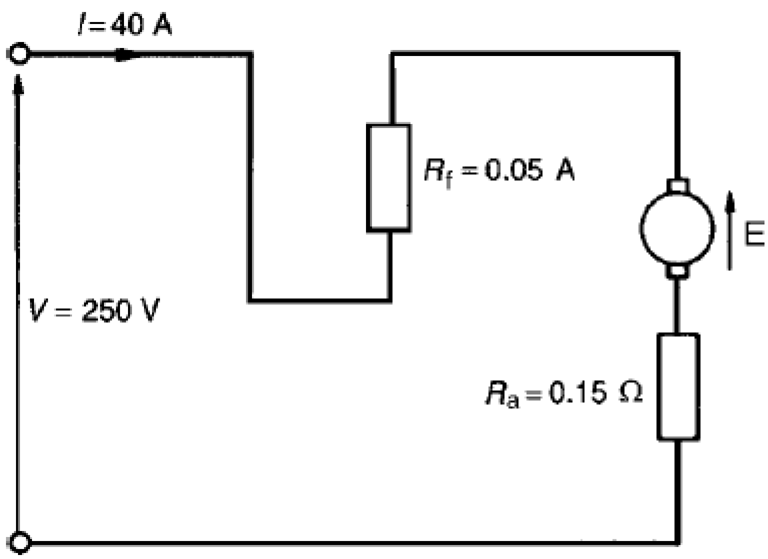


Fig 8.20

From equation (10), efficiency,

$\eta = VI - I_a{}^2 R_a - I_f V - C / VI \times 100\%$

However for a series motor, $I_f = 0$ and the $I_a{}^2 R_a$ loss needs to be $I^2(R_a + R_f)$ Hence efficiency, $\eta = VI - I^2(R_a + R_f) - C/VI \times 100$

For maximum efficiency $I^2(R_a + R_f) = C$. Hence efficiency,

$\eta = VI - 2I^2(R_a + R_f)/VI \times 100\%$

$=[(250)(40) - 2(40)^2(0.15 + 0.05)]/(250)(40) \times 100\%$

$=[10\,000 - 640]/10\,000 \times 100\%$

$=9360/10\,000 \times 100\% =$ **93.6%**

Problem 31. A 200V d.c. motor develops a shaft torque of 15Nm at 1200 rev/min. If the efficiency is 80 per cent, determine the current supplied to the motor.

Sol.:

The efficiency of a motor= output power/input power × 100%.

The output power of a motor is the power available to do work at its shaft and is given by $T\omega$ or $T(2\pi n)$ watts, where T is the torque in Nm and n is the speed of rotation in rev/s. The input power is the electrical power in watts supplied to the motor, i.e. VI watts.

Thus for a motor, efficiency, $\eta = T(2\pi n)/VI \times 100\%$

i.e. $80 = [[(15)(2\pi)(1200/60)]/(200)(I)] \times 100$

Thus the current supplied,

$I = (15)(2\pi)(20)(100)/(200)(80) =$ **11.8A**

Problem 32. A d.c. series motor drives a load at 30 rev/s and takes a current of 10A when the supply voltage is 400V. If the total resistance of the motor is 2Ω and the iron, friction and windage losses amount to 300W, determine the efficiency of the motor.

Sol.:

Efficiency, $\eta = VI - I^2R - C/VI \times 100\%$

$= [(400)(10) - (10)^2(2) - 300]/(400)(10) \times 100\%$

$= [4000 - 200 - 300]/4000 \times 100\%$

$= 3500/4000 \times 100\% =$ **87.5%**

Example 33: A 500V shunt motor takes a total current of 100A and runs at 1200 rev/min. If the shunt field resistance is 50Ω, the armature resistance is 0.25Ω and the iron, friction and windage losses amount to 2kW, determine the overall efficiency of the motor.

Sol.:

V=500 V, I=100 A , n=1200 rev/m, Rf=50 Ω, Ra=0.25 Ω, C=2KW

If=V/Rf=500/50=10 A

Ia=I-If=100-10=90 A

$\eta = VI - I_a{}^2 R_a - I_f V - C / VI \times 100\%$

$=[500*100-90^2*0.25-10*500-2000]/(500*100) \times 100\%$

=81.95%

Example 34: series-wound motor is running at 500 rev/min and its shaft torque is 130 Nm. If its efficiency at this load is 88 per cent, and the current taken from the supply is 30.99 A, find voltage supply?

Sol.:

n=500 r/m, T=130 Nm, $\eta=88\%$, I=30.99 A

$\eta = T(2\pi n)/VI \times 100\%$

$0.88=[130*2\pi*(500/60)]/(V*30.99)$

$V=249.59$ V

Example 35: A series motor drives a load at 1500 rev/min and takes a current of 20A when the supply voltage is 250V. If the total resistance of the motor is 1.5Ω and the iron, friction and windage losses amount to 400W, determine the efficiency of the motor.

Sol:

n=1500 r.p.m , I=20 A, V=250 V,R=1.5 Ω, C=400 W

Efficiency,$\eta = [VI - I_a{}^2 R - C]/VI \times 100\%$

$=(250*20-20^2*1.5-400)/(250*20) \times 100\%$

=80%

H.W:

1. A 240V shunt motor takes a total current of 80A. If the field winding resistance is 120Ωand the armature resistance is 0.4Ω, determine (a) the current in the armature, and (b) the back e.m.f. [(a) 78A (b) 208.8V]

2. A d.c. motor has a speed of 900 rev/min when connected to a 460V supply. Find the approximate value of the speed of the motor when connected to a 200Vsupply, assuming the flux decreases by 30 per cent and neglecting the armature volt drop. [559 rev/min]

3. A series motor having a series field resistance of 0.25Ωand an armature resistance of 0.15Ω, is connected to a 220V supply and at a particular load runs at 20 rev/s when drawing 20A from the supply. Calculate the e.m.f. generated at this load. Determine also the speed of the motor when the load is changed such that the current increases to 25A. Assume the flux increases by 25 per cent [212V, 15.85 rev/s]

[30.94A]

4.A 300V series motor draws a current of 50A. The field resistance is 40mΩ and the armature resistance is 0.2Ω. Determine the maximum efficiency of the motor. [92 per cent]

5. A series motor drives a load at 1500 rev/min and takes a current of 20A when the supply voltage is 250V. If the total resistance of the motor is 1.5Ω and the iron, friction and windage losses amount to 400W, determine the efficiency of the motor. [80 per cent]

6. A series-wound motor is connected to a d.c. supply and develops full-load torque when the current is 30A and speed is 1000 rev/min. If the flux per pole is proportional to the current flowing, find the current and speed at half full-load torque, when connected to the same supply. [21.2A, 1415 rev/min]