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College of Engineering
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ENGINEERING ECONOMY

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2nd Grade

Introduction to Economics

<u>The Decision-Making Process-Classifying Problems</u>

One method of classifying problems is by degree of complexity. Problems can be classified as simple, intermediate, or complex.

An example **simple problem is the decision to select a bus or taxi for travel within a city.** An example intermediate problem is deciding which one of several competing automatic packaging machines to purchase for a manufacturing facility. An example complex problem is the selection of a site for a new manufacturing facility.

Intermediate problems and the economic aspects of complex problems are best suited for solution by engineering economic analysis because of the following characteristics:

a. The problem is sufficiently important to justify the commitment of a nontrivial amount of resources.

b. The problem is sufficiently complex that it be well organized for solution.

c. The problem involves significant economic considerations.

The Decision-Making Process—Classifying Problems

<u>Question 1.</u>

Which one of the following problems is best suited for solution by engineering economic analysis?

Choose an answer by clicking on one of the letters below,

A Choosing between a new or used copy of a textbook.

B Deciding to buy or to lease vehicles for a company's sales force.

C Writing a computer simulation model of an automobile assembly plant.

D Selecting the best location for a daily walk.

Question 2.

A possible objective for NASA might be to support private-sector tourism in space by 2020. Which of the following issues is best suited for solution by engineering economic analysis? Choose an answer by clicking on one of the letters below:

A Estimating the cost of sending a person on a space trip.

B Choosing the ethical values of possible tourists.

C Determining the proper international partners for space tourism.

D Evaluating the benefits to developing countries.

What is Engineering Economics?

(1) **Engineering economics**, can be defined as the science that deals with techniques of quantitative analysis useful for selecting a preferable alternative from several technically viable ones.

(2) **What is Engineering Economy**? Is the application of economic techniques to evaluate engineering alternatives.

The role of engineering economics is to assess the appropriateness of a given project, estimate its value, and justify it from an engineering standpoint

The Decision-Making Process-Rational Decision Making

Making Economic Decisions

Selecting an appropriate criterion (or criteria) for selecting among competing alternatives is a critical step in engineering decision making.

If a problem involves fixed input among the possible alternatives, then the appropriate general criterion is to maximize output. For example, a company may be considering the purchase of a new office copy machine. If two competing alternatives have the same cost (fixed input), then the appropriate criterion would be to select the copy machine that in some appropriate measure has the higher output. The criterion could be as simple as the copy rate of the machine (pages per minute) or as complex as the copy rate with some measure of special features availability.

If a problem involves fixed output among the possible alternatives, then the appropriate general criterion is to minimize input. For example, a company may be considering the installation of a new elevator. If two competing alternatives have the same output as measured in load capacity, operating speed, and so on, then the appropriate criterion would be to install the elevator with the lower cost (minimum input).

If neither input nor output is fixed among the alternatives being considered, then the appropriate criterion is to maximize (output) minimize (input) or, stated more simply, to maximize profit. For example, if one of two competing production machines will be purchased, and if the machines differ in both initial cost and output rate, then the appropriate criterion is to select the machine that will generate the higher profit (benefits derived from machine output - costs).

Rational Decision-Making Process عملية صنع القرارالمنطقي او الرشيد

1. Recognize a decision problem ماهي المشكلة التي تحتاج لقرار
2. Define the goals or objectives
3. Collect all the relevant information
4. Identify a set of feasible decision alternatives
5. Select the decision criterion to use
6. Select the best alternative.

Making Economic Decisions

Example 3 A College of Engineering freshman is required to purchase a notebook PC upon or before enrollment. Two competing models are available through the college at the same attractive, discounted price. Both models meet the minimum performance specifications of the college, and they have identical warranty plans. They also have identical weights and footprints.

What economic criterion should the freshman use in selecting which PC to buy?

A Choose either PC.

B Choose the PC with the lower cost.

C Choose the PC with the greater performance (output).

D Choose the PC with the lower (benefits - cost).

Question 2.

A distribution center must purchase a new fork truck, and three competing candidates have been identified. The costs of the three alternatives vary, as do the benefits (e.g., maximum payload).

What economic criterion should be used in selecting a fork truck for purchase?

A Choose the fork truck with the lowest cost.

B Choose the fork truck with the highest benefits.

C Choose the fork truck with the highest (cost - benefits).

D Choose the fork truck with the highest (benefits - cost).

Question 3.

A manufacturing facility provides uniforms and uniform cleaning for its production employees. Either one of two local vendors can provide the identical service. What economic criterion should be used in selecting a uniform provider?

A Choose the service provider with the lower cost

B Choose the service provider with the higher service level

C Choose either service provider

D Alternate the selection of the provider from month to month

Engineering Decision Making for Current Costs

Some of the easiest forms of engineering decision making deal with problems related to alternative designs, methods, or materials. If results of the decision occur in a very short period of time, one can quickly add up the costs and benefits for each alternative. Then, using suitable economic criterion, the best alternative can be identified. (Note that this course is mostly concerned with matters that deal with the effect of time on money. This approach ignores the time value of money.)

Example

Question 4.

Farmer Jones must decide what combination of seed, water, fertilizer, and pest control will be most profitable for the coming year. The local agricultural college did a study of this farmer's situation and prepared the table below.

Plan	Income/Acre	Cost/Acre
A	$ 800	$ 600
B	2600	2300
C	2250	1800

D	750	650

Help Farmer Jones make his decision; figure out which plan he should follow.

What criterion should you use?

1.A Minimize Income/Acre.

1.B Maximize Income/Acre.

1.C Maximize profit.

1.D Minimize Cost/Acre.

2. Now figure out which plan to use.

Answer	Plan	Income/Acre	Cost/Acre
2.A	A	$ 800	$ 600
2.B	B	2600	2300
2.C	C	2250	1800
2.D	D	750	650

Engineering Costs and Cost Estimating

An engineering economic analysis may involve many types of costs. Here is a list of cost types, including definitions and examples.

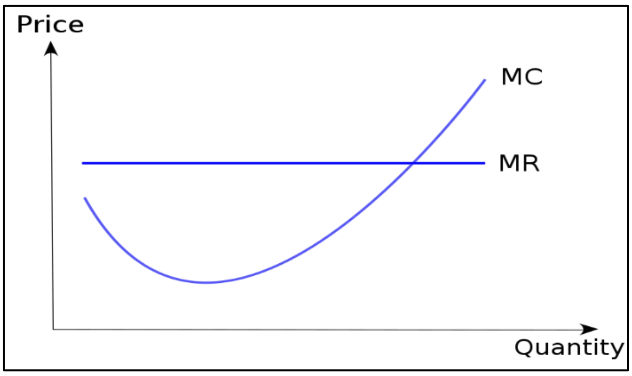
A **fixed cost** is constant, independent of the output or activity level. The annual cost of property taxes for a production facility is a fixed cost, independent of the production level and number of employees.

A **variable cost** does depend on the output or activity level. The raw material cost for a production facility is a variable cost because it varies directly with the level of production.

The **total cost** to provide a product or service over some period of time or production volume is the total fixed cost plus the total variable cost, where:

Total variable cost = (Variable cost per unit) (Total number of units)

A **marginal cost** is the variable cost associated with one additional unit of output or activity. A direct labor marginal cost of $2.50 to produce one additional production unit is an example marginal cost.



M**arginal cost** curve

The **average cost** is the total cost of an output or activity divided by the total output or activity in units. If the total direct cost of producing 400,000 is $3.2 million, then the average total direct cost per unit is $8.00.

The **breakeven point** is the output level at which total revenue is equal to total cost. It can be calculated as follows:

BEP = FC/(SP - VC)

Where BEP = breakeven point

FC = fixed costs

SP = selling price per unit

VC = variable cost per unit

A **sunk cost** is a past cost that cannot be changed and is therefore irrelevant غير ذي صله in engineering economic analysis. One exception is that the cost basis of an asset installed in the past will likely affect the depreciation schedule that is part of an after-tax economic analysis. Although depreciation is not a cash flow, it does affect income tax cash flow. Three years ago, an engineering student purchased a notebook PC for $2,800. The student now wishes to sell the computer. The $2,800 initial cost is an irrelevant, sunk cost that should play no part in how the student establishes the minimum selling price for the PC. كيف سيحدد الطالب الحد الأدنى لسعر البيع للكمبيوتر.

An **opportunity cost** is the cost associated with an opportunity that is declined. It represents the benefit that would have been received if the opportunity were accepted. Suppose a product distributor decides to construct a new distribution center instead of leasing a building. Leasing a building immediately would have resulted in a $12,000 product distribution cost savings during the next 6 months while the new warehouse is being constructed. By forgoing the warehouse leasing alternative, the distributor experiences an opportunity cost of $12,000.

A **recurring cost** المتكرره is one that occurs at regular intervals and is anticipated المتوقع. The cost to provide electricity to a production facility is a recurring cost.

A **nonrecurring cost** is one that occurs at irregular intervals and is not generally anticipated. The cost to replace a company vehicle damaged beyond repair in an accident is a nonrecurring cost.

An **incremental cost** represents the difference between some type of cost for two alternatives. Suppose that A and B are mutually exclusive investment alternatives. If A has an initial cost of $10,000 while B has an initial cost of $12,000, the incremental initial cost of (B - A) is $2,000. In engineering economic analysis we focus on the differences among alternatives, thus incremental costs play a significant role in such analyses.

A **cash cost** is a cash transactionالمعاملات , or cash flowالتدفق النقدي . If a company purchases an assetالاصول , it realizes a cash cost.

A **book cost** is not a cash flow, but it is an accounting entry إدخال محاسبي that represents some change in value. When a company records a depreciation charge of $4 million in a tax year, no money changes hands. However, the company is saying in effect that the market value of its physical, depreciable assets has decreased by $4 million during the year.

Life-cycle costs refer to costs that occur over the various phases of a product or service life cycle, from needs assessment through design, production, and operation to decline and retirement.

Question 1.

A company produces a single, high-volume product. One year its production volume was 780,000 units, its fixed costs were $3.2 million and its variable costs were $16 per unit. What was the company's total cost for the year?

Choose an answer by clicking on one of the letters below,

A $3,200,000

B $3,200,016

C $12,480,000

D $15,680,000

Question 2.

A company produces a single, high-volume product. One year its production volume was 780,000 units, its fixed costs were $3.2 million and its variable costs were $16 per unit. What was the company's average cost per unit produced?

A $20.10

B $4.10

C $16.00

D $36.10

Question 3.

A manufacturer purchased and installed a shrink-wrap machine 4 years ago at a cost of $4,000. A new machine is now needed, and one is available for $7,000 less a $1,000 trade-in allowance for the old machine. The market value of the old machine without trade-in on a new model is $500. Which of the four values above is a sunk cost in engineering economic analysis?

A $500

B $1,000

C $4,000

D $7,000

Question 4.

A manufacturer purchased and installed a production machine 6 years ago at a cost of $40,000. Since then the machine has been depreciated for tax purposes to a value of $7,000 and it now requires replacement. A new machine will be purchased for $60,000 and the old machine sold to a used equipment dealer for $10,000.

Which of the four dollar values above is a book cost, ?

A $7,000

B $10,000

C $40,000

D $60,000

Question 5.

A manufacturer produces and sells exactly 600,000 units of a single product annually. The fixed cost of the company is $3.6 million per year, and the variable cost is $47 per unit. In the coming year, the company is selling its product at a price of $56 per unit. Calculate the breakeven point (BEP) in units for the coming year.

A BEP is about 77,000 units

B BEP is about 64,000 units

C BEP is 400,000 units

D BEP is 600,000 units

Since there is a $9 profit on each unit ($56 - $47), you divided the fixed cost for all units manufactured, $3.6 million, by the unit profit to get 400,000 units.

PRE GENERAL OVERALL DESIGN CONSIDERATIONS

The development of the overall design project involves many different design considerations. Failure to include these considerations in the overall design project may, in many instances, alter the entire economic situation so drastically جذريا as to make the ventureالمشروع unprofitable. **Some of the factors involved in the development of a complete plant design** include:

- plant location,
- plant layout,

- materials of construction,
- structural design,
- utilities,
- buildings,
- storage,
- materials handling,
- safety,
- waste disposal,
- federal, state, and local laws or codes,
- patentsبراءات الاختراع .
- Record keeping and accountingالمحاسبه procedures

OPTIMUM DESIGN

In almost every case encountered by an engineer, there are several alternative methods which can be used for any given process or operation. For example, formaldehyde can be produced by catalytic dehydrogenation of methanol, by controlled oxidation of natural gas, or by direct reaction between CO and H, under special conditions of catalyst, temperature, and pressure.

Each of these processes contains many possible alternatives involving variables such as gas-mixture composition, temperature, pressure, and choice of catalyst.

It is the responsibility of the engineer, in this case, to choose the best process and to incorporate into the design the equipment and methods which will give the best results.

Optimum Economic Design

If there are two or more methods for obtaining exactly equivalent final results, the preferred method would be the one involving the least total cost. This is the basis of an optimum economic design. One typical example of an optimum economic design is determining the pipe diameter to use when pumping a given amount of fluid from one point to another. Here the same result (i.e., a set amount of fluid pumped between two given points) can be accomplished by using an infinite number of different pipe diameters. However, an economic balance will show that one particular pipe diameter gives the least total cost.

The total cost includes the cost for pumping the liquid and the cost (i.e., fixed charges) for the installed piping system.

A graphical representation showing the meaning of an optimum economic pipe diameter is presented in Fig. l-l. As shown in this figure, the pumping cost increases with decreased size of pipe diameter because of frictional effects, while the fixed charges for the pipeline become lower when smaller pipe diameters are used because of the reduced capital investment. The optimum economic diameter is located where the sum of the pumping costs and fixed costs for the pipeline becomes a minimum, since this represents the point of least total cost. In Fig. l-l, this point is represented by E.

When the engineer speaks of an optimum economic design, it ordinarily means the cheapest one selected from a number of equivalent designs.

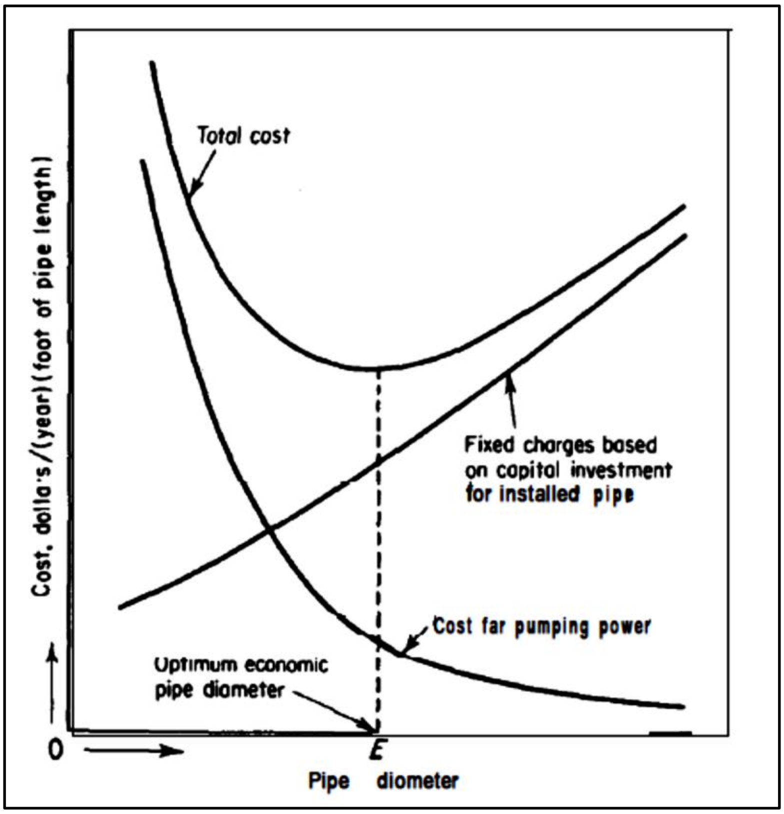


FIGURE 1.1 Determination of optimum economic pipe diameter for constant mass-throughput rate.

PRACTICAL CONSIDERATIONS IN DESIGN

Any engineer must never lose sight of the practical limitations involved in a design. It may be possible to determine an exact pipe diameter for an optimum economic design, but this does not mean that this exact size must be used in the final design. Suppose the optimum diameter were, 3.43 in. (8.71 cm).

It would be impractical to have a special pipe fabricated with an inside diameter of 3.43 in. Instead, the engineer would choose a standard pipe size, which could be purchased at regular market prices. In this case, the recommended pipe size would probably be a standard $3\frac{1}{2}$ in.-diameter pipe having an inside diameter of 3.55 in. (9.02 cm).

If the engineer happened to be very thoughtful about getting an adequate return on all investments. He or she might say, "A standard 3-in.- diameter pipe would require less investment and would probably only increase the total cost slightly; therefore, we should compare the costs with a 3-in. pipe to the costs with the $3\frac{1}{2}$ -in. pipe before making a final decision."

Theoretically, this engineer is correct in this case. Suppose the total cost of the installed $3\frac{1}{2}$ in. pipe is $5000 and the total cost of the installed 3-in. pipe is $4500. If the total yearly savings on power and fixed charges, using the $3\frac{1}{2}$ -in. pipe instead of the 3-in. pipe, were $25, the yearly percent return on the extra $500 investment would be only 5 percent. Since it should be possible to invest the extra $500 elsewhere to give more than a 5 percent return, it would appear that the 3 in. diameter pipe would be preferred over the $3\frac{1}{2}$ in.-diameter pipe.

The logic presented in the preceding example is perfectly sound.

Even though the optimum economic diameter was 3.43 in. the good engineer knows that this diameter is only an exact mathematical number and may vary from month to month as prices or operating conditions change. Therefore, all one expects to obtain from this particular **optimum economic calculation** is (**a good estimation).**

The preceding examples typify the type of practical problems the engineer encounters. In design work, theoretical and economic principles must be combined with an understanding of the common practical problems that will arise when the process finally comes to life in the form of a complete plant or a complete unit.

2nd Lecture

INTEREST AND INVESTMENT COSTS

According to the classical definition, interest is the money returned to the owners of capital for use of their capital. This would mean that any profit obtained through the uses of capital could be considered as interest. Modern economists seldom نادرا ما adhere to the classical definition. Instead, they prefer to substitute the term ***return on capital*** or ***return on investment*** for the classical ***interest.***
Engineers define interest as the ***compensation paid for the use of borrowed capital.***

Interest can be distinguished by:

1. The rate at which interest will be paid is usually fixed at the time the capital is borrowed.
2. A guarantee is made to return the capital at some set time in the future or on an agreed upon pay off schedule.

The observation that a dollar today is worth more than a dollar in the future means that people must be compensated for lending money. The compensation for loaning money is in the form of an interest payment.

Interest is the difference between the amount of money lent and the amount of money later repaid. It is the compensation for giving up the use of the money for the duration of the loan.

Or: **Interest is the rental value of money; it is the amount of money or fee paid for using somebody else's money.**

An amount of money today called the **principal amount** (P), can be related to a **future amount** (F) by the interest amount **interest rate (**I), and can be expressed as $F = P + I$. The interest I can also be expressed as an interest rate i with respect to the principal amount so that $I = Pi$. Thus

$F = P + Pi = P(1 + i)$

Time Value of Money

The following are reasons why today money is worth more than one year later money

1. Inflation
2. Risk
3. Cost of money

Of these three items, the cost of money is the most predictable, and hence, it is the essential component of economic analysis. Cost of money is represented by: either by money paid for the use of borrowed money or the return on investment.

Cost of money is determined by an interest rate.

Time value of money is defined as the time-dependent value of money due to both changes in the purchasing power of money (inflation or deflation) and from the real earning potential of the alternative investments over time وتعرف القيمة الزمنيه للنقود بأنها قيمه الأموال المعتمدة علي الوقت بسبب التغيرات في القوه الشرائية للأموال (التضخم أو الانكماش) وعلى فرص الكسب الحقيقية للاستثمارات البديلة مع مرور الوقت

EXAMPLE 1:

A man bought a one-year guaranteed investment certificate for $5000 from a bank on May 15 last year. The bank was paying 10 percent on one-year GIC at the time. (A **GIC**: **guaranteed investment certificate** is a safe and secure investment with very little risk. You don't have to worry about losing your money because it is guaranteed. A GIC works like a savings account in that you deposit money into it and earn interest on that money.)

Solution

One year later, the man cashed in his certificate $5500.

Interest Periods

The most commonly used interest period is one year. If we say, for example, “6 percent interest” without specifying an interest period, the assumption is that 6 percent interest is paid for a one-year period. However, interest periods can be of any duration. Here are some other common interest periods:

Interest Period Interest is calculated:

Semiannually: Twice per year, or once every six months

Quarterly: Four times a year, or once every three months

Monthly : 12 times per year

Weekly : 52 times per year

Daily : 365 times per year

Continuous For infinitesimally small periods

TYPES OF INTEREST

Simple Interest

To illustrate the basic concept of interest, an additional notation will be used

F(N)= Future sum of money after N periods

Then for simple interest $F(1)= P+(P)(i)= P(1+i)$

and $F(N)= P+(P)(N)(i)= P(1+Ni)$ الفائدة لكذا من السنين

Interest = (Principal) (Number of periods) (Interest rate)…………….(1)

$I=P(N)(i)$

For **example,** if $100 were the compensation demanded for giving someone the use of $1000 for a period of one year, the principal would be $1000, and the rate of interest would be l00 (profit)/l000(P) = 0.1 or 10 percent/year.

The simplest form of interest requires compensation payment at a constant interest rate based only on the original principal. Thus, if $1000 were loaned for a total time of 4 years at a constant interest rate of 10 percent/year, the simple interest earned would be

$1000 x 0.1 x 4 = 400 $

Example 1

A 100$ at a 10% per year for 5 years yields?

Solution

F(5)=100{1+(5) (0.1)}=150 $

Example 2

A 100$ is loaned for 3 years at a simple interest rate of 10% per annum what will be the interest earned?

Solution

I= (100) (0.1)(3)= 30$

While the total amount earned at the end of 3 years would be 100$+30$=130$

Example 3

An employer borrow 10 000$ on last April and must repay 10 700 $ 1 year later. Determine the interest amount and the interest rate paid? Note: use the simple interest law.

Solution

I(interest)= Amount owned now (F)- Original amount (P)

Interest= 10700-10000=700$

Interest paid for one year=

Interest rate= $\frac{\textit{Interest accurred per time unit}}{\textit{Original amount}}$**x100**

$=\frac{700}{10000}$ **x**100= 7% per year

Example 4

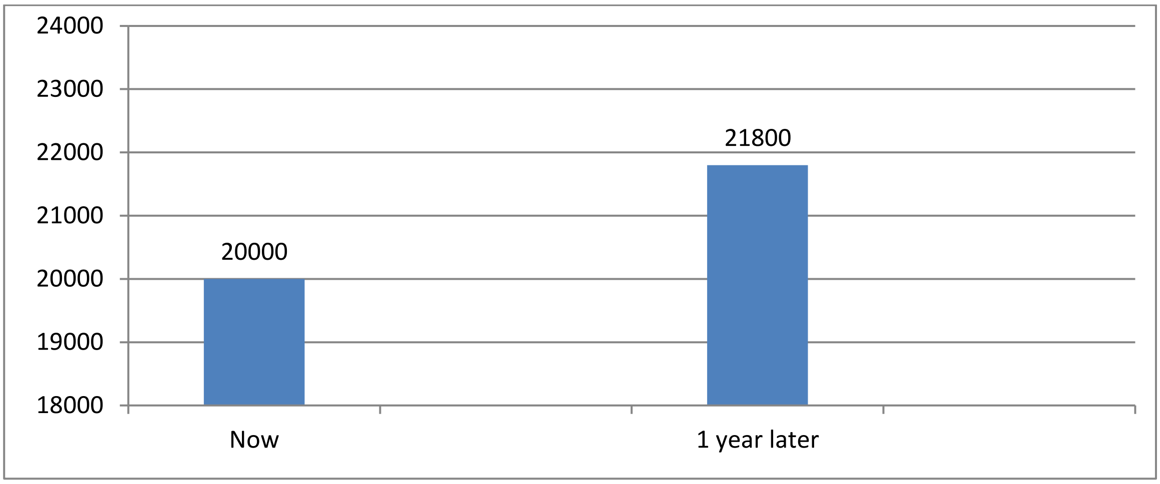
A contractor plans to borrow 20 000 $from a bank for one year at 9% interest to buy a new shovel equipment. A) Compute the interest and the total amount due to after 1 year. B) Construct a column graph that shows the original amount and total amount due to after one year used to compute the loan interest rate 9% per year.

Solution

A) I = NPi=1 X 20000 X 0.09X=1 800 $

The total amount due to after 1 year **20,000+ 1,800=21, 800 $**

B)



Example 5

a. Calculate the amount of deposit 1 year ago to have 1000 $ now at an interest rate of 5% per year.
b. Calculate the amount of interest earned during this time period.

<u>Solution</u>

a. F=**P + NiP** **1000= X+0.05X**

X=952.38 $

b. i=**F-P** i= **(1000-**952.38)$

i= 47.62$

Example 6

A contractor loaned money to design a wastewater treatment plant. The loan was 10000 $ for three years at 5% per year simple interest. How much money will the contractor repay at the end of the 3 years?

F= 10000+ 3 x 0.05 x 10000

F =11500$

Example (7)

Find out simple interest of a loan of (100 CU) for January, Feb., and March at (10%)

Solution:

F=P (1 + i n)

= 100(1 + 0.1(3/12) = 102.5 CU

If we want to find the exact value the solution will be:

F = 100(1 + 0.1 (31+28+31)/365)= 102.46 CU

Note: CU is currency unit وحده العملة which may be Iraqi Dinar or Dollar or Yen etc.

Compound Interest

Interest capital, has a time value. If the interest were paid at the end of each time unit, the receiver could put this money to use for earning additional returns. Compound interest is to include interest on the interest. Thus, an initial loan of $1000 at an annual interest rate of 10 percent would require payment of $100 as interest at the end of the first year. If this payment were not made, the interest for the second year would be ($1000 + $100)(0.10) = $110,

and the total ***compound amount*** due after 2 years would be

$1000 + $100 + $110 = $1210

The effect of compounding of interest can be shown in the following calculation:

$F(1) = P + Pi = P(1+i)$

$F(2) = F(1) + F(1)i = F(1)(1+i) = P(1+i)(1+i) = P(1+i)2$

$F(3) = F(2) + F(2)i = F(2)(1+i) = P(1+i)2(1+i) = P(1+i)3$

$F(4) = F(3) + F(3)i = F(3)(1+i) = P(1+i)^3(1+i) = P(1+i)^4$

$F(n) = P(1+i)^n$

$S = P(l + i)^n$

Interest= (Principal +all accruedالمستحقه interest)x(interest rate)

Example 8

A man had $800 stashed مخباه under his mattress for 30 years. How much money has he lost by not putting it in a bank account at 8 percent annual compound interest all these years?

Solution

We can think of the $800 as a present amount and the amount in 30 years as the future amount, Given: P = $800, i = 0.08 per year, N = 30 years

$F = P(1 + i)^N = 800(1 + 0.08)^{30} = \$\ 8050.13$

He suffered an opportunity cost of $8050.13 – $800 = $7250.13 by not investing the money

Example 9

An engineer deposited amount of money 1000$ in a bank for three years at an interest rate of 10%. Calculate the cumulative amount of money for each year (compound interest).

Solution

F(3)= 1000(1+0.10)^3 = 1331$

The table below describes the cumulative amount of money each year

End of year	Amount of money owed at the beginning of year$	Interest charge for year $	Amount owed at the end of year$
1	1000	0.1x10000=100	1100
2	1100	0.1x 1100=110	1100+110=1210
3	1210	0.1x1210=121	1210+121= 1331

Example 10

What will $100 yield at 10% per year for 5 years (compound interest)?

F(5)=100(1+0.1)^5 = $ 161.05

Example 11

What will the $ 1000 be at the end of a 3 year investment period at an interest of 8%

Solution

F=$ 1000 (1+.08)^3

F=$ 1259.71

Example 12

Determine the present amount that is economically equivalent to $3000 in five years, given the investment potential of 8% per year

Solution

F=p(1+.i)^n

$3000=P(1+.08)^5

P=$ 2042

Compound Amount Factor

In the formula for finding the future value of a sum of money with compound interest, the mathematical expression

$(1+i)^n$ is referred to as the compound amount factor, represented by the functional format (F/P, i, n) and the future amount will be F= P(F/P, i , n).

Interest Tables

Values of the compound amount factor, present worth, and other factors, are tabulated for a variety of interest rates and number of periods in most text on engineering economy. Example tables are presented in the associated appendix.

Cash-Flow Diagrams

Cash flow is the sum of money recorded as receipts(مبالغ مستلمة ايصالات)or disbursements المدفوعات in a financial records. A cash flow diagram presents the flow of cash as arrows on time line scaled to the magnitude of the cash where expenses are down arrows and receipts are up arrows.

Cash flow notes:

1. The horizontal line is a time scale with progression of time moving from left to right. The period (mostly year) labels are applied to intervals of time rather than points on time scale. For example the end of period 2 is coincident with the beginning of period 3. Only if specific dates are employed should the points in time rather than periods be labeled.
2. The arrows signify cash flows. If a distinction needs to be made, downwards represents disbursements (negative cash flows or cash outflows) and upward arrows represent receipt (positive cash flow or cash inflow).
3. The cash flow diagram is dependent on point of view. For example, the situation shown in figures below were based on cash flow as seen by the lender. If the direction of all arrows had been reversed, the problem would have been diagrammed from borrower s viewpoint.

Example 13

If you had \$2000 now and invested it at 10% interest compounded annually, how much would it be worth in eight years?

Solution

This problem can be solved in three ways:

1. Using a calculator

$F=\$ 2000(1+.1)^8$

$F=\$ 4287.18$

2. Using compound interest tables: The interest tables can be used to locate compound amount factor for i = 10% and n=8. The number you get can be substituted into the equation.

   F=\$2000 (F/P, 10%, 8)= \$2000(2.1435)=\$4287.20

Example 14

A person wants to receive amount of money equal to \$8500 after 5 years. How much money has to deposit now in bank with compound rate interest 7%?

Solution

$F=P(1+i)^n$

$\$8500=P(1+0.07)^5$ P= \$ 6060.38

Or using the tables

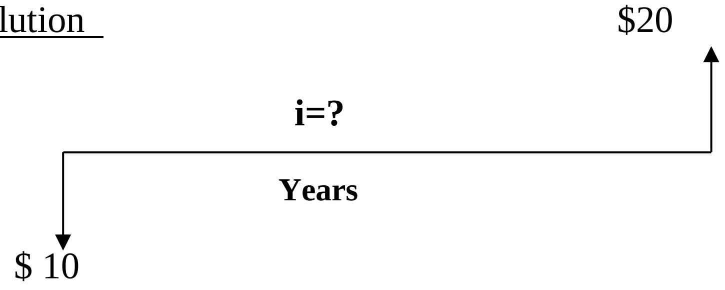
i=7%, n=5 the factor P/F=0.71298= 0.7130

P=8500 (0.7130)=\$6060.5

Example 15

Suppose you buy a share of stock for \$10 and sell it for \$20 your profit is thus \$10. If that happens within a year 5, your rate return is an impressive 100% (Profit/ present amount). If it takes five years, what would be the rate interest on your investment?

Solution



$\$ 20=\$10(1+i)^5$

Solve for i by one of the following methods

Method 1. Go through a trial and error process in which you insert different values of i into the equation until you find a value works in the sense that the right hand side of the equation equals $20. The solution value is i= 14.87%. The trial and error procedure is extremely tedious and inefficient for most problems, so it is not widely practiced in the real world.

Method 2. Using the interest table, starts with equation:

$\$20=\$10(1+i)^5$

$2=1(1+i)^5$

Now look across the n= 5 row under the (F/P,I , n=5) column until you can locate the value of 2 . This value is approximated in the 15% interest table at (F/P, 15%, 5) = 2.0114, so the interest rate at which $10 grows to 20$ over five years is very close to 15%. This procedure will be very tedious for fractional interest rates or when n is not a whole integer, as you may have to approximate the solution by linear interpolation.

Method 3.The most practical approach is to use either a financial calculator or electronic spreadsheet such as Excel. A financial function such as ***Rate (N;0;p;F)*** allows us to calculate an unknown interest rate. The precise command statement would be as follows:

=RATE(5;0;-10;20)= 15%

Note that we enter the present P value as a negative number in order to indicate a cash outflow in Excel.

Example 16

A contractor wants to borrow $20000 for buying a new equipment but he needs to select the bank that impose a small rate interest, he likes to repay this loan after 10 years equal to 35000. Which is the best i interest for this decision?

Solution

1. Use the trial and error, assume i=8%

$35000=20000(1+.08)^{10} = \$43178.5 \neq \$35000$

Use a lower rate interest 6%

$35000=20000(1+.06)^{10}$ = \$35816.95

Almost i=6%

2. Use the Excel sheet and use the function RATE.

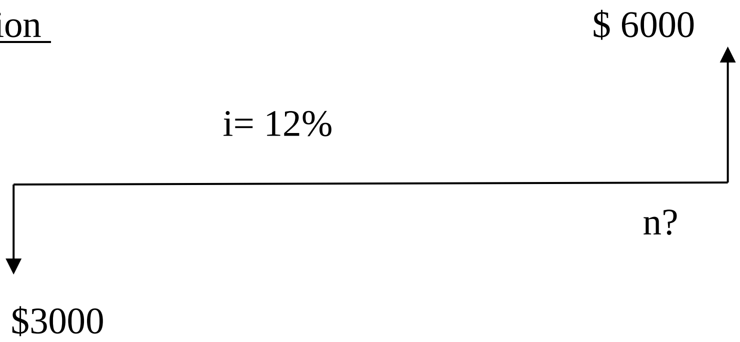
   =RATE(n;0;-P;F)

   =RATE(10;0;-20000;35000)= 6%

Example 17

You have just purchased 100 shares of General Electric stock at \$30 per share. You will sell the stock when its market price doubles. If you expect the stock price to increase 12% per year, how long do you expect to wait before selling the stock?

Solution



$F=P(1+i)^n$ $\quad$ $6000=3000(1+.12)^n$ $\quad$ $2=(1.12)^n$

1. **Using a calculator**

Log 2=n log 1.12 $\quad$ n=log2/log 1.12 = 6.11 ≈ 6 years

2. **Using spread sheet program**: The excel command would look like this:

   =NPER(i;0;-P;F)

   =NPER(0.12;0;-3000;6000) = 6.11≈ 6 years

3rd Lecture
INTEREST AND INVESTMENT COSTS
Equivalent Uniform and Uneven annual series

Compound Amount Factor: Find F, Given A, i and n

Suppose we are interested in the future amount F of a fund, to which we contribute المساهمه (A)dollars each period and on which we earn interest at a rate of i per period. The contributions are made at the end of each of the N periods. These transaction المعاملات are graphically illustrated in Fig. .Looking at this diagram, we see that if an amount A is invested at the end of each period for n periods, the total amount F that can be withdrawn at the end of n periods will be the sum of the compound amounts of the individual deposits.

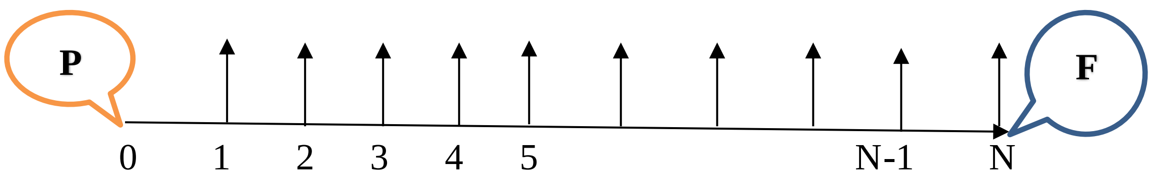


Fig. Equal series find equivalent P or F.

As shown in Fig., the A dollars we put into the fund at the end of the first period will be worth $A(1+i)^{n-1}$ at the end of n periods. The A dollars we put into the fund at the end of the second period will be worth $A(1+i)^{n-2}$ and so forth, Finally , the last A dollars that we contribute at the end of the nth period will be worth exactly A dollars at that time. This means we have a series in the form

$$F = A(1+i)^{n-1} + A(1+i)^{n-2} + \dots\dots\dots\dots A(1+i)^{n-n-1} + A(1+i)^{n-n} \quad (1)$$

Or expressed alternatively,

$$F = A + A(1+i) + A(1+i)^{2} + \dots\dots\dots\dots A(1+i)^{n-1} \quad (2)$$

Multiplying Eq (2) by (1+i) results in

$F(1+i) = A(1+i) + A(1+i)^2 + A(1+i)^3 + \ldots\ldots\ldots\ldots\ldots\ldots\ A(1+i)^n$ (3)

Subtracting 2 from 3 to eliminate common terms gives us:

$F(1+i) - F = -A + A(1+i)^n$, $F + Fi - F = A[(1+i)^n - 1]$

$F = A\left[\frac{(1+i)^n - 1}{i}\right] = A\ (F/A, I, n)$ A: هو حجم الدفعة المنتظمة F: هوالمبلغ المستقبلي لهذه الدفعات

The bracted term is called **equal- payment-series compound amount factor** or the **uniform-series compound- amount factor (معامل التسلسل لمجموع الكميات المتراكبة).** This interest factor has been calculated for various combinations of I and n in the tables associated with this lecture.

Example 3.1

Suppose you make an annual contribution of $5000 to your savings account at the **end** of each year for five years. If your savings account earn 6% interest annually, how much can be withdrawn at the end of five years?

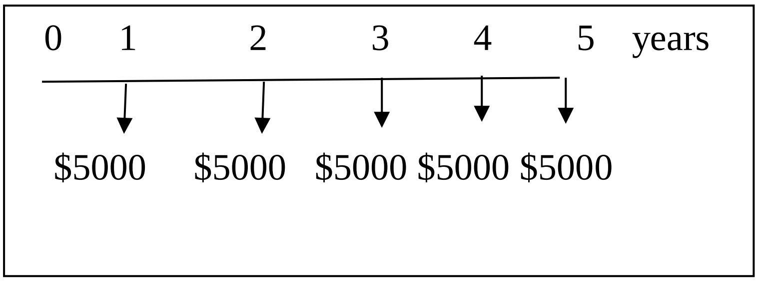


Figure. Equal Payment series compound amount of saving money

Solution:

F= $A\left[\frac{(1+i)^n - 1}{i}\right] = \$5000\left[\frac{(1+.06)^5 - 1}{.06}\right] = \$5000 \times 5.63709 = \$28185.46$

Or we can find by using the equal –payment –series compound –amount factor, we obtain:

F= $5000 (F/A, 6%, 5) = $5000(5.6371) = $28,185.46

To obtain the future value of the annuity on Excel, we may use the following financial command: =FV (.06;5;5000;0).

Example 3.2

A fund is setup with an initial investment of $1000 to provide uniform year-end payment each year for six year. How much will be the payment amount to; if the investment is made at an interest rate of 3% and fund is to be completely exhausted by the sixth payment?

Solution

$A=P\left[\frac{(1+i)^n\, i}{(1+i)^n-1}\right]=\$1000\left[\frac{(1+.03)^6 X\, .03}{(1+.03)^6-1}\right]=\$\ 184.60$

Example 3.3

A uniform annual investment is to be made into a sinking fund to provide a capital at the end of 7 years for the replacement of a contractor. An interest rate of 2.5% is available. What is the annual investment to provide the $ 15, 000 ?

Solution

$\mathbf{A=F}\left[\frac{i}{(1+i)^n-1}\right]=\mathbf{\$15000}\left[\frac{.025}{(1+.025)^7-1}\right]=\mathbf{\$1987.5}$

Example 4.4

Given an interest of 5% per year what sum of money would be accumulated after 6 years if $200 were invested at the end of each year for the 6 years?

Solution

$\mathbf{F=}\ A\left[\frac{(1+i)^n-1}{i}\right]=\ \$200\left[\frac{(1+.05)^6-1}{.05}\right]=\$1360$

Example 3.5

With an interest of 6% what uniform end of period pay must be made for 10 years to repay an initiated debt of $2000

Solution

$A=P\left[\frac{(1+i)^n\, i}{(1+i)^n-1}\right]=\$2000\left[\frac{(1+.06)^{10} X\, .06}{(1+.06)^{10}-1}\right]=\$\ 271.72$

XXXXXXxxExample 3.6

A. $10000 at the end of ten years from now was equivalent to x if the interest rate is 5% what is x

Solution

$\mathbf{P=F(1+i)^{-n}\ =\$10000\ (1+.05)^{-10}}$

P= $6139= x **كانت مكتوبة F=**

B. **X at the end of 5 years**

X=F= p(1+i)^n= $6139(1+ .05)^5 = $7833.40

Example 3.7

If the initial cost to purchase a machine is $ 10000 and the annual maintenance cost is $200 for 8 years (its useful life). If the annual interest rate i= 6% and the machine had no salvage value (ليس له سعر عند اعادته او تدويره). What is the cost of the equivalent annual mechanism? (ماهي القيمة المكافئة لقيمة تدهور الجهاز سنويا) Ignore the values of fees and labor costs.

Solution

$$A=P\left[\frac{(1+i)^n\, i}{(1+i)^n-1}\right]= \$10000\left[\frac{(1+.06)^8 X\, .06}{(1+.06)^8-1}\right]= \$\ 1610.30$$

The total equivalent cost =$ 1610.30 + $200 = $1810.30/ year

Example 3.8

A man wants to save money for his children that they can have $10000 at the end of period of 15 years from now. He paid $500 for 15 years each year starting a year later from now. What should the interest be?

Solution

$$F= A\left[\frac{(1+i)^n-1}{i}\right]$$

$$\$10000= \$500\left[\frac{(1+i)^{15}-1}{i}\right] \qquad \text{divide by 500}$$

$20 i= (1+i)^{15} -1

Solve either by trial and error or by logs

Log 20 + log i= 15[log 1+ log i] - log 1

Log 20 = 15 log i –logi = 14 log i

1.310=14 logi

0.929= log i

I=4%

Present Value of an Uneven Series by Decomposition into Single Payments

Example 3.9

Wilson Technology, a growing machine shop, wishes to set aside money now to invest over the next four years in automating its customer service department. The company can earn 10% on a lump sum deposited now, and it wishes to withdraw the money in the following increments:

Year 1: \$25,000, to purchase a computer and database software designed for customer service use;

Year 2: \$3,000, to purchase additional hardware to accommodate anticipated growth in use of the system;

Year 3: No expenses; and

Year 4: \$5,000, to purchase software upgrades.

How much money must be deposited now to cover the anticipated payments over the next 4 years?

<u>Solution</u>

Calculate the equivalent present value of each single cash flow and then to sum the present values to find P. In other words, the cash flow is broken into three parts as shown in figures below.

i=10% per year

P1 + P2 +P3 +P4

$P1=\$25000/(1+.1)^1 =\$22727.2$

$P2= \$3000/(1+0.1)^2 =\$2479.3$

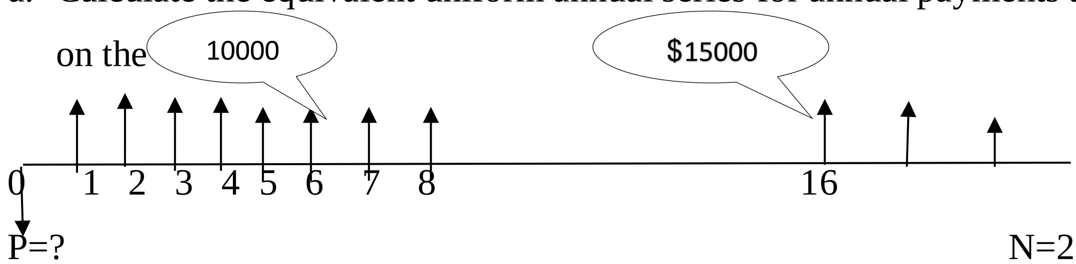
$P4=\$5000/(1+0.1)^4= \$3415.3$

22727.2 +\$2479.3+ \$3415.3=\$ 28622

To see if \$ 28622 is indeed a sufficient amount lets calculate the balance at the end of each year. If you deposit \$ 28622 now, it will grow to $F= P (1+0.1)^1 = \$28622(1.1) = \$31484$ at the end of year one. From this balance, you pay out \$25000. The remaining balance \$6484 will again grow to \$6484(1.1) = 7132 at the end of year two. Now you make the second payment \$3000 out of this balance which will leave you with \$4132 at the end of year two. Since no payment occurs in year three the balance will grow to \$4132(1.1) =\$5000 at the end of year four. The final withdrawal in the amount of \$5000 will deplete the balance completely.

Example 3.10

a. Calculate the equivalent uniform annual series for annual payments as shown on the



i= 6%,

b. We wish to have 10000\$ at year 6
c. We wish to have 15000\$ at year 16
d. wish to gain annual payment of A= 2000\$ for 20 years

Solution

$P1=F6\,(1+i)^{-n}$ = \$10000$(1+.06)^{-6}$ = 7049.6 \$ نريد الحصول على \$10000 في السنة 6

$P2=F16(1+.06)^{-n}$ = 15000\$$(1+.06)^{-16}$ =5904. 69 \$ نريد الحصول على \$15000 في السنة 16

$P_A= A\left[\frac{(1+i)^n-1}{(1+i)^n i}\right]= \$2000\left[\frac{(1+.06)^{20}-1}{(1+.06)^{20}\,.06}\right]= \$\,229398.4$ نريد الحصول على دخل سنوي \$2000

Pt= P1 + P2 + P3 = 7049.6 \$ +5904. 69 \$ +\$ 229398.4 = \$ 242352

لايجاد سلسلة الدفعات السنوية المكافئة

$A=P\left[\frac{(1+i)^n\, i}{(1+i)^n-1}\right]= \$242352\left[\frac{(1+.06)^{20} X\, .06}{(1+.06)^{20}-1}\right]= \$\,21\,129.4\$$

That is to have 21 129.4 \$ each year for 20 years.

Example 3.11

Calculate the equivalent annual uniform payments for an amount of future money of 10000\$ after 10 yrs. Where i =0.06

$A=F\left[\frac{i}{(1+i)^n-1}\right]=\$10000\left[\frac{.06}{(1+.06)^{10}-1}\right]=$ <u>\$759.49</u>

Or

P= $10000(1+.06)$^{-10}$ =5583.94$

A=P $\left[\frac{(1+i)^n\, i}{(1+i)^n - 1}\right]$ **= 5583.94** $\left[\frac{(1+.06)^{10} X .06}{(1+.06)^{10} - 1}\right]$ **=** **<u>$ 758.67</u>**

Checking

F= A $\left[\frac{(1+i)^n - 1}{i}\right]$ **= $759.45** $\left[\frac{(1+.06)^{10} - 1}{.06}\right]$ **= $9999.951**

Handling Time Shifts in a Uniform Series

As mentioned above, the first deposit of the five –deposit series was made at the end of period one and the remaining four deposits were made at the end of each following period. Suppose that all deposits were made at the beginning of each period instead. How would you compute the balance at the end of period five?

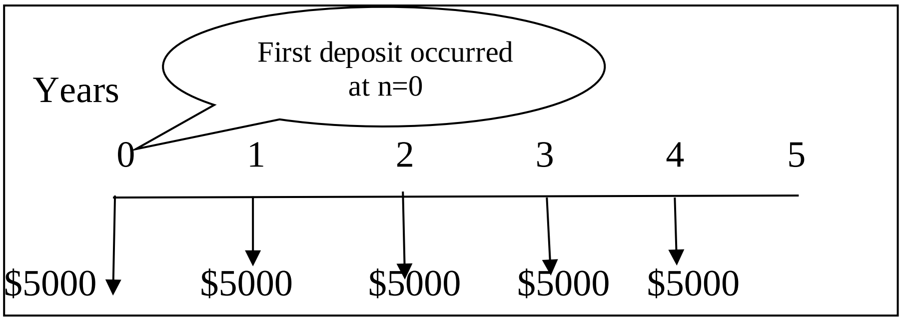


Figure Handling Time Shifts in a Uniform series.

Compare the figure above each payment had been shifted one year earlier; thus, each payment is compounded for one extra year. Note that with **<u>end- of- year</u>** deposit the ending balance F was $28185.46. While with **<u>beginning –of-year</u>** deposit, the same balance accumulates by the end of period four. This balance can earn interest for one additional year. Therefore, we can easily calculate the resulting balance as

F5=$28185.46(.06+1)^{1} = $29876.59

Or

Following financial command available on Excel

=FV (6%; 5; 5000; 1)

Or

F total= \$28185.45+\$5000$(1+.06)^1$ - \$5000=\$29876.59

By adding the \$5000 deposit at period zero to the original cash flow and subtracting the \$5000 deposit at the end of period five, we obtain the second cash flow.

Sinking –Fund Factor:

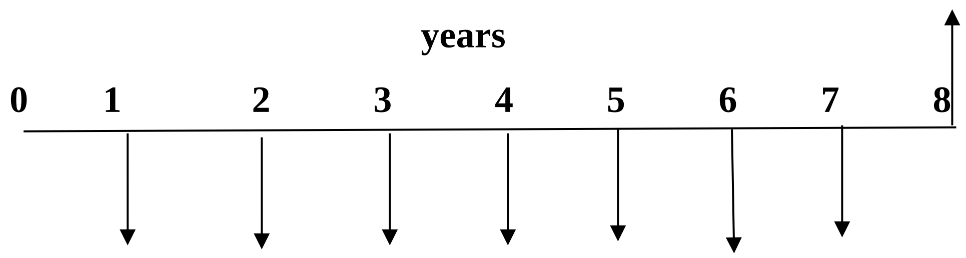
$$\mathbf{F=A}\left[\frac{(1+i)^n-1}{i}\right] = A(F/A, i, n)$$

$$\mathbf{A=F}\left[\frac{i}{(1+i)^n-1}\right] = F\ (A/F, i, n)$$

The term within the brackets is called the equal –payment –series sinking-fund factor or just sinking – fund factor. A sinking fund is an interest-bearing account into which a fixed sum is deposited each interest period, it is commonly established for the purpose of replacing fixed assets.

Example 2.12,

You were asked by your manager to set up a saving plan for your organization. Your organization needs a comprehensive maintenance and will be executed after 8 years from now. The maintenance plan will need at least \$ 100,000 in the bank. How much do you need to save each year in order to have the necessary funds if the current rate of interest is 7%? Assume that end –of –year payments are made.



I=.07, A=?

Solution

$$\mathbf{A=F}\left[\frac{i}{(1+i)^n-1}\right] = \mathbf{\$\ 100{,}000}\left[\frac{.07}{(1+.07)^8-1}\right] = \$9,746.77$$

Or using the sinking –fund factors, we obtain

A=$ 100,000 (A/F; .07;8)

=$9,746.78

Example 3.13

You want to purchase a new shovel, there is two alternatives the first one is to pay the present cost equal to 45, 000 and the second one is to pay the annual payments to collect the future amount is $50, 000 after 5 years with rate interest 6%. Which alternative will be selected? Discuss them.

Solution

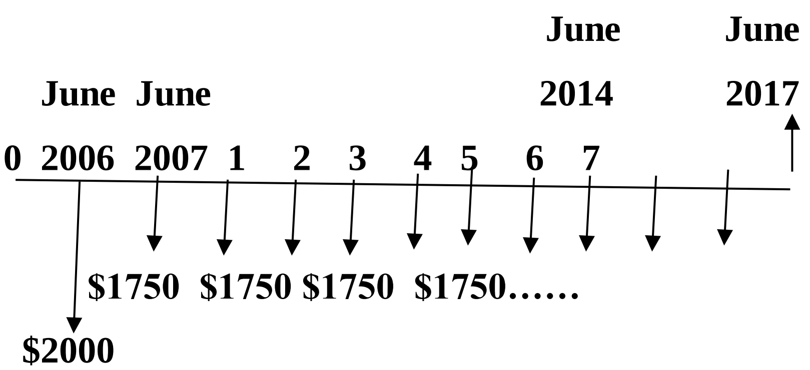
$A=F\left[\frac{i}{(1+i)^{n}-1}\right]=$

$A=50000\left[\frac{.06}{(1+.06)^{5}-1}\right]=\$50000\ (0.17739)=\$\ 8869.5$

The second alternative will be selected for saving the initial price although it pays $ 8869.5 each year in the second alternative.

Example 3.14

A contractor deposited in the bank at June of 2006 amount of money equal $2000, and in the next year June of 2007 he begun to deposit every year amount of money for seven year (till to 2014) a uniform series payment equal to $1750 then he planned to withdraw the total money at the June 2017. How much money he can receive? The rate interest of the bank=7%.



Solution

$F1= P(1+i)^{n} = 2000(1+.07)^{8} =\$3436.2$

F2= $A\left[\frac{(1+i)^n-1}{i}\right] = \$1750\ \left[\frac{(1+.07)^8-1}{.07}\right] = \$17954.65$

F1+F2=\$3436.2 +\$17954.65= \$21, 390.85

F Total= $P(1+i)^n$ = \$21, 390.85 $(1+.07)^3$= \$26, 204.71

Capital- Recovery Factor (Annuity Factor):

We can determine the amount of a periodic payment, A if we know P, I, and n.

$F=P(1+i)^n$ by replacing F we get

$$F= A\left[\frac{(1+i)^n-1}{i}\right]$$

$$P(1+i)^n = A\left[\frac{(1+i)^n-1}{i}\right]$$

$$A= P(1+i)^n\left[\frac{i}{i(1+i)^n-1}\right]$$

$$A=P\left[\frac{(1+i)^n\, i}{(1+i)^n-1}\right] \qquad (12)$$

$$P= A\left[\frac{(1+i)^n-1}{i(1+i)^n}\right]$$

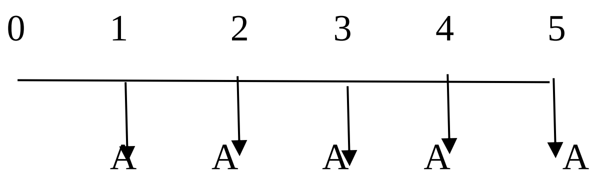
Now we have an eq. for determining the value of the series of end of period payments, A , when the present sum P is known. The portion within the brackets is called the **equal- payment series capital recovery factor,** or simply **capital recovery factor** which is designed (A/P, I, n). In finance, this A/P factor is referred to as the **annuity factor**. The annuity factor indicates a series of payments of a fixed or constant, amount for a specified number of periods.

Paying off an Educational Loss: Find A, Given P/I and i

<u>Example 3.15</u>

You borrowed \$ 21061.82 to finance the educational expense for your senior year of college. The loan will be paid off over five years. The loan carriers an interest rate of 6% per year and is to be repaid in equal annual installments اقساط over the next five years. Assume that the money was borrowed at the beginning of your senior year and that the first installment will be due a year later. Compute the amount of the annual installations.

I=.06



P= \$21061.82   i= .06   n= 5   A=?

A=P$\left[\frac{(1+i)^n\, i}{(1+i)^n-1}\right]$= \$21061.82 $\left[\frac{(1+.06)^5\ .06}{(1+.06)^5-1}\right]$= \$5,000

Or using the capital –recovery factor from table, we obtain

A=\$21,061.82 (A/P, 6%, 5)

=\$21061.82 (0.2374) = \$5000

Or the Excel solution using annuityالقسط السنوي function commands as follows:

=PMT (I; n; P) = PMT(.06; 5; 21061.82)

The result of this formula is \$5000

Example 3.16

The primary calculation – performed by contractor- of the expenses for leasing a construction plantتاجير مصنع للبناء equal to \$ 850 per year this expenses involved the rent and maintenance for 5 years (duration of project life) with rate of interest is 4%. Now the contractor has amount of money \$ 3000. How much the total of the present amount has to have which be equivalent to annual payment?

Solution

P= A$\left[\frac{(1+i)^n-1}{i(1+i)^n}\right]$ = \$850 $\left[\frac{(1+.04)^5-1}{.04(1+.04)^5}\right]$ = \$ 3784.03

The additional amount that should be existing with the contractor in addition to the existing money is

\$3784.03- \$3000= \$784.03

Example 3.17

Below the different payments were deposited in the bank with their dates, what is the total future amount for them at the end of 9th year the compound rate interest is 5%. Draw also the cash flow diagram

{500+750(P/A, 5%, 5)+1000(P/F, 5%, 6)}[F/P, 5%, 9]

Solution

{500+ Eq1+ Eq2}Eq3

$P=[A\left[\frac{(1+.05)^5-1}{.05(1+.05)^5}\right] = \$750\left[\frac{(1+.05)^5-1}{.05(1+.05)^5}\right]$

$P=F\left(\frac{1}{(1+i)n}\right)= 1000(1/(1+.05)^6]* Eq3$

$Eq3=F= P(1+i)^n = (1+.05)^9$

[(500+ 750 x 4.3294 + 1000 x 0.74621) * 1.5513= $6970.30

Example 3.18

Someone wanted to invest amount of money that will be paid in regular equal payments of $500 for the next 11 yrs, he has now $2 400 what total present worth should be ? and how much equivalent in the 7th yr., i= .08

Solution

$P=A(500)\left[\frac{(1+.08)^{11}-1}{.08(1+.08)^{11}}\right]$ = 3569 I.D

3569+ 2400= 5969 I.D

$F=5969(1+.08)^7$

Nominal i and Effective i

Interest rates may be stated for some period, such as a year, while the computation of interest is based on shorter compounding subperiods such as months. In this section we consider the relation between the *nominal* interest rate that is stated for the full period and the *effective* interest rate that results from the compounding based on the subperiods.

This relation between nominal and effective interest rates must be understood to answer questions such as: How would you choose between two investments, one bearing 12 percent per year interest compounded yearly and another bearing 1 percent per month interest compounded monthly? Are they the same?

Nominal interest rate is the conventional method of stating the annual interest rate. It is calculated by multiplying the interest rate per compounding period by the number of compounding periods per year.

Many banks for example state the interest arrangement for credit cards in the following manner " % 18 " compounded monthly

This statement means simply that each month the bank will charge (1.5% interest 12 months per year= 18% per year)

Now 18% is the nominal interest or annual percentage rate (APR) and that the compounding frequency is monthly (12 times a year), although the APR is commonly used by financial institutions and is familiar to many customers, it does not explain precisely the amount of interest that will accumulate in a year.

To explain the true effect of more frequent compounding on annual interest amounts, we will introduce the term effective interest rate, commonly known as annual effective yield, or annual percentage yield (APY).

Annual Effective Yield or Effective Interest Rate is the actual but not usually stated interest rate, found by converting a given interest rate with an arbitrary compounding period (normally less than a year) to an equivalent interest rate with a one-year compounding period, or it is the annual effective yield that truly represents the interest earned in a year. On a yearly basis you are looking for a cumulative rate 1.5% each month for 12 times. This cumulative rate predicts the actual interest payment on your outstanding credit card balance

We could calculate the total annual interest payment for a credit card debt of $1000 by using the formula given

If p= $1000, i= 18%, n =12, the rate interest compounded monthly, is to find ieffective

i effective = 0.18/ 12= .015

$F= P(1+i)^n = \$1000(1+.015)^{12}$ $\$1000(1+.18)= 1180.$

F= $1 195.62

Clearly, the bank is earning more than 18% on your original credit card debt. In fact $195.62. The implicationالاثار is that, for each dollar owed, you are paying an equivalent annual interest of 19.56 cents.

Time period: the basic time unit of the interest rate. This the n statement of i% per time period. The time unit of one year is by far the most common. It is assumed when not stated otherwise.

Compounding Period (CP): The time unit used to determine the effect of interest. It is defined by the compounding term in the interest rate statement. If it is not stated it is assumed to be one year.

$F = P(1 + i_s)^m$ i_s: interest for subperiod

We want to find the effective interest rate, i_e, that yields the same future amount F at the end of the full period from the present amount P.

$P(1 + i_s)^m = P(1 + i_e)$

Then

$(1 + i_s)^m = (1 + i_e)$

$$i_e = (1 + i_s)^m - 1 \qquad (2.3)$$

Note that Equation (2.3) allows the conversion between the interest rate over a compounding subperiod, i_s, and the effective interest rate over a longer period, i_e, by using the number of subperiods, m, in the longer period.

Example 3.12

A man wants to buy a machine $ 100 000 by paying uniform series every 3 months for 10 yrs. Find the annuity payments if i was 5% compounded monthly

Solution

$$\mathbf{i_e = (1 + i_s)^m - 1} \qquad \mathbf{(2.3)}$$

عدد فترات الفائدة/عدد الدفعات

$$\textbf{Ieff} = \left(1+\frac{.05}{12}\right)^{12/4} - 1 = \$\ 0.01255 = 1.25\%$$

N=4x10= 40

$$\textbf{A} = \textbf{P}\left[\frac{(1+.0125)^{40} x 0.0125}{(1+.0125)-1}\right. = \$\ 3192.4$$

Example 3.13

The different bank loan rates for three separate equipment projects are listed below. Determine the effective rate on the basis of compounding period for each statement

a. 9% per year compounded quarterly فصليا
b. 9% per year compounded monthly
c. 9% per year compounded weekly

Solution

$$\text{i subperiod } i_s = \frac{i\,\%\,per\ time\ period\ n}{m\ compunding\ per\ periods\ per\ n} = \frac{i}{m}$$

Nominal i (r) per n	Compounding period	m	i subperiod i_s=r/m	Distribution over Time Period
a. 9% per year	Quarterly	4	9/4=2.25	Four time a year
b. 9% per year	monthly	12	9/12=0.75	12 time/year
c.9% per year (4.5% per 6 months)	week	52 (26)	9/52=0.173 (4.5/26=0.175)	52 time/year (26 time /year)

Example 26

If there is an amount of money equal to $1000 is deposited in a bank with rate interest 18%, find the effective rate interest if compounded **a. Semiannually**نصف سنوي **b. monthly**

Solution

a. i effective = $\frac{i}{m} = \frac{0.18}{2}$ =0.09

i effective rate interest per year=$(1+.09)^2$ -1= 0.1881= 18.81%

F= $P(1+i)^n$ = $1000(1+0.1881) = $1188.1

b. i effective = $\frac{i}{m} = \frac{0.18}{12}$ =0.015

i effective rate interest per year=$(1+.015)^{12}$ -1= 0.19562= 19.56%

F= $P(1+i)^n$ = $1000(1+0.19562) = $1195.61

Example 27

What is the annual effective interest rate equivalent to a nominal rate of 12 percent a year?

The nominal interest rate is given as r = 12 percent, and the number of corresponding periods per year is m = 12.

This gives i_s = r/m = 0.12/12 = 0.01

$i_e = (1 + i_s)^m - 1$

$= (1 + 0.01)^{12} - 1$ = 0.126825 = 0.127 or 12.7%

An interest rate of 1 percent per month, compounded monthly, is equivalent to an effective rate of approximately 12.7 percent per year, compounded yearly.

Example 28

What is the largest rate of interest 3% compounded monthly or 3.5 % compounded Semiannually?

Solution

1. $i_s = \frac{r}{m} = \frac{.03}{12} = .0025$ *interest subperiod or interest per period*

$i_e = (1+ i_s)^m - 1 = (1+.0025)^{12} - 1 = .0304 = 3.04\%$ Effective interest per year compounded monthly

2. $i_s = \frac{r}{m} = \frac{.035}{2} = .0175$ *interest subperiod or interest per period*

$i_e = (1+ i_s)^m - 1 = (1+.0175)^2 - 1 = .0353 = 3.53\%$ Effective interest per year compounded semiannually

Example 29

A man deposited a $ 2000 in the bank with rate interest 8% compounded daily calculate the cumulative amount after one year?

Solution

$i_s = \frac{.08}{365} = 2.19 \times 10^{-4}$ interest per subperiod

$i_e = (1+2.19 \times 10^{-4})^{365} - 1 = 0.08327$

F= $2000(1+0.08327)= $ 2166.54

Example

What is the effective interest rate equivalent to a nominal rate of 6 percent compounded quarterly?

Solution

$$\mathbf{i_e = (1+ i_s)^m - 1 = (1+ \frac{.06}{4})^4 - 1 = 0.06136 \times 100 = 6.13\%}$$

Example 33

A man deposited amount of 500$ every 6 month for a period of 7 years. What is the amount will be accumulated after the last payment if the interest rate is 8%, which compounded quarterly?

Solution

i= 8% compounded quarterly

i subperiod= $i_s = \frac{.08}{4} = .02$ interest per subperiod

because the deposit every 6 months or two times in one year

$i_e = (1+.02)^2 - 1 = 0.0404 = 4.04\%$

There is two deposit every year, thus n= no.of years X no. of payments

7 years 14 payments

F= A $\left[\frac{(1+i)^n -1}{i}\right]$ = $**500** $\left[\frac{(1+.0404)^{14} -1}{.0404}\right]$ = $ 9172.12

Example 34

A contractor wants to purchase a new construction equipment by $ 100 000 there is an agreement with the supplier side for covering this cost by paying a uniform series each 3 months for 10 years . Find the uniform payments if the rate interest 5% compounded monthly ?

Solution

I_s=.05/ 12= .004166 per period

No. of periods = $\frac{no.of\ interest\ during\ the\ year}{no.of\ payments} = \frac{12}{4} = 3$

Ie= $(1+.004166)^3$-1=.01255

There are 4 payments during the year for 10 years= 4 X 10=40

A=P$\left[\frac{(1+i)^n\ i}{(1+i)^n-1}\right]$= $100 000 $\left[\frac{(1+.01255)^{40} X\ .01255}{(1+.01255)^{40}-1}\right]$= $ 1610.30

Example 35

What is the effective interest rate of a nominal interest rate of 6% compounded quarterly for one year?

Solution

i subperiod=.06/4= 0.015

i_e= $(1+ 0.015)^4$ -1= 0.06136= 6.136%

Example 36

A man has deposited $ 1000 at now, after 4 year he also deposited $3000, and after 6 years, he deposited again $1500 with a rate of interest of 6% compounded semiannually. How much will the amount become after 10 years?

Solution

i subperiod= .06/2=.03

i effective=$(1+.03)^2$ -1= .0609= 6.09%

F1+ F2+ F3 = 1000(1+.0609)^{10}$ + 3000(1+.0609)^6$ + $1500(($(1+.0609)^4$= $7983.54

Example 37

A savings and loan offers a 5.25 %rate per annum compound daily over 365 days per year. What is the effective annual rate ?

Solution

i subperiod= .0525/365=1.438 x 10^{-4}

i effective=$(1+1.438 \times 10^{-4})^{365}-1=0.0539=5.39\%$

How much money will be in a bank account at the end of 15 years if $100 is invested today and the nominal interest rate is 8 percent compounded semiannually?
Solution

$$i_e = \left(1 + \frac{r}{m}\right)^m - 1$$

where i_e = the effective annual interest rate
r = the nominal rate per year
m = the number of periods in a year
$i_e = (1 + 0.08/2)^2 - 1 = 0.0816$

where $r = 0.08$
$m = 2$

When the effective yearly rate for each of 15 years is appli
tation, the future worth is

$$\begin{aligned} F &= P(F/P,i,N) \\ &= P(1+i)^N \\ &= 100(1+0.0816)^{15} \\ &= 324.34 \end{aligned}$$

Once again, we conclude that the balance will be $324.34.

Deferred Annuities (Uniform Series) or Deferred Loan Repayment

All annuitiesالمعاشات (uniform series) discussed to this point involved the first cash flow being made at the end of the first period and they are called **ordinary annuities.**
If the cash flow does not being payed until some later date the annuity is known as deferredالمؤجله annuity.

What Is a Deferred Annuity?
A deferred annuity is like a contract with an insurance company that promises to pay the owner a regular income, or a lump sum, at some future date. Investors often use deferred annuities to supplement their other retirement التقاعد income, such as Social Security. Deferred annuities differ from immediate annuities, which begin making payments right away. If the annuity is deferred j periods, the situation is as portrayed in figure below. It should be noted in this figure that the entire framed ordinary annuity has been moved forward from time present or time zero by J periods. It must be remembered that in an annuity deferred for J periods the first payment is made at the end of (J+1) period, assuming that all periods involved are equal in length.

Example 4-1

Suppose that a father, on the day his son is born, wishes to determine what lump sum amount would have to be paid into an account bearing interest of 12% per year to provide withdrawals of $2,000 on each of the son's 18th, 19th, 20th, and 21st birthdays.

Solution

The present worth of this annuity occurs at the 17th birthday hence:

Present amount at the end of year 17=P_{17}= $A\left[\frac{(1+i)^n-1}{i(1+i)^n}\right]$ = $\$2000\left[\frac{(1+.12)^4-1}{.12(1+.12)^4}\right]$

P_{17}=$2000(3.037493467) = $6074.698693

Now P17 is known the next step is to calculate P_0

P0=$F(1+i)^{-n}$=6074.69869(1+.12)^{-17}$=$6074.698693(0.1456443409)=$884.74587

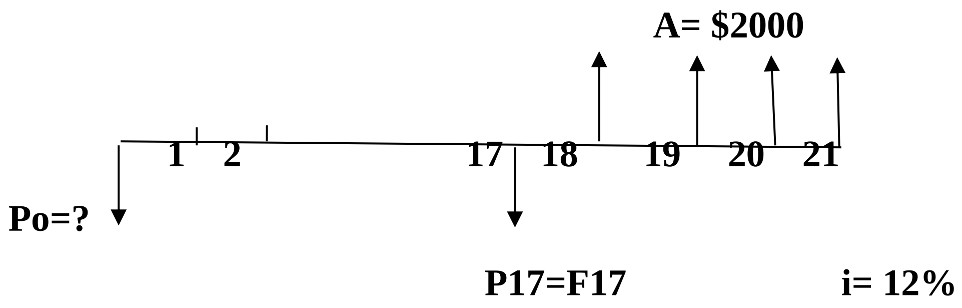


Figure ( ) Cash flow diagram of deferred annuity.

Example 4-2

As an addition to the previous problem, suppose that it is desired to determine the equivalent worth of the four payments as of the sons 24^{th} birthday. This could mean that the four payments never were withdrawn or that possibly the son took them and immediately redeposited them in an account also earning interest at 12% compounded annuity. Using subscript system we desire to calculate F24 as shown in figure ( )

Figure ( ) Cash flow diagram of deferred annuity.

Solution

One way to work this is to calculate F21

$F21= A\left[\frac{(1+i)^n -1}{i}\right] = \$2000\left[\frac{(1+.12)^4 -1}{.12}\right]$=$2000(4.779328)= $9558.656

$F24=P(1+.12)^3$ =$9558.656(1.404928)=$ 13429.0885

Another way to work the problem is

P17=$6074.69869 and P0=$884.74587 are each equivalent to the four payment.

Hence one can find F24 directly given P17 or P0 as such:

$F24=P17(1.12)^{7}$==\$6074.69869(2.2106)=\$ 13429.0052 or

$F24=P0(1.12)^{24}$ =\$884.74587(15.1786)=\$ 13429.229

Which check closely with each other of the previous answers. The different in the result numbers can be attributed to round –off error in the interest factors.

Example 4-3

You borrowed \$ 21061.82 to finance the educational expenses for your senior year of college. The loan will be paid off over five years. The loan carries an interest rate of 6% per year and is to be repaid in equal annual installments over the next five years. Assume that the money was borrowed at the beginning of your senior year and that the first installment will be due a year later. Compute the amount of the annual installments. Now; suppose that you had wanted to negotiate with the bank to defer the first loan installment until the end of year two (but still desire to make five equal installments at 6%). If the bank wishes to earn the same profit as in the previous case, what should be the annual installment?

Figure ( ) Cash flow diagram of deferred annuity .

Solution

In deferring one year the bank will add the interest accruedالمستحقه during the first year to the principal. In other words we need to find the equivalent worth of \$ 21061.82 at the end of year 1.

$F1=P(1+i)^{n}$ = \$21061.82(1.06)=\$ 22325.5292

$A= P\left[\frac{(1+i)^{n}\, i}{(1+i)^{n}-1}\right]$ =\$ 22325.5292 $\left[\frac{(1+.06)^{5}\,.06}{(1+.06)^{5}-1}\right]$=\$5302 Payment each year

Net Cash Value

In many economic calculations, we need to know expenditures or expenses values against revenues, returns, or interest. This is done by unifying these financial values according to the years of their expenditures or revenues, and then the compulsory collection is made one o them is positive and the other is negative.

Example 4.4

An investment of $10000 can be made that will produce uniform annual revenue of $ 5310 for five years and then have a positive salvage value of $2000 at the end of year 5. Annual expenses will be $ 3000 at the end of each year for operating and maintaining the project. Draw a cash flow diagram for the 5 year life of project.

Solution

Note that the beginning of a given year is the end of preceding year, for example, the beginning of year 1 is the end of year 0. Cumulative cash flow is shown in this tabulation

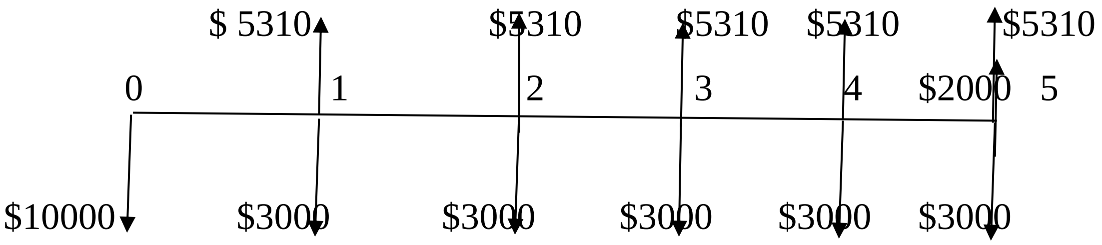


Table Cumulative cash flow

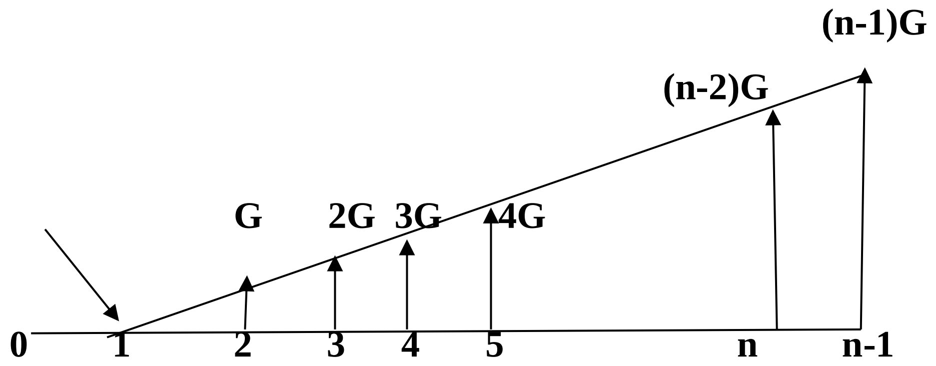
End of year	Net cash flow$	Cumulative cash flow$
0	10000	10000
1	2310	7960 7690
2	2310	5300 5380
3	2310	3070
4	2310	760
5	4310	3550

Example 4-5

A mechanical device will cost

HANDLING LINEAR GRADIENT SERIES

Sometimes cash flows will vary linearly, that is, they increase or decrease by a set amount G the gradient amount. This type of series is known as a strict gradient series, as seen in the figure below. Note that each payment is A= (n-1)G. Note also that the series begins with zero cash flow at the end of the period zero. If G> 0, the series is referred to as increasing gradient. If G< 0 , it is referred to as a decreasing gradient series.



Cash flow diagram of a series gradient series.

Arithmetic Gradient Factor (P/G and A/G)

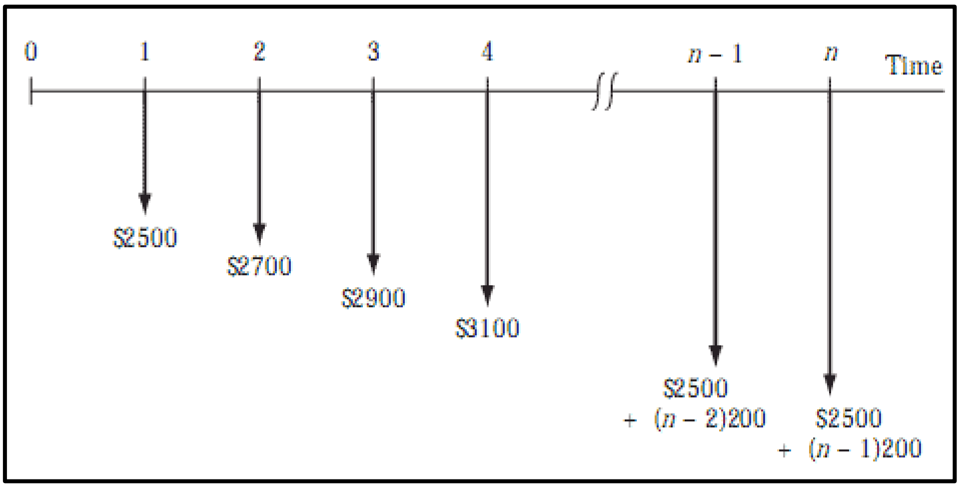
An arithmetic gradient is a cash flow series that either increase or decrease by a constant amount. The cash flow whether income or disbursement, changes by the same arithmetic amount each period the amount of the increase or decrease is called the gradient. Assume a manufacturing engineer predicts that the cost of maintaining a robot will increase by $5000 per year until the machine is retired. The cash flow series of maintenance costs involves a constant gradient, which is 5000 per year.

Formulas previously developed for an A series have year-end amounts of equal value. In the case of a gradient, each year-end cash flow is different, so new formulas must be derived. First, assume that the cash flow at the end of year 1 is the base amount of the cash flow series and, therefore, not part of the gradient series. This is convenient because in actual applications, the base amount is usually significantly different in size compared to the gradient. For example, if you purchased a used car with a 1-year warranty, you might expect to pay the gasoline and insurance costs during the first year of operation. Assume these cost $2500; that is, $2500 is the base amount. After the first year, you absorb the cost of repairs, which can be expected to increase each year. If you estimate that total costs will increase by $200 each year, the amount the second year is $2700, the third $2900, and so on to year n , when the total cost is 2500 (n - 1)200. The cash flow diagram is shown in **Figure below.** Note that the gradient ($200) is first observed between year 1 and year 2, and the base amount ($2500 in year 1) is not equal to the gradient.

Define the symbols G for gradient and CFn for cash flow in year n as follows.

G : constant arithmetic change in cash flows from one time period to the next; G may be positive or negative.

CFn= base amount +(1-n)G



If the base amount is ignored, a generalized arithmetic (increasing) gradient cash flow diagram is as shown in **Figure ( )**. Note that the gradient begins between years 1 and 2. This is called a conventional gradient.

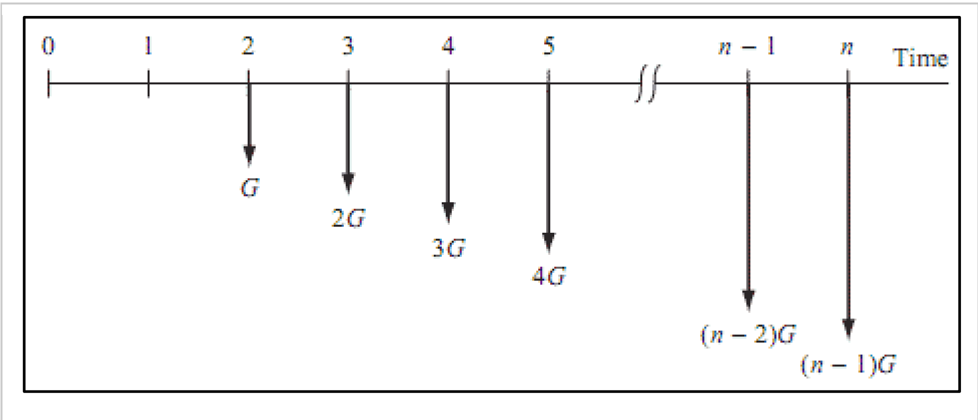


Diagram of an arithmetic gradient series with a base amount of $1500 and a gradient of $50.

Example

A manufacturing of Equipment Company has initiated a new program for renting the equipment. It expects to realize a revenue of $ 80000 in fees next year. Fees are expected to increase uniformly to a level of $ 200000 in 9 years. Determine the arithmetic gradient and construct the cash flow diagram.

Solution

The base amount is $80000 and the total revenue increase

200000 -80000= $120000

120000/9-1= 120000/8= $15000

G=$15000

In this text, three factors are derived for arithmetic gradient: the P/G factor for present worth (what is P knowing the gradients), the A/G factor for annual series (what is P knowing the annuities), and the F/G factor for future worth (what is F knowing the gradients). There are several ways to derive them. We use the single-payment present worth factor (P/F , i , n), but the same result can be obtained by using the F/P , F/A , or P/A factor using tables. In the Figure above, the present worth at year 0 of only the gradient is equal to the sum of the present worth of the individual cash flows, where each value is considered a future amount.

$$P = G(P/F,i,2) + 2G(P/F,i,3) + 3G(P/F,i,4) + \cdots + [(n-2)G](P/F,i,n-1) + [(n-1)G](P/F,i,n)$$

Factor out G and use the P/F formula.

$$P = G\left[\frac{1}{(1+i)^2} + \frac{2}{(1+i)^3} + \frac{3}{(1+i)^4} + \cdots + \frac{n-2}{(1+i)^{n-1}} + \frac{n-1}{(1+i)^n}\right] \qquad [2.21]$$

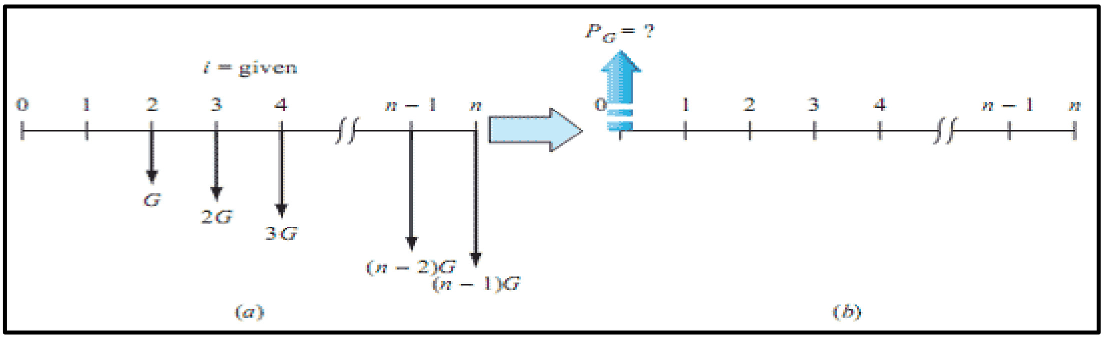
Multiplying both sides of Equation [2.21] by $(1+i)^1$ yields

$$P(1+i)^1 = G\left[\frac{1}{(1+i)^1} + \frac{2}{(1+i)^2} + \frac{3}{(1+i)^3} + \cdots + \frac{n-2}{(1+i)^{n-2}} + \frac{n-1}{(1+i)^{n-1}}\right] \qquad [2.22]$$

Subtract Equation [2.21] from Equation [2.22] and simplify.

$$iP = G\left[\frac{1}{(1+i)^1} + \frac{1}{(1+i)^2} + \cdots + \frac{1}{(1+i)^{n-1}} + \frac{1}{(1+i)^n}\right] - G\left[\frac{n}{(1+i)^n}\right] \qquad [2.23]$$

The left bracketed expression is the same as that contained in Equation [2.6], where the P/A factor was derived. Substitute the closed-end form of the P/A factor from Equation [2.8]



Conversion diagram from an arithmetic gradient to a present worth.

Simplify to solve for P/G, the present worth of the gradient series only.

$$P_G = \frac{G}{i}\left[\frac{(1+i)^n - 1}{i(1+i)^n} - \frac{n}{(1+i)^n}\right] \qquad [2.24]$$

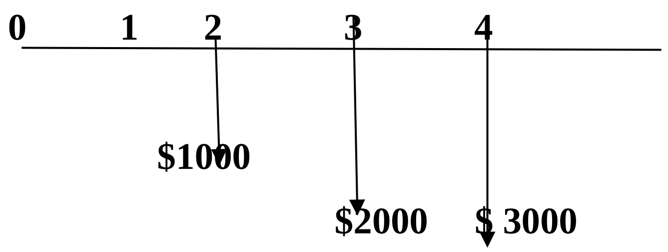
Remember the conventional arithmetic gradient starts in year 2 and P is located in year zero.

Example

It is expected that maintenance costs for the first year equal to zero of the construction equipment, at the end of the second year will be $ 1000 and $2000 for the third year and $3000 for the fourth year. The annual rate interest is 15% Calculate

1. The present value of these expenses
2. The annual amount of the uniform series for four years

Solution



1.

$$P = \frac{G}{i}\left[\frac{(1+i)^n - 1}{i(1+i)^n} - \frac{n}{(1+i)^n}\right]$$

$$= \frac{\$1000}{0.15}\left[\frac{(1+0.15)4-1}{0.15(1+0.15)4} - \frac{4}{1+.15)4}\right] = \$3790$$

2.

$$(A/G,i,n) = \left[\frac{1}{i} - \frac{n}{(1+i)^n - 1}\right]$$

$$A = \$1000\left[\frac{1}{0.15} - \frac{4}{(1+.15)4}\right] = \$\ 1326.27$$

Example

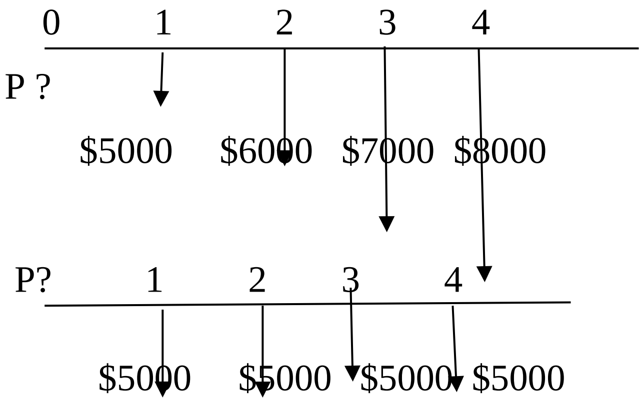
Below the following payments are the installmentsاقساط for purchasing the new shovel, if the contractor wants to pay these installments at now how much cost will be ? with rate interest equal to 15% for 4 years

End of year	Payments$
1	$5000

2	$6000
3	$7000
4	$8000

Solution

To solve the problem with gradient form it needs to divide to 2 parts for conforming مطابقه this



Total P amount= The amount of regular payments+ The amount of gradient

$P_A = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$ = P=$ 5000$\left[\frac{(1+0.15)^4 - 1}{0.15(1+0.15)^4}\right]$ =$14274.89

$$P = \frac{G}{i}\left[\frac{(1+i)^n - 1}{i(1+i)^n} - \frac{n}{(1+i)^n}\right]$$

$= \frac{\$1000}{0.15}\left[\frac{(1+0.15)4-1}{0.15(1+0.15)4} - \frac{4}{1+.15)4}\right]$ **= $3824**

P_{total}=P_A +P= $ 14274.89+$3824=$18098.82

For the equivalent payment as annual payments

A=P$\left[\frac{(1+i)^n\, i}{(1+i)^n - 1}\right]$

$$P_A= A\left[\frac{(1+i)^n -1}{i(1+i)^n}\right] = P=\$\ 8000\left[\frac{(1+0.15)^4 -1}{0.15(1+0.15)^4}\right] =\$22874.6$$

$$P = \frac{G}{i}\left[\frac{(1+i)^n -1}{i(1+i)^n} - \frac{n}{(1+i)^n}\right]$$

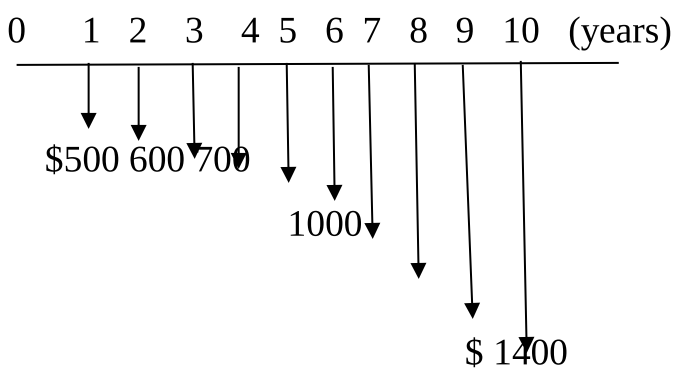
$$=\frac{\$1000}{0.15}\ \left[\ \frac{(1+0.15)4-1}{0.15(1+0.15)4} - \frac{4}{(1+.15)4}\right] = \$3824$$

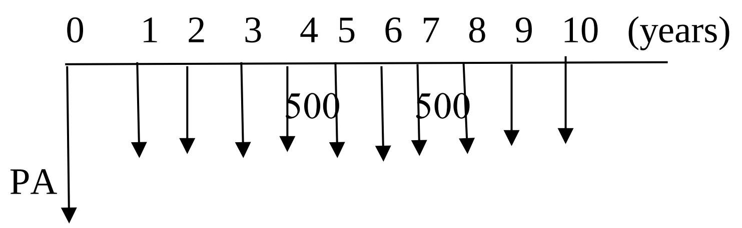
Pnet=P_A-P_G= $ 22874.6 - 3824=$19047.87

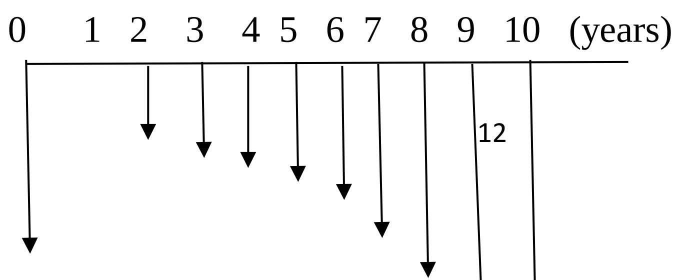
Example

A man plans to invest the money by depositing $500/year from now. He has ensured that this deposit will increase by $100 yearly for ten years. What is the present value of this investment and the rate of interest 5% per year? What is the value of the annual amounts equivalent to this annually investment?

Solution







100 200 400

700

800 900

$P_A = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$ = P=\$ 500$\left[\frac{(1+0.05)^{10} - 1}{0.05(1+0.05)^{10}}\right]$ =\$3861.26

$$P = \frac{G}{i}\left[\frac{(1+i)^n - 1}{i(1+i)^n} - \frac{n}{(1+i)^n}\right]$$

$=\frac{\$100}{0.05}\left[\frac{(1+0.05)10-1}{0.05(1+0.05)10} - \frac{4}{(1+.15)10}\right]$ **= \$3168**

Ptotal= 3861.26+3168= \$7029.26

The value of the annual equivalent is the result of sum of an equal amount to A and an equivalent amount to G

A=A1+ G, A1=\$500

$$(A/G,i,n) = \left[\frac{1}{i} - \frac{n}{(1+i)^n - 1}\right]$$

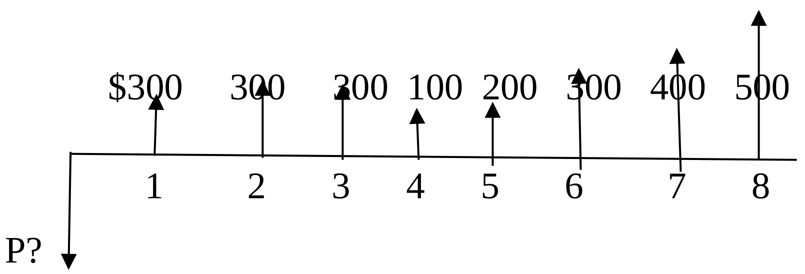
A_G= $\left[\frac{1}{0.05} - \frac{10}{(1+.05)10-1}\right]$ = \$410.2

A= 500+410.2=\$910.2

The total value of P is \$ 7029.26 then apply the law A =P$\left[\frac{(1+i)^n\, i}{(1+i)^n - 1}\right]$ and the output will be \$910 it is the annual equivalent of the current effective amount.

Example

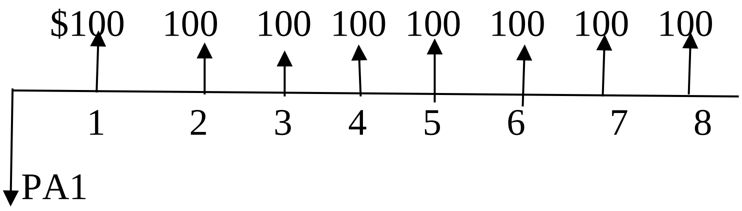
Find the present value of the following cash flow below with rate interest is 15%



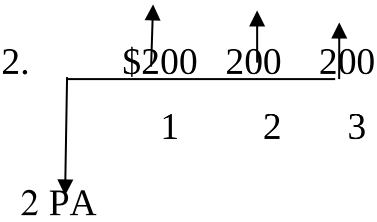
Solution

We need to divide the payments into three parts

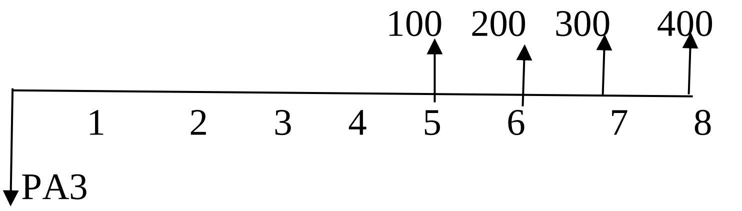
1.



2.



3.



P_{A1}= $A\left[\frac{(1+i)^n-1}{i(1+i)^n}\right]$ = P=\$ $100\left[\frac{(1+0.15)^8-1}{0.15(1+0.15)^8}\right]$ =\$448.73

P_A2= $A\left[\frac{(1+i)^n-1}{i(1+i)^n}\right]$ = P=\$ $200\left[\frac{(1+0.15)^3-1}{0.15(1+0.15)^3}\right]$ =\$1156.14

$$P=\frac{G}{i}\left[\frac{(1+i)^n-1}{i(1+i)^n}-\frac{n}{(1+i)^n}\right]$$

PG=$\frac{\$100}{0.15}\left[\frac{(1+0.15)5-1}{0.15(1+0.15)5}-\frac{5}{(1+.15)5}\right]$ = \$577.5

P=F ($\frac{1}{(1+i)n}$)= 577.5(1/ (1+.15)3]= \$379.96

Ptotal=448.73+1156.14+379.73=\$1282.73