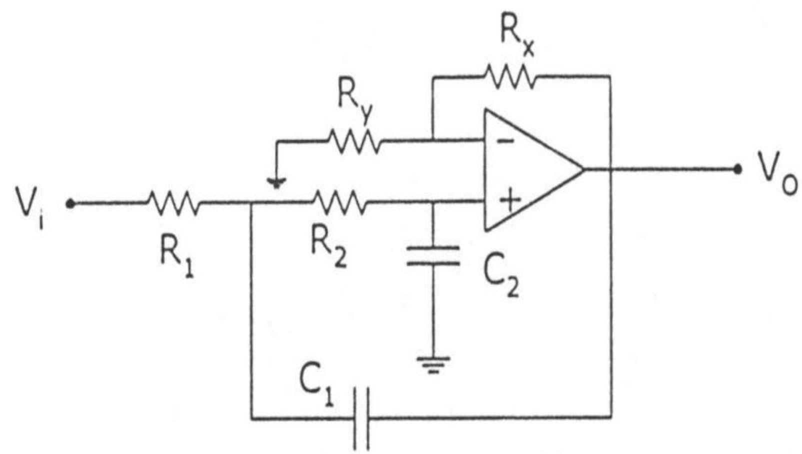
Active Filters

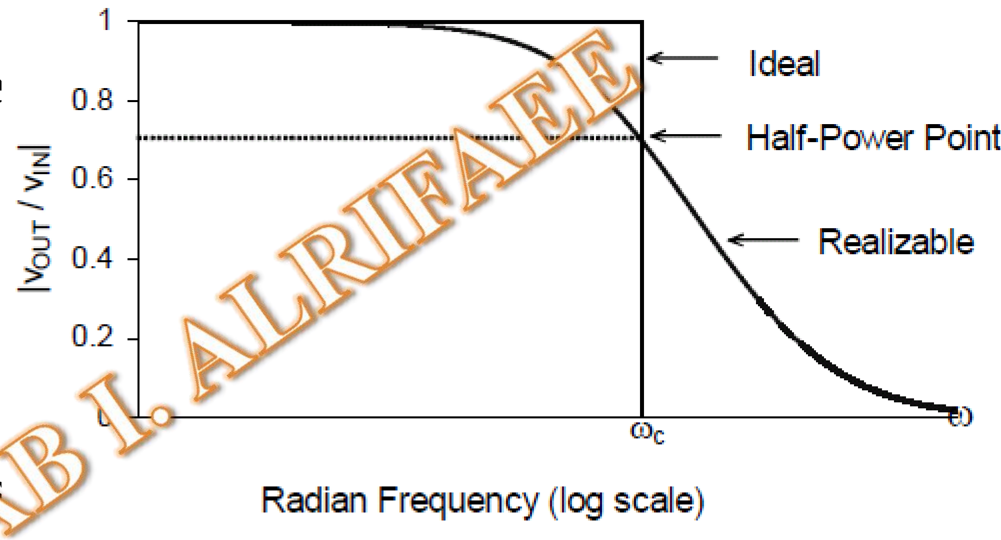
* A*ctive filter* is an electronic circuit consisting of an amplifier and other devices such as resistors and capacitors. In contrast, a *passive filter* is a circuit which consists of passive devices such as resistors, capacitors and inductors. Operational amplifiers are used extensively as active filters.

*A ***low-pass filter*** transmits (passes) all frequencies below a *critical* (*cutoff*) frequency denoted as ,w_c,

and *attenuates* (blocks) all frequencies above this cutoff frequency. An op amp low-pass filter is shown in the fig.:



* The frequency response *for amplitude of* a LPF. is shown in the figure below:



The straight vertical and horizontal lines represent the ideal (unrealizable and the smooth curve represents the practical (realizable) low-pass filter characteristics. The vertical scale represents the magnitude of the ratio of output-to-input voltage , $\frac{V_O}{V_I}$, that is the gain G_v . The cutoff frequency w_c is the frequency at which the maximum value of $\frac{V_O}{V_I}$ which is $G_{v_{max}}$, (when $G_{v_{max}} = 1$, then the responce is shown as in the fig. above) falls to 0.707 $G_{v_{max}}$, and this is the *half power* or the -3db point.

Filter design

- A general expression for the transfer function a 1st-order low pass filter can be written as: $A_v(s) = \frac{A_v(0)}{1+\frac{S}{\omega_\circ}}$

The response to a steady state sinusoid is: $A_v\,(j\boldsymbol{\omega}) = \frac{A_{v(0)}}{1+\frac{j\omega}{\omega_\circ}}$

$$|A_v\,(j\omega)| = \frac{A_{v(0)}}{\left[1+\left(\frac{\omega}{\omega_\circ}\right)^2\right]^{1/2}}$$

$$\emptyset = -\tan^{-1}\left(\frac{\omega}{\omega_\circ}\right)$$

<u>low pass filter</u>

2nd-order low pass filter transfer function is given by

$$A_v(S) = \frac{A_v\,(0)}{\frac{S^2}{\omega_\circ{}^2}+\frac{S}{Q\omega_\circ}+1} \qquad \text{.........} \; (1)$$

$$|A_v(j\omega)| = \frac{A_{v(0)}}{\left[1-\left(\frac{\omega^2}{\omega_\circ^2}\right)^2+\frac{\omega^2}{\omega_\circ^2 Q^2}\right]^{1/2}} \quad ....(2)$$

* The poles are the roots of the denominator polynomial given by

$$S_{1,2} = -\frac{\omega_\circ}{2Q} \pm \omega_\circ \left[\left(\frac{1}{2Q}\right)^2 - 1\right]^{1/2}$$

To design a 2nd-order LPF shown in the figure above, The transfer function of this circuit is given by:

$$A_v(S) = \frac{V_o}{V_i} = \frac{A_{v_o}}{S^2 R_1 R_2 C_1 C_2 + S[R_2 C_2 + R_1 C_2 + R_1 C_1(1-A_{v_o})]+1} \quad .... (3)$$

From comparing equation (1) and (3) and equating the coefficient of denominators, it’s seen that

$$\omega_\circ = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} \quad ....(4)$$

$$\frac{1}{Q} = \frac{R_2C_2+R_1C_2+R_1C_1\left(1-A_{v_o}\right)}{\sqrt{R_1R_2C_1C_2}} \quad \text{........ (5)}$$

* Now, if $R_1 = R_2$, $C_1 = C_2$ then from equation (5):

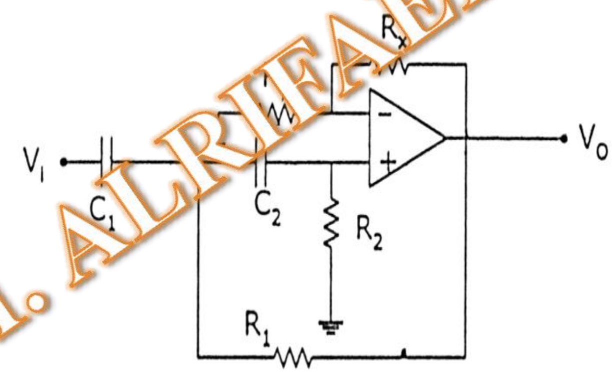
$\frac{1}{Q} = 3 - A_{V_o}$, for Butter worth response: $\frac{1}{Q} = \sqrt{2}$,So

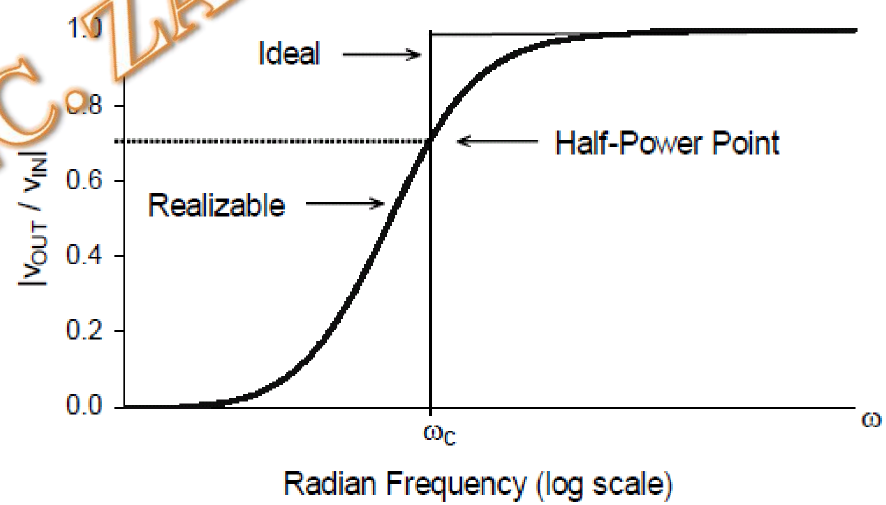
$1.414 = 3 - A_{V_o}$ and $A_{V_o} = 1.6$

* The OP-AMP's gain can be set to give this ratio. However, if the gain of the amplifier is set at unity and $R_1=R_2$ and $\frac{1}{Q} = \sqrt{2}$, from equation (3): $C_1 = ?C_2$.

* **High Pass Filter**

- A *high-pass filter* transmits (passes) all frequencies above a critical (cutoff) frequency w_c , and attenuates (blocks) all frequencies below the cutoff frequency. An op amp high-pass filter is shown in the Fig. and its frequency response in the fig.





HPF. Design

- A high pass 2nd-order filter can be obtained from the low pass 2nd-order of equation (1) by applying the transformation

$$\left.\frac{S}{\omega_\circ}\right|_{LP} \rightarrow \left.\frac{\omega_\circ}{S}\right|_{HP}$$

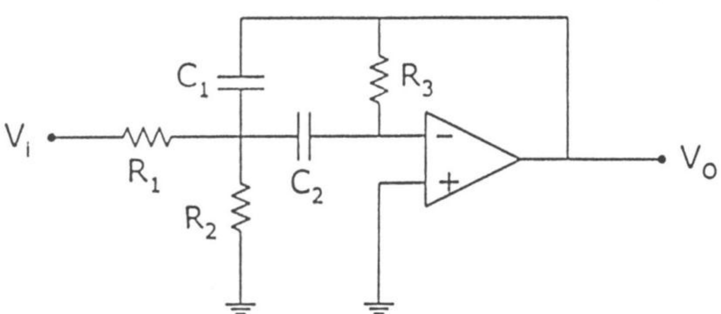
And the same design procedure is followed as before.

* Band Pass Filter

The general transfer function of a band pass filter is given by

$$A_v(S) = \frac{A_{V_o}\frac{\omega_\circ}{Q}S}{S^2+\frac{\omega_\circ}{Q}S+\omega_\circ^2} \quad \ldots\ldots\ldots (1)$$

$$Q = \frac{center\ frequency}{bandwidth} = \frac{\omega_\circ}{\omega_1-\omega_2}$$



where $\boldsymbol{\omega}_1$ and $\boldsymbol{\omega}_2$ are the upper and lower 3-dB points.

$$\left|A_{V(j\omega)}\right| = \frac{A_{V_o}}{\left[1+Q^2\left(\frac{\omega}{\omega_\circ}-\frac{\omega_\circ}{\omega}\right)^2\right]^{1/2}} \;, \text{ And } \emptyset = -\tan^{-1}\left[Q\left(\frac{\omega}{\omega_\circ}-\frac{\omega_\circ}{\omega}\right)\right]$$

If the band pass filter as on the fig. above is taken, the transfer function is given by

$$A_v(S) = \frac{\frac{S}{R_1C_1}}{S^2+\frac{C_1+C_2}{R_3C_1C_2}S+\frac{1}{\acute{R}R_3C_1C_2}} \ldots\ldots(2)$$

Where R´ = R1/ / R2

- Equating the coefficients of equations (1) and (2) yields

$$R_1 C_1 = \frac{Q}{\omega_\circ A_{V\circ}} \quad \text{........(3)}$$

$$R_3 \frac{C_1 C_2}{C_1 + C_2} = \frac{Q}{\omega_\circ} \quad \text{.........(4)}$$

$$\acute{R} R_3 C_1 C_2 = \frac{1}{\omega_\circ{}^2} \quad \text{.......(5)}$$

From equations (8) to (10) the values of R_1, R_3, R', C_1 and C_2 can be obtained from specific value of $\boldsymbol{\omega}_\circ$, Q, and $A_{v\circ}$.

Since we have only three equations for the five parameters, two of these (say C_1 and C_2) may be chosen arbitrary.

*Band Reject Filter

- A band reject filter can be constructed by connecting the low and high pass filter sections in parallel and summing their outputs, as shown in the figure below.