1. Interpolation

Interpolation is the estimation of a variable value between data points. MATLAB provides the following function for one dimensional interpolation which is used to interpolate a function of one variable (e.g. y=f(x)):

```
interp1(x,y,x_int,'method')
```

- x,y: your data points
- x_int: vector contains the values to be interpolated
- 'method': interpolation method:
  - linear
  - spline

For two dimensional interpolation, which is used to interpolate a function of two variables (e.g. z=f(x,y)), MATLAB provides the following function:

```
interp2(x,y,z,x_int,y_int,'method')
```

- x,y,z: your data points
- x_int,y_int: vector contains the values to be interpolated
- 'method': interpolation method:
  - linear
  - spline

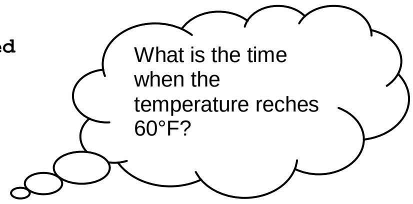
- Linear interpolation applies straight lines on the data point to obtain an estimate for the interpolated values.
- Spline interpolation applies a cubic polynomial between each pair of data to obtain an estimate for the interpolated values.
- If you omit the interpolation method, MATLAB will use the linear interpolation as a default.

Example 1: The following data shows the temperature measurement with time. Find the temperature at 8 and 10 hr.

Time (hr)	7	9	11	12
Temperature (°F)	49	57	71	75

<u>MATLAB session:</u>

```
>> x=[7 9 11 12];
>> y=[49 57 71 75];
>> x_int=[8 10]; % the values to be interpolated
>> interp1(x,y,x_int,'linear')
ans =
    53    64
>> interp1(x,y,x_int,'spline')
ans =
   51.2000   64.3000
```

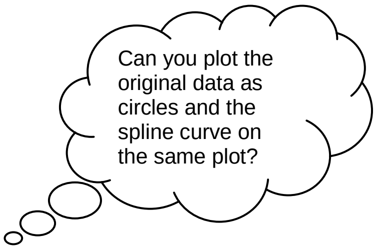


Example 2: The values of the viscosity of ethylene glycol, μ (lb/ft hr), at various temperature were shown in the following table. Find the value of the viscosity at 40, 70, 125, and 160°F using linear and cubic interpolation.

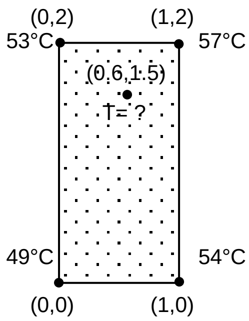
T (°F)	0	50	100	150	200
μ (lb/ft hr)	242	82.1	30.5	12.6	5.57

MATLAB session:

```
>> T=[0:50:200];
>> mu=[242 82.1 30.5 12.6 5.57];
>> T_int=[40 70 125 160];
>> interp1(T,mu,T_int)
ans =
  114.0800   61.4600   21.5500   11.1940
>> interp1(T,mu,T_int,'spline')
ans =
  102.6146   53.3664   19.5733   10.6408
```



Example 3: The temperature distribution within a flat plate is shown below. Estimate the temperature at the point whose coordinates are (0.6,1.5).



MATLAB session:

```
>> x=[0 1];y=[0 2];
>> z=[49 54;53 57];
>> interp2(x,y,z,0.6,1.5)
ans =
   54.5500
```

Example 4: The vapor pressure of ammonia is given in the table below according to the temperature and molal concentration percentage of ammonia. Estimate the vapor pressure at (7%,65°F), (13%,75°F), and (17%,90°F).

	Molal concentration (%)		
Temperature (°F)	0	10	20
60	0.26	1.42	3.51
80	0.51	2.43	5.85
100	0.95	4.05	9.34

MATLAB session:

```
>> conc=[0 10 20];
>> temp=[60 80 100];
>> vp=[0.26 1.42 3.51;0.51 2.43 5.85;0.95 4.05 9.34];
>> c_int=[7 13 17];
>> t_int=[65 75 90];
>> interp2(conc,temp,vp,c_int,t_int)
ans =
    1.2675    3.1037    6.2885
>> interp2(conc,temp,vp,c_int,T_int,'spline')
ans =
    1.1107    2.8900    5.9729
```

2. Integration

One of the methods which MATLAB implements to find the approximate value of the integral is the trapezoidal integration with the **trapz(x,y)** function. The array y contain the function values at the points contained in the array x.

Example 1: Compute the integral $\int_0^{\pi} \sin x \cos x \, dx$.

MATLAB session:

```
>> x=[0:0.01:pi];
>> y=sin(x).*cos(x);
>> trapz(x,y)
ans =
    1.2682e-006
```

Example 2: An immersed electrical heater is used to raise the temperature of a liquid initially at 20°C. The mass of the liquid in the container is 250 kg, and the mean heat capacity of the system is 4 kJ/kg°C. Use the following data from a chart recorder to find the final temperature of the liquid after 300 sec.

t (s)	0	30	60	90	120	150	180	210	240	270	300
Q (kW)	33	33	34	35	37	39	41	44	47	50	54

MATLAB session:

```
>> T1=20;m=250;Cp=4;
>> t=[0:30:300];
>> qdot=[33 33 34 35 37 39 41 44 47 50 54];
>> q=trapz(t,qdot)
q =
   12105
>> T2=q/(m*Cp)+T1
T2 =
   32.1050
```

$$\dot{q} = \frac{dq}{dt}$$

$$\therefore q = \int_{t1}^{t2} \dot{q} dt = mCp(T_2 - T_1)$$

Example 3: The top product leaving a batch distillation column is being continuously reprocessed so that it is difficult to measure directly the amount produced. However, the flow rate can be continuously measured and samples are taken at hourly intervals giving the following results:

Time, hrs	0	1	2	3	4	5	6	7	8
Flow rate, kg/min	60	62	63	62	62	60	59	57	57
More volatile component (%)	90	92	91	91	89	87	84	80	75

a. What is the total amount produced during the period of 8 hr?
b. What proportion of this total is the more volatile component?

MATLAB session:

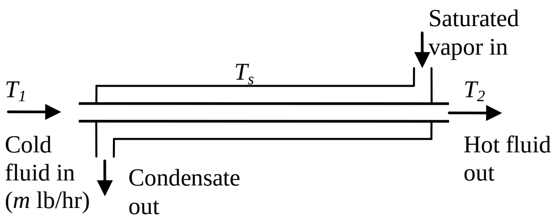
```
>> t=[0:8];
>> mdot=[60 62 63 62 62 60 59 57 57]*60;
>> x=[90 92 91 91 89 87 84 80 75]/100;
>> a=trapz(t,mdot)
a =
  29010
>> b=trapz(t,mdot.*x)/a
b =
    0.8722
```

$$\dot{m} = \frac{dm}{dt}$$

$$\therefore m = \int_{t1}^{t2} \dot{m} dt$$

Example 4: A shell and tube heat exchanger is employed for heating a steady state stream of m lb/hr of a fluid from an inlet temperature T_1 to an exit temperature T_2. This is achieved by continuously condensing a saturated vapor in the shell, maintaining its temperature at Ts.

The required exchanger length is found to be:



$$L = \frac{m}{\pi D}\int_{T_1}^{T_2} \frac{C_p dT}{h(T_s - T)}$$

Where D is the tube diameter. The local heat transfer coefficient h is given by the correlation:

$$h = \frac{0.023k}{D}\left(\frac{4m}{\pi D\mu}\right)^{0.8}\left(\frac{\mu C_p}{k}\right)^{0.4}$$

Where Cp, μ, and k (the specific heat, viscosity, and thermal conductivity of the fluid respectively) are functions of temperature. For the following case, write a MATLAB program to compute the required exchanger length, L given: m = 45000 lb/hr, T_1 = 0°F, T_2 = 180°F, Ts=250°F, D = 1.032 in, k = 0.153 BTU/hr ft °F

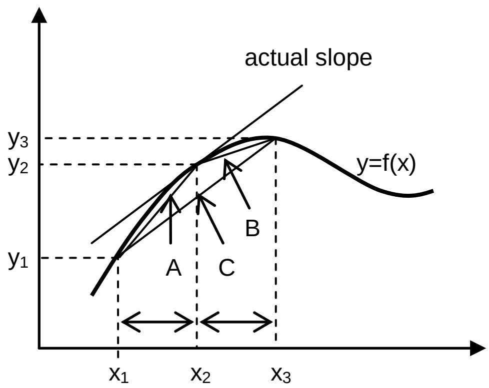
$C_p = 0.53 + 0.00056T$ BTU/lb°F, and $\mu = \begin{Bmatrix} 0°F & 242 \\ 50 & 82.1 \\ 100 & 30.5 \\ 150 & 12.6 \\ 200 & 5.57 \end{Bmatrix}$ lb/ft-hr, and

<u>MATLAB session:</u>

```
>> D=1.032/12; % convert from inch to ft
>> m=45000;T1=0;T2=180;Ts=250;
>> T=linspace(T1,T2,1000);
>> Cp=0.53+0.00056*T;
>> k=0.153;
>> temp=[0:50:200];
>> mu=[242 82.1 30.5 12.6 5.57];
>> mu_int=interp1(temp,mu,T,'spline');
>> h=(0.023*k/D)*(4*m./(pi*D*mu_int)).^0.8.*(mu_int.*Cp/k).^0.4;
>> L=(m/(pi*D))*trapz(T,Cp./(h.*(Ts-T)))
L =
  157.4526
```

3. Differentiation

To estimate the derivative of a function at a given point (e.g. point x_2 in the figure), you should estimate the slope using nearby data points. This can be obtained using the backward difference (slope A), forward difference (slope B), and central difference (slope C):



$$m_A = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$ backward difference

$$m_B = \frac{y_3 - y_2}{x_3 - x_2}$$ forward difference

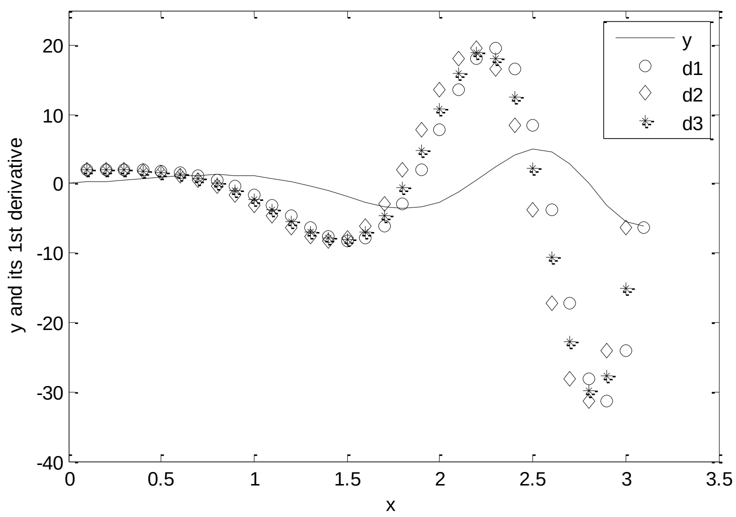
$$m_C = \frac{y_3 - y_1}{x_3 - x_1}$$ central difference

MATLAB provides the **diff** function to use for computing derivative estimates. Its syntax is **diff(x)** where x is a vector of values, and the result is a vector containing the differences between adjacent elements in x. that is if x has n elements, the result will have n-1 elements. For example, if **x=[5 7 12 -20]**, then **diff(x)** returns the vector **[2 5 -32]**.

Example 1: Plot the estimate of the derivative dy/dx for the function: $y = 2x\cos x^2$ for $0 \le x \le \pi$ using backward, forward, and central differences.

<u>MATLAB session:</u>

```
>> x=[0:0.1:pi];
>> y=2*x.*cos(x.^2);
>> d1=diff(y)./diff(x); % backward difference
>> d2=(y(3:end)-y(2:end-1))./(x(3:end)-x(2:end-1)); % forward difference
>> d3=(y(3:end)-y(1:end-2))./(x(3:end)-x(1:end-2)); % central difference
>> plot(x,y,x(2:end),d1,'o',x(2:end-1),d2,'d',x(2:end-1),d3,'*')
>> xlabel('x');ylabel('y and its 1st derivative')
>> legend('y','d1','d2','d3')
```



PROBLEMS

1. The following data is the measured temperature T of water flowing from a hot water faucet after it is turned on at time =0.
   a. Plot the data first.
   b. Estimate the temperature values at the following times using linear and spline interpolation: t= 0.6, 2.5, 4.7, and 8.9.

t (sec)	T (°F)	t (sec)	T (°F)
0	72.5	6	109.3
1	78.1	7	110.2
2	86.4	8	110.5
3	92.3	9	109.9
4	110.6	10	110.2
5	111.5		

2. The latent heat of vaporization of liquid propane is given in the given table. Interpolate to find the temperature and latent heat of vaporization corresponding to a vapor pressure of 37.5 atm.

T (°K)	Δh (cal/mol)	Vapor pressure (atm)
341.71	2443	25
351.23	2069	30
359.61	1615	35
367.18	912	40
370.00	0	42.1

3. The specific volume v (ft^3/lb) of superheated methane at various temperatures and pressures. Estimate the specific volume of methane at (56.4°F, 12.7 psia), (56.4,22.7), (112,12.7), (200,15), and (17,54.3).

Temp (°F)	Pressure (psia)				
	10	20	30	40	60
-200	17.15	8.47	5.57	4.12	2.678
-100	23.97	11.94	7.91	5.91	3.91
0	30.72	15.32	10.19	7.63	5.06
100	37.44	18.7	12.44	9.33	6.21
200	44.13	22.07	14.7	11.03	7.34

4. Compute the Fresnel's cosine integral, $\int_0^b \cos x^2 dx$, when the upper limit is $b = \sqrt{2\pi}$.

5. A certain object moves with velocity v(t) given in the following table. Determine object's position x(t) at t=10 seconds if x(0)=3.

Time (sec)	0	1	2	3	4	5	6	7	8	9	10
Velocity (m/s)	0	2	5	7	9	12	15	18	22	20	17

6. A tank having vertical sides and a bottom area of 100 ft^2 stores water. To fill the tank, water is pumped into the top at the rate given in the following table. Determine the water height h(t) at t=10 min.

Time (min)	0	1	2	3	4	5	6	7	8	9	10
Flow rate (ft^3/min)	0	80	130	150	150	160	165	170	160	140	120

7. The equation for the voltage v(t) across a capacitor as a function of time is:

$$v(t) = \frac{1}{C}\int_0^t i(t)dt + Q_o$$

where i(t) is the applied current and Q_o is the initial charge. Suppose the C=10^{-6} Farad and that Q_o=0. The applied current is $i(t) = \left[0.01 + 0.3e^{-5t}\sin(25\pi t)\right] \times 10^{-3}$ amperes. Calculate the voltage after 0.3 s.

8. The heat capacity of a gas is tabulated at a series of temperatures

T (°C)	20	50	80	110	140	170	200	230
Cp (J/mol°C)	28.95	29.13	29.3	29.48	29.65	29.82	29.99	30.16

Calculate the change in enthalpy for 3 mol of this gas going from 20°C to 230°C.

9. For real gases, the fugacity *f* is given by:

$$\ln \frac{f}{P_T} = \int_1^{P_T} \frac{C-1}{P} dp$$

P (atm)	C	P (atm)	C
1	0.9940	80	
10	0.9370	120	
20	0.8683	160	0.5252
40	0.7043	250	0.7468
60	0.4515	400	1.0980

where C is the experimentally determined compressibility factor. For methane, values of C are given in the table at -70°C. Write a MATLAB session that computes the fugacity for: (a) 60 atm and (b) 120 atm.

10. Consider the condensation of a pure vapor on the outside of a single cooled horizontal tube. According to the Nusselt theory of film condensation, the mean heat transfer coefficient h is given by:

$$h = \left(\frac{k^3 \rho g \lambda}{\nu \, r \Delta T}\right)^{1/4} \left(\frac{2^{3/2}}{3\pi}\right) I^{3/4}$$

where $I = \int_0^{\pi} (\sin\beta)^{1/3} d\beta$. Here, k, ρ, and ν are the thermal conductivity, density, and kinematic viscosity of the condensed liquid film, r is the tube radius, λ is the latent heat of condensing vapor, g is the gravitational acceleration, ΔT is the difference between the vapor saturation temperature (T_v) and the tube wall temperature (T_w), and all these quantities are in consistent units.

For water, the group $\phi = (k^3 \rho g \lambda / \nu)$, in BTU^4/hr^4 °F ft^7, varies with temperature T(°F) as follows:

T	100	110	120	130	140	150	160	170
$\phi \times 10^{-14}$	0.481	0.536	0.606	0.670	0.748	0.820	0.892	0.976
T	180	190	200	210	220	230	240	250
$\phi \times 10^{-14}$	1.051	1.130	1.218	1.280	1.327	1.376	1.430	1.503

The above formula for h, ϕ should be evaluated at the mean film temperature $\bar{T} = (T_V + T_W)/2$. Write a MATLAB program that uses the above equations to compute h. The input data should include values for T_v, T_w, and d (tube diameter in inches). The program should then compute and print values of the integral I, the mean film temperature $\bar{T}$, the corresponding value of ϕ, and the resulting heat transfer coefficient h. Suggested data are:

T_V	T_W	d
212	208	0.75
212	208	2.00
212	210	0.75
120	116	0.75

11. The volume of a liquid in a spherical tank of radius r as a function of the liquid's height h above the tank bottom is given by:

$$V(h) = \pi r h^2 - \pi \frac{h^3}{3}$$

a. Determine the volume rate of change dv/dh with respect to height.

b. Determine the volume rate of change dv/dt with respect to time.

12. Use the backward, forward, and central difference methods for estimating the derivative of the following function: $y(x) = e^{-x}\sin(3x)$. Use 101 points from $x = 0$ to $x = 4$.

13. Plot the estimate of the derivative dy/dx from the following data. Do this using forward, backward, and central differences. Compare the results.

x	0	1	2	3	4	5	6	7	8	9	10
y	0	2	5	7	9	12	15	18	22	20	17

14. A certain object's position as a function of time is given by $x(t) = 6t\sin(5t)$. Plot its velocity and acceleration as functions of time for $0 \le t \le 5$ (Apply central differences only).