Operational Amplifiers (Op Amps)

Introduction

* An operational amplifier is modeled as a voltage controlled voltage source.

* An operational amplifier has a very high input impedance and a very high gain.

Use of Op Amps

* Op amps can be configured in many different ways using resistors and other components.

* Most configurations use feedback.

* It can also be used in making a voltage- or current-controlled current source.

* An op amp can sum signals, amplify a signal, integrate it, or differentiate it. The ability of the op amp to perform these mathematical operations is the reason it is called an *operational amplifier.*

Applications of Op Amps

* Amplifiers provide gains in voltage or current.

* Op amps can convert current to voltage.

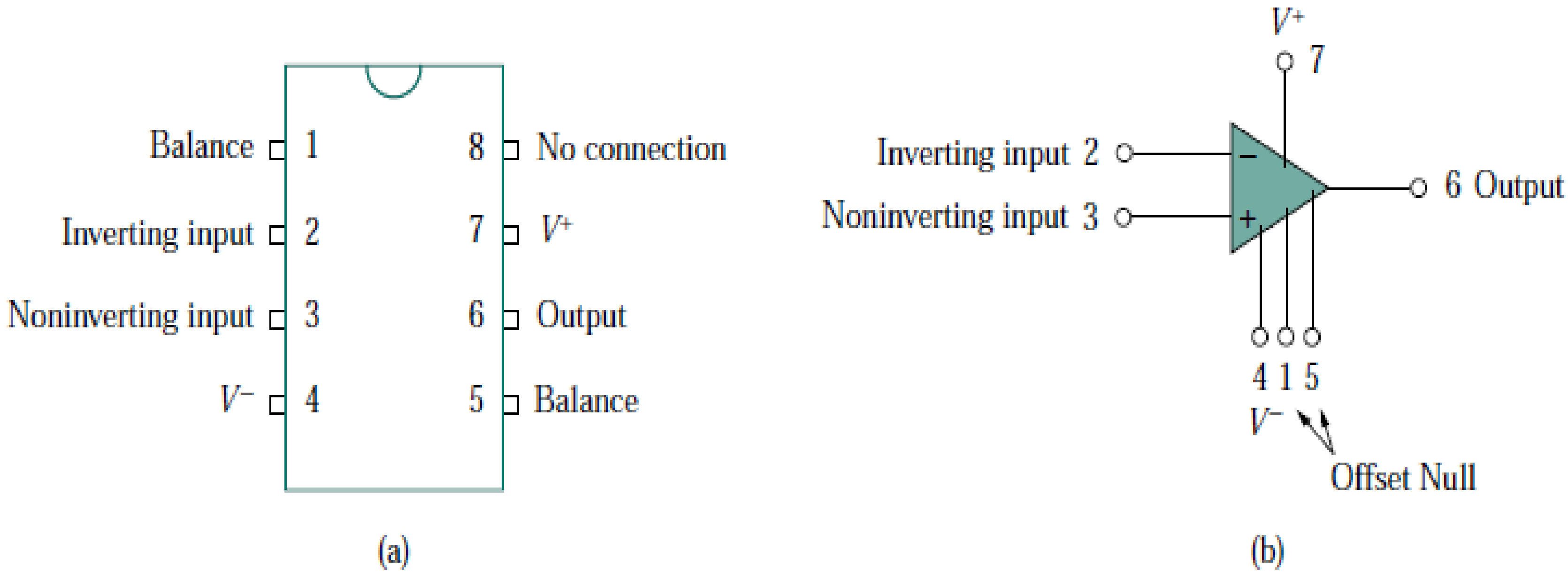
* Op amps can provide a buffer between two circuits.

* Op amps can be used to implement integrators and differentiators.

*More applications are Low-pass, High-pass, Band-pass and Band-reject filters.

Op Amp Symbol

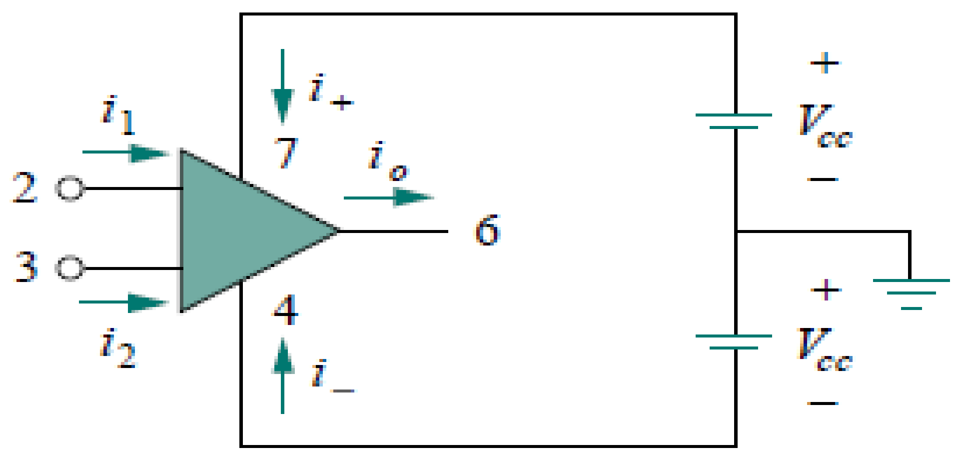
* The Op Amp: (a) pin configuration, (b) circuit symbol.



The Op Amp Model

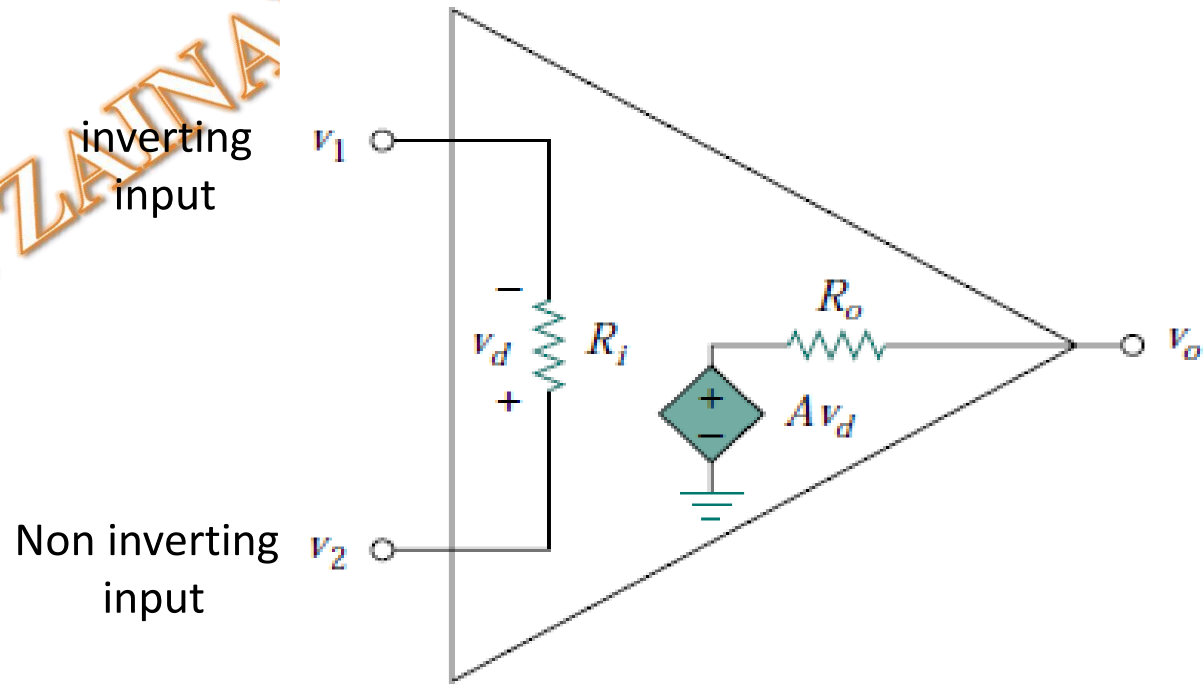
As an active element, the op amp must be powered by a voltage supply as typically shown in the Fig. below. Although the power supplies are often ignored in op amp circuit diagrams for the sake of simplicity, the power supply currents must not be overlooked. By KCL,

$$i_o = i_1 + i_2 + i_+ + i_-$$



The equivalent circuit model of an op amp is shown in the Fig. below. The output section consists of a voltage-controlled source in series with the output resistance *Ro*. It is evident from the Fig., that the input resistance *Ri* is the Thevenin equivalent resistance seen at the input terminals, while the output resistance *Ro* is the Thevenin equivalent resistance seen at the output. The differential input voltage *vd* is given by

$$v_d = v_2 - v_1$$



- where v_1 is the voltage between the inverting terminal and ground and v_2 is the voltage between the non-inverting terminal and ground. The op amp senses the difference between the two inputs, multiplies it by the gain A, and causes the resulting voltage to appear at the output. Thus, the output v_o is given by

$$v_o = Av_d = A(v_2 - v_1)$$

- *A* is called the *open-loop voltage gain* because it is the gain of the op amp without any external feedback from output to input. The table below shows typical values of voltage gain *A*, input resistance *Ri* , output resistance *Ro*, and supply voltage *VCC*.

Parameter	Typical Range	Ideal Values
Open-loop gain, A	10^5 to 10^8	∞
Input resistance, R_{in}	10^6 to 10^{13}	∞
Output resistance, R_o	10 to 100	0 Ω
Supply voltage, v_{cc}	5 to 24	

- The concept of feedback is crucial to our understanding of op-amp circuits. A negative feedback is achieved when the output is fed back to the inverting terminal of the op-amp., when there is a feedback path from output to input, the ratio of the output voltage to the input voltage is called the <u>*closed-loop gain*</u>. As a result of the negative feedback, it can be shown that the closed-loop gain is almost insensitive to the open-loop gain *A* of the op-amp. For this reason, op-amps are used in circuits with feedback paths.

Consequences of the Ideal Op Amp

* Infinite input resistance means the current into the inverting input is zero:

$$i_1 = 0 = i_2$$

* Infinite gain means the difference between *v*+ and *v*- is zero:

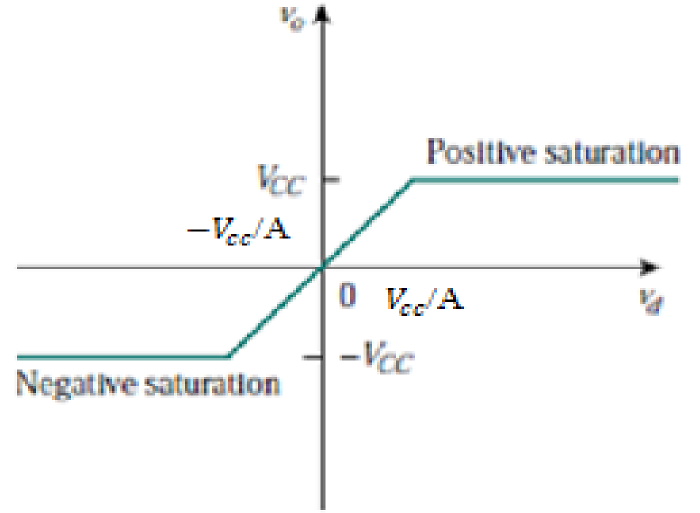
$$v_2 - v_1 = 0$$

Practical limitation of the op amp

* That is means the magnitude of its output voltage cannot exceed $|V_{cc}|$, i.e., the output voltage is dependent on and is limited by the power supply voltage. This figure illustrates that the op amp can operate in three modes, depending on the differential input voltage v_d, *where* $v_d = v_2 \text{-} v_1$:

1. Positive saturation, $v_o = V_{cc}$.
2. Linear region, $-V_{cc} \le v_o = A\, v_d \le V_{cc}$
3. Negative saturation, $v_o = -V_{cc}$.

* If we attempt to increase v_d beyond the linear range, the op amp becomes saturated and yields $v_o = V_{cc}$ or $v_o = -V_{cc}$.



INVERTING AMPLIFIER

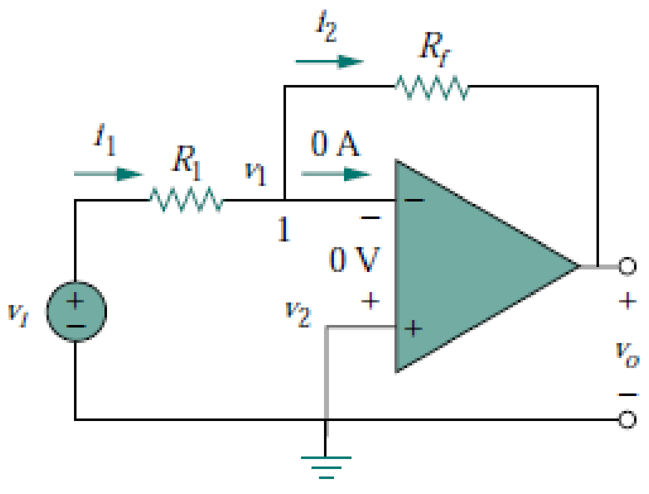
For Ideal Operational Amplifier:

$$i_1 = i_2 \qquad \Longrightarrow \qquad \frac{v_i - v_1}{R_1} = \frac{v_1 - v_o}{R_f}$$

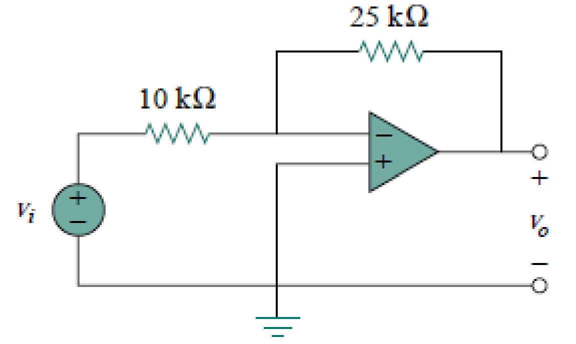
$$\frac{v_i}{R_1} = -\frac{v_o}{R_f}$$

$$v_o = -\frac{R_f}{R_1} v_i$$

$$A_v = v_o/v_i = -R_f/R_1$$



- Refer to the op amp in the Fig. shown, if *vi* = 0.5 V, calculate: (a) the output voltage *vo*, and (b) the current in the 10 k resistor.



- **Solution:**

(a)

$$\frac{v_o}{v_i} = -\frac{R_f}{R_1} = -\frac{25}{10} = -2.5$$

$$v_o = -2.5v_i = -2.5(0.5) = -1.25 \text{ V}$$

(b) The current through the 10-kΩ resistor is

$$i = \frac{v_i - 0}{R_1} = \frac{0.5 - 0}{10 \times 10^3} = 50\ \mu\text{A}$$

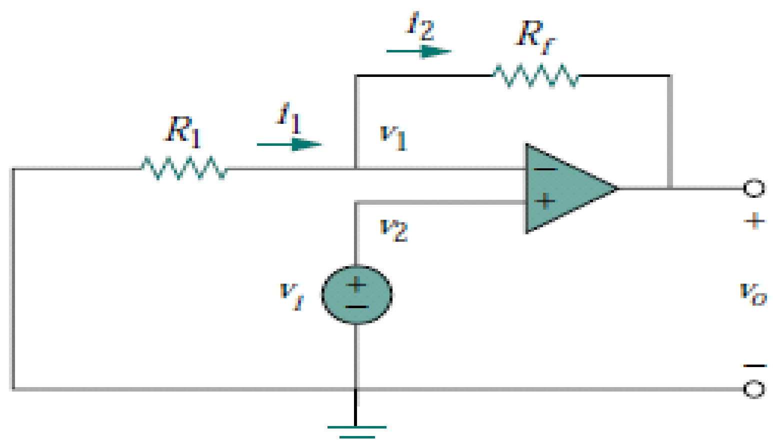
NONINVERTING AMPLIFIER

For Ideal Operational Amplifier:

$$i_1 = i_2 \qquad \Longrightarrow \qquad \frac{0 - v_1}{R_1} = \frac{v_1 - v_o}{R_f}$$

$$\frac{-v_i}{R_1} = \frac{v_i - v_o}{R_f}$$

$$v_o = \left(1 + \frac{R_f}{R_1}\right) v_i$$



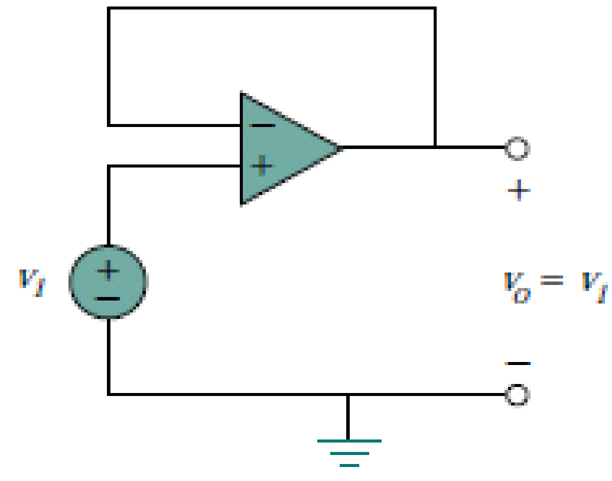
$$A_v = v_o / v_i = 1 + R_f / R_1$$

voltage follower:

For Ideal Operational Amplifier:

* In a voltage follower (or unity gain amplifier), the output follows the input. Thus,

$$v_o = v_i$$



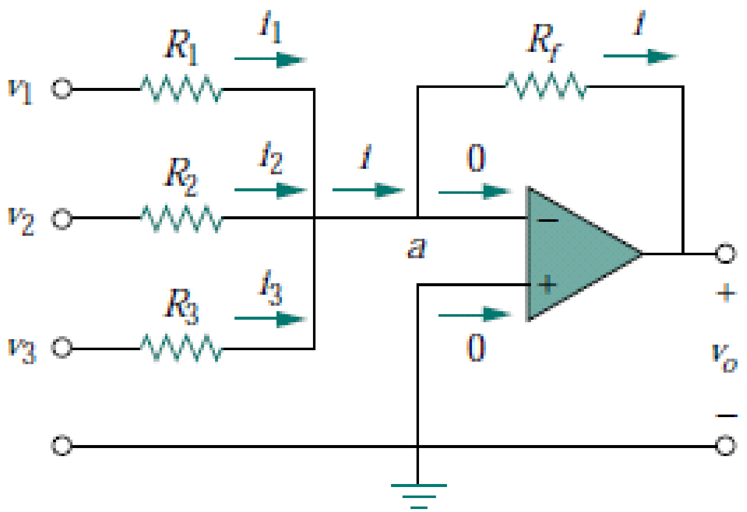
* Such a circuit has a very high input impedance and is therefore useful as an intermediate-stage (or buffer) amplifier to isolate one circuit from another.

SUMMING AMPLIFIER

$$i = i_1 + i_2 + i_3$$

$$i_1 = \frac{v_1 - v_a}{R_1}, \qquad i_2 = \frac{v_2 - v_a}{R_2}$$

$$i_3 = \frac{v_3 - v_a}{R_3}, \qquad i = \frac{v_a - v_o}{R_f}$$



$$v_o = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3\right)$$

DIFFERENCE AMPLIFIER

- Applying KCL to node *a*,

$$\frac{v_1 - v_a}{R_1} = \frac{v_a - v_o}{R_2}$$

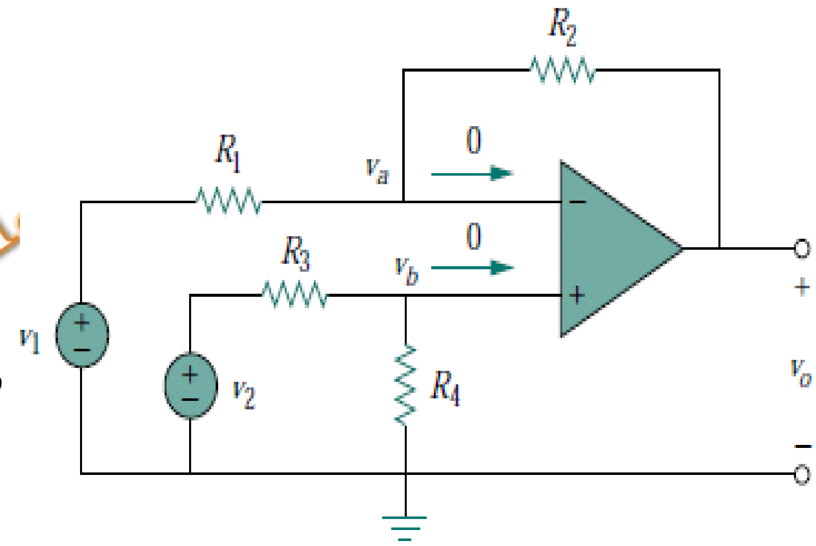
$$v_o = \left(\frac{R_2}{R_1} + 1\right) v_a - \frac{R_2}{R_1} v_1$$

- Applying KCL to node *b*,

$$\frac{v_2 - v_b}{R_3} = \frac{v_b - 0}{R_4}$$

$$v_b = \frac{R_4}{R_3 + R_4} v_2$$

- But, $v_a = v_b$



$$v_o = \left(\frac{R_2}{R_1} + 1\right)\frac{R_4}{R_3 + R_4}v_2 - \frac{R_2}{R_1}v_1$$

$$v_o = \frac{R_2}{R_1}\frac{(1 + R_1/R_2)}{(1 + R_3/R_4)}v_2 - \frac{R_2}{R_1}v_1$$

- Since a difference amplifier must reject a signal common to the two inputs, the amplifier must have the property that $v_o = 0$ when $v_1 = v_2$. This property exists when

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

* Thus, when the op amp circuit is a difference amplifier, then,

$$v_o = \frac{R_2}{R_1}(v_2 - v_1)$$

* If $R2 = R1$ and $R3 = R4$, the difference amplifier becomes a *subtractor*, with the output

$$v_o = v_2 - v_1$$

Example:

Calculate v_o and i_o in the op amp circuit in Fig.

1.

This is a summer with two inputs.

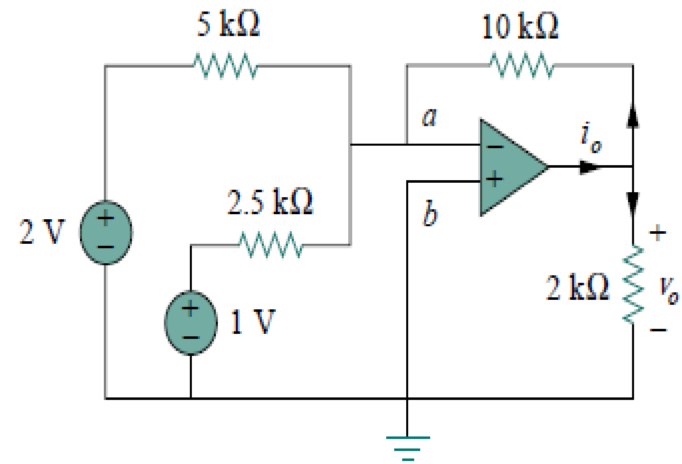


Fig. 1

$$v_o = -\left[\frac{10}{5}(2) + \frac{10}{2.5}(1)\right] = -(4+4) = -8\text{ V}$$

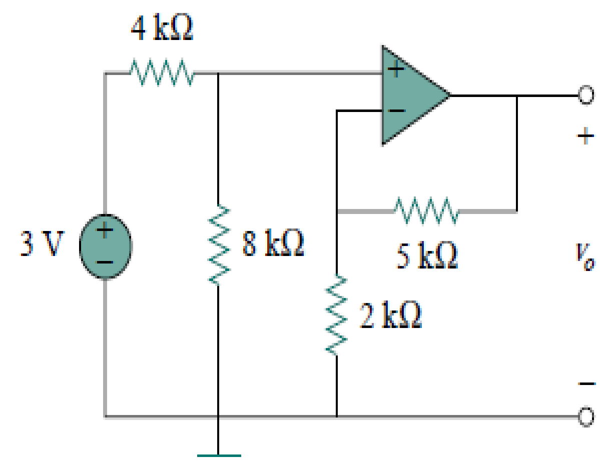
The current i_o is the sum of the currents through the 10-kΩ and 2-kΩ resistors. Both of these resistors have voltage $v_o = -8$ V across them, since $v_a = v_b = 0$. Hence,

$$i_o = \frac{v_o - 0}{10} + \frac{v_o - 0}{2}\text{ mA} = -0.8 - 0.4 = -1.2\text{ mA}$$

H.W.:

Calculate v_o in the circuit in Fig.

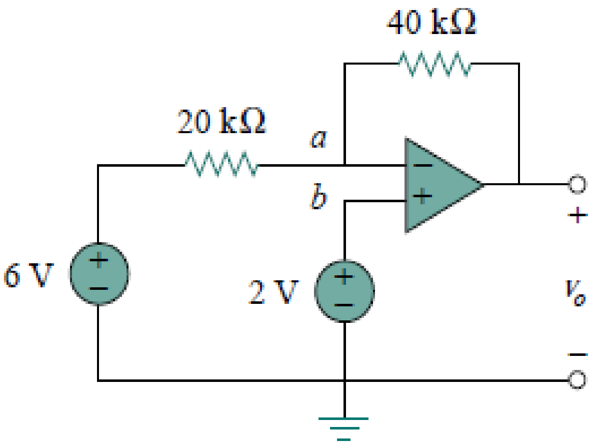
Answer: 7 V.



Example:

Determine *vo* in the op amp circuit

Solution:



Applying KCL at node a,

$$\frac{v_a - v_o}{40} = \frac{6 - v_a}{20}$$

$$v_a - v_o = 12 - 2v_a \quad \Longrightarrow \quad v_o = 3v_a - 12$$

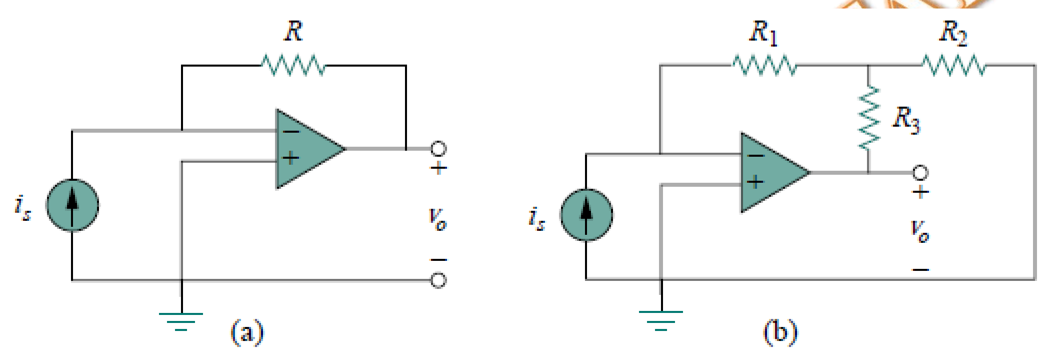
But $v_a = v_b = 2$ V for an ideal op amp, because of the zero voltage drop across the input terminals of the op amp. Hence,

$$v_o = 6 - 12 = -6 \text{ V}$$

Notice that if $v_b = 0 = v_a$, then $v_o = -12$ V

H.W.:

- Two kinds of current-to-voltage converters (also known as *trans-resistance amplifiers*) are shown in Fig. a& Fig. b



- (a) Show that for the converter in Fig.a

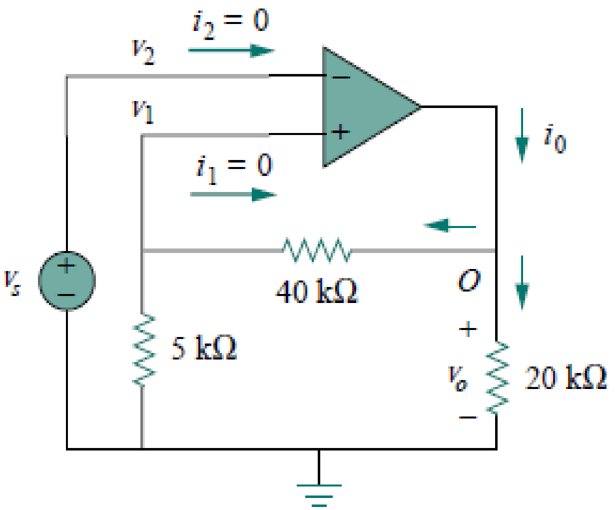
$$\frac{v_o}{i_s} = -R$$

- (b) Show that for the converter in Fig.b

$$\frac{v_o}{i_s} = -R_1\left(1 + \frac{R_3}{R_1} + \frac{R_3}{R_2}\right)$$

Example:

A 741 op-amp is used in the circuit of the Fig. shown, Calculate the closed-loop gain *vo/vs* . Find *io* when *vs* = 1 V. (using the ideal op amp model).



sol.:

$$v_2 = v_s$$

Since $i_1 = 0$, the 40-kΩ and 5-kΩ resistors are in series because the same current flows through them. v_1 is the voltage across the 5-kΩ resistor. Hence, using the voltage division principle,

$$v_1 = \frac{5}{5+40}v_o = \frac{v_o}{9} \qquad v_2 = v_1$$

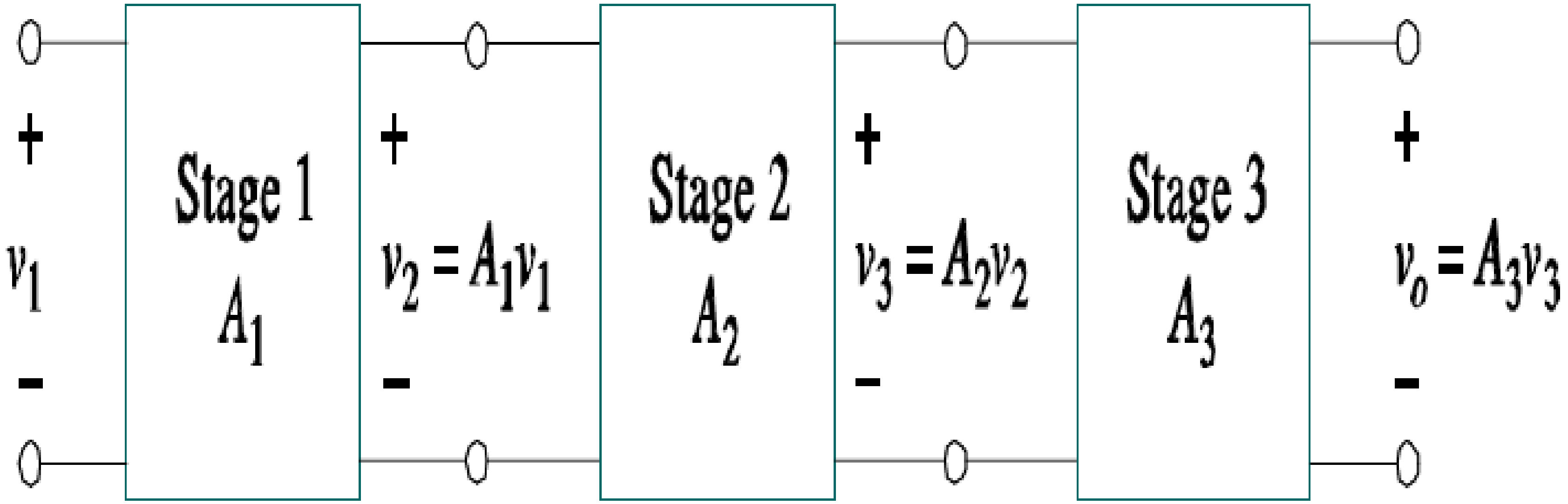
$$v_s = \frac{v_o}{9} \quad \Longrightarrow \quad \frac{v_o}{v_s} = 9$$

At node O,

$$i_o = \frac{v_o}{40+5} + \frac{v_o}{20} \text{ mA}$$

when $v_s = 1$ V, $v_o = 9$ V. $\qquad i_o = 0.2 + 0.45 = 0.65$ mA

CASCADED OP - AMP

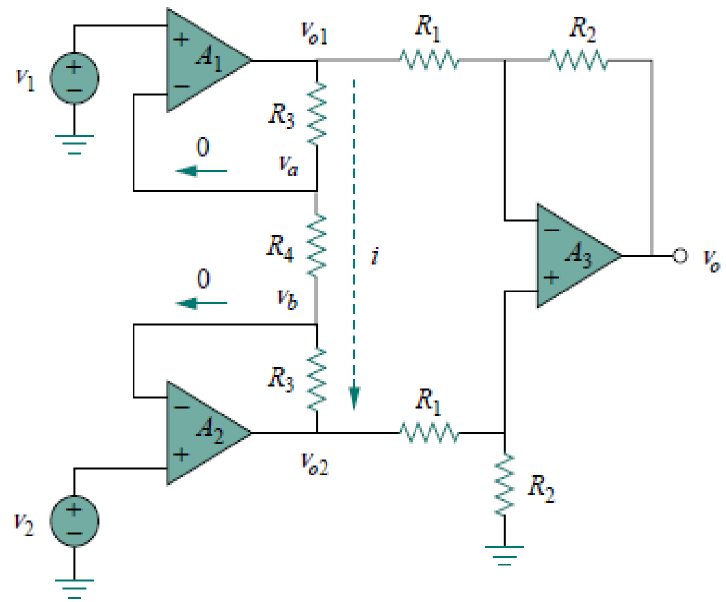


A three-stage cascaded connection.

$$A = A_1 A_2 A_3$$

Example:

An *instrumentation amplifier* shown, is an amplifier of low-level signals used in process control or measurement applications and commercially available in single-package units. Show that



$$v_o = \frac{R_2}{R_1}\left(1 + \frac{2R_3}{R_4}\right)(v_2 - v_1)$$

Solution:

We recognize that the amplifier A_3 is a difference amplifier.

$$v_o = \frac{R_2}{R_1}(v_{o2} - v_{o1})$$

Since the op amps A_1 and A_2 draw no current, current i flows through the three resistors as though they were in series. Hence,

$$v_{o1} - v_{o2} = i(R_3 + R_4 + R_3) = i(2R_3 + R_4)$$

But

$$i = \frac{v_a - v_b}{R_4}$$

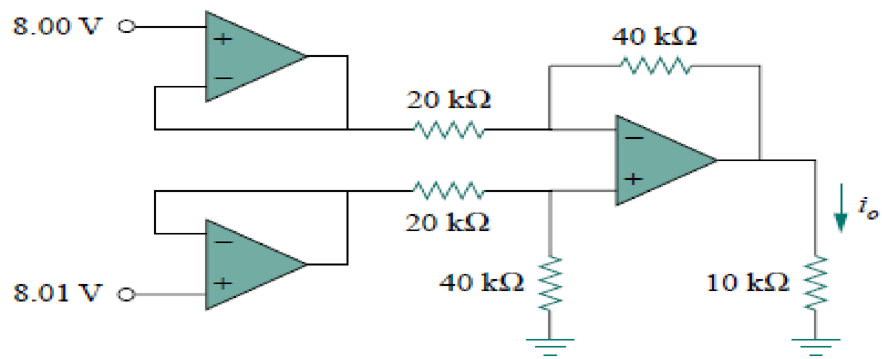
and $v_a = v_1, \quad v_b = v_2$. Therefore,

$$i = \frac{v_1 - v_2}{R_4}$$

$$v_o = \frac{R_2}{R_1}\left(1 + \frac{2R_3}{R_4}\right)(v_2 - v_1)$$

H.W.:

Obtain i_o in the instrumentation amplifier circuit



Example.:

Design an op amp circuit with inputs v_1 and v_2 such that $v_o = -5v_1 + 3v_2$.

Solution:

The circuit requires that

$$v_o = 3v_2 - 5v_1$$

This circuit can be realized in two ways.

DESIGN 1 If we desire to use only one op amp,

$$v_o = \frac{R_2}{R_1}\frac{(1 + R_1/R_2)}{(1 + R_3/R_4)}v_2 - \frac{R_2}{R_1}v_1$$

$$\frac{R_2}{R_1} = 5 \quad \Longrightarrow \quad R_2 = 5R_1$$

Also,

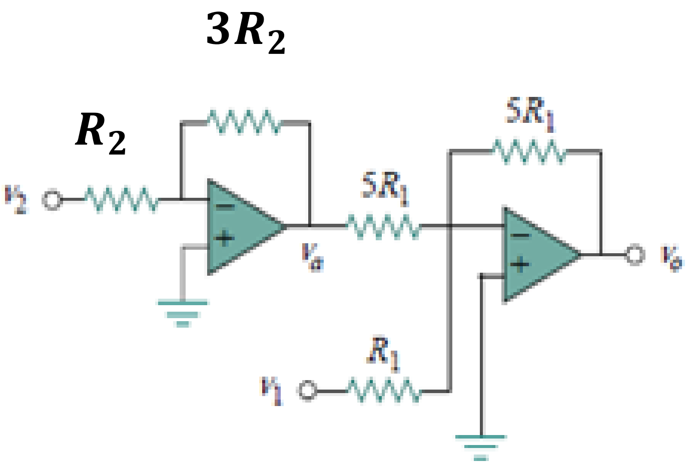
$$5\frac{(1 + R_1/R_2)}{(1 + R_3/R_4)} = 3 \quad \Longrightarrow \quad \frac{\frac{6}{5}}{1 + R_3/R_4} = \frac{3}{5}$$

or

$$2 = 1 + \frac{R_3}{R_4} \quad \Longrightarrow \quad R_3 = R_4$$

If we choose $R_1 = 10$ kΩ and $R_3 = 20$ kΩ, then $R_2 = 50$ kΩ and $R_4 = 20$ kΩ.

DESIGN 2 If we desire to use more than one op amp, we may cascade an inverting amplifier and a two-input inverting summer, as shown in Fig.



$$v_o = -v_a - 5v_1$$

and for the inverter,

$$v_a = -3v_2$$

$$v_o = 3v_2 - 5v_1$$

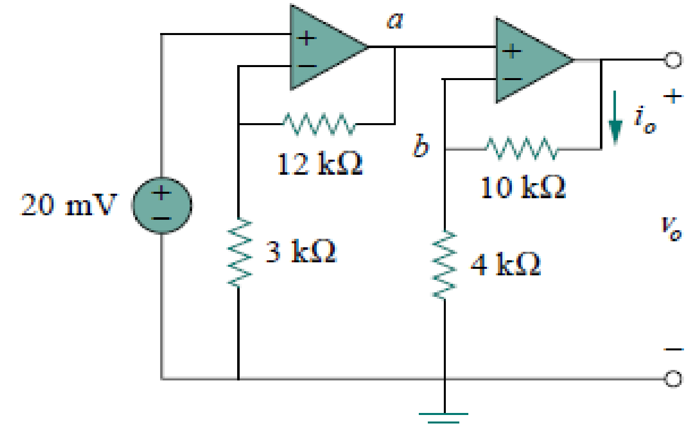
we may select $R_1 = 10\ \text{k}\Omega$ and $R_2 = 20\ \text{k}\Omega$ or $R_1 = R_2 = 10\ \text{k}\Omega$.

H.W.:

Design a difference amplifier with gain 4.

Answer: Typical: $R_1 = R_3 = 10\ \text{k}\Omega$, $R_2 = R_4 = 40\ \text{k}\Omega$.

Example: Find v_o and i_o in the circuit



Solution:

This circuit consists of two noninverting amplifiers cascaded. At the output of the first op amp,

$$v_a = \left(1 + \frac{12}{3}\right)(20) = 100 \text{ mV}$$

At the output of the second op amp,

$$v_o = \left(1 + \frac{10}{4}\right)v_a = (1 + 2.5)100 = 350 \text{ mV}$$

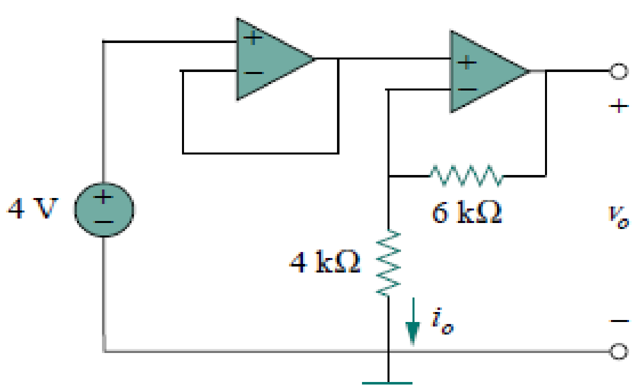
The required current i_o is the current through the 10-kΩ resistor.

$$i_o = \frac{v_o - v_b}{10} \text{ mA}$$

But $v_b = v_a = 100$ mV. Hence,

$$i_o = \frac{(350 - 100) \times 10^{-3}}{10 \times 10^3} = 25\ \mu\text{A}$$

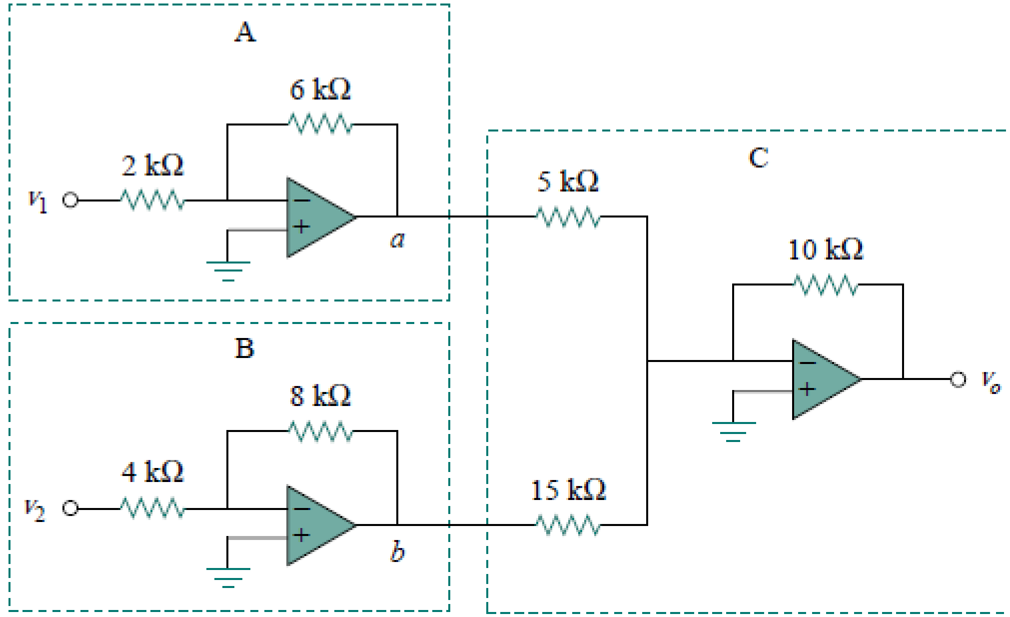
H.W.:



Determine v_o and i_o in the op amp circuit

Answer: 10 V, 1 mA.

Example: If $v_1 = 1$ V and $v_2 = 2$ V, find v_o in the op amp circuit



Solution:

The circuit consists of two inverters A and B and a summer C as shown

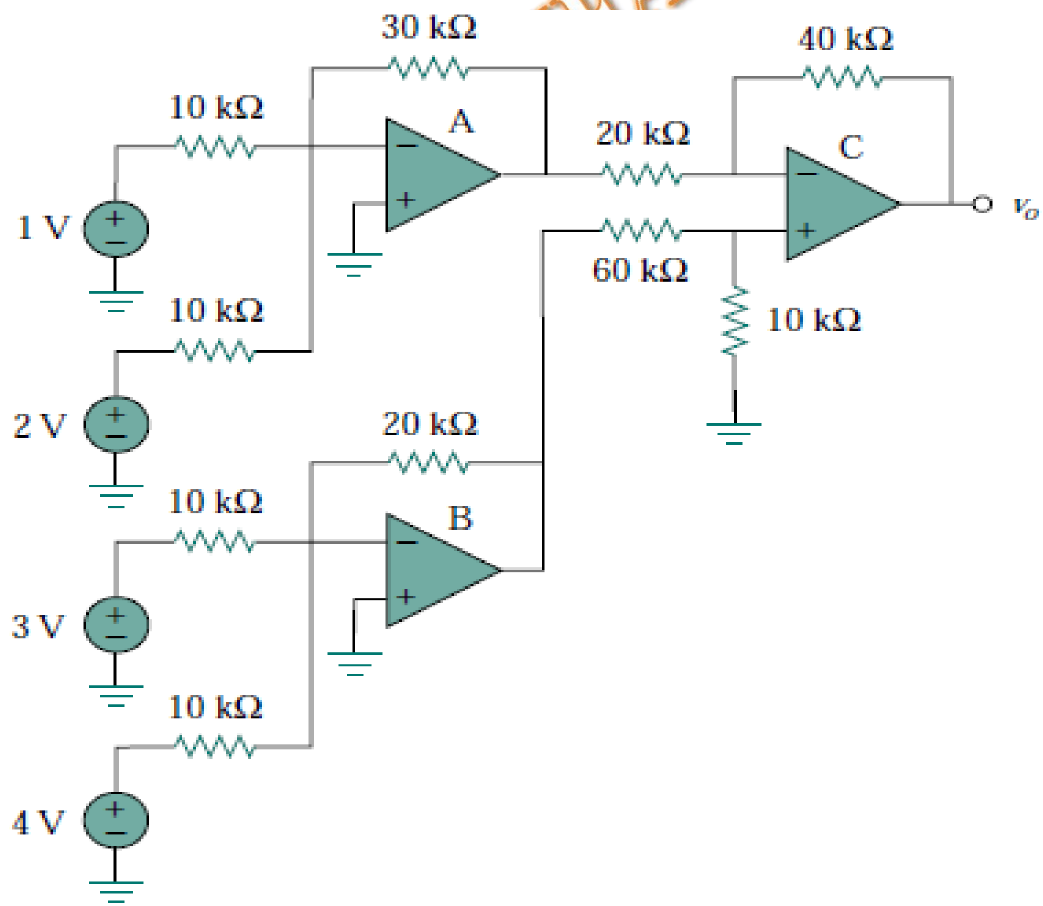
$$v_a = -\frac{6}{2}(v_1) = -3(1) = -3 \text{ V}, \qquad v_b = -\frac{8}{4}(v_2) = -2(2) = -4 \text{ V}$$

These become the inputs to the summer so that the output is obtained as

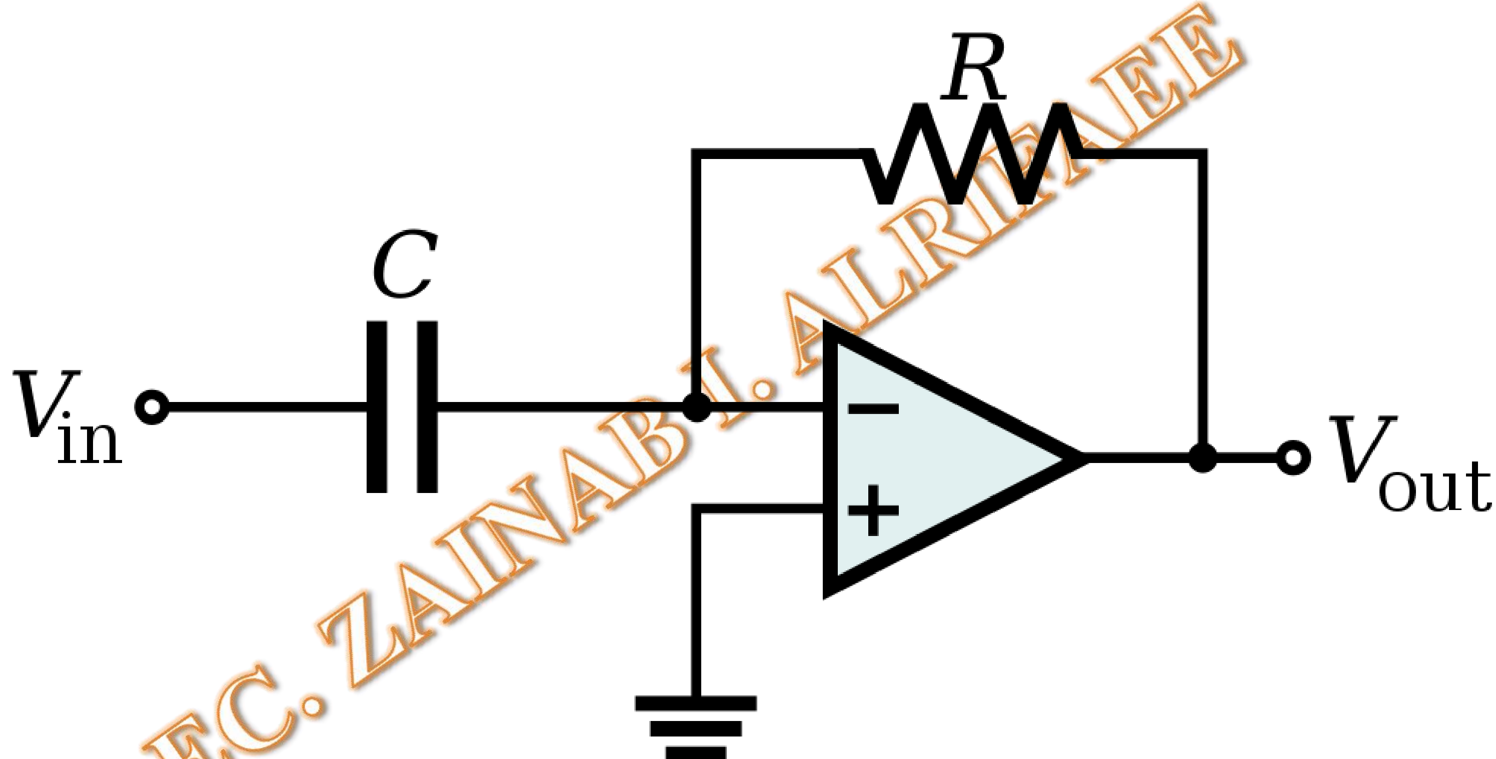
$$v_o = -\left(\frac{10}{5}v_a + \frac{10}{15}v_b\right) = -\left[2(-3) + \frac{2}{3}(-4)\right] = 8.333 \text{ V}$$

Example

* Find the output voltage for the following circuit:

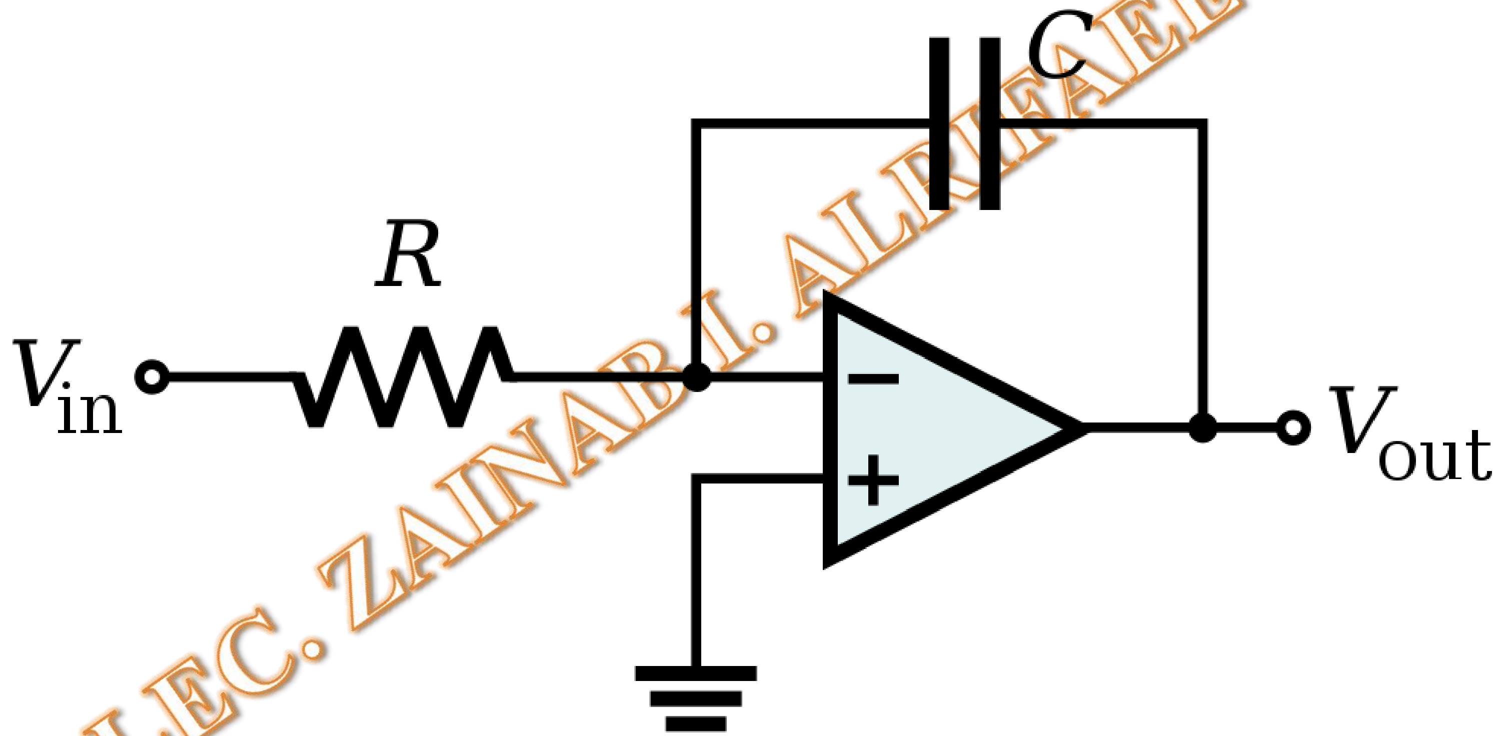


Op-Amp Differentiator



$$V_{\text{out}} = -RC\,\frac{\mathrm{d}V_{\text{in}}}{\mathrm{d}t}$$

Op-Amp Integrator

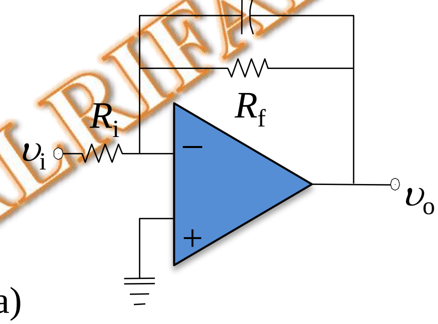


$$V_{\text{out}} = -\int_0^t \frac{V_{\text{in}}}{RC}\,\mathrm{d}t + V_{\text{initial}}$$

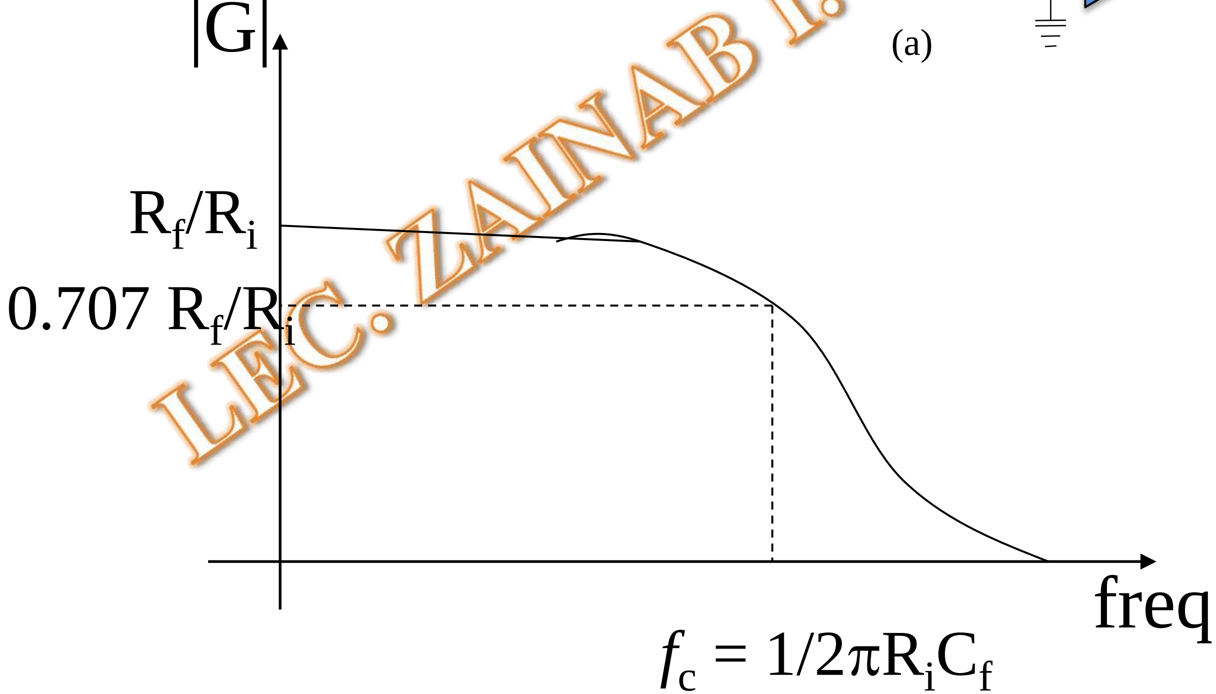
Active Filters- Low-Pass Filter

- A low-pass filter attenuates high frequencies

$$\text{Gain} = G = \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{-R_f}{R_i}\,\frac{1}{1 + j\omega R_f C_f}$$



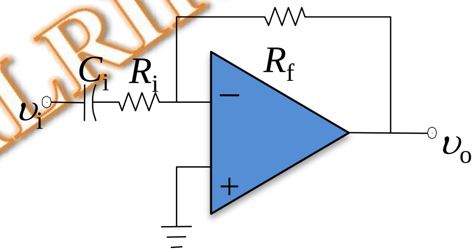
(a)



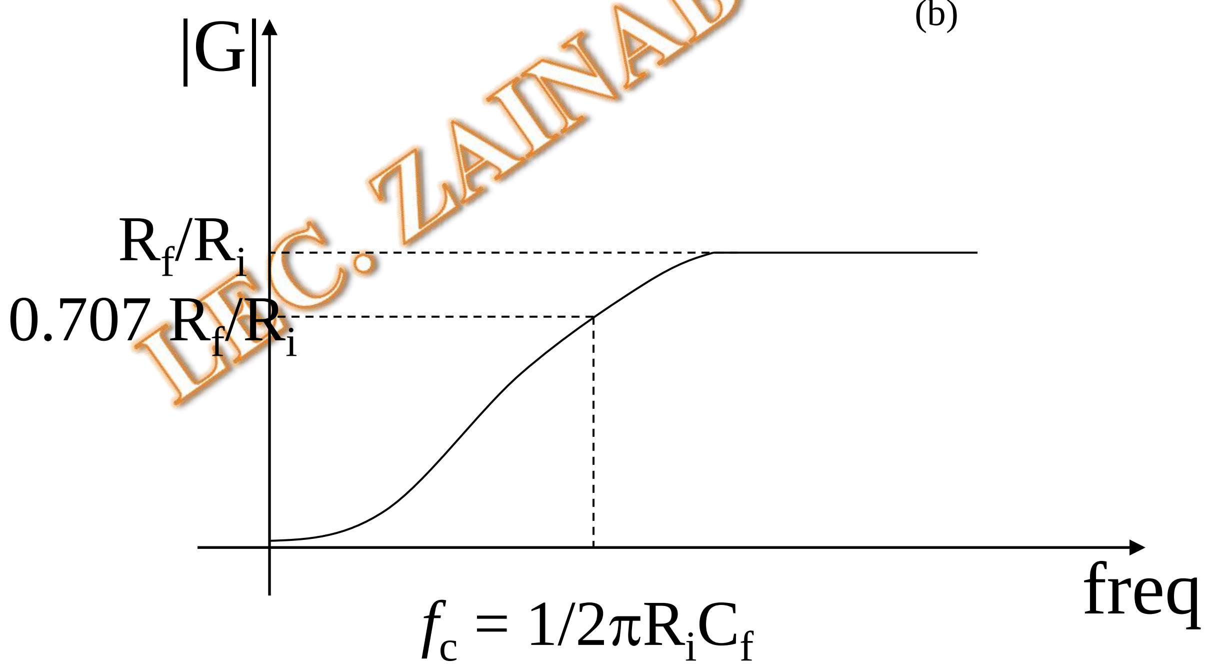
Active Filters (High-Pass Filter)

- A high-pass filter attenuates low frequencies and blocks dc.

$$\text{Gain} = G = \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{-R_f}{R_i}\,\frac{j\omega R_i C_i}{1 + j\omega R_i C_i}$$



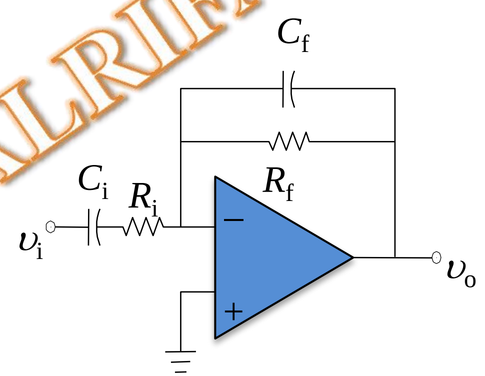
(b)



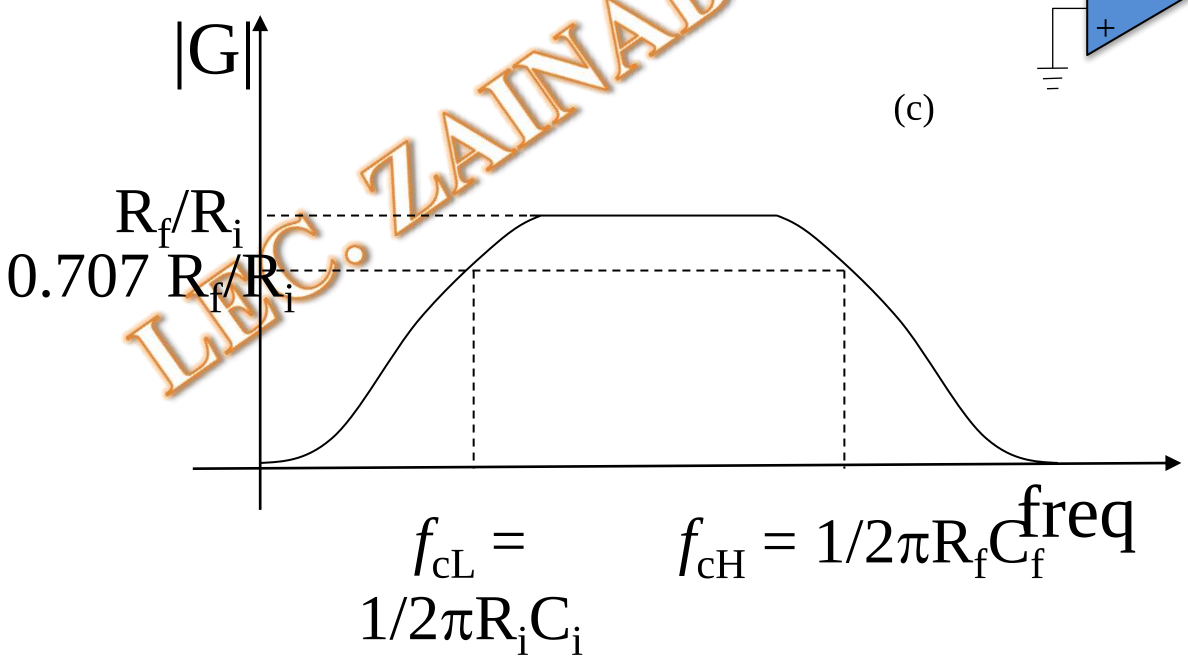
Active Filters (Band-Pass Filter)

- A bandpass filter attenuates both low and high frequencies.

$$\frac{V_o(j\omega)}{V_i(j\omega)} = \frac{-j\omega R_f C_i}{(1 + j\omega R_f C_f)(1 + j\omega R_i C_i)}$$



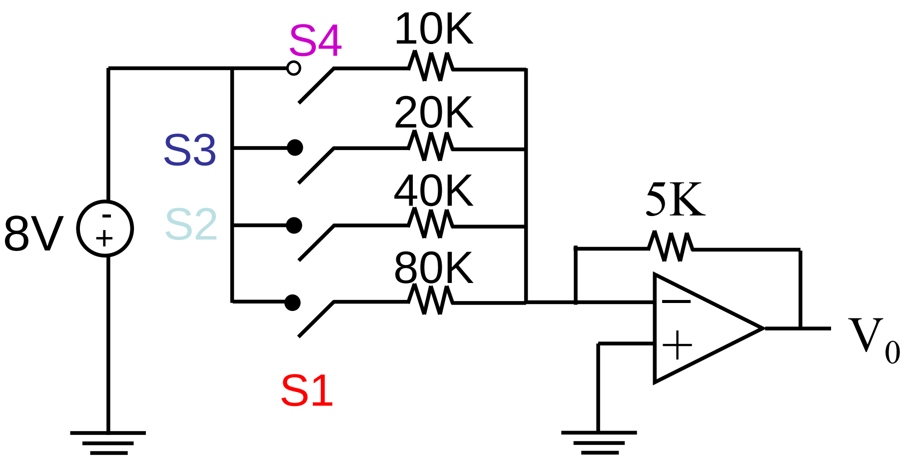
(c)



Application: Digital-to-Analog Conversion

A DAC can be used to convert the digital representation of an audio signal into an analog voltage that is then used to drive speakers -- so that you can hear it!

"Weighted-adder D/A converter"



4-Bit D/A

(Transistors are used as electronic switches)

S1 closed if LSB =1
S2 " if next bit = 1
S3 " if " " = 1
S4 " if MSB = 1

Binary number	Analog output (volts)
0 0 0 0	0
0 0 0 1	.5
0 0 1 0	1
0 0 1 1	1.5
0 1 0 0	2
0 1 0 1	2.5
0 1 1 0	3
0 1 1 1	3.5
1 0 0 0	4
1 0 0 1	4.5
1 0 1 0	5
1 0 1 1	5.5
1 1 0 0	6
1 1 0 1	6.5
1 1 1 0	7
1 1 1 1	7.5

↑ MSB ↑ LSB

Characteristic of 4-Bit DAC