Lecture one ((Review on Vectors))

<u>Scalar and Vector</u>: The term scalar refers to a quantity represented by single real number, like mass, density, pressure.

الكميات غير المتجهة تشير او تمثل برقم واحد حقيقي مثل الكتله او الكثافة او الضغط.

Vector quantity has both magnitude and direction, like force, velocity, acceleration

الكميات المتجهه لها قيمة واتجاه مثل القوة / السرعة / التعجيل.

<u>Vector algebra</u>:

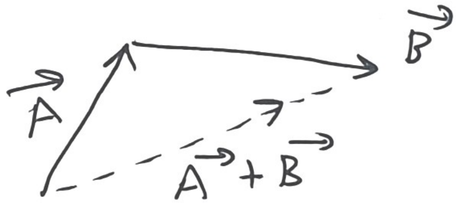
Let $\vec{A}$, $\vec{B}$ & $\vec{C}$ are vectors, then

① $\vec{A} + \vec{B} = \vec{B} + \vec{A}$

② $\vec{A} + (\vec{B} + \vec{C}) = (\vec{A} + \vec{B}) + \vec{C}$

③ $(r+s)(\vec{A} + \vec{B}) = r(\vec{A} + \vec{B}) + s(\vec{A} + \vec{B})$

$= r\vec{A} + r\vec{B} + s\vec{A} + s\vec{B}$



where r & s are scalar quantity

④ $\vec{A} = \vec{B}$ if and only if $\vec{A} - \vec{B} = \text{Zero}$

Rectangular coordinate system

let point $P(1,2,3)$ and $Q(2,-2,1)$ are as shown in Fig.(1), then the vector $\overrightarrow{OP}$ is the vector pointing from the origin (point O) to point P

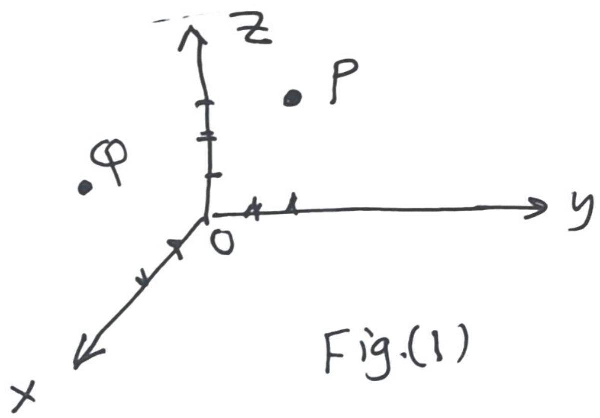


Fig.(1)

$$\overrightarrow{OP} = \underbrace{(1-0)}_{\text{احداثيات } x \text{ لنقطة } p} a_x + \underbrace{(2-0)}_{\text{احداثيات } y \text{ لنقطة } p} a_y + \underbrace{(3-0)}_{\text{احداثيات } z \text{ لنقطة } p} a_z$$

similarly $\overrightarrow{OQ} = (2-0)a_x + (-2-0)a_y + (1-0)a_z$

$$\Rightarrow \quad \overrightarrow{OP} = a_x + 2a_y + 3a_z$$

$$\overrightarrow{OQ} = 2a_x - 2a_y + a_z$$

However $\overrightarrow{PQ} = (2-1)a_x + (-2-2)a_y + (1-3)a_z$

$$\overrightarrow{PQ} = a_x - 4a_y - 2a_z$$

where a_x = unit vector in x-direction
متجهة طولها واحد للدلالة على اتجاه محور السينات

a_y = unit vector in y-direction

a_z = unit vector in z-direction

If the vector $\vec{B} = B_x a_x + B_y a_y + B_z a_z$

then the length of $\vec{B}$ is $|\vec{B}|$

$$|\vec{B}| = \sqrt{B_x^2 + B_y^2 + B_z^2}$$ and

unit vector of $\vec{B}$ (direction of $\vec{B}$) $= \dfrac{\vec{B}}{|\vec{B}|}$

Ex: Find the vector extending from
مثال
الممتد
the origin to point $G(2,-2,-1)$
and then determine its unit vector.

Solution: المطلوب ايجاد المتجه $\vec{OG}$
الحل

$$\vec{OG} = (2-0)a_x + (-2-0)a_y + (-1-0)a_z$$

$$\vec{OG} = 2a_x - 2a_y - a_z$$

unit vector of $\vec{OG} = \vec{a}_{OG} = \dfrac{\vec{OG}}{|\vec{OG}|}$

$$\vec{a}_{OG} = \frac{2a_x - 2a_y - a_z}{\sqrt{(2)^2 + (-2)^2 + (-1)^2}} = \frac{2a_x - 2a_y - a_z}{\sqrt{9}}$$

$$\vec{a}_{OG} = \frac{2}{3}a_x - \frac{2}{3}a_y - \frac{a_z}{3}$$

sometimes, we denote the unit vector of $\vec{OG}$ as : $\vec{a}_{OG}$ or $\vec{U}_{OG}$ or $\vec{I}_{OG}$

Ex: Given $M(-1,2,1)$, $N(3,-3,0)$ and $P(-2,-3,4)$, find ⓐ $\vec{R}_{MN}$
ⓑ $\vec{R}_{MN} + \vec{R}_{MP}$ ⓒ $|\vec{r}_M|$ ⓓ $\vec{a}_{MP}$

ⓒ $|2\vec{r}_p - 3\vec{r}_N|$

solution: ⓐ $\vec{R}_{MN}$ is the vector $\overrightarrow{MN}$
الحل

$$\vec{R}_{MN} = (3-(-1))a_x + (-3-2)a_y + (0-1)a_z$$

$$\vec{R}_{MN} = 4a_x - 5a_y - a_z$$

ⓑ $\vec{R}_{MN} + \vec{R}_{MP}$

$$\vec{R}_{MP} = (-2-(-1))a_x + (-3-2)a_y + (4-1)a_z$$

$$\vec{R}_{MP} = -a_x - 5a_y + 3a_z$$

$$\therefore \vec{R}_{MN} + \vec{R}_{MP} = \underbrace{4a_x - 5a_y - a_z}_{\vec{R}_{MN}} \; \underbrace{- a_x - 5a_y + 3a_z}_{\vec{R}_{MP}}$$

$$\vec{R}_{MN} + \vec{R}_{MP} = 3a_x - 10a_y + 2a_z$$

ⓒ $|\vec{r}_M|$ is length of the vector $\vec{r}_M$.

$\vec{r}_M$ is another notation to $\overrightarrow{OM}$

$\therefore$ $|\vec{r}_M|$ = length of the vector $\overrightarrow{OM}$

$$\therefore \overrightarrow{OM} = (-1-0)a_x + (2-0)a_y + (1-0)a_z$$

$$\overrightarrow{OM} = -a_x + 2a_y + a_z$$

$$|\overrightarrow{OM}| = \sqrt{(-1)^2 + (2)^2 + (1)^2} = \sqrt{1+4+1} = \sqrt{6}$$

$$\therefore |\vec{r}_M| = |\overrightarrow{OM}| = \sqrt{6}$$

ⓓ $\vec{a}_{MP}$ is unit vector of $\vec{MP}$

$\because$ $\vec{MP}$ is the same vector as $\vec{R}_{MP}$

$$\vec{MP} = -a_x - 5a_y + 3a_z$$

$$\vec{a}_{MP} = \frac{\vec{MP}}{|\vec{MP}|} = \frac{-a_x - 5a_y + 3a_z}{\sqrt{1+25+9}}$$

$$\vec{a}_{MP} = \frac{1}{\sqrt{35}}(-a_x - 5a_y + 3a_z)$$

ⓔ $|2\vec{r_p} - 3\vec{r}_N|$ is the length of the vector $2\vec{r_p} - 3\vec{r}_N$ or length of the ~~vecto~~ vector $2\vec{OP} - 3\vec{ON}$

$$2\vec{OP} - 3\vec{ON} = 2[-2a_x - 3a_y + 4a_z] - 3[3a_x - 3a_y]$$

$$= -4a_x - 6a_y + 8a_z - 9a_x + 9a_y$$

$$= -13a_x + 3a_y + 8a_z$$

$$|2\vec{OP} - 3\vec{ON}| = \sqrt{(-13)^2 + (3)^2 + (8)^2}$$

$$= \sqrt{169+9+64} = \sqrt{242}$$

ملاحظة :

① يستعمل الرمز $\vec{r_p}$ للدلالة عن المتجهة $\vec{OP}$

② يستعمل الرمز $\vec{r_N}$ للدلالة عن المتجه $\vec{ON}$

③ يستعمل الرمز $\vec{R}_{MP}$ للدلالة عن المتجه $\vec{MP}$

The Vector field

We define a vector field as vector function. Each components as a function of (x,y,z).

ال vector field هي متجهة تكون المركبة x و y و z هي دالة لـ (x,y,z) مثال

$$\vec{V} = V_x a_x + V_y a_y + V_z a_z$$

المركبه x   المركبه y   المركبة z

هذه المركبات هي دالة لـ x,y,z وليست ثوابت

$$V_x = V_x(x,y,z)$$
$$V_y = V_y(x,y,z)$$
$$V_z = V_z(x,y,z)$$

Ex: The vector field $\vec{S} = (x-1)^2 a_x + (x+xyz)a_y + (z^2+x^2-y)a_z$

(a) Evaluate S at point $(1,2,3)$

(b) Determine unit vector of $\vec{S}$ at point $(1,2,3)$

solution: $\vec{S}$ is vector field لأنه مركباتها دالة لـ x,y,z

$$\vec{S}_{\text{at point }(1,2,3)} = (1-1)^2 a_x + (1\times2\times$$

$$= (1-1)^2 a_x + (1+1\times2\times3)a_y + (3^2+1^2-2^2)a_z$$

$$\Rightarrow \vec{S} = 7a_y + 6a_z$$

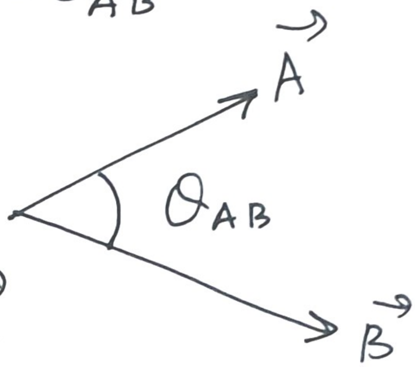
unit vector of S at point (1,2,3) is

$$|\vec{S}| = \sqrt{7^2+6^2} = \sqrt{49+36} = \sqrt{85}$$

Dot product (scalar product)

$$\vec{A} \cdot \vec{B} = |\vec{A}| \times |\vec{B}| \cos\theta_{AB}$$

dot



where θ_{AB} angle between vector $\vec{A}$ & $\vec{B}$

Remarks

① $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$

② if $\vec{A} = A_x a_x + A_y a_y + A_z a_z$

and $\vec{B} = B_x a_x + B_y a_y + B_z a_z$

then $\vec{A} \cdot \vec{B} = A_x \cdot B_x + A_y B_y + A_z B_z$

النتيجه او الجواب scalar ولذلك يسمى

scalar product

③ $\vec{A} \cdot \vec{A} = |\vec{A}|\,|\vec{A}| \cos 0^\circ$

$= |\vec{A}|^2$

$\therefore \vec{A} \cdot \vec{A} =$ مربع طول المتجهة $\vec{A}$

Ex: $\vec{A} = 3a_x + 4a_y - 2a_z$

$\vec{B} = 2a_x + 3a_y + 10a_z$

find $|\vec{A}|$, $|\vec{B}|$ and angle between $\vec{A}$ & $\vec{B}$

solution:

$\because \vec{A} = 3a_x + 4a_y - 2a_z \Rightarrow |\vec{A}| = \sqrt{9+16+4} = \sqrt{29}$

$\because \vec{B} = 2a_x + 3a_y + 10a_z \Rightarrow \vec{B} = \sqrt{4+9+100} = \sqrt{113}$

$\because \vec{A} \cdot \vec{B} = |A| \cdot |B| \cos\theta_{AB}$

$$\underbrace{(3a_x + 4a_y - 2a_z)}_{A} \cdot \underbrace{(2a_x + 3a_y + 10a_z)}_{B} = |A||B| \cos\theta_{AB}$$

$$6 + 12 - 20 = \sqrt{29} \cdot \sqrt{113} \cos\theta_{AB}$$

$$\cos\theta_{AB} = -\frac{2}{\sqrt{29} \cdot \sqrt{113}} = -0.034$$

$$\theta_{AB} \cong 92^{\circ}$$

ملاحظة

Remark: Dot product is used to find angle between any two vector

يستخدم لايجاد الزاوية بين اي متجهتين.