

## Chapter Six

### Three-phase systems

#### Introduction

Generation, transmission and distribution of electricity via the National Grid system is accomplished by three phase alternating currents. The voltage induced by a single coil when rotated in a uniform magnetic field and is known as a **single-phase voltage**. Most consumers are fed by means of a single-phase a.c. supply. Two wires are used, one called the live conductor (usually coloured red) and the other is called the neutral conductor (usually coloured black). The neutral is usually connected via protective gear to earth, the earth wire being coloured green.

#### Three-phase supply

**A three-phase supply** is generated when three coils are placed  $120^\circ$  apart and the whole rotated in a uniform magnetic field. The result is three independent supplies of equal voltages which are each displaced by  $120^\circ$  from each other. (i) The convention adopted to identify each of the phase voltages is: R-red, Y-yellow, and B-blue.

(ii) The phase-sequence is given by the sequence in which the conductors pass the point initially taken by the red conductor. The national standard phase sequence is R, Y, B. A three-phase a.c. supply is carried by three conductors, called 'lines' which are coloured red, yellow and blue. The currents in these conductors are known as line currents ( $I_L$ ) and the p.d.'s between them are known as line voltages ( $V_L$ ). A fourth conductor, called the neutral (coloured black, and connected through protective devices to earth) is often used with a three-phase supply. If the three-phase windings are kept independent then six wires are needed to connect a supply source (such as a generator) to a load (such as motor). To reduce the number of wires it is usual to interconnect the three phases. There are two ways in which this can be done, these being:

(a) a star connection, and (b) a delta, or mesh, connection.

## Star connection

(i) A star-connected load is shown in Fig. 6.1 where the three line conductors are each connected to a load and the outlets from the loads are joined together at N to form what is termed the neutral point or the star point.

(ii) The voltages,  $V_R$ ,  $V_Y$  and  $V_B$  are called phase voltages or line to neutral voltages. Phase voltages are generally denoted by  $V_p$ .

(iii) The voltages,  $V_{RY}$ ,  $V_{YB}$  and  $V_{BR}$  are called line voltages.

(iv) From Fig. 6.1 it can be seen that the phase currents (generally denoted by  $I_p$ ) are equal to their respective line currents  $I_R$ ,  $I_Y$  and  $I_B$ , i.e. for a star connection:

$$I_L = I_p$$

(v) For a balanced system:

$$I_R = I_Y = I_B, V_R = V_Y = V_B$$

$$V_{RY} = V_{YB} = V_{BR}, Z_R = Z_Y = Z_B$$

and the current in the neutral conductor,  $I_N = 0$  When a star-connected system is balanced, then the neutral conductor is unnecessary and is often omitted.

(vi) The line voltage,  $V_{RY}$ , shown in Fig. 6.2(a) is given by  $V_{RY} = V_R - V_Y$  ( $V_Y$  is negative since it is in the opposite direction to  $V_{RY}$ ). In the phasor diagram of Fig. 6.2(b), phasor  $V_Y$  is reversed (shown by the broken line) and then added phasorially to  $V_R$  (i.e.  $V_{RY} = V_R + (-V_Y)$ ). By trigonometry, or by measurement,  $V_{RY} = \sqrt{3} V_R$ , i.e. for a balanced star connection:

$$V_L = \sqrt{3} V_p$$

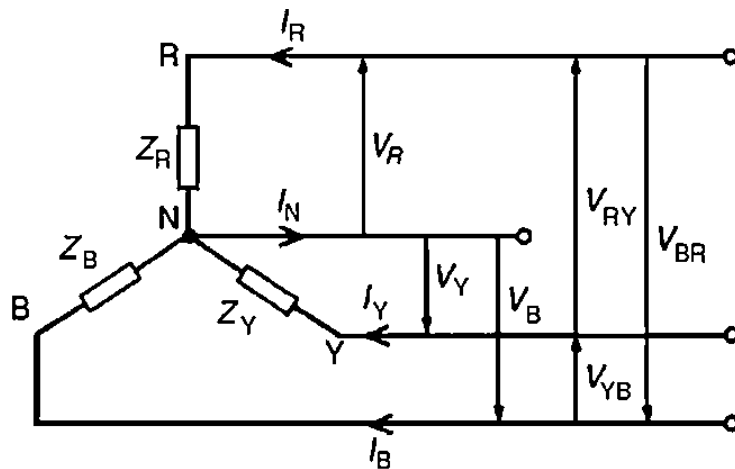
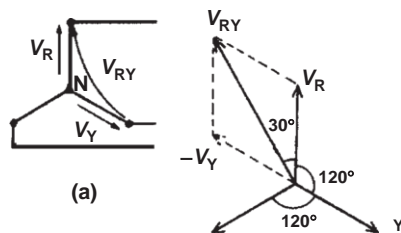


Fig 6.1



**Figure 6.2**

For most of the 20th century, the supply voltage in the UK in domestic premises has been 240V a.c. (r.m.s.) at 50 Hz. In 1988, a European-wide agreement was reached to change the various national voltages, which ranged at the time from 220V to 240V, to a common European standard of 230V. As a result, the standard nominal supply voltage in domestic single-phase 50 Hz installations in the UK has been 230V since 1995. However, as an interim measure, electricity suppliers can work with an asymmetric voltage tolerance of 230V +10%/−6% (i.e. 216.2V to 253V). The old standard was 240V ±6% (i.e. 225.6V to 254.4 V), which is mostly contained within the new range, and so in practice suppliers have had no reason to actually change voltages. Similarly, the three-phase voltage in the UK had been for many years 415V ±6% (i.e. 390V to 440V). European harmonisation required this to

be changed to 400V +10%/−6% (i.e. 376V to 440V). Again, since the present supply voltage of 415V lies within this range, supply companies are unlikely to reduce their voltages in the near future. Many of the calculations following are based on the 240V/415V supply voltages which have applied for many years and are likely to continue to do so.

**Problem 1.** Three loads, each of resistance 30 are connected in star to a 415V, 3-phase supply. Determine (a) the system phase voltage, (b) the phase current and (c) the line current.

Sol.:

A '415V, 3-phase supply' means that 415V is the line voltage,  $V_L$

(a) For a star connection,  $V_L = \sqrt{3} V_p$ . Hence phase voltage,

$$V_p = V_L / \sqrt{3} = 415 / \sqrt{3} = 239.6V$$

or 240V, correct to 3 significant figures.

(b) Phase current,  $I_p = V_p / R_p = 240 / 30 = 8A$

(c) For a star connection,  $I_p = I_L$  hence the line current,  $I_L = 8A$

**Problem 2.** A star-connected load consists of three identical coils each of resistance 30 and inductance 127.3 mH. If the line current is 5.08A, calculate the line voltage if the supply frequency is 50 Hz.

Sol.:

Inductive reactance

$$X_L = 2\pi fL = 2\pi(50)(127.3 \times 10^{-3}) = 40 \Omega$$

Impedance of each phase

$$Z_p = \sqrt{R^2 + X_L^2} = \sqrt{30^2 + 40^2} = 50 \Omega$$

For a star connection

$$I_L = I_p = V_p / Z_p$$

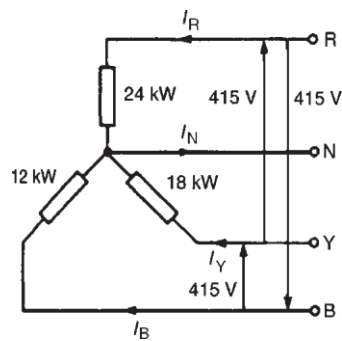
Hence phase voltage,

$$V_p = I_p Z_p = (5.08)(50) = 254V$$

Line voltage

$$V_L = \sqrt{3} V_p = \sqrt{3}(254) = 440V$$

**Problem 3.** A 415V, 3-phase, 4 wire, star-connected system supplies three resistive loads as shown in Fig. 6.3. Determine the current in each line



**Figure 6.3**

Sol.:

(a) For a star-connected system  $V_L = \sqrt{3} V_p$ , hence  $V_p = V_L / \sqrt{3}$   
 $= 415 / \sqrt{3} = 240V$

Since current  $I = \text{power } P / \text{voltage } V$  for a resistive load then

$$I_R = P_R / V_R = 24\,000 / 240 = \mathbf{100A}$$

$$I_Y = P_Y / V_Y = 18\,000 / 240 = \mathbf{75A}$$

$$\text{and } I_B = P_B / V_B = 12\,000 / 240 = \mathbf{50A}$$

### **Delta connection**

(i) A delta (or mesh) connected load is shown in Fig. 6.4 where the end of one load is connected to the start of the next load.

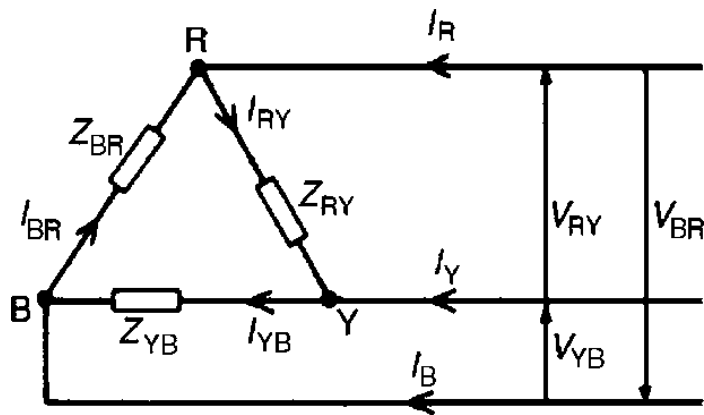


Figure 6.4

(ii) From Fig. 6.4, it can be seen that the line voltages  $V_{RY}$ ,  $V_{YB}$  and  $V_{BR}$  are the respective phase voltages, i.e. for a delta connection:

$$V_L = V_p$$

(iii) Using Kirchhoff's current law in Fig. 6.4,  $I_R = I_{RY} - I_{BR} = I_{RY} + (-I_{BR})$  From the phasor diagram shown in Fig. 6.5, by trigonometry or by measurement,  $I_R = \sqrt{3} I_{RY}$ , i.e. for a delta connection:

$$I_L = \sqrt{3} I_p$$

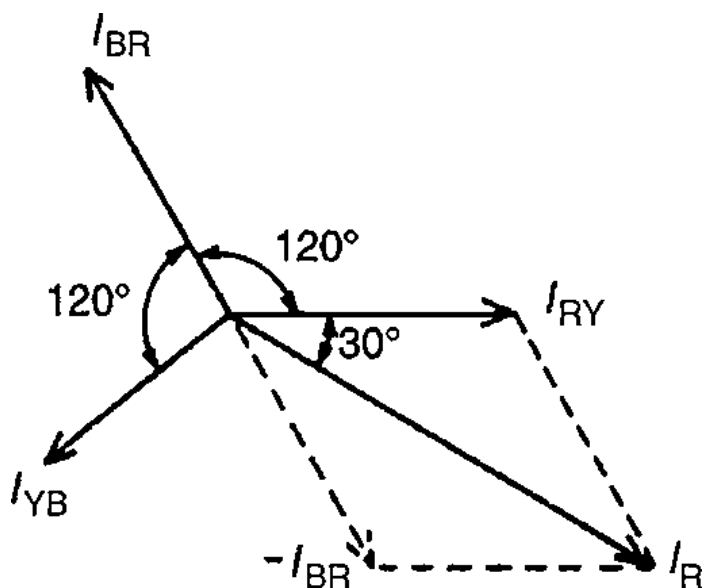


Figure 6.5

**Problem 4.** Three identical coils each of resistance 30 and inductance 127.3mH are connected in delta to a 440V, 50 Hz, 3-phase supply. Determine (a) the phase current, and (b) the line current.

Sol.:

Phase impedance,  $Z_p = 50$  (from Problem 2) and for a delta connection,  $V_p = V_L$

(a) Phase current,  $I_p = V_p / Z_p = V_L / Z_p = 440 / 50 = 8.8A$

(b) For a delta connection,

$$I_L = \sqrt{3} I_p = \sqrt{3}(8.8) = 15.24A$$

**Problem 5.** Three coils each having resistance 3  $\Omega$  and inductive reactance 4  $\Omega$  are connected (i) in star and (ii) in delta to a 415V, 3-phase supply. Calculate for each connection (a) the line and phase voltages and (b) the phase and line currents.

Sol.:

(i) For a star connection:  $I_L = I_p$  and  $V_L = \sqrt{3} V_p$ .

(a) A 415V, 3-phase supply means that the line voltage,  $V_L = 415V$

Phase voltage,

$$V_p = V_L / \sqrt{3} = 415 / \sqrt{3} = 240V$$

(b) Impedance per phase,

$$Z_p = \sqrt{R^2 + X_L^2} = \sqrt{3^2 + 4^2} = 5 \Omega$$

Phase current,

$$I_p = V_p / Z_p = 240 / 5 = 48A$$

Line current,

$$I_L = I_p = 48A$$

(ii) For a delta connection:  $V_L = V_p$  and  $I_L = \sqrt{3} I_p$ .

(a) Line voltage,  $V_L = 415V$

Phase voltage,  $V_p = V_L = 415V$

(b) Phase current,

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$$I_p = V_p / Z_p = 415 / 5 = 83 \text{ A}$$

$$\text{Line current, } I_L = \sqrt{3} I_p = \sqrt{3}(83) = 144 \text{ A}$$

### Power in three-phase systems

The power dissipated in a three-phase load is given by the sum of the power dissipated in each phase. If a load is balanced then the total power  $P$  is given by:

$$P = 3 \times \text{power consumed by one phase.}$$

The power consumed in one phase  $= I_p^2 R_p$  or  $V_p I_p \cos \phi$  (where  $\phi$  is the phase angle between  $V_p$  and  $I_p$ ).

For a star connection:

$$V_p = V_L / \sqrt{3} \text{ and } I_p = I_L$$

$$\text{Hence } P = 3 V_L / \sqrt{3} I_L \cos \phi = \sqrt{3} V_L I_L \cos \phi$$

For a delta connection:

$$V_p = V_L \text{ and } I_p = I_L / \sqrt{3}$$

$$\text{Hence } P = 3 V_L I_L / \sqrt{3} \cos \phi = \sqrt{3} V_L I_L \cos \phi$$

Hence for either a star or a delta balanced connection the total power  $P$  is given by:

$$P = \sqrt{3} V_L I_L \cos \phi \text{ watts}$$

$$\text{or } P = 3 I_p^2 R_p \text{ watts}$$

Total volt-amperes

$$S = \sqrt{3} V_L I_L \text{ volt-amperes}$$

**Problem 6.** Three  $12 \Omega$  resistors are connected in star to a 415V, 3-phase supply. Determine the total power dissipated by the resistors.

Sol.:

$$\text{Power dissipated, } P = \sqrt{3} V_L I_L \cos \phi \text{ or } P = 3 I_p^2 R_p$$



Line voltage,  $V_L = 415\text{V}$  and phase voltage  $V_p = 415 / \sqrt{3} = 240\text{V}$

(since the resistors are star-connected). Phase current,  $I_p = V_p / Z_p = V_p / R_p = 240 / 12 = 20\text{A}$

For a star connection

$$I_L = I_p = 20\text{A}$$

For a purely resistive load, the power factor =  $\cos \phi = 1$

Hence power

$$P = \sqrt{3} V_L I_L \cos \phi = \sqrt{3} (415)(20)(1) = 14.4\text{kW}$$

$$\text{or power } P = 3 I_p^2 R_p = 3 (20)^2 (12) = 14.4\text{kW}$$

**Problem 7.** The input power to a 3-phase a.c. motor is measured as 5kW. If the voltage and current to the motor are 400V and 8.6A respectively, determine the power factor of the system.

Sol.:

Power  $P = 5000\text{W}$ , line voltage  $V_L = 400\text{V}$ , line current,  $I_L = 8.6\text{A}$  and

$$\text{power, } P = \sqrt{3} V_L I_L \cos \phi$$

Hence

$$\text{power factor} = \cos \phi = P / (\sqrt{3} V_L I_L) = 5000 / (\sqrt{3} (400)(8.6)) = 0.839$$

**Problem 8.** Three identical coils, each of resistance  $10\ \Omega$  and inductance  $42\text{mH}$  are connected (a) in star and (b) in delta to a 415V, 50 Hz, 3-phase supply. Determine the total power dissipated in each case.

Sol.:

(a) Star connection

Inductive reactance,

$$X_L = 2\pi f L = 2\pi (50)(42 \times 10^{-3}) = 13.19\ \Omega.$$

$$\text{Phase impedance, } Z_p = \sqrt{R^2 + X_L^2} = \sqrt{10^2 + 13.19^2} = 16.55\ \Omega.$$

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Line voltage,  $V_L = 415 \text{ V}$  and phase voltage,  $V_P = V_L/\sqrt{3} = 415/\sqrt{3} = 240 \text{ V}$ .

Phase current,  $I_P = V_P/Z_P = 240/16.55 = 14.50 \text{ A}$ .

Line current,  $I_L = I_P = 14.50 \text{ A}$ .

Power factor =  $\cos \phi = R_P/Z_P = 10/16.55 = 0.6042$  lagging.

Power dissipated,  $P = \sqrt{3} V_L I_L \cos \phi = \sqrt{3} (415)(14.50)(0.6042) = 6.3 \text{ kW}$  (Alternatively,  $P = 3I_P^2 R_P = 3(14.50)^2(10) = 6.3 \text{ kW}$ )

(b) Delta connection

$V_L = V_P = 415 \text{ V}$ ,

$Z_P = 16.55$ ,  $\cos \phi = 0.6042$  lagging (from above).

Phase current,  $I_P = V_P/Z_P = 415/16.55 = 25.08 \text{ A}$ .

Line current,  $I_L = \sqrt{3} I_P = \sqrt{3} (25.08) = 43.44 \text{ A}$ .

Power dissipated,  $P = \sqrt{3} V_L I_L \cos \phi$

$= \sqrt{3} (415) (43.44) (0.6042) = 18.87 \text{ kW}$  (Alternatively,  $P = 3I_P^2 R_P = 3(25.08)^2(10) = 18.87 \text{ kW}$ )

Hence loads connected in delta dissipate three times the power than when connected in star, and also take a line current three times greater.

**Problem 9.** A 415V, 3-phase a.c. motor has a power output of 12.75kW and operates at a power factor of 0.77 lagging and with an efficiency of 85 per cent. If the motor is delta-connected, determine (a) the power input, (b) the line current and (c) the phase current.

Sol.:

(a) Efficiency = power output/power input. Hence  $85/100 = 12750/\text{power input}$  from which,

power input =  $12\,750 \times 100/85 = 15\,000 \text{ W}$  or  $15 \text{ kW}$

(b) Power,  $P = \sqrt{3} V_L I_L \cos \phi$ , hence line current,  $I_L = P / (\sqrt{3}(415)(0.77))$

$= 15\,000 / (\sqrt{3} (415)(0.77)) = 27.10 \text{ A}$

(c) For a delta connection,  $I_L = \sqrt{3} I_p$ , hence phase current,  $I_p = I_L / \sqrt{3} = 27.10 / \sqrt{3} = 15.65A$

### Measurement of power in three-phase systems

Power in three-phase loads may be measured by the following methods:

#### (i) One-wattmeter method for a balanced load

Wattmeter connections for both star and delta are shown in Fig. 6.6

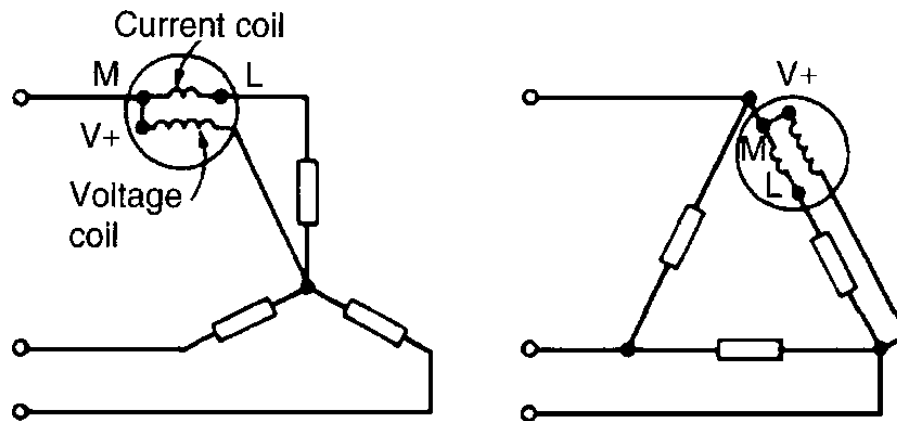


Figure 6.6

Total power =  $3 \times$  wattmeter reading

#### (ii) Two-wattmeter method for balanced or unbalanced loads

A connection diagram for this method is shown in Fig. 6.7 for a star-connected load. Similar connections are made for a delta-connected load.

Total power = sum of wattmeter readings =  $P_1 + P_2$

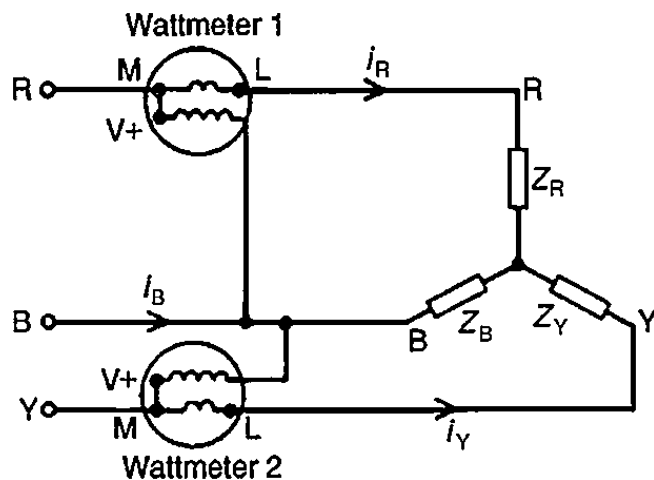


Figure 6.7

The power factor may be determined from:

$$\tan \phi = \frac{\sqrt{3}(P_1 - P_2)}{(P_1 + P_2)}$$

It is possible, depending on the load power factor, for one wattmeter to have to be 'reversed' to obtain a reading. In this case it is taken as a negative reading (see Problem 15).

### (iii) Three-wattmeter method for a three-phase, 4-wire system for balanced and unbalanced loads

(see Fig. 6.8). Total power =  $P_1 + P_2 + P_3$

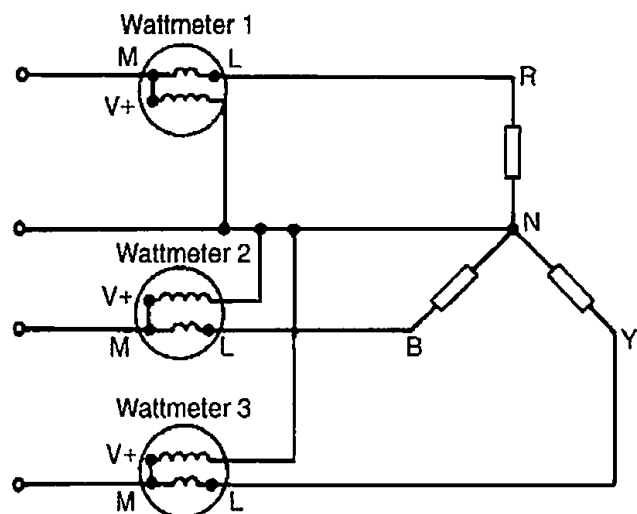


Figure 6.8

**Problem 10.** (a) Show that the total power in a 3-phase, 3-wire system using the two wattmeter method of measurement is given by the sum of the wattmeter readings. Draw a connection diagram. (b) Draw a phasor diagram for the two-wattmeter method for a balanced load.

Sol.:

(a) A connection diagram for the two-wattmeter method of a power measurement is shown in Fig. 6.9 for a star-connected load.

Total instantaneous power,  $p = e_R i_R + e_Y i_Y + e_B i_B$  and in any 3-phase system  $i_R + i_Y + i_B = 0$ ; hence  $i_B = -i_R - i_Y$ . Thus,  $p = e_R i_R + e_Y i_Y + e_B (-i_R - i_Y)$

$$= (e_R - e_B) i_R + (e_Y - e_B) i_Y$$

However,  $(e_R - e_B)$  is the p.d. across wattmeter 1 in Fig. 6.9 and  $(e_Y - e_B)$  is the p.d. across wattmeter 2

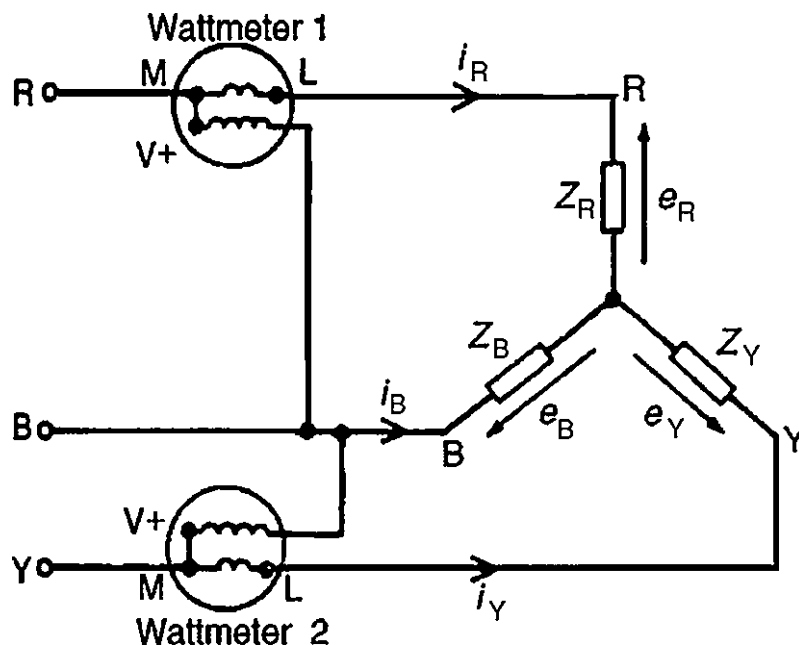


Figure 6.9

Hence total instantaneous power,  $p = (\text{wattmeter 1 reading}) + (\text{wattmeter 2 reading})$   
 $= p_1 + p_2$

(b) The phasor diagram for the two-wattmeter method for a balanced load having a lagging current is shown in Fig. 6.10, where  $V_{RB} = V_R - V_B$  and  $V_{YB} = V_Y - V_B$  (phasorially).

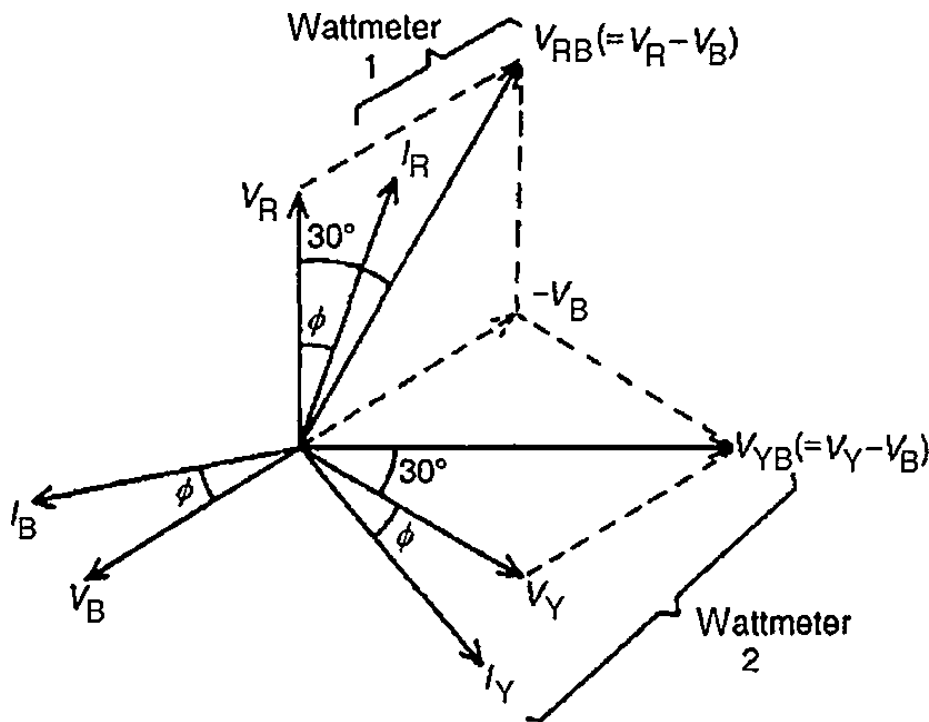


Figure 6.10

**Problem 11.** A 400V, 3-phase star connected alternator supplies a delta-connected load, each phase of which has a resistance of  $30\ \Omega$  and inductive reactance  $40\ \Omega$ . Calculate (a) the current supplied by the alternator and (b) the output power and the kVA of the alternator, neglecting losses in the line between the alternator and load.

Sol.:

A circuit diagram of the alternator and load is shown in Fig. 6.11

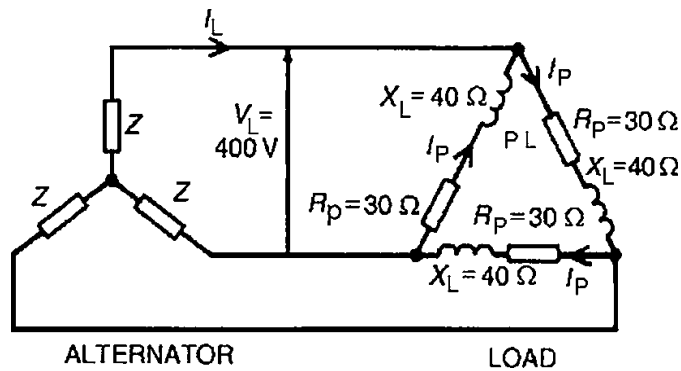
(a) Considering the load:

Phase current,  $I_p = V_p / Z_p$

$V_p = V_L$  for a delta connection, hence  $V_p = 400\text{V}$ .

Phase impedance,  $Z_p = \sqrt{R_p^2 + X_L^2} = \sqrt{30^2 + 40^2} = 50\ \Omega$ .

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**Figure 6.11**

Hence  $I_p = V_p / Z_p = 400 / 50 = 8\text{A}$ .

For a delta-connection, line current,

$$I_L = \sqrt{3} I_p = \sqrt{3}(8) = 13.86 \text{ A.}$$

Hence 13.86A is the current supplied by the alternator.

(b) Alternator output power is equal to the power dissipated by the load i.e.

$$P = \sqrt{3} V_L I_L \cos \phi,$$

$$\text{Where } \cos \phi = R_p / Z_p = 30 / 50 = 0.6.$$

$$\text{Hence } P = \sqrt{3} (400)(13.86)(0.6) = 5.76\text{kW.}$$

$$\text{Alternator output kVA, } S = \sqrt{3} V_L I_L = \sqrt{3}(400)(13.86) = 9.60 \text{ kVA.}$$

**Problem 12.** Each phase of a delta-connected load comprises a resistance of 30 and an  $80 \mu\text{F}$  capacitor in series. The load is connected to a 400V, 50 Hz, 3-phase supply. Calculate (a) the phase current, (b) the line current, (c) the total power dissipated and (d) the kVA rating of the load.

Sol.:

$$(a) \text{ Capacitive reactance, } X_C = 1 / 2\pi fC = 1 / 2\pi(50)(80 \times 10^{-6}) = 39.79\Omega$$

$$\text{Phase impedance, } Z_p = \sqrt{R_p^2 + X_C^2} = \sqrt{30^2 + 39.79^2} = 49.83\Omega.$$

$$\text{Power factor} = \cos \phi = R_p / Z_p = 30 / 49.83 = 0.602$$

$$\text{Hence } \phi = \cos^{-1} 0.602 = 52.99^\circ \text{ leading.}$$

Phase current,  $I_p = V_p / Z_p$  and  $V_p = V_L$  for a delta connection. Hence

$$I_p = 400/49.83 = 8.027A$$

(b) Line current,  $I_L = \sqrt{3} I_p$  for a delta-connection. Hence  $I_L = \sqrt{3}(8.027) = 13.90A$

(c) Total power dissipated,

$$P = \sqrt{3} V_{LL} \cos \phi = \sqrt{3}(400)(13.90)(0.602) = 5.797kW$$

(d) Total kVA,  $S = \sqrt{3} V_{LL} I_L = \sqrt{3}(400)(13.90) = 9.630 \text{ kVA}$

**Problem 13.** Two wattmeter's are connected to measure the input power to a balanced 3-phase load by the two-wattmeter method. If the instrument readings are 8kW and 4kW, determine (a) the total power input and (b) the load power factor.

Sol:

(a) Total input power,  $P = P_1 + P_2 = 8 + 4 = 12kW$

(b)  $\tan \phi = \sqrt{3}(P_1 - P_2)/(P_1 + P_2)$

$$= \sqrt{3}(8 - 4)/(8 + 4) = \sqrt{3} * 4/12 = 1/\sqrt{3}$$

$$\text{Hence } \phi = \tan^{-1} 1/\sqrt{3} = 30^\circ$$

$$\text{Power factor} = \cos \phi = \cos 30^\circ = 0.866$$

**Problem 14.** Two wattmeters connected to a 3-phase motor indicate the total power input to be 12kW. The power factor is 0.6. Determine the readings of each wattmeter.

Sol.:

If the two wattmeters indicate  $P_1$  and  $P_2$  respectively then  $P_1 + P_2 = 12kW$  .....(1)

$\tan \phi = \sqrt{3}(P_1 - P_2)/(P_1 + P_2)$  and power factor  $= 0.6 = \cos \phi$ . Angle  $\phi = \cos^{-1} 0.6 = 53.13^\circ$  and  $\tan 53.13^\circ = 1.3333$ . Hence  $1.3333 = \sqrt{3}(P_1 - P_2)/12$  from which,  $P_1 - P_2 = 12(1.3333)/\sqrt{3}$

i.e.  $P_1 - P_2 = 9.237kW$  ..... (2)

Adding Equations (1) and (2) gives:  $2P_1 = 21.237$  i.e.  $P_1 = 21.237/2 = 10.62kW$



Hence wattmeter 1 reads 10.62kW From Equation (1), wattmeter 2 reads  $(12-10.62)=1.38\text{kW}$

**Problem 15.** Two wattmeters indicate 10kW and 3kW respectively when connected to measure the input power to a 3-phase balanced load, the reverse switch being operated on the meter indicating the 3kW reading. Determine (a) the input power and (b) the load power factor.

Sol.

Since the reversing switch on the wattmeter had to be operated the 3kW reading is taken as -3 kW

(a) Total input power,

$$P=P_1+P_2=10+(-3)=7\text{kW}$$

$$(b) \tan \phi = \sqrt{3}(P_1 - P_2)/(P_1 + P_2) = \sqrt{3}(10 - (-3))/(10 + (-3))$$

$$= \sqrt{3} \cdot 13/7 = 3.2167$$

$$\text{Angle } \phi = \tan^{-1} 3.2167 = 72.73^\circ$$

$$\text{Power factor} = \cos \phi = \cos 72.73^\circ = 0.297$$

**Problem 16.** Three similar coils, each having a resistance of  $8 \Omega$  and an inductive reactance of  $8 \Omega$  are connected (a) in star and (b) in delta, across a 415V, 3-phase supply. Calculate for each connection the readings on each of two wattmeters connected to measure the power by the two-wattmeter method.

Sol.:

(a) Star connection:  $V_L = \sqrt{3} V_p$  and  $I_L = I_p$

Phase voltage,  $V_p = V_L/\sqrt{3} = 415/\sqrt{3}$  and phase impedance,  $Z_p = \sqrt{R_p^2 + X_L^2}$

$$= \sqrt{8^2 + 8^2} = 11.31\Omega$$

Hence phase current,  $I_p = V_p/Z_p = 415/11.31 = 21.18\text{A}$

$$\text{Total power, } P = 3I_p^2 R_p = 3(21.18)^2(8) = 10\,766\text{W}$$

If wattmeter readings are  $P_1$  and  $P_2$  then:  $P_1 + P_2 = 10\,766 \dots\dots\dots(1)$

Since  $R_p = 8$  and  $X_L = 8$ , then phase angle  $\phi = 45^\circ$  (from impedance triangle).

$$\tan \phi = \sqrt{3}(P_1 - P_2)/(P_1 + P_2)$$

$$\text{hence } \tan 45^\circ = \sqrt{3}(P_1 - P_2)/10\,766$$

$$\text{from which } P_1 - P_2 = (10\,766)(1)/\sqrt{3} = 6216\text{W} \dots\dots(2)$$

Adding Equations (1) and (2) gives:

$$2P_1 = 10\,766 + 6216 = 16\,982\text{W}$$

$$\text{Hence } P_1 = 8491\text{W}$$

$$\text{From Equation (1), } P_2 = 10\,766 - 8491 = 2275\text{W}.$$

When the coils are star-connected the wattmeter readings are thus 8.491kW and 2.275kW

(b) Delta connection:  $V_L = V_p$  and  $I_L = \sqrt{3} I_p$

$$\text{Phase current, } I_p = V_p/Z_p = 415/11.31 = 36.69\text{A}.$$

$$\text{Total power, } P = 3I_p^2 R_p = 3(36.69)^2(8) = 32\,310\text{W}$$

$$\text{Hence } P_1 + P_2 = 32\,310\text{W} \dots\dots\dots(3)$$

$$\tan \phi = \sqrt{3}(P_1 - P_2)/(P_1 + P_2) \quad \text{thus } 1 = \sqrt{3}(P_1 - P_2)/32\,310$$

$$\text{from which, } P_1 - P_2 = 32\,310/\sqrt{3} = 18\,650\text{W} \dots\dots (4)$$

$$\text{Adding Equations (3) and (4) gives: } 2P_1 = 50\,960 \text{ from which } P_1 = 25\,480\text{W}.$$

$$\text{From Equation (3), } P_2 = 32\,310 - 25\,480 = 6830\text{W}$$

When the coils are delta-connected the wattmeter readings are thus 25.48kW and 6.83kW

**If  $c = a + jb$**

$$\text{Magnitude of } c = M_c = \sqrt{a^2 + b^2}$$

$$\text{Phase of } c = \theta = \tan^{-1} \frac{b}{a}$$

$$a = M_c \cos \theta$$

$$b = M_c \sin \theta$$

**Problem 17.** Three coils each having resistance  $3\Omega$  and inductive reactance  $4\Omega$  are connected in star to a 415V, 3-phase supply. Calculate (a) the line and phase voltages and (b) the phase and line currents. (c) the current in the neutral conductor.

Sol.:

For a star connection:  $I_L = I_p$  and  $V_L = \sqrt{3} V_p$ .

(a) A 415V, 3-phase supply means that the line voltage,  $V_L = 415V$

Phase voltage,

$$V_p = V_L / \sqrt{3} = 415 / \sqrt{3} = 240V$$

(b) Impedance per phase,

$$Z_p = \sqrt{R^2 + X_L^2} = \sqrt{3^2 + 4^2} = 5\Omega$$

Phase current,

$$I_p = V_p / Z_p = 240 / 5 = 48A$$

Line current,

$$I_L = I_p = 48A$$

$$c) I_R = V_R / Z$$

$$V_R = 240 \angle 0^\circ$$

$$Z = 5, \theta = \tan^{-1} \frac{4}{3} = 53.13^\circ$$

$$I_R = \frac{240 \angle 0^\circ}{5 \angle 53.13^\circ} = 48 \angle -53.13^\circ$$

$$I_Y = \frac{240 \angle 120^\circ}{5 \angle 53.13^\circ} = 48 \angle 66.87^\circ$$

$$I_B = \frac{240 \angle 240^\circ}{5 \angle 53.13^\circ} = 48 \angle 186.87^\circ$$

$$I_N = I_R + I_Y + I_B$$

$$= 48 \angle -53.13^\circ + 48 \angle 66.87^\circ + 48 \angle 186.87^\circ$$

$$= 28.8 - j38.4 + 18.85 + j44.1 - 47.65 - j5.7 = 0 + j0 = 0$$