Even & odd function: (signals)

$x(t) = x(-t)$ even function

$x(t) = -x(-t)$ odd function.

exp:-

$\cos t, t^2, t^4,$ even function

$\sin t, t^3, t, t^5, ---$ odd function.

Notes:-

1- Sum of two or (more) even signals, ~~produce~~

$t^2 + t^4 + \cos t =$ even function

$(-t)^2 + (-t)^4 + \cos -t = t^2 + t^4 + \cos t$

or product of two or (more) even signals,

$t^2 \cos t = (-t^2) \cdot \cos(-t) = t^2 \cos t$

or product of even ~~function~~ number of odd signal result even signal.

$t^3 \sin t = (-t)^3 \cdot \sin -t$

$= -t^3 \cdot -\sin t$

$\boxed{t^3 \sin t = t^3 \cdot \sin t}$

2- Sum of two or more odd signal

$t^3 + t^7 + \sin t$

or product of odd number of odd signal result odd signals.

$t^3 \cdot \sin t \cdot \sin 2t = (-t)^3 \cdot \sin -t \cdot \sin -2t$

$= -t^3 \cdot -\sin t \cdot -\sin 2t$

$t \quad = -(t^3 \cdot \sin t \cdot \sin 2t)$

$$\boxed{\therefore x(t) = -x(-t)}$$

Even & odd componets of a signal:-

$$x(t) = x_e(t) + x_o(t) \quad \text{--- } (1)$$

$x_e(t)$ = even component

$x_o(t)$ = odd component.

$$x(-t) = x_e(t) - x_o(t) \quad \text{--- } (2)$$

$\therefore$ sum eq (1) with eq (2), we obtain:-

$$x(t) + x(-t) = 2x_e(t)$$

$$\boxed{\therefore x_e(t) = \frac{x(t) + x(-t)}{2}}$$

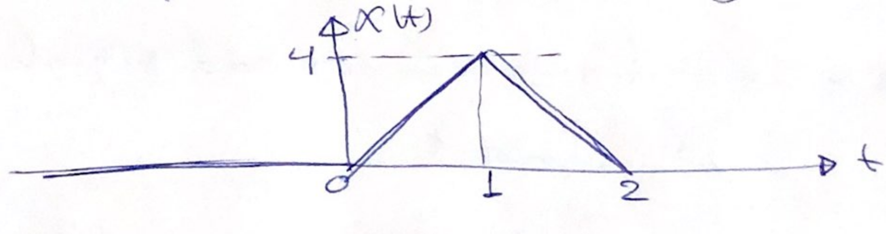
if subtr. eq (2) from eq (1), we obtain:-

$$x(t) - x(-t) = 2x_o(t)$$

$$\boxed{\therefore x_o(t) = \frac{x(t) - x(-t)}{2}}$$

Exp1:-

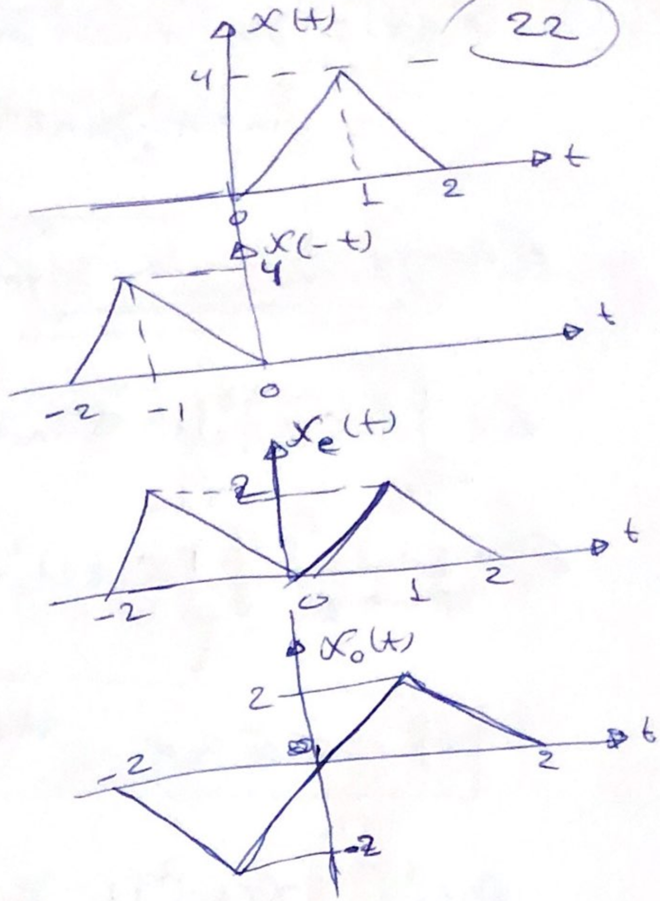
what is the even or odd of the signal?



Sol:-

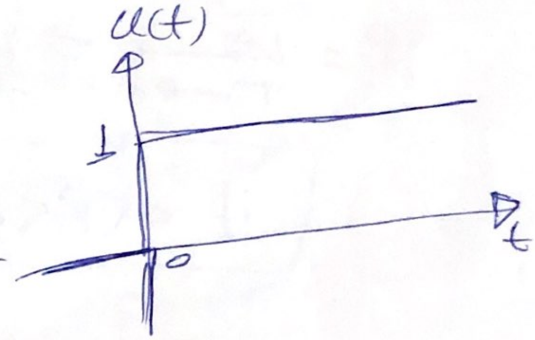
$x_e(t) = \frac{x_e(t) + x_o(t)}{2}$

$x_o(t) = \frac{x_e(t) - x_o(t)}{2}$



H-w
Find the even & odd component of single unit step?

unit steps $u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & \text{otherwise} \end{cases}$



Exp2:-
Find the even & odd component
$x(t) = \cos t + \sin t + \cos t \sin t + \sin^2 t$.

Sol:-
$x_e(t) = \cos t + \sin^2 t = \cos t + \frac{1}{2} + \frac{1}{2}\cos 2t$

Exp1 :- check energy signal & power signal.

$X_6(t) = \sin t + \cos t \sin t$

$= \sin t + \frac{1}{2} \sin 2t$

Energy & power signal :-

$$E = \int_{-T/2}^{T/2} |x(t)|^2 dt \quad \text{finite duration.}$$

$$E = \lim_{T \to \infty} \int_{-\frac{T}{2}}^{T/2} |x(t)|^2 dt \quad \text{infinite duration.}$$

$$\boxed{\text{if } 0 < E < \infty \therefore \text{its energy signal}}$$

$$P = \frac{1}{T} \int_{-\frac{T}{2}}^{T/2} |x(t)|^2 dt \quad \text{finite duration.}$$

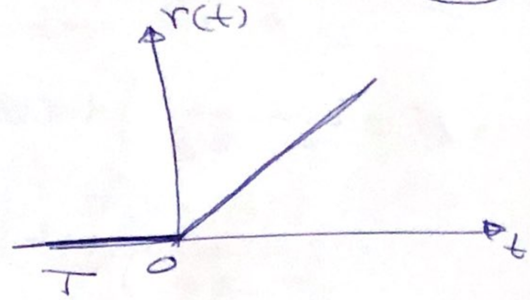
$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt \quad \text{infinite duration.}$$

$$\boxed{\text{if } 0 < P < \infty \therefore \text{its power signal}}$$

Exp1:- check energy signal & power signal.

Sol:-

$$r(t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases}$$



$$E = \lim_{T \to \infty} \int_{-\frac{T}{2}}^{T/2} |x(t)|^2 dt = \lim_{T \to \infty} \int_{0}^{T} |x(t)|^2 dt$$

$$\therefore E = \lim_{T \to \infty} \int_{0}^{T} (t)^2 dt = \lim_{T \to \infty} \left[\frac{t^3}{3}\right]_0^T = \lim_{T \to \infty} \frac{t^3}{3}$$

$$\therefore E = \frac{(\infty)^3}{3} = \infty$$ $\therefore$ its not energy signal.

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{0}^{T} |x(t)|^2 dt = \lim_{T \to \infty} \frac{1}{T} \int_{0}^{T} t^2 dt$$

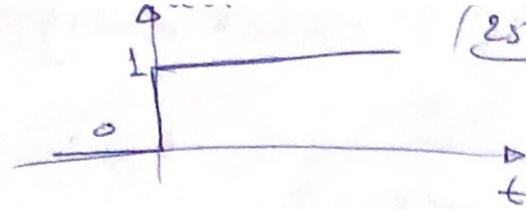
$$= \lim_{T \to \infty} \frac{1}{T} \left[\frac{t^3}{3}\right]_0^T = \lim_{T \to \infty} \frac{1}{\cancel{T}} \cdot \frac{\cancel{T}^3}{3} =$$

$$= \lim_{T \to \infty} \frac{T^2}{3} = \infty$$

$\boxed{\therefore \text{it's not power signal.}}$

$\therefore$ The signal ~~not~~ niether energy or nor power.

Exp



$$E = \lim_{T \to \infty} \int_0^T |x(t)|^2 \, dt$$

$$= \lim_{T \to \infty} \int_0^T (1)^2 \, dt = \lim_{T \to \infty} [t]_0^T = \lim_{T \to \infty} [T] = \infty$$

$$P = \lim_{T \to \infty} \frac{1}{T} \int_0^T |x(t)|^2 \, dt = \lim_{T \to \infty} \frac{1}{T} \int_0^T (1)^2 \, dt$$

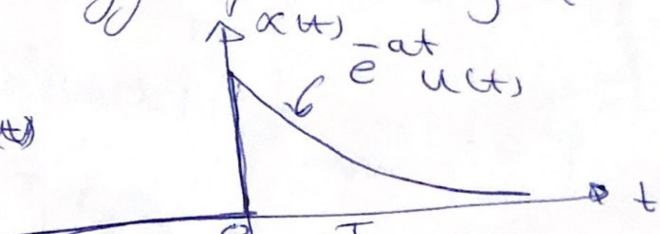
$$= \lim_{T \to \infty} [\cancel{T}] \frac{1}{\cancel{T}} = 1$$

$0 < P < \infty$ ∴ power signal.

Exp3:- check the energy or power signal.

(1) if $a > 0$

∴ $x(t) = e^{-at} u(t)$



Sol:-

$$\therefore E = \lim_{T \to \infty} \int_{-T/2}^{T/2} |x(t)|^2 \, dt = \lim_{T \to \infty} \int_0^T |x(t)|^2 \, dt$$

$$= \lim_{T \to \infty} \int_0^T e^{-2at} \, dt = \lim_{T \to \infty} \left[\frac{e^{-2at}}{-2a} \right]_0^T$$

$$= \lim_{T \to \infty} \frac{-1}{2a} \left[e^{-2aT} - 1 \right] = \boxed{\frac{1}{2a}}$$

∴ energy signal since $0 < E < \infty$

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt = \lim_{T \to \infty} \frac{1}{T} \int_0^T |x(t)|^2 \, dt$$

$\therefore P = \lim_{T \to \infty} \frac{1}{T} \int_0^T e^{-2at} \, dt = \lim_{T \to \infty} \frac{1}{T} \left[\frac{e^{-2at}}{-2a} \right]_0^T$

$P = \lim_{T \to \infty} \frac{1}{T} \left[\frac{-1}{2a} \left(e^{-2aT} - 1 \right) \right] = \frac{1}{2a(\infty)} \left\{ 1 - \overset{\approx 0}{e^{-\infty}} \right\}$

$= 0 \cdot 1 = 0$

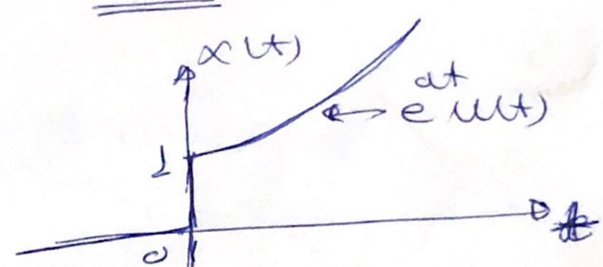
not power signal.

(2) if $a = 0$

$\therefore x(t) = u(t)$

$\therefore$ power signal from pervious example (exp 2.)

(3) if $a < 0$



$\therefore x(t) = e^{at} u(t)$

$\therefore E = \lim_{T \to \infty} \int_0^T |x(t)|^2 dt = \lim_{T \to \infty} \int_0^T e^{2at} dt$

$= \lim_{T \to \infty} \frac{1}{2a} \left[e^{2aT} - 1 \right] = \infty.$

$P = \lim_{T \to \infty} \frac{1}{T} \int_0^T e^{+2at} dt = \lim_{T \to \infty} \frac{1}{T} \left[\frac{e^{2at}}{2a} \right]_0^T$

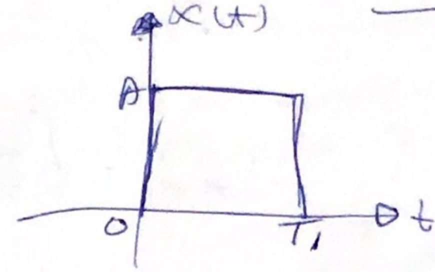
$P = \lim_{T \to \infty} \frac{1}{2aT} \left[\overset{\infty}{e^{2aT}} - 1 \right] = \frac{1}{2a\infty} [\infty]$

$= 0 \cdot \infty = 0$

$\therefore$ niether energy nor power

Exp4:- Check is energy or power signal?

$$x(t) = \begin{cases} A & 0 \le t \le T \\ 0 & \text{otherwise} \end{cases}$$



Sol:-

$$E = \int_0^T |x(t)|^2 dt = \int_0^T A^2 dt = A^2 \int_0^T dt$$

$$\boxed{E = A^2 T} \text{ Joule}$$

finite

$\therefore 0 \le E < \infty \quad \therefore$ energy signal.

$$P = \frac{1}{T}\int_0^T |x(t)|^2 dt = \frac{1}{T}\int_0^T A^2 dt = \frac{A^2}{\infty}[T_1 - 0] = \frac{A^2 T_1}{\infty}$$

$= 0$ watt

$\therefore$ power signal

H-W check the energy or power signal.

① $x_2(t) = x_1(-t)$ if $x_1(t) = e^{at} u(t)$

② $x_3(t) = x_1(t) + x_2(t)$

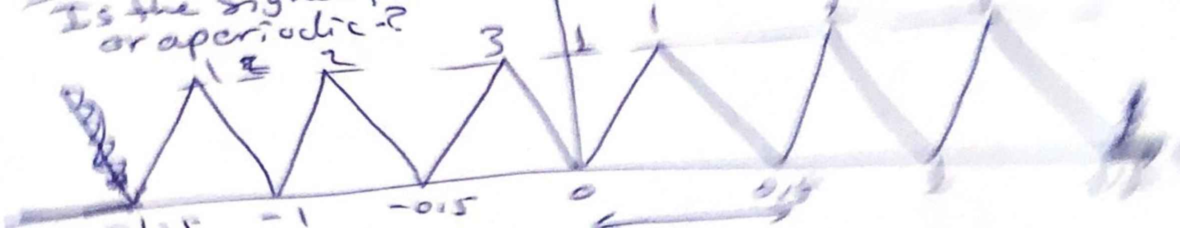
... is called fundamental time period ...

* when two signals of same frequency are added the resultant signal is also sinusoidal signal is periodic.

* when two signals of different frequency are added the resultant signal is maybe periodic or non periodic

<u>Exp 1: -</u>

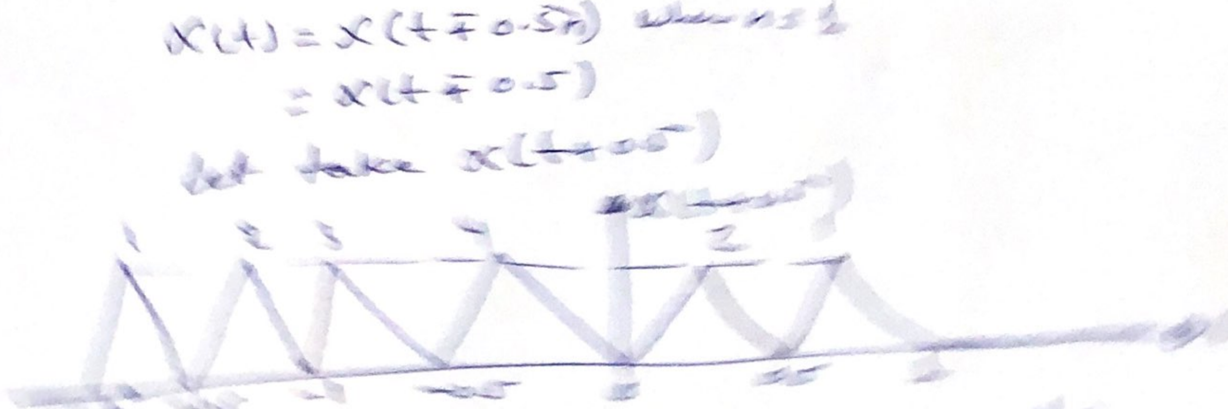
Is the signal periodic or aperiodic -?



$x(t) = x(t \mp 0.5n)$ when $n = 1$

$= x(t \mp 0.5)$

let take $x(t+0.5)$



$\therefore x(t) = x(t + 0.5)$ ∴ the signal is periodic ...

periodic signal & aperiodic signal :-

* A signal $x(t)$ is said to be periodic if it satisfies the condition :- at $-\infty$ to ∞

$$x(t) = x(t \mp nT_0) \;,\; x(n) = x(n \mp mN)$$

fundemental

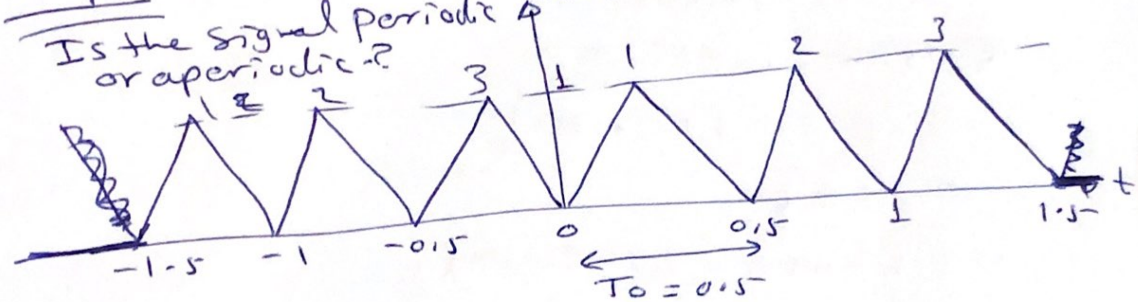
* The smallest value of T_0 which satisfies the above condition is called fundemental time periode (T_0)

$$f_0 = \frac{1}{T_0} \;,\; \omega_0 = 2\pi f = \frac{2\pi}{T_0} \;,\; T_0 \neq 0,\; T_0 \neq \infty,\; T_0 = \text{positive.}$$

* when two signals of same frequency are added the resultant signal is also sinusoidal signal is periodic.
* when two signals of different frequency are added the resultant signal is maybe periodic or non periodic

Exp :-

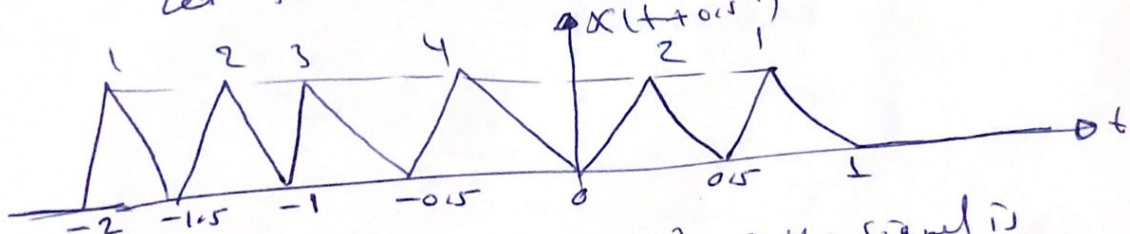
Is the signal periodic or aperiodic ?



$x(t) = x(t \mp 0.5n)$ when $n = 1$

$= x(t \mp 0.5)$

let take $x(t + 0.5)$



$\therefore x(t) \neq x(t + 0.5)$ $\therefore$ the signal is aperiodic or non periodic signal.

Exp2:- Calculation of fundementalperiod ($T_0=?$)

$x(t) = A_0 e^{jw_0t}$

$x(t+T_0) = A_0 e^{jw_0(t+T_0)}$

$x(t) \overset{?}{=} x(t+T_0)$

$\cancel{A_0 e^{jw_0t}} = \cancel{A_0 e^{jw_0t}} \cdot e^{jw_0T_0}$

$e^{jw_0T_0} = 1 = e^{j2\pi k}$

$w_0T_0 = 2\pi k$

$\Rightarrow T_0 = \frac{2\pi k}{w_0}$, if $k=1$

$\boxed{\Rightarrow T_0 = \frac{2\pi}{w_0}}$

$e^{jx} = \cos x + j\sin x$

$x = 2\pi k$

$e^{j2\pi k} = \cos 2\pi k + j \sin 2\pi k$ ($\cos 2\pi k = 1$, $\sin 2\pi k = 0$)

$e^{j2\pi k} = 1$

$e^{j2\pi} = \cos 2\pi + j\sin 2\pi$ ($\cos 2\pi = 1$, $\sin 2\pi = 0$)

$= 1$

Exp3:

$x(t) = \sin^2 4\pi t$

$\cos 2\theta = 1 - 2\sin^2\theta$

$\Rightarrow \sin^2\theta = \frac{1}{2}[1-\cos 2\theta]$

$\theta = 4\pi t$

$\Rightarrow \sin^2 4\pi t = \frac{1}{2}[1-\cos 8\pi t]$

$x(t) = \frac{1}{2} - \frac{\cos 8\pi t}{2}$

dc
Value
periodic
$T_0 \rightarrow$ undefind

$w_0 = 8\pi \Rightarrow T_0 = \frac{2\pi}{w_0} = \frac{2\pi}{8\pi} = \frac{1}{4}$ se

$\Rightarrow \boxed{T_0 = \frac{1}{4} \text{ se}} = 0{,}25$ see

$\Rightarrow f_0 = \frac{1}{T_0} = 4Hz$. of $x(t)$

Exp4:-

$$x(t) = \underbrace{\sin 6\pi t}_{x_1(t)} + \underbrace{\cos 5\pi t}_{x_2(t)}$$

$x_1(t) = \sin \underline{\underline{6\pi}}_{\omega_1} t$, $\omega_1 = 6\pi$, $\therefore T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{6\pi} = \frac{1}{3}$ s

$x_2(t) = \cos 5\pi t$, $\omega_2 = 5\pi$, $T_2 = \frac{2\pi}{5\pi} = \frac{2}{5}$

$\therefore \frac{T_1}{T_2} = \frac{1/3}{2/5} = \frac{5}{6} = 0.8333\underline{\underline{3}}$ [Retional]

$\therefore T_0 = LCM(T_1, T_2) = (\frac{1}{3}, \frac{2}{5})$

$$T_0 = \frac{LCM((1,2)}{HCF(3,5)} = \frac{2}{1} = 2\,M\sec.$$

$\therefore f_0 = \frac{1}{2} = 0.5\,Hz.$

Real & Imaginary signals:-

* A signal is said to be real when it satisfies the Condition.

$$x(t) = x^*(t)$$

Exp:- ① $x(t) = \cos \omega_0 t$.

$x^*(t) = \cos \omega_0 t$

$\therefore x(t) = x^*(t)$

② $x(t) = at$

$x^*(t) = at$

* A signal is said to be imaginary when it satisfies the Condition

$$x(t) = -x^*(t)$$

Exp:-

$$x(t) = ibt$$

$$x^*(t) = (ibt)^* = -ibt$$

$$\Rightarrow x(t)^* = -x(t).$$

$$\left.\begin{aligned} x(t) &= e^{Jwot} = \cos wot + J\sin wot \\ x(t) &= e^{-Jwot} = \cos wot - J\sin wot \end{aligned}\right\} \text{Complex signal.}$$