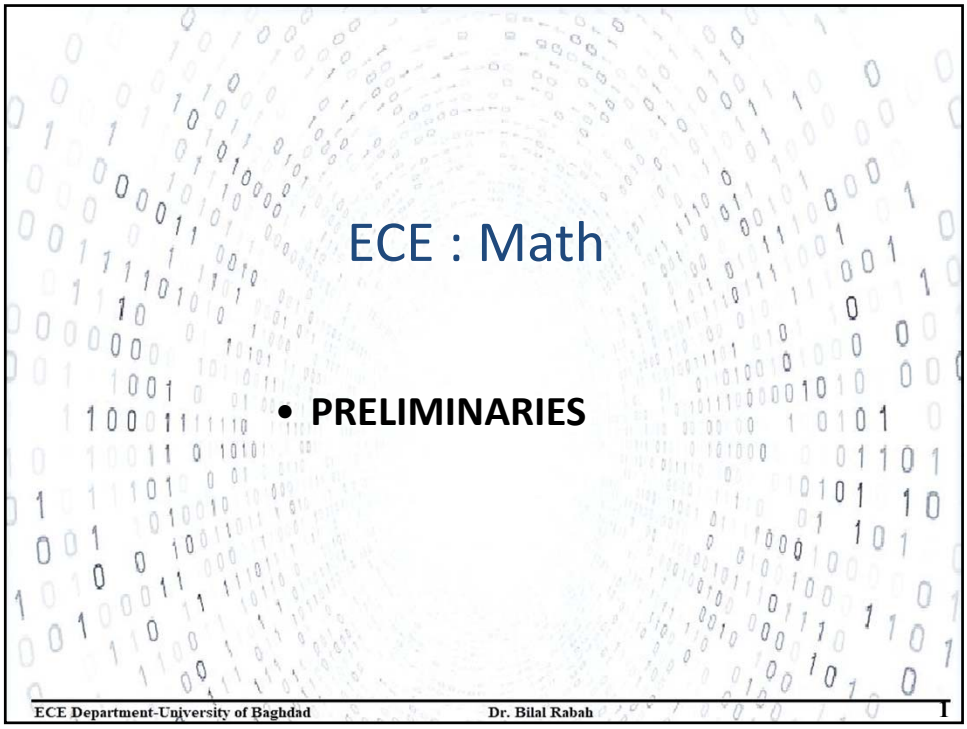
ECE : Math
• PRELIMINARIES
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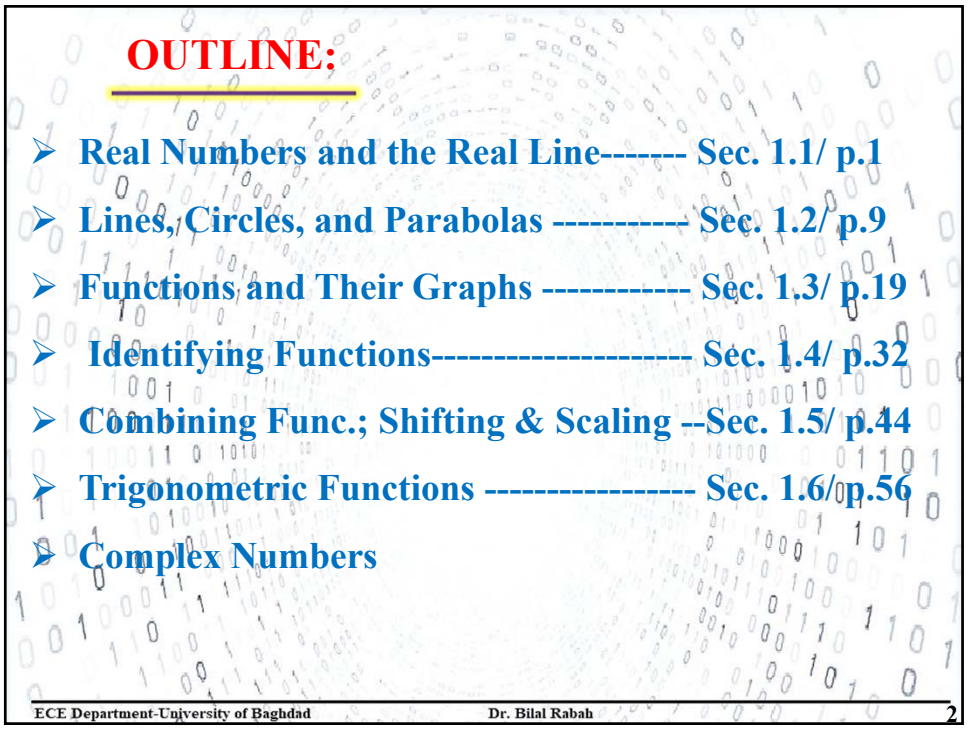
1

1


OUTLINE:
Real Numbers and the Real Line------- Sec. 1.1/ p.1
Lines, Circles, and Parabolas ---------- Sec. 1.2/ p.9
Functions and Their Graphs ---------- Sec. 1.3/ p.19
Identifying Functions---------- Sec. 1.4/ p.32
Combining Func.; Shifting & Scaling --Sec. 1.5/ p.44
Trigonometric Functions ---------- Sec. 1.6/p.56
Complex Numbers
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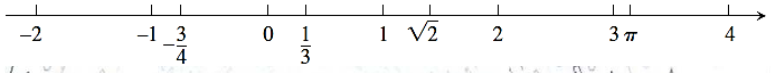
2

2

Real Numbers

- Real numbers are numbers that can be expressed as decimals, such as



- The symbol denotes $\mathbb{R}$ the real number system.
- The algebraic properties say that the real numbers can be added, subtracted, multiplied, and divided (except by 0, you can never divide by 0).
- We distinguish four special subsets of real numbers.
  1. The natural numbers, namely 1, 2, 3, 4, …
  2. The integers, namely 0, ± 1, ± 2, ± 3, ± 4, …
  3. The rational numbers, namely the numbers that can be expressed in the form of a fraction, namely ¾
  4. The irrational numbers, Real numbers that are not rational, namely, π, $\sqrt[4]{5}$, $\log_3 7$.

3

Real Numbers

- A set is a collection of objects, and these objects are the elements of the set. If S is a set, the notation $a \in S$ means that a is an element of S, and $a \notin S$ means that a is not an element of S.
- If S and T are sets, then $S \cup T$ is their union and consists of all elements belonging either to S or T (or to both S and T).
- The $S \cap T$ intersection consists of all elements belonging to both S and T.
- The empty set $\emptyset$ is the set that contains no elements. For example, the intersection of the rational numbers and the irrational numbers is the empty set.
- A finite interval is said to be closed if it contains both of its endpoints, half-open if it contains one endpoint but not the other, and open if it contains neither endpoint.
- The endpoints are also called boundary points; they make up the interval's boundary. The remaining points of the interval are interior points and together comprise the interval's interior.

Table 1.1, p. 4 Example 1, p. 4

4

Absolute Value

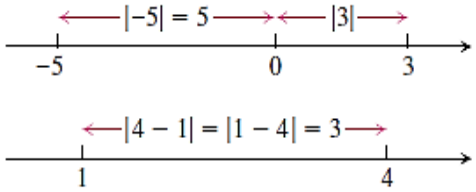
- **The absolute value of a number x, is defined by the formula**

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0. \end{cases}$$

Example 2, p. 5

- **Geometrically, the absolute value of x is the distance from x to 0 on the real number line. Since distances are always positive or 0**

$|x - y|$ = the distance between x and y



5

Absolute Value

- **It is important to remember that $\sqrt{a^2} = |a|$. Do not write $\sqrt{a^2} = a$ unless you already know that $a > 0$.**

Absolute Value Properties

1.	$\lvert -a \rvert = \lvert a \rvert$	A number and its additive inverse or negative have the same absolute value.
2.	$\lvert ab \rvert = \lvert a \rvert \lvert b \rvert$	The absolute value of a product is the product of the absolute values.
3.	$\left\lvert \frac{a}{b} \right\rvert = \frac{\lvert a \rvert}{\lvert b \rvert}$	The absolute value of a quotient is the quotient of the absolute values.
4.	$\lvert a + b \rvert \leq \lvert a \rvert + \lvert b \rvert$	The **triangle inequality**. The absolute value of the sum of two numbers is less than or equal to the sum of their absolute values.

See Exercise 1.1, p. 8
(1-54)

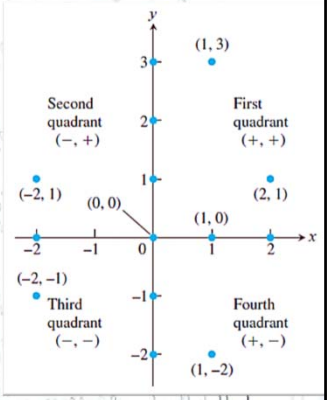
- **For the questions with T, understand the idea and ignore the graphing**

Example 3-6 p. 6

6

Lines, Circles, and Parabolas

Cartesian Coordinates in the Plane



- These lines are called coordinate axes in the plane.
- The origin *O*, also labeled 0, of the coordinate system is the point in the plane where x and y are both zero.
- The coordinate axes of this coordinate or Cartesian plane divide the plane into four regions called quadrants.
- When a particle moves from one point in the plane to another, the net changes in its coordinates are called increments.
- They are calculated by subtracting the coordinates of the starting point from the coordinates of the ending point. If x changes from x_1 to x_2 the increment in x is

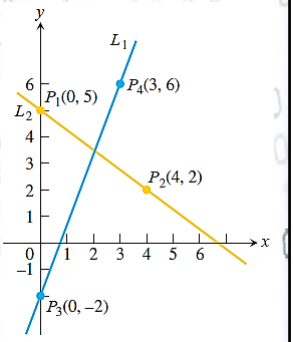
$$\Delta x = x_2 - x_1$$

7

Lines, Circles, and Parabolas

- Any nonvertical line in the plane has the property that the ratio

$$m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$



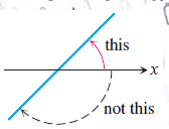
- The slope tells us the direction (uphill, downhill) and steepness of a line. A line with positive slope rises uphill to the right (Blue line); one with negative slope falls downhill to the right (Yellow line).
- The greater the absolute value of the slope, the more rapid the rise or fall, the high value for steepness.
- The slope of a vertical line is undefined. Since the run (x_2-x_1) is zero for a vertical line, we cannot evaluate the slope ratio *m*.

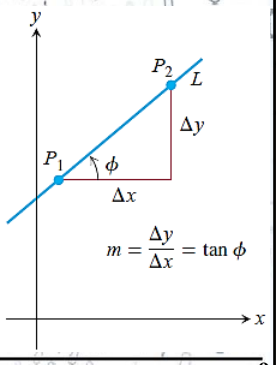
8

Lines, Circles, and Parabolas

- The angle of inclination of a line that crosses the x-axis is the smallest counterclockwise angle from the x-axis to the line.
- The inclination of a horizontal line is 0°. The inclination of a vertical line is 90°.
- If (phi, ∅) is the inclination of a line,
- then $0 \leq \emptyset < 180°$.
- The relationship between the slope m of a nonvertical line and the line's angle of inclination ∅ is

  $m = \tan\emptyset$
- The equation for a nonvertical straight line, where m and the coordinate of only one point is known, can be used to find the equation of the line. If P(x, y) is any other point on line, then





$$y = y_1 + m(x - x_1)$$

The equation is the **point-slope equation**

9

Lines, Circles, and Parabolas

The equation

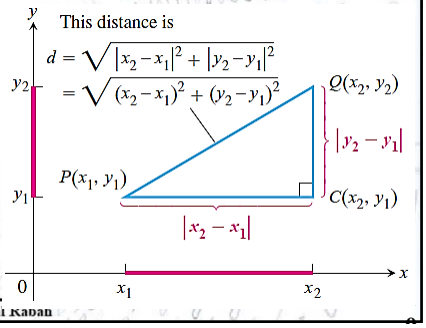
$$y = mx + b$$

is called the **slope-intercept equation** of the line with slope m and y-intercept b.

- Lines that are parallel have equal angles of inclination, so they have the same slope (if they are not vertical).
- If two nonvertical lines are perpendicular, their slopes m_1 and m_2 satisfy $m_1 * m_2 = -1$, so each slope is the negative reciprocal of the other:

$$m_1 = -\frac{1}{m_2}, \qquad m_2 = -\frac{1}{m_1}$$

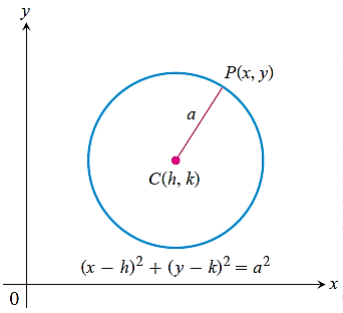
- The distance between points in the plane is calculated with a formula that comes from the Pythagorean theorem

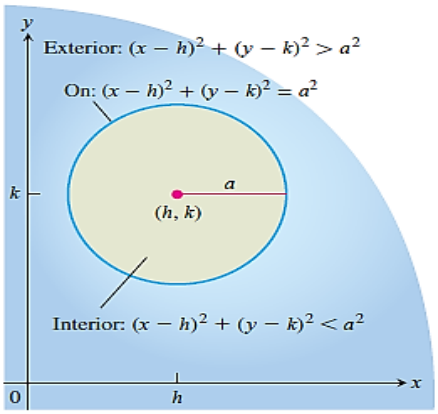


10

Lines, Circles, and Parabolas

- Circle of radius a is the set of all points whose distance from center equals a. From the distance formula:
- The circle of radius a =1 and centered at the origin is the unit circle with equation $x^2+y^2 = 1$.

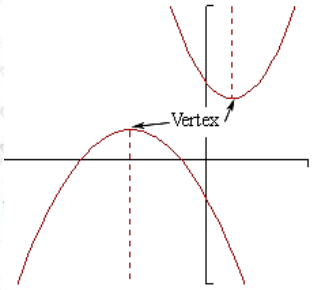




11

Lines, Circles, and Parabolas

- The graphs of quadratic functions are called parabolas.
- All parabolas are with "U" shaped and they will have a highest or lowest point that is called the vertex.
- The parabola can be used for: satellite dishes and radar dishes.



The Graph of $y = ax^2 + bx + c$, $a \neq 0$

The graph of the equation $y = ax^2 + bx + c, a \neq 0$, is a parabola. The parabola opens upward if $a > 0$ and downward if $a < 0$. The axis is the line

$$x = -\frac{b}{2a}. \qquad (2)$$

The vertex of the parabola is the point where the axis and parabola intersect. Its x-coordinate is $x = -b/2a$; its y-coordinate is found by substituting $x = -b/2a$ in the parabola's equation.

12

Lines, Circles, and Parabolas

Example 1-9, p. 10

See Exercise 1.2, p. 16
(1-80)

- For the questions with T, understand the idea and ignore the graphing

13

Functions and Their Graphs

- The value of one variable quantity, which we might call *y*, depends on the value of another variable quantity, which we might call x.
- Since the value of y is completely determined by the value of x, we say that y is a function of x. $y = f(x)$ ("y equals f of x")
- The letter x, called the independent variable, represents the input value of *f*, and y, the dependent variable, represents the corresponding output value of *f* at x.
- The set of all possible input values is called the domain of the function. The set of all values of *f*(x) is called the range of the function.
- Think of a function *f* as a kind of machine that produces an output value *f*(x) in its range whenever we feed it an input value x from its domain.

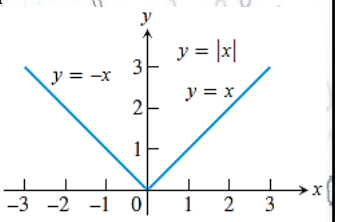
Example 1,2, p. 23

14

Functions and Their Graphs

- **Piecewise-Defined Functions: Sometimes a function is described by using different formulas on different parts of its domain.**
- **One example is the absolute value function**

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0, \end{cases}$$



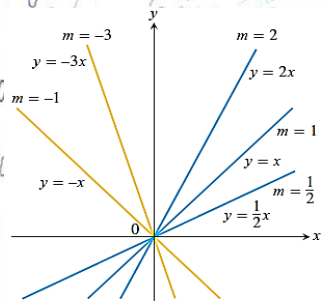
Example 5-8, p. 27

See Exercise 1.3, p. 29
(1-6, 23-27, 33-36)

- **For the questions with T, understand the idea and ignore the graphing**

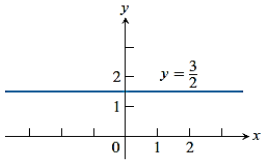
Identifying Functions

- **There are a number of important types of functions frequently encountered in calculus. We identify and briefly summarize them here.**

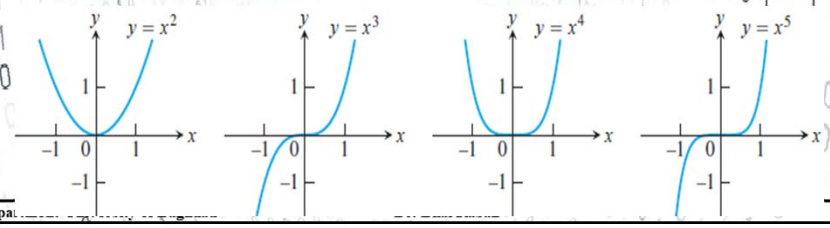


Linear Functions

$$f(x) = mx + b$$

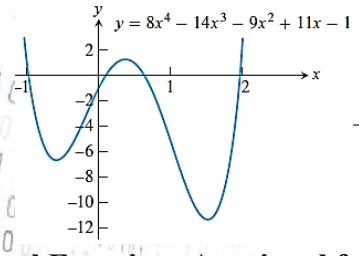


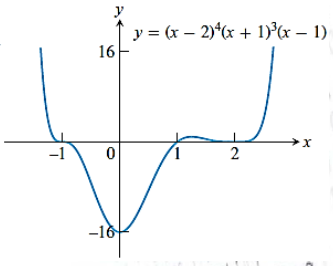
Power Functions



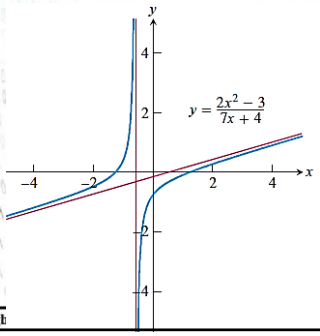
Identifying Functions

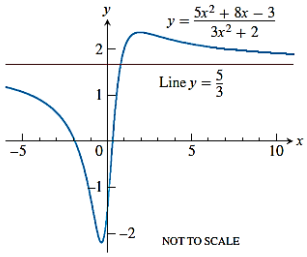
Polynomials Functions $p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$





Rational Functions A rational function is a quotient or ratio of two polynomials



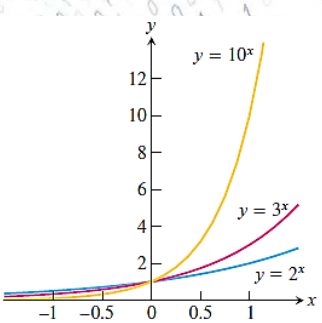


17

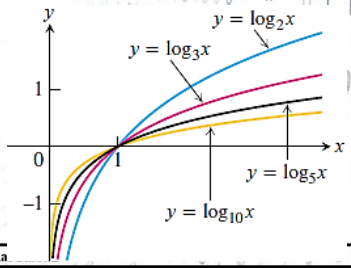
Identifying Functions

Trigonometric Functions: the sine and cosine functions.

Exponential Functions:
functions of the form $y = a^x$



Logarithmic Functions: These are the functions $y = \log_a x$,



Example 1, p. 37

18

Identifying Functions

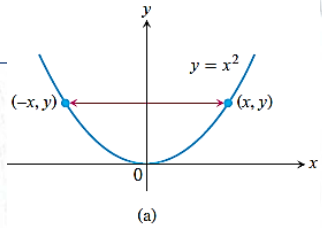
DEFINITIONS Even Function, Odd Function

A function $y = f(x)$ is an

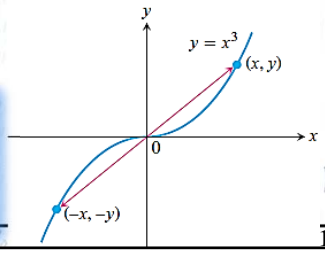
even function of x if $f(-x) = f(x)$,
odd function of x if $f(-x) = -f(x)$,

for every x in the function's domain.

Example 2, p. 38



(a)



- **The graph of an even function is symmetric about the *y*-axis.**
- **The graph of an odd function is symmetric about the origin.**

See Exercise 1.4, p. 42
(1-6, 19-30)

- **For the questions with T, understand the idea and ignore the graphing**

19

Combining Function; Shifting & Scaling

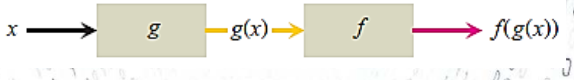
- **Like numbers, functions can be added, subtracted, multiplied, and divided (except where the denominator is zero) to produce new functions.**

$$(f + g)(x) = f(x) + g(x).$$
$$(f - g)(x) = f(x) - g(x).$$
$$(fg)(x) = f(x)g(x).$$

- **Composite Functions: Composition is another method for combining functions.**

If f and g are functions, the **composite** function $f \circ g$ ("f composed with g") is defined by

$$(f \circ g)(x) = f(g(x)).$$



Example 3, p. 47

20

Combining Function; Shifting & Scaling

- **Shifting a Graph of a Function**

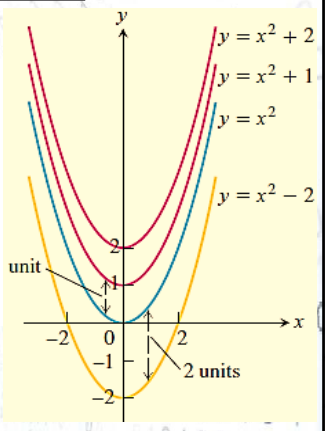
Shift Formulas

Vertical Shifts

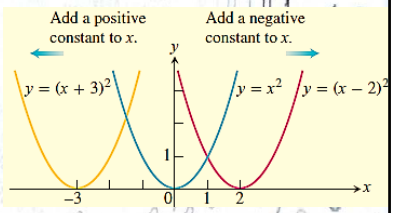
$y = f(x) + k$ — Shifts the graph of f *up* k units if $k > 0$
Shifts it *down* $|k|$ units if $k < 0$

Horizontal Shifts

$y = f(x + h)$ — Shifts the graph of f *left* h units if $h > 0$
Shifts it *right* $|h|$ units if $h < 0$



Example 4, p. 48



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Combining Function; Shifting & Scaling

Vertical and Horizontal Scaling and Reflecting Formulas

For $c > 1$,

$y = cf(x)$ — Stretches the graph of f vertically by a factor of c.

$y = \frac{1}{c} f(x)$ — Compresses the graph of f vertically by a factor of c.

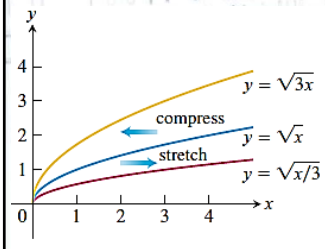
$y = f(cx)$ — Compresses the graph of f horizontally by a factor of c.

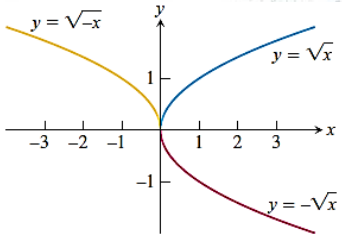
$y = f(x/c)$ — Stretches the graph of f horizontally by a factor of c.

For $c = -1$,

$y = -f(x)$ — Reflects the graph of f across the x-axis.

$y = f(-x)$ — Reflects the graph of f across the y-axis.





Example 5-6, p. 49

22

Combining Function; Shifting & Scaling

See Exercise 1.5, p. 52
(1-60, 79)

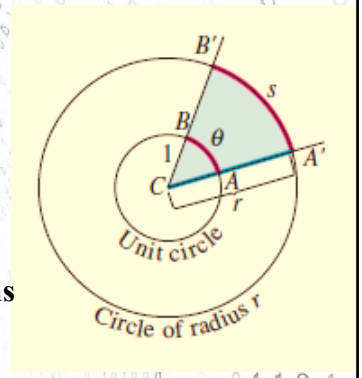
- For the questions with T, understand the idea and ignore the graphing

23

Trigonometric Functions

- **Radian Measure:**

- The radian measure of the angle *ACB* at the center of the unit circle equals the length of the arc that *ACB* cuts from the unit circle.
- Since the circumference of the circle is 2π and one complete revolution of a circle is 360°, the relation between radians and degrees is given by π radians = 180°.



Conversion Formulas

1 degree $= \frac{\pi}{180}$ (≈0.02) radians

Degrees to radians: multiply by $\frac{\pi}{180}$

1 radian $= \frac{180}{\pi}$ (≈57) degrees

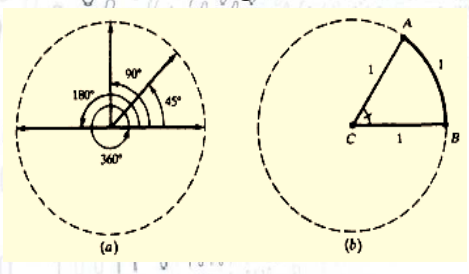
Radians to degrees: multiply by $\frac{180}{\pi}$

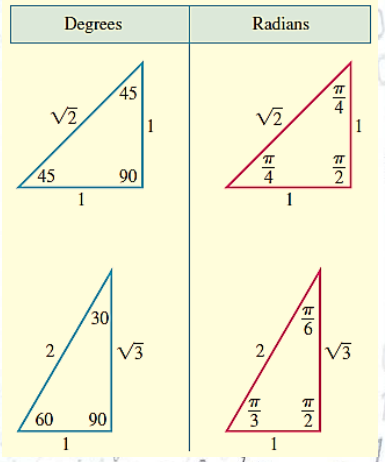
24

Trigonometric Functions

- **Radian Measure:**

- **Consider a circle with a radius of one unit. Let the center be C, and let CA and CB be two radii for which the intercepted arc AB of the circle has length 1. Then the central angle ACB is taken to be the unit of measure, one radian.**





- **From now on it is assumed that all angles are measured in radians.**
- **When we talk about the angle $\frac{\pi}{3}$, we mean $\frac{\pi}{3}$ radians (which is 60°), not degrees.**

25

Trigonometric Functions

- **Radian Measure:**

Express in degrees an angle of: (*a*) $5\pi/12$ radians; (*b*) 0.3π radians; (*c*) 3 radians.

Use (*25.1*).

(*a*) $\frac{5\pi}{12}$ radians $= \frac{5\pi}{12}\left(\frac{180}{\pi}\text{ degrees}\right) = \frac{5}{12} \times 180° = 75°$

(*b*) 0.3π radians $= \frac{0.3\pi}{\pi} \times 180° = 54°$

(*c*) 3 radians $= \frac{3}{\pi} \times 180° = \left(\frac{540}{\pi}\right)° \approx 172°$

Since $\pi \approx 3.14$.

(*a*) In a circle of radius 5 centimeters, what arc length along the circumference is intercepted by a central angle of $\pi/3$ radians?

(*b*) In a circle of radius 12 feet, what arc length along the circumference is intercepted by a central angle of 30°?

Use (*25.3*): $s = r\theta$.

(*a*) $s = 5 \times \frac{\pi}{3} = \frac{5\pi}{3}$ centimeters

(*b*) The central angle must be changed to radian measure. By Table 25-1,

$$30° = \frac{\pi}{6}\text{ radians} \quad \text{and so} \quad s = 12 \times \frac{\pi}{6} = 2\pi\text{ feet}$$

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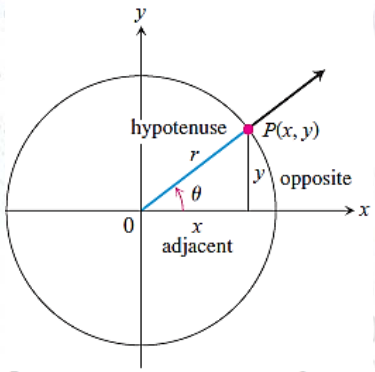
Trigonometric Functions

- **Basic Trigonometric Functions**

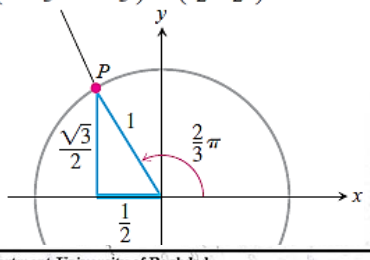
$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}}$$



$$\left(\cos\frac{2\pi}{3}, \sin\frac{2\pi}{3}\right) = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$$



$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$\sec\theta = \frac{1}{\cos\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

Example 1, p. 59

Trigonometric Functions

- **Periodicity of the Trigonometric Functions**

The cosine and sine functions are *periodic,* of period 2π that is, for all θ,

$$\cos(\theta + 2\pi) = \cos\theta \qquad \sin(\theta + 2\pi) = \sin\theta \qquad \tan(\theta + 2\pi) = \tan\theta$$
$$\sec(\theta + 2\pi) = \sec\theta \qquad \csc(\theta + 2\pi) = \csc\theta \qquad \cot(\theta + 2\pi) = \cot\theta$$

Similarly, $\cos(\theta - 2\pi) = \cos\theta$, $\sin(\theta - 2\pi) = \sin\theta$, and so on. We describe this repeating behavior by saying that the six basic trigonometric functions are *periodic*.

EXAMPLES

(a) $\sin\frac{7\pi}{3} = \sin\left(2\pi + \frac{\pi}{3}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

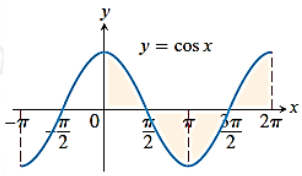
(b) $\cos 5\pi = \cos(3\pi + 2\pi) = \cos 3\pi = \cos(\pi + 2\pi) = \cos\pi = -1$

(c) $\cos 390° = \cos(30° + 360°) = \cos 30° = \frac{\sqrt{3}}{2}$

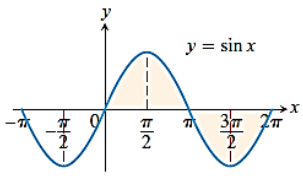
(d) $\sin 405° = \sin(45° + 360°) = \sin 45° = \frac{\sqrt{2}}{2}$

Trigonometric Functions

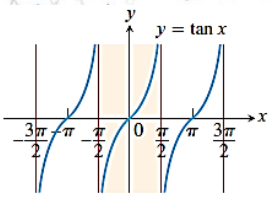
- **Graph of the Trigonometric Functions**



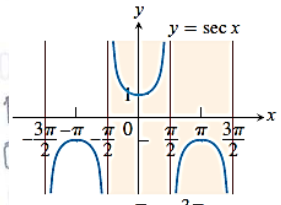
Domain: $-\infty < x < \infty$
Range: $-1 \le y \le 1$
Period: 2π
(a)



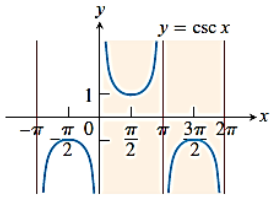
Domain: $-\infty < x < \infty$
Range: $-1 \le y \le 1$
Period: 2π
(b)



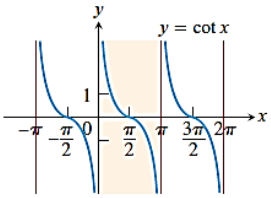
Domain: $x \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \ldots$
Range: $-\infty < y < \infty$
Period: π
(c)



Domain: $x \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \ldots$
Range: $y \le -1$ and $y \ge 1$
Period: 2π
(d)



Domain: $x \neq 0, \pm\pi, \pm 2\pi, \ldots$
Range: $y \le -1$ and $y \ge 1$
Period: 2π
(e)



Domain: $x \neq 0, \pm\pi, \pm 2\pi, \ldots$
Range: $-\infty < y < \infty$
Period: π
(f)

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Trigonometric Functions

- **Properties of the Trigonometric Functions**

Periods of Trigonometric Functions

Period π:
$\tan(x + \pi) = \tan x$
$\cot(x + \pi) = \cot x$

Period 2π:
$\sin(x + 2\pi) = \sin x$
$\cos(x + 2\pi) = \cos x$
$\sec(x + 2\pi) = \sec x$
$\csc(x + 2\pi) = \csc x$

Even	Odd
$\cos(-x) = \cos x$	$\sin(-x) = -\sin x$
$\sec(-x) = \sec x$	$\tan(-x) = -\tan x$
	$\csc(-x) = -\csc x$
	$\cot(-x) = -\cot x$

- **Trigonometric Identities**

$$\cos^2\theta + \sin^2\theta = 1.$$

$$1 + \tan^2\theta = \sec^2\theta.$$

$$1 + \cot^2\theta = \csc^2\theta.$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos^2\theta = \frac{1 + \cos 2\theta}{2}$$

$$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$$

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Trigonometric Functions

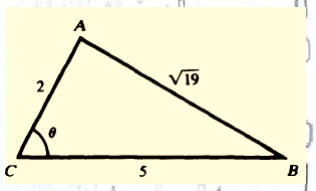
- **Properties of the Trigonometric Functions**

EXAMPLE Find angle θ in the triangle of Fig. 26-8.

Solving the law of cosines for $\cos\theta$,

$$\cos\theta = \frac{a^2+b^2-c^2}{2ab} = \frac{(5)^2+(2)^2-(\sqrt{19})^2}{2(5)(2)} = \frac{25+4-19}{20} = \frac{1}{2}$$

Then, from Table 26-1, $\theta = \pi/3$.



(*Sum and Difference Formulas*):

(*a*) $\cos(u+v) = \cos u \cos v - \sin u \sin v$

(*b*) $\cos(u-v) = \cos u \cos v + \sin u \sin v$

(*c*) $\sin(u+v) = \sin u \cos v + \cos u \sin v$

(*d*) $\sin(u-v) = \sin u \cos v - \cos u \sin v$

$$\cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta \text{ and } \sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta;$$

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Trigonometric Functions

- **Properties of the Trigonometric Functions**

EXAMPLES

(*a*) $$\cos\frac{\pi}{12} = \cos\left(\frac{\pi}{3}-\frac{\pi}{4}\right) = \cos\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\sin\frac{\pi}{4}$$
$$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \frac{\sqrt{2}+\sqrt{6}}{4}$$

(*b*) $$\cos 135^\circ = \cos(90^\circ + 45^\circ) = \cos 90^\circ \cos 45^\circ - \sin 90^\circ \sin 45^\circ$$
$$= (0)\left(\frac{\sqrt{2}}{2}\right) - (1)\left(\frac{\sqrt{2}}{2}\right) = -\frac{\sqrt{2}}{2}$$

(*c*) $$\sin\frac{7\pi}{12} = \sin\left(\frac{\pi}{2}+\frac{\pi}{12}\right) = \sin\frac{\pi}{2}\cos\frac{\pi}{12} + \cos\frac{\pi}{2}\sin\frac{\pi}{12}$$
$$= (1)\left(\frac{\sqrt{2}+\sqrt{6}}{4}\right) + (0)\left(\sin\frac{\pi}{12}\right) \quad \text{[by example (a)]}$$
$$= \frac{\sqrt{2}+\sqrt{6}}{4}$$

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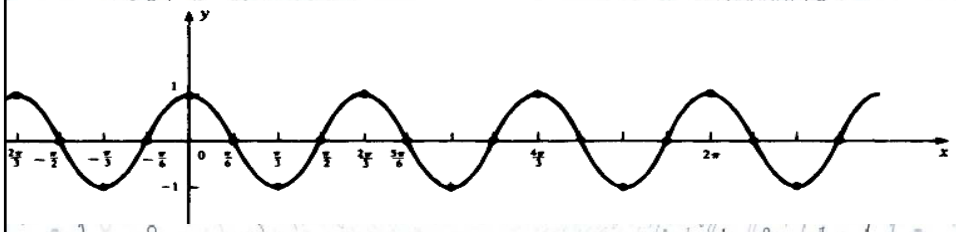
Trigonometric Functions

- **Transformations of Trigonometric Graphs**

Example: For y = cos 3*x*, graph this function

$$\cos 3\left(x + \frac{2\pi}{3}\right) = \cos(3x + 2\pi) = \cos 3x$$

the function is of period $p = 2\pi/3$. Hence, the length of each wave is $2\pi/3$. The number of waves over an interval of length 2π (corresponding to one complete revolution of the ray determining angle x) is 3. This number is called the *frequency f* of cos 3x. In general,

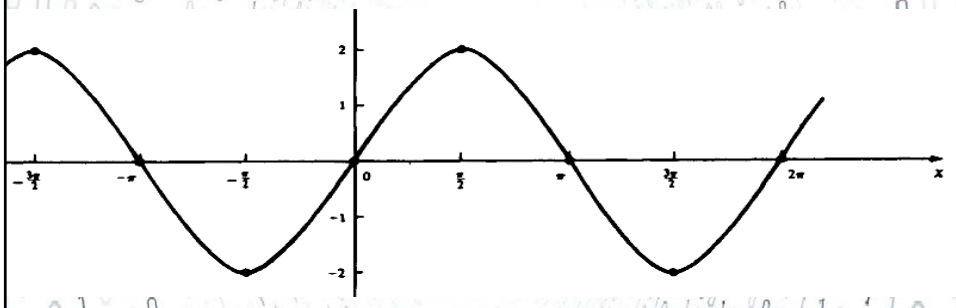


Trigonometric Functions

- **Transformations of Trigonometric Graphs**

- **Graph 2sin x.**

The period (wavelength) and the frequency of the function 2 sin x are the same as those of the function sin x. But the amplitude, the maximum height of each wave, of 2 sin x is 2, or twice the amplitude of sin x.



Trigonometric Functions

- **Transformations of Trigonometric Graphs**

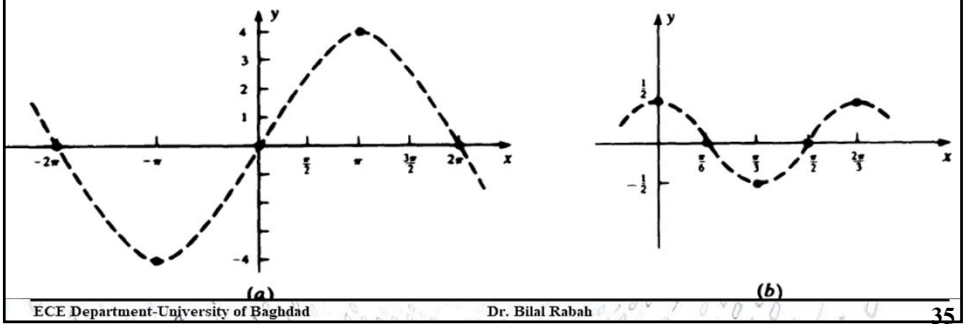
Find the period, frequency, and amplitude of:

(*a*) $4 \sin \frac{x}{2}$ (*b*) $\frac{1}{2} \cos 3x$

and sketch their graphs.

(*a*) For $4 \sin \frac{1}{2}x \equiv A \sin bx$, the frequency is $f = b = \frac{1}{2}$, the period is $p = \frac{2\pi}{f} = \frac{2\pi}{\frac{1}{2}} = 4\pi$, and the amplitude is $A = 4$. The graph is sketched in Fig. 27-6(*a*).

(*b*) For $\frac{1}{2} \cos 3x \equiv A \cos bx$, the frequency f is $b = 3$, the period is $p = \frac{2\pi}{b} = \frac{2\pi}{3}$, and the amplitude is $A = \frac{1}{2}$. The graph is sketched in Fig. 27-6(*b*).



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Trigonometric Functions

- **Transformations of Trigonometric Graphs**

Vertical stretch or compression; reflection about x-axis if negative

Vertical shift

$$y = af(b(x + c)) + d$$

Horizontal stretch or compression; reflection about y-axis if negative

Horizontal shift

See Exercise 1.6, p. 65
(7-26, 56-59, 61-64)

- **For the questions with T, understand the idea and ignore the graphing**

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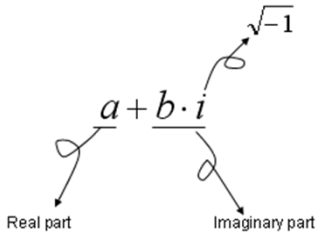
Complex Numbers-

What is a Complex Number ?

Whenever you come across a new concept, your first question would be "what is it ?". So you would now have a question "What is a Complex Number ?".

Mathematical definition is simple. A complex number is any number that can be expressed in the following format



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Complex Numbers-

What is the meaning of the complex number ?

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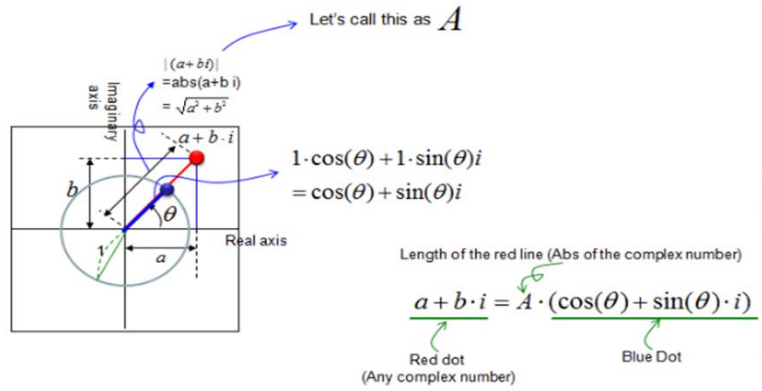
Complex Numbers-

There's another form of representing a complex number. It is illustrated as below and I think you can understand the meaning of the expression as long as you have basic knowledge of trigonometry.



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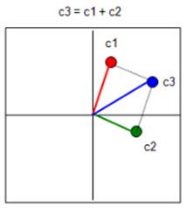
Complex Numbers-

Mathematical Operations for Complex Number ?

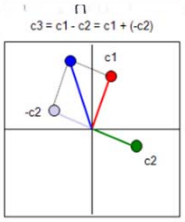
If I define two complex number c1 = a + bi, c2 = c + di, the addition of these two numbers is as follows.

c3 = c1 + c2 = (a + b i) + (c + d i) = (a + c) + (b + d) i

You would notice that this is exactly the same result as summation of two vectors [a, b], [c, d].



c3 = c1 - c2 = (a + b i) - (c + d i) = (a - c) + (b - d) i



< Multiplication >

c3 = c1 x c2

=(a + b i) x (c + d i) = a c + a d i + b c i + b d i^2 = (a c - b d) + (a d + b c) i

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Complex Numbers-

Euler Equation

One of the most famous form of complex number which is used in engineering would be what we call 'Euler form' or 'Euler Equation'. It is represented as follows.

$$e^{i\theta} = \cos(\theta) + i\sin(\theta)$$

Just by looking at the equation itself, you would notice that
i) Euler equation would tell you how to interpret the exponential form of a complex number
ii) Euler Equation would tell you the exponential form of a complex number would represents a certain cyclic/periodic behavior since cos(), sin() represents cyclic/periodc behavior).
iii) Amplitude/Maginitude of e^(i theta) is always "1" regardless of the value theta.

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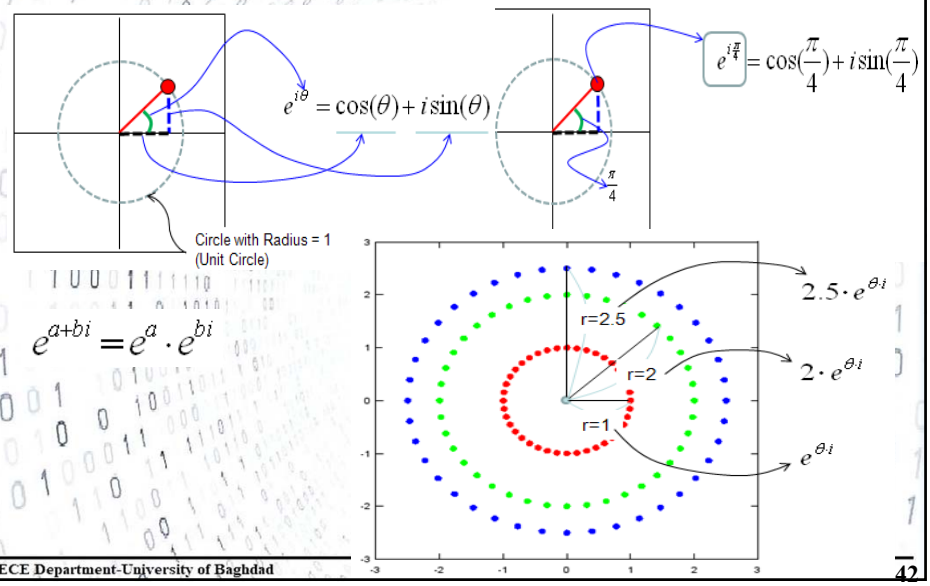
Complex Numbers-

Euler Equation



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