Antennas and Propagation: Lecture 1

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Static Electric Fields

Static electric fields are the types of fields that do not change in magnitude or direction with time.

Many ancient civilizations have recorded many observations on static electricity. around 600 BCE, the Greek philosopher Thales noted that amber (كهرمان) after being rubbed on silk (حرير), could pick up pieces of straw (القش). This was one of the earliest documented accounts of static electricity [1].

Coulomb's Law

In the late 18th century, the French engineer Charles Augustus Coulomb found that the force between charges is proportional to the product of the two charges, inversely proportional to the square of the distance between the charges, and acts in a line containing them.

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[1] The term (Electric) originates from the Latin word (electricus), which means (like amber). Similarly, the Arabic word (كهرباء) comes from the same origin (كهرمان).
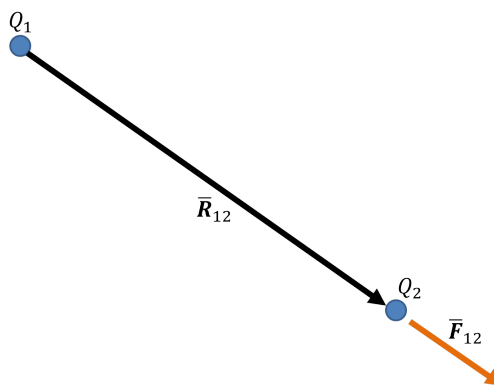
Figure 1: Coulomb's law eample showing the vector force $\bar{F}_{12}$ acting on Q_1 and Q_2.

The proportionality constant was found to be $\frac{1}{4\pi\epsilon_0}$, where ϵ_0 is the free space permittivity $= 8.85 \times 10^{-12} F/m$.

Consider Figure (1), where Q_1 and Q_2 have charge quantities given in coulombs (C). A distance vector $\bar{R}_{12} = R_{12}\bar{a}_{12}$ of magnitude R_{12} (in meters) and direction $\bar{a}_{12}$ can be drawn between the two charges. Charge Q_1 exerts a vector force $\bar{F}_{12}$, in newtons (N), on charge Q_2 that is given by Coulomb's law:

$$\bar{F} = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{12}^2} \mathbf{a}_{\bar{\mathbf{12}}} \tag{1}$$

Electric Field Intensity

If we consider the charge Q_2 to be a test charge, **the electric field intensity**, $\bar{E}$, is defined as the coulomb force due to a fixed charge (Q_1) on a unit test charge (Q_2), or

$$\bar{E} = \frac{\bar{F}}{Q_2} \tag{2}$$

Hence, electric field intensity due to a certain charge in the free space is expressed by:

$$\bar{E} = \frac{Q}{4\pi\epsilon_0 R^2} \mathbf{a}_{\bar{\mathbf{R}}} \tag{3}$$

where Q is the value of the charge (in coulombs), and R is the distance between the charge and the testing point (in meters). Here, $\mathbf{a}_{\bar{\mathbf{R}}}$ is a **unit vector** directed *from the charge to the testing point*, which is the direction of the electric field intensity vector $\bar{E}$. The units of the electric field intensity are Volts per meters, or V/m.

Field Lines

The behavior of the field can be visualized by using *field lines*. These lines follow the direction of field vectors in space, as illustrated in Figure (2). In Figure (2-a), the field vectors are
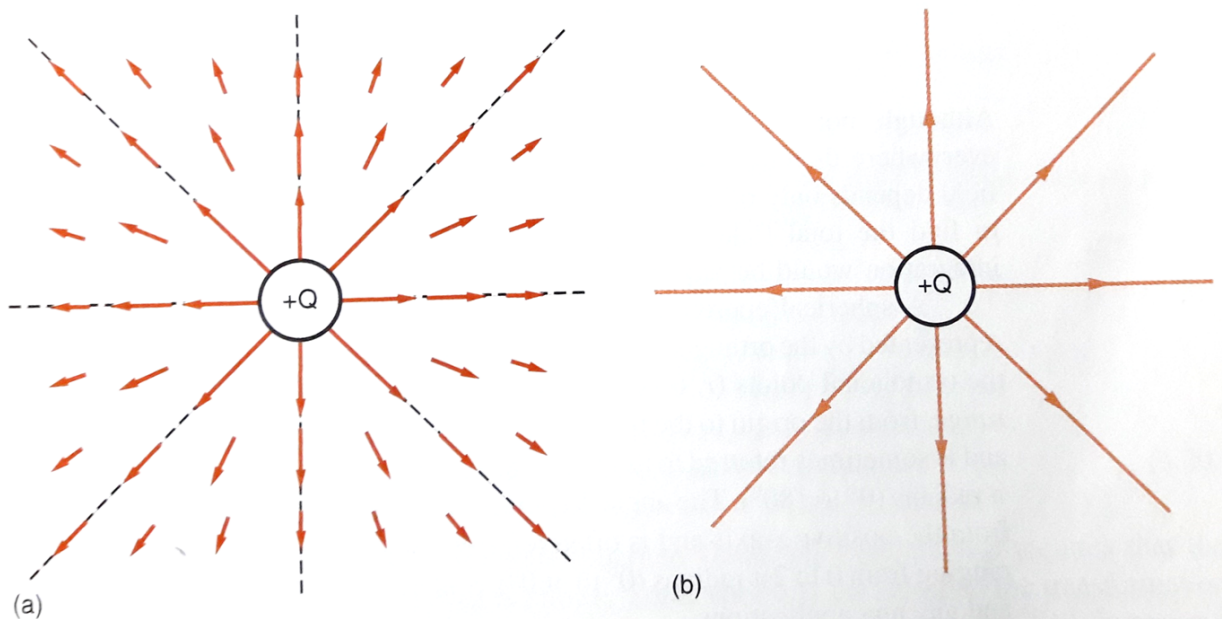
Figure 2:

found within a regular grid in two-dimensional space surrounding a point charge. Some of these field vectors can easily be joined by field lines, as shown in Figure (2-b), that radiate away from the positive charge.

Field strength at a point is related to how close together the field lines are near that point. The field lines provide a convenient visual aid to understand what the fields are doing.

Electric Flux Density

Now, let's consider the two metallic spheres shown in Figure (3). The inner sphere is charged with a positive charge, and then it was placed inside a *neutral* (not charged) outer sphere with about 2 cm of dielectric material between them. A negative charge will be induced on the outer sphere with magnitude equal to the charge on the inside sphere *regardless* of the dielectric material that separates the two spheres. This relationship between the two spheres that is independent on the medium is called the **electric flux**, or Ψ, and it is measured in coulombs.

Let's look again at Figure (3), at the surface of the inner sphere, Ψ coulombs of electric flux are produced by the charge Q coulombs distributued uniformly over a surface having an area of $4\pi a^2$. The *density* of the flux at this surface is $\Psi/4\pi a^2$. This is an important quantity called the **Electric flux density**, or $\bar{D}$, and it is measured in coulombs per square meters.

Now, if we have a single charge (Q) in the free space, the flux density at any point R meters away from the charge will be:

$$\bar{D} = \frac{Q}{4\pi R^2}\bar{\mathbf{a_R}} \tag{4}$$

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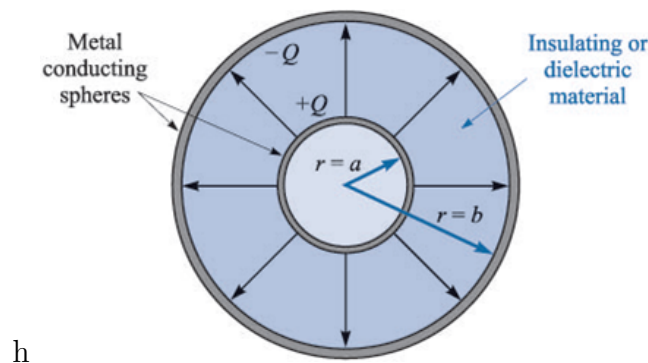Figure 3: Electric flux in the region betwen two charged spheres.

By comparing this expression to that in equation (3), we can conclude that:

$$\bar{D} = \epsilon_0 \bar{E} \tag{5}$$

This relation between $\bar{D}$ and $\bar{E}$ is only valid in free space. For general media, as we will see later, the relation becomes $\bar{D} = \varepsilon \bar{E}$, where ε is the material's permittivity.

The advantage of using electric flux density ($\bar{D}$) rather than electric field intensity ($\bar{E}$) is that the former relates to the number of flux lines emanating from one set of charge and terminating on the other, **independent of the media**.

We can conclude from the experiments with concentric spheres shown in Figure (3) that the electric flux passing through any imaginary sphere between the two metallic spheres is equal to the charge enclosed by the surface of that imaginary sphere. This concept can be generalized by saying that *the electric flux passing through any closed surface is equal to the total charge enclosed by that surface.* This is known as **Gauss's law**. In mathematical form, this law is written as:

$$\oint \bar{D}.d\bar{S} = Q_{enc.} \tag{6}$$

where the circle on the integral indicates the integration is performed over a closed surface. Equation (6) is called the *integral form* of Gauss's law.

Divergence and the Point Form of Gauss's Law

The concept of divergence is related to Gauss's law, where net flux is evaluated exiting a closed surface. The divergence of a vector field at a particular point in space is a spatial derivative of the field indicating to what degree the field emanates (or *diverges* يتباعد)from the point. Its value, a scalar quantity, says whether the point contains a source or a sink of field.

The divergence of the vector field $\bar{A} = A_x \bar{a}_x + A_y \bar{a}_y + A_z \bar{a}_z$ in *cartesian* coordinates is:

$$\nabla.\bar{A} = \frac{\partial}{\partial x} A_x + \frac{\partial}{\partial y} A_y + \frac{\partial}{\partial z} A_z \tag{7}$$

For *cylindrical* coordinates, the divergence of a vector field $\bar{B} = B_\rho \bar{a}_\rho + B_\phi \bar{a}_\phi + B_z \bar{a}_z$ is:

$$\nabla.\bar{B} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho\bar{B}_\rho) + \frac{1}{\rho}\frac{\partial}{\partial\phi}\bar{B}_\phi + \frac{\partial}{\partial z}B_z \tag{8}$$

and for *spherical* coordinates, the divergence of a vector field $\bar{C} = C_r \bar{a}_r + C_\theta \bar{a}_\theta + C_\phi \bar{a}_\phi$ is:

$$\nabla.\bar{C} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 C_r) + \frac{1}{rsin\ \theta}\frac{\partial}{\partial\theta}(C_\theta\ sin\ \theta) + \frac{1}{r\ sin\ \theta}\frac{\partial}{\partial\phi}C_\phi \tag{9}$$

The concept of divergence will also lead to the very useful differential form of Gauss's law, as follows:

$$\nabla.\bar{D} = \rho_v \tag{10}$$

Here, ρ_v is the *volume charge density* in Coulomb per cubic meters, or C/m^3. Equation (10) is the first of Maxwell's four equations.

Electric Potential

We are all familiar with the concept of electric potential, which is usually known as *voltage* in circuit analysis. Here, we will develop the concept of electric potential and show the relationship to the electric field intensity.

When force is applied to move an object, *work* is the product of the force and the distance the object travels in the direction of the force. Mathematically, in moving the object from point a to point b, the work can be expressed as:

$$W = \int_a^b \bar{F}.d\bar{L} \tag{11}$$

where $d\bar{L}$ is a differential length vector along some portion of the path between a and b.
We know from Coulomb's law that the force exerted on a charge Q by an electric field $\bar{E}$ is $\bar{F} = QE$. The work done *by the field* in moving the charge from point a to point b is then:

$$W_E = Q\int_a^b \bar{E}.d\bar{L} \tag{12}$$

If an external force moves the charge against the field, the work done is the *negative* of W_E, or:

$$W = -Q\int_a^b \bar{E}.d\bar{L} \tag{13}$$

Now, we can define the *electric potential difference* V_{ab} as the work done by an external source to move a charge from point a to point b in an electric field divided by the amount of charge moved:

$$V_{ba} = \frac{W}{Q} = -\int_a^b \bar{E}.d\bar{L} \tag{14}$$

The potential difference can also be related to absolute potentials, or *electrostatic potentials*, at points a and b:

$$V_{ba} = V_b - V_a \tag{15}$$

Finding the absolute potential at some point requires that we have a reference potential. Often a ground plane is chosen as the zero potential reference.

Now, if we choose a closed path, the integral will return zero potential difference:

$$\oint \bar{E}.d\bar{L} = 0 \tag{16}$$

which is very similar to *Kirchhoff's voltage law*.

Let's calculate the potential difference between two points in space resulting from the field of a point charge located at the origin. Since the electric field intensity is radially directed, only movement in the radial direction will influence the potential. If we move from radius a to radius b, we have:

$$V_{ba} = -\int_a^b \bar{E}.d\bar{L} = -\int_a^b \frac{Q}{4\pi\varepsilon_0 r^2}\bar{a}_r.dr\bar{a}_r$$

which upon evaluating the integral yields:

$$V_{ba} = \left.\frac{Q}{4\pi\varepsilon_0 r}\right|_{r=a}^{r=b} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{b} - \frac{1}{a}\right) = V_b - V_a$$

Now, if we set a reference voltage of zero at an infinite radius, then the absolute potential at some finite radius (r) from a point charge fixed at the origin is:

$$V = \frac{Q}{4\pi\varepsilon_0 r} \tag{17}$$

Gradient

Figure (4) shows a plot of the electrostatic potential superimposed over the field lines for a point charge. The electrostatic potential contours form *equipotential surfaces* surrounding the point charge. All points on such a surface have the same potential.
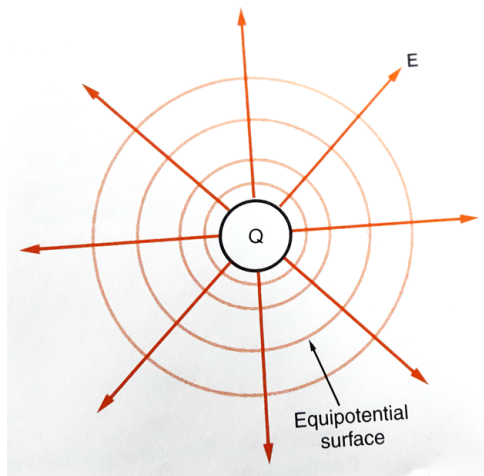

Figure 4: Equipotential lines are shown orthogonal to field lines for a point charge.

It is evident that these surfaces are always orthogonal to the field lines. In fact, if the behavior of the potential is known, the electric field can be determined by finding the maximum rate and direction of spatial change of the potential field. We can use the del operator again, this time in the *gradient equation*:

$$\bar{E} = -\nabla V \tag{18}$$

where the negative sign indicates that the field is pointing in the direction of decreasing potential. In Cartesian coordinates, the gradiant equation for a scalar potential V is:

$$\nabla V = \frac{\partial}{\partial x} V \bar{a}_x + \frac{\partial}{\partial y} V \bar{a}_y + \frac{\partial}{\partial z} V \bar{a}_z \tag{19}$$

and in cylindrical coordinates, ∇V is:

$$\nabla V = \frac{\partial}{\partial \rho} V \bar{a}_\rho + \frac{1}{\rho} \frac{\partial}{\partial \phi} V \bar{a}_\phi + \frac{\partial}{\partial z} V \bar{a}_z \tag{20}$$

and in spherical coordinates:

$$\nabla V = \frac{\partial}{\partial r} V \bar{a}_r + \frac{1}{r} \frac{\partial}{\partial \theta} V \bar{a}_\theta + \frac{1}{r \, sin \, \theta} \frac{\partial}{\partial \phi} V \bar{a}_\phi \tag{21}$$